Non-Destructive Evaluation of the Volumes of Egg Shell and Interior: Theoretical Approach

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Abstract

Describing the properties of table eggs requires the development of methods enabling to look inside the egg without destroying it, suggesting a thorough theoretical study including the formulation of theoretical aspects of this advanced egg-related research area. For this purpose, we developed a mathematical assay for computing the volumes of shell and interior of a chicken egg using, as input data, its main external geometric dimensions (length, maximum breadth, and the value of its shift from the centre of the horizontal axis) as well as the thickness of the shell. The shell volume can be determined as the product of the average thickness by the surface area estimated along the midline of shell section. We obtained theoretical dependences of the midline-based estimate of surface area on the values of the average shell thickness and the outer surface area of the egg. Since the volume of egg interior, in addition to the volumes of the entire egg and shell, is also affected by air cell volume, we derived theoretical formulae for computing this indicator. To calculate it, in addition to the values of the basic geometric dimensions of the egg, data on the diameter of the air cell or its height should be used, which is quite simple to measure with conventional measuring instruments like an ovoscope.

Keywords: Eggshell volume; volume of egg interior; air cell volume; non-destructive measurement; chicken egg

Nomenclature (as expanded from Narushin et al., 2021b)

a_1, a_2, b_1, b_2, c_2	Coefficients used for simplifying the solution of equations for calculating the surface area and volume of the air cell
B	Egg maximum breadth
B_m	Egg maximum breadth corrected for the midline of the shell
d	Diameter of the air cell
h	Height of the air cell
k_{ac}	Coefficient used for deducing the equation to calculate the air cell volume
L	Egg length
L_m	Egg length corrected for the midline of the shell
S	Egg surface area
S_m	Egg surface area corrected for the midline of the shell
SI	Egg shape index, i.e., B to L ratio
T	Average shell thickness
V	Egg volume
V_{ac}	Air cell volume
V_i	Volume of the egg interior
V_s	Shell volume
w	Parameter that corresponds to a distance between two vertical axes, one of which coincides with B and the other one is crossing the egg at the point of $L/2$

1. Introduction

Table eggs are generally recognized as very nutritious food items containing protein, lipids, vitamins, and micronutrients (e.g., Chambers *et al.*, 2017; Réhault-Godbert *et al.*, 2019; Tamiru *et al.*, 2019), while certain egg components may be even augmented to optimize human nutrition and boost health (e.g., Surai *et al.*, 2000; Surai, 2001). Taking this into account, the weight of the egg interior is much more substantial and valuable for the consumer than the whole egg. As a whole structure, a chicken egg can be conditionally divided into two main components: the shell and the interior. Despite the fact that the shell plays a crucial role in keeping the egg safe, the size of the internal component is more important for table eggs. Currently, the volume of interior can be estimated: (1) indirectly using the weight and/or linear dimensions of a whole egg (Narushin, 1994; Khurshid *et al.*, 2003), which can be easily measured by conventional measurements; or (2) through direct measurements after breaking the egg. Nevertheless, development of new approaches to research methods in the field of poultry genetics and breeding, assessment of food quality and the engineering of novel high throughput egg sorting technologies poses the challenge of creating non-destructive methods for determining the volumetric characteristics of the morphological/structural egg components (Narushin, 1997).

Because any bird's egg can conventionally be represented as the sum of two main components, the shell and the interior, the volume of interior can be judged by the difference in the volume of egg and its shell. The only thing is that the air cell introduces a certain bias, and its volume should also be taken into account in these computations. Air cell measurement is part of the standard egg quality determination procedure prescribed in many countries (e.g., USDA, 2000). indicates the age of the egg, the shelf life and, accordingly, the nutritional properties.Измерения величины воздушной камеры входит в состав стандартных процедур определения качества яйца во многих странах (к примеру, USDA, 2000), т.к. свидетельствует о возрасте яйца, сроках его хранения и, соответственно, пищевых свойствах. There is sufficiently proven procedure for determining the air cell parameters, i.e., its height and diameter, by assessing the egg under an ovoscope using conventional measuring devices, for example, a micrometre (Samli *et al.*, 2005) or an air cell gauge (USDA, 2000) as well as more sophisticated methods, like ultrasound beams (Aboonajmi *et al.*, 2010), dielectric techniques (Ragni *et al.*, 2007), and machine imaging (Brand *et al.*, 2013). However, a method of evaluating the air cell volume based on these measurements has not been worked out yet. The only calculation model proposed by Phillips *et al.* (1992) was based on linear measurements of the air cell diameter as well as egg length and breadth. Nevertheless, since this method depended on a constant obtained by the authors experimentally and taken as an average based on the results of daily measurements of eggs during incubation, it can hardly be accepted for solving this problem.

The shell volume is also of great importance in the study and assessment of the quality of table eggs. Atanasov (2019) defined the ratio of the volume of a whole egg to its shell volume as a universal index for predicting the optimal shelf life of table eggs. Concerning the methods for estimating the volume of eggshell or, rather, its weight, by volume of which one can indirectly judge the shell volume due to sufficiently stable density of the shell material (Carter, 1968a; Harms *et al.*, 1990; Harms, 1991), a number of studies were carried out that can be conditionally grouped as follows:

1. The shell weight can be figured out via the weight of the whole egg (Rahn and Paganelli, 1989; Narushin, 1994; Seker, 2004).
2. The shell weight is calculated using more than one parameter, for example, egg weight and basic linear dimensions (Khurshid *et al.*, 2003; Shafey *et al.*, 2014) or egg weight and egg density (Nordstrom and Ousterhout, 1982; Harms *et al.*, 1990; Harms, 1991).

In those works, the authors used data obtained as a result of direct measurements of a certain sample of eggs, often not exceeding 200 pieces.

Thus, we can summarize that the studies carried out so far in this research area have been empirical and resulted in obtaining dependences that were adequate only to a definite sample of eggs the authors worked with. On the other hand, there have not been deeper theoretical investigations to identify proper mathematical solutions. In this regard, our study was aimed at generating substantiated mathematical dependencies enabling to identify the volumes of the shell and interior of poultry eggs without destroying it.

2. Methodology

A hen's egg can be accurately described with a Hügelschäffer's model that relies on three linear egg parameters: length, *L*, maximum breadth, *B*, and a parameter *w* equals to OO_1 (Fig. 1), i.e., a difference between a distance from the egg pointy end to a vertical axis, which corresponds to the egg maximum diameter, *B* and the half length of the egg, *L*/2 (Petrović and Obradović, 2010; Petrović *et al.*, 2011; Narushin *et al.*, 2020b).

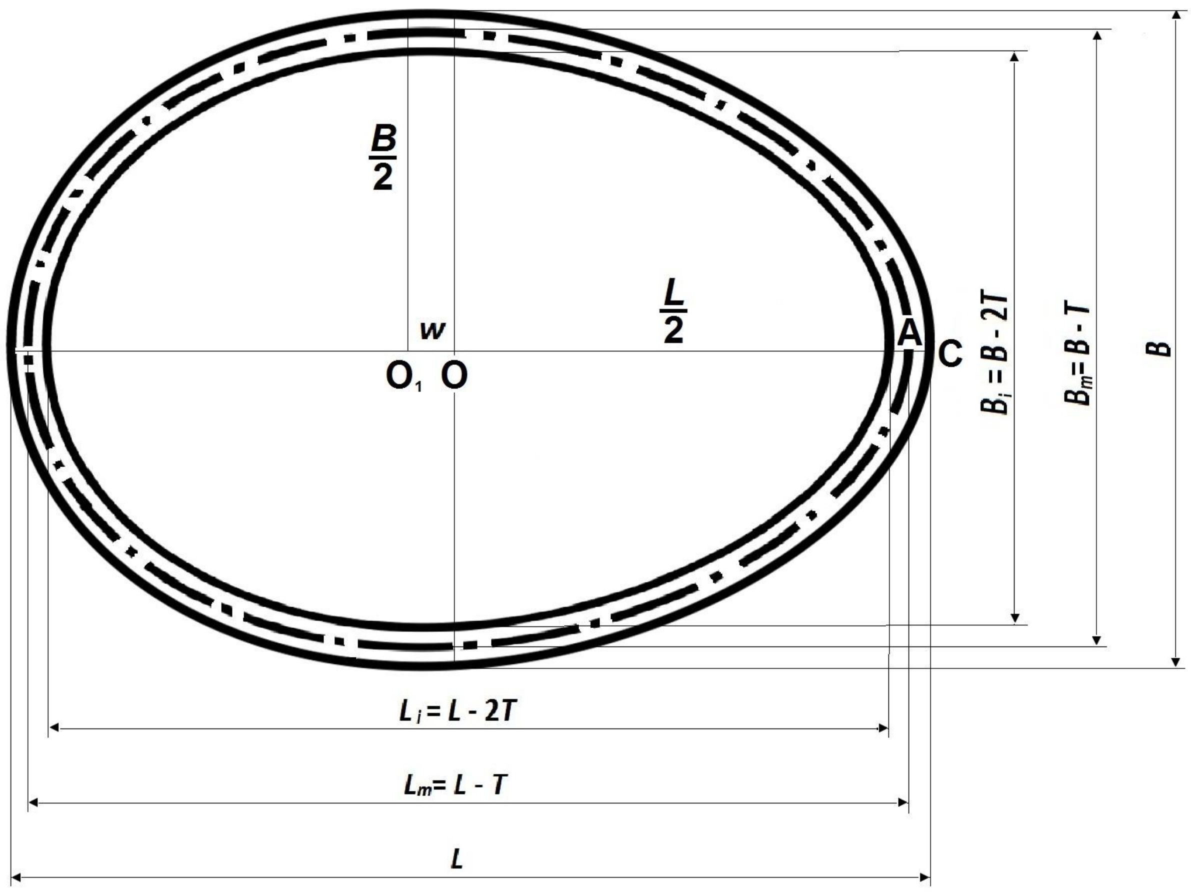


Fig. 1. Schematic image of the eggshell.

To undertake the simulation, we decided to be limited with the data of hen's eggs only, so the following ranges of the linear parameters mostly typical for such eggs were considered in accordance with Romanoff and Romanoff (1949) and our previous studies (Narushin, 1994; Narushin, 2001; Narushin *et al.,* 2020a): (1) egg length, L = 5.2…6.4 cm; (2) shape index, SI = B/L = 0.70…0.78, with a corresponding recalculation of the values of $B = SI{\cdot}L$; and (3) w = 0.01…0.50. All possible combinations of L, B and w were substituted into the formula for calculating S using the Hügelschäffer's egg model (Narushin *et al.,* 2020b) that enabled generating the data of surface areas for 837 simulated egg profiles.

For further calculations, we will use such a parameter as surface area of the shell measured along its midline, S_m, as shown in Fig. 1 dash-dotted curve. Midline is similar to the term the neutral line, borrowed from industrial engineering, exactly sheet bending process, where it is used as an imaginary line that has the same length after bending as it had before bending. The neutral line does not always pass directly in the very center of a bent beam, and its location is largely due to many parameters; nevertheless, according to a number of authors (Diegel, 2002; Betts, 2010; Stewart, 2016), it can be safely assumed to be equidistant from the outer and inner layers, especially for thin-walled vessels. According to Diegel (2002), these include those in which the radius of the wall exceeds its threefold thickness, which is quite consistent with the shell of chicken eggs. This condition can be verified by practical calculations using the formulas we derived earlier (Narushin et al., 2021a).

To define the values of S_m, the egg linear parameters L and B were reduced by the value of the average shell thickness, T (Fig. 1):

$$L_m = L - T \text{ and } B_m = B - T$$

where L_m and B_m are corresponding to the length and maximum breadth of the egg being measured according to the midline of the shell.

To check if the parameter w changes when the egg profile would be uniformly contracted, the following estimations were undertaken using the scheme in Fig. 1:

$$w = \mathrm{O_1C} - \frac{L}{2} \qquad (1)$$

$$w_m = \mathrm{O_1A} - \frac{L_m}{2} = \mathrm{O_1C} - \frac{T}{2} - \frac{L-T}{2} = \mathrm{O_1C} - \frac{L}{2} = w \qquad (2)$$

The above calculations suggest that the value of the parameter w remains unchangeable if the egg profile is contracted.

To run the simulation for determining S_m, the values of L_m, B_m and w were added using a dataset of the variable T = 0.025…0.050 cm. This range excessively covers all possible variations for the hen's eggs. The S_m values were obtained using the respective formula for the egg surface area from Narushin *et al.* (2021b):

$$S = \pi BL\left(\left(0.043\frac{w}{L}+0.292\right)\frac{B}{L}-0.061\frac{w}{L}+0.704\right), \quad (3)$$

resulting in

$$S_m = \pi(B-T)(L-T)\left(\left(0.043\frac{w}{L-T}+0.292\right)\frac{B-T}{L-T}-0.061\frac{w}{L-T}+0.704\right) \quad (4)$$

3. Theory

3.1. Eggshell volume

If we consider an egg representation (Fig. 1) with the shell conditionally shown with evenly allocated thickness, by analogy with calculating the volume of cylindrical shells (Stewart, 2016) it is possible to state that the shell volume, V_s, equals to a product of its area measured over a middle shell surface, S_m (shown in Fig. 1 with a dash-dot line), and the average thickness, T:

$$V_s = S_m \cdot T \quad (5)$$

The methodological approach we have chosen (Eq. 5) in calculating V_s, in our opinion, is simpler and more convenient than use of integral calculus for finding this parameter, since it can cause certain difficulties and, as a consequence, inaccuracies in the result obtained, which was demonstrated by us earlier (Narushin et al., 2020b; 2021a; 2021b).

In our case, to estimate the V_s value indirectly, we would need to measure T and recalculate the shell surface area S_m over the midline of its shell.

Currently, accurate measurement of the shell thickness without breaking the egg can be performed using, for example, a commercial ultrasonic device produced by ORKA (2020) or a non-destructive deformation device by Stable Micro Systems (2020).

Thus, the idea of our investigations on the eggshell volume was to focus on a comparison of the values of S_m and S and an estimation of a possible dependence between them, $S_m = f(S)$, in order to provide the appropriate mathematical recalculations of V_s.

3.2. Air cell volume

Conventionally, the air cell of any egg can be represented in the form of a rotation figure, with the height, h, and base diameter, d, as shown in Fig. 2 by the straight line AB.

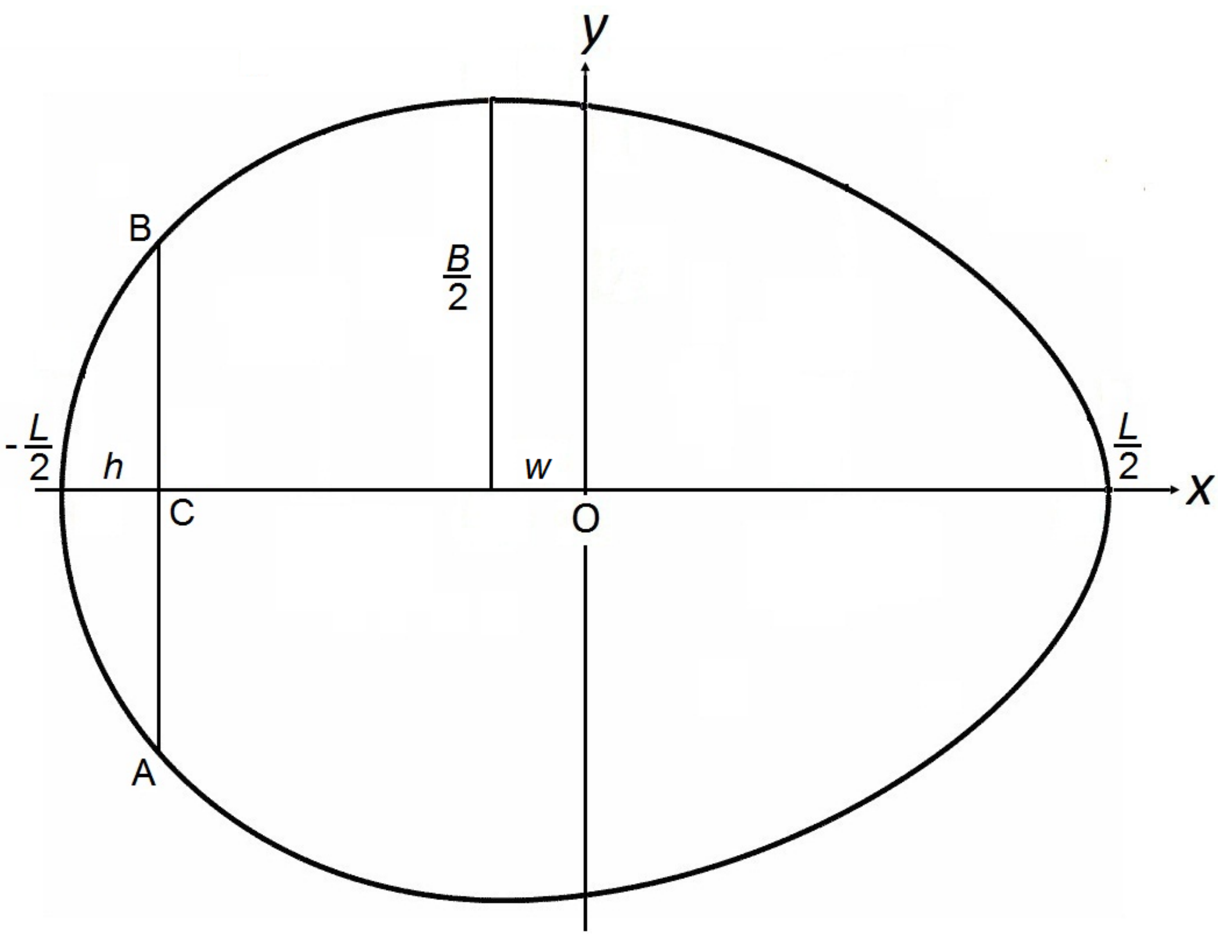


Fig. 2. Geometrical interpretation of the air cell inside the egg.

A volume, *V*, of any figure of revolution can be estimated using the following formula of integral geometry:

$$V = \pi \int_{x_1}^{x_2} y^2 \, \mathrm{d}\, x \qquad (6)$$

where x_1 and x_2 are the limits of a function *y*.

The coordinate of point C is determined from the condition: $-L/2 + h$. Then, the limits of the integral (6) will correspond to:

$x_1 = -\frac{L}{2}$ and $x_2 = h - \frac{L}{2}$,

with y matching the egg shape profile that was previously described by us with the Hügelschäffer's model (Narushin *et al.*, 2020b):

$$y = \pm \frac{B}{2}\sqrt{\frac{L^2 - 4x^2}{L^2 + 8wx + 4w^2}} \qquad (7)$$

Hence, in our case, the volume of the air cell, V_{ac}, can be presented as

$$V_{ac} = \pi \cdot \frac{B^2}{4} \int_{-\frac{L}{2}}^{h-\frac{L}{2}} \frac{L^2 - 4x^2}{L^2 + 8wx + 4w^2}\,\mathrm{d}\,x \qquad (8)$$

The deduction of the integral (8) was demonstrated in detail in Narushin *et al.* (2020b) when estimating the volume of the whole egg. Thus, omitting the basic part of the mathematical transformation, we were able to proceed with the following computations:

$$V_{ac} = \frac{\pi B^2 L^2}{4} \cdot \frac{1}{8w} \ln\left|L^2 + 8wx + 4w^2\right| \Bigg|_{-\frac{L}{2}}^{h-\frac{L}{2}} -$$

$$- \frac{\pi B^2}{8w} \left(\frac{\left(x + \frac{L^2}{8w} + \frac{w}{2} \right)^2}{2} - \left(\frac{L^2}{4w} + w \right)\left(x + \frac{L^2}{8w} + \frac{w}{2} \right) + \left(\frac{L^2}{8w} + \frac{w}{2} \right)^2 \cdot \ln\left| x + \frac{L^2}{8w} + \frac{w}{2} \right| \right) \Bigg|_{-\frac{L}{2}}^{h-\frac{L}{2}} \qquad (9)$$

that resulted in the final formula:

$$V_{ac}=\frac{\pi B^2 L}{32}\left(\left(\frac{w}{L}\right)^{-1}\cdot\ln\left|1+\frac{8\frac{w}{L}\cdot\frac{h}{L}}{\left(1-2\frac{w}{L}\right)^2}\right|-\frac{1}{8}\cdot\left(\frac{w}{L}\right)^{-3}\cdot\right.$$

$$\left.\cdot\left(4\frac{w}{L}\cdot\frac{h}{L}\cdot\left(\left(1-2\frac{w}{L}\right)^2+4\frac{w}{L}\cdot\frac{h}{L}\right)-8\frac{w}{L}\cdot\frac{h}{L}\left(1+4\left(\frac{w}{L}\right)^2\right)+\frac{1}{2}\cdot\left(1+4\left(\frac{w}{L}\right)^2\right)^2\cdot\ln\left|1+\frac{8\frac{w}{L}\cdot\frac{h}{L}}{\left(1-2\frac{w}{L}\right)^2}\right|\right)\right) \quad (10)$$

The detailed transformations of Eq. (9) are provided in Appendix A.

Eventually, we can consider Eq. (10) as follows:

$$V_{ac}=\frac{\pi B^2 L}{32}\cdot k_{ac} \quad (11)$$

where k_{ac} is a coefficient expressed with a following equation:

$$k_{ac}=\left(\frac{w}{L}\right)^{-1}\cdot\ln\left|1+\frac{8\frac{w}{L}\cdot\frac{h}{L}}{\left(1-2\frac{w}{L}\right)^2}\right|-\frac{1}{8}\cdot\left(\frac{w}{L}\right)^{-3}\cdot$$

$$\cdot\left(4\frac{w}{L}\cdot\frac{h}{L}\cdot\left(\left(1-2\frac{w}{L}\right)^2+4\frac{w}{L}\cdot\frac{h}{L}\right)-8\frac{w}{L}\cdot\frac{h}{L}\left(1+4\left(\frac{w}{L}\right)^2\right)+\frac{1}{2}\cdot\left(1+4\left(\frac{w}{L}\right)^2\right)^2\cdot\ln\left|1+\frac{8\frac{w}{L}\cdot\frac{h}{L}}{\left(1-2\frac{w}{L}\right)^2}\right|\right) \quad (12)$$

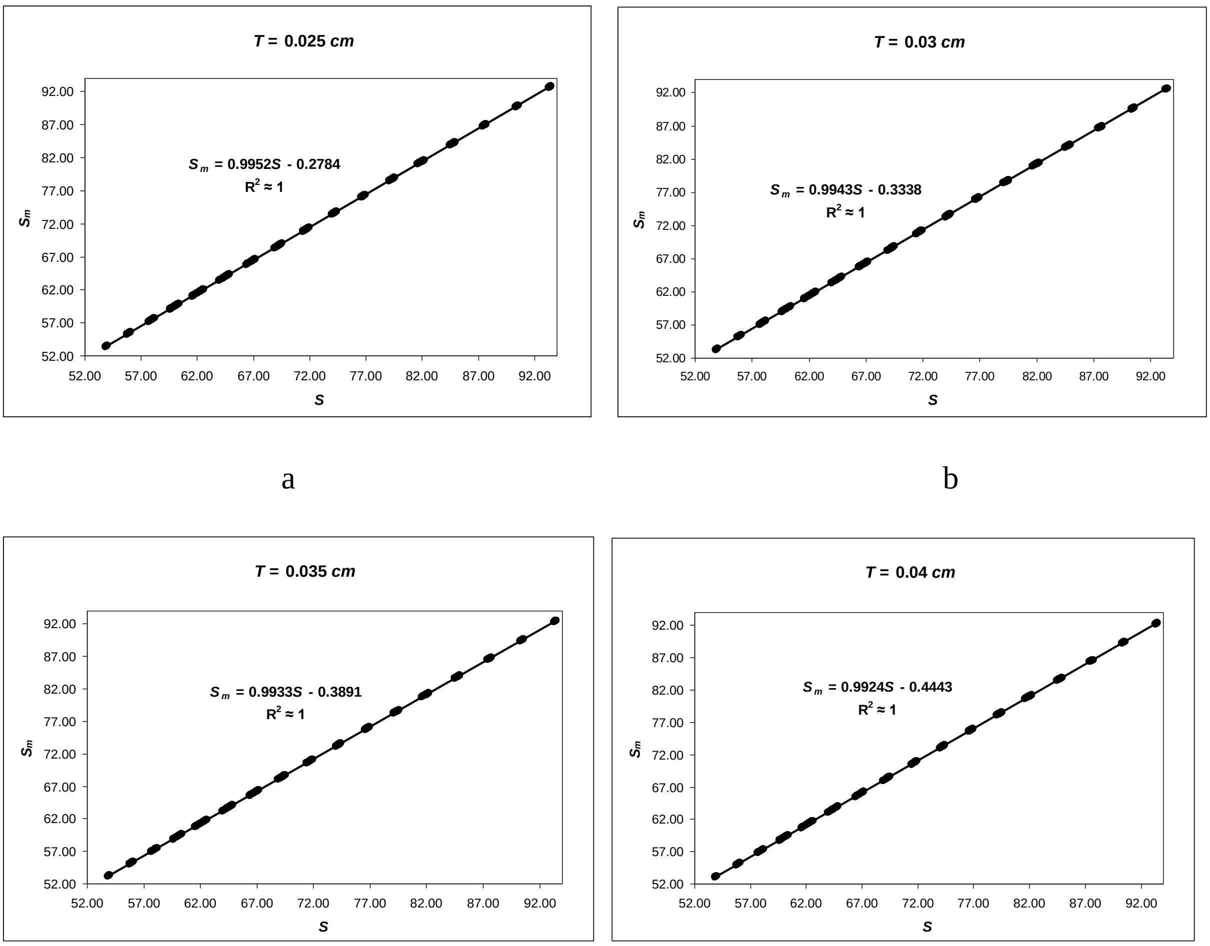
3.3. Volume of egg interior

With the resulting formulas to determine the structural constituents of an egg, namely the shell volume (Eq. 5) and air cell (Eq. 11), it is easy to calculate the egg interior volume, V_i, by simply subtracting the data Eqs. 5 and 11 from whole egg volume measurements.

4. Results

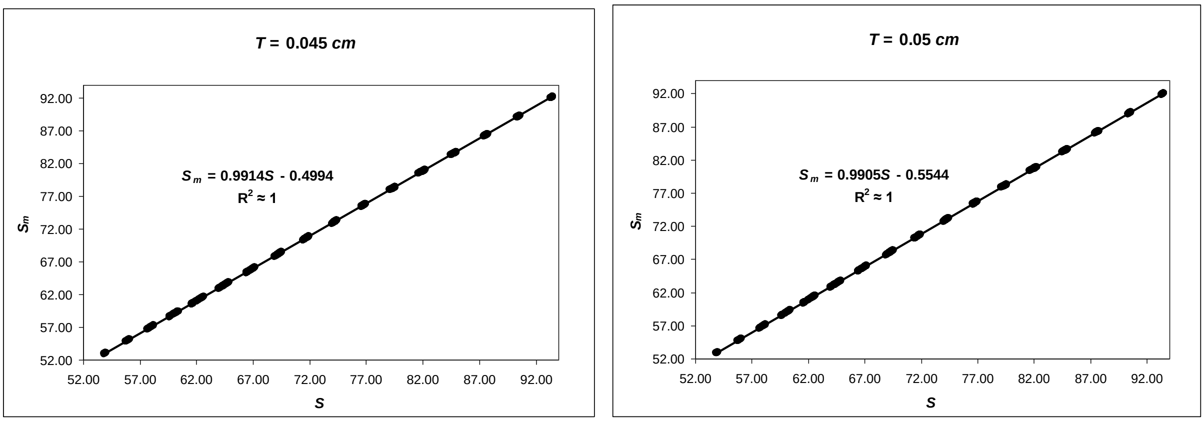
4.1. Eggshell volume

We computed the values of S and S_m and presented them in a form of graphic dependences (Fig. 3) reflecting changes of T in increment of 0.005 cm, each of which being approximated with linear dependencies.



a b

c d



e f

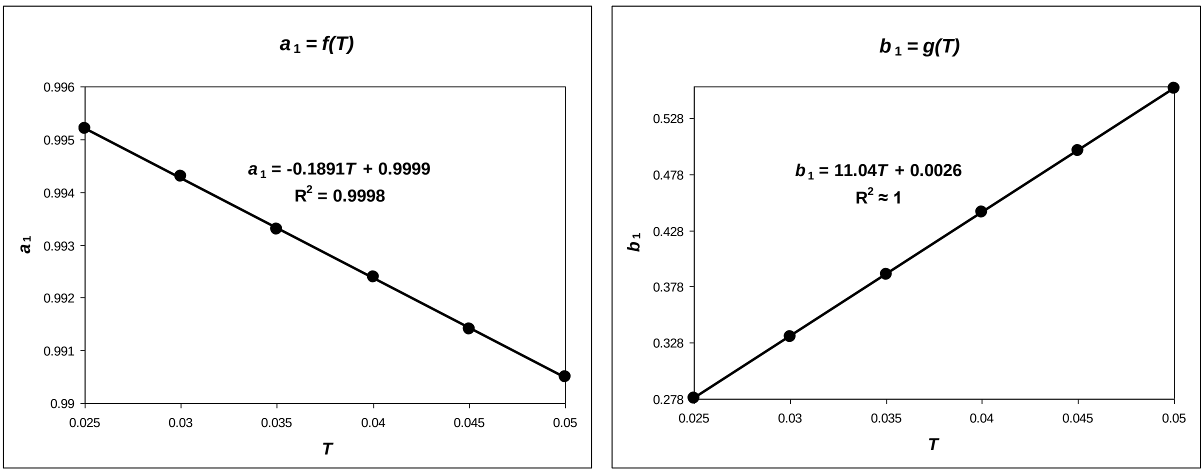
Fig. 3. Graphic dependences of $S_m = f(S)$ when T equals to: (a) 0.025 cm, (b) 0.03 cm, (c) 0.035 cm, (d) 0.04 cm, (e) 0.045 cm, and (f) 0.05 cm.

All the obtained equations (Fig. 3) have the following form:

$$S_m = a_1 S - b_1 \tag{13}$$

where a_1 and b_1 are coefficients.

The values of both coefficients a and b in Eq. (13) were approximated by the dependences $a_1 = f(T)$ and $b_1 = g(T)$ that are presented in Fig. 4.



a b

Fig. 4. The results of approximating the values of the coefficients a_1 and b_1 by the functions $f(T)$ and $g(T)$.

Substituting these data in Eq. (13) and rounding up to two decimals, we finally obtain:

$$S_m = (1 - 0.19T)S - 11.04T \quad (14)$$

Eventually, considering Eq. (5), the shell volume can be determined as follows:

$$V_s = ((1 - 0.19T)S - 11.04T)T \quad (15)$$

4.2. Air cell volume

We tried to simplify Eq. (12) to make it more suitable for both the computations and possible mathematical transformations. For that, we considered the possible variations of w/L from 0 to 0.25, as it was shown by Narushin *et al.* (2021a) to be adequate for any avian egg; and h/L from 0 to 0.15. These data were supported by the studies of Liu *et al.* (2017), Aboonajmi *et al.* (2010), Ragni *et al.* (2007), Samli *et al.* (2005) and others who showed that even the long-time storage of table eggs (in some investigations even more than 1 month) did not tend to increase the air cell height by more than 15% of the egg length. Substituting the values of w/L in increment of 0.05 and those of h/L in increment of 0.03 into Eq. (11), we produced six graphic dependences (Fig. 5), each of which approximated with polynomials.

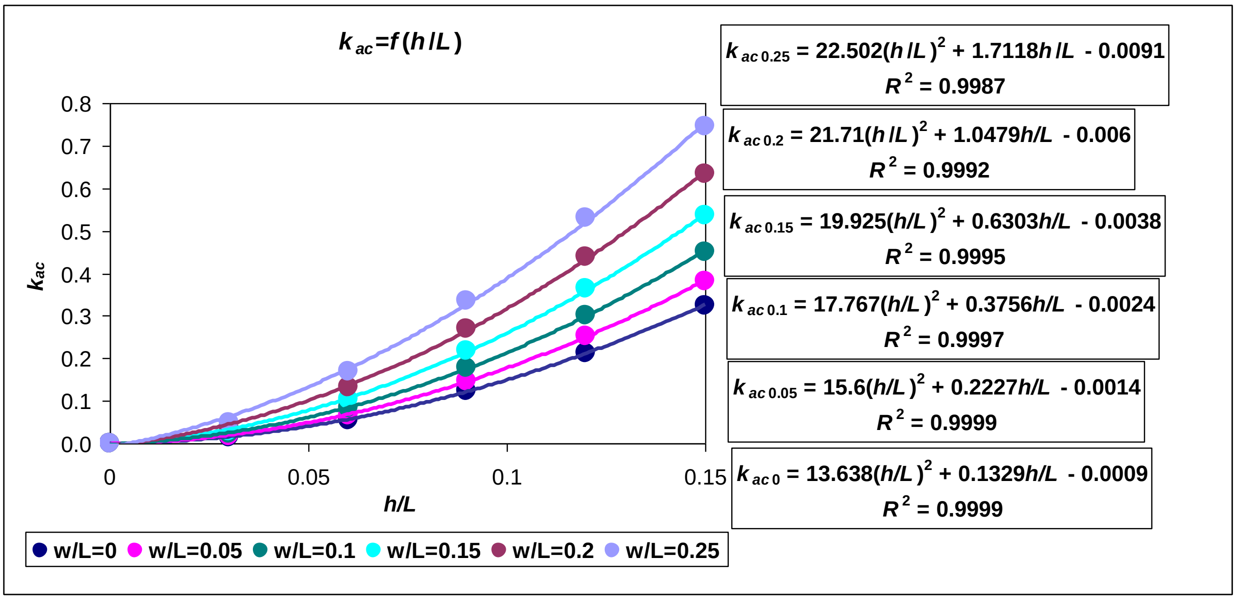


Fig. 5. Graphic dependences of $k_{ac} = f(h/L)$ when w/L equals to: 0; 0.05; 0.1; 0.15; 0.2 and 0.25, respectively.

All these approximating regressions were of the same type that can be generally expressed as follows:

$$k_{ac} = a_2\left(\frac{h}{L}\right)^2 + b_2\frac{h}{L} - c_2 \tag{16}$$

where a_2, b_2 and c_2 are coefficients of the corresponding equations in Fig. 5.

Due to minor values of the coefficient c_2 that did not have any influence on the results, only the coefficients a_2 and b_2 were considered for further evaluation of their dependences on the varied values of w/L. The respective graphic functions and approximating formulae are shown in Fig. 6.

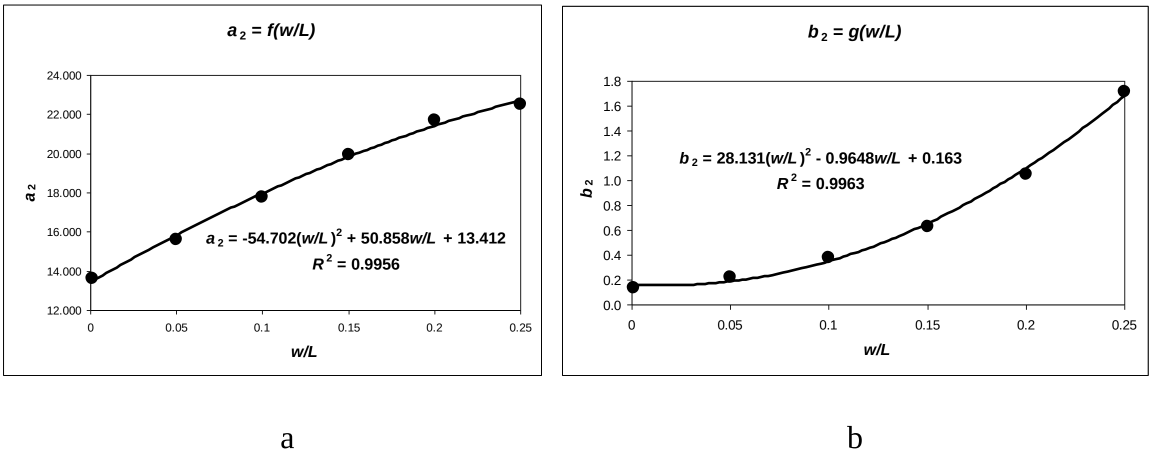


Fig. 6. The results of approximating the values of the coefficients a_2 and b_2 by the functions $f(w/L)$ and $g(w/L)$.

Substituting these data into Eq. (16), we obtained:

$$k_{ac} = 13.41\frac{h}{L}\left(\frac{h}{L}\left(1+3.79\frac{w}{L}-4.08\left(\frac{w}{L}\right)^2\right)+0.012\left(1-5.92\frac{w}{L}+172.58\left(\frac{w}{L}\right)^2\right)\right) \quad (17)$$

Comparison of the results of evaluating k_{ac} using Eqs. (12) and (17) showed their practically complete agreement: the correlation coefficient was equal to 0.9996. We also applied the approximation coefficient found by the following formula of Makridakis *et al.* (1982):

$$\varepsilon = \frac{1}{n}\cdot\sum_{1}^{n}\left|\frac{v_1 - v_2}{v_1}\right|\cdot 100\% \quad (18)$$

where n is a number of samples in the calculations, and ν_1 and ν_2 are the values of k_{ac} defined correspondingly by Eqs. (12) and (17). The computed approximation coefficient was equal to 6.1%, meaning that almost 94% of the results corresponds to each other.

Transforming Eq. (17) into a more convenient form and substituting it into Eq. (11), we finally obtained the V_{ac} estimation formula:

$$V_{ac} = \frac{1.32B^2h}{L^3}(h(L^2+3.79Lw-4.08w^2)+0.012L(L^2-5.92Lw+172.58w^2)) \quad (19)$$

In some cases, it is practically easier to measure the diameter, *d*, of the air cell than its height, *h*. Therefore, we decided to define a way of recalculating each parameter from the other one. In Fig. 2, *d* corresponds to the distance AB, which can be defined from the Hügelschäffer's model (Eq. (6)), considering *x* in the point C equals to *h* – *L*/2.

Then, accounting *d* = AC+BC = 2BC:

$$d = 2\frac{B}{2}\sqrt{\frac{L^2-4\left(h-\frac{L}{2}\right)^2}{L^2+8w\left(h-\frac{L}{2}\right)+4w^2}} = 2B\sqrt{\frac{h(L-h)}{(L-2w)^2+8hw}} \quad (20)$$

To figure out the function *h* = *f*(*d*), we considered Eq. (20) as the two following formulae:

$$d = 2B\sqrt{\frac{\frac{h}{L}\left(1-\frac{h}{L}\right)}{\left(1-2\frac{w}{L}\right)^2+8\frac{h}{L}\cdot\frac{w}{L}}} \quad (21)$$

$$h^2+\left(\frac{2d^2w}{B^2}-L\right)\cdot h+\frac{d^2(L-2w)^2}{4B^2}=0 \quad (22)$$

wherefrom

$$h = \frac{LB^2-2d^2w-\sqrt{(2d^2w-LB^2)^2-d^2B^2(L-2w)^2}}{2B^2} \quad (23)$$

The detailed solution of Eq22 is provided in Appendix B.

4.3. Volume of egg interior

Considering the basic formula for identifying the volume of the egg interior, V_i:

$$V_i = V - V_s - V_{ac}, \tag{24}$$

we can infer the following resultant equation for the computation of this parameter:

$$V_i = V - ((1 - 0.19T)S - 11.04T)T - \\ - \frac{1.32B^2 h}{L^3}(h(L^2 + 3.79Lw - 4.08w^2) + 0.012L(L^2 - 5.92Lw + 172.58w^2)) \tag{25}$$

In the case when the cell diameter is measured, the recalculation of h is performed using Eq. (23).

5. Discussion

Both in practice and research work involving table eggs, there may be situations when it would be much more relevant to determine not only characteristics of the whole egg but also parameters of the egg interior. At the same time, it is important to leave the egg intact, without causing any damage. Such a non-invasive technique would be highly desired, for example, in the food industry, when predicting nutritional value, or when developing a technology for saturating eggs with nutritious and/or health-promoting ingredients (e.g., Surai & Sparks, 2001; Surai & MacPherson, 2002). In poultry industry, it would be in demand for research related to incubation, poultry farming, *in ovo* vaccination, etc. In this regard, the method of non-destructive estimation of the volume of interior is of considerable importance. We have made an attempt to create a theoretical basis for such a methodology, taking into account that it is the theoretical premises that lay the basis on which any scientific doctrine is subsequently built.

To solve the problem of determining the volume of interior of any poultry egg, it is necessary to first measure a number of parameters. Linear dimensions as well as the estimation of egg volume and surface area are quite straightforward as was discussed by Narushin *et al.* (2020b). The height and diameter of air cell can also be determined, since these measurements are widely used in the standard assessment of the quality of edible and hatching eggs. Considering that the membrane bordering the rear wall of the air cell is most often curved, it is advisable to take several measurements of its height and / or diameter, after which the average result is used in the calculations.

The possible complexity of non-destructive measurement can be represented by the shell thickness parameter. Commercially available apparatuses for testing shell thickness, like the ultrasonic device by ORKA (2020) or the non-destructive deformation device by Stable Micro Systems (2020), cannot guarantee an accurate determination of this parameter due to the rather small measurement value. Therefore, this issue should be addressed further, and the solution to this problem can be the use of a whole complex of measurements. For example, Narushin *et al.* (2004) proposed to use a combination of basic egg measurements including egg weight, volume and surface area that in some cases can be supplemented by infrared spectroscopy data.

In earlier studies, Carter (1968b) suggested a calculation formula for the shell thickness based on measurements of its elastic deformation carried out at several points and a series of linear measurements of the whole egg. Thus, selecting additional parameters of non-destructive measurements, in addition to commercially available instrumentation for indirect testing the shell thickness, it is feasible to raise the accuracy of its determination up to the required level.

6. Conclusions

Since any scientific idea requires *ab ovo* its thorough theoretical study, in this article we attempted to deliver precisely the theoretical aspects of a new research area aimed at solving an engineering problem of "how to look inside an egg without destroying it." At the current stage of this research project, we have suggested a novel approach for estimating the volumes of shell and interior of a chicken egg. As a result, we can conclude that such a unique and enigmatic natural object as a bird's egg has fewer and fewer obstacles that prevent us from looking into what is inside. A symbiosis of measuring technology and mathematical calculations, as we demonstrated here, can facilitate a fairly accurate evaluation of the egg interior characteristics, while leaving the outer shell intact. The proposed mathematical solutions supplement a toolbox for non-destructive assessment of table and hatching eggs that can be used further in egg-related research, food engineering and poultry industry.

Appendices A and B. Supplementary data

Supplementary data to this article can be found online at

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Figure captions

Fig. 1. Schematic image of the eggshell.

Fig. 2. Geometrical interpretation of the air cell inside the egg.

Fig. 3. Graphic dependences of $S_m = f(S)$ when T equals to: (a) 0.025 cm, (b) 0.03 cm, (c) 0.035 cm, (d) 0.04 cm, (e) 0.045 cm, and (f) 0.05 cm.

Fig. 4. The results of approximating the values of the coefficients a_1 and b_1 by the functions $f(T)$.

Fig. 5. Graphic dependences of $k_{ac} = f(h/L)$ when w/L equals to: 0; 0.05; 0.1; 0.15; 0.2 and 0.25, respectively.

Fig. 6. The results of approximating the values of the coefficients a_2 and b_2 by the functions $f(w/L)$.