Non-destructive measurement of chicken egg characteristics: improved formulae for calculating egg volume and surface area

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Abstract

The key external characteristics of chicken eggs are volume (*V*) and surface area (*S*) that should be accurately estimated for various applications in poultry research, food engineering and technology development areas, although many researchers often use approximate calculation formulae proposed back in the middle of the last century. These were based on speculative constants and prone to computational errors. Using the Hügelschäffer's model and simulation modelling, we generated 1820 various combinations of eggs, according to which *V* and *S* were computed with more accurate and simplified formulae that we have improved here. As a result, dependencies were obtained for a simplified calculation of these parameters using only non-destructive measurement data on the egg length and maximum breadth, with the average error in calculating *V* being 1.1% and that for *S* 0.3%. The produced equations for *V* and *S* of chicken eggs can be used in non-invasive measurement of egg characteristics. Further improvement of these formulae does not seem feasible, suggesting closing for now the polemic about more cromulent calculation of these egg variables and relevant computation errors and constants.

Keywords: Non-destructive measurement; Egg volume; Egg surface area; Computation error; Hügelschäffer's model; Simulation modelling

Nomenclature

a, *b*	Coefficients used for simplifying the solution of the integral $I_{w/L}$
B	Egg maximum breadth
$I_{w/L}$	Integral for recalculating the surface area *S* depending on the *w* to *L* ratio
k_S	Coefficient that is used for the recalculation of S_V
k_V	Coefficient for the recalculation of the egg volume, *V*, using the egg geometrical measurements, *B* and *L*
L	Egg length
S	Surface area of an ovoid which shape corresponds to the Hügelschäffer's model that was taken as a geometrical prototype of an actual chicken egg
S_{el}	Surface area of an ellipsoid
S_s	Egg surface area recalculated through the simplified formula under the measurements of the egg geometrical measurements, *B* and *L*
S_V	Egg surface area recalculated through the meaning of the egg volume
$S_{w=0}$	Egg surface area recalculated from Eqn4 after substituting $w = 0$
SI	Egg shape index, i.e., *B* to *L* ratio
V	Volume of an ovoid which shape corresponds to the Hügelschäffer's model that was taken as a geometrical prototype of an actual chicken egg
V_{el}	Volume of an ellipsoid
w	Parameter that corresponds to a distance between two vertical axes, one of which coincides with *B* and the other one is crossing the egg at the point of *L*/2

1. Introduction

The volume and surface area of a hen's egg, a traditional and valuable food product, are main oomorphological parameters characterizing its quantitative and qualitative properties. Therefore, both the accuracy and convenience of their calculations are needed to use them further in various applications in poultry research, food engineering and technology design. There is, however, considerable controversy and polemic in the published studies on this aspect started back to the mid 20th century. The first to raise this issue were Romanoff and Romanoff (1949), who, for determining the egg volume, suggested as the initial calculation formula the one for ellipsoids, V_{el}, using the length, *L*, and maximum breadth, *B*, of the eggs, while having proposed the surface area, S_V, to be computed based on the egg volume, *V*:

$$V_{el} = \frac{\pi LB^2}{6} = 0.5236LB^2, \quad (1)$$

$$S_V = k_S V^{\frac{2}{3}}, \quad (2)$$

where k_S is a speculative constant, and those authors suggested the value $k = 4.831$ to ensure the smallest error in non-destructive measuring chicken eggs.

Also, Romanoff and Romanoff (1949), revisiting and analysing the previous research conducted in this area, presented several versions of the constants, both for Eqns 1 and 2, resulting in computation errors up to 15%. This, obviously, depended on the sample size and particular properties of eggs with which the authors of those studies had worked. The search for the right constants ensuring the accuracy of calculation of these indicators continued after 1949, with varied values of the above constants being suggested (as reviewed, for example, in Narushin, 1997).

In the course of theoretical investigations, Narushin (2005) and Narushin et al. (2020a) demonstrated that *k* is not a constant, but a function of the linear parameters of the egg, i.e., its length, *L*, and maximum breadth, *B*. Recently, Narushin et al. (2020b) showed that the contours of a chicken egg can be ideally described with the Hügelschäffer’s model (Ursinus, 1944;

Schmidbauer, 1948), and a parameter *w* that corresponds to a distance between two vertical axes, one of which coincides with *B* and the other one is crossing the egg at the point of *L*/2, has an additional effect on the coefficient *k*. Accordingly, the following novel formulae were derived for determining the volume, *V*, and the surface area, *S*, for the Hügelschäffer's ovoid (Narushin et al., 2020b):

$$V = \frac{\pi B^2}{256w^3}\left(4wL(L^2+4w^2)-(L^2-4w^2)^2\cdot\ln\left|\frac{L+2w}{L-2w}\right|\right) \quad (3)$$

$$S \approx \frac{\pi BL^2}{12}\left(-\frac{8BLw}{(L^2-4w^2)^2}+\frac{2\sqrt{3\left(L^2-2wL+4w^2\right)^3+B^2\left(5wL-L^2-4w^2\right)^2}}{\left(L^2-2wL+4w^2\right)^2}+\right.$$

$$\left.+\frac{2\sqrt{3\left(L^2+2wL+4w^2\right)^3+B^2\left(5wL+L^2+4w^2\right)^2}}{\left(L^2+2wL+4w^2\right)^2}+\frac{2\sqrt{\left(L^2+4w^2\right)^3+4B^2L^2w^2}}{\left(L^2+4w^2\right)^2}\right) \quad (4)$$

Although the above theoretical formulae (Eqns 3 and 4) for calculating *V* and *S* would provide a high accuracy in determining these indicators, the measurement of the parameter *w* would create a definite difficulty in their use. Moreover, in contrast to the exact theoretical dependence of calculating the volume of the Hügelschäffer's ovoid (Eqn3), the formula for determining its surface area, *S* (Eqn4), was only derived in an approximate way (Narushin et al., 2020b), and this was due to the complexity of the integral calculation.

Since in the current non-destructive measurement practice there are no precise direct methods for measuring the surface area, we estimated the accuracy of the obtained dependence (Eqn4) by substituting the value *w* = 0. According to Petrović and Obradović (2010), in this case the Hügelschäffer's ovoid is transformed into an ellipsoid. Then, the correspondingly modified Eqn4, i.e., after giving zero values to *w*, will have the following form, which should correspond to the surface area of ellipsoids:

$$S_{w=0} \approx \frac{\pi B\left(2\sqrt{3L^2 + B^2} + L\right)}{6} \tag{5}$$

in which $S_{w=0}$ means the egg surface area being recalculated from Eqn4 after substituting $w = 0$.

Eqn5 should be similar to the formula for calculating the surface area of ellipsoids, S_{el}, for which the major axis is L and the minor axis is B and, in accordance with Narushin et al. (2020a), matches:

$$S_{el} = \frac{\pi B}{2}\left(L\frac{\arcsin\sqrt{1-SI^2}}{\sqrt{1-SI^2}} + B\right) \tag{6}$$

in which SI is egg shape index that equals to the B to L ratio.

Nevertheless, the results of calculating the surface area of the ellipsoids using Eqn5 clearly differ from the data obtained using Eqn6, while the difference reaches 75%, which is unacceptable for practical purposes. Obviously, this inconsistency is caused by the use of approximate methods for solving the classical integral used to find the surface area of any body of revolution, and in our case, this has the following form (Narushin et al., 2020b):

$$S = \pi B \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\sqrt{(L^2 + 8wx + 4w^2)^3(L^2 - 4x^2) + 4B^2(4wx^2 + (L^2 + 4w^2)x + wL^2)^2}}{(L^2 + 8wx + 4w^2)^2}\,\mathrm{d}\,x \tag{7}$$

In this regard, the objective of this study was to infer simplified computation formulae for chicken egg volume, V, and surface area, S, that would be as accurate as possible. We expected that the sought equation for V would be only based on two direct linear measurements of the eggs, i.e., their length, *L*, and maximum breadth, *B*, and that for *S* would be deduced using both data on *L*, *B* and the parameter *w*, and a simpler calculation option based on measurements of the egg volume. In the course of this investigation, we also employed a simulation method to derive the desired formulae.

2. Methodology

The simulation modelling technique was exploited in this study in order to avoid limitation caused by any certain sample size of eggs, the corresponding limited measurement results of which were previously transferred by the other investigators to all possible variation in eggs, i.e., all combinations of their dimensions and shapes that are present in nature. The entirety of such combinations can be assured only by going simulatively through all feasible main egg parameters. This approach would make it possible to take into account all variants of value combinations for three parameters, *L*, *B*, and *w*, applicable to chicken eggs and used in the appropriate calculation formulae for determining *V* and *S* (Narushin et al., 2020b). On the other hand, we also tested if it would be more convenient to use in these formulae the ratios of the well-known egg shape index, $SI = B/L$, instead of using just *B*, and the ratio w/L, instead of *w*.

Based on egg measurement data from Romanoff and Romanoff (1949), the following variation limits of *L* and *SI* were taken for a simulation trial:

$L = 5.2…6.4$ cm,

$SI = B/L = 0.66…0.84$.

For determining the limits for w/L, we used the theoretical background of Obradović et al. (2013) who studied modifications of the geometric contours of Hügelschäffer’s model as well as our own results (Narushin et al., 2020c). Accordingly, the minimal value of *w* is 0 (in this case the Hügelschäffer’s ovoid is transformed in the ellipsoid) and the maximum one is not more than

$w_{max} = (L - B)/2$, wherefrom the maximum possible value of w/L for any avian egg does not exceed 0.25. Considering the above, the possible variations of w/L were taken as:

$w/L = 0…0.25$.

Thus, the above data served as the basis for modelling a wide variety of eggs. By changing the values for L in increments of 0.2 cm, those for SI in increments of 0.02, and those for w/L in increments of 0.05, we simulated 1820 combinations typical for the entire feasible variety of chicken eggs. These were then tested to calculate the actual egg volume, V, using Eqn3 and surface area, S, using Eqn7 for which we also exploited a numerical method in MS Excel as was proposed elsewhere (Piessens et al., 1983).

After that, the obtained data for V were compared with those calculated with the formula for ellipsoids V_{el} (Eqn1). The values of V and S were also used to estimate the coefficient k_S in Eqn2 and compare with those deduced for simplified calculations under the measurements of L and B.

3. Results and Discussion

3.1. Egg volume

Eqn1 can be presented for ellipsoids as $V_{el} = k_V LB^2$, where we have the constant $k_V = 0.5236$. In the sample of simulated 1820 eggs, the results of comparing the calculated data for V (Eqn3) and V_{el} (Eqn1) showed a computation error when using Eqn1 that ranged between 0 and 5.1%, with an average value of 1.4%. This would be quite acceptable in performing studies that do not require a very high accuracy. Alternatively, after calculating the ratio V/LB^2, we obtained the average value of the constant in Eqn1 equal to 0.5163 ± 0.0065. Using this new constant, the modified formula for representing the volume of chicken eggs, V, can be rewritten as follows:

$$V = 0.5163LB^2 \qquad (8)$$

that enabled to lower the computation error variation in the range 0–3.7%, with an average value being 1.1%.

We made further attempts to increase the calculation accuracy and approximated the data for *V* (Eqn3) and V_{el} (Eqn1) by a corresponding function (Fig. 1), which approximation gave us the following dependence:

$$V = 0.9936V_{el} - 0.4065, \qquad (9)$$

with $R^2 = 0.9974$.

As a result, after the respective transformation, we deduced the following formula that can be used to calculate the volume of chicken eggs:

$$V = 0.5202LB^2 - 0.4065 \qquad (10)$$

Using the improved formula (Eqn10), the results of calculations somewhat approached the actual value of the volume, although its computation error practically did not change in comparison with Eqn8, being in the range 0–3.6% with an average value of 1.1%.

Our next step to better the accuracy was to find an adequate functional dependence of k_V on the values of *L* and *B* taken alone or in combination. We found that the prediction of k_V was very low in all cases, being the same for the multiple function $k_V = f(L,B)$ and when only the parameter *L* is considered. Approximation of the dependence $k_V = f(L)$ (Fig. 2) was performed using the following dependence:

$$k_V = 0.0047L + 0.9594 \quad (11)$$

with R^2 = 0.0228,

and its inputting into $V = k_V LB^2$ led to the following formula:

$$V = (0.005L + 0.959)LB^2 \quad (12)$$

Applying Eqn12, we obtained a further slight improvement of the egg volume calculation accuracy: the respective computation error was in the range 0.06–3.40%, with an average value being 1.07%.

Thus, when calculating the egg volume and if the research data do not imply a very high measurement accuracy, we propose using Eqn12 that ensures the simplicity of measurement of the initial egg parameters and the sufficient accuracy of the obtained results. Alternatively, simpler and more conventional Eqns 8 and 10 can also be used.

3.2. Shell surface area

Eqn7 was modified in such a way as to replace the linear dimensions *L*, *B*, and *w* with their ratios, $SI = B/L$ and w/L, and the following alteration of the variable was produced in the integral:

$$t = \frac{x}{L}, \quad (13)$$

resulting in the following changes:

$$\mathrm{d}x = L\mathrm{d}t$$

and the limits of integration:

$$x_{\min} = -\frac{L}{2}$$

$$t_{\min} = \left(-\frac{L}{2}\right) \cdot \frac{1}{L} = -\frac{1}{2}$$

$$x_{\max} = \frac{L}{2}$$

$$t_{\max} = \frac{L}{2} \cdot \frac{1}{L} = \frac{1}{2}.$$

Finally, Eqn7 was transformed into the following one:

$$S = \pi BL \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\sqrt{\left(1+8\frac{w}{L}t+4\left(\frac{w}{L}\right)^2\right)^3 \cdot (1-4t^2) + 4SI^2 \cdot \left(4\frac{w}{L}t^2 + \left(1+4\left(\frac{w}{L}\right)^2\right)t + \frac{w}{L}\right)^2}}{\left(1+8\frac{w}{L}t+4\left(\frac{w}{L}\right)^2\right)^2} \mathrm{d}t \qquad (14)$$

To solve the integral in Eqn14, denoting it for convenience as $I_{w/L}$, we substituted the possible values of w/L in increments of 0.05, resulting in the undergoing equations.

1. When $w/L = 0$, the Hügelschäffer's ovoid equals to the ellipsoid, and in this case the integral from Eqn7 is rewritten as follows:

$$I_0 = \int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{1-4t^2(1-SI^2)}\, \mathrm{d}t \qquad (15)$$

2. When w/L = 0.05,

$$I_{0.05} = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\sqrt{(1.01+0.4t)^3 \cdot (1-4t^2) + 4SI^2 \cdot (0.2t^2 + 1.01t + 0.05)^2}}{(1.01+0.4t)^2} \mathrm{d}t \tag{16}$$

3. When w/L = 0.1,

$$I_{0.1} = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\sqrt{(1.04+0.8t)^3 \cdot (1-4t^2) + 4SI^2 \cdot (0.4t^2 + 1.04t + 0.1)^2}}{(1.04+0.8t)^2} \mathrm{d}t \tag{17}$$

4. When w/L = 0.15,

$$I_{0.15} = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\sqrt{(1.09+1.2t)^3 \cdot (1-4t^2) + 4SI^2 \cdot (0.6t^2 + 1.09t + 0.15)^2}}{(1.09+1.2t)^2} \mathrm{d}t \tag{18}$$

5. When w/L = 0.2,

$$I_{0.2} = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\sqrt{(1.16+1.6t)^3 \cdot (1-4t^2) + 4SI^2 \cdot (0.8t^2 + 1.16t + 0.2)^2}}{(1.16+1.6t)^2} \mathrm{d}t \tag{19}$$

6. When w/L = 0.25,

$$I_{0.25} = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\sqrt{(1.25+2t)^3 \cdot (1-4t^2) + 4SI^2 \cdot (t^2 + 1.25t + 0.25)^2}}{(1.25+2t)^2} \mathrm{d}t \quad (20)$$

Each of the integrals (Eqns 15–20) was determined by numerical methods in MS Excel at different values of *SI* = 0.65...0.85, while each of the obtained dependencies (Fig. 3) was approximated by the corresponding equation:

$$I_0 = 0.2933SI + 0.7018 \quad (21)$$

with R^2 = 0.9996,

$$I_{0.05} = 0.2938SI + 0.7012 \quad (22)$$

with R^2 = 0.9996,

$$I_{0.1} = 0.2952SI + 0.6993 \quad (23)$$

with R^2 = 0.9997,

$$I_{0.15} = 0.2974SI + 0.6963 \quad (24)$$

with R^2 = 0.9997,

$$I_{0.2} = 0.3004SI + 0.6921 \quad (25)$$

with R^2 = 0.9997,

$$I_{0.25} = 0.304SI + 0.6867 \quad (26)$$

with R^2 = 0.9998.

All obtained dependences (Eqns21–26) have a single linear form:

$$I_{w/L} = aSI + b \quad (27)$$

where a and b are the coefficients, which in turn were investigated for the functional dependence on the value of the ratio w/L (Fig. 4).

The result of approximating the coefficients a and b gives the following dependences:

$$a = 0.043\frac{w}{L} + 0.292, \quad (28)$$

$$b = 0.704 - 0.061\frac{w}{L}, \quad (29)$$

which made it possible to obtain the resulting formula for the value of the integral from Eqn27:

$$I_{w/L} = \left(0.043\frac{w}{L} + 0.292\right)SI - 0.061\frac{w}{L} + 0.704 \quad (30)$$

As a result, the formula for calculating S (Eqn14) can be presented in its final form:

$$S = \pi BL\left(\left(0.043\frac{w}{L} + 0.292\right)SI - 0.061\frac{w}{L} + 0.704\right) \quad (31)$$

As follows from Fig. 4, the coefficients a and b remain practically unchanged with the average values of a_m = 0.297 and b_m = 0.696. Then, the simplified formula for calculating the surface area after substituting the values for a_m and b_m in Eqn27 will be rewritten as:

$$S_s = \pi BL(0.297SI + 0.696) = 0.933B(B + 2.343L) \quad (32)$$

in which S_s means that the surface area was recalculated according to the simplified function.

Considering that there is no a direct accurate method for measuring *S*, we estimated the accuracy of calculating integrals (Eqns 15–20) and, accordingly, the values of *S* (Eqn31), analysing the particular case when the Hügelschäffer's ovoid transforms into an ellipsoid, i.e. for $w = 0$. In this case, Eqn31 should give identical values calculated by Eqn6 with substitution of any *SI* variable values. For the convenience of substitutions, we presented Eqn6 for calculating the surface area of ellipsoids as follows:

$$S_{el} = \frac{\pi BL}{2}\left(\frac{\arcsin\sqrt{1-SI^2}}{\sqrt{1-SI^2}} + SI\right) \tag{33}$$

Comparative analysis showed full agreement of the results using Eqn31 (after the corresponding substitution of $w = 0$), with a computation error being 0.00002%.

Thus, we concluded that Eqn31 can be taken as the base for making accurate calculations of the surface area of eggs, the shape of which corresponds to the Hügelschäffer's model.

The next stage of our research was the analysis of data obtained by means of simulation modelling of the whole possible variety of chicken eggs. At the same time, the calculation of the egg volume was carried out according to the previously derived formula (Eqn3) for ovoids described by the Hügelschäffer's model (Narushin et al., 2020b). The *S* values of Eqn31 were the criteria for comparative analysis with other derived formulae, i.e., with simplified S_s (Eqn32) and S_{el} (Eqn33).

A comparative analysis of the generated 1820 values of chicken eggs showed the average computation error for S_s equals to 1.2%, with variation from 0 to 5.7%. For S_{el} (according to Eqn33), the average error was 0.3%, with variation from 0 to 0.9%.

Thus, the use of the formula for calculating the surface area of ellipsoids (Eqn33) is quite acceptable in the study of the surface area of chicken eggs.

Using Eqn2 and correspondingly dividing the values of S (according to Eqn31) by V (according to Eqn3) in the power 2/3, the respective data were obtained for the coefficient k_S, the average value of which being 4.944 ± 0.048. The obtained value of k_S turned out to be a bit higher than those published by Romanoff and Romanoff (1949), 4.831, but very close to the one reported by Paganelli et al. (1974), i.e., 4.951. Replacing k_S in Eqn2 with its newly found average value of 4.944, we produced the following formula:

$$S_V = 4.944V^{\frac{2}{3}} \qquad (34)$$

The computation error of measurements carried out using the modified Eqn34 in comparison with S according to Eqn31 was in the range 0–3.1%, with an average value being 0.8%.

In this study, we have tried to put an end to the issue of simplified formulae for calculating the volume of chicken eggs. Previously, due to the approximate calculation of volume and surface area, there was a problem in calculating these main egg characteristics quickly and accurately.

As a result of the present investigation, a methodological approach was developed, with the help of which it is possible to relatively simply assess the adequacy of existing or newly created methods for the simplified calculation of the external parameters of bird eggs. This approach is based on the principle of simulation modelling of the parameters that are inherent in eggs of the given species, and on advanced mathematical formulae, which can be used to accurately calculate the parameter of interest.

Importantly, the significance and adequacy of the obtained formulae for calculating the egg volume (Eqns 8–10, 12) and surface area (Eqns 31–34) rely on computation advantages provided by simulating all possible variations in the geometric parameters characteristic of chicken eggs, which covers any experimental sample size in producing non-destructive measurements of actual eggs.

4. Conclusions

In summary, the calculation of egg volume by exploiting only two straightforward linear parameters *L* and *B* and employing the obtained simplified dependencies (Eqns 8–10,12) can be safely taken as a basis for practical usage in poultry research, food engineering and technology development areas. At the same time, if it needs a very high accuracy in evaluating this parameter, we recommend that in addition to measuring *L* and *B*, the parameter *w* be used according to the formula (3) previously deduced by (Narushin et al., 2020b). For the computation of the surface, we suggest implementing three formulae, i.e., Eqn31 for using of which the measurements of the egg basic linear parameters *L*, *B* and *w* are needed, Eqns32 and 33, which are exploiting only two straightforward parameters *L* and *B*, and Eqn34 obtained via the egg volume. Eqn31 considers all drawbacks of our previous theoretical findings (Narushin et al., 2020b, 2021) and may be recommended for evaluating *S*, if it needs a very high accuracy. As shown by the results of simulation modelling, the surface area of chicken eggs is practically identical to this parameter calculated for ellipsoids having the same dimensions of their main axes. Given the new simpler formulae obtained for the egg volume (Eqns 8–10, 12) and surface area (Eqns 31–34), we believe that their further improvement is hardly feasible and that further polemics on this issue can be closed for now.

Declaration of Competing Interest

The authors declare no competing financial interest.

References

Narushin, V. G. (1997). Non-destructive measurements of egg parameters and quality characteristics. *World's Poultry Science Journal*, *53*(2), 141–153.

Narushin, V. G. (2005). Egg geometry calculation using the measurements of length and breadth. *Poultry Science*, *84*(3), 482–484.

Narushin, V. G., Lu, G., Cugley, J., Romanov, M. N., & Griffin, D. K. (2020a). A 2-D imaging-assisted geometrical transformation method for non-destructive evaluation of the volume and surface area of avian eggs. *Food Control*, *112*, Article 107112.

Narushin, V. G., Romanov, M. N., Lu, G., Cugley, J., & Griffin, D. K. (2020b). Digital imaging assisted geometry of chicken eggs using Hügelschäffer's model. *Biosystems Engineering*, *197*, 45–55.

Narushin, V. G., Romanov, M. N., Lu, G., Cugley, J., & Griffin, D. K. (2020c). A universal formula for avian egg shape. *bioRxiv*, Article 2020.08.15.252148. doi: https://doi.org/10.1101/2020.08.15.252148.

Narushin, V. G., Romanov, M. N., Lu, G., Cugley, J., & Griffin, D. K. (2021). How oviform is the chicken egg? New mathematical insight into the old oomorphological problem. *Food Control*, *119*, Article 107484.

Obradović, M., Malešević, B., Petrović, M., & Đukanovic, G. (2013). Generating curves of higher order using the generalisation of Hügelschäffer's egg curve construction. *Buletinul Ştiinţific al Universităţii „Politehnica" din Timişoara: Transactions on Hydrotechnics*, *58*(72), 110–114.

Paganelli, C. V., Olszowka, A., & Ar, A. (1974). The avian egg: Surface area, volume and density. *Condor*, *76*(3), 319–325.

Petrović, M., & Obradović, M. (2010). The complement of the Hugelschaffer's construction of the egg curve. In M. Nestorović (Ed.), *Proceedings of the 25th National and 2nd International Scientific Conference moNGeometrija 2010* (pp. 520–531). Belgrade: Faculty of Architecture in Belgrade, Serbian Society for Geometry and Graphics.

Piessens, R., Doncker-Kapenga, E. de, Überhuber, C. W., & Kahaner, D. K. (1983). *QUADPACK: A subroutine package for automatic integration*. Berlin, Heidelberg: Springer-Verlag.

Romanoff, A. L., & Romanoff, A. J. (1949). *The Avian Egg*. New York: John Wiley & Sons Inc.

Schmidbauer, H. (1948). Eine exakte Eierkurvenkonstruktion mit technischen Anwendungen. *Elemente der Mathematik*, *3*(3), 67–68.

Ursinus, O. (Ed.). (1944). Kurvenkonstruktionen für den Flugzeugentwurf. *Flugsport*, *36*(9), Merkblätter 15–18.

Figure captions

Fig. 1. The relationship between the volume of a chicken egg, *V*, and the volume of an ellipsoid, V_{el}, that has similar geometric dimensions of length and maximum breadth, with the dependence line being described with the function $V = 0.9936\ V_{el} – 0.4065$ ($R^2 = 0.9974$).

Fig. 2. The relationship between the coefficient k_V and the egg length, *L* ($k_V = 0.0047\ L + 0.9594$; $R^2 = 0.0228$).

Fig. 3. Values of integrals from Eqn12 depending on possible values of *w*/*L* and *SI*: $I_0 = 0.2933\ SI + 0.7018$ ($R^2 = 0.9996$); $I_{0.05} = 0.2938\ SI + 0.7012$ ($R^2 = 0.9996$); $I_{0.1} = 0.2952\ SI + 0.6993$ ($R^2 = 0.9997$); $I_{0.15} = 0.2974\ SI + 0.6963$ ($R^2 = 0.9997$); $I_{0.2} = 0.3004\ SI + 0.6921$ ($R^2 = 0.9997$); $I_{0.25} = 0.304\ SI + 0.6867$ ($R^2 = 0.9998$).

Fig. 4. Functional dependence of the coefficients *a* and *b* in Eqn19 on possible values of *w*/*L*, where $a = 0.0431\ w/L + 0.292$ ($R^2 = 0.9343$) and $b = –0.0605\ w/L + 0.7038$ ($R^2 = 0.9242$).