Joint Optimisation on Maintenance Policy and Resources for Multi-unit Parallel Production System and its Applications[1]

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Abstract: Multi-unit parallel production systems (MuPPSs) widely exist in many industries. A general requirement for a MuPPS is that each production system should continuously operate and its downtime should be as short as feasible. For such systems, optimising preventive maintenance (PM) policy is crucial so that resources such as the number of spare units and that of repairmen can be optimally used. On this basis, this paper develops an optimal model jointly optimising the PM policy and the number of repairmen and spare units, for which the genetic algorithm is used. A real case study of a train working line is presented to demonstrate the necessity of the proposed model. Finally, sensitivity analysis is conducted to assess the influence of some parameters on the optimal results.
Key words: Joint optimisation; Maintenance policy; Maintenance resource; Multi-unit parallel production system; transportation system.

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1.Introduction

1.1Motivation

Multi-unit parallel production systems (MuPPSs) widely exist in the continuous production industries, such as manufacturing, railway and civil aviation. A general requirement for a MuPPS is that each production line should continuously operate and its downtime should be minimised. To this end, one may consider two alternative solutions on maintenance: two types of maintenance approaches are performed:

- The first type regards each production line as a multi-unit series system, where some units can be maintained on the line, and the production line can continuously operate by relying on the buffer storage. The upstream and downstream of the buffer storage for the unit is the focus for maintaining the operation. Many research were conducted to address this issue (Gan et al., 2013; Wu & Do, 2017; Zhou et al., 2018).
- The second type is that the production line is viewed as one unit, which can be replaced by a spare part and maintained off-line to keep the production operating. In this type, the allocation of the maintenance resource should jointly consider the PM policy and the number of spare units and repairmen/teams(Zhang et al., 2018).

From an economic viewpoint, resource allocation is a crucial issue for some firms, especially in the case of large MUPPSs. On the one hand, a large number of spare units and repairmen/teams can result in the waste of resources. On the other hand, the availability of a production system can be affected because of a lack of sufficient maintenance. Consequently, with the enhancement of operating units in reliability and economy, a challenge for these production systems is that each production line or operating unit should avoid stoppages as much as possible; even one stoppage is disallowed in some normal operating systems. The joint consideration of PM policy and the numbers of operating units and repairmen/teams are a vital issue for some enterprises; these aspects are traditionally treated in isolation, while these aspects have been traditionally treated almost in isolation. Based on this consideration, this paper focuses on the second type and discusses its application in a continuous production system. Therefore, the purpose of this paper is to determine how to jointly optimise the PM policy and the number of spare units and repairmen/teams for MuPPSs with fixed production lines.

1.2 Releated works

Several studies were performed to establish optimal maintenance policies based on different point of views. Most of them focused on maintenance optimisation and devoted little attention towards the configuration of the maintenance resource for the manufacturing system. Several works suggested that the group maintenance for the multi-unit system can save maintenance cost(Chiu et al., 2018). However, a dilemma is present between maintenance and production for MuPPSs in the continuous production industry. To resolve this dilemma, maintenance, the number of operating units,, and the number of repairmen should be jointly considered in decision-making. Ni et al. (2015) emphasized that production is often interrupted by a prescheduled PM without considering the throughput target and PM tasks are not conducted in a cost-effective manner. They investigated the extra hidden opportunities for PMs during production time without violating the system throughput requirement.

Zhou et al. (2016) studied the maintenance optimisation of a parallel-series system by considering the stochastic and economic dependences amongst the components, as well as a limited maintenance capacity. Wu (2019) proposed an exponential smoothing of intensity model and investigated its special case model, both of which model the failure process of a series system composed of multiple components. Kumar & Lad (2017) considered the interdependence of production and maintenance plan and the direct relationship between product quality and maintenance plan and proposed an integration method for the production and maintenance plans of a parallel production system. Their method aims to determine the best production and maintenance plans to minimise the overall operating cost. Yoo & Lee (2016) searched a coordinated scheduling scheme for the operations and maintenance activities of parallel machines to minimise the scheduling cost represented by the completion time, weighted sum of completion times, maximum completion delay and sum of completion delays. Wu & Scarf (2017) modeled the repair effect in multi-unit systems with a stochastic process, and proposed two models for the failure process of a series system that overcome the limitations of existing models that are either restrictive or require knowledge of the failure process for individual components. Zhao et al. (2019) developed a maintenance modeling and optimization framework for single-unit systems with atypical degradation path of which the pattern is influenced by inspections. The paper assumed that the system degradation is decreased to a random value instantaneously after each inspection and the degrading rate is elevated due to the inspection. Zahedi-Hosseini et al. (2018) established a model of joint inspection and spare part inventory policy for maintenance units in a parallel system, where simultaneous shutdowns seriously affect the production performance and incur losses. Pargar et al. (2017) studied PM and renewal scheduling for multi-unit systems and developed an integrated optimization method to schedule PM and renewal projects by grouping them and simultaneously finding the optimal balance between them. Sun et al. (2020) quantified the benefit of incorporating the reallocation into the CBM framework on series systems, and the optimal control limits for reallocation and preventive replacement are investigated based on the periodic inspection framework. Dui et al. (2019) proposed an extended joint integrated importance measure (JIIM) to effectively guide the selection of PM components, aiming to maximize gains of the system performance. A multi-valued decision diagram based method is then developed to evaluate the proposed JIIM for general repairable systems. Zhao et al. (2020)presented a joint inspection and spare ordering policy for a single-unit system with two levels of defective states, and introduced a threshold level to decide whether to place an emergency order or wait for the normal order when the normal ordered spare hasn't been delivered. Liu et al. (2017) developed a maintenance policy for a multi-unit system subject to hidden failures and the inspection intervals for each component was determined by minimizing the long-run cost rate. Cheng et al. (2017)presented a joint optimization model of production lot sizing and CBM for a multi-unit production system which produces the products to meet the demand in a finite time horizon. Gao et al. (2020) investigated the joint optimization of lot sizing and maintenance policy for a multi-product production system subject to a soft or hard failure modes. de Jonge & Scarf (2019) reviewed maintenance optimisation models for multi-unit systems and pointed out that the joint optimization of spare units ordering and maintenance is mainly considered.

From the above review, it is found that little research on jointly considering normal production, the number of repairmen and spare units to optimise PM policy has been devoted. Nevertheless, it is vital to determine some maintenance recourse configurations for enterprisers. Too many repairmen and spare units for a fixed production line can result in resource waste, whereas too few may lead to some stoppages due to needs of repairmen or spare units. Consequently, how to jointly consider normal production, the number of repairmen and spare units to optimise PM policy is a meaningful issue.

Based on this consideration, this paper mainly orients the MuPPS in continuous production systems and develops an optimisation model to obtain the optimal PM and the number of repairmen and spare units. Furthermore, it determines the optimal displacement sequence of spare units using the genetic algorithm (GA) according to the optimal PM policy and the number of repairmen and spare units. To illustrate the proposed model and method, a real case study of a train working line is exampled and sensitive analysis of some parameters are discussed in the last section.

1.3Novelty and contributions

The novelty and contributions of this work are: 1) it presents a modelling method that depicts the cycle operating process of the MuPPS in continuous production systems to be a single-periodic operating diagram; 2) it considers the joint optimisation of PM and the number of repairmen and spare units, whereas this problem is much applied; 3) it proposes an optimal method of the displacement sequence for spare units according to the optimal results of PM and the number of repairmen and spare units, this method can solve the operating order of spare units and repairmen.

The remainder of this paper is organized as follows: Section 2 introduces maintenance and its assumptions; Section 3 details the modeling and optimization; Section 4 gives a real case study; and Section 4 concludes this paper.

2 Maintenance and assumptions

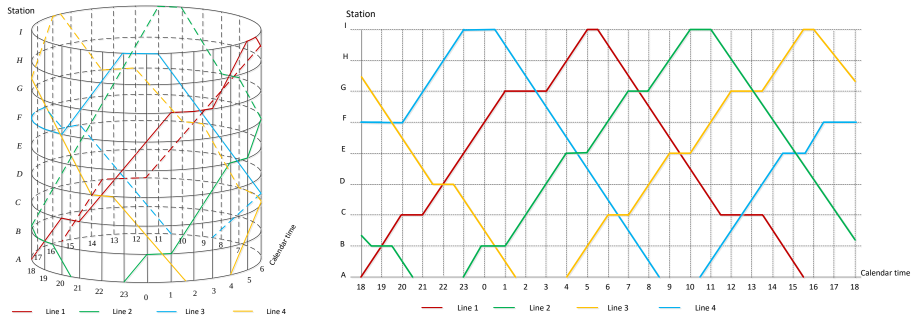
On the basis of the requirements of MuPPSs in continuous production systems, we investigate the joint optimisation of the PM policy and the number of spare units and repairmen/teams for MuPPSs with fixed production lines to introduce the operating and maintenance processes. The modelling assumptions are discussed in this section.

2.1 Maintenance process

A MuPPS in continuous production systems with M production lines requires M units to keep operating. To satisfy this requirement, one or more spare units and repairmen/teams are needed to maintain or replace operating units. For the convenience of analysis, this study regards repairman as a repair team, and *overhaul* is an exchangeable term of *replacement*. PM is periodically executed on the operating units at kT (k=1, 2,…, N-1), and the spare units are preventively maintained at an accumulated operational time T_{st}. Preventive replacement for the operating units is performed at NT, followed by that for the spare ones. Minimal repair is conducted upon unit failures. If more than one unit in the system is under PM, the number of repairmen and spare units can be increased to prevent the production line from stopping operating.

Railway transportation is a typical MuPPS in continuous production systems. To understand the maintenance mechanism, we take the transportation of locomotives in a firm in China as an example to illustrate the relevant operating and maintenance processes. A typical example of a one-way train working diagram is shown in Fig. 1, which includes the cyclic graph of the diagram (Fig. 1(a)) and the expanded graph (Fig. 1(b)). The four operation lines are marked with different colours, and letters A–I denote the nine stations, respectively. The X-axis is the calendar time. Herein, locomotives are viewed as units of a MuPPS. For instance, at 18 PM, locomotive 1 starts operation at line 1, locomotive 2 operates from station C to station B, locomotive 3 operates from station H to station G and locomotive 4 is waiting at station F. The cycle iterates. The four lines are typical MuPPSs with four operating units, in which each production line must continue operating as much as possible. The production tasks are performed by different locomotives, which include operating and spare units.

The operating unit is replaced by the spare unit when the former requires PM, and one or more repairmen are hired to perform the maintenance.

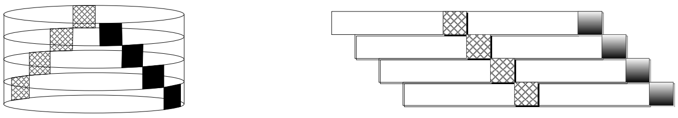


(a) Cyclic graph of train working diagram

(b) Expended graph of train working diagram

Fig. 1 Train working diagram

The operating cycle of the transportation of the locomotives is displayed in Fig. 2(a). The expanded graph presented in Fig. 2(b) illustrates a typical MuPPS in a continuous production system that includes four operating lines and can therefore be viewed as a four-unit MuPPS. This MuPPS is assumed to comprise one repairman, one spare unit, one PM activity, and one preventive replacement. The PM interval, PM time, and preventive replacement time are denoted by T, T_p and T_r, respectively, and T_p=T_r. One belt denotes one production line, and the three different rectangles represent the operating, PM and preventive replacement time, respectively. Spare units can replace the operating units if the latter is being preventively maintained. The operating process of the four-unit parallel system can be regarded as an infinite loop from the beginning of the operation to system replacement if the system maintains a long-term operation.



(a) Cycle graph of four-unit production system

(b) Expanded graph of four-unit production system

□ Operating time ■ PM time ▩ Preventive replace time

Fig. 2 Operation process graph of MuPPS

2.2 Assumptions

Based on the above-described operating and maintenance process, we make the following assumptions.

(1) The MuPPS includes M production lines and operating units, M_s spare units and M_r repairmen. The operating and spare units are identical. The spare units are only used to replace the operating units when the latter requires PM, and a repairman can only repair one unit at a time point.

(2) The failure of each unit can be immediately detected and undergo minimal repair, and time

on minimal repair can be neglected.

(3) For a new unit, the failure intensity function $h(t)$ monotonically increases in the operating time. A PM not only brings $h_i(t)$ to 0, but also alters $h_i(t)$: $h_i(t)=a_ih(t)$ before the ith PM (i=1, 2, …, N), where $1=a_1<a_2<\ldots<a_N$ (Nakagawa, 1988).

(4) A PM interval for spare units T_s and that for operating units T are different and are a fixed values, where $\alpha_1 T\le T_s\le\alpha_2 T$ ($1<\alpha_1$, $\alpha_2>1$). Let n denote the mean PM times for a spare unit within each PM interval of operating units. Therefore, n relates to N and M. The value of n is an array$\left\{\frac{1}{N},\frac{1}{N-1},\cdots,1,\frac{M}{M-1},\frac{M}{M-2},\cdots,M\right\}$. The mean T_s is $T_{st}=MT_p/n$, where T_p is the PM time and is constant.

(5) The costs of the minimal repair, PM and replacement are denoted by C_f, C_p, and C_r, respectively. They are constant and $C_r>C_f>C_p$. The cost rate C_d , incurred due to the waiting time for the spare units and C_w on repairmen are also constant.

In practice, time on minimal repair is normally shorter than time on PM to avoid stopping operating. It depends onengineers' experience and the availability of spare parts, while time on PM is relatively constant. For instance, minimal repair on an engine of one diesel locomotive may take several hours (usually less than 3 hours), while time on PM may typically take 3 days. As such, the minimal repair time is neglected in this study.

3 Modeling and optimization

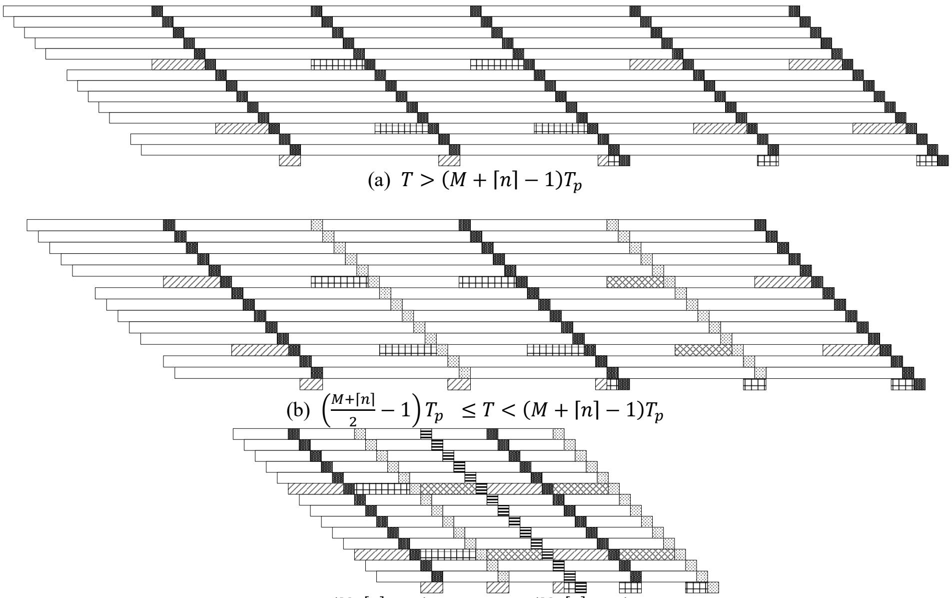
This section provides the details of the joint optimisation modelling of the PM policy, the number of spare units, and the number of repairmen. An example is presented to elaborate the modelling process of the MuPPS, and the specific steps of the optimisation are discussed.

3.1 Modeling

For the MuPPS in continuous production systems, its modelling process based on the expanded graph is illustrated as follows. According to the analysis of the maintenance process, both the numbers of spare units M_s and repairmen M_r relate to the length of the PM interval T, the number of operating units M, PM time T_p, and the mean PM interval for spare units T_{st}. T is a main factor for the determination of other parameters, and thus the modelling is discussed based on the situations of different ranges T and values of n.

For illustration purposes, a MuPPS with parameters M=12 and N=5 is presented as an example to introduce the modelling process. According to the definition of n, n={1/5, 1/4 ,1/3, 1/2, 1, 12/11, 12/10,…, 12}. The operating processes of n=12/5 at three different ranges of T is illustrated in Fig. 3, as an example to illustrate the model process.

All units of the MuPPS illustrated in Fig. 3 are new at the beginning. The expression n=12/5 means that the spare unit is preventively maintained for n=2.4 times in each PM interval, and $T_{st}=12\times T_p\times 5/12=5T_p$. Selection of M_r with different range of T is shown in Table 1. One spare unit is infeasible under the condition, thus two spare units and one repairman are considered in subplot (a). Two spare units and repairmen are considered in subplot (b), and three spare units and repairmen are considered in subplot (c) of Fig.3.



(a) $T > (M + \lceil n \rceil - 1)T_p$

(b) $\left(\frac{M+\lceil n \rceil}{2} - 1\right)T_p \leq T < (M + \lceil n \rceil - 1)T_p$

(c) $\left(\frac{M+\lceil n \rceil}{3} - 1\right)T_p \leq T < \left(\frac{M+\lceil n \rceil}{2} - 1\right)T_p$

repairman 1 repairman 2 repairman 3 spare unit 1 spare unit 2 spare unit 3

Fig.3 Operating process of MuPPS (n=12/5)

In the first PM interval (Fig. 3[a]), the operating units start to run, while the spare units are in the cold standby state, and the repairman is available. Suppose that the operating units have been operated for some time, and then units 1~5 are preventively maintained in sequence. Spare unit 1 replaces operating units 1~5 when each of them undergoes PM. Further, spare unit 1 is preventively maintained and takes the places of operating units 6~10 when each of them is in PM. Subsequently, spare unit 1 is preventively maintained again and then replaces operating units 11 and 12 when each unit undergoes PM. The following PM intervals are the same as the first one. In the last PM interval, preventive replacement is performed for all operating and spare units, and a new cycle continues. In Figs. 3(b) and 3(c), the operating and maintenance processes are the same as those illustrated in subplot (a) of Fig.3.

The numbers of the repairmen and spare units are the key to the effective allocation of maintenance resources and control of the maintenance cost, so is the waiting time of the repairmen. Therefore, the maintenance model is closely related to these parameters. The method for determining these parameters are explained as follows.

(1) M_r and T_r

The number of repairmen M_r increases if two or more units are at the PM state at the same time. As shown in Figs. 3 and 4, M_r is related to the length of T, which is affected by M, N, n and T_p., respectively. For a long T, the PM tasks can be dispersed within the PM interval, whereas repairmen need increase to implement the PM tasks for a short T. Additionally, a small M means less Mr, a small n denotes less PM times for spare units, and a short T_p can arrange more PM tasks. Consequently, M_r is closely related to T. The relationship between T and M_r is expressed in Table.1.

Table.1 Relationship between T and M_r

T	M_r
$\left[(M+\lceil n\rceil-1)T_p\ ,+\infty\right]$	1
$\left[\left(\frac{M+\lceil n\rceil}{2}-1\right)T_p\ ,(M+\lceil n\rceil-1)T_p\right]$	2
$\left[\left(\frac{M+\lceil n\rceil}{3}-1\right)T_p\ ,\left(\frac{M+\lceil n\rceil}{2}-1\right)T_p\right]$	3
$\vdots$	$\vdots$
$\left[0\ ,\left(\frac{M+\lceil n\rceil}{k-1}-1\right)T_p\right]$	k

The total waiting time of the repairmen T_r is related to the starting time and completion time of the system replacement, as well as to the work time of the repairman. Therefore, T_r can be expressed as

$$T_r = M_r\left(N\left(T+T_p\right)+(M+\lceil n\rceil-1)T_p\right)-(MN+\lceil Nn\rceil)T_p$$

(2) M_s and T_s

The number of spare units M_s is related to the values of M and n, and their relationship is explained as follows.

i) If n is an integer and n<1, M_s=M_r.

ii) If n is not an integer and n>1, a shortage of repairmen occurs at the end of a PM cycle, as shown at the end of the third PM interval in Fig. 3(b). Therefore, $I = M\backslash\left(\frac{M}{n}\right)$, where '\' indicates taking the remainder. The number of spare units is 2 if the PM interval is sufficiently large. M_s is determined as follows.

iii) If $\left(\frac{M}{n}\right)\backslash I = 0$,

$$M_s = \begin{cases} 2 & T \geq (M+\lceil n\rceil-1)T_p \\ M_r & otherwise \end{cases} \tag{1}$$

iv) If $\left(\frac{M}{n}\right)\backslash I \neq 0$,

$$M_s = \begin{cases} 2 & T \geq (M+\lceil n\rceil-1)T_p \\ M_r & N < \lceil (M/n)/I\rceil \ and\ T < (M+\lceil n\rceil-1)T_p \\ M_r+1 & N \geq \lceil (M/n)/I\rceil \ and\ T < (M+\lceil n\rceil-1)T_p \end{cases} \tag{2}$$

The waiting time of the spare unit T_s is related to the starting time of the system and the completion time of the spare unit replacement, as well as to the work time of the spare units. Therefore, T_s is determined as

$$T_s = M_sN\left(T+T_p\right)-(MN+\lceil Nn\rceil)T_p \tag{3}$$

After the determination of the parameters, the long-term operating cost rate of the system can be established. The renewal cycle of the MuPPS is a loop between the starting and completion times of the replacement for every unit. Y denotes the length of the renewal cycle. The cost of the minimal repairs, PM, preventive replacement, waiting time of spare units, and waiting of repairmen will be incurred during the renewal cycle. Denote the total cost as C. Hence, C and Y form a renewal reward process. According to the renewal reward theorem, the long-term operating cost rate of the system is expressed as

$$g(T,n,N,M_s,M_r)=\frac{E[C]}{E[Y]}. \tag{4}$$

The cost of minimal repairs includes the costs of the operating and spare units (CF_1 and CF_2, respectively) when both units are operating.

$$CF_1 = C_f M \sum_{i=1}^{N} H_i(T), \tag{6}$$

and

$$CF_2 = C_f M_s \left\{\sum_{i=1}^{\lfloor NMT_p/(M_sT_{st})\rfloor} H_i(T_{st}) + H_{\lceil NMT_p/(M_sT_{st})\rceil}\left(NMT_p/M_s - T_{st}\, NMT_p/(M_sT_{st})\right)\right\}, \tag{7}$$

where $H_i(T)=\int_0^T h_i(t)\,dt$.

The cost of PM CP is determined by the PM times for the operating and spare units, that is,

$$CP(n) = C_p\left(M(N-1)+N_p\right), \tag{8}$$

where N_p is the number of PM for spare units during a renewal cycle, which can be determined as

$$N_p=\begin{cases}0 & Nn \le M_s\\ \lceil Nn\rceil - M_s & otherwise\end{cases} \tag{9}$$

The replacement cost CR is related to the number of operating and spare units and can be defined by

$$CR = C_r(M+M_s) \tag{10}$$

Lastly, the cost incurred due to the waiting time includes the cost of the repairmen CW and the cost of spare units CD, which can be expressed as

$$\begin{aligned}CW &= C_wT_r\\ CD &= C_dT_s\end{aligned} \tag{11}$$

The specific expression of the long-term operating cost rate of the system $g(T,n,N,M_s,M_r)$ is written as

$$g(T,n,N,M_s,M_r)=\frac{CF_1+CF_2+CW+CD+CP(n)+CR}{NM\left(T+T_p\right)} \tag{12}$$

3.2 Optimization

In Eq.(11), $g(\bullet)$ has five variables, among which only T is a continuously variable and the rest is discrete variable. According to Eq.(12), given an initial value for N=1, M_r=1, M_s=1 and n, one can use the function fminbnd(•) in Matlab to find the minimal value of $g(T,n,N,M_s,M_r)$: g_{min}. One can then increase each discrete variable and search g_{min}. A set of g_{min}'s can be sought and a minimum g_{min} can be determined. A brief optimisation process is introduced by the N-S flowchart shown in Fig. 4.

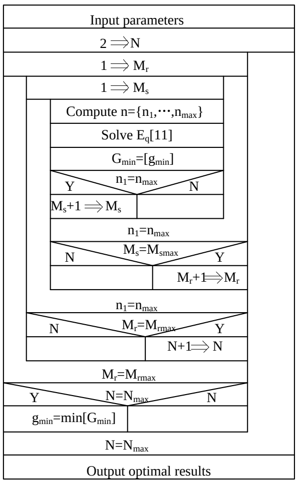


Fig.4 Optimisation process

3.3 Determination of displacement sequence

PM time is a fixed interval and n^* usually is fraction. It is therefore difficult to determine the sequence of spare units to displace operating units when they are in the PM state. Based on the PM plan for each operating unit, there are two situations on the displacing process of spare units: One is without an overlapping area and the other is with an overlapping area, which are illustrated in Fig. 5 and Fig. 7, respectively. The system has one repairman and spare unit in Fig. 5, and two spare units and repairmen, as shown in Fig. 7. For a system without an overlapping area, one spare unit is arranged to displace the operating unit, whereas two are needed in an overlapping area. Meanwhile, PM is limited by the number, M_r, of repairmen, which makes PM units less than or equal to M_r and M_s. With this character, we discrete the displacing time of spare units with a mean length T_{st}, and then use the optimization algorithms to determine the displacement sequence of spare units operating time. Herein, the genetic algorithm (GA) is used since it is widely used to seek the global optima (Sriskandarajah et al., 1998).

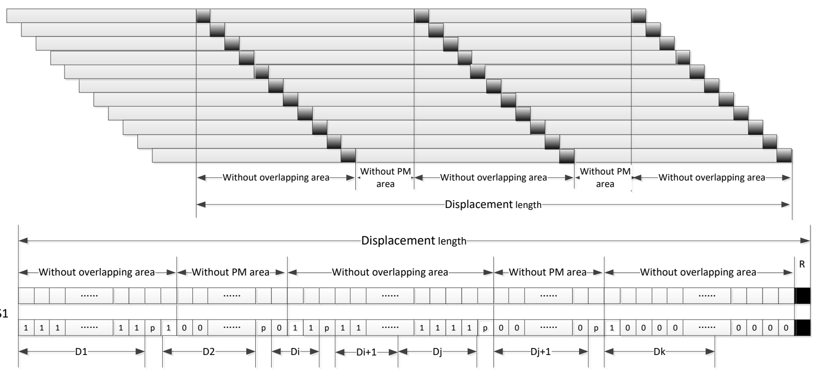


Fig. 5 No overlapping area

To explain the optimisation of the displacement sequence of spare units, we present some definitions: *Displacement length* is a period from operating of the first spare unit to the replacement of the last one, and which includes of some *bytes* d_i. The value of d_i is $\{0, 1, p\}$, 0 standing for no PM, 1 for displacement for one operating unit and p for PM for one spare unit. The length of one byte equals to T_p. *Displacing length* includes some D_i's, which is the total operating length of a spare unit after a PM and displayed in Fig.6. The total PM and replacement times for spare units is K, as defined in Eq.(12).

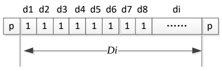


Fig. 6 Plot of D_i

S_i denotes the ith spare unit, the total length of S_i is the displacement length. For two spare units shown as Fog.7, the value of each byte in S_2 is determined according to S_1. The value d_i in no overlapping area of S_2 is 0 or p if it is 1 at corresponding byte of S_1, otherwise d_i is 1 in S_2 if corresponding byte of S_1 is 0 or p. In overlapping area, d_is are all 1 in S_1 and S_2 because the system has two repairmen and spare units.

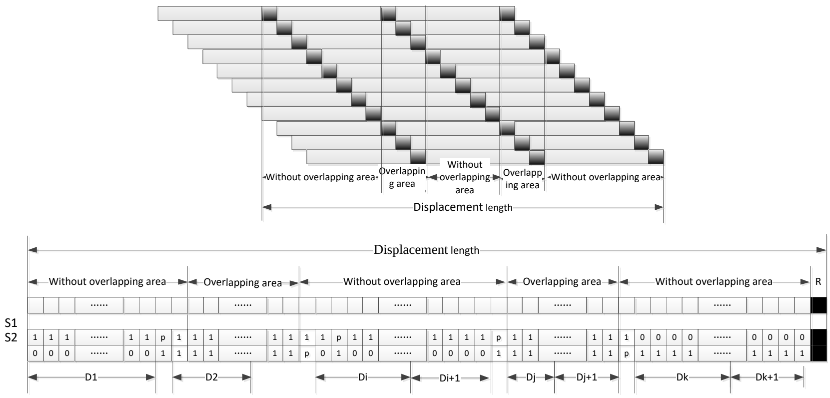


Fig.7 Overlapping area>0

According to the definition of n^*, D_0 is defined in Eq.(14). There are many combinations of D_i under the limitation of T_{st}, and the optimal displacement sequence is close to D_0 because it is limited by the spare units and repairmen. Thus, we define the objective function P_{min} as bellow. Then, use GA to obtain the optimal displacement sequence of spare units.

Objective function: $P_{min} = \sqrt{\sum_{i=1}^{K}(D_i - D_0)^2}$ (13)

where $K = \lceil nN \rceil,\ D_0 = \begin{cases} M(N-i) & \text{if } n = \left\{\frac{1}{N}, \frac{1}{N-1}, \cdots, \frac{1}{N-i}, \cdots 1\right\} \\ M-i & \text{if } n = \left\{\frac{M}{M-1}, \frac{M}{M-2}, \cdots, \frac{M}{M-i}, \cdots, M\right\} \end{cases}$ and $D_i = \sum_{i=1}^{m} d_i$

(14)

4 Application in transportation systems

This section presents a real case study of the railway transportation line in area district in Northwest China to illustrate the application of the proposed model. Sensitivity analysis is also conducted to determine the influence of the parameters on the optimal results.

4.1 Introduction of railway production line

The roundtrip transportation task of a railway line in one area is fulfilled by several fixed locomotives, and thus it is a typical MuPPS. To illustrate the application of the proposed model, Fig. 8 displays the working diagram of a passenger train in a district in Northwest China. In this working diagram, six carriages of a passenger train travel back and forth between stations A and D. That is, 12 locomotives serve as the operating unit roles to drive the passenger trains. For example, assume that the locomotive driving passenger train T9534 is L, starts from station A at 13:45 and arrives in station D at 6:20. Then, it drives train K9786 from station D at 18:30 and arrives in station A at 13:40 the next day. The six pairs of locomotives are arranged to repeatedly drive the six carriages of passenger trains until one of them requires PM. The locomotive that needs PM is overhauled by a spare one. The distance between each two stations is 500 km, and the round trip for one locomotive is 3000km. To ensure normal operation, the system needs several spare locomotives and repairmen. Spare locomotives remain at the standby state, and the repairmen are free when the operating locomotives do not require PM. Determining the PM policy and the required number of spare locomotives and repairmen is therefore important for the firm.

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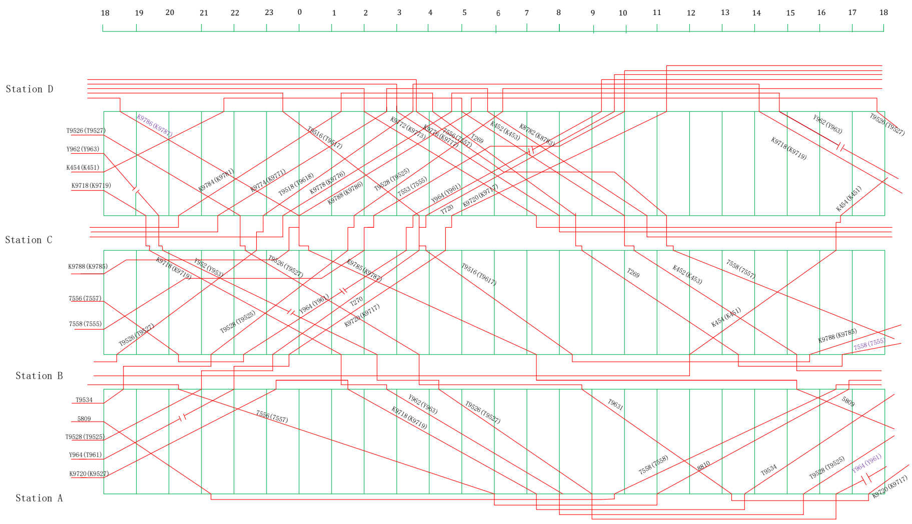


Fig. 8 Train working diagram in one Chinese area

The distance between each two stations is 500km, and the round trip for one locomotive is 3000km, marked as T_0=3000km. Thus, the operating time is an integer multiple of T_0, and the Eq.(12) can be adjusted as Eq.(15).

$$g(k,n,N,M_s,M_r) = \frac{CF_1+CF_2+CW+CD+CP(n)+CR}{NM(kT_0+T_p)} \tag{15}$$

where k=1,2,…

According to (Z. Zhang et al., 2012), the failure intensity $h(t)$ of the diesel locomotive in this area follows the two-fold Weibull competing risk model. The expression is stated as below and it is used in the following optimisation modelling.

$$h(t) = 2.84 \times 10^{-5}\left(\frac{t}{30239}\right)^{-0.14} + 1.17 \times 10^{-4}\left(\frac{t}{26519}\right)^{2.1032}$$

4.2 Optimization

To optimise the PM policy and the number of spare units and repairmen, the following additional assumptions are made.

1) The spare locomotives serve as replacements when the operating locomotives need to be preventively maintained or overhauled.

2) The result of the overhaul is the same as that of replacement, and the former can render the locomotive to a ‘good as new’ state. After the ith PM, $a_i=4i/(3i+1)$, where i=1, 2, …, N.

The other assumptions remain the same as those enumerated in Section 2. The corresponding parameters in this case are listed in Table 2.

Table 2 Parameters

Items	Parameters	Items	Parameters
M	12	C_d	10¥/km
T_p	3000km	C_w	50¥/km
C_p	70,000¥	C_r	90,000¥
C_f	80,000¥		

The optimal results after minimising Eq.(12) through the proposed optimisation method are summarised in Table 3.

Table 3 Optimal results

Items	Value	Items	Value
g_{min}	9.331	M_r^*	2
N^*	6	M_s^*	2
k^*	7	n^*	12/8

Table 3 shows that the long-term operating cost rate of the system is 9.331¥/km, and the railway transportation system from station A to station D requires 12 operating locomotives, two repairmen and two spare locomotives. The operating locomotives are preventively maintained in sequence with a PM interval of seven round trips (21,000 km), and overhauls are performed on the operating and spare locomotives at the fourth PM. The PM for spare locomotives is implemented with a mean interval of $T_{st} = MT_p/n = 24{,}000$km. PM and preventive replacement times for spare units is nN=12/8×6=9, thus one spare unit is 4 and another is 5.

According to the optimal results, n^*=12/8 and N^*=6, which means K=9 and D_0=8. $P_{min} = \sqrt{8}$ can be obtained using GA. For the first spare unit, D_1=(1111111), D_2=(111100011111), D_3=(000111100011111) and D_4=(1111111110000). For another spare unit, D_1=(000000011111111), D_2=(1111111), D_3=(1111111) and D_4=(1111111), D_5=(11110011111). Based on these results, the detailed workflow flowchart is shown as Fig.9.

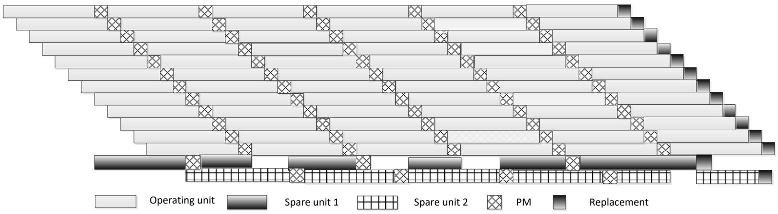


Fig.9 Workflow flowchart

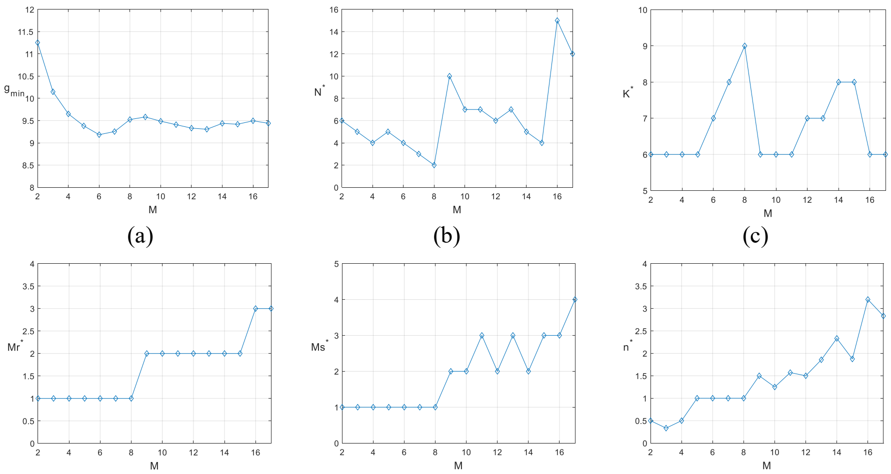
According to the above works, the optimal configuration for the train transportation system can be determined. This work can inspire a new idea for PM and maintenance recourse configuration of train transportation and some similar MuPPSs.

4.3 Sensitive analysis

M, T_p and a_i are significant parameters of the MuPPS because they affect the number and waiting time of repairmen and spare units. Therefore, the influences of these factors should be analysed.

a) Influence of M on the optimal results

The number of operating units M characterises the scale of the MuPPS. Fig.10 presents the optimal results with different M values. The minimum values of g_{min} as M increases are illustrated in Fig. 10(a). The value of g_{min} steadily changes with the increase in M. Under these circumstances, the optimal Mr^* and Ms^* maintain a stepwise growth. Zhang et al. (2018) found that an optimal M exists for one repairman and spare unit. Figs. 10(b), 10(c) and 10(e) exhibit the optimal values of N^*, k^* and n^* with the increase in M. These figures show the unclear tendency of the change of M and therefore suggest that the optimal number of repairmen and spare units and the optimal PM policy are closely related to M.



(a) (b) (c)

(d) (e) (f)

Fig. 10 Sensitive analysis of M

b) Influence of T_p on optimal results

Table.4 The optimal results with different T_p

T_p	2500	2600	2700	2800	2900	3000	3100	3200	3300	3400	3500
g_{min}	9.376	9.353	9.313	9.313	9.297	9.331	9.313	9.292	9.273	9.254	9.495
N^*	6	6	6	6	6	6	6	6	7	7	5
k^*	6	6	6	6	6	7	7	7	7	7	8
Mr^*	2	2	2	2	2	2	2	2	2	2	2
Ms^*	2	2	2	2	2	2	2	2	2	2	2
n^*	12/8	12/8	12/8	12/8	12/8	12/8	12/6	12/6	12/6	12/6	12/6

Table 4 indicates that T_p influences k^* and n^* and causes them to demonstrate stepwise increase. By contrast, the former has no effect on Mr^* or Ms^*. In addition, the influence of T_p on g_{min}, k^* and N^* is the same, and the regularity is unclear.

c) Influence of a_i on optimal results

Table 5 Optimal results for different a_i

$a_i(\beta,\eta)$	(2.02,1.8)	$4i/(3i+1)$	(1.5,1.2)
g_{min}	3.738	9.331	4.374
N^*	1	6	2
k^*	16	7	13
Mr^*	1	2	1
Ms^*	1	2	1
n	12/24	12/8	12/12

The value of a_i describes the effect of PM. In general, a_i increases with the increase in i, and a_1=1 for a new system. The relationship of a_i and i is expressed as:

$$a_i = \frac{\beta}{\eta}\left(\frac{i}{\eta}\right)^{\beta-1} \quad (\eta>0, \beta>0)$$

The following three cases are considered.

1) a_i is a rigid, monotonically increasing and convex function with respect to i.

2) a_i is a rigid, monotonically increasing and concave function with respect to i.

3) a_i is a linearly increasing function with respect to i. This case is the same as the above assumptions.

The other parameters are the same as those in the case study. Three groups of optimal results are presented in Table 5. The first case shows the lowest g_{min}, whereas the last one displays the highest value. N^* demonstrates the same tendency as n^*, whereas k^* exhibits the opposite trend. Moreover, Mr^* and Ms^* remain constant. Therefore, improving the PM effect from the viewpoint of g_{min} is beneficial for the users.

5. Conclusions

This study analysed the mechanisms of MuPPSs in continuous production systems with multiple repairmen and spare units. It investigated the joint optimisation of the PM policy, the number of repairmen, and the number of spare units. The optimal model of the long-term operating cost rate was established, and the optimal PM policy and number of repairmen and spare units were obtained by minimising the system cost rate. The genetic algorithm is used to determine the optimal displacement sequence of spare units. A real case study of a train working diagram was analysed. The sensitivity analysis of several parameters was conducted to expound the modelling and optimisation processes, which demonstrated that the number of operating units and the PM time of the operating unit have a noticeable effect on the optimal results. The number of repairmen is directly proportional to the number of operating units and the length of PM time, respectively. The findings could guide practitioners in determining the optimal number of repairmen and spare units and establishing a reasonable PM policy for different production scales and maintenance capabilities.

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