VOLATILITY RELATIONS IN STOCKS, DIVIDENDS AND LIFETIME INCOME

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Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university.

Chapter 3 of this thesis has received much input from journal referee comments as well as suggestions from academics in conference presentations. The results in this chapter was presented at the 12th International Conference on Computational and Financial Econometrics (CFE 2018) at the University of Pisa, Italy, 14-16 December 2018, the OR61: 61st Annual Operations Research Conference, University of Kent, Sibson, Park Wood Road, Canterbury, UK, 3-5 September 2019, and the 13th International Conference on Computational and Financial Econometrics (CFE 2019) at the Senate House, University of London, UK. 14-16 December 2019. The chapter is under *revision* with Quantitative Finance Journal.

Chapter 4 and Chapter 5 have been modified following academic and practitioner comments and suggestions. The two chapters are closely related to an industry-led research project on "UK Equity Release Mortgages: a review of the No Negative Equity Guarantee" commissioned by the Actuarial Research Centre (ARC) of the Institute and Faculty of Actuaries (IFoA) and the Association of British Insurers (ABI) available at https://www.actuaries.org.uk/documents/review-no-negative-equity-guarantee. More specifically, comments were received from the research project review committee, seminar presentations with academics at the University of Kent Business School. The thesis draws from essential inputs received from both academic and industry practitioners. Chapter 4, and Chapter 5 have been presented as a PhD Job Market Presentation at the BAFA – South East Area Group – 2020 Conference, Chancellor's Hall, Senate House, University of London. I am currently in the process of merging Chapter 4 and Chapter 5 for submission to the Journal of Economic Dynamics and Control. Chapter 2 of this thesis will be submitted to the Management Science Journal.

Acknowledgements

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Summary

The aim of this thesis is to investigate financial solutions for improving the quality of life for retired seniors. The main pillars of research are associated with equity release mortgages (ERMs) and dividend payments, as a cash-flow related solution which can boost pensioners income.

This thesis investigates: i) international evidence of the dividend volatility puzzle with empirical focus on understanding dividend and stock price volatility issues in the present value relation by examining the variance bound inequality introduced by Shiller (1981) and LeRoy & Porter (1981); ii) variance bound inequality by examining implied dividend volatility estimated from novel data emanating from index dividend future options and stock index options respectively. iii) pricing mechanism for non-negative equity guarantee (NNEG) clause in ERM contracts with detailed analysis on the model risk sensitivity, parameter estimate sensitivities, and borrower characteristics. iv) from a portfolio viewpoint, the joint dynamics of funding cost, long term effect of house price risk, and property impairment characterising NNEG valuation.

Chapter 2, contributes to the emerging debate on stock-dividend volatility by providing a multi-national assessment on the long-run behaviour of dividend volatility within financial markets in developed economies. This was done vis-a-vis equity market volatility, utilising stock-index data spanning from 1800s to December 2018. An examination of Robert Shiller's stock-dividend volatility puzzle was conducted by replicating his argument within extended financial market data on the S&P index spanning January 1871- December 2018. The purpose was to capture Shiller's procedure for forecasting long-run dividend growth and subsequently compare the performance of the replicated study under newly improved long-run regression-based methods. This allows for an examination of the consistency of the stock-dividend volatility puzzle across different market indices in various financial markets. The findings support the continuance of the puzzle; however, the magnitude of volatility

violation showed a significant decrease over the long-run.

Chapter 3, provides new evidence on the stock-dividend volatility puzzle by exploring implied volatilities on stock and dividend markets in novel financial derivative products. There was a comparison of information on the implied volatility surface of stock-index to the corresponding implied volatility surface of index dividend futures on the stock index. The thesis outlined a computational procedure for aggregating implied volatility estimates based on the Black-Scholes, Black model, and the model-free approach. Our findings illustrate how implied volatility term-structure of STOXX 50 with time-to-maturity exceeding 9-months moves enough to be justified by subsequent dividend fluctuations. Options with maturities between 1-9 months lead to implied volatilities that move too much to be justified by forward-looking changes in dividends. The implied volatility term structure of stock consistently exceeds that of index dividend futures thereby confirming Shiller's dividend puzzle under novel financial data and instruments. However, the magnitude of excess implied volatility declines with long-dated time-to-maturity, suggesting that discrepancies between the two are influenced by the investment horizon. Further, the thesis designed a set of trading strategies using implied volatility and the ratio of implied volatilities as a trading signal that proves to be successful for STOXX50 equity space.

Chapter 4 focuses on understanding the issues associated with the pricing mechanisms for the non-negative equity guarantee (NNEG) clause in ERM contracts, taking a detailed look at model risk. More specifically, this chapter compared the NNEG valuation principles under the Black-Scholes model that is recommended by the regulations in the UK with a more academic approach based on time-series modelling for gauging the underlying house price risk. The chapter also investigates parameter sensitivities under both models and further determine the conditions under which the two models can produce similar outcomes. Our preliminary findings on NNEG valuation in both loan-by-loan and portfolio viewpoints show that the GBM model consistently overestimates the NNEG cost. The rental-yield parameter turns out to be the key risk driver in the NNEG valuation.

Chapter 5 extends the context of the NNEG cost pricing study by investigating the fundamental effect of idiosyncratic or dilapidation risk of the collateral house in the ERM contracts. The chapter specifically investigates the characteristics of the NNEG values under the requisite and well-known empirical features of UK house price data that are often absent in perfectly rational models. The thesis focuses on the cash flow implications of the two models introduced and discussed in chapter 4. The final sections of the chapter are dedicated to analysing and parameterising property impairment that the loan issuer automatically inherits via the ERM loan contract. The model for the impairment to collateral house uses a value compressing factor that adjusts the market value of the underlying house prices for the impact of dereliction/dilapidation of property maintenance. This allows the loan issuer to isolate and analyse the resulting pool of impaired collateral properties. Shiller & Weiss (2000) attributes this potential reduction in property to lack of maintenance by the ERM borrower who may face a lessened financial interest in the collateral house. Modelling the impairment factor allows us to further investigate the issue of hefty discounting applied to home values as mentioned in Warshawsky & Zohrabyan (2016). The findings are indicative of a positive relationship between implied volatility and the impairment factor in ERM portfolios. Also, house price pathways under the GBM model display more variability compared to that of the ARMA-EGARCH model; thereby suggesting a higher likelihood for house prices to be overall lower at long term horizons. The loan issuer's earnings at risk are larger under the GBM model, leading to higher reserve liabilities for loan contracts. The time series model seems to provide an opportunity for the loan issuer to capture extreme tail observations in the earnings at risk, thereby allowing us to closely observe the long term effect of collateral house price idiosyncratic risk.

Chapter 6 outlines final discussions that conclude the thesis, and point towards new directions in which this research might take to fill the existing gaps in the literature. The Appendices at the end of the thesis presents further results and materials which are separated from the main text.

Table of Notations

Table 1: Table of Notations

Dividend Volatility Notations

D_t	=	real dividends level accruing to the stock index in year t
P_t	=	real stock price level in year t
p_t	=	log of real stock price in year t
d_t	=	log of real dividends accruing to stock index in year t
g_t	=	annual log dividend growth rate $g_t = d_t - d_{t-1}$
r_t	=	log index returns at time t
y_t	=	log dividend yield $y_t = d_t - p_t$
$P^m_{t,k}$	=	real price level in month k of year t
$D^m_{t,k}$	=	real dividend level in the k-th month of year t
$y^m_{t,k}$	=	log dividend yield in the k-th month of year t
$p^m_{t,k}$	=	log of real price level in the k-th month of year t
$d^m_{t,k}$	=	log of real dividend level in the k-th month of year t
$\widetilde{p}_t$	=	real detrended stock price level in year t
$\widetilde{d}_t$	=	real detrended dividend level accruing to the stock index in year t
$\widetilde{\gamma}$	=	one period real discount factor for all detrended series $\widetilde{\gamma} = \lambda(1+r)$
r	=	one period real discount rate for all detrended series
λ	=	trend factor for price and dividend time series
$\widetilde{p}^*_t$	=	perfect forecast price (ex-post rational price level)
E_t	=	conditional expectation operator given information available at time t
Δ	=	first difference operator $\Delta x_t \equiv x_t - x_{t-1}$
T	=	subscript for base year used for detrending t
K	=	strike price of stock index call and put options
$\tilde{K}$	=	strike price of index dividend futures call and put options
$\tilde{C}$	=	market price of index dividend futures call options
$\tilde{P}$	=	market price of index dividend futures put options

C	=	market price of stock index call option
P	=	market price of stock index put option
Δ_c	=	call option delta
Δ_p	=	put option delta
$\sigma^{(mf)}$	=	model-free implied volatility for stock
$\tilde{\sigma}^{(mf)}$	=	model-free implied volatility of dividend
F_t	=	market price of futures contract
$\mathcal{V}_k(t)$	=	conditional total variance of implied volatility of stock
$\widetilde{\mathcal{V}}_k(t)$	=	conditional total variance of implied volatility of dividend
θ	=	implied volatility ratio (IVR)
Θ	=	implied volatility differences (IVD)
$MA(\widetilde{N})(\Phi)_t$	=	$\widetilde{N}$-step moving average of the series $\Phi_{t\geqslant 0}$
Φ_t	=	surface of all correlation coefficients between Θ and moves in the stock log-return
N	=	Normal cumulative distribution function

Equity Release Mortgage Notations

g	=	continuously compounded rental yield
H_t	=	house price index at time t
K_t	=	accumulated loan balance, usually equal to $K_t = L_0 e^{Rt}$
L_0	=	initial loan value
μ	=	expected growth rate for house price returns under GBM model
η	=	limit total number of months to be considered
ν	=	percentage giving the LTV ratio
$q(t)$	=	ERM loan survival probability at time t
R	=	roll-up rate charged on the loan; this is the rate at which the loan balance grows
r	=	risk free discount rate
σ	=	volatility parameter for the house price series

Acronyms

ARMA-EGARCH Autoregressive Moving Average Exponential Autoregressive Conditional Heteroskedasticity model. V–VII, X, XV, XVI, 107, 123, 124, 130, 131, 133, 142, 143, 145–147, 152, 153, 155–158, 161, 167–170, 185, 186, 201–212, 219, 220

ARMA-EGARCH-rn Autoregressive Moving Average Exponential Autoregressive Conditional Heteroskedasticity model under the risk-neutral measure. 133, 150–153, 156, 157

ARMA-EGARCH-rw Autoregressive Moving Average Exponential Autoregressive Conditional Heteroskedasticity model under the real-world measure. 133

ARMA-GJR Autoregressive Moving Average with Glosten, Jagannathan, and Runkle model under risk-neutral measure. VI, XV, 143, 153, 154

BGS Balance Guaranteed Swaps. 139

CEQ Certainty Equivalence. 95

CPI Consumer Price Index. 26, 27, 66

CWM Continuous Workout Mortages. 221

DP Discussion Papers. 118

ERM Equity Release Mortgages. V–VII, XI, XIII, XV, XVI, 6, 12, 13, 104–108, 111–129, 131, 133, 138, 139, 150–154, 156, 161, 164, 172, 175, 176, 181–186, 188, 189, 193, 195, 200, 202, 204, 207, 208, 211, 217, 219–222

GBM Geometric Brownian Motion. VI, VII, X, XIV, XVI, 106, 107, 122, 123, 135, 136, 140–147, 150, 152, 155–157, 159, 161, 178, 182–186, 196, 198, 200, 201, 203–212, 219, 220

GBM-rn Geometric Brownian Motion under risk neutral measure. XV, 150–152, 154–157, 178, 220

GFD Global Financial Database. 26, 65

GMM Generalized Method of Moments. 146

IDF Index Dividend Futures Options. 12, 74, 76, 77, 85, 96

IV Implied Volatility. 11, 12, 74, 76–78, 81, 85, 88, 93, 96, 218

IVD Implied Volatility Differences. XIII, 12, 82, 86, 90, 91, 93, 95

IVR Implied Volatilty Ratio. XIII, 82, 86, 94

LTC Long Term Care. 115, 116, 175

LTV Loan to collateral house value ratio. VI, VII, X, XV, 106, 116, 121, 151–155, 161, 171, 185–187, 203

MBIVD Model-based Implied Volatility Difference. 93, 95

MDR Minimum Deferment Rate. 119

MFIVD Model-free Implied Volatility Difference. 89–93, 95

MIDAS Mixed Frequency Data Sampling Regressions. 15, 27, 38–44, 46, 48, 50, 52, 54

MLE Maximum likelihood Estimation. X, 140, 146

MM Method of Moments. X, 140

NNEG Non-negative Equity Guarantee. VI, X, XV, 12, 13, 105–108, 113, 114, 116–122, 124–126, 128, 131–138, 141, 142, 146, 150–161, 164, 167–170, 177–179, 182–185, 187–189, 193, 194, 196, 200–204, 211, 212, 219–221

OECD Organisation for Economic Co-operation and Development. 1, 4

PRA Prudential Regulatory Authority. 113, 116–120, 126, 128, 148, 155, 158, 159, 169

PS Policy Statement. 118

SSD Second-order Stochastic Dominance. 52–54, 84, 92, 93, 95

VP Volatility Parameter. 119

CHAPTER 1

Introduction

Global economies are beset with challenges posed by population longevity and ageing. Population projections by the UK Office of National Statistics (2016) and the US Census Bureau (2014) suggest rapid aging as the size of senior population category increases, due to life expectancy improvement. The ageing-population phenomenon in these two countries may be attributed to their respective age structures, where baby-boom[1] live births are entering older ages, along with lower birthrates. The UK senior population (i.e. age 65 and over) accounts for 17.7% of the total population, and it is expected to have a 6.6 percentage point increase by 2039 (UK Office of National Statistics, 2016). A demographic study by Ortman et al. (2014) showed that more than 20% of US residents will be aged 65 and over by 2030; compared with 13% in 2010 and 9.8% in 1970.

Population growth, ageing structure and longevity are a critical concern when few within the economically active age need to support a larger than expected number of seniors. An emphasised finding in the 2016 UK national statistics report is the anticipated decline in the traditional working-age population although it has remained stable over the last 40 years. According to the March 2019 labour force survey report by the office of national statistics, UK's employment rate comprises of labour force aged years from 16 to 64 years in paid work. In the European Statistical Office (Eurostat) population projections Muszyńska & Rau (2012) used a working age range of 15 - 64. The working age has generally remained unchanged for a many countries. Extending the working age to 70 or 75 years results in long working years and lower pension/retirement period thereby shortchanging labour force of the country since they enjoy less periods of

[1] 1960 in the case of UK / 1950 in the case of the US. According to Rouzet et al. (2019), the working-age population is projected to shrink by about 25 million people in Japan, 12 million people in Germany, and 300 million people in China. Population growth projections for OECD countries show that the population aged over 80 years will grow from 4% of total population in 2010 to 10% of total population by 2050 (OECD 2011).

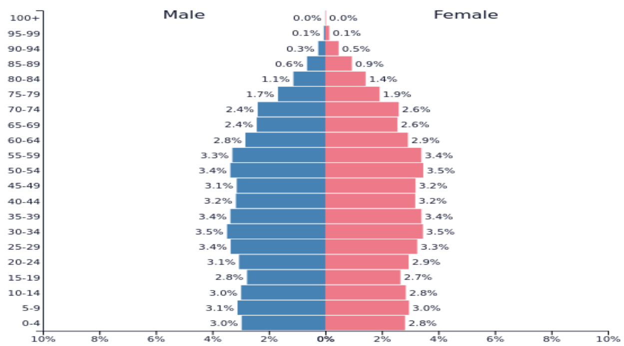
pension benefits (see Smeaton & McKay 2003, Barnes et al. 2004).

Longevity challenges may persist even when the retirement age is extended through parametric[2] pension reforms. Increasing the retirement age suggests longer working period and shorter retirement period. On this basis, people with age-at-death below the average life expectancy would have worked to their grave without any retirement benefit. Households can improve their work-life financial planning horizon when retirement age is extended. Increasing retirement age may also amplify incentives to explore other viable post-retirement funding solutions. Feasibility of such a policy may result in lower demand for home equity release in the long term. On the other hand, the cost implications of an extended retirement age also needs some consideration. Health complications of old age may result in high premiums for workers compensation benefit plans. The quality of living standards of working and retired population may eventually be undermined when retirement age extension is not feasible.

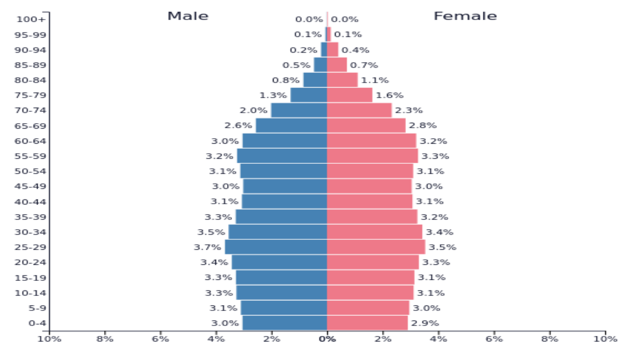
The narrative is no different in the global socioeconomic space, with an illustration of how the ageing population is more profound in Asia in Figure 1.1. Another general observation relates to the increasing size of the old-age group and the resulting loss of the pyramidal shape that depicts a positive interaction between fertility, mortality and variations in migration. The ageing population-effect is profound in Japan and China compared to the US and the UK with other regions of the world gradually move towards ageing societies.

The management of longevity risk poses critical challenges to lifetime income funding around the world. The imbalance between longevity, fertility and socio-economic advancement within a given population ultimately beset adverse social, economic, and political challenges on policy-makers (Von Weizsäcker 1996, Davis 1997*a*, Faruqee & Mühleisen 2003). Policy options on long-term care, retirement programs and other government welfare systems designed for the ageing population ultimately face funding complications; cost explosion; thereby becoming stifled in the long term. Studies by Mitchell & Piggott (2004), Scholz et al. (2006), Munnell et al. (2008), Bingzheng et al. (2014), Alai et al. (2014), and Boyer et al. (2019) have expressed collective fears about the sustainability of social security frameworks and the adequacy of retirement income replacement cash flows in the face of the ageing population. In a recent study that builds on public statistics across fourteen (14) European countries, Defau & De Moor (2020) reported that pension funds face additional costs which emanate from inactive participants. This expresses the tendency of seniors receiving lifetime income to outlive expectations, thereby becoming an extra burden. This scenario also characterises the mismanagement of longevity risk, that deteriorates finances, creates bankruptcy and exposes retired individuals to the risk of losing lifetime incomes (Antolin & Mosher 2014). Possible solutions aimed at mitigating such complex challenges are discussed in Vogel et al. (2017); they include: increasing retirement

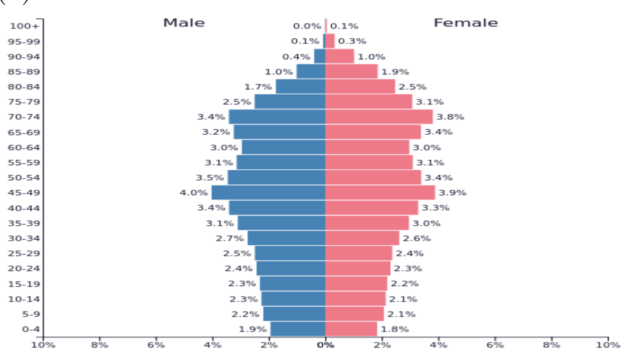
[2]Parametric pension reforms involve altering parameter values of a country's pensions program.



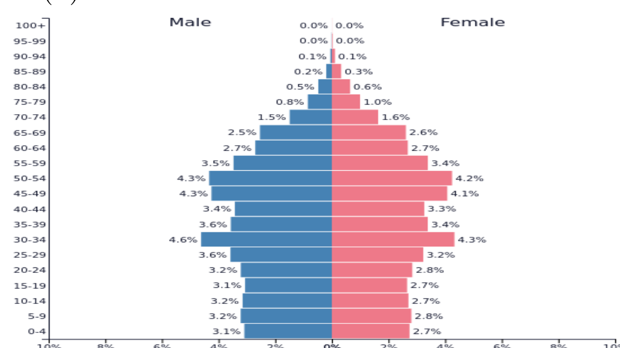
(a) UK



(b) US



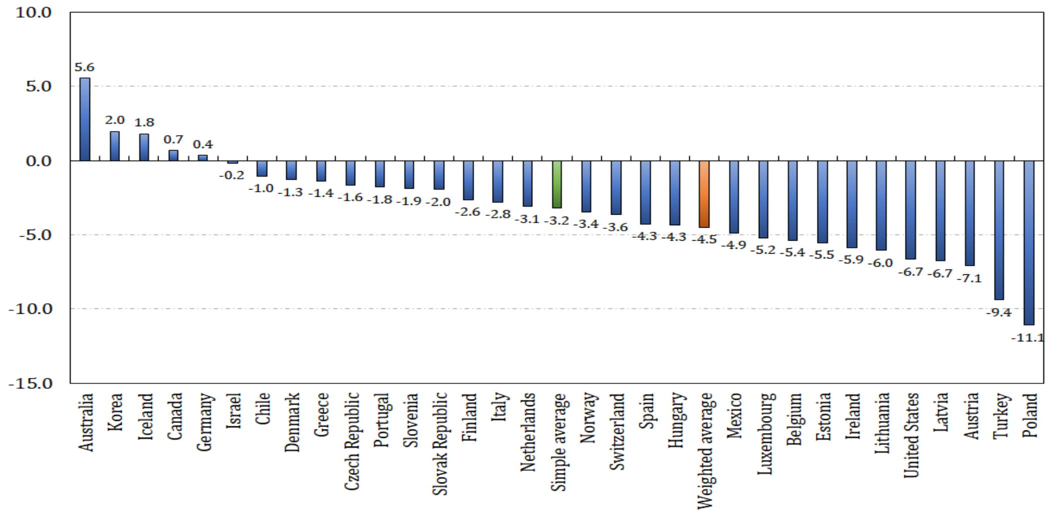
(c) Japan



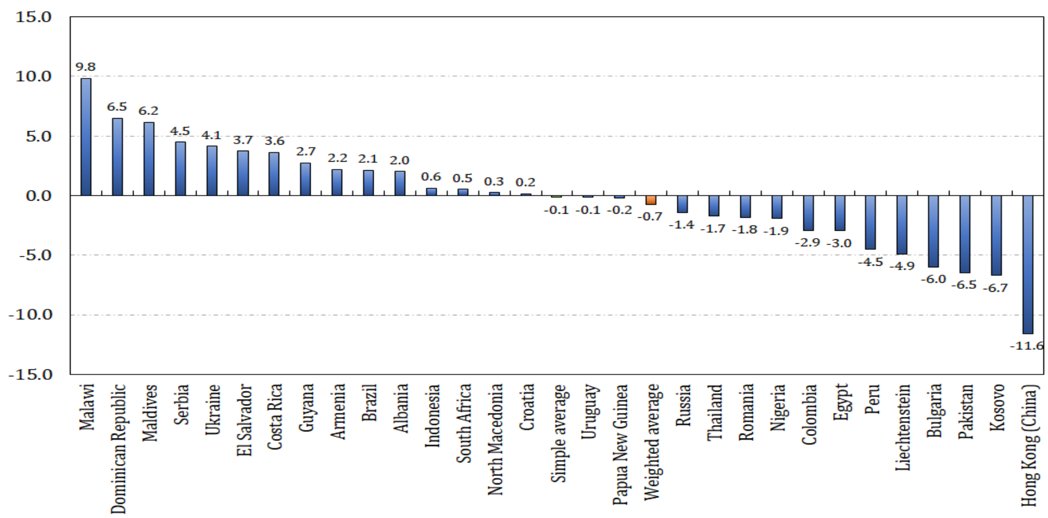
(d) China

Figure 1.1: Age distribution of population for UK, US, Japan and China in 2019.

Source: United Nations, Department of Economic and Social Affairs, Population Division. World Population Prospects: The 2019 Revision. (Medium variant) age, investing in foreign markets, and targeting endogenous human capital formation. Unfortunately, the outcome and impact of these recommendations are not short-term while being cost intensive.



(a) OECD selected countries



(b) Countries from other jurisdictions

Figure 1.2: Annual real investment rates of return (in %) of funded and private pension plans, net of investment expenses, 2018.

Notes: All the annual returns are computed over the period December 2017 - December 2018 with an exception to Australia (June 2017 - June 2018). Source: OECD Global Pension Statistics.

Pension funds and pension plans across the world have experienced investment losses. The OECD (2019), report on pension markets shows a decline in real investment rates of return by an average of -3.2% in OECD countries in their 2017-2018 year review. Figure 1.2 illustrates how 26 out of the 31 sampled OECD countries experienced losses in real investment rates of return. The report mainly attributed these losses to bearish equity market events that occurred around the last quarter of 2018. Hennecke et al. (2017) outlined instances when return on retirement savings are subdued by low interest rates on capital markets. This threat is further heightened as retired seniors live longer. The joint effect of impaired investment returns and mortality improvement creates adverse impact on pension funding patterns. A direct impact of longevity is captured in the 2017 World Economic Forum (WEF) report, where the retirement savings gap is expected to grow by five percent (5%) each year by 2050. World Economic Forum (2017) also reported that the United States (US) retirement savings gap was at $70 trillion, being 1.5 times larger than the annual GDP recorded across 7 countries[3]

So far, these observed statistics characterise the state of global retirement systems prior to the COVID-19 pandemic in 2020. The financial environment has experienced significant dislocations from the norm. The impact of the COVID-19 pandemic has been severe on global economies in 2020 with a recession despite extensive fiscal measures to contain its negative impact across the global economy (see Mogaji 2020). The outbreak can potentially lead to prolonged incidence of long-term unemployment, increase in government debt levels, lower growth rates, decline in capital market values, and larger number of unemployed labour force (see OECD 2020). The uncertainty surrounding time-to-recovery from negative impact of COVID-19 is further worsened by non-existent vaccines and lack of reliable treatment. Jordà et al. (2020) critically analyses reasons why the long-run effects of the pandemic is expected to last approximately 40 years with economic environments characterised by substantially low real rates of return. The resulting long-term challenges faced by governments include among many, rising ageing-related spending on pensions, health care cost, and long-term care. It is unclear, how governments around the world will fund stimulus efforts to avoid recession. Tackling population morbidity remains an active part of government strategy to fight COVID-19 related mortality however, Muszyńska & Rau (2012) demonstrated how health improvements and progressive prevention of disability will not by themselves compensate for the ageing of the labour force.

The short-term impact of the COVID-19 pandemic on pension systems can be associated with the United States equity market crash in February 24, 2020 which preceded a $20 trillion global equity market loss (see Mitchell 2020). Amidst the substantially low real rate of return environment, Jordà et al. (2020) and Mitchell (2020) predict increases in real wage rates in labour markets. Meanwhile, the pensions market space is not fully protected from the negative impact of the pandemic and pre-COVID-19 complications still persist with a tendency to worsen over time. Mitchell (2020) discusses how funding rate for the Dutch retirement plan has decreased from 105% (before pandemic) to below 70%. Discussions in Mitchell (2020) also suggest defined benefit (DB) pension schemes will face substantial under-funding complications. Early

[3]The countries studied in the 2017 World Economic Forum report are Australia, Canada, China, India, Japan, Netherlands, United Kingdom, and United States. included in the survey.

or voluntary retirement may also increase if unemployment persists for long. The direct impact of the pandemic on country-specific funding rate is not immediately observed as various countries rely on regulatory-based valuation methods of pension liabilities. Defined contribution (DC) pension plans are expected to be impacted by the pandemic as level of joblessness increases in the near-term.

It has consequently become necessary to work towards ensuring retirement protection for senior population groups. Key to this step are efforts made toward the preservation and improvement of the economic well-being of pensioners while ensuring uninterrupted funding frameworks for social security (Choua et al. 2004).

Among other critical ageing needs, the seniors struggle with insufficient incomes & retirement savings and larger debt[4] (La Grange & Lock 2002, Chou et al. 2006, Twomey 2015, Boyer et al. 2019). This results from the use of leverage in financing daily living, against the need for capital preservation, further income generation, and longevity. Reliable funding for post-retirement income liquidity is a growing concern for both senior population and public policy-makers. Proposed solutions to retirement liquidity constraints traditionally explore the use of state pensions, retirement plan annuities, personal savings/liquid assets and other alternative discretionary wealth e.g. sale of homes, renting or downsizing homes, etc. Recent past events show how the senior population is at risk of losing significant value in their pension plans, home equity, and depreciation in assets (Bhuyan 2010). According to Hennecke et al. (2017), alternative forms of housing equity liquidation impose much higher financial and psychological burden on the elderly; some of these alternatives include sale of one's home or renting out, and downsizing (moving to smaller homes). Currently, equity release mortgages (ERM) constitute private savings that; boost retirement income security, provide a viable medium for smoothing lifetime income, and support alleviation of challenges presented by ageing population on public budgets (see Fornero et al. 2016).

Equity Release Mortgages (ERM) as they are known in the US or reverse mortgages (RM) in the UK, are collateralised loans that allow senior borrowers to convert equity that is locked in their houses into lifetime income while ageing-in-place; thereby retaining the "possession-value" of the house. ERMs fall under a special class of lifetime mortgages with common identifiable features which include; (i) sale to senior population members typically aged 62 and older Equity Release Council (2017) although Boehm & Ehrhardt (1994) sets average borrower age at 75 years, (ii) embedded with non-negative equity guarantees

[4]Per the 2014 income distribution study by the UK Department of Work & Pensions, the oldest pensioner age group in the UK is most likely to be in relatively low income (i.e. before housing cost) group. Such individuals may also be exposed to rising healthcare costs and difficulties in maintaining financial independence usually relying on credit cards to cover basic living expenses. Employers' inclination to shift from defined-benefit (guaranteeing retirement benefits) to defined-contribution plans (match employee contribution) primarily transfers the retirement savings responsibility and associated risks solely to the employee, thereby inflicting retiring households with massive inadequate savings (Chatterjee 2016).

and non-recourse clauses (in the UK), (iii) contract duration is not fixed-term, (iv) issued loans demand no regular repayments from borrowers. As a lifetime loan contract on the borrower's house repayment of the accumulated loan becomes due when the borrower dies, prepays early, or moves into permanent long-term care.

This doctoral thesis focuses on providing a comprehensive study on sustainable lifetime income as pertains to the retired senior population within worldwide economic systems. The pillars of this research study are impinged on volatility relations in stocks, dividends and equity release mortgages as cash-flow related solutions that can boost volatility management in lifetime income and help fund recurring out-of-pocket costs for retired seniors aged over 55 years.

Dividend investments fall under discretionary financial wealth available to the retired senior. They are payouts from company earnings which also provide a medium through which market participants can focus on fundamentals that drive the value of equities. This feature of dividend investment enables fund managers to diversify against risk associated with pure equity exposure. Interestingly, dividends also provide substantial hedge against inflationary pressures. According to LeRoy & Porter (1981), Shiller (1981), Engel (2005), LeRoy (2010), Van Binsbergen et al. (2012) and Lansing (2016) dividends exhibit lower volatility when compared with equities. Currently, dividend derivative contracts are offered over long-dated horizon thereby providing substantial upside benefits from both dividends cycle and dividends market as a whole.

Investment in dividend derivatives[5] serve as a safe bet on the future of company earnings where profit/loss can be determined independent of the market context. This is in contrast with equity investments where profits are exposed to the company's financial health, earnings, and market risk factors.

[5]The Eurex exchange introduced option contracts on the EURO STOXX 50® DVP (dividend points index) during the second quarter of 2010. These contracts settle in cash into the realized dividends that is paid during their respective settlement periods. Thus, the final settlement value is the sum of all paid gross dividends in the settlement period. The futures contracts are completely collateralised by the index notional with interest that accrue at the EONIA. By the third-quarter of 2017, the Eurex exchange recorded high liquidity with an open interest of about 1.1 million contracts and an average daily volume of at least 3100 contracts (see, https://www.eurexchange.com/resource/blob/81092/196052c5bdb64477f2dda5d608f5ad55/data/factsheet_eurex_index-dividend-futures.pdf). Both single stock and equity index dividend futures are traded on the Eurex, allowing market participants to target specific stocks. There is also, index dividend futures option contracts which are written on the index dividend futures. These products altogether provide tools for managing and hedging dividend exposures.

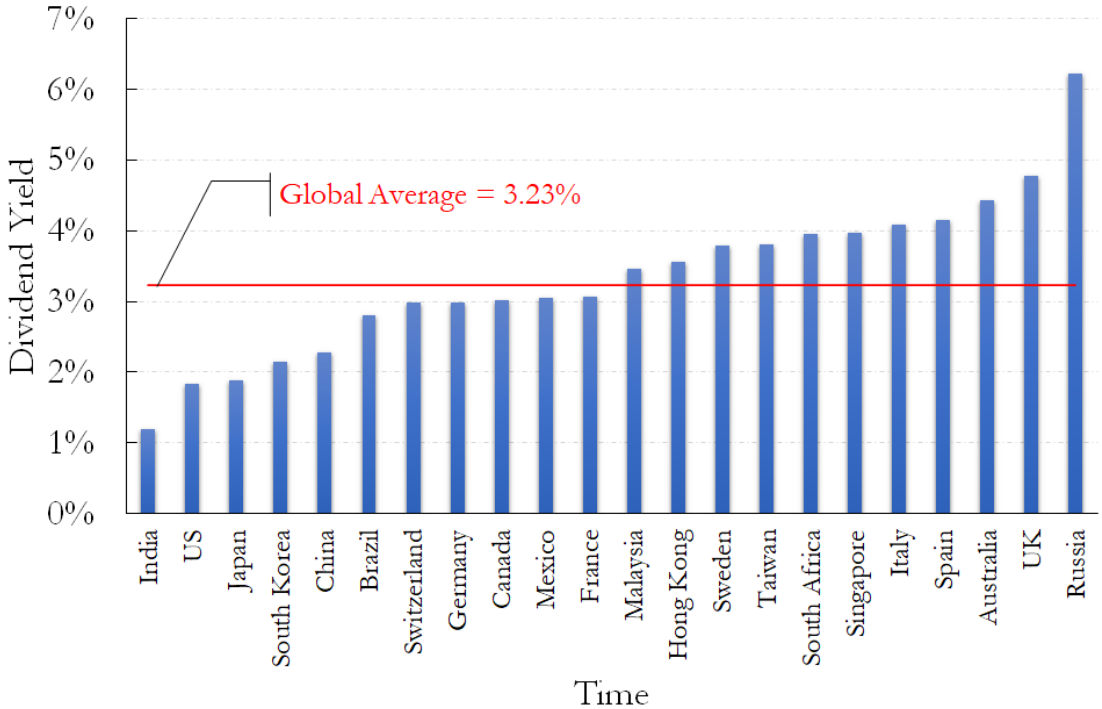


Figure 1.3: Distribution of global dividend yields across a sample of 22 economies

Recent market data on dividends across international markets also provide evidence of dividend yield stability. Figure 1.3 illustrates the market outlook for dividend yields across 22 selected economies around the world. About half of these countries have average dividend yield that exceed the global average of 3.23% as at December 25, 2019. Figure 1.4 presents the historical evolution of dividend yield across 10 globally selected economies i.e. Australia, China, India, United States (US), Canada, France, United Kingdom (UK), Germany, Japan, and Spain. The dividend yield time series is quarterly, starting from Q4-2015 and ending Q4-2019. Australia, Spain and the UK reported dividend yield levels above the 3.23% global average.

I also present the distribution of excess dividend yield over the yield to maturities (YTMs) of long-term government bonds in figure 1.5. More than half of the countries reported positive excess dividend yields over the YTMs of their respective 10-year government bonds; thereby confirming that dividend yields have a tendency to do better compared with medium to long term risk-free returns.

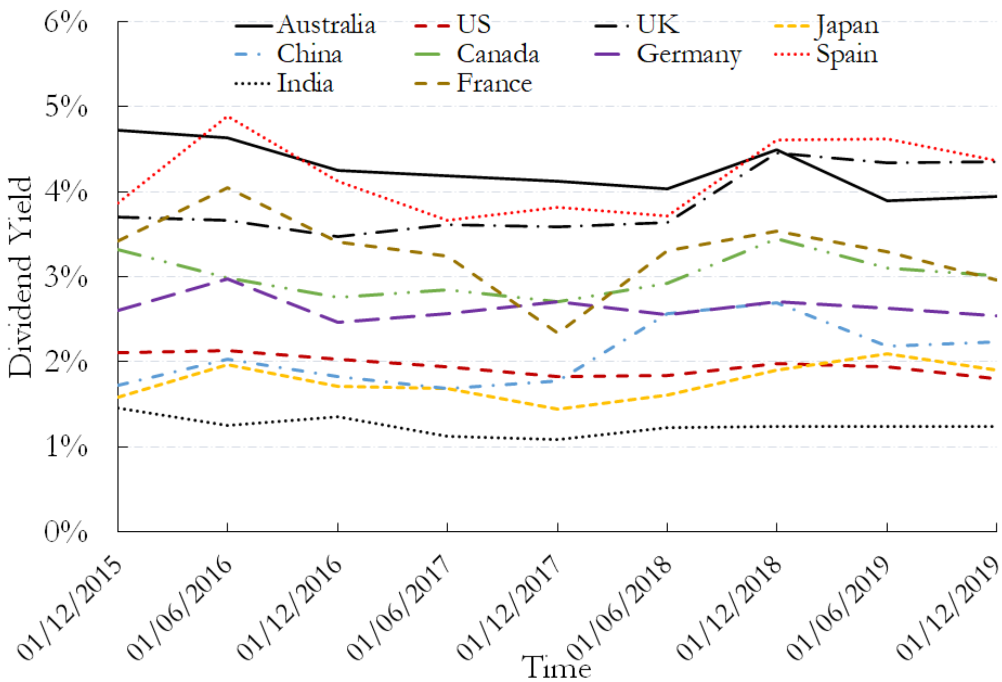


Figure 1.4: Dividend yield trends across selected economies.

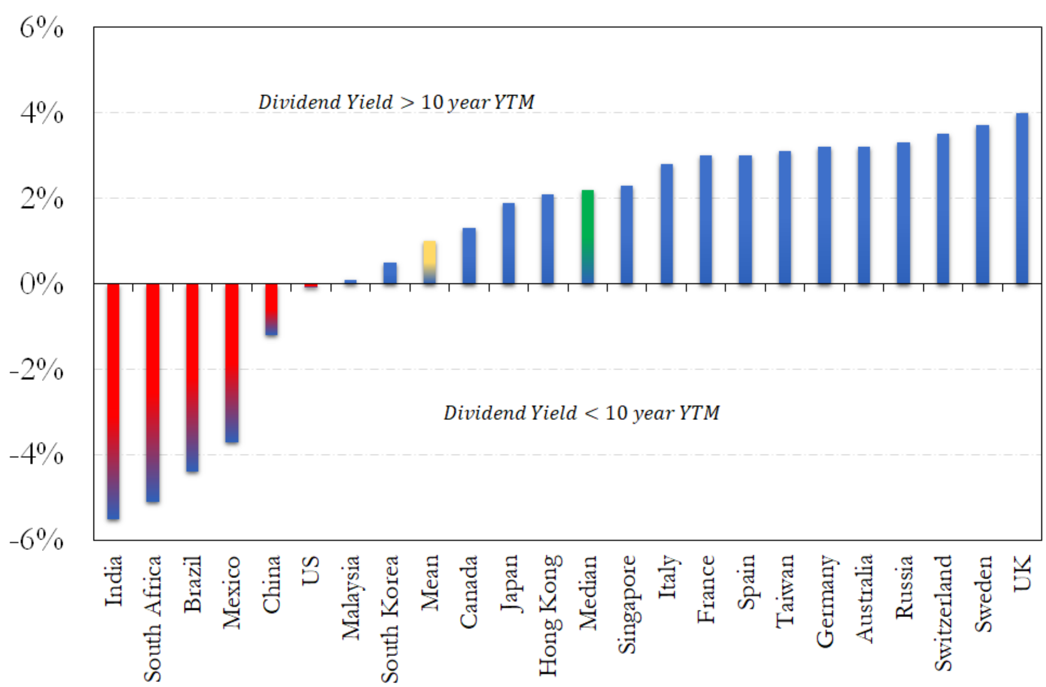


Figure 1.5: Distribution of cross-country excess dividend yield over 10-year sovereign bond yield to maturities

While focusing on funding adequacy and high dividend yield in long-run horizon simulations, Fong

(2016) found evidence that dividend investment strategies significantly outperform diversified equity portfolios. Minbatiwala (2012) also argued for the need to consider growth in dividend payment rather than high dividend yields, when investigating high income generating tendencies over lengthy horizons. This has to do with the fact that capital preservation is a key consideration when analysing lifetime income. The slightest misconception on volatility has the potential to create a wide distribution in the final values of the retirement portfolios. Despite this, there is a widely held notion that dividends inherently have lower exposures to risk (volatility) while contributing to the expected total return. This alludes to the low dividend volatility concept that academics and practitioners hold (see LeRoy & Porter 1981, Shiller 1981, Gilles & LeRoy 1991, Bulkley & Tonks 1992, Engel 2005, Lansing & LeRoy 2014, Lansing 2016).

In relation to equity prices, Shiller (1981) and LeRoy & Porter (1981) observed that when actual prices p_t at any given time t are determined as the discounted value of future dividends p_t^*, the variance of p_t is at least 5 that of p_t^* thereby suggesting $Var(p_t) \geqslant Var(p_t^*)$, an inequality that was alternatively termed the variance bound puzzle. Other variations of the variance bounds relation can be found in the works of West (1988*b*), Kleidon (1986), Marsh & Merton (1986),Bulkley & Tonks (1989), LeRoy & Parke (1992), Bulkley & Tonks (1992), Engel (2005), and Lansing (2016).

The first segment of the thesis, looks at international evidence of excess volatility puzzle using fifty (50) globally selected financial markets. There is dearth in existing literature investigating the empirical evidence for the puzzle on such a large scale. The analysis took a critical look at variance bound inequalities as proposed by Shiller (1981), West (1988*b*), LeRoy & Parke (1992), and Engel (2005); using realised volatility that was estimated from historical equity index data over long-range datasets. The simple notion that rational investors determine current stock prices as the sum of future expected dividends has attracted attention in empirical literature. The seminal Shiller (1981) study, in particular, has sparked lively debates on whether the higher volatility of realized prices relative to prices computed as the sum of discounted future dividends can be used as evidence to reject the hypothesis of investor rationality. Given that previous literature focused almost exclusively on the US, the thesis provides a maiden study of *excess volatility puzzle* in an international context.

The thesis goes a step further to explore whether the observed excess volatilities provided enough evidence to reject the hypothesis of weak form rationality in terms of allowing for exploitable profit opportunities. The objective is to test whether the observed excess volatility in realized prices can be attributed to revisions in the parameters of the dividend model selected by market participants. Bulkley & Tonks (1992) argued that rational investors are inclined to use unbiased methodologies when forming their expectations about future dividend growth. This goes to suggests that investors dynamically revise the parameters of their unbiased models thereby resulting in some excess volatility of realized prices relative to that of expected prices based on future discounted dividends. The test is based on a simple ”buy low - sell high” trading strategy that uses currently available data to set the parameters of the trading rule. Shiller (1981), Ackley (1983) and Bulkley & Tonks (1992) discussed cases where large volatility of stock returns would suggest the existence of trading rules that may dominate the classical buy-and-hold strategy.

The findings in the first chapter of the thesis confirm the commonly reported observation in the US of realized price volatility exceeding the volatility of ex-post rational prices also applies to an international sample of 50 countries. Although excess volatility in realized prices was confirmed across sampled countries, its magnitude appeared to be substantially higher in developing countries compared to their developed counterparts. The thesis presents evidence that the observed difference is most likely driven by the length of available data in each country rather than a reflection of the fundamental relationship between excess volatility and a country's state of economic development. The trading strategy is second order dominant over the buy-and-hold rule across all countries, suggesting that the observed excess volatility is driven by the dividend process used by the investor.

Another striking finding is the sensitivity of variance bounds test results to the specification utilized to obtain expectations of future dividends. For instance, using mixed-frequency regressions to obtain a measure of expected dividend growth results in substantially less pronounced excess volatility on average; compared to the standard Shiller (1981) approach of obtaining a price trend via regression against time. Though the *magnitude* of the effect varies, all measures of ex-post rational prices consistently result in excess volatility across the vast majority of sampled countries.

The results further provides empirical support for the argument that evaluation of stock market rationality is heavily dependent on the test's specific assumptions about the dividend process. In similarity to previous findings from the US market, the chapter documents that variance bounds tests are characterized by the same challenges and nuances with respect to dividend stationarity when applied to other countries. In pursuance to this, inferences about stock market rationality will ultimately depend on the assumptions that one is willing to make about the underlying dividend process. Further research should consider replacing variance tests with model-free or model-based orthogonality tests that have higher power than the benchmark returns test and circumvent the nuisance parameter problem as described by LeRoy & Steigerwald (1995).

The second part of the research sought to examine whether the observed variance bound relation associated with realized volatility retain its features and arguments when estimated with implied volatility (IV). Currently, financial market professionals pricing dividend derivative contracts rely on a forward-looking view on the total sum of dividends likely to be paid by leading companies. The adopted approach in the second segment of the research study focused on the analysis of the relationship between the implied volatility of stocks and the implied dividend volatility using data on index stock options and index dividend futures (IDF) options. Subsequent upon emerging findings on empirical evidence of the variance bounds theory, the research proceeded to setup and optimize potential trading strategies for pension funds while focusing on the capital preservation for sustainable a lifetime income.

The approach illustrates how implied volatility term structure of options contracts with time-to-maturity exceeding "9-months" are justified by subsequent fluctuations in dividends. However, stock and dividend volatility is far apart in options contracts with time-to-maturities around "1-month", "1-3 months" and "3-9 months". Such periods coincide with dividend announcement dates of index constituents, hence associated with lower levels of dividend uncertainties. This phenomenon clearly makes the dividend puzzle effect more prominent when IVs for corresponding stock-index options and IDF options do not decrease together.

The analysis showed IV of stock index options consistently exceeded that of IDF, thus confirming previous criticism about novel financial data and instruments. The magnitude of excess implied volatilities declines with long-dated time-to-maturity, suggesting that the discrepancy between the two IV is sensitive to the investment horizon. This result holds in both model-free and model-based cases. The evidence for the dividend puzzle inferred from expectations on future realizable dividends has a term-structure feature, being almost negligible in the long-run and strongly evident in the short horizons. This implies that inferences about the dividend puzzle for forward-looking purposes, cannot be directly obtained from that observed from historical data without reference to the investment horizon.

For the first time in literature, the thesis constructs an implied volatility index for STOXX 50 dividend futures contracts and further, add to knowledge about the dividend puzzle by using data from both stock and dividend derivative markets. The evolution of this index clearly showed that in recent years there has been a lot more volatility on dividend markets. The trading strategy results also showed that market participants can improve returns by combining a bet on the relationship of implied index dividend volatility and implied volatility of the index rather than using a directional bet on the underlying asset. Both model-free and model-based implied volatility differences (IVD) trading signals outperformed the long-only trading baseline model portfolio.

The final part of the research is directed towards providing a comprehensive appraisal of equity release mortgages. As highlighted, there is a need to study a unified model that integrates regulatory, provider and borrower dimensions of ERM. This will provide a good platform to decide on the best methodological approach for the non-negative equity guarantee (NNEG) that is still hotly debated between regulators and practitioners. Under the research conceptualization theme, the various design products existing in the ERM market space were studied to design an optimal solution for borrowers, loan providers and regulators to create a useful debt instrument capable of solving cash-flow related problems in retirement. Relevant literature in sustainable finance and pensions planning point towards retirement cash flow enhancement through equity release mortgages (Hancock 2000, Reed & Gibler 2003, Weber & Chang 2006, Rowlingson 2006, Chou et al. 2006, Dilnot 2011, Sacks & Sacks 2012, Pfeiffer & Harold Evensky CFP 2012, Andrews & Oberoi 2015, Cocco & Lopes 2015, Hanewald et al. 2016, 2020). Surveys on the ERM product market space show how demand is driven by changing debt attitudes, mortality improvement and decreasing savings rates among the population (Luiz & Stobie 2010, Hanewald et al. 2020).

Andrews (2009), Chou et al. (2006), and Gibb et al. (2007) suggested that ERM cash flows have the potential to improve the sustainability of retirement income cash flows, ease financial burdens on government and decrease government pension liabilities. Bodie et al. (2007) considered ERM as an efficient channel which helps with the transfer of inter-generational wealth in a hassle-free way for seniors. Its potential to improve poverty among senior homeowners in the US is also well accounted for with Kutty (1998) empirically estimating a 3 percentage point decrease in poverty among elderly homeowners in the US by use of ERM plans. In a recent study, Han et al. (2017) argues that ERM potentially provide for stable funds[6] for living expenses among borrowers until maturity or termination of the contract.

The thesis is organized as follows: the first section of this chapter is dedicated to literature review on dividend volatility and the second section reviews literature on ERM and NNEG. The study on dividend volatility is in two parts. Chapter 2 is on international evidence of the dividend volatility puzzle. The empirical study focuses on understanding the issues around stock price and dividend volatility in the present value relation by looking in detail at their variance bound inequalities. Chapter 3 presents the second part of the dividend research which extends the variance bound inequality tests by looking at implied dividend volatility which is estimated from novel data from index dividend future options and that of stock index options. Chapter 4 focuses on understanding the issues associated with the pricing mechanisms for the NNEG clause in ERM contracts, with specifics on model risk sensitivity, parameter estimate sensitivities, and borrower characteristic sensitivities in detail. Chapter 5 extends the context of the NNEG cost pricing study by investigating the fundamental effect of idiosyncratic house price risk on ERM contracts. The chapter specifically investigates the characteristics of the NNEG values under the requisite and well-known empirical features of UK house price data that are often absent in perfectly rational models. The thesis focuses on the cash flow implications of the two models introduced and discussed in the previous chapter. The final section of the chapter are dedicated to analysing and parameterising property impairment that the loan issuer automatically inherits via the ERM loan contract. This allows the loan issuer to isolate and

[6]Pfau (2015) argued that the non-linear nature of ERM cash flows potentially allows borrowers to spend more on their home values; this is attributable to the observed synergies emerging from reduced termination risk with deferred loan repayment and the embedded non-recourse clauses which potentially allow a borrower to spend more than their home value.

analyse the resulting pool of impaired collateral properties. Each chapter of the thesis has an introduction, the methodology applied to address the research question we pose, and the findings of the empirical analysis before concluding. Chapter 6 outlines final discussions that conclude the thesis, and point towards new directions in which this research might take to fill the existing gaps in the literature. The Appendices at the end of each thesis chapter presents supplementary results and materials which are separated from the main text.

CHAPTER 2

Variance Bounds Test of Stocks & Dividends

2.1 Introduction

Not very many topics in finance and economics have attracted as much attention as the question of what determines stock prices and, in particular, whether or not stock prices reflect the fundamental value of the underlying firms. The debate surrounding this question has run the full spectrum from Keynes' views about stock markets operating as Casinos for the lucky to the *Efficient Market Hypothesis* of Fama and Samuelson. One specific approach that has been adopted to examine the rationality, or otherwise, of stock markets refers to the variance bounds tests that were first introduced by LeRoy & Porter (1981) and Shiller (1981).

This chapter of the thesis, contributes to the ongoing debate about stock market rationality and variance bounds tests by expanding the analysis to an international sample of 50 countries. The payment of dividends are noted to have in-built seasonality with peak activities around the periods of April-June. This chapter develops a new methodology for testing the excess volatility puzzle, accounting for intra-year seasonality through a mixed-frequency data sampling (MIDAS) regression setting. Under this new modelling approach, the chapter provides evidence that stock markets with a short history are inefficient but, as the sample size increases, the volatility ratio which is the basis of variance bound tests approaches a threshold level of 1 that is consistent with market efficiency.

The chapter contributes to the debate on the excess volatility puzzle primarily by shifting the focus from the US to a large sample of international stock markets. The previous literature on variance bounds tests of stock market rationality has focused exclusively on the US stock market.[1] While the emphasis on the US is certainly understandable, we believe that expanding the analysis in an international context can provide valuable insights into the existence and general character of the excess volatility puzzle. To this end, we perform a set of variance bounds tests using data on stock index prices and dividends across 50 countries. In order to understand the general applicability of the excess volatility puzzle, we examined a mix of sample countries that is quite disperse, in terms of both geographical location as well as state of economic development. To the best of our knowledge, this is the first study on the excess volatility puzzle and stock market rationality across multiple international markets.

Variance bounds tests are based on the simple theoretical concept that the current price of a stock represents the consensus forecast among rational investors of the discounted sum of its expected dividends. Assuming that dividends follow a stationary process, Shiller (1981) shows that the variance of observed stock prices cannot, in theory, exceed the variance of the object that it forecasts, i.e. the variance of the discounted sum of expected dividends. Focusing on the S&P 500 index, Shiller (1981) reports a substantial violation of this variance upper bound, with the volatility of realized stock index prices exceeding the volatility of subsequent discounted dividend payments by a ratio of 5 to 1, while LeRoy & Porter (1981) report a similar finding with respect to a small sample of individual US stocks. Shiller (1981) and LeRoy & Porter (1981) further argued that these empirical violations of the upper bound of realized price variance constitute strong evidence of investor irrationality in the US stock market. This finding of realized prices exhibiting much higher volatility than what would have been expected from the volatility of dividends has been often referred to as the *excess volatility puzzle*, and it has led to the emergence of a substantial literature that attempts to resolve it.

Subsequent studies have expressed concerns about the suitability of the upper bounds test to detect violations of market rationality. For instance, Flavin (1983) examines the properties of the test in small samples and demonstrates that it is biased towards detecting violations of the variance's upper bound, resulting in unreasonably frequent rejections of the hypothesis of efficient markets. Kleidon (1986) and West (1988*b*) make a similar argument about the test's small sample bias and suggest that the violation of US stock market rationality reported in the Shiller (1981) seminal study could be affected by this limitation of the test, although the magnitude of the gross excess volatility on the US stock market is too high to be accounted for only by small sample bias West (1988*a*). In addition, both Flavin (1983) and Kleidon (1986) provided examples of stochastic processes for which violation of the variance inequalities is likely but extreme violations are unlikely.

[1] A notable exception is Bulkley & Tonks (1989) who retested the variance bound tests proposed in Shiller (1981) and some new ones they introduced for the UK market.

Another feature of the original variance bounds test that has been called into question is the assumption of stationary dividends. On this issue, Marsh & Merton (1986) argued that dividends are most likely non-stationary because of the general tendency of firms to smooth dividends over time.

Shiller (1986) refuted those arguments and pointed out to several examples where violations of variance inequalities persisted even of stationarity of dividends was not imposed. Furthermore, Dejong & Whiteman (1991) use Bayesian analysis to show that dividends and prices are more likely to be trend-stationary than integrated and, thus, provides support in favor of the approach presented in Shiller (1981).

The first generation variance bounds test proposed by Shiller (1981) were further improved into a number of alternative *second generation* variance bounds tests. First, Mankiw et al. (1985) relaxed the assumption of dividend stationarity and demonstrated that their alternative variance bounds test does not suffer from small sample bias. Other researchers pointed out deficiencies of the simple present value model of stock prices and the variance bound tests as proof of market inefficiency. Joerding (1988) showed by testing a Euler equation for more than one iteration that the present value model is misspecified but could not identify the source of misspecification. West (1988*b*) further extends the analysis on non-stationary dividends and derives an upper bounds test with respect to the variance of innovations in the stock price, which theoretically must be lower than the variance of innovations in the corresponding dividends. In a similar manner, Engel (2005) derived a bounds test on the variance of first differences in prices, allowing for dividends to follow a stationary or a unit-root process, the arithmetic price-change variance is a monotonically decreasing function of investors' information on future dividends and showing that the excess variance inequality could be in fact reversed. However, Lansing & LeRoy (2014) proved that when investors are risk averse log return variance is not a monotonic decreasing function of investors' information on future dividends. Lansing (2016) modified the framework of Engel (2005) regarding investors' risk preferences and information about future dividends to provide an alternative bounds test on the variance of first differences in prices.

One line of research identified the importance of learning about the model parameters as more data becomes available. Bulkley & Tonks (1989) focused on testing the weak form of market efficiency and proposed a modified test using expectations computed from a model estimated using only data available up to the time of expectation. They identified a simple trading rule providing positive excess return which can be interpreted as evidence rejecting the weak efficient market hypothesis. Focusing on a simple learning model for dividends, Timmermann (1993) showed that the distribution of the variance ratio is modified substantially when learning is accounted for into the model.

The reluctance of managers to decrease dividend payments to avoid sending bad signals to shareholders has a stabilising effect to smooth dividends thereby making its volatility low. Negative information still tends to be incorporated into prices via channels other than dividend announcements. Observed stock prices therefore appear more volatility compared to dividends. Some relevant studies Chang et al. (2006), Bharath et al. (2009), and Leary & Roberts (2010), directly attribute managers equity decisions to asymmetric information. Management decision and asymmetric information may constitute another paradigm to explain equity price and dividend variations. Chen et al. (2012) discussed how dividend smoothing tends to bury dividend predictability. Using net payout and earnings, alternative measures less subject to dividend smoothing the study showed how cash flow news impacts price variations more than news on discount rate. There is also empirical evidence to show how dividend smoothing makes dividend yield more persistent Cochrane (2008). Dividend smoothing can however be mitigated in finitely long samples thereby making it difficult to assess the extent to which dividend policy with varying degrees of smoothing affects price and dividend variations (see Chen et al. 2012).

The impact of dividend signal in literature has been mixed. The dividend indifference proposition of Modigliani & Miller (1959) explains how dividend policy has no impact on firm discount rate or firm value, ceteris paribus. Grullon et al. (2005) discussed instances where changes in future profitability of companies are not signalled by dividend changes. Empirical studies on the information content of dividends present instances where negative market reactions are driven by dividend omissions and decreases and vice versa (see Elfakhani 1998, McCluskey et al. 2006). Miller & Modigliani (1961) rationalised the effect of dividend changes on stock returns via new information that signal future cash flow. Similarly, the *risk information hypothesis* of Howatt et al. (2009) characterises the signalling of earnings risk through dividend policy. According to Howatt et al. (2009) positive changes in the average real Earnings Per Share (REPS) are associated with positive changes in dividends and significant changes in the variance of REPS occur after change in dividend. The concept describes the potential stickiness of increases and decreases in dividend in relation to corporate earnings. The impact of dividend signalling is mainly driven by management efforts to control size of dividend increase to avoid sending bad signals to shareholders in future; this practice may increase variability in future earnings when dividends are gradually raised within a financial environment characterised by variable earnings. The dividend signalling concept however constitute an empirical question that is beyond the focus of my thesis.

Drawing a conclusion on the stock market's rationality, or lack of it, will naturally depend on the specific test used to examine this hypothesis and, in particular, on the assumptions that a given test is based on. As such, empirical studies of US stock market rationality via variance bounds tests have produced mixed results. West (1988*a*), Gilles & LeRoy (1991), Shiller (2003) and LeRoy (2010) provided a comprehensive review of the econometric problems related to variance bounds tests of market efficiency.

Empirical studies have tended to fall in one of two camps. For instance, Mankiw et al. (1985), Campbell & Shiller (1987*a*), West (1988*b*), Shiller (1988), Bulkley & Tonks (1989), Cochrane (1991), Dejong & Whiteman (1991), LeRoy & Parke (1992) and Timmermann (1993) argued that results from variance bounds tests are indicative of stock market irrationality, while Kleidon (1986), Shea (1989), Ackert & Smith (1993), and Lansing (2016) suggested that their empirical findings are compatible with the hypothesis of stock market efficiency. Cochrane (1992) appeared to hold the relative middle ground by concluding that, even though his empirical results do not directly reject the present-value model for stock prices, the discount rates used in the tests "must possess some unusual characteristics" in order to be compatible with the hypothesis of stock market efficiency.

The chapter also contribute to the literature by exploring two alternative approaches for obtaining expectations of dividend growth. Given the present value model's prediction that realized stock prices reflect investors' expectations about future dividends, the choice of a specific approach to construct the time-series of expected dividends (by determining a value for expected dividend growth) is likely to have an impact on the results of variance bounds tests. Starting from the seminal Shiller (1981) study subsequent papers have explored different ways of producing more accurate expectations of dividend growth (see Cochrane 1992, for a more detailed discussion). I contribute to this debate by applying relatively more recent techniques from the literature on dividend predictability. More specifically, we obtain dividend trend factors via regressions of realized dividend growth against the lagged dividend-price ratio, either in a simple time-series regression setting (as in Cochrane 2008) or in a mixed-frequency regression setting (as in Asimakopoulos et al. 2017).

I find strong evidence of excess volatility in our sample of international stock markets when using first generation variance bounds tests, consistent with the excess volatility puzzle in the US. More specifically, the volatility of realized prices exceeds the volatility of ex-post rational prices computed from the subsequent dividends in all sample countries, with only one exception. Furthermore, in the majority of cases, the ratio of realized price volatility over the volatility of ex-post rational prices exceeds the value of 5:1 that was reported in the original Shiller (1981) study for the US market, suggesting that deviations from the hypothesis of market rationality could potentially be even more pronounced when examined in an international context.

The chapter also document that the magnitude of excess volatility varies in a non-random way across our sample. Countries with developing economies and shorter time-series of available data are consistently found to be characterized by larger volatility ratios (realized over ex-post rational prices) compared to developed countries with longer available datasets. While it might be intuitively appealing to conclude from this relationship that stock markets in developing countries tend to deviate from investor rationality more significantly compared to their developed counterparts, we actually find evidence that the differences in volatility ratios are most likely driven by the length of the available data series rather than the countries' particular state of economic development. This finding is compatible with the small sample bias of the first generation variance bounds test discussed in Flavin (1983), Kleidon (1986), and West (1988*b*). Nevertheless, the fact that we ultimately detect substantial excess volatility even in countries with particularly long series of available data suggests that small sample bias is unlikely to explain away the presence of excess volatility in our sample, even though it is likely to have an impact on its reported magnitude. An interesting case is Spain, where the estimated volatility ratio is less than unity.

The results suggest that volatility ratios are sensitive to the specific approach adopted for computing ex-post rational prices, even though the qualitative conclusion of excess volatility does not change. In particular, we find that two alternative approaches to obtaining trend factors produce lower volatility ratios compared to the ones obtained under the original Shiller (1981) approach. Following the Cochrane (2008) approach of obtaining expected dividend growth via simple time-series regressions of realized dividend growth against the lagged dividend yield gives lower volatility ratios, while estimating these regressions in a mixed-frequency setting (following Asimakopoulos et al. 2017) results in substantially lower volatility ratios. In any case, the volatility of realized prices consistently exceeds what would have been expected based on the subsequent dividends across all three approaches, suggesting that the excess volatility that we observe is robust to several approaches used to infer ex-post rational prices.

Finally, the chapter reports evidence of dividends deviating significantly from stationarity in the vast majority of our sample countries. I adjust our analysis for non-stationarity by performing the variance bounds test proposed in Engel (2005). In contrast to our previous results, the volatility ratios that are computed based on this second generation test support the hypothesis of market rationality across all countries, irrespective of the approach adopted to compute ex-post rational prices. Although a comprehensive examination of how to model best the dividend process lies outside the scope of this chapter, the results suggest that the ongoing debate on the use of alternative variance bounds test in the US market also applies in an international context. In this sense, we provide evidence from a large sample of countries that, similarly to existing evidence from the US, any conclusion about the hypothesis of stock market rationality will depend on the specific assumptions made about the nature of the dividend process.

The remaining of the chapter is organized as follows. Section 2.3 discusses the data used in the empirical analysis and the construction of the main variables of interest. Section 2.4 presents the empirical results of the first generation variance bounds tests, while Section 2.5 reports the results under alternative approaches of obtaining trend factors. Section 2.6 discusses the issue of dividend stationarity and presents the results of second generation variance bounds tests, while Section 2.8 concludes.

2.2 Literature Review

This section presents a detailed review of relevant literature that back the research study on the international evidence of the variance bounds test. The review of literature starts by defining and contextualising the concept of variance bounds tests for equity index prices and dividend payments. The discussion sheds more light on the empirical and theoretical aspects of the relationship between price and dividend volatilities. More specifically, we explore both the empirical and theoretical underpinnings on the subject by surveying relevant literature that well identified with the variance bounds tests; tracing the history right from its first generation Shiller (1981) and LeRoy & Porter (1981), and the second generation discussions in Mankiw et al. (1985), Marsh & Merton (1986), Merton (1986), West (1988*b*), Mankiw et al. (1991), LeRoy & Parke (1992), Engel (2005), and more recently Lansing (2016). This allows us to position the contribution we make to the important discussion of variance bounds theory and empirical studies.

Asset prices dynamics has been the focus of most research in finance. Gordon (1962), Shiller (1981), LeRoy & Porter (1981), Van Binsbergen et al. (2012) and many others who study the prices of aggregate stock market, determine price as the sum of discounted future dividend payments. However, Van Binsbergen et al. (2012) differs from the other studies when they rather characterise the dynamics of the individual components that are in the sum of discounted dividends of the index. They show that the volatilities of the individual components are higher than that of the total sum they are taken from. This implies that, market practitioners and academics stand the risk of underestimating volatility when they determine prices by taking a forward-looking market view on total sum of expected dividends likely to be paid by leading index constituents.

Ang & Liu (2007) provide an excellent characterisation of the joint dynamics of dividends, expected returns, prices, and stochastic volatility. More specifically, they describe the restrictions on the dynamics of prices, expected return and stochastic volatility by specifying the dividend process. Since the stock price returns is equal to the sum of the dividend yield and capital gains, there exist a relationship between price and dividend when the process that drives expected return is specified.

I can establish a link between stock price fluctuations and that of dividends when prices are determined as the present value of current and expected future dividends. Shiller (1981),LeRoy & Porter (1981),West (1988*b*), Kleidon (1986),LeRoy & Parke (1992), and Engel (2005) derive a volatility relation between actual stock prices and that of ex-post[2] rational prices. These studies differ by the assumptions on whether dividend is stationary or nonstationary. Brealey et al. (2011) also discusses conditions where the stock price

[2]The ex-post rational prices is defined as the realised present value of current and future dividends.

fluctuations are driven by changes in the expected discount present value of dividends.

2.2.1 Variance bounds tests

The variance bound debate in literature has mainly been econometric in nature. Most research studies use empirical and simulation methods to demonstrate results, when analytical issues need clarification. The estimation of volatility may take two forms; It is either directly calculated as "realised volatility" of the underlying assets or as the "implied volatility" that is estimated from some innovations in the asset or from some subset of the market's information set. For any given asset, the term structure of volatility provides an efficient reflection of the market expectations across different horizons. As these expectations vary of over time, so will dynamics of asset prices and its associated returns also change.

The question on what drives equity price dynamics, including subsequent information contained within observed fluctuations has been widely investigated. On this subject matter, most discussions in the literature typically seek to explain why aggregate stock prices, a highly volatile series turns out to be the expected present value of a corresponding dividend series that is much smoother. Some notable works include those of Lewis & Whiteman (2006), Binsbergen & Koijen (2010), Lacerda & Santa-Clara (2010), Van Binsbergen & Koijen (2011), Van Binsbergen et al. (2012), Chen et al. (2012), Rangvid et al. (2014), and Jagannathan & Liu (2019). These studies are linked by the present-value relation, which typically plays a key role in valuation of financial assets.

In its generalized form, the simple present-value relation, equates the time t realisation of a dependent variable time series x_t to the present-value of discounted expected value of $\{y_t\}$, given currently known information I_t. This is written as

$$x_t = \sum_{k=0}^{n} \alpha^k E(y_{t+k}|I_t) \tag{2.1}$$

where, $\{y_t\}$ is a scalar[3] time-series and information on y_t and x_t is available up until, and including time t. α is the scalar $\alpha < 1$.

The present value relation has been extensively applied[4] in varied research studies Grossman & Shiller (1981), Shiller (1981), LeRoy & Porter (1981), LeRoy & Parke (1992), Campbell et al. (1998), Cochrane (2008), Binsbergen & Koijen (2010), Golez & Koudijs (2018), and Jagannathan & Liu (2019). There is

[3] $\{y_t\}$ is a scalar time-series jointly generated with another time-series vector $\{\underline{v}_t\}$, as an independent stationary linear bivariate or multivariate stochastic process. With this outline, the independent bivariate series i.e.$\{y_t, \underline{v}_t\}$ take their distributions exogenously.

[4](2.1) depicts the expectations theory of term structure when y_t is short-term interest rate and x_t is long-term interest rate. Likewise, permanent income hypothesis results when y_t is short-term income and x_t is long-term average income

also strong empirical base supporting the explanatory power of the dividend yield effect in depicting the true economic value of stocks in efficient markets. Current extensions of (2.1) captures the role of market anticipation of future economic variables in stock price dynamics Cochrane (2008), Van Binsbergen et al. (2012), Li & Yang (2013), Lansing (2016), Golez & Koudijs (2018), and Jagannathan & Liu (2019). Using a latent variable model for dividends within present-value relations, Van Binsbergen & Koijen (2011) finds improved dividend growth predictability. Jagannathan & Liu (2019) also shows further improvement in dividend predictability when the effects of learning on dividend dynamics is captured in the (2.1) for long-run risk models.

Shiller (1981), LeRoy & Porter (1981), Mankiw et al. (1985), Marsh & Merton (1986), and Kleidon (1986) present evidence where aggregate stock prices are more volatile than implied by the present-value relation. In Shiller's model-free variance bound test, stocks are at least 5 times more volatile than their corresponding dividends. This constitutes the volatility puzzle that has received a lot of attention in literature. The first-generation method in Shiller (1981) and LeRoy & Porter (1981) does not, investigate the statistical significance of its point estimates due to the lack of a dividend model in the specification. The model-based alternatives proposed by LeRoy & Porter (1981), Campbell & Shiller (1987*b*, 1989) seek to reconcile several critical distinctions that, although present in the model-free, were not emphasized adequately or developed there.

A lot of studies attempt to rationalise and resolve the excess volatility associated with stock prices. In line with Shiller's variance bounds test, Gilles & LeRoy (1991) argues for the need to account for market frictions by parameterising preferences of market participants that are more general compared to what has already been done. Perhaps, the approach can help researchers to adequately explore the volatility puzzle and further engage formal statistical tests to assertions when working with model specifications that link expected returns, return volatility and the price-dividend ratio.

The basic idea of the volatility puzzle is to test whether the variance relations of pay-offs conform to bounds derived from the present value relation. It is sometimes mentioned hand in hand with orthogonality tests, which has to do with testing whether actual prices and their ex-post counterpart are related just as implied by the present value equation. LeRoy & Parke (1992) discusses the difference between the two; in that, orthogonality tests focuses on validating the equality implication of the present value relation while the bounds test seeks to determine the validity of the present value inequality relation.

The variance bounds test presented in Shiller (1981) has seen a good number of criticisms, starting with Flavin (1983), West (1988*b*), Kleidon (1986) and more recently Engel (2005) and Lansing (2016). Flavin points to the sample variance being a biased[5] estimate of the population variance, thereby affecting the test

[5]Baised estimates result form the fact that the correction for degrees of freedom is insufficient.

results. Stock price time series in the bounds test requires a trend adjustment of the time series in order to attain stationarity; Kleidon (1986) shows that the trend-adjusted series in Shiller's paper is non-stationary. Kleidon's assertion, opened up a new issue that underlie subsequent enquiries on the variance bounds theory. This has to do with whether prices are stationary[6] after applying first generation detrending procedure[7].

A number of studies attempt to bypass the trend-adjustment procedure in varied ways. For instance, LeRoy & Parke (1992) suggest using the dividend-price ratio series rather than actual prices themselves, hence deriving the variance bounds relation in terms of price-dividend ratio instead. In their specification, ex-post rational price of the stock is defined as $p_t^* = \beta d_{t+1} + \beta^2 d_{t+2} + \cdots$ where β denotes the discount factor. From the present value relation, actual prices of the stock is written as $p_t = E(p_t^*|I_t)$ where I_t is the available information up to time t. The approach described in LeRoy & Parke (1992) presents an alternative specification of the variance bounds test using the dividend price ratio. Here, $p_t/d_t = E(p_t^*/d_t|I_t)$ with a variance bound relation written in the form $Var(p_t/d_t) \leqslant Var(p_t^*/d_t)$. The final results are largely in the same direction as that of the first generation variance bounds test, and the estimated volatilities also remain in the same order of magnitude. Campbell & Shiller (1988) and Cochrane (2008) also suggest other corrective methods for trend adjustments. One also comes across a model-free variance bounds and orthogonality test in LeRoy & Parke (1992), where an argument is made for the need to specify an appropriate dividend forecasting model.

There has also been discussions on developing appropriate formal tests for the variance bounds theory. This argument is found in discussions introduced by Flavin (1983), and they are based on the fact that the underlying distribution that drives the sample variance in the bounds tests is unknown. Such a model-based test is important and LeRoy & Parke (1992) attest to this, but also express concerns about the implication of its results. For instance, rejecting the null will suggest a rejection of the present-value relation. However, failure to reject the null could be driven by a case of misspecified dividend model.

2.2.2 Objectives of the chapter

In order to contribute to the excess volatility puzzle debate, the thesis shifts the focus from the US to a larger sample of globally selected stock markets. This allows us to add to previous literature on variance bounds tests of stock market rationality that has exclusively focused on the US stock market. This chapter of the

[6] If stationarity is not achieved, the variance of the prices will be time-varying.

[7] Detrending is done by fitting a loglinear trend to stock prices. Shiller (1981) uses the regression coefficient to calculate the growth rate of the prices series which is used to detrend the prices. LeRoy & Porter (1981) also assumes that the trend in prices is attributed to inflation and retained earnings and proceed to reverse the effects of these factors on the stock prices.

thesis performs a set of variance bounds tests using prices of stock indices and their corresponding dividends across 50 countries. This provides an essential support structure to understand the general applicability of the excess volatility puzzle. The 50 sampled countries are at different states of economic development thereby allowing for a robust investigation of the theoretical concepts outlined in relevant literature. This is the first research study on the excess volatility puzzle and stock market rationality across multiple international markets which seeks to:

i) address the analysis of the variance bounds puzzle in an international context,

ii) investigate valuable new insights on its existence, and explore the generalizability of the excess volatility puzzle.

iii) analyse the impact of recent approaches for obtaining expectations of dividend growth on results of variance bounds test.

2.2.3 Research questions

Given the present value model's prediction that realized stock prices reflect investors' expectations about future dividends, the choice of a specific approach to construct the time-series of expected dividends (by determining a value for expected dividend growth) is likely to have an impact on the results of variance bounds tests. Starting from the seminal Shiller (1981) study, where expected dividend growth is essentially obtained via a simple regression of prices against time, subsequent papers have explored different ways of producing more accurate expectations of dividend growth; Cochrane (1991) provides more detailed discussion. The contribution to this debate is by applying relatively more recent techniques from the literature on dividend predictability. I obtained dividend trend factors via regressions of realized dividend growth against the lagged dividend-price ratio, either in a simple time-series regression setting as in Cochrane (2008) or in a mixed-frequency regression setting (as in Asimakopoulos et al. 2017). More specifically, the research focuses on the first and second generation variance bounds tests and intends to answer the following questions:

i) Is there a strong international evidence of excess volatility to support the conclusions of variance bounds tests?

ii) Can the first generation variance bounds test be improved with a robust approach for obtaining expectations of dividend growth?

iii) Are the results from variance bounds tests indicative of stock market irrationality or of stock market efficiency?

2.3 Data sources and main variables

I obtain international data from the Global Financial Database (GFD). The dataset covers 50 countries and it includes monthly observations for the nominal index price, index dividend, the Consumer Price Index (CPI), and the risk-free rate for each sample country. The overall sample period runs from January 1840 to December 2018, for a total of 179 years (2,148 months). However, given that coverage in GFD begins at different times for different countries, the number of available observations varies across the sample countries. For example, data for the French equity market is available from 1840 while, at the other end of the spectrum, coverage for Bulgaria only begins in 2001. Some countries had individual stock price data but started calculating and publishing stock price index late. For instance, Canada started calculating and publishing their stock price index about a century before the United Kingdom. Section 2.6.2 of the Appendix presents summary metadata on the FTSE100 index from the Global Financial Database (GFD).

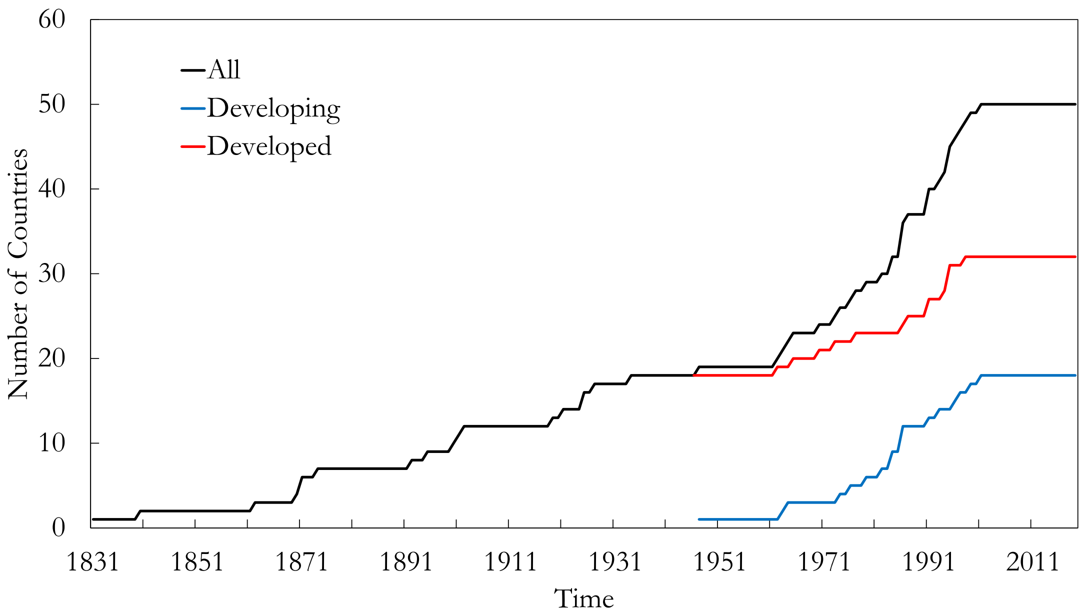


Figure 2.1: Data coverage

Notes: This figure plots the number of countries with available stock index price, and dividend data from 1840 to 2018. Categorisation into developing and developed countries is based on the International Monetary Fund, from 2018.

I use P_t and D_t to denote the index price level and the level of index dividends, respectively, for a given country at time t at an annual frequency. I use the lower case variables p_t and d_t to denote the corresponding logarithmic prices and dividends, respectively. Then, we compute the annual log dividend growth rate g_t, log index returns r_t, and the log dividend yield y_t at time t as follows

$$g_t = ln(D_t) - ln(D_{t-1}) = d_t - d_{t-1} \tag{2.2}$$

$$r_t = ln\left(\frac{P_t + D_t}{P_{t-1}}\right) \tag{2.3}$$

$$y_t = ln\left(\frac{D_t}{P_t}\right) = d_t - p_t \tag{2.4}$$

For each country, we obtain annual observations by aggregating monthly dividends and by using end-of-year prices. More specifically, we construct the time-series of annual dividends D_t as the sum of the 12 monthly dividends that were paid during a particular year, while the time-series of annual index prices P_t refers to the prices observed on the last day of December in each year. Finally, we compute real annual prices and dividends by deflating the nominal time-series by the respective CPI[8].

The aggregation of monthly dividends in D_t and the use of end-of-year prices in P_t naturally results in some loss of information, since the resulting dividend-price ratio y_t will not take into account any variation in prices or dividends that occurred within a particular year. In order to address this concern, we also employ a mixed-frequency data sampling (MIDAS) technique which allows us to use information from prices and dividends at a higher (monthly) frequency, similarly to Asimakopoulos et al. (2017). To this end, we use the monthly quoted variables $P^m_{t,k}$ and $D^m_{t,k}$, which denote the price levels and dividend levels, respectively, observed in year t and on month k, as well as the corresponding logarithmic variables $p^m_{t,k}$, $d^m_{t,k}$, and $y^m_{t,k}$.

Table 2.1 reports descriptive statistics for the time-series of annual log index returns r_t, log dividend growth g_t, and the dividend yield y_t in each country, with the sample countries sorted in ascending order according to the number of available observations. Unsurprisingly, mean dividend growth has been positive in all countries, with the only exception of Egypt which experienced a -0.015 mean log-change in annual dividends during the sample period. Despite its almost universally positive sign, mean dividend growth varies substantially across different countries, ranging from a minimum of 0.013 (Japan) to a maximum of 0.465 (Argentina). Interestingly, countries with shorter available data series generally have a higher mean and standard deviation of dividend growth compared to countries with more available data. For example, the highest standard deviation of dividend growth is exhibited by Bulgaria (0.994) which only has 17 years of data in our sample. Similarly, the mean log return also seems to be negatively related to the number of available observations across countries, and it remains predominantly positive (with the exception of

[8] I follow the approach in Shiller (1981) to deflate nominal variables. More specifically, we use the mean annual CPI to deflate monthly variables for observations before 1900, while we use the monthly CPI to deflate observations after 1900.

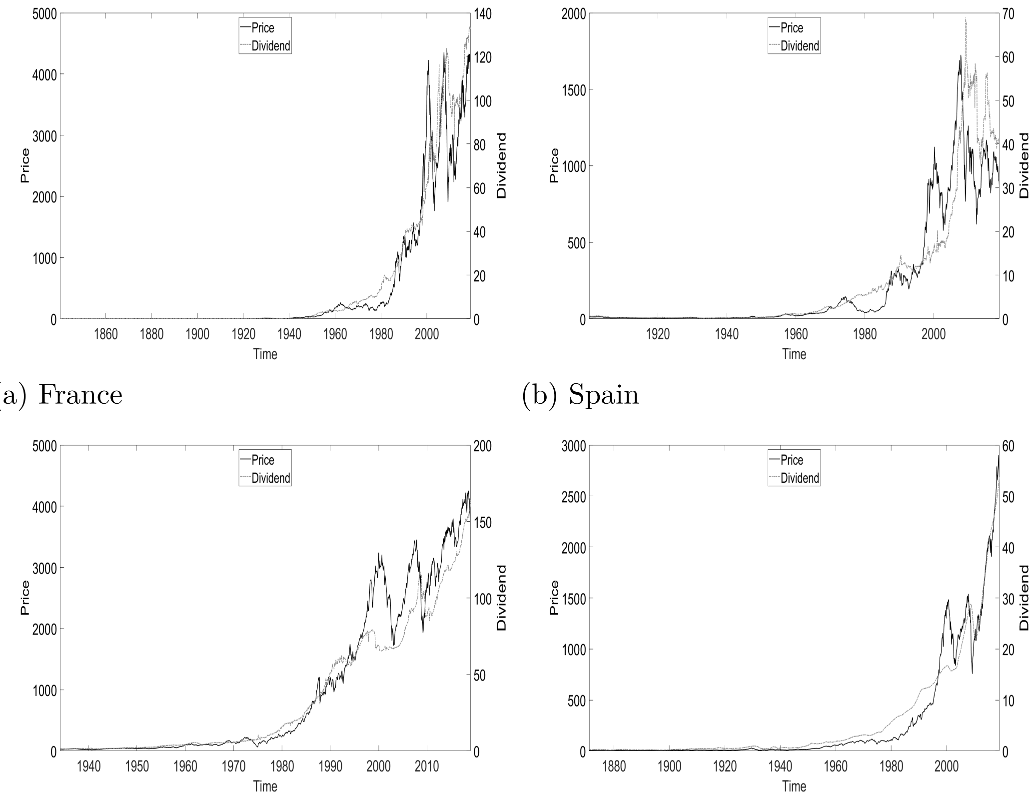
Portugal).

Table 2.1: Descriptive statistics

Country	t_0	n	Return (r)		Dividends (d)		Dividend Yield (y)	
			Mean	St.Dev.	Mean	St.Dev.	Mean	St.Dev.
BGR	2001	17	0.094	0.508	0.126	0.994	0.278	0.426
ROU	1999	19	0.146	0.402	0.229	0.575	0.486	0.378
RUS	1998	20	0.219	0.501	0.315	0.483	0.281	0.200
TUN	1997	21	0.094	0.147	0.107	0.187	0.398	0.131
BRA	1996	22	0.143	0.330	0.152	0.200	0.427	0.097
CZE	1995	23	0.035	0.246	0.107	0.640	0.523	0.277
HUN	1995	23	0.125	0.344	0.119	0.246	0.264	0.129
POL	1995	23	0.045	0.275	0.053	0.322	0.297	0.157
ISR	1994	24	0.078	0.255	0.083	0.279	0.338	0.118
EGY	1993	25	0.154	0.465	-0.015	0.445	0.928	1.256
CHN	1991	27	0.079	0.418	0.110	0.444	0.269	0.135
IDN	1991	27	0.119	0.339	0.198	0.342	0.238	0.076
IRL	1991	27	0.051	0.300	0.049	0.270	0.259	0.158
PRT	1987	31	-0.003	0.266	0.041	0.180	0.395	0.165
COL	1986	32	0.178	0.357	0.163	0.463	0.371	0.172
NGA	1986	32	0.164	0.309	0.159	0.277	0.604	0.181
TWN	1986	32	0.070	0.373	0.101	0.422	0.273	0.180
TUR	1986	32	0.340	0.653	0.318	0.275	0.358	0.278
KEN	1984	34	0.059	0.290	0.032	0.363	0.912	0.465
MAR	1984	34	0.099	0.178	0.076	0.110	0.505	0.269
PHL	1982	36	0.105	0.390	0.049	0.349	0.335	0.432
JOR	1979	39	0.041	0.224	0.053	0.271	0.404	0.161
GRC	1977	41	0.047	0.407	0.013	0.400	0.698	0.657
THA	1976	42	0.070	0.369	0.050	0.287	0.496	0.291
CHL	1974	44	0.272	0.443	0.270	0.490	0.476	0.220
MYS	1974	44	0.076	0.271	0.067	0.157	0.308	0.087
SGP	1973	45	0.060	0.277	0.076	0.095	0.287	0.112
NOR	1970	48	0.059	0.273	0.073	0.178	0.378	0.156
HKG	1965	53	0.109	0.368	0.099	0.245	0.420	0.164
ZAF	1964	54	0.113	0.201	0.113	0.159	0.402	0.131
KOR	1963	55	0.108	0.294	0.071	0.239	0.764	0.776
FIN	1962	56	0.082	0.292	0.079	0.181	0.516	0.222
ARG	1947	71	0.473	0.852	0.465	1.039	0.381	0.222
GBR	1934	84	0.057	0.204	0.061	0.076	0.530	0.159
NZL	1927	91	0.045	0.201	0.040	0.223	0.603	0.166
AUT	1925	93	0.053	0.277	0.047	0.214	0.376	0.136
ITA	1925	93	0.064	0.269	0.058	0.620	0.443	0.203
IND	1921	97	0.061	0.223	0.043	0.269	0.641	0.442
CHE	1919	99	0.040	0.197	0.032	0.209	0.352	0.187
SWE	1902	116	0.059	0.215	0.059	0.179	0.483	0.146
JPN	1901	117	0.024	0.318	0.013	0.221	0.543	0.425
ESP	1900	118	0.036	0.203	0.036	0.141	0.546	0.276
NLD	1892	126	0.030	0.193	0.027	0.215	0.559	0.236
DNK	1874	144	0.034	0.170	0.035	0.150	0.553	0.242
BEL	1871	147	0.031	0.192	0.032	0.306	0.452	0.172
USA	1871	147	0.043	0.179	0.036	0.101	0.519	0.216
DEU	1870	148	0.022	0.330	0.017	0.256	0.495	0.222
AUS	1862	156	0.042	0.147	0.041	0.127	0.671	0.158
FRA	1840	178	0.053	0.192	0.052	0.134	0.476	0.170
CAN	1831	187	0.037	0.161	0.039	0.252	0.486	0.191

Notes: This Table presents a set of descriptive statistics for the main variables of interest, namely stock index returns (r), index dividends (d), and the dividend yield (y). All variables are annualized. The Table reports the mean and standard deviation of each time-series, as well as the first year of available observations (t_0) and the total number of observations in years (n). Statistics are tabulated separately for each country, across 50 countries in total. Countries are sorted in ascending order based on the number of years with available data (n).

Figure 2.2 plots the time-evolution of the index price levels and the corresponding dividends for four developed economies, namely the France, Japan, US, and UK. Consistent with the positive means reported in Table 2.1, Figure 2.2 depicts an upward trend in the index price and the associated dividends, with the a significant degree of co-movement between the two time-series.[9]



(a) France (b) Spain

(c) UK (d) US

Figure 2.2: Time-evolution of index level and dividends

Notes: This Figure plots the time-series of index price levels and index dividends for a subsample of four countries, namely France, Japan, UK and US.

2.4 Variance bounds tests

The log dividend yield can be expressed as the difference between expected stock returns and the expected dividend growth plus a constant (Campbell & Shiller 1988)

[9]Albeit unreported to conserve space, the respective plots for the remaining 46 sample countries reveal similar patterns to the ones reported in Figure 2.2.

$$y_t = \alpha + E_t[\sum_{j=0}^{\infty} \rho^{j-1} r_{t+1+j}] - E_t[\sum_{j=0}^{\infty} \rho^{j-1} \Delta g_{t+1+j}] \tag{2.5}$$

where ρ denotes an autoregressive parameter vector. I follow Cochrane (2008) and estimate ρ via a Vector Autoregressive (VAR) specification based on the changes in dividend growth Δg_t, stock returns r_t, and the dividend yield y_t. This specification can serve as a reasonable starting point which allows us to obtain ex-post rational prices by using the dividend yield and expected dividend growth. While the dividend yield is readily observable at time t, we still need a meaningful measure of expected dividend growth based on information available at t. Hence, the empirical analysis starts by following the standard approach of estimating the growth trend via a regression of price levels against time, as was originally proposed in Shiller (1981). More specifically, in order to compute the de-trended real log price $\tilde{p}_t$ and dividend $\tilde{d}_t$, we begin by regressing the real log price p_t against a deterministic trend

$$p_t = \alpha + \beta t + \epsilon_t \tag{2.6}$$

The long-run exponential growth rate that is used to de-trend the price and dividend series is $\lambda = exp(\beta)$. Thus, the real de-trended time-series are given by

$$\tilde{p}_t = \frac{p_t}{\lambda^{t-T}} \tag{2.7}$$

$$\tilde{d}_t = \frac{d_t}{\lambda^{t+1-T}} \tag{2.8}$$

Finally, similarly to Shiller (1981), we compute the de-trended ex-post rational price $\tilde{p}_t^*$ recursively from the terminal date T using the equation

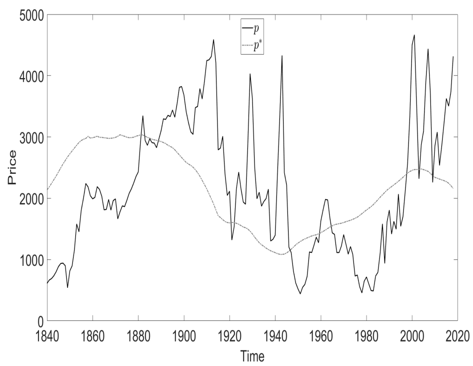
$$\tilde{p}_t^* = \bar{\gamma}(\tilde{p}_{t+1}^* + \tilde{d}_t) \tag{2.9}$$

where $\bar{\gamma} = \lambda(1 + r)$ is a discount factor and r denotes the one-year risk-free rate of interest. In order to solve the recursive problem in (2.9), we set the terminal value $\tilde{p}_T^*$ of the ex-post rational price $\tilde{p}_t^*$ equal to the average de-trended realized price over the sample period, i.e $\tilde{p}_T^* = \frac{1}{T}\sum_{t=1}^{T} \tilde{p}_t$.

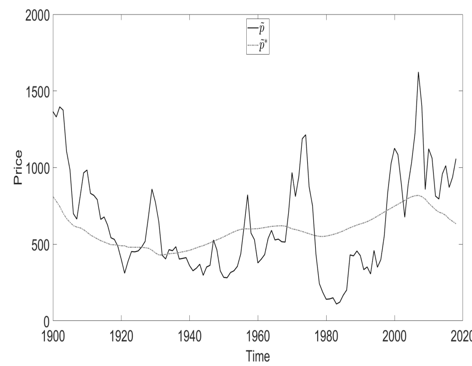
The efficient markets model implies that, since rational investors determine current stock prices by discounting future dividends, $\tilde{p}_t$ represents an optimal forecast of $\tilde{p}_t^*$, in other words $\tilde{p}_t = E_t[\tilde{p}_t^*]$. Then it follows that the forecast error $u_t = \tilde{p}_t^* - \tilde{p}_t$ must be uncorrelated with the forecast itself. This means that $var(\tilde{p}_t^*) = var(\tilde{p}_t + u_t) = var(\tilde{p}_t) + var(u_t)$. Since variances are obviously non-negative, this relationship results in the following expression for the standard bounds test of Shiller (1981) and LeRoy & Porter (1981)

$$\sigma(\tilde{p}_t) \leqslant \sigma(\tilde{p}_t^*) \tag{2.10}$$

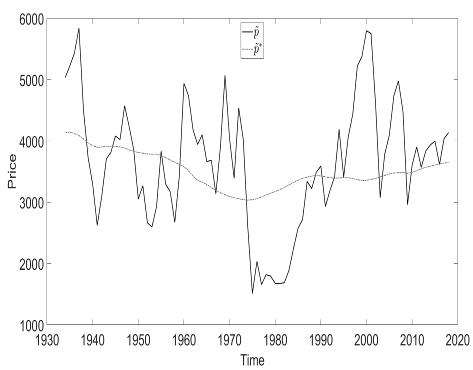
The excess volatility puzzle refers to the commonly reported empirical finding of the volatility of realized stock prices exceeding substantially the volatility of expected stock prices that are based on expected dividend payments. In order to get a preliminary idea about the magnitude of the excess volatility puzzle across different markets, we begin by plotting in Figure 2.3 the historical evolution of the de-trended stock price $\tilde{p}_t$ and the corresponding ex-post rational price $\tilde{p}_t^*$ for a subset of four developed countries, namely France, Spain, US and UK. The resulting plots are consistent with the hypothesis of excess volatility in the time-series of realized prices compared to that of dividend-based expected prices, as evidenced by $\tilde{p}_t^*$ consistently plotting as a much smoother and more stable series compared to that of its respective $\tilde{p}_t$. In this sense, Figure 2.3 provides some preliminary evidence against the rational expectations hypothesis.



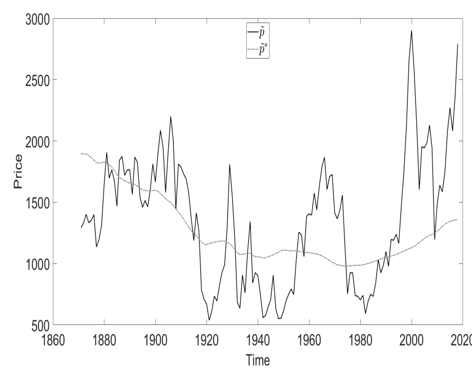
(a) France



(b) Spain



(c) UK



(d) US

Figure 2.3: Time-evolution of realized price and ex-post rational price

Notes: This Figure plots the time-series of de-trended realized prices $\tilde{p}_t$ and de-trended ex-post rational prices $\tilde{p}_t^*$ for a subsample of four countries, namely France, Spain, UK and US.

Next, the chapter explores the magnitude of the puzzle across all markets by plotting the volatility of the realized price $\sigma(\tilde{p}_t)$ against the volatility of the ex-post rational price $\sigma(\tilde{p}_t^*)$ for each of the 50 sample countries. The first thing to notice on Figure 2.4 is that, *in all cases*, $\sigma(\tilde{p}_t)$ is indeed higher than the volatility $\sigma(\tilde{p}_t^*)$ that would have been expected conditional on dividends, confirming the existence of the excess volatility puzzle across all of our sample countries. The magnitude of the puzzle in many countries appears to be considerably higher compared to the commonly quoted ratio of 5:1 in the US market that was reported in the seminal Shiller (1981) study.

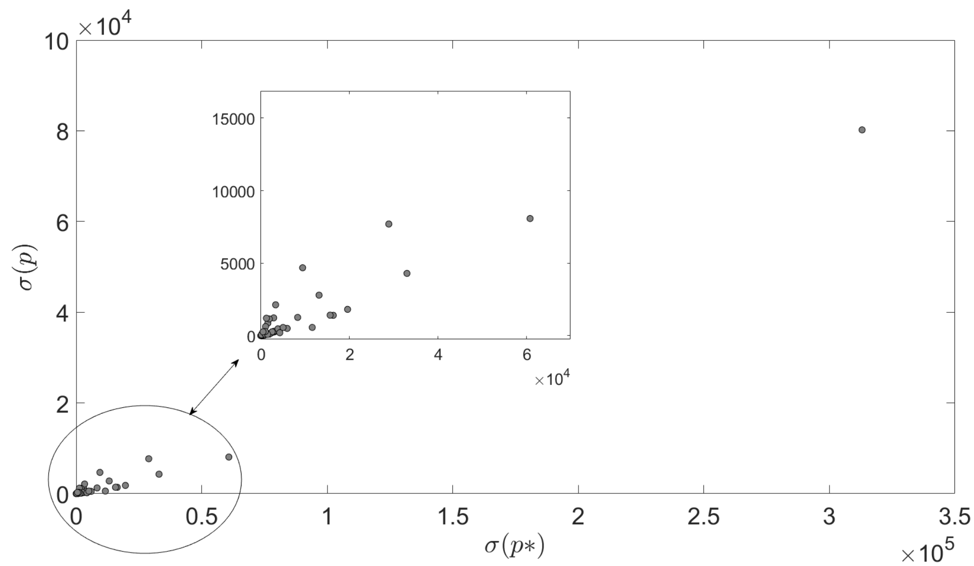


Figure 2.4: Volatility of realized vs ex-post rational prices

Notes: This Figure plots the realized stock price volatility $\sigma(p)$ against the volatility of the ex-post rational price $\sigma(p^*)$, across a sample of 50 countries.

As can be seen from the first column of Table 2.2, the volatility ratio $\frac{\sigma(\tilde{p}_t)}{\sigma(\tilde{p}_t^*)}$ ranges from a minimum of 0.5 (Spain) to a maximum of 22.7 (Malaysia), with a mean (median) value of 7.5 (6.0). Overall, in addition to the ratio exceeding one (i.e. $\sigma(\tilde{p}_t)$ exceeding $\sigma(\tilde{p}_t^*)$) across almost all countries, the volatility ratio exceeds the value of 5 for the majority of sample countries (27 out of 50).

Table 2.2: Variance bounds tests

	Full sample	Subsamples formed on time-series length		
		$L < 30$	$30 < L < 60$	$60 < L$
mean	7.5	12.5	9.2	2.4
median	6.0	11.7	8.8	2.3
min	0.5	3.1	1.1	0.5
max	22.7	22.3	22.7	3.9
$n > 5$	27	12	15	0
n	50	13	18	19

Notes: This Table presents the results of standard Shiller (1981) variance bounds tests. Each volatility ratio $\frac{\sigma(\tilde{p}_t)}{\sigma(\tilde{p}_t^*)}$ is computed separately for each sample country, across 50 countries in total. The Table reports the mean, median, minimum, and maximum values of each ratio across the 50 sample countries, the number of cases where the ratio exceeds the value of 5:1 reported in Shiller (1981), and the number of countries n. The first column reports the results across the full sample of 50 countries, while the last three columns report the results from subsamples that have been formed based on the countries' number of years of available data L.

Importantly, the magnitude of the puzzle seems to be inversely related to the length of the available time-series. To get a better idea about the magnitude of this relationship, Figure 2.5 plots the $\frac{\sigma(\tilde{p}_t)}{\sigma(\tilde{p}_t^*)}$ ratio against the number of available years in the dataset, while the last three columns of Table 2.2 report descriptives for the volatility ratio separately for three subsamples of countries where the length of the available time-series is either below 30, between 30 and 60, or above 60 years. Both Figure 2.5 and Table 2.2 show that countries with the shortest available time-series tend to have markedly higher volatility ratios compared to countries with longer available time-series. For instance, countries with time-series that are shorter than 30 years have a mean volatility ratio of 12.5, compared to a mean ratio of only 2.4 for countries with time-series that are longer than 60 years. Furthermore, 12 out of 13 countries in the first subsample (sample size $<$ 30) have a volatility ratio that is higher than 5, compared to 15 out of 18 countries in the second subsample (30 $<$ sample size $<$ 60) and to 0 out of 19 countries in the last subsample (60 $<$ sample size).

In addition to the upper bound of the variance of stock prices that is described in (2.10), Shiller (1981) also derives the maximum value of the variance of changes in price for a given variance of dividends. Assuming that dividends are stationary, it is shown that

$$\sigma(\Delta\tilde{p}) \leqslant \frac{\sigma(\tilde{d})}{\sqrt{2r}} \tag{2.11}$$

The main intuition behind the proof of the inequality in (2.11) is that the variance of changes in price is larger when information about future dividends is revealed more smoothly across time, as opposed to future dividends being known either many years before or just before they are paid[10].

[10]Shiller (1981) uses a standard first-order autoregressive specification for future dividends to derive the

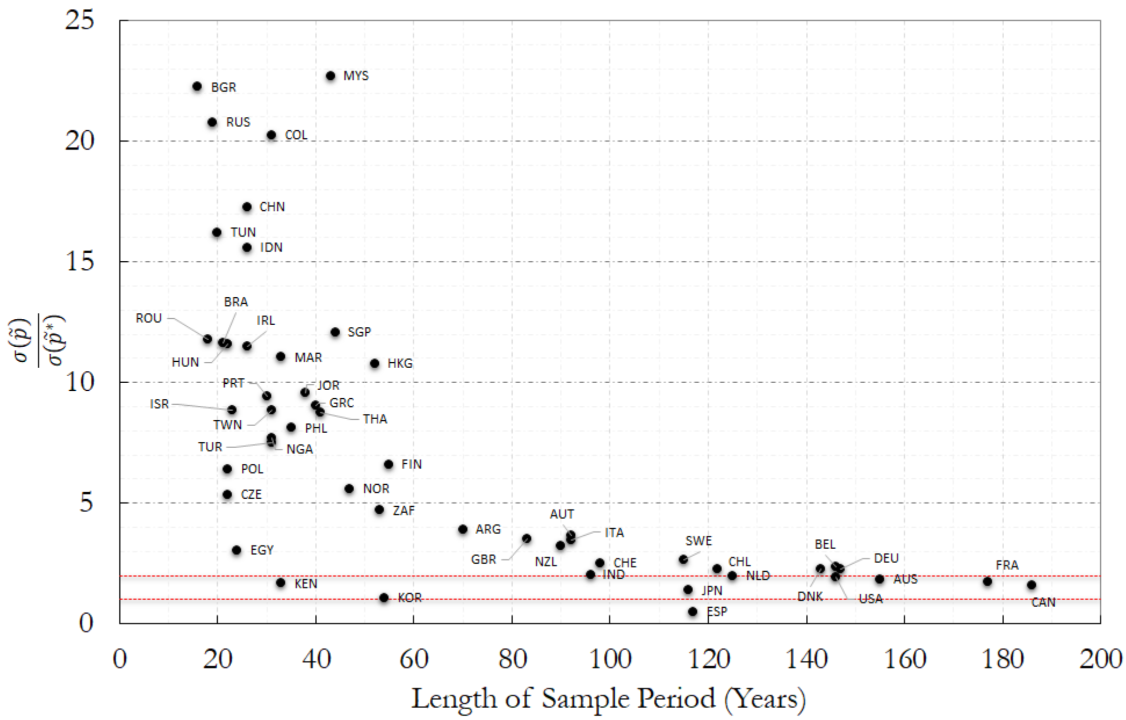


Figure 2.5: Volatility ratios and sample period length

Notes: This Figure plots the ratio $\frac{\sigma(\tilde{p}_t)}{\sigma(\tilde{p}_t^*)}$ of the volatility of the realized price by the volatility of the ex-post rational price, across a sample of 50 countries. The volatility ratios have been ordered (on the horizontal axis) in ascending order based on the number of years of available data for each country. See summary description on data availability in Appendix 2.6.2

Finally, we also examine a slightly different version of the variance inequality that was also introduced in Shiller (1981)

$$\sigma(\Delta\tilde{p} + \tilde{d}_{t-1} + r\tilde{p}_{t-1}) \leqslant \frac{\sigma(\tilde{d})}{\sqrt{2r}} \tag{2.12}$$

Table 2.3 reports descriptive statistics for the volatility ratios that relate to inequalities (2.11) and (2.12) in Panels A and B, respectively. These results confirm the hypothesis of excess volatility across all sample countries, although the alternative volatility ratios are now closer to 5, with their mean values in the full sample equal to 5.8 and 5.6 under inequalities (2.11) and (2.12), respectively. Interestingly, the volatility ratios that are based on price changes are not monotonically decreasing across the three size-based country groups. Nevertheless, a clear difference is still found between the volatility ratios of countries with the longest time-series (on average 6.9 and 6.8 for inequalities (2.11) and (2.12), respectively) and those of countries

maximum value for the variance of changes in price. I drop the time subscripts from the variance inequality in (2.11) because the unconditional co-variance between $\tilde{d}_t$ and any information variable will depend on k but not on t.

with the shortest time-series (2.9 and 2.8, respectively).

Table 2.3: Upper bounds tests of price change variance

Panel A: $\frac{\sigma(\Delta\tilde{p})}{\frac{\sigma(\tilde{d})}{\sqrt{2\bar{r}}}}$				
	Full sample	Subsamples formed on time-series length		
		$L < 30$	$30 < L < 60$	$60 < L$
mean	5.8	6.9	8.0	2.9
median	5.1	6.4	7.3	2.8
min	1.0	3.3	1.8	1.0
max	18.9	12.9	18.9	5.4
$n > 5$	26	9	15	2
n	50	13	18	19
Panel B: $\frac{\sigma(\Delta\tilde{p}+\tilde{d}_{t-1}+\bar{r}\tilde{d}_{t-1})}{\frac{\sigma(\tilde{d})}{\sqrt{\bar{r}_2}}}$				
	Full sample	Subsamples formed on time-series length		
		$L < 30$	$30 < L < 60$	$60 < L$
mean	5.6	6.8	7.8	2.8
median	5.0	6.3	7.2	2.8
min	0.5	3.1	1.7	0.5
max	18.5	12.8	18.5	5.3
$n > 5$	25	10	14	1
n	50	13	18	19

Notes: This Table presents the results of variance bounds tests on the variance of price changes (Shiller 1981). Each volatility ratio is computed separately for each sample country, across 50 countries in total. The Table reports the mean, median, minimum, and maximum values of each ratio across the 50 sample countries, the number of cases where the ratio exceeds the value of 5:1 reported in Shiller (1981), and the number of countries n. The first column of each panel reports the results across the full sample of 50 countries, while the last three columns report the results from subsamples that have been formed based on the countries' number of years of available data L.

This general tendency of developing countries having higher volatility ratios than developed countries could conceivably reflect a fundamental difference among countries at different stages of development. However, the negative relationship between volatility ratios and sample length could simply be a statistical artefact caused by computing volatility ratios over small vs large samples, unrelated to any fundamental characteristics of the countries in question. On this particular issue, Flavin (1983), Kleidon (1986) and West (1988*b*) argue that, in small samples, the variance bounds test developed by Shiller (1981) is biased towards finding realized price volatility exceeding that of ex-post rational prices. Data from the Global Financial Database (GFD) is quite exhaustive since it is aggregated form varied sources. I checked for completeness to eliminate all forms of truncation. The data in Chapter 2 are based on availability with the GFD database and it is impossible to re-compute the missing stock price indices for countries with "short" data. The last date for the sample period is December 31, 2018 for all samples although the start data varies across countries. Empirical analysis to explore the statistical artifact by including country specific features/characteristics as dummy variables is beyond the variance bounds test analysis.

Table 2.4: Variance bounds tests -Capped data length

Panel A: Data length capped at 100 years				
	Full sample	Subsamples formed on *original* time-series length		
		$L < 30$	$30 < L < 60$	$60 < L$
mean	8.0	12.5	8.9	3.7
median	6.5	11.7	8.8	3.3
min	1.1	3.1	1.1	1.4
max	22.7	22.3	22.7	16.8
$n > 5$	28	12	15	1
n	50	13	18	19
Panel B: Data length capped at 60 years				
	Full sample	Subsamples formed on *original* time-series length		
		$L < 30$	$30 < L < 60$	$60 < L$
mean	8.6	12.5	8.9	5.5
median	7.7	11.7	8.8	5.1
min	1.1	3.1	1.1	1.6
max	22.7	22.3	22.7	10.5
$n > 5$	36	12	15	9
n	50	13	18	19
Panel C: Data length capped at 30 years				
	Full sample	Subsamples formed on *original* time-series length		
		$L < 30$	$30 < L < 60$	$60 < L$
mean	13.6	12.5	11.8	16.2
median	11.6	11.7	10.3	13.5
min	1.9	3.1	1.9	7.3
max	54.2	22.3	29.5	54.2
$n > 5$	48	12	18	18
n	50	13	18	19

Notes: This Table presents the results of variance bounds tests when the time-series length has been capped to the100, 60, and 30 most recent years (in Panels A, B, and C, respectively). Each volatility ratio $\frac{\sigma(\tilde{p}_t)}{\sigma(\tilde{p}_t^*)}$ is computed separately for each sample country, across 50 countries in total. The Table reports the mean, median, minimum, and maximum values of the volatility ratio across the 50 sample countries, the number of cases where the ratio exceeds the value of 5:1 reported in Shiller (1981), and the number of countries n. The first column reports the results across the full sample of 50 countries, while the last three columns report the results from subsamples that have been formed based on the countries' *original* number of years of available data L.

I explore whether small sample bias is affecting our previous results of variance bounds tests in the following way. First, we cap all countries' datasets at 100, 60, and 30 years (most recent ones), and then we re-compute the volatility ratio $\frac{\sigma(\tilde{p}_t)}{\sigma(\tilde{p}_t^*)}$ for the capped time-series. As can be seen from Table 2.4, the results are indeed consistent with a small sample bias affecting the variance bounds test results. This bias towards rejecting stock market efficiency is evident from the fact that volatility ratios tend to increase the more strictly we cap a country's data length. For instance, simply capping the data length to 100 years results in a mean volatility ratio of 3.7 across countries with the longest original data length ($60 < L$), compared to a mean of 2.4 when no cap was imposed (Table 2.2). When we apply a stricter cap of 30 years, these countries with the longest original data length experience an even more dramatic increase of volatility ratios, with their mean ratio now being equal to 16.2. Furthermore, when we focus on the extreme case of capping data length at 30 years (Panel C of Table 2.4), the previously reported strong negative relationship between the magnitude of excess volatility and the length of the original dataset can no longer be observed. Somewhat surprisingly, countries with the shortest original data length actually have a lower mean volatility ratio than that of countries with the longest original data length (12.5 compared to 16.2, respectively).

2.5 Expected dividend growth

2.5.1 Dividend predictability regressions

Since rationally expected future dividends are unobservable by nature it is important to use an appropriate rate of expected dividend growth when we compute the rational stock price as the discounted value of future dividends. Our previous empirical results were based on following the original approach in Shiller (1981), where the expected growth rate for stock prices (and, by extension, for dividends) is given by the trend factor from regressing stock prices against time (equation (2.6)). More recent studies on dividend predictability suggest that alternative measures of expected dividend growth might be better suited to forecast future dividends and, hence, could provide a more efficient ex-post rational price. To this end, we explore the predictability of dividend growth and, more specifically, the extent to which it can be forecasted using the dividend-price ratio (Cochrane 2008, Binsbergen & Koijen 2010, Asimakopoulos et al. 2017, Golez & Koudijs 2018). For each country in our sample, we run separately a time-series regression of dividend growth on the previous period's dividend-price ratio

$$\Delta g_{t+1} = \beta_{0,g} + \beta_{1,g} y_t + \epsilon_{g,t+1} \tag{2.13}$$

where the subscript t denotes time at an annual frequency. Following Cochrane (2008), the standard errors are GMM-corrected for heteroscedasticity. Similarly to Ang & Bekaert (2007), we begin by estimating these time-series regressions using aggregated annual data, ignoring seasonality issues. Our objective in this exercise is to obtain an improved de-trending factor $\lambda = exp(\beta)$, with the slope from equation (2.13) replacing the β that was originally obtained via regressing stock prices against time in equation (2.6).

A potential concern at this point is that the aggregation procedure involved in computing annual prices and dividends will lead to some loss of within-year information. I address this concern by employing a mixed-frequency regression framework MIDAS, where the lower-frequency (annual) dependent variable Δg_{t+1} is regressed against a higher-frequency (monthly) independent variable y_t^m. The main aspects of the MIDAS framework are discussed in the following section.

2.5.2 MIDAS Regressions

The MIDAS approach was introduced in Ghysels et al. (2004), Ghysels et al. (2005), and Ghysels et al. (2006) as a framework to estimate regression specifications where the dependent and the independent variables are sampled at different frequencies. Subsequent empirical studies have used the MIDAS approach in a variery of applications where the dependent variable is typically quoted at a lower frequency compared to the independent variables. For example, Forsberg & Ghysels (2006) estimate a MIDAS regression specification in the context of forecasting index volatility at longer horizons using absolute daily returns, while Clements & Galvão (2008) use monthly macroeconomic and financial indicators to forecast quarterly GDP growth. Our use of the MIDAS approach is more closely related to Asimakopoulos et al. (2017) who explore the predictive ability of the monthly dividend-price ratio over subsequent annual dividend growth.

In our study, the MIDAS framework is applied to estimate the effect of the higher frequency data of the log dividend yield y_t^m (monthly, i.e. $m = 12$) on the lower frequency data of the log dividend growth rate Δg_{t+1} (annual). The regression model can be written as

$$\Delta g_{t+1} = \widehat{\beta}_{0,g} + \widehat{\beta}_{1,g} B(L^{1/m}; \boldsymbol{\theta}) y_t^m + \widehat{\epsilon}_{g,t+1} \tag{2.14}$$

for $t = 1, \ldots, T$, where $L^{1/m}$ denotes the lag operator of the log dividend yield data. The term $B(L^{1/m}; \boldsymbol{\theta}) = \sum_{k=0}^{K-1} \omega_k(\boldsymbol{\theta}) L^{k/m}$ denotes a known polynomial function of $L^{1/m}$ whose coefficients depend on a small dimensional vector of parameters θ, while $L^{k/m}$ is the lag operator of y_t^m for k/m periods. The maximum length of the polynomial function is $K - 1$. In this setting, the overall impact of the lagged y_t^m on Δg_{t+1} is given by the coefficient $\widehat{\beta}_{1,g}$, which can be obtained by normalizing the weights $\omega_k(\boldsymbol{\theta})$ so that they sum up to one[11]. The general form equation (2.14) can be rewritten as

[11] See Ghysels et al. (2006) for a more detailed discussion on the benefits of weight normalization in MIDAS applications.

$$\Delta g_{t+1} = \widehat{\beta}_{0,g} + \widehat{\beta}_{1,g}\omega\widehat{y}_t^m + \widehat{\epsilon}_{g,t+1} \tag{2.15}$$

where $\omega\widehat{y}_t^m = \sum_{k=0}^{11}\omega_k y_{t,k}^m$. Equation (2.15) can be thought of as a projection of the annual dividend growth variable Δg_{t+1} onto the monthly log dividend yield variable y_t^m using up to $K-1$ monthly lags (i.e. 11 lags in our case).

The specific shape of the weighting scheme ω will depend on the chosen specification for the polynomial function. On this issue, Ghysels et al. (2007) discuss a number of alternative weighting schemes. In this study, we consider four alternative specifications for the polynomial function, each of which will result in a different weighting scheme and, by extension, a different effect of y_t^m on Δg_{t+1}. More specifically, our pool of candidate weighting schemes consists of a polynomial with a step-function, an exponential Almon lag polynomial, a normalized beta lag polynomial, and an Almon lag polynomial of order P. Instead of selecting a single weighting scheme universally across time and different countries, we adopt a more flexible approach where we dynamically select the optimal weighting scheme at each time t and separately for each country i.

The first specification is a polynomial with a step-function. Under this alternative, a regressor X_t can be expressed as the partial sum of the higher frequency x^m, so that $X_t(K,m) = \sum_{j=1}^{K} x_{t-\frac{j}{m}}^m$. Then, the MIDAS regression with M steps can be estimated as a simple Ordinary Least Squares (OLS) regression of the lower-frequency dependent variable against the regressor $X_t(K_i,m)$, where $K_1 < \cdots < K_M$. The impact of x^m on the dependent variable can be measured by the sum of all coefficients in the OLS regression (i.e. $\sum_{i=1}^{M}\beta_i$), since it appears in all the partial sums. A more detailed discussion of MIDAS with step-functions can be found in Forsberg & Ghysels (2006).

The second specification is an exponential Almon lag polynomial, which has also been used in by Asimakopoulos et al. (2017). Interestingly, Ghysels et al. (2007) argue that this scheme can be thought of as the most general weighting scheme, as it has the most flexible shape. In its unrestricted version, the exponential Almon lag polynomial is fully determined by its two parameters θ_1 and θ_2, with the corresponding weights computed as

$$\omega_k(\theta_1,\theta_2) = \frac{e^{\theta_1 k+\theta_2 k^2}}{\sum_{k=1}^{K} e^{\theta_1 k+\theta_2 k^2}} \tag{2.16}$$

The third specification is a normalized beta lag polynomial. Based on the beta function, this polynomial is fully determined by the three parameters θ_1, θ_2 and θ_3. The beta function is very flexible, as it allows for weights that can take a variety of different shapes. For instance, Ghysels et al. (2007) show that larger values of θ_2 result in faster declining weights, with the rate of weight decline essentially determining how many lags will be included in the MIDAS specification. The weights under the normalized beta lag polynomial can be computed as

$$\omega_k(\theta_1, \theta_2, \theta_3) = \frac{h_k^{\theta_1 - 1}(1 - h_k)^{\theta_2 - 1}}{\sum_{k=1}^{K} h_k^{\theta_1 - 1}(1 - h_k)^{\theta_2 - 1}} + \theta_3 \tag{2.17}$$

where $h_k = (k-1)/(K-1)$. In addition to its unrestricted version above, we also consider a restricted version where the last lag is set to zero, i.e. $\omega_k(\theta_1, \theta_2, 0)$.

The final specification is the non-normalized Almon lag polynomial of order P, first introduced in Almon (1965). The corresponding weights for each lag k can be computed as

$$\omega_k(\theta_0, \ldots, \theta_P) = \sum_{p=0}^{P} \theta_p k^p \tag{2.18}$$

The weights in (2.18) are obtained via a non-linear least squares estimation, where the optimal lag order is selected using the AIC/BIC of the least squares estimation (for more details, see Ghysels et al. 2007). This approach assumes that the successive weights lie on a polynomial, estimating a few points on the curve as regression coefficients and then using polynomial interpolation to interpolate between them for the remaining points.

I allow the specification of ω_k to be determined endogenously by identifying the optimal scheme dynamically from this pool of four schemes. Our dynamic selection of the the optimal weighting scheme is based on an out-of-sample evaluation of the forecasts produced by the candidate schemes. For each sample country, we begin by constructing a 2-year validation period at the beginning of the available data series, in order to produce an one-month ahead forecast of dividend growth under each candidate scheme (similar to Andreou et al. 2013). Then, we continue producing one-month ahead forecasts of dividend growth based on a recursive validation period, with each country's first sample month as the fixed starting point of the expanding validation window. This approach results in the construction of four time-series of one-month ahead forecasts (i.e. one time-series per candidate scheme) for each country. Finally, we dynamically select the optimal scheme at time t, separately for each sample country, as the one that produces the lowest Root Mean Squared Error (RMSE).

Overall, one significant advantage of the MIDAS approach is its flexibility in that it does not impose any particular assumptions about the effect of different lags. Instead, the optimal weighting scheme is driven entirely by the data. Asimakopoulos et al. (2017) further highlight that the use of non-linear lag polynomials under a MIDAS regression results in a more parsimonious estimation with a lower sensitivity to specification errors, compared to the alternatives of state-space models or mixed-frequency vector autoregression (VAR) models. Finally, the MIDAS approach avoids the issue of parameter proliferation, which could have more pronounced consequences in some of our sample countries with relatively small datasets.

2.5.3 Empirical Evidence

Table 2.5 reports the regression results from the three alternative approaches to obtaining measures of expected dividend growth, namely via regressions of stock prices against time as in Shiller (1981), regressions of changes in dividend growth against the lagged dividend-price ratio at an annual frequency in Cochrane (2008) and then as well, MIDAS regressions of changes in dividend growth against the lagged dividend-price ratio similarly in Asimakopoulos et al. (2017). Estimating the standard Shiller-type regressions of log-price against time in (2.6) results in universally positive slopes across all sample countries, with all 50 coefficients being statistically significant at the 1% level. Moreover, the magnitude of these slopes is generally lower for developed countries with longer time-series relative to developing countries with shorter time-series. For instance, Japan has 117 years of available data and it is found to have the lowest slope ($\beta = 0.03$), compared to Romania which has only 19 years of data and the highest slope ($\beta = 0.64$). This relationship suggests that, unsurprisingly, developed countries experience on average a lower stock price growth compared to developing countries.

Regressing changes in dividend growth against the lagged dividend-price ratio produces predominantly positive slope coefficients. More specifically, when we use annual dividend and price figures to estimate the regression specification in (2.13), 35 out of 50 countries are found to have a positive $\tilde{\beta}_{1,g}$, with 11 of these cases being statistically significant at the 5% level. By comparison, out of the 15 negative slopes, only 3 are found to be statistically significant at the 5% level (namely for Austria, Finland, and Japan). When we account for intra-year seasonality by estimating the MIDAS specification in (2.15), the resulting slope coefficients of the dividend-price ratio are now universally positive, with 15 out of 50 positive slopes also being statistically significant at the 5% level.

In terms of in-sample predictability, accounting for seasonality by using a mixed frequency regression setting seems to improve the measure of expected dividend growth considerably relative to using annually-aggregated values. In particular, the goodness-of-fit of the MIDAS regressions in (2.15) is substantially higher relative to that of the Cochrane-type regressions in (2.13), with a mean R-square of 55% in the former compared to only 4% in the latter. Based on this difference in predictive power, we would expect the de-trending factor $\lambda = e^{\beta}$ to be more accurate when β is proxied by the slopes from the MIDAS regressions compared to those from the Cochrane-type regressions. Interestingly, regressing stock prices against time also results in a very high in-sample fit, with a mean R-square of 53%. Nevertheless, these R-square values

Table 2.5: Predictive regressions of dividend growth on the dividend yield

Country	n	Shiller-type regressions slope	R^2	Cochrane-type regressions slope	R^2	MIDAS regressions slope	R^2
BGR	17	0.51***	0.50	0.42***	0.14	0.09	0.90
ROU	19	0.64***	0.53	0.21*	0.09	0.07	0.85
RUS	20	0.58***	0.56	0.01	0.00	0.06	0.78
TUN	21	0.57***	0.55	0.13	0.04	0.05	0.74
BRA	22	0.64***	0.56	0.10	0.01	0.05	0.53
CZE	23	0.44***	0.51	0.31**	0.15	0.05	0.96
HUN	23	0.54***	0.52	-0.09*	0.03	0.02	0.55
POL	23	0.48***	0.50	0.06	0.01	0.06	0.88
ISR	24	0.4***	0.57	0.21	0.07	0.08	0.70
EGY	25	0.36***	0.58	0.03	0.02	0.04	0.86
CHN	27	0.41***	0.55	-0.02	0.00	0.07	0.35
IDN	27	0.43***	0.52	-0.25	0.05	0.08	0.75
IRL	27	0.45***	0.52	0.01	0.00	0.05	0.67
PRT	31	0.41***	0.48	-0.05	0.01	0.04**	0.44
COL	32	0.40***	0.58	0.21*	0.04	0.07	0.64
NGA	32	0.49***	0.51	0.40**	0.20	0.05**	0.49
TWM	32	0.40***	0.52	0.13	0.06	0.07	0.72
TUR	32	0.51***	0.52	0.15**	0.10	0.03***	0.44
KEN	34	0.38***	0.45	0.16**	0.06	0.07*	0.58
MAR	34	0.41***	0.59	-0.04	0.03	0.01	0.31
PHL	36	0.34***	0.53	0.02	0.00	0.06	0.45
JOR	39	0.31***	0.52	0.16*	0.06	0.06	0.68
GRC	41	0.26***	0.49	0.13**	0.08	0.05	0.62
THA	42	0.24***	0.53	0.08	0.02	0.06	0.56
CHL	44	0.31***	0.65	0.29**	0.06	0.10***	0.36
MYS	44	0.24***	0.56	0.03	0.00	0.03	0.32
SGP	45	0.25***	0.57	0.05**	0.05	0.02	0.23
NOR	48	0.17***	0.58	0.08*	0.04	0.04	0.60
HKG	53	0.26***	0.60	0.15**	0.06	0.05	0.49
ZAF	54	0.27***	0.55	0.06	0.01	0.03	0.37
KOR	55	0.19***	0.58	0.05*	0.04	0.06***	0.54
FIN	56	0.22***	0.61	-0.09**	0.07	0.02**	0.66
ARG	71	0.26***	0.50	0.01	0.00	0.06	0.45
GBR	84	0.14***	0.55	0.06**	0.04	0.01*	0.16
NZL	91	0.11***	0.49	0.08	0.01	0.06***	0.64
AUT	93	0.10***	0.53	-0.10**	0.04	0.04	0.16
ITA	93	0.12***	0.45	0.15	0.04	0.08***	0.92
IND	97	0.14***	0.52	-0.02	0.00	0.06***	0.65
CHE	99	0.08***	0.61	-0.07	0.02	0.08	0.58
SWE	116	0.05***	0.64	-0.10*	0.04	0.05*	0.62
JPN	117	0.03***	0.33	-0.05**	0.05	0.02*	0.09
ESP	118	0.08***	0.45	-0.02	0.00	0.02*	0.16
NLD	126	0.07***	0.50	0.01	0.00	0.06**	0.58
DNK	144	0.05***	0.45	-0.03	0.01	0.05***	0.52
BEL	147	0.09***	0.43	0.09*	0.02	0.07**	0.74
USA	147	0.06***	0.63	-0.03*	0.02	0.03**	0.25
DEU	148	0.08***	0.20	-0.08	0.02	0.07	0.31
AUS	156	0.08***	0.58	0.00	0.00	0.05**	0.39
FRA	178	0.06***	0.48	-0.02	0.00	0.04**	0.34
CAN	187	0.07***	0.61	0.12**	0.04	0.07*	0.73

Notes: This Table presents the results from predictive least squares regressions of dividend growth. The Table reports the estimated intercept and slope coefficients, and the regressions' R-square. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *, respectively. The first panel reports the results from regressing stock prices against time, as in Shiller (1981). The second panel reports the results from regressing dividend growth against the lagged dividend-price ratio at an annual frequency, (Cochrane 2008). The third panel reports the results from mixed-frequency data sampling (MIDAS) regressions of dividend growth against the lagged dividend-price ratio. The second column tabulates the total number of observations in years (n). Results are tabulated separately for each country, across 50 countries in total. Countries are sorted in ascending order based on the number of years with available data (n).

are not directly comparable to those obtained when estimating the predictive regressions in (2.13) and (2.15), since they refer to regressions with different dependent variables.

Tables 2.6 and 2.7 report the results of variance bounds tests when expected dividend growth is obtained via Cochrane-type regressions and MIDAS regressions, respectively, while Figure 2.6 plots the volatility ratios $\frac{\sigma(p)}{\sigma(p*)}$ under the Shiller, Cochrane and MIDAS approaches to computing the de-trending factor. As can be seen from Table 2.6, when we obtain the de-trending factor via Cochrane-style regressions instead of Shiller-type regressions, our results on the excess volatility puzzle remain largely the same. The mean volatility ratio $\frac{\sigma(p)}{\sigma(p*)}$ across all 50 countries is somewhat lower than the one presented in Table 2.2 (6.0 compared to 7.5), but it is still higher than the value of 5. Furthermore, the magnitude of excess volatility is still found to be negatively related to the length of the available time-series, as countries with the shortest series of available data (less than 30 years) have the highest mean volatility ratio (9.1) and countries with the longest time-series (more than 60 years) have the lowest mean volatility ratio (3.7). The results for the two additional volatility ratios $\frac{\sigma(\Delta\tilde{p})}{\frac{\sigma(\tilde{d})}{\sqrt{2\bar{r}}}}$ and $\frac{\sigma(\Delta\tilde{p}+\tilde{d}_{t-1}+\bar{r}\tilde{d}_{t-1})}{\frac{\sigma(\tilde{d})}{\sqrt{\bar{r}_2}}}$ (Table 2.6, Panels B and C) are qualitatively the same as those reported in Table 2.2.

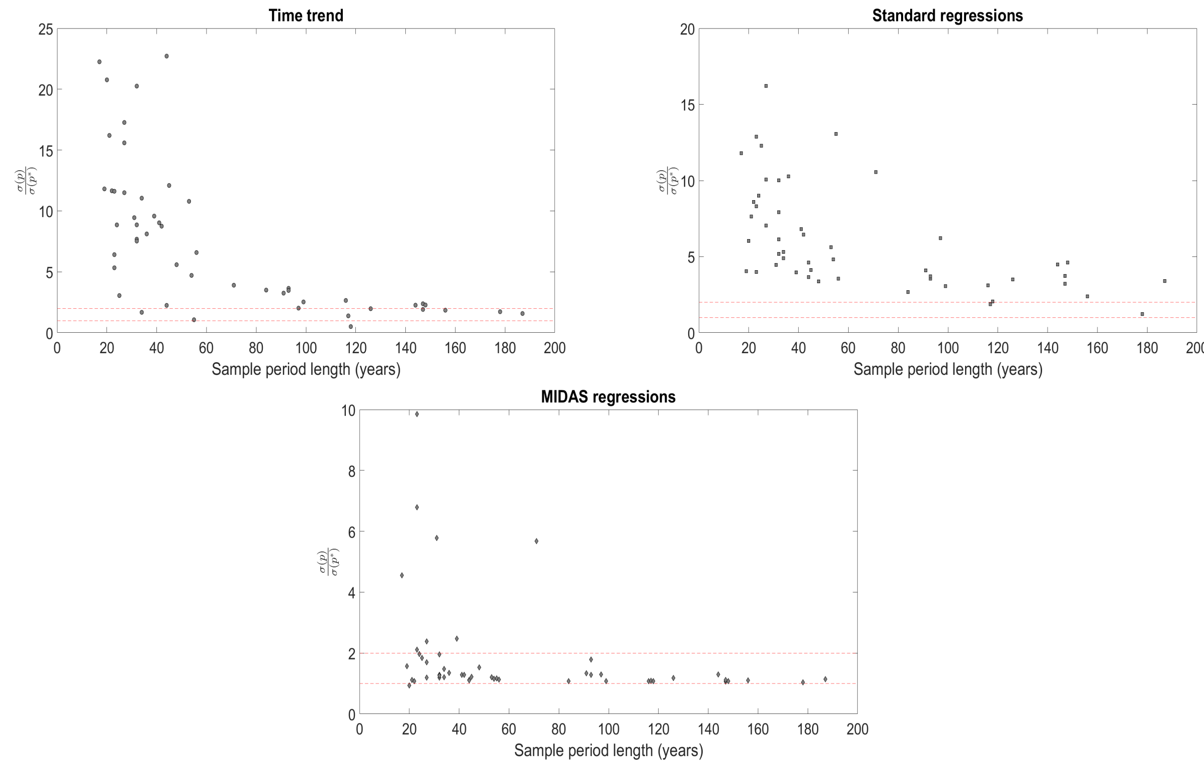


Figure 2.6: Volatility ratios under alternative expected growth measures

Notes: This Figure plots the ratio of the volatility of the realized price against the volatility of the ex-post rational price, across a sample of 50 countries. The upper graph plots the volatility ratios when the expected dividend growth is obtained via regressions of the stock price against time. The bottom-left graph plots the respective volatility ratios when expected dividend growth is obtained via standard regressions of changes in dividend growth against the lagged dividend yield at an annual frequency. The bottom-right graph plots the respective volatility ratios when expected dividend growth is obtained via MIDAS regressions of changes in dividend growth against the lagged dividend yield.

In contrast, the empirical results are noticeably different when expected dividend growth is obtained via the MIDAS regressions in (2.15). Even though the $\sigma(p) \leqslant \sigma(p^*)$ inequality is violated in almost all sample countries (with the only exception of Russia), the volatility ratio $\frac{\sigma(p)}{\sigma(p*)}$ exceeds the value of five in only 4 out of 50 countries (namely Argentina, Czech Republic, Poland and Portugal). By comparison, the volatility ratio exceeded the value of five in 26 and 23 countries under the Shiller and Cochrane frameworks, respectively. Moreover, under the MIDAS-based de-trending factor, the mean volatility ratio is now equal to 1.9 in the full sample, which is lower than the mean ratios under the Shiller and Cochrane approaches (7.5 and 6.0, respectively) but still supports the volatility puzzle. Finally, the magnitude of excess volatility is again found to be negatively related to the length of the available time-series, but the differences among subsamples of different length are less pronounced compared to our previous results.

Table 2.6: Upper bounds tests - Expected growth via Cochrane-type regressions

	Full sample	Subsamples formed on time-series length		
Panel A: $\frac{\sigma(p)}{\sigma(p*)}$				
		$L < 30$	$30 < L < 60$	$60 < L$
mean	6.0	9.1	6.0	3.7
median	4.7	8.6	5.2	3.5
min	1.2	4.0	3.4	1.2
max	16.2	16.2	13.1	10.6
$n > 5$	23	11	10	2
n	50	13	18	19
Panel B: $\frac{\sigma(\Delta\tilde{p})}{\frac{\sigma(\tilde{d})}{\sqrt{2\bar{r}}}}$				
	Full sample	Subsamples formed on time-series length		
		$L < 30$	$30 < L < 60$	$60 < L$
mean	2.8	2.8	3.5	2.0
median	2.2	2.6	2.6	1.8
min	1.0	1.5	1.3	1.0
max	12.6	6.5	12.6	4.7
$n > 5$	3	1	2	0
n	50	13	18	19
Panel C: $\frac{\sigma(\Delta\tilde{p}+\tilde{\tilde{d}}_{t-1}+\bar{r}\tilde{\tilde{d}}_{t-1})}{\frac{\sigma(\tilde{d})}{\sqrt{\bar{r_2}}}}$				
	Full sample	Subsamples formed on time-series length		
		$L < 30$	$30 < L < 60$	$60 < L$
mean	2.7	2.8	3.4	2.0
median	2.2	2.5	2.6	1.8
min	0.9	1.4	1.3	0.9
max	12.4	6.4	12.4	4.7
$n > 5$	3	1	2	0
n	50	13	18	19

Notes: This Table presents the results of variance bounds tests when expected dividend growth is obtained via Cochrane-type regressions of changes in dividend growth against the lagged dividend-price ratio at an annual frequency. Each volatility ratio is computed separately for each sample country, across 50 countries in total. The Table reports the mean, median, minimum, and maximum values of each ratio across the 50 sample countries, the number of cases where the ratio exceeds the value of 5:1 reported in Shiller (1981), and the number of countries n. The first column of each panel reports the results across the full sample of 50 countries, while the last three columns report the results from subsamples that have been formed based on the countries' number of years of available data L.

Table 2.7: Upper bounds tests - Expected growth via MIDAS regressions

	Panel A: $\frac{\sigma(p)}{\sigma(p*)}$			
	Full sample	Subsamples formed on time-series length		
		$L < 30$	$30 < L < 60$	$60 < L$
mean	1.9	2.9	1.6	1.4
median	1.2	1.8	1.3	1.1
min	0.9	0.9	1.1	1.0
max	9.8	9.8	5.8	5.7
$n > 5$	4	2	1	1
n	50	13	18	19

	Panel B: $\frac{\sigma(\Delta\tilde{p})}{\frac{\sigma(\tilde{d})}{\sqrt{2\bar{r}}}}$			
	Full sample	Subsamples formed on time-series length		
		$L < 30$	$30 < L < 60$	$60 < L$
mean	1.4	1.2	2.0	0.8
median	1.0	1.0	1.1	0.7
min	0.4	0.6	0.5	0.4
max	8.0	2.4	8.0	1.5
$n > 5$	2	0	2	0
n	50	13	18	19

	Panel C: $\frac{\sigma(\Delta\tilde{p}+\tilde{d}_{t-1}+\bar{r}\tilde{d}_{t-1})}{\frac{\sigma(\tilde{d})}{\sqrt{\bar{r}_2}}}$			
	Full sample	Subsamples formed on time-series length		
		$L < 30$	$30 < L < 60$	$60 < L$
mean	1.0	1.0	1.5	0.6
median	0.8	0.9	1.0	0.5
min	0.2	0.4	0.4	0.2
max	6.6	1.8	6.6	1.4
$n > 5$	1	0	1	0
n	50	13	18	19

Notes: This Table presents the results of variance bounds tests when expected dividend growth is obtained via MIDAS regressions of changes in dividend growth against the lagged dividend-price ratio. Each volatility ratio is computed separately for each sample country, across 50 countries in total. The Table reports the mean, median, minimum, and maximum values of each ratio across the 50 sample countries, the number of cases where the ratio exceeds the value of 5:1 reported in Shiller (1981), and the number of countries n. The first column of each panel reports the results across the full sample of 50 countries, while the last three columns report the results from subsamples that have been formed based on the countries' number of years of available data L.

2.6 Variance bounds tests and stationarity of dividends

The theoretical variance inequality in (2.10) was proposed by Shiller (1981) based on the assumption that dividends follow a stationary process. Consequently, if dividends deviate from stationarity, then observing that the variance of the realized price exceeds the variance of the ex-post rational price does not necessarily imply a violation of the efficient market hypothesis. Rather, such violations of the upper variance bound could be driven by dividends simply following a non-stationary process.

In order to account for the common empirical finding of dividends deviating from stationarity, Mankiw et al. (1985) and West (1988*b*) propose alternative variance bounds tests to evaluate stock market efficiency. More specifically, West (1988*b*) shows that, under relatively weak assumptions including potential non-stationarity in dividends, the variance of innovations in the stock price must be lower than the variance of innovations in the corresponding dividend.[12] Engel (2005) further extends the analysis deriving a new variance bound on the first difference of stock prices, under the assumption that dividends are stationary or that they follow a unit-root process.[13] Working with prices expressed in first differences, Engel (2005) argues that the excess volatility inequality (2.10) is reversed to

$$var(\Delta\tilde{p}_t) \geqslant var(\Delta\tilde{p}_t^*) \tag{2.19}$$

where $\Delta\tilde{p}_t = \tilde{p}_t - \tilde{p}_{t-1}$ and $\Delta\tilde{p}_t^* = \tilde{p}_t^* - \tilde{p}_{t-1}^*$.

In order to understand whether the previous results of the variance bounds tests are driven by persistence in dividends, we perform a number of stationarity and unit-root tests on our main variables of interest. More specifically, we run the Augmented Dickey-Fuller (ADF) test, the Phillips-Perron (PP) test, and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test, with the results reported in Table 2.8. Our results are consistent with those reported in previous studies about dividends deviating from stationarity, with the KPSS test rejecting the null hypothesis of dividends following an $I(0)$ process across all 50 countries at the 10% level, while the null of stationarity is rejected at the 5% level in 34 countries. Moreover, the results

[12] West (1988*b*) computes innovations, i.e. unexpected changes, in the stock price and the corresponding dividend series as the residuals from fitting ARIMA (q, s, r) specifications on these series. In addition to relaxing the assumption of stationarity of dividends, the variance inequality proposed in West (1988*b*) also attempts to address the issue of small sample bias that has been attributed to the Shiller (1981) variance bounds test (see also Flavin 1983, Kleidon 1986, Marsh & Merton 1986).

[13] Lansing (2016) expands the analysis further by deriving alternative variance bounds for changes in stock prices by allowing for various degrees of information available to investors about future dividends. The framework proposed by Lansing (2016) allows for ex-dividend stock prices and risk-averse investors, in contrast to cum-dividend stock prices and risk-neutral investors that are assumed under the Engel (2005) framework.

of the ADF test suggest that dividends generally follow an $I(1)$ process, with the null of a unit root being rejected at the 5% level in only 4 countries (Chile, Malaysia, Italy, and Canada). The PP test results are very similar, with the null of a unit root being rejected at the 5% level in only 5 out of 50 countries (Taiwan, New Zealand, Italy, Australia, and Canada).

Since dividends seem to follow a unit root process in the majority of our sample countries, we proceed by evaluating the Engel (2005) bounds test on the variance of first differences in prices, given in inequality (2.19). Table 2.9 reports a set of summary statistics of the Engel (2005) volatility ratio $\frac{\sigma(\tilde{p}_t - \tilde{p}_{t-1})}{\sigma(\tilde{p}^*_t - \tilde{p}^*_{t-1})}$, computed separately under each of the three previously discussed approaches of obtaining a measure of expected dividend growth. When we focus on the relationship between the variance of first differences in realized and ex-post rational prices, as opposed to the variance of price levels, the empirical evidence in fact strongly supports the hypothesis of market efficiency. Irrespective of the specific measure of expected dividend growth used, the volatility ratio exceeds unity across all sample countries (with the single exception of the Czech Republic, where the volatility ratio takes the fairly borderline value of 0.97 when expected dividend growth is computed via MIDAS regressions). In other words, the volatility of first differences in realized prices consistently exceeds that of first differences in ex-post rational prices, in line with the reversed inequality in Engel (2005) and, ultimately, in support of market efficiency. Interestingly, the volatility ratios that are based on first differences in prices are substantially higher than the corresponding ratios that are based on price levels. For example, the mean ratio $\frac{\sigma(\tilde{p}_t - \tilde{p}_{t-1})}{\sigma(\tilde{p}^*_t - \tilde{p}^*_{t-1})}$, is equal to 25.1, compared to a mean ratio $\frac{\sigma(\tilde{p}_t)}{\sigma(\tilde{p}^*_t)}$ of 7.5 (under Shiller-type regressions for expected dividend growth). Furthermore, the specific approach used to obtain expected dividend growth has a substantial impact on the magnitude of the volatility ratio, with Shiller-type regressions resulting in the highest volatility ratios and MIDAS regressions resulting in the lowest one (mean ratios are 25.1 and 2.8, respectively). Finally, the relationship between the magnitude of the volatility ratio and the length of the available dataset is not as clear as the strong negative relationship that was observed in our previous analysis of the standard Shiller (1981) volatility ratio. For instance, when we estimate the Engel (2005) volatility ratio, it is the middle group of countries with dataset lengths between 30 and 60 years that is found to have the highest mean volatility ratio (under the Shiller- and MIDAS-based measures of expected dividend growth), rather than the group of countries with the shortest datasets as was the case with the Shiller (1981) volatility ratio.

Clearly, assumptions about the dividend process directly affect how the respective variance bounds test is formulated and, by extension, they will have a substantial impact on whether the test is likely to support or reject the hypothesis of stock market rationality. In the case of stationary dividends, Shiller (1981) demonstrates that the variance of dividends provides an *upper* bound for the variance of stock prices and that of stock price innovations, described in inequalities (2.11) and (2.12), respectively. However, when

Table 2.8: Unit root and stationarity tests

Country	n	$\tilde{d}_t$ ADF test $H_0 : I(1)$	$\tilde{d}_t$ PP test $H_0 : I(1)$	$\tilde{d}_t$ KPSS $H_0 : I(0)$	$\tilde{p}_t$ ADF test $H_0 : I(1)$	$\tilde{p}_t$ PP test $H_0 : I(1)$	$\tilde{p}_t$ KPSS $H_0 : I(0)$
BGR	17	-1.881	-3.016	0.091*	-2.537	-2.089	0.104*
ROU	19	-2.419	-3.441*	0.075*	-2.34	-2.086	0.112*
RUS	20	-2.384	-2.332	0.147**	-1.541	-2.402	0.185**
TUN	21	-3.611*	-2.865	0.053*	-1.504	-2.000	0.098*
BRA	22	-1.413	-1.851	0.206**	-1.325	-1.997	0.173**
CZE	23	-1.617	-2.911	0.147**	-2.224	-2.283	0.100*
HUN	23	-1.588	-1.351	0.193**	-3.005	-3.692**	0.125*
POL	23	-2.275	-1.676	0.107*	-2.597	-3.359*	0.072*
ISR	24	-0.209	-1.074	0.197**	-2.859	-4.947***	0.086*
EGY	25	-2.364	-2.499	0.096*	-2.737	-2.501	0.093*
CHN	27	-2.807	-3.502*	0.077*	-2.465	-4.745***	0.132*
IDN	27	-3.081	-3.579*	0.075*	-1.267	-2.152	0.200**
IRL	27	-2.236	-1.831	0.154**	-2.534	-2.141	0.151**
PRT	31	-2.835	-2.341	0.145*	-2.460	-2.570	0.140*
COL	32	-2.784	-2.744	0.106*	-2.479	-2.487	0.075*
NGA	32	-3.525*	-2.482	0.181**	-1.259	-2.180	0.208**
TUR	32	-1.752	-1.431	0.222***	-3.571**	-6.787***	0.076*
TWN	32	-2.880	-4.195**	0.106*	-6.568***	-4.708***	0.092*
KEN	34	-2.761	-2.832	0.196**	-2.819	-2.610	0.146*
MAR	34	-2.426	-2.524	0.159**	-2.082	-1.600	0.219***
PHL	36	-2.238	-2.760	0.289***	-2.335	-2.411	0.125*
JOR	39	-1.712	-2.154	0.267***	-2.556	-2.036	0.148**
GRC	41	-1.851	-1.921	0.187**	-2.086	-1.980	0.186**
THA	42	-2.380	-2.111	0.136*	-2.517	-2.252	0.112*
CHL	44	-3.607**	-3.128	0.222***	-3.518*	-2.149	0.222***
MYS	44	-3.692**	-2.816	0.055*	-2.581	-2.967	0.158**
SGP	45	-2.500	-2.129	0.141*	-1.979	-4.226***	0.232***
NOR	48	-1.689	-1.975	0.387***	-2.885	-2.981	0.183**
HKG	53	-1.630	-3.482*	0.289***	-3.294*	-4.176***	0.147**
ZAF	54	-1.876	-2.180	0.326***	-2.508	-3.029	0.305***
KOR	55	-1.626	-1.483	0.208**	-3.465*	-3.033	0.101*
FIN	56	-2.232	-2.003	0.323***	-2.772	-2.634	0.180**
ARG	71	-2.038	-2.514	0.424***	-1.280	-1.553	0.564***
GBR	84	-2.252	-2.345	0.251***	-2.731	-2.787	0.284***
NZL	91	-2.925	-3.937**	0.085*	-3.539**	-3.817**	0.048*
AUT	93	-3.235*	-2.519	0.297***	-3.057	-3.029	0.307***
ITA	93	-3.595**	-3.511**	0.138*	-2.683	-2.731	0.207**
IND	97	-1.617	-2.185	0.425***	-1.180	-1.370	0.703***
CHE	99	-2.019	-2.479	0.371***	-2.594	-2.830	0.236***
SWE	116	-1.028	-0.978	0.814***	-1.349	-1.450	0.805***
JPN	117	-1.930	-1.668	0.563***	-2.036	-1.925	0.577***
ESP	118	-2.824	-2.227	0.495***	-3.087	-2.609	0.377***
NLD	126	-1.712	-1.753	0.84***	-1.737	-1.638	0.775***
DNK	144	-0.728	-1.12	0.456***	0.357	-0.014	0.898***
BEL	147	-2.952	-2.779	0.808***	-1.382	-1.529	0.932***
USA	147	-2.774	-3.105	0.269***	-1.883	-2.217	0.569***
DEU	148	-1.255	-1.177	0.789***	-1.321	-1.167	0.773***
AUS	156	-3.284*	-3.647**	0.425***	-3.289*	-3.783**	0.263***
FRA	178	-2.644	-2.06	0.565***	-2.691	-2.634	0.537***
CAN	187	-4.628***	-4.806***	0.341***	-4.610***	-6.430	0.163**

Notes: This Table presents the results from a set of unit root and stationarity tests on the time-series of dividends $\tilde{d}_t$ and stock prices $\tilde{p}_t$. The Table reports the test statistics computed in the Augmented Dickey Fuller (ADF) test, the Phillips-Perron (PP) test, and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test. The null hypothesis in the ADF test and the PP test is that the respective time-series contains a unit root, while the null hypothesis in the KPSS test is that the time-series is stationary. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *, respectively. The second column tabulates the total number of observations in years (n). Results are tabulated separately for each country, across 50 countries in total. Countries are sorted in ascending order based on the number of years with available data (n).

Table 2.9: Variance bounds tests - Engel volatility ratio

	Full sample	Subsamples formed on time-series length		
Panel A: Expected growth via Shiller-type regressions				
		$L < 30$	$30 < L < 60$	$60 < L$
mean	25.1	31.3	32.6	12.6
median	20.4	29.1	29.5	11.9
min	6.1	10.4	6.3	6.1
max	78.4	63.6	78.4	21.7
$n > 1$	50	13	18	19
n	50	13	18	19
Panel B: Expected growth via Cochrane-type regressions				
	Full sample	Subsamples formed on time-series length		
		$L < 30$	$30 < L < 60$	$60 < L$
mean	14.1	15.3	15.0	12.4
median	10.5	10.5	11.6	9.2
min	4.7	4.7	5.5	4.7
max	60.2	32.9	58.1	60.2
$n > 1$	50	13	18	19
n	50	13	18	19
Panel C: Expected growth via MIDAS regressions				
	Full sample	Subsamples formed on time-series length		
		$L < 30$	$30 < L < 60$	$60 < L$
mean	2.8	2.3	3.6	2.2
median	2.2	2.4	2.8	1.7
min	1.0	1.0	1.2	1.0
max	14.5	4.4	14.5	9.0
$n > 1$	49	12	18	19
n	50	13	18	19

Notes: This Table presents the results of computing the Engel (2005) volatility ratio $\frac{\sigma(\Delta\tilde{p}_t)}{\sigma(\Delta\tilde{p}_t^*)}$. Panel A reports the results when expected dividend growth is obtained via Shiller-type regressions of price against time. Panel B reports the results when expected dividend growth is obtained via Cochrane-type regressions of changes in dividend growth against the lagged dividend-price ratio, while Panel C reports the results when expected dividend growth is obtained via MIDAS regressions of changes in dividend growth against the lagged dividend-price ratio. Each volatility ratio is computed separately for each sample country, across 50 countries in total. The Table reports the mean, median, minimum, and maximum values of each ratio across the 50 sample countries, the number of cases where the ratio exceeds the value of 1, and the number of countries n. The first column of each panel reports the results across the full sample of 50 countries, while the last three columns report the results from subsamples that have been formed based on the countries' number of years of available data L.

dividends are non-stationary, the same variance of dividends results in a *lower* bound for the variance of stock price changes, described in inequality (2.19).

Overall, our empirical results are representative of the debate in the literature about the sensitivity of market efficiency tests to the underlying assumptions about the dividend process. Under the assumption of dividend stationarity, the "first generation" variance bounds tests proposed by Shiller (1981) consistently reject the hypothesis of market efficiency across 50 international stock markets, independent of the particular framework used to produce expectations about future dividend growth. However, allowing for non-stationary dividends, the "second generation" test proposed by Engel (2005) in fact provides support for the hypothesis of efficient markets where stock price changes are driven by investors' rational expectations about future dividends. In the end, any conclusion on the question of stock market efficiency will ultimately depend on one's views about the likelihood of dividends following a stationary process. While a comprehensive analysis of the most appropriate way to model the dividend process remains outside the scope of this chapter, our empirical evidence of dividends' significant deviations from stationarity in the majority of sample countries appear to cast some doubt on the common rejection of market efficiency when using the original upper variance bounds test proposed by Shiller (1981).

2.7 Trading strategy

Our empirical results thus far are consistent with significant violations of the conventional variance bounds across multiple international markets. I now proceed to investigate whether this excess volatility of realized prices relative to expected prices constitutes evidence to reject the hypothesis of weak form rationality in terms of allowing for exploitable profit opportunities.

Bulkley & Tonks (1992) argue that some part of realized prices' excess volatility can be attributed to revisions in the parameters of the model that market participants use for the dividend process. In this setting, investors are rational and they use unbiased techniques when forming their expectations about future dividend growth. However, the fact that investors dynamically revise the parameters of their unbiased model will result in some excess volatility of realized prices relative to that of expected prices based on future disocunted dividends. Motivated by this argument, we explore whether the relatively high volatility of p could potentially be attributed to market participants revising the parameters of their unbiased structural model for dividends. To this end, we construct a simple "buy low - sell high" trading strategy that uses currently available data to set the parameters of the trading rule.[14]

[14]Shiller (1981), Ackley (1983) and Bulkley & Tonks (1992) discuss cases where large excess volatility of stock returns would suggest the existence of trading rules that may dominate the standard buy-and-hold

Our strategy is based on the simple trading rule adopted in Bulkley & Tonks (1992). More specifically, we consider an investor who can choose to invest either in the index or in a risk-free bond. At any point in time t, the investor will decide how to invest by comparing the actual index price p against the perfect-foresight price p^*. If the realized price of the index is at least $\Delta\%$ higher compared to the expected price, then the investor will choose to sell the index and buy the risk-free bond. The bond is then held until the realized price falls at least $\Delta\%$ below the expected price, at which point the investor will buy back into the index. The investor receives dividend payments when holding the index and the 1-year risk-free rate when holding the bond, reinvesting all returns.

Under this simple trading rule, the investor will sell overpriced shares when their price exceeds the present value of future dividends (i.e when $p > p^* \times (1 + \Delta)$) and buy them back when they become underpriced in terms of their price falling below the present value of expected dividends (i.e. when $p < p^* \times (1-\Delta)$). If markets are efficient, then deviations of realized prices from their expected values would not be large enough for this trading rule to dominate a simple buy-and-hold strategy, especially considering the unavoidable model error around expected prices and the low rate of return of the risk-free asset (consistent with the well-documented equity premium puzzle discussed in Mehra 2003). Bulkley & Tonks (1992) provide a comprehensive discussion of the rationale behind this trading rule.

The investor starts with an initial wealth of \$100 across any sample country, and the decision to switch in or out of the index is taken on December 31st of each year. I adopt a dynamic threshold Δ_t for the investor's choice to switch between the index and the risk-free bond. In particular, at each time t we select a value for Δ_t that would have ex post maximized the investor's profits from the trading strategy during the period $(0, t-1)$.

Table 2.10 presents the performance of this trading strategy against that of the simple buy-and-hold benchmark.[15] The Table reports a set of measures that reflect each strategy's performance, separately for each of the 50 sample countries. More specifically, we report each strategy's annual volatility, mean annual return, Sharpe Ratio, Treynor Ratio, Sortino Ratio, and the 5% Value-at-Risk (computed parametrically). In addition to these measures, we report the p-value from the Linton et al. (2005) second order stochastic dominance (SSD) test of the trading strategy against the buy-and-hold benchmark. The null hypothesis of the SSD test is that the trading strategy TS stochastically dominates the buy-and-hold BH (i.e. $H_0 : TS \succ_2 BH$), which represents the case where every risk-averse investor would prefer TS over BH, irrespective of the

strategy.

[15]Expected prices p^* are computed via the MIDAS regressions approach. For robustness, we have replicated this exercise with expected prices that have been obtained via Shiller-type and Cochrane-type regressions. The results are qualitatively similar, and thus unreported for brevity, but they are available upon request.

specifics of their risk preferences. Therefore, a rejection of the null hypothesis would constitute evidence in favor of market efficiency, since the trading strategy based on excess volatility would not lead to a performance that is universally preferred by every investor, with the ordering of TS relative to BH being instead dependent on individual investors' particular utility functions.

As can be seen from Table 2.10, the buy-and-hold benchmark offers annual returns that are on average higher (46 out of 50 countries) and have a higher level of volatility (all 50 countries) relative to the trading strategy. This finding is unsurprising, since the trading strategy involves some periods where the investor switches from risky stocks to holding risk-free bonds that offer lower returns. In terms of risk-adjusted performance measures, which would be more meaningful when comparing the two strategies, the results are somewhat mixed. For instance, the trading strategy offers a Sharpe Ratio that exceeds that of the benchmark in 20 out of 50 countries, suggesting that the trade-off between mean returns and volatility tends to generally be better under the simple buy-and-hold strategy. Performance is more balanced between the trading strategy and the benchmark when measured in terms of the Treynor Ratio and the Sortino Ratio, with the trading strategy outperforming the buy-and-hold in 22 and 24 countries, respectively. Finally, the trading strategy loads consistently less on downside risk, as evidenced by a lower VaR relative to the benchmark across all 50 countries. Similarly to the lower levels of volatility, a lower VaR is also to be expected given that the trading strategy substitutes bonds for riskier stocks during some periods.

Importantly, the SSD test fails to reject the null of the trading strategy dominating the buy-and-hold benchmark in almost every sample country (with the only exception of Argentina). These results suggest that any risk-averse investor in almost every country would prefer trading based on the difference between realized prices and expected prices, compared to simply buying and holding the index, irrespective of their specific utility function. In other words, the excess volatility of realized prices consistently results in profitable investment opportunities that dominate the buy-and-hold strategy. Therefore, investors revising the parameters of their unbiased model for dividends seems to be an unlikely explanation for the magnitude of excess volatility that is observed across multiple countries, given the extent of the trading strategy's stochastic dominance over the benchmark. Overall, while risk-adjusted performance measures provide relatively mixed results, the SSD test provides strong evidence against the hypothesis of efficient markets, consistent with the arguments in Shiller (1981) and Bulkley & Tonks (1992).

Table 2.10: Trading strategy performance

Country	Volatility		Return		Sharpe Ratio		Treynor Ratio		Sortino Ratio		VaR		SSD Test
	TS	BH	TS	BH	TS	BH	TS	BH	TS	BH	TS	BH	TS$>_2$BH
ARG	1.41	6.72	0.34	1.82	0.23	0.47	0.24	0.49	5.37	9.32	-0.37	-0.83	0.00
AUS	0.09	0.14	0.05	0.05	-0.05	-0.08	-0.05	-0.08	-0.59	-0.79	-0.09	-0.18	0.78
AUT	0.02	0.32	0.06	0.09	-0.07	-0.04	-0.15	-0.04	-0.22	-0.35	0.03	-0.38	0.77
BEL	0.06	0.22	0.06	0.05	0.05	-0.09	0.05	-0.09	0.61	-1.02	-0.04	-0.29	0.79
BRA	0.08	0.34	0.18	0.21	-0.09	-0.05	-0.11	-0.05	-0.17	-0.30	0.06	-0.31	0.73
BGR	0.18	0.44	0.09	0.23	0.22	0.13	0.23	0.13	2.84	0.20	-0.14	-0.76	0.51
CAN	0.11	0.17	0.05	0.05	-0.08	-0.10	-0.08	-0.10	-0.95	-1.12	-0.14	-0.23	0.79
CHL	0.01	0.20	0.06	0.11	-0.05	0.20	-0.05	0.20	-0.03	0.64	0.04	-0.21	0.42
CHN	0.01	0.35	0.04	0.08	-0.29	-0.05	-0.18	-0.05	-0.12	-0.10	0.03	-0.57	0.62
COL	0.03	0.45	0.09	0.20	0.06	0.15	0.08	0.16	0.09	0.84	0.04	-0.40	0.37
CZE	0.02	0.25	0.03	0.09	0.12	0.10	0.24	0.10	0.28	0.31	0.01	-0.35	0.59
DNK	0.08	0.19	0.04	0.05	-0.28	-0.14	-0.31	-0.14	-3.52	-2.02	-0.09	-0.23	0.77
EGY	0.01	0.64	0.16	0.32	-1.00	0.06	-0.57	0.07	-0.11	0.11	0.13	-0.62	0.47
FIN	0.07	0.35	0.09	0.14	0.09	0.04	0.09	0.04	0.54	0.25	-0.03	-0.39	0.62
FRA	0.09	0.21	0.05	0.07	0.00	0.02	0.00	0.02	-0.05	0.27	-0.08	-0.25	0.53
DEU	0.04	0.48	0.05	0.08	-0.10	-0.08	-0.11	-0.08	-0.69	-0.74	-0.02	-0.53	0.77
GRC	0.09	0.42	0.14	0.14	-0.05	-0.19	-0.06	-0.18	-0.23	-0.82	0.00	-0.57	0.72
HKG	0.03	0.27	0.04	0.08	0.06	0.02	0.11	0.02	0.20	0.06	0.00	-0.40	0.69
HUN	0.02	0.29	0.07	0.11	0.06	0.02	0.08	0.01	0.06	0.06	0.04	-0.39	0.64
IND	0.05	0.27	0.06	0.09	-0.13	-0.03	-0.14	-0.03	-0.86	-0.27	-0.03	-0.31	0.72
IDN	0.02	0.14	0.08	0.13	0.09	0.37	0.06	0.35	0.04	0.68	0.05	-0.10	0.31
IRL	0.03	0.25	0.05	0.10	0.07	0.06	0.10	0.06	0.16	0.14	0.01	-0.40	0.51
ISR	0.02	0.27	0.05	0.11	-0.01	0.14	-0.01	0.14	-0.02	0.63	0.02	-0.30	0.37
ITA	0.07	0.35	0.06	0.11	-0.20	-0.02	-0.24	-0.02	-1.51	-0.19	-0.05	-0.38	0.51
JPN	0.18	0.30	0.05	0.07	-0.10	-0.09	-0.10	-0.09	-0.88	-0.73	-0.25	-0.48	0.73
JOR	0.02	0.27	0.06	0.08	-0.12	-0.06	-0.21	-0.06	-0.17	-0.43	0.03	-0.33	0.78
KEN	0.03	0.25	0.13	0.05	-0.46	-0.46	-0.31	-0.43	-0.22	-1.03	0.09	-0.40	0.63
KOR	0.09	0.32	0.07	0.15	-0.20	0.07	-0.20	0.07	-1.44	0.63	-0.08	-0.32	0.26
MYS	0.21	0.28	0.08	0.10	0.00	0.00	0.00	0.00	0.01	0.02	-0.25	-0.36	0.72
MAR	0.01	0.21	0.05	0.08	0.09	0.09	0.19	0.09	0.08	0.28	0.03	-0.26	0.45
NLD	0.05	0.19	0.04	0.05	-0.15	-0.09	-0.15	-0.08	-1.41	-0.76	-0.04	-0.26	0.77
NZL	0.08	0.22	0.06	0.06	-0.09	-0.09	-0.10	-0.09	-0.81	-0.84	-0.06	-0.26	0.75
NGA	0.15	0.35	0.20	0.25	0.28	0.10	0.31	0.10	1.45	0.39	-0.02	-0.36	0.71
NOR	0.04	0.30	0.07	0.11	-0.06	0.00	-0.19	0.00	-0.24	-0.02	0.01	-0.39	0.74
PHL	0.05	0.28	0.10	0.08	-0.04	-0.18	-0.07	-0.17	-0.11	-0.71	0.01	-0.41	0.70
POL	0.09	0.31	0.10	0.11	0.05	-0.09	0.14	-0.09	0.35	-0.38	-0.03	-0.41	0.67
PRT	0.05	0.31	0.08	0.06	0.01	-0.18	0.01	-0.17	0.03	-0.83	0.00	-0.44	0.74
ROU	0.06	0.54	0.09	0.27	0.18	0.17	0.34	0.17	0.47	0.26	0.00	-0.62	0.49
RUS	0.08	1.05	0.11	0.45	-0.09	0.19	-0.07	0.19	-0.27	1.07	0.00	-0.67	0.25
SGP	0.01	0.27	0.03	0.08	0.09	0.09	0.11	0.09	0.11	0.25	0.01	-0.37	0.52
ZAF	0.14	0.23	0.13	0.15	0.07	0.04	0.07	0.04	0.57	0.28	-0.08	-0.21	0.81
ESP	0.04	0.22	0.06	0.06	-0.10	-0.14	-0.18	-0.14	-0.70	-1.41	0.00	-0.29	0.77
SWE	0.12	0.23	0.06	0.09	0.01	0.03	0.01	0.03	0.06	0.24	-0.13	-0.28	0.54
CHE	0.06	0.20	0.03	0.06	-0.20	0.00	-0.20	0.00	-1.25	0.03	-0.09	-0.28	0.35
TWN	0.02	0.29	0.03	0.06	0.10	-0.01	0.30	-0.01	0.47	-0.03	0.00	-0.43	0.68
THA	0.25	0.32	0.12	0.12	0.09	-0.02	0.09	-0.02	0.50	-0.11	-0.25	-0.42	0.76
TUN	0.01	0.20	0.07	0.12	-0.06	0.20	-0.10	0.20	-0.03	1.53	0.05	-0.17	0.23
TUR	0.19	1.68	0.38	0.84	-0.39	-0.01	-0.73	-0.01	-1.04	-0.14	0.08	-0.72	0.49
GBR	0.09	0.17	0.07	0.07	0.03	-0.03	0.03	-0.03	0.27	-0.19	-0.07	-0.21	0.80
USA	0.15	0.17	0.05	0.06	-0.03	0.00	-0.03	0.00	-0.25	-0.04	-0.20	-0.24	0.54

Notes: This Table reports the performance of a trading strategy that is based on the difference between realized prices p and and expected prices p^*, with the latter estimated via MIDAS regressions of dividend growth against the lagged dividend-price ratio. The performance of the trading strategy TS is compared against that of the simple buy-and-hold benchmark BH. The Table reports several measures of performance for each strategy, namely annual Volatility (in %), mean annual Return (in %), Sharpe Ratio, Treynor Ratio, Sortino Ratio, and 5% Value-at-Risk (VaR). The last column of the Table reports the p-value of the Linton et al. (2005) second order stochastic dominance (SSD) test. The null hypothesis of the SSD test is that the trading strategy stochastically dominates the buy-and-hold benchmark (i.e. $H_0 : TS >_2 BH$). The results are reported separately for each sample country, across 50 countries in total.

2.8 Chapter Summary

The simple notion that rational investors determine current stock prices as the sum of future expected dividends has attracted a lot of attention in the empirical literature. The seminal Shiller (1981) study, in particular, has sparked a lively debate on whether the higher volatility of realized prices relative to that of prices computed as the sum of discounted future dividends can be used as evidence to reject the hypothesis of investor rationality. Given that the previous literature has focused almost exclusively on the US, the current article provides the first study of this *excess volatility puzzle* in an international context.

I confirm that the commonly reported finding in the US of realized price volatility exceeding the volatility of ex-post rational prices also applies to a large international sample of 50 countries. Although excess volatility in realized prices is confirmed across almost every sample country, its magnitude appears to be substantially higher in developing countries compared to their developed counterparts. However, we present evidence that this difference is most likely driven by the length of available data in each country rather than reflecting a fundamental relationship between excess volatility and a country's state of economic development.

Another important finding refers to the sensitivity of variance bounds test results to the specification used in order to obtain expectations of future dividends. For instance, using mixed-frequency regressions to obtain a measure of expected dividend growth results in substantially less pronounced excess volatility on average compared to that observed when using the standard Shiller (1981) approach of obtaining a price trend via regressions against time. Nevertheless, even though the *magnitude* of the effect varies, all measures of ex-post rational prices consistently result in excess volatility across the vast majority of sample countries.

Finally, our results provide further empirical support for the argument that evaluating stock market rationality is heavily dependent on the test's specific assumptions about the dividend process. Similarly to previous findings from the US market, we document that variance bounds tests are characterized by the same challenges and nuances with respect to dividend stationarity when applied to other countries as well. Consequently, inferences about stock market rationality will ultimately depend on the assumptions that one is willing to make about the underlying dividend process. A promising line of further research would be to consider replacing variance tests with model-free or model-based orthogonality tests that have higher power than the benchmark returns test and circumvent the nuisance parameter problem as described by LeRoy & Steigerwald (1995).

Appendices

APPENDIX A

Additional Material for Chapter 2

A.1 Variance Bounds Determination

This section begins with a replication of the Shiller (1981) simple efficient markets model. Within this model, earnings serves as an indicator for future dividends see (see also Shiller 2015).

A.1.1 The volatility puzzle

A widely held viewpoint concerns the fact that; $p_t = E_t(p_t^*)$ where p_t^* is optimally forecast such that it provides a representation of all available information at time t. Suppose an observable forecast error ε_t can be defined as;

$$\varepsilon_t = p_t^* - p_t$$

where $cov(\varepsilon_t, p_t) = 0$ indicates no correlation between forecasted values and forecast error, then the unconditional variance of the p^*; $var(p^*) = var(\varepsilon) + var(p)$ for which $var(p) \leqslant var(p^*)$ since we strictly expect $var(\varepsilon) \geqslant 0$.

$$\sigma(p) \leqslant \sigma(p^*) \tag{A.1}$$

Relating to inequality (A.1), the theory explores varied related theoretical questions as well as other inequalities which impose a lower and an upper bound on $\sigma(\delta_t)$ and $\sigma(\Delta p)$. With respect to $\sigma(p)$, it is argued that $\sigma(p)$ attains its highest with smooth dividend information announcement and dips to its minimum otherwise; the series also attains high kurtosis with irregular dividend information.

A.1.2 A limit on the innovations and the first difference in price

Here, the thesis closely follow Shiller (1981) and outline the theoretical framework of the volatility puzzle relation. I discuss how the first-generation formulation differ from other versions suggest by Kleidon (1986), West (1988*b*), LeRoy & Parke (1992) and Engel (2005). In what immediately follows, the study presents a breakdown of the variance bound theory in steps i.e.

Step 1a (An expression for P_t): Shiller (1981)'s simple efficient market model estimates real share price P_t using the relation;

$$P_t = \sum_{k=0}^{\infty} \gamma^{k+1} E_t D_{t+k} \tag{A.2}$$

where $0 < \gamma < 1$, (refer to table 1.1 for the definition of terms). Information at time t generally include P_t and D_t, their lags, and all other necessary variables.

Step 1b An expression for the holding-period return[1] H_t): The one-period holding period return for any given period of time t to $t+1$ under the simple efficient market model is equivalent to $\frac{P_{t+1} - P_t + D_t}{P_t}$.

Step 1c (An expression for p_t): Using the long-run growth factor $\lambda^{t-T} = (1+g)^{t-T}$, the real detrended stock price p_t and real detrended dividends d_t series are realisable as; $p_t = P_t/\lambda^{(t-T)}$ and $d_t = D_t/\lambda^{(t+1-T)}$. The growth factor $\lambda = \exp(\beta)$, where β is from the regression $\ln(p_t) = \alpha + \beta(\text{Time})$.

Suppose we proceed with a trivial modification of model (A.2) i.e. dividing both sides by λ^{t-T};

$$\frac{P_t}{\lambda^{t-T}} = \frac{\sum_{k=0}^{\infty} \gamma^{k+1} E_t D_{t+k}}{\lambda^{t-T}} \times \frac{\lambda^{k+1}}{\lambda^{k+1}}$$

$$p_t = \frac{\sum_{k=0}^{\infty} (\lambda\gamma)^{k+1} E_t D_{t+k}}{\lambda^{(t+k)+1-T}}$$

since $d_{t+k} = \frac{D_{t+k}}{\lambda^{t+k+1-T}}$

$$p_t = \sum_{k=0}^{\infty} (\lambda\gamma)^{k+1} E_t d_{t+k}$$

and the real discount factor[2] for the detrended series is $\lambda\gamma = \frac{1+g}{1+r} \equiv \bar{\gamma}$

$$p_t = \sum_{k=0}^{\infty} (\bar{\gamma})^{k+1} E_t d_{t+k} \tag{A.3}$$

I require $g < r$ to obtain finite stock prices thus, $\bar{\gamma} \equiv \lambda\gamma < 1$. A necessary assumption is that; d_t is

[1] Under the model expressed in 1.2, $E_t(H_t) = r$.

[2] $\bar{r}$ is defined by $\bar{\gamma} \equiv \frac{1}{1+\bar{r}}$. An appropriate discount rate for p_t and d_t

jointly stationary with information. $var(p)$ can be expressed without time subscripts when the unconditional covariance between d_t and any information variable z_t depends only on k and not t. Given this assumption and taking the unconditional expectation across model (A.3) results in;

$$E(p_t) = E\left[\sum_{k=0}^{\infty}(\bar{\gamma})^{k+1}E_t d_{t+k}\right] = E(p) = E(d)\left[\sum_{k=0}^{\infty}(\bar{\gamma})^{k+1}\right]$$

$$E(p) = E(d)\left(\frac{\bar{\gamma}}{1-\bar{\gamma}}\right)$$

Substituting $\bar{\gamma} = 1/(1+\bar{r})$ and solving for $\bar{r}$ shows clearly that the discount rate is equivalent to the ratio of $E(d)$ and $E(p)$.

Step 1d (An expression for p_t^*): Shiller (1981) expresses the *ex-post* rational price p_t^* as the expected present value of actual future dividends:

$$p_t = E_t(p_t^*) \tag{A.4}$$

where $p_t^* = \sum_{k=0}^{\infty}\bar{\gamma}^{k+1}d_{t+k}$. According to Shiller (1981), p_t^* is approximately observable (within some error margin) when available dividend series are long enough. If the terminal value is known, the p_t^* series is recursively estimable by the relation;

$$p_t^* = \bar{\gamma}(p_{t+1}^* + d_t) \tag{A.5}$$

In his 1981 study, Shiller (1981) arbitrarily sets the terminal-value of p_t^* to $E(p)$. Any corresponding assumption made on the estimated terminal-value will mandate adjustment p^* for exponential trend.

Step 2a (Deriving the limits on $\delta(p)$ and $\Delta(p)$): I define a necessary innovation operator δ i.e. change in conditional expectation made in response to new information arriving between $t-1$ to t is such that;

$$\delta_t p_t = p_t - p_{t-1} + d_t - \bar{r}p_{t-1}$$

According to Shiller (1981) found that $\delta_t p_t \approx \Delta p_t$ and $\delta_t p_t$ is observable, contrary Samuelson (1973), Granger (1975) suggestion that Δp_t by efficient markets is unforecastable (empirical results confirm the equivalence). Based on this, the relation between price and dividend in terms innovations[3] is can be expressed as

$$\delta_t p_t = \sum_{k=0}^{\infty}\bar{\gamma}^{k+1}\delta_t d_{t+k} \tag{A.6}$$

Assumptions are required to be made on the nature of accrual of dividend information pattern in order to

[3] $\delta_t d_{t+k}$ denotes the time when dividends are announced publicly. This term is clearly not directly unobservable

derive inequalities on $\delta_t p_t$. Under perfect positive correlation of time t innovations in equation (A.5), the $\sigma^2(\delta_t p_t)$ attains its maximum and $\sigma^2(\delta p) = \left(\sum_{k=0}^{\infty} \bar{\gamma}^{k+1}\sigma_k\right)^2$.

Step 2b (Deriving the limits on $\delta(p)$ and $\Delta(p)$): The next relation we review is:

$$d_t = E(d) + \sum_{k=0}^{\infty} \delta_{t-k} d_t \tag{A.7}$$

Comparing (A.6) to (A.7) suggests that $\forall t \geqslant 0$, d_t in 1.7 represent varied linear combinations of dividend innovations as given in equation 1.5. A relation between $\sigma^2(\delta p)$ and $\sigma^2(d)$ is established such that:

$$\sigma^2(d) = \sum_{k=0}^{\infty} \sigma^2(\delta d_k) = \left(\sum_{k=0}^{\infty} \sigma_k^2\right) \tag{A.8}$$

σ^2, $(\delta_{t-k} d_t)$, $\sigma^2(\delta d_k)$, and σ_k^2 remain in equivalence under stationarity assumption. With no autocorrelation in the innovation series, forecasted dividend series maintain an ARIMA form making it possible to maximize $\sigma^2(dp)$. Equation (A.7) can be rewritten as:

$$d_t - E(d) = \sum_{k=0}^{\infty} \delta_{t-k} d_t$$

Hence,

$$d_t - E(d) \equiv \hat{d}_t = \sum_{k=0}^{\infty} \delta_{t-k} d_t = \sum_{k=0}^{\infty} a_k \varepsilon_{t-k}$$

$\varepsilon_t \equiv \delta_t d_t$.

Step 2b (The limit on $\sigma(\Delta p_t)$): This concerns the upper bound behaviour of $\sigma(\Delta p)$ given $\sigma(d)$. Recall that:

$$\Delta p_t = \delta_t p_t + \bar{r} p_{t-1} - d_{t-1}$$

with which we can have the first-difference[4] in real detrended stock price index;

$$\Delta p_t = \left(\sum_{k=0}^{\infty} \bar{\gamma}^{k+1} \delta_t d_{t+k} + \bar{r} \sum_{j=1}^{\infty} \delta_{t-j} \sum_{k=0}^{\infty} \bar{\gamma}^{k+1} d_{t+k-1} - \sum_{j=1}^{\infty} \delta_{t-j} d_{t-1}\right) \tag{A.9}$$

with all order-conditions satisfied for maximisation, we find a slightly different inequality:

$$\sigma(\Delta p) \leqslant \frac{\sigma(d)}{\sqrt{2\bar{r}}} \tag{A.10}$$

[4]innovations at time t in d are perfectly correlated while those of different times are necessarily uncorrelated. This allows us to express forecasted dividend process as ARIMA process. Futher, the upper limit is obtained form an optimally forecasted first-order autoregressive dividend when $\hat{\delta}_t = (1-\bar{r})\hat{\delta}_{t-1} + \varepsilon_t$ and $E_t d_{t+k} = (1-\bar{r})^k d_t$.

is realised at the maximum point.

A.2 Parameter estimation process

Step 1: Generating the real share price P_t and real dividend D_t. The following necessary adjustments were made using the Bureau of Labor Statistics whole price index (WPI):

For real price P_t series[5] starting 1900:

$$P_{(\mathrm{t}\geqslant 1900,\mathrm{T}=1979)} = \frac{(\text{Nominal Share Price})_t}{(\text{WPI for January})_t} \times (\text{WPI})_T \tag{A.11}$$

For real price P_t series prior to 1900:

$$P_{(\mathrm{t}<1900,\mathrm{T}=1979)} = \frac{(\text{Nominal Share Price})_t}{(\text{Annual Average WPI})_t} \times (\text{WPI})_T \tag{A.12}$$

For real dividend series D_t:

$$D_{(\text{all t},\mathrm{T}=1979)} = \frac{(\text{Nominal Dividend})_t}{(\text{Annual Average WPI})_t} \times (\text{WPI})_T \tag{A.13}$$

Step 2: Generate H_t, the holding period returns using the relation

$$H_t \equiv \frac{P_{t+1} - P_t + D_t}{P_t}$$

Step 3: Estimate the long-run exponential growth rate $\lambda \equiv (1+g)$ i.e. the trend factor for detrending P_t and D_t. Shiller (1981) estimates this through the following process:

$$\ln(P_t) = \alpha + \beta(Time)$$

$$\beta = \ln\lambda$$

[5]The data used for the empirical analysis and replication of Shiller (1981) study is obtained from (Shiller 1992, chp.26)[6]. The set comprises monthly actual dividend payment and price series on the S&P500 index spanning 1871 to 1979. The replication process directly adopts calculations and descriptions provided by Robert Shiller. The required adjustment to generate P_t and D_t from the nominal price and dividend series is outlined in "Step 1a". See appendix A for respective proofs. I use S&P500 index price and dividend data series covering 1871-2017 to verify whether Shiller's prior findings still hold. Then, we extend the verification study to test for consistency in the case of fifty (50) globally selected financial stock markets. The dataset consist of monthly index price and actual dividend payment series from Global Financial Data https://www.globalfinancialdata.com/

$$\lambda \equiv \exp(\beta) \tag{A.14}$$

Step 4 Estimate the real detrended stock price p_t and real detrended dividend d_t

$$p_t = \frac{P_t}{\lambda^{t-T}} \tag{A.15}$$

$$d_t \equiv \frac{D_t}{\lambda^{t+1-T}} \tag{A.16}$$

Step 5 Estimate the *ex-post* rational stock price index p_t^* by setting the terminal value $p_T^* \equiv \mathrm{E}(p_t)$ i.e. average over the entire sample. The series is generated using equation 1.5 and $\bar{\lambda} \equiv 1/(1+\bar{r})$.

A.3 Model-free variance bounds test

The rationale of the Model-free variance bounds is to produce a methodology that will enable us explore and determine whether the rejection of the present-value relation is statistically significant (see LeRoy & Parke 1992).

The procedure follows the following steps

1. The geometric random walk is used to simulate the actual ex-post rational stock prices.

$$d_{t+1} = d_t \varepsilon_{t+1}$$

   where ε is IID with mean μ and variance σ^2. This suggest that model-free test is model-free only in the sense that the test statistic is constructed without specifying a dividends model. The choice of dividend model does not affect the size of the rejection region.

   Suppose dividends evolve through time according to the model (see alos in Kleidon 1986, Froot & Obstfeld 1991, LeRoy & Parke 1992)

$$\ln D_{t+1} = \mu + \ln D_t + \sigma \epsilon_{t+1}$$

   where ϵ_{t+1} is a Gaussian white-noise process with $E(\epsilon_t) = E[\epsilon_t^2] = 0$.

2. The Monte Carlo experiments are implemented the parameters of the geometric random walk has parameters estimated from real-world data.

A.4 Economic history of Spain

Eizaguirre et al. (2004) considered the Spanish economic and financial history with a focus on the volatility of the stock markets, over the period 1941:01 to 2001:12. They suggest that there were several distinctive periods in the Spanish economy that influenced that volatility on their stock market. The first period in their study covers the autarhic behaviour imposed by the Civil War and World War 2, up to the "Stabilization Plan" of 1959 when the economy opened up to foreign investment.

The second period (1973-1985) was marked by oil crises hit and the intense instability of the 1970s and 1980s. The third period (1986-2001) of their study is associated with the integration in the European environment. The analysis in Eizaguirre et al. (2004) point out to a change in behaviour of Spanish stock market volatility which happened in the 1970s based on the economic development and the end of Franco's dictatorship and not in the 1980s when financial development occurred. Inn more recent times, Spanish economy has become more internationally integrated and therefore international instability is transmitted to Spain. There were also periods of abnormally high volatility in the 1987, 1991 and 1999 and the volatility after the breaks was less persistent.

The evolution of the stock prices in the post-war era is in stark contrast with the behaviour of stock markets in Spain prior to 1945. Figure A.1 shows the evolution of the real stock prices between 1900 and 2018, on a log scale. The poor performance of stock prices in Spain for the first half of this period can be attributed to the fact that there were several stock exchanges in Spain distributed geographically and b) the introduction of the regulation of stock markets by transfer of French legislation in 1830 that created a stock market environment where French style stock markets coexisted with Anglo-Saxon style free markets and other small traditional systems, the existence in parallel of different institutional settings generating significant distortions on Spanish financial markets (see Cagigal 2008, for more details).

The historical evolution of the Spanish economy is strongly connected to its social and political structures. Figure A.1 attempts to present a periodization of Spanish history and the evolution of real stock price and dividend time series. This provides a good basis to partially investigate and rationalize observed lower values for Spain in relation to the phases of development experienced by her economy. Harrison (1990), Casares et al. (2000), and Vives (2015) provide excellent panoramic view on the Spanish economy. More specifically, we highlight the six unique phases of Spain's economy discussed in Vives (2015). First the *primitive colonial economy* which characterises the beginning of Hispanic economy amidst colonization by the Eastern Mediterranean people. This period is centred on control of trade routes for metals when the Hispanic economy was in close association with the Mediterranean world. The second is *feudal and Seigniorial econ-*

omy-8th to 12th century where the collapse of mercantile and monetary economy ushers in the family-type economy in Christian Spain with economic activities based on wheat and sheep. Commercial trading persisted within the Muslim Spain. Third, is the *commercial expansion of the Bourgeois patriciate*-12th century; here urban development led to new economic systems and living standards. Barcelona became the economic hub of Spain from the 12th to the 15the century. Gold, spices, slaves and clothing manufacturing became the main commodities of trade. The fourth is the *Mercantile period* - 16th to 17th century when the discovery of the new world and more importantly the inception of the Spanish monarchy. According to Vives (2015), the monarchy era was characterised by survival economic mentality and openness to the American market which was experiencing substantial growth. Arrival of the Mexican and Peruvian silver led to capitalism as the old veil of feudal system broke apart. Efforts to maintain a large army against limited resources led to a financial collapse in 1680. *Economic transformation phase* - 18th to 19th century is fifth. This phase is characterised by economic activities and a new reformation agenda where Spain aligned with the rest of Europe. This led to the establishment of commercial enterprise and large-scale capital ideology. Development came speedily as Spain opened to commerce and industry, thereby attaining her share of industrial revolution in the 19th century as industrialisation from England spilled over into the region. The sixth and final economic period discussed by Vives (2015) is the *contemporary economy phase* -beginning in 1917. The social and political space faced contentions between views on socialism and capitalism. The most important decision in this phase is based on how the economy balanced agricultural, industrial and technological revolution against notable failures as Spain worked match her development to that of the western world (see also Harrison 1990, Casares et al. 2000). I do acknowledge that economic progress around these periods are slow within the social and political structures that evolve over time.

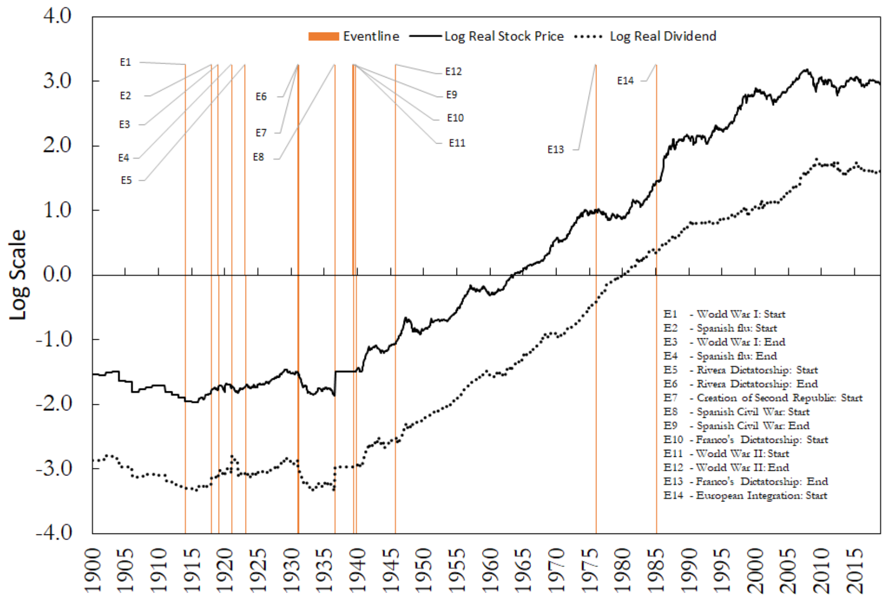


Figure A.1: Spain Real Stock Prices 1900-2018

Notes: This Figure plots the log values of real stock index prices for the Spanish stock market since 1900. The interwar and civil unrest price series illustrates how real stock prices decline by more than one-half from 1900 to 1960.Real stock price values started to pickup after the second world war.

A.5 International evidence of dividend volatility

A.5.1 Cross-country dividend data structure

Data used for our empirical analysis and replication study is from the Global Financial Database (GFD). I have a sample of fifty (50) stock markets spread globally. Specifically, we use data from Argentina, Australia, Austria, Belgium, Brazil, Bulgaria, Canada, Chile, China, Colombia, Ecuador, Czech Republic, Denmark, Egypt, Finland, France, Germany, Greece, Hong Kong, Hungary, India, Indonesia, Ireland, Israel, Italy, Japan, Jordan, Kenya, Korea, Malaysia, Morocco, Netherlands, New Zealand, Nigeria, Norway, Philippines, Poland, Portugal, Romania, Russia, Singapore, South Africa, Spain, Sweden, Switzerland, Taiwan, Thailand, Tunisia, Turkey, United Kingdom, and United States.

The GFD datasets are monthly nominal index prices, the aggregate nominal dividends paid by index constituents and the consumer price index for each sampled country. The last date for the sample period is December 31, 2018 for all samples although the start data varies across countries.

A.6 Base Year Effect on Variance Bounds

Table A.1: Base Year Effect on Dividend Puzzle

		$\frac{\sigma(p)}{\sigma(p^*)}$		$\frac{\sigma(\Delta p)}{\sigma(d)/\sqrt{2\bar{r}}}$		$\frac{\sigma(\Delta p+d_{t-1}-\bar{r}p_{t-1})}{\sigma(d)/\sqrt{\bar{r}_2}}$	
Country	**No. of Obs**	**Base Yr: 2002**	**Base Yr: 2018**	**Base Yr: 2002**	**Base Yr: 2018**	**Base Yr: 2002**	**Base Yr: 2018**
BGR	17	13.56	22.85	3.81	5.04	3.94	5.13
ROU	19	4.93	11.81	3.60	5.37	3.42	5.27
RUS	20	5.07	20.77	5.68	11.12	5.39	10.99
TUN	21	8.49	16.20	5.11	6.99	4.93	6.87
BRA	22	5.02	11.66	5.00	7.85	4.64	7.63
CZE	23	3.99	5.33	3.50	4.00	3.29	3.82
HUN	23	7.28	11.62	5.00	6.40	4.92	6.33
POL	23	4.74	6.41	3.89	4.54	3.84	4.50
ISR	24	8.28	8.86	7.13	7.38	6.91	7.17
EGY	25	0.91	3.05	2.01	3.30	1.37	3.12
CHN	27	11.66	17.27	10.69	12.93	10.49	12.77
IDN	27	5.61	15.59	5.71	9.14	5.39	8.95
IRL	27	9.24	11.50	5.35	5.96	5.29	5.91
PRT	31	7.34	9.45	6.73	7.64	6.60	7.53
COL	32	10.15	20.25	6.71	9.41	6.37	9.18
NGA	32	2.59	7.68	4.54	9.03	3.52	8.67
TWN	32	7.41	8.86	9.16	9.96	8.97	9.79
TUR	32	2.11	7.53	5.71	10.98	5.10	10.70
KEN	34	0.81	1.68	1.58	2.59	1.13	2.46
MAR	34	8.60	11.05	4.53	5.11	4.38	4.98
PHL	36	4.69	8.11	2.83	3.84	2.72	3.77
JOR	39	5.92	9.58	5.68	7.40	5.50	7.27
GRC	41	6.88	9.03	4.77	5.43	4.65	5.33
THA	42	6.29	8.75	5.76	6.80	5.56	6.64
CHL	44	2.50	3.75	2.49	3.10	2.30	2.96
MYS	44	15.27	22.72	10.82	13.05	10.54	12.82
SGP	45	9.09	12.09	11.64	13.32	11.36	13.08
NOR	48	4.32	5.59	6.18	7.17	6.02	7.04
HKG	53	8.15	10.79	16.11	18.92	15.66	18.54
ZAF	54	2.32	4.71	3.85	5.81	3.69	5.72
KOR	55	0.81	1.07	1.51	1.78	1.40	1.71
FIN	56	5.48	6.59	5.54	6.15	5.41	6.03
ARG	71	1.32	3.90	1.43	3.08	0.96	2.99
GBR	84	2.74	3.50	4.59	5.39	4.41	5.25
NZL	91	2.68	3.25	4.36	5.10	4.23	4.99
AUT	93	2.89	3.65	3.30	3.79	3.22	3.73
ITA	93	2.83	3.47	2.86	3.24	2.79	3.18
IND	97	1.23	2.02	1.67	2.63	1.53	2.59
CHE	99	2.41	2.53	2.68	2.75	2.63	2.70
SWE	116	2.36	2.65	2.20	2.41	2.17	2.37
JPN	117	1.18	1.38	1.95	2.18	1.89	2.14
ESP	118	2.69	3.18	2.67	3.06	2.61	3.00
NLD	126	1.68	1.97	1.72	1.94	1.70	1.93
DNK	144	1.99	2.26	1.90	2.15	1.93	2.18
BEL	147	2.02	2.38	2.39	2.77	2.37	2.75
USA	147	1.63	1.91	2.81	3.29	2.74	3.24
DEU	148	2.02	2.28	1.60	1.77	1.55	1.72
AUS	156	1.60	1.85	2.69	3.21	2.56	3.11
FRA	178	1.56	1.73	2.66	2.94	2.57	2.86
CAN	187	1.40	1.58	2.53	2.89	2.44	2.81

Notes: This table reports and compares the results of the volatility bounds test which are estimated form different base year consumer price index (CPI). I compare the impact of changing the base year from 2018 to 2002 when calculating the real stock index price and dividends series. The dividend puzzle still exist, but at different magnitudes across countries.

2.6.1 Additional stationarity test results

Table 2.2: Stationarity test on transformed price and dividend series

	P_t/D_t			p_t/d_t			$\widetilde{p}_t/\widetilde{d}_t$		
Country	PP	KPSS	ADF	PP	KPSS	ADF	PP	KPSS	ADF
ARG	-8.009***	0.116*	-3.757**	-5.491***	0.088*	-3.87**	-8.009***	0.116*	-3.757**
AUS	-5.594***	0.251***	-3.761**	-5.549***	0.219***	-3.791**	-5.594***	0.251***	-3.761**
AUT	-4.444***	0.154**	-3.162*	-4.657***	0.143*	-3.233*	-4.444***	0.154**	-3.162*
BEL	-6.746***	0.137*	-4.923***	-4.605***	0.176**	-4.556***	-6.746***	0.137*	-4.923***
BRA	-4.384**	0.072*	-3.199	-4.838***	0.07*	-3.869**	-4.384**	0.072*	-3.199
BGR	-3.764**	0.079*	-2.849	-2.968	0.075*	-2.837	-3.764**	0.079*	-2.849
CAN	-5.918***	0.504***	-3.97**	-5.184***	0.506***	-3.677**	-5.918***	0.504***	-3.97**
CHL	-4.321***	0.088*	-3.161	-3.944**	0.097*	-2.844	-4.321***	0.088*	-3.161
CHN	-2.968	0.102*	-2.426	-2.795	0.101*	-2.284	-2.968	0.102*	-2.426
COL	-4.322***	0.099*	-3.069	-4.083**	0.124*	-2.949	-4.322***	0.099*	-3.069
CZE	-4.754***	0.109*	-2.128	-3.614*	0.113*	-2.051	-4.754***	0.109*	-2.128
DNK	-3.665**	0.796***	-2.63	-3.431*	0.836***	-2.631	-3.665**	0.796***	-2.63
EGY	-4.459***	0.094*	-1.517	-2.263	0.079*	-2.325	-4.459***	0.094*	-1.517
FIN	-3.045	0.245***	-2.306	-2.485	0.258***	-2.006	-3.045	0.245***	-2.306
FRA	-4.177***	0.209**	-5.561***	-4.192***	0.227***	-4.606***	-4.177***	0.209**	-5.561***
DEU	-12.145***	0.071*	-6.979***	-9.579***	0.107*	-6.294***	-12.145***	0.071*	-6.979***
GRC	-4.442***	0.073*	-3.691**	-3.841**	0.064*	-3.404*	-4.442***	0.073*	-3.691**
HKG	-5.601***	0.067*	-3.157	-5.498***	0.056*	-3.462*	-5.601***	0.067*	-3.157
HUN	-3.16	0.104*	-2.191	-3.189	0.109*	-1.902	-3.16	0.104*	-2.191
IND	-3.687**	0.457***	-3.463**	-2.722	0.556***	-2.342	-3.687**	0.457***	-3.463**
IDN	-23.003***	0.15**	-1.938	-6.222***	0.178**	-1.699	-23.003***	0.15**	-1.938
IRL	-2.977	0.077*	-2.528	-3.182	0.078*	-2.645	-2.977	0.077*	-2.528
ISR	-2.736	0.192**	-2.014	-2.346	0.182**	-1.572	-2.736	0.192**	-2.014
ITA	-9.458***	0.049*	-5.527***	-4.694***	0.07*	-4.283***	-9.458***	0.049*	-5.527***
JPN	-2.32	0.254***	-2.661	-2.043	0.36***	-1.707	-2.32	0.254***	-2.661
JOR	-2.876	0.128*	-2.987	-2.888	0.136*	-2.428	-2.876	0.128*	-2.987
KEN	-4.81***	0.105*	-1.901	-3.734**	0.097*	-2.663	-4.81***	0.105*	-1.901
KOR	-2.138	0.211**	-1.827	-1.768	0.266***	-1.431	-2.138	0.211**	-1.827
MYS	-3.489*	0.225***	-1.882	-3.536**	0.23***	-1.831	-3.489*	0.225***	-1.882
MAR	-1.81	0.216***	-1.951	-1.636	0.243***	-1.972	-1.81	0.216***	-1.951
NLD	-3.549**	0.189**	-2.767	-3.542**	0.199**	-2.5	-3.549**	0.189**	-2.767
NZL	-6.449***	0.092*	-3.558**	-5.745***	0.099*	-3.45*	-6.449***	0.092*	-3.558**
NGA	-3.443*	0.12*	-1.722	-3.291*	0.113*	-2.357	-3.443*	0.12*	-1.722
NOR	-2.596	0.235***	-1.659	-2.367	0.235***	-1.595	-2.596	0.235***	-1.659
PHL	-2.325	0.198**	-2.049	-2.073	0.243***	-2.053	-2.325	0.198**	-2.049
POL	-2.514	0.092*	-2.093	-2.307	0.107*	-1.473	-2.514	0.092*	-2.093
PRT	-4.53***	0.056*	-3.963**	-3.694**	0.073*	-3.024	-4.53***	0.056*	-3.963**
ROU	-2.876	0.082*	-2.761	-2.747	0.109*	-2.287	-2.876	0.082*	-2.761
RUS	-3.532*	0.063*	-2.824	-3.135	0.081*	-2.146	-3.532*	0.063*	-2.824
SGP	-4.177**	0.242***	-1.687	-3.393*	0.279***	-1.577	-4.177**	0.242***	-1.687
ZAF	-3.98**	0.117*	-2.66	-3.667**	0.117*	-2.544	-3.98**	0.117*	-2.66
ESP	-3.319*	0.117*	-3.616**	-3.185*	0.141*	-3.381*	-3.319*	0.117*	-3.616**
SWE	-4.596***	0.199**	-3.39*	-4.484***	0.208**	-3.392*	-4.596***	0.199**	-3.39*
CHE	-2.563	0.261***	-1.883	-2.501	0.355***	-1.913	-2.563	0.261***	-1.883
TWN	-5.441***	0.059*	-4.236**	-3.845**	0.089*	-4.201**	-5.441***	0.059*	-4.236**
THA	-2.621	0.211**	-2.31	-2.235	0.248***	-1.903	-2.621	0.211**	-2.31
TUN	-2.649	0.097*	-1.824	-2.555	0.093*	-2.054	-2.649	0.097*	-1.824
TUR	-2.823	0.191**	-2.015	-3.632**	0.252***	-1.248	-2.823	0.191**	-2.015
GBR	-3.993**	0.202**	-3.375*	-4.095***	0.215**	-3.484**	-3.993**	0.202**	-3.375*
USA	-2.772	0.531***	-2.467	-3.103	0.549***	-2.399	-2.772	0.531***	-2.467

Table 2.3: Variance bounds test with sample size limited to 30

Country		MIDAS			Cochrane-type			First Gen.		
	n	$\frac{\sigma(p)}{\sigma(p^*)}$	$\frac{\sigma(\Delta\tilde{p})}{\frac{\sigma(\tilde{d})}{\sqrt{2\bar{r}}}}$	$\frac{\sigma(\Delta\tilde{p}+\tilde{d}_{t-1}+\bar{r}\tilde{d}_{t-1})}{\frac{\sigma(\tilde{d})}{\sqrt{\bar{r_2}}}}$	$\frac{\sigma(p)}{\sigma(p^*)}$	$\frac{\sigma(\Delta\tilde{p})}{\frac{\sigma(\tilde{d})}{\sqrt{2\bar{r}}}}$	$\frac{\sigma(\Delta\tilde{p}+\tilde{d}_{t-1}+\bar{r}\tilde{d}_{t-1})}{\frac{\sigma(\tilde{d})}{\sqrt{\bar{r_2}}}}$	$\frac{\sigma(p)}{\sigma(p^*)}$	$\frac{\sigma(\Delta\tilde{p})}{\frac{\sigma(\tilde{d})}{\sqrt{2\bar{r}}}}$	$\frac{\sigma(\Delta\tilde{p}+\tilde{d}_{t-1}+\bar{r}\tilde{d}_{t-1})}{\frac{\sigma(\tilde{d})}{\sqrt{\bar{r_2}}}}$
ARG	30	4.84	1.35	1.39	16.82	4.82	4.84	9.00	5.81	5.71
AUS	30	1.92	0.71	0.58	4.98	1.69	1.66	11.41	8.17	8.00
AUT	30	1.38	2.30	1.99	8.54	2.75	2.72	15.07	10.70	10.59
BEL	30	1.28	1.28	1.05	7.34	2.13	2.10	14.27	6.33	6.27
BRA	22	2.16	1.05	0.92	8.60	3.27	3.23	11.66	7.85	7.63
BGR	17	2.34	1.13	1.46	11.80	3.98	3.89	22.85	5.04	5.13
CAN	30	1.01	1.15	0.94	6.75	1.52	1.49	7.31	6.09	6.03
CHL	30	2.23	1.15	1.02	7.67	2.66	2.63	10.28	4.93	4.78
CHN	27	1.71	1.04	0.86	7.04	2.44	2.39	17.27	12.93	12.77
COL	30	2.37	1.09	0.97	6.88	2.97	2.94	18.89	9.07	8.87
CZE	23	1.19	0.82	0.38	3.98	1.54	1.45	5.33	4.00	3.82
DNK	30	1.61	1.18	1.01	11.42	2.02	2.00	22.20	10.30	10.19
EGY	25	1.03	2.36	1.34	12.29	6.53	6.43	3.05	3.30	3.12
FIN	30	1.34	2.07	1.66	5.92	2.03	1.99	15.03	10.77	10.63
FRA	30	1.55	2.16	1.69	6.51	1.80	1.76	20.80	14.17	13.93
DEU	30	2.06	0.86	0.76	7.82	1.96	1.94	22.73	11.16	11.01
GRC	30	1.55	1.50	1.33	9.79	5.55	5.48	11.09	7.18	7.07
HKG	30	1.46	1.08	0.92	7.48	3.36	3.31	15.73	15.50	15.16
HUN	23	2.03	1.00	0.92	12.88	3.38	3.36	11.62	6.40	6.33
IND	30	1.95	1.32	1.20	17.84	5.12	5.07	54.20	17.85	17.65
IDN	27	1.36	2.32	1.81	10.06	2.94	2.91	15.59	9.14	8.95
IRL	27	2.79	0.97	0.94	16.21	3.28	3.27	11.50	5.96	5.91
ISR	24	1.08	1.69	1.22	9.00	2.01	1.97	8.86	7.38	7.17
ITA	30	1.95	0.85	0.69	6.39	2.39	2.35	7.33	4.64	4.63
JPN	30	2.75	3.22	2.78	36.74	11.43	11.32	10.82	5.22	5.16
JOR	30	1.40	1.94	1.44	4.66	1.42	1.38	14.40	7.55	7.42
KEN	30	0.82	0.89	0.41	13.39	5.95	5.73	1.94	2.94	2.80
KOR	30	2.50	1.02	0.94	13.99	2.39	2.37	19.80	9.35	9.25
MYS	30	1.26	0.60	0.44	6.01	2.64	2.58	9.84	9.71	9.53
MAR	30	1.26	1.19	0.88	5.41	1.65	1.61	29.53	9.22	9.00
NLD	30	1.99	0.74	0.69	8.71	2.10	2.09	12.78	7.75	7.64
NZL	30	1.84	1.59	1.17	6.24	2.94	2.85	9.65	5.86	5.74
NGA	30	2.61	4.73	2.91	3.93	4.47	4.26	8.93	9.71	9.33
NOR	30	0.98	0.77	0.53	4.42	2.25	2.21	5.23	6.43	6.34
PHL	30	1.28	1.61	1.31	11.28	3.18	3.13	13.61	8.64	8.53
POL	23	1.38	0.86	0.72	8.30	2.56	2.53	6.41	4.54	4.50
PRT	30	1.75	0.82	0.60	4.62	1.86	1.80	9.74	7.00	6.88
ROU	19	1.49	0.92	0.44	4.04	1.54	1.44	11.81	5.37	5.27
RUS	20	1.75	0.62	0.47	6.04	1.56	1.49	20.77	11.12	10.99
SGP	30	0.94	0.99	0.72	4.81	3.25	3.19	5.36	6.28	6.19
ZAF	30	1.26	0.74	0.62	7.37	2.31	2.28	8.58	6.43	6.34
ESP	30	1.25	0.63	0.43	4.28	1.53	1.49	9.71	6.71	6.63
SWE	30	2.14	0.79	0.67	6.09	1.84	1.81	10.97	6.87	6.78
CHE	30	1.22	0.62	0.41	6.45	1.85	1.80	17.99	10.55	10.45
TWN	30	3.09	6.82	5.64	5.47	1.78	1.73	5.52	7.37	7.23
THA	30	1.90	0.87	0.75	7.17	3.92	3.82	10.62	7.00	6.83
TUN	21	1.88	1.32	0.96	7.64	1.93	1.88	16.20	6.99	6.87
TUR	30	1.72	3.85	3.02	9.56	12.84	12.58	9.56	12.47	12.21
GBR	30	2.15	4.03	2.71	11.92	7.09	6.90	15.39	11.16	10.85
USA	30	1.84	1.22	1.00	16.43	5.36	5.28	19.16	9.22	9.04

Table 2.4: Variance bounds test with sample size limited to 60

Country		MIDAS			Cochrane-type			First Gen.		
	n	$\frac{\sigma(p)}{\sigma(p*)}$	$\frac{\sigma(\Delta\tilde{p})}{\frac{\sigma(\tilde{d})}{\sqrt{2\bar{r}}}}$	$\frac{\sigma(\Delta\tilde{p}+\tilde{d}_{t-1}+\bar{r}\tilde{d}_{t-1})}{\frac{\sigma(\tilde{d})}{\sqrt{\bar{r_2}}}}$	$\frac{\sigma(p)}{\sigma(p*)}$	$\frac{\sigma(\Delta\tilde{p})}{\frac{\sigma(\tilde{d})}{\sqrt{2\bar{r}}}}$	$\frac{\sigma(\Delta\tilde{p}+\tilde{d}_{t-1}+\bar{r}\tilde{d}_{t-1})}{\frac{\sigma(\tilde{d})}{\sqrt{\bar{r_2}}}}$	$\frac{\sigma(p)}{\sigma(p*)}$	$\frac{\sigma(\Delta\tilde{p})}{\frac{\sigma(\tilde{d})}{\sqrt{2\bar{r}}}}$	$\frac{\sigma(\Delta\tilde{p}+\tilde{d}_{t-1}+\bar{r}\tilde{d}_{t-1})}{\frac{\sigma(\tilde{d})}{\sqrt{\bar{r_2}}}}$
ARG	60	4.43	1.33	1.38	11.76	4.76	4.77	3.31	2.21	2.14
AUS	60	1.39	0.47	0.37	2.89	1.19	1.16	2.83	4.03	3.90
AUT	60	1.57	2.06	1.71	4.74	2.47	2.43	4.98	5.11	5.03
BEL	60	1.49	1.21	0.95	4.46	1.58	1.57	6.83	5.00	4.94
BRA	22	2.16	1.05	0.92	8.60	3.27	3.23	11.66	7.85	7.63
BGR	17	2.34	1.13	1.46	11.80	3.98	3.89	22.85	5.04	5.13
CAN	60	1.41	1.72	1.33	4.03	1.40	1.37	3.45	4.37	4.32
CHL	44	1.81	1.58	1.04	3.65	3.05	2.91	3.75	3.10	2.96
CHN	27	1.71	1.04	0.86	7.04	2.44	2.39	17.27	12.93	12.77
COL	32	2.16	1.03	0.92	6.14	2.56	2.54	20.25	9.41	9.18
CZE	23	1.19	0.82	0.38	3.98	1.54	1.45	5.33	4.00	3.82
DNK	60	1.99	1.38	1.09	6.92	1.89	1.87	2.38	2.09	2.07
EGY	25	1.03	2.36	1.34	12.29	6.53	6.43	3.05	3.30	3.12
FIN	56	1.28	1.53	1.20	3.55	1.57	1.54	6.59	6.15	6.03
FRA	60	1.46	1.62	1.23	3.52	1.63	1.60	9.71	7.96	7.82
DEU	60	1.62	0.65	0.56	4.40	1.75	1.73	9.93	9.23	9.09
GRC	41	1.31	1.37	1.20	6.81	4.43	4.39	9.03	5.43	5.33
HKG	53	2.20	1.07	0.97	5.61	3.08	3.05	10.79	18.92	18.54
HUN	23	2.03	1.00	0.92	12.88	3.38	3.36	11.62	6.40	6.33
IND	60	1.64	1.11	1.02	8.90	3.64	3.61	1.63	1.26	1.23
IDN	27	1.36	2.32	1.81	10.06	2.94	2.91	15.59	9.14	8.95
IRL	27	2.79	0.97	0.94	16.21	3.28	3.27	11.50	5.96	5.91
ISR	24	1.08	1.69	1.22	9.00	2.01	1.97	8.86	7.38	7.17
ITA	60	1.10	1.29	1.00	4.18	2.18	2.14	4.06	3.88	3.83
JPN	60	2.59	3.11	2.57	8.26	4.15	4.11	7.77	5.80	5.72
JOR	39	1.34	1.77	1.31	3.96	1.31	1.27	9.58	7.40	7.27
KEN	34	0.83	0.90	0.42	5.30	4.30	4.11	1.68	2.59	2.46
KOR	55	2.63	1.52	1.11	13.06	3.98	3.97	1.07	1.78	1.71
MYS	44	1.43	0.50	0.43	4.61	2.32	2.27	22.72	13.05	12.82
MAR	34	1.33	1.14	0.83	4.90	1.67	1.62	11.05	5.11	4.98
NLD	60	1.41	0.58	0.54	4.08	1.93	1.92	10.51	6.66	6.56
NZL	60	1.48	1.32	1.02	4.97	2.79	2.71	5.41	5.51	5.40
NGA	32	2.61	4.59	2.82	7.91	9.18	8.82	7.68	9.03	8.67
NOR	48	0.92	1.07	0.79	3.37	1.45	1.41	5.59	7.17	7.04
PHL	36	1.71	0.86	0.77	10.28	2.73	2.70	8.11	3.84	3.77
POL	23	1.38	0.86	0.72	8.30	2.56	2.53	6.41	4.54	4.50
PRT	31	1.69	2.90	2.28	4.45	1.85	1.79	9.45	7.64	7.53
ROU	19	1.49	0.92	0.44	4.04	1.54	1.44	11.81	5.37	5.27
RUS	20	1.75	0.62	0.47	6.04	1.56	1.49	20.77	11.12	10.99
SGP	45	1.01	0.82	0.59	4.12	2.61	2.56	12.09	13.32	13.08
ZAF	54	1.30	0.76	0.63	4.83	1.79	1.77	4.71	5.81	5.72
ESP	60	1.09	0.62	0.41	2.66	1.52	1.48	4.86	4.84	4.73
SWE	60	1.88	0.68	0.58	4.14	1.70	1.68	4.63	4.73	4.66
CHE	60	1.28	0.55	0.39	3.99	1.65	1.62	6.34	5.78	5.70
TWN	32	4.09	7.95	6.65	5.19	1.76	1.70	8.86	9.96	9.79
THA	42	1.54	0.75	0.61	6.44	3.55	3.47	8.75	6.80	6.64
TUN	21	1.88	1.32	0.96	7.64	1.93	1.88	16.20	6.99	6.87
TUR	32	1.79	5.13	3.86	10.00	12.64	12.36	7.53	10.98	10.70
GBR	60	1.78	1.68	1.11	3.76	2.36	2.30	5.42	4.83	4.68
USA	60	1.64	0.81	0.70	6.76	2.56	2.54	5.14	4.34	4.26

Table 2.5: Variance bounds test with sample size limited to 100

Country		MIDAS			Cochrane-type			First Gen.		
	n	$\frac{\sigma(p)}{\sigma(p*)}$	$\frac{\sigma(\Delta\tilde{p})}{\frac{\sigma(\tilde{d})}{\sqrt{2\bar{r}}}}$	$\frac{\sigma(\Delta\tilde{p}+d_{t-1}+\bar{r}d_{t-1})}{\frac{\sigma(\tilde{d})}{\sqrt{\bar{r_2}}}}$	$\frac{\sigma(p)}{\sigma(p*)}$	$\frac{\sigma(\Delta\tilde{p})}{\frac{\sigma(\tilde{d})}{\sqrt{2\bar{r}}}}$	$\frac{\sigma(\Delta\tilde{p}+d_{t-1}+\bar{r}d_{t-1})}{\frac{\sigma(\tilde{d})}{\sqrt{\bar{r_2}}}}$	$\frac{\sigma(p)}{\sigma(p*)}$	$\frac{\sigma(\Delta\tilde{p})}{\frac{\sigma(\tilde{d})}{\sqrt{2\bar{r}}}}$	$\frac{\sigma(\Delta\tilde{p}+d_{t-1}+\bar{r}d_{t-1})}{\frac{\sigma(\tilde{d})}{\sqrt{\bar{r_2}}}}$
ARG	71	4.03	1.31	1.37	10.55	4.72	4.74	3.90	3.08	2.99
AUS	100	1.46	1.33	0.86	2.71	1.28	1.25	2.16	3.56	3.47
AUT	93	1.02	1.48	1.12	3.54	2.48	2.44	3.65	3.79	3.73
BEL	100	1.31	1.24	0.99	3.84	1.63	1.62	3.98	3.50	3.47
BRA	22	2.16	1.05	0.92	8.60	3.27	3.23	11.66	7.85	7.63
BGR	17	2.34	1.13	1.46	11.80	3.98	3.89	22.85	5.04	5.13
CAN	100	1.25	0.48	0.38	2.96	1.40	1.38	1.44	2.91	2.86
CHL	44	1.81	1.58	1.04	3.65	3.05	2.91	3.75	3.10	2.96
CHN	27	1.71	1.04	0.86	7.04	2.44	2.39	17.27	12.93	12.77
COL	32	2.16	1.03	0.92	6.14	2.56	2.54	20.25	9.41	9.18
CZE	23	1.19	0.82	0.38	3.98	1.54	1.45	5.33	4.00	3.82
DNK	100	0.99	0.92	0.56	5.22	1.88	1.86	2.48	1.88	1.92
EGY	25	1.03	2.36	1.34	12.29	6.53	6.43	3.05	3.30	3.12
FIN	56	1.28	1.53	1.20	3.55	1.57	1.54	6.59	6.15	6.03
FRA	100	1.26	1.51	1.10	3.46	2.47	2.42	3.42	4.91	4.80
DEU	100	3.60	1.87	1.79	20.77	5.27	5.25	16.80	4.27	4.24
GRC	41	1.31	1.37	1.20	6.81	4.43	4.39	9.03	5.43	5.33
HKG	53	2.20	1.07	0.97	5.61	3.08	3.05	10.79	18.92	18.54
HUN	23	2.03	1.00	0.92	12.88	3.38	3.36	11.62	6.40	6.33
IND	97	1.59	1.04	0.96	6.22	3.38	3.36	2.02	2.63	2.59
IDN	27	1.36	2.32	1.81	10.06	2.94	2.91	15.59	9.14	8.95
IRL	27	2.79	0.97	0.94	16.21	3.28	3.27	11.50	5.96	5.91
ISR	24	1.08	1.69	1.22	9.00	2.01	1.97	8.86	7.38	7.17
ITA	93	1.64	0.71	0.57	3.70	2.16	2.12	3.47	3.24	3.18
JPN	100	0.69	0.77	0.42	3.83	3.36	3.32	1.60	1.73	1.71
JOR	39	1.34	1.77	1.31	3.96	1.31	1.27	9.58	7.40	7.27
KEN	34	0.83	0.90	0.42	5.30	4.30	4.11	1.68	2.59	2.46
KOR	55	2.63	1.52	1.11	13.06	3.98	3.97	1.07	1.78	1.71
MYS	44	1.43	0.50	0.43	4.61	2.32	2.27	22.72	13.05	12.82
MAR	34	1.33	1.14	0.83	4.90	1.67	1.62	11.05	5.11	4.98
NLD	100	1.65	1.54	1.00	3.64	1.89	1.88	3.82	2.01	1.98
NZL	91	1.36	1.07	0.82	4.09	2.37	2.30	3.25	5.10	4.99
NGA	32	2.61	4.59	2.82	7.91	9.18	8.82	7.68	9.03	8.67
NOR	48	0.92	1.07	0.79	3.37	1.45	1.41	5.59	7.17	7.04
PHL	36	1.71	0.86	0.77	10.28	2.73	2.70	8.11	3.84	3.77
POL	23	1.38	0.86	0.72	8.30	2.56	2.53	6.41	4.54	4.50
PRT	31	1.69	2.90	2.28	4.45	1.85	1.79	9.45	7.64	7.53
ROU	19	1.49	0.92	0.44	4.04	1.54	1.44	11.81	5.37	5.27
RUS	20	1.75	0.62	0.47	6.04	1.56	1.49	20.77	11.12	10.99
SGP	45	1.01	0.82	0.59	4.12	2.61	2.56	12.09	13.32	13.08
ZAF	54	1.30	0.76	0.63	4.83	1.79	1.77	4.71	5.81	5.72
ESP	100	1.11	0.58	0.38	2.15	1.48	1.45	4.51	4.40	4.30
SWE	100	1.69	0.59	0.50	3.22	1.49	1.47	2.65	3.41	3.35
CHE	99	1.28	0.54	0.38	3.07	1.34	1.30	2.53	2.75	2.70
TWN	32	4.09	7.95	6.65	5.19	1.76	1.70	8.86	9.96	9.79
THA	42	1.54	0.75	0.61	6.44	3.55	3.47	8.75	6.80	6.64
TUN	21	1.88	1.32	0.96	7.64	1.93	1.88	16.20	6.99	6.87
TUR	32	1.79	5.13	3.86	10.00	12.64	12.36	7.53	10.98	10.70
GBR	84	1.75	1.49	0.96	2.67	1.42	1.39	3.50	5.39	5.25
USA	100	1.48	0.65	0.58	4.07	2.04	2.03	1.95	3.38	3.32

Table 2.6: Cross-country mean squared forecast errors (MSFE) for the regression analysis used in the variance bounds test

Country	Shiller	Cochrane	MIDAS
ARG	1.614	1.498	0.851
AUS	1.451	0.198	0.135
AUT	0.781	0.254	0.233
BEL	2.261	0.530	0.418
BRA	2.016	0.306	0.177
BGR	0.228	0.538	0.496
CAN	1.947	0.398	0.246
CHL	3.579	0.423	30.955
CHN	1.360	0.512	0.446
COL	2.127	0.851	0.535
CZE	0.605	0.256	0.517
DNK	1.354	0.236	0.155
EGY	1.842	0.588	0.616
FIN	1.985	0.205	0.168
FRA	0.982	0.170	0.136
DEU	26.904	5.931	2.055
GRC	1.011	0.627	0.532
HKG	2.339	0.175	0.096
HUN	1.019	0.220	0.247
IND	1.108	0.449	0.299
IDN	0.902	0.291	0.293
IRL	1.034	0.387	0.365
ISR	1.351	0.204	0.208
ITA	1.344	0.771	0.728
JPN	1.772	0.292	0.269
JOR	0.720	0.467	0.439
KEN	1.112	0.633	0.404
KOR	1.579	0.293	0.288
MYS	1.274	0.182	0.171
MAR	2.428	0.170	0.130
NLD	1.118	0.359	0.228
NZL	0.282	0.364	0.233
NGA	0.777	0.281	0.240
NOR	1.062	0.233	0.188
PHL	1.243	0.282	0.224
POL	0.270	0.302	0.445
PRT	0.443	0.214	0.176
ROU	1.090	0.890	0.651
RUS	2.041	0.174	0.403
SGP	1.547	0.104	0.099
ZAF	1.096	0.216	0.177
ESP	1.446	0.181	0.144
SWE	1.552	0.215	0.187
CHE	1.282	0.214	0.213
TWN	0.740	0.221	0.264
THA	1.033	0.281	0.338
TUN	1.227	0.233	0.187
TUR	0.971	0.308	0.288
GBR	0.812	0.081	0.079
USA	1.498	0.113	0.094

2.6.2 Global Financial Database Metadata on FT All-Share Index

Sources include Thorold Rogers, A History of Prices in England, (1693-1697), Larry Neal, The Rise of Financial Capitalism: International Capital Markets in the Age of Reason, New York: Cambridge Univ. Press, 1990 (1698-January 1811), W. W. Rostow and Anna J. Schwartz, The Growth and Fluctuation of the British Economy 1790-1850), (2 vols.), Oxford: Oxford U.P., 1953, p. 368, (February 1811-December 1850), Hayek as given in Rostow, ibid., p. 456 (January 1851-June 1867), K.C. Smith and G.F. Horne, An Index Number of Securities, 1867-1914, London and Cambridge Economic Service Special Memorandum No. 37, (July 1867-December 1906), Banker's Magazine (January 1907-May 1933), Economist (1933-1962), Financial Times (1950-). Volume data are provided for the London Stock Exchange. The number of bargains is provided in the FT 30 Industrials file, the Value of shares traded is provided in the FT All-Share Index, and the total shares traded is provided in the FT-500 Non-Financial Index. The All-Share index contains the historical data for the United Kingdom. East Indies Stock is used for 1693. The index is an unweighted arithmetic average of Bank of England and East Indies stock from 1694 to August 1711, and of Bank of England, East Indies and South Sea stock from September 1711 to January 1811. Rostow's Total Index of Share Prices is used from 1811 to 1850. Hayek's index was taken from Rostow and excludes banks, insurance, and bridge stocks, but includes industrial stocks. This index is linked to the London and Cambridge Economic Service index, which begins in July 1867 and continues until 1906. The L&CES index consisted of 25 stocks in 1867 and had grown to 75 stocks by 1914. The Banker's Magazine kept a capitalization-weighted index of 287 stocks, which gave the total capital values of the companies that were included. This was the broadest index of London shares at the time and the index is used beginning in 1907. Although this index was calculated beginning in 1887, the Banker's Magazine usually omitted calculating the index for one month during the summer, and for this reason it is excluded until 1907 when calculations were made for every month. The London market closed in August 1914 and reopened in January 1915. The Banker's Magazine Index is used through May 1933. Beginning in June 1933, the Actuaries General Index is used. This index included financial stocks, commodities, and utilities, but excluded debentures and preferred shares. Beginning in April 1962, the Financial Times-Actuaries All-Share Index is used. All indexes have been chain linked to one another to create a continuous index with the All-Share index's base of April 10, 1962 used as the base for the entire index. The All-Share Index is a capitalization-weighted price index and covers about 98-99% of the capital value of all UK companies. It uses the Paasche formula, adjust for capitalization changes, and has its components reviewed in December. It combines the FT-SE 100, FT-SE Mid-250 and FT-SE Small Cap indices, but excludes the Fledgling and AIM index components. Data are weekly from 1965 to 1968 and daily thereafter.

CHAPTER 3

Implied Volatility of Stock and Dividend Derivatives

3.1 Introduction

Aggregate stock market valuation models typically set equity prices equal to the present-value of their expected future dividend payments. To a large extent, such valuation principles depend on accurately forecasting realised future dividend payments but there are challenges[1] to attain error-free future dividend forecasts.

Important studies Shiller (1981), LeRoy & Porter (1981), West (1988*b*) on dividend and stock volatility found that movements in stock index prices are more volatile than movements in actual dividend, indicating a stock dividend puzzle. These conclusions are conceptually based on volatility relations derived from expected present-value models using predictive regressions[2]. Other recent extensions to the Shiller present value model consider learning effects on stock price dynamics. Jagannathan & Liu (2019) use learning effects in a latent variable present-value model; their findings point out to some possible market-agent aversion to long-run risk and learning impact on volatility of stock-prices. Li & Yang (2013) showed that dividend volatility positively predicts future asset returns, with predictive power improving with the forecasting horizon. A no-arbitrage technique to compute the value of discrete dividend payments using only the information from market prices

[1] Regarding long-horizon return-forecasting regressions, Fama & French (1988), Cochrane (2008) and other literature emphasise low out-of-sample forecasting R^2 including the fact that R^2 values only increase with length of horizon. On out-of-sample predictability, Jagannathan & Liu (2019) found 25.3%-27.1% predictability in total variation of annual aggregates stock returns coming from dividends.

[2] Cochrane (2008) however argued that such models suffer from an "errors-in-variables" problem.

of options is presented in Desmettre et al. (2017).

In a different approach Pang et al. (2008) departed from the usual assumption that dividends may be characterised by a normal distribution and proposed employing the beta distribution for dividend yields; they use the coefficient of variation for studying the stability of dividend yields. Li & Yang (2013) also showed that dividend volatility positively predicts future asset returns, with predictive power improving with the forecasting horizon. There are also nonparametric predictive inference (NPI) option pricing techniques to exploit price volatilities within short term investment horizons. I find in He et al. (2018) and Chen et al. (2019) that the performance of such methods strongly depend on long historical data.

Derivative markets currently trade in stand-alone dividend derivative contracts that come with direct exposure to dividend volatilities. In this regard, relevant literature hold the view that such markets have the tendency to advance and support rational pricing of assets in stock markets (see Brennan 1998). Some of these stand-alone contracts are based on index dividends paid by the index constituents with maturities that extend to about a decade. Dividend derivatives are described and discussed in Wilkens & Wimschulte (2010); Mixon & Onur (2016); Tunaru (2018); Filipović & Willems (2019). An interesting dividend derivative product development is described in Brown & Davis (2004) as the Australian "endowment warrant," which is a long-term contract with strike price corrected downward every time the underlying stock pays a dividend. Other important works on pricing matters when assuming that dividends are stochastic are the seminal paper by Geske (1978), the papers by Lioui (2005) and Lioui (2006) who revisited Black-Scholes and put-call parity under stochastic dividends, Korn & Rogers (2005) who deal with the stochastic evolution of discrete dividend payments assuming their timing is known and Nielsen (2007) who studies the role of dividends in pricing derivatives securities more generally.

Dividend swaps have emerged recently as a useful innovation to extract forward looking information for futures equity returns, see Van Binsbergen et al. (2013) and Golez (2014). A two-factor model estimated on dividend futures data only is proposed by Kragt et al. (2020) who show that the model can cover a significant part of the observed daily stock market returns. The predictability in patterns of implied volatility (IV) and option prices is statistically possible, as shown in Goncalves & Guidolin (2006) among others. An excellent summary of the general structure of systems widely used for equity portfolio analytics is also presented in Pachamanova & Fabozzi (2014).

EURO STOXX 50® Index Dividend Futures Options (IDF) contracts are settled in cash into the realized dividends paid during their settlement period; commencing a day after the third Friday in December and ending on the third Friday of December the following year. The cash settlement is the sum of all dividends which are paid over the settlement period. The dividend payment category takes into account those paid in either cash or shares. There are ten annual maturities available on Eurex Exchange at any given time.

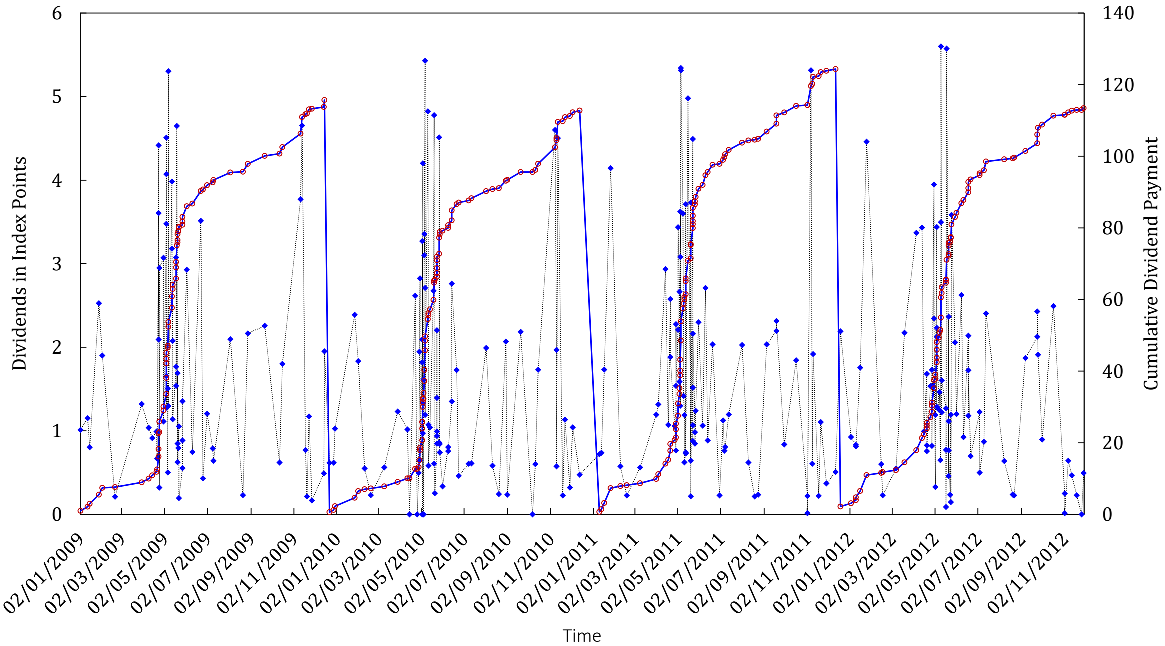


Figure 3.1: Historical evolution of cumulative dividend and daily dividends in index points on the Dow Jones Euro STOXX50®.
*Notes:*The time series is at a daily frequency, from December 22, 2008 to December 17, 2012. The blue markets are index dividend points and the red-dotted line is cumulative dividends.

Index dividend futures option contracts provides an efficient means of measuring the market participants' *ex ante* assessment of implied index dividend volatility. Since dividend derivatives have become stand-alone contracts, it will be important to investigate issues about implied dividend volatility and also uncover its subsequent relationship with that implied by index stock options.

In Figure 3.1, I present two ways of reporting the Dow Jones Euro STOXX50® dividends within calendar years, using either cumulative cash dividends or index dividend points. The cumulative quantity produces a sigmoidal shape illustrated in Figure 3.1. The non-decreasing cumulative dividend payment can be dealt with a diffusive logistic growth process. This approach however gives no guarantee to obtaining a monotonically increasing dividend. One way to deal with this shortcoming is to impose a functional form that produces strictly positive curvature, since dividends are non-negative. The underlying asset in this study is EURO STOXX 50® DVP (Dividend Points Index)[3]. Figure 3.1 shows how the time series of dividend points resembles a jump process, similar to the case of (Tunaru 2018, Filipović & Willems 2019). The jump sizes

[3]The contract value is EUR 100 per index dividend point of the underlying and the price quotation is in points. Minimum price change of 0.01, equivalent to a value of EUR 1 per contract. The dividend point time-series is not strictly non-decreasing. The options settle to the realized value of dividend payments in a future period.

can be modelled with a beta distribution to ensure strictly positive payments occur.

In this study, I investigate the stock and dividend volatility puzzle from a derivatives perspective. I analyse the IV of index-dividend futures options and stock-index option contracts. For the first time in the literature, I take advantage of a recent set of derivative instruments, the stock and IDF options, to investigate jointly the implied volatilities for stock and for stock index dividend. The novelty of my approach consists in having a *forward looking* view and being able to determine the IV associated with each side, stock and dividends, from different sets of options. In this context, I outline conditions under which the dividend puzzle effects inferred from historical data are safely exchangeable with future expectations of it. I explicitly allow for variability in time-to-maturity of option contracts and interest rates, while outlying a computational approach that allows aggregation of volatility measures under the Black-Scholes formula; Black (1976) formula and the Bakshi et al. (2003) model-free IV approach. I empirically show that the stock dividend puzzle still exists in this new context but the discrepancy between the two volatilities declines with the horizon. Furthermore, I explore how to identify profitable trading strategies using trading signals generated from the time-varying implied volatility ratios that define the dividend puzzle.

This study is organised as follows. Section 3.2 describes the methodology of the study. Here, I discuss the computational approach and numerical methods to derive the dividend puzzle from option prices data. Section 3.3, presents the data sets and discusses the main results and their implications. I show further numerical investigations on trading strategies designed around the mean reversion characteristic of IV and the implied mean-reversion of the ratio of IV for stock and dividend futures. In Section 3.4, I summarize the findings and conclude the study.

3.1.1 Variance bounds test for implied dividend volatility

Aggregate stock market valuation models typically set equity prices equal to the present value of their expected future dividend payments. To a large extent, such valuation principles depend on accurately forecasting realised future dividend payments but there are challenges[4] to attain error-free future dividend forecasts.

Important studies Shiller (1981), LeRoy & Porter (1981), West (1988*b*) on dividend and stock volatility found that movements in stock index prices are more volatile than movements in actual dividend, indicating a stock dividend puzzle. These conclusions are conceptually based on volatility relations derived from expected

[4]Regarding long-horizon return-forecasting regressions, Fama & French (1988), Cochrane (2008) and other literature emphasise low out-of-sample forecasting R^2 including the fact that R^2 values only increase with the length of the horizon. On out-of-sample predictability, Jagannathan & Liu (2019) found 25.3%-27.1% predictability in the total variation of annual aggregates stock returns coming from dividends.

present-value models using predictive regressions. Van Binsbergen & Koijen (2011) however argued that such models suffer from an "errors-in-variables" problem. In addition, known regression-based approaches are using historical asset prices (Shiller 1981, LeRoy & Porter 1981, Golez 2014, Jagannathan & Liu 2019) thereby setting a retrospective view on the concept of dividend market volatility.

In this paper, for the first time in the literature, I take advantage of a recent set of derivative instruments, the stock index dividend options, to investigate jointly the implied volatilities for stock and for stock index dividend. The novelty of my approach consists in having a proper *forward looking* view and being able to determine the implied volatilities associated with each side, stock and dividends, from different sets of options. I will show that the stock dividend puzzle still exists in this new context but the discrepancy between the two volatilities declines with the horizon. Furthermore, I explore how to identify profitable trading strategies based on the implied volatilities and their ratios as trading signals.

Geske (1978) pointed out that assuming that dividends are known when in fact they are not, has the effect to misestimate the volatility. There is increasing literature related to dividend derivatives. LeRoy & LaCivita (1981), Michener (1982) argued that observed violations in expected-present value volatility bounds are due to assumptions on the choice of dividend discount factor. Both studies found extra volatility within stock prices when using a stochastic discount. Flavin (1983) investigate whether sampling variability in empirical studies may be the root cause of violations observed in variance bounds. Other studies (see Marsh & Merton 1986, Dejong & Whiteman 1991) also investigate the possibility of observing a reversal in Shiller's present-value variance relation under different time-series treatment for dividends.

Taking a different approach Pang et al. (2008) departed from the usual assumption that dividends may be characterised by a normal distribution and proposed employing the beta distribution for dividends and the coefficient of variation of investing the stability of dividend yields. Recent extensions to the Shiller present value model consider learning effects on stock price dynamics. Jagannathan & Liu (2019) use learning effects in a latent variable present-value model; their findings show that market-agent aversion to long-run risk and learning impact stock-prices. These conclusions provide further support to Shiller (2015) irrational exuberance argument and further suggest a need to consider a forward-looking approach to explore *why* prices are more volatile than dividends.

Manley & Mueller-Glissmann (2008) provide excellent description of the applications of dividends for improvement of investment strategies. Dividend derivatives are discussed in Wilkens & Wimschulte (2010); Mixon & Onur (2016); Tunaru (2018); Filipović & Willems (2019).

Novel market data from new financial instruments could provide salient information on the relation between stock and dividend volatility. I adopt a forward-looking approach that explores discussions on stock and dividend volatility from the perspective of a derivative. This study analyses IV of IDF option and stock-index option contracts. Thus a forward-looking view of implied volatilities of the underlying assets is feasible.

The predictability in patterns of IV and option prices is statistically possible, as shown in Harvey & Whaley (1992); Cont et al. (2002); Goncalves & Guidolin (2006); Gatheral (2011) among others. This lays a steady foundation for us to conceptualise the IV surface as a stochastic state variable accounting for the evolution of the underlying asset price process. I explicitly allow for variability in the time-to-maturity of the option contracts, while outlying a computational approach that allows aggregation of volatility measures under Black & Scholes (1973); Black (1976) including the Bakshi et al. (2003) model-free IV approach. Figure 3.2 illustrates the idea behind the study.

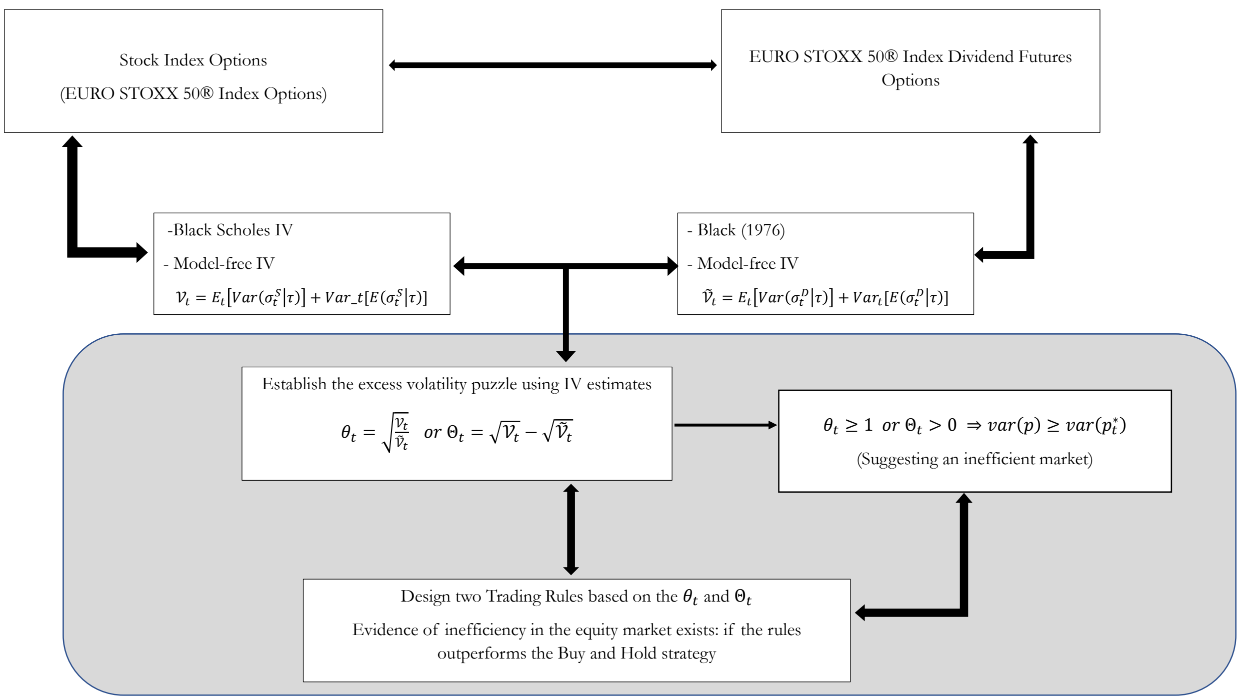


Figure 3.2: Conceptualisation of variance bounds inequalities in derivatives markets

3.2 Models and Methods

In this section, I discuss two different implied volatility estimation methods: the first is Bakshi et al. (2003) *model-free* and the Black-Scholes and Black (1976) *model-based* approaches. Under the model-based approach, I employ Brent (2013) derivative-free method to solve the convergence problem in the numerical iterations.

3.2.1 The model-free implied volatility

Our model-free IV estimation procedure closely follow Bakshi et al. (2003) using DeMiguel et al. (2013) interpolation procedure to increase my approximation accuracy.

I let S_t denote the stock price at time t and define the τ-period log-return $R_{(t,\tau)} = \ln\left(S_{(t+\tau)}/S_t\right)$. I further let $C_{(t,\tau,K)}$ and $P_{(t,\tau,K)}$ denote the stock-index call and put options prices; K and τ, respectively denote the strike price and time to maturity. For the IDF option market, $\widetilde{C}_{(t,\tau,\tilde{K})}, \widetilde{P}_{(t,\tau,\tilde{K})}, \widetilde{K}$, and τ denote call price, put price, strike and time to maturity respectively. The fair value of the variance contract in Bakshi et al. (2003) is $V_{(t,\tau)} \equiv E_t^*\{\exp(-r\tau)R^2_{(t,\tau)}\}$ with a price given by

$$V_{(t,\tau)} = \int_{S_t}^{\infty} \frac{2\left(1-\log\left(\frac{K}{S_t}\right)\right)}{K^2} \cdot C_{(t,\tau,K)} dK + \int_0^{S_t} \frac{2\left(1-\log\left(\frac{K}{S_t}\right)\right)}{K^2} \cdot P_{(t,\tau,K)} dK \tag{3.1}$$

$$\widetilde{V}_{(t,\tau)} = \int_{\widetilde{S}_t}^{\infty} \frac{2\left(1-\log\left(\frac{\widetilde{K}}{\widetilde{S}_t}\right)\right)}{\widetilde{K}^2} \cdot \widetilde{C}_{(t,\tau,\widetilde{K})} d\widetilde{K} + \int_0^{\widetilde{S}_t} \frac{2\left(1-\log\left(\frac{\widetilde{K}}{\widetilde{S}_t}\right)\right)}{\widetilde{K}^2} \cdot \widetilde{P}_{(t,\tau,\widetilde{K})} d\widetilde{K} \tag{3.2}$$

$\widetilde{V}_{(t,\tau)}$ denotes the variance contract in the dividend case with $\widetilde{S}_t \equiv F_t$, the stock index dividend futures with the same time to maturity τ as the option. From (3.1) the τ-period model-free IV can be calculated as $\sigma^{(mf)} = (V_{(t,\tau)})^{1/2}$ for stocks and $\widetilde{\sigma}^{(mf)} = (\widetilde{V}_{(t,\tau)})^{1/2}$, for dividends. The computation of IV uses out-of-the-money calls and puts with delta $\Delta_c < 0.5$ and $\Delta_p > -0.5$ respectively (see Britten-Jones & Neuberger 2000, DeMiguel et al. 2013).

Equations 3.1 and 3.2 uses the cross-section of both call and put option prices. The derivation first assumes the market agent always pays to take a long position in the volatility contract. To unwind the position, all out-of-the money puts and calls will be weighted by their strike-price-dependent values so that $2(1-\ln(K/S_t))/K^2$. The integral in (3.1) is approximated from the available option data after discretisation. For each maturity in my option data, an interpolation and extrapolation procedure for out-of-the-money call and put IV is done through cubic splines[5]. Given maturities and their corresponding interest rates, I estimate the final model-free IV's from the option prices calculated from the interpolated volatilities. There are two challenges to calculating the model-free implied volatilities, i.e. 1) truncation of the integration domain and 2) discretising the respective integrals while using a continuum of option prices. This continuum ranges from S_t to ∞. Options market prices are available for just a subset of this required range, and this is where the

[5]According to DeMiguel et al. (2013) using a narrower grid deteriorates accuracy of the approximations. I use 1001 points for the integral approximation.

truncation comes in. Jiang & Tian (2005) point to instances where options market prices are available in intervals of \$5.00 and options prices are uniquely identified by the available strikes. This observation also supports the use of discretisation and integration with respect to strike prices. Jiang & Tian (2005) and DeMiguel et al. (2013) also discuss how to deal with these two problems. The left-point rule of numerical integration can also be used. Integration of strike price allows us to calculate a single implied volatility for a range of strike prices compared to the Black-Scholes which generates one implied volatility for each available strike.

3.2.2 Implied volatilities from Black-Scholes and Black Models

An interesting approach to estimating volatility directly from recent data and then accounting for estimation uncertainty is described in Popovic & Goldsman (2012).

A wide range of closed-form approximations were also proposed over the years, with the method in Li (2008) as one of the best providing accurate computations for both at-the-money and out-of-the-money options, faster than standard solver algorithms. Homescu (2011) provided a survey of methodologies proposed for constructing implied volatility surfaces from options prices.

The theoretical Black-Scholes price of the stock index option is:

$$\nu_{S,\tau}^{(j)} = \chi(j) S_t e^{-q\tau} \Phi(\chi(j) d_1) - \chi(j) K e^{-r\tau} \Phi(\chi(j) d_2) \tag{3.3}$$

$$\chi(j) = \begin{cases} 1, & \text{if } j = c \\ -1, & \text{if } j = p \end{cases}$$

where $j \in \{c, p\}$, with c labelling the call option and p the put option, S is the price of the underlying asset, K is the option strike price, q is the dividend yield, r is the risk-free interest rate and $\tau = T - t$ denotes the time-to-maturity for both calls and puts. $\Phi(\cdot)$ is the cumulative normal distribution function. $d_1 = \frac{\ln(\frac{S}{K}) + (r - q + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}$ and $d_2 = d_1 - \sigma\sqrt{\tau}$.

For index dividend futures options, I employ the Black (1976) model leading to the formulae:

$$\widetilde{\nu}_{S,\tau}^{(j)} = e^{-r\tau} \left(\chi(j) F_t \Phi(\chi(j) \widetilde{d}_1) - \chi(j) K \Phi(\chi(j) \widetilde{d}_2) \right) \tag{3.4}$$

where F_t is futures[6] price of the underlying asset at time t, K is the option strike price, r is the risk-free interest rate and τ denotes the time-to-maturity for both cases. For Black (1976), $\widetilde{d}_1 = \frac{\ln(\frac{F}{K}) + \frac{1}{2}\sigma^2\tau}{\widetilde{\sigma}\sqrt{\tau}}$ and

[6] I set the dividend yield equal to the risk-free rate and the futures price as the stock price. Because the present value of the futures price is the prepaid forward price for the futures contract.

$\widetilde{d}_2 = \widetilde{d}_1 - \widetilde{\sigma}\sqrt{\tau}$. By definition, I expect the respective implied volatility $\sigma_t(K,T)$ and $\widetilde{\sigma}_t(K,T)$ to produce a theoretical Black-Scholes and Black (1976) option price which equals their market prices. Additionally, I note that the Black-Scholes call (put) option pricing formula is a decreasing function of implied volatility such that for all option prices ranging from $\lim_{\sigma \to 0^+} \nu^{(j)}_{(S,K,t;\sigma)} \equiv \max[0, S - Ke^{-r(\tau)}]$ to $\lim_{\sigma \to \infty} \nu^{(j)}_{(S,K,t;\sigma)} \equiv S$, Hence, a uniquely identifiable implied volatility is estimable.

The IV will equate the market price of the option to the Black-Scholes value and the Black (1976) value, respectively. IV is obtained by respectively inverting $\int_\Omega e^{r\tau}(S-K)^+ q[S]dS = \nu^{(j)}_{S,\tau}$ and $\int_\Omega e^{r\tau}(S - K)^+ q[F]dS = \widetilde{\nu}^{(j)}_{F,\tau}$; where $q(\cdot)$ is the risk-neutral density and $j = c$ in $\chi(j)$. Since I have no analytic solution to this, IV is calculated via numerical iterative technique where $h(\sigma^{(BS)}) = \nu_{(S,K,\sigma^{(BS)},q,r,t)} - \nu_{(mkt)} = 0$ for equity and $h(\widetilde{\sigma}^{(B)}) = \widetilde{\nu}_{(S,K,\widetilde{\sigma}^{(BS)},r,t)} - \widetilde{\nu}_{(mkt)} = 0$ for index dividend options. $\nu_{(S,K,\sigma^{(BS)},q,r,t)}$ and $\widetilde{\nu}_{(S,K,\widetilde{\sigma}^{(B)},r,t)}$ are the respective theoretical Black-Scholes and Black (1976) option prices, and $\nu_{(mkt)}$ and $\widetilde{\nu}_{(mkt)}$ are their respective market prices. I solve the convergence problem by using Brent (2013) derivative-free method, where

$$\begin{aligned}\sigma_{t+1} = &\frac{\sigma_t h(\sigma_{t-1})h(\sigma_{t-2})}{(h(\sigma_t) - h(\sigma_{t-1}))(h(\sigma_t) - h(\sigma_{t-2}))}\\ &+ \frac{\sigma_{t-1} h(\sigma_{t-2})h(\sigma_t)}{(h(\sigma_{t-1}) - h(\sigma_{t-2}))(h(\sigma_{t-1}) - h(\sigma_t))}\\ &+ \frac{\sigma_{t-2} h(\sigma_{t-1})h(\sigma_t)}{(h(\sigma_{t-2}) - h(\sigma_{t-1}))(h(\sigma_{t-2}) - h(\sigma_t))}\end{aligned} \tag{3.5}$$

in my procedure, the secant method

$$\sigma_{(t+1)} = \sigma_{(t-1)} - h(\sigma_{(t-1)}) \frac{\sigma_{(t-1)} - \sigma_{(t-2)}}{h(\sigma_{(t-1)}) - h(\sigma_{(t-2)})} \tag{3.6}$$

replaces the quadratic interpolation when $\sigma_t = \sigma_{t-1}$ in consecutive approximations.

3.2.3 Implied volatilities of aggregated data

I aim to provide a convenient approach by which information contained within multiple IVs can be efficiently aggregated and compared. Noting the fact that IV surface cross-section is a function of moneyness (m) and time-to-maturity ($\tau = T - t$), I also recognise that estimated IV could be conditioned on the same parameters at any given time. It is possible to have a situation whereby moneyness in the stock index option market does not directly correspond to moneyness in the index dividend option market. Conditioning on well defined categorisation of option maturity term-structure will permit aggregation and comparison of implied volatilities across different option market classes (see, Goncalves & Guidolin 2006, Bernales &

Guidolin 2014).

For each underlying asset, stock index or dividend index futures, I consider all European options with maturities categorised by the length of τ into three classes, short-term maturity,medium maturity and long-term maturity. In each class there is a varying but finite set of option prices with different strikes. I denote by $\mathcal{V}_t(\tau)$ and $\widetilde{\mathcal{V}}_t(\tau)$ the conditional total variance at time t of implied volatility of stock index and dividend index futures from respective τ category. Hence $\mathcal{V}_t(\tau)$ and $\widetilde{\mathcal{V}}_t(\tau)$ can be decomposed into two components:

$$\mathcal{V}_t = E_t[Var(\sigma_t^S|\tau)] + Var_t[E(\sigma_t^S|\tau)] \tag{3.7}$$

$$\widetilde{\mathcal{V}}_t = E_t[Var(\sigma_t^D|\tau)] + Var_t[E(\sigma_t^D|\tau)] \tag{3.8}$$

From (3.7) and (3.8), ${\mathcal{V}_t}^{1/2}$ and $\widetilde{\mathcal{V}}_t^{1/2}$ appropriately suffice as the standard deviation of total variance of implied volatilities. I define the Implied Volatilty Ratio (IVR) θ as;

$$\theta_t = \sqrt{\frac{\mathcal{V}_t}{\widetilde{\mathcal{V}}_t}} \tag{3.9}$$

which allows us to compare volatility across option maturity in the two markets. It is also possible to use Implied Volatility Differences (IVD) Θ as a basis to compare volatility across option maturities.

$$\Theta_t = \sqrt{\mathcal{V}_t} - \sqrt{\widetilde{\mathcal{V}}_t} \tag{3.10}$$

3.2.4 Trading strategies using the implied volatilities

One important argument supporting the theory that stock markets are not efficient is the existence of trading strategies that are superior to a buy and hold strategy. Volatility derivatives markets are the latest fast expanding markets in the derivatives asset class spectrum. General principles and many detailed insights into trading volatility are presented in Gatheral (2011) for a more quantitative approach and Rhoads (2011) for a more investment strategy approach.

The trading strategy outlined in this section will be applied to my in-sample trading dataset and its performance will be assessed using an out-of-sample dataset. The criteria for splitting the data is described in the results section. The motivation for this section is to investigate whether the information on implied volatility ratios defined above holds useful trading information. In this regard, I first ascertain the single trading horizon at which point the correlation between the trading signal and the trading threshold is optimal.

For any given signal[7] Ψ_t where $t = 1, \ldots, T$, I calculate the surface of all correlation coefficients between $\left(1 - \frac{\Psi_t}{MA(N)(\Psi)_t}\right)$ and moves in the stock log-return r_{t+k}, where $MA(N)(\Psi)_t$ denotes the N step moving average of the series $\{\Psi\}_{t \geqslant 0}$ at time t.

Note that by taking 1 minus the trading signal process, a positive correlation is in fact equivalent to a negative correlation between implied volatility ratio (signal versus threshold) as a trading vehicle and corresponding stock return. I let $r_t^{strategy}$ denote the time series of returns generated by the implemented trading strategy, where $t = 1, \ldots, T$ and T is the size of calculation window for the returns space.

The analysis of the annualized performance of each trading strategy follows DeMiguel et al. (2013) and is based on: (i) out-of-sample trading strategy volatility, and (ii) out-of-sample Sharpe ratio(SR), (iii) average return $\widehat{\mu}$ multiplied by 252, (iv) certainty-equivalent (CE) return, and (v) the drawdown-based Calmar ratio (CR) performance measure:

$$\widehat{\sigma}^{strategy} = \left[\frac{252}{T-1} \sum_{t=1}^{T} \left(r_t^{strategy} - \widehat{\mu}^{strategy}\right)\right]^{1/2} \tag{3.11}$$

$$\widehat{SR}^{strategy} = \frac{\widehat{\mu}^{strategy}}{\widehat{\sigma}^{strategy}} \tag{3.12}$$

$$\widehat{\mu}^{strategy} = \frac{252}{T} \sum_{t=1}^{T} r_t^{strategy} \tag{3.13}$$

$$\widehat{CE}^{strategy} = u^{-1}\left(\frac{1}{T} \sum_{t=1}^{T} u\left(r_t^{strategy}\right)\right) \tag{3.14}$$

$$\widehat{CR}^{strategy} = \frac{r^{strategy} - r_f}{-\max\{Drawdown\}} \tag{3.15}$$

where u is the power utility function, with a relative risk aversion value of 1 and $Drawdown$ is the number of negative returns.

For each strategy, I also analyse value at risk performance measures using the standard $\text{VaR}_\alpha = -\left[r_t + z_\alpha \cdot \hat{\sigma}\right]$. In addition, I also test whether the difference between the Sharpe ratios of the respective trading strategies are statistically significantly better than the benchmark strategy. This test is based on the Ledoit & Wolf (2008) statistic that provides robustness when returns are time series in nature or have heavier tail than the normal distribution[8]. The null hypothesis is $H_0 : \Delta = 0$ where $\Delta = \mu_j/\sigma_j - \mu_n/\sigma_n$ is the difference between two Sharpe ratios. The two trading strategies are denoted by j and n, with the buy and hold strategy denoted by n, and their respective sample average returns and standard deviations are $\widehat{\mu_j}$, $\widehat{\mu_n}$ and

[7]The trading signals are generated from the time-varying model-free and model-based IV differences which are computed from the series of implied volatility surfaces that span the sample period.

[8]Ledoit & Wolf (2008) provide detailed explanations on how to improve robustness of the Sharpe ratio test statistic using bootstrapping and Heteroscedastic autocorrelated consistent (HCA) covariance matrix estimation inference. There are other implementations of the Sharpe ratio test also in Bilson et al. (2015) which is based on Jobson & Korkie (1981) Z-test statistic, Barras et al. (2010), and Ardia & Boudt (2018).

$\widehat{\sigma}_j, \widehat{\sigma}_n$. The test statistic d is specified as

$$d = \frac{|\widehat{\Delta}|}{s(\widehat{\Delta})} \tag{3.16}$$

where the standard error of $\widehat{\Delta}$ is $s(\widehat{\Delta})$ is well defined and elaborated in Ledoit & Wolf (2008). DeMiguel & Nogales (2009) and Ardia & Boudt (2018) respectively adopted the approach to assess stability properties of portfolio selection methods and performance of hedge funds. Suppose I denote by $\widehat{F}$, the empirical distribution function of the B bootstrap pairs that correspond to $\widehat{\mu}_j/\widehat{\sigma}_j - \widehat{\mu}_n/\widehat{\sigma}_n$; then a two-sided p-value[9] for (3.16) is given by $\widehat{p} = 2\widehat{F}(0)$. Ledoit & Wolf (2008) showed that the bootstrapping procedure that is based on sampling with replacement allows us to resample from the observed return series thereby offering superior benefits over some other existing methods that only offer null-restrictions to data.

Finally, I calculate the bootstrap[10] test statistic for the Linton et al. (2005) second-order stochastic dominance (SSD) test. The purpose is to determine the trading strategy return series that is preferred by any given risk averse investor irrespective of their preferences. The null hypothesis of the SSD test is $H_0 : j \succ_2 n$ where $\succ_2$ indicates that trading strategy j stochastically dominates the buy and hold strategy n. A rejection of this null suggests that any strong ordering by j and n that correspond to specific utility functions $\mathcal{U}_j$ and $\mathcal{U}_n$ will not enjoy general acceptance (see Linton et al. 2005).

3.3 Computational Results

This section presents the results of the computational procedure outlined in section 3.2. The section also implements and assess the performance of trading strategies that depend on the information held in the term structure implied volatility ratios.

3.3.1 Data sets

The data used in this analysis consists of daily trading information on the Dow Jones Euro Stoxx50®, which is called in this paper EURO STOXX 50 for brevity, the EURO STOXX 50 dividend futures option prices and the EURO STOXX 50 option prices. The time series are from May 20, 2010 to August 14, 2018, giving a

[9] Ledoit & Wolf (2008) also developed a studentized time series bootstrap confidence interval for Δ such the two ratios are different if zero is not contained in the obtained interval

[10] Linton et al. (2005) provide a nice discussion on the reasons why subsampling improves the power of the SSD test.

total of 2,098 trading days[11]. The index dividend futures option data were obtained from the Eurex exchange while the stock index option data is from OptionMetrics.

I exclude options prices with extreme moneyness, and those that violate the known no-arbitrage conditions. Our filtering conditions follow Bakshi et al. (1997), DeMiguel et al. (2013) and Fabozzi et al. (2014). I also filter out observations were $1.7 < \frac{C_t(K,\tau)-P_t(K,\tau)}{F_t(\tau)-K} < 0.3$ and $F_t(\tau) = K$. This allows us to control for impact of noise from unreliable sample points based on moneyness[12]. Both index dividend and stock index options have moneyness definition set at $m = K/F_t - 1$ following Fabozzi et al. (2014). In addition, I exclude options with implied volatilities iv less than 1% and greater than 110%; options with less than 7 days-to-maturity(DTM (τ), hereafter) are also filtered out since they have stronger sensitivity to the slightest error in option prices.

Following Goncalves & Guidolin (2006), Fabozzi et al. (2014), Wang et al. (2017), I construct option subsamples based on time-to-maturity range $16 < \tau \leqslant 24$, $24 < \tau \leqslant 60$, $60 < \tau \leqslant 180$, and $180 < \tau \leqslant 360$ for the index dividend and stock index option datasets. The resulting output is a cross-section of call and put option contracts with different strike prices observed for any selected trading day. I use variable interest rates which are linearly interpolated from zero-coupon Euro-currency denominated Treasury yields from OptionMetrics. Interest rates are continuously compounded.

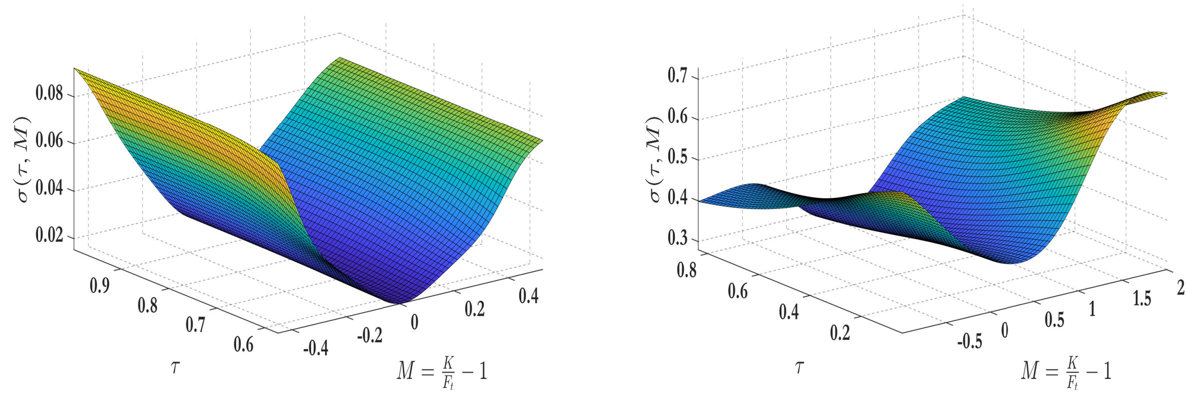
3.3.2 Implied volatility surface plots

Figure 3.3 presents implied volatility surface plots of a randomly selected mid-year trading day from my sample. Black (1976) model implied volatility plots for May 21, 2011 presented in Figure 3.3a and the Black-Scholes implied volatility surface plots are presented in Figure 3.3b using same selected day The expected smoothness in IDF implied volatility is evident in the plots. The surface plot of index-dividend futures is smoother with less humps.

Periods half-way around June indicate the point of inflection on the sigmoidal nature of historical plots of index dividend points. This is a feature of the dividend markets, with the majority of the companies paying dividends around the end of the financial tax year. Figure 3.3 illustrate how IV surface of equity consistently lies above its dividend counterpart for a given day. A test for the presence of mean reversion

[11] I excluded data in the range $t < 2010$ because it was unavailable

[12] Moneyness classification (m) is constructed as follows: Deeper-out-of-the-money ($m < -0.15$ for puts and $m > 0.15$ for calls); Deep-out-of-the-money ($(-0.15, -0.10]$ for puts and $(0.10, 0.15]$ for calls); for Out-of-the-money ($(-0.10, -0.05]$ for puts and $(0.05, 0.1]$ for calls); at-the-money ($[-0.05, 0.05]$); in-the-money ($(0.05, 0.1]$ for puts and $(-0.10, -0.05]$ for calls); deep-in-the-money ($(0.10, 0.15]$ for puts and $(-0.15, -0.10]$ for calls) and deeper-in-the-money ($m > 0.15$ for puts and $m < -0.15$ for calls). Goncalves & Guidolin (2006), Fabozzi et al. (2014), Wang et al. (2017).



(a) Index-dividend: (21/05/2010)

(b) Stock-index: (21/05/2010)

Figure 3.3: Implied volatility surface for equity and dividends associated with STOXX 50 using daily data between May 21, 2010 to June 08, 2011.

Notes: Graphical plots (a) and (b) are index-dividend futures implied volatilities surface and stock-index implied volatility surface for 21/05/2010. The trading day was randomly selected from the sampling period May 21, 2010 - June 08, 2011. (a) is from Black (1976) model and (b) is from Black-Scholes model

effect in the ratios confirms the mean-reversion characteristic. Only ATM implied volatilities are used in the estimation process. Most of the action is in the first half of the year with the ratio falling to low flat levels for six months horizon.

Figure 3.5 plots the surface graph of IVD (Θ) for two randomly selected trading dates in the dataset i.e May 21, 2010 and June 08, 2011. The corresponding IVR (θ) surface plots for the same trading dates are presented in Figure 3.4. The implied volatility ratio values are strictly greater than 1 with respective maximum of 25.09 and 6.58 in May 21, 2011 and June 8, 2011. The respective minimum values for both days are 4.33 and 1.89. Table 3.1 presents summary results describing the characteristics of volatility ratio equation in (3.9) for model-based implied volatilities. For nearest maturity options, the variability in stock prices are at least 2 times larger than the variability in dividends.

The magnitude of the ratio rises gently through medium-term maturity options (i.e. 1-3 month maturity options); and reaches its maximum-point around (3-9 month maturity options), where variability in stocks is at least 5.3 times larger than the variability in dividends. Furthermore, I observe a decline in the θ for long-dated options and it is clear that $0 \leqslant \theta \leqslant 1$ for options with maturities exceeding 9 months-to-maturity. The model-free implied volatility ratios are presented alongside the model-based results as bold figures.

The results on the model-free implied volatility shows that the variability in stock prices could be at least 32 times larger than the variability in dividends in short-dated maturities, thereby suggesting that stock index implied volatility moves too much to be associated with index dividend implied volatilities. The magnitude of the ratio remains high for 1-3 and 3-9 month maturity options hence suggesting that findings

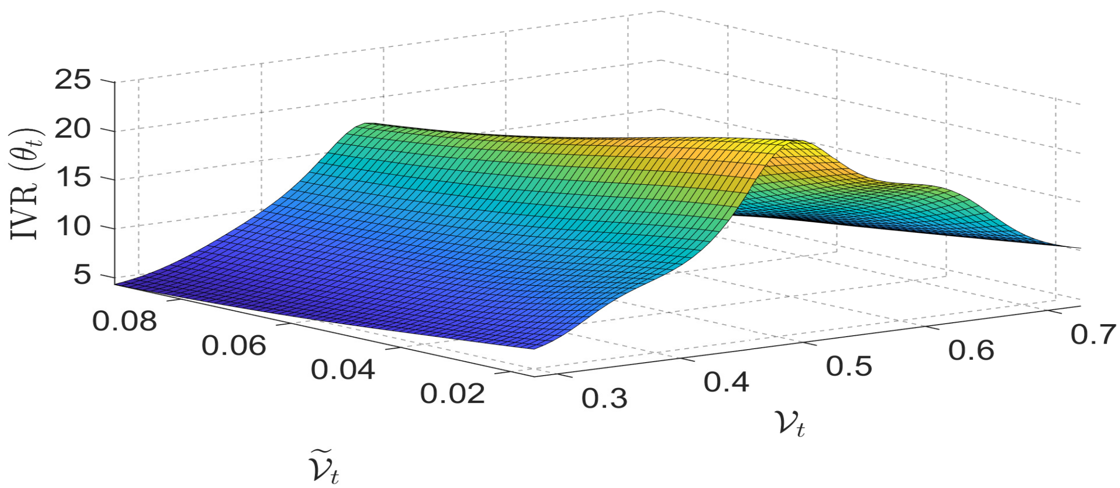


Figure 3.4: Surface plot of the ratio of model-based implied volatilities of stocks to dividends, using daily data between May 21, 2010 to June 8, 2018.
Notes: The IVR (θ) are obtained from (3.9) using the implied Black-Scholes volatility for numerator and implied Black (1976) model volatility for denominator. Only ATM implied volatilities are used.

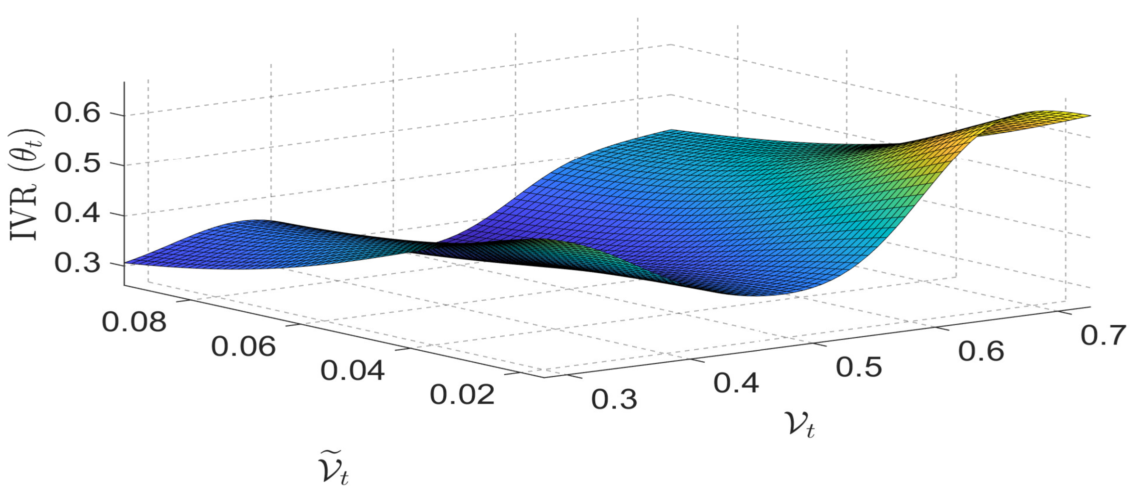


Figure 3.5: Surface plot of the ratio of model-based implied volatilities of stocks to dividends, using daily data between May 21, 2010 to June 8, 2018.
Notes: The IVD (Θ) are obtained from (3.10) using the implied Black-Scholes volatility for numerator and implied Black (1976) model volatility for denominator. Only ATM implied volatilities are used.

in short-dated maturities still persist in the medium-term maturity case. Despite this observation, I find a dramatic decline in the volatility ratio for long-dated options; suggesting that derivatives investors expect dividend volatility to be more in tune with the volatilities of stock index option.

Table 3.1: Implied volatility ratios between stock and dividend markets.

	Stock		**Dividend**	
Panel A: Comparing volatility across τ using model-based and model-free implied volatilities				
TTM	$\sqrt{\mathcal{V}}$	$\sqrt{\mathcal{V}^{(mf)}}$	$\sqrt{\widetilde{\mathcal{V}}}$	$\sqrt{\widetilde{\mathcal{V}^{(mf)}}}$
$16 < \tau \leqslant 24$	0.154	0.090	0.080	0.001
$24 < \tau \leqslant 60$	0.160	0.087	0.050	0.002
$60 < \tau \leqslant 180$	0.156	0.073	0.029	0.034
$180 < \tau < 360$	0.128	0.054	0.214	0.202
Panel B: Implied Volatility Differences (IVD) and Bootstrap Confidence Intervals on Θ				
	IVD (Θ)		**Bootstrap Conf. interval for IVD**	
TTM	Θ	$\Theta^{(mf)}$	**95% CI for** Θ	**95% CI. for** $\Theta^{(mf)}$
$16 < \tau \leqslant 24$	0.074	0.089	$[0.066, 0.083]$	$[0.087, 0.089]$
$24 < \tau \leqslant 60$	0.111	0.085	$[0.107, 0.114]$	$[0.084, 0.085]$
$60 < \tau \leqslant 180$	0.128	0.039	$[0.126, 0.130]$	$[[0.036, 0.041]$
$180 < \tau < 360$	-0.086	-0.148	$[-0.087, -0.086]$	$[-0.149, -0.147]$

Notes: $\mathcal{V}(t)$ and $\widetilde{\mathcal{V}(t)}$ respectively denote the total variance of IV for stock and dividend markets defined in (3.10). Superscript (mf) denotes corresponding model-free estimates. The respective bootstrap confidence intervals for implied volatility differences Θ are reported.

The model-based implied volatilities are observed to be significantly higher than the model-free estimates for index options as well as dividend options. This is because I partition[13] the option dataset into four (4) non-overlapping categories using TTM τ. On each given day of the dataset, the model-based IV procedure generates an implied volatility estimate for each unique strike price that is available in each group G, while the model-free IV technique integrates across all the strike prices available within a given group $G_j,\ j = 1, \ldots, 4$, thereby generating a single IV. In this regard, the total variability of IV $\mathcal{V}_j$ within each $G_j,\ j = 1, \ldots, 4$ is higher for the model-based IV compared to the model-free case. Goncalves & Guidolin (2006), Fabozzi et al. (2014), Wang et al. (2017) present similar partitioning scheme for IV of stock index options.

Table 3.1 also displays the model-free implied volatility of dividend options steadily declining as TTM falls from six months to 16 days. The model-based estimate has a similar pattern. The excess volatility puzzle identified by Shiller (1981) can be interpreted as evidence against the efficiency of trading on stock markets. The lack of market efficiency opens the door for identifying profitable trading strategies. In the next section I will try to identify some trading strategies using the implied volatility information as signals

[13]As outlined previously, the partitioning spans contracts with $16 < \tau \leqslant 24, 24 < \tau \leqslant 60, 60 < \tau \leqslant 180$, and $180 < \tau \leqslant 360$ TTM thereby creating four groups denoted by G_1, G_2, G_3, and G_4. The total number of contracts in each group is respectively written as $N_{g_1}, N_{g_2}, N_{g_3}$, and N_{g_4}. There are $\tilde{k}$ unique strike prices in each $G_j, j = 1, \ldots, 4$.

and compare those strategies with a standard buy and hold benchmark. If my strategy, based on a forward looking signal extracted from options, beats the buy and hold strategy this is further support for the excess volatility puzzle.

Table 3.2: Normality test on the log of index dividend futures prices.

Day	30/11/2011		25/11/2012		26/11/2013	
Test/ TTM	17	24	17	24	17	24
KS Limiting Form	3.21***	3.735***	5.512***	5.453***	2.145***	2.346***
KS Stephens Modification	3.224***	3.885***	5.529***	5.47***	2.253***	2.445***
KS Marsaglia Method	3.21***	3.735***	5.512***	5.453***	2.145***	2.346***
KS Lilliefors Modification	0.252***	0.835***	0.36***	0.363***	0.536***	0.538***
Anderson-Darling Test	16.308***	20.504***	29.254***	28.352***	5.911***	7.093***
Shapiro-Wilk Test	0.741***	0.013***	0.719***	0.707***	0.273***	0.244***
Shapiro-Francia Test	0.747***	0.113***	0.723***	0.711***	0.243***	0.216***
Jarque-Bera Test	37.459***	6.667***	37.932***	55.634***	116.492***	207.243***
Dagostino & Pearson Test	27.42***	55.869***	34.996***	38.83***	42.972***	49.528***
Day	24/04/2014		05/11/2015		21/08/2018	
Test/ TTM	17	24	17	24	17	24
KS Limiting Form	2.334***	1.837***	3.317***	3.865***	2.752***	2.281***
KS Stephens Modification	2.574***	1.962***	3.386***	3.922***	2.957***	2.383***
KS Marsaglia Method	2.334***	1.837***	3.317***	3.865***	2.752***	2.281***
KS Lilliefors Modification	0.825***	0.53***	0.538***	0.536***	0.83***	0.538***
Anderson-Darling Test	8.456***	4.323***	14.509***	19.944***	11.446***	6.699***
Shapiro-Wilk Test	0.281***	0.327***	0.152***	0.122***	0.091***	0.253***
Shapiro-Francia Test	0.5***	0.295***	0.133***	0.106***	0.091***	0.224***
Jarque-Bera Test	2.667	43.322***	1948.913***	5214.998***	3.667	173.076***
Dagostino & Pearson Test	11.413***	33.648***	88.242***	116.267***	17.439***	47.372***

Notes: 17 and 24 days are randomly selected time-to-maturity (TTM).

I present the normality test results for the log of index dividend futures prices in Table 3.2 where Normality is consistently rejected by a large set of normality tests in each trading day near maturity of contract. Rejection of normality suggest that the Black (1976) may not accurately display the set of remarkable characteristics of the index dividend futures options like the 'pull-to-par' effect reflected on implied volatility.

3.3.3 Model-free trading Strategy for predictive peak for difference in implied volatilities

The predictive peak is determined numerically and it is shown in Figure 3.6 occurs with $N = 111$ model-free implied volatility differences (MFIVD) moving average (trading) days and $K = 50$ (trading) days of STOXX50 index log-returns.

Using the identified correlation peak, I implement the first trading strategy by focusing on periods when the current MFIVD crosses its own moving average threshold for 111 days. I go long in the stock and

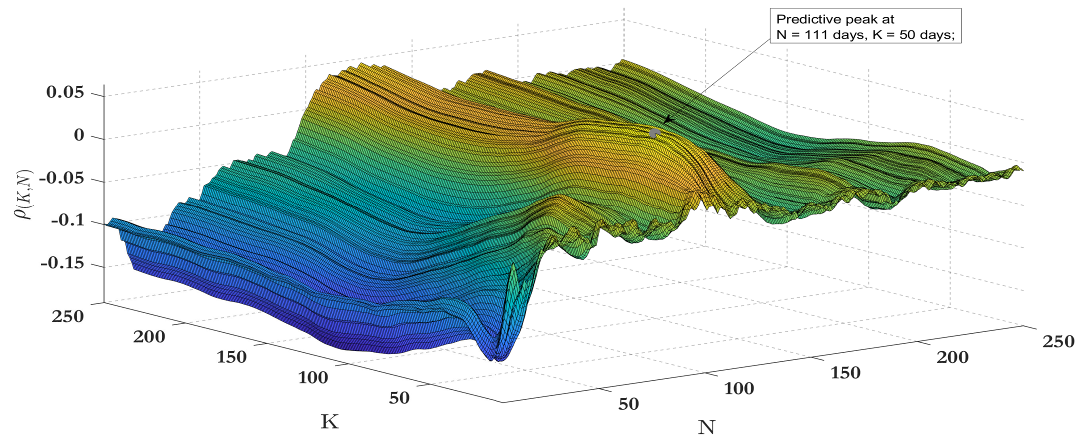


Figure 3.6: Correlation of model-free IVD level relative to N-day moving average versus K-days moving average for STOXX 50 log-returns.

Notes: Relative level is defined as the ratio of the model-based IVD to its own N-day moving average minus one. K is the k-day out log-return of STOXX 50 index which I define as $r_{t+k} = ln(\frac{P_{t+k}}{P_t})$ where $t = 1, \ldots, T$ indicates daily trading days and $k = 1, \ldots, 250$ is the index for observed number of days out. N is the n-day moving average for the model-based implied volatility difference (IVD) signal

short in the futures whenever the MFIVD crosses below its moving average and close the long position and go short in the stock and long in futures contract as soon as the MFIVD crosses above its moving average (see similar strategies also in Brock et al. 1992, Williams 2011). This is based on the observed negative correlation between IV and the stock index. One can see the evolution of MFIVD, its moving average and stock in Figure 3.7

In order to avoid problems of not being able to pay for closing trades, every trade is carried out for a proportion Q of the total balance in the money market account I hold for these trades. For all trading strategies considered, the signal is denoted by $\{\Psi_t\}_{t\geqslant 0}$. The dynamic threshold is $\{\Gamma_t\}_{t\geqslant 0}$ alongside a tradeable liquid asset $\{A_t\}_{t\geqslant 0}$. Let's denote by $\tau_1, \tau_2, \ldots, \tau_n$ the grid of points where $\Psi_t - \Gamma_t$ changes signs, that is when the signal crosses over the threshold. All trading strategy will have an initial endowment of 10 million EUR that will be put into a money account $\{C_t\}_{t\geqslant 0}$ accruing money market interest r. I consider the 12 month Euribor rates in the analysis for interest rate calculations.

The trading strategy is based on the idea that MFIVD and STOXX 50 are negatively correlated. Hence by monitoring current MFIVD versus its moving average, I can see when the difference in implied volatilities of stock and dividend is high or low. If it is high then that means the stock price is low now and at some time in the near future the IV of stocks will go down so the MFIVD will be low and hence the stock price will be high. This means that when MFIVD is above its threshold I buy the stock, and hedge at the same time by going short in the nearest maturity index dividend futures contract. I will unwind this position

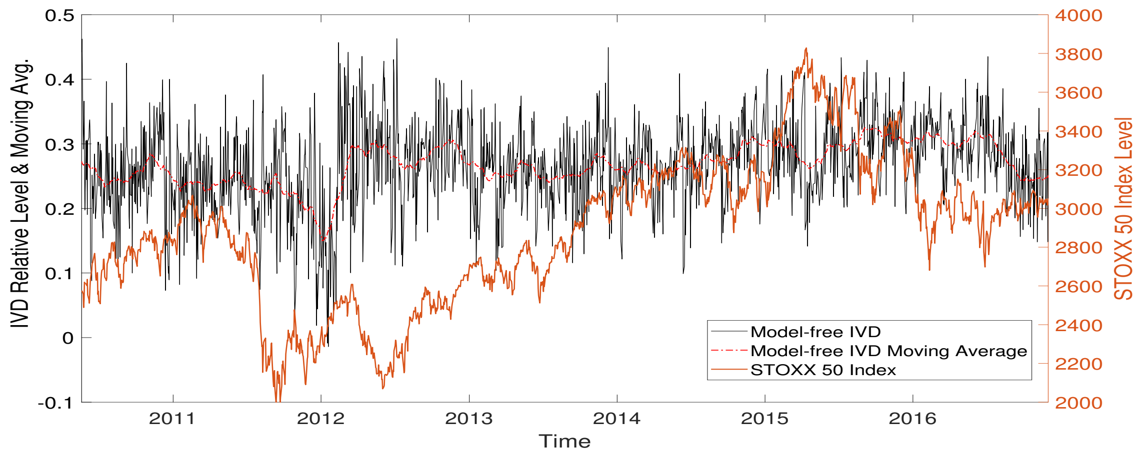


Figure 3.7: Comparison of Model-free IVD and STOXX 50 using daily data from May 20, 2010 to December 31, 2016

and sell the stock later on when the MFIVD goes below its threshold; at this point I immediately close the hedge and go long in the nearest maturity futures contract. Thus, the investor could go long the asset A by buying stock equivalent to 10% of the value of in the money account at the time of transaction, τ_i, hence $0.1 \times C_{\tau_i}$, if $\Psi_{\tau_i} - \Gamma_{\tau_i} > 0$ and initiate a short hedge using the nearest maturity index dividend futures contract. To unwind, the investor will sell the entire stock position at τ_i and also close out the futures hedge at the same time. This involves going short by selling $0.1 \times C_{\tau_i}$ in stock if $\Psi_{\tau_i} - \Gamma_{\tau_i} < 0$ and then unwind at τ_{i+1} by buying back stock to cancel their short position. Suppose the spot prices of the index at τ_i and τ_i are respectively denoted by S_{τ_i} and $S_{\tau_{i+1}}$ and the corresponding futures prices are F_{τ_i} and $F_{\tau_{i+1}}$. The futures position is $F_{\tau_i} - F_{\tau_{i+1}}$, and the stock position is $S_{\tau_i} e^{r(\tau_{i+1}-\tau_i)} - S_{\tau_{i+1}}$.

The transaction is followed through only when the absolute difference $|\Psi_{\tau_i} - \Gamma_{\tau_i}| > 0.01$ in order to avoid transaction costs for trades that are not economically significant. Unlike similar excess volatility trading strategies in Bulkley & Tonks (1989) and Bulkley & Tonks (1992), I are able to use *a priori* trading strategies that involve going *short*. Here, the basis risk between borrowing and lending rates is not significant since the trading is done daily and hedged with a Futures trade[14]. I test the quality of the trading strategies for the MFIVD by comparing the performance of their associated portfolios to that of a benchmark[15] portfolio which is based on a long-only trading strategy. The buy and hold strategy of the benchmark becomes a base model that allows us to evaluate performance only in terms of the trading signal dynamics (see Brock et al. 1992, Lo & Patel 2008).

[14]The transaction fees for cash settlement is on the Eurex Exchange is EURO 0.35 per contract. Position closing adjustments is EUR 0.70 per contract.

[15]The initial trade is for a proportion of Q of the total balance in the money account I hold. At inception, the investor goes long on the asset, and buys stock equivalent to Q% of the total value held in the money account; $Q \times C_{\tau_0}$, where $0.1 \leqslant Q \leqslant 1$. This initial position is held till end of the sample period.

Table 3.3 reports the standard comparative performance measures for various variants of this strategy under unique market regimes spanned by risk-free rate values and proportions of investments. The annualized return of wealth realised under the MFIVD trading signal consistently exceed that of the buy and hold benchmark. The annualized volatility of returns generated by the MFIVD trading strategy is lower compared to the buy and hold. This observation remains consistent across varying proportions of investment except for cases where $Q = 5\%$. The annualized Sharpe ratios calculated under buy and hold benchmark consistently lie below that of MFIVD trading rule across all Q's.

I use the Shape ratio test proposed by Ledoit & Wolf (2008) to verify whether differences in Sharpe ratios generated by the two strategies are statistically significant. Table 3.3 shows that the annualized Sharpe ratio of the MFIVD trading strategy is significantly higher than those generated by the buy and hold benchmark when $Q \leqslant 50\%$. I also report test statistics of Linton et al. (2005) second order stochastic dominance (SSD) test. The SSD test is to determine which trading rule generates stochastically dominant wealth preferred by any risk averse investor, irrespective of their preferences. I fail to reject the null hypothesis of the SSD test results, suggesting that the MFIVD strategy stochastically dominates the buy and hold benchmark. The statistical dominance of the MFIVD is robust for varying Q's.

Table 3.3 also presents the Calmar ratios of the MFIVD and buy and hold across different investment proportions. The MFIVD performs better on risk adjusted basis compared to the buy and hold. The certainty equivalent (CEQ) values of the MFIVD strategy are consistently lower, compared with the annualized return of the MFIVD strategy across all Q's thereby indicating a *risk averse* market agent. The difference between the strategy's annualized return and the market agent's CEQ provides information about the risk premium, which ranges between 0.1% and 0.2%.

Table 3.3: Comparison of trading strategy performance and risk using data the differences in model-free IV as trading signal.

Performance Measure	Series	$Q = 5\%$	$Q = 20.0\%$	$Q = 35.0\%$	$Q = 50.0\%$	Q=65.0%	$Q = 80.0\%$	$Q = 95.0\%$
Ann. Return	BH	0.005	0.008	0.012	0.015	0.019	0.022	0.022
	$MFIVD$	0.062	0.063	0.054	0.062	0.055	0.057	0.058
Volatility	BH	0.011	0.042	0.075	0.108	0.141	0.174	0.206
	$MFIVD$	0.033	0.034	0.044	0.060	0.074	0.088	0.102
Historical VaR	BH	-0.001	-0.004	-0.008	-0.011	-0.015	-0.018	-0.022
	$MFIVD$	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Sharpe Ratio	BH	0.422	0.201	0.159	0.142	0.134	0.128	0.105
	$MFIVD$	1.859	1.857	1.212	1.033	0.745	0.643	0.567
Calmar Ratio	BH	0.189	0.117	0.095	0.085	0.080	0.078	0.065
	$MFIVD$	15.240	3.238	1.015	0.549	0.381	0.291	0.251
CEQ	BH	0.005	0.008	0.012	0.015	0.019	0.022	0.021
	$MFIVD$	0.060	0.061	0.052	0.060	0.054	0.055	0.056
Sharpe Ratio Test	d	1.322*	2.964***	1.865**	1.542*	1.031	0.838	0.720
SSD Test	$MFIVD >_2 BH$	0.723	0.817	0.843	0.847	0.837	0.787	0.803

Notes: Standard summary performance measures for the trading strategy using Implied Volatility Differences (IVD) i.e. $MFIVD = MFIV_s - MFIV_d$ as signal. The trading signal is based on a 111-day moving average of the model-free IVD. Q is a proportion of the total balance in the money account I hold. I report the p-values of the Linton et al. (2005) second order stochastic dominance (SSD) ratio test statistic. The null hypotheses under the SSD test is $H_0 : MFIVD >_2 BH$ suggesting that the trading strategy $MFIVD$ based on model-free IVD second order stochastic dominates the buy and hold strategy BH. The test statistic of the Ledoit & Wolf (2008) Sharpe ratio test is $d = |\widehat{\Delta}|/s(\widehat{\Delta})$ where Δ denotes the difference between the Sharpe ratios of MFIVD and buy and hold benchmark.

3.3.4 Model-based trading Strategy for predictive peak for difference in implied volatilities

I repeat the steps followed above for model-free signal using the difference of the model-based IV of stock to dividend as my trading signal. The predictive peak of the correlation matrix shown in Figure 3.8 occurred for $N = 88$ days of the moving average and $K = 156$ days for the returns horizon. It is known from mathematical works of Jacod & Shiryaev (2013), that predicting or calling out the actual peak is difficult if not impossible while observing the actual random data flow in real time. The trading strategy tries to select such best minimal and maximal price points to buy and sell and make an arbitrage profit. Figure 3.9 shows the evolution of this new trading signal versus its threshold and the STOXX 50. The trading strategy in this case is based on selling STOXX 50 when the MBIVD is below its 88-days moving average and unwind the trade when the order between the two reverses. The values in Table 3.3 indicate that, depending on the period and the style of investing, prudent versus aggressive. Periods of substantially high MBIVD in Figure 3.9 are associated with lower Sharpe ratios in the trading strategy.

In Table 3.4 I report the comparative performance measures under various variants of this trading

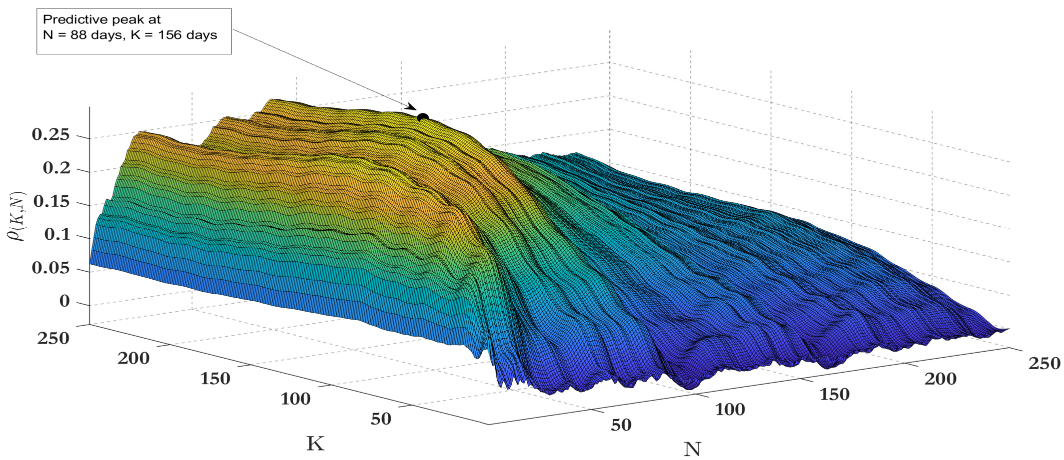


Figure 3.8: Correlation of model-based IVR level relative to N-day moving average versus K-days moving average for STOXX 50 log-returns.

Notes: Relative level is defined as the ratio of the model-based IVR to its own N-day moving average minus one. K is the k-day out log-return of STOXX 50 index which I define as $r_{t+k} = ln(\frac{P_{t+k}}{P_t})$ where $t = 1, \ldots, T$ indicates daily trading days and $k = 1, \ldots, 250$ is the index for observed number of days out. N is the n-day moving average for the model-based implied volatility ratio (IVR) signal

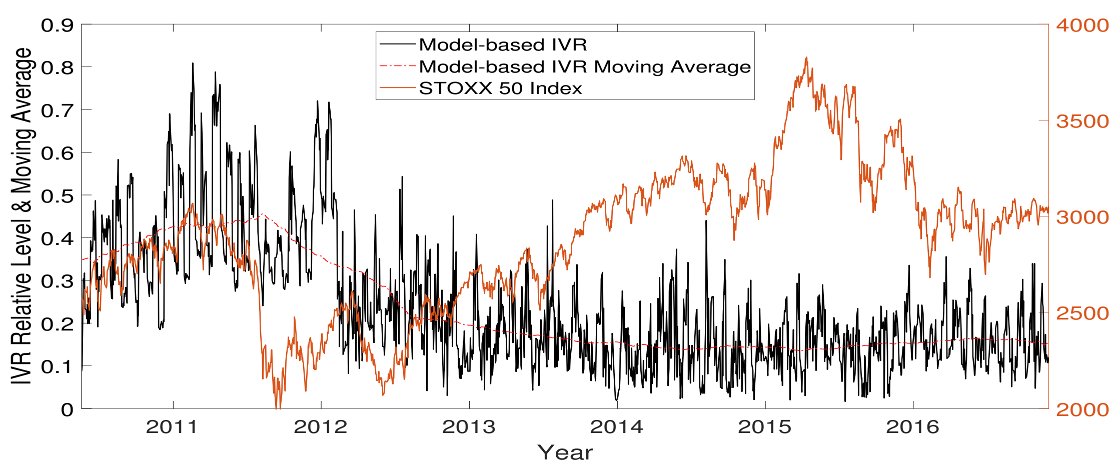


Figure 3.9: Comparison of Model-based IVR and STOXX 50 using daily data from May 20, 2010 to December 31, 2016

Notes: The graph illustrates historical evolution of Black-Scholes implied volatility ratio together with its 136-day moving average. This is plotted alongside the STOXX 50 index level. The data is from May 20, 2010 to December 31, 2016.

strategy. The results are largely consistent with that of the MFIVD trading strategy. I do not reject the null hypothesis of the SSD test results across all investment proportions. Hence, the MBIVD trading strategy stochastically dominates the buy and hold benchmark. The annualized certainty equivalence (CEQ) of wealth generated under the MBIVD trading rule generally exceed that of the buy and hold. The market agent's risk premium has a minimum of 2% and a maximum of 3%. The Sharpe ratio's of the MBIVD trading strategy exceeds that of the buy and hold benchmark across all investment proportions. Except for $Q = 95\%$, the Ledoit & Wolf (2008) test statistic shows annualized Sharpe ratio of MBIVD is significantly higher than that of the buy and hold benchmark. Average annual returns of realized wealth under the MBIVD are substantially higher when compared to the buy and hold. The annualized volatility of the buy and hold benchmark is approximately 1.4 to 2.5 times larger than the annualized volatility of the MBIVD strategy.

Table 3.4: Comparison of trading strategy performance and risk using data the differences in model-based IV as trading signal.

Performance measure	Series	$Q = 5\%$	$Q = 20\%$	$Q = 35\%$	$Q = 50\%$	$Q = 65\%$	$Q = 80\%$	$Q = 95\%$
Ann. Return	BH	0.005	0.008	0.012	0.015	0.019	0.022	0.022
	$MBIVD$	0.064	0.055	0.066	0.072	0.069	0.078	0.076
Volatility	BH	0.011	0.042	0.075	0.108	0.141	0.174	0.206
	$MBIVD$	0.032	0.031	0.041	0.050	0.062	0.074	0.082
Historical VaR	BH	-0.001	-0.004	-0.008	-0.011	-0.015	-0.018	-0.022
	$MBIVD$	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Sharpe Ratio	BH	0.422	0.201	0.159	0.142	0.134	0.128	0.105
	$MBIVD$	1.973	1.773	1.620	1.442	1.124	1.055	0.927
Calmar Ratio	BH	0.189	0.117	0.095	0.085	0.080	0.078	0.065
	$MBIVD$	57.709	4.926	4.154	3.245	1.472	1.601	1.196
Certainty Equivalence	BH	0.005	0.008	0.012	0.015	0.019	0.022	0.021
	$MBIVD$	0.062	0.053	0.064	0.069	0.067	0.075	0.073
Sharpe Ratio Test	d	1.378*	2.798***	2.576***	2.259**	1.694*	1.563*	1.351
SSD Test	$MBIVD \succ_2 BH$	0.763	0.830	0.817	0.810	0.843	0.830	0.827

Notes: Standard summary performance measures for the trading strategy using implied volatility differences (IVD) i.e. $MBIVD = MBIV_s - MBIV_d$ as signal. Every trade is carried out for a proportion Q of the total balance in the money account held for respective trades. The trading signal is based on a 88-day moving average of the model-based IVD. I report the p-values of the Linton et al. (2005) second order stochastic dominance (SSD) ratio test statistic. The null hypotheses under the SSD test is $H_0 : MBIVD \succ_2 BH$ suggesting that the trading strategy $MBIVD$ based on model-based IVD second order stochastic dominates the buy and hold strategy BH. The test statistic of the Ledoit & Wolf (2008) Sharpe ratio test is $d = |\widehat{\Delta}|/s(\widehat{\Delta})$ where Δ denotes the difference between the Sharpe ratios of MBIVD and buy and hold benchmark.

3.4 Chapter Summary

The study has compared the information in the implied volatility of stock index options to the information in the implied volatility of index dividend options. The approach illustrates how the implied volatility term-structure of option contracts with time-to-maturity exceeding "9-months" are justified by subsequent fluctuations in dividends although contracts with time-to-maturities around "1-month", "1-3 months" and "3-9 months" stock volatility and dividend volatility are far apart. Such periods coincide with dividend announcement dates of index constituents, hence associated with lower levels of dividend uncertainties. This finding may as well be driven by the impact of averaging like in an Asian option is less volatile than the asset. Longer maturity implies more dividend dates hence more stable time series, thereby allowing market agents to apply the similar weights to stock and dividend uncertainties. This phenomenon clearly makes the dividend puzzle effect more prominent when IV for corresponding stock-index options and IDF options do not decrease together.

The analysis shows IV of stock index options consistently exceeds that of index dividend options, thereby confirming previous criticism based on novel financial data and instruments. However, the magnitude of excess implied volatilities declines with long-dated time-to-maturities, suggesting that the discrepancy between the two IV is sensitive to the investment horizon. This result holds in both model-free and model-based cases. The evidence for the dividend puzzle inferred from expectations on future realizable dividends has a term-structure feature, being almost negligible in the long-run and strongly evident in the short horizons. This implies that inferences about the dividend puzzle for forward-looking purposes, cannot be directly obtained from that observed from historical data without reference to the investment horizon.

For the first time in the literature I constructed an implied volatility index for the STOXX 50 dividend futures contract. This is then employed as a vehicle to help us learn more about the properties of the volatility excess puzzle using information from the dividend derivatives markets. The evolution of this index shows clearly that in recent years there has been a lot more volatility on dividend markets. The trading strategy results also indicate that market participants can improve returns by considering trading signals based on stock and dividend IV differences; which both outperform the long-only trading baseline model portfolio.

Further investigations shall be focused on detecting calendar spread arbitrage and butterfly arbitrage opportunities, in both equity and dividend derivatives markets, using the characterization results of static arbitrage presented in Gatheral & Jacquier (2014) for implied volatility surfaces, perhaps also combining this with the computational shortcut approach discussed in Fengler (2009).

Appendices

APPENDIX B

Additional Material for Chapter 3

B.1 Summary Statistics for Stock and Dividend Implied Volatility

Table B.1: Summary statistics for stock implied volatilities across moneyness

Moneyness	Statistic	1-month Call	1-month Put	1-3 months Call	1-3 months Put	3-9 months Call	3-9 months Put	9-12 months Call	9-12 months Put	Total
$m < -0.15$	mean	0.555	0.538	0.463	0.465	0.377	0.388	0.329	0.344	0.426
	stdev.	0.076	0.066	0.055	0.051	0.035	0.037	0.022	0.026	0.045
$(-0.15, -0.10]$	mean	0.334	0.313	0.278	0.283	0.245	0.253	0.224	0.246	0.271
	stdev.	0.037	0.030	0.023	0.023	0.015	0.018	0.009	0.011	0.021
$(-0.10, -0.05]$	mean	0.303	0.285	0.262	0.263	0.234	0.241	0.217	0.235	0.255
	stdev.	0.033	0.030	0.022	0.024	0.016	0.020	0.009	0.011	0.021
$(-0.05, 0.00]$	mean	0.271	0.248	0.254	0.252	0.228	0.239	0.209	0.234	0.242
	stdev.	0.032	0.029	0.023	0.023	0.016	0.019	0.008	0.012	0.020
$(0.00, 0.05]$	mean	0.236	0.224	0.233	0.235	0.221	0.229	0.206	0.238	0.228
	stdev.	0.030	0.032	0.019	0.021	0.017	0.020	0.009	0.014	0.020
$(0.05, 0.1]$	mean	0.250	0.230	0.233	0.230	0.213	0.228	0.200	0.230	0.226
	stdev.	0.027	0.028	0.020	0.022	0.016	0.018	0.008	0.013	0.019
$(0.10, 0.15]$	mean	0.278	0.264	0.235	0.230	0.211	0.219	0.192	0.226	0.232
	stdev.	0.029	0.030	0.020	0.023	0.014	0.017	0.009	0.013	0.020
$m > 0.15$	mean	0.540	0.499	0.414	0.396	0.271	0.300	0.222	0.297	0.372
	stdev.	0.071	0.059	0.057	0.051	0.034	0.038	0.019	0.027	0.046
Total	mean	0.382	0.354	0.324	0.322	0.268	0.282	0.240	0.270	0.305
	stdev.	0.048	0.042	0.035	0.034	0.023	0.026	0.013	0.018	0.030

Notes:The values in the table are based on moneyness classification used ourthe paper. Moneyness is defined as $m = K/F_t - 1$. Moneyness classification m is constructed as follows: deep-out-of-the-money (DOTM) is assigned if $m < -0.15$, out-of-the-money (OTM) if $-0.15 \leqslant m \leqslant -0.05$ and $-0.05 < m < 0.05$ if at-the-money (ATM). Likewise, in-the-money (ITM) is assigned if $0.05 \leqslant m \leqslant 0.15$ and deep-in-the-money (DITM) for $m > 0.15$ following Rosenberg & Engle (2002) and Wang et al. (2017).

Table B.2: Summary statistics for dividend implied volatility across moneyness

Moneyness	Statistic	1-month		1-3 months		3-9 months		9-12 months		Total
		Call	Put	Call	Put	Call	Put	Call	Put	
$m < -0.15$	mean	0.100	0.063	0.092	0.041	0.082	0.040	0.214	0.081	0.129
	stdev.	0.007	0.008	0.005	0.005	0.001	0.003	0.030	0.023	0.024
$(-0.15, -0.10]$	mean	0.066	0.040	0.056	0.026	0.066	0.041	0.115	0.073	0.088
	stdev.	0.003	0.005	0.003	0.003	0.002	0.004	0.016	0.019	0.016
$(-0.10, -0.15]$	mean	0.039	0.025	0.034	0.021	0.056	0.042	0.098	0.066	0.077
	stdev.	0.002	0.003	0.001	0.000	0.003	0.004	0.015	0.020	0.017
$(-0.05, 0.00]$	mean	0.021	0.020	0.023	0.021	0.045	0.043	0.071	0.056	0.062
	stdev.	0.001	0.000	0.001	0.000	0.007	0.008	0.014	0.018	0.016
$(0.00, 0.05]$	mean	0.020	0.022	0.021	0.023	0.042	0.046	0.054	0.068	0.059
	stdev.	0.000	0.000	0.000	0.000	0.007	0.008	0.012	0.019	0.014
$(0.05, 0.10]$	mean	0.022	0.036	0.021	0.031	0.041	0.051	0.051	0.083	0.061
	stdev.	0.003	0.002	0.000	0.001	0.007	0.007	0.012	0.018	0.014
$(0.10, 0.15]$	mean	0.034	0.060	0.023	0.048	0.040	0.058	0.058	0.112	0.074
	stdev.	0.004	0.004	0.002	0.003	0.007	0.005	0.014	0.021	0.016
$m > 0.15$	mean	0.051	0.093	0.033	0.075	0.038	0.073	0.070	0.169	0.099
	stdev.	0.006	0.006	0.004	0.004	0.006	0.002	0.016	0.039	0.022
Total	mean	0.050	0.051	0.043	0.040	0.054	0.049	0.083	0.085	0.082
	stdev.	0.004	0.004	0.003	0.003	0.004	0.004	0.016	0.022	0.018

Notes:The values in the table are based on moneyness classification used ourthe paper. Moneyness is defined as $m = K/F_t - 1$. Moneyness classification m is constructed as follows: deep-out-of-the-money (DOTM) is assigned if $m < -0.15$, out-of-the-money (OTM) if $-0.15 \leqslant m \leqslant -0.05$ and $-0.05 < m < 0.05$ if at-the-money (ATM). Likewise, in-the-money (ITM) is assigned if $0.05 \leqslant m \leqslant 0.15$ and deep-in-the-money (DITM) for $m > 0.15$ following Rosenberg & Engle (2002) and Wang et al. (2017).

B.1.1 Trading strategy for predictive peak for VSTOXX

The predictive peak is determined numerically and it is shown in Figure B.1 occurs with $N = 31$ VSTOXX moving average (trading) days and $K = 13$ (trading) days of STOXX50 index log-returns.

Using the identified correlation peak, I implement the first trading strategy by focusing on periods when the current implied volatility crosses its own moving average threshold for 31 days. I go long in the stock whenever the VSTOXX crosses below its moving average and close the long position and go short in the stock as soon as the VSTOXX crosses above its moving average (see similar strategies also in Brock et al. 1992, Williams 2011, Guobuzaite & Martellini 2012). This is based on the observed negative correlation between VSTOXX and the stock index. One can see the evolution of VSTOXX, its moving average and stock in Figure B.2

In order to avoid problems of not being able to pay for closing trades, every trade is carried out for a proportion Q of the total balance in the money market account I hold for these trades. For all trading strategies considered, the signal is denoted by $\{\Psi_t\}_{t\geqslant 0}$. The dynamic threshold is $\{\Gamma_t\}_{t\geqslant 0}$ alongside a tradeable liquid asset $\{A_t\}_{t\geqslant 0}$. Let's denote by $\tau_1, \tau_2, \ldots, \tau_n$ the grid of points where $\Psi_t - \Gamma_t$ changes signs,

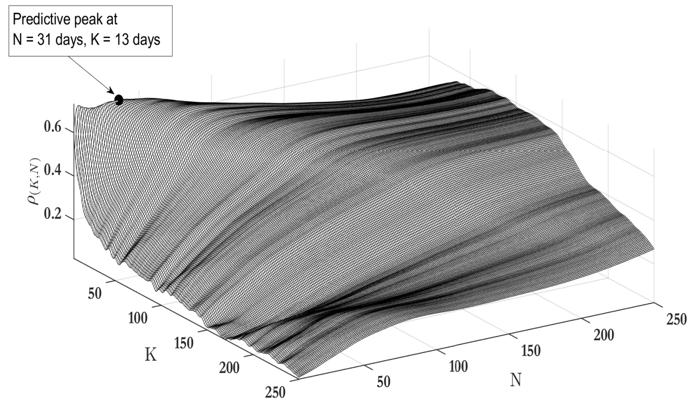


Figure B.1: Correlation of VSTOXX relative level to subsequent moves in the STOXX 50 log-returns.
Notes: Relative level is defined as the ratio of the VSTOXX to its own N-day moving average minus one. K is the number of days out fo log-return of STOXX 50 index which I calculate as $r_{t+k} = ln(\frac{P_{t+k}}{P_t})$ where $t = 1, \ldots, T$ indicates daily trading days and $k = 1, \ldots, 250$ is the index for observed number of days out. N is the n-day moving average for the VSTOXX signal.

that is when the signal crosses over the threshold. All trading strategy will have an initial endowment of 10 million EUR that will be put into a money account $\{C_t\}_{t\geqslant 0}$ accruing money market interest r. I consider the 3 month Euribor rates in the analysis for interest rate calculations.

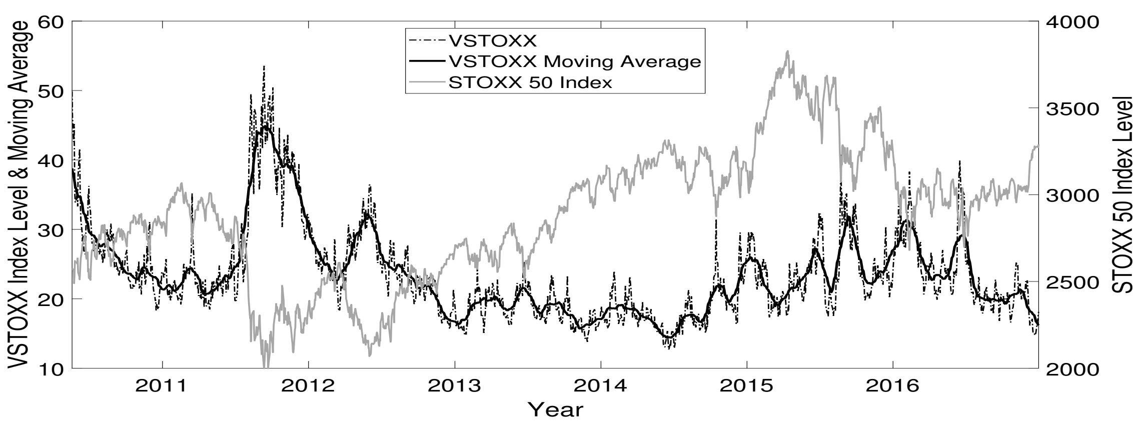


Figure B.2: Comparison of VSTOXX and STOXX 50 using daily data from May 20, 2010 to December 31, 2016
Notes: Graph (a) Illustrates the historical evolution of VSTOXX together with its 31-day moving average. This is plotted alongside the STOXX 50 index level.

The trading strategy is based on the idea that VSTOXX and STOXX 50 are negatively correlated. Hence by monitoring current VSTOXX versus its moving average, I can see when VSTOXX is high or low. If it is high then that means the stock price is low now and at some time in the near future VSTOXX will go down so it will be low and hence the stock price will be high. This means that when VSTOXX is above its threshold I buy the stock which I sell later on when VSTOXX goes below its threshold. Thus, the investor could go long the asset A by buying stock equivalent to 10% of the value of in the money account at the time of transaction, τ_i, hence $0.1 \times C_{\tau_i}$, if $\Psi_{\tau_i} - \Gamma_{\tau_i} > 0$ and then unwind, i.e. sell the entire stock position at τ_{i+1}; and it will go short by selling $0.1 \times C_{\tau_i}$ in stock if $\Psi_{\tau_i} - \Gamma_{\tau_i} < 0$ and then unwind at τ_{i+1} by buying back stock to cancel their short position. The transaction is followed through only when the absolute difference $|\Psi_{\tau_i} - \Gamma_{\tau_i}| > 0.01$ in order to avoid transaction costs for trades that are not economically significant. I test the quality of the trading strategies for the VSTOXX and the IV ratios by comparing the performance of their associated portfolios to that of a benchmark[1] portfolio which is based on a long-only trading strategy. The buy and hold strategy of the benchmark becomes a base model that allows us to evaluate performance only in terms of the trading signal dynamics (see Brock et al. 1992, Lo & Patel 2008).

Table B.3 reports the standard comparative performance measures for various variants of this strategy under various market regimes spanned by risk-free rate values and different proportions of investments. The annual return increases with the proportion of endowment used for investment but so does its volatility. The Sharpe ratio decreases with the proportion so using a more conservative approach would produce larger Sharpe ratios. The strategy will produce clear superior returns in an economic environment with larger risk-free interest rates. Panel C reports the performance based on the benchmark strategy. Comparing with Panel A and B it is clear that my strategy is superior to this benchmark.

[1] The initial trade is for a proportion of Q of the total balance in the money account I hold. At inception, the investor goes long on the asset, and buys stock equivalent to Q% of the total value held in the money account; $Q \times C_{\tau_0}$, where $0.1 \leqslant Q \leqslant 1$. This initial position is held till end of the sample period.

Table B.3: Comparison of trading strategy performance and risk using daily data from May 20, 2010 to December 31, 2016.

Weight Q(%)	10	20	30	40	50	60	70	80	90
Panel A: Performance Measures on In-Sample Data (May 20, 2010 - Dec. 31, 2016)									
Historical VaR (95%)	-0.0003	-0.0007	-0.0011	-0.0014	-0.0018	-0.0021	-0.0025	-0.0028	-0.0032
Sharpe Ratio (Rf=-0.7%)	1.1479	0.7573	0.6154	0.5356	0.4805	0.4376	0.4015	0.3696	0.3405
Calmar Ratio	0.4754	0.3649	0.3144	0.2832	0.2598	0.2402	0.2228	0.2067	0.1916
Annualized CEQ Return	0.0196	0.0258	0.0314	0.0364	0.0407	0.0445	0.0477	0.0502	0.0521
Panel B: Performance Measures on Out-Sample Data (Jan. 1, 2017 - Aug. 14, 2018)									
Historical VaR (95%)	-0.001	-0.002	-0.003	-0.004	-0.0051	-0.0061	-0.0071	-0.0081	-0.0091
Sharpe Ratio (Rf=-0.7%)	2.4121	2.224	2.1694	2.148	2.1399	2.1384	2.1405	2.1449	2.1507
Calmar Ratio	3.4825	3.7525	3.8561	3.9178	3.9624	3.9983	4.0292	4.0567	4.082
Annualized CEQ Return	0.0266	0.0485	0.0703	0.0918	0.1132	0.1343	0.1553	0.176	0.1966
Panel C: Performance Measures on Benchmark Strategy (May 20, 2010 - Dec. 31, 2016)									
Historical VaR (95%)	-0.0023	-0.0046	-0.0069	-0.0091	-0.0114	-0.0138	-0.0161	-0.0185	-0.0207
Sharpe Ratio (Rf=-0.7%)	0.6551	0.3604	0.2546	0.1961	0.1564	0.1262	0.1014	0.0799	0.0607
Calmar Ratio	0.1841	0.1118	0.0841	0.0673	0.0532	0.0418	0.0318	0.0228	0.0142
Annualized CEQ Return	0.0147	0.0160	0.0169	0.0172	0.0171	0.0165	0.0154	0.0139	0.0118

Notes: In this table, I report values for the standard summary performance measures for the trading strategy using VSTOXX as signal. The trading signal is based on a 31-day moving average of the VSTOXX, obtained from the predictive peak in the correlation of VSTOXX and STOXX 50 index return.

CHAPTER 4

NNEG Valuation in ERM Contracts

4.1 Introduction

Equity release mortgages (ERM) as known in the United Kingdom (UK) are designed and sold by insurers to borrowers aged above 55 years. ERMs are offered to the borrower either as lump-sum upfront loan or leveled payment received over an agreed period. By design, the borrowers house is the collateral on the loan and the issued loan is not repaid until contract termination, a condition triggered either by borrower's demise or transition into long-term care or a voluntary repayment initiated by the borrower.

Reifner et al. (2009) provided an excellent country-based grouping of ERM schemes across European Union (EU) member states. The criteria for the proposed grouping were population demographics, pension provisions, state of mortgage markets, property market statistics, and survey results on the use and market sentiments on ERM contracts. More specifically, four (4) groups were identified; United Kingdom, Ireland, and Spain constituted the *significant ERM market* group (i.e first group); Sweden, France, Austria, Italy, Finland, and Hungary had *less developed loan model ERM market* (i.e. second group); Romania, Bulgaria, and Germany were in the third category with *less developed sale model ERM markets.* The fourth group included member states *without ERM markets.* Recently, Hennecke et al. (2017) found favourable equity release market conditions in the United Kingdom and the Netherlands, while Ireland and Germany had the exact opposite. The ERM market space in the United Kingdom has significantly matured through some reputation issues in her history. Examples include the shared appreciation mortgages (SAM) scandal in the late 80's which also received substantial negative press up until the early 1990's. Working to prevent future possible scandals, the Financial Conduct Authority (FCA) and the Bank of England (BOE) Prudential Regulatory Authority (PRA) progressively aim to address and/or prevent such reputation issues by using rule-based regulations. For instance, the United Kingdom currently has the nonnegative equity guarantee clause, and Solvency II matching adjustment rules for illiquid unrated assets and ERMs. These regulatory interventions sets the UK equity release market space within the *significant ERM market* group. These recent market categorisations jointly inform my decision to focus on UK equity release market space, which has consistently remained in the significant group.

As of 2013, Netherlands had the highest mortgage debt per capita in the European Union (see Toussaint 2013). The country also possessed four striking observations regarding the role of housing wealth: 1) citizens can build affluence via housing wealth (Haffner & De Vries 2010, Toussaint 2013), 2) there is little evidence for outright home ownership as almost homeowners tend to retain mortgage until their old age (Van der Schors et al. 2007, Toussaint 2013), 3) younger households are inclined to take additional mortgage debt in order to fund post retirement lifetime expenses (Toussaint 2013), and 4) a significant proportion of the Dutch population rent their homes while the high-income class are likely to be owner-occupiers (Mulder et al. 2004).

There are a number of important discussions on the potential and impact of ERMs. On the issue of potential gain, Moscarola et al. (2015) showed that well structured ERMs could reduce old age economic vulnerability by taking persons aged 65 over out of the lowest tail of income distribution. In a recent survey across the EU, Megyeri (2018) also found that ERMs could reduce the impact of poverty risk of the elderly in society. Hennecke et al. (2017) also showed that equity release mortgages provide some form of pension insurance while the borrower liquidates her owner-occupied property.

In the UK, the ERM product space is under active prudential regulation. A notable stipulation under current regulatory standards transfers to the loan issuer the excess cost created when the accumulated loan value is over the market-value of the collateral house at termination. This is the no-negative equity guarantee (NNEG) clause that is hotly debated among academics, practitioners and the regulator. Since the NNEG condition is embedded into the ERM contract by design, I should much as well expect the value of ERMs to be directly dependent on the resulting NNEG value. Hence, risks associated with the NNEG valuation will spill over to the ERM value.

The risk factors that drive the NNEG value include the volatility of house prices, the contract rate by which the issued loan is accumulated, borrower-specific decrement rates (i.e. mortality rate, long-term care incidence rates and voluntary prepayment) that in effect randomizes the time-to-termination of the contract, the risk-free interest rate and the service flow rate. Hosty et al. (2008), Li et al. (2010), Ji et al. (2012), Alai et al. (2014), Kogure et al. (2014), Prudential Regulation Authority (2019*b*), Prudential Regulation Authority (2019*a*), Dowd et al. (2019), and Huang et al. (2020) provide excellent discussion on the factors that affect NNEG values in ERM contracts.

The main risk embedded in ERM is the collateral-effect channelled by house price risk. Therefore, the behaviour of the main stakeholder in this financial market depends on their understanding of house price risk. Relevant literature on ERMs tend to address different facets of the ERM risk-management process. Ma & Deng (2013) presented an actuarial based model for pricing the Korean ERM with constant monthly payments and also with graduated monthly payments indexed to the growth rate of consumer prices. They found that any shock to house prices may impact younger borrowers more severely. The Ma & Deng (2013) sensitivity test results on contract maturity (termination) showed how call option values decrease with maturity, suggesting higher ERM values at younger ages. The study also reported a positive relationship between house price volatility and call option value. Thus higher volatility will impact younger borrowers more. Younger borrowers include borrowers within lower age range profile i.e. 55 - 60 years (see Li et al. 2010, Ma & Deng 2013).

Shao et al. (2015) consider that there are only two main risks that insurers selling ERMs face, real-estate risk and longevity risk and investigated the joint effect of the two on the pricing and risk profile of ERM loans. Their stochastic multi-period model was based on a new hybrid hedonic/repeat-sales pricing model and a stochastic mortality model with cohort trends (the Wills-Sherris model). They concluded that using an aggregate house price index and not considering cohort trends in mortality may lead to an underestimation of total risk in ERMs.

Wang et al. (2014) developed an analytical formula for calculating the *feasible* loan-to-value (LTV) ratio in an adjusted-rate ERM applied to regular tenure payments. In their model, interest rates are modelled jointly with the adjustable-rate ERM, and the housing price follows a jump diffusion process with a stochastic interest rate. Assuming the loan interest rate is adjusted instantaneously with the short rate given by a CIR model, they show that the LTV ratio is independent of the term structure of interest rates, even when the housing prices follow an exponential Lévy process. Concerns have been raised about the sustainability of the ERMs at high levels of housing price volatility. Regarding ERMs, these studies tend to support the assertion by He et al. (2015), and Lim (2018) that housing market price dynamics impacts the entry-exit and expansion-contraction decisions of the loan issuers through the collateral channel.

The NNEG clause is usually conceptualised as a put option in Prudential Regulation Authority (2018, 2019*a*,*b*) and relevant literature (see Li et al. 2010, Dowd et al. 2019). Szymanoski (1994) argued that the dynamics of house prices is well represented by a geometric Brownian motion (GBM). Studies that used geometric Brownian motion for house prices related to ERM modelling are Hosty et al. (2008), Kau et al. (1992), Huang et al. (2011), Ji et al. (2012), Pu et al. (2014).

This is in contradiction with the findings of Case & Shiller (1989) and a large body of empirical evidence (Tunaru 2017) where: well-documented serial correlation of returns of property prices is not captured; the variance for a GBM increases infinitely with the time horizon GBM.

Recent studies accept that house price time-series exhibit serial correlation that invalidates the GBM assumption (Kogure et al. 2014). Li et al. (2010) considered the Nationwide House Price index and they remarked that, for this property index, a) there is a strong positive autocorrelation effect among the log-returns, b) the volatility of the log-returns varies with time, c) a leverage effect is present in the log-return series. All these three properties present critical challenges to the use of the GBM for house prices. The characteristics of large movements in house prices presented in Sun & Tsang (2019) also affects the GBM process as a data-generating process for house prices. There is a need to consider pricing models that provide valuable improvement to account for the properties of house prices while presenting a valuable extension to the GBM. It will also be beneficial to ensure the new model contains the GBM as a special case.

The NNEG valuation principles under the Black-Scholes model that is recommended by the regulator in the UK was compared with a more academic approach based on a ARMA-EGARCH model; a more suitable methodology for gauging house price risk. The Black-Scholes model overestimates in relative terms the NNEG by comparison with the ARMA-EGARCH during calm times of low interest rates and positive growth of house prices, and underestimates comparatively NNEG values during turbulent times of higher interest rates and downfall of property prices. This impact could significantly affect efforts geared toward a secured financial footing for ERMs. The choice of the right model may help improve the stability of the ERM market and benefit ultimately the people who need this financial instrument, given that any additional cost to the insurer imposed via the regulatory channel is ultimately passed to the consumer. The impact of house-price-specific risk-sensitivities on NNEG valuation was investigated by focusing on the long-term effect of house price risk. Finally, I compared the impact on the portfolio of ERM loans of valuing the NNEG by the regulator's approach versus the financial economics academic approach.

The findings show that the GBM model recommended by the regulator in the UK produces much higher values of the NNEG when compared with a best fit ARMA-EGARCH model selected on the basis of forecasting house prices well. Utilising an inappropriate model in the context of reverse mortgage loan market may in the end stifle this market by imposing very high capital reserve requirements on insurers. This is very important since there is no diversification benefit for an insurer issuing ERM loans with each loan being valued separately for NNEG calculations purposes. Inflating the volatility parameter will automatically imply a high variance of house prices at long maturities for the GBM model, thereby impacting directly on ERMs loan characteristics for the younger borrowers who would benefit the most from this new asset class.

The study finds evidence to suggest that service flow rate parameter is not the key driver of underlying house prices in UK. With majority of house prices not paying rents, which can be verified in the Office of National Statistics, it would be wrong in this perspective to assume that all houses have prices driven by rents. According to the 2016-17 English Housing Survey report of the Ministry of Housing Communities, & Local Government (2018), 14.4 million (63%) households are owner occupied in England. The proportion increased gradually from the 80's, peaking at 71% in 2003. The proportion may be different from country to country but there is no known country where all houses generate rental income. An overestimation of the service flow rate induces downward trending house prices in the long run that ultimately inflates the NNEG values.

While the ERMs may offer a viable solution to long term care and pension boosting to the elderly generation in most developed economies, there is a general lack of development of this market world-wide. A possible explanation could be that the interaction between consumers, insurers and regulator needs to be improved in order to allow for injected capital to work more efficiently.

The remaining of the Chapter is structured as follows: Section 4.2, presents a detailed discussion on ERM and NNEG literature, shedding light on the background of proposed pricing models and regulatory standards for ERMs in the UK; Section 4.3, reviews the modelling approaches that will be investigated in the study; Section 4.4, describes the data and some preliminary analysis including parameter estimation for the models in the previous section; Section 4.5, presents a comparative sensitivity analysis of the NNEG valuation indicating the effect of changes on the risk drivers onto NNEG values and indirectly on ERMs loan values. The section further presents an analysis on the risk exposure characteristics for a lender or equity release mortgage loans and how those risks vary when using the regulatory imposed approach or my proposed approach; Section 4.6, summarizes the conclusions of the study.

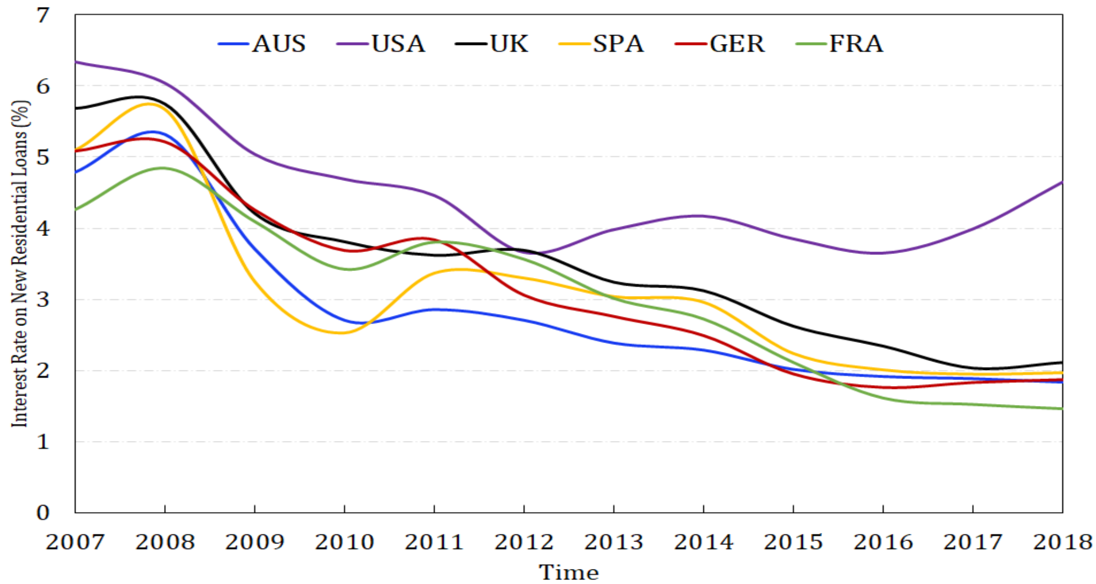
4.2 Literature Review

Mortgage loans and their amortization remain critical over the lifetime of individuals within any given economic environment. Although repayment for such loans often become substantial constituent of a borrower's lifetime liability, mortgages provide a medium for wealth/equity accretion among households. To a large extent residential mortgage markets around the world are exposed to systemic risk which is typically linked to house price deviations. House price deviations involve tendency of property prices to move away from their fundamentals; exposures of this nature are detrimental to lending institutions. House price deviations also refer to the excess of actual house prices over rational house price (i.e. equilibrium price levels based on fundamentals). Case & Shiller (2003) proxy housing market fundamentals by state-level per capital personal income and house price indices from Fiserv CSW spanning 1985 - 2002. McCarthy & Peach (2004) also adopted the same proxy. Mikhed & Zemčík (2009) adopts fundamental factors such as personal income, population, house rent, stock market wealth, building costs, and mortgage rate. In addition to housing market fundamentals in Case & Shiller (2003), Himmelberg et al. (2005) also used the house price growth rates, price-to-income ratio, and rent-to-price ratio. All variables are observable. Regarding this matter, Shiller et al. (2013) stipulates a need to link housing prices to mortgage values and payments with a downward adjustment to prevent negative equity. One can associate effects of short and long term house price deviations to systemic risk and market crisis. Whereas short term deviations are related to low liquidity that is inherent in housing market, long term deviations are associated with instances of irrational behaviour[1] of traders on the market.

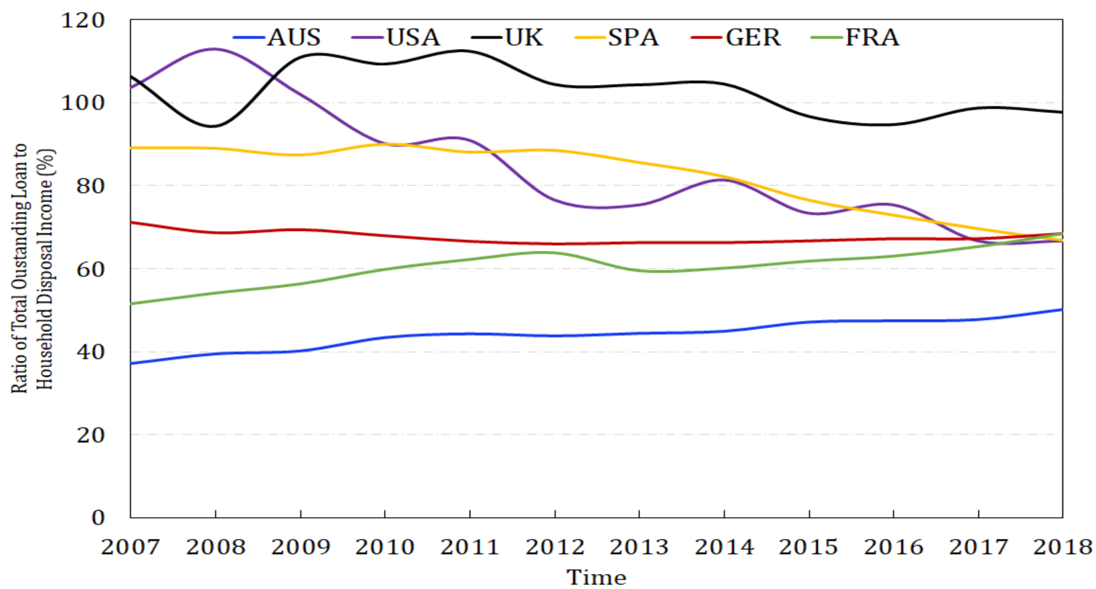
Creation of housing wealth is necessary to maintain living standard among economically active individuals, thus mortgage markets could provide a good reflection of the state of the economy. On this basis, social benefits of mortgage markets can be improved when risk exposures of financial institutions within the market space are efficiently managed. Shiller et al. (2013) argued a need to share house price risk between the lender and borrower via automatic adjustments pertaining to house price levels. Recently Campbell et al. (2020) found adjustable rate mortgage that provide payment adjustment during economic recessions could stabilize consumption growth. The proposed adjustment introduced in Campbell et al. (2020) involved use of interest-only payment during recessionary periods. This mostly pertain to the US market where plain vanilla adjustable rate mortgage loans are popular. Relevant literature on mortgage design mostly agree on the need to improve on plain vanilla adjustable rate mortgages to provide relief to household budget. More recently, Shiller et al. (2019) discussed an excellent implementation of Continuous Workout Mortgage (CWM) design which cuts out all expensive workout associated with defaults in plain vanilla mortgage contracts, while sharing house price risk with the lender. CWMs are also capable of relieving the government of the cost of debt relief schemes required during period of financial recessions.

Mortgage interest and ratio of outstanding loan to household disposable income are critical concepts that underlie design of mortgage loans. In Figure 4.1 I present a market-specific overview that depicts evolution of these two factors across selected countries. Figure 4.1(a) presents the evolution of mortgage lending interest rates across five (5) selected countries i.e. Austria(AUS), United States of America (USA), United Kingdom (UK), Spain (SPA), Germany (GER), and France (FRA).

[1] According to Shiller (2015) investor enthusiasm possesses a psychological basis that explain speculative bubbles on financial markets. This phenomenon is popularly termed as irrational exuberance. Shiller (2015) further cites a scenario whereby investor enthusiasm is spurred by news of asset price increases which subsequently spreads by psychological contagion from trader to trader.



(a) Interest rates on new loans



(b) Total outstanding loan to disposable income ratio

Figure 4.1: Evolution of representative interest rates on new residential loans and ratio of outstanding loan to household disposable income across selected countries from 2007 to 2018

Notes: The interest rates are annual averages based on monthly figures. The time series for United Kingdom (UK), Austria (AUS), and France (FRA) are weighted average interest rate on loans to households for house purchase. The series for United States(USA) is based on initial fixed period interest rate of over 10-years on loans for house purchase. Time series for Spain (SPA) is based on initial fixed period interest rate up to 1 year on loans for house purchase. The interest rate series of Germany (GER) is calculated as the initial fixed period interest rate over 5 and up to 10 years on loans for house purchase.
Source: European Mortgage Federation National Experts, European Central Bank, National Central Banks, Federal Reserve.

The representative mortgage rate for periods ranging from 2007 to mid-2008 lie above 4% across all five countries. Post 2008 representative mortgage lending rate declined steadily across all selected countries although that for USA was above 4%. The clustering of the mortgage rates of selected European countries in Figure 4.1 could refer to an existing mutual relationship between loan design within the European Union zone. On the other hand, Figure 4.1(b) shows how the ratio of total outstanding loan to household disposable income has evolved from 2007 to 2018. Interestingly the ratio tended to be less variable over the sample period possibly depicting how rates are adjusted in order to manage default risk, price volatility, and consumption volatility.

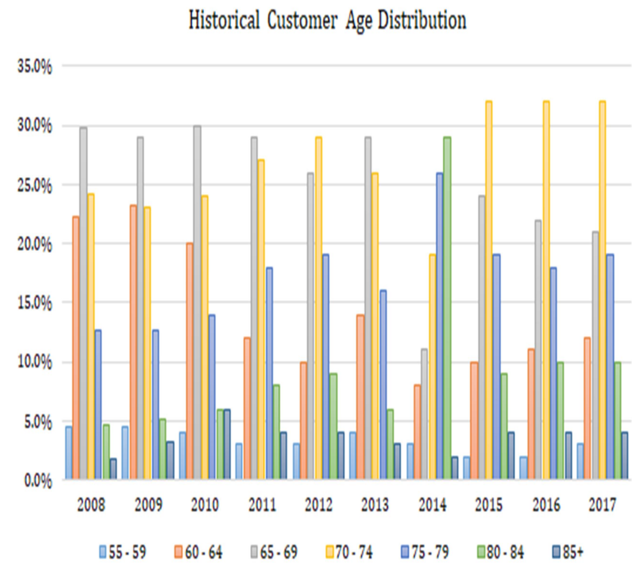
Borrower-specific characteristics may also play a key role in driving risk exposures during market recessions Campbell (2013). For instance, retired seniors with significant debt pose higher probability of default during market down turn. Despite this possibility, a well structured mortgage scheme may set an efficient basis for home equity release participation during retirement. Farnham & Sevak (2007) found a fast rate of transition into retirement when house prices are improved. Likewise, long term mortgage loan borrowers also inherit protection against deteriorations in creditworthiness (see Campbell 2013). Regarding life time income sustainability, retired seniors who are cash-poor but house-rich, could create substantial lifetime income when they safely release equity held up in their homes. Hence, housing wealth during old age provides an opportunity for life time improvement through home equity release mortgages.

4.2.1 Equity Release Mortgages

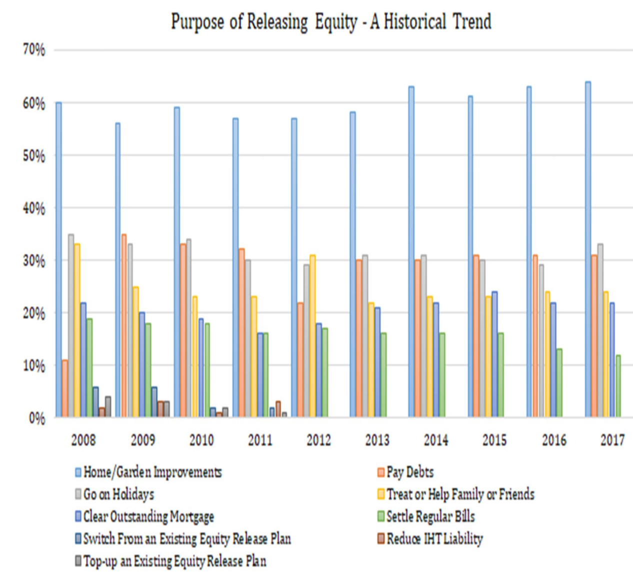
As a financial instrument, ERMs can be traced back into the mid- to late 1980s in the United Kingdom and 1960s in the United States where they are called Reverse Mortgages. The Reverse Mortgage market saw significant product revision during the 1980s under the Federal Home Loan Bank Board in the United States. The financial instrument slowly spread across the global economies gaining more popularity in Australia, Japan, Singapore, Hong Kong, France and some European countries, (see Addae-Dapaah & Leong 1996, Chou et al. 2006, Ma & Deng 2013, Mitchell & Piggott 2004, Merton & Lai 2016).

While the differences are apparent, similarities exist when it comes to the purpose for releasing equity, the age distribution of buyers and the product mix within the market space. With respect to the product type, most market practitioners specifically use design features as criteria for classification[2] of ERMs.

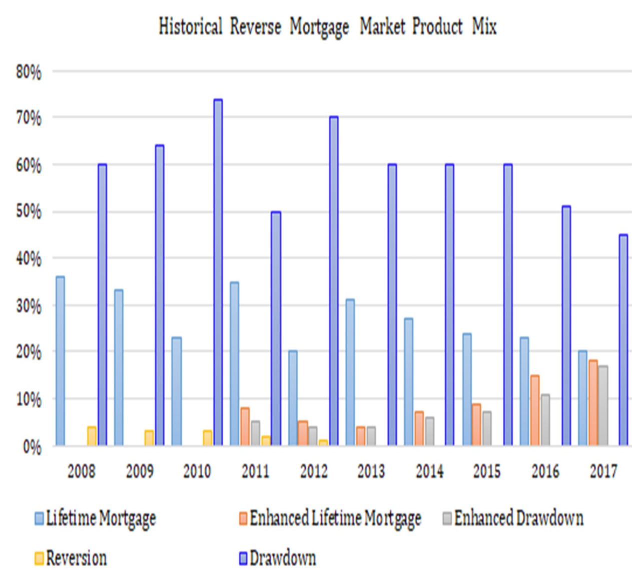
[2]The following products are readily distinguishable: Voluntary/Partial Repayment, which permits adhoc or regular repayments of up to a maximum of 10% of initial loan per annum. Early repayment charges are non-existent. Drawdown Facilities allows for the withdrawal of funds in stages rather than fixed lump-sum. Interest is applied upon withdrawals. Inheritance guarantee has a maximum loan amount that is reduced, while a fixed portion of the property value is ring-fenced as a minimum inheritance irrespective of the total interest accrued to the loan outlay. The fixed ERC design has charges that form a fixed percentage of the



(a)



(b)



(c)

Figure 4.2: ERM product market features in the United Kingdom from 2008 - 2017

Source: Equity Release Council

initial loan for a stipulated period. They are specifically designed to be decreasing in nature. Early payment charges are approximately zero after the stipulated period. Downsizing protection allows borrowers to scale down to a smaller property in order to repay the accrued loan amount. It usually has a five-year qualifying period in practice. The sheltered/age restricted accommodation design imposes a lender's requirement wear-in for a given period, some ERMs can be protected against sheltered or age restricted properties. Some products have embedded interest payments, thereby permitting a monthly interest settlement either in full or partially. With this, interest accrued on the loan decreases. Borrower's have the option to return to the normal interest roll-up at any period. Bishop & Shan (2008) provide further description on the product.

Figure 4.2 presents a summary of market features for the UK equity release market space. A link can be clearly drawn between the purpose of releasing equity and the most patronised product design. The summarised market data shows that most customers choose drawdown ERM and subsequently tend to use the proceeds to clear outstanding mortgage debt. According to the UK Equity Release Council market report, more than 50% of borrowers released equity in order to improve their homes/gardens for the period spanning from 2008 to 2017. Post 2015 market data also showed that more than 30% of borrowers are aged 70 - 74 years, whereas less than 5% are aged 85 and over.

Despite the variability in design, the bottom-line fact still remains that developed economies are working towards improving home equity extraction schemes. According to Merton & Lai (2016) the success of reverse mortgage schemes to become a sustainable source of lifetime income strongly depends on availability of funds from supply side. With respect to the US home equity conversion mortgage (HECM), Merton & Lai (2016) argued the need for privatising the institutional provision for reverse mortgages. The minimum age for a borrower should be 60 in Korea for the Korean government-insured ERM program. Per the Equity Release Council (2019) market report, there is no lifetime mortgage or home reversion plan provider catering to under 55 age market; "re-mortgages" are the closest to an ERM for borrowers under 55. In a lifetime mortgage, it is also beneficial to know the risk associated with borrowers in the minimum age. In Italy, the minimum age by law is 65. Dowd (2018) takes 70 as the base case scenario. The minimum age for ERMs is generally around 55/60/65. The UK has the lifetime mortgage for buyers aged 55 years and over and home reversions for buyers aged 65 years and over. France, Japan and South Korea have buyers aged over 60 years, Spain works with buyers above 55 years while Canada and Australia respectively sell to 55 - 62 years and 60 - 65 years (see Bhuyan 2010, Bridge et al. 2010, Kobayashi et al. 2017).

In the United Kingdom, ERMs are not sold without the no-negative equity guarantee (NNEG), a regulatory clause which compels the loan issuer to write off any excess of the accrued loan amount above the market value of the collateral property after contract termination. The NNEG clause is subject to other conditions stipulated in the prudential regulatory authority (PRA) Policy Statements which otherwise makes their implementation complex. The NNEG clause is also used in other countries like Australia, Japan, and Canada. The age profile of the target market slightly varies across countries.

The NNEG clause has so far become the fundamental concern to the pricing of ERMs in the United Kingdom. The funding for the ERM loan portfolio on the side of the issuer also comes against a host of challenges due to the long and uncertain maturity profile of the constituent assets. ERM cash flow uncertainty fundamentally borders around its timing and prospective "net-liquidation-value" to be realised at maturity. Per design, ERMs require adequate and flexible long-term funding to sustain parity with timing, frequency and severity of the cash flow process generated by the contract. So far, existing literature on valuation and risk management of ERM contracts converge to a common unison that the components of cash flow uncertainty may well be driven by house-price volatility, mortality rate, risk-free rate, the contract rate applied on the initial loan, prepayment and long-term care incidence, regulatory changes (see Chinloy & Megbolugbe 1994, Zhai et al. 2000, Wang et al. 2008, Bhuyan 2010, Huang et al. 2011, Tunaru 2017). These observed risk-factors drive the tendency of house-prices to be lower or higher than accumulated loan value observed at the end of a given contract period. Chinloy & Megbolugbe (1994) described this uncertainty as the "cross-over effect".

To the ERM issuer, the safety and reliability of the contract relies on the event that the net-liquidation-value of the underlying property at expiration adequately matches outstanding debt-related liabilities generated from the extended loan. While this event remains uncertain in some countries, Japan and the United Kingdom, for example, have the regulator fixing some key parameters in the calculation of the NNEG. Issues concerning the pricing of the NNEG, treatment of contract fees and charges including the maximum loan limit constitute key NNEG related problems highlighted in relevant ERM literature.

Although the long-term benefits and observed positive outlook of ERM schemes are conceivable, Hosty et al. (2008), Siu-Hang Li et al. (2010), Andrews & Oberoi (2015), Kobayashi et al. (2017) found that low take-up rates of ERMs are partly due to NNEG clauses which potentially makes them unattractive and financially onerous to lenders. From borrower's standpoint, Chou et al. (2006), Overton (2010), and Alai et al. (2014) also found that pricing of ERM consistently makes take-up unappealing to potential borrowers who consider initial loan amounts to be unrepresentative of home-value. Using house price return time series, Siu-Hang Li et al. (2010) and Lee et al. (2012) separately showed that NNEG clauses can be a significantly risky financial burden hence a need for dynamic risk hedging strategies whenever NNEG's are educed. Common to the outline of these studies is the observation of NNEG valuation-related complications when it comes to determining suitable rates for discounting prospective cash flows, establishing a credible basis for estimating exit, prepayment likelihoods and drawdown rates, setting the appropriate basis for the potential evolution of underlying house price process and model-calibration for pricing the NNEG risk.

The OECD reported that by 2050 there will be approximately one senior dependent over the age of 65 for any two people of working age. Worldwide, governments are faced with two major problems, increasing pension deficits and increasing long-term care costs due to improvement in life expectancy of the population and lower birth rates. Following the recommendations of the Dilnot Commision, the Care Act of 2014 in the U.K. requires local government to put a cap on out-of-pocket care spending (see Mayhew et al. 2017). The Dilnot Commission investigated the fair funding on care and support in the UK highlighting the need to reform pre 2011 funding systems. According to the Commission, the then existing 60-year-old legislation on social care finding in UK was complex and unsustainable. Under the new Care Act 2014, an individual in need of care and support undergoes a needs assessment process with local authorities to ascertain required needs impact well-being expected outcomes they intend to achieve. In the United States long term care costs account for about 9% of total health expenditures with upward trends. Brown & Finkelstein (2011) discussed varied reasons why the insurance of long-term care (LTC) sector does not provide efficient solutions for these complexities. The European Commission's 2018 ageing report projects public expenditure on LTC to increase from 1.6% to 2.7% of GDP between 2016 and 2070 (Spasova et al. 2018).

The ageing[3] population across developed countries suggest a potential increase in future demand for formal pensions and LTC, thereby compelling various governments to direct their fiscal policies towards identifying sustainable replacement. On this background, ERMs[4] are designed to offer feasible solutions to complexities of sustainable lifetime income facing society since they allow senior borrowers to convert equity locked in their houses into cash or lifetime income while ageing-in-place. In societies where housing wealth is concentrated in the hands of senior citizens[5], ERMs can serve as solution to ease demographic and fiscal pressures of ageing population (see Malpass 2008, Doling & Ronald 2010).

Under new regulations, ERMs have been endorsed by Robert Merton as a viable source of funding for the elderly. Merton (2007) argued that ERMs could be an efficient vehicle to transfer intergenerational wealth, in a hassle-free way for elderly homeowners. Moreover, Merton & Lai (2016) discussed a structural design of ERMs that is meant to improve the risk sharing between the borrower and the lender while also highlighting the important role of the regulator. Cocco & Lopes (2015) outlined improvements to ERM design that may help to expand this market for those in need. At the same time, education plays a major factor that links low demand for ERMs with end user's lack of knowledge or understanding of the financial product characteristics (Davidoff et al. 2017). People would benefit from simple rules to make long-term decisions related to retirement (Binswanger & Carman 2012).

The size of ERM market has not seen uniform growth worldwide across developed countries. Nakajima & Telyukova (2017) highlighted that only approximately 2% of eligible homeowners had an ERM in 2011. This proportion has increased only slightly by 2017 with 55,000 senior borrowers taking ERMs out of the total 2 million population of adults over 65. This is surprising since ERM, together with LTC insurance, carry

[3]By 2050, 44% of the world's population will live in relatively aged countries with at least 20% of population aged 60 and over (United Nations 2015)

[4]Reverse mortgages as they are termed in the USA or Equity Release Mortgages in UK, are collateralised loans with a stochastic maturity that is tied to either the death, transition into long-term care or the early prepayment of the borrower. The loans are issued only to borrowers aged over 55 years. By design, the initial loan is a stipulated percentage of the initial market price of the house at inception of the contract; and there is strictly no repayment until maturity of the loan contract. The product is a roll-up mortgage under which initial lump sum loan accumulates interest at the contract rate interest rate until the borrower either dies, moves into long-term care, or voluntarily cancels the contract.

[5]Over $500 billion of Australian's home equity is concentrated in the hands of senior citizens aged 65 or over.

consumption transition benefits to senior adults (Davidoff 2009). ERMs in UK have increased substantially between 2012 and 2019, reaching about four billion sterling pound in outstanding loans notional Equity Release Council (2018). In Australia, the market size grew from $0.9 billion in 2005 to $3.32 billion in 2011, with the total number of outstanding loans increasing from 14,584 in 2005 to 42,410 in 2011, (Deloitte Australia 2018).

The stifled growth in take-up rates of ERM among elderly citizens could be design-specific when looking at the supply-side. For example, in the U.S., I can identify the issue of high premia charged to consumer; while the UK market has the non-negative equity guarantee (NNEG) clause stipulating that any excess of the accrued loan amount above the sale value of the property after the exit event will be written off by the lender, subject to certain conditions. Other possible demand-side explanations relate to low product knowledge on the ERM risks (Davidoff 2015, Davidoff et al. 2017), moral hazard perhaps highlighted through default in payment of property taxes (Shiller & Weiss 1999) and homeowners insurance (Moulton et al. 2015).

4.2.2 Regulatory standards on ERMs

Since the NNEG clause is predominantly associated with the ERM market in UK, the conceptual framework will be situated on current requirements in Product Standards(PS) within the Statement of Principles of the Equity Release Council[6] and Bank of England's Prudential Regulatory Authority (PRA). An issuer of a ERM has to consider many factors that contribute to the price dynamics of the ERM and subsequently, other cashflow valuations. The main factors are age of borrower(s), initial house price, loan-to-value (LTV), house price growth, risk-free rate, contract rate to be applied on the disbursed loan, mortality rate of borrower(s), long term care (LTC) incidence, early prepayment rates, current yield curve, forward yield curve, funding issues if necessary, idiosyncratic risk due to postcode house price differences, ratings requirements if any, regulatory requirements (Solvency II) and most likely the list is not exhaustive.

For UK reinsurers, the PRA's rules on valuation are described in Valuation 2.1 of the PRA Rulebook. For fair valuation the requirement is to value assets at the amount for which they could be exchanged between knowledgeable willing parties in an arm's length transaction. Companies reporting under UK GAAP, FRS 102 additionally employ the fair value as the amount for which a liability settled, or an equity instrument granted could be exchanged, between knowledgeable, willing parties in an arm's length transaction. On the other hand, IFRS 13 defines fair value as the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date.

In his letter to Mark Carney on 5 December 2018, the Governor of Bank of England, Philip Hammond

[6]See www.equityreleasecouncil.com/ship-standards/statement-of-principles

from HM Treasury, stated:

> In discharging its general functions, the PRA must also have regard to the regulatory principles set out in Section 3B of the Act, which are:[...]
>
> the principle that a burden or restriction which is imposed on a person, or on the carrying on of an activity, should be proportionate to the benefits, considered in general terms, which are expected to result from the imposition of that burden or restriction.

The key word here is "proportionate" and this is why it is imperative to allow insurers to conduct internal calculations on the risks associated with ERMs. In their document Prudential Regulation Authority (2017), the PRA stated that they will assess the capital reserve made for the NNEG risk against their own view of the underlying risks retained by the insurer. Their assessment is spanned by the following four principles,

1. Securitisation where firms hold all tranches do not result in a reduction of risk to the firm.
2. The economic value of ERM cash flows cannot be greater than either the value of an equivalent loan without an NNEG of the present value of deferred possession of the property providing collateral.
3. The present value of deferred possession of property should be less than the value of immediate possession.
4. The compensation for the risks retained by a firm as a result of the NNEG must comprise more than the best estimate cost of the NNEG.

There is enough clarity in the first two points with the second point reflecting the concept introduced in the PRA's discussion paper DP1/16. According to the PRA , the best estimate cost of the NNEG is "the mean of a stochastic distribution of possible future guarantee costs, where random variables used in the stochastic projection have been calibrated based on a best estimate of their true distributions."

In Prudential Regulation Authority (2018) there is a substantial section on feedback to responses received on various risk-calculation issues on ERMs. On point 2.29 the PRA considers that the Black-Scholes formula is still appropriate for NNEG put option valuation, but in CP13/18 they also made it clear that other option pricing frameworks may be used as long as it can be demonstrated that valuations meet the four principles enumerated above. Black-Scholes formula has been reiterated in Prudential Regulation Authority (2019*b*), that describes the final methodology for managing illiquid unrated assets and equity release mortgages. The formula is described with two fixed values for the main two parameters that are difficult to estimate, the volatility of the house price $\sigma = 13\%$ and the **minimum** deferment[7] rate $q = 1\%$.

The ERMs can be seen as a portfolio of an income security and a crossover put option that is automatically applied at termination, effectively posting the house as collateral in the loan back to the lender even if the accumulated outstanding balance is larger. The termination time is obviously determined by one of the following four types of risk: mortality, long-term care, prepayment and refinancing (Szymanoski 1994). Depending on the market conditions, a longer termination time increases the possibility of something bad happening, as well as generate higher administrative costs. The advantage or disadvantage position in the contract is also contingent on the relative *cumulative* growth of house prices versus loan balance. Longer contracts may allow risks associated with stochastic house price movement to crystalise.

Moral hazard may increase the NNEG risk if the borrowers forfeit on their obligations to maintain the state of the property (Shiller & Weiss 1999). To minimise this risk most ERMs products require borrowers to maintain the insurance for the property, pay the running fees on the house, maintain the property, be the sole residents in the property, do not leave the property unoccupied for longer than six months.

4.2.3 ERM consultation issues in the UK

The PRA maintains an active market monitoring process in regulating the ERM market space. The discussion here takes a look at the implication of new changes to the PRA Solvency Statements for insurers and reinsurers who hold ERM products on their balance sheet. The changes respectively affect the following parameters; real interest rates (long-term rates), deferment rate, and house price volatility. The responses considered here in relation to discussion papers (DP) in Prudential Regulation Authority (2019*b*) and Prudential Regulation Authority (2019*a*) policy statements (PS), which constitute current improvements that follow from Bank of England (2016), Prudential Regulation Authority (2017), and Prudential Regulation Authority (2018).

The PRA's analysis distinctively considers the real interest rates to be related to the minimum deferment rate. The Prudential Regulation Authority (2018) report sets the "best estimate" of the deferment rate at 2% p.a. The deferment rate accounts for the extent to which the fair value for deferred possession of the collateral property should vary from the price paid for immediate possession. For example, if the current market value of the collateral property is £150,000; then a 2% deferment suggest that a rational individual will be prepared to pay £147,058.82 to own the property. The deferment rate is used to calculate the deferred future possession value of the property. One may consider the annualised deferment rate as the discount rate

[7]The PRA's 1% deferment rate becomes effective from December 31, 2021 where it completely phases out the current nil deferment rate regime.

applied on the current market value of collateral property to set the discounted price equal to the current price a rational individual is willing to pay today for deferred future possession.

Pertaining to the *Reviewing and updating the minimum deferment rate*, the PRA asserts that their main objective is to allow for efficient management of the NNEG costs to interest rate sensitivity. The minimum deferment rate (MDR) for ERMs together with the volatility parameter (VP) will be reviewed often[8]. The MDR is strictly positive, and in absolute terms, its expected change will not be less than 0.5 percentage points. Changes in the MDR is tied to material changes in long-term real risk-free interest rates. This suggest a need to establish the link between long-term real interest rates and the MDR, and how transparent the review process will be.

The link between the two is probably resulting from the fact that the PRA considers property as a real[9] asset hence bearing a link with implied inflation rates. There is however, a need to assess how the frequency of MDR reviews will impact ERM pricing; including the possible imperfect relationship between nominal and real interest rates. Industry practitioners express the need to work towards reducing the sensitivity of the NNEG to interest rates. The PRA imposes the Black-Scholes option pricing model as the appropriate model for the valuation of the NNEG clause. Given the fact that the classical Black-Scholes model depends on risk-neutralisation methods, there is no need for loan issuers to consider inflation or property growth expectations. The service flow rate may in fact, be a better parameter to use. This further goes to suggest that the MDR should rather be linked to the service flow rate and not interest rates. Looking at the assumptions under the Black-Scholes model, the PRA's procedure presents an inappropriate hybrid of real-world and risk-neutral approaches in the ERM market space.

The *Deferment rate estimation*[10] is based on the Sportelli formula and is determined as follows: define π to denoted the property return which can be decomposed in two ways:

$$\pi = g + k \tag{4.1}$$

$$\pi = r_f + \tilde{r} \tag{4.2}$$

where g is the deferment rate, k is the long-term capital growth rate, r_f is long-term nominal risk-free rate and $\tilde{r}$ is a given risk premium. Rearrangement of the system of equations in (4.1) results in $g = r_f + \tilde{r} - k$;

[8]the minimum deferment rate will be reviewed twice a year i.e. March and September.

[9]In Prudential Regulation Authority (2019*a*), the PRA agrees with some ERM loan issuers who value the NNEG under real-world probabilities with nominal property growth calculated as inflation plus a stable assumption.

[10]The justification is based on the Sportelli formula, which determines market value of real assets in relation to the premium a freeholder attaches to deferred possession. The formula is inherently closer to the real interest rates rather than market view (see Prudential Regulation Authority 2019*a*).

thereby suggesting that changes in the g are driven by changes in r_f. The real deferment rate is obtained by deducting long-term inflation from the both sides of nominal deferment rate equation.

$$g - i = r_f + \tilde{r} - k - i \tag{4.3}$$

where i is the long-term inflation rate. The PRA further suggests this to be the minimum estimate of the deferment rate, assuming the estimation produced can be made more robust when k and $\tilde{r}$ are thoroughly calibrated. Rental yields are only seen as reasonable starting point for calculating deferment rates in the short-run. Market professionals somehow prefer to use the service flow rate because of the Sportelli formula may introduce inflation volatility risk into the balance sheets. This will happen when insurers estimate the deferment rate in nominal terms. Prudential Regulation Authority (2018) notes that the term structure of net service flow rate departs significantly from the deferment rates over the long-run. With the ERM being a long-term loan that experiences liability growth with time, the service flow rate may not be a true reflection of the borrower's foregone income.

The following are other active engagements the PRA undertakes: reviewing and updating the volatility parameter, treatment of ERM loans with uncertain principal and interest, and general issues around ERM valuation for firms. The PRA consistently engage the ERM service providers to ensure rule-based regulatory requirements are met. This is in line with prudential regulations required under the IFRS solvency II standards.

4.2.4 Design and pricing mechanics of ERMs

In its basic form, the ERM loan is required to be the primary debt that is written against the collateral house. With an exception to payments for maintenance cost, property taxes, and other related loan service charge(s), the borrower has no other credit obligations. The initial loan paid out at the inception of the contract is also determined in direct relation to the market value of the house. Both academia and practitioners contextualise the ERM contract as a portfolio that comprises of income security and a crossover put option that kicks-in when any of the termination events occur. In this event, the NNEG automatically sets the collateral house to the loan issuer even when the accumulated loan balance exceeds the market value of the house.

Rufenacht (2012) presents an efficient market consistent valuation procedure for pricing the embedded put option. Andrews & Oberoi (2015), Pfau (2015), Merton & Lai (2016) also discuss issues related to securitisation, risk management and design of ERM schemes. Pricing valuations from the borrower's perspective is also outlined in Nakajima & Telyukova (2017) and Blevins et al. (2017). The pricing complexities of the

NNEGs in ERM contracts mainly draw to frictional divergence in views on the best model for its pricing and calibration. This thesis argues for the need to investigate alternative NNEG valuation methodologies and argue for the need to focus on pricing the NNEG from a model-risk perspective. They expand on the work of Hosty et al. (2008) and present a detailed discussion on the key issues that affect the NNEG pricing including analysis on the sensitivities of parameter estimates, and issuer and borrower characteristics. This thesis also proposes the need to take a portfolio view of the NNEG valuation procedure while considering idiosyncratic risk due to house price. Previous studies Addae-Dapaah & Leong (1996), Li (2010), Wang et al. (2008) have solely focused on an individual pricing approach.

Majority of existing studies consider the option-based view relating to which, the NNEG is conceived as a written put-options on the collateral house price (see Chinloy & Megbolugbe 1994, Miceli & Sirmans 1994, Boehm & Ehrhardt 1994, Siu-Hang Li et al. 2010, Tsay et al. 2014). With regards to this approach, some of the first generation models Addae-Dapaah & Leong (1996) make use of static mortality tables, largely ignoring the effects of mortality variations over time. In order to account for asymmetric jump effects, Chen, Chang, Lin & Shyu (2010) make use of a generalised Lee-Carter model for borrower mortality; while assuming house price growth has an autoregressive moving average mean (ARMA) model with a variance that follows a generalized autoregressive conditional heteroskedasticity (GARCH). The joint effect of age and house price risk on ERM is also emphasised in Ma & Deng (2013) who use an actuarial-based valuation for Korean-based ERM products.

Shao et al. (2015) has also shown that using an aggregate house price index without consideration for cohort trends in mortality will result in understating ERM values. Their model discusses the relative effect of longevity and house-price risk within a multi-period Wills-Sherris mortality model and a new hybrid repeated-sales pricing model for house prices. Wang et al. (2014) on the other hand shows that the initial loan-to-value (LTV) ratio is independent of the term structure of loan interest rates. Wang et al. (2014) present a methodology for pricing a variable-rate reverse mortgage where the instantaneously adjustable interest rate process for loans is modelled by a CIR model for interest rates. The ERM products under this approach are not sustainable at higher house price volatilities according to the base scenario. House price volatility increases with an increase in respective model parameters, thereby causing an increase in the cost of ERM insurance. Wang et al. (2014) further showed how the present value of premiums mobilised under the Federal Housing Administration falls below the present value of total expected cash outflow triggered by claim payments.

The valuation process for NNEG has to clear some important hurdles:

A1 Identify a suitable economic scenario generator including the house price index under the real-world or physical measure P. This can be useful for other risk-management calculations such as value-at-risk or expected-shortfall.

A2 Identify a suitable mechanism for switching from real-world measure P to risk-neutral measure Q. This step is called *risk-neutralisation* of valuation calculations.

B Specify the model for the random maturity which is determined by multiple decrement probabilities for the ERM; this incorporates mortality, move to long-term care and prepayment.

C Risk-neutral valuation of the contract (ERM or NNEG).

For step A1 insurers can select their preferred ESG (subject to regulatory approval). The NNEG put option in A2 has random maturity and strike price. The strike price depends on the accumulated value of the lump sum loan. This makes the market incomplete since a unique martingale measure fails to exist. There is a need consider some flexibility when it comes to selecting a method for risk-neutralisation. Possible methods include conditional Esscher Wang et al. (2017) and Buhlman et al. (1996), Geometric Lévy Process as shown in Miyahara (2011), and Transaction Cost Models in Davis (1997*b*). Approaches vary in terms of theoretical and computational complexity. Insurers also have great flexibility over the choice of maturity distribution model including possible future mortality improvements, prepayment and so on as implemented in this chapter.

4.2.5 Real-world pricing approach

The works that involve a real-world/physical measure in their valuation methodologies include Chinloy & Megbolugbe (1994), Ortiz et al. (2013), Lew & Ma (2012) and Ma & Deng (2013). These studies mostly implement a deterministic pricing technique to model house price growth which makes them a bit misleading when calibrating the dynamics and sensitivities of the valuation parameters. Hosty et al. (2008) model the house price process by a geometric Brownian motion (GBM) under the real-world measure and hint that the technique assumes the future house price index return is independent of prior periods. The volatility term remains the same as that of the risk-neutral approach, but their respective drift-terms are different. However, there is always dependency in levels even if returns are independent. Hosty et al. (2008) suggested a mean-reverting model will suffice as an appropriate approach. Tunaru & Quaye (2019) also suggest that the real-world measure becomes a fitting alternative whenever one can specify a good econometric model for the underlying house prices.

In general, the expected rate of growth in a house price index is expected to be higher than the risk-free rate. With equal volatilities under risk-neutral measure and real-world measure, for a GBM specification, a put option evaluation under the risk-neutral measure will be larger, ceteris paribus, than a real-world valuation. This relative order relationship will be obviously reversed if the trend for house prices goes negative (or at least with a drift rate lower than the risk-free rate). From works such as Boness (1964), the valuations could be the same but only under risk-neutrality one can use the same discount rate. The Boness (1964) formula is mathematically identical to the Black-Scholes 1973 formula, the approach requires two rates: one for discounting the stock (house or HPI here) and a separate rate to discount the expected (under actual measure) PV of the option payoffs, call or (here) put.

4.2.6 Risk-neutral pricing approach

The risk-neutral valuation technique for ERMs has approach has equally received much attention. Some of such works include Hosty et al. (2008), Kogure et al. (2014), Alai et al. (2014), Ji et al. (2012),Lee et al. (2012),Wang et al. (2014),Li et al. (2010), Chen, Cox & Wang (2010). The corresponding risk-neutralisation techniques usually employed to risk-neutralise the predictive distributions include the: Esscher transform Shao et al. (2015), Alai et al. (2014), Wang transform Wang et al. (2014), Li (2010), Bayesian entropy technique Kim & Li (2017), Kogure et al. (2014), and Ang & Piazzesi (2003), Ang et al. (2006) stochastic discount factor model which is applied in Alai et al. (2014) and Shao et al. (2015).

Hosty et al. (2008) describes another version of the risk-neutralisation approach that fits a lognormal model to the Nationwide Average House Price. Tunaru & Quaye (2019) review and replication of Hosty et al. (2008) show that the data generating process assumed under the GBM model possesses a risk-neutralised drift term that is estimated as the difference between the yield on government stock less the service flow rate which is calibrated from the IPD residential property index. Arguing on the basis of a market-consistent approach, they scale up the house price volatility from 5% p.a. to 11% p.a. using the Booth & Marcato (2004) de-smoothing technique. The downside to this approach has to do with the fact that the Booth & Marcato (2004) method is for commercial properties, while ERMs are written on residential properties.

There are also econometric models proposed for modelling the underlying house price indices in the ERM pricing. Kim & Li (2017) for instance fitted a VAR-DCC/GARCH, Chen, Cox & Wang (2010), Li et al. (2010), Kogure et al. (2014), Yang (2011) and Lee et al. (2012) fitted an ARMA-EGARCH time series model. Shao et al. (2015) and Alai et al. (2014) also work with a VAR model for the house price process. The time series version does well to factor in the stylised features of the return series of the underlying house price index. Clearly, these models draw many benefits including the ease with which I can change the distribution of the error process and the specification for the conditional mean and variance.

It is worth noting that the other continuous-time models are specified under the real-world measure before switching to their risk-neutral versions. For example the mean-reverting process in Fabozzi et al. (2012) or in Knapcsek & Vaschetti (2007), the jump-diffusion process as in Wang et al. (2014), Lee et al. (2012) or Knapcsek & Vaschetti (2007), and the Lévy process also in Wang et al. (2014).

4.2.7 Objectives of the study

Following the discussion on the research gaps, the research study aims to:

i) formulate a framework that incorporates actual house price features and ensures fair valuation of the NNEG concept in ERM contracts.

ii) extensively investigate the sensitivity of model parameters to the NNEG liability cash flows in ERM contracts.

iii) present a general model that allows ERM loan issuers to explore the impact of multiple decrement conditions of ERM contract termination.

4.2.8 Research questions of the study

There is a need to amplify the research question connected to the valuation of the NNEG clause in ERMs and more expediently provide answers to them. More specifically, the following research questions will be addressed:

i) Does the ARMA-EGARCH framework transparently calibrate and ensure fair valuation of the NNEG concept in ERM contracts?

ii) What are the key drivers of the NNEG liability cash flows in ERM contracts?

iii) Is there a general model that efficiently calibrate the multiple termination modes of ERM loans?

4.3 Models and Methods

This section of the thesis will conceptualise and establish the pricing model for the ERM loan. The focus is to propose an improved model than the one which has been adopted by practitioners in the UK. The calculations in general are done for an ERM loan based on a lump-sum contract, the commonly issued loan on the market. Similar to Hosty et al. (2008) and Li et al. (2010), I use quarterly frequency for all variables and I follow the cash-flows at the end of each period $i \in \{1, \ldots, \eta\}$ where η is an acceptable provisional end maturity given by survival to 100 years. Actuarial modelling typically assumes a maximum age of 120 rather than 100. NNEG calculations for borrowers aged 100 years and above are zero. The Office of National Statistics (ONS) life table has a limiting age of 100, survival probability of death are approximately zero at that point. I assume loan termination is independent of interest rate and house prices in order to simplify calculations. Hosty et al. (2008) adopt the same assumption. I shall denote by $Y_t = \ln\left(\frac{H_t}{H_{t-1}}\right)$ the log-return of the house price index at time t.

Henceforth $V(t)$ denotes the value at time of an ERM. Without loss of generality I focus on the case of a lump-sum loan L_0 as a percentage of the current value H_0 of the collateral house owned by the borrower. The fixed rate R is the accruing rate on the loan until τ and $\{r_t\}_{t\geqslant 0}$ is the risk-free (or the issuer funding rate, OIS or similar. Interest rates are assumed to be deterministic in my analysis) in the economy for discounting cash-flows by the issuer. The analysis for a floating interest rate environment is also considered in the discussion section.

The accrued balance on the loan at time τ is $L_\tau = L_0 e^{R\tau}$. In order to avoid mathematical complications I assume that $r_t \equiv r$, so the risk-free rate is constant. Then the market-consistent value of the ERM at time t to the issuer is computed under a risk-neutral pricing measure as

$$V(t) = E_t^Q[e^{-r(\tau-t)} \min(L_\tau, H_\tau)] \tag{4.4}$$

This formula can be further decomposed as in credit markets

$$\begin{aligned} V_t &= E_t^Q[e^{-r(\tau-t)}(H_\tau - \max(H_\tau - L_\tau, 0))] && (4.5)\\ &= E_t^Q[e^{-r(\tau-t)}(L_\tau - \max(L_\tau - H_\tau, 0))] && (4.6) \end{aligned}$$

$$V_t = e^{-r(\tau-t)} E_t^Q[L_\tau] - e^{-r(\tau-t)} E_t^Q[\max(L_\tau - H_\tau, 0)] \tag{4.7}$$

whereas formula (4.5) shows that the value of the ERM can be also seen as the value today of future house possession and selling a call option on the house with the strike price L_τ

$$V_t = \underbrace{e^{-r(\tau-t)} E_t^Q[H_\tau]}_{\text{prepaid forward on house}} - \underbrace{e^{-r(\tau-t)} E_t^Q[\max(H_\tau - L_\tau, 0)]}_{\text{call option on house}} \tag{4.8}$$

Since τ is stochastic, a direct valuation of the above formulae is not facile. However, I can use a conditioning argument, working with an annual grid and rewrite

$$V_t = \sum_{j=t+1}^{T} E_t^Q[e^{-r(\tau-t)}L_\tau \times 1\{\tau = j\}] - \sum_{j=t+1}^{T} E_t^Q[e^{-r(\tau-t)}\max(L_\tau - H_\tau, 0) \times 1\{\tau = j\}] \tag{4.9}$$

where T represents the maximum age that the borrower may reach, taken typically as 100 or 120 in ERM markets. A monthly grid is also feasible. Denoting $L_j = L_0 e^{Rj}$ and H_j the house price at time j, under the assumption that the termination time τ is independent of the house price process $\{H_t\}_{t\geqslant 0}$ and applying the deferred probability of termination at time t as ${}_{t|j}q_x$

$$V_t = \sum_{j=t+1}^{T} {}_{t|j}q_x E_t^Q[e^{-r(\tau-t)}L_j] - \sum_{j=t+1}^{T} {}_{t|j}q_x E_t^Q[e^{-r(\tau-t)}\max(L_j - H_j, 0)] \tag{4.10}$$

or

$$V_t = \sum_{j=t+1}^{T} \Big({}_{t|j}q_x L_0 e^{(R-r)(j-t)} - {}_{t|j}q_x E_t^Q[e^{-r(j-t)}\max(L_j - H_j, 0)]\Big) \tag{4.11}$$

An ERM is then a portfolio of ERM yearly components, derived as the difference between a fixed income bullet bond and a put option with the exercise price L_j and contingent on the collateral house price, and then weighted by ${}_{t|j}q_x$ the deferred termination probability. x is the age of the borrower at contract inception. The calculation of ${}_{t|j}q_x$ is the probability that a borrower aged x terminates a contract between time $x+t$ and $x+t+j$. I present a formal derivation below. Once the NNEG value embedded in the loan is determined the valuation model becomes easily computable since the value of the loan repayment is nothing but a zero-coupon bond, and lenders must have robust valuation tools for the latter. Dowd et al. (2019) argue that the Black (1976) model can be applied to value the NNEG put option. The UK Prudential Regulatory Authority (PRA) as well, recommends a Black-Scholes pricing formula (see Prudential Regulation Authority 2019*b*).

I use a multiple decrement model to evaluate the conditional probability of termination. This is similar to Dickson et al. (2013); in Figure 4.3, ${}_tq_x^{(1)}$ denotes the probability that an active borrower aged x dies between age x and $x+t$; ${}_tq_x^{(2)}$ is the probability that an active borrower aged x voluntarily prepays between age x and $x+t$, and ${}_tq_x^{(3)}$ denotes the probability that an active borrower aged x moves into long-term care between age x and $x+t$. An active ERM loan contract is considered to be in state (0) i.e. the *Active state*. For any decrement $d = 1, 2, 3$, ${}_tq_x^{(d)}$ denotes the probability that a borrower aged x at inception of the contract fails within t years due to decrement (d).

This suggest that the (x) moves out of the active state, thereby causing the ERM contract to lapse. All the three states of decrements are mutually exclusive, in this regard I denote the probability of failing due to

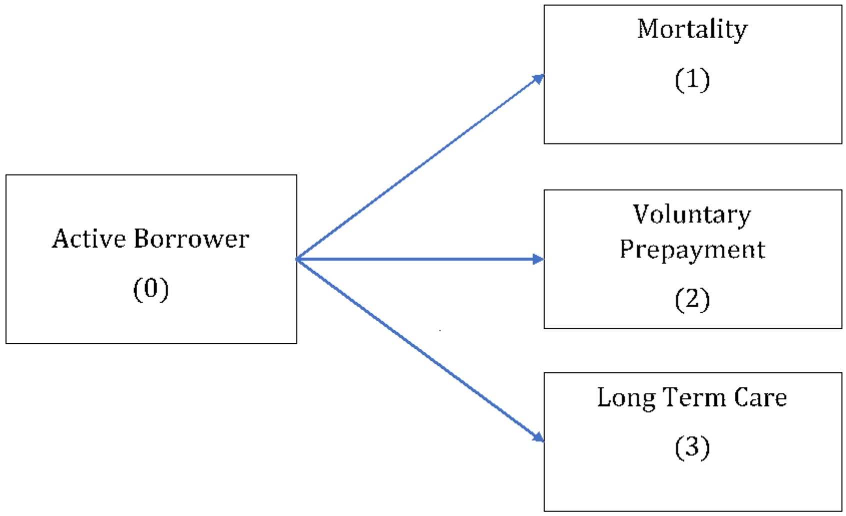


Figure 4.3: Discrete probabilities of ERM termination

any given decrement by $_t\widehat{q_x}$ which is the sum of the individual mutually exclusive probabilities of transition between the three states. For the three-state decrement model in Figure 4.3,

$$_t\widehat{q_x} = {}_tq_x^{(1)} + {}_tq_x^{(2)} + {}_tq_x^{(3)} \tag{4.12}$$

The probability of remaining in the active state is $_t\widehat{p_x} = 1 - {}_t\widehat{q_x}$. On this basis, the probability that any contract will be terminated within t years due to a specific decrement depends on (x) being in the active state before the failure year. More specifically, (x) survives $t-1$ years before failing in the t-th year. The probability that an ERM contract issued to (x) terminates due to decrement d within t years is

$$\begin{aligned} {}_tq_x^{(d)} &= \sum_{k=0}^{t-1} {}_kp_x^{(\tau)}\, q_{x+k}^{(d)} \\ &= {}_{k|}q_x^{(d)} \end{aligned} \tag{4.13}$$

This can be extended to the form:

$$_{t|j}q_x^{(d)} = \sum_{k=t}^{t+j-1} {}_kp_x^{(\tau)}\, {}_jq_{x+t}^{(d)} \tag{4.14}$$

which denotes the probability that a borrower aged x terminates a contract due to decrement d between time $x+t$ and $x+t+j$. In many instances the loan is given to a living couple. The loan will survive as long as one of the couple survives. One common assumption is to use for a borrowing couple a 95% adjustment factor of the base mortality table for the male and female.

Knapcsek & Vaschetti (2007) calculate the joint *cumulative* probability of death after t years for a couple (x), and (y) with the formula

$$ {}_tq_{xy} = {}_tp_x \times {}_tp_y \tag{4.15} $$

where ${}_tp_x$ is the cumulative probability of death by year t for (x). The same goes for ${}_tp_y$. There is also a possible correlation built-in as couples can take care of each other and survive longer.

In this regard the ERM contract survives t years and fails within the next u years due to some decrement d. In the multiple decrement setup, the NNEG and ERM value in (4.11)

$$ V_t = \sum_{j=t+1}^{T} \left({}_{t|u}q_x^{(d)} L_0 e^{(R-r)(j-t)} - {}_{t|u}q_x^{(d)} E_t^Q [e^{-r(j-t)} \max(L_j - H_j, 0)] \right) \tag{4.16} $$

4.3.1 The Deferment Rate

If H_t is the house price today the deferment price to get the house at a future time T is denoted by $\overleftarrow{F}_t(T)$ and the deferment rate q is defined by the equation

$$ \overleftarrow{F}_t(T) = H_t e^{-q(T-t)} \tag{4.17} $$

In derivatives terms $\overleftarrow{F}_t(T)$ is the prepaid forward price on the collateral house, which is linked to the forward price directly through interest compounding

$$ F_t(T) = e^{r(T-t)} \overleftarrow{F}_t(T) \tag{4.18} $$

Combining (4.17) and (4.18)

$$ F_t(T) = H_t e^{(r-q)(T-t)}. \tag{4.19} $$

Computationally, if $r - q < 0$ then $\{F_t(T)\}_{T \geqslant 0}$ decreases with T so the forward house price curve will be in backwardation. Vice versa, if $r - q > 0$ then $\{F_t(T)\}_{T \geqslant 0}$ increases with T so the forward house price curve will be in contango. Formula (4.19) simply ignores the fact that it is not possible to short sell house and that transaction costs are relevant in this market.

The PRA condition is requiring $\overleftarrow{F}_t(T) < H_0$ which from (4.17) is equivalent to $q > 0$. At the same time, the same condition is equivalent to

$$ F_t(T) < H_t e^{r(T-t)} \tag{4.20} $$

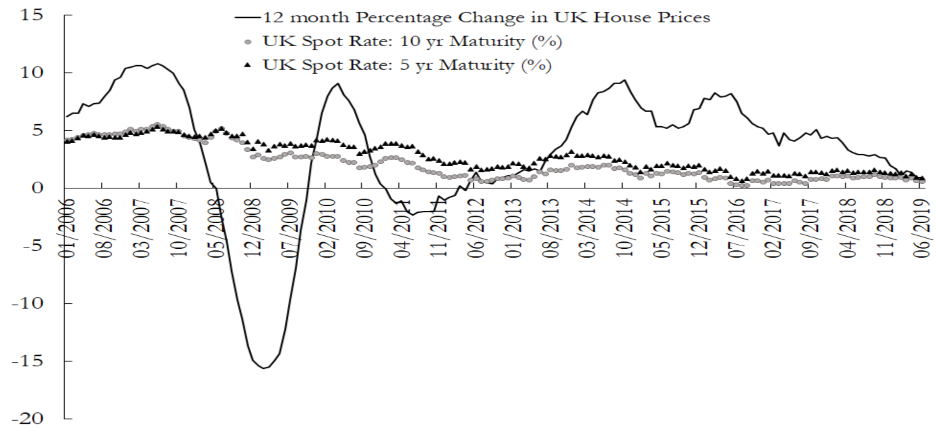


Figure 4.4: UK House Price Growth Vs 5-year and 10-year Risk Free Yields from January 2006 to June 2019.

Source: House prices are from HM Land Registry, Registers of Scotland, Land and Property Services Northern Ireland, Office for National Statistics – UK House Price Index. The monthly risk free series is from the Bank of England

which is equivalent to say that the forward curve on house prices will be bounded by the current house price inflated at the risk-free rate. One can remark that the upper boundary for the house price forward presented in (4.20) is too tight compared with the proper no-arbitrage upper boundary derived in Syz & Vanini (2011) who provide a more realistic set-up for property forward valuations.

While I agree that (4.20) is likely to hold during normal times, hence almost all of the time, I disagree that it ought to be true *all* the time. In the aftermath of a property market crash such as the subprime crisis, the house that may be the collateral in an ERM loan has the value impaired by the market collapse. However, sellers who may like to sell forward a house say in five or ten years time, may not agree to discount even further the house for their prepaid price. It is more likely then that the property sellers may consider that the property prices will recover from the market collapse in the medium to long term run and the house inflation rate is likely to exceed the risk free rate. Between 2008Q1 and 2009Q4, the UK property prices collapsed with an average annual price change of -6.44%. Property prices recovered by 1.13% p.a. between 2009Q1 and 2014Q4 and 2.67% p.a. between 2009Q1 to 2019Q3 while the 5 year and 10 year spot rates averaged 1.40% p.a. and 2.26% p.a. in the UK, respectively over the ten year period.

Another argument invoked by Dowd et al. (2019) in favor of the Black '76 model is the ability to work with the Black 1973 formula even when the increments of the data generating process returns are autocorrelated and they point to Cornalba et al. (2002) as the theoretical work supporting this rationale. However, Cornalba et al. (2002) work strictly with Black-Scholes formula and not with Black 1973 and moreover, their technique is applied when the increments are negatively autocorrelated near horizon. This is in contradiction with house price historical series that exhibit positive autocorrelation near horizon and negative autocorrelation long horizon, a well known empirical feature of real-estate prices time series (Tunaru 2017). Furthermore, the excellent technique otherwise presented in Cornalba et al. (2002) requires the possibility to hedge or trade long and short the underlying asset.

4.3.2 ARMA-EGARCH model

The ARMA-EGARCH model is answering two problems encountered when modelling house prices. First I have serial correlation. The ARMA part of the model should be able to capture efficiently this effect. Secondly, negative and positive innovations may have different effects on the conditional volatility, allowing financial markets to react asymmetrically to bad and good news, even though in absolute value those innovations may have the same magnitude (Patterson 2000). The model specification should also ensure that the conditional volatility or variance is always positive.

This model is built as a submodel for log-returns and a submodel for conditional volatilities. Hence, as in Li et al. (2010), first I specify an ARMA(m,M)

$$Y_t = c + \sum_{i=1}^{m} \phi_i Y_{t-i} + \sum_{j=1}^{M} \theta_j \epsilon_{t-j} + \epsilon_t \tag{4.21}$$

where $\epsilon_t \sim N(0, h_t)$, c is the drift term, m and M are the maximum lag order of the GARCH term and the ARCH term. Y_t is the log return of house price at time t and ϕ_i and θ_j denote the dependence of Y_t on the Y_{t-1} and the j-th innovation ϵ_{t-j}. A required condition is for $1 - \phi_1 B - \phi_2 B^2 - \cdots \phi_m B^m$ and $1 - \theta_1 B - \theta_2 B^2 - \cdots \theta_M B^M$ to possess common factors with absolute value of the zeros being greater 1. For the conditional variance h_t the EGARCH(P,Q) model is specified

$$\ln(h_t) = k + \sum_{i=1}^{P} \alpha_i \ln(h_{t-i}) + \sum_{j=1}^{Q} \beta_j [|\widetilde{\epsilon_{t-j}}| - E|\widetilde{\epsilon_{t-j}}|] + \sum_{j=1}^{Q} \gamma_j \widetilde{\epsilon_{t-j}} \tag{4.22}$$

with $\tilde{\epsilon}_t = \frac{\epsilon_t}{\sqrt{h_t}}$ is the standardized innovation at time t. The respective weights on $\ln(h_{t-i})$ and the standardized innovations $\widetilde{\epsilon_{t-j}}$ are α_i and β_i for $i = 1, \ldots, P$. The leverage[11] effect in the EGARCH process is captured by the leverage parameter γ_j for $j = 1, 2, \cdots, Q$. To ensure Equation 4.22 is weakly stationary, $1 - \alpha_1 B - \alpha_2 B^2 - \cdots \alpha_p B^P$ and $1 - \beta_1 B - \beta_2 B^2 - \cdots \beta_Q B^Q$ must have no common factors and the absolute value of the zeros must be greater than 1. Based on parameter count, the ARMA-EGARCH model gives 9 degrees of freedom to fit. This is considerable compared to GBM which needs just 2 parameters i.e. μ and

[11]The leverage effects allows the conditional variances to respond asymmetrically to positive and negative innovations. $\gamma_j > 0$ implies that a positive $\widetilde{\epsilon_{t-j}}$ adds more conditional variance than its negative counterpart of same magnitude.

σ. I expected ARMA-EGARCH model to immediately give advantage in terms of better fit.

Here t follows the discrete time grid dictated by the data, either monthly or quarterly for Nationwide average house price (non-seasonally adjusted).

My main interest is to use this model and obtain forecasts of volatility to long horizons that can be used for NNEG valuation. It follows then that, under the real-world measure $\mathbb{P}_t$

$$Y_t|\mathcal{F}_{t-1} \sim N(\mu_t, h_t) \tag{4.23}$$

where $\mu_t = c + \sum_{i=1}^{m} \phi_i Y_{t-i} + \sum_{j=1}^{M} \theta_j \epsilon_{t-j}$. The ARMA and EGARCH exact specifications are selected based on goodness-of-fit diagnostic statistics. The risk-neutralisation procedure of the ARMA-EGARCH model is presented below.

4.3.3 Esscher transform risk-neutralisation for ARMA-EGARCH

Let T be the longest possible maturity for the ERM product; as an example, for a 65 years old if I consider 100 the longest survivor age then $T = 35$ and let $\mathbb{P}$ be the probability measure associated with the information set $\mathcal{F}_T$. Consider $\mathbb{P}_t$ be the projected measure $\mathbb{P}$ on the smaller information set $\mathcal{F}_t$. Following Buhlman et al. (1996), Siu et al. (2004) and Li et al. (2010), for a given sequence of t constants $\lambda_1, \lambda_2, \ldots, \lambda_t$, the distribution function of Y_t under the measure $\widetilde{\mathbb{P}}_t$ given F_{t-1} through

$$F_{\widetilde{\mathbb{P}}_t}(y; \lambda_t|\mathcal{F}_t) = \frac{\int_{-\infty}^{y} e^{\lambda_t x} dF_{\mathbb{P}_t}(x|\mathcal{F}_t)}{E_{\mathbb{P}_t}(e^{\lambda_t Y_t}|\mathcal{F}_t)} \tag{4.24}$$

The key to the risk-neutralisation under the conditional Esscher measure is to observe that the moment generating function of Y_t given $\mathcal{F}_{t-1}$ under $\widetilde{\mathbb{P}}_t$ is calculated from

$$E_{\widetilde{\mathbb{P}_t}}(e^{zY_t}; \lambda_t|\mathcal{F}_{t-1}) = \frac{E_{\mathbb{P}_t}(e^{(z+\lambda_t)Y_t}|\mathcal{F}_{t-1})}{E_{\mathbb{P}_t}(e^{\lambda_t Y_t}|\mathcal{F}_{t-1})} \tag{4.25}$$

Because $Y_t|\mathcal{F}_{t-1} \sim N(\mu_t, h_t)$ so then it can be proved that

$$E_{\widetilde{\mathbb{P}_t}}(e^{zY_t}; \lambda_t|\mathcal{F}_{t-1}) = e^{(\mu_t + h_t\lambda_t)z + \frac{1}{2}h_t z^2} \tag{4.26}$$

The risk-neutral-measure is identified from the local martingale condition by finding those λ_t^q such that

$$E_{\widetilde{\mathbb{P}}_t}(e^{Y_t};\lambda_t^q|\mathcal{F}_{t-1}) = e^{r-g} \tag{4.27}$$

with r the risk-free rate and g the service flow rate[12]. This gives the risk-neutralising constants

$$\lambda_t^q = \frac{r-g-\mu_t-\frac{1}{2}h_t}{h_t} \tag{4.28}$$

Combining things together gives the sequence of risk-neutral measures Q_t such that

$$E_{\widetilde{\mathbb{Q}}_t}(e^{zY_t};\lambda_t^q|\mathcal{F}_{t-1}) = e^{(r-g-\frac{1}{2}h_t)z+\frac{1}{2}h_t z^2} \tag{4.29}$$

which shows that the risk-neutralization effect is to keep the same type of normal distribution but change by translation the parameters. Thus, under $\mathbb{Q}_t$, I have that

$$Y_t|\mathcal{F}_{t-1} \sim N(r-g-\frac{1}{2}h_t, h_t) \tag{4.30}$$

For pricing the NNEG I need to calculate the following risk-neutral expectation, see Li et al. (2010),

$$e^{-r(t+0.5+\delta)}E_{\mathbb{Q}}[\Big(L_0 e^{R(t+0.5+\delta)} - H_{t+0.5+\delta}\Big)^+] \tag{4.31}$$

For simplicity let us denote by $\tau = t+0.5+\delta$ which is the known maturity given by the termination of the RM, and $K = L_0 e^{R(t+0.5+\delta)}$ is the accrued balance at τ which is known. Hence the option above is a put option on H_τ. Now, a correct approach will have to take a path-dependent approach and build recursively the chain of conditional volatilities (variances) to the required maturity. For example, for maturity τ, the house price H_τ can be calculated as

$$H_\tau = H_0 \exp(\sum_{i=1}^{i=\tau} Y_i)$$

[12]In the absence of market prices for forwards/futures or total return swaps on property, selecting a martingale measure is done with the conditional Esscher transform. An excellent discussion of technical issues involved with using the conditional Esscher transform to identify a martingale measure under the incomplete market setting, in relation to GARCH models, can be found in Siu et al. (2004). For calibration purposes the local martingale condition described here arises by constructing a self-financing portfolio with one unit of the asset S and all rental cum-income invested in the bank account. Under the bank account numeraire, the portfolio discounted is a martingale. In other words, under the martingale measure as identified above, the normalised gains process is a martingale, see Bjork (2009) for an exposition how to deal with dividend income in asset pricing. The rental income should be considered *net of running costs* where possible. Another difficulty with rental income is that when using pounds rental income, for option pricing purposes the present value of *all future* rental income is needed and calculating that looks very difficult, particularly for long horizons.

using Monte Carlo simulation based on (4.30) for the risk-neutral measure and based on (4.23) under the real world measure. I shall refer to this Monte Carlo simulation approach[13] as the ARMA-EGARCH risk neutral (ARMA-EGARCH-rn) for the former and the ARMA-EGARCH real world (ARMA-EGARCH-rw).

4.3.4 Simulating the house price return process

The procedure to simulate the house price process is a straightforward one (see examples in Hardy 2003). Given the lognormal (LN) distribution with parameters μ and σ per time unit, and an appropriate random number generator[14]; I proceed with the following steps:

(i) Generate z_1, a standard normal deviate

(ii) Calculate the log-return of house prices at the first time unit $Y_1 = \mu + \sigma z_1$, and then use Y_1 to generate $H_1 = H_0 \exp(Y_1)$ the house price at time $t = 1$.

(iii) Repeat steps (i) and (ii) for each $t = 2, 3, \ldots, n$ where n denotes the projection period in each simulation.

(iv) Continue by repeating steps (i) and (iii) for some G scenarios, for which I choose G in order to increase accuracy in the analysis.

4.3.5 ARMA option pricing model for NNEG

This section develops a closed form formula for the NNEG put option defined for ERM contracts when underlying house price returns follow an autoregressive moving average (ARMA) process. The model considers an instance where the drift term of the ARMA process has an AR effect and the diffusion term an MA effect. Wang et al. (2012) defined a martingale transformation required for the discounted house price following this framework.

House price historical series in empirical data exhibit positive autocorrelation near horizon and negative autocorrelation at the long horizon (see Tunaru 2017), thereby setting the ARMA model an alternate good fit since ERMs are likewise long-term collateralised loans. The ARMA model is specified as follows

$$\widetilde{Y}_t = \omega dt + \sum_{i=1}^{\widetilde{m}} \alpha_i d \ln H_{t-ih} + \sum_{j=0}^{\widetilde{M}} \sigma \beta_j dW^P_{t-jh} \tag{4.32}$$

[13] A similar procedure applies for the ARMA-GARCH family of models.
[14] Here I used *twister*, the default MATLAB random number generator to increase the speed of the large simulation exercise.

where, α_i and β_j the coefficients[15] of the autogressive and moving average terms. $\tilde{m}$ and $\widetilde{M}$ are the respective AR and MA orders. $\widetilde{Y}_t$ is the house price log return under the ARMA model. The constant term is ω, σ is the constant volatility coefficient, $h > 0$ is a small arbitrarily fixed constant and $dt > 0$. W_t^P in the model is a one-dimensional standard Brownian motion defined over a filtered probability space where $dW_{t-ih}^P, i = 1, 2, \ldots, N$ are the consecutive instantaneous increments of the standard Brownian motion at $t - ih$. h is set equal to the frequency of the data set when working in an empirical setting.

Wang et al. (2012) showed that (4.32) does not allow for arbitrage opportunities and further possesses a closed-form solution for ARMA($\tilde{m}, \widetilde{M}$)-type options in a martingale pricing approach (see, Appendix C.1). Recall previously that the NNEG value at the maturity date T is $V(T) = \max[K_T - H_T, 0]$, where K_T is the accumulated loan balance and H_T is the house price at time T. The value of the put option P_{t_0} at time-t_0 is given by

$$P_{t_0} = e^{-r(T-t_0)} E^Q \Big[\max(K_T - H_T, 0) | \mathcal{F}_{t_0} \Big] \tag{4.33}$$

The corresponding closed-form solution for (4.33) is written as

$$P_{t_0} = K e^{-r(T-t_0)} \Phi\big(- d_{2N}(t_0, T)\big) - H_{t_0} e^{g(T-t_0)} \Phi\big(- d_{1N}(t_0, T)\big) \tag{4.34}$$

where

$$d_{1z}(t_0, s) = \frac{\ln(H_{t_0}/K) + \Big(r - g + 0.5\sigma_z^2(t_0, s)\Big)(s - t_0)}{\sigma_z(t_0, s)\sqrt{(s - t_0)}}$$

and

$$d_{2z}(t_0, s) = d_{1z}(t_0, s) - \sigma_z(t_0, s)\sqrt{(s - t_0)}$$

$$\sigma_z^2(t_0, s) = V_z(t_0, s)/(s - t_0)$$

$z = [(s - t_0)/h]$ for any $s > t_0$, where $[x]$ denotes the integer part of any x. The cdf of the standard normal distribution is $\Phi(\cdot)$. The closed-form formula expressed in (4.34) is the Black-Scholes formula which has a volatility function that depends on α and β in (4.32). The ARMA-type Black (1976) futures option pricing version can be obtained from (4.34) when H_{t_0} is replaced with Fe^{T-t_0}, the futures house price at time t_0, where T is the settlement date. (4.34) reduces to the Black-Scholes model as $(s - t_0) \to 0$ (see Heston & Nandi 2000, Liao & Chen 2006, Wang et al. 2012). Setting $\varPhi_0$ to 1 in Appendix (C.8), results in a conditional

[15] $\beta_0 = 1$

variance as follows

$$V_n(t_0,t) = \sigma^2\Big(1_{(n>0)}\sum_{j=0}^{n-1}\big(1+\sum_{i=1}^{j}\Psi_i\big)^2 h + \big(1+\sum_{i=1}^{n}\Psi_i\big)^2(t-t_n)\Big),\ \forall t \in [t_0,T] \tag{4.35}$$

where $n = [(t-t_0)/h]$ and Ψ are parameter coefficients that depend on the moving average (MA) orders in Equation (4.38) . Additionally, if $\Psi_i = 0$ for all $i = 1, 2, \ldots, n$, then I obtain the same variance used in the Black-Scholes model:

$$\sigma_n^2(t_0,t) = \frac{V_n(t_0,t)}{t-t_0} = \frac{\sigma^2\Big(1_{(n>0)}\sum_{j=0}^{n-1} h + (t-t_n)\Big)}{t-t_0} = \sigma^2 \tag{4.36}$$

This implies that using the Black-Scholes model when $\Psi_i \neq 0$ for $i = 1, 2, \ldots, n$ will result in undervaluing the NNEG. Wang et al. (2012) found that the Black-Scholes model will overvalue the option when $\alpha_1 + \beta_1 < 0$ and further confirms that the AR effect is more significant in driving option prices compared with the MA effect. This is contrary to earlier findings in Liao & Chen (2006) who assert the opposite in stock option contracts. Equation 4.32 can be rewritten using lag operators $L^k\chi_t = \chi_{t-kh}$ as follows

$$\big(1-\alpha_1 L - \cdots - \alpha_{\widetilde{m}}L^{\widetilde{m}}\big)\widetilde{Y}_t = \omega dt + \sigma\big(1+\beta_1 L + \cdots + \beta_{\widetilde{M}}L^{\widetilde{M}}\big)dW_t^P \tag{4.37}$$

Equation (4.37) reduces to the GBM model when the AR and MA orders both equal zero. In this sense, the approach I introduce and the GBM are consistent and additionally, I get a better higher order approximation for the house price volatility. Ψ_j is estimated by evaluating

$$\Psi_j = \begin{cases} \sum_{i=1}^{\widetilde{m}}\Big(c_i\Big(\sum_{k=0}^{j}\beta_{j-k}\lambda_i^k\Big)\Big) & ,\ \text{if } j \leqslant \widetilde{M} \\ \sum_{i=1}^{\widetilde{m}}\Big(c_i\lambda_i^{j-\widetilde{M}}\Big(\sum_{k=0}^{\widetilde{M}}\beta_{\widetilde{M}-k}\lambda_i^k\Big)\Big) & ,\ \text{if } j \geqslant \widetilde{M} \end{cases} \tag{4.38}$$

where $\sum_{i=1}^{\widetilde{m}} c_i = 1$ for $i = 1, \ldots, \widetilde{m}$ and

$$c_i = \frac{\lambda_i^{\widetilde{m}-1}}{\Pi_{k=1}^{\widetilde{m}}(\lambda_i-\lambda_k)},\quad i = 1,\ldots,\widetilde{m}, k \neq i$$

The variance of the instantaneous house price returns R_t at any given time t, conditioned on information set available at time t_0 is given by

$$Var_{t_0}(R_t) = \sigma^2\Big(\sum_{j=0}^{n}\big(\Psi_j^2\big)\Big)dt,\quad \forall t \in [t_n, t_{(n-1)}) \tag{4.39}$$

where $n = [(t - t_0)/h]$, and $R_t \equiv \widetilde{Y}_t$. Likewise, the conditional autocorrelation coefficient for house price returns is:

$$Corr_{t_0}\big(R_t, R_{(t+gh)}\big) = \frac{\sum_{j=0}^{n} \Psi_j \Psi_{(j+g)}}{\sqrt{\sum_{i=0}^{n} (\Psi_i)^2} \sqrt{\sum_{i=0}^{n+g} (\Psi_i)^2}}. \tag{4.40}$$

Thus, there will be no autocorrelation in the absence of the autoregressive and moving average parameters. Appendix C.2 presents the procedure to obtain Ψ_j.

4.3.6 ARIMA-GARCH option pricing model for NNEG

This subsection will describe and present the NNEG put option pricing using an ARIMA-GARCH model for underlying house price returns. Recall that the house price return series are characterised by negative autocorrelation in the long term. In order to allow for an efficient description of the actual house price return dynamics, I need to adjust the error distribution of the conditional mean model including the specification of the autoregressive conditional heteroscedastic (ARCH) component. The presence of heteroscedasticity suggest that the market is incomplete. Badescu et al. (2011) and Ogneva & Golembiovskii (2018) also suggested a likelihood of non-replicable contingent liabilities.

To ensure I maintain parsimony in the model specification, the thesis opted for a flexible error distribution model. The ARIMA-GARCH is one of such models with this feature, which allows for the conditional variance of the return series to be modelled with a generalised ARCH (GARCH) and subsequently, the conditional mean series by the autoregressive integrated moving average (ARIMA).

As shown earlier, the volatility of the real residential market does not follow GBM the essential precondition required under the Black-Scholes model. Additional time series characteristics of residential property returns for the UK is outlined in Section 5.3.

The mathematical formulation for the NNEG value is based on the no-arbitrage principle, which relies on a risk-neutralization measure. The market is not complete under the ARCH model, hence the non-existence of a unique equivalent martingale measures. Following Duan (1995), Föllmer & Schied (2011), and Ogneva & Golembiovskii (2018), the house price at time t is given by $H_t = H_0 e^{y_t}$ where y_t denotes the continuously compounded rate of return and $y_0 = 0,\ t = 0, \ldots, T$ in the ARIMA-GARCH case.

Suppose I denote the rate of return increment by δ, and specify a GARCH model for house price prices as in Heston & Nandi (2000) where

$$\ln H_t = \ln H_{t-\delta} + r + \lambda\sigma_t^2 + \sigma_t\varepsilon_t \tag{4.41}$$

$$\tilde{\sigma}_t^2 = \tilde{\sigma}_0 + \sum_{j=1}^{\widehat{m}} \tilde{\alpha}_j \tilde{\sigma}_{t-j\delta}^2 + \sum_{j=1}^{\widehat{M}} \tilde{\beta}_j \left(\tilde{\varepsilon}_{t-j\delta} - \tilde{\gamma}_j \tilde{\sigma}_{t-j\delta}\right)^2 \tag{4.42}$$

The option price under this model is estimated numerically as the mean of the present value of option payoffs $\tilde{x}_t$ under risk neutral measure Q, where

$$E^Q\left[\tilde{x}_t|\mathcal{F}_{t_0}\right] = \tilde{x}_{t_0}, \quad t_0 < t \tag{4.43}$$

$$E^Q\left[e^{y_t}|\mathcal{F}_{t_0-1}\right] = e^r \tag{4.44}$$

The estimation of (4.43) and (4.44) for my GARCH process follows Duan (1995) local risk-neutral estimation procedure. While satisfying Camara (2003) sufficient conditions, I assume the errors of the GARCH process follow a normal distribution and the conditional variance is constant over one period in relation to the variations in the risk-neutral measure. Under this procedure, the real-world house price returns are given by:

$$y_t = r + \lambda\sigma_t - \frac{1}{2}\sigma_t^2 + \varepsilon_t \quad \varepsilon_t|\mathcal{F}_{t-1} \sim i.i.d.\mathcal{N}(0, \sigma_t^2) \tag{4.45}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{\widehat{m}} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{\widehat{M}} \beta_j \sigma_{t-j}^2 \tag{4.46}$$

where λ is the coefficient of the risk-premium, r is the one period risk-free rate of interest. The corresponding risk-neutralized versions of (4.45) and (4.46) are given by

$$y_t = r + \lambda\sigma_t - \frac{1}{2}\sigma_t^2 + \xi_t \quad \xi_t|\mathcal{F}_{t-1} \sim i.i.d.\mathcal{N}(0, \sigma_t^2) \tag{4.47}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{\widehat{m}} \alpha_i \left(\xi_{t-i} - \lambda\sigma_{t-1}\right)^2 + \sum_{j=1}^{\widehat{M}} \beta_j \sigma_{t-j}^2 \tag{4.48}$$

The value of the house together with the NNEG put option under the risk-neutral measure Q is written as

$$H_T = H_t \exp\left(\left(T-t\right)r - \frac{1}{2}\sum_{i=t+1}^{T} \sigma_i^2 + \sum_{i=t+1}^{T} \xi_i\right) \tag{4.49}$$

$$p_t = e^{-(T-t)} E^Q\left[\max(K_T - H_T, 0)|\mathcal{F}_t\right] \tag{4.50}$$

where K_T the strike price, is the accumulated value of the initial loan at time T. The termination of the ERM contract occurs at T when the borrower dies, transition into long term care or early prepays.

4.3.7 Interest rate risk

The most evident risk affecting ERMs is interest rate risk. Given the long and uncertain maturity of these loans, one needs to rely on models to simulate future paths for interest rates. Lenders of ERMs use, in general, two types of rate. The contract rate R is the rate charged on the loan. This is the rate at which the loan balance grows. Secondly, there is the discount rate $\{r_t : t \geqslant 0\}$ which is the discount rate used to calculate the present value of the mortgage loan. Very often in the literature $r_t \equiv r$, which is a constant risk-free rate used for discounting purposes. There is evidence that, where choice is available, borrowers will prefer adjustable rates to fixed rates.

From a lender perspective, Cho et al. (2013) advocated using a multi-period cash-flow model incorporating house price risk, interest rate risk and termination delay. They argue that the lump sum mortgages are more profitable and less risky than the tenure ERMs. One possible explanation is that the analytical valuation of an ERM with tenure payments is far more complex than that for a lump sum mortgage. A valuation framework that takes into consideration the mortality risk, interest rate risk and housing price risk is also detailed in Lee et al. (2012).

A very interesting observation (Pfau 2016) linked to interest rates is how the line of credit of an ERM grows. The loan balance typically grows at a rate given by the reference interest rate, say one-month LIBOR, a fixed spread reflecting the lender's profit margin plus a fixed mortgage insurance premium. The sum rate is called the effective rate and is applied to project the growth of the loan balance. The same rate is also applied to increase the overall principal limit, which for line-of-credit ERM contracts is equal to the balance of the line-of-credit plus the loan balance and plus set-asides. The design arbitrage is that interest and insurance premiums are charged only to the loan balance. The line-of-credit and set-aside accrue under the effective rate *as if* these rates are also charged to these ledgers.

Li et al. (2010) assumed a risk-free continuously compounded rate $r = 4.56\%$ that is the average yield from the 20-year nominal zero-coupon British government securities in the year 2007; and a contract rate $R = 6.39\%$ continuously compounded obtained as the average contract rate for the top 10 UK equity release providers in May 2007. The lump sum loan in the ERM contract is usually accumulated at a constant contract rate Ji et al. (2012) used for the UK the following parameters for NNEG valuation: $r = 4.75\%$; rate $R = 7.5\%$ while Dowd (2018) takes as the base case $r = 1.5\%$ (and decreasing to 0.5% for a stress scenario).

For the Korean market, Lew & Ma (2012) reported that the average value of the 10-year government bond rates was 5.12% between 2002 and 2007 so the expected interest rate was calculated as 7.12% after adding 2% lender's margin. Those values were adjusted in Feb 2012 to be 3.3.% for house prices and 6.33% for the expected interest rate.

Some articles assume independent evolution between house prices and interest rates, Chinloy & Megbolugbe (1994), Wang et al. (2008). Others assume a two-factor model correlating house prices and interest rate dynamics (Huang et al. 2011), or a multidimensional regression model as in Chang et al. (2012) and a VAR approach as in Alai et al. (2014).

The interest rates can be fixed, and many borrowers seem to prefer this route, but it can be very steep, in some cases the rates being in double digit figures such as 12% or 15%. Annually-adjustable rates can be used to link the payments on the ERMs to a reference interest rate. The reference rates that have been used on the market are the 1-year constant maturity treasury, the 1-month and 1-year LIBOR, the 10-year Treasury rate in the USA, and the certificate of deposit (CD) rate in Korea. In order to avoid liquidity pressures, this rate is usually not allowed to vary by more than few percentage points within a year.

4.3.8 Costs of funds

Hosty et al. (2008) stated the following annualised funding costs based on the information from the wholesale banking markets at the time of their research: average swap rate 5.10%; funder's margin over LIBOR 0.40%, redemption profile insurance and risk premium 0.25%, cost of solvency capital 0.07%.

The cost of redemption profile insurance is discussed in more detail in Hosty et al. (2008). The idea is that funders of ERMs will buy insurance to rematch earning LIBOR on the full outstanding balance of the portfolio, irrespective if the portfolio level falls below best estimate such as if multiple decrements are faster than predicted, in which case there is a cost of breaking swaps, or if the portfolio level is higher than the best estimate such as in the case of delayed multiple decrements due to cohort behaviour, innovation in medicine etc. in which case more swaps must be added. The financial instrument that helps managing this risk is the Balance Guaranteed Swap (BGS). Hosty et al. (2008) mentions that before 2007 a full cover BGS had a cost of 70 bps p.a. and buyers of BGS usually reduced this hedging costs by using a narrower confidence interval around expected redemption portfolio profile.

The pricing and management of the BGS has been discussed in Fabozzi et al. (2009), Fabozzi et al. (2010) and, more recently, in more detail in Tunaru (2017). The standard pricing is done based on a portfolio of swaptions or amortising swaptions. One problem frequently ignored by BGS market makers is that when a swaption is exercised, the inherited swap, although contributing positively towards hedging the desired risk short term may change later on into a liability. Pricing can be done also with a portfolio of caps or with a portfolio of floors. In a world without transaction costs it should not matter for the final result which way the pricing is done, so whether one type of derivative carries or not downside risk should not matter. However, the pricing of portfolio of floors is not central focus of this chapter. These are more expensive than swaptions but they do not carry any downside.

Another problem here is that the notional is not always amortising or accreting (negative amortisation). The outstanding balance depends on remaining loans, individual loan balance growth, house prices and age of borrowers. Therefore, the BGS price will be more difficult to calculate than the usual BGS price related to forward mortgages.

4.4 Data and Preliminary Analysis

Here I focus on parameter estimation using data for the house price time-series in the United Kingdom. Table 4.1 reports the estimates for rate of growth of house prices μ and volatility σ, under GBM, across various regions, and using two methods of estimation.

Table 4.1: Estimation of annualised drift and volatility parameters from Nationwide average house price quarterly time series 1974-2019 (non-seasonally adjusted) for the entire UK and also across regions, using three methods of estimation: maximum likelihood estimation (MLE), and method of moments (MM).
Notes: YorksHside is Yorkshire and Humberside

	Period 1974-2019				**Period 2007-2019**			
	MLE		MM		MLE		MM	
Region	μ	σ	μ	σ	μ	σ	μ	σ
North	6.20%	6.62%	6.42%	6.64%	0.01%	4.22%	0.10%	4.26%
YorksHside	6.20%	6.64%	6.42%	6.66%	0.43%	4.58%	0.54%	4.63%
North West	6.62%	5.57%	6.77%	5.59%	0.41%	4.11%	0.50%	4.15%
East Midlands	6.73%	5.99%	6.92%	6.01%	1.58%	4.07%	1.66%	4.11%
West Midlands	6.65%	6.11%	6.84%	6.12%	1.54%	4.05%	1.61%	4.09%
East Anglia	6.89%	6.86%	7.12%	6.88%	2.11%	5.06%	2.25%	5.12%
Outer South East	7.11%	6.23%	7.31%	6.25%	2.50%	4.98%	2.62%	5.03%
Outer Met	7.32%	5.87%	7.49%	5.89%	3.11%	5.10%	3.25%	5.15%
London	7.89%	6.40%	8.10%	6.42%	4.13%	6.10%	4.31%	6.16%
South West	7.12%	6.14%	7.31%	6.15%	1.84%	4.50%	1.95%	4.54%
Wales	6.34%	6.60%	6.56%	6.61%	0.54%	5.74%	0.71%	5.79%
Scotland	6.17%	5.39%	6.32%	5.40%	0.44%	4.42%	0.54%	4.47%
Northern Ireland	6.36%	8.04%	6.68%	8.06%	-2.88%	9.04%	-2.46%	9.13%
UK	6.81%	5.08%	6.93%	5.09%	1.69%	4.30%	1.78%	4.35%

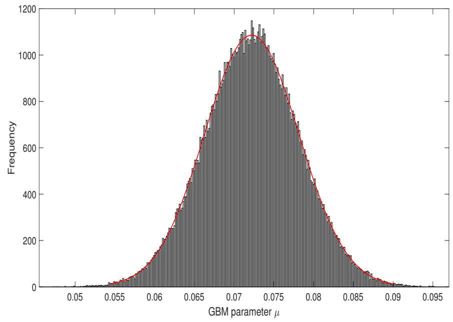
There is variability in the estimates of house price expected growth rate and volatility, across regions, depending on the period of estimation and method of estimation. In a separate analysis for monthly data of the same Nationwide series, I found slight differences as well. One can note that post subprime crisis the volatility is smaller than before the crisis and the expected growth rate decreased also substantially.

The Nationwide average house price series do not incorporate any adjustments for the fact that old houses are replaced by new and much more expensive houses. There is no clear mechanism on how old houses are replaced by new ones. The longest time series on house prices in the UK goes back to 1952. It is difficult to capture precisely this effect. Insurers can take a conservative view and apply a dilapidation discount as a haircut at the termination of the contract. The dilapidation discount rates should increase with time.

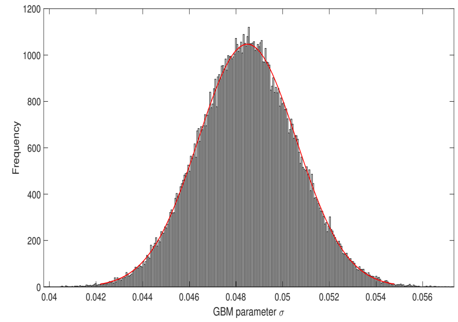
An important remark;[16] the estimated drift term is done only when I locally know the future direction of the housing market prices. This cannot be done without errors, but the solution can be improved by deriving the distribution of the GBM parameters μ and σ. When the sample variance of the empirical distribution is known, (Jacod 1994, p.2) demonstrates that μ can be consistently estimated with a high probability of convergence to the true value of the drift term if $M \times dt \to \infty$ where M is the number of observations and dt is the time step. Based on this, I benefit by letting the number of observations increase with a fixed dt

In a numerical study, I simulate the sampling distribution of the parameters and use iterative methods to build a random distribution of each parameter. I propose taking a Value-at-risk approach where I focus on selected percentiles of the house price distribution. Drawing from the fact that the marginal distribution of the house price is lognormal I can simulate the distribution of percentiles given the parameter distributions. Based on this technique, a Monte Carlo pricing procedure for the NNEG put option where H_t in (4.5) and (4.6) is replaced by the percentile of house price.

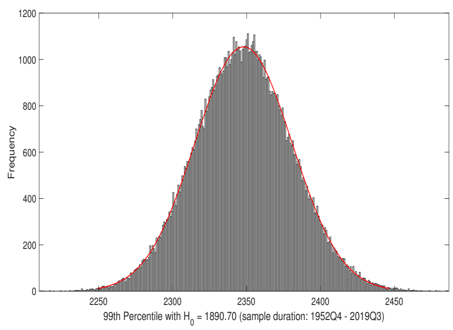
[16] Given the log return $Y_i = \log h(t_i) - \log h(t_{i-1})$ where h is an observed sample house price time series. I estimate the sample mean using the initial and final points as the relevant information while neglecting all other remaining observations.



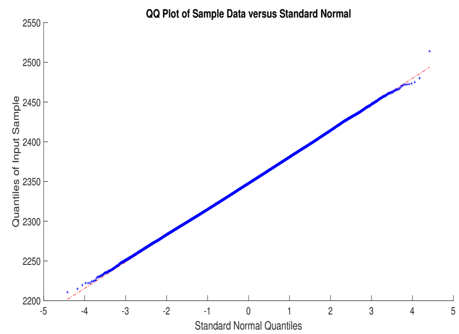
(a) Sampling distribution of μ



(b) Sampling distribution of σ



(c) Sampling distribution of 99th percentile of house prices



(d) q-q Plot for 99th percentile of house prices

Figure 4.5: Exploring the sampling distribution for GBM parameters and that of percentiles of house prices over 1952Q4 - 2019Q3.
Notes: The sampling distributions are obtained from 100,000 Monte Carlo simulations. The percentiles reported in (c) are computed from the log normal distribution using the set of parameters simulated in (a) and (b).

The q-q Plot in Figure 4.5 shows that sampling distribution of the 99th quantile place more weight on the lower values and less weight on higher values. The Monte Carlo procedure is implemented for the NNEG put option.

4.4.1 Finding the ARMA-EGARCH model

For the ARMA-EGARCH models I consider a forward model selection procedure. From all models that fit well data I select the model using an Occam's razor approach, looking for the simplest possible model (i.e. the smallest number of parameters) that has significant parameters but that also provides a very good fit to the data. The model with superior AIC and BIC goodness-of-fit is preferred.

The model I selected is the ARMA(4,3)-EGARCH(1,1) with parameters in Table 4.2. There could be other ARMA-GARCH-type models that may provide a better fit than the model I have identified. The universe of ARMA-GARCH-type models is very large and it is outside the scope of this research to search through the entire universe of this model class.

Table 4.2: Parameters estimates for the ARMA(4,3)-EGARCH(1,1) model over the monthly Nationwide average house price time series between 1952Q4 and 2019Q1 (non-seasonally adjusted).

Parameter	**Estimate**	**Std. Error**	**t-value**	**Pvalue**
c	0.0156	0.0033	4.7681	0.0000
ϕ_1	-0.2197	0.0456	-4.8158	0.0000
ϕ_2	-0.2706	0.0333	-8.1271	0.0000
ϕ_3	-0.2450	0.0466	-5.2580	0.0000
ϕ_4	0.7260	0.0356	20.4060	0.0000
θ_1	0.8621	0.0608	14.1850	0.0000
θ_2	0.9539	0.0141	67.7490	0.0000
θ_3	0.9126	0.0591	15.4410	0.0000
k	-1.5497	0.4902	-3.1616	0.0016
α_1	0.8167	0.0571	14.2970	0.0000
β_1	0.4837	0.1032	4.6870	0.0000
γ_1	0.1476	0.0540	2.7357	0.0062

4.4.2 Forecasting comparison

Ultimately, a good model for house price returns should have good forecasting power, at least at short and medium horizon. I retained the out-of-sample period of 2016-2018, monthly, to compare the forecastability of various models. For the short forecasting horizon of two years ARMA(4,3)-EGARCH(1,1) model produces similar forecasts with the GBM specification, under any of the three parameter estimation method.

In Figure 4.6, I illustrate the forecasting error for the out-of-sample Nationwide monthly time series for the last two years. My results show that it is possible, at a given point in time and for a given forecasting horizon, to have very different models that give similar futures house price values.

One may argue that 2016-2018 was a very benign period for house prices in the UK. I have also redone the analysis for a five year out-of-sample period, with quarterly data (also monthly in the appendix), the latter going back to 1952. To this end, I employed the quarterly Nationwide average house price data because it goes back to 1952, so it may have more chances to capture more extreme movements. I conduct a forecasting exercise with five years out of sample data. The results are presented in Figure 4.7 and they show the superiority of the ARMA-EGARCH model in terms of forecasting future house prices.

Furthermore, I test more formally which model has superior forecasting capability or whether the GBM model (preferred by the regulator) produces similar forecasts as my proposed model. The results of Diebold-Mariano tests are illustrated in Table 4.3 where the same analysis is carried out with quarterly data. Clearly, for any of the three methods of estimation of parameters (MLE, MM, and GMM), the ARMA-EGARCH model is, overall, superior to the GBM model.

I have also redone the analysis using the ARMA-GJR model. Here are the main outputs showing that

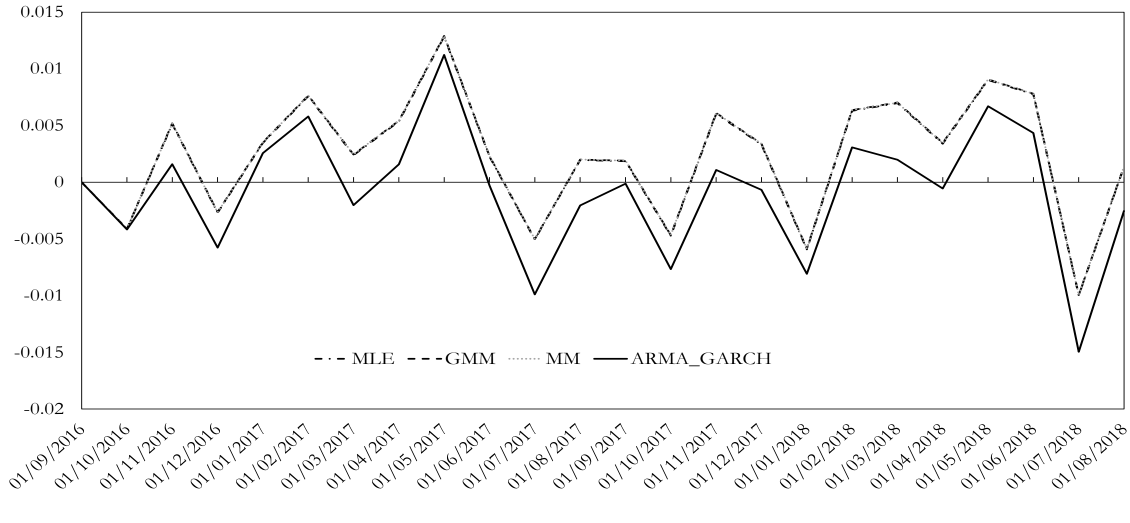


Figure 4.6: 2-year forecasting for ARMA(4,3)-EGARCH(1,1)
Notes: Comparison of out-of-sample forecasting error (actual minus forecast) for Nationwide Average House Price Monthly (non-seasonally adjusted) for ARMA(4,3)-EGARCH(1,1) and GBM model specifications, over the out-of-sample period Oct 2016 to Sep 2018.

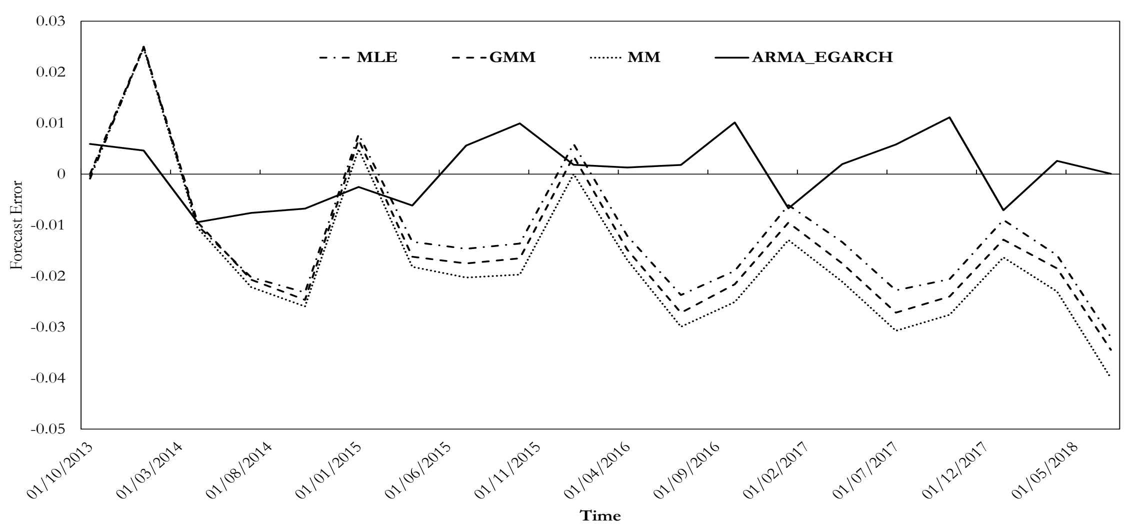


Figure 4.7: 5-year forecasting for ARMA(4,3)-EGARCH(1,1)
Notes: Comparison of out-of-sample forecasting error (actual minus forecast) for Nationwide Average House Price Quarterly (non-seasonally adjusted) for ARMA(4,3)-EGARCH(1,1) and GBM model specifications, over the out-of-sample period Q4 2013 to Q3 2018.

Table 4.3: Comparing forecasting (quarterly) under the GBM model with different estimation methods versus the ARMA(4,3)-EGARCH(1,1) model with Diebold Mariano test over the out-sample of 20 quarters (Q4 2012-Q3 2018).

MODEL	RMSE	MAE	
GBM-MLE	0.0147	0.0129	
GBM-GMM	0.0176	0.0158	
GBM-MM	0.0189	0.0170	
ARMA(4,3)-EGARCH(1,1)	0.0063	0.0055	
Diebold-Mariano Forecast Accuracy Testing			
MODEL 1	MODEL 2	STATISTIC	P-VALUE
GBM-MLE	GBM-GMM	-4.0356	0.0007
GBM-MLE	GBM-MM	-3.9256	0.0009
GBM-MLE	**ARMA(4,3)-EGARCH(1,1)**	3.6009	0.0019
GBM-GMM	GBM-MM	-3.4400	0.0027
GBM-GMM	**ARMA(4,3)-EGARCH(1,1)**	4.3231	0.0004
GBM-MM	**ARMA(4,3)-EGARCH(1,1)**	4.3598	0.0003

Notes: I report some measures of forecasting accuracy such as root mean squared error (RMSE) and mean average error (MAE) as well as the Diebold-Mariano test for comparing GBM model under different estimation methods with the selected ARMA-EGARCH model, based on the out-of-sample data for monthly Nationwide time series. The models in bold provide superior forecasting performance by comparison with the paired model. The test statistic is compared with critical values of standard normal distribution $N(0, 1)$. If I fail to reject the null, i.e. the p-value is between 0.05 and 0.95 at 90% confidence level, then the two models compared produced similar forecasts. Otherwise, the model in the direction of the statistic (1 if negative, 2 if positive) will give better forecasts.

again a model of this type would be preferable to the GBM model. The graphs in Figures 4.8 and 4.9 indicate that the ARMA(4,2)-GJR(1,1) model will outperform the GBM model for two year and five year forecasting horizon.

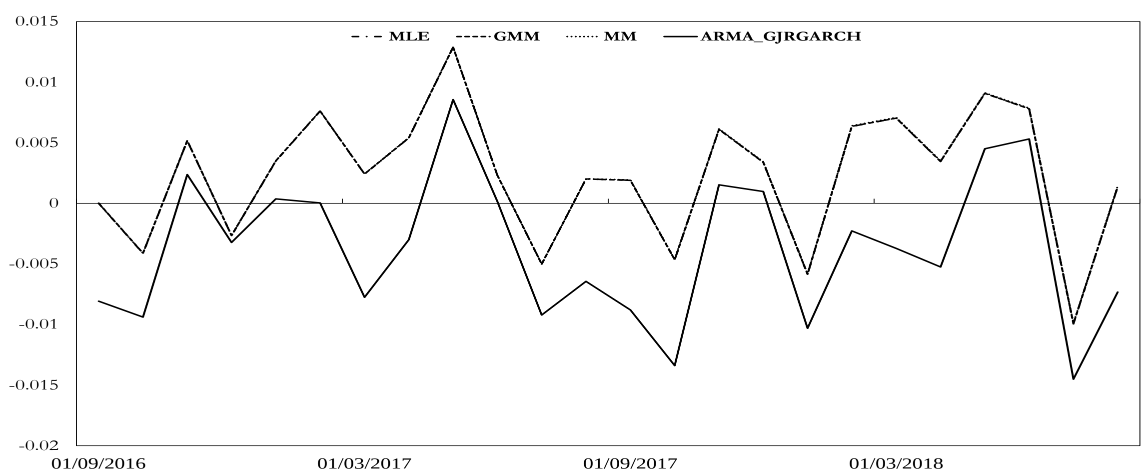


Figure 4.8: 2-year forecasting for ARMA(4,2)-GJR(1,1)

Notes: Comparison of out-of-sample forecasting error (actual minus forecast) for Nationwide House Price Index Monthly for ARMA(4,2)-GJR(1,1) and GBM model specifications, over the out-of-sample period Oct 2016 to Sep 2018.

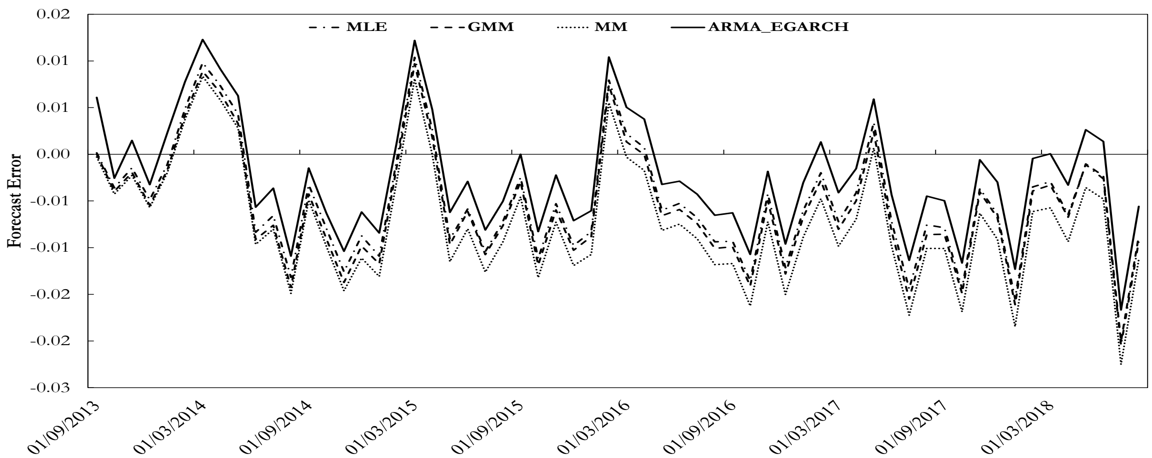


Figure 4.9: 5-year forecasting for ARMA(4,2)-GJR(1,1)

Notes: Comparison of out-of-sample forecasting error (actual minus forecast) for Nationwide House Price Index Monthly for ARMA(4,2)-GJR(1,1) and GBM model specifications, over the out-of-sample period Oct 2013 to Sep 2018.

Regarding the NNEG valuations, the NNEG values depicted in Figure 4.14 for the baseline scenario shows a similar performance to the ARMA-EGARCH, suggesting consistency of the findings.

4.4.3 Forecasting power of ARMA-EGARCH and GBM models

In Figure 4.10, I redo the same analysis for the forecasting error for the out-of-sample Nationwide monthly time series with five years out of sample. Now, the ARMA(4,3)-EGARCH(1,1) outperforms the GBM house price forecasting. Moreover, now the MLE estimates for GBM dominates the MLE and GMM method, confirming that there is substantial parameter estimation risk even for such a simple model as GBM.

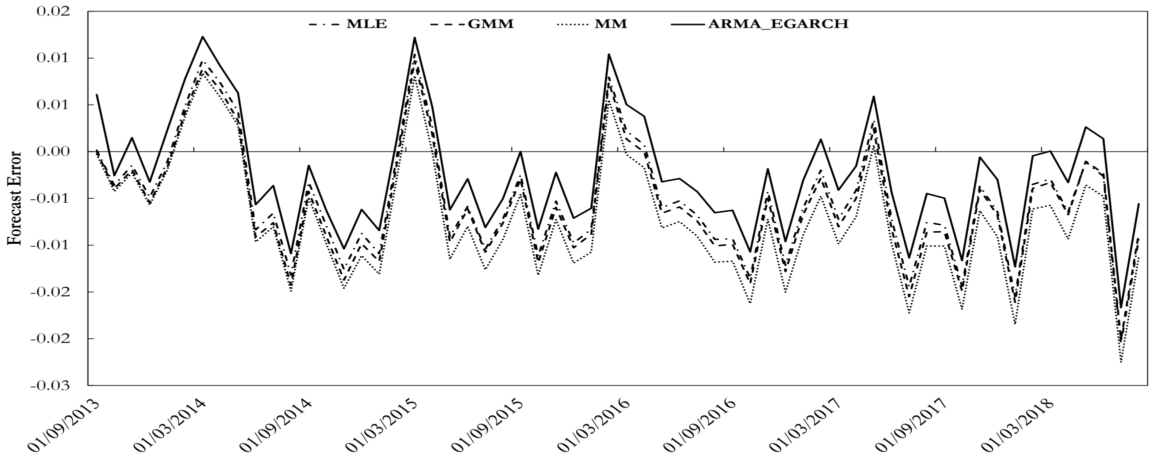


Figure 4.10: Comparison of out-of-sample forecasting error (actual minus forecast) for Nationwide Average House Price Monthly (non-seasonally adjusted) for ARMA(4,3)-EGARCH(1,1) and GBM model specifications, over the out-of-sample period Oct 2013 to Sep 2018.

In Table 4.4, I present the forecasting testing result based on monthly frequency and refitted models.[17] Even for this much longer period the forecasting under the ARMA-EGARCH model is superior to the forecasting under GBM.

Table 4.4: Comparing forecasting (monthly) under the GBM model with different estimation methods versus the ARMA(4,3)-EGARCH(1,1) model with Diebold Mariano test over the out-sample of 60 months (Oct 2012-Sep 2018).

MODEL	RMSE	MAE	
GBM-MLE	0.0079	0.0067	
GBM-GMM	0.0081	0.0069	
GBM-MM	0.0090	0.0078	
ARMA(4,3)-EGARCH(1,1)	0.0063	0.0051	
Diebold-Mariano Forecast Accuracy Testing			
MODEL 1	MODEL 2	STATISTIC	P-VALUE
GBM-MLE	GBM-GMM	-3.9838	0.0002
GBM-MLE	GBM-MM	-6.7823	0.0000
GBM-MLE	**ARMA(4,3)-EGARCH(1,1)**	3.7681	0.0004
GBM-GMM	GBM-MM	-6.0371	0.0000
GBM-GMM	**ARMA(4,3)-EGARCH(1,1)**	3.9739	0.0002
GBM-MM	**ARMA(4,3)-EGARCH(1,1)**	4.8545	0.0000

4.4.4 Service flow rate

Any additional income produced by the collateral house needs to be adjusted for any contingent claim calculations under the risk-neutral (market valuation) approach. For the majority of buyers, houses play

[17]For ease of comparison I retained the GBM model with the three estimation methods and ARMA(4,3)-EGARCH(1,1) that again provides a good fit to the data in-sample.

the role of a consumption asset and not that of an investment asset. There is no evidence that service flow rates are driving future house prices so the *expected* house prices at various future long horizons cannot be determined with growth models in the same way expected share prices may be determined with growth models linked to dividends. The PRA (see PRA CP13-18, paras 2.12-2.15) arrive at the 2% value for the *net service flow rate* calculated as the gross service flow rate (5%) minus maintenance costs, management costs, voids, with central estimate for net service flow rate as 2% but 1% permitted as a *minimum* value. Ji et al. (2012) used for UK the service flow rate $g = 2\%$ while Dowd (2018) mentions $g = 2\%$ and $g = 3\%$ as a base case rate, increasing to $g = 4\%$ as a stress test, and varying between $1\%, 0\%$ and -2.75% as well. Hosty et al. (2008) used 3.3%.

The more precise calculations are challenging because the buy-to-let percentage of houses portfolio is relatively small and it varies geographically, with London and South-East as the main areas. Hence, the idiosyncratic component of service flow rates is quite large. This spatial lack of homogeneity of buy-to-let activity, together with the fact that less than 20% of a housing portfolio may be considered to be associated with rented properties, makes it very difficult to consider service flow rates as the main drivers of house prices.

4.4.5 Estimating service flow rate with rental income data

The Office for National Statistics has been gathering data on service flow rates for a 10% of all properties rented out. From their data I have calculated the monthly sterling rental values average for England taking into account the weights and income given by property type.[18]. The monthly service flow rate for England is then calculated by dividing the average sterling rental sum to the average property price in England in that month. In addition, I also calculated *proxy average* quartiles estimates for service flow rates using weighted averages of lower, median and upper quartile of monthly sterling rental figures.[19] Figure 4.11 displays monthly series, average, proxy median and proxy lower and upper quartiles for England. The mean average monthly service flow rate over this period is 0.4315% (5.1776% annualised) while the mean proxy upper quartile is 0.48% (5.76% annualised). Note that this service flow rate corresponds only to the pool of properties rented out.

There seems to be a lot of variation in the evolution of service flow rates over time, with a large drop observed at the end of 2009 and first half of 2010. There is also great variation across regions in terms

[18]I left out the rents coming from room only.

[19]The proxy quartiles do not represent actual quartiles since weight averaging the medians will not necessarily produce the median, for example. I produced these proxies to have a rough idea of distribution of service flow rates.

of service flow rates[20] evolution that needs to be managed idiosyncratically similar to the same issue for volatility.

According to the Office for National Statistics, there were about 26.4 million households in the UK in the 2012 (following 2011 census) out of which approximately 5 million lived in rented out properties.[21] Hence less than 20% of properties are rented out. This means that a rough calculation would give a total service flow rate, weighted by the 20% representing the actual renting market, of 1.03% (5.1776% × 20%) per annum.

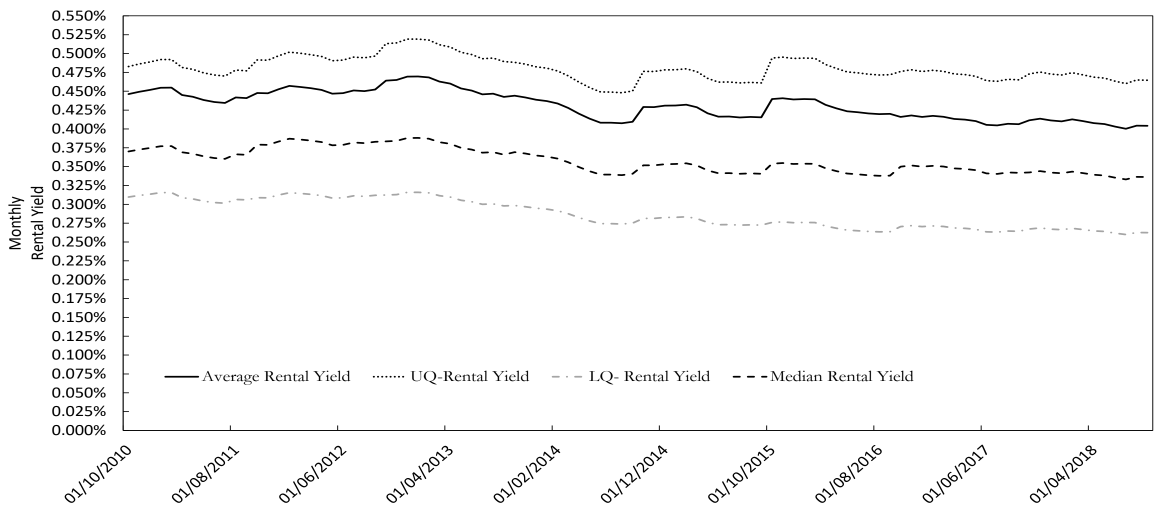


Figure 4.11: Monthly series, average, proxy median and proxy lower and upper quartiles for England between Oct 2010 and Sep 2018. Source: Author's calculation based on data from the Office for National Statistics.

An even more precise calculation should take into account the *net service flow rate* which is calculated as the service flow rate net of running costs. The latter is calculated taking into account three elements.[22] First, the voids, defined as the number of months per year the property stays unrented. The usual rule of thumb is to assume one month's loss of gross rental income per annum, so the sterling pound average rental income will be multiplied by 11/12. Then, letting agent's fees in the range 10-15% of the rental income plus VAT (12%-18% including VAT) at the current rate of 20%. I can take the mid-value of 15% that needs to be deducted from the resulting sum after applying the voids. The third component refers to maintenance

[20]The Global Property Guide 2018, projects lower gross annual service flow rate in London. It estimates gross service flow rate for houses in prime central London to be around 3.2%. https://www.globalpropertyguide.com/Europe/United-Kingdom#rental-yields

[21]Personal communication with Rhys Lewis from the Office of National Statistics.

[22]The values for these elements were selected upon consultation with specialists in the field.

costs that are typically around 15% of the gross rental income, inclusive of any VAT. Hence, agents' fee and maintenance cost together will erode the rental income by 30%. The average net service flow rate then following from the above calculations will give an annualised net service flow rate of 0.66%. In this study I used an average value of 1% as representative for 2018 in the UK for baseline scenarios, and I considered higher and lower values (including negative) for sensitivity scenarios discussed later on.

There are few other important points regarding the relationship between ERM and net service flow rate. By contractual terms, the collateral houses in the ERMs cannot be rented out. This implies a service flow rate of 0. The service flow rate was calculated from the rental income that is representative across the properties in the index. If more than 20% of properties become available for renting the rents are likely to decrease because of supply and demand. It is not clear what will happen with the house prices then, so I cannot say either way what will be the effect overall on the house market.

Recall that the 20% of the houses that produce rental income is not just a sample of from the total population of houses that produce rental income. It is the full subset of the population of houses in the UK. Hence, the 80% remaining will not have one house that will pay rent. Since I are trying to determine the dynamics of the data-generating process, at the moment, any house price index will have to adjust rental income over the entire population. Likewise, if 80% of the houses will produce that rental income then I would multiply $5\% \times 0.80$ to get the relevant service flow rate, and if all houses are rented out producing 5% rental income then 5% is the service flow rate on the index.

However, while those issues are important in themselves, my modelling is using a data-generating process for a house price index. I envisage that the NNEG valuations obtained in this way are only "indicative", say for a house that has exactly the same price as the index. The data-generating process, say GBM, requires the additional income part to be taken into account at the risk-neutralisation stage. Rental yield is needed for GBM but also for any other model employing the conditional Esscher martingale measure for risk-neutral pricing purposes.

4.5 Risk Sensitivity Analysis

This section of the study presents a detailed analysis on the sensitivity of the NNEG and ERM cost sensitivity respective model parameters. Both the ARMA-EGARCH-rn[23] and GBM-rn[24] are presented side-by-side while I analyse the degree of responsiveness of the cost estimates. The ARMA-EGARCH-rn is based on 100,000 Monte Carlo Simulations while the GBM-rn is based on the Black-Scholes closed-form formula.

[23]NNEG estimates under risk neutral ARMA-EGARCH model
[24]NNEG estimates under risk neutral GBM model

The large simulation allows us to improve accuracy and efficiency (see Hardy 2003). The summary results produced from the sensitivity exercise for risk-free rate, contract rate , service flow rate, mortality improvement/deterioration, and early prepayment is presented in Table 4.5 and Table 4.6 for both ARMA-EGARCH-rn and GBM-rn respectively. The LTV sensitivity results are in Figure 4.15.

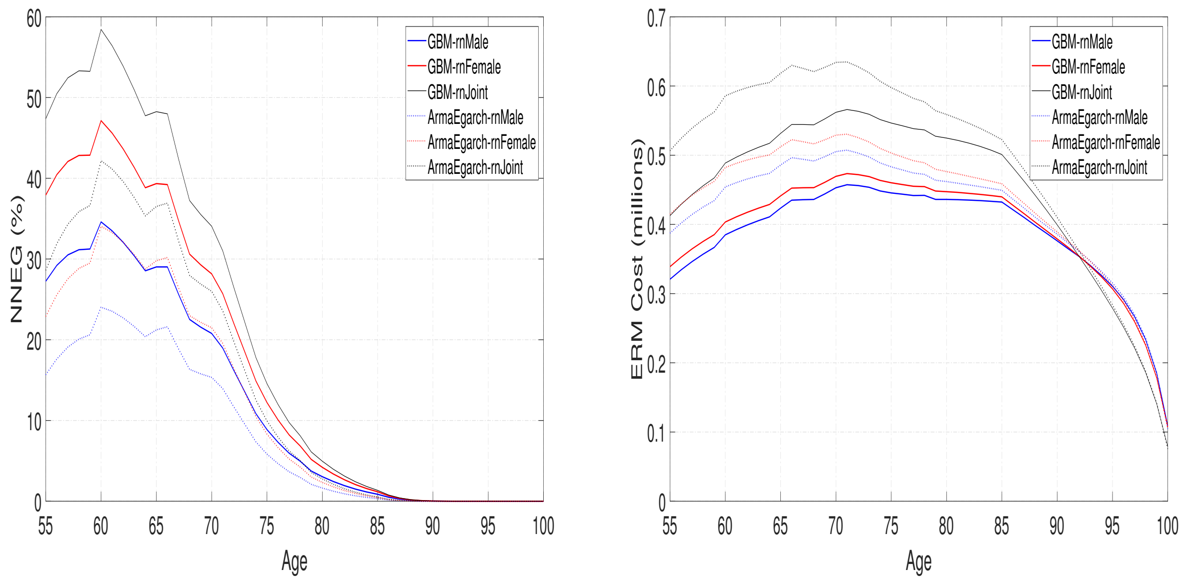
4.5.1 The NNEG and ERM baseline scenario

There is currently no market price benchmark on the UK ERM market space. An efficient way to conduct a good market valuation involves the use of sensitivity analysis, where a pre-specified benchmark scenario is carefully defined. In this section, I specify the baseline values for key parameters that drive the NNEG values. The termination conditions are mortality, early pre-payment, and transition into longterm care, suggesting a multiple decrement model. The study makes provisions to also investigate issues that pertain to single decrement (mortality only) in relation to the normal practice of relevant literature. The single decrement case is a special case that allows one to simplify assumptions relating to the pricing procedure. The baseline scenario uses ($r = 0.57\%, R = 4.91\%, g = 1\%, \sigma = 4.88\%$ and Flexible loan-to-value (LTV) rates). The initial[25] house price is assumed to be $H_0 = £310000$ for borrower-age ranging from 55 years to 100 years.

The comparative NNEG valuations are illustrated in Figure 4.12. The NNEG values are reported as a percentage of the initial loan advanced to the borrower at the inception of the contract. According to the Equity Release Council (ERC) Autumn 2019 market report, the average contract rate fell below 5% for the first time in July 2019. The current average contract rate is 4.91%. The sensitivity test results allows the study to investigate the effect of previous market conditions as much as possible. Figure 4.12 also illustrates the term structure of the ERM cost to the loan issuer.

The baseline scenario illustrates high NNEG to loan ratio and the GBM-rn model values consistently lie above the ARMA-EGARCH-rn values for male, female and joint borrower's altogether. The scenario also possess a positive but relatively small risk-neutral drift $r - g = -0.03\%$ and volatility level (4.88%). These two attributes suggest that the underlying house prices gradually decreases in relation to the accumulated loan balance over the lifetime of the borrower, thereby corroborating Warshawsky & Zohrabyan (2016) assertion that house prices have heavy discounting in standard equity release products.

[25] My initial house price level is based on Equity Release Council (2019) market report.



(a) NNEG Cost at Baseline

(b) ERM Cost at Baseline

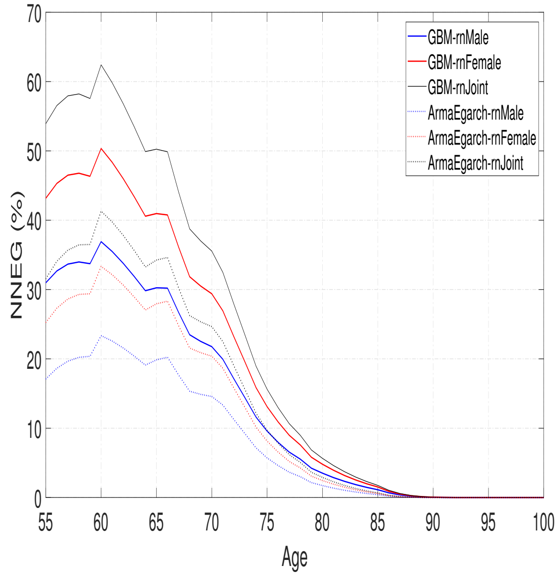
Figure 4.12: NNEG values as percentage of initial lump sum loan for the two baseline scenarios.

Notes:The results are for both the GBM-rn and ARMA-EGARCH-rn , under multiple decrement rates for the two baseline scenario with $r = 0.57\%, g = 0.5\%, \sigma = 4.86\%$, and $R = 4.91\%$ with Flexible LTV pricing

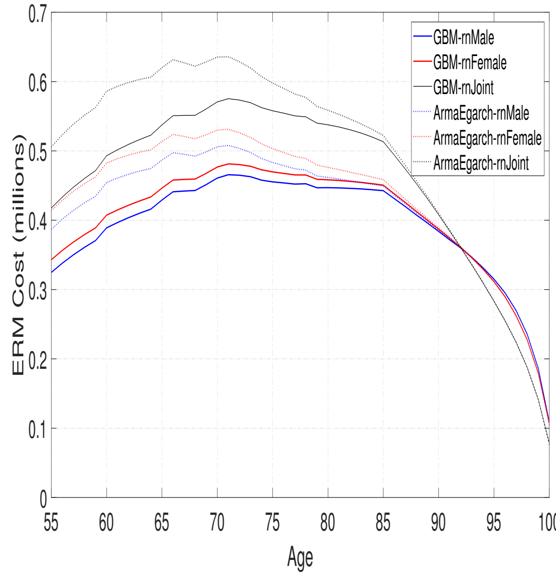
The NNEG to initial loan ratio remains above 0% for all borrowers up until age 85 years. There is a sharp decline in NNEG values after age 60 years. The ARMA-EGARCH-rn pricing model produces lower NNEGs compared to GBM-rn, although the direction of responsiveness to an increase in the contract rate lead to upward shift of NNEGs in both models. The NNEG values appear to rise faster at lower age ranges i.e. 55-60 years and peaks at age 60 in both models.

Li et al. (2010) observe a reversal of this ordering in NNEG due mainly to the nature of the underlying assumptions. First, their study is based on a fixed initial loan for all borrowers at £30,000. This creates a different LTV structure when borrowers have different initial house price values. The volatility estimates in their GBM valuation process is also different because the sampling period of the house price index data ends in 2008 for their study. The risk-free rate is about more than twice the service flow rate while the contract rate is at $R = 6.39\%$. Based on these fundamental differences, I hasten to add that my analysis and discussions does not propose an expected ordering the GBM and ARMA-EGARCH NNEG valuation procedures. It is possible to observe NNEG values that boarder around the GBM version, when one adopts another model that fits and forecasts house price series well. Having a market benchmark value from the ERM spaces can help with calibration against this observed variations.

Figure 4.13 shows the NNEG values for the baseline scenarios under a full risk-free curve described in Appendix C.6.8 for 3rd May 2020. The calculations indicate higher NNEG values under a full risk-free curve. Notice that ARMA-EGARCH-rn values are re-simulated with adjusted drifts every month based on changing risk-free rates sourced from the full term structure of risk-free rates.



(a) NNEG Cost under floating risk-free curve



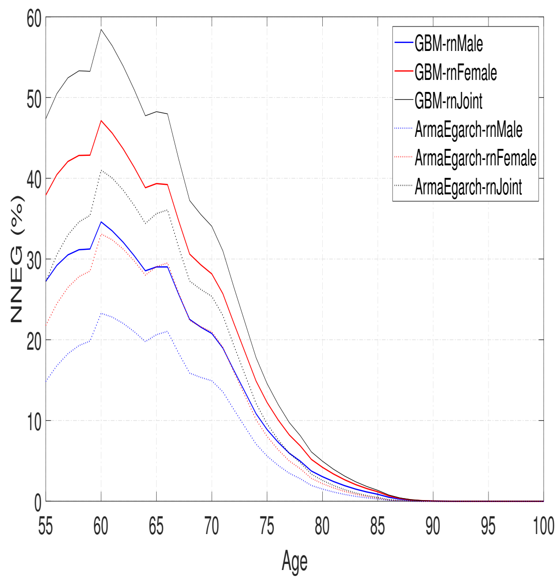
(b) ERM Cost under floating risk-free curve

Figure 4.13: Calculations of NNEG and ERM cost for baseline scenarios w.r.t. risk-free curve and the baseline scenarios $R = 4.91\%, g = 1\%, \sigma = 4.88\%$.

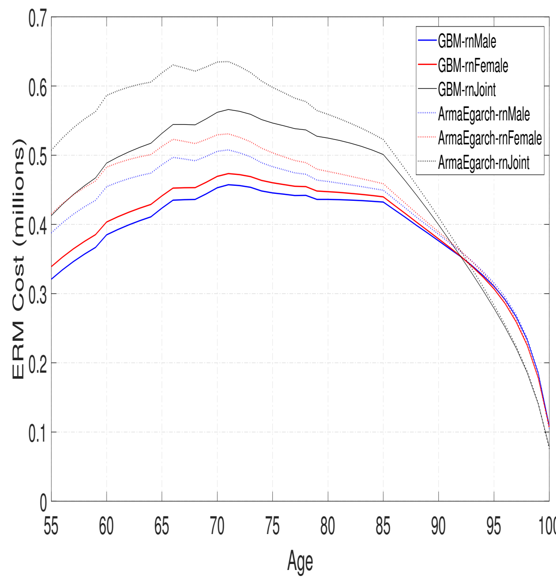
Overall, I can see that there are very small marginal changes in the levels of NNEG, but the profiles of NNEG vectors under the two baseline scenarios remain the same. The reason for that is that the only sizeable difference in risk-free rates between my constant rate of $r = 0.57\%$ and the rates indicated in Appendix C.9 is at the front end of the curve. However, the NNEG values at ages 88 and over are zero due to LTV protection. Also, loan accretion does not have enough time to exceed the market value of the collateral house when borrowers are aged over 88 years.

4.5.2 ARMA-GJR Results

Figure 4.14 presents the NNEG and ERM cost curves over the lifetime of the loan contract of a female borrower aged 55 at inception. The ARMA-GJR results are consistent with the earlier results observed for the ARMA-EGARCH model. This is not surprising, particularly when the fitted ARMA-GJR model is of the same order as the fitted ARMA-EGARCH model.



(a) NNEG Cost at Baseline



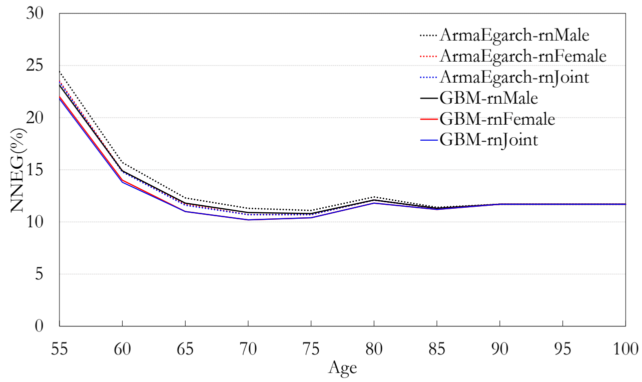
(b) ERM Cost at Baseline

Figure 4.14: NNEG valuations as percentage of lump sum for GBM-rn and ARMA-GJR, under multiple decrement rates for the two baseline scenario with $r = 0.57\%, g = 1\%, \sigma = 4.88\%$ and standard Flexible LTV vector valuations.

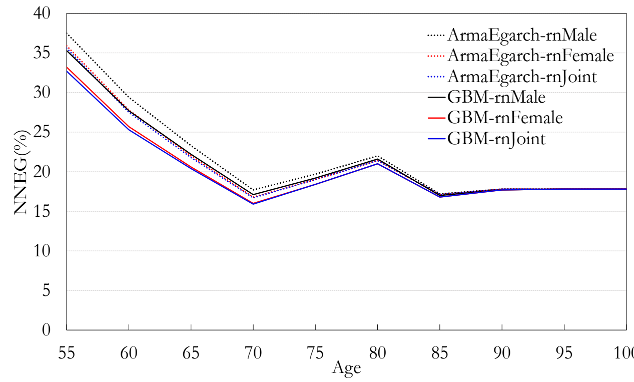
4.5.3 Sensitivity of NNEG main baseline scenario to LTV variations

The LTVs play a fundamental role in NNEG valuations and they can change the profile of NNEG vectors as well as their overall magnitude level. Figure 4.15 illustrates the comparison of the baseline scenarios under various market LTV loadings. The LTV has a great influence on the final value of the NNEG and one can argue that one of the most efficient methods to manage the NNEG risk is to consider the sensitivity of NNEG to LTV variations.

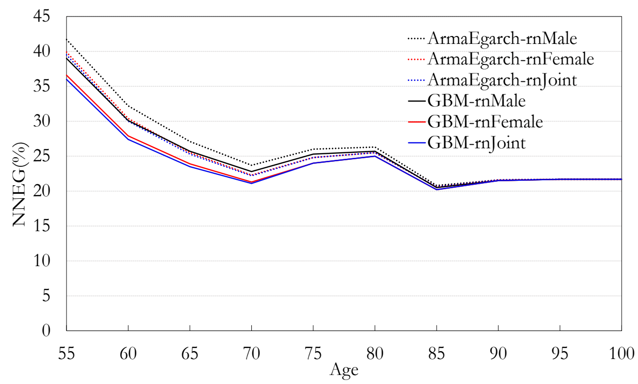
The comparative NNEG calculations presented in Figure 4.15 indicate that higher LTVs lead to a steepening of the NNEG values. Market practitioners may consider downward (upward) adjustment to the LTVs based on the high (low) house price volatility expectations. This partly allows the loan issuer some control over the basis risk discussed in Andrews & Oberoi (2015).



(a) Flexible Plus LTV



(b) Flexible Max LTV



(c) Flexible MaxPlus LTV

Figure 4.15: NNEG sensitivities with respect to loan-to-value (LTV) ratios

Notes: The sensitivity results are for LTV variations. Subplot (a) illustrates the sensitivity results for the ARMA-EGARCH versus GBM model under Flexible Plus LTV. Subplot (b) illustrates the sensitivity results for the ARMA-EGARCH versus GBM model under Flexible Max LTV. Subplot (c) illustrates the sensitivity results for the ARMA-EGARCH versus GBM model under Flexible Max Plus LTV. The baseline parameter values are $r = 0.57\%$, $R = 4.91\%$, and $g = 1.00\%$ using Flexible LTV. The service flow rate R is strictly positive under PRA requirements. In each case, the sensitivity results indicate the degree of responsiveness of NNEG cost to changes in the model parameter. In each case, the sensitivity results indicate the degree of responsiveness of NNEG cost to changes in the model parameter. The values reported in the table are computed by varying the respective sensitivity parameter by 1% either upward or downward with exception to mortality and prepayment which considers 20% upward or downward movement.

4.5.4 NNEG sensitivities with respect to risk-free rate

The risk-free rate (r) impacts NNEG calculations in two ways. The obvious way is via discount factors, a lower risk-free rate keeping the back end NNEG put payoffs still high, or, to put it the other way, a high risk-free rate dampening the back end NNEG values. The second channel of interference is the calibration of the conditional Esscher martingale measure using $r - g$ as the local drift, which also appears in the GBM-rn.

This risk-neutral local drift makes NNEG put values move in the opposite direction to risk-free rates, the larger the risk-free value the lower the NNEG, and the lower the risk-free rate r the higher the NNEG put value.

The analysis presented in Table 4.5 and Table 4.6 shows that both ARMA-EGARCH-rn and GBM-rn NNEG values decrease when r increases and the NNEGs increase when r decreases. This is intuitively correct, *ceteris paribus* a larger r will shift the projected future house price values in a risk-neutral world upwards, while a lower r will project lower house prices. Around current levels of risk-free rate, the responsiveness of ARMA-EGARCH-rn valuations is substantially higher than that of the GBM-rn valuations. This suggest that one can efficiently account for the degree of responsiveness in risk-free changes when the calibration of the house price process is improved. The improvement in the calibration process associated with the ARMA-EGARCH risk-neutral model over that of the GBM is reported in Table 4.3.

4.5.5 NNEG sensitivities with respect to contract rate

The contract rate (R) is decisive in the NNEG ending up in the money. It is evident that even a slight increase in the contract rate , compounded monthly to 45-55 years, may inflate the loan accumulated balance to very high values. Hence, another good tool for risk-managing the NNEG levels attached to ERMs is having contract rate s as low as possible. The sensitivities of the NNEG calculations with respect to variations in the contract rate are reported in Table 4.5 and Table 4.6. The NNEG values increase dramatically with the increase in the contract rate R and decrease rapidly with the decrease in the contract rate. When time to maturity T is high, the lump sum loan gets enough accumulation time while the tendency of impairment/dilapidation to the collateral house also increases.

4.5.6 NNEG sensitivities with respect to service flow rate rate

The service flow rate (g) is another important lever for influencing NNEG calculations. A pessimistic investor may consider potential high service flow rates as influencing house price growth and argue that high service flow rates are equivalent to low house prices. This may not always be the case for UK since, there has been a structural imbalance of new property build for the last decades. Furthermore, a relatively small percentage of people prefer renting to buying property. Based purely on economic considerations it is expensive to rent in the UK. People who rent tend to do so because they probably have no other choice e.g. cannot buy because they did not save for a deposit.

In the models I investigate in this study, the service flow rate plays the opposite role to r, such that a high g will computationally give a lower risk-neutral drift (even negative) so the pathway of house prices is trending down, boosting the NNEG put values. The opposite is true for low or even negative g.

The sensitivity of NNEG valuations with respect to the service flow rate g under various scenarios are presented in Tables 4.5 and 4.6. The NNEG values increase with an increase in service flow rate and decrease with the decrease in service flow rate, for both the GBM-rn and ARMA-EGARCH-rn. Intuitively this is correct, since a larger value for g implies a low or even negative drift in the risk-neutral world so the projected house prices will be lower in the future, implying a higher NNEG value. Both risk-neutral and actual worlds are affected by changes in g. Large value of g affects the real-world and risk-neutral world in the same way, i.e., producing a negative drift. In the real-world we have $\mu - g$ as drift term and in the real-world we have $r - g$ as the drift term. The behavior of the drift term depends on the magnitude of μ and r. A smaller value for g or even zero, as some insurers are using, leads to a more positive drift in the GBM-rn model that will give increased house prices in the future and hence lower NNEGs.

4.5.7 NNEG sensitivities with respect to house price volatility

The NNEG valuations with respect to changes in the volatility (σ) of house prices that impact the GBM-rn approach is presented in Table 4.6. The volatility of the data generating process employed for house prices plays a key role in any option-type valuation. It is known from option theory that higher volatility will imply higher values for the NNEG put.

For the ARMA-EGARCH volatility parameters as estimated on the Nationwide monthly historical time series, I used an almost identical multiplication factor as coming out from the ratio of GBM volatility in the stressed scenario versus the baseline scenario. For example, when I stressed $\sigma_{GBM} = 10\%$, the ratio to the baseline volatility of $\sigma_{GBM} = 3.9\%$ is about 2.5. Hence, I multiply the series of ARMA-EGARCH volatilities by a factor of 2.5 when redoing the NNEG calculations for the ARMA-EGARCH-rn. Likewise, for other sensitivity values of σ, I multiply the entire vector of ARMA-EGARCH volatilities with the appropriate multiplication factor to preserve the same ratio taken for GBM volatility values. The ARMA-EGARCH-rn sensitivity test results are reported in Table 4.5.

I also find that changes in volatility $\sigma \pm 1\%$ the ARMA-EGARCH NNEG sensitivities are *below* the GBM NNEG sensitivities. Thus, in a recession period I can expect the ARMA-EGARCH NNEG values to be higher because the volatility is not constant low and the random variation may generate also occasionally larger volatility values that may produce higher NNEGs. These occasional higher values will drag the average Monte Carlo average NNEG estimate upwards. By contrast, the GBM-rn will give the same valuation given by the analytical formula.

Table 4.5: Sensitivity test of the NNEG cost under ARMA-EGARCH

Baseline Values	Age	55	60	65	70	75	80	85	90
	LTV	0.115	0.17	0.225	0.285	0.324	0.365	0.415	0.415
	Initial Loan	35650	52700	69750	88350	100440	113150	128650	128650
PANEL A: NNEG SENSITIVITY TO RISK FREE RATE									
$r =\uparrow 1\%$	Male	-0.974	-0.956	-0.949	-0.948	-0.959	-0.969	-0.982	-1.000
	Female	-0.972	-0.953	-0.946	-0.945	-0.957	-0.968	-0.975	-1.000
	Joint Life	-0.972	-0.953	-0.946	-0.945	-0.956	-0.968	-0.979	-1.000
$r = 0\%$	Male	2.653	0.842	0.223	-0.108	-0.170	-0.188	-0.175	-1.000
	Female	2.503	0.776	0.185	-0.133	-0.190	-0.205	-0.191	1.084
	Joint Life	2.499	0.776	0.184	-0.134	-0.191	-0.205	-0.188	0.945
$r =\downarrow 1\%$	Male	6.670	2.315	1.062	0.444	0.394	0.451	0.624	1.908
	Female	6.140	2.119	0.956	0.379	0.338	0.400	0.568	3.167
	Joint Life	6.138	2.122	0.956	0.377	0.333	0.396	0.576	2.890
PANEL B: NNEG SENSITIVITY TO ROLL-UP INTEREST RATE									
$R =\uparrow 1\%$	Male	6.090	2.113	0.949	0.370	0.318	0.362	0.501	1.908
	Female	5.622	1.937	0.853	0.311	0.267	0.317	0.463	3.167
	Joint Life	5.619	1.939	0.853	0.310	0.262	0.314	0.469	2.890
$R =\downarrow 1\%$	Male	-0.969	-0.950	-0.944	-0.943	-0.955	-0.967	-0.974	-1.000
	Female	-0.967	-0.947	-0.941	-0.940	-0.953	-0.965	-0.975	-1.000
	Joint Life	-0.967	-0.947	-0.941	-0.940	-0.953	-0.966	-0.973	-1.000
PANEL C: NNEG SENSITIVITY TO RENTAL YIELD									
$g =\uparrow 1\%$	Male	4.160	1.353	0.534	0.122	0.123	0.211	0.405	1.908
	Female	3.782	1.204	0.449	0.068	0.075	0.167	0.358	3.167
	Joint Life	3.781	1.207	0.449	0.066	0.070	0.162	0.362	2.890
$g = 0\%$	Male	-0.962	-0.938	-0.932	-0.933	-0.949	-0.963	-0.974	-1.000
	Female	-0.959	-0.933	-0.928	-0.929	-0.947	-0.962	-0.975	-1.000
	Joint Life	-0.959	-0.933	-0.927	-0.929	-0.946	-0.962	-0.973	-1.000
PANEL D: NNEG SENSITIVITY TO HOUSE PRICE VOLATILITY									
$\sigma =\uparrow 1\%$	Male	0.264	0.226	0.407	0.540	0.572	0.581	0.550	1.000
	Female	0.213	0.253	0.428	0.546	0.582	0.589	0.566	1.084
	Joint Life	0.213	0.253	0.428	0.546	0.582	0.589	0.566	0.945
$\sigma =\downarrow 1\%$	Male	-0.214	-0.425	-0.556	-0.643	-0.714	-0.789	-0.858	-1.000
	Female	-0.203	-0.403	-0.513	-0.658	-0.679	-0.782	-0.836	-1.000
	Joint Life	-0.202	-0.403	-0.513	-0.658	-0.679	-0.782	-0.836	-1.000
PANEL E: NNEG SENSITIVITY TO MORTALITY CHANGES									
mortality is 20% heavier	Male	-0.228	-0.473	-0.602	-0.687	-0.729	-0.764	-0.789	-1.000
	Female	-0.250	-0.486	-0.611	-0.694	-0.735	-0.768	-0.796	-1.000
	Joint Life	-0.321	-0.537	-0.652	-0.728	-0.768	-0.800	-0.829	-1.000
mortality is 20% lighter	Male	0.563	0.033	0.222	0.384	0.442	0.492	0.552	1.000
	Female	0.560	0.032	0.223	0.385	0.446	0.498	0.562	1.000
	Joint Life	0.738	0.153	0.128	0.305	0.368	0.420	0.487	1.000
PANEL F: NNEG SENSITIVITY TO VOLUNTARY PREPAYMENT									
Prepayment is 20% heavier	Male	-0.228	-0.472	-0.600	-0.683	-0.723	-0.754	-0.781	-1.000
	Female	-0.186	-0.446	-0.581	-0.669	-0.711	-0.743	-0.772	-1.000
	Joint Life	-0.129	-0.408	-0.551	-0.644	-0.686	-0.721	-0.749	-1.000
Prepayment is 20% lighter	Male	0.501	0.121	0.216	0.394	0.464	0.522	0.608	1.000
	Female	0.461	0.128	0.238	0.439	0.492	0.554	0.610	1.000
	Joint Life	0.291	0.210	0.400	0.491	0.518	0.621	0.655	1.000

Notes:The sensitivity results are for the risk-free rate r, contract rate R, service flow rate g, and house price volatility σ. The baseline parameter values are $r = 0.57\%$, $R = 4.91\%$, and $g = 1.00\%$. The service flow rate is strictly positive under PRA requirements. In each case, the sensitivity results indicate the degree of responsiveness of NNEG cost to changes in the model parameter. The values reported in the table are computed by varying the respective sensitivity parameter by 1% either upward or downward with exception to mortality and prepayment which considers 20% upward or downward movement.

Table 4.6: Sensitivity test of the NNEG cost under GBM

Baseline Values	Age	55	60	65	70	75	80	85	90
	LTV	0.115	0.17	0.225	0.285	0.324	0.365	0.415	0.415
	Initial Loan	35650	52700	69750	88350	100440	113150	128650	128650
PANEL A: NNEG SENSITIVITY TO RISK FREE RATE									
$r =\uparrow 1\%$	Male	-0.601	-0.860	-0.819	-0.782	-0.785	-0.793	-0.802	-1.000
	Female	-0.972	-0.854	-0.812	-0.775	-0.780	-0.788	-0.795	-1.000
	Joint Life	-0.974	-0.854	-0.812	-0.775	-0.779	-0.788	-0.793	-1.000
$r = 0\%$	Male	1.768	1.277	1.070	0.939	0.979	1.046	1.116	1.500
	Female	1.694	1.225	1.027	0.904	0.947	1.019	1.094	1.333
	Joint Life	1.696	1.226	1.028	0.905	0.946	1.014	1.096	1.667
$r =\downarrow 1\%$	Male	4.181	2.807	2.279	1.962	2.072	2.260	2.465	4.500
	Female	3.945	2.658	2.165	1.872	1.989	2.186	2.410	4.000
	Joint Life	3.952	2.663	2.168	1.873	1.985	2.175	2.422	4.333
PANEL B: NNEG SENSITIVITY TO ROLL-UP INTEREST RATE									
$R =\uparrow 1\%$	Male	3.840	2.599	2.118	1.826	1.927	2.095	2.279	4.000
	Female	3.630	2.465	2.014	1.745	1.850	2.029	2.222	3.667
	Joint Life	3.635	2.469	2.017	1.745	1.847	2.018	2.237	4.000
$R =\downarrow 1\%$	Male	-0.906	-0.849	-0.807	-0.769	-0.772	-0.780	-0.791	-1.000
	Female	-0.902	-0.843	-0.800	-0.762	-0.767	-0.776	-0.786	-1.000
	Joint Life	-0.902	-0.843	-0.800	-0.762	-0.766	-0.776	-0.785	-1.000
PANEL C: NNEG SENSITIVITY TO RENTAL YIELD									
$g =\uparrow 1\%$	Male	2.512	1.719	1.453	1.314	1.490	1.737	2.023	4.000
	Female	2.334	1.598	1.356	1.234	1.414	1.669	1.966	3.333
	Joint Life	2.339	1.602	1.358	1.235	1.411	1.659	1.970	3.667
$g = 0\%$	Male	-0.875	-0.804	-0.758	-0.722	-0.736	-0.753	-0.767	-1.000
	Female	-0.869	-0.795	-0.748	-0.712	-0.728	-0.750	-0.769	-1.000
	Joint Life	-0.869	-0.795	-0.748	-0.711	-0.728	-0.750	-0.770	-1.000
PANEL D: NNEG SENSITIVITY TO HOUSE PRICE VOLATILITY									
$\sigma =\uparrow 1\%$	Male	0.336	0.229	0.203	0.200	0.273	0.398	0.605	2.000
	Female	0.303	0.204	0.180	0.178	0.248	0.367	0.564	1.667
	Joint Life	0.304	0.205	0.181	0.179	0.247	0.363	0.570	2.000
$\sigma =\downarrow 1\%$	Male	-0.280	-0.199	-0.177	-0.174	-0.230	-0.316	-0.442	-1.000
	Female	-0.259	-0.180	-0.159	-0.157	-0.213	-0.300	-0.427	-1.000
	Joint Life	-0.260	-0.181	-0.160	-0.156	-0.212	-0.300	-0.430	-1.000
PANEL E: NNEG SENSITIVITY TO MORTALITY CHANGES									
mort. = 20% heavier	Male	-0.215	-0.211	-0.211	-0.214	-0.224	-0.234	-0.244	-0.500
	Female	-0.234	-0.228	-0.229	-0.231	-0.240	-0.248	-0.256	-0.333
	Joint Life	-0.307	-0.304	-0.308	-0.316	-0.332	-0.351	-0.363	-0.333
mort. = 20% lighter	Male	0.506	0.486	0.484	0.490	0.523	0.559	0.558	0.500
	Female	0.508	0.488	0.486	0.490	0.517	0.545	0.538	0.333
	Joint Life	0.676	0.659	0.664	0.681	0.729	0.778	0.800	0.667
PANEL F: NNEG SENSITIVITY TO VOLUNTARY PREPAYMENT									
Prepay. is 20% heavier	Male	-0.216	-0.211	-0.208	-0.208	-0.210	-0.207	-0.198	0.000
	Female	-0.179	-0.177	-0.176	-0.177	-0.179	-0.181	-0.179	-0.333
	Joint Life	-0.125	-0.122	-0.120	-0.117	-0.114	-0.109	-0.096	0.000
Prepay. is 20% lighter	Male	0.507	0.484	0.479	0.478	0.497	0.510	0.488	0.500
	Female	0.411	0.398	0.396	0.397	0.411	0.424	0.410	0.333
	Joint Life	0.295	0.285	0.280	0.277	0.280	0.274	0.259	0.000

Notes:The sensitivity results are for the risk-free rate r, contract rate R, service flow rate g, and house price volatility σ. The baseline parameter values are $r = 0.57\%$, $R = 4.91\%$, and $g = 1.00\%$. The service flow rate is strictly positive under PRA requirements. In each case, the sensitivity results indicate the degree of responsiveness of NNEG cost to changes in the model parameter. The values reported in the table are computed by varying the respective sensitivity parameter by 1% either upward or downward with exception to mortality and prepayment which considers 20% upward or downward movement.

It can be noticed that the NNEG values increase with the increase in volatility and decrease with the decrease in volatility. Doubling the volatility level seems to increase the NNEGs by about 75%. The NNEG values increase almost 7 times fold when switching from $\sigma = 5\%$ to $\sigma = 12\%$.

4.5.8 NNEG sensitivities to mortality rates

Here I analyse the effect of ramping up mortality rates by 20% (mortality deterioration) or slowing down mortality rates by 20% (mortality improvement) for the baseline scenario. Increasing the mortality rate will bring forward the termination of the loans, which in turn will diminish the NNEG values because the front months values are weighted with larger multiple decrement probabilities. When the mortality rates decrease the NNEG values at the back end are weighted with higher multiple decrement probabilities, and so the total NNEG values will be higher than the baseline scenarios. The impact of 20% increase/decrease of mortality rates on NNEG values is quite small. In order to observe larger changes, very large changes in mortality rates must occur. All things being equal NNEG values will be low in COVID situation when mortality incidence rate is high compared with post-COVID situation when borrowers have improved survival rate. This is because, mortality and NNEG values are negatively related.

4.5.9 NNEG sensitivities to prepayment rates

Here I analyse the effect of ramping up prepayments by 20% or slowing down prepayment by 20% for each of the two baseline scenarios. Overall, prepayment rate changes do not change very much the NNEG profile. Although the NNEG profile does not change much as prepayment rates change, the value does when prepayments change significantly.

From discussion with few insurers, the current evidence on prepayments is rather mixed. For older vintages the prepayment rates are virtually zero. For younger vintages the prepayment[26] rates used for risk-management are higher than those reported previously in Hosty et al. (2008). The prepayment due to refinancing is farther from full capacity. The main driver for prepayment seems to be downsizing when one member of the couple, usually the husband, dies and the surviving borrower decides to move into a smaller property.

[26] Prepayment penalties may be charged by loan providers when borrowers voluntarily repay some or all of the amount borrowed before the contract travels its full length. According to the Equity Release Council (2019), there is no prepayment penalty when borrowers transition into long-term care. The penalties are used to offset expenses associated with the early termination e.g. if the loan provider obtained the said lump sum from another lending agreement. Typically providers will issue out affordable lump sums to the ERM borrower(s) while making assumptions about potential length of time contract may travel.

Prepayment rates should increase in economic times characterised by recessions with decreasing interest rate regimes following booming periods with high interest rate regimes. For NNEGs, high prepayments and decreasing contract rate s are offset by low risk-free rates r. Prepayments should be of concern when risk-free rates are low and contract rate s are high, that is in the aftermath of a crisis such as the subprime crisis. However, most lenders will try to ramp-up their portfolios in those times and ERM borrowers will not switch shortly after getting their loan due to ERCs and other psychological factors. In the aftermath of a crisis one may expect house prices to be low which will reduce the incentive for refinancing due to the LTV constraint.

4.6 Chapter Summary

In this chapter of the thesis, I show that the GBM model recommended by the regulator in the UK produces much higher values of the NNEG when compared with a best fit ARMA-EGARCH model selected on the basis of forecasting house prices well. Utilising an inappropriate model in the context of reverse mortgage loan market may in the end stifle this market by imposing very high capital reserve requirements in insurers. This is very important since there is no diversification benefit for an insurer issuing ERM loans, each loan being valued separately for NNEG calculations purposes. Furthermore, inflating the volatility parameter will automatically imply a high variance of house prices at long maturities for the GBM model, therefore impacting directly ERMs loan characteristics for the younger borrowers who would benefit the most from this new asset class.

The study also finds evidence to suggest that service flow rate parameter is not the key driver of underlying house prices in UK. If the majority of house prices do not pay rents, and this is verified in the Office of National Statistics, it would be wrong to assume that all houses have prices driven by rents. The proportion may be different from country to country but there is no known country where all houses generate a rental income. An overestimation of the service flow rate induces downward trending house prices in the long run that ultimately will inflate the NNEG values.

While the ERMs may offer a viable solution to long term care and pension boosting to the elderly generation in most developed economies, there is a general lack of development of this market world-wide. One possible explanation is that the interaction between the consumers, the insurers and the regulator needs to be improved and the capital should work more efficiently.

The application of Black 1973 option pricing formula for NNEG valuation is theoretically not sound and it can lead to important misvaluations for ERM, depending on the levels of risk-free rates. This may have a detrimental effect on the development of this important financial product for society.

The Black 76 model would be applicable if there was a house price futures contract traded. This is not the case currently but the research in this area highlights the importance of introducing futures contracts for hedging house price risk in financial and insurance markets. UK insurers may also consider hedging their house price risk with CME house price futures, while simultaneously insulating from foreign exchange risk by also using FX USD/GBP futures or options. House price futures contracts may also be introduced at some point in future on the UK market, thereby making the Black 76 applicable.

Appendices

APPENDIX C

Additional Material for Chapter 4

C.1 The ARMA process Martingale property

The valuation of the NNEG clause in ERM contracts has been specified as an European put option on the collateral house price. The resulting derivative contract is therefore written on the house price H. The proof is an adaptation of that of Wang et al. (2012) and let the continuously compounded risk-free interest rate $r_t \equiv r,\ \forall t \in [0,T]$, so that the risk-free rate is constant in order to avoid mathematical complications. Suppose further that there exists a savings account denoted $\mathcal{B}$ such that

$$d\mathcal{B}_u = r\mathcal{B}_u du,\ \forall u \in [0,T] \tag{C.1}$$

where $\mathcal{B}_0 = 1$ and $\mathcal{B}_t = e^r t$. The house price dynamics under the ARMA process can be rewritten as:

$$\ln H_t = \ln H_{t_0} + r(t-t_0) - 0.5V_n(t_0,t) + Z_n^P(t_0,t), \forall t \in [t_n, t_{n+1}) \tag{C.2}$$

where

$$\begin{aligned} A_n(t_0,t) = (\mu - r)(t-t_0) - 0.5V_n(t_0,t) + \sigma \sum_{j=n+1}^{\infty} \left(\Psi_j(W_{t-jh}^P - W_{t_0-jh}^P)\right) \\ +\sigma \sum_{i=0}^{n} \left(\Psi_j(W_{t_0}^P - W_{t_0-ih}^P)\right), \quad \forall t \in [t_n, t_{n+1}) \end{aligned} \tag{C.3}$$

$$Z_n^P(t_0,t) = \sigma \sum_{j=0}^{n} \left(\Psi_j(W_{t-jh}^P - W_{t_0}^P)\right), \quad \forall t \in [t_n, t_{n+1}) \tag{C.4}$$

$$V_n(t_0,t) \equiv Var_p\big(\ln(H_t/H_{t_0})|\mathcal{F}_{t_0}\big), \quad \forall t \in [t_n, t_{n+1}) \tag{C.5}$$

where $n = [(t-t_0)/h]$.

$$Z_n^P(t_0,t) = \sigma\Big[\Psi_0 \int_{t_0}^{t} dW_u^P + \Psi_1 \int_{t_0}^{t-h} dW_u^P + \cdots + \Psi_n \int_{t_0}^{t-nh} dW_u^P\Big] \tag{C.6}$$

$$= 1_{(n>0)}\Big(\sigma \sum_{j=0}^{n-1} \big(\big(\sum_{i=0}^{j} \Psi_i\big) \int_{t-(j+1)h}^{t-jh} dW_u^P\big)\Big) + \sigma\Big(\sum_{i=0}^{n} \Psi_i\Big) \int_{t_0}^{t-nh} dW_u^P \tag{C.7}$$

which denotes a rearrangement of Z_n^P as the sum of independent increments of Brownian motion. The same property results in conditional variance

$$V_n(t_0,t) \equiv Var_p\big(\ln(H_t/H_{t_0}|\mathcal{F}_{t_0})\big) = \sigma^2\Big(1_{(n>0)} \sum_{j=0}^{n-1} \big(\sum_{i=0}^{j} \Psi_i\big)^2 h + \big(\sum_{i=0}^{n} \Psi_i\big)^2 (t-t_n)\Big) \tag{C.8}$$

the conditioning indicates that prior to time t_0, the house price and the Brownian motion are known values[1]. Under the martingale probability measure Q, I transform the discounted house price into a Q-martingale and write it as

$$E^Q\big(H_t|\mathcal{F}_{t_0}\big) = H_{t_0} e^{(r(t-t_0))}, \ \forall t \in [t_0, T] \tag{C.9}$$

Transformation to a Q-martingale is provided in Wang et al. (2012). Suppose house price dynamics follow (4.32), and its roots $1 - \alpha_1 z - \cdots - \alpha_p z^P = 0$ lies outside a unit circle and φ satisfies

$$A_n(t_0,t) + \sigma \sum_{i=0}^{n} \big(\Psi_i \int_{t_0}^{t-ih} \varphi(s) ds\big) = 0, \quad \forall t \in [t_n, t_{n+1}) \tag{C.10}$$

where $n = [(t-t_0)/h]$ and $dW_t^Q = dW_t^P - \phi(t)dt$ is a one-dimensional Brownian motion. The resulting house price process under Q is

$$\ln H_t = \ln H_{t_0} + r(t-t_0) - 0.5 V_n(t_0,t) + Z_N^Q(t_0,t), \ \forall t \in [t_n, t_{n+1}) \tag{C.11}$$

where $n = [(t-t_0)/h]$ and $Z_N^Q(t_0,t) = \sigma \sum_{j=0}^{n} \Psi_j \big(W_{t-jh}^Q - W_{t_0}^Q\big)$.

[1] The house prices together with the Brownian motion within this interval are also assumed to be bounded.

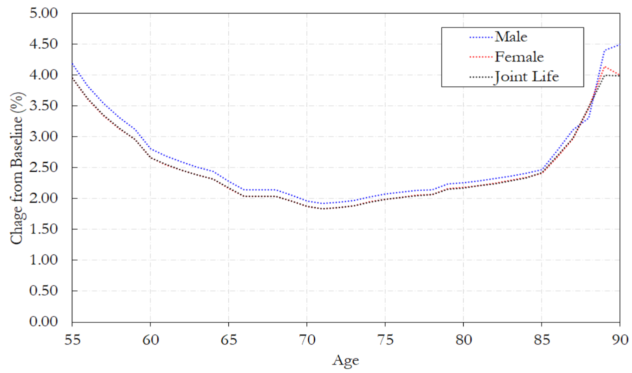
C.2 Determining the information set for ARMA model

In MATLAB, ψ_j is directly obtained by using the following procedure: *LagOp*, *mrdivide* and *toCellArray*. More specifically, if $C = (1 - 0.5L - 0.38L)$ and $D = (1 + 0.15L + 0.2L2)$ for an ARMA(2,2) model with $\alpha_1 = 0.5$, $\beta_1 = 0.15$ and $\alpha_2 = 0.38$, $\beta_2 = 0.2$, I execute $C = LagOp(1 - 0.5 - 0.38)$, $D = LagOp(1\ 0.15\ 0.2)$ and

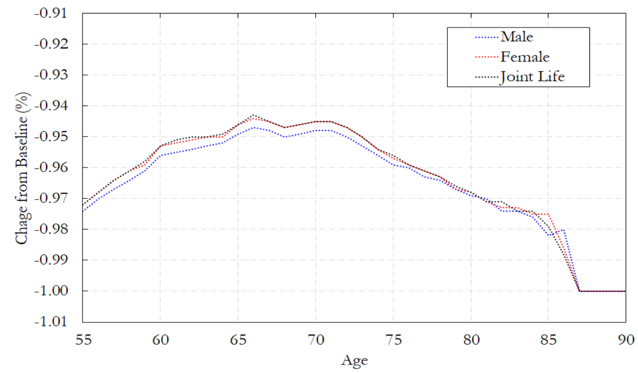
$$\psi = toCellArray(mrdivide(D, C,' RelTol', 0, Degree, 6))$$

in MATLAB. Then, ψ is equal to $[\psi_0, \ldots, \psi_6] = [1, 0.65, 0.91, 0.70, 0.69, 0.61, 0.57]$

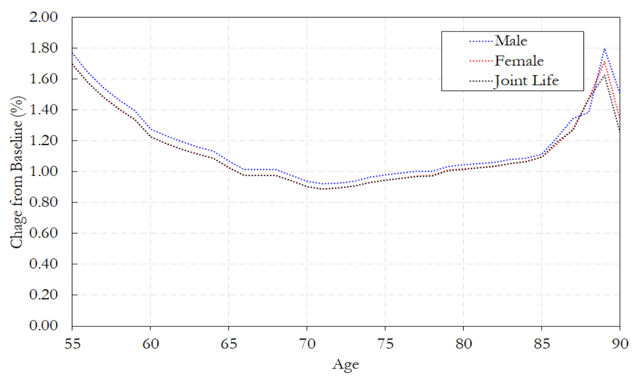
C.3 NNEG sensitivity to the risk free rate



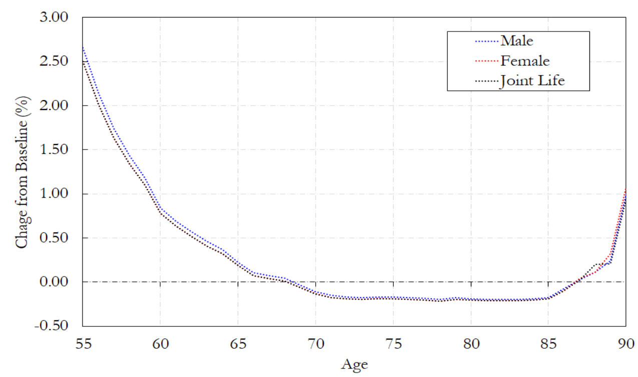
(a) GBM: r up 1%



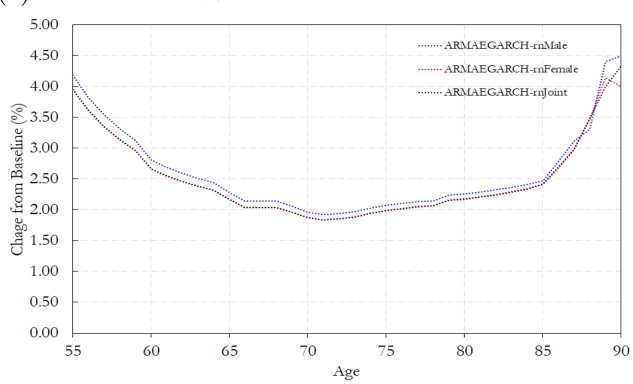
(b) ARMA-EGARCH: r up 1%



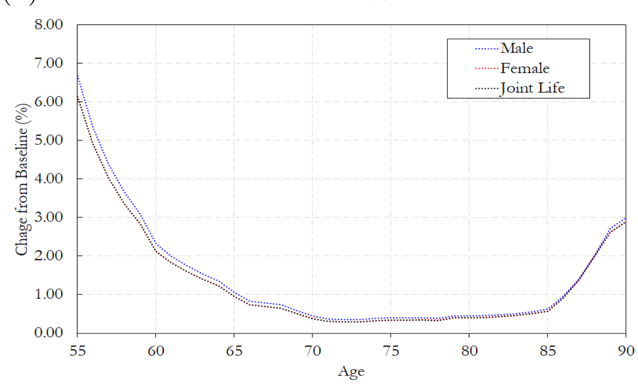
(c) GBM: $r = 0\%$



(d) ARMA-EGARCH: $r = 0\%$



(e) GBM: r down 1%

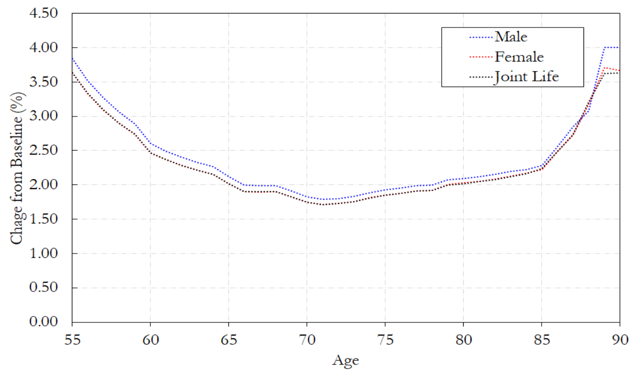


(f) ARMA-EGARCH: r down 1%

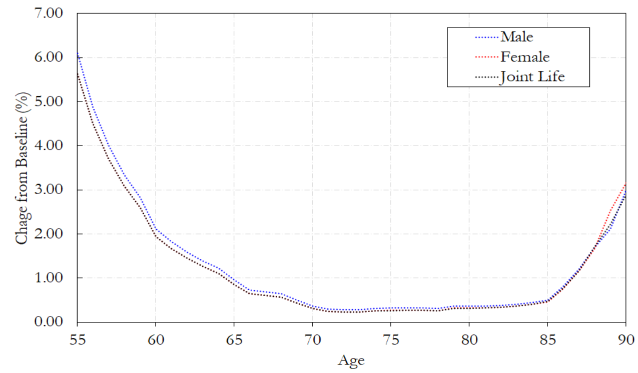
Figure C.1: Sensitivity test for changes in risk free rate

Notes:The sensitivity results are for the risk-free rate r. In each case, the sensitivity results indicate the degree of responsiveness of NNEG cost to changes in the model parameter. This values reported in the table is computed as $(NNEG_{(1)} - NNEG_{(0)})/NNEG_{(1)}$, where $NNEG_{(0)}$ is the NNEG cost estimated from the baseline parameters and $NNEG_{(1)}$ is the new NNEG cost estimated by varying the sensitivity parameter by 1% either upward or downward.

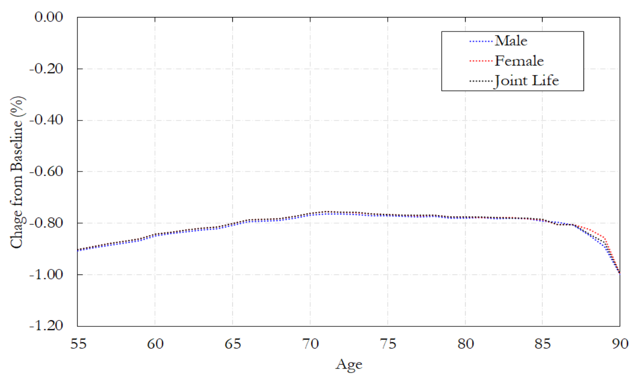
C.4 NNEG sensitivity to the contract rate



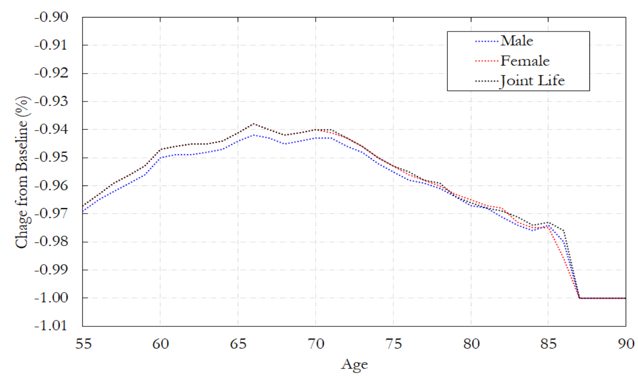
(a) GBM: R up 1%



(b) ARMA-EGARCH: R up 1%



(c) GBM: R down 1%

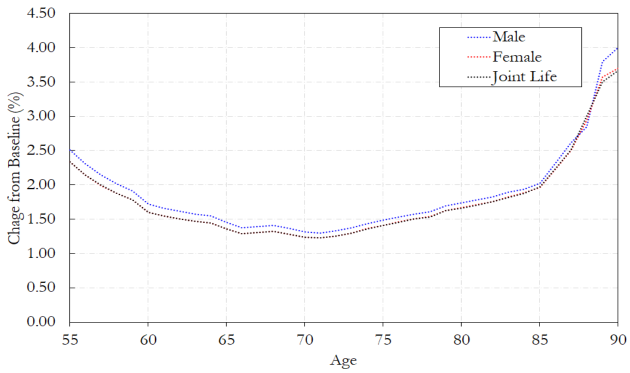


(d) ARMA-EGARCH: R down 1%

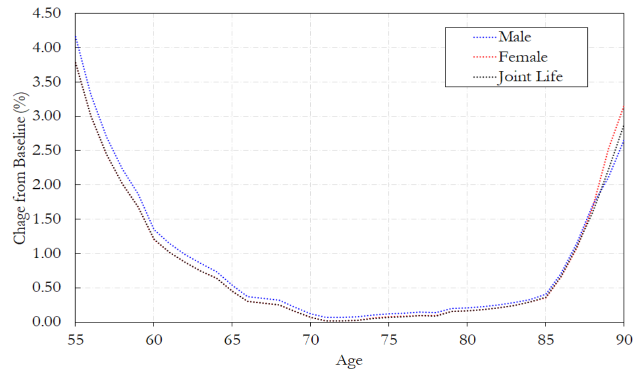
Figure C.2: Sensitivity test for changes in Roll Up rate

Notes:The sensitivity results are for the contract rate R applied to the loan. The baseline parameter values are $r = 0.57\%$, $R = 4.91\%$, and $g = 1.00\%$. The market contract rate is at all time low of 4.91% for the first half of 2019 Equity Release Council (2019). In each case, the sensitivity results indicate the degree of responsiveness of NNEG cost to changes in the model parameter. This values reported in the table is computed as $(NNEG_{(1)} - NNEG_{(0)})/NNEG_{(1)}$, where $NNEG_{(0)}$ is the NNEG cost estimated from the baseline parameters and $NNEG_{(1)}$ is the new NNEG cost estimated by varying the sensitivity parameter by 1% either upward or downward.

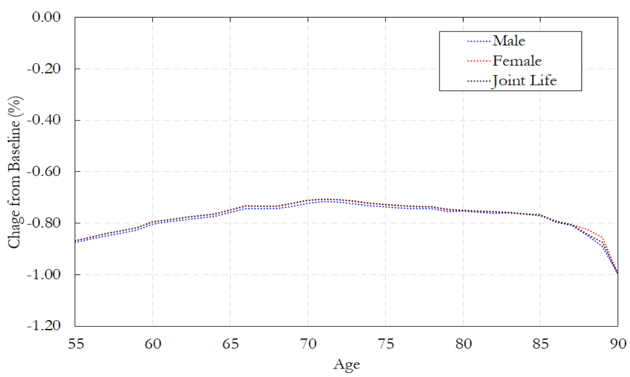
C.5 NNEG sensitivity to the service flow rate



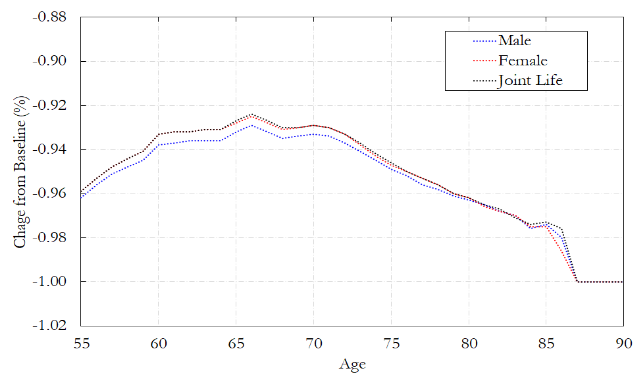
(a) GBM: g up 1%



(b) ARMA-EGARCH: g up 1%



(c) GBM: g down 1%

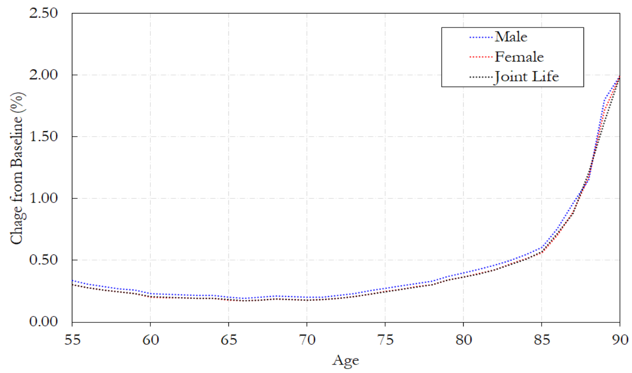


(d) ARMA-EGARCH: g down 1%

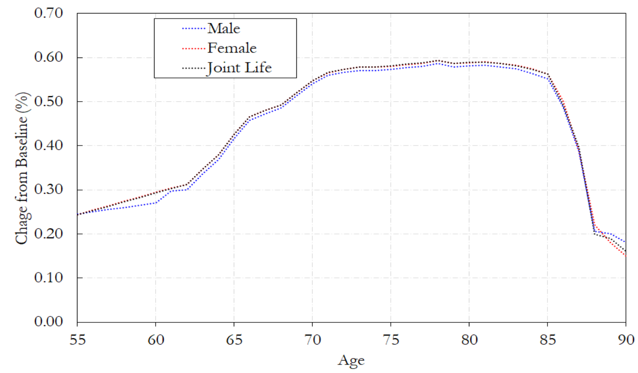
Figure C.3: Sensitivity test for changes in Rental Yield

Notes:The sensitivity results are for the service flow rate g. The baseline parameter values are $r = 0.57\%$, $R = 4.91\%$, and $g = 1.00\%$. The service flow rate is strictly positive under PRA requirements. In each case, the sensitivity results indicate the degree of responsiveness of NNEG cost to changes in the model parameter. This values reported in the table is computed as $(NNEG_{(1)} - NNEG_{(0)})/NNEG_{(1)}$, where $NNEG_{(0)}$ is the NNEG cost estimated from the baseline parameters and $NNEG_{(1)}$ is the new NNEG cost estimated by varying the sensitivity parameter by 1% either upward or downward.

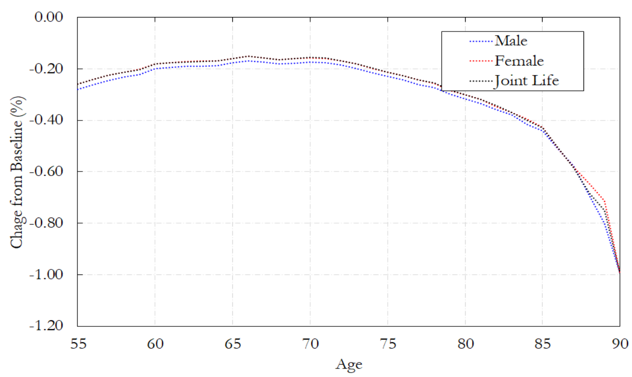
C.6 NNEG sensitivity to the volatility house prices



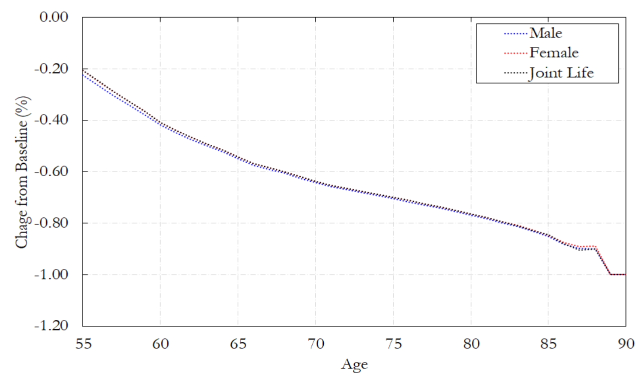
(a) GBM: σ up 1%



(b) ARMA-EGARCH: σ up 1%



(c) GBM: σ down 1%



(d) ARMA-EGARCH: σ down 1%

Figure C.4: Sensitivity test to volatility rate

Notes:The sensitivity results are for house price volatility σ. The baseline parameter values are $r = 0.57\%$, $R = 4.91\%$, and $g = 1.00\%$. In each instance, the sensitivity results indicate the degree of responsiveness of NNEG cost to changes in the model parameter. This values reported in the table is computed as $(NNEG_{(1)} - NNEG_{(0)})/NNEG_{(1)}$, where $NNEG_{(0)}$ is the NNEG cost estimated from the baseline parameters and $NNEG_{(1)}$ is the new NNEG cost estimated by varying the sensitivity parameter by 1% either upward or downward.

C.6.1 General characteristics

C.6.2 Rental yield

Ji et al. (2012) used for UK the service flow rate $g = 2\%$ while Dowd (2018) mentions $g = 2\%$ and $g = 3\%$ as a base case rate, increasing to $g = 4\%$ as a stress test, and varying between $1\%, 0\%$ and -2.75% as well. Hosty et al. (2008) used 3.3%.

C.6.3 Loan-to-Value Ratio (LTV)

Table C.1: Loan to values (LTVs) for various equity release mortgages issued 29/11/2018.

Age	Flexible	Flexible Plus	Flexible Max	Flexible Max Plus
55	11.50%	16.00%	21.20%	24.00%
56	12.50%	17.00%	22.40%	25.00%
57	13.50%	18.00%	23.60%	26.00%
58	14.50%	19.00%	24.80%	27.00%
59	15.50%	20.00%	26.00%	28.00%
60	17.00%	21.00%	27.50%	29.50%
61	18.00%	22.00%	28.50%	31.00%
62	19.00%	23.00%	29.50%	32.00%
63	20.00%	24.00%	30.50%	33.00%
64	21.00%	25.00%	31.40%	34.00%
65	22.50%	26.50%	32.20%	35.10%
66	24.00%	28.00%	32.50%	36.20%
67	24.80%	29.00%	33.50%	37.30%
68	25.60%	30.00%	34.50%	38.40%
69	27.00%	31.50%	35.50%	39.50%
70	28.50%	33.00%	36.60%	41.10%
71	29.70%	34.00%	37.70%	42.20%
72	30.50%	35.00%	39.00%	43.40%
73	31.20%	35.50%	40.00%	44.60%
74	31.70%	36.00%	41.00%	45.80%
75	32.40%	37.00%	42.00%	47.00%
76	33.20%	38.00%	43.00%	48.00%
77	34.00%	39.00%	44.00%	49.00%
78	35.00%	40.00%	45.50%	50.00%
79	35.50%	41.00%	46.50%	50.50%
80	36.50%	42.00%	48.00%	51.50%
81	37.50%	43.00%	49.00%	52.50%
82	38.50%	44.00%	49.40%	53.00%
83	39.50%	45.00%	49.80%	53.00%
84	40.50%	46.00%	50.20%	53.00%
85 and over	41.50%	47.00%	50.50%	53.00%

Source: Legal & General

For the LTV I also point to 40% (Dowd 2018), to 27% for new drawdowns and 32% for new lump sum plans as reported in the Equity Release Council (ERC) 2017 Report.

Hosty et al. (2008) is using an initial loan advanced as £20,000 while LTV starts from 15% at age 55 and increases by 1% each year up to 50% at age 90. The minimum house price is £70,000. This procedure sets the maximum house price[2] to 133,333 at age 55. Similarly, Li et al. (2010) has a minimum house price of £60,000, starts from 17% at age 60 and increases by 1% each year up to 50% at age 90 and an initial loan

[2]There seems to be a typo in Hosty et al. (2008) who give 233,333.

at £30,000.

An overall average value of 20% seems to be representative, but a more refined table taken into account age is also useful.

C.6.4 Longevity or mortality risk

The sellers of ERMs have considered for a long time that longevity risk is diversifiable. Hence, by pooling a large numbers of loans I could use mortality tables to determine the terminations of loans. The same idea applies to long-term care risk and prepayment risk.

Table C.2: Longevity expectations based on Immediate Annuities Male and Female Lives.

Year	**Expectation of life at birth**		**Expectation of life at age 65**	
	Male	**Female**	**Male**	**Female**
1841	40	42	11	12
1900	49	52	11	12
2000	76	80	16	19
2020	79	83	18	21

Notes: Derived from Continuous Mortality Investigation Research 00 tables.

The mortality data used in Hosty et al. (2008) is derived from the Continuous Mortality Investigation Research (CMI "00") mortality tables. The tables are referred to as Immediate Annuities Male Lives (IML "00") and the Immediate Annuities Female Lives (IFL "00"), adjusted for cohort effects (i.e. where rates of improvement in mortality have been different for people born in different periods historically). The tables show the probability of death during any year for an individual of a particular age who is alive at the start of that year.

Table C.3: Mortality of different socio-economic classes as a percentage of population mortality.

Class	**Ages 50-64**
I	72%
II	77%
IIIN	104%
IIIM	130%
IV	120%
V	180%

Source: Hosty et al. (2008).

Hosty et al. (2008) discussed how to adjust mortality rates for different socio-economic classes and by property value. Table C.3 shows the adjustment factors that occur due to different socio-economic conditions while Table C.4 indicates the adjustment factor by the type of property.

Table C.4: Mortality assumptions by property value

Property Value	Mortality Assumption
up to GBP130k	120% base
GBP130k -GBP250 k	100% base table
GBP250 k-GBP750 k	85% base table
GBP750k +	55% base table

As an additional stress scenario, Dowd (2018) considers the expected (mean) longevity increased by two years.

One may also use the T08 series of term mortality tables, based on 2007-2010 data collected by the CMI. I use the Office for National Statistics mortality tables (ONS tables for 2015-2017).

C.6.5 Mortality Table: Office for National Statistics(ONS)

Table C.5: Mortality Table Office for National Statistics 2015-2017

Age	Male	Female
x	q_x	qx
55	0.56%	0.4%
56	0.61%	0.4%
57	0.61%	0.4%
58	0.68%	0.5%
59	0.75%	0.6%
60	0.85%	0.6%
61	0.96%	0.6%
62	1.02%	0.7%
63	1.12%	0.8%
64	1.22%	0.8%
65	1.26%	0.9%
66	1.47%	1.0%
67	1.64%	1.1%
68	1.58%	1.1%
69	1.89%	1.3%
70	2.09%	1.4%
71	2.28%	1.5%
72	2.57%	1.7%
73	2.72%	2.0%
74	3.18%	2.1%
75	3.53%	2.2%
76	3.82%	2.6%
77	4.27%	2.9%
78	4.60%	3.3%
79	5.42%	3.6%
80	5.72%	4.2%
81	6.38%	4.5%
82	7.21%	5.3%
83	7.89%	6.0%
84	9.07%	6.8%
85	10.42%	7.8%
86	11.49%	8.9%
87	12.06%	9.6%
88	14.22%	11.2%
89	15.38%	12.6%
90	17.01%	14.2%
91	20.26%	16.3%
92	20.79%	17.5%
93	22.32%	19.9%
94	25.25%	22.1%
95	27.05%	23.9%
96	28.71%	25.4%
97	30.49%	27.5%
98	35.92%	28.3%
99	38.74%	31.4%
100	34.60%	32.8%

C.6.6 Long term care risk

Long Term Care (LTC) is defined the inability to carry out at least two activities of daily living ("ADLs"). There is very little data available on the movement of people into long-term care as a result of their inability to perform ADLs and making it difficult to accurately predict the rate of morbidity which will affect the timing of the underlying cash flows entering the transaction.

When premiums were originally set for the HECM[3] loans, there was no actual exit data so the assumption made was that loan exits would occur at 1.3 times the rate of mortality, see Rodda et al. (2004). The actuarial market practice in the UK calculates morbidity rate as a factor of the mortality rate.

Table C.6: Percentage loading to base mortality due to long term care entry.

Age	Male(%)	Female(%)
$\leqslant 70$	2	3
$(70, 80]$	4	12
$(80, 90]$	5	13
$(90, 100]$	4	8

Source: Hosty et al. (2008).

For multi-state modelling considering the interaction between long-term care entry and mortality is paramount because there is significantly higher mortality experienced by long-term care residents compared to "at home" mortality means that to maintain the same aggregate assumption for mortality by age lighter than average mortality should be assumed for "at home" lives. Table C.6, from Hosty et al. (2008), shows the long-term care net impact of additional decrements, offset by reductions in at-home mortality, taken to be the uplifts to base mortality, with intermediate values by linear interpolation.

C.6.7 Prepayment risk

Not very much is known about the values of the prepayment rate for ERMs. In the US in the early days of the HECM programme, a flat prepayment rate of 0.3 times the mortality rate of the youngest borrower in the family was used. In Korea, a prepayment rate of 0.2 times the 2010 mortality rate for females was chosen based on Korean demographic data.

[3]The only reverse mortgage insured by the US Federal Government is called a Home Equity Conversion Mortgage (HECM), and is only available through an Federal Housing Administration (FHA) approved lender. If you are a homeowner age 62 or older and have paid off your mortgage or paid down a considerable amount, and are currently living in the home, you may participate in FHA's HECM program. The HECM is FHA's reverse mortgage program that enables you to withdraw a portion of your home's equity.

Table C.8: Prepayment rates reported in Ji et al. (2012)

Year	Prepayment rate (%)
1	0.0
2	0.0
3	0.15
4	0.3
5	0.3
6+	0.75

Source:Institute of Actuaries (2005)

Prepayment risk is usually managed with early redemption charges (ERC). Hosty et al. (2008) describe this feature that varies by different providers. The ERC can be fixed rate charge or marked to market. In August 2007, the fixed charge scales ranged from 3% flat for the first 5 years and nil thereafter, to 7% initially stepping down to nil after 10 years and some providers applied charges for the first 20 years. Many large providers were charging mark to market penalties with the ERC applied depending on interest rate movements between inception and repayment. The ERCs were capped (currently at either 20% or 25%).

Hosty et al. (2008) considered the following prepayment rates. The first set was taken from the Norwich Union prospectus for Equity Release Funding (no.5) plc, August 2005, as follows: ERF1, 4.4% p.a.; ERF2, 3.7%; ERF3, 2.5%; ERF4, 1.4% (prepayment rates given by number of loans). In addition, the prepayment rates in Table C.7 were noted from Bell & Bain Ltd, Glasgow.

Table C.7: Prepayment rates assumptions

Year	Prepayment rate (%)
1-2	1.00
3	2.0
4-5	2.5
6-8	2.0
9-10	1.0
11-20	0.5
21+	0.25

Source:Hosty et al. (2008)

Ji et al. (2012) separated prepayment rates into two sources, Table C.7 for remortgaging of ERMs and Table C.8 for prepayment arising from changes in personal circumstances.

C.6.8 Discount factors

One issue that is often neglected in NNEG valuation is the choice of discount factors. Quite often the discount factors are derived from a unique constant risk-free rate, accepted in the framework described by Knapcsek & Vaschetti (2007), see also Dowd (2018). Kogure et al. (2014) used $df(t) = (1 + r)^{-t}$ with $r = 0.5\%$ for the Japanese market. Li et al. (2010) use returns from Treasury-bills as a proxy for short-term interest rate and Kim & Li (2017) employed the 91-day certificate of deposit as a proxy for the same risk-free rate. Hosty et al. (2008) used a constant risk-free rate equal to 4.5% but considers the discount rate as 4.75%, effectively extracting the NNEG risk premium by applying the same time invariant risk premium of 0.25%.

Wang et al. (2014) and Lee et al. (2012) employ a CIR short-rate model for discount curves, which will also fit in the framework described in Knapcsek & Vaschetti (2007).

I used in my calculations the risk-free curve on 6 April 2020, downloaded from Bloomberg. This is described in Table C.9.

Table C.9: GBP Risk-free term structure of interest rates on 26 December 2018.

Maturity	03/05/2020
1M	0.12%
3M	0.14%
6M	0.17%
1Y	0.06%
2Y	0.00%
3Y	0.03%
4Y	0.05%
5Y	0.07%
6Y	0.09%
7Y	0.10%
8Y	0.13%
9Y	0.20%
10Y	0.23%
12Y	0.32%
15Y	0.41%
20Y	0.52%
25Y	0.54%
30Y	0.55%
40Y	0.48%
50Y	0.41%

C.7 Valuation Under the Geometric Brownian Motion

The GBM dynamics is specified under the real-world measure using the equation

$$dH_t = \mu H_t dt + \sigma H_t dW_t \tag{C.12}$$

For simplicity, I denote by $K = L_0 e^{RT}$ the exercise price of my NNEG put option at maturity T.

C.7.1 Risk-neutral world GBM pricing

Under risk-neutral world the dynamics changes only in the drift to

$$dH_t = (r - g)H_t dt + \sigma H_t dW_t \tag{C.13}$$

where g is the service flow rate.[4].

The Black-Scholes formula behind the NNEG put option is

$$Put(H_0, K, T) = e^{-rT} E^Q \left(\max[K - H_T, 0]\right) \tag{C.14}$$

where Q is the risk-neutral measure implied by the Black-Scholes model. Then

$$Put(H_0, K, T) = Ke^{-rT}\Phi(-d_2) - H_0 e^{-gt}\Phi(-d_1) \tag{C.15}$$

where $d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln(H_0/K) + (r - g + 0.5\sigma^2)T\right]$ and $d_2 = d_1 - \sigma\sqrt{T}$.

C.7.2 Real-world GBM pricing

Under this method securities are priced using real-world probabilities derived from the historical information and a risk-neutral (funding rate) discount rate.

This would be valued under real-world measure as

$$Put(H_0, K, T) = e^{-r^* T} E^P \left(\max[K - H_T, 0]\right) \tag{C.16}$$

[4]I consider service flow rate here in order to be able to compare GBM-rn as used by some insurers with other approaches. I do not necessarily agree that $g \neq 0$

where r^* should be the risk-adjusted interest rate reflecting the premium charged for investing in this market.

Using the usual trick that

$$\begin{aligned} E^P\left(\max[K - H_T, 0]\right) &= E^P\left((K - H_T)1_{\{H_T<K\}}\right) \\ &= E^P\left(K1_{\{H_T<K\}}\right) - E^P\left(H_T1_{\{H_T<K\}}\right) \\ &= KP(H_T < K) - E^P\left(H_T1_{\{H_T<K\}}\right) \end{aligned} \tag{C.17}$$

One can show with standard calculations that

$$P\left(H_T < K\right) = \Phi\left(-\frac{1}{\sigma\sqrt{T}}\left[\ln(H_0/K) + (\mu - 0.5\sigma^2)T\right]\right)$$

and

$$E^P\left(H_T1_{\{H_T<K\}}\right) = H_0e^{\mu T}\Phi\left(-\frac{1}{\sigma\sqrt{T}}\left[\ln(H_0/K) + (\mu + 0.5\sigma^2)T\right]\right)$$

Thus

$$Put(H_0, K, T) = e^{-r^*T}\left[K\Phi\left(-d_2\right) - H_0e^{\mu T}\Phi\left(-d_1\right)\right] \tag{C.18}$$

where $d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln(H_0/K) + (\mu + 0.5\sigma^2)T\right]$ and $d_2 = d_1 - \sigma\sqrt{T}$.

C.7.3 Black 1973 model

Some argued that the "correct" approach is to use the Black (1976) formula for pricing the NNEG. Under this model pricing the NNEG would be done with the formula

$$Put = e^{-rT}[K_TN(-d_2) - F(T)N(-d_1)] \tag{C.19}$$

with

$$d_1 = \frac{\ln(F(T)/K_T) + 0.5\sigma^2\tau}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

where r is the risk-free rate of interest, K_T is the strike price for period T calculated as $K_T = L_0e^{R\times T}$ (here L_0 is the initial loan value) and $F(T)$ is the *forward* house price for year T, which *also* has the formula

$$F(T) = H_0e^{(r-g)T} \tag{C.20}$$

where g is the house rental rate and H_0 is the current house price.

C.7.4 Forecasting measures

The root mean squared error (RMSE) is defined as the squared root of the average squared forecasting errors. The mean absolute error (MAE) is defined as the average of the absolute values of forecasting errors. A lower RMSE or MAE indicates a better forecasting method or model. The forecasting measures may be higher because of a couple of really bad forecasts or outliers.

An improved approach for comparing forecasting methods (models) is the Diebold Mariano test Diebold & Mariano (1995). This test is based on a loss function L for the forecasting error e_t and it calculates the loss differential between two methods 1 and 2 as

$$d_{12t} = L(e_{1t}) - L(e_{2t})$$

Under the appropriate technical assumptions the Diebold-Mariano statistic is defined as

$$DM_{12} = \frac{\overline{d_{12}}}{\hat{\sigma}_{\overline{d_{12}}}}$$

where $\overline{d_{12}} = \frac{1}{T}\sum_{t=1}^{t=T} d_{12t}$ and $DM \to N(0,1)$.

The null hypothesis is that the two models produce equal expected forecast loss. The alternative is that one model has a superior (lower) expected forecast loss than the other one. Usually a quadratic loss function is used, i.e. $L(e_t) = e_t^2$.

CHAPTER 5

House Price Volatility and ERM Portfolio Cash Flows

5.1 Introduction

In this Chapter of the thesis, I investigate the long-term effects of idiosyncratic house price risk on ERM portfolio cash flow. Case & Shiller (1987), Case & Shiller (1990), Case et al. (2000), Geltner (2007), and Tunaru (2017) present excellent discussions on derivative instruments designed on real-estate indices which serve as efficient reference for house price data. The work of Case & Shiller (1987) identified two categories of real-estate indices i.e. transactions-based index and the valuation-based (appraisal based) index. The different variants of house price indexes are identified by the underlying methodology, from which they created.

The transaction-based indices is linked to actual transaction price data obtained over the period of interest. After controlling for index return volatility and disparities due to differing trading periods in granular information obtained regarding this category of indexes Hoag (1980), Miles et al. (1990), Webb et al. (1992), Fisher et al. (2004), Geltner (2007), and Tunaru (2017) suggest transaction-based indices is ideal choice for the underlying asset in derivative contracts designed on real-estate properties. Extensive literature addressing the well-identified statistical complexities on this class of house-price data brings forth the hedonic-value and repeat-sales regression methodologies (see Adelman & Griliches 1961, Bailey et al. 1963, Rosen 1974, Shiller 2008). Case & Shiller (1989) argued that repeat-sales indices extends additional benefits of not being affected by mixed sales and other related changes. A key difference between the transaction-based and appraisal-based indices has to do with the treatment of capital improvement expenditures as it relates to the index return appreciations. Recent studies highlight model risk effects in hedonic methods. Tunaru (2017) discussed instances where the hedonic valuation method inherits the pitfalls associated with statistical regression methods, e.g multicollinearity, parameter estimation errors, and model misspecification.

The appraisal-based/valuation-based indices are dependent on continual updates of individual real-estate characteristics. For each period, the classical appraisal-based index returns is estimated using a simple aggregation of regular appraised values. Tunaru (2017) outlines the difficulty in accurate maintenance of this class of indices for very large populated regions. Construction of appraisal-based methods are typically beset with "valuation smoothing" since the methodology is purely subject to the valuer's opinions year-on-year (Hager & Lord 1985, Clayton et al. 2001). The valuation smoothing arises from the fact that year-on-year appraisals creates valuation estimates which are tied to prior sales prices for the properties. This flaw becomes more persistent when the real-estate market tends to be rapid in advancement. There is also a dampening effect on the randomized error term, hence the volatility of the valuation-based historical price series tends to be lower compared to the volatility of actual property price series or hedonic price series (Siu-Hang Li et al. 2010).

The classical geometric Brownian motion (GBM) is employed with an intention to capture the diffusion dynamics in house price volatility when pricing ERM contracts (see Hosty et al. 2008, Li et al. 2010). The downside to the GBM model has been thoroughly addressed in Chapter 4. When pricing ERM contracts, most time series models adopt a two-part conditional heteroscedastic model, one for fitting the conditional mean and another for the conditional variance. Model specifications in Chen, Cox & Wang (2010) and Siu-Hang Li et al. (2010) including those outlined in Equations (4.21) and (4.22) of the thesis are typical examples. Although this modelling approach sometimes suffers from some degree of weak-stationarity problems Tsay (2005) and Wei et al. (2006) showed that the process is more likely to capture the observed autocorrelation effect inherent in property price return series. This is so because, the model allows the estimated conditional log-returns to be equally depend on innovations of its autocorrelation and moving average coefficients. In this same regard, the second portion of the two-part specification in (4.22) accounts for the innovations in the conditional variance. Siu-Hang Li et al. (2010) specified an exponential GARCH (EGARCH hereafter) model for the conditional variance which accounts for the leverage effect in the return series. Huang et al. (2020) also worked with ARMA-GARCH jump model that can capture the characteristics of jump persistence, volatility clustering and autocorrelation.

Relating to the stochastic modelling of property prices in UK, Siu-Hang Li et al. (2010) discussed a need to account for the statistical properties of the selected index when pricing NNEGs. They argue that real-estate performance models developed by Wilkie (1995), Booth & Walsh (2001), and Booth & Marcato

(2004) are unlikely to capture instances of high serial correlation, leverage effect and the heteroscedasticity of log-return volatility, hence not ideal for modelling . As mentioned earlier, the geometric Brownian motion (GBM) model which is another alternative, has serious shortfalls. Its assumptions inherently besets it with attributes[1] which makes them inappropriate for the house price time series.

Some degree of subjectivity exists in house price volatility modelling in relevant ERM literature. Fair value pricing of ERM contracts depends on the efficient modelling of the underlying property in the contract (Siu et al. 2004, Hosty et al. 2008, Li et al. 2010, Kogure et al. 2014, Cocco & Lopes 2015, Dowd et al. 2019). Lew & Ma (2012) used a housing price growth rate of 3.5% per annum, reflecting the average house price growth rate in Korea between 1986 and 2006. With respect to the UK, Hosty et al. (2008) argued that the house price inflation growth rate should be between RPI and the economic growth plus a "bit", and he gives 2.5% to 5.5% as a confidence interval for the house price inflation, "with either extreme difficult to justify". Ji et al. (2012) assumed initial house price values: 176500, 111000, 81000, and 60000 respectively linked to the age of the younger spouse 60, 70, 80 and 90 years. Hosty et al. (2008) analysed house price inflation (growth) for various U.K. regions over the period 1974-2007 and observed that average annual U.K. growth was 8.8% for the period with the lowest-growth area being 0.5% lower (Scotland at 8.3% p.a.) and the highest-growth area 0.5% higher (London at 9.3%). For the United Kingdom Hosty et al. (2008) used a pricing basis of 4.5% HPI (mean level) with 8% volatility and then loaded the volatility by 3% to cover the shift from Index to individual properties (idiosyncratic risk), giving a final assumption of 11% volatility. These values are calculated relative to a geometric Brownian motion. Importantly, using de-smoothing process pioneered by Geltner increases volatility to 17.01% per annum. Dowd (2018) applied a volatility of 10% per annum for the house prices and mentioned stress tests conducted to have house prices fall by 30% and 40% although he also indicates that the future values of prices cannot enter considerations for pricing NNEG. It is unclear how the reduction of 30% or 40% in house prices is achieved; one can only presume that this is applied directly to the current house price.

Alai et al. (2014) argued for the need to account for property dilapidation over the life time of the ERM contract since the loan issuer is likely to inherit a collateral house that is impaired in value. A careful consideration will show that the property dilapidation rate is the parameter which will adjust the market value of the underlying house price in order to account for the impact of dereliction in property maintenance. Shiller & Weiss (2000) earlier attributed the incidence of this to potential reduction in property maintenance to instances where the ERM borrower may face a lessened financial interest in the collateral house. When

[1]The GBM assumptions imposes the following implications i.e. $\forall t > 0$, single-period returns follow a lognormal distribution parametrised μ and σ which are time-invariant. This presupposes a no conditional heteroscedasticity and leverage effect. Also, $\forall t \neq u$, the return series are independent, indicating a no autocorrelation in log-return series

the impairment parameter is calibrated, the loan issuer will have an opportunity to analyse any resulting pool of dilapidated residential properties, as this class of collateral will have lower values compared to the general real estate market prices. The impairment factor will also capture the resulting basis[2] risk in the event where individual property prices in the loan issuer's portfolio fall below the house price index changes. The impairment factor is used to parameterise the degree of dilapidation suffered by the collateral house from inception of the ERM contract to its termination. It will account for dereliction idiosyncratic effects, and capital improvement expenditure. The idea of the impairment factor is based on the normal wear and tear of the collateral property. The dynamics of the impairment parameter may also provide additional information that can improve the calibration of the house price volatility.

In this chapter I investigate the long term effects of idiosyncratic house price risk on ERM portfolio cashflows. The discussion argues for the need to explain the characteristics of the NNEG portfolio values when the well-known features[3] of house price time series are well calibrated in the contract valuation. The analysis formulates a framework that transparently calibrates and integrates property impairment and idiosyncratic house price risk into ERM contracts to ensure fair value creation in ERM portfolios. Within the same setup, the analysis also investigates the cash flow implications of the idiosyncratic house price risk when using the GBM or time series based models in pricing ERM contracts. The chapter presents a closed form formula for the impairment factor with which I explore the relationship between the term structure of impairment factors and the volatility estimate from the house price model. More specifically, the analysis identifies the corresponding impairment factors for various ages of a borrower that should be allied in order to match the NNEG valuation without idiosyncratic risk. For example, using a volatility of $\sigma = 13\%$ in the Black-Scholes put option NNEG valuation tends to produce NNEG values equivalent to a 2.6% annual impairment factor in the NNEG valuation under the adjusted Black-Scholes formula[4]. This means that the collateral house will lose in value about 13% after after fives just because of dilapidation and dereliction idiosyncratic effects.

Based on the current market trends, I considered the portfolio effects of house price risk when looking at the cash flow analysis from a lender perspective. We constructed an ERM portfolio that takes the view that all borrowers live exactly to their expected lifetime. In setting up the profile of the borrowers, the study recognises that the future lifetime expectancy of a 60 year old will differ from the lifetime expectancy of a 70 year old and so on, and it will also be different between males and females. Another approach randomises

[2] Andrews & Oberoi (2015) is one of the early studies that document the term-structure of basis risk in ERM contracts.

[3] Glaeser & Nathanson (2017) presented an excellent discussion of house price time series features within a perfectly rational model.

[4] The adjusted Black-Scholes formula equals the original NNEG Black-Scholes pricing formula adjusted for property impairment or property dilapidation.

arrival of termination event between the current age of the borrower and 100, so for example a 65 year old female borrower's time until termination is a random number between 1 and 35. Drawing 4 as the random number means that the female borrower has 4 years until termination as an extreme portfolio I also consider cash-flows for a portfolio where all borrowers go to 100 years.

The portfolio has 10,000 loan contracts in total. The total portfolio size is made of 4927 females and 5073 males. The sampling procedure follows from Hosty et al. (2008). Based on the loan to value ratios (LTVs) introduced in Appendix C.1 the study used initial property values distributed as follows: 2500 borrowers on Flexible LTV with 100,000 loan; 2500 borrowers on Flexible LTV with 200,000 loan, 2500 borrowers on Flexible LTV with 310,000 loan and 2500 borrowers on Flexible Max Plus LTV with 950,000 loan. In order to analyse the portfolio NNEG cash flows, I considered the following components of the loan portfolio: the total expected value of the house collateral, portfolio cash generated, total portfolio accrued cash in money account, the annual *expected* payment upon borrower termination, the four-year ahead total portfolio *expected* payments, and the total NNEG exposure at risk (EAR) due to house collateral. The calculations were based on the assumption that the loans are terminated at a random time before the expected future lifetime maturities, for male and female borrowers. The mortality rates are estimated from the Office of National Statistics life table for 2017.

The study presents the evolution of the portfolio outstanding balance and the evolution of generated cash from ERM loan terminations under the ARMA conditional mean and an EGARCH conditional variance model (ARMA-EGARCH, hereafter) for underlying property prices. The spread between quantiles of portfolio generated cash is larger under the GBM model. This implies riskier cash-flow projections that would automatically require larger capital reserves under Solvency 2 set of regulations which supports the conservative nature of the GBM model. The first 2-5 years of the portfolio lifetime shows an approximately equal spread between the quantiles of portfolio generated cash under both models. This supports the earlier observation regarding the 2-year out-sample forecast analysis in Chapter 4 where both GBM and ARMA-EGARCH models give similar future house price values.

When compared to the ARMA-EGARCH model, uncertainty levels in portfolio generated cash flow is much larger under the GBM model. This observation is consistently associated with scenarios where time-to-termination of portfolio loan contracts exceed 10-years. With respect to the comparative ledgers such as the expected payments that the portfolio will generate next year versus the payments generated over the next four years. There is a high plateau of sustainable payments of the money inflow under the ARMA-EGARCH simulations between 15-22 years. Under the GBM model there is a peak around 20 years after which the expected cash flows decrease much faster than in the case of the ARMA-EGARCH simulations.

The probability distribution of the EAR measures at various time horizons for the ARMA-EGARCH model and the GBM model, respectively show that essentially all histograms are entirely on the negative domain. This is explained by the fact that the simulated house prices exceed the outstanding balance for the respective loans in the portfolio. At all horizons, the distribution of EAR values coming from the Monte Carlo simulation exercise in the GBM case has longer negative tail than the distribution generated for the corresponding ARMA-EGARCH case, confirming that GBM simulations are overly conservative from a regulatory perspective and leading to excess capital reserves costs that will make the RM market less efficient on a capital efficiency basis and less appealing to consumers who will ultimately absorb this extra cost.

The range of ERM portfolio exposure at risk (EAR) is consistently larger in the GBM model compared to the ARMA-EGARCH. The negative EAR values I observed is because the individual collateral house consistently lies above the accumulated loan amounts. For periods beyond 20-years, the frequency of EAR values generated under the ARMA-EGARCH model is double that of the GBM model. The ARMA-EGARCH model seems to provide an opportunity for to capture tail extreme tail observations in EAR; thereby allowing issuers to explain the long term effect of house price risk.

The remainder of the chapter is organized as follows: Section 5.2, describes the design structure of the ERM portfolio and the parametrisation of portfolio cash flows; Section 5.3, presents results on empirical analysis of UK house price index data. The investigation outlines both national and regional specific features of the Nationwide house price index series for the UK; Section 5.5, reports the results under alternative approaches of setting up the ERM portfolio comparing cash flows under both the GBM and the ARMA-EGARCH models for underlying house prices. The analysis on the term structure of impairment factors is presented in Section 5.4; Section 5.6 concludes.

5.2 Models and Methods

In this section, I describe and setup an ERM portfolio that closely capture current market information, trying to stay as close as possible with the portfolio assumptions in Hosty et al. (2008) but linking the termination probabilities to current mortality tables. The new market statistics include actual practitioner data on contract rate, risk-free interest rate, mortality improvement factors, long-term care incidence rate etc. The main ERM portfolio takes the view that all borrowers live exactly to their expected lifetime. Note that the lifetime expectancy of a 60 year old will differ from the lifetime expectancy of a 70 year old and so on, and it will be also different between males and females.

The portfolio has 10,000 loan contracts in total, 4927 female and 5073 males. The initial property value is distributed as follows: 100k - 2500 borrowers on Flexible LTV, 200k - 2500 borrowers on Flexible LTV, 310k - 2500 borrowers on Flexible LTV, and 950k - 2500 borrowers on Flexible Max Plus LTV.

The research uses the following cash flow variables, where i denotes the loan number and t the year ahead.

- $\tau_t^{(i)}$ = indicator variable if termination for loan i arrives in year t (taken as 1) or not (taken as 0)
- $\omega_t^{(i)}$ = 1 if the loan (i) is still active, and is equal to 0 if it is not active.
- $K_t^{(i)} = L_0^{(i)} e^{Rt}$ accumulated balance for loan i at time t
- $\widetilde{K_t^{(i)}} = K_t^{(i)} \times \omega_t^{(i)}$ accumulated balance for loan i at time t, if the borrower survives to 100 years (and $t \geqslant 100$.).
- $\widetilde{K_t} = \sum_i \widetilde{K_t^{(i)}}$ is the portfolio outstanding balance at time t
- $C_t^{(i)} = min(H_t^{(i)}, K_t^{(i)}) \times \tau_t^{(i)}$ is the cash generated in year t from loan i
- $C_t = \sum_i C_t^{(i)}$ total portfolio new cash generated by loans terminating in year t
- AC_t = total portfolio accrued cash in money account by time t; this is calculated recursively $AC_t = AC_{t-1} \times e^r + C_t$.
- $P_t^{(i)} = E(C_t^{(i)}) = E(min(H_t^{(i)}, K_t^{(i)}) \times \tau_t^{(i)}) = E(min(H_t^{(i)}, K_t^{(i)})) \times E(\tau_t^{(i)})$; is the payment *expected* from loan (i) in year t under risk-neutral probability. This would be clearly zero in all years except the year when borrower is expected to terminate[5]. In that year, that is when $\tau_t^{(i)}) = 1$, $P_t^{(i)} = E(min(H_t^{(i)}, K_t^{(i)}))$.
- $P_t = \sum_i P_t^{(i)}$ is the total payment expected on the portfolio in year t,
- $P_t^{4Y} = P_{t+1} + P_{t+2} + P_{t+3} + P_{t+4}$ is the total portfolio *expected* payments over the *next* four[6] years.
- $\widetilde{F_t^{(i)}} = L_0^{(i)} e^{rt} \times \omega_t^{(i)}$ accumulated funding balance for loan i up to time t
- $\widetilde{F_t} = \sum_i \widetilde{F_t^{(i)}}$ is the portfolio outstanding funding balance at time t
- $NetP_t = P_t - (\widetilde{F_t} - \widetilde{F_{t-1}})$ is the portfolio total expected payment on the portfolio in year t, *net* of interest payment for that year.
- $NetP_t^{4Y} = P_t^{4Y} - (\widetilde{F_{t+4}} - \widetilde{F_t})$ is the total portfolio *expected* payments over the *next* four years, net of interest payments during those four years.
- $H_t = \sum_i H_t^{(i)}$ = total expected value of house collateral at time t
- $EAR_t^{(i)} = K_t^{(i)} - H_t^{(i)}$ NNEG exposure at risk for loan (i) at time t
- $EAR_t = \sum_i EAR_t^{(i)}$ = total NNEG exposure at risk due to house collateral
- $E_t^{(i)} = E(K_t^{(i)} - H_t^{(i)}) = K_t^{(i)} - E(H_t^{(i)})$ is the expected exposure for loan (i) at time t
- $E_t = \sum_i E_t^{(i)}$ = total expected exposure due to house collateral
- $\Gamma_t^{(i)} = \Gamma_{t-1}^{(i)} e^r - C_t; \Gamma_0 = L_0$ is the oustanding liability net of funding costs.

[5] My main portfolio takes the view that all borrowers live exactly to their expected lifetime. Note that the lifetime expectancy of a 60-year-old will differ from the lifetime expectancy of a 70-year-old and so on, and it will be also different between males and females.

[6] The 2-year out-of-sample forecasting error for the GBM and ARMA-EGARCH models in 4.7 shows how both models produce similar future house price values. The ARMA-EGARCH performs better in a 5-year out-of-sample forecasting error analysis. Hence, the total portfolio payments over the next four years provides an opportunity to compare portfolio cash-flow implications of the two models within a time frame where both models possess equal performance in predictability.

5.2.1 Modelling House Price Idiosyncratic Risk in ERM Loans

Recall the NNEG liability to the ERM loan issuer is valued as a put option in (4.6) that matures in, say, T years with a terminal value $\max(K_T - H_T, 0)$. Following Alai et al. (2014), I assume the collateral house in the ERM contract is possibly impaired in value at termination. We model the impairment using a value compressing factor of $\wp$. From another view point, $\wp$ is the parameter that adjusts the market value of the underlying house prices for the impact of dereliction/dilapidation of property maintenance.

Shiller & Weiss (2000) attributed this potential reduction in property to lack of maintenance by the ERM borrower who may face a lessened financial interest in the collateral house. The $\wp$ parameter will allow the loan issuer to analyse the resulting pool of impaired residential properties, as this class of houses will have lower values compared to the general real estate market prices. Furthermore, the $\wp$ will also capture the resulting basis risk between individual property prices that are collateral in the loan issuer's portfolio and the general house price index. We are also able to further investigate the issue of hefty discounting applied to home values as mentioned in Warshawsky & Zohrabyan (2016).

Since the impairment parameter affects only the house prices at maturity, I have $\bar{H}_T = (1-\wp)^T H_T$, where $\wp$ denotes the annual rate of impairment charged on the collateral house price. It is evident that I should impose that $0 \leqslant \wp < 1$ while for practical purposes $\wp$ is closer to zero. The valuation of the NNEG put option price is readjusted under a Black-Scholes model[7] as follows

$$\begin{aligned}
\bar{A}(0) &= E^Q\left[e^{-rT}\left(K_T - \max(K_T - \bar{H}_T, 0)\right)\right] && (5.1)\\
&= E^Q\left[e^{-rT}\left(K_T - \max(K_T - (1-\wp)^T H_T, 0)\right)\right] && (5.2)\\
&= (1-\wp)^T\{e^{-rT}E^Q\left[\left(K_T(1-\wp)^{-T} - \max(K_T(1-\wp)^{-T} - H_T, 0)\right)\right]\} && (5.3)
\end{aligned}$$

Hence, formulae (5.2) and (5.3) give the Black-Scholes price of the NNEG while allowing for the impaired value adjustment. E^Q denotes the risk-neutral expectation of the NNEG put option payoff and Q denotes the risk-neutral measure. When using (5.2), I replace H_T with $\bar{H}_T$; in (5.3) I will multiply te Black-Scholes formula by $(1-\wp)^T$ and adjust K_T to $K_T(1-\wp)^T$.

The readjusted Black-Scholes NNEG formula accounting for the idiosyncratic risk at time $t = 0$ is

$$BSPut_{adj}(T) = K_T e^{-rT}\Phi(-d_2^*) - H_0(1-\wp)^T e^{-gT}\Phi(-d_1^*) \quad (5.4)$$

[7]The Black-Scholes model is used here mainly for simplicity of exposition given that the NNEG valuation can be carried out in closed-form solution. The thesis shows numerous instances using the Black-Scholes model for the NNEG valuation.

where $d_1^* = \frac{1}{\sigma\sqrt{T}}\left[\ln(H_0/K) + (r - g + \ln(1-\wp) + 0.5\sigma^2)T\right]$ and $d_2^* = d_1^* - \sigma\sqrt{T}$.

Consider ${}_Tp_x^\tau$ the multiple decrement survival probability and denote by $\mu_x^{(d)}, \mu_x^{(v)}$ and $\mu_x^{(e)}$ respectively the force of decrement due to: mortality, voluntary prepayment and entry into long term care for a borrower aged x at inception of the ERM contract. The cost of the NNEG with a given maximum term say η years (such as $\eta = 45$ years for a 55 years old borrower) is the value of the portfolio of put options in (4.6) and (5.2).

$$\Pi(0) = \sum_{t=1}^{\eta} BSPut_{adj}(t)_{t-1}p_x^\tau {}_1q_{x,t-1}^d \tag{5.5}$$

where ${}_{t-1}p_x^\tau {}_1q_{x,t-1}^d$ is the probability that (x) survives t and dies within the next $t+1$. Replacing with ${}_1q_{x,t-1}^v$ and ${}_1q_{x,t-1}^e$ will allow us to account for termination by voluntary prepayment or entry into long term care. The three forces are mutually exclusive.

The formula (5.5) can be used to extract the equivalent implied volatility to use in the plain Black-Scholes NNEG formula that matches the idiosyncratic risk adjusted Black-Scholes NNEG valuation. In this way one can answer the question “what is the volatility parameter to use in the Black-Scholes formula such that the NNEG put option value matches another NNEG put option which is based on real-world estimated volatility with impaired adjusted house value at termination?”. In other words, how can I transfer the idiosyncratic house price risk, under any suitable model, into a volatility adjustment under Black-Scholes put option price formula?.

In order to get a feeling for the difference between the idiosyncratic house price risk adjusted NNEG and a no idiosyncratic NNEG with specified volatility $\bar{\sigma}$, under the Black-Scholes set-up, since the formula (5.5) is just a weighted average of individual annual or monthly put option prices, I need to consider only the difference in formula (5.4) and the Black-Scholes put option price derived earlier which is equal to

$$\Lambda(\wp) = K_T e^{-rT}\left[\Phi\big(-d_2^*\big) - \Phi(-\widetilde{d}_2)\right] - H_0 e^{-gT}\left[(1-\wp)^T \Phi\big(-d_1^*\big) - \Phi(-\widetilde{d}_1)\right] \tag{5.6}$$

where

$$d_1^* = d_1 + \frac{T}{\sigma}\ln(1-\wp), \quad d_2^* = d_2 + \frac{\sqrt{T}}{\sigma}\ln(1-\wp)$$

and

$$\widetilde{d}_1 = \frac{1}{\widetilde{\sigma}\sqrt{T}}[\ln(H_0/K_T) + (r - g + 0.5\widetilde{\sigma}^2)T]$$

and $\widetilde{d}_2 = \widetilde{d}_1 - \widetilde{\sigma}\sqrt{T}$. Remarking that

$$\Lambda(0) = -\big[K_T e^{-rT}\Phi(-\widetilde{d}_2) - H_0 e^{-gT}\Phi(-\widetilde{d}_1)\big] \tag{5.7}$$

which is negative being minus the price of a put option, and

$$\lim_{\wp \nearrow 1} \Lambda(\wp) = K_T e^{-rT}(1 - \Phi(-\widetilde{d}_2)) + H_0 e^{-gT}\Phi(-\widetilde{d}_1) \tag{5.8}$$

$$= K_T e^{-rT}\Phi(\widetilde{d}_2) + H_0 e^{-gT}\Phi(-\widetilde{d}_1) \tag{5.9}$$

which is positive, the continuity of Λ as a function of $\wp$ implies that there is a solution to the equation $\Lambda(\wp) = 0$ for any $H_0, K_T, r, g, \widetilde{\sigma},$ and σ.

5.3 Empirical properties of UK house price index

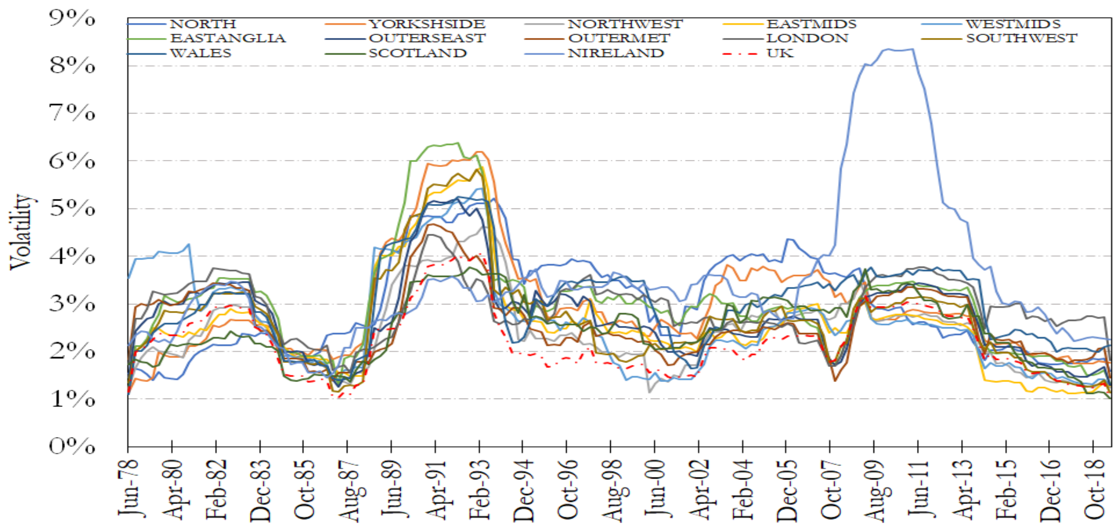
This section analyses the house price index across the entire UK in order to determine cross-similarities and differences between the regional and national property price time series. We let Y_t denote the log-return of UK house price index and calculate the j-horizon compounded return as the sum of quarterly returns where

$$\begin{aligned} Y_{t+1:t+j} &= \ln H_{t+j} - \ln H_t \\ &= \sum_{j=1}^{J} \Big[h_{t+j} - h_{t+j-1} \Big] \end{aligned} \tag{5.10}$$

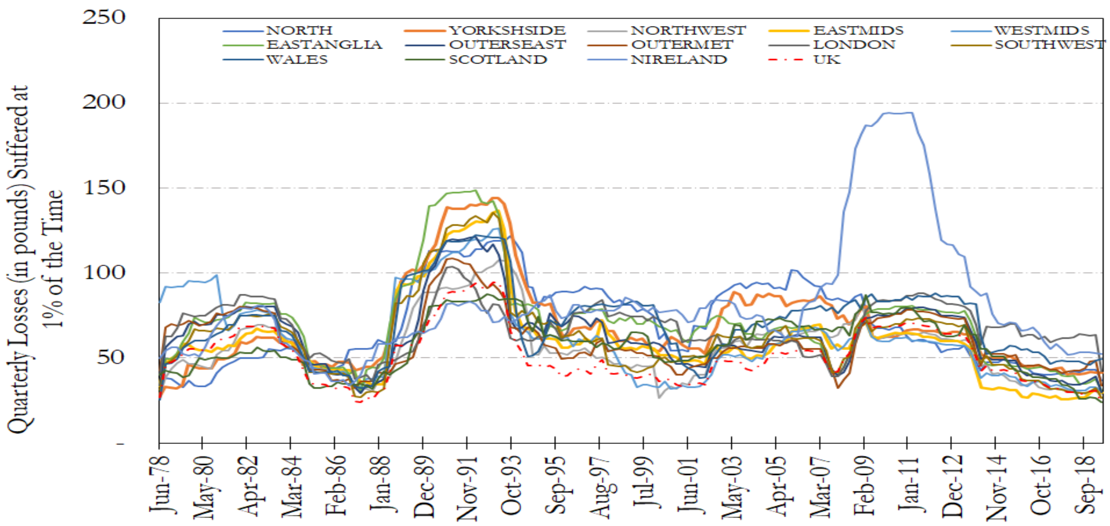
where h_t is the log of house price at time t. The data is from 1973Q4 - 2019Q3 and plots for house price index log-returns at $j = 5$ and $j = 10$ are presented in Figure 5.1 and Figure 5.2 respectively. The corresponding 99%-VaR is calculated for each rolling volatility estimate in order to explore the characteristics of quarterly losses suffered at 1% of the time assuming the value of investment is £1000.

Figure 5.1 shows how the volatility of the 5-year horizon log-returns lies between 1% and 8.5% across the entire sample period. Between February 1993 and August 1998, the rolling volatility for the national house price index (the red dotted line) is below that of the other regions. For post 1994 periods, I observe that the 5-year rolling volatility series for Northern Ireland consistently lies above the calculations in the other regions. There is a downward trend in volatility for periods after 2015 across the whole UK. Clearly, the regional indices appear to increase and decrease together together with the price index at the national level.

The 10-year rolling volatility plots presented in Figure 5.2 shows lower levels of volatility, ranging between 2% and 4% (without Northern Ireland) and between 2% and 6% when Northern Ireland is included.



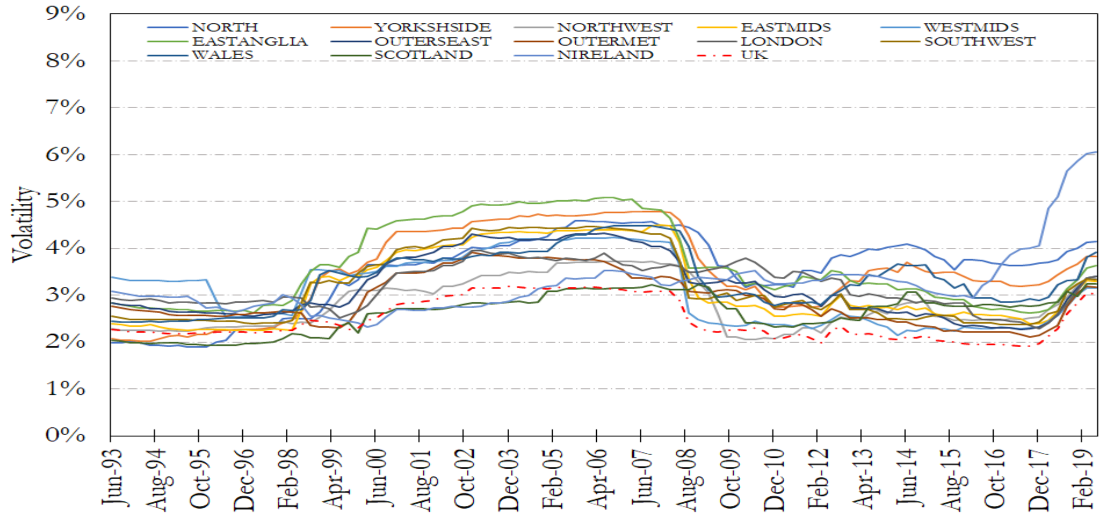
(a) 20-quarter (5 year) rolling volatilities



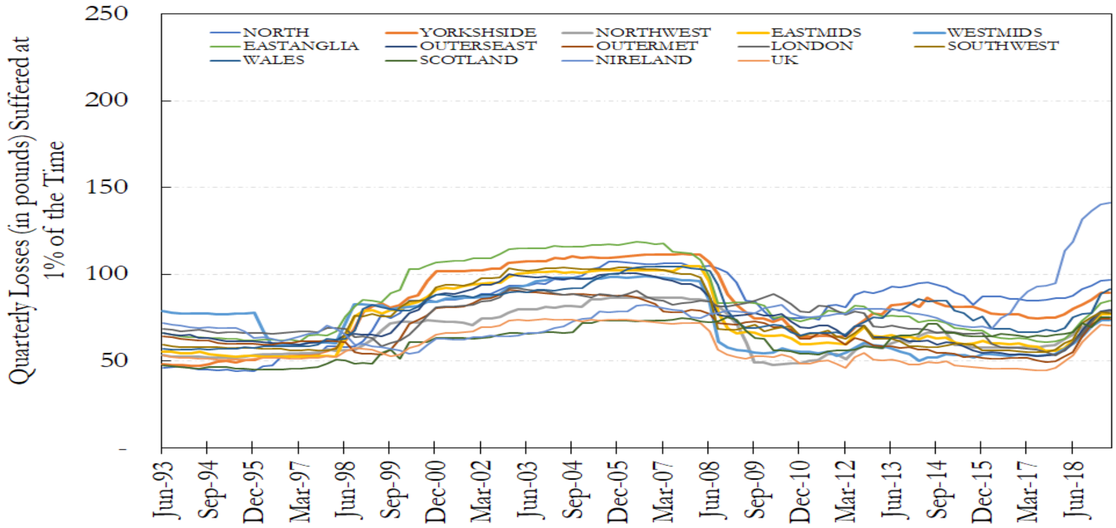
(b) 20-quarter (5 year) rolling historical VaR

Figure 5.1: Evolution of 20-quarter rolling volatility and 99%-VaR.

Notes: Figure (a) presents the 20-quarter rolling volatilities of house price returns and Figure (b) illustrates the evolution 99%-VaR using a 20-quarter (5-year) rolling window of house price returns. The value of investment in the VaR calculations is £1000. Each σ_τ is based on sample size $\tau, \tau-1, \tau-2, \ldots, \tau-20$, where τ is at a quarterly frequency.



(a) 40-quarter (10 year) rolling volatilities



(b) 40-quarter (10 year) rolling historical VaR

Figure 5.2: Evolution of 40-quarter rolling volatility and 99%-VaR.

Notes: Figure (a) presents the 40-quarter rolling volatilities of house price returns and Figure (b) illustrates the evolution 99%-VaR using a 40-quarter (10-year) rolling window of house price returns. The value of investment in the VaR calculations is £1000. Each σ_τ is based on sample size $\tau, \tau-1, \tau-2, \ldots, \tau-40$. τ is at a quarterly frequency.

Here I observe lesser variability in the rolling volatility. The 99%-VaR is less variable over the longer-horizon with a £100 maximum (without Northern Ireland). The VaR levels appears to move in the same direction as the rolling-volatilities. The number of observations in the 10-year rolling window volatility calculation is large enough to always incorporate more of the post-2015 upward trend observed in the house price series. This is in contrast to the 5-year rolling window volatility calculation which slowly adds on larger observations post-2015.

5.3.1 Multi-sample test of equal variance across regions

We implement a multi-sample test for equal variances across the regional house price returns series. This is a test of homogeneity of variance across regional house price returns against the alternative hypothesis that not all regional log-returns series have the same variance. This will allow loan issuers to verify whether the volatility used in pricing the NNEG clause in ERMs should be region-specific. The test produces a Bartlett's[8] statistic of 62.247 with a low p-value, $p = 2.0758e^{-8}$, suggesting a rejection of the null hypothesis that the variances are homogeneous across all regions. This implies at least one region has significantly different volatility in log-returns.

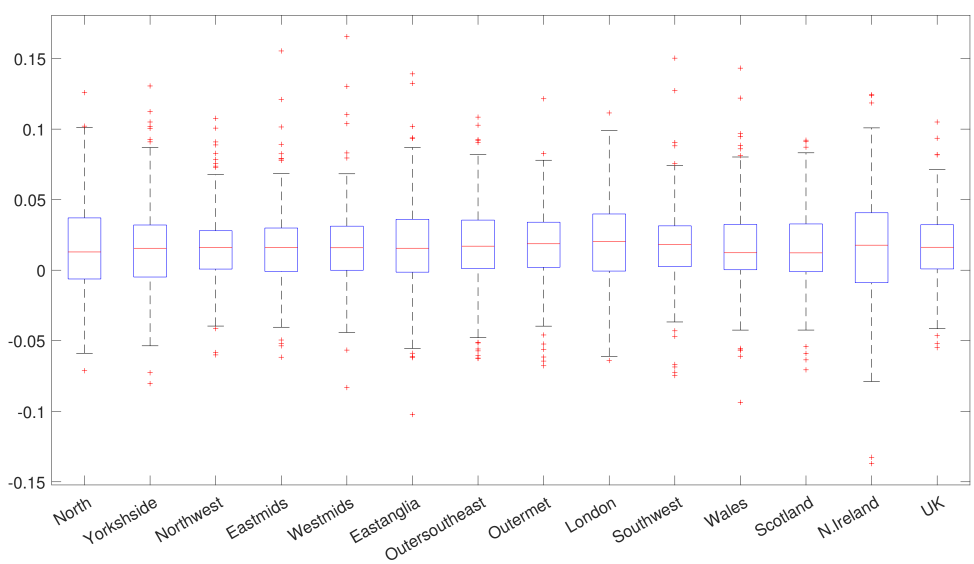


Figure 5.3: Box-plot for Bartlett's test of equal variance across regional house price index in UK.

Notes: The test is conducted on quarterly log-returns of UK house price indices using a sample period spanning 1973Q4 to 2019Q3.

[8]The Bartlett's test is a test of homogeneity of the variances across regions. This allows us to verify whether the regional house price samples have equal variances. An alternative test is the Levene test.

5.3.2 Distributional properties of regional house prices

Figure 5.4 presents a plot of the volatility of realized house price return against the annual average price returns across UK. Northern Ireland recorded the largest (8.12%) volatility of returns while Scotland recorded the least (5.40%). The highest average return was observed in the London house price series (1.96%), while the North reported the lowest average return of 1.53%.

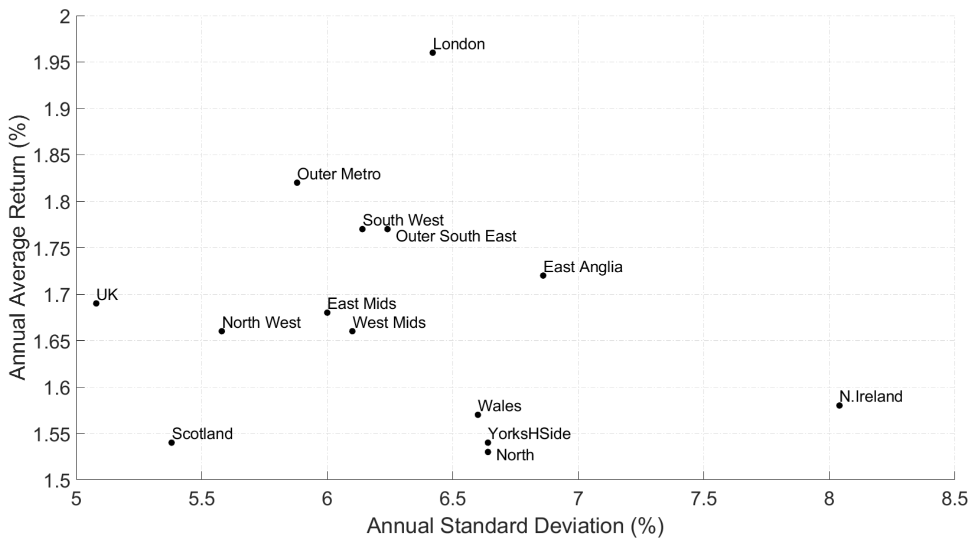


Figure 5.4: Volatility and average house price index for United Kingdom.

Notes: This Figure plots the volatility of the realized house prices against the annual average house price returns across the UK. The illustration summarises the house price index mean-variance relation across all regions in the UK.

We use the Kolmogorov-Smirnov (KS) test to verify whether the observed distributional differences in returns across regions is significant. Most NNEG pricing models in literature tend to use volatility of national house prices index as proxy for all loans, notwithstanding the location of the underlying house collateral (see Hosty et al. 2008, Li et al. 2010, Dowd 2018, Dowd et al. 2019). This may lead to model risk exposures when region-specific variations in house prices are completely ignored and volatility is kept constant at the national level. The KS test enables us to identify whether the underlying region-specific differences in UK house price variations are significant.

A common practice to make calculations more facile is to invoke the independent-and-identically-distributed (i.i.d) assumption, where the regional and national house price returns are assumed to come from the same distribution (see Li et al. 2010, Dowd et al. 2019). Huang et al. (2020) considered the effect of region-specific variations in house price risk and fitted an ARIMA-GARCH model with jump calibrations house price return series of four regions in the UK. Empirical results presented in Table 5.1 suggest that (i.i.d) assumption may not hold under different return horizons. A possible solution to this issue is to determine the best transformation to apply to the regional return-series $Y_{t+1:t+j}$ to produce a corresponding version $\widetilde{Y}_{t+1:t+j}$ which possesses the same distribution as the national house price return series $Y_{t+1:t+j}$.

Table 5.1: Two-Sample Kolmogorov-Smirnov Test Using Regional and National House Price Index Return Series.

Region	$J=1$	$J=4$	$J=5$	$J=10$
North	0.142**	0.167**	0.165**	0.194**
YorksHside	0.109	0.156**	0.146*	0.174**
NorthWest	0.093	0.161**	0.104	0.188**
EastMids	0.066	0.094	0.098	0.132
WestMids	0.066	0.139*	0.061	0.201*
EastAnglia	0.06	0.083	0.140*	0.125
OuterSouthEast	0.077	0.089	0.146*	0.139
OuterMet	0.044	0.111	0.134*	0.146
London	0.06	0.161**	0.177**	0.201**
SouthWest	0.055	0.067	0.11	0.097
Wales	0.109	0.161**	0.152**	0.181**
Scotland	0.098	0.139*	0.177**	0.222**
Nireland	0.142**	0.061	0.165**	0.153

Notes: The test decision is based on the null hypothesis that the regional and national house price index return series are from the same distribution. Significance at 5% level is indicated by $**$ while $*$ indicates significance at 10% level. We conduct the test for $J = 1, 4, 5,$ and 10 quarter-horizon.

These preliminary results further suggest that prudent ERM issuers need to explore diversification strategies for region-specific distributional variations together with their impact on regulatory capital requirements. In an instance where ERMs are priced on a loan-by-loan basis as per regulatory requirement; the loan issuer would be expected to set a house price volatility which ensures fair value pricing. This choice would be the real-world house price volatility estimated from either the national or regional house price index (see an example in Huang et al. 2020). The loan issuer may also settle on a mathematically adjusted real-world volatility (see Hosty et al. 2008, Li et al. 2010). Results in Table 5.1 also confirm the need to consider region-specific variations in the volatility of UK house prices essentially when there is more than one-period return involved.

5.3.3 Accounting for regional house price variations

We have so far explored formal statistical tests which jointly confirm distributional differences between regional and national house price index. We proceed to investigate possible transformations that can be applied on the regional log-returns in order to mimic the distributional properties of the log-returns on the national house price series. Suppose, I let $Y_{t+1:t+j}$ and $\widetilde{Y}_{t+1:t+j}$ respectively denote the regional and national house price return series at time t with $J-$period horizon and implement the transformation

$$E\left[\frac{\widetilde{Y}_{t+1:t+j} - a_1}{a_2}\right] = E[Y_{t+1:t+j}] \tag{5.11}$$

$$Var\left[\frac{\widetilde{Y}_{t+1:t+j} - a_1}{a_2}\right] = Var[Y_{t+1:t+j}] \tag{5.12}$$

where $E[\cdot]$ and $Var[\cdot]$ are the respective unconditional expectation and variance of the series. Then a_1 and a_2 are the unknown constants required to transform the regional return series such that

$$E\left[Y_{t+1:t+j}\right] = a_2^{-1}E\left[\widetilde{Y}_{t+1:t+j}\right] - a_1 \cdot a_2^{-1} \tag{5.13}$$

$$Var\left[Y_{t+1:t+j}\right] = a_2^{-2}Var\left[\widetilde{Y}_{t+1:t+j}\right] \tag{5.14}$$

Table 5.2 reports the values of a_1 and a_2 that correspond to each $J-$horizon return across the UK regional house price indices. The results shows that a_1 is generally negative at lower horizon returns i.e $j \leqslant 4$ while a_2 is strictly positive across all return horizons. $a_1 > 0$ for higher horizons.

The values of a_1 and a_2 are used to create the region-specific transformed log-return series. It is worth noting that the transformed series generated in the transformation exercise are now dependent samples. In this regard, I will be unable to use the traditional Kolmogorov-Smirnov (KS) test that based on independent samples. We adopt a bootstrap version of the univariate KS test proposed in Abadie (2002) in order to correct for the case where the distributions being compared are not independent. Table 5.3 shows how the proposed transformation procedure increases the likelihood of obtaining regional house price log return series which are from the same family of distribution as the national house price return series. The findings suggests that the scaling constants applied to the volatility parameter in the GBM pricing model for NNEG valuation must be carefully selected.

Table 5.2: Estimates of Scaling Constants

Region	$J = 1$		$J = 4$		$J = 5$		$J = 10$	
	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
North	-0.007	1.306	-0.013	1.105	-0.063	1.076	-0.087	1.040
YorksHside	-0.007	1.308	-0.020	1.199	-0.081	1.118	-0.135	1.095
NorthWest	-0.002	1.097	-0.008	1.088	-0.044	1.094	-0.032	1.026
EastMids	-0.003	1.179	-0.010	1.140	-0.039	1.103	-0.058	1.082
WestMids	-0.004	1.202	-0.007	1.078	-0.026	1.041	-0.001	0.976
EastAnglia	-0.006	1.350	-0.016	1.257	-0.062	1.202	-0.130	1.211
OuterSouthEast	-0.003	1.226	-0.011	1.218	-0.039	1.182	-0.080	1.184
OuterMet	-0.001	1.157	-0.006	1.167	-0.012	1.133	-0.031	1.130
London	-0.002	1.262	-0.003	1.207	0.014	1.154	0.063	1.092
SouthWest	-0.003	1.208	-0.007	1.140	-0.025	1.116	-0.052	1.125
Wales	-0.006	1.299	-0.016	1.154	-0.059	1.076	-0.076	1.033
Scotland	-0.003	1.060	0.004	0.850	0.008	0.842	0.027	0.841
Nireland	-0.011	1.582	-0.033	1.406	-0.143	1.242	-0.245	1.193

Notes: This table presents the estimates of the respective constants a_1 and $a - 2$ needed to transform the regional series in order to obtain the same distribution as the national house price return series.

Table 5.3: Two-Sample KS Test with Abadie Correction

Region	$J = 1$	$J = 4$	$J = 5$	$J = 10$
North	0.038	0.061	0.073	0.061
YorksHSide	0.038	0.072	0.073	0.072
NorthWest	0.044	0.056	0.061	0.056
EastMids	0.033	0.067	0.049	0.067
WestMids	0.055	0.044	0.055	0.044
EastAnglia	0.049	0.078	0.061	0.078
OuterSouthEast	0.055	0.061	0.085	0.061
OuterMetro	0.06	0.061	0.098	0.061
London	0.055	0.078	0.122	0.078
SouthWest	0.06	0.056	0.03	0.056
Wales	0.055	0.067	0.067	0.067
Scotland	0.066	0.067	0.085	0.067
NIreland	0.055	0.122*	0.152**	0.122*

Notes: The test is carried out between each transformed regional house price return series, and the UK national house price index return series. The reported values are the test statistic of each test. The Abadie correction enables us to implement the KS test for pairs that are not independent and identically distributed. The test-statistic of the two-sample KS test with Abadie (2002) correction are reported. ** denotes significance at 95%.

5.3.4 Predictability of house prices

In order to account for the dynamic behaviour of the log-return of house price time series, I specify a model that will describe the temporary deviations about the trend of log house price h_t

$$h_t = \Upsilon_t + \sum_{j=0}^{\infty} \upsilon_j \varepsilon_{t-j} \tag{5.15}$$

where Υ_t describes the trend and ε_t is a random disturbance. Fluctuations in h_t will be temporary if $\sum_{j=0}^{\infty} \upsilon_j \varepsilon_{t-j}$ is stationary stochastic process. h_t is *trend stationary* as $\upsilon_j \to 0$ for large j.

This ensures that a decline in house price index below trend today has no effect on forecasts of the index value $E_t(h_{t+j})$ in distant future. Thus growth rates for any given house price index must rise above their historical average for several periods until the trend line is re-established. The random walk equation to capture permanent house price index fluctuations is specified as

$$h_t = \mu + h_{t-1} + \varepsilon_t \tag{5.16}$$

The size of the random walk component in the house price index is measured by the variance of its log differences $Var(h_t - h_{t-q}) = q\sigma^2$. Gu (2002) showed that a constant variance suggests that the natural log of house price index is a pure random walk, q is the number of differences taken. As $Var(h_t - h_{t-q}) \to 2\sigma_h^2$ the natural log of house price index is stationary about a trend as specified in Equation 5.15. If h_t is a random walk, then $(1/q) \times Var(h_t - h_{t-q})$, which is the variance ratio[9] expressed as a function of q-difference would be a constant at σ^2 (see Gu 2002).

Empirical results in Gu (2002) showed that fluctuations in house price index can be partially temporary with a small component of random walk. In the long run, this structure allows the impact of small shocks to be reversed. Hence, a shock that impacts house prices today will be slowly reversed in the long run at a rate which a simple parametric model e.g. GBM may not be able to capture. Appendix D.2 presents the standard normal test statistic for the variance ratio test specified for the hypothesis of homoscedasticity and heteroscedasticity consistent estimators.

[9]The variance ratio declines to zero if h_t is trend-stationary and negatively correlated. It increases with q if h_t is not trend stationary and positively correlated. The variance ratio will equal the variance of the shock to the random walk component if the house price index prices is partly permanent and partly temporary. The modelling will then be a combination of a stationary series and a random walk.

Table 5.4: Estimated variance ratios and heteroscedasticity consistent Z-values

Region	$q(2)VR$	$Z^*(q)$	$q(4)VR$	$Z^*(q)$	$q(8)VR$	$Z^*(q)$	$q(16)VR$	$Z^*(q)$
North	0.48	-4.74	0.28	-3.69	0.14	-3.00	0.09	-2.30
YorksHside	0.61	-4.06	0.36	-3.90	0.22	-3.14	0.12	-2.42
Northwest	0.68	-3.33	0.38	-3.74	0.26	-2.98	0.17	-2.30
EastMids	0.67	-3.23	0.42	-3.21	0.26	-2.72	0.16	-2.19
WestMids	0.63	-2.35	0.29	-2.83	0.21	-2.43	0.12	-2.14
EastAnglia	0.62	-4.60	0.33	-4.36	0.22	-3.12	0.12	-2.34
OuterSoutheast	0.79	-2.35	0.43	-3.59	0.29	-2.80	0.18	-2.22
OuterMetro	0.82	-2.07	0.51	-3.19	0.32	-2.80	0.19	-2.28
London	0.64	-4.04	0.37	-4.02	0.25	-3.08	0.15	-2.40
SouthWest	0.81	-2.08	0.36	-3.72	0.24	-2.79	0.15	-2.14
Wales	0.48	-4.22	0.32	-3.14	0.20	-2.59	0.11	-2.20
Scotland	0.59	-3.98	0.24	-4.28	0.13	-3.38	0.07	-2.58
Nireland	0.51	-5.33	0.30	-4.28	0.19	-3.22	0.11	-2.41
uk	0.91	-1.15	0.46	-3.65	0.31	-2.92	0.18	-2.36

Notes:This table presents the variance ratios estimated across the regional house price indexes in the UK over q-differences of log-returns. The data is quarterly and q is set at (2, 4, 8, and 16). A unit variance ratio is indicative of a series that follows random walk. A ratio greater than unity indicates that variances of the log-return series grow more than proportionally with time and this indicates a positive autocorrelation. A variance ratio that is less than unity suggests that the variance of the log-return series grow less than proportionally with time indicating a negative autocorrelation. $Z^*(q) > 1.645$ indicates statistical significance at 10% level, $Z^*(q) > 1.96$ indicates significance at 5% level, and $Z^*(q) > 2.575$ denotes significance at 1% level.

Table 5.4 report the variance ratio test results for UK house price index series. The variance ratios are smaller than unity indicating that variances of house price returns grow less than proportionally with time. Variance ratio values less than unity are suggestive of a negative autocorrelation for the index return series. Furthermore, the test show statistically significant autocorrelations at 5% level across all regions including Wales, Scotland and Northern Ireland. More specifically, the results suggest that house price returns are not random across UK, hence price increase in one quarter are likely to be followed by a price decrease in another and vice versa[10]. Negative autocorrelation is consistently observed when q is increased from 2 to 16 providing evidence that house price changes are not random.

5.3.5 Volatility, return and autocorrelation

The thesis further examined the empirical relationship between autocorrelation, volatility, and house price log-returns. Similar to Gu (2002), I specify regression equation

$$\rho_{iq} = \alpha + \beta_1 \sigma_i + \beta_2 \sigma_i^2 + \beta_3 Y_i + \varepsilon_{i,t} \tag{5.17}$$

[10]The level of seasonality could also play a role in driving this observation.

where ρ_q is level of autocorrelation in the i^{th} house price index with q-difference. $\rho_q = |\widehat{VR} - 1|$, allowing us to measure the extent of deviation from randomness with respect to the same scale. σ_i and σ_i^2 respectively denote the standard deviation and variance of log of house price. Y_i is average quarterly log returns. We use σ_i and σ_i^2 as explanatory variables to account for any nonlinear relationship that may exist between volatility and autocorrelation. The regression results are presented in Table 5.5.

Table 5.5: Estimated relations between volatility, return and autocorrelation

$\|VR-1\|$	α	σ	σ^2	Y	**Adj.**R^2
$q(2)$	12.84 (4.41)**	-21.04 (8.68)**	13.06 (5.02)**	-253.1 (136.44)*	0.48
$q(4)$	6.21 (2.79)*	-10.19 (5.50)*	5.69 (3.18)**	-61.83 (86.38)	0.36
$q(8)$	5.43 (1.71)***	-8.52 (3.37)**	4.72 (1.95)**	-49.71 (52.99)	0.58
$q(16)$	4.00 (1.14)***	-6.00 (2.24)**	3.21 (1.29)**	-20.67 (35.19)	0.58

Notes: The table presents the regression coefficients depicting the relationship between volatility, return and autocorrelation at $q = 2, 4, 8$ and 16. The t-statistics are in parenthesis. *, ** and *** respectively denote the significance at 10%, 5%, and 1% levels. The sample period is from 1973Q4 -2019Q3.

The relationship between volatility and autocorrelation is negative and statistically significant. This observation is consistent with earlier results observed in Gu (2002) who used United States house price index. The relationship between Y and ρ_q is negative statistically insignificant when q is greater than 2. In practice, the prudential regulatory authority (PRA) uses an inflated volatility (about, 13%) when pricing the NNEG in ERMs. This suggests the PRA assigns a higher probability to the incidence of the crossover risk. Some market practitioners believe that for shorter durations, the presence of serial correlation might mean that the Black-Scholes volatility needed to be higher than historic volatility in order to replicate an ARMA result that did pick up that serial correlation. Given the results in Table 5.5, it is not clear to how increasing volatility for a process that does not have serial correlation, such as the GBM, will induce or recover values as if serial correlation existed. This is an ad-hoc procedure that does not seem to have any grounding into statistical modelling. Once again, the volatility relations seem to suggest that values such as 11% or 13% may be used as stress values and not as current data generated volatility markers.

5.4 Property Impairment and Implied Volatilities

Figure 5.5 presents the corresponding impairment factors for various ages of a borrower that should be applied in order to match; the NNEG valuation *without* idiosyncratic risk and 13% volatility under Black-

Scholes formula with corresponding NNEG valuation *with* idiosyncratic house price risk under Black-Scholes formula and ARMA-EGARCH method. For example, using a volatility of 13% in the Black-Scholes put option NNEG valuation is equivalent to assuming a 2.6% annual impairment factor in the NNEG valuation under the adjusted Black-Scholes formula. This means that the collateral house will lose in value about 13% after five years, 23% after ten years, 33% after 15 years and 41% after 20 years because of dilapidation and dereliction idiosyncratic factors. We can notice that for the UK, the idiosyncratic risk adjustment rate to be applied for the ARMA-EGARCH is always below the corresponding rate for the Black-Scholes, with the largest difference of almost 1% per annum, for borrowers with an age between 85 and 90. The intuition for this results is that an ARMA-EGRACH model produces house price returns within a smaller band the GBM for which variance of returns increases linearly with time and therefore small impairment factors are implied under this model to match a given target NNEG.

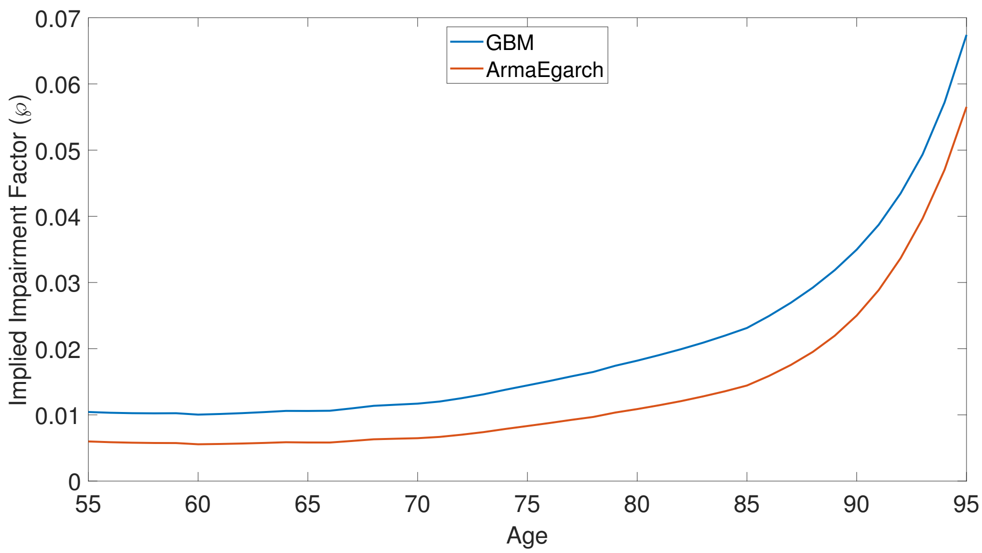
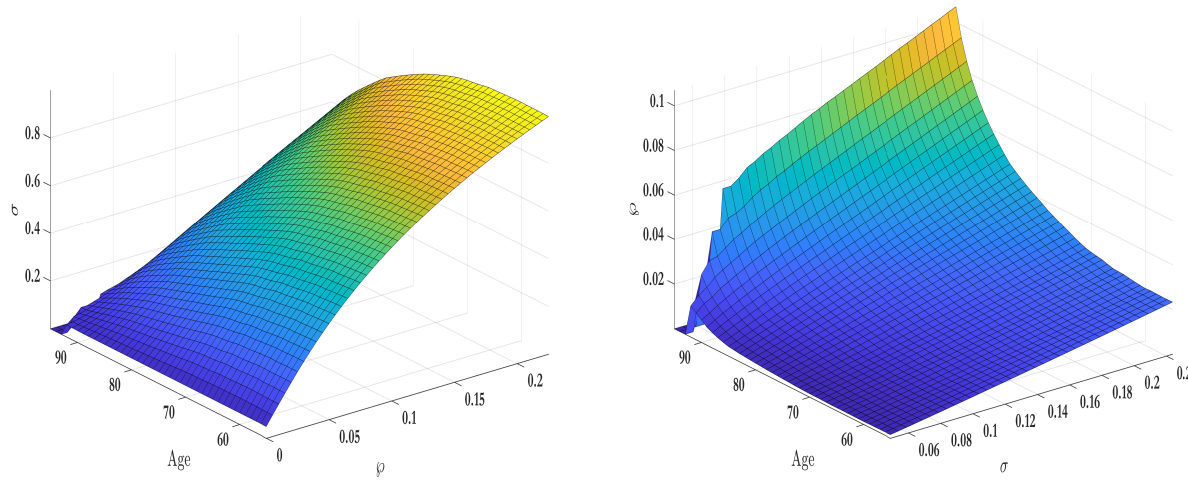


Figure 5.5: Comparison of the implied impairment factor across borrower's age for NNEG valuation under the Black-Scholes and ARMA-EGARCH models

Notes: The implied volatility calculations are for a female borrower aged 55 years at inception and standard market values for other inputs $H_0 = 310,000, r = 0.175\%, R = 5.25\%, g = 1\%$ at the end of 2019.

The graphs depicted in Figure 5.6 show the implied volatility obtained by varying the impairment factor on panel (a) and the implied impairment factor given by varying the volatility parameter on panel (b), in (5.5) where the left side is taken as the NNEG value obtained as recommended by the PRA, using the Black-Scholes formula with an inflated volatility of $\sigma = 13\%$ and a rental yield equal to $g = 1\%$. Both implied volatility surfaces are quite steep, reflecting the characteristics of the Black-Scholes formula.

Since the ARMA-EGARCH model is more robust for modelling house prices under the physical measure,



(a) implied volatility

(b) implied impairment factor

Figure 5.6: Sensitivities related to idiosyncratic risk relative to the baseline scenario using calculations for a Female borrower aged 55 years at inception and standard market values for other inputs $H_0 = 310, r = 0.57\%, R = 4.91\%, g = 1\%$ at the end of 2019.

it is useful to observe the sensitivity of the impairment factor when applied to the NNEG valuation under the ARAM-EGARCH to match the NNEG valuation under Black-Scholes with various volatility parameters σ. This exercise leads to the results presented in Figure 5.7. If an insurer would like to match the PRA Black-Scholes NNEG with 13% volatility while using internally the ARMA(4,3)-EGARCH(1,1) model then the values along the curve extracted from Figure 5.7 for all respective borrower's age are quite small up to the age of 80, while increasing rapidly afterwards. In other words, under the ARMA-EGARCH model, the idiosyncratic risk manifests itself only for quite old borrowers who are indeed unlikely to carry out repairs of the house. By contrast, the same mechanism applied under the Black-Scholes model marks idiosyncratic risk for "younger" ages, over-penalising in our opinion this category of borrowers from this point of view.

5.5 ERM Portfolio Cash Flows Analysis

In this section, I will consider the portfolio effects of house price risk when looking at the cash flows analysis from a lender perspective. My main portfolio takes the view that all borrowers live exactly to their expected lifetime. Note that the lifetime expectancy of a 60 year old will differ from the lifetime expectancy of a 70 year old and so on, and it will be also different between males and females.[11]

[11] Another portfolio randomises arrival of termination event between the current age of the borrower and 100, so for example for a 65 year old female I draw a random number between 1 and 35. As an extreme

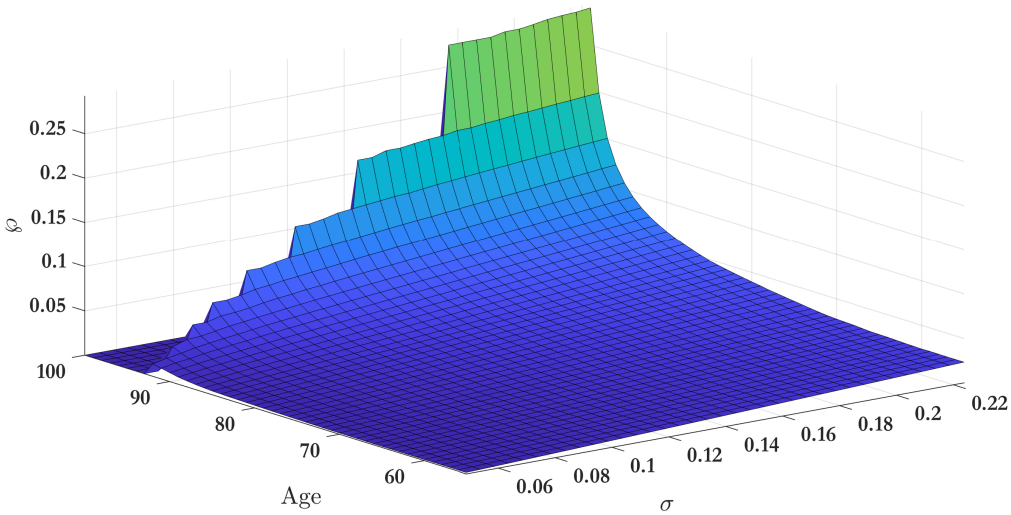


Figure 5.7: The implied impairment factor across borrower's age when matching the NNEG valuation under the Black-Scholes with various σ and ARMA-EGARCH model.

Notes: The implied volatility calculations are for a female borrower aged 55 years at inception and standard market values for other inputs $H_0 = 310,000, r = 0.175\%, R = 5.25\%, g = 1\%$ at the end of 2019.

The portfolio has 10,000 loan contracts in total, 4927 female and 5073 males. The initial property value is distributed as follows: 2500 borrowers on Flexible LTV with 100,000 loan; 2500 borrowers on Flexible LTV with 200,000 loan, 2500 borrowers on Flexible LTV with 310,000 loan and 2500 borrowers on Flexible Max Plus LTV with 950,000 loan.

The following notations are used here.

If $\omega_t^{(i)} = 1$ if the loan (i) is still active, and is equal to 0 if it is not active then $K_t^{(i)} = L_0^{(i)} e^{Rt}$ is the accumulated balance for loan i at time t and $\widetilde{K_t^{(i)}} = K_t^{(i)} \times \omega_t^{(i)}$ is the accumulated outstanding balance for loan i at time t, if the borrower survives to t; $\widetilde{K_t} = \sum_i \widetilde{K_t^{(i)}}$ is the portfolio outstanding balance at time t.

The evolution of accumulated loan balance $L_0^{(i)} e^{Rt} \times \omega_t^{(i)}$ in Figure 5.9a is the same for both GBM and ARMA-EGARCH models. There seems to be an inflection point just after 20 years for the portfolio accumulated balance. The tipping point for the generated cash is around 20 years. Thereafter, projected cash flows in the second part of the life of the portfolio decreases at a faster rate. Under both models the portfolio runs down at 40 years.

In order to analyse the portfolio NNEG cash flows, I let $H_t = \sum_i H_t^{(i)}$ denote the total expected value of house collateral at time t. $C_t^{(i)} = min(H_t^{(i)}, K_t^{(i)}) \times \tau_t^{(i)}$ is the cash generated in year t from loan i and

portfolio I also consider cash-flows for a portfolio where all borrowers go to 100 years. The results for this latter portfolio are not reported here but they are available from authors upon request.

Table 5.6: Descriptive statistics for a hypothetical ERM portfolio.

Age Band	Prop. Value (mil GBP)	Initial Loan (mil GBP)	Weight	Average Maturity
Panel A: Profile of male borrowers				
55-60	16.74	2.30	0.85%	27.69
60-64	204.29	38.80	11.28%	24.59
65-69	440.82	109.20	26.30%	21.01
70-74	447.95	135.87	28.73%	15.78
75-79	264.12	89.63	18.09%	8.02
80-84	141.98	54.52	10.36%	3.00
84+	56.73	23.54	4.39%	3.00
Panel B: Profile of female borrowers				
55-60	14.26	1.92	0.75%	28.20
60-64	198.71	37.79	11.31%	25.19
65-69	427.18	105.76	26.26%	22.41
70-74	451.05	136.99	29.83%	18.35
75-79	262.88	89.51	18.58%	12.55
80-84	121.52	46.80	9.15%	4.57
84+	51.77	21.48	4.13%	3.00

$C_t = \sum_i C_t^{(i)}$ represents the total portfolio new cash generated by loans terminating in year t; AC_t is the total portfolio accrued cash in money account by time t and it is calculated recursively $AC_t = AC_{t-1} \times e^r + C_t$.

$P_t^{(i)} = E(C_t^{(i)}) = E(min(H_t^{(i)}, K_t^{(i)}) \times \tau_t^{(i)}) = E(min(H_t^{(i)}, K_t^{(i)})) \times E(\tau_t^{(i)})$; is the payment *expected* from loan (i) in year t. This would be clearly zero in all years except the year when borrower is expected to terminate. In that year, that is when $\tau_t^{(i)} = 1$, $P_t^{(i)} = E(min(H_t^{(i)}, K_t^{(i)}))$.

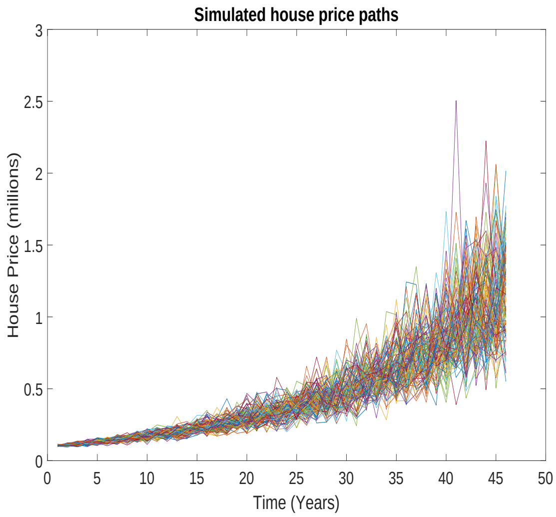
$P_t = \sum_i P_t^{(i)}$ is the total payment expected on the portfolio in year t, while $P_t^{4Y} = P_{t+1} + P_{t+2} + P_{t+3} + P_{t+4}$ is the total portfolio *expected* payments over the *next* four years.

$EAR_t^{(i)} = K_t^{(i)} - H_t^{(i)}$ NNEG exposure at risk for loan (i) at time t and then $EAR_t = \sum_i EAR_t^{(i)} =$ total NNEG exposure at risk due to house collateral. The calculations are based on the assumption that the loans are terminated at a random time before the expected future lifetime maturities, for male and female borrowers.

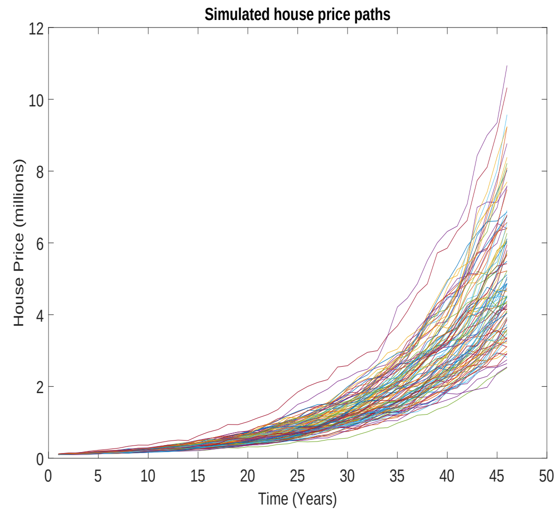
5.5.1 Portfolio calculations with GBM and ARMA-EGARCH

The calculations are based on the assumption that the loans are terminated at a random time before the expected future lifetime maturities, for male and female borrowers. A separate set of calculations with loan terminated at exactly the expected future lifetimes and another one at random times up to 100 years, are available upon request from authors.

The graphs in Figure 5.8 illustrate house price pathways under the GBM and the ARMA-EGARCH model, under the physical measure. The GBM price paths display more variability, indicating the possibility of house prices to be overall lower at long term horizons.



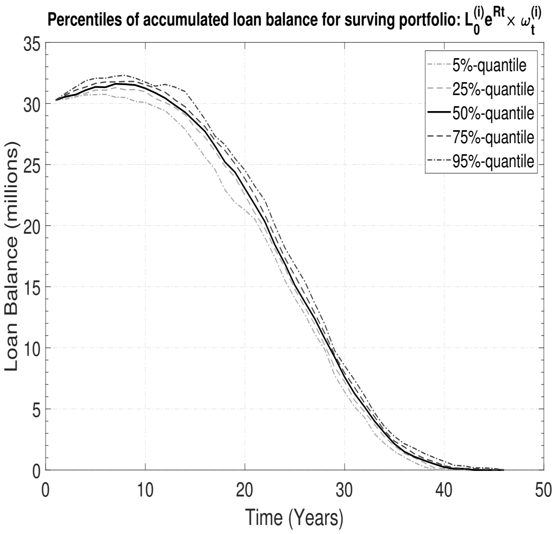
(a) GBM scenarios



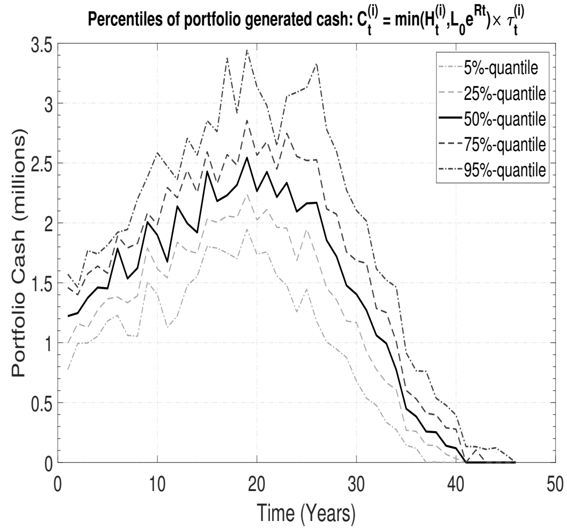
(b) ARMA-EGARCH scenarios

Figure 5.8: House price pathways up to 46 years under GBM and ARMA-EGARCH models.

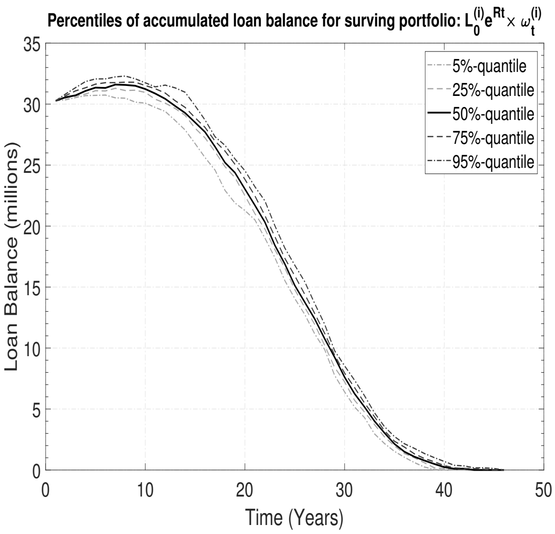
In Figure 5.9, I illustrate the evolution of the portfolio outstanding balance under both GBM and ARMA-EGARCH model, on the left side graphs, and the evolution of generated cash from loan terminations, again under each GBM and ARMA-EGARCH models. The evolution is quite similar under both models for each of the five important quantiles. There seems to be an inflection point between 10-15 years for the portfolio accumulated balance, which may be explained by the peak around 20 years horizon for the portfolio generated cash.



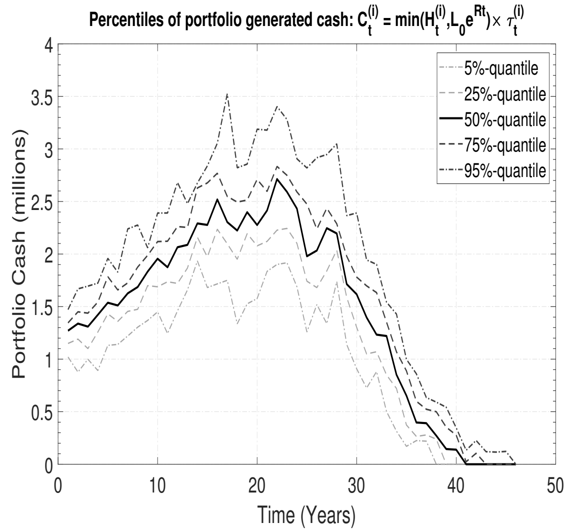
(a) GBM



(b) GBM



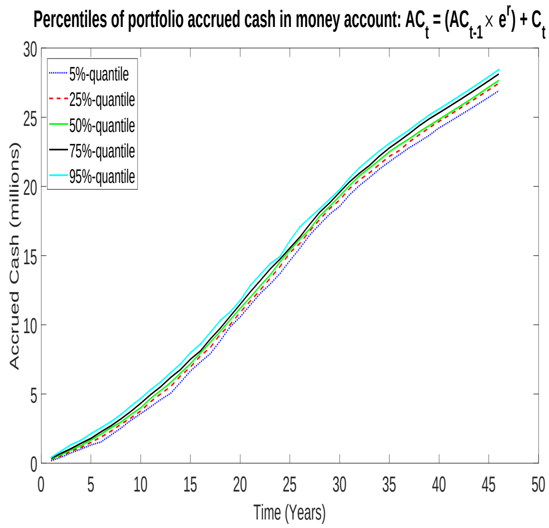
(c) ARMA-EGARCH



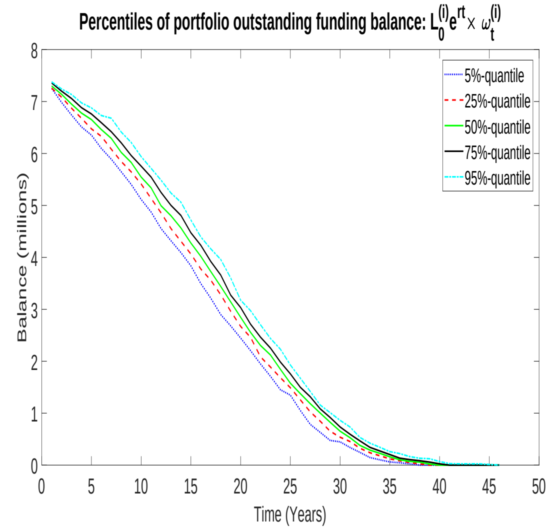
(d) ARMA-EGARCH

Figure 5.9: Evolution of outstanding portfolio loan balance.

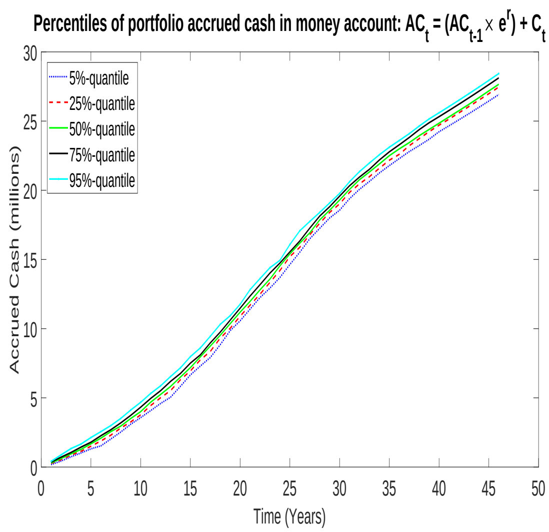
The portfolio accrued cash account grows as depicted in Figure 5.10 where I also illustrate the portfolio outstanding funding balance. The evolution is almost symmetric, as the cash generated grows rapidly the funding liability balance decreases substantially beyond 25 years.



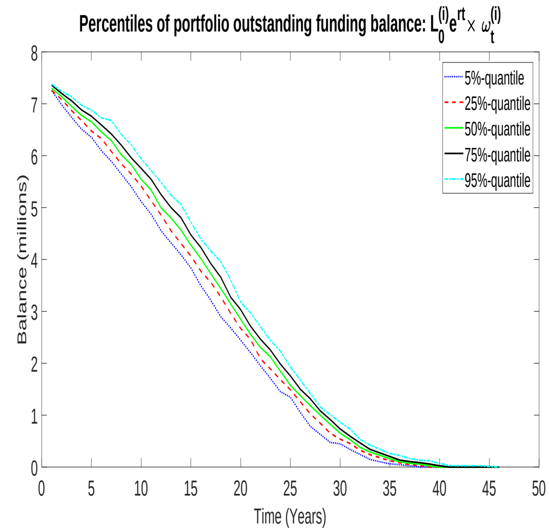
(a) GBM



(b) GBM



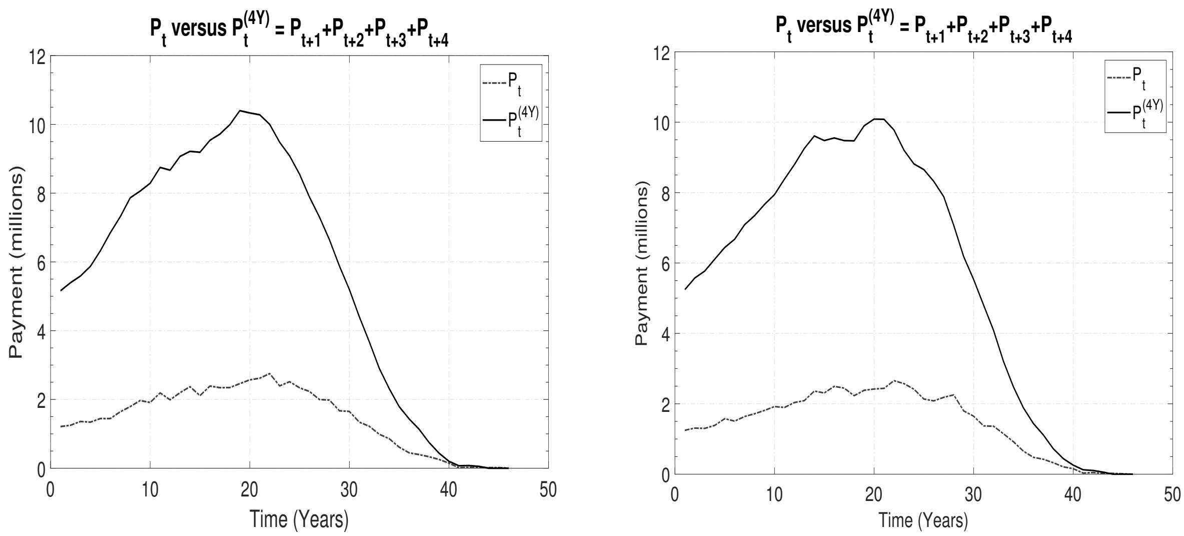
(c) ARMA-EGARCH



(d) ARMA-EGARCH

Figure 5.10: Evolution of cash generated by ERM portfolio and outstanding funding balance.

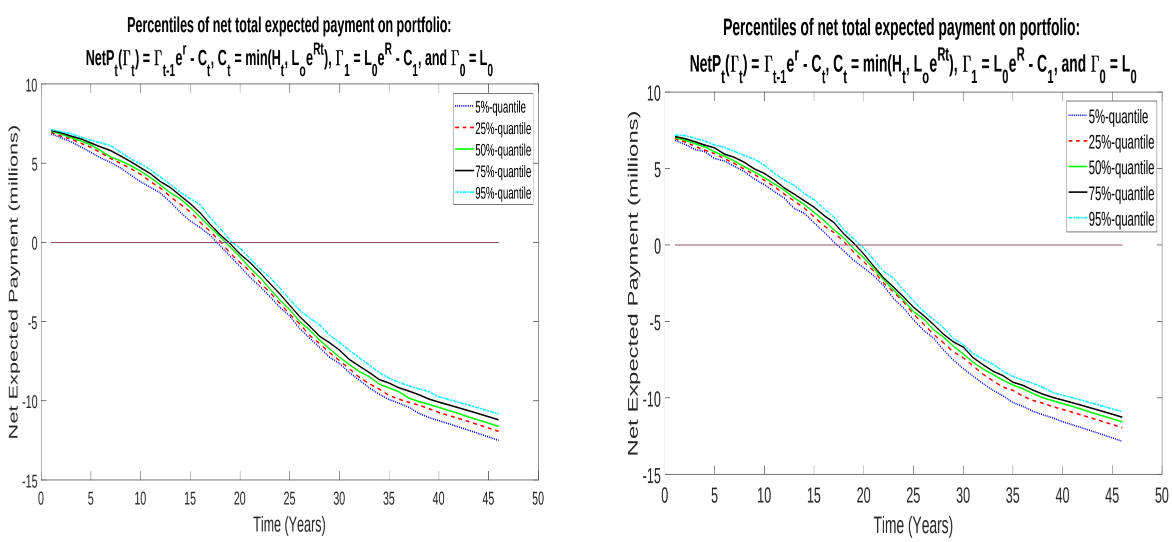
Figure 5.11 shows various comparative ledgers. On the left side I display the expected payments that the portfolio will generate next year versus the payments generated over the next four years. The peak of the money inflow will occur, under the GBM model between 20-25 years and under the ARMA-EGARCH model, between 15-25 years. The portfolio outstanding balance dominates the funding balance, over the entire portfolio life.



(a) GBM (b) ARMA-EGARCH

Figure 5.11: Comparison of various balance ledgers measures.

The graphs displayed in Figure 5.12 show the portfolio expected payment, net of funding costs, under both models. The evolution and percentiles indicate a similar evolution, with the break-even zero cross-over point just before 20 years. This suggests that, under our simulated scenarios, the risk exposure declines over time, and only after 20 years the portfolio of ERMs becomes truly profitable.



(a) GBM (b) ARMA-EGARCH

Figure 5.12: Evolution of net balance. A negative balance indicates profits.

The probability distribution of the EAR measures are calculated at various time horizons (5,10,15,20,25,30) and described by the graphs in Figure 5.13 for the GBM model and by the graphs in Figure 5.14 for the

ARMA-EGARCH model. We observe that essentially all histograms are entirely on the non positive domain. This is explained by the fact that the simulated house prices exceed the outstanding balance for the respective loans in the portfolio. With a larger number of simulated scenarios is possible that some loans will have accumulated balances exceeding the price of the collateral house.

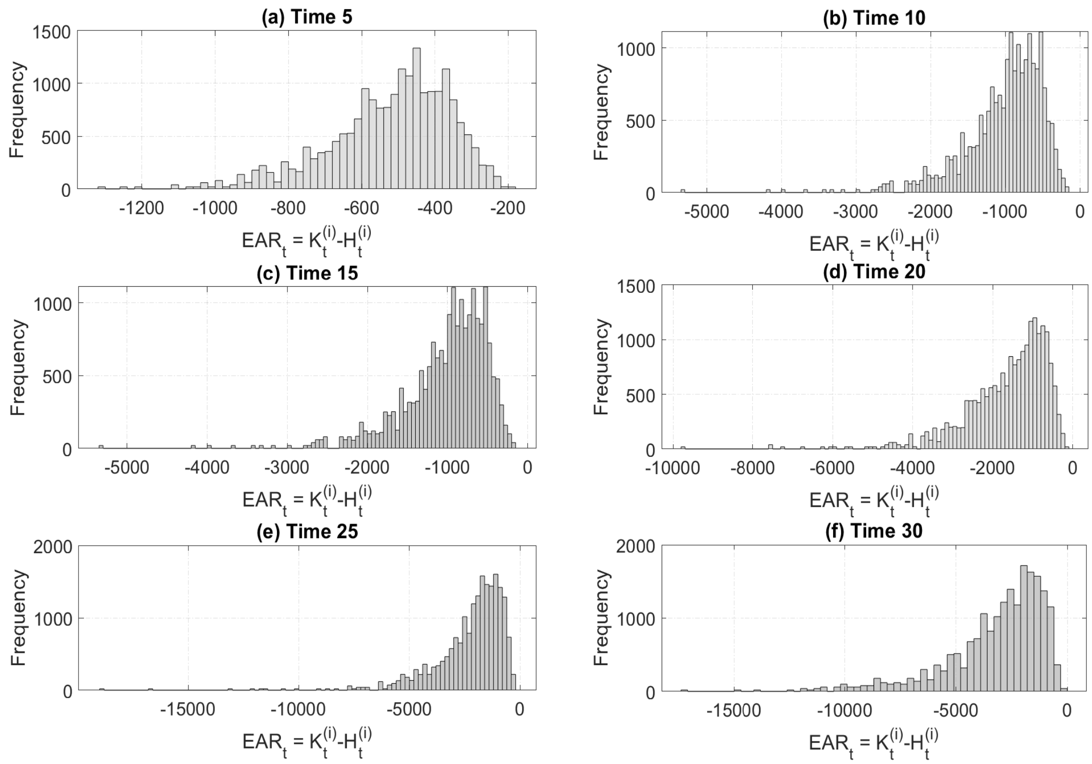


Figure 5.13: EAR portfolio measures at various points in time under GBM model.

Figure 5.13 and Figure 5.14 shows the percentiles of the expected exposure due to collateral risk, under both GBM and the ARMA-EGARCH model. While the profile looks very similar for the two models, as order of magnitude is larger for the latter model. The fact that the exposure is actually negative is a good characteristic, indicating in reality that there is very little risk due to the exposure to house prices.

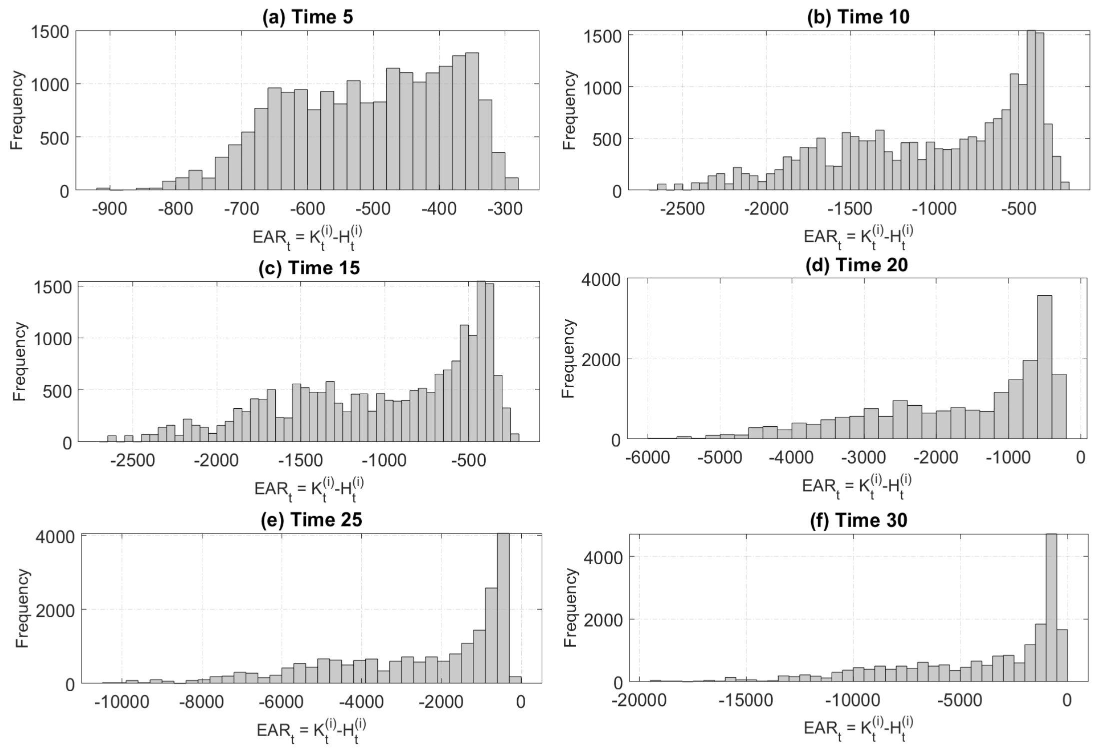
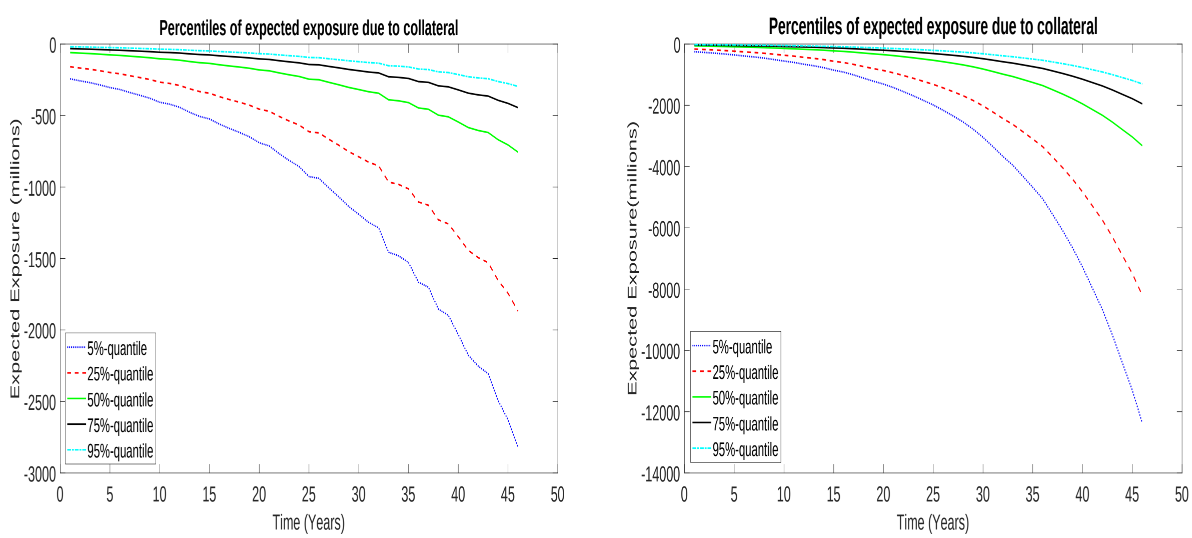


Figure 5.14: EAR portfolio measures at various points in time under ARMA-EGARCH model.



(a) GBM (b) ARMA-EGARCH

Figure 5.15: Profile of expected exposure based on evolution of collateral house prices.

5.6 Chapter Summary

The valuation of the NNEG is a hotly debated topic among practitioners and academics. So far, the ERM market space determine the NNEG cost as a put option on the collateral house with a random time-to-maturity that is contingent on the borrower(s) mortality, early prepayment or transition into long term care. The strike price of the put option is the accumulated value of the unamortised loan at the maturity date. The PRA still considers that the Black-Scholes formula is appropriate for the NNEG put option valuation. Other option pricing techniques can be used so far as they satisfy a set of principles in the PRA policy statement. So far, the best estimate of the NNEG cost is "the mean of a stochastic distribution of possible future guarantee costs, where random variables used in the stochastic projection have been calibrated based on a best estimate of their true distributions". The valuation procedure described for the Black-Scholes formula comes with fixed values of the volatility of the underlying house price to 13% and a minimum deferment rate of 1%.

This research study undertakes a detailed investigation on the stylised properties of regional house price indices in the UK. The findings suggest that house price increments exhibit positive autocorrelation in the near horizon and negative autocorrelation in the long horizon. The investigation on the relationship between autocorrelation and volatility shows that autocorrelation is significant and negatively related to volatility across all regions in the UK, consistent with Gu (2002) and Tunaru (2017). The sampling distribution of the house price volatility in the GBM model ranges between 4.0% - 5.6% which is at least two times lower than the 13% volatility that is fixed by the PRA.

Another important finding refers to the relationship between the property impairment factor ($\wp$) and the volatility (σ) of the underlying house prices over the lifetime of the ERM loan contract. $\wp$ and σ are positively related, and the scenario used in this research study shows that an implied volatility of 13% is equivalent to assuming a 2.6% annual impairment factor in the NNEG valuation. Thus, the market value of the collateral house $\bar{H}_2 0 = (1-\wp)^T H_2 0$ will lose about 41% of its value (without) impairment by the 20th year of the ERM contract. The maximum likelihood estimate (MLE) of the volatility in the GBM model used in this study is 4.88%, which is equivalent to an impairment rate of 0.8% in Figure 5.6. In this case, the market value of the collateral house at the 20th year loses 15% of its market value.

Finally, the results show how the house price pathways under the GBM model display more variability compared to the ARMA-EGARCH model. This suggests that the is a higher possibility for house prices to be overall lower at long term horizons. The term structure of the portfolio outstanding funding balance is symmetric, as cash the generated grows rapidly the funding liability balance decreases substantially beyond

25 years. Regarding the expected payments that the portfolio generates between the next year and the next four years, I find that the peak of the money inflow will occur, under the GBM model between 20-25 years and 15-25 years under the ARMA-EGARCH. The portfolio outstanding balance consistently exceeds the funding balance over the entire portfolio life. More importantly I find that the cross-over point occurs just before 20 years, suggesting that the risk exposure declines over time and the certainty of portfolio profits greatly improves after 20 years.

The NNEG value can be considered at the portfolio level or at the loan level. In this research I focus our analysis on individual loan NNEG calculation. Future research could continue with investigations on NNEG valuation at portfolio level and the degree of capital savings that can be made due to diversification of portfolios and possible NNEG calculation at portfolio level.

Appendices

APPENDIX D

Additional Material for Chapter 5

D.1 De-smoothing approach

One approach to deal with serial-correlation in house prices that is apparently being used by life actuaries working on annuities is to use a desmoothing procedure and get the modelling that way. While I do not fully agree with the standard desmoothing procedure that is normally applied to commercial real estate valuations because the indices there are appraisal based, a potentially good line of modelling in the context of real estate derivatives is described in Van Bragt et al. (2015) and Tunaru (2017). They consider the observed real estate price index as the convex combination of an "efficient market" price or true market price $y(t)$ and the previously observed market price $a(t-1)$

$$a(t) = Ky(t) + (1-K)a(t-1)$$

with K a confidence parameter linking the two. This model is equivalent to an exponentially weighted moving average (EWMA) model that is well-known in financial risk management. To account properly for time value of money the model is adjusted using an expected annual return π

$$a(t) = Ky(t) + (1-K)(1+\pi)a(t-1)$$

Van Bragt et al. (2015) assume that the underlying market returns follow a random walk process with drift. For a total return real estate index, they prove that the price of a forward contract would then be equal to

$$F_t(T) = \frac{1}{df(t,T)}[y(t)(1-\alpha_{K,T}(t) + a(t)\alpha_{K,T}(t)]$$

where $\alpha_{K,T}(t) = (1-K)^{T-t}$.

Moreover, Van Bragt et al. (2015) derive an approximate formula for the forward *and* a European put option contingent on a real estate index, using techniques developed for pricing Asian options and based on calculate the first moment M_1 and the second moment M_2 of $a(t)$, under the risk-neutral measure. Thus, the forward price formula is

$$F_t(T) = M_1;$$

while the European put option formula for strike X is

$$p(t) = df(t,T)[\Phi(-d_2) - F_t(T)\Phi(-d_1)]$$

with $\sigma = \sqrt{\frac{1}{T-t}\ln\left(\frac{M_2}{M_1}\right)}$, $d_1 = \frac{\ln(F_t(T)/X)+0.5\sigma^2(T-t)}{\sigma\sqrt{T-t}}$, $d_2 = d_1 - \sigma\sqrt{T-t}$.

The model developed by Van Bragt et al. (2015) can also be adapted to include seasonality effects and there are analytical formulae for pricing swaps on real-estate index as well.

D.2 The variance-ratio test for house prices

The test statistic of the variance-ratio test at q-differences is denoted $VR(q)$ and is given by

$$VR(q) = \frac{\sigma^2(q)}{\sigma^2(1)} \tag{D.1}$$

where $\sigma^2(q)$ is $1/q$ times the variance of the q-differences, $\sigma^2(1)$ is the variance of the first differences.

$$\sigma^2(q) = \frac{1}{m}\sum_{t=q}^{nq}(h_t - h_{t-q} - q\hat{\mu}) \tag{D.2}$$

where, $m = q(nq-q+1)\left(1-\frac{q}{nq}\right)$. and also

$$\sigma^2(1) = \frac{1}{(nq-1)}\sum_{t-1}^{nq}\left(h_t - h_{t-1} - \hat{\mu}\right)^2 \tag{D.3}$$

where $\hat{\mu} = \frac{1}{nq}\left(h_{nq} - h_0\right)$ and h_0 and h_{nq} are the first and last observations of the time series.

Asymptotic standard normal test statistic for variance-ratio under the hypothesis of homoscedasticity

$$z(q) = \frac{VR(q) - 1}{\phi(q)^{0.5}} \sim N(0,1) \tag{D.4}$$

where $\phi(q) = \frac{2(2q-1)(q-1)}{3q(nq)}$. Asymptotic standard normal test statistic for the heteroscedasticity-consistent estimator is

$$z^*(q) = \frac{VR(q) - 1}{\phi^*(q)^{0.5}} \sim N(0,1) \tag{D.5}$$

where $\phi^*(q) = \sum_{j=1}^{q-1} \left[\frac{2(q-j)}{q}\right]^2 \hat{\delta}(j)$ and

$$\hat{\delta}(j) = \frac{\sum (h_t - h_{t-1} - \hat{\mu})^2 (h_{t-j} - h)}{\left[\sum (h_t - h_{t-1} - \hat{\mu})^2\right]^2} \tag{D.6}$$

CHAPTER 6

Conclusion

The thesis has attempted to address two issues surrounding financial solutions for improving the lifetime income of retired seniors. Dividend and equity release mortgage (ERM) cash flow volatility was studied to investigate characteristics which make them suitable alternatives for post retirement income. Pressures resulting from population demography including risk of post retirement poverty and pension income adequacy are the main problems. Both dividends and ERMs constitute private savings towards retirement income adequacy, as most economic environments experience persistent low interest rate environment. As a possible solution to lifetime income smoothing, we have shown that dividend and equity release mortgage investments have a potential to produce positive short-run realisable outcomes at lower cost.

Firstly, we have shown from our findings on dividends that; the excess of realized price volatility over its ex-post rational counterpart does not only pertain to the US but also extends to a large international sample of 50 countries. We found the magnitude of excess volatility to be substantially higher in developing countries compared to their developed counterparts. However, we present evidence that this difference is most likely driven by the length of available data in each country rather than reflecting a fundamental relationship between excess volatility and a country's state of economic development.

The findings also showed the sensitivity of variance bounds test results to the specification used in order to obtain expectations of future dividends. For instance, using mixed-frequency regressions to obtain a measure of expected dividend growth results in substantially less pronounced excess volatility on average compared to that observed when using the first generation variance bounds test of obtaining a price trend via regressions against time. Nevertheless, even though the *magnitude* of the effect varies, all measures of ex-post rational prices consistently result in excess volatility across the vast majority of sample countries.

More specifically, the results provide further empirical support for the argument that evaluating stock market rationality is heavily dependent on the test's specific assumptions about the dividend process. Similar to previous findings from the US market, we document that variance bounds tests are characterized by the same challenges and nuances with respect to dividend stationarity when applied to other countries as well. Consequently, inferences about stock market rationality will ultimately depend on the assumptions that one is willing to make about the underlying dividend process.

Secondly, the thesis extended the variance bounds literature by taking a forward-looking view of dividend volatility. We compared the information in the implied volatility (IV) of stock index options to the information in the IV of index dividend futures options. The findings showed how the variability in IV term-structure of option contracts with long-dated time-to-maturity can be justified by subsequent fluctuations in dividends. On the other hand, near-term maturity options have stock IVs which exceed that of dividends. In practice, such near-maturity index dividend futures options have expiry dates that coincide with dividend announcement dates of index constituents, hence associated with lower levels of dividend uncertainties. This phenomenon clearly makes the dividend puzzle effect more prominent when IV for corresponding stock-index options and index dividend futures options do not decrease together. The analysis showed that IV of stock index options consistently exceed that of index dividend options, thereby confirming previous criticism based on novel financial data and instruments. Interestingly, the magnitude of excess volatility declines with long-dated time-to-maturities, suggesting that the discrepancy between the two IV is sensitive to the investment horizon. This result holds irrespective of how IV is calculated i.e. either using the model-free or model-based approach. For the first time in the literature the thesis has constructed an IV index for the STOXX 50 dividend futures contract. This presents in the way to help us learn more about the properties of the dividend puzzle in dividend derivatives markets. The evolution of this index shows clearly that in recent years there has been a lot more volatility on dividend markets. The trading strategy results also showed that market participants can improve returns by combining a bet on the relationship of implied index dividend volatility and implied volatility of the index rather than use a directional bet on the underlying asset. Trading signals generated with the IV ratio and IV differences both outperform the long-only trading baseline model portfolio.

One striking implication of these joint findings on excess volatility puzzle relates to the fact that; expectations about future realizable dividends has a term-structure feature, being almost negligible in the long-run and strongly evident in the short horizons. Excess volatility puzzle inferences which are made from forward-looking data may not be directly replicated by using observations and techniques based on historical data, without considering investment horizon. It is likely to lose macro-market efficiency when index (firm aggregated) dividend data is forecasted very far into the future[1]; likewise, inferences from historical data

[1]Future dividend expectations are forecasted by making specific assumptions about the dividend gener-

are may be driven by sample size. Dividend derivatives are currently considered on a long-dated horizon, providing investors with synthetic exposure to gross returns realized on Euro STOXX 50 DVP futures. We have empirically shown they exhibit lower volatility when compared with equities. Over the long-run fund managers could draw diversification benefits against substantial equity exposure by using dividend derivatives.

Thirdly, regarding the ERMs, we show that the GBM model recommended by the regulator in the UK produces much higher values of the NNEG when compared with a best fit ARMA-EGARCH model selected on the basis of forecasting house prices well. Utilising an inappropriate model in the context of reverse mortgage loan market may in the end stifle this market by imposing very high capital reserve requirements in insurers. This is very important since there is no diversification benefit for an insurer issuing ERM loans, each loan being valued separately for NNEG calculations purposes. Furthermore, inflating the volatility parameter will automatically imply a high variance of house prices at long maturities for the GBM model, therefore impacting directly ERMs loan characteristics for the younger borrowers who would benefit the most from the this new asset class. An overestimation of the rental yield induces downward trending house prices in the long run that ultimately will inflate the NNEG values.

The application of Black '76 option pricing formula for NNEG valuation is theoretically not sound and it can lead to important miscalculations for ERM, depending on the levels of risk-free rates. This may have a detrimental effect on the development of this important financial product for society. The model would be applicable if there was a house price futures contract traded. This is not the case currently but the research in this area highlights the importance of introducing futures contracts for hedging house price risk in financial and insurance markets. As the underlying market is incomplete, pricing NNEG based on risk-neutral concepts will always be controversial. It might also be worthwhile to consider the full distribution of the outcomes, based on the real-world projections of the underlying economic variables. It will be beneficial to explore other finance/economic/actuarial models suitable for this purpose.

The current results point out to some early important points. In the absence of market prices or recognised benchmark prices, it is difficult to identify the best model (method) with reference to ERM prices. The best that can be done under current circumstances is to (a) look for a model that has good forecasting power of house prices; and (b) compare various models across a large set of scenarios, from standard baseline to stressed scenarios.

ARMA-EGARCH family of models outperforms the GBM model under real-world measure in terms of forecasting short- and medium-term house prices. This is not surprising since the theoretical properties of the GBM model are in contradiction with the empirical stylised features of house price time series. The

ating process.

GBM (Black-Scholes) model is simple to implement but it lacks theoretical support for this asset class. It may inflate in relative terms the NNEG for the young age borrowers due to high variance of house prices at long maturities.

The method of parameter estimation may give different results. For GBM the GMM parameter estimation method may produce superior forecasting results versus MLE and MM. At the same time, the estimates under GMM can be substantially different from the parameter values estimated by MLE or MM. Black 76 model theoretical formula for pricing European options is based on the futures of the underlying asset- house prices in this case. In the absence of a residential house futures contract, one cannot switch modelling from spot house prices onto futures.

Moreover, one common mistake in some papers covering NNEG valuation is to refer to the forward house prices in relation to the Black 76 model. In addition, if the forward house prices are produced using a formula that has not been proved from the first principles, the risk management for ERMs may become unreliable. Using multiple decrements will always deflate NNEG values due to earlier termination. It is possible to get similar NNEG vectors under very different models. Adjusting volatilities (one value for each maturity) under GBM-rn may lead to a matching of NNEG values produced by an ARMA-EGARCH model. The ARMA-EGARCH model produces NNEG values that depend on the risk-neutralisation method. The minimal entropy risk-neutral measure can also be applied in conjunction to ARMA-EGARCH house price modelling. The NNEG formulae under minimal entropy method need to be refined if net rental income ought to be included.

The analysis shows that NNEG values produced by the GBM cannot be considered lower bounds for NNEGs calculated from more appropriate models, nor upper bounds. The only exception is the regions of high NNEG risk when the two classes of models (GBM on one part and the ARMA-EGARCH on the other) come out almost the same.

The valuations under ARMA-EGARCH models may look more complicated computationally but theoretically they are more robust. There is still a question about what is the best way to risk-neutralise the results. The current method of risk-neutralisation for the ARMA-EGARCH using the conditional Esscher transform still makes use of the $r - g$ in the martingale calibration and this is slightly concerning. The minimal entropy risk-neutral procedure, which can be used to bypass difficulties related to the rental yield estimation is largely unstable when extensions are made to include the incidence of mortality.

6.1 Future research

The next stage of research for the non-negative equity guarantee clause in ERMs is very crucial due to recent increase in market demand. Volatility of house prices is key driving factor of NNEG values, hence the need to adopt valuation models which are robust to downturns in the economic environment. The future conceptual framework for ERM contracts must seek a balance borrower and issuer expectations to ensure survival of unanticipated property price decline. This thesis sets the pace by investigating the relationship between house price volatility and property dilapidation or dereliction. This approach integrates borrower participation and property depreciation events in the NNEG valuation, with an aim to reduce risk of moral hazard. Using house price index at postcode level will also allow ERM loan issuers to manage moral hazard emanating from the borrower's failure to maintain the property. Huang et al. (2020) found substantial differences in NNEG values for different regions in UK.

Future research may also consider adapting the design mechanism of Continuous Workout Mortgage (CWM) to fair valuations of ERMs. CWM products bear close resemblance to ERM contracts via the negative equity insurance component. Per its dual design, CWMs have a fixed rate home loan and a negative equity insurance embedded with automatic adjustments for price changes in the underlying collateral house. The benefit here is to share underlying risk while the ERM loan issuer awaits the NNEG liability. Automatic adjustments for house price changes over contract lifetime will provide for better management of the NNEG liability. Shiller et al. (2013) and Shiller et al. (2019) present excellent discussions on CWMs and its viability for mitigating financial fragility. Initial loan issued in an ERM contract strictly accretes over time without any borrower repayment until termination. Continuous workout mortgages may be adapted to obtain closed form formulae for an actuarially fair contract rate to accumulate the initial loan and also for NNEG values with a focus on estimating key parameters of the contract. One could also explore how CWMs can be used to enhance financial resilience by mitigating against unanticipated systemic risk. This will improve sustainability of ERM cash flow for the elderly in our society.

In further work beyond the thesis timeline, one may consider modelling the NNEG put option by using a jump-diffusion option pricing model with serially correlated jumps. Here jump sizes could be modelled with an autoregressive process while exploring the underlying house price return volatility. This approach is efficient towards investigating the relationship between observed jump sizes, house price return, and NNEG cost. The mixed frequency data sampling GARCH (MIDAS-GARCH)is an alternative time series model that can be used to explore the relationship between macroeconomic indicators i.e. GDP, inflation, interest rate, unemployment etc with short-run and long-run house price volatility. This technique will allow future studies to consider the impact of systemic exogenous variables on house price volatility when pricing ERMs. Last but not the least, there is a need to consider new data driven nonparametric procedures that has potential to generate similar results to historical simulation in risk management.

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