Estimating maintenance effectiveness of a repairable system under time-based preventive maintenance[1]

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Abstract

Corrective maintenance (CM) is carried out to correct failures while preventive maintenance (PM) is to avert failure. They both play an important role in asset management. Accurately estimating the effectiveness of PM is needed as it has an impact on system health management. Given failure times and maintenance related data, it is possible to estimate maintenance effectiveness. This paper proposes PM and CM models based on the different combinations of the type of maintenance carried out, estimates the parameters in those models, simulates their failure intensities and then studies maintenance effectiveness using maintenance data of industrial equipment.

Keywords: repairable system, preventive maintenance, corrective maintenance, PM and CM models, imperfect repair, effectiveness of maintenance

Acronyms

AIC	Akaike information criterion
ARA	Arithmetic reduction of age
ARI	Arithmetic reduction of intensity
BIC	Bayesian information criterion
CM	Corrective maintenance
CP	Calabria-Pulcini
GFRR	Geometric failure rate reduction
GRA – CP	Geometric reduction of age – Calabria Pulcini
LLP	Log linear process

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NHPP	Nonhomogeneous Poisson process
PLP	Power law process
PM	Preventive maintenance

Notation

$\lambda(t\|H_{t^-})$	Intensity of the failure process of the repairable system given H_{t^-}
H_{t^-}	History of the failure process prior to time t which includes the number of failures / PMs, and failure times / PM times
C_k	kth maintenance time
W_k	Time between the kth and the $(k-1)$th maintenance
K_{t^-}	Number of maintenance actions (PMs and CMs) before time t
T_i	ith failure / CM time
X_i	time between the ith and the $(i-1)$th failures / CMs,
N_{t^-}	number of failures before time t
τ_j	jth PM time
χ_j	time between the jth and the $(j-1)$th PMs
M_{τ^-}	number of PMs before time t
U_k	$U_k = 0$ if the k th maintenance time is CM and $U_k = 1$ if the kth maintenance time is PM
ρ	Degree of maintenance for imperfect maintenance models, ρ_C for CM and ρ_P for PM

1. Introduction

1.1 Background

Corrective maintenance (CM) or repair is carried out to bring a failed system back to its operating status. In the reliability community, maintenance effectiveness is categorized into perfect, imperfect, and minimal. A perfect maintenance brings the system to a good-as-new status, a minimal maintenance restores the system to the status immediately before its failure (or as bad as old), and an imperfect maintenance brings the system to the status between good as new and bad as old (Syamsundar and Naikan, 2009; Syamsundar et al., 2011; and Doyen et al., 2017). Sometimes maintenance can lead to a situation that leaves the system worse than it was before, which may largely be due to failures being ill-maintained by undertrained maintenance professionals, or to a situation that improves the reliability of the system better than that of a new system, which is normally due to technological advance.

Preventive maintenance (PM) can be time- or condition- based. Time-based PM is carried out at scheduled times and condition-based PM is conducted based on the condition of the item under study. Time based PM can comprise of minor, medium or major maintenances, or can be routine activities such as inspection, cleaning and bolt tightening.

PM is needed to reduce the probability of failures so that the overall costs can be optimized. If no PM is conducted and only CM is carried out upon failures, the incurred cost will include maintenance cost and other failure related costs such as lost production on account of untimely disruption and increased delivery times leading to a loss of reputation. This may lead to unviable operations due to the relevant high cost and is only possible if the PM is effective.

Obtaining an accurate estimate of the effectiveness of PM is vital for understanding its impact and for deciding the maintenance strategy so that the health of the technical system is improved and the relevant costs can be minimized. This is only possible with appropriate modelling of the PM and CM process.

1.2 Related Literature

The failure process of a repairable system is usually modelled by stochastic processes, which can be categorized into two classes:

a) local time based, in which a model is a function of the time since the last failure, and
b) global time based, in which a model is a function of the time since the inception of the failure process.

Examples of the local time based models include, the renewal process model, the Kijima models I & II (Kijima and Sumita, 1986; Kijima, 1989; Brown et al., 1983), the geometric process (Lam, 1988), the geometric failure rate reduction (GFRR) model (Finkelstein, 2008), and the doubly geometric process (Wu, 2018).

Examples of the global time based models include:

- the non-homogeneous Poisson process (Syamsundar and Naikan, 2009),
- arithmetic reduction of age (ARA) and arithmetic reduction of intensity (ARI) models (Doyen and Gaudoin, 2004),
- the Calabria-Pulcini (CP) model (Calabria and Pulcini, 1999),
- the proportional intensity model (Percy and Alkali, 2007),
- the geometric reduction of age (GRA) model (Doyen et al., 2017), and
- failure process models with the exponential smoothing of intensity functions (Wu, 2019).

As time-based PMs are pre-scheduled, they do not form stochastic processes. Modelling of the effectiveness of PM started with the formulation of PM policies, which are extensively covered in review papers, see Wang (2002), Wu and Zuo (2010), and Tadj et al. (2011), for example. Optimal PM schedules are arrived at using various PM policies by minimizing the expected cost or maximizing the reliability as the criterion. Examples of research dealing with PM optimisation under various conditions include Liu et al. (2012), Wang et al. (2017), Cao et al. (2018), Levitin et al. (2018), Shen et al. (2019), Sun et al. (2019) and Yang et al. (2019). Between adjacent PM activities, CM is conducted.

Liu et al. (2012) assume various levels of the maintenance effectiveness of imperfect Kijima I & II, Nakagawa and non-linear PM models where CM actions are assumed minimal. Wang et al. (2017) develop an optimal preventive maintenance policy using a generalised geometric process and assume the generalised maintenance effectiveness as $exp(0.01n)$. Cao et al. (2018) discuss a selective maintenance model where imperfect maintenance is possible, however, they do not consider the effectiveness of imperfect maintenance. Levitin et al. (2018) propose a preventive policy for a 1 out of N: G warm standby system subject to internal failures and external shocks with a linear cumulative exposure model. They assume the effectiveness parameter to be 0 to 0.4 in increments of 0.1 for their example. Shen at al. (2019) develop an improvement factor model for degrading systems in a dynamic environment and assume that the system is subject to imperfect maintenance actions before replacement. They develop an optimal maintenance policy for the system by assuming a maintenance effectiveness of 0.8. Sun et al. (2019) consider the saturation effect while scheduling preventive maintenance with a virtual age model and assume maintenance effectiveness as an s-shaped function with a lower limit of 0.7. Yang et al. (2019) investigate a novel two-phase preventive maintenance policy for a single-component system with an objective of maximizing the revenue generated by the performance-based contracting (PBC) using a proportional age reduction model. They consider a maintenance effectiveness of 0.4 in their case study.

In existing research, optimisation of PM policies is carried out under assumptions that PM is subject to a level of maintenance effectiveness, while CM is usually considered to be minimal. Neither modelling of the PM-CM process under different maintenance assumptions nor actual estimation of the model parameters for real technical systems data has been carried out, which will lead to sub-optimisation of PM policy.

Time-based PM models and CM processes with application to fit failure data of repairable systems are covered in Percy and Alkali (2007) and Doyen and Gaudoin (2011), respectively. Doyen and Gaudoin (2011) propose PM-CM models with PM and CM being different levels of maintenance.

However, they apply ARA_1 and ARI_∞ models to two data sets. The proportional Intensity imperfect maintenance model is proposed in Percy and Alkali (2007) and applied to a dataset of PM and CM times. Other combinations of imperfect maintenance models have not been investigated, nor are the choice of models and the best fit model. Apart from this, there is little research estimating maintenance effectiveness in a model with a consideration of both PM and CM. properly researching into it will lead to more cost-effective decisions on maintenance policies.

Estimation of maintenance effectiveness is model dependent. Unless a correct choice of the model is made, a correct estimation of maintenance effectiveness is not possible. For this reason, we need to apply models with all possible combinations of maintenance types to obtain proper estimates. This presupposes that modelling of the PM-CM process has been carried out covering all the combinations.

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1.3 Novelty and Contribution

This work deals with the estimation of maintenance effectiveness of a repairable system. The novelty and contributions of the paper are listed below.

- Firstly, various combinations of types of PM and CM maintenance activities are listed and their implications explained;
- Secondly, modelling of the PM-CM process for all combinations of maintenance activities is carried out. In addition, a simple method for converting any imperfect CM model to a PM-CM model is proposed. This method has not been explored in existing literature This also leads to a new definition of the ARA_1 and ARA_∞ PM-CM models, which is different from those models developed in Doyen and Gaudoin (2011).
- Thirdly, the estimates of maintenance effectiveness for the particular dataset of PM and CM times is obtained. A model selection procedure based on the corrected Akaike Information Criterion and a goodness of fit test are proposed to obtain the best fit model to the dataset of PM and CM times. This validates the estimate of maintenance effectiveness derived from system failure datasets.

1.4 Overview

This remainder of the paper is structured as follows. Section 2 describes the PM-CM process models and their parameter estimation. Section 3 simulates intensities with various PM-CM models. Section 4 applies the models to repairable systems for assessment of their PM-CM process carried out. Section 5 concludes the work.

2. PM and CM models and their inference

This section describes a typical PM-CM process along with notation.

2.1 PM-CM Process

It is known that PM is planned and CM is conducted upon failures. A maintenance process, which includes both PM and CM, therefore forms a PM model and CM stochastic process, denoted by a PM-CM process.

PM and CM times and the PM-CM process along with notation are described in Fig. 1. The centre line represents that CMs and PMs are carried out on the system at specific instances of time.

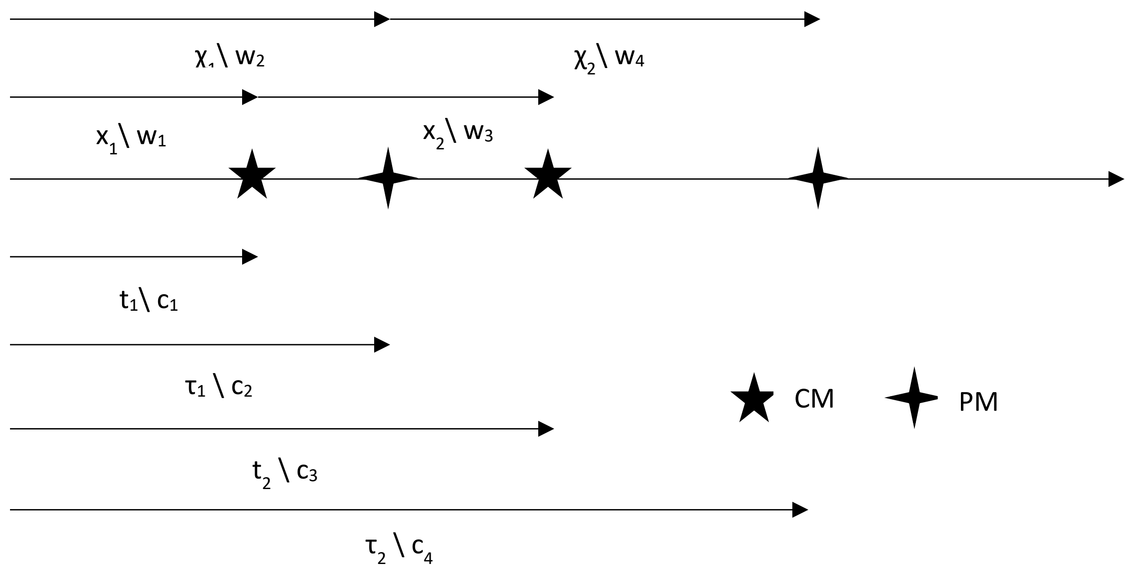


Fig. 1: PM and CM times and the PM-CM process

Failures followed by CMs occur at times T_i , $i = 1, 2, 3, \ldots, n$ with $T_0 = 0$, and inter failure times are denoted by $X_{i+1} = T_{i+1} - T_i$. N_t– denotes the number of failure just before time t. In Fig. 1, t_1 and t_2 represent the times to failures and hence CMs 1 and 2 since inception while x_1 and x_2 represent the inter failure times.

PMs occur at times $\tau_j, j = 1,2,3 \ldots m$ with $\tau_0 = 0$, and inter PM times are denoted by $\chi_{j+1} = \tau_{j+1} - \tau_j$. M_τ– denotes the number of failures before time τ. In Fig. 1, τ_1 and τ_2 represent the times to PMs 1 and 2 since inception while χ_1 and χ_2 represent the inter PM times.

A combination of PM and CM maintenance events forms the PM-CM process, in which the maintenance events occur at times $C_k, k = 1,2,3, \ldots (m + n)$ with $C_0 = 0$, and inter event times are denoted by $W_{k+1} = C_{k+1} - C_k$. K_t– denotes the number of maintenance events before time t. In Fig.

1, c_1 , c_2 , c_3 and c_4 represent the times to maintenance events, i.e., PMs 1 and 2 and CMs 1 and 2 since inception while w_1 , w_2 , w_3 and w_4 represent the inter maintenance event times.

2.2 PM and CM Models

It is incomprehensive to investigate a couple of typical levels of maintenance effectiveness in research as the real practice can have all possible scenarios. As such, this section considers all possible combinations of maintenance effectiveness on CM and PM.

PM and CM models can be built considering perfect, minimal and imperfect maintenance. Nine different combinations of such repairs are possible as given in Table 1 along with their usefulness.

Table 1: Different combinations of PM and CM Maintenances

Sl. No.	Type of maintenance at CM	Type of maintenance at PM	Practical implication
1	Perfect	Perfect	When complete replacement takes place at PM and CM e.g., replacement of components, sub-systems
2	Perfect	Minimal	When complete replacement takes place at CM but minor maintenance at PM
3	Minimal	Minimal	When minor maintenance take place and the failure intensity remains the same after maintenance
4	Minimal	Perfect	When minor maintenance takes place at CM and complete replacement at PM
5	Imperfect	Imperfect	When partial replacement and maintenances take place at both CM and PM
6	Imperfect	Perfect	When partial replacement and maintenances take place at CM and complete replacement at PM
7	Perfect	Imperfect	When complete replacement takes place at CM partial replacement and maintenances take place at PM
8	Imperfect	Minimal	When partial replacement and maintenances take place at CM and minor maintenance at PM
9	Minimal	Imperfect	When minor maintenance take place at CM and partial replacement and maintenances take place at PM

Different combinations of maintenances lead to different types of PM and CM models as given in the sub-sections below.

PM-CM models can be categorized as local and global models based on whether their intensity functions are based on local times or global times. Local time is defined as the time since previous maintenance and is designated by $t - C_{K_{t^-}}$, and global time as time since the inception of the system and is designated by t.

Denote U_K as an indicator such that, $U_K = 0$ if the kth maintenance is a CM and $U_K = 1$ if the kth maintenance is a PM, ρ_C as the parameter representing the effectiveness of CM, and ρ_P as the parameter representing the effectiveness of PM. The parameter ρ is chosen so that $\rho_C, \rho_P = 0$ represents minimal maintenance and $\rho_C, \rho_P = 1$ represents perfect maintenance.

$\lambda(t|H_{t^-})$ denotes the intensity of the failure process of the repairable system, given H_{t^-} the history of the failure process prior to time t, which includes the number of failures / PMs, and failure times / PM times

2.2.1 Perfect and Minimal PM and CM Models

PM and CM models that are generated to model different maintenance types of combinations 1 to 4 in Table 1 are given below.

If the maintenance effectiveness of both CM and PM is perfect, both parameters ρ_C and ρ_P are equal to 1. The intensity for this process is a function of the local or inter failure times, $t - C_{K_{t^-}}$, and is given by Doyen and Gaudoin (2011):

$$\lambda(t|H_{t^-}) = \lambda(t - C_{K_{t^-}}) \quad (1)$$

When a renewal takes place at CM and minimal maintenance is carried out at PM, we get a perfect CM and minimal PM, where ρ_C = 1 and ρ_P = 0. The intensity for this process is a function of the local or inter failure times for CM, $t - T_{N_{t^-}}$ and is given by Doyen and Gaudoin (2011):

$$\lambda(t|H_{t^-}) = \lambda(t - T_{N_{t^-}}) \quad (2)$$

Widely considered models for the renewal process include the models with the inter-failure times following the Weibull distribution and the Gumbel distribution, respectively.

If the maintenance effectiveness of both CM and PM are minimal, both parameters ρ_C and ρ_P equal to 0. The intensity for this process is a function of the global time or times to failure and is given by Doyen and Gaudoin (2011):

$$\lambda(t|H_{t^-}) = \lambda(t) \quad (3)$$

When CM is minimal and PM is perfect, then parameter ρ_C = 0 and ρ_P = 1. The intensity for this process is given by Doyen and Gaudoin (2011):

$$\lambda(t|H_{t^-}) = \lambda(t - \tau_{M_{t^-}}) \quad (4)$$

However, as the intensity of the process is defined in terms of the failure process, the intensity for the process with minimal maintenance CM interspersed between maximal PMs is given by:

$$\lambda(t|H_{t^-}) = \lambda(t - \tau_{M_{t^-}}) = \prod_{j=1}^{M_t} \lambda_j(t) \quad (5)$$

The NHPP (non-homogeneous Poisson process) with the power law model and the NHPP with the log linear model can be used to model the failure process with minimal maintenance in the models (3), (4) and (5).

All the perfect and minimal maintenance PM and CM models are summarized in Table 2.

Table 2: Perfect and Minimal Maintenances PM and CM Model

Sl. No.	Type of maintenance at CM	Type of maintenance at PM	Intensity of failure	Source	Model properties
1	Perfect	Perfect	$\lambda(t\|H_{t^-}) = \lambda(t - C_{K_{t^-}})$	Doyen and Gaudoin (2011)	$\rho_C = 1$ $\rho_P = 1$
2	Perfect	Minimal	$\lambda(t\|H_{t^-}) = \lambda(t - T_{N_{t^-}})$	Doyen and Gaudoin (2011)	$\rho_C = 1$ $\rho_P = 0$
3	Minimal	Minimal	$\lambda(t\|H_{t^-}) = \lambda(t)$	Doyen and Gaudoin (2011)	$\rho_C = 0$ $\rho_P = 0$
4	Minimal	Perfect	$\lambda(t - \tau_{M_{t^-}}) = \prod_{j=1}^{M_t} \lambda_j(t)$	Doyen and Gaudoin (2011)	$\rho_C = 0$ $\rho_P = 1$

2.2.2 Imperfect PM and CM Models

A simple method is used to develop the imperfect PM and CM models with indicator U_K and effectiveness parameters ρ_C and ρ_P, respectively. Then we have the following analyses.

- If the kth maintenance is a CM, $U_K = 0$ and the maintenance effectiveness parameter is $(\rho_P)^0(\rho_C)^{1-0}$, i.e., ρ_C.
- If the kth maintenance time is a PM, $U_K = 1$ and the maintenance effectiveness parameter is $(\rho_P)^1(\rho_C)^{1-1}$, i.e., ρ_P.

That is, a model developed for an imperfect CM process can be extended to a PM-CM model by replacing the parameter ρ in the model with $(\rho_P)^{U_K}(\rho_C)^{1-U_K}$.

PM and CM models generated for combination 5 of Table 1 are given below.

Examples of imperfect maintenance processes considered in local time include Kijima model I (Kijima and Sumita, 1986; Kijima. 1989), Kijima model II (Kijima, 1989; Brown et al., 1983), geometric process (Lam, 1988), and GFRR (Finkelstein, 2008). In all these processes, we replace the maintenance effectiveness parameter ρ with $(\rho_P)^{U_K}(\rho_C)^{1-U_K}$.

The failure intensity of the Kijima I model (Kijima and Sumita, 1986; Kijima. 1989) is given by;

$$\lambda(t|H_{t^-}) = \lambda\left(t - T_{N_{T^-}} + (1-\rho)\sum_{i=1}^{N_{t^-}} x_i\right) \quad (5)$$

By replacing (1-ρ) in the model with $(1-\rho_P)^{U_K}(1-\rho_C)^{1-U_K}$, $T_{N_{T^-}}$ with $C_{K_{t^-}}$, and x_i with w_i, the failure intensity for the Kijima I with imperfect CM and imperfect PM is given by;

$$\lambda(t|H_{t^-}) = \lambda\big(t - C_{K_{t^-}} + \textstyle\sum_{i=1}^{K_{t^-}}(1-\rho_P)^{U_i}(1-\rho_C)^{1-U_i}w_i\big). \tag{6}$$

The failure intensity of the Kijima model II (Kijima, 1989; Brown et al., 1983) is given by;

$$\lambda(t|H_{t^-}) = \lambda(t - T_{N_{T^-}} + \textstyle\sum_{i=1}^{N_{t^-}}(1-\rho)^{N_{t^-}+1-i}x_i) \tag{7}$$

Here $N_{t^-} + 1 - i = N_{t^-} - (i-1)$ becomes $M_{t^-} - M_{C_{i-1}}$ for PM actions and $N_{t_-} - N_{C_{i-1}}$ for CM actions. By replacing $(1-\rho)^{N_{t^-}+1-i}$ with $(1-\rho_P)^{M_{t^-}-M_{C_{i-1}}}(1-\rho_C)^{N_{t_-}-N_{C_{i-1}}}$, $T_{N_{T^-}}$ with $C_{K_{t^-}}$, and x_i with w_i the failure intensity for the Kijima II model with imperfect CM and imperfect PM is given by;

$$\lambda(t|H_{t^-}) = \lambda\big(t - C_{K_{t^-}} + \textstyle\sum_{i=1}^{K_{t^-}}(1-\rho_P)^{M_{t^-}-M_{C_{i-1}}}(1-\rho_C)^{N_{t_-}-N_{C_{i-1}}}w_i\big). \tag{8}$$

For $\rho_C, \rho_P = 1$, models (6) and (8) reduce to (1) and for $\rho_C, \rho_P = 0$ the models reduce to (3).

The failure intensity of the GP model (Lam, 1988) is given by;

$$\lambda(t|H_{t^-}) = \rho^{N_{t^-}}\lambda\left(\rho^{N_{t^-}}\big(t - T_{N_{t^-}}\big)\right). \tag{9}$$

By replacing ρ with $(\rho_P)^{U_K}(\rho_C)^{1-U_K}$, N_{t_-} with K_{t^-}, $T_{N_{t^-}}$ with $C_{K_{t^-}}$, the failure intensity for GP with imperfect CM and imperfect PM is given by;

$$\lambda(t|H_{t^-}) = \lambda\left(\textstyle\prod_{i=1}^{K_{t^-}}(\rho_P)^{U_i}(\rho_C)^{1-U_i}\left(t - C_{K_{t^-}}\right)\right) \tag{10}$$

$$= \lambda\left((\rho_P)^{M_{t^-}}(\rho_C)^{N_{t^-}}\big(t - C_{K_{t^-}}\big)\right)$$

The failure intensity of the GFRR model (Finkelstein, 2008) is given by;

$$\lambda(t|H_{t^-}) = \rho^{N_{t^-}}\lambda\big(t - T_{N_{t^-}}\big). \tag{11}$$

By replacing ρ with $(\rho_P)^{U_K}(\rho_C)^{1-U_K}$, N_{t_-} with K_{t^-}, $T_{N_{t^-}}$ with $C_{K_{t^-}}$, the failure intensity for GFRR with imperfect CM and imperfect PM is given by;

$$\lambda(t|H_{t^-}) = \textstyle\prod_{i=1}^{K_{t^-}}(\rho_P)^{U_i}(\rho_C)^{1-U_i}\,\lambda\big(t - C_{K_{t^-}}\big) \tag{12}$$

$$= (\rho_P)^{M_{t^-}}(\rho_C)^{N_{t^-}}\lambda(t - C_{K_{t^-}})$$

For $\rho_C, \rho_P = 1$, models (10) and (12) reduce to (1). For the models at (10) and (12) minimal maintenance is not defined.

Widely considered models for the renewal process are the with power law and log linear processes, which also form the baseline process for local imperfect maintenance models.

Examples of imperfect maintenance processes considered in global time include ARA_1 , ARA_∞ , ARI_1 , ARI_∞ (Doyen and Gaudoin, 2004), CP (Calabria and Pulcini, 1999; Percy and Alkali, 2007), and GRA – CP (Doyen et al., 2017). In all these processes, we replace the maintenance effectiveness parameter ρ with $(\rho_P)^{U_K}(\rho_C)^{1-U_K}$.

The failure intensity of the ARA$_1$ model (Doyen and Gaudoin, 2004) is given by:

$$\lambda(t|H_{t^-}) = \lambda\left(t - \rho T_{N_{t^-}}\right). \tag{13}$$

By replacing ρ with $(\rho_P)^{U_K}(\rho_C)^{1-U_K}$, and $T_{N_{T^-}}$ with $C_{K_{t^-}}$, the failure intensity for the ARA$_1$-PM and ARA$_1$-CM is given by:

$$\lambda(t|H_{t^-}) = \lambda\left(t - (\rho_P)^{U_i}(\rho_C)^{1-U_i} C_{K_{t^-}}\right). \tag{14}$$

The failure intensity of the ARA$_\infty$ model (Doyen and Gaudoin, 2004) is given by;

$$\lambda(t|H_{t^-}) = \lambda\left(t - \rho \sum_{j=0}^{N_{t^-}-1}(1-\rho)^{N_{t^-}-j} T_j)\right), \tag{15}$$

where $N_{t^-} - j$ with j starting from 0 becomes $M_{t^-} - M_{C_{j-1}}$ for PM actions and $N_{t_-} - N_{C_{j-1}}$ with j starting from 1 for CM actions. By replacing ρ with $(\rho_P)^{U_K}(\rho_C)^{1-U_K}$, $(1-\rho)^{N_{t^-}-j}$ with $(1-\rho_P)^{M_{t^-}-M_{C_{j-1}}}(1-\rho_C)^{N_{t_-}-N_{C_{j-1}}}$, and T_j with C_j the failure intensity for ARA$_\infty$-PM and ARA$_\infty$-CM is given by:

$$\lambda(t|H_{t^-}) = \lambda\left(t - (\rho_P)^{U_{K_{t^-}}}(\rho_C)^{1-U_{K_{t^-}}} \sum_{i=1}^{K_{t^-}-1} \prod_{j=1}^{i}\left(1 - \left((\rho_P)^{M_i - M_{C_{j-1}}}(\rho_C)^{N_i - N_{C_{j-1}}}\right)\right) C_j\right). \tag{16}$$

For $\rho_C, \rho_P = 1$, models (14) and (16) reduce to (1) and for $\rho_C, \rho_P = 0$ and the models reduce to (3).

The failure intensity of the ARI$_1$ model (Doyen and Gaudoin, 2004) is given by:

$$\lambda(t|H_{t^-}) = \lambda(t) - \rho\left(\lambda\left(T_{N_{t^-}}\right) - \lambda(0)\right) \tag{17}$$

By replacing ρ with $(\rho_P)^{U_K}(\rho_C)^{1-U_K}$, and $T_{N_{T^-}}$ with $C_{K_{t^-}}$ the failure intensity for ARI$_1$ with imperfect CM and imperfect PM is given by;

$$\lambda(t|H_{t^-}) = \lambda(t) - (\rho_P)^{U_i}(\rho_C)^{1-U_i}\left(\lambda\left(C_{K_{t^-}}\right) - \lambda(0)\right) \tag{18}$$

The failure intensity of the ARI$_\infty$ model (Doyen and Gaudoin, 2004) is given by:

$$\lambda(t|H_{t^-}) = \lambda(t) - \rho \sum_{j=0}^{N_{t^-}-1}(1-\rho)^{N_{t^-}-j}\left(\lambda\left(T_j\right) - \lambda(0)\right) \tag{19}$$

Here $N_{t^-} - j$ with j starting from 0 becomes $M_{t^-} - M_{C_{j-1}}$ for PM actions and $N_{t_-} - N_{C_{j-1}}$ with j starting from 1 for CM actions. By replacing ρ with $(\rho_P)^{U_K}(\rho_C)^{1-U_K}$, $(1-\rho)^{N_{t^-}-j}$ with $(1-\rho_P)^{M_{t^-}-M_{C_{j-1}}}(1-\rho_C)^{N_{t_-}-N_{C_{j-1}}}$, and T_j with C_j the failure intensity ARI$_\infty$-PM and ARI$_\infty$-CM is given by:

$$\lambda(t|H_{t^-}) = \lambda(t) - (\rho_P)^{U_{K_{t^-}}}(\rho_C)^{1-U_{K_{t^-}}} \sum_{i=1}^{K_{t^-}-1} \prod_{j=1}^{i}\left(1 - \left((\rho_P)^{M_i - M_{C_{j-1}}}(\rho_C)^{N_i - N_{C_{j-1}}}\right)\right)$$

$$\left(\lambda\left(C_j\right) - \lambda(0)\right) \tag{20}$$

For $\rho_C, \rho_P = 0$, models (18) and (20) reduce to (3).

The failure intensity of the GRA-CP model (Doyen et al. 2011) is given by;

$$\lambda(t|H_{t^-}) = (1-\rho)^{N_{t^-}}\lambda((1-\rho)^{N_{t^-}}t) \quad (21)$$

By replacing $(1-\rho)$ with $(1-\rho_P)^{U_K}(1-\rho_C)^{1-U_K}$, and $N_{t\_}$ with K_{t^-}, the failure intensity for GRA – CP with imperfect CM and imperfect PM is given by;

$$\lambda(t|H_{t^-}) = \lambda\left(\prod_{i=1}^{K_{t^-}}(1-\rho_P)^{U_i}(1-\rho_C)^{1-U_i}\,t\right) = \lambda((1-\rho_P)^{M_{t^-}}(1-\rho_C)^{N_{t^-}}t) \quad (22)$$

The failure intensity of the CP model (Calabria and Pulcini, 1999) is given by;

$$\lambda(t|H_{t^-}) = (1-\rho)^{N_{t^-}}\lambda(t) \quad (23)$$

By replacing $(1-\rho)$ with $(1-\rho_P)^{U_K}(1-\rho_C)^{1-U_K}$, and $N_{t\_}$ with K_{t^-}, the failure intensity for CP with imperfect CM and imperfect PM is given by [26];

$$\lambda(t|H_{t^-}) = \prod_{i=1}^{K_{t^-}}(1-\rho_P)^{U_i}(1-\rho_C)^{1-U_i}\,\lambda(t) = (1-\rho_P)^{M_{t^-}}(1-\rho_C)^{N_{t^-}}\lambda(t) \quad (24)$$

For $\rho_C, \rho_P = 0$, models (22) and (24) reduce to (3). For the models at (22) and (24), maximal maintenance is not defined.

All the imperfect maintenance PM and CM models are summarized in Table 3.

Table 3: Imperfect Maintenance PM and CM Models

Sl. No.	CM Model	PM Model	Intensity of failure	Model Properties
5	Kijima 1	Kijima 1	$\lambda\left(t - C_{K_{t^-}} + \sum_{i=1}^{K_{t^-}}(1-\rho_P)^{U_i}(1-\rho_C)^{1-U_i}w_i\right)$	For ρ_C = 1, ρ_P = 1 the models reduce to (1), for ρ_C = 0, ρ_P = 0 the models reduce to (3)
	Kijima 2	Kijima 2	$\lambda\left(t - C_{K_{t^-}} + \sum_{i=1}^{K_{t^-}}(1-\rho_P)^{M_{t^-}-M_{C_{i-1}}}(1-\rho_C)^{N_{t\_}-N_{C_{i-1}}}w_i\right)$	
	GP	GP	$\lambda\left((\rho_P)^{M_{t^-}}(\rho_C)^{N_{t^-}}\left(t - C_{K_{t^-}}\right)\right)$	For ρ_C = 1, ρ_P = 1 the models reduce to (1)
	GFRR	GFRR	$(\rho_P)^{M_{t^-}}(\rho_C)^{N_{t^-}}\lambda(t - C_{K_{t^-}})$	
	ARA_1	ARA_1	$\lambda\left(t - (\rho_P)^{U_i}(\rho_C)^{1-U_i}C_{K_{t^-}}\right)$	For ρ_C = 1, ρ_P = 1 the models reduce to (1), for ρ_C = 0, ρ_P = 0 the models reduce to (3)
	ARA_∞	ARA_∞	$\lambda\left(t - (\rho_P)^{U_{K_{t^-}}}(\rho_C)^{1-U_{K_{t^-}}}\sum_{i=1}^{K_{t^-}-1}\prod_{j=1}^{i}\left(1 - \left((\rho_P)^{M_i-M_{C_{j-1}}}(\rho_C)^{N_i-N_{C_{j-1}}}\right)\right)C_j\right)$	
	ARI_1	ARI_1	$\lambda(t) - (\rho_P)^{U_i}(\rho_C)^{1-U_i}\left(\lambda\left(C_{K_{t^-}}\right) - \lambda(0)\right)$	For ρ_C = 0, ρ_P = 0 the models reduce to (3)
	ARI_∞	ARI_∞	$\lambda(t) - (\rho_P)^{U_{K_{t^-}}}(\rho_C)^{1-U_{K_{t^-}}}\sum_{i=1}^{K_{t^-}-1}\prod_{j=1}^{i}\left(1 - \left((\rho_P)^{M_i-M_{C_{j-1}}}(\rho_C)^{N_i-N_{C_{j-1}}}\right)\right)\left(\lambda\left(C_j\right) - \lambda(0)\right)$	
	GRA-CP	GRA-CP	$\lambda((1-\rho_P)^{M_{t^-}}(1-\rho_C)^{N_{t^-}}t)$	For ρ_C = 0, ρ_P = 0 the models reduce to (3)
	CP	CP	$(1-\rho_P)^{M_{t^-}}(1-\rho_C)^{N_{t^-}}\lambda(t)$	

2.2.3 Other Combinations of PM and CM Models

Other combinations such as imperfect CM with either perfect or minimal PM and either perfect or minimal CM with imperfect PM for type combinations 5 to 8 of Table 1 can also be worked out. A few typical PM and CM models are given below.

The failure intensity for an imperfect Kijima I CM and perfect maintenance PM model is given by;

$$\lambda(t|H_{t^-}) = \prod_{j=1}^{M_\tau} \lambda_j\left(t - T_{N_{t^-}} + \sum_{i=1}^{N_{t^-}} (1-\rho_C)^{1-U_i} x_i\right). \quad (25)$$

The failure intensity for the perfect CM and imperfect Kijima I PM model is given by;

$$\lambda(t|H_{t^-}) = \lambda\left(t - C_{K_{t^-}} + \sum_{i=1}^{K_{t^-}} (1-\rho_P) U_i w_i\right). \quad (26)$$

The failure intensity for an imperfect ARA_1-CM and minimal PM model is given by:

$$\lambda(t|H_{t^-}) = \lambda\left(t - \rho_C T_{N_{t^-}}\right) \quad (27)$$

The failure intensity for the minimal CM and imperfect ARA_1-PM model is given by:

$$\lambda(t|H_{t^-}) = \lambda\left(t - \rho_P u_i C_{K_{t^-}}\right) \quad (28)$$

Some other combinations of PM and CM models are summarized in Table 4 below. These are further not considered here as they are special cases of imperfect CM and imperfect PM models.

Table 4: Other combinations of PM and CM Models

Sl. No.	Type of maintenance at CM / Model	Type of maintenance at PM / Model	Intensity of failure	Model Properties
6	Imperfect / Kijima 1	Perfect	$\prod_{j=1}^{M_\tau} \lambda_j\left(t - T_{N_{t^-}} + \sum_{i=1}^{N_{t^-}} (1-\rho_C)^{1-U_i} x_i\right)$	ρ_C = Imperfect ρ_P = 1
7	Perfect	Imperfect / Kijima 1	$\lambda\left(t - C_{K_{t^-}} + \sum_{i=1}^{K_{t^-}} (1-\rho_P) U_i W_i\right)$	ρ_C = 1 ρ_P = Imperfect
8	Imperfect / ARA_1	Minimal	$\lambda\left(t - \rho_C T_{N_{t^-}}\right)$	ρ_C = Imperfect ρ_P = 0
9	Minimal	Imperfect / ARA_1	$\lambda\left(t - \rho_P U_i C_{K_{t^-}}\right)$	ρ_C = 0 ρ_P = Imperfect

2.3 Parameter estimation

The most commonly used method of inferring the parameters of the failure process of a repairable system is the method of maximum likelihood estimation as given in Lindqvist (2006), as this method is easily tractable and possesses the some good statistical properties.

The likelihood (Doyen and Gaudoin, 2011) of the PM-CM process is given by:

$$L_t(\theta) = \prod_{i=1}^{K_{t^-}} \left(\lambda_{C_i}(i-1, W_{i-1}, U_{i-1})\right)^{1-U_i} exp\left(-\sum_{j=1}^{K_{t^-}+1} \int_{c_{j-1}}^{c_j} \lambda_s\,(j-1, W_{j-1}, U_{j-1}) ds\right) \quad (29)$$

where $C_{K_{t^-}+1}$ is set equal to t.

The likelihood for the minimal CM and minimal PM process at (3) with a power law as the initial failure intensity is given by:

$$L(\theta) = \prod_{i=1}^{k} \left(\alpha\beta t_i^{\beta-1}\right)^{1-u_i} exp(-\alpha t_n^{\beta}) \tag{30}$$

The likelihood for the ARA_1 CM and ARA_1 PM process at (14) with a power law process as the initial failure intensity is given by:

$$L(\theta) = \prod_{i=1}^{k}(\alpha\beta)^{(1-u_i)} {c_1}^{(1-u_1)(\beta-1)} \prod_{i=2}^{k}\left((c_i - \rho_P^{u_i}\rho_C^{(1-u_i)}c_j)^{(1-u_i)}\right)^{\beta-1} exp\left(-\alpha {c_1}^{\beta} - \alpha \sum_{i=2}^{k}\begin{pmatrix}(c_i - \rho_P^{u_i}\rho_C^{(1-u_i)}c_j)^{\beta} \\ -(c_{i-1} - \rho_P^{u_i}\rho_C^{(1-u_i)}c_j)^{\beta}\end{pmatrix}\right) \tag{31}$$

The model with the maximum log likelihood function may provide the model with the best fit among the alternatives chosen. A better check for models will be the Akaike likelihood criterion (AIC), which favours models with large likelihood function and the small number of parameters. The criterion is given by:

$$AIC(k) = -2\, ln\, L + 2k \tag{32}$$

where k is the number of parameters of the model.

The model with the minimum AIC estimate is considered as the model with a better fit.

However, when p is large as compared to n the sample size, a corrected version of the AIC, called AIC_C should be used for obtaining better model fit. The AIC_C is given by:

$$AIC_C(k) = -2\, ln\, L + 2k + \frac{2k(k+1)}{(n-k-1)}. \tag{33}$$

The goodness-of-fit test for obtaining the model with the best fit using the residual process can be use, see Wu,(2019), for example. The residual process for the above models is given by;

$$\hat{\varepsilon}(t) = N(t) - \Lambda(t|H_{t^-}). \tag{34}$$

The process $\{\varepsilon(t), t \geq 0\}$ should follow the normal distribution and have uncorrelated increments if the failure process model with intensity $\lambda(t|H_{t^-})$ is correctly specified. The Cramer von Mises (CvM) test can be used to test for normality and the Breusch-Godfrey (BG) test to test the serial correlation of the increments of the error process.

A two step methodology is proposed to obtain the model with the best fit to the data set. First the log-likelihood, AIC and AIC_C values of the model are estimated and the model with the least AIC_C value is chosen and checked for goodness of fit. If the goodness of fit test is passed, the model is chosen as the best fit model to the dataset. If the goodness of fit test fails, the model with the next least AIC_C value is chosen and again checked for the goodness of fit. This process is repeated till a model passes the goodness of fit test and this model is chosen as the model with the best fit to the dataset.

The best-fit model and its estimated parameters are used to understand the features of the PM-CM process of the system and then to optimise its maintenance policies.

The parameters of interest to the maintenance personnel such as the expected times to failure and the expected number of failures can be obtained through simulation of the best-fit model using the inverse transform method. The probability $F(t_i)$ is generated as a uniform random variable in (0, 1).

The failure time t_{i+1} for the GRA-CP CM and GRA-CP PM process at Eq. (21) with a log linear process as its initial failure intensity is then given by:

$$t_{i+1} = \frac{ln\left(-\beta \prod_{j=1}^{i} (\rho_P)^{U_j} (\rho_C)^{1-U_j} \, ln(1-u) + exp\left(\alpha + \beta \prod_{j=1}^{i} (\rho_P)^{U_j} (\rho_C)^{1-U_j} t_i\right)\right) - \alpha}{\beta \prod_{j=1}^{i} (\rho_P)^{U_j} (\rho_C)^{1-U_j}}. \quad (35)$$

As PM times are planned, the failure times obtained through simulation are compared to the next planned PM time. If the simulated failure time is greater than the PM time, then the PM time is considered in its place and the simulation continued. To obtain the expected times to failures, the average of 1000 sets of simulated failure times is considered.

3. Simulation

All the PM-CM process models proposed in the previous sections are simulated and the failure intensities are plotted in Figs. 2 to 15. Only the NHPP with the power law intensity function with shape parameter β=3 is considered with all the models, the renewal process with inter-failure times following the Gumbel distribution $\alpha 3x^2$ with local time PM and CM models and power law NHPP $\alpha 3t^2$ with global time PM and CM models. The scale parameter α is so chosen as to have the timeline within 30 units of time maximum for all the models. Planned PM times are considered at every 5 time units for the simulated data sets i.e., at 5, 10, 15, 20, 25 units of time. It is also considered that the effectiveness of PM is better than that of CM. Hence $\rho_p > \rho_C$ in all cases. A vertical dashed line is used to indicate the PM times in the failure intensity plots. The parameters considered for plotting the intensities are given in Table 5.

Table 5: Parameters of PM and CM models used for simulating their failure intensities

Parameter/ Model No	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Apha	0.2	0.2	0.02	0.02	0.01	0.01	1	1	0.02	0.02	0.02	0.02	0.02	0.02

Beta	3	3	3	3	3	3	3	3	3	3	3	3	3	3
Rho_C	1	1	0	0	0.3	0.3	0.7	0.7	0.5	0.5	0.5	0.5	0.05	0.1
Rho_P	1	0	0	1	0.7	0.7	0.9	0.9	0.7	0.7	0.7	0.7	0.3	0.3

All CM actions are carried out upon failures while PM actions are scheduled. and time based and censor the failure process.

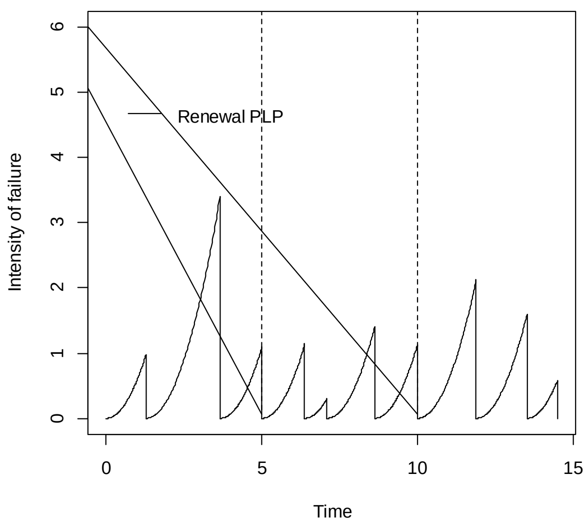


Fig. 2: Intensity of failure vs times to failure for simulated data with PM and CM Model 1 with Renewal Process PLP

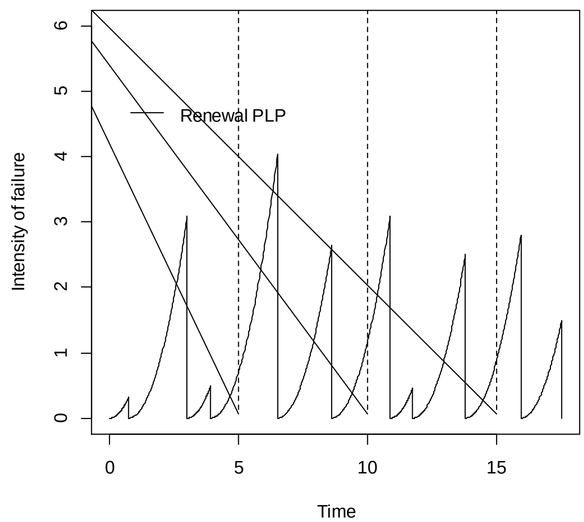
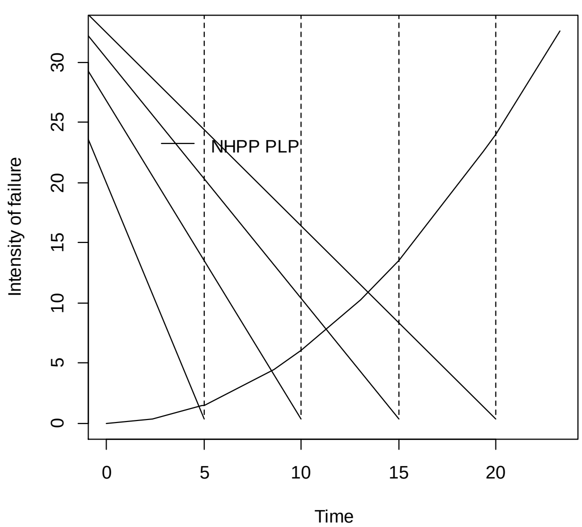


Fig. 3: Intensity of failure vs times to failure for simulate data with PM and CM Model 2 with Renewal Process PLP



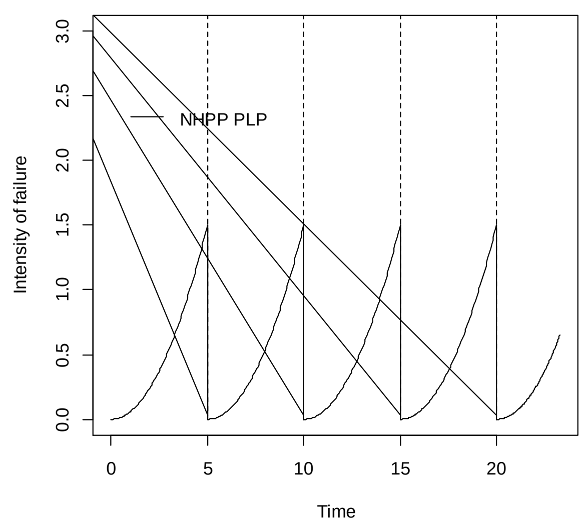
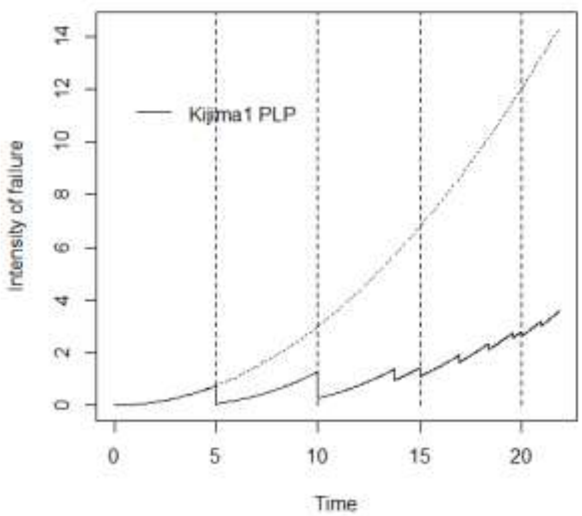


Fig. 4: Intensity of failure vs times to failure for simulated data with PM and CM Model 7 with NHPP PLP

Fig. 5: Intensity of failure vs times to failure for simulate data with PM and CM Model 8 with NHPP PLP



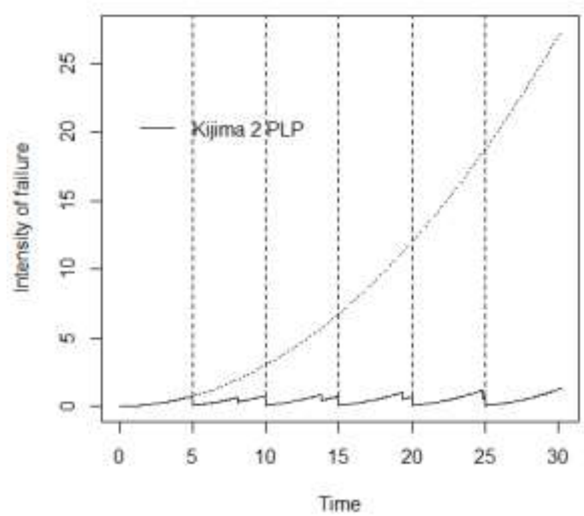
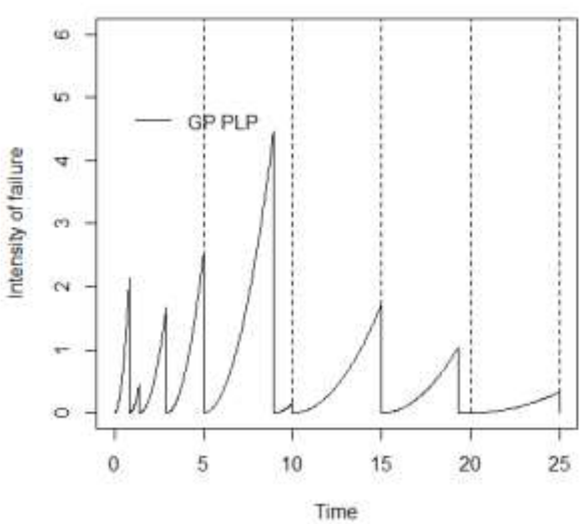


Fig. 6: Intensity of failure vs times to failure for simulated data with PM and CM Model 3 with Kijima 1 Process PLP

Fig. 7: Intensity of failure vs times to failure for simulate data with PM and CM Model 4 with Kijima 2 Process PLP



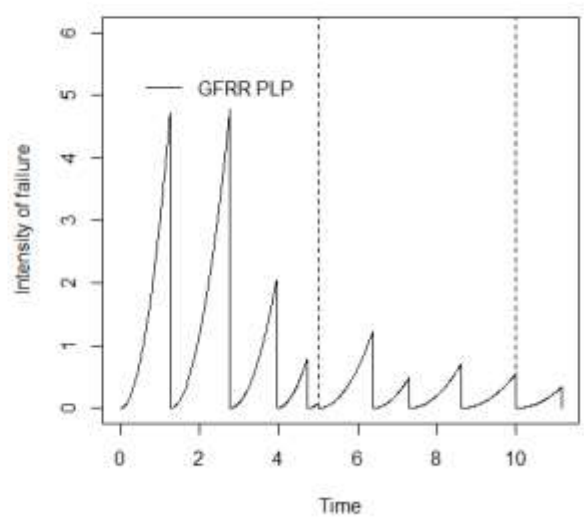
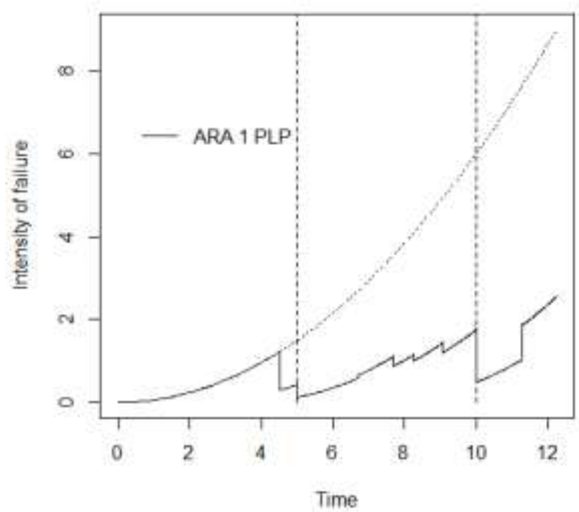


Fig. 8: Intensity of failure vs times to failure for simulated data with PM and CM Model 5 with GP PLP

Fig. 9: Intensity of failure vs times to failure for simulate data with PM and CM Model 6 with GFRR PLP



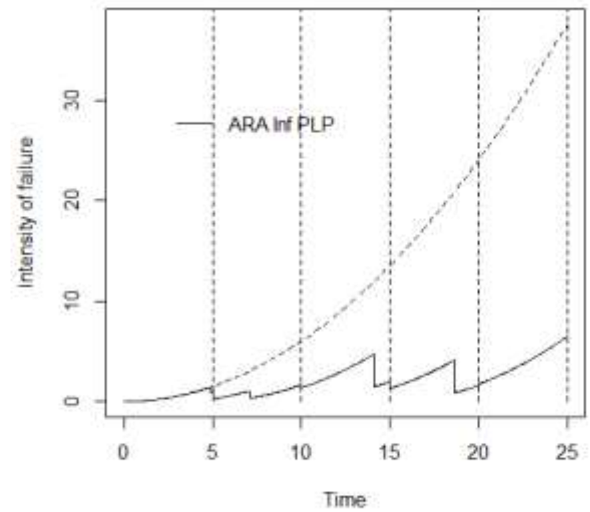
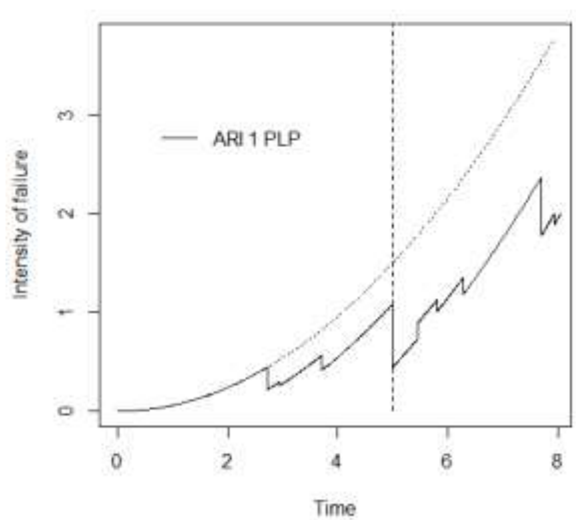


Fig. 10: Intensity of failure vs times to failure for simulated data with PM and CM Model 9 with ARA 1 PLP

Fig. 11: Intensity of failure vs times to failure for simulate data with PM and CM Model 10 with ARA Inf PLP



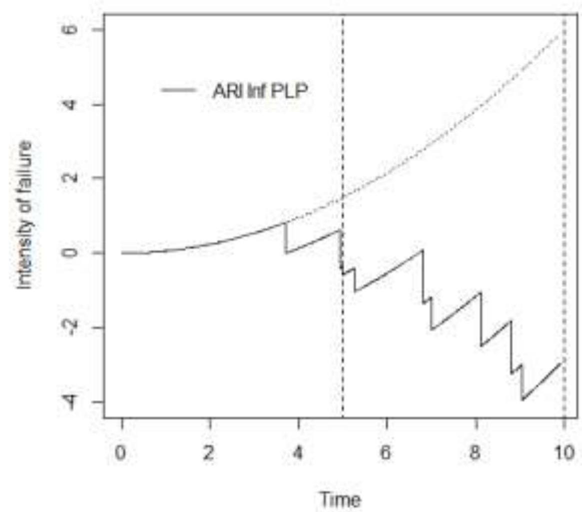
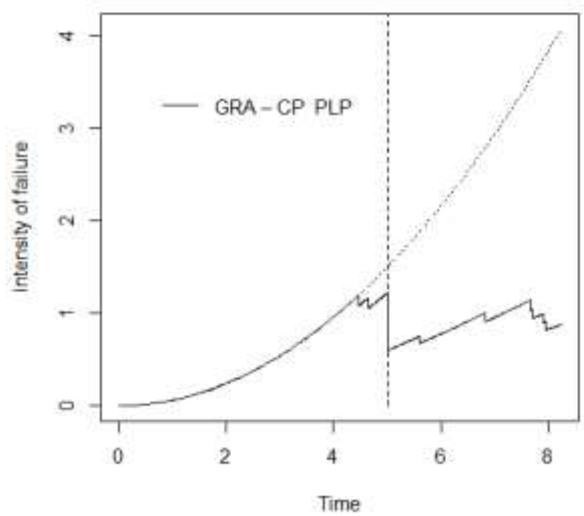


Fig. 12: Intensity of failure vs times to failure for simulated data with PM and CM Model 11 with ARI 1 PLP

Fig. 13: Intensity of failure vs times to failure for simulate data with PM and CM Model 12 with ARI Inf PLP



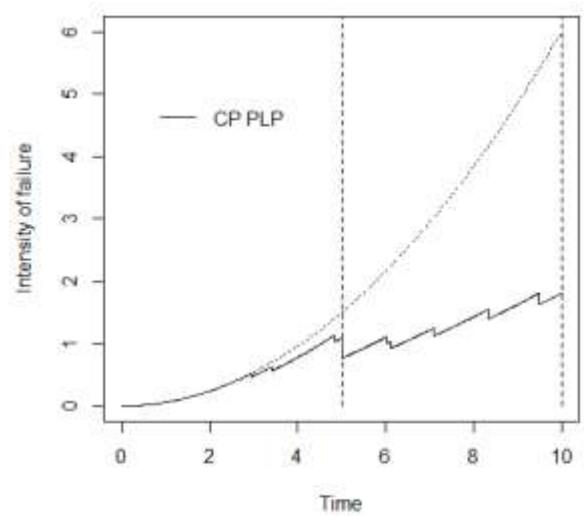


Fig. 14: Intensity of failure vs times to failure for simulated data with PM and CM Model 13 with GRA-CP PLP

Fig. 15: Intensity of failure vs times to failure for simulate data with PM and CM Model 14 with CP PLP

Fig. 2 depicts the failure intensity where both CM and PM are perfect. The failure intensity starts at zero and proceeds at an increasing rate till either CM or PM is carried out when it is reset to zero after a perfect maintenance. This occurs usually when a complete replacement takes place at CM or PM.

Fig. 3 depicts the failure intensity where CM is perfect and PM is minimal. The failure intensity starts at zero and proceeds at an increasing rate till a CM occurs when it is reset to zero after a perfect maintenance. Scheduled PM actions take place in between but do not affect the failure process that continues after PM at the same level at which it was prior to the PM. This occurs usually when a complete replacement takes place at CM but only minor maintenances are carried out at PM.

Fig. 4 depicts the failure intensity where both CM and PM are minimal. The failure intensity starts at zero and proceeds at an increasing rate. Neither CM nor PM affects the failure intensity, which remains at the same level as it was prior to the CM or PM action. This occurs usually when only minor maintenances take place at CM or PM.

Fig. 5 depicts the failure intensity where CM is minimal and PM is perfect. The failure intensity starts at zero and proceeds at an increasing rate till a PM occurs when it is reset to zero after a perfect maintenance. CM takes place in between when any failure occurs but does not affect the failure process that continues after CM at the same level at which it was prior to the CM. This occurs usually when a complete replacement takes place at PM but only minor maintenances are carried out at CM.

Figs. 6 to 15 depict the failure intensities of different types of imperfect maintenance models applied to the PM and CM maintenance processes. The PM maintenance actions have been assumed to be more effective as compared to the CM maintenance actions as given in Table 5. Figs. 6 to 9 represent local time imperfect maintenance processes where the time to first failure follows a renewal process. Figs. 10 to 15 depict global time imperfect maintenance processes.

Figs. 6 and 7 depict the failure intensity of the Kijima 1 and Kijima 2 PM and CM processes respectively. Here the maintenance factor acts linearly on the process and the failure intensity drops to a lower level after maintenance at PM times as compared to CM times as PM has been assumed to be more effective and improves the virtual age to a younger level.

Figs. 8 and 9 depict the failure intensity of the geometric process and geometric failure rate reduction process, respectively. Here as the maintenance factor acts geometrically on the process and the maintenance action at PM being more effective slows down the failure process as compared to the maintenance action at CM time.

Figs. 10 to 13 depict the failure intensity of the ARA_1, ARA_∞, ARI_1 and ARI_∞ imperfect maintenance models, respectively. Here the maintenance factor acts linearly on the process and in all the cases the PM action causes the failure intensity to drop to a lower level as compared to the CM action. It can be seen that this sometimes causes the intensity of the failure process to increase after a CM action in ARA_1 and ARI_1 imperfect maintenance models thus increasing its virtual age.

Figs. 14 and 15 depict the failure intensity of the GRA-CP and CP imperfect maintenance models, respectively. Here again the maintenance factor acts geometrically on the process and the PM action being more effective not only lowers but also improves the failure intensity, as compared to the CM maintenance action.

Similar figures can be developed for failure intensities with log linear process also.

4. Case studies

Three datasets of PM and CM times from repairable systems are considered for analysis and assessment of maintenance effectiveness in this section. The three datasets are, as shown in Table 3

- Stubs within a heat exchanger that warms up the feed water of a fossil fired thermal power plant (Doyen and Gaudoin, 2011),
- Component 2 used in a continuous process industry (Ascher and Kobbacy, 1995) and
- Roller Mill of a Cement Plant (Love and Guo, 1991).

Table 3: Data Sets of Repairable Systems used in this paper

Sl. No.	Dataset	No of CMs and PMs	Data Source
1	Stubs	7 CM + 4 PM	Doyen and Gaudoin (2011)
2	Component 2	11 CM + 18 PM	Ascher and Kobbacy (1995)
3	Roller Mill	18 CM + 13 PM	Love and Guo (1991)

All the models given above with different combinations of PM and CM effects are applied to these datasets. The results and analysis are presented in the sub-section below.

4.1 Analysis of Stubs Dataset

The dataset of PM and CM in the number of cold starts given in Krit (2006) and Doyen and Gaudoin (2011) of stubs within a heat exchanger that warms up the feed water of a fossil fired thermal power plant covering a period from 1997 to 2006 is considered for analysis. This dataset consists of 7 CM and 4 PM actions and is a very small dataset. Stubs are joined by a welded connection that are subjected to thermal fatigue, especially during cold starts, and tend to crack and leak. This necessitates the boiler to be shutdown for repairing the leaks and can lead to costly outages of the power plant. To mitigate this, inspections are carried out for detecting cracks, which, if found, are gouged out and preventively repaired. As cold starts have a great bearing on the development of cracks due to differential thermal expansion between the shell and tubes, the CM and PM actions are measured in terms of the number of cold starts.

Applying all the PM and CM models at (1) to (14) to this dataset, the results are tabulated in Table 4. No convergence is obtained for ARI_1 model (11) with this data set.

Table 4: Parameters of Models fitted to the dataset of PMs and CMs of stubs

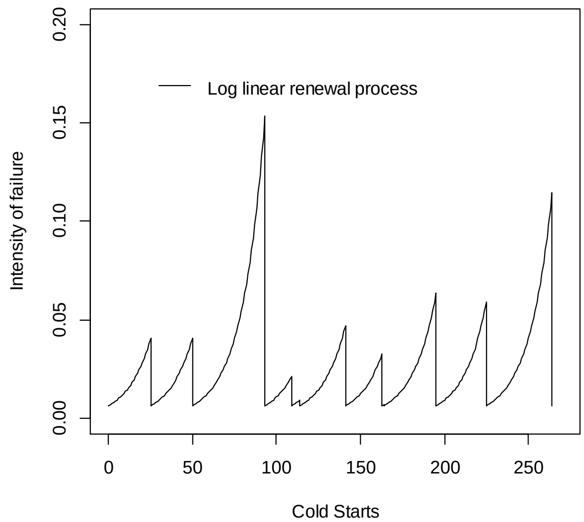
Sl No	CM Maintenance Effect	PM Maintenance Effect	Maintenance Model / Baseline	Log Likeli-hood	AIC	AIC_C	Parameter Estimates			
							α	β	ρ_c	ρ_p
1	Perfect	Perfect	Renewal / Weibull	31.6	67.2	68.7	3.48×10^{-3}	1.60	--	--
2	Perfect	Perfect	Renewal / Log Linear	30.17	64.3	65.8	-5.05	7.40×10^{-2}	--	--
3	Perfect	Minimal	Renewal / Weibull	32.3	68.6	70.1	0.013	1.20	--	--
4	Perfect	Minimal	Renewal / Log Linear	32.1	68.2	69.7	-4.09	0.018	--	--
5	Minimal	Minimal	NHPP / PLP	32.2	68.4	75.07	5.65×10^{-3}	1.28	--	--
6	Minimal	Minimal	NHPP / LLP	32.4	68.8	75.47	-3.60	-2.21×10^{-4}	--	--
7	Minimal	Perfect	NHPP / PLP	31.2	66.4	73.07	0.001	1.74	--	--
8	Minimal	Perfect	NHPP / LLP	31.8	67.6	74.27	-4.25	0.013	--	--
9	Imperfect	Imperfect	Kijima 1 / Weibull	30.7	69.4	76.07	1.76×10^{-4}	2.36	0.94	1
10	Imperfect	Imperfect	Kijima 1 / Log Linear	29.8	67.6	74.27	-4.64	0.085	0.96	1.39

Sl No	CM Maintenance Effect	PM Maintenance Effect	Maintenance Model / Baseline	Log Likeli-hood	AIC	AIC_C	Parameter Estimates			
							α	β	ρ_c	ρ_p
11	Imperfect	Imperfect	Kijima 2 / Weibull	29.5	67	73.67	$1.09\text{x}10^{-5}$	3.06	0.56	1
12	Imperfect	Imperfect	Kijima 2 / Log Linear	29.5	67	73.67	-5.26	0.078	0.73	1.29
13	Imperfect	Imperfect	GP / Weibull	31.3	70.6	77.27	$3.22\text{x}10^{-3}$	1.59	0.87	1.4
14	Imperfect	Imperfect	GP / Log Linear	29.7	67.4	74.07	-5.02	0.065	0.89	1.31
15	Imperfect	Imperfect	GFRR / Weibull	31.3	70.6	77.27	$3.22\text{x}10^{-3}$	1.59	0.79	1.7
16	Imperfect	Imperfect	GFRR / Log Linear	29.9	67.8	74.47	-5.03	0.072	0.81	1.55
17	Imperfect	Imperfect	ARA_1 / PLP	29.7	67.4	74.07	$2.35\text{ x}10^{-5}$	2.88	0.91	1
18	Imperfect	Imperfect	ARA_1 / LLP	29.3	66.6	73.27	-5.13	0.08	0.98	1.07
19	Imperfect	Imperfect	ARA_∞ / PLP	28.8	65.6	72.27	$6.11\text{x}10^{-8}$	4.15	0.41	0.53
20	Imperfect	Imperfect	ARA_∞ / LLP	28.8	65.6	72.27	-7.02	$7.79\text{x}10^{-2}$	0.38	0.49
21	Imperfect	Imperfect	ARI_∞ / PLP	29.5	67	73.67	$4.78\text{x}10^{-4}$	1.99	0.40	0.74
22	Imperfect	Imperfect	ARI_∞ / LLP	29.5	67	73.67	-2.25	$9.59\text{x}10^{-3}$	0.75	0.19
23	Imperfect	Imperfect	GRA-CP / PLP	28.4	64.8	71.47	$1.7\text{ x}10^{-13}$	7.05	0.07	0.12
24	Imperfect	Imperfect	GRA-CP / LLP	28.19	64.4	71.07	-11.6	0.14	0.16	0.12
25	Imperfect	Imperfect	CP / PLP	28.4	64.8	71.47	$1.7\text{ x}10^{-13}$	7.05	0.68	0.53
26	Imperfect	Imperfect	CP / LLP	29.4	66.8	73.47	-5.5	0.074	0.82	0.75
27	Imperfect	Perfect	Kijima 2 / Weibull	29.5	65	68.43	$1.16\text{x}10^{-5}$	3.05	0.57	1

This dataset has been analysed by Doyen and Gaudoin (2011). They applied only ARA_1-PM and ARA_1-CM, which are equivalent to Kijima 1-PM and Kijima 2-CM (5), and ARA_∞-PM and ARA_∞-CM, which are equivalent to Kijima 2-PM and Kijima-2 CM (6) to this dataset. They arrived at the conclusion that PM is perfect and renews the intensity while CM renews the intensity by half. They proposed that a perfect PM with imperfect Kijima 2 CM will suit the dataset best. The estimated parameters with this model is given at Sl. No. 27 of Table 4.

As can be seen from the table, however, the minimum AIC_C value is obtained for the log Linear model with perfect CM and perfect PM combination and hence this model provides the best fit to the data. Both indicate perfect maintenance. The residuals from the models were checked for normality and serial correlation. The CvM test gives a p-value of 0.95 and p-value with BG test for order 1 is 0.40. Hence, the model provides a good fit to the data.

The intensity of failures and cumulative intensity of failures with perfect maintenance log linear renewal process is given in Figs. 16 and 17, respectively.



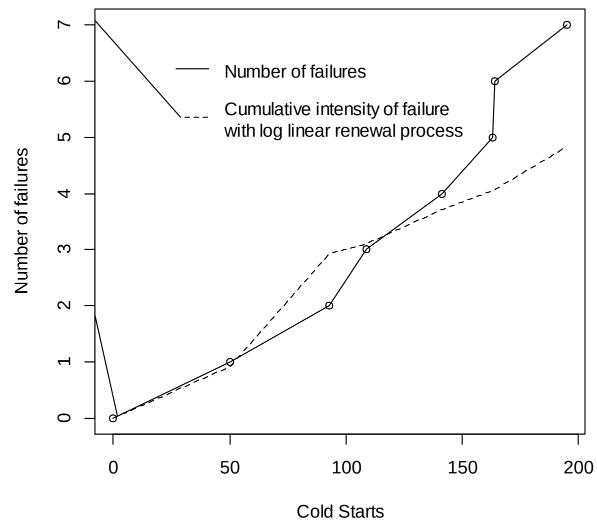


Fig. 16: Intensity of failure vs cold starts to failure for Stubs failure data with log linear renewal process considering CM and PM

Fig. 17: Cumulative intensity of failure vs cold starts to failure for Stubs failure data with log linear renewal process considering CM and PM

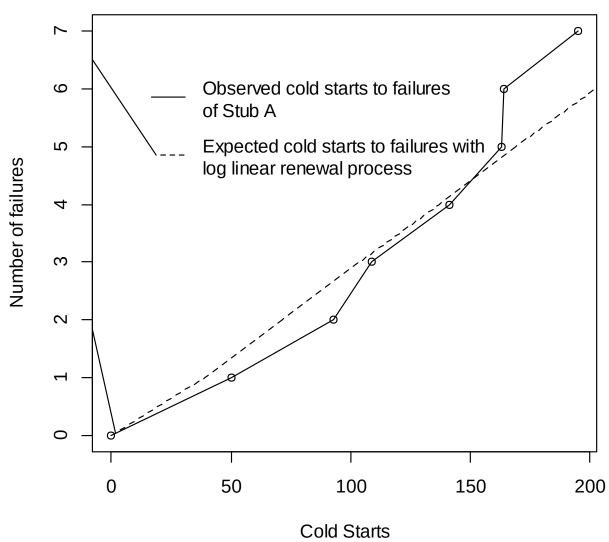


Fig. 18: Expected cold starts to failure vs observed cold starts to failure for Stubs failure data with log linear renewal process considering CM and PM

The expected cold starts to failure are obtained through simulation. The expected cold starts are obtained as the average of 1000 simulations. The expected cold starts to failures with the log linear renewal process for Stubs failure data with CM and PM is given in Fig. 18. It can be seen that it is good fit to the observed cold starts to failure.

If two PMs are considered to be carried out at 330 and 396, i.e. 66 cold starts each from the last PM considered at 264 cold starts, the expected number of cold starts obtained as the average of 1000 simulations are given in Table 5. It can be seen that each of the two CMs occur between the PMs. Inspection may be further enhanced in a planned manner and PM may be carried out earlier, to reduce the number of failures.

Table 5: Expected number of cold starts considering PM every 66 cold starts

Sl. No.	Expected number of cold starts with planned PM
1	264(PM), 274, 311, 330(PM), 346, 380, 396(PM)

The best-fit model being perfect CM and perfect PM changes the perspective with which the maintenance process has been viewed in Doyen and Gaudoin (2011) and has implications on the further maintenance strategy to be followed.

In the first place the dataset is probably too small to fit an imperfect maintenance model which is brought out by the AIC_C values. Though, Doyen and Gaudoin (2011) propose the models as given at (1) to (4) and listed in Table 2, they have not used these models to analyse the dataset.

On analysing the estimated values of parameters, it can be seen that the $\beta' s$ value indicates that the stubs are in the wear out phase, thus carrying out PM should be effective in this case. Here CM is also found to be perfect along with PM and is as effective as PM and not half as effective as given in Doyen and Gaudoin (2011). Given that both CM and PM have the same effect, either can be used for maintenance of these stubs. However, considering disruption in production and the costly resultant outage of the plant, if a failure takes place during operating time, it may be better to optimize the inspection schedule of the stubs so as to reduce the failures. Presently, the PMs are being carried out at an average of approximately 66 cold starts. Inspection may be further enhanced in a planned manner and PM may be carried out earlier, to reduce the number of failures.

4.2 Analysis of Component 2 Dataset

The dataset of PM and CM times of Component 2, as given in Ascher and Kobbacy (1995), is considered for analysis. Component 2 is used in a continuous-process industry where the equipment is run continuously for twenty four hours a day and only stopped for CM or PM. This dataset consists of 11 CM and 18 PM actions. The type of components or the nature of PM and CM actions carried out on the system are not known in this case. All that is known is that all PM actions are of one hour duration and carried out at an average of 206 hours. CM durations are of varying times ranging from 4 to 29 hours.

Applying all the PM and CM models at (1) to (14) to this dataset in Ascher and Kobbacy (1995), the results are shown in Table 6. No convergence is obtained for ARI_∞ model (12) with a log linear baseline for this data set.

Table 6: Parameters of Models fitted to the dataset of PMs and CMs Of Component 2

Sl No	CM Maintenance Effect	PM Maintenance Effect	Maintenance Model / Baseline	Log Likeli-hood	AIC	AIC_C	Parameter Estimates			
							α	β	ρ_c	ρ_p
1	Perfect	Perfect	Renewal / Weibull	72.6	149.2	149.66	0.025	0.58	--	--
2	Perfect	Perfect	Renewal / Log Linear	73.16	150.32	150.78	-5.073	-9.86×10^{-3}	--	--
3	Perfect	Minimal	Renewal / Weibull	69.86	143.72	144.18	0.067	0.50	--	--
4	Perfect	Minimal	Renewal / Log Linear	72.63	149.26	149.72	-5.054	-2.57×10^{-3}	--	--
5	Minimal	Minimal	NHPP / PLP	74.21	152.42	152.88	0.0367	0.69	--	--
6	Minimal	Minimal	NHPP / LLP	71.35	146.7	147.16	-4.625	-8.53×10^{-4}	--	--
7	Minimal	Perfect	NHPP / PLP	75.03	154.06	154.52	0.0025	1.029	--	--
8	Minimal	Perfect	NHPP / LLP	74.5	153	153.46	-5.4	-3.79×10^{-3}	--	--
9	Imperfect	Imperfect	Kijima 1 / Weibull	71.43	150.86	152.53	0.213	0.287	0.96	0.995
10	Imperfect	Imperfect	Kijima 1 / Log Linear	70.22	148.44	150.11	-4.235	-8.37×10^{-3}	0.71	0.98
11	Imperfect	Imperfect	Kijima 2 / Weibull	69.54	147.08	148.75	0.11	0.403	0.998	0.43
12	Imperfect	Imperfect	Kijima 2 / Log Linear	70.92	149.84	151.51	-4.307	-0.0101	0.78	0.53
13	Imperfect	Imperfect	GP / Weibull	70.16	148.32	149.99	0.054	0.624	0.92	0.86
14	Imperfect	Imperfect	GP / Log Linear	68.63	145.26	146.93	-2.788	-0.057	0.8	0.85
15	Imperfect	Imperfect	GFRR / Weibull	70.16	148.32	149.99	0.054	0.624	0.95	0.91
16	Imperfect	Imperfect	GFRR / Log Linear	70.73	149.46	151.13	-4.197	-8.85×10^{-3}	0.95	0.91
17	Imperfect	Imperfect	ARA_1 / PLP	68.9	145.8	147.47	0.22	0.31	1	0.9
18	Imperfect	Imperfect	ARA_1 / LLP	70.2	148.4	150.07	-4.23	-8.89×10^{-3}	0.94	0.91
19	Imperfect	Imperfect	ARA_∞ / PLP	71.2	150.4	152.07	0.22	0.28	0.86	0.84

Sl No	CM Maintenance Effect	PM Maintenance Effect	Maintenance Model / Baseline	Log Likeli-hood	AIC	AIC_C	Parameter Estimates			
							α	β	ρ_c	ρ_p
20	Imperfect	Imperfect	ARA∞ / LLP	71.2	150.4	152.07	-4.2	-1.11×10^{-2}	0.61	0.60
21	Imperfect	Imperfect	ARI_1 / PLP	73.7	155.4	157.07	0.028	0.77	0	0.46
22	Imperfect	Imperfect	ARI_1 / LLP	73.2	154.4	156.07	-3.77	-0.95	0.24	0.09
23	Imperfect	Imperfect	ARI∞ / PLP	71.9	151.8	153.47	9.42×10^{-3}	0.99	0.07	0.08
24	Imperfect	Imperfect	GRA-CP / PLP	67.33	142.66	144.33	9.61×10^{-14}	5.16	0.12	0.11
25	Imperfect	Imperfect	GRA-CP / LLP	67.24	142.48	144.15	-12.31	0.024	0.10	0.11
26	Imperfect	Imperfect	CP / PLP	67.33	142.66	144.33	9.61×10^{-14}	5.16	0.41	0.38
27	Imperfect	Imperfect	CP / LLP	67.57	143.14	144.81	-4.605	-0.0101	-0.15	-5.19

Minimal maintenance is considered for CM and PM schedules are optimised considering perfect maintenance (4) and minimal maintenance (3) for PM in Ascher and Kobbacy (1995). However, as can be seen from the table, the minimum AIC_C value is obtained for the GRA – CP model with imperfect CM and imperfect PM combination for a log linear baseline process. The residuals from the models were checked for normality and serial correlation. The CvM test gives a p-value of 0.73 and the p-value with BG test for order 1 is 0.77. Hence, the model provides a good fit to the data.

The intensity of failures and cumulative intensity of failures with the GRA – CP model for a log linear baseline process is given in Figs. 19 and 20 respectively.

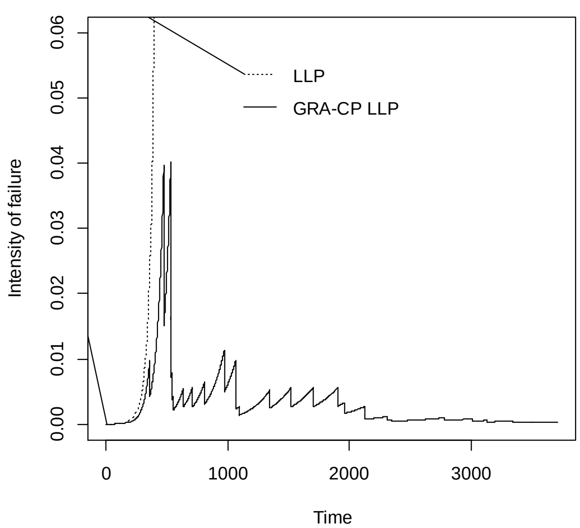


Fig. 19: Intensity of failure vs times to failure for Component 2 failure data with

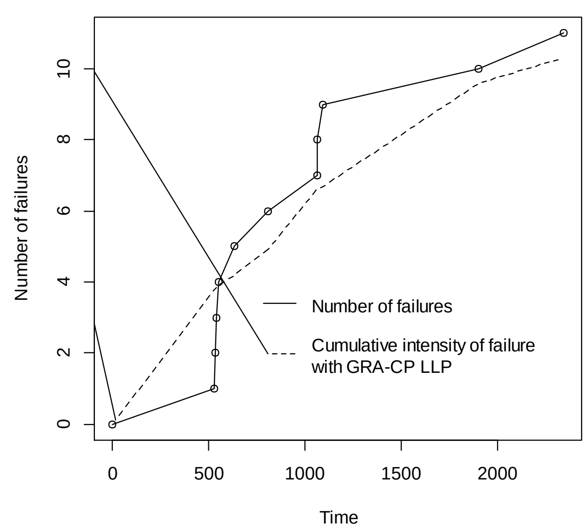


Fig. 20: Cumulative intensity of failure vs times to failure for Component 2 failure

GRA – CP LLP considering PM and CM

data with GRA – CP LLP considering PM and CM

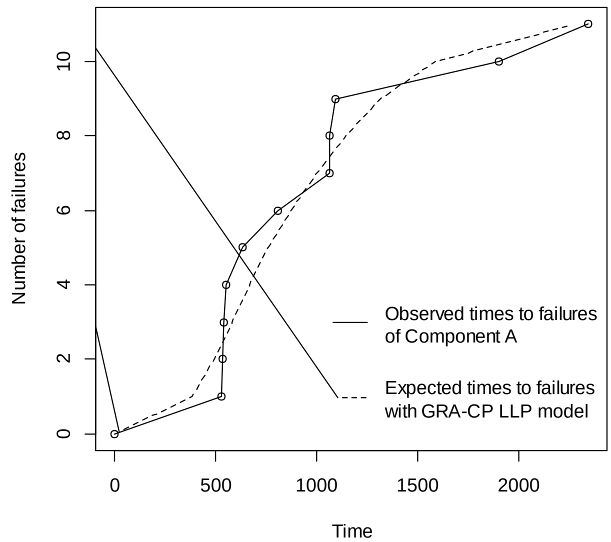


Fig. 21: Expected times to failure vs observed times to failure for Component 2 failure data with GRA – CP LLP considering CM and PM

The expected times to failure are obtained through simulation with (25). The expected times are obtained as the average of 1000 simulations. The expected times to failures with the GRA CP – LLP model for Component 2 failure data considering CM and PM is given in Fig. 21. It can be seen that it is close match to the observed times to failure.

Presently the PMs are being carried out at an average of approximately 206 days. Hence, considering two PMs to be carried out at 3919 and 4125 i.e., 206 days each from the last PM time 3713, the expected times to failures obtained as the average of 1000 simulations are given in Table 7. It can be seen that no CMs before either PM. The next CM is expected at 4157 days. This further confirms the improving trend in the maintenance of Component 2.

Table 7: Expected times to failure considering PM every 206 days

Sl. No.	**Expected times to failures with planned PM times**
1	3919(PM), 4125(PM), 4157

Analysis of the estimated parameter values indicates a β value, which shows a slightly deteriorating system. However, in combination with ρ_c and ρ_p values, the β value for this model indicates an improving system. This can also be seen in general from the β values obtained with the other models. Figs. 20, 21 and 22 show an improving trend for the failure intensity. Both PM and CM are equally effective. However, considering disruption in production, if a failure takes place during operating time, it may be better to preventively maintain component 2 to an optimum schedule so as to reduce the failures.

4.3 Analysis of Roller Mill Dataset

The dataset of PM and CM times given in Love and Guo (1991) of a Roller Mill in a Cement Plant is considered for analysis. The data consist of 13 PM and 18 CM observations for the period November 1988 to March 1989. Applying all the PM and CM models at (1) to (14) to this dataset, the results are tabulated in Table 8.

Table 8: Parameters of Models fitted to the dataset of PMs and CMs of Roller Mill

Sl No	CM Maintenance Effect	PM Maintenance Effect	Maintenance Model / Baseline	Log Likeli-hood	AIC	AIC$_C$	Parameter Estimates			
							α	β	ρ_c	ρ_p
1	Perfect	Perfect	Renewal / Weibull	106.4	216.8	217.23	$1.26x10^{-4}$	0.87	--	--
2	Perfect	Perfect	Renewal / Log Linear	108.3	220.6	221.03	-5.73	$-7.95x10^{-3}$	--	--
3	Perfect	Minimal	Renewal / Weibull	101.3	206.6	207.03	$1.38x10^{-6}$	0.77	--	--
4	Perfect	Minimal	Renewal / Log Linear	102.3	208.6	209.03	-6.66	$-4.24x10^{-3}$	--	--
5	Minimal	Minimal	NHPP / PLP	109.5	223	223.43	$4.77x10^{-4}$	1.61	--	--
6	Minimal	Minimal	NHPP / LLP	109.6	223.2	223.63	-5.58	$5.09x10^{-4}$	--	--
7	Minimal	Perfect	NHPP / PLP	110.1	224.2	224.63	$7.21x10^{-3}$	1.22	--	--
8	Minimal	Perfect	NHPP / LLP	109.5	223	223.43	-4.78	$2.82x10^{-3}$	--	--
9	Imperfect	Imperfect	Kijima 1 / Weibull	106.2	220.4	221.94	$7.37x10^{-6}$	0.83	1	1
10	Imperfect	Imperfect	Kijima 1 / Log Linear	105.5	219	220.54	-6.98	$-2.3x10^{-3}$	0.25	2.95
11	Imperfect	Imperfect	Kijima 2 / Weibull	100.7	209.4	210.94	$7.81x10^{-9}$	2.42	0.04	0.01
12	Imperfect	Imperfect	Kijima 2 / Log Linear	101.6	211.2	212.74	-7.52	$-6.56x10^{-3}$	0.84	0.16
13	Imperfect	Imperfect	GP / Weibull	104.3	216.6	218.14	$3.33x10^{-5}$	0.91	0.91	1.47
14	Imperfect	Imperfect	GP / Log Linear	106.3	220.6	222.14	-6.50	$-2.17x10^{-4}$	0.91	1.47
15	Imperfect	Imperfect	GFRR / Weibull	104.3	216.6	218.14	$3.33x10^{-5}$	0.91	0.92	1.43
16	Imperfect	Imperfect	GFRR / Log Linear	106.4	220.8	222.34	-6.88	$-4.75x10^{-3}$	0.92	1.42

Sl No	CM Maintenance Effect	PM Maintenance Effect	Maintenance Model / Baseline	Log Likeli-hood	AIC	AIC_C	Parameter Estimates			
							α	β	ρ_c	ρ_p
17	Imperfect	Imperfect	ARA_1 / PLP	101.5	211	212.54	$5.14\text{x}10^{-7}$	1.57	0	0.42
18	Imperfect	Imperfect	ARA_1 / LLP	103.3	214.6	216.14	-6.66	$-5.13\text{x}10^{-3}$	1.09	1.08
19	Imperfect	Imperfect	ARA_∞ / PLP	104.0	216	217.54	0.014	2.47	0.041	0.040
20	Imperfect	Imperfect	ARA_∞ / LLP	102.3	212.6	214.14	-6.89	$6.43\text{x}10^{-3}$	0.051	0.050
21	Imperfect	Imperfect	ARI_1 / PLP	101.5	211	212.54	$2.19\text{x}10^{-6}$	1.56	0	0.26
22	Imperfect	Imperfect	ARI_1 / LLP	103.9	215.8	217.34	-5.81	-0.95	0.44	0.24
23	Imperfect	Imperfect	ARI_∞ / PLP	102.1	212.2	213.74	$6.51\text{x}10^{-6}$	2.1	0.039	0.037
24	Imperfect	Imperfect	ARI_∞ / LLP	108.7	225.4	226.94	-6.03	$-2.13\text{x}10^{-3}$	0.16	0.32
25	Imperfect	Imperfect	GRA-CP / PLP	108.6	225.2	226.74	$2.33\text{x}10^{-4}$	1.85	0.09	-0.33
26	Imperfect	Imperfect	GRA-CP / LLP	102.9	213.8	215.34	-5.62	$2.25\text{x}10^{-3}$	0.05	-0.1
27	Imperfect	Imperfect	CP / PLP	108.4	224.8	226.34	$3.20\text{x}10^{-3}$	1.85	0.08	-0.26
28	Imperfect	Imperfect	CP / LLP	100.9	209.8	211.34	-8.31	$9.59\text{x}10^{-3}$	0.26	0.28

Love and Guo (1991) analysed the data considering perfect PM with perfect CM and minimal CM with other covariates to optimize the PM schedules, respectively.

As can be seen from the table, the minimum AIC_C value is obtained for perfect CM with Weibull Renewal process and Minimal PM model. The residuals from the models were checked for normality and serial correlation. The CvM test gives a p-value of 0.65 and p-value with BG test for order 1 is 0.81. Hence, the model provides a good fit to the data.

The intensity of failures and cumulative intensity of failures with perfect CM with Weibull renewal process and minimal PM model are given in Figs. 22 and 23, respectively.

The expected times to failure are obtained through simulation. The expected times are obtained as the average of 1000 simulations. The expected times to failures with perfect CM with the Weibull renewal process and minimal PM model for Roller Mill failure data considering CM and PM is given in Fig. 24. It can be seen that it is a reasonably close match to the observed times to failure.

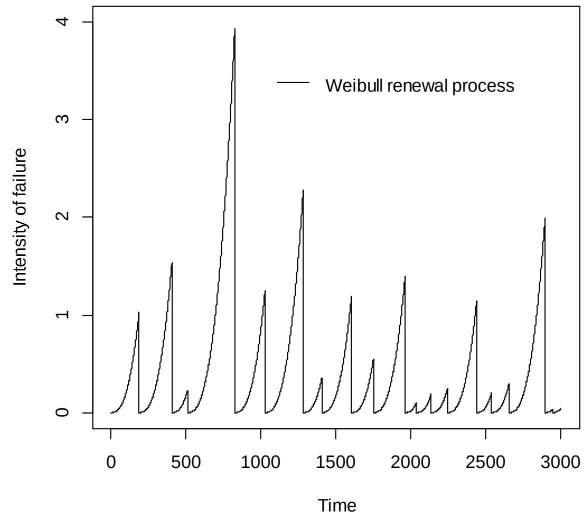


Fig. 22: Intensity of failure vs times to failure for Roller Mill failure data with perfect CM and minimal PM

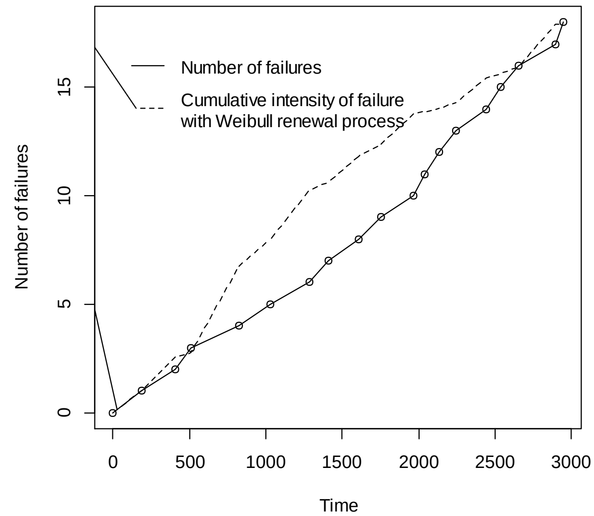


Fig. 23: Cumulative intensity of failure vs times to failure for Roller Mill failure data with perfect CM and minimal PM

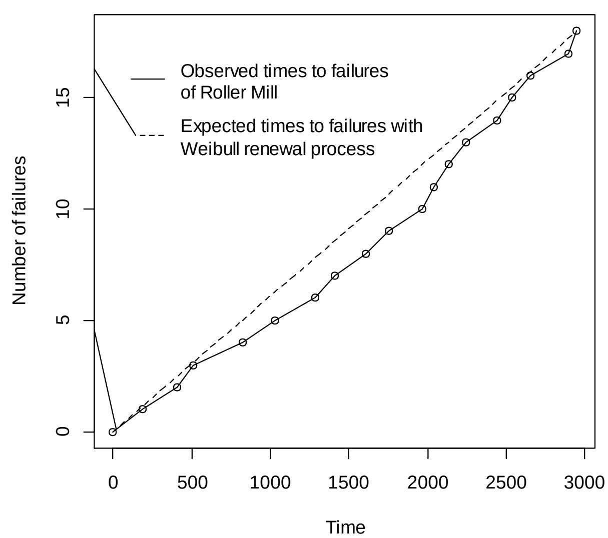


Fig. 24: Expected times to failure vs observed times to failure for Roller Mill failure data perfect CM and minimal PM

Presently the PMs are being carried out at an average of approximately 231 days. Hence considering two PMs to be carried out at 3234 and 3465 i.e., 231 days each from the last PM time 3003, the expected times to failures obtained as the average of 1000 simulations are given in Table 9. It can be seen that one CM before PM at 3234 and two CMs before PM at 3465 are expected.

Table 9: Expected times to failure considering PM every 231 days

Sl. No.	Expected times to failures with planned PM times
1	3003(PM), 3114.3, 3234(PM), 3278.5, 3441.7, 3465(PM)

The best fit model being GRA-CP CM and PM changes the perspective with which the maintenance process has been viewed in Love and Guo (1991) and has implications on the further maintenance strategy to be followed. It can be seen from the table that for the models where PM has been considered perfect as given at sl. No. 1, 2, 7 and 8 the AIC_C values are much higher. The effectiveness of PM is minimal and considering PM perfect as in Love and Guo (1991) will provide the wrong results.

Analyzing the estimated values of parameters it can be seen that the β value indicates that the failure intensity of the Roller Mill is increasing with time. Considering the fact that CM is perfect, and the effectiveness of PM is minimal, here CM can be used a strategy for carrying out maintenance if the cost of failure and consequent disruption of production is not high. PM work can be dropped all together. In case the cost of failure is not acceptable, then a much improved and more effective PM work has to be designed for the Roller Mill.

5. Conclusion

A systematic process has been adopted to assess the maintenance effectiveness with PM and CM for a repairable system. Once a good-fit model is arrived at, the model output can be interpreted to obtain an understanding of the maintenance effectiveness of PMs and CMs being carried out on the system.

The case studies clearly indicate that assumptions regarding models to be used for PM and CM data and maintenance effectiveness lead to sub-optimal results when optimizing maintenance. In all the case studies, the models originally fitted to the datasets proved to be different to the models arrived at using the methodology in this paper. Consequently, the estimation of maintenance effectiveness has also been changed. This has resulted in a changed maintenance strategy to be followed for the repairable systems under consideration.

Here only statistical fit of the models to the data is considered for understanding maintenance effectiveness with PM and CM models. The results are to be correlated with the feedback from the engineers / maintainers at site. Additionally, the properties of the models and their estimators are to be studied to obtain a better understanding of the modelling process. The estimated models can be further used to obtain quantities such as the expected number of failures based on the PM policy, which may be interesting to maintenance engineers . Only time based scheduled PM is considered here. This can be extended to condition based maintenance. There can be different types of PMs on the same system consisting of different sets of activities e.g., PM 1 consisting of inspection, cleaning, lubrication and routine maintenances, PM 2 consisting of replacement of some parts, and PM 3 consisting of thorough overhaul. PM 2 may include PM 1 activities and PM 3 may include PM 1 and PM 2 activities. The PM and CM models developed above can also be extended to such situations.

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