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Reliability of integrated electricity and gas supply system with performance substitution and sharing mechanisms*

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Abstract: The reliability of a system that shares a single type of resource with others has been studied in the literature. In reality, a system may require different types of resources. For example, a building service system may be powered by electricity (for lighting) and gas (for heating). However, in the literature, research in this area is scarce. This paper therefore investigates the reliability of a system with multiple nodes. Each node has demand requirement for two types of resources, which can be shared among the system nodes subject to their bandwidths, respectively. In addition, the resources may be able to substitute each other. This paper considers both unidirectional and bidirectional substitutions. The system is said failed if either resource supply in a node is smaller than its demand even after performance substitution and sharing. A universal generating function technique is proposed to evaluate the system reliability. Numerical examples are presented to illustrate the applicability of the model. The influences of bandwidths and substitution rates on system reliability are also discussed.

Keywords: Reliability, performance sharing, performance substitution, universal generating function, mixed energy

1. Introduction

Many recent papers have studied the reliability modeling and optimization of systems with performance sharing mechanisms, due to its wide applications (Levitin, 2011, Yu et al. 2014, Qiu and Ming, 2019). Xiao and Peng (2014) considered the

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optimal allocation and maintenance strategies for multi-state elements in series-parallel systems. They employed a universal generating function-based algorithm to evaluate the system availability and the genetic algorithm to solve the cost minimization problem. Xiao et al. (2016) considered the performance sharing problem in multi-state loading systems. Specifically, they integrated the effect of loading and external impact protection to maximize system availability. Zhai et al. (2017) further incorporated the internal impact of a system and modeled an attack-defense game, considering performance sharing. Yu et al. (2017) analyzed the reliability of a non-repairable phased-mission system with common cause failures. Recently, Wang et al. (2018) proposed a reliability model for multi-state systems and assumed that the surplus performance of each unit can only be shared by its adjacent units. Zhao et al. (2018) extended the model to a multi-state k-out-of-n system. Besides, the common bus system state is divided into multiple levels. Wu et al. (2019) considered the performance sharing mechanism in a capacitated system, assuming that some systems may allow certain extent of performance deficiency. Levitin et al. (2019) incorporated the components with imperfect repair in a common bus system and evaluated its availability and performance deficiency. Works are also done in different practical systems, i.e., Xiao et al. (2020) modeled the reliability of a linear sliding window system. They also proposed a reliability evaluation algorithm and analyzed the optimal element allocation problem based on the genetic algorithm. Yan et al. (2020) considered the reliability of the power grid system composed of generators and nodes, where both the performance and demand for each generator and node are random variables. Additionally, many researchers considered the performance sharing with energy management jointly. Akter et al. (2020) considered the energy sharing problem in residual microgrids and showed that the distributed energy management scheme improves the self-reliance while minimizing the total cost of the system at the same time. Both Kusakana (2019) and Klein et al. (2020) analyzed the peer-to-peer energy sharing between prosumers and end-user management, respectively.

In some real-world applications, components may share surplus performance with other components that have performance deficiency, subject to a transmission bandwidth. However, existing research is restricted to the assumption that merely a single type of resource is shared. In reality, many practical systems need multiple types of resources to meet its demand. For example, an energy company may simultaneously supply electricity and gas to households and plants in order to meet their demands. In such scenarios, the supply and demand for the two resources in each respective node are random. To be more realistic, we also consider the probability that one resource may be able to substitute another in some degree. For instance, Wu et al. (2020) investigated the reliability of a production line with the dual sourcing and product substitution strategies and found that the employment of performance sharing can significantly increase the reliability. In fact, a gas-powered system can be substituted by corresponding electricity power equipment. For example, an electric cooker can replace a gas cooker, and a gas heater can also be replaced by an electric heater. Two separate performance sharing mechanisms corresponding to electricity and gas are available to share the performances among the nodes subject to respective bandwidth constraints. If both the electricity supply and gas supply are no less than the demands after performance substitution and sharing, the system is regarded as reliable. For ease of illustration, we present a simple example in Table 1.

Table 1 An illustrative Example

Substitution rate=0.5	Node 1	Node 2	Bandwidth
Electricity (Supply/Demand)	6/4	12/6	2
Gas (Supply/Demand)	2/x	5/4	1

Without loss of generality, we assume that only electricity can substitute gas in this illustrative example, but not vice versa. We should note that the energy substitution between nodes 1 and 2 is limited by the bandwidth and the substitution of electricity to gas is limited by the substitution rate. When $x=0,1$ and 2, the system is obviously reliable. When $x=3$, the system is reliable since one unit of gas can be compensated by two units of electricity from node 1 or by one unit of gas shared from node 2. When $x=4$, the system is reliable if both possible actions for $x=3$ are taken. When $x=5$, the system is reliable since besides actions taken for $x=4$, one unit of gas can be compensated by two units of electricity from node 2 (no matter shared with node 1 and convert to gas or convert to gas at node 2 and share with node 1). When $x > 5$, the system is said failed due to the limitations of bandwidth even if there is still surplus electricity performance at node 2.

Before proceeding to a brief literature review, we discuss the main difference between the electricity system and the gas system and highlight our main concentration in this paper. In fact, this paper does not assume that the equipment under investigation must have two types of resource simultaneously. The motivation of this paper actually came from the observation that some functions of equipment are accomplished by the consumption of gas while the others are accomplished by the consumption of electricity. In other words, to some extent, these two types of resource can be substituted. While some functions tend to consume gas more than electricity, and vice versa. A typical example can be found in home heating where both gas and electricity are regarded as potential energy. When the electricity system fails, gas can be used as a temporary source of energy to assure the functionality of the system.

In the existing literature, many researchers focused on the reliability evaluation of different types of systems by incorporating different modelling methods, i.e., Zhao and Cui (2010) considered a k-out-of-n system and proposed a finite Markov chain imbedding approach to evaluating the reliability. Zhai et al. (2018) incorporated an aggregated combinational model to evaluate the reliability of a non-repairable parallel phased-mission system. Yang et al. (2020) examined the operations and maintenance policy of wind turbines integrating both wind and aging information. For a systematic review, the reader is referred to Wu et al. (2019). Among all methods, the universal generating function (UGF) is widely employed in evaluating the reliability of a multi-state system (Levitin, 2005, Li et al. 2017) since it has great computational advantages, especially in multi-state components that have different performance levels and several failure modes. In addition, the UGF is more flexible and convenient for modeling reliability of multi-state systems. For instance, Bao et al. (2019) considered the reliability assessment of integrated gas and power systems, Bisht and Singh (2019) conducted reliability analysis in complex bridge networks, and Gao et al. (2020) analyzed a reconfigurable manufacturing system based on the UGF method. However, none of the above works evaluated the reliability of a mixed energy supply system with two performance transmission mechanisms. Therefore, in this paper, we propose a model to evaluate the reliability of an integrated electricity and gas supply system with both performance sharing and performance substitution mechanisms.

The rest of this paper is organized as follows. Section 2 describes the system. Section 3 models the reliability based on universal generating function. Both the cases of single directional substitution and mutual substitution are considered. Section 4 presents examples to illustrate the model. Section 5 conducts sensitivity analysis and discusses the facilities locating problem. Section 6 concludes and discusses future works.

Notation List

Notations	Descriptions
Gl_j, $l \in \{E,G\}$	Discrete random performances for electricity and gas, respectively
n	Number of independent components in the system
Dl_j, $l \in \{E,G\}$	Discrete random demand for electricity and gas, respectively
C_k, $k \in \{E,G\}$	Bandwidth of the performance sharing mechanism for electricity and gas, respectively
λ_{GE} , λ_{EG}	Electricity-gas substitution rate and gas-electricity substitution rate
TS_l, $l \in \{E,G\}$	Total amount of sufficiency for electricity and gas, respectively
TF_l, $l \in \{E,G\}$	Total amount of deficiency for electricity and gas, respectively
R_l, $l \in \{E,G\}$	Remaining electricity and gas, respectively
$ge_{j,k}$, $gg_{j,k}$, $de_{j,k}$, $dg_{j,k}$, cl_{ol}	Specific realization of GE_j, GG_j, DE_j, DG_j and C , respectively
$\Delta l_j(z)$,$l \in \{E,G\}$	UGF representing the PMF of performance and demand of component
$\pi l_{j,r}$	Probabilities of joint events
$\overrightarrow{GE_j}$, $\overrightarrow{GG_j}$	Set of possible realizations of GE_j and GG_j, respectively
$\overrightarrow{Cl}$,$\mathrm{l} \in \{\mathrm{E,G}\}$	Set of possible realizations of C_e and C_g, respectively

R	System reliability
$UE_S(z), UG_S(z)$	UGF representing the PMF of performance and demand of electricity and gas, respectively
$U_S^B(z)$	UGF representing the PMF of performance, demand and bandwidth of electricity and gas, respectively
$\overrightarrow{DE_j}, \overrightarrow{DG_j}$	Set of possible realizations of DE_j and DG_j, respectively

2. System Description

Consider an integrated electricity and gas supply system consisting of n nodes, where node j is characterized by two discrete random performances (GE_j for electricity and GG_j for gas) with a given probability mass functions (PMF) and subject to random demand requirements (DE_j for electricity and DG_j for gas) with given PMFs. Two independent performance sharing mechanisms are employed to re-distribute the node with surplus resource to the other one with deficiency. The bandwidths of these two mechanisms are subjected to random variables C_E and C_G, respectively, with given PMFs. We assume that an electricity-gas substitution rate λ_{EG} and a homogeneous gas-electricity substitution rate λ_{GE} in each node are independent of time. It is worth noting that both the cases of unidirectional and bidirectional substitutions are discussed in this paper.

The amount of sufficiency and deficiency in both channels before performance sharing and substitution can be obtained by

$$TS_l = \sum_{j=1}^{n} \mathrm{Max}(Gl_j - Dl_j, 0), l \in \{E, G\}, \tag{1}$$

and

$$TF_l = \sum_{j=1}^{n} \mathrm{Max}(Dl_j - Gl_j, 0), l \in \{E, G\}, \tag{2}$$

respectively, where $\mathrm{Max}(Gl_j - Dl_j, 0)$ returns the performance sufficiency of node j and $\mathrm{Max}(Dl_j - Gl_j, 0)$ denotes the performance deficiency of node j.

2.1 Only electricity-gas substitution

The reliability indicator can be formulated in accordance with three cases.

a) Compare TF_E with $\mathrm{Max}(TS_E, C_E)$. When $TF_E > \mathrm{Min}(TS_E, C_E)$, the system fails since it will experience electricity deficiency even after performance sharing. Otherwise, calculate $R_E = \mathrm{Min}(TS_E, C_E) - TF_E$ and refer to cases b). Note that R_E is the remaining electricity that can be shared after satisfying the electricity demand in all nodes.

b) Compare TF_G with $\mathrm{Min}(TS_G, C_G)$. When $TF_G \le \mathrm{Min}(TS_G, C_G)$, the system operates as both electricity and gas demands are satisfied. Otherwise, refer to case c).

c) Consider the substitution of gas performance by the remaining electricity that can be shared with substitution rate λ_{EG}. Update the surplus of gas as $TS_G + \lambda_{EG} R_E$ and the bandwidth capacity of gas as $\lambda_{EG}(C_E - TF_E) + C_G$. Compare TF_G with $\mathrm{Min}(TS_G + \lambda_{EG} R_E, \lambda_{EG}(C_E - TF_E) + C_G)$. Specifically, when $TF_G \le \mathrm{Min}(TS_G + \lambda_{EG} R_E, \lambda_{EG}(C_E - TF_E) + C_G)$, the system is regarded as reliable. Otherwise, the system is said failed.

While all possible cases being considered, the system state for any given combination of $(TF_E, TF_G, TS_E, TS_G, C_E, C_G)$ can be defined using the indicator function as

$$
\begin{aligned}
&1_{EG}(TF_E \le \mathrm{Min}(TS_E, C_E), TF_G \\
&\le \mathrm{Min}(TS_G + \lambda_{EG} R_E, \lambda_{EG}(C_E - TF_E) + C_G) \,|\, TF_E, TF_G, TS_E, TS_G, C_E, C_G).
\end{aligned} \tag{3}
$$

where $1_{EG}(TRUE) = 1$ represents system success and $1_{EG}(FALSE) = 0$ represents system failure. The reliability can thereby be obtained by

$$
R_{EG} = \Pr(TF_E \le \mathrm{Min}(TS_E, C_E), TF_G \le \mathrm{Min}(TS_G + \lambda_{EG} R_E, \lambda_{EG}(C_E - TF_E) + C_G)). \tag{4}
$$

where $TF_E \le \mathrm{Min}(TS_E, C_E)$ ensures that the electricity demand is satisfied by the generated electricity and the term $TF_G \le \text{Min}(TS_G + \lambda_{EG} R_E, \lambda_{EG}(C_E - TF_E) + C_G)$ ensures that the gas demand is satisfied with generated gas and those gas transferred from electricity.

2.2 Only gas-electricity substitution

The reliability indicator under this case works in the same manner as subsection 2.1 except that "*E*" and "*G*" should be exchanged. We directly perform the indicator function, given any combination $(TF_E, TF_G, TS_E, TS_G, C_E, C_G)$ as follows

$$\begin{aligned} &1_{GE}(TF_G \le \text{Min}(TS_G, C_G), TF_E \\ &\le \text{Min}(TS_E + \lambda_{GE} R_G, \lambda_{GE}(C_G - TF_G) + C_E) \mid TF_E, TF_G, TS_E, TS_G, C_E, C_G). \end{aligned} \tag{5}$$

where λ_{GE} is the gas-electricity substitution rate. The reliability under the gas-electricity substitution can be similarly denoted by

$$R_{GE} = \Pr(TF_G \le \text{Min}(TS_G, C_G), TF_E \le \text{Min}(TS_E + \lambda_{GE} R_G, \lambda_{GE}(C_G - TF_G) + C_E)). \tag{6}$$

In Eq. (6), $TF_G \le \text{Min}(TS_G, C_G)$ ensures that the gas demand is satisfied by the generated gas and the term $TF_E \le \text{Min}(TS_E + \lambda_{GE} R_G, \lambda_{GE}(C_G - TF_G) + C_E)$ ensures that the electricity demand is satisfied with generated electricity and those electricity transferred from gas.

2.3 Mutual substitution

Now we consider a case where electricity and gas can substitute each other with an electricity-gas substitution rate λ_{EG} and a gas-electricity rate λ_{GE}. The reliability indicator under this case can be modified in accordance with the four following cases.

a) Compare TF_l with $\text{Min}(TS_l, C_l)$. When $TF_l \le \text{Min}(TS_l, C_l)$ with both $l = E$ and $l = G$, the system succeeds as both electricity and gas demands are satisfied. Otherwise, refer to case b).

b) When $TF_l > \text{Min}(TS_l, C_l)$ with both $l = E$ and $l = G$, the system fails. Otherwise, refer to case c).

c) When $TF_E \le \text{Min}(TS_E, C_E)$ but $TF_G > \text{Min}(TS_G, C_G)$, calculate the remaining of electricity $R_E = \text{Min}(TS_E, C_E) - TF_E$. Update the surplus of gas as $TS_G + \lambda_{EG} R_E$ and the bandwidth capacity of gas as $\lambda_{EG}(C_E - TF_E) + C_G$. Compare TF_G with $\text{Min}(TS_G + \lambda_{EG} R_E, \lambda_{EG}(C_E - TF_E) + C_G)$. When $TF_G \le \text{Min}(TS_G + \lambda_{EG} R_E, \lambda_{EG}(C_E - TF_E) + C_G)$, the system is regarded as reliable. Otherwise, refer to case d).

d) When $TF_G \le \text{Min}(TS_G, C_G)$ but $TF_E > \text{Min}(TS_E, C_E)$, calculate the remaining of electricity $R_G = \text{Min}(TS_G, C_G) - TF_G$. Update the surplus of gas as $TS_E + \lambda_{GE} R_G$ and the bandwidth capacity of gas as $\lambda_{GE}(C_G - TF_G) + C_E$. Compare TF_G with $\text{Min}(TS_E + \lambda_{GE} R_G, \lambda_{GE}(C_G - TF_G) + C_E)$. When $TF_E \le \text{Min}(TS_E + \lambda_{GE} R_G, \lambda_{GE}(C_G - TF_G) + C_E)$, the system is regarded as reliable. Otherwise, the system is said failed.

Taking all possible cases into consideration, the indicator function for any given combination of $(TF_E, TF_G, TS_E, TS_G, C_E, C_G)$ can be defined as

$$1_{MU}\Big(\bigcup_{i,j\in\{G,E\},i\neq j} TF_i \le \text{Min}(TS_i, C_i), TF_j \le \text{Min}(TS_j + \lambda_{ij} R_i, \lambda_{ij}(C_i - TF_i) + C_j) \\ | TF_E, TF_G, TS_E, TS_G, C_E, C_G\Big). \tag{7}$$

The reliability under this case can be updated to

$$R_{MU} = \Pr\Big(\bigcup_{i,j\in\{G,E\},i\neq j} TF_i \le \text{Min}(TS_i, C_i), TF_j \le \text{Min}(TS_j + \lambda_{ij} R_i, \lambda_{ij}(C_i - TF_i) + C_j)\Big). \tag{8}$$

In Eq. (8), $TF_i \le \text{Min}(TS_i, C_i)$ ensures that the demand of at least one type of resource is satisfied on its own and $TF_j \le \text{Min}(TS_j + \lambda_{ij} R_i, \lambda_{ij}(C_i - TF_i) + C_j)$ ensures that the demand of the other resource is satisfied by its own generation and those transferred from the first type or resource. It can be seen that both case 2.1 and case 2.2 are special cases of case 2.3 when $\lambda_{GE} = 0$ and, $\lambda_{EG} = 0$, respectively.

3. Reliability Modelling

The UGF technique generates the PMF of any discrete random variable X_j as

$$u_j(z) = \sum_{h=1}^{k_j} \alpha_{j,h} z^{x_{j,h}}. \tag{9}$$

The random variable X_j has k_j possible values with probability $\alpha_{j,h} = \Pr(X_j = x_{j,h})$. By introducing a composition operator $\underset{\varphi}{\otimes}$, the UGF of n independent random variables $\varphi(X_1,...,X_n)$ can be further denoted as

$$\begin{aligned} U(z) &= \underset{\varphi}{\otimes}(u_1(z),...,u_n(z)) = \underset{\varphi}{\otimes}\left(\sum_{h_1=1}^{k_1} \alpha_{1,h_1} z^{x_{1,h_1}},...,\sum_{h_n=1}^{k_n} \alpha_{n,h_n} z^{x_{n,h_n}}\right) \\ &= \sum_{h_1=1}^{k_1} \cdots \sum_{h_n=1}^{k_n} \left(\prod_{i=1}^{n} \alpha_{j,h_i} z^{\varphi(x_{1,h_1},...,x_{n,h_n})}\right). \end{aligned} \tag{10}$$

Similar to the work by Zhai et al. (2018) and Zhao et al. (2020), we assume the random performance GE_j and GG_j take the values from given sets $\overrightarrow{GE_j} = \{ge_{j,1},...,ge_{j,HE_j}\}$ and $\overrightarrow{GG_j} = \{gg_{j,1},...,gg_{j,HG_j}\}$, respectively. On the demand side, the random demands DE_j and DG_j are assumed to take the values from given sets $\overrightarrow{DE_j} = \{de_{j,1},...,de_{j,KE_j}\}$ and $\overrightarrow{DG_j} = \{dg_{j,1},...,dg_{j,KG_j}\}$, respectively. Therefore, the PMF of performance and demand for any component can be defined as

$$ul_j(z) = \sum_{h=1}^{Hl_j} ml_{j,h} z^{gl_{j,h}}, l \in \{E,G\}, \tag{11}$$

and

$$wl_j(z) = \sum_{k=1}^{Kl_j} nl_{j,h} z^{dl_{j,h}}, l \in \{E,G\}, \tag{12}$$

respectively.

Specifically, we have $ml_{j,h} = \Pr(\overrightarrow{Gl_j} = gl_{j,h})$ and $nl_{j,k} = \Pr(\overrightarrow{Dl_j} = dl_{j,k})$. The PMF, representing the bandwidth of the performance sharing mechanism, can be denoted as

$$\eta l(z) = \sum_{o=1}^{Ol} \beta l_o z^{cl_o}, \tag{13}$$

where $\beta l_o = \Pr(Cl = cl_o)$ and the bandwidth should take the value from the given set

$\overrightarrow{Cl} = \{cl_1, ..., cl_{Ol}\}$.

By introducing another composition operator $\underset{\Leftrightarrow}{\otimes}$, we obtain the UGF of electricity's and gas's supply and demand in each node by combining the UGF of $\overrightarrow{Gl_j}$ and $\overrightarrow{Dl_j}$ as follows

$$\Delta l_j(z) = ul_j(z) \underset{\Leftrightarrow}{\otimes} wl_j(z) = \sum_{h=1}^{Hl_j} \sum_{k=1}^{Kl_j} ml_{j,h} nl_{j,h} z^{gl_{j,h}, dl_{j,h}} = \sum_{r=1}^{Al_j} \pi l_{j,r} z^{gl'_{j,r}, dl'_{j,r}}. \tag{14}$$

where $Al_j = Hl_j \cdot Kl_j$, $\pi l_{j,r} = ml_{j,\lfloor (r-1)/Hl_j \rfloor + 1} \cdot nl_{j,\mathrm{mod}(r-1,Kl_j)+1}$, $gl'_{j,r} = gl_{j,\lfloor (r-1)/Hl_j \rfloor + 1}$ and $dl'_{j,r} = dl_{j,\mathrm{mod}(r-1,Kl_j)+1}$, $\lfloor x \rfloor$ denotes the maximal integer that is not larger than and $\mathrm{mod}(x, y)$ returns the remainder of the division of parameter x by parameter y.

The UGF of the whole system can be obtained using the following procedures:

a). Use Ω to represent the nodes considered so far and let $\Omega = \varnothing$ in the beginning. Assign the initial system UGF as $Ul_\Omega(z) = Ul_\varnothing(z) = z^\varnothing$.

b). For each node i ranging from 1 to n, repeat $Ul_{\Omega \cup i}(z) = Ul_\Omega(z) \underset{+}{\otimes} \Delta l_j(z)$ and update Ω as $\Omega \cup \{i\}$, where $\underset{+}{\otimes}$ is a composition operator that obtains the multiplication of coefficients and the union of exponents for each pair of terms from the two UGFs to be combined.

Finally, the UGFs of the electricity and gas channels can be denoted as

$$Ul_S(z) = Ul_\varnothing(z) \underset{+}{\otimes} \Delta l_1(z) \underset{+}{\otimes}, ..., \underset{+}{\otimes} \Delta l_n(z) = \sum_{r_n=1}^{Al_n} ... \sum_{r_1}^{Al_1} \prod_{j=1}^{n} \pi l_{j,r_j} z^{\bigcup_{j=1}^{n} gl'_{j,r_j}, \bigcup_{j=1}^{n} dl'_{j,r_j}}. \tag{15}$$

Now we construct the UGF of the system as

$$U_S(z) = UE_S(z) \underset{+}{\otimes} UG_S(z) = \sum_{rg_{n=1}}^{AG_n} ... \sum_{rg_1=1}^{AG_1} \sum_{re_n=1}^{AE_n} ... \sum_{re_1=1}^{AE_1} \prod_{j=1}^{n} \pi E_{j,re_j} \pi G_{j,rg_j} z^{\{\bigcup_{j=1}^{n} gE'_{j,re_j}, \bigcup_{j=1}^{n} gG'_{j,rg_j}\}, \{\bigcup_{j=1}^{n} dE'_{j,re_j}, \bigcup_{j=1}^{n} dG'_{j,rg_j}\}}.$$

(16)

Limited by the capacity of bandwidth, we construct the UGF incorporating C_E and C_G as

$$U_S^B(z) = U_S(z) \underset{B}{\otimes} \eta E(z) \underset{B}{\otimes} \eta G(z) =$$

$$\sum_{og=1}^{OG} \sum_{oe=1}^{OE} \sum_{rg_n=1}^{AG_n} \dots \sum_{rg_1=1}^{AG_1} \sum_{re_n=1}^{AE_n} \dots \sum_{re_1=1}^{AE_1} \beta E_{oe} \beta G_{og} \prod_{j=1}^{n} \pi E_{j,re_j} \pi G_{j,rg_j}$$

$$z^{\{\bigcup_{j=1}^{n} gE'_{j,re_j}, \bigcup_{j=1}^{n} gG'_{j,rg_j}\},\{\bigcup_{j=1}^{n} dE'_{j,re_j}, \bigcup_{j=1}^{n} dG'_{j,rg_j}\},\{cE_{oe},cG_{og}\}} \quad (17)$$

The reliability of the whole system under the three cases (only electricity-gas substitution - R_{EG}, only gas-electricity substitution - R_{GE}, and mutual substitution - R_{MU}) can be obtained by summing up the coefficients of the updated system UGF by using the reliability indicator function as shown in Eqs. (4), (6), and (8) as

$$R_{EG} = \sum_{og=1}^{OG} \sum_{oe=1}^{OE} \sum_{rg_n=1}^{AG_n} \dots \sum_{rg_1=1}^{AG_1} \sum_{re_n=1}^{AE_n} \dots \sum_{re_1=1}^{AE_1} 1_{EG}(TF_E \le \mathrm{Min}(TS_E, C_E), TF_G$$
$$\le \mathrm{Min}(TS_G + \lambda_{EG} R_E, \lambda_{EG}(C_E - TF_E) + C_G) | TF_E, TF_G, TS_E, TS_G, C_E, C_G)$$
$$\beta E_{oe} \beta G_{og} \prod_{j=1}^{n} \pi E_{j,re_j} \pi G_{j,rg_j} z^{\{\bigcup_{j=1}^{n} gE'_{j,re_j}, \bigcup_{j=1}^{n} gG'_{j,rg_j}\},\{\bigcup_{j=1}^{n} dE'_{j,re_j}, \bigcup_{j=1}^{n} dG'_{j,rg_j}\},\{cE_{oe},cG_{og}\}},$$

(18)

$$R_{GE} = \sum_{og=1}^{OG} \sum_{oe=1}^{OE} \sum_{rg_n=1}^{AG_n} \dots \sum_{rg_1=1}^{AG_1} \sum_{re_n=1}^{AE_n} \dots \sum_{re_1=1}^{AE_1} 1_{GE}(TF_G \le \mathrm{Min}(TS_G, C_G), TF_E$$
$$\le \mathrm{Min}(TS_E + \lambda_{GE} R_G, \lambda_{GE}(C_G - TF_G) + C_E) | TF_E, TF_G, TS_E, TS_G, C_E, C_G)$$
$$\beta E_{oe} \beta G_{og} \prod_{j=1}^{n} \pi E_{j,re_j} \pi G_{j,rg_j} z^{\{\bigcup_{j=1}^{n} gE'_{j,re_j}, \bigcup_{j=1}^{n} gG'_{j,rg_j}\},\{\bigcup_{j=1}^{n} dE'_{j,re_j}, \bigcup_{j=1}^{n} dG'_{j,rg_j}\},\{cE_{oe},cG_{og}\}},$$

(19)

and

$$R_{MU} = \sum_{og=1}^{OG} \sum_{oe=1}^{OE} \sum_{rg_n=1}^{AG_n} \dots \sum_{rg_1=1}^{AG_1} \sum_{re_n=1}^{AE_n} \dots \sum_{re_1=1}^{AE_1} 1_{MU}(\bigcup_{i,j \in \{G,E\}, i \ne j} TF_i \le \mathrm{Min}(TS_i, C_i), TF_j$$
$$\le \mathrm{Min}(TS_j + \lambda_{ij} R_i, \lambda_{ij}(C_i - TF_i) + C_j) \, | TF_E, TF_G, TS_E, TS_G, C_E, C_G)$$
$$\beta E_{oe} \beta G_{og} \prod_{j=1}^{n} \pi E_{j,re_j} \pi G_{j,rg_j} z^{\{\bigcup_{j=1}^{n} gE'_{j,re_j}, \bigcup_{j=1}^{n} gG'_{j,rg_j}\},\{\bigcup_{j=1}^{n} dE'_{j,re_j}, \bigcup_{j=1}^{n} dG'_{j,rg_j}\},\{cE_{oe},cG_{og}\}},$$

(20)

where

$$TF_E = \sum_{j=1}^{n} (\mathrm{Max}\left(dE'_{j,re_j} - gE'_{j,re_j}\right), 0), \quad (21)$$

$$TS_E = \sum_{j=1}^{n} (\mathrm{Max}\left(gE'_{j,re_j} - dE'_{j,re_j}\right), 0), \quad (22)$$

$$TF_G = \sum_{j=1}^{n}(\text{Max}\left(dG'_{j,rg_j} - gG'_{j,rg_j}\right), 0), \tag{23}$$

$$TS_G = \sum_{j=1}^{n}(\text{Max}\left(gG'_{j,rg_j} - dG'_{j,rg_j}\right), 0), \tag{24}$$

$$C_E = cE_{oe}, \tag{25}$$

and

$$C_G = cG_{og}. \tag{26}$$

We elaborate the evaluation of Eqs. (18) and (19) as follows. After the construction of UGF, for each UGF term, we should first calculate the $TF_E, TS_E, TF_G, TS_G, C_E$ and C_G of the system based on the numbers on the exponent. After that, we should check if the system is functioning under the given combination of $TF_E, TS_E, TF_G, TS_G, C_E$ and C_G according to the indicator function. Finally, the reliability of the system can be obtained by summating the coefficients of all combinations of parameters that makes the system functional.

4. Examples

4.1. Illustrative examples

The PMFs of the random performance and random demand for electricity and gas in two nodes are shown in Table 2.

Table 2 PMFs of two nodes

Node/Resource	Performance Set	Probability	Demand Set	Probability
1/Electricity	(8,4)	(0.5,0.5)	(6,2)	(0.6,0.4)
1/Gas	(6,4)	(0.6,0.4)	(5,3)	(0.8,0.2)
2/Electricity	(4,3)	(0.6,0.4)	(2,1)	(0.5,0.5)
2/Gas	(3,1)	(0.5,0.5)	(3,2)	(0.5,0.5)

4.1.1 Only electricity-gas substitution

The electricity-gas substitution rate is assumed to be $\lambda_{EG} = 0.8$. The bandwidths of performance sharing have PMF (4,0) with probability (0.5,0.5) for electricity and (2,0) with probability (0.6,0.4) for gas. The UGFs of electricity and gas in two nodes can be obtained by

$$uE_1(z) = 0.5z^8 + 0.5z^4, wE_1(z) = 0.6z^6 + 0.4z^2,$$

$$uG_1(z) = 0.6z^6 + 0.4z^4, wG_1(z) = 0.8z^5 + 0.2z^3,$$

$$uE_2(z) = 0.6z^4 + 0.4z^3, wE_2(z) = 0.5z^2 + 0.5z^1,$$

and

$$uG_2(z) = 0.5z^3 + 0.5z^1, wG_1(z) = 0.5z^3 + 0.5z^2.$$

The UGFs of the performance sharing system are

$$\eta E(z) = 0.5z^4 + 0.5z^0,$$

and

$$\eta G(z) = 0.6z^2 + 0.4z^0.$$

The UGFs of the electricity's and gas's performance and demand can be obtained by

$$\Delta E_1(z) = 0.3z^{\{8\},\{6\}} + 0.3z^{\{4\},\{6\}} + 0.2z^{\{8\},\{2\}} + 0.2z^{\{4\},\{2\}},$$
$$\Delta G_1(z) = 0.48z^{\{6\},\{5\}} + 0.32z^{\{4\},\{5\}} + 0.12z^{\{6\},\{3\}} + 0.08z^{\{4\},\{3\}},$$
$$\Delta E_2(z) = 0.3z^{\{4\},\{2\}} + 0.3z^{\{4\},\{1\}} + 0.2z^{\{3\},\{2\}} + 0.2z^{\{3\},\{1\}},$$

and

$$\Delta G_2(z) = 0.25z^{\{3\},\{3\}} + 0.25z^{\{3\},\{2\}} + 0.25z^{\{1\},\{3\}} + 0.25z^{\{1\},\{2\}}.$$

The UGFs of the electricity and gas channel can be obtained by

$$\begin{aligned} UE_S(z) = {} & 0.09z^{\{8,4\},\{6,2\}} + 0.09z^{\{4,4\},\{6,2\}} + 0.09z^{\{8,4\},\{6,1\}} + 0.09z^{\{4,4\},\{6,1\}} \\ & +0.06z^{\{8,3\},\{6,2\}} + 0.06z^{\{8,3\},\{6,1\}} + 0.06z^{\{4,3\},\{6,2\}} + 0.06z^{\{4,3\},\{6,1\}} \\ & +0.06z^{\{8,4\},\{2,2\}} + 0.06z^{\{8,4\},\{2,1\}} + 0.06z^{\{4,4\},\{2,2\}} + 0.06z^{\{4,4\},\{2,1\}} \\ & +0.04z^{\{8,3\},\{2,2\}} + 0.04z^{\{8,3\},\{2,1\}} + 0.04z^{\{4,3\},\{2,2\}} + 0.04z^{\{4,3\},\{2,1\}}, \end{aligned}$$

$$\begin{aligned} UG_S(z) = {} & 0.12z^{\{6,3\},\{5,3\}} + 0.12z^{\{6,3\},\{5,2\}} + 0.12z^{\{6,1\},\{5,3\}} + 0.12z^{\{6,1\},\{5,2\}} \\ & +0.08z^{\{4,3\},\{5,3\}} + 0.08z^{\{4,3\},\{5,2\}} + 0.08z^{\{4,1\},\{5,3\}} + 0.08z^{\{4,1\},\{5,2\}} \\ & +0.03z^{\{6,3\},\{3,3\}} + 0.03z^{\{6,3\},\{3,2\}} + 0.03z^{\{6,1\},\{3,3\}} + 0.03z^{\{6,1\},\{3,2\}} \\ & +0.02z^{\{4,3\},\{3,3\}} + 0.02z^{\{4,3\},\{3,2\}} + 0.02z^{\{4,1\},\{3,3\}} + 0.02z^{\{4,1\},\{3,2\}}. \end{aligned}$$

The $U_S(Z)$ has 256 terms as

$$\begin{aligned} U_S(z) = {} & 0.0108z^{\{8,4,6,3\},\{6,2,5,3\}} + 0.0108z^{\{8,4,6,3\},\{6,2,5,2\}} + 0.0027z^{\{8,4,6,3\},\{6,1,3,3\}} \\ & +0.0012z^{\{8,3,4,3\},\{6,2,3,2\}} + 0.0072z^{\{4,4,4,3\},\{6,1,5,3\}} + \cdots + 0.0012z^{\{8,3,6,1\},\{2,2,3,2\}}. \end{aligned}$$

The $U_S^B(Z)$ has 1024 terms as

$$\begin{aligned} U_S^B(Z) = {} & 0.00324z^{\{8,4,6,3\},\{6,2,5,3\},\{4,2\}} + 0.00216z^{\{8,4,6,3\},\{6,2,5,3\},\{4,0\}} \\ & +0.00144z^{\{8,4,4,1\},\{6,2,5,2\},\{4,0\}} + 0.00216z^{\{8,4,6,3\},\{6,1,5,2\},\{4,0\}} \\ & +0.00036z^{\{4,4,4,1\},\{6,2,3,2\},\{0,0\}} + 0.00054z^{\{8,2,6,1\},\{6,1,3,3\},\{4,2\}} \end{aligned}$$

$$+\cdots+0.00081z^{\{4,4,6,3\},\{6,2,3,3\},\{4,2\}}.$$

Among all possible cases, 796 terms indicating system success. In effect, the system reliability can be obtained by (all results are kept to four decimal places):

$$R_{EG}=Pr(\,TF_E\le \mathrm{Min}(TS_E,C_E),TF_G\le \mathrm{Min}(TS_G+\lambda_{EG}R_E,\lambda_{EG}(C_E-TF_E)+C_G))$$
$$=0.7578.$$

4.1.2 Only gas- electricity substitution

The gas- electricity substitution rate is assumed to be $\lambda_{GE}=0.6$, where all other parameters remain the same. There are 424 terms indicating system success, and the system reliability can be obtained by:

$$R_{GE}=Pr(\,TF_G\le \mathrm{Min}(TS_G,C_G),TF_E\le \mathrm{Min}(TS_E+\lambda_{GE}R_G,\lambda_{GE}(C_G-TF_G)+C_E))$$
$$=0.3562.$$

4.1.3 Mutual Substitution

The substitution rate is assumed to be $\lambda_{EG}=0.8$ and $\lambda_{GE}=0.6$ where all other parameters remain the same. There are 812 terms indicating system success, and the system reliability can be obtained by

$$R_{MU}=Pr(\bigcup_{i,j\in\{G,E\},i\neq j}TF_i\le \mathrm{Min}(TS_i,C_i),TF_j$$
$$\le \mathrm{Min}(TS_j+\lambda_{ij}R_i,\lambda_{ij}(C_i-TF_i)+C_j))=0.7698.$$

4.2. Case Study

Now we consider a more complex case where two resources can be substituted and shared among four nodes. The PMF of each node is performed in Table 3.

We also conduct sensitivity analysis on the bandwidths of two channels and substitution rate λ_{EG} and λ_{GE}.

Table 3 PMFs of four nodes

Node/Resource	Performance Set	Probability	Demand Set	Probability
1/Electricity	(8,4)	(0.5,0.5)	(6,2)	(0.6,0.4)
1/Gas	(6,4)	(0.6,0.4)	(5,3)	(0.8,0.2)
2/Electricity	(4,3)	(0.6,0.4)	(2,1)	(0.5,0.5)
2/Gas	(3,1)	(0.5,0.5)	(3,2)	(0.5,0.5)
3/Electricity	(6,4)	(0.5,0.5)	(6,2)	(0.6,0.4)
3/Gas	(6,4)	(0.6,0.4)	(5,3)	(0.8,0.2)

4/Electricity	(2,1)	(0.6,0.4)	(2,1)	(0.5,0.5)
4/Gas	(3,1)	(0.5,0.5)	(3,2)	(0.5,0.5)

4.2.1 Only electricity-gas substitution

The electricity-gas substitution rate is assumed to be $\lambda_{EG}=0.8$. The bandwidths of performance sharing have PMF (4,2) with probability (0.5,0.5) for electricity and (3,6) with probability (0.6,0.4) for gas. The UGFs of electricity and gas in two nodes can be obtained by

$$uE_1(z) = 0.5z^8 + 0.5z^4, wE_1(z) = 0.6z^6 + 0.4z^2,$$
$$uG_1(z) = 0.6z^6 + 0.4z^4, wG_1(z) = 0.8z^5 + 0.2z^3,$$
$$uE_2(z) = 0.6z^4 + 0.4z^3, wE_2(z) = 0.5z^2 + 0.5z^1,$$
$$uG_2(z) = 0.5z^3 + 0.5z^1, wG_2(z) = 0.5z^3 + 0.5z^2.$$
$$uE_3(z) = 0.5z^6 + 0.5z^4, wE_3(z) = 0.6z^6 + 0.4z^2,$$
$$uG_3(z) = 0.6z^6 + 0.4z^4, wG_3(z) = 0.8z^5 + 0.2z^3,$$
$$uE_4(z) = 0.6z^2 + 0.4z^1, wE_4(z) = 0.5z^2 + 0.5z^1,$$

and

$$uG_4(z) = 0.5z^3 + 0.5z^1, wG_4(z) = 0.5z^3 + 0.5z^2.$$

The UGFs of the performance sharing mechanisms for electricity and gas are

$$\eta E(z) = 0.5z^4 + 0.5z^2,$$
$$\eta G(z) = 0.6z^3 + 0.4z^6.$$

The UGFs of the electricity's and gas's performance and demand can be obtained by

$$\Delta E_1(z) = 0.3z^{\{8\},\{6\}} + 0.3z^{\{4\},\{6\}} + 0.2z^{\{8\},\{2\}} + 0.2z^{\{4\},\{2\}},$$
$$\Delta G_1(z) = 0.48z^{\{6\},\{5\}} + 0.32z^{\{4\},\{5\}} + 0.12z^{\{6\},\{3\}} + 0.08z^{\{4\},\{3\}},$$
$$\Delta E_2(z) = 0.3z^{\{4\},\{2\}} + 0.3z^{\{4\},\{1\}} + 0.2z^{\{3\},\{2\}} + 0.2z^{\{3\},\{1\}},$$
$$\Delta G_2(z) = 0.25z^{\{3\},\{3\}} + 0.25z^{\{3\},\{2\}} + 0.25z^{\{1\},\{3\}} + 0.25z^{\{1\},\{2\}}.$$
$$\Delta E_3(z) = 0.3z^{\{6\},\{6\}} + 0.3z^{\{4\},\{6\}} + 0.2z^{\{6\},\{2\}} + 0.2z^{\{4\},\{2\}},$$
$$\Delta G_3(z) = 0.48z^{\{6\},\{5\}} + 0.32z^{\{4\},\{5\}} + 0.12z^{\{6\},\{3\}} + 0.08z^{\{4\},\{3\}},$$
$$\Delta E_4(z) = 0.3z^{\{2\},\{2\}} + 0.3z^{\{2\},\{1\}} + 0.2z^{\{1\},\{2\}} + 0.2z^{\{1\},\{1\}},$$
$$\Delta G_4(z) = 0.25z^{\{3\},\{3\}} + 0.25z^{\{3\},\{2\}} + 0.25z^{\{1\},\{3\}} + 0.25z^{\{1\},\{2\}}.$$

The UGFs of the electricity and gas channel can be obtained by

$UE_S(z) = 0.0081z^{\{8,4,6,2\},\{6,2,6,2\}} + 0.0081z^{\{8,4,6,2\},\{6,2,6,1\}} +$

$+ \cdots 0.0081z^{\{4,4,6,2\},\{6,2,6,2\}} + 0.0036z^{\{4,4,6,1\},\{6,1,2,1\}} + 0.0036z^{\{4,3,6,2\},\{6,1,2,2\}}$.

and

$UG_S(z) = 0.0144z^{\{6,3,6,3\},\{5,3,5,3\}} + 0.0144z^{\{6,3,6,3\},\{5,3,5,2\}} +$

$+ \cdots 0.006z^{\{6,3,4,3\},\{3,2,3,3\}} + 0.0036z^{\{6,1,6,1\},\{3,3,5,2\}} + 0.0006z^{\{6,1,4,3\},\{3,3,3,2\}}$.

Thus, the $U_S(Z)$ has 65536 terms as

$U_S(z) = 5.76 \times 10^{-6} z^{\{4,4,6,1,6,1,4,3\},\{2,2,2,2,3,2,5,3\}}$

$+ \cdots 3.456 \times 10^{-5} z^{\{4,3,6,1,6,3,6,3\},\{2,1,6,1,5,3,5,2\}} + 5.76 \times 10^{-6} z^{\{4,3,4,2,4,1,4,1\},\{2,2,6,1,3,3,5,3\}}$.

The $U_S^B(Z)$ has 262144 terms as

$U_S^B(z) = 1.152 \times 10^{-6} z^{\{4,4,4,1,6,1,4,3\},\{2,2,1,1,3,3,5,3\},\{2,6\}}$

$+1.728 \times 10^{-6} z^{\{4,3,6,2,6,1,4,1\},\{2,2,1,1,3,3,5,2\},\{2,6\}}$

$+ \cdots$

$+7.680 \times 10^{-6} z^{\{4,3,6,2,4,3,4,1\},\{2,1,2,2,5,2,3,2\},\{2,6\}}$.

There are 216764 terms indicating system success, and the system reliability can be obtained by

$$R_{EG} = Pr(TF_E \leq \mathrm{Min}(TS_E, C_E), TF_G \leq \mathrm{Min}(TS_G + \lambda_{EG} R_E, \lambda_{EG}(C_E - TF_E) + C_G)) = 0.7744.$$

4.2.2 Only gas- electricity substitution

The gas-electricity substitution rate is assumed to be $\lambda_{GE} = 0.6$, where all other parameters remain the same. There are 171048 terms indicating system success, and the system reliability can be obtained by:

$$R_{GE} = Pr(TF_G \leq \mathrm{Min}(TS_G, C_G), TF_E \leq \mathrm{Min}(TS_E + \lambda_{GE} R_G, \lambda_{GE}(C_G - TF_G) + C_E)) = 0.5469.$$

4.2.3 Mutual Substitution

The substitution rate is assumed to be $\lambda_{EG} = 0.8$ and $\lambda_{GE} = 0.6$, where all other parameters remain the same. There are 225852 terms indicating system success, and the system reliability can be obtained as:

$$R_{MU} = Pr\left(\bigcup_{i,j\in\{G,E\},i\neq j} TF_i \leq \mathrm{Min}(TS_i, C_i), TF_j \leq \mathrm{Min}(TS_j + \lambda_{ij}R_i, \lambda_{ij}(C_i - TF_i) + C_j)\right) = 0.8023.$$

5. Impacts of different parameters on system reliability

There are many factors affecting system reliability, i.e., the performance distributions of nodes, the demand of nodes, the substitution rates, the bandwidths, and the positioning of the facilities.

5.1 The impact of electricity-gas substitution rate

When the electricity-gas substitution rate changes from 0 to 1 while all other parameters remain the same, the reliability of the system will change accordingly, as shown in Figure 1. EG-reliability refers to the probability of the case where electricity can substitute gas but vice versa. MU-reliability means the probability that the two resources can substitute each other. It can be seen that the MU-reliability curve is in parallel to the EG-reliability curve. In fact, even if mutual substitution is allowed, for each given combination of states for system nodes performance and demand, the substitution will at most happen at one direction. For all cases where electricity is substituted for gas, the increase of the electricity-gas substitution rate has the same effect on system reliability, no matter which type of substitution is allowed.

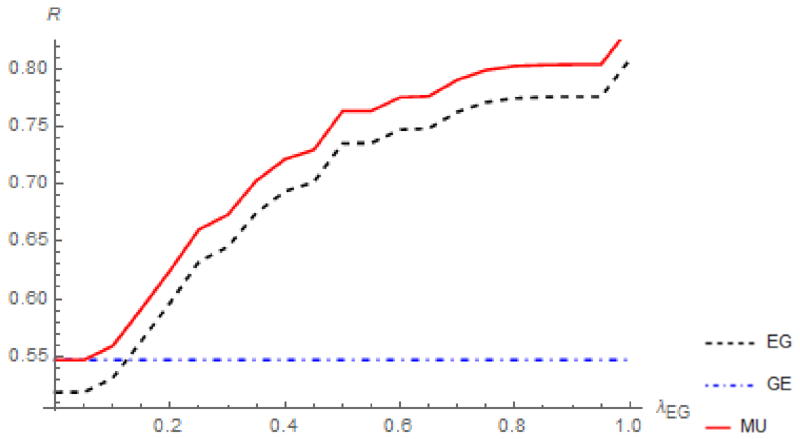

Figure 1 Reliability alterations with respect to λ_{EG}

As shown in Figure 1, when the substitution rate between electricity and gas is of low level, electricity substitution is inefficient. This makes the reliability under GE larger than the reliability under EG. The augment of λ_{EG} represents the fact that the same unit of electricity can now substitute more gas than before, leading to the increase in system reliability under EG.

5.2. The impact of gas- electricity substitution rate

When the gas-electricity substitution rate changes from 0 to 1 and the other parameters remain the same, the reliability of the system will correspondingly change, as shown in Figure 2. Similar to Figure.1, it can be seen that the gas-electricity substitution-reliability curve is in parallel with the mutual reliability curve.

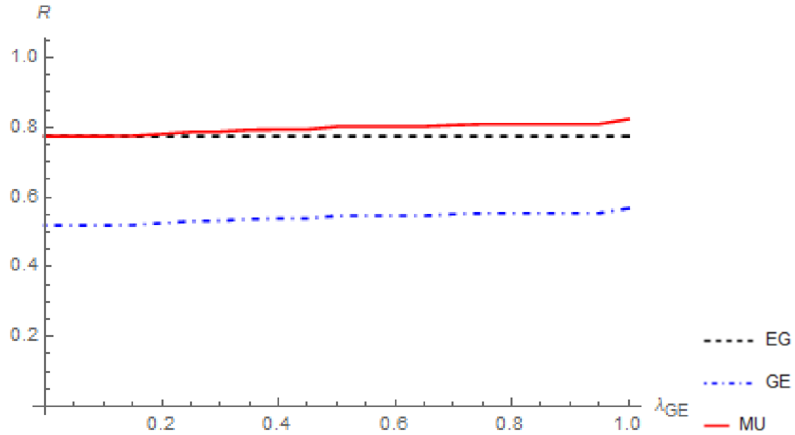

Figure 2 Reliability alterations with respect to λ_{GE}

Similarly, in Figure 2, when the substitution rate between gas and electricity is of low level, the gas substitution is inefficient. The increase in λ_{GE} leads to the slight increase in system reliability under GE since the same unit of gas can now replace more electricity than before. This represents that in most cases the electricity is abundant, making the increase of the gas-electricity substitution rate do not considerably improve system reliability.

5.3. The impact of electricity's bandwidth

When the bandwidth of electricity alters from (4,0) to (4,4) with step 0.25, the reliability under different performance substitution mechanisms is shown in Figure 3.

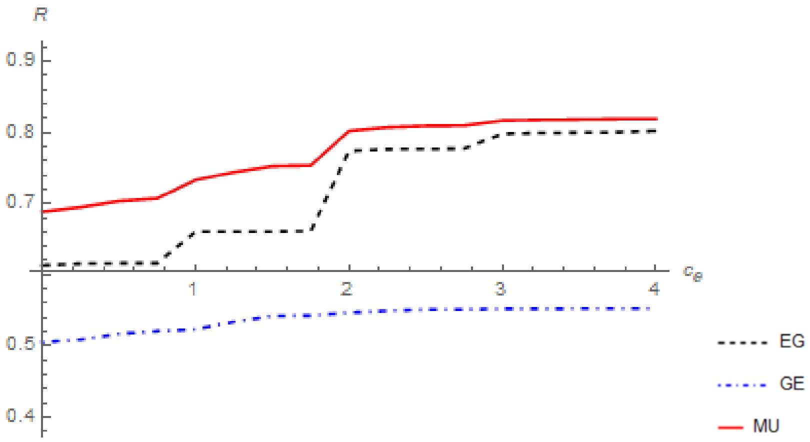

Figure 3 Reliability alterations when electricity bandwidth changes from (4,0) to (4,4)

Figure 3 shows that the increase of electricity's bandwidth facilitates the sharing of electricity for all three cases since the shared electricity now includes not only the performance that used to substitute gas but also those transferred from gas directly. Since electricity is abundant based on our setting, the reliability augment in EG-reliability and MU-reliability is greater than the case of GE.

Similarly, when the bandwidth of electricity alters from (0,2) to (4,2) with step 0.25, the results are shown in Figure 4. The results can be explained similarly to Figure 3.

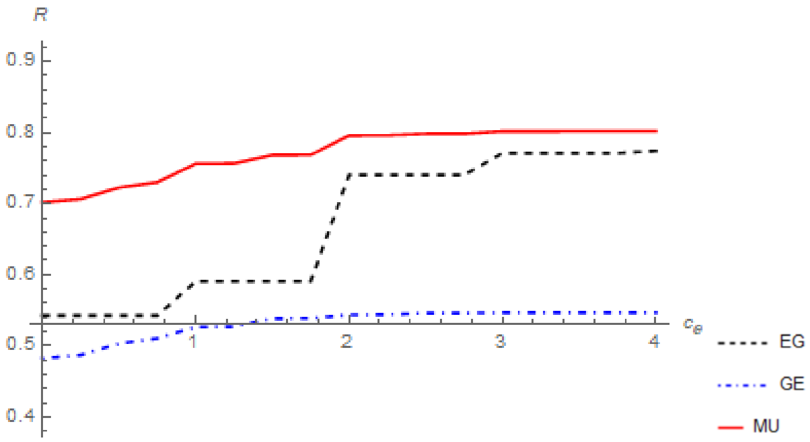

Figure 4. Reliability alterations when electricity bandwidth changes from (0,2) to (4,2).

5.4. The impact of gas's bandwidth

When the bandwidth of electricity alters from (3,0) to (3,4) with step 0.25, the results are shown in Figure 5.

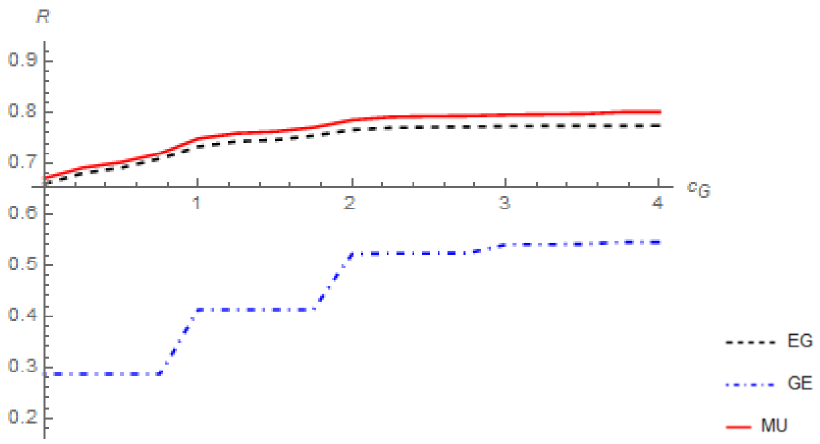

Figure 5. Reliability alterations when gas bandwidth changes from (3,0) to (3,4).

Figure 5 demonstrates that the increase of gas's bandwidth facilitates the sharing of gas for all three cases since the shared electricity consists of the performance used to substitute electricity and those transferred from electricity. Again, the reliability increases in both EG case and mutual case are greater than the GE case since electricity is abundant in our benchmark.

When the bandwidth of electricity alters from (0,6) to (4,6) with step 0.25, the results are shown in Figure 6 and can be explained similar to Figure 5.

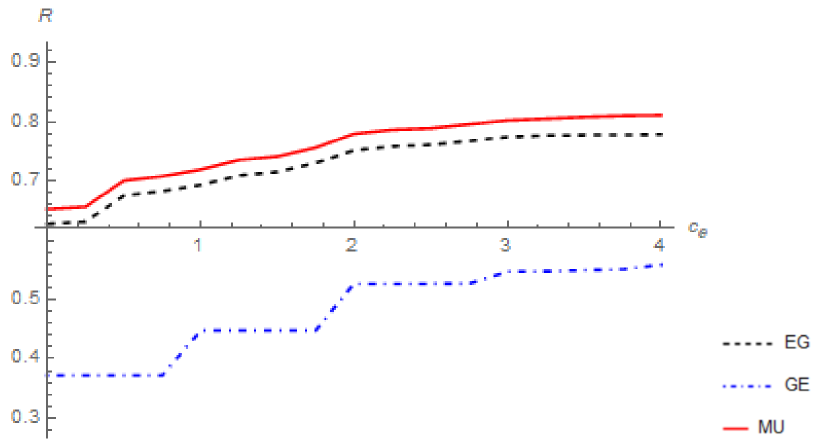

Figure 6 Reliability alterations when electricity bandwidth changes from (0,6) to (4,6)

5.5. The optimization of facilities positioning

As each system node may have a different electricity demand and a different gas demand, the positioning of electricity and gas facilities may influence the system reliability. For ease of discussion, this paper assumes that each node must have exactly one electricity facility and one gas facility. The performance distributions of the facilities under their initial positions are similar in Table 3. The optimal facility locations which maximizes the system reliability is thereby studied in this section. Specifically, we consider three situations as follows.

5.5.1 Optimization of electrical facility location

Considering the positioning of four facilities into four nodes, 24 situations may emerge. The positions of the facilities can be denoted by a vector ***HE***={*he*(1), *he*(2), *he*(3), *he*(4)}, where *he*(*i*) denotes the initial position of the electricity facility.

Under such a setting, the initial position can be denoted as ***HE***={1,2,3,4} and the system reliability under the parameters given in section 4.2 is already calculated as R_{EG}= 0.7744, R_{GE}= 0.5469, and R_{MU}= 0.8023.

Through enumerating the reliability under different situations, we obtain the optimal positioning strategy that optimizes the system reliability in the case of EG, GE, and mutual, respectively. For each positioning strategy, the system reliability for all three cases are shown as in Table 4.

Table 4 Optimization of electrical facility location

HE	R_{EG}	R_{GE}	R_{MU}	Objective
{3,2,4,1}	0.77700	0.54790	0.80590	$R_{EG\|\mathrm{MAX}}$
{1,3,2,4}	0.77057	0.54625	0.79781	$R_{EG\|\mathrm{MIN}}$
{1,4,3,2}	0.77435	0.54867	0.80402	$R_{GE\|\mathrm{MAX}}$
{1,3,2,4}	0.77057	0.54624	0.79781	$R_{GE\|\mathrm{MIN}}$
{3,1,4,2}	0.77668	0.54864	0.80631	$R_{MU\|\mathrm{MAX}}$
{1,3,2,4}	0.77057	0.54624	0.79781	$R_{MU\|MIN}$

5.5.2 Optimization of gas facility locations

Similarly, 24 situations may occur in a gas facility location. The positions of the facility can be denoted by a vector ***HG***={*hg*(1),*hg*(2),*hg*(3),*hg*(4)}, where *hg*(*i*) denotes the initial position of the gas facility.

Under such a setting, the initial position can be denoted by ***HG***={1,2,3,4} and the system reliability under the parameters given in section 4.2 is already calculated as R_{EG}= 0.7744, R_{GE}= 0.5469, R_{MU}= 0.8023.

Through enumerating the reliability under different situations, we obtain the optimal positioning strategy that optimizes system reliability in the case of EG, GE, and both as shown in Table 5.

Table 5 Optimization of gas facility location

HE	R_{EG}	R_{GE}	R_{MU}	Objective
{1,2,4,3}	0.78095	0.56068	0.81494	$R_{EG\|}$MAX
{1,3,2,4}	0.77056	0.54624	0.79781	$R_{EG\|}$MIN
{1,2,4,3}	0.78095	0.56068	0.81493	$R_{GE\|}$MAX
{1,3,2,4}	0.77056	0.54624	0.79781	$R_{GE\|}$MIN
{1,2,4,3}	0.78095	0.56068	0.81493	$R_{MU\|}$MAX
{1,3,2,4}	0.77056	0.54624	0.79781	$R_{MU\|}$MIN

5.5.3 Optimization of electrical and gas facility location

Considering the positioning of four electric facilities and four gas facilities into four nodes, 576 situations may occur. The positions of the facilities can be denoted by a vector ***HEG***={*he*(1), *he*(2), *he*(3), *he*(4), *hg*(1), *hg*(2), *hg*(3), *hg*(4)}, where *he*(*i*) denotes the initial position of the electricity facility and *hg*(*i*) denotes the initial position of the gas facility.

The initial position can be denoted as ***HEG***={1,2,3,4,1,2,3,4} and the system reliability is already calculated as R_{EG}= 0.7744, R_{GE}= 0.5469, R_{MU}= 0.8023. The optimal positioning strategy that optimizes the system reliability under three cases are shown in Table 6.

Table 6 Optimization of electrical and gas facility location

HE	R_{EG}	R_{GE}	R_{MU}	Objective
{1,2,3,4,1,2,4,3}	0.78095	0.56068	0.81493	$R_{EG\|}$MAX
{1,2,3,4,1,3,2,4}	0.77056	0.54624	0.79781	$R_{EG\|}$MIN
{1,2,4,3,2,1,4,3}	0.78095	0.56075	0.81500	$R_{GE\|}$MAX
{1,2,3,4,1,3,2,4}	0.77056	0.54625	0.79781	$R_{GE\|}$MIN
{1,2,4,3,2,1,4,3}	0.78095	0.56075	0.81500	$R_{MU\|}$MAX
{1,2,3,4,1,3,2,4}	0.77056	0.54624	0.79781	$R_{MU\|}$MIN

6. Conclusions

This paper investigated the reliability of a system powered by two types of energy resource. It incorporated performance substitution and sharing mechanisms to maximize its reliability. Two types of resource, electricity and gas, can be shared among nodes but restricted by bandwidth, respectively. Both unidirectional and bidirectional substitutions were considered in this paper. A universal generating function technique is proposed to evaluate the system reliability. An illustrative example and one case study were presented to demonstrate the applicability of the proposed model. To better understand the impact of substitution rate between two resource and bandwidths in each channel on the system reliability, we conducted sensitivity analysis. Results showed that the reliability under mutual substitution is higher than or at least equal to the reliability under unidirectional substitution. We further showed that the combination of substitution and performance sharing can significantly increase the system reliability. Our proposed model also shed light on facility location optimization. Aiming at maximizing the reliability of the entire system, managers can pre-locate their facility, considering both mechanisms proposed in this paper.

For future works, one can analyze a more complex system where both performance substitution and sharing are employed. Additionally, using the proposed model to analyze system reliability can be time-consuming if there are unstable nodes (components) or the distribution (more complex performance set, demand set, and corresponding probability). In effect, some heuristic algorithms or simulation techniques can be applied to deal with large-scale systems to obtain a near-optimal solution in practice (see Xiao et al. 2020, for example). Additionally, one can construct a network or employ multi-state multi-valued decision diagram (Li et al. 2017) to depict a more complicated situation. When the front node has impact on the rear node, the state probability of the rear node should be correspondingly altered to the conditional probability of the front node. Huang and An (2008) considered the conditional UGF technique and both Zhou et al. (2020) and Yang et al. (2020) modeled cascading failure scenarios. Furthermore, the substitution rate can be determined by the survey to the plant as well as the resident. By conducting a survey in a given area, one can easily find the type of equipment requires gas and electricity, respectively. Additionally, the survey can show that which type of equipment requires, i.e., gas or electricity more than the other but can be compensated by the other. As for the equipment requires both types of resource, we can have a progressive inquiry on the specific amounts of resource needed per day. Based on all these results, one can find that when gas/electricity is of deficient, how much the substitutive resource is needed.

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