# New MAXCAP Related Problems: Formulation and Model Solutions

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**Abstract**

In this paper three related problems of the maximum capture (MAXCAP) model are proposed. These include the case where facilities provide a certain amount of service level for the customers, the possibility where customers do not allocate their demand completely to one facility but prorate their demand based on the service level, and finally we explore the situation where customers will not opt for sharing their demand irrespective of the service level if the next attractive facility is too far way which we express by a distance threshold. These models are put forward to mimic realistic situations related to customer behavior when it comes to selecting a facility. Their respective mathematical formulations are put forward and tested on a case study and also over a range of larger data sets.

*Keywords*: Competitive location, customer behavior, ILP formulations.

## 1. Introduction

Most industries and retailing in particular operate in a harsh competitive market where efficiency and responsiveness dictate the success as well as the sustainability of a firm. Among the factors that make a retailing business successful in a competitive environment include product quality, pricing policies and the ability for the firm to maintain and improve its market share. A good location of a facility will provide an added competitive advantage when compared to a less attractive location as the latter yields an extra cost which makes it harder to compete in prices. Besides, this financial loss will restrict the investment in terms of enhancing product quality, training and innovation. In other words, a firm with less attractive locations will suffer to retain its market share even if the other logistical drivers such as transport and inventory management are efficiently conducted.

One of the significant contributions to the field of competitive location was by ReVelle (1986) who developed a model named Maximum Capture Model (MAXCAP). The model finds optimal sites seeking the maximum share of the market in the presence of competition from an existing firm. This paper sparked the research in this field and several models were developed in the lines of the MAXCAP model, many being significantly practical in nature. Several modifications were then made considering factors other than distances such as the consumer choices as customers were found to trade off the cost of travel and the attractiveness of alternative shopping opportunities.

In this paper we present ILP formulations to related problems of the MAXCAP by integrating the service level factor in the customer's decision making. Without loss of generality we suppose that there are two competing firms in the market, a firm *A* that is entering the market and that has already spotted a list of potential sites for opening new branches and a firm *B* that is already present in the market for which its locations are already known. Our aim is to maximize the total demand or market share captured by firm *A* by finding the optimal or best locations of its new *p* facilities.

The main interaction between a customer and any outlet will be measured through its distance to the open site that is closer than the existing site and the level of service that will be available to the customer. In case of two firms sharing a common location area, which means that they are at equal distances to a customer, this customer will patronize the outlet that offers better services. The proposed models follow the same line as the Multiplicative Competitive Interaction models of Nakanishi & Cooper (1974) except that we consider that the customer choices will not just rely on the distance to the outlet but also on the quality and the variety of sideline services he/she expects to receive. It is understood that all clients will not use all services during their regular visit, but it confers an added value for the client to know that the services are available to him/her if need be.

The paper is organized as follows: In the next section we present the MAXCAP and its ILP formulation followed by a brief review on related competitive location problems. Our three extensions and their mathematical formulations will be given in Section 3. A real case study is then presented in Section 4 followed by an intensive computational experiment using larger instances in Section 5. The last section summarizes our conclusions and points out to some interesting suggestions that could be worth exploring in the future.

## 2. A review on the MAXCAP and its related problems

We first provide the main assumptions used in the MAXCAP and its formulation as this will form the basis for our investigation.

### The MAXCAP Model

In this model, the new firm *A* looks to open *p* new facilities, given the presence of *q* existing outlets. Without loss of generality it can be assumed that all existing facilities belong to the same firm, say *B*. Let the market be represented by a connected and undirected network $G = (V, E)$ where *V* is the set of vertices and *E* the set of edges. The objective is to open *p* new facilities on a subset of potential sites $J \subset V$ in order to maximize the market capture for the entering firm.

*Assumptions*

1. The spatial market is defined by a graph. Demands from customers are aggregated to the nodes of the network and the demand can be doubly served if necessary. This means that if a certain criterion is met, any given demand node can be served by either of the existing or entering firm or can be even shared by both.
2. Potential sites for the new locations are pre-specified (this includes the possibility that all the vertices of the network can be considered as potential sites).
3. Both firms are demand maximizing.
4. It is assumed that the opening costs for all outlets are the same although the impact of the initial investment will contribute to define the service level of the facility.
5. The product sold is homogeneous which means that the customers will purchase the same product from all facilities.
6. The customers' decision on patronizing the store is based on the distance and not on the price (as price is assumed to be the same in all stores).
7. There is no reaction from the competitors to the entrance of new outlets.
8. There is no uncertainty in the parameters of the model.

*The MAXCAP Formulation*

The MAXCAP problem ($P_0$) can be formulated as follows.

$$P_0 = \begin{cases} Max \ \sum_{i \in I} \sum_{j \in J} a_i \rho_{ij} x_{ij} & (1) \\ subject \ \ to & \\ x_{ij} \le y_j, \quad \forall i \in I, \forall j \in J, & (2) \\ \sum_{j \in J} x_{ij} = 1, \quad \forall i \in I & (3) \\ \sum_{j \in J} y_j = p, & (4) \\ x_{ij}, y_j \in \{0,1\}. & (5) \end{cases}$$

Where

$i, I$ = Index and the set of existing demand assumed to be located at the vertices of the graph.

$j, J$ = Index and the set of potential locations.

$d_{ij}$ = Distance between customer $i$ and facility $j$.

$p$ = Number of outlets to be effectively open by firm *A*.

$q$ = Number of existing outlets of firm *B*.

$a_i$ = Demand at node *i*.

$b_i$ = The nearest facility to customer *i* among the *q* firm *B*'s outlets.

$$\rho_{ij} = \begin{cases} 1 & \text{if } d_{ij} < d_{ib_i} \\ 0 & \textit{Otherwise} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{if demand node } i \text{ is assigned to node } j \\ 0 & \text{Otherwise} \end{cases}$$

$$y_j = \begin{cases} 1 & \text{if a new outlet is opened at node } j \\ 0 & \text{Otherwise} \end{cases}$$

The objective function (1) is to maximize the total demand attracted by the new firm A directly from the customers that have patronized its outlets ( $\rho_{ij} x_{ij} = 1$). The constraint set (2) forces each demand node (*i*) to be allocated to only one open site at a time, constraints (3) guarantee that the allocation can only be made if an outlet is opened at node *j* whereas constraint (4) fixes the number of new outlets to be exactly *p* and (5) refer to the nature of the decision variables. ReVelle (1986) in his model also included a second term in the objective function which is the shared demand between the existing and the entering firm if both happen to be equidistant from a customer.

## Related MAXCAP problems

Eiselt & Laporte (1989) extended the MAXCAP model to include outlets with different sizes. Serra *et al.* (1992) also examined this extension by considering that the different servers are organized in a hierarchical fashion. There is much literature on competitive location that considers the attraction of the facility based on the size and the diversity of the service, see Drezner (1994), Drezner & Drezner (1996), and Berman & Krass (2002). Depending on the community growth or the economic vitality there can be uncertainty in the model parameters like the variability in the demand. In other words, some customers' demand may not be known in advance to the entering firm. This problem was tackled by Serra & ReVelle (1996) using a scenario type approach by generating various levels of demands in each of the selected scenarios.

Consider a situation where a firm *A* (entering firm) already has some outlets in the market and it is planning to enter with more outlets, unlike the general case of the MAXCAP where there are no already existing outlets for the entering firm. The objective still being the maximum capture of the market but the existing outlets are allowed to relocate within the market to increase profit. This location problem is termed as the Maximum Capture Problem with Relocation (MAXRELOC) and was studied by ReVelle & Serra (1991) and further reformulated by Serra *et al*. (1992).

Another related problem is about a firm entering a new spatial market where there are no current competitors, but some will enter the market in the future, which may affect the capture of the existing firm. The locations of the competitors are not known but the number of outlets of the competitors is known. When the competitor enters the market, it will obviously try to attract demand, which is captured by the existing firm. Therefore the existing firm should preempt the competitors bid to capture the market share. This problem is called the Preemptive Capture Problem (PRECAP) and it is studied by Serra & ReVelle (1994). Hakimi (1983) previously defined this problem as the (*r/p* – centroid) problem. Serra & ReVelle (1995) have also extended the PRECAP model by relaxing the assumption that the number of outlets to be placed is known in advance.

Most of the location models assume that the customers patronize the closest facility. Karkazis (1989) introduced the size of the facility into the model as a criterion for the customer's preference besides the distance. In that study, it was assumed that there were existing facilities in the market and the new locations and the optimum number of facilities to be located is to be determined. Eiselt & Laporte (1989) generalized the ReVelle's model to include the parameters of the gravity model and Voronoi diagrams. Serra *et al.* (1999a) presented two models by including customer choice rules in the MAXCAP whereas Serra *et al.* (1999b) extended the MAXCAP model to include the threshold constraint where the entrance of new facilities can actually force some of the existing facilities to leave the market due to the minimum threshold. In line with incorporating customer choice into the MAXCAP model, Serra & Colomé (2001) modified the key parameter of the model to represent the consumer behavior with respect to distance. They presented a metaheuristic based on GRASP and a tabu search to solve small networks for a comparative study. For an overview of heuristic search, see Salhi (2006).

Colomé *et al.* (2003) introduced a stochastic threshold constraint while their approach was different in two ways. First, the capture was based on the gravity model by Huff (1964), and not just on the closeness of the facilities to the customers, and second, the facility is opened if it meets the threshold by a desired probability. Two meta-heuristics namely an ant system and a tabu search are developed. Pelegrín *et al.* (2007) put forward a Genetic Algorithm that has the potential of finding a predefined set of multiple good or 'optimal' solutions for three related MAXCAP problems. Silva & Serra (2007) included the waiting time in the problem, giving a new direction to the choice of the customers for selecting facilities such as fast foods, ATM machines, and retail stores. The authors used an ant-based heuristic to solve the problem.

Sáiz *et al*. (2010) considered the situation where two competing firms decide on location and quality of a new facility in a new market. They used a two stage game where on the lower level one chooses the quality and on more strategic level suppliers choose the location. Very recently, Pelegrin *et al.* (2011) studied an interesting variant of the problem which aims at determining a location equilibria in the presence of delivered prices for competing firms. A two-stage method was proposed by Saidania *et al*. (2012) that take into consideration the reaction of competitors already in the market while locating facilities. In the first stage they determine quality and in the second stage determine the best location of the new facility using an interval based global optimization method. For general competitive location including the strategic choice of improving or creating new facilities with the presence of budget restriction can be found in the recent paper by Drezner *et al.* (2012).

## 3. The New Modified Maximum Capture Models

The first basic model which we develop is called the MAXCAP with service level (MAXCAP-SL). We assume that for an open outlet the total attracted demand is prorated based on the level of service, and a residual demand is left to be allocated to other competitors. This second variant which includes both the service level and the residual demand is referred to as the MAXCAP-SLR. As the residual demand is allocated to the competitor firm, the distance traveled by the customer is not taken into consideration. Practically a customer who opts to go to the competitor site will go only if the competitor site is not too far from this customer's closest facility. Therefore, a distance limit can be set, which if not met will make him/her shop from his/her closest facility. This further extension of distance threshold is incorporated in the third and final problem which we called the MAXCAP-SLRT.

### The MAXCAP with Service Level

It is assumed from now on that the competition between the different firms will be based on their distances to customers' location and the service level they offer. If a demand node happens to have his/her two nearest open outlets at equal distance, the outlet which offers the higher service level will be patronized by this customer. This assumption does not follow the traditional approach where the oldest outlet in the market is usually favored.

For each existing or potential outlet we define a service level denoted by $\alpha_j$ that takes into consideration a set of customer's decision criteria. For example, in the case of the petrol station location problem as studied in our case study (see Section 4), this can be the number of delivery points such as pumps, road accessibility, onsite car servicing (car wash, car repair, etc.), customer service satisfaction, driving factor (supermarket, general public service, etc.) and other miscellaneous services such as cash point, catering service, among others. For the existing outlets, customers' perception about the current service level can be obtained through a survey. Each criterion is evaluated individually and an aggregation measure is used to form the service level (SL) which we define as $\alpha_j$ where $0 \le \alpha_j \le 1$. For each potential site, different service levels can be tested to reflect the company's strategy and therefore the level of the initial investment. Firms may have different strategies in different areas of the market and can offer different level of services according to the targeted customer area. This approach offers the possibility for the management to eventually adjust their overall strategy to optimally or at best identify those parameters to be modified to increase their market penetration and therefore their market share.

The additional feature of the proposed model is that each outlet will attract a prorated percentage of the demand that is closer than the competitor site based on the service level $\alpha_j$, and the rest will be allocated to the nearest competitor. This applies to both existing and entering firms. For an open outlet that offers high rated services (i.e. $\alpha_j \cong 1$), it will attract all demand present at any node that is allocated to it. For the special case where $\alpha_j = 1$ for all facilities, our proposed model reduces to the original ReVelle's MAXCAP model. In addition to the above assumptions and in case of equal service levels, we consider the existing outlet to be selected.

The MAXCAP with service level which we denote by MAXCAP-SL, is formulated as follows ($P_1$).

$$P_1 = \begin{cases} Max \sum_{i \in I} \sum_{j \in J} a_i \alpha_j \rho_{ij} x_{ij} \\ subject \quad to \quad (2)-(5) \end{cases} \tag{6}$$

Where the additional and modified notation is given as follows

$\alpha_j$ = Service level, given for all existing and potential outlets, where $j \in J$ .

$b_i$ = The nearest facility to customer *i* among firm B's *q* outlets, where $i \in I$ .

$$\rho_{ij} = \begin{cases} 1 \quad \text{if} \quad \begin{cases} d_{ij} < d_{ib_i} \\ or \\ d_{ij} = d_{ib_i} \text{ and } \alpha_j > \alpha_{b_i} \end{cases} \\ 0 \quad \text{otherwise} \end{cases}$$

The two parameters $\alpha_j$ and $\rho_{ij}$ in the objective function (6) differentiate the model from the existing MAXCAP model (*P0*). Based on the service level $\alpha_j$, the total demand attracted by the new firm *A* from the customers that directly have patronized its outlets ($\rho_{ij} x_{ij} = 1$) is obtained. The constraint sets are similar to those of the regular MAXCAP model.

Compared to the traditional MAXCAP, this model differs in two ways. In case of competing firms located at equal distances to a customer, the latter will choose to shop at the one that offers the highest service level and not necessarily the oldest one. Also, if a facility happens to be closer than its competitor with respect to a given demand area, a prorated demand which equals to its attractiveness will only be patronized and the rest will choose to use the closest competitor.

## The MAXCAP with Service Level and Residual Demand

The Maximum Capture Model with Service Level and Residual Demand which we refer to as the MAXCAP-SLR model is based on the fact that the customer demand captured is prorated based on the service level and hence its demand is not completely assigned to its corresponding closest firm. In other words, if a firm *j* attracts customer *i*, not all the demand $a_i$ will be assigned to firm *j* but only a proportion which we define by $a_i \alpha_j$ . The remaining proportion of the demand of this customer, which we refer to as the residual demand, will thus be allocated to the next closest facility, which could be either from the competitor firm or the next closest facility of the entering firm. However, for the current study we assume that if a node is captured, the prorated demand is given to the patronizing firm and the residual to the competing firm. In other words, if a proportion of the demand at node *j* , say $\alpha_j$, is captured by firm *A*, then the proportion of the residual demand of customer *j*, denoted by (1-$\alpha_j$), will be assigned to firm *B*. Using the same token, if the demand is not captured by firm *A*, that means it is captured by firm *B*, and the residual is captured by firm *A*, still increasing the capture of the demand by firm *A* in a given market but with a lesser amount. The MAXCAP-SLR model is now presented as (*P2*) in the following.

$$P_2 = \begin{cases} Max\sum_{i\in I}\sum_{j\in J} a_i\alpha_j\rho_{ij}x_{ij} + \sum_{i\in I} a_i\left(1-\alpha_{b_i}\right)\left(1-\sum_{j\in J}\rho_{ij}x_{ij}\right) \\ subject \quad to \quad (2)-(5) \end{cases} \qquad (7)$$

The objective function (7) has two components. The first is the total demand attracted by the new firm *A* from the customers that directly have patronized its outlets ($\rho_{ij}x_{ij} = 1$). The second term is the demand attracted from its competitor firm *B*. That is for any demand node that is not allocated to the new firm *A* as ($\sum_{j\in J}\rho_{ij}x_{ij} = 0$), a residual demand of $a_i(1-\alpha_{b_i})$ from firm *B* will be allocated to firm *A*. The other constraints are the same as those of the regular MAXCAP model.

**The MAXCAP with Service Level, Residual Demand and Threshold Distance**

In the above MAXCAP-SLR model, the competitor will always receive a small proportion of the demand of a given node but this assumption is too soft as the competitor may be situated too far away from the customer site. To imitate human behavior choices, we incorporate a distance threshold into the model where the customer will be fully assigned to the entering firm if the competitor happens to be situated beyond a prescribed distance threshold. In brief, this is a practical extension of the MAXCAP-SLR where the residual of the existing firm is attracted by the entering firm if and only if a distance threshold is satisfied; otherwise the residual demand cannot be attracted and will remain with the existing firm. If the entering firm captures a demand area, the attracted demand is $\alpha_j a_i$ and the residual is $(1-\alpha_j)a_i$ which can be attracted by the existing firm. The attraction of the total demand by the existing/competing firm is subject to the fulfillment of the distance threshold. We refer to this problem as the MAXCAP-SLRT. To achieve this, we redefine the demand area *j* as follows:

The total demand captured by the model in the entire region will be a combination of the prorated demand based on the service level and the residual demands. Therefore the total demand captured from the selected nodes can possibly include any of the following combinations.

$$a_i = \begin{cases} a_i^{AS} & \text{if threshold distance criteria is not met by both entering or competitor firms and the firm A captures demand prorated to its service level.} \\ a_i^{AR} + a_{b_i}^{BR} & \text{if threshold distance criteria is met by firm A to acquire added residual demand of competitor firm.} \\ a_i^{AS} + a_i^{AR} & \text{if threshold distance criteria is not met by firm A to capture residual of firm B and firm B fails to meet the distance threshold to capture the demand from firm A.} \\ a_i^{AS} + a_{b_i}^{BR} + a_i^{AR} & \text{if threshold distance criteria is met by firm A to capture residual of firm B and firm B fails to meet the distance threshold to capture the demand from firm A.} \end{cases}$$

Where

$a_i^{AS} =$ Prorated demand of node $i$ captured by the firm $A$ as per service level offered by firm $A$.

$a_i^{AR} =$ Prorated demand of node $i$ left by firm $A$ that can be captured by firm $B$.

$a_i^{BS} =$ Prorated demand of node $i$ captured by the firm $B$ as per service level offered by firm $B$.

$a_i^{BR} =$ Prorated demand of node $i$ left by firm $B$ that can be captured by firm $A$.

The model will attract a prorated percentage of the demand that is closest to it, based on the service level $\alpha_j$. The rest of the demand will be subject to a different selection provided that the distance to the closest open competitor is not greater than a given threshold distance. This aspect of the customers selection process will be captured through the parameters $\rho'_{ij}$ and $\rho''_{ij}$, which are defined as follows.

$$\rho'_{ij} = \begin{cases} 1 \text{ if } 0 \le d_{ij} - d_{ib_i} \le T \\ 0 \text{ otherwise} \end{cases}$$

and

$$\rho''_{ij} = \begin{cases} 1 \text{ if } 0 \le d_{ib_i} - d_{ij} > T \\ 0 \text{ otherwise} \end{cases}$$

Where $T$ is the threshold distance. The parameter $\rho'_{ij}$ takes a value 1 if the node $i$ is closer to a potential site than the nearest existing site and $\rho''_{ij}$ takes a value 1 if the existing site is closer than the potential site.

For each residual demand at a given node, the distances to the closest competing open sites will be compared. In addition, if the difference is less than a given threshold distance then customers will select the competing firm. For an open outlet that offers high rated services (i.e. $\alpha_j \cong 1$), it will attract all demand present at any node that is allocated to it. If $\alpha_j = 1$ and if $\rho_{ij} = 1$ for all sites in the market, then our proposed model coincides with the original MAXCAP model. The capture of the residual demand depends on the values taken by the parameters $\rho'_{ij}$ and $\rho''_{ij}$ which are obtained by comparing the threshold distance from each demand point.

*<u>An illustrative example</u>*

We provide here a small example to illustrate the different customer assignments of the two models namely with and without distance thresholds. In this small network, the assignment of the demand to the existing/competitor site (Circle) and the entering site (Hexagon) based on distance alone (MAXCAP-SLR) is given in Figure 1. The capture of an existing site is shown in vertical lines (red color) and the capture that goes to the entering firm in horizontal lines (blue color) according to the service level. The direct capture as per the service level is shown by continuous line connections and the assignment of the residual demand by dashed lines. We briefly explain how these results are obtained in Figures 2a-2d:

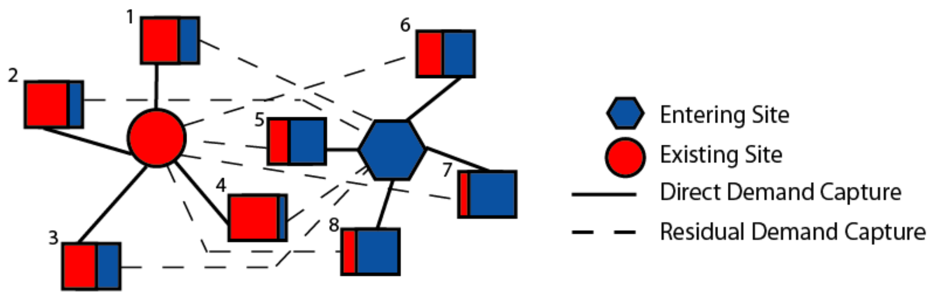
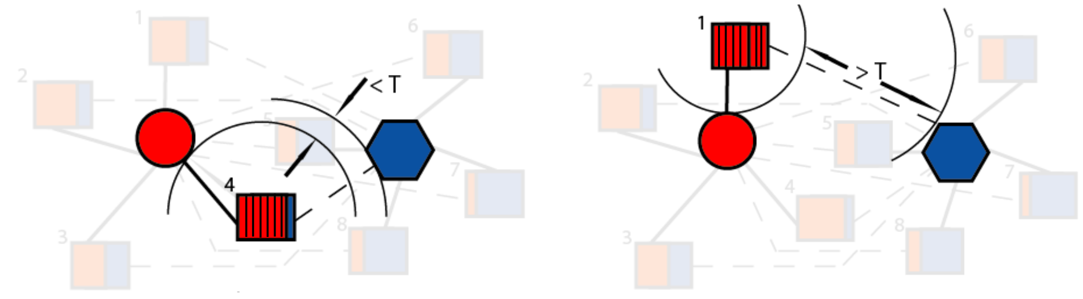
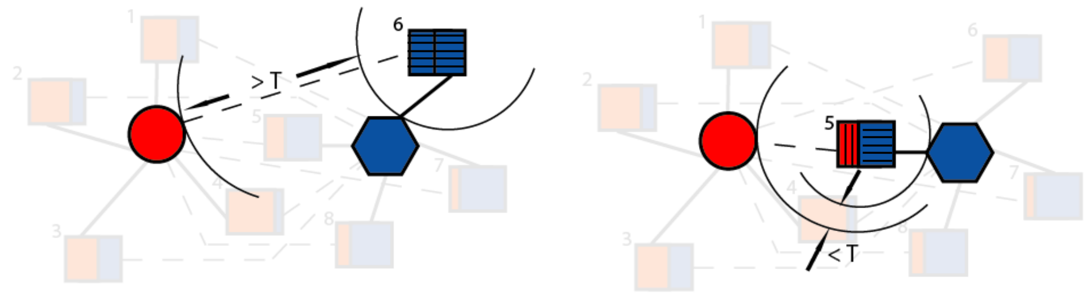



**Figure 1**. Demand capture by MAXCAP-SLR.




(a) $\rho'_{ij} = 1$ (b) $\rho'_{ij} = 0$




(c) $\rho''_{ij} = 1$ (d) $\rho''_{ij} = 0$

**Figure 2**. Effect of the Threshold distance over capture of residual demand.

If we consider node 4 that is captured by the existing site in Figure 2a, we get $\rho'_{ij} = 1$, which means that we attract the residual. If we consider node 1 (Figure 2b), we get $\rho'_{ij} = 0$, which means that we will lose the residual demand and thus the demand at node will completely go to the existing site. In a similar manner, now consider site 6 (Figure 2c) that is captured by the entering facility. Here, we get a value $\rho''_{ij} = 1$ and thus the residual will not go to the existing facility and node 6 will completely be captured by the entering firm. On the other hand if we consider node 5 (Figure 2d), we get $\rho''_{ij} = 0$ which means that we cannot get the residual demand of this node and this will be taken by the existing site.

The overall impact of the threshold distance on this small network will thus result in some demand nodes being completely captured by either the existing or the entering facilities and some customers will be sharing their residual demand with each other. The demand captured by MAXCAP-SLRT is different from MAXCAP-SLR as shown in Figure 3.

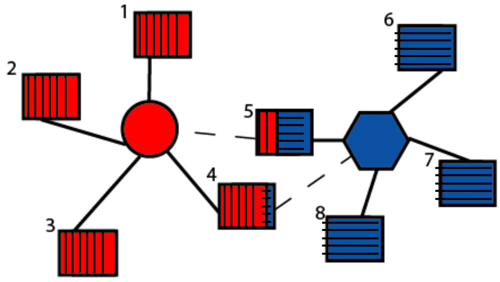



**Figure 3**. Demand capture by MAXCAP-SLRT.

The MAXCAP-SLRT model is now presented as formulation $P_3$.

$$P_3 = \begin{cases} Max \sum_{i\in I} a_i \sum_{j\in J} \alpha_j \rho_{ij} x_{ij} + \sum_{i\in I} a_i \left(1-\alpha_{b_i}\right)\left(\sum_{j\in J} \rho'_{ij} x_{ij}\right)\left(1-\sum_{j\in J} \rho_{ij} x_{ij}\right) \\ \quad + \sum_{i\in I} a_i \left(\sum_{j\in J} \rho''_{ij} x_{ij}\right) \sum_{j\in J} (1-\alpha_j) \rho_{ij} x_{ij} \\ subject \quad to \quad (2)-(5) \end{cases} \qquad (8)$$

The objective function (8) has three terms. The first is the total demand attracted by the new firm *A* from the customers that have directly patronized its outlets ( $\rho_{ij} x_{ij} = 1$). The second term is the demand attracted from its competitor firm *B*. That is for any demand node that is not allocated to the new firm *A* (i.e., $\sum_{j\in J} \rho_{ij} x_{ij} = 0$), a residual demand of $a_i(1-\alpha_{b_i})$ from firm *B* will be allocated to *A* if the extra distance the customer has to cover to reach the competing location does not exceed a certain threshold ( $\sum_{j\in J} \rho'_{ij} x_{ij}$ ) , otherwise it stays with the closest facility. The third term is the residual of the demand node captured by the entering firm. The residual can be attracted by the competitor only if the threshold distance is satisfied and it remains with the capturing firm otherwise. A point to be noted is that, given constraint set (2) the component $\sum_{j\in J} \rho'_{ij} x_{ij}$ will eventually have only one non-zero term and that is for the selected potential node *j* satisfying the threshold distance. A similar observation is also valid for the term $\sum_{j\in J} \rho''_{ij} x_{ij}$ .

The objective function defines the total market share for the entering firm once the *p* locations are found. The other constraints remain similar to those of the earlier models. If we assume a large threshold distance, the problem is then reduced to the MAXCAP-SLR as the residual demands from all nodes will automatically select the closest competitor irrespective of the extra distance they have to cover.

The objective function can be rewritten in a simplified form (as explained in the next subsection) in Equation (9).

$$Z = \sum_{i \in I} a_i \left( \sum_{j \in J} A_{ij} x_{ij} + \sum_{j \in J} B_{ij} x_{ij}^2 \right) \tag{9}$$

Where $A_{ij} = \left( \alpha_j \rho_{ij} + (1 - \alpha_{b_i}) \rho'_{ij} \right)$ and $B_{ij} = \left( \rho''_{ij} \rho_{ij} (1 - \alpha_j) - \rho'_{ij} \rho_{ij} (1 - \alpha_{b_i}) \right)$

The only concern with this model is that it has quadratic variables and thus could take a large computation time to obtain the optimal solutions as it is solved as a 0-1 non linear problem. This particular model, as we will show later, can be rewritten as a linear program. In the following, we first provide the derivation of $A_{ij}$ and $B_{ij}$ as used in (9) and then present the corresponding linear MAXCAP-SLRT model which will be used to solve this problem.

***The derivation steps for $A_{ij}$ and $B_{ij}$.***

The objective function of the MAXCAP-SLRT model (9) can be rewritten as

$$Max \sum_{i \in I} a_i \sum_{j \in J} \alpha_j \rho_{ij} x_{ij} + \sum_{i \in I} a_i \left(1 - \alpha_{b_i}\right) \left( \sum_{j \in J} \rho'_{ij} x_{ij} \right) \left( 1 - \sum_{j \in J} \rho_{ij} x_{ij} \right)$$
$$+ \sum_{i \in I} a_i \left( \sum_{j \in J} \rho''_{ij} x_{ij} \right) \sum_{j \in J} (1 - \alpha_j) \rho_{ij} x_{ij}$$

Let

$$Z = Z_1 + Z_2 + Z_3 \tag{10}$$

where

$$Z_1 = \sum_{i \in I} a_i \sum_{j \in J} \alpha_j \rho_{ij} x_{ij}$$

$$Z_2 = \sum_{i \in I} a_i \left(1 - \alpha_{b_i}\right) \left( \sum_{j \in J} \rho'_{ij} x_{ij} \right) \left( 1 - \sum_{j \in J} \rho_{ij} x_{ij} \right)$$

$$Z_3 = \sum_{i \in I} a_i \left( \sum_{j \in J} \rho''_{ij} x_{ij} \right) \sum_{j \in J} (1 - \alpha_j) \rho_{ij} x_{ij}$$

Since the term $Z_1$ needs no further simplification, it is only the terms $Z_2$ and $Z_3$ that could be simplified. Let us start with $Z_2$.

$$Z_2 = \sum_{i \in I} a_i \left(1 - \alpha_{b_i}\right) \left( \sum_{j \in J} \rho'_{ij} x_{ij} \right) - \sum_{i \in I} a_i \left(1 - \alpha_{b_i}\right) \left( \sum_{j \in J} \rho'_{ij} x_{ij} \right) \left( \sum_{j \in J} \rho_{ij} x_{ij} \right)$$

Let L = $\left( \sum_{j \in J} \rho'_{ij} x_{ij} \right) \left( \sum_{k \in J} \rho_{ik} x_{ik} \right) = \left( \rho'_{i1} x_{i1} + ... + \rho'_{iJ} x_{iJ} \right) \left( \rho_{i1} x_{i1} + ... + \rho_{iJ} x_{iJ} \right)$

Since $\sum_{j \in J} x_{ij} = 1$ (constraint set 3), the product $x_{ij}.x_{ik} = 0$ for $j \neq k$ and thus L can be simplified as

$$L = \sum_{j} \rho'_{ij}\rho_{ij}x_{ij}^2 \text{ (taking only the terms } x_{ij}.x_{ik} \text{ for } j = k\text{)}$$

Therefore $Z_2$ can be written as

$$Z_2 = \sum_{i\in I} a_i\left(1-\alpha_{b_i}\right)\left(\sum_{j\in J}\rho'_{ij}x_{ij}\right) - \sum_{i\in I} a_i\left(1-\alpha_{b_i}\right)\left(\sum_{j\in J}\rho'_{ij}\rho_{ij}x_{ij}^2\right)$$

Using the same above argument, $Z_3$ can be simplified in the same way to produce

$$Z_3 = \sum_{i\in I} a_i\left(\sum_{j\in J}\rho''_{ij}x_{ij}\sum_{j\in J}(1-\alpha_j)\rho_{ij}x_{ij}\right) = \sum_{i\in I} a_i\left(\sum_{j\in J}\rho''_{ij}\rho_{ij}(1-\alpha_j)x_{ij}^2\right)$$

Substituting $Z_1$, $Z_2$ and $Z_3$ in (10), we get the following expression of $Z$

$$Z = \sum_{i\in I} a_i \sum_{j\in J}\alpha_j\rho_{ij}x_{ij} + \sum_{i\in I} a_i\left(1-\alpha_{b_i}\right)\left(\sum_{j\in J}\rho'_{ij}x_{ij}\right) - \sum_{i\in I} a_i\left(1-\alpha_{b_i}\right)\left(\sum_{j\in J}\rho'_{ij}\rho_{ij}x_{ij}^2\right)$$
$$+\sum_{i\in I} a_i\left(\sum_{j\in J}\rho''_{ij}\rho_{ij}(1-\alpha_j)x_{ij}^2\right)$$

$$Z = \sum_{i\in I} a_i \sum_{j\in J}\underbrace{\left(\alpha_j\rho_{ij} + (1-\alpha_{b_i})\rho'_{ij}\right)}_{A_{ij}}x_{ij} + \sum_{i\in I} a_i\left(\sum_{j\in J}\underbrace{\left(\rho''_{ij}\rho_{ij}(1-\alpha_j) - \rho'_{ij}\rho_{ij}(1-\alpha_{b_i})\right)}_{B_{ij}}x_{ij}^2\right)$$

In a simplified form Z can be written as given in (9)

$$Z = \sum_{i\in I} a_i\left(\sum_{j\in J}A_{ij}x_{ij} + \sum_{j\in J}B_{ij}x_{ij}^2\right)$$

■

**The Linear MAXCAP-SLRT Model**

Since the MAXCAP-SLRT model has a quadratic term it is expected to consume a relatively large computation time when solving reasonable sized instances as will be shown in the computational result section. However, we do not need to solve this model blindly as it stands. It can be shown, for this special case, that if $x_{ij}^2$ is replaced by $x_{ij}$, the result would not change for all possible values of the variables. In other words, the optimal solution value of $Z$ under the quadratic form is exactly the same as the one under its linear counterpart. This assumption is valid due to the binary nature of the decision variables $x_{ij}$ and the linearity of the constraints. Therefore our formulation reduces to an ILP form whose computation times can be reduced significantly as will be shown in our computational results section. The objective function (9) can be rewritten as

$$Z = \sum_{i\in I} a_i\left(\sum_{j\in J}\left(\alpha_j\rho_{ij} + (1-\alpha_{b_i})\rho'_{ij}\right)x_{ij} + \sum_{j\in J}\left(\rho''_{ij}\rho_{ij}(1-\alpha_j) - \rho'_{ij}\rho_{ij}(1-\alpha_{b_i})\right)x_{ij}\right)$$

This, after simple mathematical manipulations, leads to Equation (11).

$$Z = \sum_{i \in I} a_i \sum_{j \in J} \Big( \rho_{ij} (\alpha_j + \rho''_{ij} (1 - \alpha_j)) + (1 - \alpha_{b_i}) \rho'_{ij} (1 - \rho_{ij}) \Big) x_{ij} \qquad (11)$$

## 4. The Case Study

This case study is related to a residential area in the city of Dhahran, Saudi Arabia where there are 4 existing petrol stations. The objective is to enter this market with new petrol stations (*p*) and find locations to maximize the demand capture. A Google map and corresponding diagram that represents the number of housing blocks is provided in Figure 4.

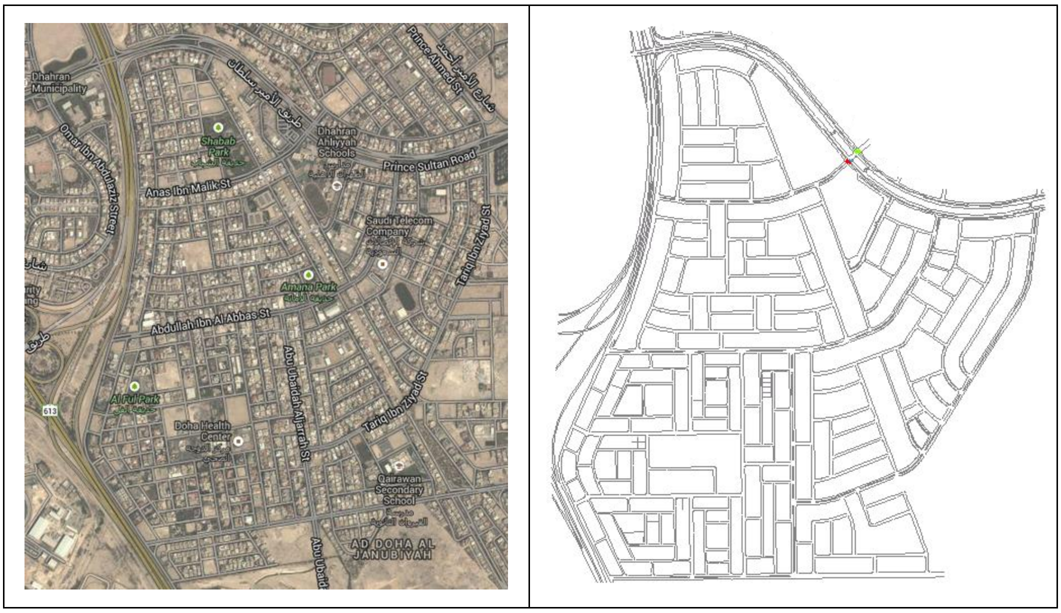



**Figure 4**. Residential Area Map and housing blocks.

There are two grades of petrol sold in Saudi Arabia, Grade 95 (0.60 Saudi Riyal (SR) per liter and Grade 91 (0.45 SR per liter). There is almost a total absence of price competition in the market and the quality of the product is uniform as they all have common supplier. The market is quite dynamic and the number of petrol stations is growing rapidly. To attract customers and to guarantee a minimum market share, the operators must give priority to customers' decision factors based on the onsite service level they will provide along with the sale of petrol.

### Input Data

The residential region was divided into 56 demand areas each aggregated to one demand node. The network is given in Figure 5. We estimated the demand in each area by counting the number of housing blocks as given in Table 1. We then used the road network to find the shortest path between any two nodes using AUTOCAD street drawings, whenever a clear path exists between them shorter than passing through any other node.

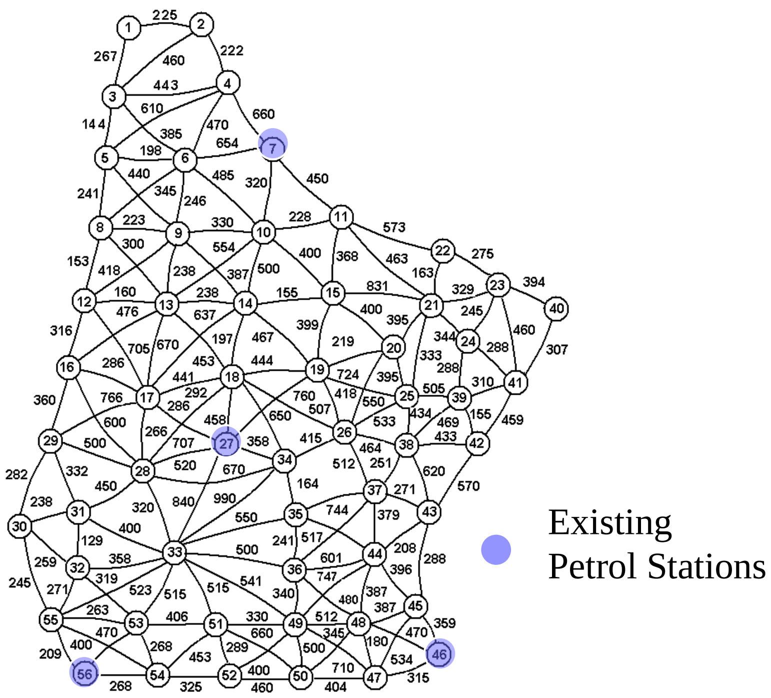



**Figure 5**. Distance Network

Table 1. Demand for the Network

| Node # | Demand | Node # | Demand | Node # | Demand | Node # | Demand | Node # | Demand | Node # | Demand |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 30 | **11** | 36 | **21** | 36 | **31** | 10 | **41** | 40 | **51** | 17 |
| **2** | 34 | **12** | 22 | **22** | 32 | **32** | 8 | **42** | 34 | **52** | 19 |
| **3** | 36 | **13** | 30 | **23** | 41 | **33** | 40 | **43** | 36 | **53** | 12 |
| **4** | 28 | **14** | 34 | **24** | 26 | **34** | 42 | **44** | 41 | **54** | 9 |
| **5** | 40 | **15** | 14 | **25** | 24 | **35** | 40 | **45** | 23 | **55** | 9 |
| **6** | 28 | **16** | 54 | **26** | 45 | **36** | 32 | **46** | 23 | **56** | 11 |
| **7** | 13 | **17** | 30 | **27** | 50 | **37** | 49 | **47** | 38 | | |
| **8** | 32 | **18** | 50 | **28** | 18 | **38** | 45 | **48** | 28 | | |
| **9** | 36 | **19** | 48 | **29** | 12 | **39** | 6 | **49** | 42 | | |
| **10** | 33 | **20** | 28 | **30** | 17 | **40** | 55 | **50** | 35 | | |

All existing petrol stations in the area, 4 in total, are assumed to belong to the same firm *B* and are located at demand areas (7, 27, 46 and 56). All their features were evaluated to estimate their corresponding service level $\alpha_j$. A survey was conducted to obtain the service levels of each existing petrol stations. A form was designed that obtains rating (1 for best to 5 for worst) for various supporting factors of the petrol station and provides a standardized value ($0 \le \alpha_j \le 1$). Figure 6 lists the factors considered for evaluating the service levels for the existing petrol stations. Note that this mechanism is found to be appropriate in our case study but other means may be more suitable for other applications.

**Category 1**
- ✓ Road Access
- ✓ Waiting Time for Service
- ✓ Ratio of No. of Pumps/Workers
- ✓ Credit Card Payment

**Category 2**
- ✓ Cleanliness
- ✓ Customer Care
- ✓ Appearance of the Station
- ✓ Gifts for filling Full Tank
- ✓ Open 24 hours

**Category 3**
- ✓ Super Market
- ✓ ATM
- ✓ Restaurant
- ✓ Drive thru/Take away food service

**Category 4**
- ✓ Garage
- ✓ Car Wash Center
- ✓ Electrical Services
- ✓ Oil Change Facility
- ✓ Mechanical Services
- ✓ Tyre Change/ Repair Services
- ✓ Battery Change/Repair Services

**Category 5**
- ✓ Public Facilities-Govt Offices
- ✓ Schools
- ✓ Hospitals
- ✓ Mosques

**Figure 6**. Categories used to evaluate service level for petrol stations.

The α values of the potential facilities were randomly and uniformly generated in the range [0.60, 0.85]. The MAXCAP problems developed were run for different values of $p$ = 2, 4, 6, and 8. The considered sets of $\alpha_j$ values for the existing firm *A* based on the survey and the entering firm *B* generated randomly are shown in Table 2 and Table 3 respectively.

Table 2. Service levels for Firm *B*

| Existing Sites | Alpha B |
|---|---|
| 7 | 0.7 |
| 27 | 0.7 |
| 46 | 0.9 |
| 56 | 0.5 |

Table 3. Randomly Generated Service levels for Firm *A* between 0.60 and 0.85.

| Node # | Service Level | Node # | Service Level | Node # | Service Level | Node # | Service Level | Node # | Service Level | Node # | Service Level |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0.79 | **11** | 0.79 | **21** | 0.74 | **31** | 0.73 | **41** | 0.7 | **51** | 0.69 |
| **2** | 0.7 | **12** | 0.85 | **22** | 0.75 | **32** | 0.8 | **42** | 0.74 | **52** | 0.83 |
| **3** | 0.69 | **13** | 0.73 | **23** | 0.72 | **33** | 0.81 | **43** | 0.83 | **53** | 0.61 |
| **4** | 0.62 | **14** | 0.85 | **24** | 0.64 | **34** | 0.83 | **44** | 0.85 | **54** | 0.67 |
| **5** | 0.79 | **15** | 0.84 | **25** | 0.79 | **35** | 0.79 | **45** | 0.62 | **55** | 0.82 |
| **6** | 0.69 | **16** | 0.72 | **26** | 0.66 | **36** | 0.63 | **46** | 0.64 | **56** | 0.69 |
| **7** | 0.69 | **17** | 0.7 | **27** | 0.64 | **37** | 0.61 | **47** | 0.61 | | |
| **8** | 0.61 | **18** | 0.84 | **28** | 0.76 | **38** | 0.68 | **48** | 0.63 | | |
| **9** | 0.63 | **19** | 0.83 | **29** | 0.75 | **39** | 0.82 | **49** | 0.74 | | |
| **10** | 0.79 | **20** | 0.7 | **30** | 0.81 | **40** | 0.63 | **50** | 0.74 | | |

**Optimal Solutions**

Table 4 shows the optimal solutions obtained for the three models, MAXCAP-SL, MAXCAP-SLR and MAXCAP-SLRT using the LINGO 6 solver. The model was run for different values of $p$ (2, 4, 6 and 8) to observe the level of capture from the market. Since the MAXCAP-SLRT model requires a threshold distance to be given to run the model, three cases of distances namely low, medium and high were considered. The sites determined along with the demand captured in the market (from the total demand of 1701) are also given.

Table 4. Solutions for different Service Level Models

| ***Model*** | ***P*** | **2** | **4** | **6** | **8** |
|---|---|---|---|---|---|
| **SL** | **Sites** | 10,35 | 10,28,34,46 | 5,15,28,34,46,55 | 5,11,14,28,35,44,46,55 |
| | **Z** | 817.65 | 1134.630 | 1240.640 | 1301.610 |
| **SLR** | **Sites** | 10,34 | 10,14,44,55 | 5,15,28,35,44,55 | 1,11,14,28,34,44,52,55 |
| | **Z** | 1039.55 | 1205.97 | 1285.33 | 1337.1 |
| **SLRT T=100** | **Sites** | 10,35 | 10,28,34,48 | 1,10,28,34,48,55 | 4,10,28,29,35,45,48,54 |
| | **Z** | 1068.02 | 1375.40 | 1502.84 | 1574.28 |
| **SLRT T=500** | **Sites** | 14,44 | 5,25,28,44 | 5,15,16,35,43,55 | 5,15,16,20,34,43,50,55 |
| | **Z** | 1007.10 | 1235.83 | 1348.78 | 1426.50 |
| **SLRT T=1000** | **Sites** | 10,34 | 10,14,44,55 | 1,15,28,35,44,55 | 1,10,14,23,28,44,52,55 |
| | **Z** | 1025.15 | 1205.97 | 1291.63 | 1347.50 |

The MAXCAP-SLR model gives, as expected, a larger capture than the MAXCAP-SL model. This is due to the ability to capture the residual demand of the demand areas which was earlier assumed to be captured by firm *B*. The capture by the MAXCAP-SLRT also depends on the residual demands captured from the existing as well as the entering firms. While the entering firm may attract the residual demand from the existing firm based on the distance threshold it is also likely that it may lose its residual demand to the existing firm. As the threshold distance increases it is clear that the chance of both firms losing their residual demands increases. In other words, we can see that as the threshold distance tends to infinity the results become similar to those of the MAXCAP-SLR. This is because the residual demand of the existing firm is completely captured by the entering firm and the residual of the entering firm is completely taken by the existing firm. In Figure 7, we illustrate the comparison of the demand captured by the original MAXCAP and the proposed models.

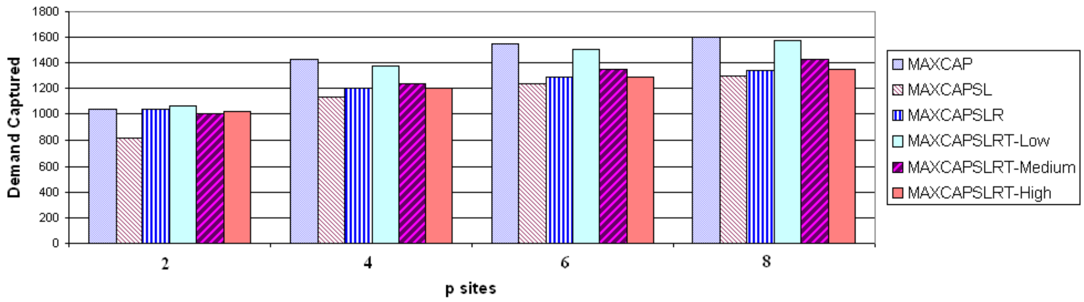



**Figure 7.** Demand capture by different MAXCAP models.

The first series plotted in Figure 7 is the demand captured by the original MAXCAP model and the rest are the demand captured by the proposed related models. It can be seen that the original model tends to capture more demand in all cases ($p$ = 2, 4, 6, 8) as it does not take into consideration the practical situations as the capture is just based on distance. As the service level is incorporated in the model the demand capture seems to decrease. The MAXCAP-SL model just captures the demand based on the service level (without residuals) and thus always has the least capture among all the models. The last three series are based on threshold distance that is an effort to get closer to customer behavior who decides based on the difference in the distance between the existing and the competitor sites. It can be observed that if the threshold distance is low the model tends to provide results closer to the original model and if the threshold distance is high it tends to get closer to the MAXCAP-SLR model.

## 5. Computational Results

In addition to the case study provided in the previous section, three larger data sets of sizes 100, 150 and 200 nodes are used to solve the MAXCAP related problems. These are extracted as the first points from the 287 fixed point data set commonly used for the multi source Weber problem (see Luis *et al.* (2009) and Brimberg *et al* (2000)), while the customer demands are uniformly generated in [0,100] for this study. Each data set is tested for different values of *p* to observe the effect on the capture.

### Optimal Results

The summary of the results for these data sets is given in Table 5 for both MAXCAP-SL and MAXCAP-SLR models. The sites selected for these models are provided in Appendix Table A-1. Table 6 gives the results for the MAXCAP-SLRT model for three cases of the threshold distance considered as low, medium and high for the 56 nodes network only. Other results are not reported because of the exponential rise in the computation times but these will be given when using the relaxed LP formulation.

Table 5. Optimal Solutions for the MAXCAP-SL Model

| **No of Nodes** | **Total Demand** | **Existing Sites** | ***p*** | **MAXCAP-SL** | | **MAXCAP-SLR** | |
|---|---|---|---|---|---|---|---|
| | | | | **Demand Captured** | **CPU (s)** | **Demand Captured** | **CPU (s)** |
| 56 | 1701 | 4 | 2 | 817.65 | 1 | 1039.55 | 1 |
| | | | 4 | 1134.63 | 1 | 1205.97 | 1 |
| | | | 6 | 1240.64 | <1 | 1285.33 | <1 |
| | | | 8 | 1301.61 | <1 | 1337.1 | <1 |
| 100 | 5101 | 5 | 2 | 2354.12 | 5 | 3067.42 | 4 |
| | | | 4 | 3299.52 | 3 | 3477.11 | 3 |
| | | | 6 | 3656.25 | 2 | 3751.55 | 1 |
| | | | 8 | 3866.32 | 2 | 3925.82 | 1 |
| | | | 10 | 3978.54 | 1 | 4014.34 | 1 |
| 150 | 7783 | 5 | 5 | 5656.54 | 5 | 5813.14 | 6 |
| | | | 10 | 6151.20 | 3 | 6180.1 | 4 |
| | | | 15 | 6297.49 | 3 | 6322.09 | 3 |
| | | | 20 | 6329.62 | 2 | 6354.22 | 3 |
| 200 | 10750 | 5 | 5 | 7318.08 | 62 | 7744.55 | 52 |
| | | | 10 | 8411.07 | 11 | 8520.44 | 12 |
| | | | 15 | 8660.44 | 6 | 8717.44 | 6 |
| | | | 20 | 8744.43 | 4 | 8788.23 | 5 |

Table 6. Optimal Solutions for the MAXCAP-SLRT Model using the Quadratic Model: Case of the 56 nodes problem.

| Threshold Level | Total Demand | Threshold Distance | p | Demand Capture | CPU (s) | Selected Sites |
|---|---|---|---|---|---|---|
| Low | 1701 | 100 | 2 | 1068.02 | 11 | 10,35 |
| | | | 4 | 1375.40 | 64 | 10,28,34,48 |
| | | | 6 | 1502.84 | 131 | 1,10,28,34,48,55 |
| | | | 8 | 1574.28 | 8 | 4,10,28,29,35,45,48,54 |
| Medium | 1701 | 500 | 2 | 1007.10 | 77 | 14,44 |
| | | | 4 | 1235.83 | 29 | 5,25,28,44 |
| | | | 6 | 1348.78 | 205 | 5,15,16,35,43,55 |
| | | | 8 | 1426.50 | 785 | 5,15,16,20,34,43,50,55 |
| High | 1701 | 1000 | 2 | 1025.15 | 32 | 10,34 |
| | | | 4 | 1205.97 | 163 | 10,14,44,55 |
| | | | 6 | 1291.63 | 75m15s | 1,15,28,35,44,55 |
| | | | 8 | 1347.50 | 110m35s | 1,10,14,23,28,44,52,55 |

Note: Existing Sites = 4

## Results for the linear model (MAXCAP-SLRT)

The relaxed model, which is relatively quicker, is run for the three cases of distance threshold. The results are given in Table 7 for low, medium and high threshold distance cases. The sites selected for each level of the threshold distance is provided in Appendix Tables A-2, A-3 and A-4 for low, medium and high threshold distances respectively.

Table 7. Optimal Solutions for MAXCAP-SLRT using the Linear Model (Low, Medium and High Threshold)

| No of Nodes | Total Demand | Existing Sites | *p* | Low threshold | | | Medium threshold | | | High threshold | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | T Dist | Demand Captured | CPU (s) | T Dist | Demand Captured | CPU (s) | T Dist | Demand Captured | CPU (s) |
| 56 | 1701 | 4 | 2 | 100 | 1068.02 | 1 | 500 | 1007.10 | 1 | 1000 | 1025.15 | 1 |
| | | | 4 | | 1375.40 | 1 | | 1235.83 | 1 | | 1205.97 | 1 |
| | | | 6 | | 1502.84 | <1 | | 1348.78 | 1 | | 1291.63 | 1 |
| | | | 8 | | 1574.28 | 1 | | 1426.50 | 1 | | 1347.50 | <1 |
| 100 | 5101 | 5 | 2 | 1 | 2815.06 | 6 | 5 | 2933.50 | 4 | 10 | 3052.12 | 4 |
| | | | 4 | | 3524.72 | 6 | | 3459.02 | 2 | | 3477.11 | 3 |
| | | | 6 | | 3984.71 | 4 | | 3753.46 | 2 | | 3751.55 | 2 |
| | | | 8 | | 4372.06 | 4 | | 3941.81 | 2 | | 3925.82 | 1 |
| | | | 10 | | 4593.94 | 3 | | 4042.57 | 2 | | 4014.34 | 1 |
| 150 | 7783 | 5 | 5 | 1 | 6464.25 | 31 | 5 | 5946.38 | 15 | 10 | 5813.14 | 5 |
| | | | 10 | | 7081.96 | 17 | | 6533.22 | 15 | | 6198.48 | 2 |
| | | | 15 | | 7301.85 | 16 | | 6735.23 | 40 | | 6341.08 | 1 |
| | | | 20 | | 7418.57 | 7 | | 6859.80 | 4 | | 6400.45 | 1 |
| 200 | 10750 | 5 | 5 | 1 | 7968.11 | 74 | 5 | 7744.55 | 101 | 10 | 7744.55 | 62 |
| | | | 10 | | 9347.22 | 30 | | 8576.85 | 9 | | 8520.44 | 12 |
| | | | 15 | | 9835.80 | 20 | | 8946.51 | 7 | | 8717.44 | 7 |
| | | | 20 | | 10100.50 | 11 | | 9111.65 | 7 | | 8821.22 | 5 |

The results when compared with the quadratic models for the corresponding cases show the same capture in both cases as demonstrated earlier. The advantage through this relaxation is the massive reduction in the computation times. For instance, for the case of the 56 node with high threshold and *p*=8, the original MAXCAP-SLRT required nearly 2 hours of execution time (110mins 35secs) whereas its relaxed LP used less than 1 sec without affecting the solution quality. This massive time saving can obviously be redirected for assessing various scenarios to yield a robust solution if need be. Although a similar capture is obtained by both models it is observed that they can provide alternative solutions in terms of opened sites. This can increase the choice for the decision maker of placing the facilities at different locations while achieving the same objective function value. For example consider the same example (case of 56 nodes with $p$ = 8) where both models yielded the same Z value of 1574.28 but the quadratic model opened the sites 4, 10, 28, 29, 35, 45, 48, 54 while the relaxed model opened the same locations except sites 4 and 48 are replaced by 6 and 47 respectively.

## 6. Conclusions and suggestions

Three related MAXCAP based models are explored. The first one allows an analysis and modeling of customers' behavior. With the proposed approach the entering firm can also tune up its strategy (by considering different service level scenarios for potential sites) to maximize its market share. A second problem which takes also into consideration, not only the service level but also the possibility that some of the customer demand may be assigned to competing firms due to service level, is put forward. The third and final problem attempts to overcome the drawback of the previous model that a distant competing firm may still attract some part of some customer demands. A distance threshold is introduced into the model that takes into consideration the distance a customer is willing to travel to reach the facility of his/her choice. Since the latter model includes quadratic variables, this is transformed into a linear model which requires a tiny fraction of its original computing time while maintaining optimality. For illustration purposes, a case study is presented where the market consists of 56 nodes with 4 existing facilities. A larger set of instances (95 in total) varying in size between 56 and 200 customers, with 4 and 5 existing facilities and $p$ varying between 2 to 20 are used to assess the performance of the proposed models. All the problems are solved optimally using LINGO.

The MAXCAP model can be further enhanced by introducing cost functions in the objective functions that provide an opportunity to make practical decisions with the presence of budget restrictions. Here the number of facilities to open can be a decision variable unlike the regular MAXCAP model where the numbers of sites to be opened are known in advance. Another research area would be to assess the impact of service level in determining the number of facilities to open. In the MAXCAP-SLR model, we assumed that the residual demand always goes to the competing firm. A study that investigates the case where the next nearest open facility also belongs to the entering firm is worth pursuing. The problem can also be modeled as a bi-objective problem (minimization of distance and maximization of customer service) where weighted programming or goal programming can be used. As the demand varies with time and locating facilities is a strategic type decision problem, robust optimization based scenario analysis for instance could be explored to determine the most robust location configuration.

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## APPENDIX

Table A-1. Optimal selected sites for the MAXCAP-SL and MAXCAP-SLR Models

| No of Nodes | *p* | MAXCAP-SL | MAXCAP-SLR |
|---|---|---|---|
| 56 | 2 | 10,35 | 10,34 |
| | 4 | 10,28,34,46 | 10,14,44,55 |
| | 6 | 5,15,28,34,46,55 | 5,15,28,35,44,55 |
| | 8 | 5,11,14,28,35,44,46,55 | 1,11,14,28,34,44,52,55 |
| 100 | 2 | 21,75 | 21,75 |
| | 4 | 17,21,45,75 | 21,44,61,75 |
| | 6 | 17,21,45,61,75,87 | 17,21,45,61,75,87 |
| | 8 | 17,18,21,45,61,64,75,87 | 17,18,21,45,61,64,75,87 |
| | 10 | 6,17,18,21,44,45,61,64,75,87 | 6,17,18,21,44,45,61,64,75,87 |
| 150 | 5 | 30,90,94,98,150 | 66,90,98,118,121 |
| | 10 | 21,30,61,66,90.98,106,117,121,150 | 28,30,61,66,90,98,106,117,121,150 |
| | 15 | 28,30,61,66,88,90,91,98, 102,106,110,118,121,135,150 | 28,30,61,66,88,90,91,98, 102,106,110,118,121,135,150 |
| | 20 | 28,30,38,61,66,88,90,91,98,102,106, 107,110,118,119,121,131,135,139,150 | 28,30,38,61,66,88,90,91,98,102,106, 107,110,116,118,119,121,135,139,150 |
| 200 | 5 | 17,66,105,127,175 | 17,66,105,127,156 |
| | 10 | 22,44,66,69,72,105,139,141,158,175 | 22,44,66,69,72,105,139,141,158,174 |
| | 15 | 17,22,44,55,66,69,72,102, 105,125,139,141,156,158,175 | 17,22,44,55,66,69,72,102,105, 125,139,141,156,158,175 |
| | 20 | 17,21,22,42,44,55,64,66,69,72,102, 105,125,139,141,145,158,171,174,175 | 17,21,22,42,44,55,64,66,69,72,102, 105,125,139,141,145,158,171,174,175 |

Table A-2. Optimal Solutions for MAXCAP-SLRT using the Linear Model (Low Threshold)

| No of Nodes | Total Demand | Existing Sites | T Dist | p | Selected Sites |
|---|---|---|---|---|---|
| 56 | 1701 | 4 | 100 | 2 | 10,35 |
| | | | | 4 | 10,28,34,48 |
| | | | | 6 | 6,10,28,34,48,55 |
| | | | | 8 | 6,10,28,29,35,45,47,54 |
| 100 | 5101 | 5 | 1 | 2 | 44,49 |
| | | | | 4 | 20,44,61,75 |
| | | | | 6 | 17,21,49,57,61,75 |
| | | | | 8 | 7,18,23,46,57,61,64,75 |
| | | | | 10 | 7,17,18,21,44,45,46,61,64,75 |
| 150 | 7783 | 5 | 1 | 5 | 30,90,94,98,118 |
| | | | | 10 | 30,45,48,61,90,94,98,116,121,150 |
| | | | | 15 | 18,30,47,59,61,76,90,98, 106,108,118,121,135,149,150 |
| | | | | 20 | 18,30,31,46,47,59,61,76,88,90,92, 98,104,106,118,121,122,135,149,150 |
| 200 | 10750 | 5 | 1 | 5 | 17,66,105,127,156 |
| | | | | 10 | 22,44,68,72,88,91,124,127,134,175 |
| | | | | 15 | 17,22,44,68,69,72,89,91, 124,136,141,142,151,158,173 |
| | | | | 20 | 17,22,42,44,68,69,72,75,105,107,110, 112,119,134,141,142,156,158,175,191 |

Table A-3. Optimal Solutions for MAXCAP-SLRT for the Relaxed Model (Medium Threshold)

| No of Nodes | Total Demand | Existing Sites | T Dist | p | Selected Sites |
|---|---|---|---|---|---|
| 56 | 1701 | 4 | 500 | 2 | 14,44 |
| | | | | 4 | 5,25,28,44 |
| | | | | 6 | 5,15,16,35,43,55 |
| | | | | 8 | 5,15,16,20,34,43,50,55 |
| 100 | 5101 | 5 | 5 | 2 | 44,49 |
| | | | | 4 | 21,44,61,75 |
| | | | | 6 | 17,21,45,61,75,87 |
| | | | | 8 | 17,18,21,45,61,64,75,100 |
| | | | | 10 | 4,11,18,22,44,45,61,64,75,100 |
| 150 | 7783 | 5 | 5 | 5 | 30,58,66,118,121 |
| | | | | 10 | 30,33,61,66,90,98,112,117,121,150 |
| | | | | 15 | 25,30,41,61,66,88,90,98, 106,112,114,118,121,135,150 |
| | | | | 20 | 8,26,30,40,58,61,66,88,90,94,96,98, 106,114,118,121,129,135,142,150 |
| 200 | 10750 | 5 | 5 | 5 | 17,66,105,127,156 |
| | | | | 10 | 22,44,66,69,72,102,105,139,141,174 |
| | | | | 15 | 22,26,30,44,66,69,72,102, 105,125,139,141,158,165,174 |
| | | | | 20 | 2,17,22,26,30,44,55,58,66,69,72, 102,105,125,139,141,156,158,165,175 |

Table A-4. Optimal Solutions for MAXCAP-SLRT for the Relaxed Model (High Threshold)

| No of Nodes | Total Demand | Existing Sites | T Dist | p | Selected Sites |
|---|---|---|---|---|---|
| 56 | 1701 | 4 | 1000 | 2 | 10,34 |
| | | | | 4 | 10,14,44,55 |
| | | | | 6 | 1,15,28,35,44,55 |
| | | | | 8 | 1,11,14,23,28,44,52,55 |
| 100 | 5101 | 5 | 10 | 2 | 21,75 |
| | | | | 4 | 21,44,61,75 |
| | | | | 6 | 17,21,45,61,75,87 |
| | | | | 8 | 17,18,21,45,61,64,75,87 |
| | | | | 10 | 6,17,18,21,44,45,61,64,75,87 |
| 150 | 7783 | 5 | 10 | 5 | 66,90,98,118,121 |
| | | | | 10 | 17,30,61,66,90,98,106,117,121,150 |
| | | | | 15 | 17,30,36,61,66,88,90,94,98, 102,106,118,121,135,150 |
| | | | | 20 | 1,15,17,30,36,38,44,61,66,88,90, 91,98,102,106,110,118,121,135,150 |
| 200 | 10750 | 5 | 10 | 5 | 17,66,105,127,156 |
| | | | | 10 | 22,44,66,69,72,105,139,141,158,174 |
| | | | | 15 | 17,22,44,55,66,69,72,102, 105,125,139,141,156,158,175 |
| | | | | 20 | 17,22,29,42,44,55,60,66,69,72,102, 105,125,139,141,156,158,175,181,182 |