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Resilience analysis of maritime transportation systems based on importance measures*

Hongyan Dui[a], Xiaoqian Zheng[a], Shaomin Wu[b]
[a]School of Management Engineering, Zhengzhou University, Zhengzhou 450001, China
[b]Kent Business School, University of Kent, Canterbury, Kent CT2 7FS, UK

Abstract: Economic development depends largely on the import and export of goods for many countries. These goods are mainly transported internationally through the maritime transportation system (MTS). In MTS, ports and ocean routes are essential for establishing and maintaining effective international trade routes. However, the ability of the ports to send and receive goods can be easily destroyed by political and natural interferences. This will cause a significant negative socio-economic impact such as port operation suspension and route disruption. Effectively implementing resilience management in MTS can therefore improve its ability to handle interruptions and minimizing losses. Based on the post-disaster analysis, this paper proposes a new method to optimize residual resilience management of ports and routes in MTS and proposes an optimal resilience model. The residual resilience is then applied to some importance measures. The Copeland method is used to comprehensively rank the importance of ports and routes. The restoration priority of interrupted ports and routes of different importance for the purpose of minimizing residual resilience is also studied. Sea routes consisting of 23 cities are used to demonstrate the applicability of the proposed method. It is found that the supply node and its connected link have a higher priority in the repair process and that the Shanghai port in the MTS is the most important node and Shanghai-Busan is the most important maritime route.

Keywords: reliability, resilience, importance measure, maritime transportation system

1. Introduction

1.1 Background

Under the trend of economic globalization, the international trade has been thriving and requiring long and complex supply chains. Maritime transportation is an

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important pillar of the international supply chain. In a long and complex supply chain system, MTS is more likely to be disrupted by man-made and natural disasters. For example, in 2004, the coast of Indonesia’s Sumatra Island was hit by a large earthquake, and the tsunami severely affected the global supply chain. The 2008 snow disaster in China caused some ports along the Yangtze river to be closed. Consequently, a large number of cargo ships in the Shanghai port were unable to berth and sail normally, and the cargo throughput of Shenzhen and Guangzhou ports dropped significantly. The 2011 Tōhoku earthquake and tsunami in Japan resulted in the destruction of many ports, which costed Japan more than $3.4 billion in maritime trade losses. The port disruption in Indonesia and the hurricane in Australia in 2017 had a tremendous impact on the Asian coal market. In 2019, a report by Nanyang Technological University and Cambridge University showed that if 15 ports in 5 Asian countries (China, Japan, South Korea, Singapore and Malaysia) were directly paralyzed by cyber-attacks, which could cause economic losses of up to US$110 billion. However, since such disasters are unpredictable, it is impossible to protect the MTS by eliminating the occurrence of disasters. The best solution may be to restore the system operation as soon as possible after the disaster. Resilience management in the MTS should therefore be used facilitate the system to "bounce back" quickly after severe disrupts. This will restore the system to its original level and minimize losses.

1.2 ***Literature reviews***

In terms of the resilience management of MTS, Mayada et al. [1] propose several schemes that improve resiliency by reducing the vulnerability of the system. Mansouri et al. [2] propose to evaluate resiliency strategies for ports using a risk management approach to defining the nature of resiliency in port infrastructure systems. Nair et al. [3] suggest measuring the resiliency of ports using the measure of intermodal resiliency. Berle et al. [4] propose a structured formal vulnerability assessment methodology, seeking to transfer the safety-oriented assessment framework into the domain of maritime supply chain vulnerability. Asadabadi and Miller-Hooks [5] propose the concept of port reliability and resilience, as well as the role of ports in supporting a larger resilient maritime system. Wan et al. [6] present a comprehensive review on transportation resilience management with emphasis on its definitions, characteristics, and research methods applied in different transportation systems. Adjetey-Bahun et al. [7] propose a simulation-based model for quantifying resilience in mass railway transportation systems by quantifying passenger delay and passenger load as the system's performance indicators. Cimellaro et al. [8] evaluate disaster resilience based on analytical functions related to the variation of functionality. Zhang et al. [9] explore resilience measures in network systems from different perspectives and analyze the characteristics of nodes and edges during failures, the matrices of node resilience and edge resilience. Cai et al. [10] propose a dynamic Bayesian network to predict the resilience value of an engineering system. Chen et al. [11] establish a model of measuring supply chain resilience based on the cost composition of the supply chain operating in the interrupted environment. Bao et al. [12] propose a tri-level model explicitly integrating the decision making on recovery strategies of disrupted facilities with the decision making on protecting facilities from intentional attacks. Xing and Levitin [13] model and study the resilience of linear consecutively connected systems with connection elements under corrective maintenance. Feng et al. [14] present some general methodologies for resilience design under internal deterioration and external shocks, and apply them into offshore wind farm.

The current research on resilience mainly focuses on complex systems. It is believed that resilience is determined by the degree and speed of performance recovery after system components fail. However, the recovery strategy after system component failure is also considered the key to managing resilience. This paper proposes using importance measures into resilience management. The purpose is to study the recovery sequence of failed components in the system, so that the system can quickly recover to its best state.

In terms of the resilience importance, Xu et al. [15] propose a new resilience-based component importance measure for networks. Fang et al. [16] propose the optimal repair time and the resilience reduction worth to measure the criticality of the components of a network system. Dui et al. [17] propose an extended joint integrated importance measure effectively to guide the selection of preventive maintenance components, aiming to maximize gains of the system performance. Dui et al. [18] study the Birnbaum importance measure, integrated importance measure, and the mean absolute deviation with respect to the changes in optimal system structure throughout the system's lifetime. Wu et al. [19] introduce an importance measure for selecting components for preventive maintenance. Levitin et al. [20] consider some commonly used importance measures in a generalized version for application to multi-state systems. Xu et al. [21] propose a new resilience-based component importance ranking measure for multi-state networks from the perspective of a post-disaster restoration process. Almoghathawi and Barker [22] propose component importance measures to analyze the variations of a network recovery. Miziula and Navarro [23] extend the Birnbaum importance measure for the case of a system with dependent components to obtain relevant properties such as connections and comparisons with other measures proposed and studied recently. Henry et al. [24] propose generic metrics and formulae for quantifying system resilience. Barker et al. [25] provide two resilience-based component importance measures, built on the extensive reliability engineering literature, for measuring component importance.

1.3 ***Motivation***

It can be seen from the above literature review that there is still little work on quantifying the resilience of MTS. The existing literature lacks a resilience measure for solving the following problems: In the MTS, how can one quantify the impact of different ports and routes on the resiliency of the MTS? If the MTS suffers from disasters, multiple ports and routes are prone to fail at the same time. In the case of limited resources, how can one determine the repair sequence of the port and routes so that the MTS can be repaired quickly in the shortest time?

This paper investigates the resilience of MTS. It proposes a new concept of residual resilience and applies it to measure the scale and speed of system performance recovery after port demand or supply interruption. The residual resilience is applied to the OPT importance, Birnbaum importance, RAW importance and RRW importance, respectively. Based on the minimum residual resilience, the priority of restoration of failed ports and routes is studied. The purpose of this method is to study the recovery priority of interrupted ports and routes from different importance based on the post-disaster MTS. It can further enrich the literature in the field of quantitative assessment of maritime resilience.

1.4 ***Overview***

This rest of this paper is structured as following. Section 2 first introduces the main international routes. The MTS network model based on the main ports and routes is established. Next, the state of the post-disaster MTS is analyzed and a concept of residual resilience is proposed. Section 3 proposes some residual resilience importance methods for the post-disaster MTS to evaluate the recovery priority of the interrupted ports and routes with the minimum residual resilience. Section 4 applies a numerical example of sea routes to verify the applicability of the proposed methods. Section 5 concludes the paper and proposes the future work.

Notations

N	Set of nodes in the logical network
L	Set of edges in the logical network
N_S	Supply nodes of MTS
N_D	Demand nodes of MTS
N_T	Transit nodes of MTS
C_0	Capacity set of the MTS
P_{ij}	Capacity of the edges
$P^S{}_i$	Capacity of the supply nodes
$P^S{}_j$	Capacity of the demand nodes
Q	Demand of all nodes in the MTS logical network
Q_0	Minimum demand value of all demand nodes when nodes and edges fail
$R(t)$	Residual resilience value of MTS
$Q^*(t)$	Desired demand
$Q(t)$	Actual demand
$q_j(t)$	Receiving traffic of the demand node j in the t-th time period
$q_{ij}(t)$	Flow from supply node i to node j at time unit t
$\mu_i\ (t)$	State of the node i at time unit t
$\mu_{ij}\ (t)$	State of the edge ij at time unit t
I_i^{OPT}	OPT residual resilience importance of failed node i
I_{ij}^{OPT}	OPT residual resilience importance of failed edge ij
I_C^B	Birnbaum residual resilience importance of node i or edge ij
I_C^{RAW}	RAW residual resilience importance of failed node i or edge ij

I_C^{RRW}	RAW residual resilience importance of failed node i or edge ij
$R(T)$	Residual resilience value of the MTS when the time unit is T
R_0	Residual resilience value of the MTS after the disaster
v^k_i	Value of the k-th importance index of node i
v^k_j	Value of the k-th importance index of node j
$C^k_{\alpha,\beta}(\alpha)$	k-th importance index of node i and node j is compared to obtain the Copeland score of node i
$C_{total}\ (\alpha)$	The Copeland total score of the node α

2. **Resilience model of MTS**

*2.1 **Build a MTS model***

The main routes for international trade in the world are the Atlantic route, the Pacific route and the Indian Ocean route. The Pacific route is chiefly for trades between developing countries and developed countries, accounting for 25% of the global freight volume and 33% of turnover [1]. The Atlantic route is mainly for trades between developed countries, accounting for 40% of the global freight volume and 67% of turnover [1]. There are few economically developed areas on the Indian Ocean route and maritime trade may therefore be underdeveloped. Because this paper mainly studies the resilience of the MTS, a port with a relatively large shipping volume is selected as the research object.

Fig. 1. The part of the international shipping route

As shown in Fig. 1, some ports and routes in maritime transportation are selected as research objects in this article. When a natural disaster occurs, the MTS will immediately fall into chaos. Many ports and shipping routes will then quickly stop normal operating and enter a suspended state. At this time, the performance of the MTS reaches its lowest state. After a natural disaster occurs, post-disaster reconstruction work is needed. Due to resource and time constraints, it is impossible to repair failed ports and routes at the same time. Therefore, how to determine the restoration sequence of failed ports and routes to restore the system performance to the greatest extent in the shortest time has become a research focus.

The logical network from the physical network is shown in Fig. 2, with the numbers of nodes listed in Table 1.

As shown in Fig. 2, the logical network of the MTS consists of nodes and connecting edges. The nodes in the logical network represent ports and the links represent the maritime routes between ports. The network flow is expressed by the throughput of ports, and the capacity of the nodes and edges is known. Ships carrying cargo sails on the maritime routes connecting the ports. The MTS in this paper is a binary system, that is, all nodes and edges have only two states: either operating or failed. The status of all nodes in the system are independent. Each failed node in the system can be repaired, and the recovery time is the same. However, no more than one failure node can be repaired at a time point.

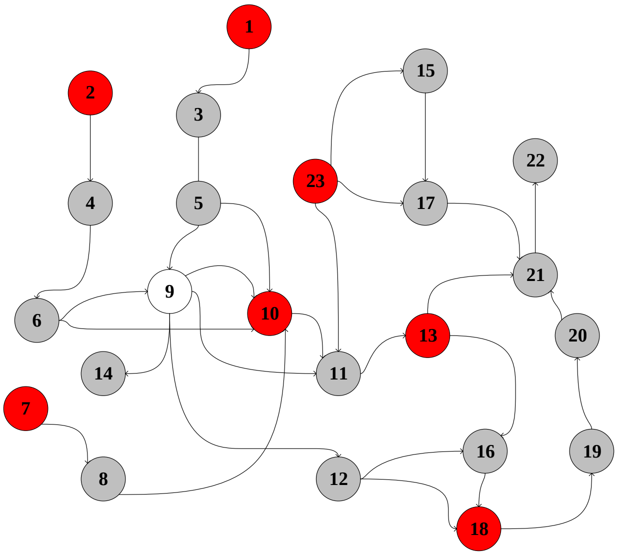

Fig. 2. Logic network of major sea routes

Table 1. Port number table

number port	1 Hamburger	2 New York	3 Rotterdam	4 Barcelona

number	5	6	7	8
port	Marsaxlokk	Genoa	Santos	Durban
number	9	10	11	12
port	KeLang	Singapore	Hong Kong	Zhoushan
number	13	14	15	16
port	Shanghai	Tianjin	Kaohsiund	Busan
number	17	18	19	20
port	Osaka	Yokohama	Seattle	Oakland
number	21	22	23	
port	Los Angeles	Kingston	Sydney	

Denote the logical network of the MTS as $G\ (N, L)$, where N represents the node set and L represents the edge set. N includes three subsets: supply node subset N_S, demand node subset N_D, and transit node subset N_T. C_0 is the capacity set of the MTS. The capacity of the edges, the supply nodes, and the demand nodes are denoted by P_{ij}, $P^S{}_i$ and $P^D{}_j \in C_0$, respectively. According to the actual port information, different nodes are selected as supply nodes, demand nodes, and transit nodes, respectively. The classification of the nodes are shown as follows.

- N_S={1, 2, 7, 10, 13, 18, 23},
- N_D={3, 4, 5, 6, 8, 11, 12, 14, 15, 16, 17, 19, 20, 21, 22},
- N_T={9},
- N={S1, S2, D3, D4, D5, D6, S7, D8, T9, S10, D11, D12, S13, D14, D15, D16, D17, S18, D19, D20, D21, D22, S23}, and
- L={1-3, 3-5, 5-9, 5-10, 2-4, 4-6, 6-9, 6-10, 7-8, 8-10, 9-10, 9-11, 9-12, 9-14, 10-11,11-13, 12-16, 12-18, 16-18, 18-19, 13-16, 13-21, 19-20, 20-21, 21-22, 23-11, 23-17, 23-15, 15-17, 17-21}.

The system function is represented by Q, which meets the demand of all nodes. The set of failed nodes is E, where $E \in N$. The set of failed edges is F, and $F \in L$. After a disaster occurs, the system function $Q(t)$ reaches the minimum value Q_0. The purpose of this paper is to determine the repair order of the failed nodes set E or failed edges F with the minimum residual resilience as the target within a given time period. A time set consisting of multiple discrete time periods is therefore needed. Let $t \in \{1,2,3...T\}$, $T \in Z^+$, and only a single fault is repaired in each time period.

2.2 Analysis of MTS states

The route change of MTS before and after the disaster can be seen from Fig. 3.

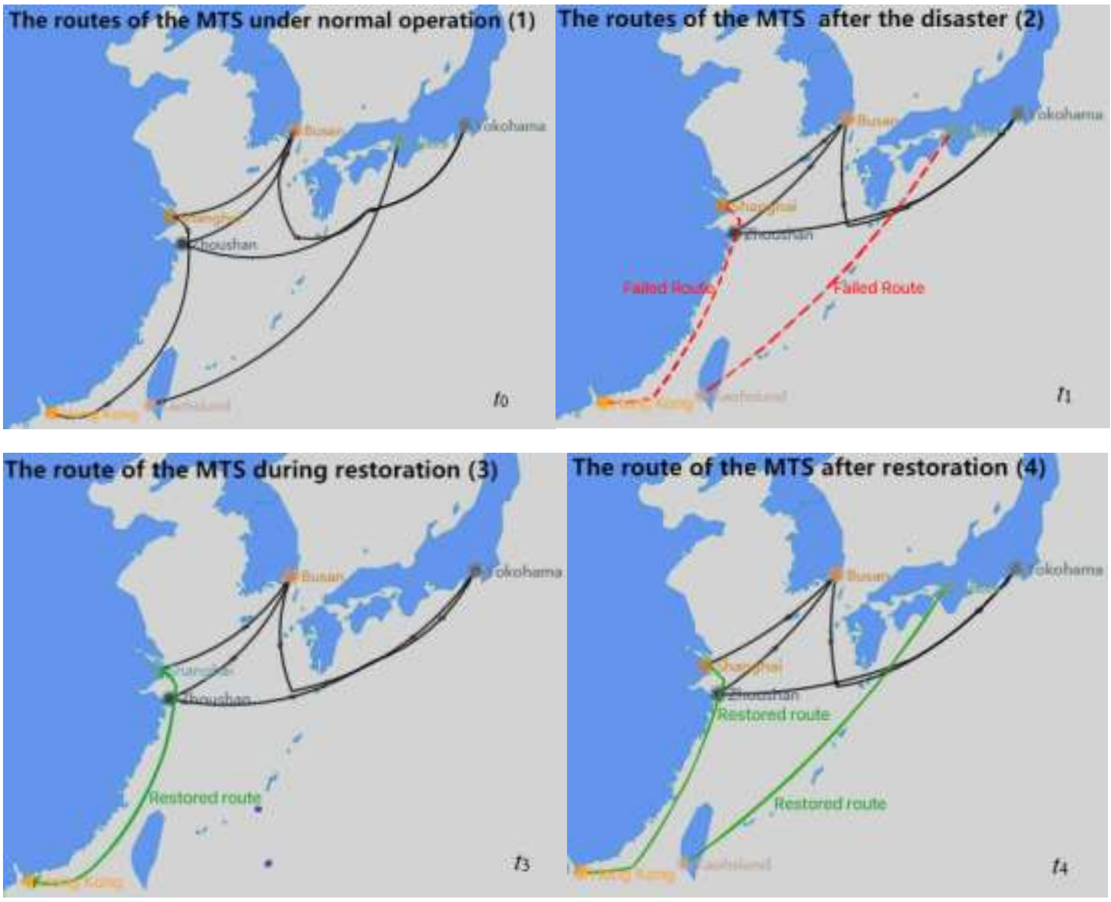

Fig. 3. The resilience process of the MTS under disaster

From t_0 to t_1, the MTS is in a normal state. The ports and routes of the MTS are operating normally. The function of the MTS is $Q(0)$. At t_1, a disaster occurred, causing the Hong Kong-Shanghai route and the Kaohsiund-Osaka route of MTS to fail. From t_1 to t_2, the function of the MTS is at its lowest state Q_0. At t_2, the MTS begins to be repaired. At t_3, the Hong Kong-Shanghai route is repaired, and the function of the MTS is $Q(t)$. At t_4, the Kaohsiund-Osaka route is repaired. The MTS completes the post-disaster restoration and the function is $Q(T)$. $Q(0)>Q(T)>\mathrm{Q}(t)>\mathrm{Q}_0$.

Therefore, the state of the MTS can be divided into the four parts.

1) The stage of disaster prevention: at this time, the MTS is operating. At this stage, advanced decision support systems can be used for disaster prevention.
2) The stage of disaster: when disaster occurs, the function of the system is affected to a certain extent. The impact of the MTS functioning depends on the severity of

disaster and the resistance of the MTS. After a disaster occurs, the disaster attack is absorbed by the system, and the system operates with a lower function. Before the system returns to the operating state, the system can be adapted to disaster attacks through a series of optimization operations.

3) The stage in which the ports and routes of the MTS recovers operation: the failed ports and routes begin to be repaired, and the function of the MTS gradually recovers.
4) The stage of stable operation of the system: the repair work of the failed ports and routes are completed, and the system gradually returns to the state of stable operation.

2.3 ***Resilience analysis of the MTS***

The resilience of the system extends the definition of reliability to the ability of the system to "bounce back" after disturbance. Much effort has been made to define and describe resilience. We define the resilience of the MTS as the ability of the MTS to resist, adapt, and quickly return to its normal and stable operating state after a disaster. In existing resilience studies, resilience is usually quantified as the ratio of the recovery value of the system function to the loss value, as recovery(t)/loss(t).

The residual resilience of the MTS is the difference between the current resilience and the optimal resilience. Therefore, the residual resilience is quantified as $R(t)$, as defined below, describes the ratio of the residual loss value (the difference between the loss value and the recovery value) to the loss value within $t > t_2$.

$$\begin{aligned} R(t) &= \frac{\text{loos}(t)\,\text{-}\,\text{recovery}(t)}{\text{loos}(t)} \\ &= \frac{\int_{t_2}^{t}(Q(0)-Q_0)dt - \int_{t_2}^{t}(Q(t)-Q_0)dt}{\int_{t_2}^{t}(Q(0)-Q_0)dt} = 1 - \frac{\int_{t_2}^{t}(Q(t)-Q_0)dt}{\int_{t_3}^{t}(Q(0)-Q_0)dt} \end{aligned} \tag{1}$$

It can be known from equation (1) that the value range of $R(t)$ is in [0, 1]. When $Q(t)=Q(t_1)$, $R(t)=1$, it implie that the function of the post-disaster MTS reaches the lowest value. When $Q(t)=Q(0)$, $R(t)=0$, it means that the function of the MTS is recovered to the ideal state. The closer the residual resilience $R(t)$ is to 0, the better the function recovery of the MTS. This definition can well quantify the scale and speed of the MTS function recovery. Because the accumulation of system recovery functions is considered, $R(t)$ in this article is not memoryless.

2.4 ***The optimal model of residual resilience in MTS***

Disaster events may cause one or more ports in the MTS to fail. When multiple ports and routes fail at the same time, the recovery strategy aims to determine the repair sequence of the ports and routes to achieve the best possible recovery within a certain period of time.

For a MTS with demand nodes, the larger the traffic received by the demand nodes, the better the capacity of the MTS. Let $q_\mathrm{j}(t)$ be the receiving traffic of the demand node j in the t-th time period, and take the maximum receiving traffic of the demand nodes as the goal.

$$Q(t)=\sum_{j\in N_D} q_j(t) \tag{2}$$

Equation (2) is applied to equation (1) to obtain the residual resilience equation (3), as shown in the following.

$$R(t)=\frac{T(\sum_{j\in N_D} P_j^D(t)-Q_0)-\sum_{t\in T}\left[\sum_{j\in N_D} q_j(t)-Q_0\right]}{T(\sum_{j\in N_D} P_j^D(t)-Q_0)} \tag{3}$$

In equation (3), $\sum_{j\in N_D} P_j(t)$ represents the demand of all nodes in the demand node set N_D being fully satisfied, that is $\sum_{j\in N_D} P_j(t)=Q^*(T)$. When t=t_3, the system begins recovering gradually. $Q(t_3)$ can be expressed by Q_0. Therefore, during the recovery time of span T, the optimization model with the minimum residual resilience as the goal is shown as follows.

$$\begin{aligned}\min R(t) &= \min\frac{T(\sum_{j\in N_D} P_j(t)-Q_0)-\sum_{t\in T}\left[\sum_{j\in N_D} q_j(t)-Q_0\right]}{T(\sum_{j\in N_D} P_j(t)-Q_0)}\\ &= \min\left\{\frac{T(\sum_{j\in N_D} P_j(t)-Q_0)-\sum_{t\in T}\left[\sum_{j\in N_D} q_{ij}(t)-\sum_{j\in N_D} q_{ji}(t)-Q_0\right]}{T(\sum_{j\in N_D} P_j(t)-Q_0)}\right\}\end{aligned} \tag{4}$$

subject to:

$$\sum_{(i,j)\in N} q_{ij}(t)-\sum_{(i,j)\in N} q_{ji}(t)\le P_i^s, i\in N_S, \forall t \tag{5}$$

$$\sum_{(i,j)\in N} q_{ij}(t)-\sum_{(i,j)\in N} q_{ji}(t)=0, i\in N_T, \forall t \tag{6}$$

$$\sum_{(i,j)\in N} q_{ij}(t) - \sum_{(i,j)\in N} q_{ji}(t) = q_j(t), j \in N_D, \forall t \tag{7}$$

$$0 \le q_j(t) \le P_j^D, j \in N_D, \forall t \tag{8}$$

$$0 \le q_j(t) \le \mu_j(t) P_j, j \in N_D, \forall t \tag{9}$$

$$0 \le q_{ij}(t) \le \mu_{ij}(t) P_{ij}, (i,j) \in N, \forall t \tag{10}$$

$$\mu_{ij}(t) - \mu_{ij}(t+1) \le 0, (i,j) \in N, \forall t \tag{11}$$

$$\mu_i(t) - \mu_i(t+1) \le 0, i \in N, \forall t \tag{12}$$

$$\sum_{i\in E} \left[\mu_i(t) - \mu_i(t-1)\right] = 1, \forall t \tag{13}$$

$$\sum_{(i,j)\in E} \left[\mu_{ij}(t) - \mu_{ij}(t-1)\right] = 1, \quad \forall t \tag{14}$$

$$\mu_{ij}(t) \in \{0,1\}, i \in N, t \in \forall t \tag{15}$$

$$\mu_i(t) \in \{0,1\}, i \in N, t \in \forall t \tag{16}$$

$$\mu_{ij}(0) = 0, (i,j) \in F \tag{17}$$

$$\mu_i(t) = 0, i \in E \tag{18}$$

In the model, $q_{ij}(t)$ represents the flow from supply node i to node j at time unit t. $\mu_{ij}(t)$ and $\mu_i(t)$ are the state of the edge ij and node i at time unit t, respectively. $\mu_{ij}(t)=1$ ($\mu_i(t)=1$) indicates that the edge ij (the node i) is running. $\mu_{ij}(t)=0$ ($\mu_i(t)=0$) means that the edge ij (the node i) is in a fault state. Constraint (5) ensures that the flow difference of supply node i ($i \in N_S$) does not exceed its supply capacity P_i^S. Constraint (6) guarantees that the net flow of the transit node i ($i \in N_T$) is zero. Constraint (7) indicates that the net flow of demand node j ($j \in N_D$) is $q_j(t)$. Constraint (8) indicates that the net flow of demand node j ($j \in N_D$) does not exceed its demand P_j^D. Constraints (9)-(10) represent that the flow of node j and edge ij cannot exceed the capacity that can be passed in the current state. Constraints (11)-(12) indicate that once the failed edge ij and node i are repaired, they will never fail again. Constraint (13) means that only one failed node can be repaired within a given time interval. Constraint (14) means that only one failed edge can be repaired within a given time interval. Constraints (15)-(18) indicate that edge ij and node i only exist in two states of operation and failure. In the initial state, all nodes in the fault set E and all edges in the fault set F are in the fault state.

3. The importance of the residual resilience of the MTS

The importance measure is used to determine the operation direction and priority related to system improvement. The purpose is to find the most effective way to maintain the system state. Generally, the importance measure is used to quantify the impact of components of the system on overall system performance. Different importance measures are developed around the residual resilience of the system. Port and route residual resilience importance models will be introduced in this section. It can lay a theoretical foundation for the application of residual resilience in the maritime system.

3.1 OPT residual resilience importance

The ports failure will affect the operation status of the MTS, so it is necessary to determine the repair sequence of the port within a certain time range to ensure that the system status returns normal. The optimal recovery time of the failed edges *ij* and failed node *i* expressed by I_{ij}^{OPT} and I_i^{OPT}, respectively. This indicator can explain the optimal time of the failed nodes and edges, so as to reduce the residual resilience value of the system to the maximum within a certain recovery time. The equation of I_{ij}^{OPT} and I_i^{OPT} is shown as follows.

$$I_C^{OPT} = \begin{cases} I_{ij}^{OPT} = 1 + \sum_{t=1}^{T}(1 - u_{ij}(t)), (i, j) \in E \\ I_i^{OPT} = 1 + \sum_{t=1}^{T}(1 - u_i(t)), i \in E \end{cases} \tag{19}$$

In equation (19), I_{ij}^{OPT} and I_i^{OPT} represent the optimal recovery time of the failed edge *ij* and the failed node *i,* respectively. μ_{ij} (t) and μ_i (t) represent the state of the failure edge *ij* and the failure node *i* in time unit *t*, respectively. T represents the required time period to recover the system function to the optimal state. This importance indicates the priority that the failed nodes should be recovered. It measures the impact of the residual resilience of the MTS once a failed node is recovered. The restoration priority of the failed nodes can be sorted according to the index value. The smaller the values of I_{ij}^{OPT} and I_i^{OPT}, the more important this node or edge is to the MTS, and the higher the recovery priority. The optimal recovery time OPT is proposed based on the optimization model in Section 3. If a node or edge fails, it can provide the optimal recovery sequence of failed nodes to minimize the residual resilience of the MTS.

*3.2 **Birnbaum residual resilience importance***

The Birnbaum importance is currently one of the most widely studied importance measures in reliability engineering. It is the difference in reliability of node *i* and edge *ij* from the working state to the failure state and measures the effect of the state change of node *i* and edge *ij* on the system state. In this section, we extend the Birnbaum importance to the study of residual resilience, which is used to measure the effect of a node and edge state on the residual resilience of the MTS. The importance is defined as $I_C{}^B$, which is converted from the original equation to the difference between the loss value and the recovery value. The definition is shown as follows.

$$I_C^B = R(T \left| \sum_{t=1}^{T} u_C = 0) - R(T \right| \sum_{t=1}^{T} u_C = 1) \tag{20}$$

where I_C^B represents the Birnbaum residual resilience importance. The failed node and edge are represented by *C*. $R(T \left| \sum_{t=1}^{T} u_C = 1) \right.$ represents the optimal residual resilience value of the MTS, where $U_C = 1$ means that the MTS is successfully recovered within the time range *T*. $R(T \left| \sum_{t=1}^{T} u_C = 0) \right.$ represents the optimal residual resilience value of the MTS with $U_C = 0$ meaning that the MTS is not recovered within the time range *T*. This importance is used to measure the potential impact of the state change of the failed node *i* and edge *ij* on the residual resilience of the MTS. The larger $I_C{}^B$ is, the greater the impact of the state of the *C* change on the MTS and the higher the priority of this node.

*3.3 **RAW residual resilience importance***

The RAW is the ratio of the actual system reliability obtained when the node *i* and edge *ij* are in the optimal operating state and the system original reliability. This importance measures the maximum possible percentage increase in system reliability due to the changes in the reliability of node *i* and edge *ij*, respectively. RAW is extended to the study of residual resilience in this paper. The importance of the RAW residual resilience is the residual resilience reduction value and is defined as the ratio of the optimal residual resilience of the MTS when only the *C* recovers within the time range *T* to the residual resilience value of the MTS after the disaster. The importance measure is expressed by $I_C{}^{RAW}$.

$$I_C^{RAW} = \frac{R(T\left|\sum_1^t u_C(t)=1\right.)}{R_0} \tag{21}$$

In equation (21), $I_C{}^{RAW}$ represents the importance of the residual resilience reduction value. The failed node and edge are represented by C. R_0 represents the residual resilience value of the MTS after the disaster. $R(T\left|\sum_1^t u_c(t)=1\right.)$ represents the residual resilience value of the MTS when only the C recovers smoothly within the time range T. This importance is used to measure the potential impact of the C on the residual resilience of the MTS once it recovers within a specified time. The smaller the RAW, the greater the impact on the residual resilience of the MTS when the C is recovered within a specified time and the higher the priority of the node.

3.4 ***RRW residual resilience importance***

The RRW is expressed by the ratio of the expected performance of the MTS to the actual performance when node i and edge ij are in the fault state. This importance is used to measure the potential damage to the MTS reliability caused by the failed node i and edge ij. RRW is extended to the study of residual resilience in this paper. The importance of RRW residual resilience is the increase in residual resilience. It is defined as the ratio of the optimal residual resilience of the MTS during recovery to the optimal residual of the MTS when the failed node i or failed edge ij is not recovered. The importance index is expressed by $I_C{}^{RRW}$, the equation is shown as follows.

$$I_C^{RRW} = \frac{R(T)}{R(T\left|\sum_{t=1}^T u_C(t)=0\right.)} \tag{22}$$

where $I_C{}^{RRW}$ represents the importance of the increase in residual resilience. The failed node i and edge ij are represented by C. $R(T)$ represents the optimal residual resilience value of the MTS when the time unit is T. $R(T\left|\sum_{t=1}^T u_C(t)=0\right.)$ represents the optimal residual resilience value of the MTS when C recovers within the time range T. This importance is used to measure the potential impact of C on the residual resilience of the MTS when it fails to recover within a specified time. The smaller $I_C{}^{RRW}$, the greater the impact on the residual resilience of the MTS when the C recovers within a specified time and the higher the priority of the node.

*3.5 **Comparisons and discussions of the importance of residual resilience***

There are many importance measures proposed in residual resilience management. The recovery sequence of failed nodes is different under different importance. The reason is that the physical meanings represented by those importance measures are different, which are explained as follows.

The importance ranking of the nodes obtained by using the OPT importance, Birnbaum importance, RAW importance, and RRW importance are different because they are proposed from different perspectives. A sole reliability importance is used to describe reliability improvement potential, and its impact on reliability loss. They measure three types of problems reliability potential, bad risk, and risk neutrality. This paper extends its meaning to residual resilience, that is, the importance of residual resilience is divided into residual resilience reduces potential, residual resilience increases negative risk, and risk neutrality measurement.

The Birnbaum residual resilience importance represents the difference between the positive and negative effects of the residual resilience of a system, it can therefore be regarded as an importance indicator of risk neutrality. A larger Birnbaum residual resilience importance value suggests a large influence of this node on the residual resilience of the system. Both the OPT residual resilience importance and the RAW residual resilience importance are focused on recovery, and are an importance indicator of the residual resilience reduce potential. The OPT residual resilience importance is an important measure of the positive impact of the node's recovery order on the residual resilience of the system. The RAW residual resilience importance measures the reduction of the residual resilience of the system by the restoration of nodes. A large OPT residual resilience importance value indicates a higher priority of the node recovery. The RRW residual resilience importance measures the negative impact on the residual resilience of the system when the node is not restored. The importance of RRW mainly considers the negative impact of nodes on the resilience of the system. It is used to identify nodes that have a potential loss on the resilience of the system. A large RRW residual resilience importance value indicates a less impact of the node on the resilience of the system.

3.6 *Resilience analysis based on the Copeland method and importance of residual resilience*

The importance ranking based on the Copeland scoring method is a single parameter ranking method. It does not require any information about the preference of the decision maker. It only needs to compare the importance of different components in the system in pairs, and then count the number of times each component beats other components. The accurate order of component importance can be obtained. The Copeland method considers the advantages, equality, and disadvantages of pairwise comparison. However, this method overemphasizes the number of "advantages" and "disadvantages", and ignores the degree of "advantages" and "disadvantages".

This method is to select two objects from the object set and compare the same index of each pair of objects. The comparison results are divided into three levels: advantages, equality, and disadvantages. The initial the Copeland score value is set to 0. If an index of an object is greater than the same index of another object, the Copeland score of the object is increased by one. If it is worse than the same index of another object, the Copeland score is decreased by one. If the same index value of the other object is equal, the Copeland score of the object is not changed. Each indicator of each pair of objects is compared, and the Copeland score value of each object is accumulated to obtain the final the Copeland score value of the object. Finally, the ranking is based on the score of each object in the object set.

The importance index set is set to $\{1, 2, 3, 4\}$. Two nodes α and β are arbitrarily selected from the system component set E. Each indicator in the indicator set is compared. Let $C_{i,j}^{k}(i)$ be the k-th importance index of the node i and node j are compared to the Copeland score of node i. The equation of $C_{i,j}^{k}(i)$ is shown as follows.

$$C_{i,j}^{k}(i)=\begin{cases} C_{i,j}^{k\text{-}1}(i)+1, v_i^k > v_i^k \\ C_{i,j}^{k\text{-}1}(i)-1, v_i^k < v_j^k \\ C_{i,j}^{k\text{-}1}(i), v_i^k = v_j^k \end{cases} \tag{23}$$

In equation (23), v_i^k and v_j^k respectively represent the value of the k-th importance index of node *i* and node *j,* respectively.

According to the definition of Al-Sharrah [26], the Copeland total score of node α is obtained by summing all the scores related to node *α*. The Copeland total score of the node *α* is defined as $C_{total}(i)$. The equation is shown as follows.

$$C_{total}(i) = \sum_{j \in E} C_{i,j}^{k}(i), i \neq j \tag{24}$$

In the equation, $C_{i,j}^{k}(i)$ represents the accumulation of the comparison results of all indexes of node *i* and node *j*. *j* represents all nodes except *i* in the system failure set *E*. The result of comparing node *i* with itself is still 0.

When the Copeland method is used to analyze resilience, it can follow the five steps below.

(1) According to the residual resilience importance equation, the importance index of each failed component is calculated.

(2) The Copeland score of each failed node under each importance index is calculated in turn.

(3) According to equation (24), the Copeland score of the failed node *i* is accumulated.

(4) The Copeland scores of all failed nodes are sorted in descending order. The higher the score, the higher the repair priority.

(5) According to the restoration priority, the failed node with higher priority is repaired. In the residual resilience optimization model, solving equation (4) is used to obtain the optimal value of residual resilience in different time periods.

4. **Result analysis**

In this section, sea routes consisting of 23 cities shown in Figs. 1 and 2 are used to demonstrate the proposed method. First, some nodes are randomly assumed to fail, and the residual resilience changes when each node is repaired. The purpose is to study the repair sequence of the failed nodes under different importance. Then all nodes are made to fail, and the repair sequence and residual resilience changes of all nodes under different importance. Finally, the repair sequence and residual resilience changes of the failed edges under different importance are studied separately when some edges and all edges fail.

4.1 *Resilience analysis of given failed nodes*

In the MTS, the sets of different failure nodes have different effects on the post-disaster network structure. In the case of the sets of different failed nodes, the repair priority of the same node is also different. Therefore, based on the data of the MTS, the importance of the given failed nodes is calculated.

According to Fig. 2, the $S7$, $S13$, $D6$, $D12$, $D15$, $D20$, $D22$ and $T9$ are selected as the failure node set E. The node capacity is expressed in the container throughput of each port in 2018, and the unit of throughput is TEU. The capacity of all edges is 3000TEU. The throughput data of each port is shown in Table 2.

Table 2. The throughput data of each port

number	1	2	3	4
port	Hamburger	New York	Rotterdam	Barcelona
throughout capacity	873	718	1451	347
number	5	6	7	8
port	Marsaxlokk	Genoa	Santos	Durban
throughout capacity	331	261	412	296
number	9	10	11	12
port	KeLang	Singapore	Hong Kong	Zhoushan
throughout capacity	1203	3660	1959	2635
number	13	14	15	16
port	Shanghai	Tianjin	Kaohsiund	Busan
throughout capacity	4201	1600	1045	2159
number	17	18	19	20
port	Osaka	Yokohama	Seattle	Oakland
throughout capacity	240	303	380	255
number	21	22	23	
port	Los Angeles	Kingston	Sydney	
throughout capacity	946	183	265	

According to the residual resilience optimization model, Q_0 after node failure is obtained. The value of Q_0 is 3205. The value of all the requirements of the demand node is 8034.

When solving the OPT importance, the limited equation is still used to ensure that a failed node must be repaired in each time period. For the solution of the importance of Birnbaum and the importance of RRW, a constraint equation also needs to be added as a constraint. According to the equation, the state set of the failed nodes is obtained, as shown in Table 3.

Table 3. The state set of the given failed nodes

failed nodes	$\boldsymbol{\mu_i(1)}$	$\boldsymbol{\mu_i(2)}$	$\boldsymbol{\mu_i(3)}$	$\boldsymbol{\mu_i(4)}$	$\boldsymbol{\mu_i(5)}$	$\boldsymbol{\mu_i(6)}$	$\boldsymbol{\mu_i(7)}$	$\boldsymbol{\mu_i(8)}$
S7	0	1	1	1	1	1	1	1
S13	1	1	1	1	1	1	1	1
D6	0	0	1	1	1	1	1	1
D12	0	0	0	0	0	0	0	1
D15	0	0	0	0	0	0	1	1
D20	0	0	0	1	1	1	1	1
D22	0	0	0	0	1	1	1	1
T9	0	0	0	0	0	1	1	1

According to the equation of different residual resilience importance, the I_C^{OPT}, I_C^{B}, I_C^{RAW} and I_C^{RRW} of failed nodes can be obtained, as shown in Fig. 4.

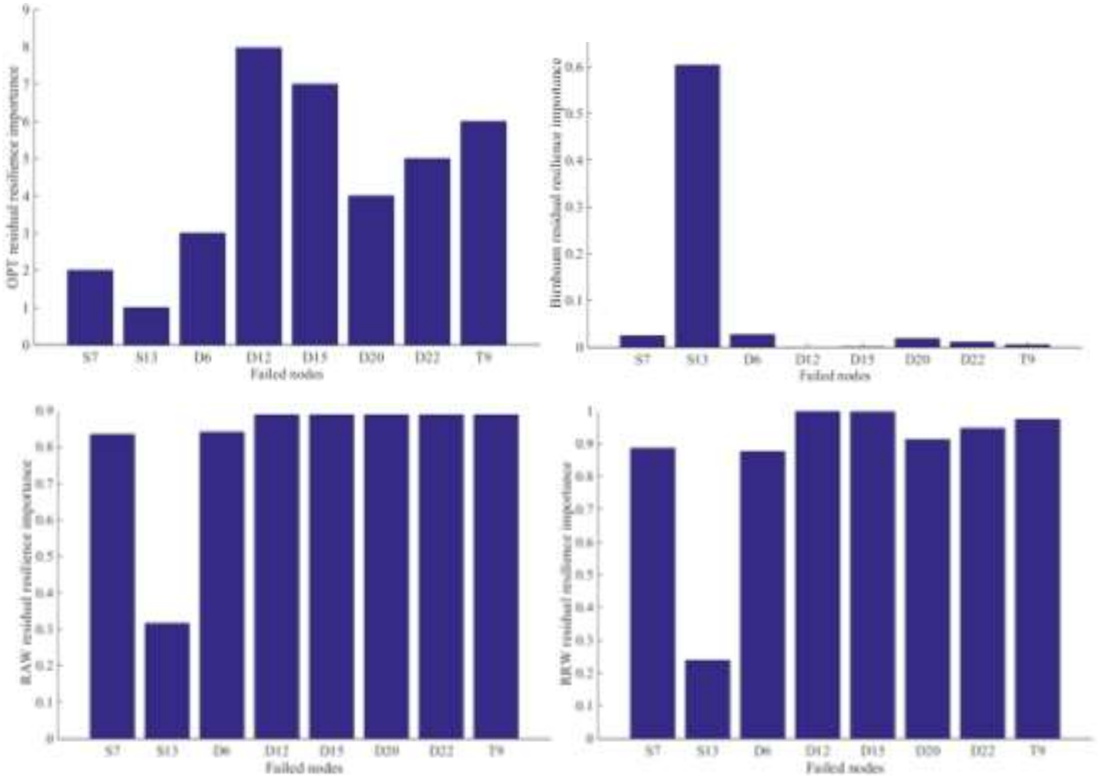

Fig. 4. The residual resilience importance of given failed nodes

From Fig. 4, one can see the following results.

- Because a large OPT residual resilience importance suggests a high repair priority of the failed node, the repair sequence of the failed nodes is {*S*13, *S*7, *D*6, *D*20, *D*22, *T*9, *D*15, *D*12}.
- Since a large value of the Birnbaum residual resilience importance indicates a high repair priority of the failed node, the repair sequence of the failed nodes is {*S*13, *D*6, *S*7, *D*20, *D*22, *T*9, *D*15, *D*12}.

- Since a large value of RAW residual resilience importance suggests a high repair priority of the failed node. The repair sequence of the failed nodes is {*S*13, *S*7, *D*6}. *T*9, *D*22, *D*20, *D*15 and *D*12 have the same repair priority.
- Since a small value of RRW residual resilience importance indicates a high repair priority of the failed node, the repair sequence of the failed nodes is {*S*13, *D*6, *S*7, *D*20, *D*22, *T*9, *D*15, *D*12}.
- Under Birnbaum importance and RRW importance, the order of nodes is the same. Therefore, when calculating the Copeland score, only one of the two importance measures is considered.

The Copeland method is used to calculate the Copeland scores of failed nodes. The comprehensive priority of failed nodes under different importance indexes is shown as follows.

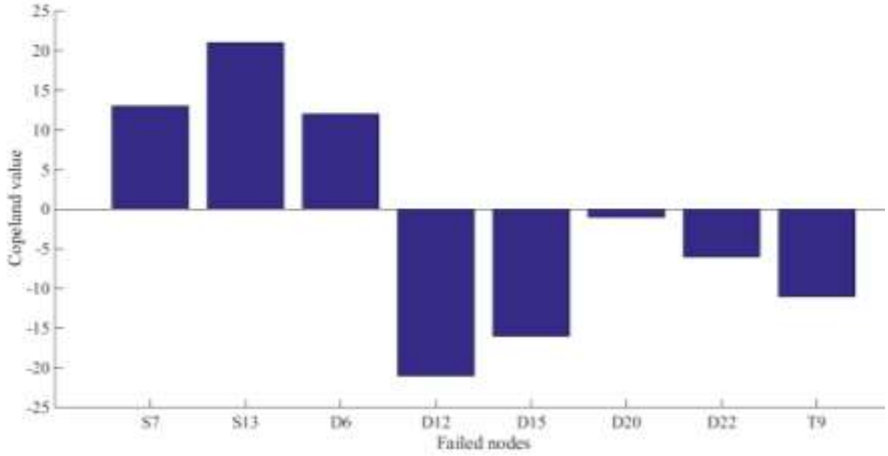

Fig. 5. The Copeland scores value of given failed nodes

Similarly, a large value of the Copeland score suggests a high repair priority of the failed node. As can be seen from Fig. 5, the repair sequence of the failed nodes is {*S*13, *S*7, *D*6, *D*20, *D*22, *T*9, *D*15, *D*12}.

The change of *R*(*t*) when the given failed nodes under different importance are repaired is shown in Fig. 6.

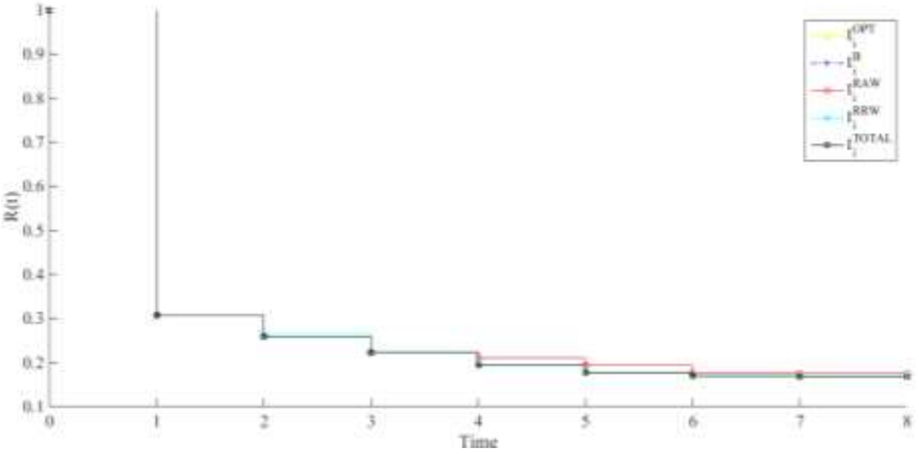

Fig. 6. The changes in residual resilience of given failed nodes

It can be seen from Fig. 6 that a failed node is repaired in each time period, and the residual resilience $R(t)$ gradually decreases with time. For the OPT importance, with the repair of the failed node, the residual resilience $R(t)$ gradually decreases from 1 to 0.1680. For the Birnbaum importance and the RRW importance, with the repair of the failed node, the residual resilience $R(t)$ gradually decreases from 1 to 0.1688. According to the priority of the failed nodes of RAW, the residual resilience $R(t)$ is finally reduced to 0.1747. At the end of the second period, $R(t)$ at the Birnbaum importance level decreases by only 0.0420, while $R(t)$ at the other importance levels decreases by 0.0477. In the entire MTS, $S7$ has a higher priority than $D6$. In the RAW residual resilience importance, although the priority of the last few nodes is the same, the repair orders of the remaining failed nodes are different, and the residual resilience changes differently. Therefore, the node priority obtained only by one importance is one-sided, and it cannot fully reflect the real priority of the failed nodes. The comprehensive repair sequence of the failed nodes is $\{S13, S7, D6, D20, D22, T9, D15, D12\}$.

*4.2 **Resilience analysis of all failure nodes***

This section assumes that all nodes have failed. According to Fig. 2, the set of failed nodes E is $\{S1, S2, D3, D4, D5, D6, S7, D8, T9, S10, D11, D12, S13, D14, D15, D16, D17, S18, D19, D20, D21, D22, S23\}$. The importance of each node in the case of full node failure is studied to determine the best repair priority.

The limiting conditions are modified based on the optimization model. It is worth noting that when some of the nodes in the system fail, $R(T\left|\sum_1^t u_C(t)=1\right.)$ in the RAW importance equation represents that only the node C is repaired and the remaining nodes are in the unrepaired state. When all nodes fail, the repair of a single node cannot satisfy the demand of the demand nodes. Therefore, the importance of RAW will not be discussed for the system where the node is completely in the failure state.

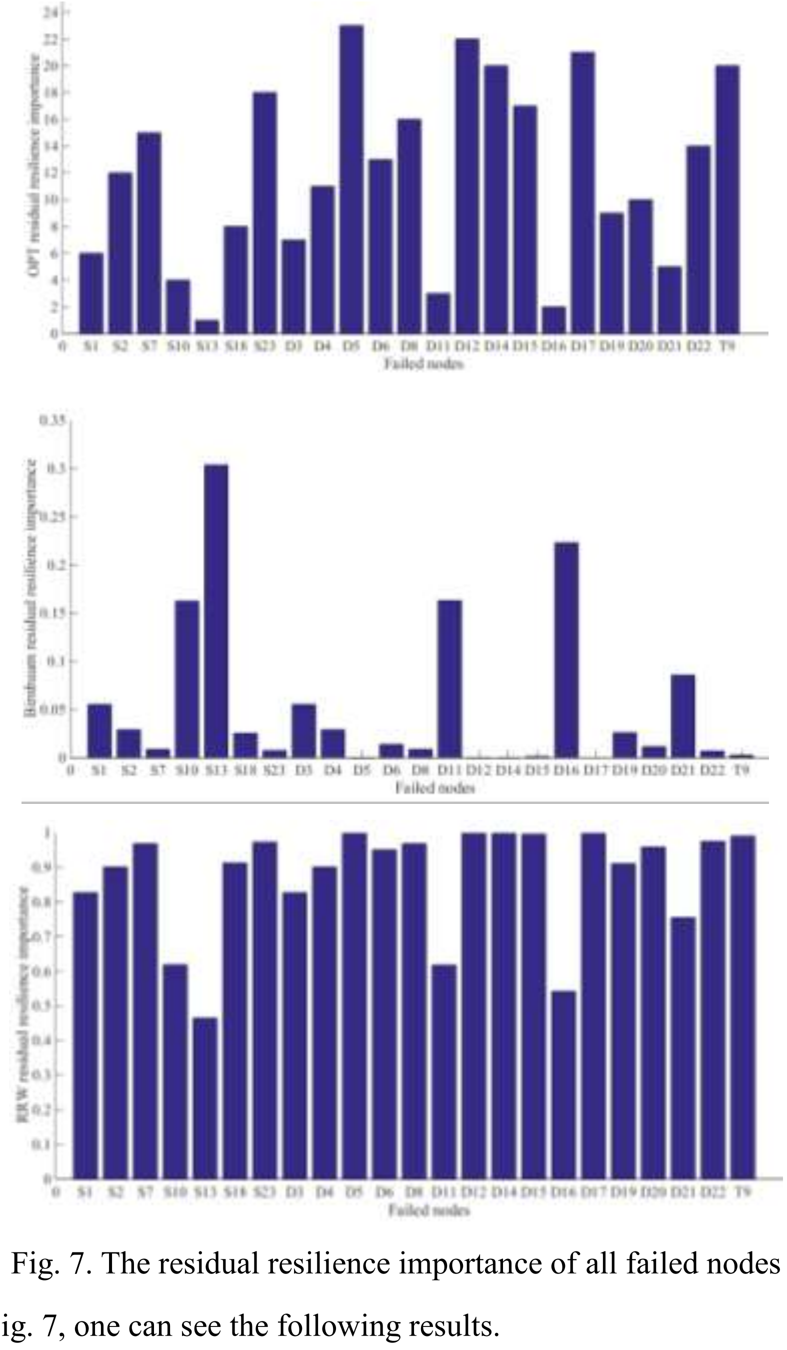

Fig. 7. The residual resilience importance of all failed nodes

From Fig. 7, one can see the following results.

- Under the OPT residual resilience importance, the repair sequence of the failed nodes is {S13, D16, D11, S10, D21, S1, D3, S18, D19, D20, D4, S2, D6, D22,

S7, D8, D15, S23, T9, D14, D17, D12, D5}.

- Under the Birnbaum residual resilience importance and the RRW residual resilience importance, the order of nodes is the same. The repair sequence of the failed nodes is {S13, D16, D11, S10, D21, {S1, D3}, {D4, S2}, D19, S18, D6, D20, {S7, D8}, S23, D22, *T*9, *D*15, *D*5, {*D*12, *D*14}, *D*17}. *S*1 and *D*3 have the same repair priority. *D*4 and *S*2 have the same repair priority. *S*7and *D*8 have the same repair priority. *D*12 and *D*14 have the same repair priority.

The Copeland method is used to calculate the Copeland score of each node. As can be seen from Fig. 8, the repair sequence of the failed nodes is {*S*13, *D*16, *D*11, *S*10, *D*21, *S*1, *D*3, *S*18, *D*19, *D*4, *S*2, *D*20, *D*6, *S*7, *D*22, *D*8, *S*23, *D*15, *T*9, *D*14, *D*5, , *D*12, *D*17 }.

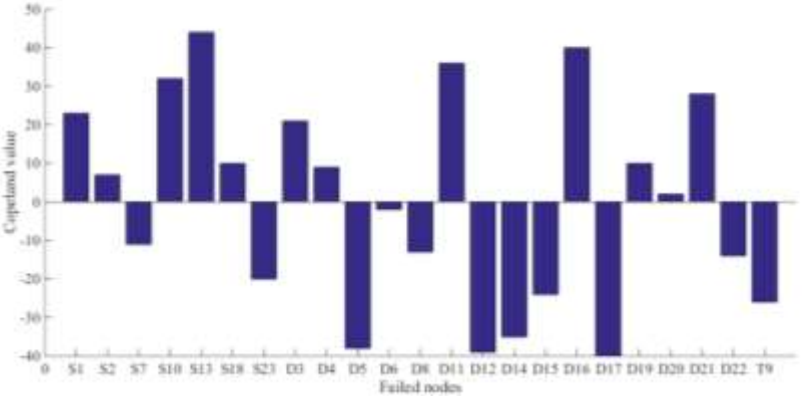

Fig. 8. The Copeland scores value of all failed nodes

When the failed nodes under different importance are repaired, the change of *R*(*t*) is shown in Fig. 9.

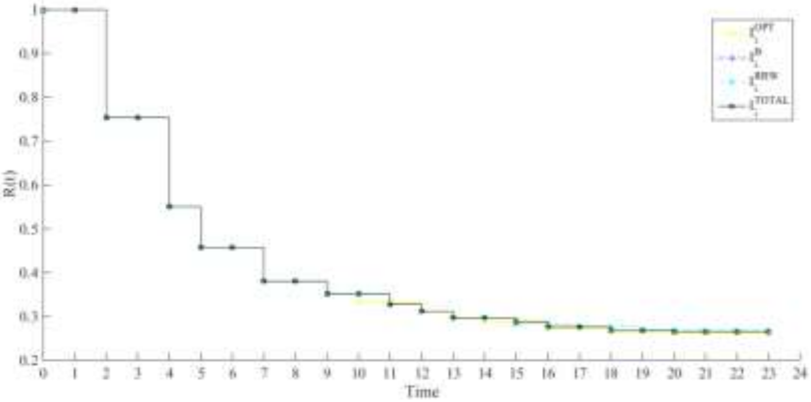

Fig. 9. The changes in residual resilience of all failed nodes

It can be seen from Fig. 9 that a failed node is repaired in each time period, and the residual resilience $R(t)$ gradually decreases with time. Before the 9th period, the changes in $R(t)$ are the same at all importance levels. At the 9th period, the $R(t)$ of the OPT importance and the comprehensive importance decreases by 0.0296, while $R(t)$ of the Birnbaum importance, RAW importance and RRW importance decrease by 0.0267. In the 10-th period, $R(t)$ did not change under the Birnbaum importance and the comprehensive importance. The reason is that in the 10th period, although $D19$ is repaired in the repair order of the Birnbaum importance, the nodes connected to the $D19$ are not repaired. Under the comprehensive importance, the D4 is repaired, but the $D4$ has no supply node to supply traffic. Under OPT importance, the $D20$ is repaired, the supply node $S18$ supplies $D20$ traffic, and $R(t)$ changes significantly. At the 11th period, under OPT importance, $R(t)$ did not change, also because the $D4$ did not have a supply node to supply traffic. $R(t)$ has obviously changed under the Birnbaum importance and the comprehensive importance, mainly due to supply node is repaired during this period. Under the comprehensive importance, there is no change in $R(t)$ in the 14-th and 17-th periods because of the same reason.

The changes of $R(t)$ under different importance levels are compared. For OPT importance, with the repair of the failed node, the residual resilience $R(t)$ gradually decreases from 1 to 0.2634. For the Birnbaum importance and the RRW importance, with the repair of the failed node, the residual resilience $R(t)$ gradually decreases from 1 to 0.2678. For the comprehensive importance, with the repair of the failed node, the residual resilience $R(t)$ gradually decreases from 1 to 0.2652. It can be seen that under the state of failure of all nodes, although all nodes are eventually repaired, using different node recovery sequences, the residual resilience varies greatly with time.

*4.3 **Resilience analysis of given failure edges***

The MTS follows the transportation of goods between multiple countries. When multiple routes fail in the MTS, it may be considered to repair the route between countries to achieve the purpose of quickly restoring transportation capacity. Therefore, the situation of given failure edges is considered to study the changes in residual resilience at different importance levels.

The edge importance of the system is discussed in this section. Under the condition that the set of failed edges is given, the repair order of the failed edges is studied. According to Fig. 2, it is assumed that the set of failure edges is {link*D*3*D*5, link*D*5*T*9, link*D*6*S*10, link*T*9*S*10, link*T*9*D*12, link*D*11*S*13, link*D*12*S*18, link*S*13*D*16, link*D*19*D*20, link*S*23*D*17}.

Table 4. The number of the given failed edges

failed edges	link*D*3*D*5	link*D*5*T*9	link*D*6*S*10	link*T*9*S*10	link*T*9*D*12
Number	1	2	3	4	5
failed edges	link*D*11*S*13	link*D*12*S*18	link*S*13*D*16	link*D*19*D*20	link*S*23*D*17
Number	6	7	8	9	10

According to the residual resilience optimization model, the Q_0 value after nodes failure is obtained. The value of Q_0 is 4487. The value of all the requirements of the demand node is 8034.

According to requirements, the limiting conditions are modified based on the optimization model. The state set of the failed edges is obtained, as shown in Table 5.

Table 5. The state set of the given failed edges

failed edges	**$\mu_{ij}(1)$**	**$\mu_{ij}(2)$**	**$\mu_{ij}(3)$**	**$\mu_{ij}(4)$**	**$\mu_{ij}(5)$**	**$\mu_{ij}(6)$**	**$\mu_{ij}(7)$**	**$\mu_{ij}(8)$**	**$\mu_{ij}(9)$**	**$\mu_{ij}(10)$**
1	0	0	0	1	1	1	1	1	1	1
2	0	0	0	0	0	0	0	0	1	1
3	0	0	0	0	1	1	1	1	1	1
4	0	0	0	0	0	1	1	1	1	1
5	0	0	0	0	0	0	0	1	1	1
6	0	0	0	0	0	0	0	0	0	1
7	0	0	0	0	0	0	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1
9	0	1	1	1	1	1	1	1	1	1
10	0	0	1	1	1	1	1	1	1	1

According to the equation of different residual resilience importance, the I_C^{OPT}, I_C^{B}, I_C^{RAW} and I_C^{RRW} of failed edges can be obtained, as shown in Fig. 9.

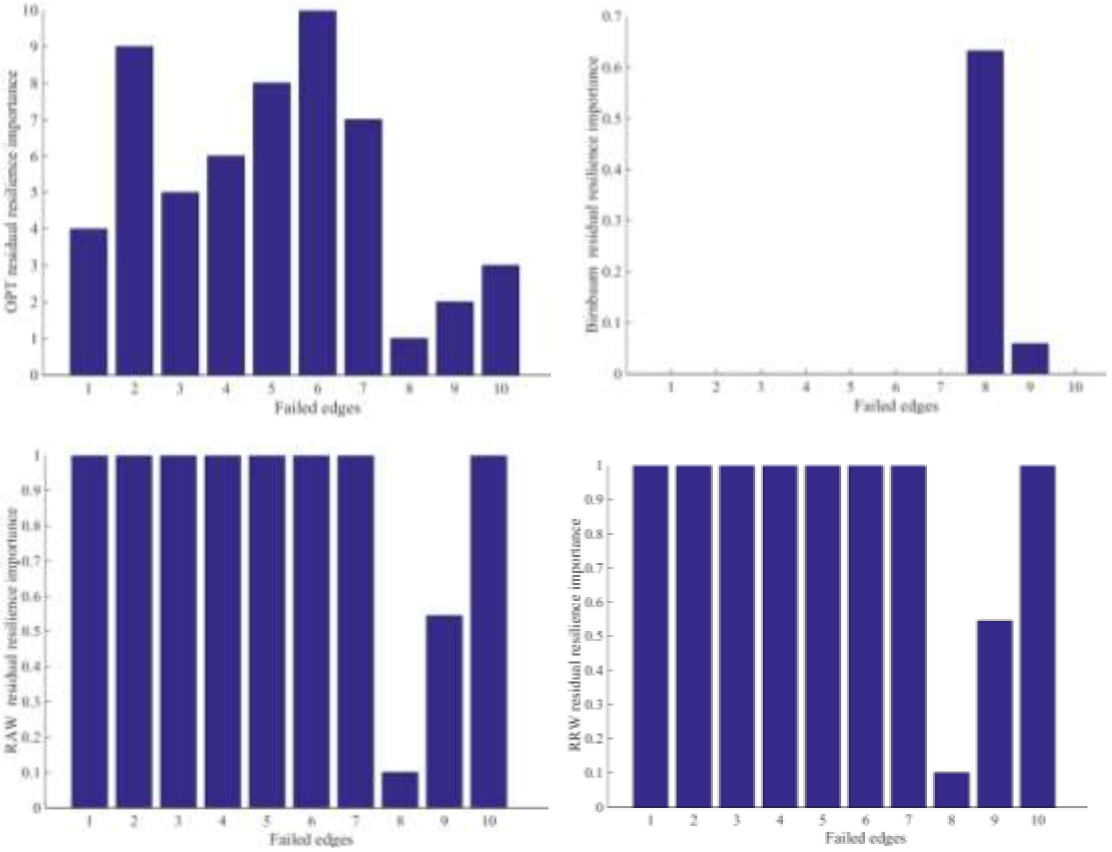

Fig. 10. The residual resilience importance of given failed edges

From Fig. 10, we can see the following results.

- Under the OPT residual resilience importance, the repair order of the failed edges is {link*S*13*D*16, link*D*19*D*20, link*S*23*D*17, link*D*3*D*5, link*D*5*T*9, link*D*6*S*10, link*D*12*S*18, link*T*9*S*10, link*D*11*S*13, link*T*9*D*12}.
- For the other importance, the repair of link*S*13*D*16 is more important for the reduction of residual resilience. The importance of link*D*19*D*20 ranks the second, and the remaining failure edges are of lower importance. The remaining failure edges have same priority.

The Copeland method is used to calculate the Copeland score of each edge. The comprehensive priority of failed edges under different importance indexes is shown as follows.

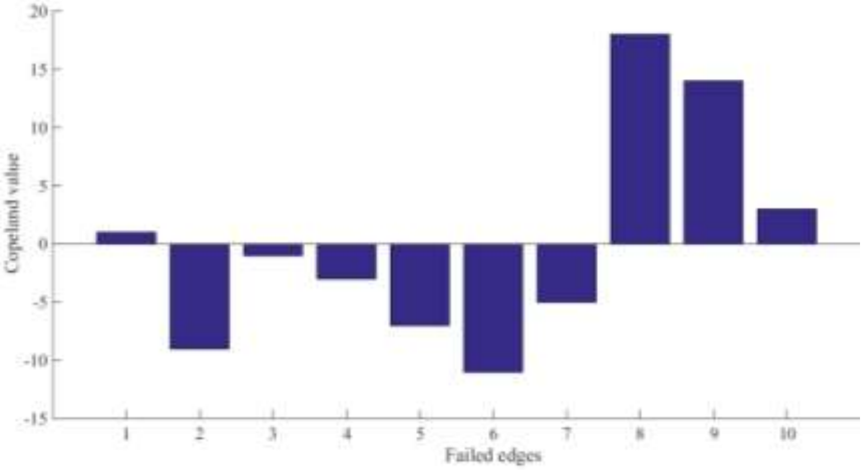

Fig. 11. The Copeland scores value of given failed edges

The larger the value of the Copeland score, the higher the repair priority of the failed edge. As can be seen from Fig. 11, the repair sequence of the given failed edges is {link$S$13D16, link$D$19D20, link$S$23D17, link$D$3D5, link$D$6S10, link$T$9S10, link$D$12S18, link$T$9D12, link$D$5T9, link$D$11S13}.

The change of $R(t)$ when the failed edges under different importance are repaired is shown in Fig. 12.

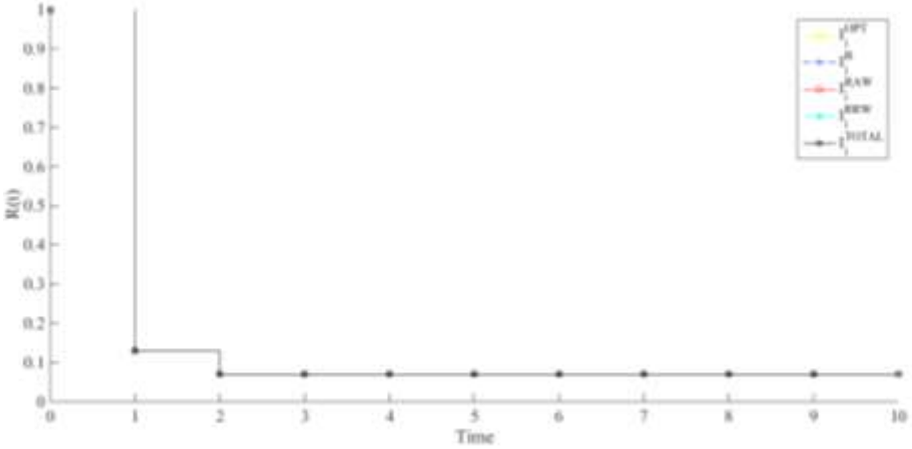

Fig. 12. The change in residual resilience of given failed edges

It can be seen from Fig. 12 that a failed edge is repaired in each time period, and the residual resilience $R(t)$ gradually decreases with time. Although the repair order of the failed edge is different under different importance, the residual resilience changes are the same. For all importance, with the repair of the failed node, the $R(t)$ gradually decreases from 1 to 0.0704. The repair sequence of the failed edges is {link$S$13D16, link$D$19D20, link$S$23D17, link$D$3D5, link$D$6S10, link$T$9S10, link$D$12S18, link$T$9D12, linkD5T9, linkD11S13}.

4.4 Resilience analysis of all failure edges

This section assumes that all edges have failed. According to Fig. 2, the set of failed edges F is {linkS1D3, linkD3D5, linkD5T9, linkD5S10, linkS2D4, linkD4D6, linkD6T9, linkD6S10, linkS7D8, linkD8S10, linkT9S10, linkT9D11, linkT9D12, linkT9D14, link S10D11, linkD11S13, linkD12D16, linkD12S18, linkD16S18, linkS18D19, linkS13 D16, linkS13D21, linkD19D20, linkD20D21, linkD21D22, linkS23D11, linkS23D17, linkS23D15, linkD15D17, linkD17D21}. The importance of each edge in the case of full edges failure is studied to determine the best repair priority.

Table 6. The number of all failed edges

failed edges	link S1D3	linkD3D5	linkD5T9	linkD5S10	linkS2D4
Number	1	2	3	4	5
failed edges	linkD4D6	linkD6T9	linkD6S10	linkS7D8	linkD8S10
Number	6	7	8	9	10
failed edges	linkT9S10	linkT9D11	linkT9D12	linkT9D14	linkS10D11
Number	11	12	13	14	15
failed edges	linkD11S13	linkD12D16	linkD12S18	linkD16S18	linkS18D19
Number	16	17	18	19	20
failed edges	linkS13D16	linkS13D21	linkD19D20	linkD20D21	linkD21D22
Number	21	22	23	24	25
failed edges	linkS23D11	linkS23D17	linkS23D15	linkD15D17	linkD17D21
Number	26	27	28	29	30

The limiting conditions are modified based on the optimization model. According to the equation of different residual resilience importance, the $I_C{}^{OPT}$, $I_C{}^{B}$, $I_C{}^{RAW}$ and $I_C{}^{RRW}$ of failed edges can be obtained, as shown in Fig. 13.

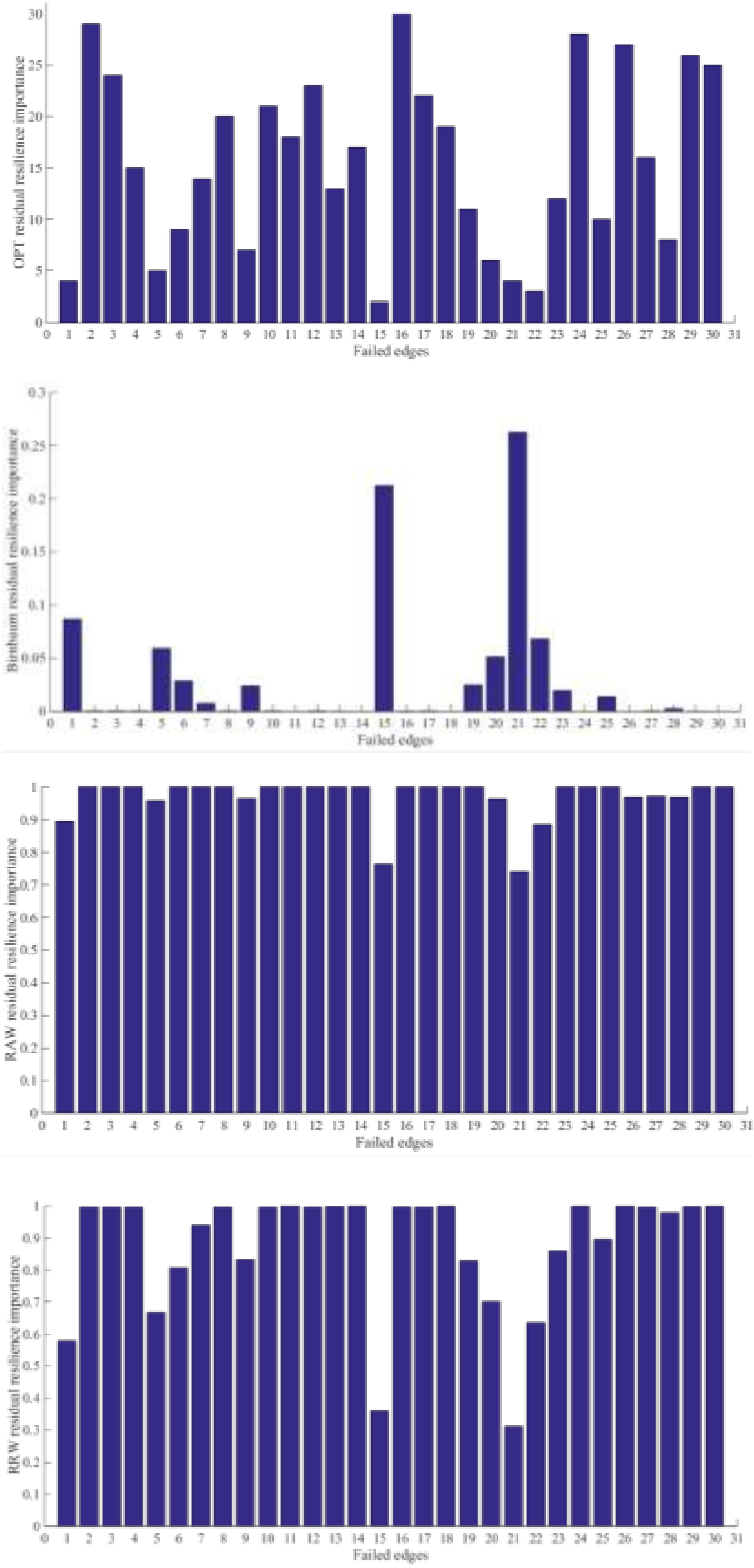

Fig. 13. The residual resilience importance of all failed edges

From Fig. 13, one can see the following results.

- Under OPT residual resilience importance, the repair order of the failed edges is {linkS13D16, linkS10D11, linkS13D21, linkS1D3, linkS2D4, linkS18D19, linkS7D8, linkS23D15, linkD4D6, linkD21D22, linkD16S18, linkD19D20, linkT9D12, linkD6T9, linkD5S10, linkS23D17, linkT9D14, linkT9S10, linkD12S18, linkD6S10, linkD6S10, linkD8S10, linkD12D16, linkT9D11, linkD5T9, linkD17D21, linkD15D17, linkS23D11, linkD20D21, linkD3D5, linkD11S13}.
- Under Birnbaum residual resilience importance, the repair order of the failed edges is {linkS13D16, linkS10D11, linkS13D21, linkS1D3, linkS2D4, linkS18D19, linkD4D6, linkD16S18, linkS7D8, linkD19D20, linkD21D22, linkD6T9, linkS23D15, {linkD3D5, linkD5T9, linkD5S10, linkD6S10, linkD8S10, linkD12D16, linkS23D17}, linkD11S13, linkD15D17, linkT9D11, {linkT9S10, linkT9D14, linkD12S18, linkD17D21, linkD20D21, linkS23D11, linkD17D21}}.
- Under RAW residual resilience importance, the repair order of the failed edges is {linkS13D16, linkS10D11, linkS13D21, linkS1D3, linkS2D4, linkS18D19, linkS7D8, {linkS23D11, linkS23D15}, linkS23D17, {linkD4D6, linkD16S18, linkD19D20, linkD21D22, linkD6T9, linkD3D5, linkD5T9, linkD5S10, linkD6S10, linkD8S10, linkD12D16, linkD11S13, linkD15D17, linkT9D11, linkT9S10, linkT9D14, linkD12S18, linkD17D21, linkD20D21, linkD17D21}}.

  Failure edges in brackets in the repair sequence have the same priority.

The Copeland method is used to calculate the Copeland score of each edge. The comprehensive priority of failed edges under different importance indexes is obtained, as shown in Fig. 14.

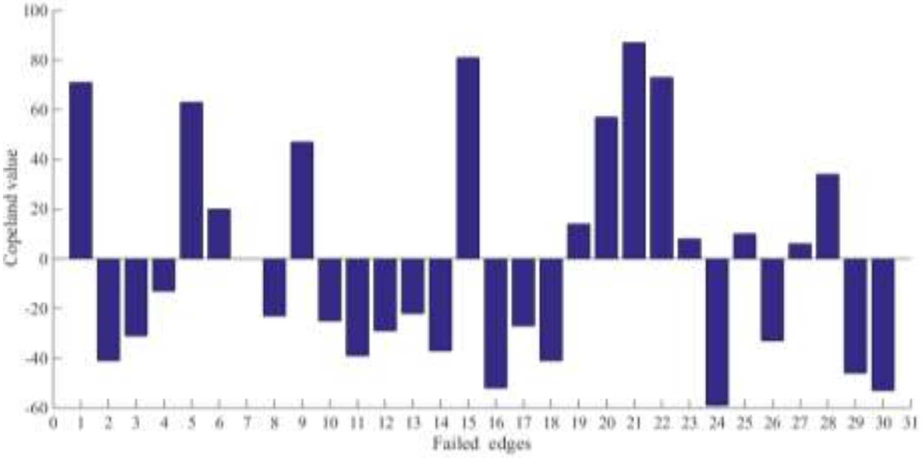

Fig. 14. The Copeland scores value of all failed edges

As can be seen from Fig. 14, the repair sequence of all failed edges is {link*S*13*D*16, link*S*10*D*11, link*S*13*D*21, link*S*1*D*3, link*S*2*D*4, link*S*18*D*19, link*S*7*D*8, link*S*23*D*15, link*D*4*D*6, link*D*16*S*18, link*D*21*D*22, link*D*19*D*20, link*S*23*D*17, link*D*6*T*9, link*D*5*S*10, link*T*9*D*12, link*D*6*S*10, link*D*8*S*10, link*D*12*D*16, link*T*9*D*11, link*D*5*T*9, link*S*23*D*11, link*T*9*D*14, link*T*9*S*10, link*D*3*D*5, link*D*12*S*18, link*D*15*D*17, link*D*11*S*13, link*D*17*D*21, link*D*20*D*21,}

The change of *R*(t) when the failed edges under different importance are repaired is shown in Fig. 15.

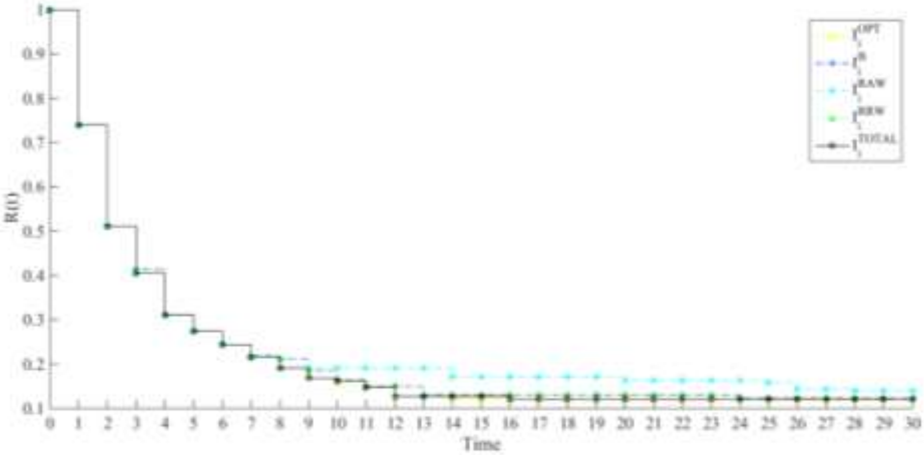

Fig. 15. The changes in residual resilience of all failed edges

From Fig. 15, a failed edge is repaired in each time period, and the residual resilience *R*(*t*) gradually decreases with time. Throughout the recovery period, the *R*(*t*) changes in the Birnbaum importance and the RRW importance are identical. From the first period to the 8-th period, the $R(t)$ changes of the OPT importance, RAW importance and the comprehensive importance are the same. Before the 3-th period, the changes in $R(t)$ are the same at all importance levels. At the 3-th period, the $R(t)$ changes of the OPT importance, the RAW importance and the comprehensive importance are the same. The $R(t)$ decreased by 0.1064, while $R(t)$ at the other importance levels decreased by 0.0981. After the 3-th period, the $R(t)$ of the Birnbaum importance and the RRW importance in each period is higher than the $R(t)$ of other importance. After the 8-th period, the $R(t)$ of the RAW importance and the comprehensive importance in each period is higher than the R(*t*) of OPT importance.

The changes of $R(t)$ under different importance levels are compared. For OPT importance, with the repair of the failed node, the residual resilience $R(t)$ gradually decreases from 1 to 0.1196. For the Birnbaum importance and the RRW importance, with the repair of the failed node, the residual resilience $R(t)$ gradually decreases from 1 to 0.1237. For the RAW importance, with the repair of the failed node, the residual resilience $R(t)$ gradually decreases from 1 to 0.1390. For the comprehensive importance，with the repair of the failed node, the residual resilience $R(t)$ gradually decreases from 1 to 0.1204. It can be seen that under the state of failure of all edges, although all edges are eventually repaired, using different recovery sequences, the residual resilience varies greatly with time. Therefore, the repair order of the failed edges is {link*S*13*D*16, link*S*10*D*11, link*S*13*D*21, link*S*1*D*3, link*S*2*D*4, link*S*18*D*19, link*S*7*D*8, link*S*23*D*15, link*D*4*D*6, link*D*21*D*22, link*D*16*S*18, link*D*19*D*20, link*T*9*D*12, link*D*6*T*9, link*D*5*S*10, link*S*23*D*17, link*T*9*D*14, link*T*9*S*10, link*D*12*S*18, link*D*6*S*10, link*D*6*S*10, link*D*8*S*10, link*D*12*D*16, link*T*9*D*11, link*D*5*T*9, link*D*17*D*21, link*D*15*D*17, link*S*23*D*11, link*D*20*D*21, link*D*3*D*5, link*D*11*S*13}.

5. **Conclusions and future work**

This paper proposed a new concept of residual resilience and applied it to different importance measures, with the purpose is to study the optimal recovery time and priority of failed nodes and edges in the MTS. This measure can provide valuable information to guide the recovery process. For nodes and edges with higher priority, sufficient recovery resources should be allocated. It is found that the supply node and the link connecting the supply node have a higher priority in the recovery process, during which recovering these types of nodes and edges is most likely to increase the total traffic received by the demanding node. The highest priority is therefore given to these nodes and edges and the system capabilities can then be quickly restored.

Different disasters have different impacts on the MTS. Therefore, in future work, we will study the impact of different disasters on the ports and routes of the MTS and the restoration cost and redundancy cost.

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References

[1] Omer M, Mostashari A, Nilchiani R, Mo Mansouri. A framework for assessing resiliency of maritime transportation systems. Maritime Policy &Management 2012; 39(7): 685-703.

[2] Mansouri M, NilchingiI R, Mostashari A. A policy making framework for resilient port infrastructure systems. Marine Policy 2010; 34(6): 1125–1134.

[3] Nair R, Avetisyan H, Miller-Hooks E. Resilience framework for ports and other intermodal components. Transportation Research Record 2010; 216: 54–65.

[4] Berle O, Asbjornslett BE, Rice JB. Formal Vulnerability Assessment of a maritime transportation system. Reliability Engineering & System Safety 2011; 96(6): 696-705.

[5] Asadabadi A, Miller-Hooks E. Maritime port network resiliency and reliability through co-opetition. Transportation Research Part E: Logistics and Transportation Review 2020; 137: 101916.

[6] Wan CP, Yang ZL, Zhang D, Yan XP, Fan SQ. Resilience in transportation systems: a systematic review and future directions. Transport Reviews 2018; 38(4):479-498.

[7] Adjetey-Bahun K, Birregah B, Châtelet E, Planchet JL. A model to quantify the resilience of mass railway transportation systems. Reliability Engineering & System Safety 2016; 153: 1-14.

[8] Cimellaro G P, Reinhorn A M, Bruneau M. Framework for analytical quantification of disaster resilience. Engineering Structures 2010; 32(11): 3639-3649.

[9] Zhang C, Xu X, Dui HY. Resilience Measure of Network Systems by Node and Edge Indicators. Reliability Engineering & System Safety 2020; 202: 107035.

[10] Cai BP, Xie M, Liu YH, Liu YL, Feng Q. Availability-based engineering resilience metric and its corresponding evaluation methodology. Reliability Engineering & System Safety 2018; 172: 216-224.

[11] Chen LW, Dui HY, Zhang C. A resilience measure for supply chain systems considering the interruption with the cyber-physical systems. Reliability Engineering & System Safety 2020; 199: 106869.

[12] Bao S, Zhang C, Ouyang M, Miao LX. An integrated tri-level model for enhancing the resilience of facilities against intentional attacks. Annals of Operations Research 2019; 283(1-2): 87-117.

[13] Xing LD, Levitin G. Connectivity modeling and optimization of linear consecutively connected systems with repairable connecting elements. European Journal of Operational Research 2018; 264: 732-741.

[14] Feng Q, Zhao X, Fan D, Cai B, Liu Y, Ren Y. Resilience design method based on meta-structure: A case study of offshore wind farm. Reliability Engineering & System Safety 2019; 186: 232-244.

[15] Xu ZP, Ramirez-Marquez JE, Liu Y, Xiahou TF. A new resilience-based component importance measure for multi-state networks. Reliability Engineering & System Safety 2020; 193: 106591.

[16] Fang YP, Pedroni N, Zio E. Resilience-Based component importance measures for critical infrastructure network systems. IEEE Transactions on Reliability 2016; 65(2): 502-512.

[17] Dui HY, Li SM, Xing LD, Liu HL. System performance-based joint importance

analysis guided maintenance for repairable systems. Reliability Engineering & System Safety 2019; 186: 162-175.

[18] Dui HY, Si SB, Yam RCM. Importance measures for optimal structure in linear consecutive-k-out-of-n systems. Reliability Engineering & System Safety 2018; 169: 339-350.

[19] Wu SM, Chen Y, Wu QT, Wang ZL. Linking component importance to optimization of preventive maintenance policy. Reliability Engineering & System Safety 2016; 146: 26-32.

[20] Levitin G, Podofillini L, Zio E. Generalised importance measures for multi-state elements based on performance level restrictions. Reliability Engineering & System Safety 2003; 82(3): 287-298.

[21] Xu ZP, Ramirez-Marquez JE, Liu Y, Xiahou TF. A New Resilience-Based Component Importance Measure for Multi-State Networks. Reliability Engineering & System Safety 2020; 193: 106591.

[22] Almoghathawi Y, Barker K. Component importance measures for interdependent infrastructure network resilience. Computers & Industrial Engineering 2019; 133: 153-164.

[23] Miziula P, Navarro J. Birnbaum importance measure for reliability systems with dependent components. IEEE Transactions on Reliability 2019; 68(2): 439-450.

[24] Henry D, Ramirez-Marquez JE. Generic metrics and quantitative approaches for system resilience as a function of time. Reliability Engineering & System Safety 2012; 99: 114-122.

[25] Barke KR, Ramirez-Marquez JE, Rocco CM. Resilience-based network component importance measures. Reliability Engineering & System Safety 2013; 117: 89-97.

[26] Al-Sharrah G. Ranking Using the Copeland Score: A Comparison with the hasse diagram. Journal of Chemical Information & Modeling 2010; 50(5): 785-791.