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RESEARCH ARTICLE

Adaptive Observer Design for Nonlinear Interconnected Systems by the Application of LaSalle's Theorem

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Summary

In this paper, a class of nonlinear interconnected systems with uncertain time varying parameters (TVPs) is considered. Both the interconnections and the isolated subsystems are nonlinear. The differences between the unknown TVPs and their corresponding nominal values are assumed to be bounded where the nominal value is not required to be known. A dynamical system is proposed and then, the error systems between the original interconnected system and the designed dynamical system are analysed. A set of conditions is developed such that the augmented systems formed by the error dynamical systems and the designed adaptive laws are uniformly ultimately bounded. Specifically, the state observation errors are asymptotically convergent to zero based on the LaSalle's Theorem while the parameter estimation errors are uniformly ultimately bounded, and the classical condition of persistent excitation is not required. A case study on a coupled inverted pendulum system is presented to demonstrate the developed methodology, and simulation shows that the proposed approach is effective and practicable.

KEYWORDS:

Nonlinear Interconnected Systems, Adaptive Observers, Time Varying Parameters, Uniformly Ultimately Boundedness.

1 | INTRODUCTION

The development of advanced technologies has produced corresponding growth in the scale of engineering systems, and thus the scale of many practical systems becomes large in order to satisfy the increasing requirement for system performance. Such systems are called large scale systems and usually can be modelled by sets of lower-order ordinary differential equations which are linked through interconnections. These systems are typically called large scale interconnected systems (see e.g. [1,2,3,4,5]).

Interconnected systems widely exist in the real world, for example, coupled inverted pendula, energy systems and biological systems etc (see e.g. [1,3,6]). Study on interconnected systems has received great attention and many results have been obtained (see e.g. [1,3,7]). Much of the existing work assumes that all system states are available in control design. However, for a practical system, only a subset of system states is usually available. In order to implement state feedback control schemes, one of possible choices is to design an observer to estimate system states, and then use the estimated states to form the feedback control loop.

Observer design has been studied for many years, and the early work can be dated back to the well known Luenberger observer. The majority of the early work about observer design is mainly for linear systems and the robust problem against various uncertainties was not considered (see [8] and references therein). However, due to the mechanical wearing and modelling errors, many practical control systems involve unknown parameters.

Recently, much literature has devoted to design adaptive observers for nonlinear systems and many different methods have been developed in order to obtain high estimation performance in the presence of parametric uncertainty and/or unstructural uncertainty. Boizot *et al* in[9] developed an adaptive observer by using extended Kalman filter to reduce the effect of perturbations. However, in terms of the parameter estimation for nonlinear systems, it is usually very difficult to analysis the stability of the extended Kalman filter. Sliding mode techniques have been applied in[10] to enhance the performance of the adaptive observer proposed by[11]. It should be noted that unknown parameters considered in these papers are constant. An observer for linear time-varying systems with known time-varying matrices affected by unknown input is designed in[12] to estimate the systems states using high order sliding mode techniques. Adaptive observer has been considered in[13] to estimate just the synthetical perturbation with unknown bounds in order to achieve a fast and accurate reusable launch vehicle attitude tracking with chattering attenuation in presence of knowing the system states and its parameters. The authors in[14] designed a state observer and an adaptive disturbance observer to estimate the system state and the disturbance, simultaneously. However, the unknown parameters and interconnected systems are not considered. An adaptive redesign of reduced order nonlinear observers is presented in[15] where the solution of a partial differential equation is required, which may not be possible in most of cases. In order to improve the quality of the current drawn from the utility grid, an adaptive nonlinear observer is designed in[16] to estimate the inductor current which is required in the closed-loop control system of power factor correction as an essential part of AC/DC converters.

An adaptive observer is designed for a class of MIMO uniformly observable nonlinear systems with linear and nonlinear parametrizations in[17] and the exponential convergence of the error dynamics for both types of parametrization is guaranteed under the persistent excitation condition. Tyukin *et al* in[18] considered the problem of asymptotic reconstruction of the state and parameter. However, in both[17] and[18], it is required that the unknown parameters are constant. The literature in[19] proposed an adaptive state estimator for a class of multi-input and multi-output non-linear systems with uncertainties in the state and the output equations, in which the systems considered are not interconnected systems. The work in[20] proposed an adaptive observer which expands the extended state observer to nonlinear disturbed systems. However, the adaptive extended state observer is linear and requires that the error dynamics can be transformed into a canonical form.

Observer design for interconnected systems has been widely studied. Observers have been proposed in[21] for linear large scale systems, where the unknown parameters are not considered. Sliding mode observers have been presented for interconnected systems in[22] where a few coordinates are required to obtain the regular form, and the parameter uncertainty is not considered. Adaptive sliding mode observer based fault reconstruction for nonlinear systems with parameters uncertainties is proposed in[23], where the unknown parameters vector considered is constant. Recently, sliding mode observers are designed in[24] for a class interconnected systems with time-varying parameters. However, it is required that the change rates of the parameters are bounded with known bounds. Moreover, the results obtained in[24] can not guarntee that the observation errors convergae to zero asymptotically. An adaptive interconnected observer is proposed for sensorless control of a synchronous motor in[25] where the system considered includes only two subsystems. In addition, the observer designed is mainly used to implement a special control task. Therefore, strong limitation is unavoidably imposed on the considered interconnected systems. Moreover, in most of the existing work, it is required that either the unknown parameters are constant (see e.g[10,26]) or the nominal values of the unknown parameters are known[27]. The corresponding observation results for large scale nonlinear interconnected systems are very limited, particularly when uncertain time varying parameters are involved.

In this paper, observers are designed for a class of nonlinear interconnected systems with uncertain time varying parameters (TVPs), in which both the isolated subsystems and the interconnections are nonlinear. The designed observers are variable structure interconnected systems but may not result in sliding motion. Under the condition that the difference between the unknown TVPs and the corresponding uncertain nominal values are bounded by constants, adaptive updating laws are proposed to estimate the parameters. The persistent of excitation condition is not required. A set of sufficient conditions are proposed such that the error dynamics formed by the system states and the designed observers are asymptotically stable while the parameters estimation errors are uniformly ultimately bounded using the LaSalle's Theorem. The results obtained are applied to a coupled inverted pendula systems, and simulation results are presented to demonstrate the effectiveness and feasibility of the developed results. The main contribution includes:

(i) Both the interconnections and isolated subsystems take nonlinear forms, which makes the developed results applicable to a wide class of interconnected systems.

(ii) The unknown parameters considered in the system are time varying and the corresponding nominal values are not required to be known. This makes the developed results are different from much of the existing work where it is required that the unknown parameters are constant.

(iii) Under a set of developed mild conditions, the asymptotic convergence of the observation error between the states of the considered systems and the states of the designed observers is guaranteed while the estimate errors of the TVPs are uniformly ultimately bounded.

2 | SYSTEM DESCRIPTION AND PROBLEM FORMULATION

Consider a nonlinear interconnected system composed of N subsystems described as follows

$$\dot{x}_i = A_i x_i + f_i(x_i, u_i) + B_i \theta_i(t) \xi_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} H_{ij}(x_j) \tag{1}$$

$$y_i = C_i x_i \tag{2}$$

where $x_i \in R^{n_i}, u_i \in U_i \in R^{m_i}$ (U_i are the admissible control set) and $y_i \in R$ are the state variables, inputs and outputs of the i-th subsystem respectively. The functions $f_i(\cdot)$ are known continuous, the scalars $\theta_i(t) \in R$ are unknown time varying parameters. The matrices $A_i \in R^{n_i \times n_i}, B_i \in R^{n_i \times 1}$ and $C_i \in R^{1 \times n_i}$ are constants, and C_i are of full rank. The terms

$$\sum_{\substack{j=1 \\ j\neq i}}^{N} H_{ij}(x_j)$$

are the known interconnections of the i-th subsystems for $i = 1, \cdots, N$. The signals $\xi_i(t) \in R$ are known regressor signals which are explained in[28,29].

Assumption 1. The matrix pairs (A_i, C_i) are observable for $i = 1, \cdots, N$.

From Assumption 1, there exist matrices L_i such that $A_i - L_i C_i$ are Hurwitz stable. This implies that, for any positive-definite matrices $Q_i \in R^{n_i \times n_i}$, the Lyapunov equations

$$(A_i - L_i C_i)^T P_i + P_i (A_i - L_i C_i) = -Q_i \tag{3}$$

have unique positive-definite solutions $P_i \in R^{n_i \times n_i}$.

Assumption 2. There exist matrices $F_i \in R$ such that solutions P_i to the Lyapunov equations (3) satisfy the constraints

$$B_i^T P_i = F_i C_i \tag{4}$$

where B_i and C_i are given in system (1)-(2) for $i = 1, \cdots, N$.

Remark 1. To solve the Lyapunov equations (3) in the presence of the constraints (4) is the well known constrained Lyapunov problem (CLP)[30]. Although there is no general solution available for this problem, associated discussion and an algorithm can be found in[31] which may help to solve the CLP for a specific system.

Assumption 3. The uncertain time varying parameters $\theta_i(t)$ satisfy

$$|\theta_i(t) - \theta_{0_i}| \leq \epsilon_{0_i} \tag{5}$$

where θ_{0_i} are unknown constant, and ϵ_{0_i} are known constant for $i = 1, \cdots, N$.

Remark 2. Assumption 3 is to specify a class of uncertainties tolerated in the observer design. The unknown constants θ_{0_i} given in (5) are called the nominal value of the uncertain TVPs $\theta_i(t)$ throughout this paper. Different from the existing work (see e.g.[27,11]), the unknown parameters $\theta_i(t)$ are time varying and the nominal values θ_{0_i} are not required to be known.

For further analysis, the terms $B_i \theta_i(t) \xi_i(t)$ in system (1) are rewritten as

$$B_i \theta_i(t) \xi_i(t) = B_i [\theta_{0_i} + \epsilon_i(t)] \xi_i(t) \tag{6}$$

where the scalers $\epsilon_i(t) = \theta_i(t) - \theta_{0_i}$.

Assumption 4. The nonlinear terms $f_i(x_i, u_i)$ satisfy the Lipschitz condition with respect to $x_i \in R^{n_i}$, and uniformly for $u_i \in U_i \in R^{m_i}$, and $H_{ij}(x_j)$ satisfy the Lipschitz condition in $x_j \in \Omega_j$ for $i = 1, 2, \cdots, N$ and $i \neq j$, that is, there exist nonnegative function ℓ_{f_i} and constant $\ell_{H_{ij}}$ such that

$$\| f_i(\hat{x}_i, u_i) - f_i(x_i, u_i) \| \leq \ell_{f_i}(u_i) \| \hat{x}_i - x_i \| \tag{7}$$

$$\| H_{ij}(\hat{x}_j) - H_{ij}(x_j) \| \leq \ell_{H_{ij}} \| \hat{x}_j - x_j \| \tag{8}$$

for $i = 1, 2, \cdots, N$ and $i \neq j$.

For further analysis, the matrix $W = [w_{ij}]_{N\times N}$ is introduced where its entries w_{ij} are defined by

$$w_{ij} = \begin{cases} \lambda_{\min}(Q_i) - 2\ell_{f_i}\|P_i\|, & i = j \\ -2\|P_i\|\ell_{H_{ij}}, & i \neq j \end{cases} \tag{9}$$

where P_i and Q_i satisfy Lyapunov equation in (3) and $\lambda_{\min}(Q_i)$ represents the minimum eigenvalue of the matrix Q_i for $i = 1, 2, \ldots, N$.

Remark 3. The Assumption 4 is the limitation to the nonlinear terms and the interconnections which is necessary to achieve the asymptotic stability of the observation error dynamics. It should be noted that in the Assumption 4, it is required that $f_i(x_i, u_i)$ satisfy the Lipschitz condition with respect to the variable x_i only. Furthermore, in order to obtain rigorous results, limitations on the matrix W defined in (9) are to be given later. It is clear to see that W involves the parameters ℓ_{f_i} and $\ell_{H_{ij}}$. Thus further limitation on the functions $f_i(\cdot)$ and $H_{ij}(x_i)$ will be provided later.

For nonlinear interconnected system (1)–(2) satisfying Assumptions 1-4, the objective of this paper is to design an observer with appropriate adaptive laws such that the states of the system (1)–(2) can be estimated asymptotically, and the estimation errors of the unknown parameters $\theta_i(t)$ in (1) are uniformly ultimately bounded.

3 | ADAPTIVE OBSERVER DESIGN WITH PARAMETERS ESTIMATION

In this section, an asymptotic observer is to be designed and the proposed adaptive laws are to be presented.

From equation (6), system (1) can be rewritten as

$$\dot{x}_i = A_i x_i + f_i(x_i, u_i) + B_i[\theta_{0_i} + \epsilon_i(t)]\xi_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij}(x_j) \tag{10}$$

$$y_i = C_i x_i \tag{11}$$

For system (10)-(11), construct dynamical systems

$$\dot{\hat{x}}_i = A_i\hat{x}_i + f_i(\hat{x}_i, u_i) + L_i(y_i - \hat{y}_i) + B_i\hat{\theta}_i(t)\xi_i(t) - 2P_i^{-1}(F_iC_i)^T|\xi_i(t)|\,\epsilon_{0_i}\psi_i(\hat{y}_i, y_i) - B_i\hat{\epsilon}_i(t)\xi_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij}(\hat{x}_j) \tag{12}$$

$$\hat{y}_i = C_i\hat{x}_i \tag{13}$$

where P_i and C_i satisfy equations (3) and (4) and the known constant ϵ_{0_i} satisfies the inequality in Assumption 3.

$$\psi_i(\hat{y}_i, y_i) = \begin{cases} \frac{F_i(\hat{y}_i - y_i)}{\|F_i(\hat{y}_i - y_i)\|}, & F_i(\hat{y}_i - y_i) \neq 0 \\ 0, & F_i(\hat{y}_i - y_i) = 0 \end{cases} \tag{14}$$

for $i = 1, 2, \cdots, N$, and $\hat{\theta}_i(t)$ is given by the adaptive law as follows

$$\dot{\hat{\theta}}_i(t) = -2\delta_i(F_i(\hat{y}_i - y_i))^T\xi_i(t) \tag{15}$$

where δ_i is a positive constant which is design parameter and $\hat{\epsilon}_i(t)$ is defined by

$$\hat{\epsilon}_i(t) = -\frac{1}{\delta_i}\hat{\theta}_i(t) \tag{16}$$

for $i = 1, 2, \cdots, N$.

Remark 4. Adaptive laws are the backbone of every adaptive control scheme. The essential idea behind adaptive law is the comparison of the measured system output with the output of observer whose structure is the same as that of the plant model. The adaptive law is formulated as a stability problem where the differential equation of the adaptive law is chosen so that certain stability conditions based on Lyapunov theory are satisfied to facilitate the analysis and design.

Remark 5. The fifth term in (12) includes a variable structure term which works as a controller to enforce the error signal to reach the designed sliding surface and achieve the stability.

Let $e_{x_i} = \hat{x}_i - x_i$. Then, from systems (10)-(11) and (12)-(13), the error dynamical systems can be described by

$$\dot{e}_{x_i} = (A_i - L_iC_i)e_{x_i} + [f_i(\hat{x}_i, u_i) - f_i(x_i, u_i)] + \sum_{\substack{j=1 \\ j\neq i}}^{N}[H_{ij}(\hat{x}_j) - H_{ij}(x_j)] + B_i\tilde{\theta}_i(t)\xi_i(t)$$
$$-B_i\hat{\epsilon}_i(t)\xi_i(t) - B_i\epsilon_i(t)\xi_i(t) - 2P_i^{-1}(F_iC_i)^T|\xi_i(t)|\epsilon_{0_i}\psi_i(\hat{y}_i, y_i) \quad (17)$$

where $\tilde{\theta}_i(t)$ is defined by

$$\tilde{\theta}_i(t) = \hat{\theta}_i(t) - \theta_{0_i} \quad (18)$$

for $i = 1, 2, \cdots, N$.

For the convenience of further analysis , let

$$\tilde{\epsilon}_i(t) = \hat{\epsilon}_i(t) - \epsilon_{0_i} \quad (19)$$

where the known constant ϵ_{0_i} satisfies the inequality (5) in Assumption 3 and $\hat{\epsilon}_i(t)$ is defined in (16), for $i = 1, 2, \cdots, N$.

The following result is ready to be presented:

Theorem 1. Under Assumptions $1-4$, the error dynamical systems (17) with adaptive law (15) are uniformly ultimately bounded if the matrix $W^T + W$ is positive definite, where the matrix $W = [w_{ij}]_{N\times N}$ and its entries w_{ij} are defined in (9). Further, the error e_{x_i} given in (17) satisfies

$$\lim_{t\to\infty}\|e_{x_i}(t)\| = 0, \qquad i = 1, 2, \dots, N \quad (20)$$

Proof. For system (15) and (17), consider the candidate Lyapunov function

$$V = \sum_{i=1}^{N} e_{x_i}^T P_i e_{x_i} + \frac{1}{2}\sum_{i=1}^{N}(\frac{1}{\delta_i}\tilde{\theta}_i^2(t) + \tilde{\epsilon}_i^2(t)) \quad (21)$$

where $\delta_i > 0$ are design parameters given in (15) for $i = 1, 2, \cdots, N$. Note that, in (21) $\tilde{\epsilon}_i(t)$ is dependent on $\tilde{\theta}_i(t)$. From (16), (18) and (19) it can be seen that the relationship between $\tilde{\epsilon}_i(t)$ and $\tilde{\theta}_i(t)$ is given by

$$\begin{aligned}\tilde{\epsilon}_i(t) &= \hat{\epsilon}_i(t) - \epsilon_{0_i} \\ &= -\frac{1}{\delta_i}\hat{\theta}_i(t) - \epsilon_{0_i} \\ &= -\frac{1}{\delta_i}(\tilde{\theta}_i(t) + \theta_{0_i}) - \epsilon_{0_i}\end{aligned}$$

Then, from (17)

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{N}(\dot{e}_{x_i}^T P_i e_{x_i} + e_{x_i}^T P_i \dot{e}_{x_i}) + \sum_{i=1}^{N}(\frac{1}{\delta_i}\tilde{\theta}_i(t)\dot{\tilde{\theta}}_i(t) + \tilde{\epsilon}_i(t)\dot{\tilde{\epsilon}}_i(t)) \\ &= \sum_{i=1}^{N}\Big\{e_{x_i}^T[(A_i - L_iC_i)^T P_i + P_i(A_i - L_iC_i)]e_{x_i} + 2e_{x_i}^T P_i[f_i(\hat{x}_i, u_i) - f_i(x_i, u_i)] \\ &+2e_{x_i}^T P_i\sum_{\substack{j=1 \\ j\neq i}}^{N}[H_{ij}(\hat{x}_j) - H_{ij}(x_j)] + 2e_{x_i}^T P_iB_i\tilde{\theta}_i(t)\xi_i(t) - 2e_{x_i}^T P_iB_i\epsilon_i(t)\xi_i(t) \\ &-2e_{x_i}^T P_iB_i\hat{\epsilon}_i(t)\xi_i(t) + \frac{1}{\delta_i}\tilde{\theta}_i(t)\dot{\tilde{\theta}}_i(t) + \tilde{\epsilon}_i(t)\dot{\tilde{\epsilon}}_i(t) - 4e_{x_i}^T P_iP_i^{-1}(F_iC_i)^T|\xi_i(t)|\epsilon_{0_i}\psi_i(\hat{y}_i, y_i)\Big\}\end{aligned} \quad (22)$$

By using condition (4) and $C_ie_{x_i} = \hat{y}_i - y_i$,

$$\begin{aligned}e_{x_i}^T P_iB_i &= ((P_iB_i)^T e_{x_i})^T = (B_i^T P_ie_{x_i})^T \\ &= (F_iC_ie_{x_i})^T = (F_i(\hat{y}_i - y_i))^T\end{aligned} \quad (23)$$

Substituting (23) into (22), it follows that

$$\begin{aligned}\dot{V} = \sum_{i=1}^{N}\Big\{ & e_{x_i}^T[(A_i - L_iC_i)^T P_i + P_i(A_i - L_iC_i)]e_{x_i} + 2e_{x_i}^T P_i[f_i(\hat{x}_i,u_i) - f_i(x_i,u_i)] \\ & +2e_{x_i}^T P_i \sum_{\substack{j=1\\ j\neq i}}^{N}[H_{ij}(\hat{x}_j) - H_{ij}(x_j)] + [2(F_i(\hat{y}_i - y_i))^T\xi_i(t) + \frac{1}{\delta_i}\dot{\tilde{\theta}}_i(t)]\tilde{\theta}_i(t) \\ & -2(F_i(\hat{y}_i - y_i))^T\epsilon_i(t)\xi_i(t) - 2(F_i(\hat{y}_i - y_i))^T\hat{\epsilon}_i(t)\xi_i(t) \\ & +\tilde{\epsilon}_i(t)\dot{\tilde{\epsilon}}_i(t) - 4(F_i(\hat{y}_i - y_i))^T|\xi_i(t)|\epsilon_{0_i}\psi_i(\hat{y}_i,y_i)\Big\} \end{aligned} \tag{24}$$

From (18), it can be seen that $\dot{\tilde{\theta}}_i(t) = \dot{\hat{\theta}}_i(t)$ because θ_{0_i} is constant. Substituting (14) and (15) into (24) gives

$$\begin{aligned}\dot{V} = \sum_{i=1}^{N}\Big\{ & e_{x_i}^T[(A_i - L_iC_i)^T P_i + P_i(A_i - L_iC_i)]e_{x_i} + 2e_{x_i}^T P_i[f_i(\hat{x}_i,u_i) - f_i(x_i,u_i)] \\ & +2e_{x_i}^T P_i \sum_{\substack{j=1\\ j\neq i}}^{N}[H_{ij}(\hat{x}_j) - H_{ij}(x_j)] - 2(F_i(\hat{y}_i - y_i))^T\epsilon_i(t)\xi_i(t) \\ & -2(F_i(\hat{y}_i - y_i))^T\hat{\epsilon}_i(t)\xi_i(t) + \tilde{\epsilon}_i(t))\dot{\tilde{\epsilon}}_i(t) - 4\|F_i(\hat{y}_i - y_i)\|\ |\xi_i(t)|\epsilon_{0_i}\Big\} \end{aligned}$$

From (19), it can be seen that $\dot{\tilde{\epsilon}}_i(t) = \dot{\hat{\epsilon}}_i(t)$.

$$\begin{aligned}\dot{V} = \sum_{i=1}^{N}\Big\{ & e_{x_i}^T[(A_i - L_iC_i)^T P_i + P_i(A_i - L_iC_i)]e_{x_i} + 2e_{x_i}^T P_i[f_i(\hat{x}_i,u_i) - f_i(x_i,u_i)] \\ & +2e_{x_i}^T P_i \sum_{\substack{j=1\\ j\neq i}}^{N}[H_{ij}(\hat{x}_j) - H_{ij}(x_j)] - 2(F_i(\hat{y}_i - y_i))^T\epsilon_i(t)\xi_i(t) \\ & -[2(F_i(\hat{y}_i - y_i))^T\xi_i(t) - \dot{\hat{\epsilon}}_i(t)]\hat{\epsilon}_i(t) - \epsilon_{0_i}\dot{\hat{\epsilon}}_i(t) - 4\|F_i(\hat{y}_i - y_i)\|\ |\xi_i(t)|\epsilon_{0_i}\Big\} \end{aligned} \tag{25}$$

Substituting (16) and (19) into (25) yields

$$\begin{aligned}\dot{V} = \sum_{i=1}^{N}\Big\{ & e_{x_i}^T[(A_i - L_iC_i)^T P_i + P_i(A_i - L_iC_i)]e_{x_i} + 2e_{x_i}^T P_i[f_i(\hat{x}_i,u_i) - f_i(x_i,u_i)] \\ & +2e_{x_i}^T P_i \sum_{\substack{j=1\\ j\neq i}}^{N}[H_{ij}(\hat{x}_j) - H_{ij}(x_j)] - 2(F_i(\hat{y}_i - y_i))^T\epsilon_i(t)\xi_i(t) \\ & -2\epsilon_{0_i}(F_i(\hat{y}_i - y_i))^T\xi_i(t) - 4\|F_i(\hat{y}_i - y_i)\||\xi_i(t)|\epsilon_{0_i}\Big\} \end{aligned}$$

It is clear from (3) that

$$\begin{aligned}\dot{V} \le & \sum_{i=1}^{N}\Big\{ - e_{x_i}^T Q_i e_{x_i} + 2\|e_{x_i}\|\|P_i\|[f_i(\hat{x}_i,u_i) - f_i(x_i,u_i)] + 2\|e_{x_i}\|\|P_i\| \sum_{\substack{j=1\\ j\neq i}}^{N}[H_{ij}(\hat{x}_j) - H_{ij}(x_j)] \\ & -2(F_i(\hat{y}_i - y_i))^T\xi_i(t)[\epsilon_i(t) + \epsilon_{0_i}] - 4\|F_i(\hat{y}_i - y_i)\|\ |\xi_i(t)|\epsilon_{0_i}\Big\} \\ \le & \sum_{i=1}^{N}\Big\{ - e_{x_i}^T Q_i e_{x_i} + 2\|e_{x_i}\|\|P_i\|[f_i(\hat{x}_i,u_i) - f_i(x_i,u_i)] + 2\|e_{x_i}\|\|P_i\| \sum_{\substack{j=1\\ j\neq i}}^{N}[H_{ij}(\hat{x}_j) \\ & -H_{ij}(x_j)] + 4\|F_i(\hat{y}_i - y_i)\||\xi_i(t)|\epsilon_{0_i} - 4\|F_i(\hat{y}_i - y_i)\||\xi_i(t)|\epsilon_{0_i}\} \end{aligned}$$

$$\leq \sum_{i=1}^{N}\Big\{-e_{x_i}^T Q_i e_{x_i} + 2\|e_{x_i}\|\|P_i\|[\ell_{f_i}\|\hat{x}_i - x_i\|] + 2\|e_{x_i}\|\|P_i\|\sum_{\substack{j=1\\ j\neq i}}^{N}[\ell_{H_{ij}}\|\hat{x}_j - x_j\|]\Big\}$$

$$\leq -\sum_{i=1}^{N}\Big\{(\lambda_{\min}(Q_i) - 2\|P_i\|\ell_{f_i})\|e_{x_i}\|^2 - \sum_{\substack{j=1\\ j\neq i}}^{N}(2\|P_i\|\ell_{H_{ij}}\|e_{x_i}\|\|e_{x_j}\|)\Big\} \tag{26}$$

Then, from the definition of the matrix W in (9) and the inequality above, it follows that

$$\dot{V} \leq -\frac{1}{2}X^T[W^T + W]X \tag{27}$$

where $X = [\|e_{x_1}\|, \|e_{x_2}\|, \cdots, \|e_{x_N}\|]^T$. From the LaSalle's Theorem (see. e.g.[32]), all the solutions of (17) are uniformly ultimately bounded and satisfy

$$\lim_{t\to\infty} X^T[W^T + W]X = 0 \tag{28}$$

Further, from the facts

$$\lambda_{\min}(W^T + W)\|X\|^2 \leq X^T(W^T + W)X$$

and

$$\|X\|^2 = \|e_{x_1}\|^2 + \|e_{x_2}\|^2 + \cdots + \|e_{x_N}\|^2$$

it is straightforward to see from (28) and the condition $W^T + W > 0$ that

$$\lim_{t\to\infty}\|e_{x_i}(t)\| = 0, \qquad i = 1, 2, \ldots, N$$

Hence the conclusion follows. △.

Remark 6. It should be noted that the constructed Lyapunov function (21) is a function of variables e_{x_i}, $\tilde{\theta}_i$ and $\tilde{\epsilon}_i$ while the right hand side of inequality (27) is a function of variables e_{x_i} only. Therefore, the condition $W^T + W$ is positive definite in Theorem 1 implies that $\dot{V}$ is semi-positive definite instead of positive definite.

Remark 7. Theorem 1 shows that the augmented systems formed by (17) and the adaptive law (15) are uniformly ultimately bounded. It should be noted that the estimated states $\hat{x}_i$ given by the observer (12) converge to the system states x_i in (1) asymptotically although the estimate error for the parameters may not be asymptotically convergent. As the uncertain parameters θ_i in system (1) are time-varying, the approaches developed in[23,11] cannot be applied to the systems considered in this paper.

Remark 8. The designed observer is a variable structure interconnected system but it may not produce a sliding motion, which is different from the work in[11]. In addition, the unknown parameters are considered as constants in[11] while in this paper they are TVPs. In addition, Assumption 3 in[11] is a limitation on uncertain parameter distributions and it is necessary for the parameter θ to be estimated as it explained clearly in Remark 2 in[11].

Remark 9. The technique used in this work to achieve stability is similar to the Vector Lyapunov Functions method explained in[33,34]. Different from[33,34], the considered systems are nonlinear with unknown time varying parameters.

Remark 10. Unlike Lyapunov stability theorems, LaSalle's theorem does not require the function in (21) to be positive definite[35,32]. The form of adaptive law (15), LaSalle's theorem and the boundedness of the parameters variation (5) guarantee that the error system (17) is uniformly ultimately bounded without the persistent excitation condition. LaSalle's theorem has been used in[36] to develop a new adaptive law for robust adaptation without the persistent excitation condition.

4 | CASE STUDY: A COUPLED INVERTED PENDULUM

In order to illustrate the method developed in this paper, case study on a coupled pendulum system is carried out in this section. Consider a system formed by two inverted pendula connected by a spring as given in Figure 1 . There are two balls are attached at the end of the two rigid rods respectively. The symbol u_1 and u_2 denote external torques imposed on the two pendula respectively which are the control inputs. The distance b between the two pendulum hinges are assumed to be changeable with respect to time t. Let $\varphi_1 = x_{11}$, $\varphi_2 = x_{21}$, $\dot{\varphi}_1 = x_{12}$, and $\dot{\varphi}_2 = x_{22}$. The coupled inverted pendulums can be modelled as (see e.g.[37,38])

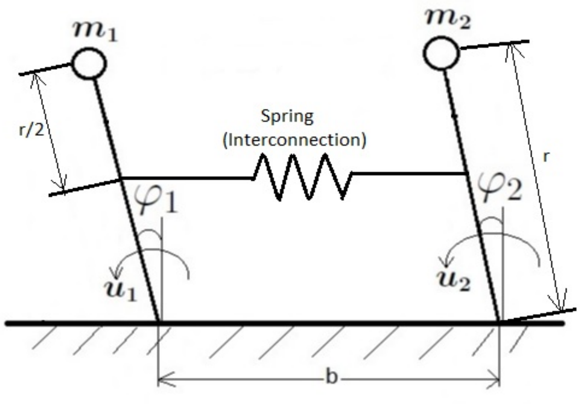

FIGURE 1 Coupled inverted pendula

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ (\frac{m_1 g r}{J_1} - \frac{k r^2}{4 J_1}) \sin(x_{11}) + \frac{1}{J_1} u_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{kr}{2J_1} \end{bmatrix} (l - b) + \begin{bmatrix} 0 \\ \frac{kr^2}{4J_1} \sin(x_{21}) \end{bmatrix} \tag{29}$$

$$y_1 = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} \tag{30}$$

$$\dot{x}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} + \begin{bmatrix} 0 \\ (\frac{m_2 g r}{J_2} - \frac{k r^2}{4 J_2}) \sin(x_{21}) + \frac{1}{J_2} u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{kr}{2J_2} \end{bmatrix} (l - b) + \begin{bmatrix} 0 \\ \frac{kr^2}{4J_2} \sin(x_{11}) \end{bmatrix} \tag{31}$$

$$y_2 = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} \tag{32}$$

The end masses of pendula are $m_1 = 0.7$ kg and $m_2 = 0.6$ kg, the moments of inertia are $J_1 = 5$ kg and $J_2 = 4$ kg, the constant of connecting spring is $k = 90$ N/m, the pendulum height is $r = 0.25$ m, and the gravitational acceleration is $g = 9.81$ m/s^2. In order to illustrate the developed theoretical results, it is assumed that $(l - b(t)) = \theta_1(t) = \theta_2(t)$ is an unknown time varying parameter for $i = 1, 2$ where l is the natural length of spring and $b(t)$ is the distance between the two pendulum hinges.

In order to avoid system states going to infinity, and for simulation purposes, the following feedback transformation is introduced

$$u_i = -k_i x_i + v_i, \qquad i = 1, 2 \tag{33}$$

$$k_1 = \begin{bmatrix} 10 & 15 \end{bmatrix} \tag{34}$$

$$k_2 = \begin{bmatrix} 8 & 12 \end{bmatrix} \tag{35}$$

Then with the given parameters above, the system (29)-(32) can be rewritten as

$$\dot{x}_1 = \underbrace{\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}}_{A_1} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0.06215 \sin(x_{11}) + \frac{1}{5} v_1 \end{bmatrix}}_{f_1(x_1, u_1)} + \underbrace{\begin{bmatrix} 0 \\ 2.25 \end{bmatrix}}_{B_1} \underbrace{(l - b(t))}_{\theta_1(t)} + \underbrace{\begin{bmatrix} 0 \\ 0.2813 \sin(x_{21}) \end{bmatrix}}_{H_{12}(x_2)} \tag{36}$$

$$y_1 = \underbrace{\begin{bmatrix} 1 & 1 \end{bmatrix}}_{C_1} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} \tag{37}$$

$$\dot{x}_2 = \underbrace{\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}}_{A_2} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0.01632 \sin(x_{21}) + \frac{1}{4} v_2 \end{bmatrix}}_{f_2(x_2, u_2)} + \underbrace{\begin{bmatrix} 0 \\ 2.8125 \end{bmatrix}}_{B_2} \underbrace{(l - b(t))}_{\theta_2(t)} + \underbrace{\begin{bmatrix} 0 \\ 0.352 \sin(x_{11}) \end{bmatrix}}_{H_{21}(x_1)} \tag{38}$$

$$y_2 = \underbrace{\begin{bmatrix} 1 & 1 \end{bmatrix}}_{C_2} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} \tag{39}$$

Choose

$$L_i = [0 \;\; 0] \qquad \text{and} \qquad Q_i = 4I$$

for $i = 1, 2$. It follows that the Lyapunov equations (3) have unique solutions:

$$P_i = \begin{bmatrix} 5 & 1 \\ 1 & 1 \end{bmatrix}, \qquad i = 1, 2 \tag{40}$$

satisfying the condition (4) with

$$F_1 = 2.25, \qquad \text{and} \qquad F_2 = 2.8125$$

For simplicity, it is assumed that

$$\xi_i(t) = 1, \qquad \epsilon_{0_i} = 1 \;\text{ and }\; \delta_i = 2$$

for $i = 1, 2$. By direct computation, it follows that the matrix $W^T + W$ is positive definite. Thus, all the conditions of Theorem 1 are satisfied. This implies that the following dynamical systems are the asymptotic observer of the nonlinear interconnected system (36)–(39):

$$\begin{aligned} \dot{\hat{x}}_1 = {} & \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} \hat{x}_{11} \\ \hat{x}_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ 0.06215 \sin(\hat{x}_{11}) + \frac{1}{5} v_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 2.25 \end{bmatrix} \hat{\theta}_1(t) - \begin{bmatrix} 0 \\ 0.4 \end{bmatrix} \frac{(\hat{y}_1 - y_1)}{\|\hat{y}_1 - y_1\|} \\ & - \begin{bmatrix} 0 \\ 2.25 \end{bmatrix} \hat{\epsilon}_1(t) + \begin{bmatrix} 0 \\ 0.2813 \sin(\hat{x}_{21}) \end{bmatrix} \end{aligned} \tag{41}$$

$$\hat{y}_1 = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_{11} \\ \hat{x}_{12} \end{bmatrix} \tag{42}$$

$$\begin{aligned} \dot{\hat{x}}_2 = {} & \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} \hat{x}_{21} \\ \hat{x}_{22} \end{bmatrix} + \begin{bmatrix} 0 \\ 0.01632 \sin(\hat{x}_{21}) + \frac{1}{4} v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2.8125 \end{bmatrix} \hat{\theta}_2(t) - \begin{bmatrix} 0 \\ 0.5 \end{bmatrix} \frac{(\hat{y}_2 - y_2)}{\|\hat{y}_2 - y_2\|} \\ & - \begin{bmatrix} 0 \\ 2.8125 \end{bmatrix} \hat{\epsilon}_2(t) + \begin{bmatrix} 0 \\ 0.352 \sin(\hat{x}_{11}) \end{bmatrix} \end{aligned} \tag{43}$$

$$\hat{y}_2 = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_{21} \\ \hat{x}_{22} \end{bmatrix} \tag{44}$$

The designed adaptive laws are given by

$$\dot{\hat{\theta}}_1(t) = -4(2.25(\hat{y}_1 - y_1))^T \tag{45}$$

$$\dot{\hat{\theta}}_2(t) = -4(2.8125(\hat{y}_2 - y_2))^T \tag{46}$$

For simulation purpose, v_1 and v_2 are chose as $v_1 = v_2 = 0.1 \sin t$, and the unknown parameters θ_{0_i} and $\theta_i(t)$ are chosen as $\theta_{0_i} = 0$ and $\theta_i(t) = 0.6 \sin t$ for $i = 1, 2$. Simulation in Figures 2 and 3 shows that the estimation error between the states of the system (29)-(32) and the states of the observer (41)-(44) converges to zero asymptotically. Figure 4 shows that the estimation of the parameters is uniformly ultimately bounded with satisfactory accuracy.

Remark 11. For a real system, the positions and/or the velocities are usually chosen as system output. However, some times, the linear combination of the position and velocity are taken as system output. Physically, such an aggregation of the output might arise in some real systems[39,40], for example, certain remote-control applications where the number of transmission and receive lines/frequencies are limited[39]. The proposed approach in this work is valid under Assumption 2. If only the positions in system (29)-(32) are chosen as system outputs, Assumption 2 does not hold, and thus the Theorem 1 does not hold either. In this case, the results developed in this paper are not applicable.

5 | CONCLUSION

In this paper, an adaptive observer design for a class of nonlinear large scale interconnected systems with unknown time varying parameters has been proposed. The unknown parameters vary within a given range. A set of sufficient conditions has been

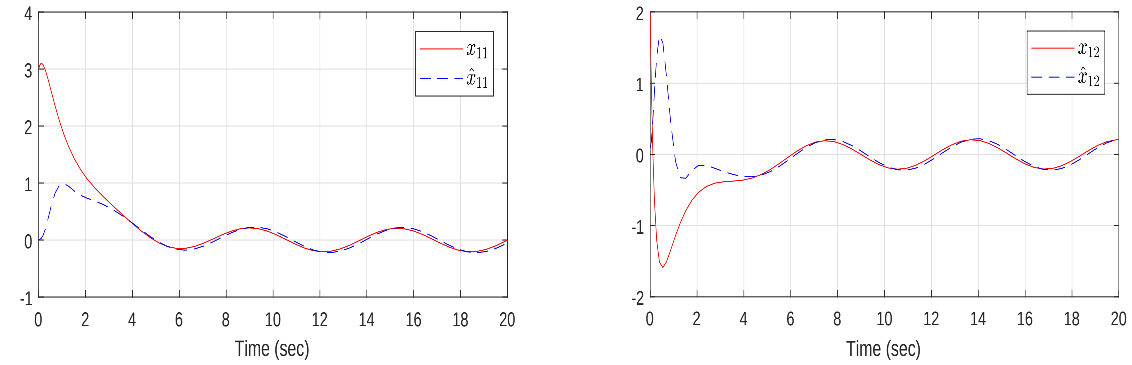

FIGURE 2 The time response of the 1st subsystem states $x_1 = \text{col}\,(x_{11}, x_{12})$ and their estimation $\hat{x}_1 = \text{col}\,(\hat{x}_{11}, \hat{x}_{12})$.

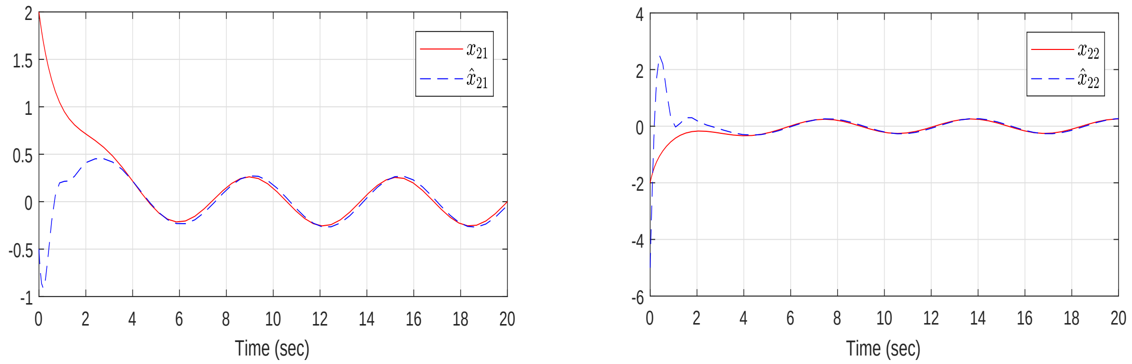

FIGURE 3 The time response of the 2nd subsystem states $x_2 = \text{col}\,(x_{21}, x_{22})$ and their estimation $\hat{x}_2 = \text{col}\,(\hat{x}_{21}, \hat{x}_{22})$.

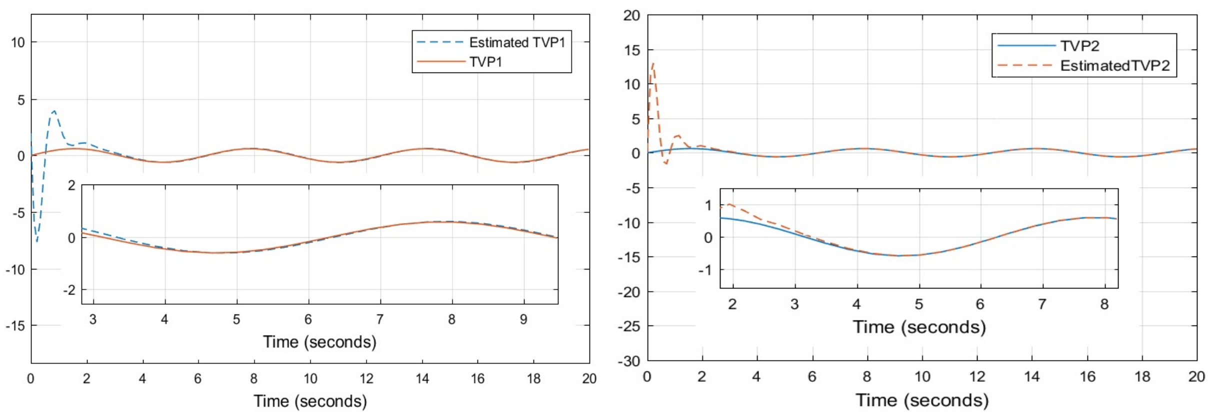

FIGURE 4 Upper: the time response of $\hat{\theta}_1(t)$ (dashed line) and $\theta_1(t)$ (solid line); Bottom: the time response of $\hat{\theta}_2(t)$ (dashed line) and $\theta_2(t)$ (solid line).

developed to guarantee that the observation error system with the proposed adaptive law is uniformly ultimately bounded. The states of the designed observer are asymptotically convergent to the original system states. Therefore, from the state estimation point of view, the designed observers are asymptotic observers. Case study on a coupled inverted pendulum system shows the practicability of the developed observer scheme for nonlinear interconnected systems. In the future, adaptive observer design for interconnected systems in the presence of measurement noises will be considered.

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