University of Kent - School of Economics

Three Essays in the Role of Firms in Macroeconomics

A Thesis Submitted in Fulfilment of the Requirements for the Degree of Doctor of Philosophy in Economics

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Declaration

I hereby declare that this thesis is my own work, as are all errors and omissions. The copyright of this thesis rests with the author. This thesis consists of approximately 21,100 words exclusive of footnotes, tables, figures and bibliography. This thesis contains no material that has been submitted previously, in whole or in part, for the award of any other academic degree.

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Abstract

This thesis consists of three chapters covering the role of firms in macroeconomics.

The first chapter focuses on the labour share of income. Theoretical and practical issues surrounding measurement are discussed and novel empirical applications are explored. Utilising household survey data, new time series are constructed for full-time equivalent employees and counter-factual proprietor labour income. These series used to generate estimates of the labour share which constitute a conceptual improvement over the existing official UK methodology. Additionally, the classification of firm intellectual property investment and its implications for the measurement of the labour share are examined. The results shown that the careful treatment of proprietor income and intellectual property products suggests there has been little to no decline in the UK labour share.

The second chapter explores the role of rising markups on optimal capital taxation. The Chamley-Judd result states that the taxation of capital income should converge to zero in the long run. However, recent empirical studies suggest that average firm-level markups have increased substantially over the past two decades; a direct implication is a rise in the share of aggregate profits. The zero long run optimal capital tax result is revisited in this context. A structural model of firm entry with monopolistic competition is developed where the fiscal authority cannot differentiate between aggregate profits and productive capital income. It is shown that when markups are sufficiently high, aggregate profits in equilibrium become large enough such that it is optimal to positively tax capital income in the long run - beyond a certain threshold the efficiency gains from taxing lump-sum profits outweigh the distortion caused by inadvertently taxing future investments in physical capital.

The third chapter examines the comovement problem in the context of firm dynamics. Empirical observations show that consumption and investment are both pro-cyclical over the business cycle. However, investment shocks in structural models typically generate opposing responses for consumption and investment. This paper develops a model of firm entry with business churn and endogenous overhead costs which produces the appropriate unconditional comovement between consumption, investment, hours and output. Moreover, this paper demonstrates that CES production technology with an elasticity of substitution well below unity, in line with empirical estimates, generates the observed comovements between macroeconomic aggregates on impact and improves model fit in terms of second moments.

Contents

1 The Labour Share: Proprietor Income and Intellectual Property 7

1.1 Introduction . . . 8

1.2 Literature Review . . . 8

1.3 Methodology . . . 12

1.4 Empirical Application . . . 18

1.5 Intellectual Property Products . . . 23

1.6 Conclusion . . . 27

1.7 Appendix . . . 28

2 Rising Firm Markups and Optimal Capital Taxation 31

2.1 Introduction . . . 32

2.2 Literature Review . . . 34

2.3 Model . . . 38

2.4 Solution and Results . . . 43

2.5 Conclusion . . . 49

2.6 Appendix . . . 51

3 Investment Shocks, Firm Dynamics and the Comovement Problem 53

3.1 Introduction . . . 54

3.2 Literature Review . . . 56

3.3 Model . . . 59

3.4 Simulations and Results . . . 65

3.5 Conclusion . . . 71

3.6 Appendix . . . 73

Bibliography 87

List of Figures

1.1 UK Labour Share (1959-2016) LS_{ONS} LS_1 LS_2 19
1.2 UK Labour Share (1959-2016) LS_1 LS_2 LS_3 20
1.3 UK Labour Share (1959-2016) LS_1 LS_2 LS_4 22
1.4 Reclassification of IPP in National Accounts 23
1.5 UK Labour Share (1959-2016) LS_{ONS} LS_{ONS}^{IPP} 25
1.6 UK Labour Share (1970-2016) LS_{ONS} LS_3^{IPP} 26
1.7 UK Propietors and Mixed Income (1959-2016) $P/(E+P)$ M/Y . . 28
1.8 LCFS: Share of UK Employees Working Part-Time (1968-2016) . . . 28

2.1 Average US Firm Markups (De Loecker and Eeckhout, 2017) 33
2.2 Long Run Optimal Tax Rates 47
2.3 Long Run Optimal Capital Tax Isoquants 48

3.1 US Data and Benchmark RBC Output 55
3.2 $\sigma = 0$ and $\sigma = 1.5$ production surfaces 65
3.3 Consumption-Leisure static δ-Φ parameter space 80
3.4 Labour-Investment Shock static δ-Φ parameter space 81
3.5 Composite of Figure 4 and 5 82
3.6 Inverse Frisch elasticity thresholds 85

List of Tables

2.1 Equilibrium Conditions 42
2.2 Steady State Expressions 44
2.3 Parameter Values 46

3.1 Equilibrium Conditions 64
3.2 Normalised Steady State Values 66
3.3 Parameter Values 66
3.4 Second Moments 70
3.5 Equilibrium Conditions 76
3.6 Steady State Expressions 76

Introduction

The study of the role of firms in economics has its origins in a seminal paper by Coase (1937). In this paper, Coase explores why firms exist and why production is not simply organised by individuals. The conclusion drawn is that firms exist to minimise the transaction costs associated with coordinating economic activity and that activity within firms can be characterised by an absence of a price mechanism - "islands of conscious power in [an] ocean of unconscious cooperation"

While there has long existed empirical studies on the role of firms, it was not until more recently that the explicit modelling of dynamic firms gained traction in macroeconomics. This strand of literature grew substantially in the decade following the Great Recession as mainstream macroeconomics reassessed itself and broadened its scope.

This has subsequently motivated a range of papers which aim to model firm behaviour at the macroeconomic level as a means of understanding economic phenomena. This thesis broadly fits into that component of the literature, in particular in the second and third chapters which build on the seminal paper of Bilbiie, et al (2012).

The first chapter of this thesis is empirical. It examines measurement issues related to the labour share of income. Particularly the role of unincorporated firm income and the classification of intellectual property products. It is argued that only through the careful treatment of qualitative assumptions can the labour share, and by extension capital and profit share, provide valuable economic insights on the macroeconomy.

The second chapter studies the impact of firms on optimal taxation. When fiscal authorities cannot differentiate between two sources of capital income, two opposing objectives must be reconciled with a single flat-rate tax on capital. From an efficiency perspective, the government wishes to tax profits, but not interest derived from productive physical capital. Modelling firms explicitly in a structural framework allows for quantification of the direct effect of the average markups on the optimal rates of taxation.

The third chapter explores the impact of investment shocks on business cycle fluctuations through the framework of dynamic firm entry and the inclusion of adjustment costs and overhead costs. Firms exhibiting CES production technology with an elasticity of substitution significantly below unity generate the observed comovements between macroeconomic aggregates in response to investment shocks and improves model fit in terms of second moments.

1 The Labour Share: Proprietor Income and Intellectual Property

Abstract

The labour share plays a fundamental role as a parameter in macroeconomic models and motivates a growing body of literature with regards to its apparent decline. Despite its crucial importance, there exists no robust consensus on how to measure the labour share in practice. This paper explores some of the conceptual and practical issues surrounding estimation and provides a novel empirical application. Utilising household survey data, more conceptually accurate estimates of proprietor labour income are constructed for the UK; additionally, the approach of Koh, et al (2018) is considered and extended to the UK economy. This paper demonstrates that the careful treatment of proprietor income and intellectual property products suggests there has been little to no decline in the UK labour share.

1.1 Introduction

The concept of the labour dates back to classical economics. However, there exists no exact consensus on how to quantify the portion of national income which accrues to labour; primarily because it is not possible to directly observe all labour income.

For example, unincorporated enterprises do not exist as separate legal entities from their owners; as such, they can freely move assets between business and personal accounts. For this reason, it is not possible to distinguish between earnings from work and the entrepreneurial profit in an unincorporated enterprise. The mixed nature of proprietor income, which contains both capital and labour remunerations, necessitates that any estimate of self-employed labour income must be imputed. As a consequence, all measures the labour share are subject to unavoidable qualitative assumptions regarding the apportioning of ambiguous income. It is these conceptual assumptions, along with novel empirical applications for the UK economy, which are further explored in this paper.

The remainder of this paper is structured as follows: section 2 consists of a literature review. Section 3 reviews several different approaches to measuring the labour share. Section 4 provides an empirical application and utilises household survey data to construct novel estimates of the labour share in the UK. Section 5 considers the role intellectual property products (IPP), as emphasised by Koh, et al (2018), and applies this approach to UK data. Section 6 concludes.

1.2 Literature Review

Classical economists wrote at length about the distribution of factor income. In fact, Ricardo (1817) claimed that 'to determine the laws which regulate this distribution, is the principal problem in political economy'. However, despite its long conceptual history, attempts to formally quantify the labour share only began in the 20th century - the first task to be undertaken by the NBER, upon its foundation in 1920, was the estimation of the 'approximate size and [functional] distribution of national income'.[1] Although early measurement efforts were often hampered by theoretical

[1]See the opening paragraph of the first chapter of King (1930).

concerns and a lack of comprehensive data, an emerging consensus grew over the decades regarding the perceived stability of the labour share. With this in mind, Kaldor (1957) fashioned several ‘stylised’ facts - the first of which emphasised the constancy of the share of national income received by labour (and capital).

However, in recent years, economists have challenged this notion and turned their attention towards the apparent protracted decline of the labour share across industrialised economies. This trend has motivated a growing body of literature devoted to accounting for the alleged decline - both from an empirical and theoretical perspective.

A seminal paper by Karabarbounis and Neiman (2014) argues that the fall in the relative price of investment goods has been a key driver of the decline in the labour share globally. They suggest that this trend has contributed to approximately 50% of the observed fall in the labour share. The intuition is that investment-specific technological change reduces the cost of capital goods relative to labour, therefore, firms substitute labour for capital and this pushes down the labour share as production becomes more capital intensive. However, this reasoning is contingent on an elasticity of factor substitution $\sigma = 1.25$; far greater than the vast majority of empirical estimates (Chirkino, 2008, Leon-Ledesma 2011). The consensus in the literature is overwhelmingly in favour of $\sigma < 1$. This suggests that there is lower substitutability between factors than is assumed by Karabarbounis and Neiman (2014), and that, contrary to their argument, capital-deepening would theoretically result in a rising labour share.

On the other hand, Elsby, et al (2013) argue that investment-specific technological change has only contributed weakly to a decline in the labour share; payroll shares observed across US industries exhibits no strong correlation with changes in their respective investment good prices. Instead, Elbsy, et al (2013) claim that the decline in the US labour share (driven by a fall in the payroll share) has its origins in globalisation and the off-shoring of labour intensive production to developing economies with lower labour costs.

Piketty (2014) advances a slightly different but more simplistic narrative. Capital-deepening rises when the rate of growth g falls; however, the average return to capital r tends to remain stable over time. The relative decline in output growth over the past few decades implies the inequality $r > g$. This causes the capital share to rise as total output grows more slowly than capital income; by construction the labour share falls. Similarly, Piketty and Zucman (2014) observe declining labour shares across industrialised economies and argue that the average return to capital has experienced a proportionally smaller fall relative to the increase capital stock. They suggest that this is due to an increase in the bargaining power of capital as a result of globalisation and the increased mobility of capital; a rise in the bargaining power of capital subsequently depresses the labour share.

However, contrary to this, Lawrence (2015) proposes a neoclassical account for the decline in the labour share, consistent with the empirical evidence of $\sigma < 1$. The claim is that the effective capital-labour ratio has fallen in most industries in the US because the increase in the physical capital-labour ratio has been offset by rapid labuor-augmenting technological change. This has increased the supply of labour available and has therefore actually decreased the effective capital-labour ratio. Utilising a standard CES production function, with gross complementary $\sigma < 1$, it follows that a decline in the effective capital-labour ratio produces a decline in the labour share of income. Lawrence (2015) suggests that this can account for most of the observed decline in the labour share in the US.

While there are other channels which may contribute to the decline in the labour shares such as, increasing mark ups, changes in the skill composition of workers, weaker bargaining power of labour and political capture, some argue that there has actually been no decline in the labour share and that measurement issues or accounting practices have been the source of the apparent decline.

For example, Bridgeman (2014) argues that the standard procedure for calculating factor shares can be problematic as it does not capture changes in the share of depreciation in output. The intuition is that there exists a portion of output which is used up in the production process and which cannot be used for current consumption or future investment; consumption of fixed capital. A rising share of depreciation in output, brought about by the increasing prevalence of IT goods (which on average depreciation more quickly than other investment goods) can create the appearance of a declining labour share. The implication is that an increasing portion of output is implicitly and erroneously being attributed to capital income - when, in fact, it is used up in production and should not be attributed to capital or labour. Applying this accounting procedure to the US economy, Bridgeman (2014) shows that the labour share exhibits no decline and is within its historical range.

Additionally, Armenter (2015) claims that changes in the classification of proprietor income and increases in the number of self-employed people have resulted in an underestimation of the labour share in the US. Until 2001 the methodology used by the Bureau of Labor Statistics (BLS) entailed that about 80% of proprietor income was attributed to labour. However, after 2001, the new methodology entailed that less than 50% of proprietor income should be classified as labour income. Retroactively applying the pre-2001 BLS methodology Armenter (2015) finds that approximately one-third of the drop in the BLS measure of the labour share since 2001 can be attributed changes in the classification of proprietor income. This point is further supported by examining the payroll share, which naturally excludes proprietor income, and shows a much smaller decline than the BLS headline measure.

Beyond the issue of proprietor income, there also exists the problem of how to classify different types of expenditure in the economy. Koh, et al (2018) note that intellectual property products have previously been considered as intermediate inputs by the BEA, but recently have been reclassified as investments. This reclassification is important from an accounting perspective as the decision to include IPP as investment revises up the value of Gross Value Added (GVA) and revises down the value of intermediate inputs.

Essentially, a portion of intermediate inputs has been reallocated to GVA. This unambiguously increases GVA and therefore increases the value of the denominator when constructing the labour share. From a purely logical basis, this results in a lower value of the labour share. To ensure that the national accounting identity holds, any income derived from IPP, which is now included in GVA, must accrue to one of the factors of production. The implicit assumption the BEA makes is that all IPP rents accrue to capital. Koh, et al (2018) states that it is this extreme assumption which generates the secular decline in the labour share - it is purely an accounting artefact and not an economic phenomenon.

1.3 Methodology

To ensure consistency between empirical estimates and analytical concepts, it is important to begin with a concrete definition of the labour share:

$$LS_D = \frac{WL}{Y}$$

Where W is average labour compensation per hour worked, L is the total number of hours worked and Y is a measure total income (Elsby, et al, 2013). Although the calculation of total income and the total number of hours worked may be relatively straightforward given current data, the quantification of average labour compensation per hour worked is more problematic. This is because it includes some labour income which cannot be observed directly; namely, a portion of proprietor income.

But what exactly is proprietor income? Following the Bureau of Economic Analysis (BEA) definition, proprietor income consists of the earnings derived from: sole proprietorships, partnerships, and any other private businesses that are organised for profit but are not classified as corporations (BEA, 2017). While corporations are obliged to record factor income for legal purposes, unincorporated enterprises are not required to keep detailed financial accounts in the same way; as such, they can freely move assets between business and personal accounts. For this reason, it is not possible to distinguish between the remuneration for work and the entrepreneurial profit in an unincorporated business. Thus, herein lies a key issue: how should we account for the labour portion of proprietor income when estimating the labour share?

Approach 1: One common method used for imputing self-employed labour income involves calculating average employee compensation (in some form) and scaling it up to the number of proprietors. This is the well-known approach suggested by Gollin (2002). It involves calculating average employee compensation and scaling it up to the total number workers in the economy (employees plus proprietors), then diving by total income to give an estimate of the labour share.

$$LS_1 = \left(\frac{C}{E}\right)\left(\frac{E+P}{Y}\right)$$

Where C is compensation of employees, E is the total number of employees and P is the total number of proprietors. Specifically here, Y represents gross value added (gross domestic product less the value of intermediate consumption: GVA). There are two main reasons for removing taxes on production less subsidies. Firstly, although net indirect taxes are levied on production, it is not precisely clear which factor bears the burden of taxation (Bridgeman, 2014). Secondly, these taxes could be used for a wide variety of purposes, which may disproportionately benefit capital or labour income (Lawrence, 2015). The burdens and purposes of indirect taxation are likely to vary substantially across time; therefore, to reduce potential bias and noise when constructing factor shares, taxes on production less subsidies are usually omitted from the denominator.

The LS_1 approach has the clear advantage of requiring minimal data to construct. However, it assumes that, on average, proprietors are compensated for their labour in equal proportion to employees; regardless of hours worked. This is a significant weakness for two reasons. Firstly, wage distributions of employees are generally skewed rightward in developed economies due to the presence of highly paid managerial and financial jobs; a sector of work in which self-employed people generally do not participate. Therefore, these types of imputation of self-employed labour income may potentially generate upward bias in estimates of the labour share.[2]

Secondly, consider a compositional shift from full-time to part-time employment in the economy. Ceteris paribus, this would lead to a decrease in total employee compensation, since the total number of hours worked by employees has fallen. However, the total number of employees, part-time and full-time, does not change. As a result, average employee compensation decreases but average employee compensation per hour worked remains unchanged. This example merely emphasises that the numerator (employee compensation) depends on both the intensive margin and the extensive margin while the denominator (number of employees) is expressed purely

[2]A priori, this could generate bias estimates in either direction if the average employee wages differs sufficiently from the (unobservable) average proprietor labour income. This issue is addressed with a novel approach in LS_4.

in terms of the extensive margin.[3]

Approach 2: A natural solution to this second issue could be to simply express everything in terms of hours worked (the favoured method of the BLS). This would fully account for the intensive and extensive margin since employee compensation would be divided by the number of total hours worked by employees; thereby absorbing the effects of both margins. This approach would instead yield average employee compensation per hour worked, which would then be scaled up to the total number of hours worked (employee hours plus proprietor hours). Dividing this estimate of total labour income by gross value added gives a modified LS_2 version of the aforementioned LS_1 measure:

$$LS_2 = \left(\frac{C}{E_H}\right)\left(\frac{E_H + P_H}{Y}\right)$$

Where E_H is the total number of hours worked by employees and P_H is the total number of hours worked by proprietors. An advantage of this approach is that it moves closer to the conceptual definition of the labour share LS_D by fully accounting for the intensive margin when imputing self-employed labour income. The assumption here is that, on average, proprietors are compensated for their labour at the same hourly rate as employees.

However, to construct LS_2 requires data on the total hours worked by proprietors; this can be difficult to gauge since those who are self-employed often do not have contractually fixed hours of work. Moreover, proprietors may have a greater predisposition to work additional unrecorded hours by allowing their work time to blur into their leisure time[4].

A disadvantage of the LS_2 approach is that it requires additional data to construct, which usually limits its historical scope. Often, it is the long run behaviour of the labour share which is of interest to economists for both empirical and theoretical reasons. Therefore, a possible solution to this issue is to find a middle ground between the LS_1 and LS_2 measures, whereby adjustments are made for the intensive

[3]This ceases to be an problem only if average hours worked per employee are always equal to average hours worked per proprietor; thereby extinguishing the effect of the intensive margin.

[4]For example, managing their business accounts while at home in the evening or communicating with clients outside of usual working hours.

margin but historical scope is not excessively restricted.

Approach 3: A potential remedy could be to modify LS_1 by utilising full-time equivalent employees, as opposed to the total number of employees. This allows for adjustments in the intensive margin by converting the *hours* worked by part-time employees into a *quantity* of full-time employees. For example, consider an economy with one full-time employee who works 32 hours per week, and two part-time employees, who both work 16 hours a week each. In this case, the number of full-time equivalent employees in the economy would be 2 since the total hours worked by the part-time employees is equivalent to one full-time employee.

For LS_3, full-time equivalent employees ($FTEE$) requires a concrete definition. The BEA defines full-time equivalent employees as: 'the product of the total number of employees [. . .] and the ratio of average weekly hours per employee for all employees to average weekly hours per employee on full-time schedules.' Where a full-time schedule is defined to be an employee who works 'on average at least 30 hours per week'. Furthermore, persons engaged in production ($PEIP$) is defined as: 'full-time equivalent employees plus the number of self-employed persons' (BEA, 2017).

$$FTEE = E\left(\frac{E_H/E}{E_H^{FT}/E^{FT}}\right) = \frac{E_H}{E_H^{FT}/E^{FT}}$$

$$PEIP = FTEE + P$$

Where E_H^{FT} is total hours worked by full-time employees and E^{FT} is the total number of full-time employees. Utilising $FTEE$ and $PEIP$ yields the following measure:

$$LS_3 = \left(\frac{C}{FTEE}\right)\left(\frac{PEIP}{Y}\right)$$

This approach is robust to the aforementioned example whereby a compositional shift in employee hours worked biases estimates of the labour share. This is because using $FTEE$ allows for adjustments to be made in the employee intensive margin through the denominator. However, no adjustments can be made for changes in the hours worked by the self-employed because of the implicit assumption that all proprietors are full-time workers (who work, on average, the same weekly hours as full-time employees). This is because employment is ultimately still expressed in terms of quantities of workers; average hours worked by part-time employees are converted into a quantity of full-time employees.

Therefore, the assumption here is that proprietors, on average, are compensated for their labour in equal proportion to full-time (equivalent) employees[5]. However, at least some proprietors are part-time workers, therefore, it is possible that this measure of the labour share could be upward bias. This is because the equivalent of a full-time employee's wage is being assigned to some part-time self-employed workers; thus inflating the estimate of total proprietor labour income.

On the other hand, we might expect proprietors to possess some type of managerial or human capital skill that employees do not possess, which would, in theory, allow them to command a higher return on their labour (Kreuger, 1999). This is also not captured here, so may downward bias the labour share. Potentially, the net affect of these biases could cancel out; however, this cannot be verified this in any quantitative sense without utilising comprehensive micro-survery data. This motivates the LS_4 approach put forward here.

Approach 4: A novel method for constructing the labour share could be to estimate the counter-factual wages of self-employed individuals using micro-level survey data and incorporating it into the LS_2 approach.[6] This LS_4 approach differs from LS_2 insofar it scales the total proprietor hours worked by an average hourly premium; this computed as follows. Firstly, a standard Mincerian wage equation is constructed for employees:

$$\ln(Ewage) = \hat{\alpha} + \hat{\beta}_1 Edu + \hat{\beta}_2 Age + \hat{\beta}_3 Age^2 + ... + \hat{\delta} X$$

Where $\ln(Ewage)$ is the average hourly wage of employees. Here, Age serves as a proxy for experience and Edu is the number of years of education received by the individual. X is a vector of additional controls (such as ethnicity, gender, marital status). The counter-factual wage of each individual proprietor i is then constructed by fitting the observed self-employed values for education and age (plus other con-

[5] $E \geq FTEE$ by definition; as a corollary $LS_3 \geq LS_1$

[6] To the best of my knowledge I have not seen this approach implemented elsewhere.

trols) into the estimated equation:

$$\ln\left(CFPwage_i\right) = \hat{\alpha} + \hat{\beta}_1 Edu_i + \hat{\beta}_2 Age_i + \hat{\beta}_3 Age_i^2 + ... + \hat{\delta} X_i$$

Adjusting for Jensen's inequality, the log hourly wage estimates for each proprietor i are transformed back into levels. Summing across all counter-factual wage estimate and dividing by the number of proprietors in the survey N_P yields the average counter-factual proprietor wage. Constructing the average employee hourly wage, where N_E is the number of employees, and taking the ratio of the two aforementioned variables gives the following.

$$\frac{\frac{\sum_i^{N_P} CFPwage_i}{N_P}}{\frac{\sum_i^{N_E} Ewage_i}{N_E}} = \frac{AvgCFPwage}{AvgEwage} = AHP$$

Where AHP is the relative average hourly premium (or penalty) to being a proprietor. This represents the difference in hourly wage that an employee receives relative to a proprietor; notably the direction of the difference is not restricted (there could potentially be a penalty in hourly wage for proprietors). The modified LS_4 approach is shown below:

$$LS_4 = \left(\frac{C}{E_H}\right)\left(\frac{E_H + P_H \times AHP}{Y}\right)$$

Ultimately, the assumption made here is that employees and proprietors, who have similar characteristics, on average, receive the same hourly compensation for their labour. This is a less restrictive assumption than all the aforementioned measures. In the special case where there is no average hourly premium to being self-employed $AHP = 1$, this measure nests that of the BEA (approach LS_2). While LS_4 constitutes a significant advantage over the aforementioned measures, one drawback is the implicit assumption of employees and self-employed individuals possessing the same unobservable characteristics; something which is likely to be untrue (Kruger, 1999). For a more robust approach to LS_4 it would be appropriate to adjust for non-random distribution of unobservables by using a Heckman selection model - this attempts to adjust for the decision of individuals to opt in to self-employment.

1.4 Empirical Application

The above measures of the labour share are constructed for the UK economy and are contrast with the official Office for National Statistics (ONS) measure. The approach favoured by the ONS simply allocates mixed proprietor income in the same portion as in the rest of the economy. In other words, the share of labour income is assumed to be the same in the unincorporated sector as it is in the incorporated sector. This measure is equivalent to removing ambiguous self-employed income from a measure of output in the denominator. Where M is mixed proprietor income, the ONS labour share is:

$$LS_{ONS} = \frac{C}{Y - M}$$

Figure 1.1 displays three measures of the UK labour share. The black line is the official ONS measure, which exhibits a clear decline. The blue and red lines represent the LS_1 and LS_2 measures, respectively, and exhibit a less protracted decline.

The official ONS measure displays a clear fall, from approximately 0.68 in the early 1960s to 0.59 in the late 1990s, thereafter it stabilises; nearly a 10 percentage point fall. The LS_1 measure tracks the ONS measure very well until approximately 1975, afterwards the estimates begin to diverge, and since the 2008 recession the average gap between these two measures has widened to approximately 5 percentage points. This gap is reflected in the weak correlation between the percentage of proprietors in the workforce and the share of mixed income in output post-1975 (shown in appendix).

The percentage of the UK workforce that was self-employed between 1959 and 1974 varied between 6.8% and 7.8%. However, after 1975 the share of self-employed people in the workforce rose; in 2016 it was 13.1%. Meanwhile, the share of mixed proprietor income in the economy has remained stable between 5.5% and 7.7% for the sample period.

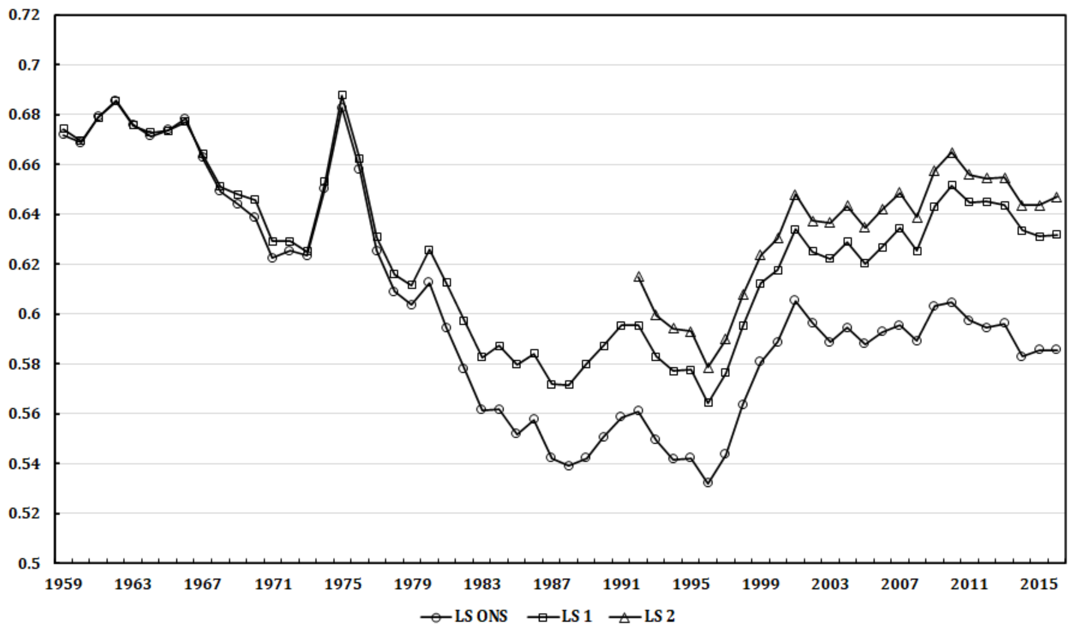

Figure 1.1: UK Labour Share (1959-2016) LS_{ONS} LS_1 LS_2

These trends could potentially be explained by the incentives and costs associated with incorporating businesses. Due to new technology, it is likely that the administrative costs to incorporation have fallen over time. Additionally, there are increased tax incentives for small and medium-sized businesses to incorporate. Moreover, the number of self-employed individuals who work part-time (less than 30 hours per week) has increased notably - see Appendix. It is likely that these factors have contributed to a reduction in the average economic output of a given self-employed proprietorship; thus keeping the share of mixed income in the economy stable, while the share of self-employed workers has risen.

Clearly, the LS_2 measure does not exhibit the same historical scope as the other two approaches (proprietor hours worked data has only been recorded in the Labour Force Survey (LFS) since 1992). Moreover, self-employed hours worked are reported in bands of time, opposed to the exact amount of hours (presumably for reasons of expediency, since it is likely that proprietor hours will vary a great deal more than employee hours with the day-to-day flow of business). To bypass this issue, a simplifying, albeit imperfect, assumption is to suppose that proprietors are normally distributed within the time bands. This means that the mid-point of each band represents the average hours worked by proprietors within that given threshold.[7]

[7]The top hours worked band is open-ended; however, it will be assumed that no individuals

Moreover, as LS_1 imputes self-employed labour income via the headcount ratio of employees to proprietors, the gap between LS_2 and LS_1 represents the intensive margin. On average, self-employed people work longer hours - this is partially due to the fact that a significant portion of employees are part-time workers, while much fewer self-employed people work part time. As a result, both measures have very similar trends but LS_2 is scaled up slightly.

Although it is not possible to extend proprietor hours beyond 1992, a conjecture (based on changing trends in part-time employment relative to full-time employment over the past few decades) is that the gap between LS_2 and LS_1 is likely to have been small in the early 1960s. If this is the case, the LS_2 measure exhibits a smaller decline, stabilising around 0.65 - which would imply only a 3% drop in the labour share. We can also support this intuition by examining the LS_3 approach.

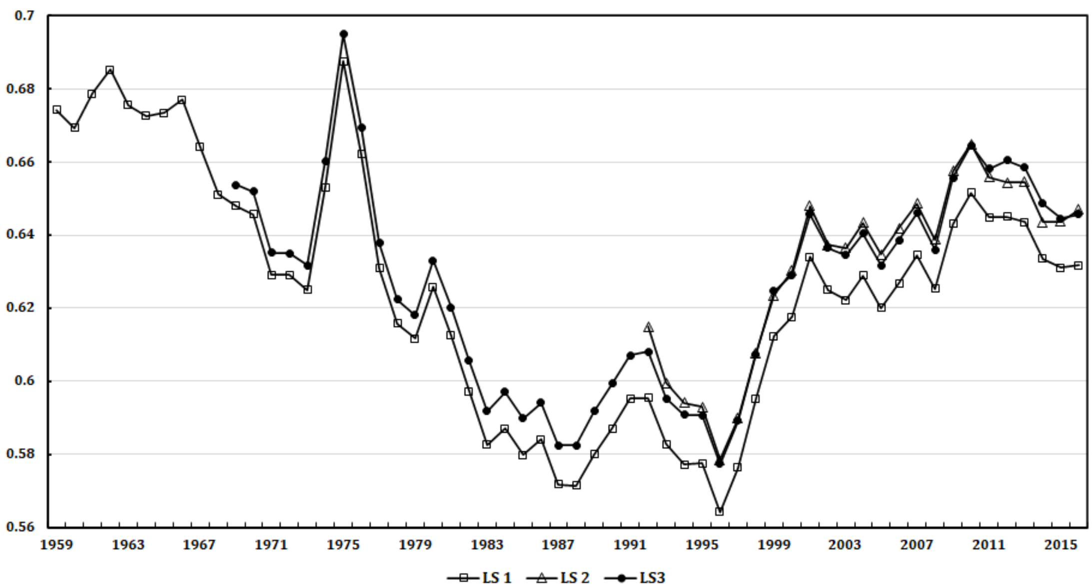

Figure 1.2: UK Labour Share (1959-2016) LS_1 LS_2 LS_3

First, LS_3 measure must be constructed for the UK via the creation of a full-time equivalent employees series. This is a series which is collected by the BEA but not the ONS. Instead, UK household survey data from the Living Costs and Food Survey (LCFS)[8] is used to construct full-time equivalent employees. This is an

exceed the UK legal maximum weekly working hours of 48 hours per week (on average over any given 17 week period).

[8]Known as the Expenditure and Food Survey (EFS) from 2001-2008 and the Family Expenditure Survey (FES) from 1957-2001.

annual cross-sectional study which collects a wide range of household-level data, including information on hours worked. The series is constructed accordingly, the LCFS asks employees their 'normal weekly hours [worked]'. Average normal weekly hours worked for all employees, part-time and full-time, is easily obtained by taking the mean of all responses to this question. However, average normal weekly hours worked for only full-time employees must also be calculated. This is done by separating out full-time employees from part-time employees based on their normal weekly hours worked. Following the previously specified BEA definition, any employees who normally works less than 30 hours a week are dropped from the sample. The mean is then calculated for those remaining in the sample in order to generate the average normal weekly hours worked for full-time employees only. The ratio of these two are then multiplied by the number of employees to give the number of full-time equivalent employees. Persons engaged in production can then be constructed simply by adding the number proprietors in the survey to this measure.

It can be seen in Figure 1.2, the LS_3 measure tracks LS_2 remarkably well for the whole series, with the slight exception of 2009 onward. This could be due to the increase in part-time self-employment after the Great Recession. Therefore, the assumption of all self-employed workers being full-time workers in the LS_3 approach may be less realistic post-Great Recession (consider the rise of the gig economy; Uber, Deliveroo, and other similar platforms). However, the main advantage of the LS_3 measure is that it has greater historical scope for the UK utilising LCFS data back until 1968. Given this, it could be a reasonable to presume that if the LS_2 measure could be extended backwards, it would likely mirror the LS_3 measure closely.

Additionally, LS_3 tracks LS_1, fairly closely, with a slight widening of the gap over time, which reflects the rise in the relative share of part-time employees in the economy. When comparing the LS_1 measure with the LS_3 measure, there is more of an observed decline owing to the fact that the LS_1 measure makes no attempt to adjust for the intensive margin, and therefore underestimates the labour share when the share of part-time employees in the economy rises (as it has done in the UK from the mid 1970s onward).

Figure 1.3 includes the novel approach of LS_4 whereby the average hourly premium to being a self-employed is constructed from LCFS survey data. Counter-factual estimates suggest that proprietors receive a slightly higher hourly compensation for their labour, on average, relative to employees. This appears to be due to compositional effects, for example, self-employed people are overwhelmingly male, older, white and married. These characteristics are associated with higher hourly wages among employees.

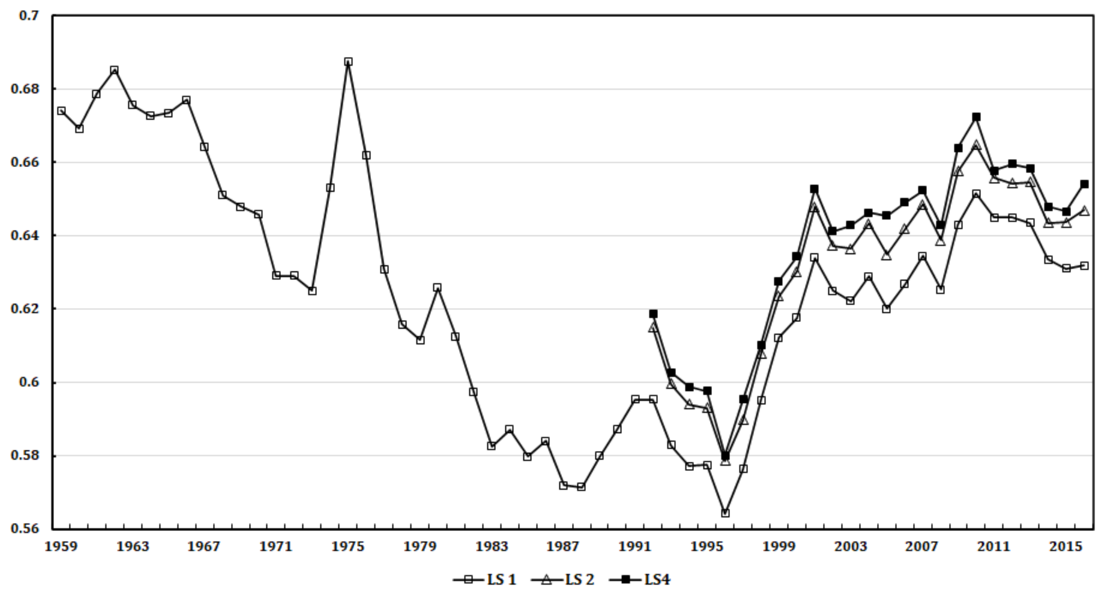

Figure 1.3: UK Labour Share (1959-2016) LS_1 LS_2 LS_4

When inferring proprietors' counter-factual wages through this method, it results in self-employed people having an average hourly premium to their labour (shown in Appendix). This could also perhaps be rationalised economically as proprietors may possess some additional managerial or human capital skills which would allow them to command an above average hourly wage in the labour market if they were to become an employee. As shown in Figure 1.3, due to the small average hourly premium, the LS_4 measure is slightly higher than the LS_2 measure. The gap between the red line and the yellow line represent the extent to which controlling for different characteristics among employees relative to self-employed impacts estimates of labour income. However, as can be seen, for the most part LS_4 closely tracks LS_2, especially before the Great Recession.

However, aside from proprietor income there are other forms of potentially ambiguous income in the economy. There is some debate surrounding how to apportion income from different sources, for example intellectual property, or more broadly put, intangible assets. This is dealt with in the next section.

1.5 Intellectual Property Products

In the context of measuring the labour share, Koh, et al (2018) raise the issue of how to conceptually classify different types of expenditure in the economy, in particular, intellectual property products. They note that IPPs have previously been considered as an intermediate input, but recently have been reclassified by the BEA as investments.

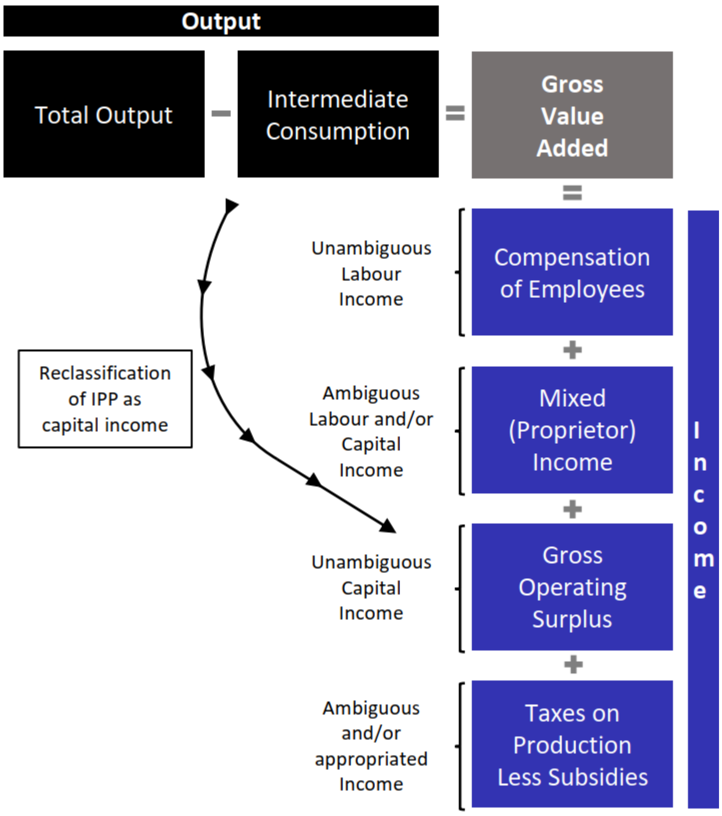

Figure 1.4: Reclassification of IPP in National Accounts

This reclassification is important because the decision to include IPP as investment revises up the value of GVA and revises down the value of intermediate inputs. A portion of intermediate inputs has been reallocated to GVA. This unambiguously increases GVA and increases the value of the denominator when constructing the labour share (from a purely accounting perspective, this lowers the value of the labour share). However, to ensure that the accounting identity in Figure 1.4 holds, any income derived from IPP, which is now included in GVA, must accrue to one of the factors of production. The portion of this reclassified income which is attributed to capital is denoted as χ. The implicit assumption the BEA makes is that all IPP rents accrue to capital $\chi = 1$ (IPP is fully capitalised). Koh, et al (2018) argue that it is this extreme assumption which creates the appearance of a decline in the labour share. The share of IPPs in gross investment in the US has been steadily increasing over the past few decades, by construction under these assumptions, the labour share has fallen.

In the UK system of national accounts IPPs (non-physical assets, formerly known as intangibles) are defined as software, research and development (R&D), artistic originals and mineral exploration. Research and development expenditure (the largest component of IPPs in the UK) has only been capitalised in ONS national account since 2014. This is in keeping with ESA and international best practice, and reflects similar accounting changes which occurred in the US for the BEA.

In the UK, the share of IPPs in aggregate gross fixed capital formation rose from approximately 4% in 1970 to 20% in 2015. The growing share of IPP emphasises their increasing importance the UK economy. However, by assuming that all IPP rents by default accrue to capital income could be misleading. Koh, et al (2018) argue that R&D workers often obtain incentive stock options as part of their compensation. Additionally, the R&D developed by workers may allow firms to be more competitive or induce higher productivity which raises workers marginal product and their wages; therefore, part of the returns to R&D could accrue to labour. Moreover, Koh, et al (2018) mentions that proprietors may spend time developing advertisement or branding their business, which could be considered as IPP, and which can generate returns for their enterprise (part of these returns ought be considered labour income).

The same approach that Koh, et al (2018) apply to the US data is applied to the UK. Below is the counter-factual case whereby IPP is removed from GVA and reclassified as an intermediate input (the capitalisation of IPP is undone). Specifically:

$$LS_{ONS}^{IPP} = \frac{C}{Y - M - IPP}$$

Where IPP represents gross investment in intellectual property products.

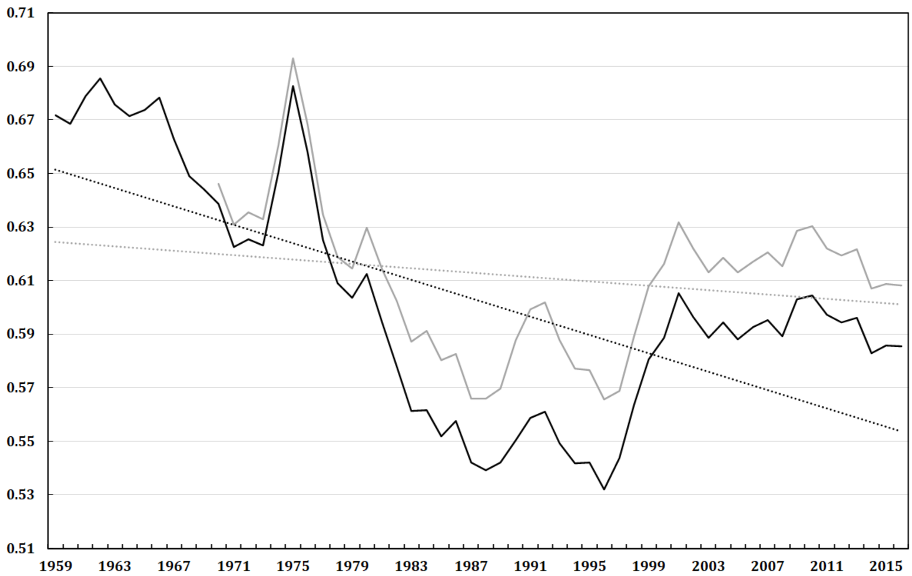

Figure 1.5: UK Labour Share (1959-2016) LS_{ONS} LS_{ONS}^{IPP}

The implicit assumption of undoing the capitalisation of IPP is that IPP rents are apportioned between capital and labour in the same ratio as the rest of the economy (thus treating them in the same manner as mixed proprietor income). As shown in Figure 1.5, the removal of IPPs from output dampens the decline in the labour share. This is reflected in the divergence between the two measures over time and the two fitted trend lines with differing gradients. From an accounting perspective this is completely logical given that the share of gross IPP expenditure in investment has been increasing steadily in the UK since 1970.

However, Koh, et al (2018) do not focus explicitly on the issue of proprietor income. Changing trends in self-employment are significant and should not be downplayed. The approach below combines the LS_3 measure with the IPP approach of Koh, et al (2018).

Firstly, the self-employed labour income is imputed, and then half of the IPP rents are attributed to labour following the benchmark assumption of McGrattan and Prescott (2014) $\chi = 0.5$. The combined effect of these two approach can be seen for the UK in Figure 1.6, specifically:

$$LS_3^{IPP} = \left(\frac{C + (1-\chi)\,IPP}{Y}\right)\left(\frac{PEIP}{FTEE}\right)$$

However, there is an additional assumption here beyond the standard LS_3 approach. The fraction of IPP rents allocated to full-time employees as assumed to be in the same proportion as self-employed persons. It can be seen that, once adjustments are made for IPP rents and proprietors, the labour share in the UK exhibits essentially no decline from 1970 until 2016, remaining on average at approximately 0.64, reflected by the dashed trend line in blue on Figure 1.6. The black line represents the LS_{ONS} measure where no additional adjustment are made; it is the same as the series in the previous figures, but truncated at 1970 (simply for convenience because data for IPP investments prior to 1970 is not available).

Figure 1.6: UK Labour Share (1970-2016) LS_{ONS} LS_3^{IPP}

Comparing the trend lines, it can be shown that the ONS measure shows a substantial decline in the labour share over the period shown, partially due to the increasing share of investments in IPPs, and partially due to the changing structure of self-employment vis-a-vis employment.

1.6 Conclusion

When empirically estimating the labour share, researchers inevitably have to make qualitative assumptions regarding the classification and apportioning of incomes. While such qualitative assumptions cannot be evaluated directly in any quantitative sense, through careful and reasonable argumentation it is possible to arrive at a conceptually cleaner estimates of the labour share. Given that the labour share plays a fundamental role as a parameter in macroeconomic models and motivates a growing body of literature, it is no longer justifiable to brush aside such broad qualitative assumptions when they clearly have profound quantitative impacts. The output of structural models, and any economic insights we derive from them are only valid insofar that analytical concepts actually correspond to what we suppose they do; as opposed to what we would like them to - usually for sake the expediency or tractability.

This paper covers some of these key issues and examines the role of proprietor income in close detail. It uses survey data from the LCFS to provide a novel approach to estimating the labour share in the UK in a more conceptually accurate way. Alongside this it also considers the role of intellectual property product classification and how the rise in the share of investment of intangibles has impacted the labour share in the UK. This paper demonstrates that the careful treatment of proprietor income and intellectual property products suggests there has been little to no decline in the labour share in the UK. This is in line with a growing body of literature, such as Bridgeman (2018) and Koh, et al (2018), which questions the credibility of the secular decline of the labour share, which has gathered significant attention in recent years.

1.7 Appendix

This appendix contains some supporting data for the labour share measures constructed in section 1.4.

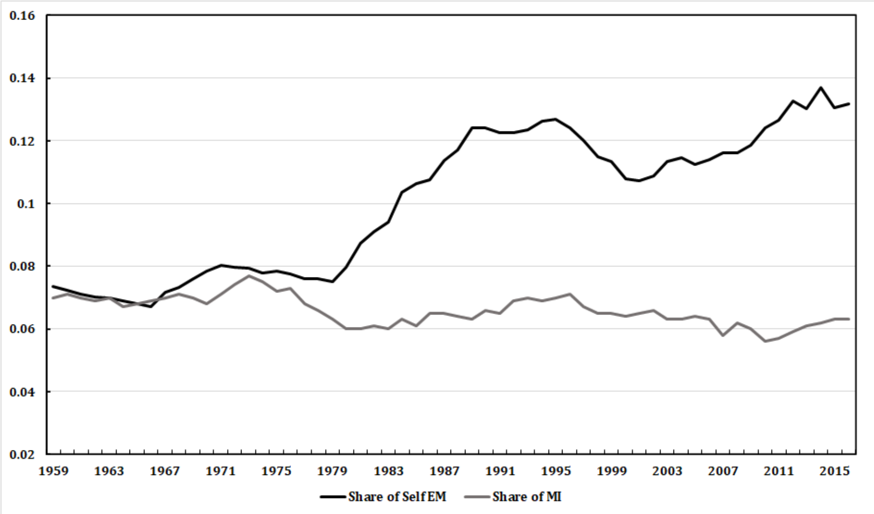

Figure 1.7: UK Propietors and Mixed Income (1959-2016) $P/(E+P)$ M/Y

Figure 1.7 shows that the share of mixed proprietor income in the economy has remained stable over time. However, the share of the total workforce who are self-employed has increased since around the mid-1970s. This suggests that the average economic output of a proprietorship has been falling since around 1975. This explains the divergence between LS_{ONS} and LS_1 shown in Figure 1.1.

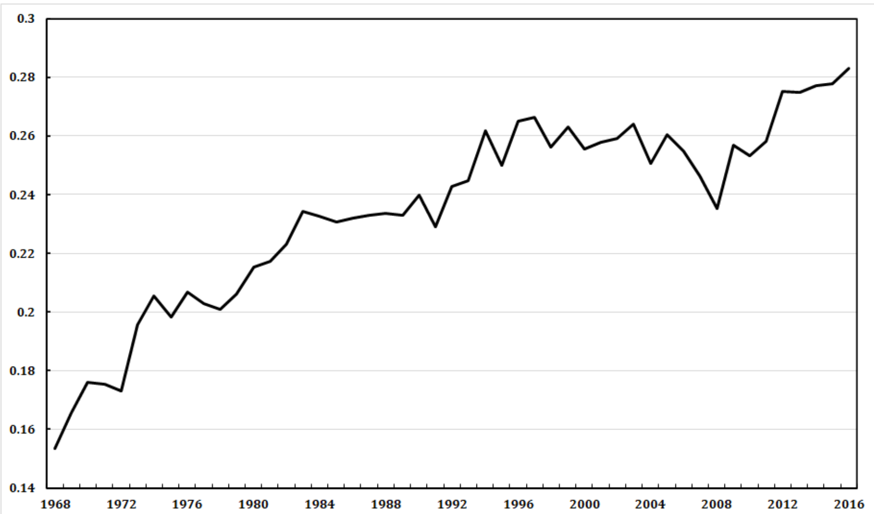

Figure 1.8: LCFS: Share of UK Employees Working Part-Time (1968-2016)

From Figure 1.8 data from the LCFS shows that the percentage of employees who work part-time has been growing steadily, increasing from approximately 15% in 1968 to 28% in 2016. This emphasises the importance of controlling for the employee intensive margin when imputing self-employed labour income - exemplified by the divergence in the LS_1 and LS_3 measures in Figure 1.2. It also provides empirical support for using full-time equivalent employees over a simple headcount measure when constructing the labour share.

2 Rising Firm Markups and Optimal Capital Taxation

Abstract

The Chamley-Judd result states that the taxation of capital income should converge to zero in the long run. This paper develops a simple model of firm entry and optimal taxation which demonstrates that, when a fiscal authority cannot differentiate between two sources of capital income, a positive capital tax may be optimal in the steady state. In this setting it is assumed there are two types of capital stock; firm capital and physical capital. The government would like to tax windfall gains as lump-sum profits derived from firms do not impact the decisions of agents at the margin. However, the government wishes to avoid the taxation of physical capital because they do not want to discourage productive future investments. These two opposing objectives must be reconciled with a single flat-rate tax on capital τ^K. Moreover, recent empirical studies suggest that average firm-level markups have increased substantially over the past two decades; a direct implication is a rise in the share of aggregate profits. In a structural framework, for an empirically plausible set of parameters, it is shown that when markups μ or firm-level decreasing returns $(1-\nu)$ are sufficiently high, aggregate profits in equilibrium become large enough such that it is optimal to positively tax capital income in the long run.

2.1 Introduction

Recent empirical studies suggest that average firm-level markups have increased substantially over the past two decades (De Loecker and Eeckhout, 2017). Rising average firm markups increase prices and reduce output; a direct implication is a rise in the share of aggregate profits (Autor, et al, 2017). Theoretically, the existence of aggregate profits indicates the presence of rent in the economy and a deviation away from the first-best. Under a structural framework, the taxation of pure rental income is non-distortionary, and therefore it is efficient to tax windfall profits at 100% for the purposes of financing public expenditures; equivalent to a lump-sum tax because it does not impact the decisions of agents at the margin (Guo and Lansing, 1999). While the theoretical appeal of pure rent taxation is widely acknowledged and understood, in practical terms it is not so straightforward (Stiglitz, 1987). A key reason for this is identification issues. Total capital income is potentially comprised of many component sources, such as; rent, corporate profits, net interest and a fraction of proprietor income; not all of which is directly observable. Distilling aggregate profits from this bundle requires qualitative assumptions to be made regarding the classification and apportioning incomes - assumptions which cannot be evaluated directly.

Given the difficulties that a government may face regarding the classification of capital income for the purposes of unmitigated taxation of windfall gains, this paper imposes the modelling notion that the fiscal authority cannot differentiate between different sources of capital income, namely; profits derived from firm capital and interest derived from physical capital. The inclusion of firm capital following Bilbiie, et al (2012) introduces monopolistic competition, markups and rental income, in the form of aggregate profits. This subsequently motivates the research question of this paper: is it efficient to tax capital in the long run when the government possess only a single fiscal tool with which to tax two differing types of capital income?

The results of Chamley (1986) and Judd (1985) both state that it is optimal to tax capital at zero percent in the long run. Their renowned result emphasises the highly distortionary nature of capital taxation and suggests that it does not appear to serve as a useful tool in equilibrium for either efficiency or redistributive purposes.

This paper revisits the Chamley-Judd result in the context of rising firm markups. Estimates of average markups weighted by firm-level sales for the US economy are reproduced below from De Loecker and Eeckhout (2017).

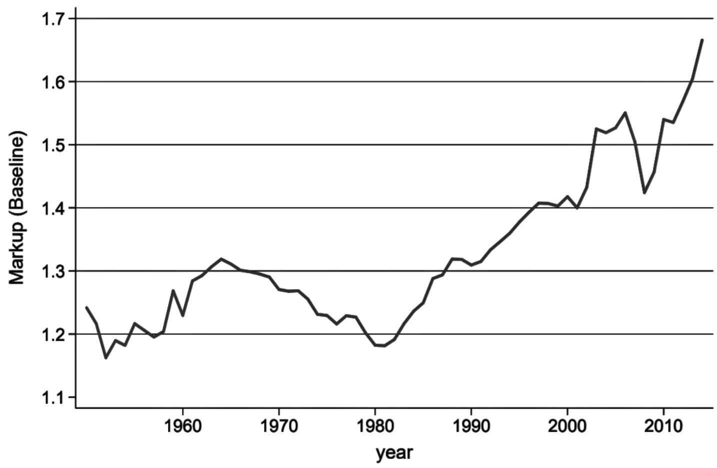

Figure 2.1: Average US Firm Markups (De Loecker and Eeckhout, 2017)

It can clearly be seen that since 1980 average markups have shown a secular upward trend; this directly affects the distribution of factor income. Ceteris paribus, markups are positively linked to aggregate profits. This has clear implication for welfare and resource allocation in a general equilibrium framework. This paper examines the impacts of rising markups on the optimal taxation of capital (and labour) income.

A single flat-rate capital tax which must be used to raise revenue from firm capital (profits) and physical capital (interest) establishes two opposing fiscal objectives Firstly, there is the 'profit effect': the government would like to tax windfall gains at 100% since lump-sum profits derived from firms do not impact the decisions of agents at the margin. Secondly, there is the 'under-investment effect': the government would like to tax physical capital at 0% since they do not wish to discourage productive future investments in physical capital.[1]

These two effects pull in opposite directions but must be reconciled with a single capital tax rate in equilibrium. The paper evaluates the direct impact of markups (and returns to scale) on the optimal taxation of capital (and labour) income in the long run. When markups rise, aggregate profits in equilibrium increase. Beyond a

[1]These two 'effect' terms are taken from Gao and Lasing (1999)

certain threshold it becomes efficient to positively tax capital in equilibrium as the 'profit effect' is dominates the 'under-investment effect'. Numerical simulations of steady state expressions yield boundary conditions for when the long run capital tax flips from zero to positive. However, if markups are low the 'under-investment effect' becomes large and the original Chamley-Judd result is recovered; capital is taxed at zero percent and labour bears the full burden of taxation.

The remainder of this paper is structured as follows. Section 2 reviews the literature. Section 3 sets up the model in the context of the aforementioned literature. Section 4 solves for steady states, the optimal capital and labour tax rates, and discusses the results. Finally, section 5 concludes. The appendix derives the conditions for the optimal tax rates in full.

2.2 Literature Review

This paper brings together two strands of literature; optimal taxation and firm dynamics. The most acclaimed and contentious finding in the literature of optimal taxation is the Chamley-Judd result. Judd (1985) questions whether capital taxation is optimal for redistributive purposes in a two-agent economy with workers and capitalists. The former supplies labour and derives income from wages and government transfers (funded by taxes on capital), the latter only invests and earns income from capital. The conclusion is that if both workers and capitalists have the same rate of time preference, the optimal redistributive tax on capital income, from the point of view of either agent, is zero in the steady state. Chamley (1986) asks whether it is efficient to tax capital in an infinite horizon representative agent framework where the government must finance an exogenous sequence of purchases with access to unconstrained debt. The outcome is that, the taxation of capital income converges to zero in the long run because any small change in its price will generate large sub-optimal responses in steady state capital accumulation. This is because the return on capital in the steady state is pinned down by the discount factor; this implies that the supply of capital is perfectly elastic with respect to its price in the long run.

Moreover, the Chamley-Judd result has been shown to hold even when some assumptions have been relaxed. For example, Atkenson, et al (1999) confirms that it holds in an economy with heterogeneous consumers, who differ in terms of consumption and labour supply, and can be taxed at different rates. Additionally, Atkenson, et al (1999) also shows that an optimal zero capital tax is consistent with an overlapping generations framework, provided the utility function exhibits homothetic preferences and is additively separable. Furthermore, Judd (1999) allows for production functions which contain public goods and non-stationarity. This set up still generates the result of an optimal zero capital tax in the long run, even if the capital stock does not converge to a unique steady state or balanced growth path.

However, the Chamley-Judd result is neither universally commended or straightforward in its interpretation. Others have challenged its robustness and demonstrate that it does not hold when differing assumptions are relaxed. For example, Jones, et al (1993) construct a Ramsey plan where government revenues g_t contribute directly to investment in capital stock, such as infrastructure or public good spending. They abstract from labour entirely to simply focus on the capital accumulation process. Investments from private agents and the government enter symmetrically into the formation of capital stock which depreciates at a single fixed rate $k_{t+1} = (1 - \delta) k_t + f(i_t, g_t)$. Under this framework, pure profits are generated due to presence of the government investment and the limiting capital tax converges to a strictly positive value in the steady state.

Moreover, Aiyagari (1995) establishes that in an economy with ex-post heterogeneity due to uninsured idiosyncratic risk, the steady state optimal tax rate on capital is positive. This is because incomplete markets create a motive for precautionary savings, in turn this leads to excessive capital accumulation as a means of self-insurance which places downward pressure on the rental rate. A positive capital tax can mitigate this ‘over-accumulation’ by pushing up the net-tax rental rate and bringing it closer in line with the unconstrained first-best rental rate, which is pinned down by the discount factor.

Correia (1996) illustrates that the Chamley-Judd result is contingent on the government’s ability to tax all factors of production. For example, given a production function homogeneous of degree one with three inputs (capital, labour and some additional factor) where restrictions are imposed on the taxation of the additional factor, the tax rate on capital income will generally be non-zero in the steady state. The sign of the tax will depend on whether the two remaining factors are complements (positive) or substitutes (negative) to capital. Furthermore, Guo and Lansing (1999) show that the introduction of firm monopoly power in product markets creates the possibility of a non-zero optimal capital tax in the long run, the sign and magnitude of which depends crucially on the degree of monopoly power and the extent to which monopoly profits can be independently taxed.

In an infinite horizon representative agent framework Abel (2007) demonstrates that, in complete contrast to the Chamley-Judd result, a constant positive capital tax and a zero labour tax can optimal for financing government purchases. However, this result is entirely due to the inclusion of capital allowances; agents can deduct capital expenditures from taxable capital income. An appropriate fixed tax on capital with contemporaneous expenditure deductions does not impact the optimal path of capital accumulation. As such, it can be used to finance government purchases in a non-distortionary way; this accounting practice essentially creates lump-sum taxation, which enables a first-best outcome.

Carlos, et al (2009) construct an overlapping generations model with uninsurable idiosyncratic risk and heterogeneity in labour productivity. They find that ex-ante (prior to ability being realised) welfare is higher with positive labour and capital taxation. Due to the life-cycle structure of the model, relative to an infinite horizon structure, capital taxation is less distortionary in the limit. This amplifies the mechanism in Aiyagari (1995) and generates significantly positive optimal capital taxation. This suggests that capital taxation is an efficient tool for redistributive purposes in the face of incomplete markets.

Straub and Werning (2015) revisit the Chamley-Judd result and demonstrate that it does not necessarily hold in either of the original frameworks if some specifications are altered slightly. Firstly, in Judd (1985), if capitalist exhibit an intertemporal elasticity of substitution (IES) below unity, the capital tax does not approach zero, but instead converges towards a positive value. This is because anticipated increases in future capital taxes raise the net-rental rate and induces higher levels of investment today. This generates a higher capital stock over the transition period which is optimally exploited with an increasing tax on capital, which eventually convergence to a stable steady state. Secondly, in Chamley (1986), if the IES is below unity and the initial stock of government debt is sufficiently large, then for any starting value K_0, the optimal solution will be to tax capital income at 100% indefinitely at the upper bound. This result can even occur when the existing stock of debt is slightly below the feasible maximum tax burden which agents can finance.

In response, Chari, et al (2016) suggests that incomplete taxation drives the findings of Straub and Werning (2015) and that significant gains to welfare, as well as the original zero capital tax result, can be recovered if a consumption tax is introduced. They argue that any large existing stock of government debt can instead be dealt with by a sizable tax on consumption immediately after the initial period. This is because a consumption tax is not subject to a natural upper bound of 100% and therefore the whole term structure with the capital tax rate binding indefinitely at the upper bound will not be exploited. In this framework, after the first few periods, labour and consumption are taxed at uniform rates and the capital tax remains at zero for all subsequent periods.

Another strand of literature, which intersects with the theme of this paper, is concerned with the role of the firm in the macroeconomy. The seminal paper of Bilbiie, et al (2012) models firm dynamics in structural framework and finds that firm markups play a key role in matching appropriate cross-correlations and second moments of macroeconomic aggregates. This has subsequently motivated studies which attempt to empirically gauge and understand the underlying causes for shifting markups.

For example, De Loecker and Eeckhout (2017) utilise firm-level panel data to construct average markups for the US economy. Markups are constructed from individual producers as a wedge between input expenditure shares and their respective output elasticities - which are obtained by estimating firm production functions. Individual firm markups are then weighted by firm sales for the US economy to obtain a measure of average markups. They find a substantial increase in markups from 1.18 in 1980 to 1.67 in 2014 and argue that this can account for perceived secular trends in macroeconomics, such a declining labour share and output growth slowdown.

Hall (2018) takes different approach by calculating marginal cost as the ratio of the adjusted expenditure on inputs to the adjusted change in output; the findings are consistent with De Loecker and Eeckhout (2017) insofar that both estimate rising markups over approximately the past two decades. However, they differ in terms of respective magnitudes; Hall (2018) estimates a lower markup ratio in 2015 of 1.38.

In addition, Barkai (2018) examines the distribution of factor income and finds that labour and capital shares have been declining in the US, while the share in pure profits, primarily driven by rising markups, has been increasing. Extending estimates of capital costs by industry, the author finds that the share of profits in gross value added has been steadily rising since the early 1980s. These findings provide a strong motivation for studying the role of markups in the context of general equilibrium optimal taxation.

2.3 Model

A standard representative agent model, in the framework of Chamley (1986) is augmented to include firm entry and imperfect taxation. One small alteration from this original set up is that the government does not have access to unconstrained borrowing. However, as this paper is concerned with the impact of firms on long run optimal taxation, this difference is trivial as government borrowing only impacts the transition path to the steady state. The model in this paper incorporates elements of Corria (1996) insofar that there exists some element of incomplete taxation. However, this due to the government being unable to discriminate between income from firm capital and physical capital, as opposed to the outright restriction of taxation of a third factor. Moreover, in Corria (1996) the third factor is remunerated at the competitive rate, whereas here the introduction of firm capital, with monopolistic competition moves, away from the framework of perfectly competitive markets. Additionally, the model developed here adopts a feature found in Guo and Lasing (1999) where the fiscal authority cannot distinguish between profits and other capital income for the purposes of taxation. However, this paper differs insofar that Guo and Lasing (1999) do not model the role of firms explicitly and do not directly explore the impact markups and firm-level returns to scale together on long run optimal taxation.

The model is kept simple in order to derive tractable analytical solutions (shown in Appendix). The consumer side of the problem begins with a simple standard log-log utility function. The infinitely lived representative agent derives utility from consumption C_t (which comprises of a basket of differentiated goods) and leisure $(1 - L_t)$ (where the total number of hours available is normalised to unity). The agent maximises lifetime utility subject to the constraints (2.1) through (2.4) where $\beta \in (0, 1)$ is the subjective discount factor and $\psi \in (0, 1)$ is the consumption-leisure weight:

$$\sum_{t=0}^{\infty} \beta^t \left\{ \psi \ln C_t + (1 - \psi) \ln (1 - L_t) \right\}$$

In any given period the consumer derives utility from a collection of goods which are aggregated into a single basket of goods C_t. The CES aggregator is as follows, where each firm produces one variety ε and γ is the elasticity of substitution between the differentiated goods.

$$C_t = \left[\int_{\varepsilon}^{n} c_t(\varepsilon)^{\frac{\gamma-1}{\gamma}} d\varepsilon \right]^{\frac{\gamma}{\gamma-1}}$$

The markup is independent of varieties produced and purely a function of the elasticity of substitution $\mu = \frac{\gamma}{\gamma-1}$. The representative agent faces the following constraints and chooses C_t, K_{t+1}, and L_t. Firm entry remains exogenous in the sense that a constant fraction of transfer income is invested into firm capital each period, therefore N_{t+1} is not a direct choice variables for the agent in the maximisation problem.

$$Y_t = C_t + I_t + G_t + T_t \tag{2.1}$$

$$G_t + T_t = w_t L_t \tau_t^l + (r_t K_t + \pi_t N_t)\,\tau_t^k \tag{2.2}$$

$$K_{t+1} = (1 - \delta_k)\,K_t + I_t \tag{2.3}$$

$$Y_t = w_t L_t + r_t K_t + \pi_t N_t \tag{2.4}$$

$$N_{t+1} = (1 - \delta_n)\,N_t + E_t \tag{2.5}$$

$$\phi T_t = E_t \tag{2.6}$$

The government raises revenue each period by taxing labour and capital in order to fund government purchases G_t and transfer payments T_t. As can be seen from equation (2.2), the capital tax rate cannot discriminate between income derived from physical capital and income derived from firm capital. Each period the government raises enough revenue to finance the exogenous sequence of purchases which does not contribute to utility and constitutes a fixed fraction of output $\frac{G_t}{Y_t} = \theta$. They also engage in taxation to fund transfers to households, a portion ϕ of which is then invested in new firms; as shown in equation (2.6); the firm entry decision is not modelled explicitly.

The reason for this is two-fold. Firstly, for simplicity, in order to derive tractable steady state expressions for tax thresholds. Secondly, keeping firm entry exogenous allows emphasis to be placed on the lump-sum aspect of profit derived from firm capital - from this a sharp and clear distinction can be drawn between the 'profit effect' and 'under-investment effect'.[2]. The first order conditions for the consumer problem are as follows:

[2]Without markups each factor gets paid its marginal product. At the firm-level there are decreasing returns $\nu < 1$. Therefore, when $\mu = 1$ (perfect substitutability $\gamma = \infty$) the aggregate income generated by firms is $(1 - \nu)\,Y$. Rent derived from profits occurs when firms have some degree of monopolistic power and factors are not renumerated at their marginal products.

$$\frac{\psi}{C_t} - \lambda_t = 0 \tag{2.7}$$

$$-\frac{(1-\psi)}{(1-L_t)} + \lambda_t w_t \left(\left(1-\tau_t^l\right) + \left(\frac{\mu}{\upsilon} - 1\right)\left(1-\tau_t^k\right)\right) = 0 \tag{2.8}$$

$$-\lambda_t + \beta\lambda_{t+1}\left(1 - \delta_k + r_{t+1}\frac{\mu}{\upsilon}\left(1-\tau_{t+1}^k\right)\right) = 0 \tag{2.9}$$

Combining equations (2.7) and (2.8) gives the labour-leisure optimality condition while substituting equations (2.8) into (2.9) yields the consumption-Euler equation (equilibrium conditions are detailed in Table 2.1 below). The capital tax appears in the labour first order condition (2.8) because the mark up redistributes some income from labour to firms due to monopoly power; as the capital tax cannot discriminate between firm capital and physical capital, this additional wedge is driven into the first order condition for (2.8).

The aggregate production technology is as follows in equation (2.10) where $\alpha \in (0,1)$ is the physical capital output elasticity, $\omega \in (0,1)$ is the labour output elasticity:[3]

$$Y_t = N_t^{1-\nu} K_t^{\alpha} L_t^{\omega} \tag{2.10}$$

However, due to the existence of decreasing returns to scale $\nu < 1$, the firm-level production technology differs from the aggregate production technology. Dividing through both sides by N_t, where lower case letters represent firm-level values, the firm level production function is $y_t = k_t^{\alpha} l_t^{\omega}$. The firm's problem involves choosing optimal capital and labour inputs for profit maximisation. First order conditions for the monopolistically competitive firm follow as standard:

$$r_t = \frac{\alpha}{\mu}\frac{y_t}{k_t} \tag{2.11}$$

$$w_t = \frac{\omega}{\mu}\frac{y_t}{l_t} \tag{2.12}$$

[3] $\nu = \alpha + \omega$ represents firm-level returns to scale. $\nu = 1$ implies constant returns; firms play no explicit role in production. $\nu < 1$ implies decreasing returns; if a given quantity of capital and labour input were to be spread over a larger number of firms it would increase output.

Firm-level profits are obtained residually by rearranging equation (2.4):

$$\pi_t = \left(1 - \frac{\upsilon}{\mu}\right) y_t \tag{2.13}$$

Collecting equations (2.1) through (2.13), the equilibrium conditions for the economy are detailed in Table 2.1. As can be seen from the government budget constraint the fiscal authority must raise a fraction of output to finance a stream of exogenous purchases and transfers, using only direct flat taxes on capital and labour. However, as there are two classes of capital stock, but only a single capital tax with which to raise revenue from these sources, two opposing fiscal objectives which must be reconciled with only a single fiscal tool.

Table 2.1: Equilibrium Conditions

Model Equations	
Labour Supply	$\frac{(1-\psi)C_t}{(1-L_t)\psi\left(\left(1-\tau_t^l\right)+\left(\frac{\mu}{\upsilon}-1\right)\left(1-\tau_t^k\right)\right)} = w_t$
Labour Demand	$w_t = \frac{\omega}{\mu}\frac{Y_t}{L_t}$
Physical Capital Supply	$\frac{C_{t+1}}{C_t} = \beta\left(1 - \delta_k + r_{t+1}\frac{\mu}{\upsilon}\left(1 - \tau_{t+1}^k\right)\right)$
Physical Capital Demand	$r_t = \frac{\alpha}{\mu}\frac{Y_t}{K_t}$
Physical Capital Law of Motion	$K_{t+1} = (1 - \delta_K) K_t + I_t$
Firm Capital Law of Motion	$N_{t+1} = (1 - \delta_N) N_t + E_t$
Entry (Firm Capital Investment)	$\phi T_t = E_t$
Production Function	$Y_t = N^{1-\nu} K^{\alpha} L^{\omega}$
Government Budget	$G_t + T_t = w_t L_t \tau_t^L + (r_t K_t + \pi_t N_t)\tau_t^K$
Income Accounting Identity	$Y_t = C_t + I_t + G_t + T_t$
Firm Level Profit	$\pi_t = \left(1 - \frac{\upsilon}{\mu}\right)\frac{Y_t}{N_t}$

The government would like to tax firm profits at confiscatory levels due to the, as previously highlighted, ‘profit effect’. However, taxation of physical capital is highly distortionary, as demonstrated by the Chamley-Judd result; a positive tax on capital income will create an 'under-investment effect'. The trade-off is between these two effects. Solving for the steady states and optimal tax rates is as follows.

2.4 Solution and Results

Dropping time subscripts for endogenous variables and rearranging equations yields the steady state expressions as functions of parameters; this is shown in Table 2.2. Note that the flat-rate capital and labour taxes are treated as exogenous up until this point, their optimal value has not yet been determined. While it is possible to make both tax rates endogenous for all t via an auxiliary maximisation problem (Ramsey plan approach; the government chooses tax rates which maximise lifetime utility subject to the households optimality conditions and constraints), this paper focuses instead on long run equilibrium outcomes. This allows for greater parameter sensitivity analysis and for optimal tax threshold expressions (see Appendix). Thus, endogenous short run tax dynamics are put to one side, in favour of a deeper evaluation of optimal tax rates in the long run. The approach put forward here is analogous to the 'golden rule' for the savings rate in the Solow model; it ensures that the long run optimal tax rates are found subject to the other parameters remaining constant; however, this approach is general enough that one could potentially examine the direct impact of any given parameter of interest on welfare maximising steady state allocations.

The long run optimal rates of taxation of capital and labour income can be obtained by maximising steady state utility. Substituting in the relevant expressions from Table 2.1 for labour and output, and rearranging, it can be seen that steady state consumption and steady state labour are both functions of the parameter set (including the tax rates; which are yet to be determined:

Table 2.2: Steady State Expressions

Steady States	
Rental Rate	$r = \left[\frac{(1-\beta+\beta\delta_k)\nu}{\beta(1-\tau^k)\mu}\right]$
Labour	$L = \left[\frac{(1-\psi)\mu}{\psi\omega\left((1-\tau^l)+\left(\frac{\mu}{\nu}-1\right)(1-\tau^k)\right)}\left[1-\frac{\delta_k\alpha\beta(1-\tau^k)}{\nu(1-\beta+\beta\delta_k)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right]+1\right]^{-1}$
Output	$Y = \left[\left(\frac{\omega}{\mu}\tau^l+\left(1-\frac{\omega}{\mu}\right)\tau^k-\theta\right)\frac{\phi}{\delta_n}\right]^{\frac{1-\nu}{\omega}}\left[\frac{\alpha\beta(1-\tau^k)}{\nu(1-\beta+\beta\delta_k)}\right]^{\frac{\alpha}{\omega}}L$
Physical Capital	$K = \left[\frac{\alpha\beta(1-\tau^k)}{\nu(1-\beta+\beta\delta_k)}\right]Y$
Investment	$I = \left[\frac{\delta_k\alpha\beta(1-\tau^k)}{\nu(1-\beta+\beta\delta_k)}\right]Y$
Consumption	$C = \left[1-\frac{\delta_k\alpha\beta(1-\tau^k)}{\nu(1-\beta+\beta\delta_k)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right]Y$
Firm Capital	$N = \left[\left(\frac{\omega}{\mu}\tau^l+\left(1-\frac{\omega}{\mu}\right)\tau^k-\theta\right)\frac{\phi}{\delta_n}\right]Y$
Firm Entry	$E = \left[\frac{\omega}{\mu}\tau^l+\left(1-\frac{\omega}{\mu}\right)\tau^k-\theta\right]\phi Y$
Firm Profit	$\pi = \left(1-\frac{\nu}{\mu}\right)\left[\left(\frac{\omega}{\mu}\tau^l+\left(1-\frac{\omega}{\mu}\right)\tau^k-\theta\right)\frac{\phi}{\delta_n}\right]^{-1}$
Wage Rate	$w = \frac{\omega}{\mu}\left[\left(\frac{\omega}{\mu}\tau^l+\left(1-\frac{\omega}{\mu}\right)\tau^k-\theta\right)\frac{\phi}{\delta_n}\right]^{\frac{1-\nu}{\omega}}\left[\frac{\alpha\beta(1-\tau^k)}{\nu(1-\beta+\beta\delta_k)}\right]^{\frac{\alpha}{\omega}}$

$$L = \frac{\psi\omega\left(\left(1-\tau^l\right)+\left(\frac{\mu}{\nu}-1\right)\left(1-\tau^k\right)\right)}{(1-\psi)\,\mu\left(1-\frac{\delta_k\alpha\beta(1-\tau^k)}{\nu(1-\beta+\beta\delta_k)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right)+\psi\omega\left((1-\tau^l)+\left(\frac{\mu}{\nu}-1\right)(1-\tau^k)\right)} \tag{2.14}$$

$$C = \frac{\left(1-\frac{\delta_k\alpha\beta(1-\tau^k)}{\nu(1-\beta+\beta\delta_k)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right)\left(\left(\frac{\omega}{\mu}\tau^l+\left(1-\frac{\omega}{\mu}\right)\tau^k-\theta\right)\frac{\phi}{\delta_n}\right)^{\frac{1-\nu}{\omega}}\left(\frac{\alpha\beta(1-\tau^k)}{\nu(1-\beta+\beta\delta_k)}\right)^{\frac{\alpha}{\omega}}}{\left(\frac{(1-\psi)\mu}{\psi\omega\left((1-\tau^l)+\left(\frac{\mu}{\nu}-1\right)(1-\tau^k)\right)}\left(1-\frac{\delta_k\alpha\beta(1-\tau^k)}{\nu(1-\beta+\beta\delta_k)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right)+1\right)} \tag{2.15}$$

Note that from the numerator in equation (2.15), that positive taxation is required to finance the exogenous sequence of purchases and guarantee the existence of a viable steady state; if both tax rates were to be set to zero, it follows clearly that $\left(\frac{\omega}{\mu}\tau^l+\left(1-\frac{\omega}{\mu}\right)\tau^k-\theta\right)<0$ and therefore $C<0$ (implying negative consumption). As steady state consumption cannot be negative, the first-best scenario of zero distortionary taxes (or purely lump-sum taxes) must be ruled out. The constrained second-best must be found to ensure the existence of equilibrium; this is approached in the following way. Substitute the analytical expressions from (2.14) and (2.15) into the steady state utility function U (dropping time subscripts). Long run welfare

U is now formulated as an explicit function of the parameter set and tax rates:

$$U = \psi \ln C + (1 - \psi) \ln (1 - L)$$
$$U = f\left(\alpha, \beta, \delta_k, \delta_n, \theta, \mu, \phi, \psi, \omega, \tau^l, \tau^k\right) \tag{2.16}$$

Amusing that all parameters are fixed constants, the steady state function for welfare can be reduced to two arguments of interest $U = f\left(\tau^l, \tau^k\right)$. In order to maximise long run welfare, the partial derivatives of (2.16) must be taken with respect to the labour tax rate τ^l and the capital tax rate τ^k. This creates two first order conditions for solving the constrained second-best; from these expressions, the welfare maximising tax rates can be recovered. Setting these two first partial derivatives equal to zero and solving the system of equations returns the two optimal tax rates. This is akin to finding optimal consumption in a simple Solow model via the 'golden rule' and it can be applied to potentially any parameter of interest.

$$\frac{\partial U}{\partial \tau^l} = 0 \quad \frac{\partial U}{\partial \tau^k} = 0 \quad \frac{\partial^2 U}{\partial \tau^{l\,2}} < 0 \quad \frac{\partial^2 U}{\partial \tau^{k\,2}} < 0$$
$$\left(\frac{\partial^2 U}{\partial \tau^{l\,2}}\right)\left(\frac{\partial^2 U}{\partial \tau^{k\,2}}\right) - \left(\frac{\partial^2 U}{\partial \tau^l \partial \tau^k}\right)^2 > 0 \tag{2.17}$$

Provided the three expressions above in (2.17) are met, the constrained second-best is obtained. The two first order conditions above ensure steady state welfare is unchanging with respect to both tax rates, the negativity of the two second order partial derivatives ensures utility is being maximised, and the last condition ensures a global maximum and not a saddle point. In order to compute the optimal tax rates and analyses the results, parameter values must first be specified.

The list of parameters and their assigned values are displayed in Table 2.3. The majority of parameter values listed are standard in the literature; most are derived from King and Rebelo (1999) or Bilbiie, et al (2012).

The remaining parameter values are as follows. The fraction of transfers ϕ which households invest in firm capital is chosen, for simplicity, as one-half (this could also be interpreted as a fixed cost of entry, where $\phi = 1$ would imply costless firm

Table 2.3: Parameter Values

Parameter		Value
β	Subjective Discount Factor	0.99
δ_K	Capital Depreciation Rate	0.025
δ_N	Firm Exit Rate	0.025
ψ	Consumption-Leisure weight	0.5
ϕ	Firm Transfers Investment Rate	0.5
θ	Exogenous Sequence Ratio	0.1
α	Physical Capital Output Elasticity	0.29-0.32
ω	Labour Output Elasticity	0.51-0.63
γ	Goods Elasticity of Substitution	2-4

entry). The exogenous sequence ratio θ is set such that ten percent of output must be raised every period by the fiscal authority for government purchases (which do not contribute to utility). The physical capital and labour output elasticities are varied between their respective values shown in Table 1.3 in order to show a range of plausible values for $(1-\nu)$, (between 0.8 and 0.95) which is determined residually as $\nu = \alpha + \omega$. The elasticity of substitution is varied between 2 and 4 to reflect the range of empirical estimates of average firm markups in the literature (implying μ between 1 and 2).

The calibration in table 2.3 refers to quarterly parameter values. However, as this paper is primarily concerned with the impact of firms on long run optimal taxation, this choice is trivial. Steady states are are ultimately unaffected by the decision to use quarterly parameter values as the subjective discount factor and the deprecation rates are adjusted such that steady states do not change (while all other parameter values remain invariant to timing denominations).

Naturally, the main two parameters of interest are the markup and the returns to scale. Here, $(1-\nu)$ represent the firm-aggregate output elasticity. This can also be interpreted as the firm-level returns to scale; as ν increases, firm-level returns to scale increases; a given quantity of capital and labour, spread over more firms, will impact aggregate output less if ν is high.

Figure 2.2 displays the results for the optimal tax rates varying these two parameters of interest. The horizontal x-axis varies the firm-level returns to scale while the shaded colours denote different values for the markup. The dashed lines correspond to the labour tax rate on the left vertical y-axis and the solid lines correspond to the capital tax rate on the right horizontal y-axis.

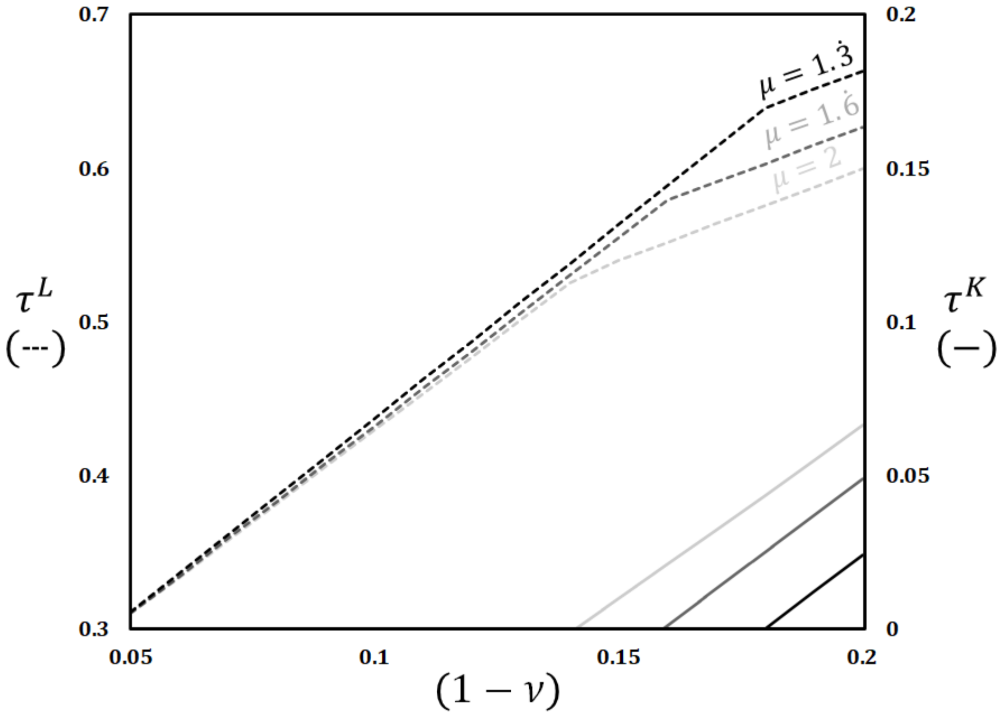

Figure 2.2: Long Run Optimal Tax Rates

As illustrated, the capital and labour tax rates both increase steadily when decreasing returns to scale intensify (moving rightward along the x-axis) with a notable kink in the function. When $(1-\nu)$ rises so does income accruing to firms; the labour and capital shares falls. In order for the government to maintain financing the exogenous sequence of purchases, the tax rate on labour must rise in order to compensate for the reduction in the labour income tax base.

As the aggregate share of profits increases with $(1-\nu)$, the marginal product of physical capital falls and the profit effect is strengthens vis-à-vis the under-investment effect. Beyond a certain threshold the efficiency gains from taxing lump-sum profits outweigh the distortion caused by inadvertently taxing future investments in physical capital. This threshold is denoted by the kink in the steady state tax functions in Figure 2.2. where the capital tax switches from zero to positive. After this point the tax rate on labour rises more slowly as its burden is shared with capital.

It can also be seen that the markup interacts with firm-level returns to scale; the capital tax becomes positive at a lower threshold of $(1-\nu)$ when the markup is higher. Three reasonable values for μ are shown, broadly in line with the empirical estimate for the US economy. From Figure 2.2. it can be seen that as firm markups rise, the relative tax burdens shift; the taxation of capital income increases while the taxation of labor income increases at a slower rate. This is because the markup is determined exclusively by the elasticity of substitution between goods. Higher markups are indicative of greater monopoly power, which directly increase the share of rental income in the economy and necessarily reduces the labour share. This further drives a wedge between the marginal product of labour and the wage rate which is subsequently offset by shifting the relative tax burden away from labour and towards capital.

Additionally, Figure 1.3 illustrates the relationship between markups, firm-level returns to scale, and the optimal capital tax rate.

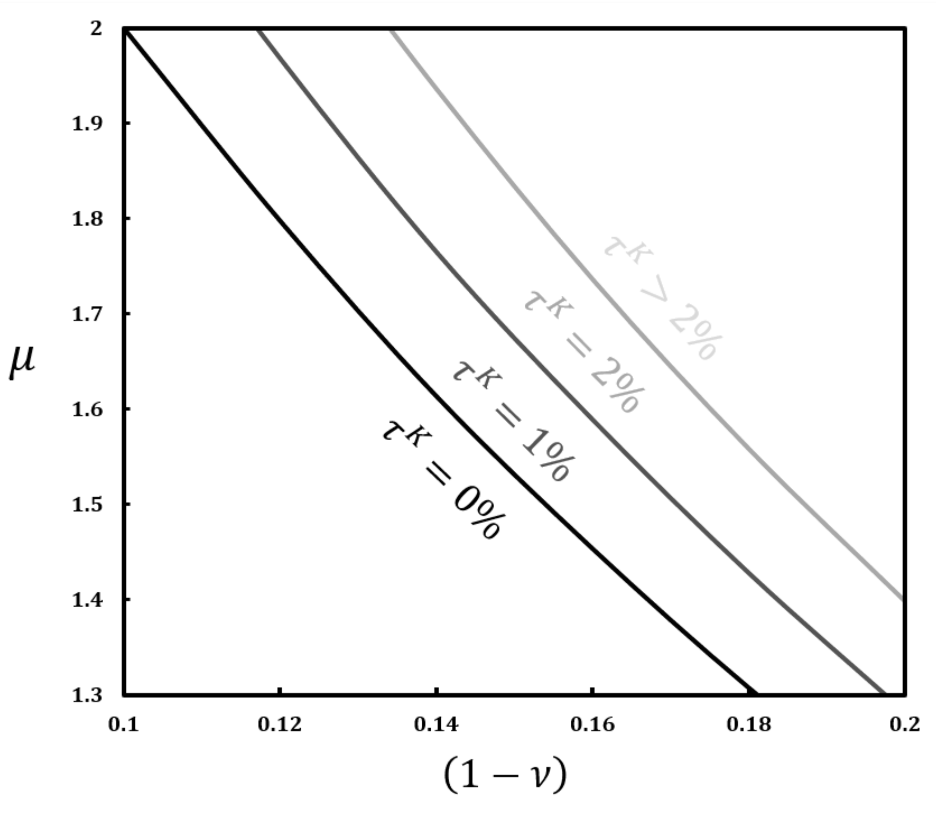

Figure 2.3: Long Run Optimal Capital Tax Isoquants

The welfare maximising capital tax rate is labelled next to each tax isoquant. From Figure 2.3 it can clearly be seen that to sustain some given optimal level of long run optimal taxation, there is a trade off between μ and returns to scale $(1-\nu)$; as both of these contribute to the aggregate profit share. As the fiscal authority cannot differentiate between profits from firm capital and interest from physical capital. For example, given the parameter values in Table 2.3, combined with $\mu = 1.6$ and $(1 - \nu) = 0.15$ (plasuable values, in line with the aforementioned literature cited for the US) is enough to generate a small positive tax on capital income in the long run.

2.5 Conclusion

The Chamely-Judd result continually proves to be a topic of interest for economists, both for theoretical and practical reasons. Recent studies such as, Chari, et al (2016) maintain that the result should be taken seriously in terms of guiding tax policy; even arguing that there are significant welfare gains, in excess of any transitional costs, of adopting a zero capital tax policy. However, important work by Straub and Werning (2019), advises against this and demonstrates the shaky foundation of the original result. Moreover, Aiyagari (1995), Correia (1996), Gao and Lansing (1999), Abel (2007), and Carlos, et al (2009) all provide modest extensions and and show why the zero capital result may not hold; subsequently, casting doubt on it as a practical policy recommendation.

This paper adds to that literature by demonstrating that if the government cannot easily differentiate between two sources of capital income; namely, profits derived from firm capital and interest derived from physical capital, a positive capital tax may be optimal in the steady state. The addition of firm capital explicitly models the role of firms in the production process. Accordingly, important observed economic phenomena, such as rising markups, can be studied in a structural setting in conjunction with optimal taxes. This paper finds that, for benchmark parameter values, empirical estimates of average firm markups in the US are sufficient to generate a positive tax on capital in equilibrium.

Therefore, despite the simple representative agent structure, a capital tax is efficient in the long run for the purpose of financing government purchases provided the profit share is sufficiently high. However, whether increases in the aggregate profits are driven by firm markups μ or firm-level returns to scale $(1 - \nu)$ determines the relative and absolute burdens of taxation between capital and labour. A ceteris paribus decrease in firm-level returns to scale causes the taxation of capital and labour income to both increases steadily. However, as firm markups rise, the relative tax burdens shift; the taxation of capital income increases while the taxation of labor income decreases. This is because the markup is determined exclusively by the elasticity of substitution between goods. Therefore, higher markups are indicative of greater monopoly power, which directly increase the share of rental income in the economy and necessarily decreases the labour share of income. This subsequently shifts the steady state tax burden away from labour and towards capital when μ rises.

Recent studies suggests that the profit share is increasing in the US economy (Gutierrez, 2017) and that this shift appears to be primarily driven by rising firm markups (Autor, et al 2019). Furthermore, De Loecker and Eeckhout (2017) empirically determine that average firm markups in the US economy have been on an upward trajectory for approximately the past few decades. If this perceived secular trend continues, the argument for the taxation of capital income is strengthened, in the context of the results established in this paper. While optimal capital levels of taxation remain a largely open question in economics for practical purposes, particularity given the current emphasis on inequality and heterogeneity in economics, acknowledging the results established here, alongside the aforementioned literature, it appears increasingly difficult to interpret the original Chamley-Judd result as being a credible result for guiding future tax policy.

2.6 Appendix

This appendix derives the analytical first order conditions which the long run welfare optimising tax rates can be recovered from. Returning to the steady state expressions, it can be seen that substituting equation (14) and (15) into (16) will formulate steady state welfare U exclusively in terms of the parameters (and the tax rates). Written explicitly:

$$U = \psi \ln \left(\frac{\left(1-\frac{\delta\alpha\beta\left(1-\tau^k\right)}{\nu\left(1-\beta+\beta\delta_k\right)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right)\left(\left(\frac{\omega}{\mu}\tau^l+\left(1-\frac{\omega}{\mu}\right)\tau^k-\theta\right)\frac{\phi}{\delta_n}\right)^{\frac{1-\nu}{\omega}}\left(\frac{\alpha\beta\left(1-\tau^k\right)}{\nu\left(1-\beta+\beta\delta_k\right)}\right)^{\frac{\alpha}{\omega}}}{\left(\frac{(1-\psi)\mu}{\psi\omega\left(\left(1-\tau^l\right)+\left(\frac{\mu}{\nu}-1\right)\left(1-\tau^k\right)\right)}\left(1-\frac{\delta\alpha\beta\left(1-\tau^k\right)}{\nu\left(1-\beta+\beta\delta_k\right)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right)+1\right)} \right) \cdots$$

$$+ (1-\psi) \ln \left(1 - \frac{\psi\omega\left(\left(1-\tau^l\right)+\left(\frac{\mu}{\nu}-1\right)\left(1-\tau^k\right)\right)}{(1-\psi)\mu\left(1-\frac{\delta\alpha\beta\left(1-\tau^k\right)}{\nu\left(1-\beta+\beta\delta_k\right)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right)+\psi\omega\left(\left(1-\tau^l\right)+\left(\frac{\mu}{\nu}-1\right)\left(1-\tau^k\right)\right)} \right)$$

Assuming that all parameters are fixed constants, the steady state function for welfare is reduced to two arguments $U = f\left(\tau^l, \tau^k\right)$. In order to maximise long run welfare, the partial derivatives of U must be taken with respect to the labour tax rate τ^l and the capital tax rate τ^k. Setting these two partial derivatives equal to zero and solving the system of equations returns the two optimal tax rates. Taking the first order partial derivative of U with respect to τ^l by repeatedly using the chain rule and quotient rule, and simplifying where possible, gives the following:

$$\frac{\partial U}{\partial \tau^l} = \left(\frac{\frac{\phi}{\delta_n}\frac{(1-\nu)}{\mu}}{\left(\frac{\omega}{\mu}\tau^l+\left(1-\frac{\omega}{\mu}\right)\tau^k-\theta\right)\frac{\phi}{\delta_n}} - \frac{\psi\omega\left(\frac{(1-\psi)\mu}{\psi\omega\left(\left(1-\tau^l\right)+\left(\frac{\mu}{\upsilon}-1\right)\left(1-\tau^k\right)\right)}\right)^2+\frac{\omega}{\mu}\left(\frac{\psi\omega\left(\left(1-\tau^l\right)+\left(\frac{\mu}{\upsilon}-1\right)\left(1-\tau^k\right)\right)}{(1-\psi)\mu}\right)^2}{\frac{(1-\psi)\mu}{\psi\omega\left(\left(1-\tau^l\right)+\left(\frac{\mu}{\upsilon}-1\right)\left(1-\tau^k\right)\right)}+\left(1-\frac{\delta\alpha\beta\left(1-\tau^k\right)}{\upsilon\left(1-\beta+\beta\delta_k\right)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right)^{-1}} \right) \psi \left(\frac{\upsilon\left(1-\beta+\beta\delta_k\right)}{\alpha\beta\left(1-\tau^k\right)}\right)^{\frac{\alpha}{\omega}} + \cdots$$

$$\frac{\left(\left(1-\tau^l\right)+\left(\frac{\mu}{\upsilon}-1\right)\left(1-\tau^k\right)\right)\psi\omega\left(\frac{1-\psi}{\psi}+\left(1-\frac{\delta\alpha\beta\left(1-\tau^k\right)}{\upsilon\left(1-\beta+\beta\delta_k\right)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right)^{-1}\right)-(1-\psi)\mu}{\left(1-\frac{\delta\alpha\beta\left(1-\tau^k\right)}{\upsilon\left(1-\beta+\beta\delta_k\right)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right)\frac{1}{\psi\omega}+\left(\left(1-\tau^l\right)+\left(\frac{\mu}{\upsilon}-1\right)\left(1-\tau^k\right)\right)\left(\frac{\left(\left(1-\tau^l\right)+\left(\frac{\mu}{\upsilon}-1\right)\left(1-\tau^k\right)\right)\psi\omega}{(1-\psi)\mu}+\left(1-\frac{\delta\alpha\beta\left(1-\tau^k\right)}{\upsilon\left(1-\beta+\beta\delta_k\right)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right)\right)^{-2}} = 0$$

Setting the above equation equal to zero ensures that steady state welfare at this point is unchanging with respect to the labour tax rate. The same steps are repeated for the second argument, the first order partial derivative of U with respect to τ^k is as follows:

$$\frac{\partial U}{\partial \tau^k} = \frac{\psi\left(\left(\frac{\omega}{\mu}\tau^l+\left(1-\frac{\omega}{\mu}\right)\tau^k-\theta\right)\frac{\phi}{\delta_n}\right)^{\frac{\nu-1}{\omega}}\left(\frac{\delta\alpha\beta}{\upsilon\left(1-\beta+\beta\delta_k\right)}+\frac{\omega}{\mu}-1\right)}{\left(1-\frac{\delta\alpha\beta\left(1-\tau^k\right)}{\nu\left(1-\beta+\beta\delta_k\right)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right)\left(\frac{\alpha\beta\left(1-\tau^k\right)}{\nu\left(1-\beta+\beta\delta_k\right)}\right)^{\frac{\alpha}{\omega}}} + \frac{\psi\left(\left(\frac{\omega}{\mu}\tau^l+\left(1-\frac{\omega}{\mu}\right)\tau^k-\theta\right)\frac{\phi}{\delta_n}\right)^{\frac{\nu-1}{\omega}}\left(\frac{(1-\psi)\mu}{\psi\omega\left(\left(1-\tau^l\right)+\left(\frac{\mu}{\nu}-1\right)\left(1-\tau^k\right)\right)}\right)^2}{\left(1-\frac{\delta\alpha\beta\left(1-\tau^k\right)}{\nu\left(1-\beta+\beta\delta_k\right)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right)\left(\frac{\alpha\beta\left(1-\tau^k\right)}{\nu\left(1-\beta+\beta\delta_k\right)}\right)^{\frac{\alpha}{\omega}}\left(\frac{\delta\alpha\beta}{\upsilon\left(1-\beta+\beta\delta_k\right)}+\frac{\omega}{\mu}-1\right)^{-1}} -$$

$$\cdots \frac{\psi\left(\left(\frac{\omega}{\mu}\tau^l+\left(1-\frac{\omega}{\mu}\right)\tau^k-\theta\right)\frac{\phi}{\delta_n}\right)^{\frac{\nu-1}{\omega}}\left(\frac{(1-\psi)\mu}{\psi\omega\left(\left(1-\tau^l\right)+\left(\frac{\mu}{\nu}-1\right)\left(1-\tau^k\right)\right)}\left(1-\frac{\delta\alpha\beta\left(1-\tau^k\right)}{\nu\left(1-\beta+\beta\delta_k\right)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right)+1\right)}{\left(1-\frac{\delta\alpha\beta\left(1-\tau^k\right)}{\nu\left(1-\beta+\beta\delta_k\right)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right)\left(\frac{\alpha\beta\left(1-\tau^k\right)}{\nu\left(1-\beta+\beta\delta_k\right)}\right)^{\frac{\alpha}{\omega}}\left(\frac{\frac{1}{\mu}+\mu(1-\psi)^2}{\psi\omega\left(\frac{\mu}{\nu}-1\right)(1-\psi)}+\frac{2\left(1-\frac{\delta\alpha\beta\left(1-\tau^k\right)}{\upsilon\left(1-\beta+\beta\delta_k\right)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right)}{\psi\omega\left(\frac{\mu}{\nu}-1\right)\left(\psi\omega\left(1-\tau^l\right)+\psi\omega\left(\frac{\mu}{\nu}-1\right)\left(1-\tau^k\right)\right)^{-1}}\right)} +$$

$$\cdots \frac{(1-\psi)\psi\omega\left(\frac{\mu}{\upsilon}-1\right)(1-\psi)\mu - \frac{(1-\psi)\psi\omega\left(\frac{\mu}{\upsilon}-1\right)(1-\psi)\mu\left(1-\frac{\delta\alpha\beta\left(1-\tau^k\right)}{\nu\left(1-\beta+\beta\delta_k\right)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right)+\left(\frac{\delta\alpha\beta+\upsilon\left(1-\beta+\beta\delta_k\right)}{\upsilon\left(1-\beta+\beta\delta_k\right)}-\frac{\psi\omega}{(1-\psi)\mu}\right)}{\psi\omega\left(\left(1-\tau^l\right)+\left(\frac{\mu}{\nu}-1\right)\left(1-\tau^k\right)\right)\left(\frac{\mu}{\omega}\left(1-\frac{\delta\alpha\beta\left(1-\tau^k\right)}{\nu\left(1-\beta+\beta\delta_k\right)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right)\left(\frac{1}{\psi}-1\right)+\left(\left(1-\tau^l\right)+\left(\frac{\mu}{\nu}-1\right)\left(1-\tau^k\right)\right)\right)^{-2}}}{(1-\psi)\mu\left(1-\frac{\delta\alpha\beta\left(1-\tau^k\right)}{\nu\left(1-\beta+\beta\delta_k\right)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right)\left((1-\psi)\mu\left(1-\frac{\delta\alpha\beta\left(1-\tau^k\right)}{\nu\left(1-\beta+\beta\delta_k\right)}-\frac{\omega}{\mu}\tau^l-\left(1-\frac{\omega}{\mu}\right)\tau^k\right)+\psi\omega\left(\left(1-\tau^l\right)+\left(\frac{\mu}{\nu}-1\right)\left(1-\tau^k\right)\right)\right)^{-1}} = 0$$

The two first order conditions above constitute a system of equations with two unknowns τ^l, τ^k. These expressions cannot be solved simply by substitution, instead numerical solution methods are used to find the pair of tax rates which simultaneously satisfy the two expressions above; from this the optimal tax thresholds can be determined, as illustrated in figure 1.3. These expressions allow for a more direct analysis of long-run optimal taxation, as the impact of each parameter on welfare maximising tax rates can be seen directly. The impact of any combination of parameters on steady state allocations can be easily and directly calculated from this point.

3 Investment Shocks, Firm Dynamics and the Comovement Problem

Abstract

Investment shocks are understood to play a key role in business cycle fluctuations. However, quantitative models typically generate opposing responses for consumption and investment with respect to these shocks. This sits in contrast with well-established observations which show that consumption and investment are both pro-cyclical over the business cycle. This paper develops a model of firm entry with business churn and endogenous overhead costs which produces the appropriate comovement between consumption and investment. Moreover, this paper demonstrates that CES production technology, with an elasticity of substitution significantly below unity, generates the observed comovements between macroeconomic aggregates on impact and improves model fit in terms of second moments.

3.1 Introduction

Studies suggest that shocks to the marginal efficiency of investment (MEI) play a significant role in business cycle fluctuations. For example, Greenwood, et al (2000) find that investment shocks explain 30% of the fluctuations in output and one-quarter of the fluctuations in hours. Moreover, Fisher (2006) determines that these shocks are responsible for 36-47% of the fluctuations in output and 42-67% of the fluctuations in hours. Similarly, Justiniano, et al. (2010) find that between 50% and 60% of fluctuations in output and hours can be attributed to investment shocks. However, in standard quantitative models, a positive (negative) shock to the marginal efficiency of investment tends to cause consumption to fall (rise) on impact while investment rises (falls). This conflicts with well-established observations which show that both consumption and investment move together and are pro-cyclical over the business cycle. This discrepancy between output and data is known in the literature as the *comovement problem*.

The origins of this problem can be traced back to the work of Barro and King (1984). In a seminal paper, they suggest that any structural model with time-separable utility and non-TFP shocks (which impact future expectations) will have difficulty in generating "empirically recognisable business cycles"; namely, the positive correlations observed between consumption, investment, output and hours in macroeconomic data. Figure 3.1 exemplifies the comovement problem, the panels on the left show US data series while the panels on the right represent output from a 'plain-vanilla' RBC model with benchmark parameter values chosen from King and Rebelo (1999). The top left panel displays an upward spike in the relative price of investment during the Great Recession (denoted by the grey bar). This implies that investment goods became more expensive during the downturn, which can be interpreted as a negative shock to investment. The bottom left panel displays the growth rates of consumption and investment for the same period. It can be seen that both of these macroeconomic variables move together in association with output and the investment shock; growth rates of consumption and investment both fall when there is an increase in the relative price of investment. In contrast, the top left shows a negative one percent shock to investment in a benchmark RBC model. From the bottom right panel, it can be seen that the model generates opposing responses, consumption rises on impact while investment falls; in conflict with the observed data over the business cycle.

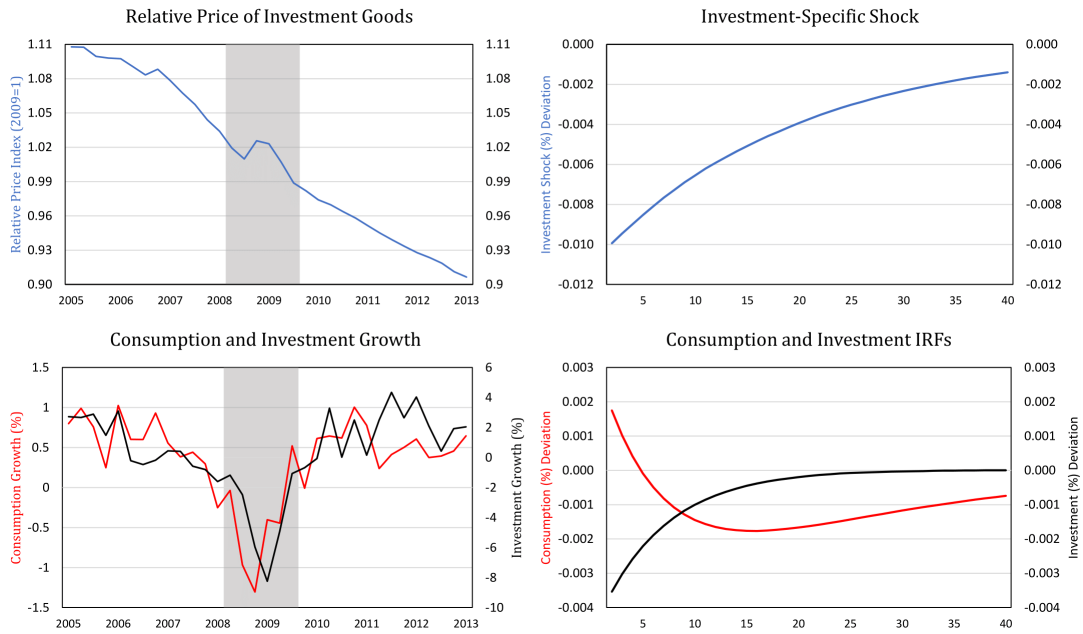

Figure 3.1: US Data and Benchmark RBC Output

Evidently, this is an issue of fundamental importance as investment shocks appear to play a large role in business cycles, yet typical structural models do not appear to generate the appropriate unconditional comovement between observable macroeconomic variables. Subsequently, this may call into question the validity of our understanding of investment shock transmission mechanisms which are derived from standard RBC models. This paper demonstrates that the inclusion of business churn and endogenous overhead costs in a model of firm entry is capable of resolving the comovement problem and improves model fit in terms of second moments. The model developed here naturally adds to the existing literature of firm dynamics pioneered by Bilbiie, et al (2012) (henceforth, BGM) through the addition of investment adjustment costs in the style of Christiano, et al (2005) and business churn with endogenous overhead costs.

The contribution of this paper is to demonstrate that the comovement problem can be resolved by emphasising the important role of firm dynamics in the economy. To date, most papers in this literature place emphasis on the consumer side and adopt a relatively standard approach to the firm side. Moreover, this paper demonstrates that lowering the elasticity of substitution enables positive comovement between consumption and investment on impact; in keeping with a range of empirical estimation which suggests σ is well below unity (see Klump, et al (2011) for a comprehensive review).

The remainder of the paper is structured as follows. Section 2 consists of a literature review. Section 3 sets up the model. Section 4 generalises the production technology and overviews the utilisation of 'normalised' CES functions. Section 5 examines the impulse response functions under different elasticities of substitution and matches moments of the model to US data. Section 6 contains concluding remarks. Lastly, the appendix simplifies the comovement problem to that of wedges and derives analytical expressions and proofs for when positive comovement is achieved.

3.2 Literature Review

It is well-known that consumption tends to moves counter-cyclically in structural models with time-separable utility (no restriction on intertemporal substitution) when there are positive shocks to the marginal efficiency of investment (Barro and King, 1984). A straightforward interpretation is that when investment in new capital goods is relatively cheaper, it becomes optimal to sacrifice some present consumption (and postpone leisure) in order to invest and obtain a higher level of output (and consumption) in the future. In other words, a fall in the relative price of investment goods induces *intertemporal* substitution which usually dominates over *intratemporal* substitution on impact. As a result, the immediate increase in investment and hours worked (and output) is such that consumption initially falls on impact and moves in the 'wrong' direction.

However, Greenwood, et al. (1988) show that it is possible to achieve positive comovement in a basic structural model. Two key aspects enable their result; non-separable utility and capacity utilisation costs. Firstly, GHH preferences are adopted, which exhibit non-separability in consumption and leisure (labour supply depends on the wage rate but not consumption; there is no wealth effect). This shuts down the intertemporal substitution channel which usually causes consumption to fall on impact. Secondly, utilisation costs allow the capital stock to be worked with greater intensity at the expense of accelerated depreciation. This makes it possible for capital input to rise immediately without an increase in physical investment. As a result, the existing capital stock is worked with greater intensity on impact, mitigating the effect of diminishing returns to labour and increasing output in response to the shock. This induces an outward shift in labour demand, which causes the wage rate and hours worked to increase. Subsequently, as there is no wealth effect, output rises along with consumption and investment. The primary disadvantage to this approach is that there is little empirical evidence (or a priori reasoning) which suggests that non-separability of consumption and leisure is a reasonable functional form to assume. The restrictive nature of GHH preferences has subsequently motivated studies which propose alternative ways of reconciling the comovement problem.

For example, in a medium-scale DSGE model, Khan and Tsoukalas (2011) demonstrate that comovement can be achieved with separable utility. Capacity utilisation with monopolistic competition and nominal rigidities are used to generate positive comovement. Nominal rigidities induce counter-cyclical wage and price mark-ups which result in an outward shift in labour demand in response to a positive investment shock. This leads to an amplified response in hours worked which causes output and consumption to rise on impact. Under this framework they emphasise that both GHH preferences and monopolistic competition with nominal rigidities are separately capable of generating this result; however, when estimating the model, the data clearly favours the latter explanation. The emphasis on the nominal rigidities channel over the weak intertemporal substitution channel also yields a superior fit for US data in terms of the second moments of consumption, output, and hours.

Furlanetto, et al (2013) propose a resolution to the comovement problem using rule-of-thumb consumers and nominal rigidities. In their setup, a portion of agents consists of rule-of-thumb consumers who do not engage in financial markets in order to smooth consumption; instead, each period they simply consume their entire income (they have a zero wealth effect). In response to a shock to the marginal efficiency of investment, it becomes optimal for agents to increase their hours worked and postpone leisure. Naturally, this leads to an increase in consumption for rule-of-thumb consumers since they do not invest. If the portion of rule-of-thumb consumers is sufficiently large (greater than approximately 25% of agents, given their specified parameter calibration), aggregate consumption rises on impact and comovement is achieved. Nominal rigidities helps reinforce positive comovement by dampening the intertemporal substitution effect; this enables the result to be achieved for a larger subset of the parameter space.

In a benchmark New Keynesian DSGE model with endogenous capital accumulation, Furlanetto and Seneca (2014) find that sufficient substitutability between leisure and consumption, along with nominal rigidities are key to resolving the comovement problem. They emphasise that GHH preferences generate comovement primarily due to the strong complementarity induced by non-separability between leisure and consumption. Using a generalised additively-separable utility function, they demonstrate that the size of the wealth effect is largely unimportant for generating the appropriate comovement of macroeconomic variables. Instead, the size of intertemporal substitution determines the magnitude of the response, while the degree of complementarity between hours and consumption determines the sign of comovement on impact.

Eusepi and Preston (2015) construct a model with heterogeneity in consumption and labour. They establish that positive comovement hinges on the consumption differential between employed and non-employed agents, and their relative labour inputs. In this framework, each household consists of a continuum of agents, and labour market participation entails a fixed cost. Households determine which members work and the duration of their work. A shock to the marginal efficiency of investment causes an increase in the intensive margin of employed agents through the usual mechanism. However, it also causes an increase in the extensive margin (share of total agents in employment) as it becomes worthwhile for non-employed agents to pay the fixed cost and enter into market production. Compositional effects arising from the increase in the employment rate leads to a substantial increase in aggregate consumption which generates positive comovement.

Ascari, et al (2016) employ a medium-scale New Keynesian model with the addition of intermediate goods in the production function to resolve the comvement problem and (what they refer to as) the "Barro-King Curse". Their 'roundabout production' approach creates an amplification mechanism whereby an increase in output raises the use of intermediate inputs and further boosts output. This enables the possibility of positive correlation between consumption and investment with respect to MEI shocks. Furthermore, they demonstrate that TFP shocks do not always generate the expected unconditional positive correlation between output and hours, as implied by Barro and King (1984). This is due to substantial intratemporal substitution combined with the aforemention amplification mehcanism which causes a fall in hours on impact of a TFP shock.

Chen and Liao (2018) address the comovement problem utilising sticky-prices in a two-sector model with consumer durables. An investment shock increases the demand for new capital goods and raises the price of consumer durables relative to non-durables. The intra-temporal substitution away from durable consumption towards non-durable consumption dominates the intertemporal substitution effect whereby current non-durable consumption is reduced in order to fund new investments. As a result, nondurable consumption rises and positive comovement is achieved. However, whether intra-temporal substitutions dominates over intertemporal substitution is primarily governed by the elasticity of substitution between durables and non-durables. Sufficient complementarity is required to achieve the desired result - an elasticity of substitution just a little below unity, with benchmark parameter values, does not generate the appropriate comovement between consumption and investment.

3.3 Model

The infinitely lived representative agent exhibits KPR preferences and derives utility from consumption (consisting of a basket of differentiated goods) and disutility from work each period. Where $\beta \in (0, 1)$ is the subjective discount factor, $\sigma_c > 0$ is the constant relative risk aversion, ${}^1/_\varphi = \eta > 0$ is the inverse Frisch elasticity and $\chi > 0$ is a scaling constant pinned down by other parameters.

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma_c}}{1-\sigma_c} - \chi \frac{L_t^{1+\eta}}{1+\eta} \right\}$$

In any given period the consumer enjoys a collection of goods which are aggregated into a single basket of consumption goods C_t. There exists a continuum of monopolistic firms which each produce their own variety ω where θ is the elasticity of substitution between the differentiated goods. The CES aggregator is as follows:

$$C_t = \left[\int_{\omega}^{n} c_t(\omega)^{\frac{\theta-1}{\theta}} \, d\omega \right]^{\frac{\theta}{\theta-1}}$$

Following the standard framework of BGM (2012) firms are atomistic and therefore the mark-up μ is independent of the number of firms (varieties produced) n. The mark-up is purely a function of the elasticity of substitution θ between the different varieties of goods $\mu = \theta/(\theta-1)$ (the substituability between goods is fixed and does not change with the number of varieties available). The representative agent maximises lifetime utility subject to the following three constraints below, where first order conditions are taken with respect to consumption C_t, labour L_t, investment I_t, capital K_{t+1}, entry E_{t+1}, and firms N_{t+1}:

$$\begin{gathered} \mathcal{L} = \textstyle\sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma_c}}{1-\sigma_c} - \chi \frac{L_t^{1+\eta}}{1+\eta} \right\} \\ -\lambda_t \left[C_t + I_t + E_t - w_t L_t - r_t K_t - \pi_t N_t \right] \\ -\varkappa_t \left[K_{t+1} - (1-\delta_K) K_t - \left(1 - \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right) I_t X_t \right] \\ -\Omega_t \left[N_{t+1} - (1-\delta_N) N_t - \left(1 - \frac{\psi}{2} \left(\frac{E_t}{E_{t-1}} - 1 \right)^2 \right) E_t \right] \end{gathered}$$

The representative agent simultaneously chooses investment and capital stock due to the presence of the investment adjustment cost in the style of Christiano, et al (2005) - the same applies for entry and firms. Where $\kappa > 0$ represents the adjustment cost for investment and $\psi > 0$ represents the adjustment cost for firm entry; this is referred to as 'business churn' (Aloi et al, 2019). Respectively, $\delta_K \in (0,1)$ and $\delta_N \in (0,1)$ represent the rate of capital depreciation and the rate of firm exit. Additionally, $\varkappa_t$ is the shadow value of an additional unit of physical capital and Ω_t is the shadow value of one more firm entering the market. Unlike BGM (2012) there is only one sector, and therefore no relative price, new firms are created out of total income, as opposed to only labour inputs. The first order conditions of the above maximisation problem are as follows:

$$\frac{\partial \mathcal{L}}{\partial C_t} = C_t^{-\sigma_c} - \lambda_t = 0 \tag{3.1}$$

$$\frac{\partial \mathcal{L}}{\partial L_t} = -\chi L_t^{\eta} - \lambda_t w_t = 0 \tag{3.2}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = \beta \lambda_{t+1} r_{t+1} - \varkappa_t + \beta \varkappa_{t+1} \left(1 - \delta_K\right) = 0 \tag{3.3}$$

$$\frac{\partial \mathcal{L}}{\partial N_{t+1}} = \beta \lambda_{t+1} \pi_{t+1} - \Omega_t + \beta \Omega_{t+1} \left(1 - \delta_N\right) = 0 \tag{3.4}$$

$$\begin{aligned} \tfrac{\partial \mathcal{L}}{\partial I_{t+1}} = -\lambda_t + \varkappa_t X_t \left(1 - \tfrac{\kappa}{2}\left(\tfrac{I_t}{I_{t-1}} - 1\right)^2 - \kappa\left(\tfrac{I_t}{I_{t-1}} - 1\right)\tfrac{I_t}{I_{t-1}}\right) \cdots \\ \cdots + \beta \varkappa_{t+1} X_{t+1} \kappa \left(\tfrac{I_{t+1}}{I_t} - 1\right)\left(\tfrac{I_{t+1}}{I_t}\right)^2 = 0 \end{aligned} \tag{3.5}$$

$$\begin{aligned} \tfrac{\partial \mathcal{L}}{\partial E_{t+1}} = -\lambda_t + \Omega_t \left(1 - \tfrac{\psi}{2}\left(\tfrac{E_t}{E_{t-1}} - 1\right)^2 - \psi\left(\tfrac{E_t}{E_{t-1}} - 1\right)\tfrac{E_t}{E_{t-1}}\right) \cdots \\ \cdots + \beta \Omega_{t+1} \psi \left(\tfrac{E_{t+1}}{E_t} - 1\right)\left(\tfrac{E_{t+1}}{E_t}\right)^2 = 0 \end{aligned} \tag{3.6}$$

Consolidating equations (3.1) and (3.2) yields the standard labour-leisure optimality condition of the household. Furthermore, combining equations (3.1) and (3.2) gives the Euler equation for capital, while combining equations (3.1) and (3.3) gives the Euler equation for firms (entry arbitrage condition). Manipulating (3.5) and (3.6) by dividing through λ_t gives the marginal Tobin's Q for capital $Q_t = \varkappa_t / \lambda_t$and the marginal Tobin's U for firms $U_t = \Omega_t / \lambda_t$ respectively (full model equations are detailed in Table 3.1 below). These conditions ensure optimal decision from the household.

The aggregate production technology is a normalised CES function with composite capital Z_t and labour L_t. The composite capital consists of physical capital stock K_t and firms N_t and has a standard Cobb-Douglas functional form:

$$Y_t = Y_0 \frac{\phi_0}{\phi_t} \left[\alpha_0 \left(\frac{Z_t}{Z_0}\right)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_0)\left(\frac{L_t}{L_0}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{3.7}$$

$$Z_t = K_t^{\nu} N_t^{1-\nu} \tag{3.8}$$

Where, $\sigma \in (0, \infty)$ is the elasticity of substitution between composite capital and labour and where the implicit elasticity of substitution between firms and physical capital is fixed at unity and $\alpha_0 \in (0, 1)$ represents the composite capital intensity in production; this also corresponds to the baseline (steady state) total capital income share.

Firms face an overhead cost ϕ_t which is paid every period and represents a fraction of their output. The overhead cost varies across the business cycle in a similar specification to a utilisation cost $\phi_t = \phi_0 U_t^{\gamma}$. This can be thought of broadly as the costs associated with operating inputs, such as the rent of the building or office and maintenance supplies. The interpretation is that when the marginal value of an existing firm is greater than the marginal value of creating a new firm $U_t > 1$ the overhead cost faced by a firm rises due to the anticipated reallocation of resources towards firm creation; positive net entry shifts firms demand for operating inputs outwards and pushes up overhead costs.

Furthermore, additional objects denoted with zero subscripts, such as Y_0, Z_0, L_0 and, ϕ_0 represent baseline (steady state) values for endogenous variables. These provide the natural benchmark around which the CES function is normalised. As part of the motivation of this paper is to examine the comovement problem under a generalised CES production technology, the effect of the elasticity of substitution on business cycle fluctuations and steady state allocations must be invariant to σ. The production function must be expressed in its 'normalised' or 'reparameterised' form where inputs and output are effectively treated as indexes - this isolates the impact of changes in σ on the business cycle and puts long run growth balanced growth dynamics to one side in favour of a deeper understanding of short run fluctuations.

Each individual firm maximises its profits by choosing physical capital and labour, subject to the constraints. The decision to invest in firm creation is chosen by the agent, hence first order conditions are taken with respect to capital and labour only. Substituting (3.8) into (3.7) and diving through by N_t, where lower case letters denote per-firm variables $z_t = k_t^{\nu}$, yields the firm-level production function. This can be used in the static profit maximisation problem as follows:

$$\Pi = y_0 \frac{\phi_0}{\phi_t} \left[\alpha_0 \left(\frac{k_t}{k_0} \right)^{\nu \frac{\sigma-1}{\sigma}} + (1-\alpha_0) \left(\frac{l_t}{l_0} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} - w_t l_t - r_t k_t$$

$$\frac{\partial \Pi}{\partial k_t} = \frac{\alpha_0}{\mu}\frac{\nu}{k_t}\left(\frac{\phi_0}{\phi_t}\frac{y_0}{z_0}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{z_t}{l_t}\right)^{\frac{1}{\sigma}} - r_t = 0 \tag{3.9}$$

$$\frac{\partial \Pi}{\partial l_t} = \frac{1-\alpha_0}{\mu}\left(\frac{\phi_0}{\phi_t}\frac{y_0}{l_0}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{y_t}{l_t}\right)^{\frac{1}{\sigma}} - w_t = 0 \tag{3.10}$$

Due to the differentiated goods the constant markup μ appears as a fixed wedge in the first order conditions for capital and labour in (3.9) and (3.10); as such each factor is not remunerated exactly at its marginal product due the presence of some monopoly power. The model is then closed with the autoregressive process where $\rho \in (0, 1)$ is persistence of the shock, where $\varepsilon_t \sim (0, \sigma^{\varepsilon})$ is the zero mean shock process and X_t is the relative price of investment goods (marginal efficiency of investment) $\ln(X_t) = \rho \ln(X_{t-1}) + \varepsilon_t$ (for now the simplifying assumption is made to have only stochastic shocks, not permanent shocks, in order to better understand the relationship between the comovement problem and the elasticity of substitution). The equilibrium equations for the model consist of the first order conditions (3.1) through (3.10) plus the additional constraints from the household optimisation problem. These conditions are detailed as follows in Table 3.1 below.

To give an overview of the normalisation procedure, Figure 3.2 below shows a plot of two production functions (and corresponding isoquants projected onto the xy-plane) which differ only in their elasticity of substitution (the red surface represents $\sigma = 1.5$ and the blue surface represents $\sigma = 0$). As can be seen, there is single ray (denoted by a dashed line) along which the two production surfaces are tangent (this can also be seen in two-dimensional space; each pair of red and blue isoquants are only tangent at a single point). At the point of tangency, the value of the elasticity of substitution will not affect the level of output. However, deviating from this point (steady state), it can clearly be seen that output will be affected when σ varies.

Normalisation requires that steady state allocations be expressed in terms of exogenous parameters which do not feature σ (endogenous variables in the steady state must not be affected by the elasticity of substitution). Following the approach of Cantore, et al (2014), output, labour (and firm overhead costs here) are

Table 3.1: Equilibrium Conditions

Model Equations	
Labour Supply	$\chi L_t^{\eta} C_t^{\sigma_c} = w_t$
Labour Demand	$w_t = \frac{1-\alpha_0}{\mu} \left(\frac{\phi_0}{\phi_t}\frac{y_0}{l_0}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{y_t}{l_t}\right)^{\frac{1}{\sigma}}$
Capital Supply (Arbitrage)	$Q_t = \beta \left(\frac{C_t}{C_{t+1}}\right)^{\sigma_c} \left(Q_{t+1}\left(1-\delta_K\right) + r_{t+1}\right)$
Capital Demand	$r_t = \frac{\alpha_0}{\mu}\frac{\nu}{k_t} \left(\frac{\phi_0}{\phi_t}\frac{y_0}{z_0}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{z_t}{l_t}\right)^{\frac{1}{\sigma}}$
Firm Entry (Arbitrage)	$U_t = \beta \left(\frac{C_t}{C_{t+1}}\right)^{\sigma_c} \left(U_{t+1}\left(1-\delta_N\right) + \pi_{t+1}\right)$
Firm Profit	$\pi_t = \frac{Y_t - w_t L_t - r_t K_t}{N_t}$
Capital Tobin's Q	$1 = Q_t X_t \left(1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \kappa\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right) \cdots$ $+\beta Q_{t+1} X_{t+1} \kappa \left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2$
Firm Tobin's U	$1 = U_t \left(1 - \frac{\psi}{2}\left(\frac{E_t}{E_{t-1}} - 1\right)^2 - \psi\left(\frac{E_t}{E_{t-1}} - 1\right)\frac{E_t}{E_{t-1}}\right) \cdots$ $+\beta U_{t+1} \psi \left(\frac{E_{t+1}}{E_t} - 1\right)\left(\frac{E_{t+1}}{E_t}\right)^2$
Firm Overhead Costs	$\phi_t = \phi_0 U_t^{\gamma}$
Production Function	$Y_t = Y_0 \frac{\phi_0}{\phi_t} \left[\alpha_0 \left(\frac{Z_t}{Z_0}\right)^{\frac{\sigma-1}{\sigma}} + \left(1-\alpha_0\right)\left(\frac{L_t}{L_0}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$
Composite Capital	$Z_t = K_t^{\nu} N_t^{1-\nu}$
Income Identity	$Y_t = C_t + I_t + E_t$
Capital Law of Motion	$K_{t+1} = \left(1-\delta_K\right) K_t + \left(1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t X_t$
Firm Law of Motion	$N_{t+1} = \left(1-\delta_N\right) N_t + \left(1 - \frac{\psi}{2}\left(\frac{E_t}{E_{t-1}} - 1\right)^2\right) E_t$
Autoregressive Process	$\ln\left(X_t\right) = \rho \ln\left(X_{t-1}\right) + \varepsilon_t$

normalised to unity. Normalising allows all endogenous variables to be identified independently of the elasticity of substitution. This ensures that steady state allocations will remain on a point of tangency between 'families' of CES production functions (transitory dynamics are altered by the elasticity of substitution, but not steady states).

Note that neither σ or η appear in any of the steady state expressions when the equilibrium equations are reduced and solved. This ensures that only transitory dynamics are effected by these two parameters; as a result, the effect these two key parameter have on business cycle moments can be evaluated directly and in isolation

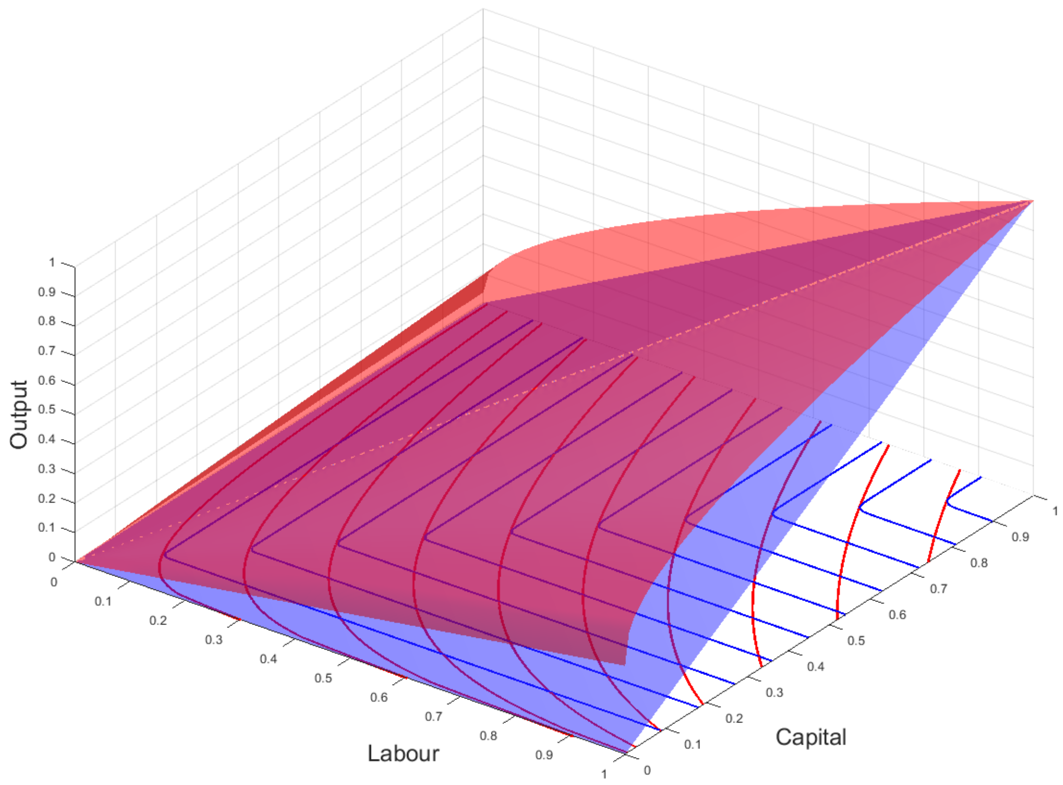

Figure 3.2: $\sigma = 0$ and $\sigma = 1.5$ production surfaces

because they have no impact on equilibrium allocations. Dropping time subscripts to denote equilibrium, the remaining steady states are obtained by consolidating and rearranging the equilibrium equations. The steady states are shown in Table 3.2.

3.4 Simulations and Results

In order to simulate the model, parameters must be chosen. The first nine parameters values in Table 2.3 are taken directly from the BGM (2012), the next four are adjustment costs which are chosen conservatively to bias against achieving positive comovement. Finally, the elasticity of factor substitution is varied in line with empirical estimates in the literature (Chirinko, 2008). It is worth noting that α_0 corresponds to the total capital share of income at all times if the production technology is Cobb-Douglas. However, when the elasticity of substitution is not equal to unity this parameter corresponds to the total capital income share only in the

Table 3.2: Normalised Steady State Values

Steady States	
Relative Price	$X = 1$
Labour	$L = 1$
Tobin's Q	$Q = 1$
Tobin's U	$U = 1$
Overhead Cost	$\phi = 1$
Output	$Y = 1$
Rental Rate	$r = \frac{1}{\beta} - 1 + \delta_K$
Capital Stock	$K = \frac{\alpha_0 \nu}{\mu r}$
Firm Profit	$\pi = \frac{1}{\beta} - 1 + \delta_N$
Wage Rate	$w = \frac{1-\alpha_0}{\mu}$
Investment	$I = \delta_K K$
Firms	$N = \frac{Y - wL - rK}{\pi}$
Entry	$E = \delta_E N$
Consumption	$C = Y - I - E$
Composite Capital	$Z = K^{\nu} N^{1-\nu}$

steady state; during transitory dynamics α_0 will differ from the capital income share when $\sigma \neq 1$. Therefore, to ensure consistency across different production technologies, α_0 is referred to as the baseline capital income share, since at the steady state, for all values of σ this holds true. All other parameters retain their standard interpretations. The list of parameters and their assigned values are displayed in table 3.3.

Table 3.3: Parameter Values

Parameter		Values
α_0	Baseline Capital Share	0.33
β	Subjective Discount Factor	0.99
δ_K	Capital Depreciation Rate	0.025
δ_N	Firm Exit Rate	0.025
σ_c	Relative Risk Aversion	1
η	Inverse Frisch Elasticity	0.25
θ	Goods Elasticitiy of Substitution	3.8
ϕ_0	Baseline Firm Overhead Costs	1
ρ	Autoregressive Coefficent	0.95
ν	Capital-Composite Elasticity	0.33
κ	Capital Adjustment Cost	1
ψ	Firm Entry Adjustment Cost	1
γ	Overhead Costs Elasticity	0.5
σ	Elasticity of Factor Substitution	$0.4 - 0.8$

The vast majority of the parameters values chosen in BGM (2012) are standard in the literature - many are derived from King and Rebelo (1999). The value of ν has been assigned to one third, this represents the physical-to-composite capital elasticity and chosen in line with the composite-capital output elasticity. The costs for capital adjustment and firm entry business churn are both assigned to unity; these are fair and reasonable estimates which do not impose excessively large costs to adjustment. Overhead firm costs dynamics function similarly to a capacity utilisation cost à la Greenwood, et al (1988) where γ is the overhead costs elasticity. This elasticity is assigned a value of one-half in order to introduce concavity into the function (as opposed to convexity, as in Greenwood, et al (1998), where working the capital stock with greater intensity has a more than proportional impact on accelerating capital depreciation). The intuition behind this concavity $1 > \gamma$ is that a marginal unit of output being reallocated to net firm creation will have a less than proportional impact on period overhead costs which firms pay for operational inputs; again, this lower value biases against positive comovement as it dampens the amplification mechanism which enables slack in the resource constraint such that consumption can rise on impact alongside investment.

Impulse Response Functions: Below displays the impulse response functions for a positive shock to the marginal efficiency of investment (with various values of σ). Firstly, it can be seen that consumption rises on impact and exhibits a positive comovement with investment, output and hours when there is sufficient gross complementarity (an elasticity in the region of $\sigma = 0.4$ and below).[1] When σ is low, a fall in the relative price of investment goods no longer causes consumption to decrease on impact; instead, it rises modestly. However, this is not due to a weak wealth effect caused by impeding the intertemporal substitution channel. Instead, it is caused by a substantial intra-temporal reallocation of resource away from net firm creation and an outward shift in labour demand. These two channels both play a key role in generating an "empirically recognisable business cycle" in regards to appropriate unconditional comovements of macroeconomic variables.

Firstly, due to the nested production technology containing two types of capital (the MEI shock applies only to physical capital), resource flow away from firm creation

[1] The comovement problem persists for higher values of σ, as the parameters such as relative risk aversion, adjustment costs, and overhead cost elasticity being chosen conservatively to bias against generating positive comovement between consumption and investment.

on impact as investment in physical capital becomes relatively cheaper. When firm entry falls $E_t \downarrow$ additional slack is immediately created in the resource constraint $Y_t = C_t \uparrow + I_t \uparrow + E_t \downarrow$ which allows either investment or consumption to rise. Secondly, firms marginal Tobin's U decreases on impact; this represents an *anticipated* reallocation of resources away from net firm entry $U_{ss+1} < 1$[2]. Subsequently, firms demand for operating inputs shifts inwards and overhead costs fall. The extent to which firm creation is curbed affects the extent to which overhead costs fall. This has the indirect effect of raising demand for factor inputs at the firm-level, enduing a rise in hours worked and output on impact and thus enabling consumption to increase alongside investment.

[2]Subscript $t = ss + 1$ represent the period the shock hits; a perturbation from the steady state.

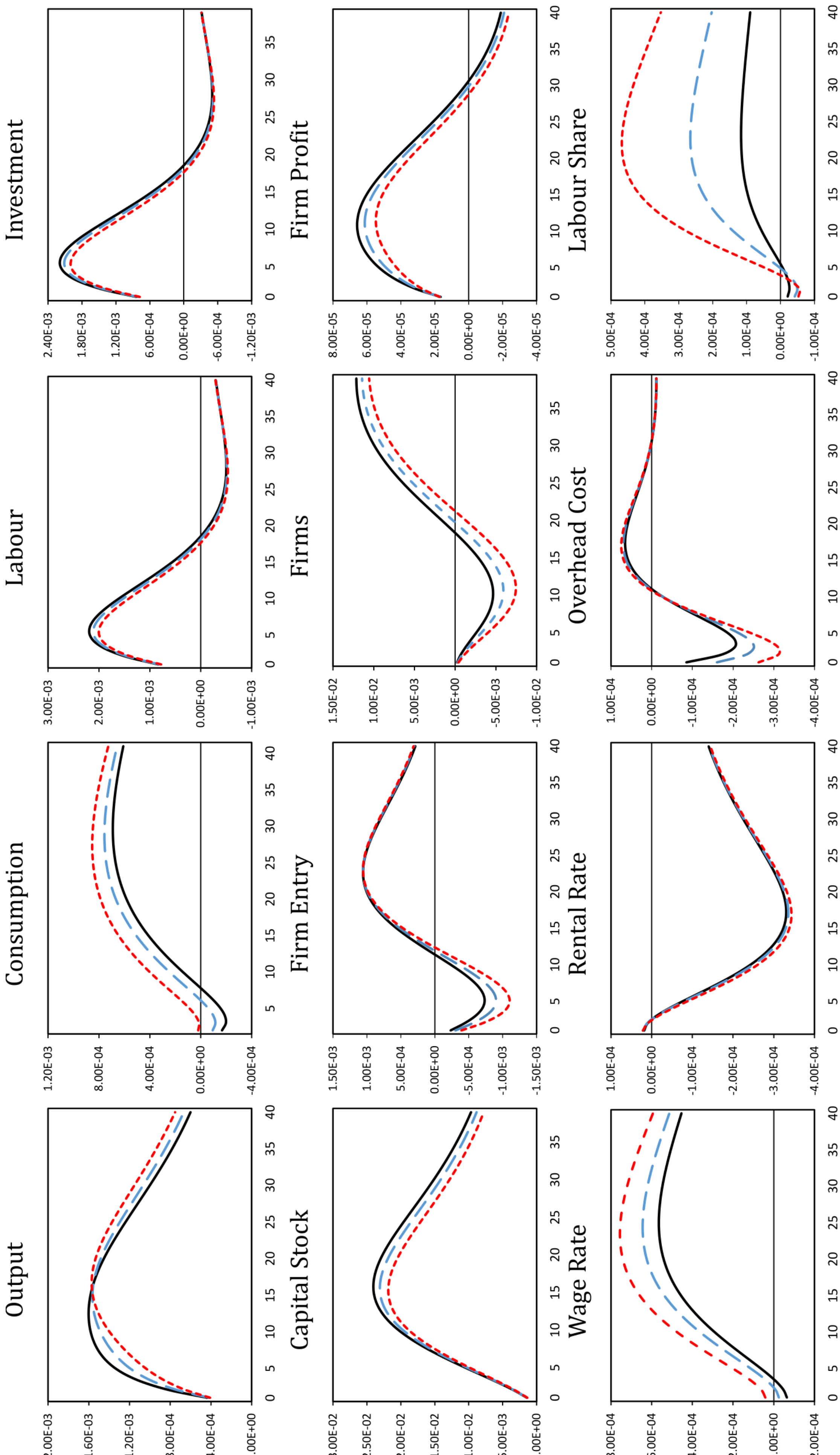

Figure 3: IRFs $\sigma = 0.4$, $\sigma = 0.6$, $\sigma = 0.8$

Matching Second Moments: Naturally, the next step is to take the model to the data to ensure the relative volatility of key macroeconomic variables are matched. The model is simulated in line with the above parameter values specified in Table 3.3 and is compared with macroeconomic data series taken from the FRED database for the US. These series are log transformed and de-trended using the Hodrick-Prescott filter. The standard deviations are then matched with the following variables displayed in Table 3.4. The model is compared with three different elasticities of substitution, which are shown in the impulse response functions in Figure 3.3.

From Table 3.4 below it can be seen that, generally speaking, lower eleasiticties of substitution better match the relative volatilities of macroeconomic variables; and are in keeping with the empirical estimates of σ in the literature. Additionally, they preserve positive comovement between consumption, investment, hours and output; which is also observed in the data.

Table 3.4: Second Moments

Variable	***Data***	$\sigma = 0.8$	$\sigma = 0.6$	$\sigma = 0.4$
	Standard Deviation (%)			
Output	1.49	0.85	0.87	0.89
	Relative Standard Deviation			
Consumption	0.59	0.50	0.56	0.62
Investment	3.14	0.80	0.76	0.71
Hours	1.27	0.54	0.56	0.60
Wages	0.61	0.41	0.48	0.59
Rental Rate	0.27	0.19	0.19	0.20
Labour Share	0.48	0.08	0.19	0.38

The second moments generated by the model for output under different elasticity specifications shown in Table 3.4 are broadly in line with the literature. For example, Justiniano, et al. (2010), claims that 50% to 60% of fluctuations in output and hours can be accounted for by investment shocks. A similar result is found here, between 57% and 60% of the volatility in output, and 43% to 47% in the volatility of hours, can be accounted for by the investment shock. Broadly speaking, the model does well in capturing the relative volailities of consumption, wages, rental rate and the labour share. On the whole, the specification of $\sigma = 0.4$ does the best job of simultaneously matching consumption, wages, rental rate and the labour share. This is consistent with Cantore, et al (2015) who claim that much lower elasticities of substitution are superior to Cobb-Douglas type specifications when analysing business cycles; in the short-run there is relatively limited substitutability between factors and a low value of σ generates the observed persistence in macroeconomic variables.

However, the model performs more poorly, for in accounting for the fluctuations in investment observed in the data.[3] This due to the fact that adjustment costs and firm entry reduce the overall variance of investment in model simulations. Lowering the adjustment costs and the relative risk aversion helps boost the relative volatility of investment such that it is greater than output; however this does not preserve the appropriate comovement between consumption and investment on impact of the MEI shock. Nevertheless, the model derived here remains a tractable flexible price model which matches the appropriate comovement between macroeconomic variables for a significant and plausible subset of the parameter space.

3.5 Conclusion

Investment shocks are understood to play a key role in business cycles. However, in typical models, in response to a positive investment shock, consumption tends to fall on impact while investment rises. This contrasts with empirical observations, which show that consumption and investment are both pro-cyclical over the business cycle. This paper demonstrates that the appropriate comovement between consumption and investment can be achieved in a simple flexible price RBC framework via the inclusion of firm entry with business churn and endogenous overhead costs. Therefore, this paper contributes to both the current firm dynamics literature as well as the recent literature on the comovement problem. Typically, the comovement problem has been dealt with by variety of approaches in structural models, usually with some combination of nominal and real frictions, typically from the consumer side of the problem. This paper instead resolves this problem from the firm side through a focus on entry dynamics and generalised CES production technology.

[3] Investment in Table 2.4 here refers to the investment in physical capital stock only, it does not include firm creation.

Analysing firm dynamics with a CES production function, it is found that the elasticity of factor substitution determines the magnitude and sign of consumption on impact in response to the investment shock. An elasticity of factor substitution well below unity (in line with empirical estimates) is required to generate the appropriate comovements. Furthermore, it is found that a lower elasticity of substitution appears to better match data in terms of second moments for the US economy: as is consistent with other studies in the literature.

This paper also provides an additional appendix which demonstrates that investment shocks can generate positive comovement between consumption and investment on impact in a standard flexible-price RBC framework through the inclusion of only a simple time-varying labour wedge. The wedge introduces a rudimentary propagation mechanism by which the volatility of the labour supply decision is amplified in response to shocks. This simple model derived in this appendix allows for tractable analytical expressions which describe when positive comovement is achieved and can be used in future research as a framework for analysing this puzzle.

3.6 Appendix

This appendix demonstrates that investment shocks can generate positive comovement between consumption and investment on impact in a standard flexible-price RBC framework through the inclusion of a simple time-varying labour wedge. The wedge introduces a rudimentary propagation mechanism by which the volatility of the labour supply decision is amplified in response to shocks; this subsequently increases the response of output on impact and provides sufficient slack in the resource constraint such that both consumption and investment can rise together. The model derived in this appendix allows for tractable analytical expressions which describe when positive comovement is achieved. With standard KPR preferences, it is shown that the Frisch elasticity, along with the relative size of the labour wedge, play a key role in determining the magnitude and sign of consumption on impact in response to an investment shock. Reducing the comovement problem (or "Barro-King Curse") down to a simple time-varying labour wedge enables straightforward and clear insights to be made regarding the transmission mechanisms which enable this result.[4]

The infinitely lived representative agent exhibits period utility of the following form and derives utility from consumption C_t and disutility from work L_t each period:

$$\sum_{t=0}^{\infty} \beta^t \left[\ln C_t - \chi \frac{L_t^{1+\eta}}{1+\eta} \right]$$

The utility function is strictly concave and continuous. Where $\beta \in (0,1)$ is the subjective discount factor, $\eta > 0$ is the inverse Frisch elasticity and $\chi > 0$ is a scaling constant which ensures that steady state labour is equal to unity. Capital stock is now written as K_{t-1} to emphasise that it is a state variable and $\delta \in (0,1)$ represent the rate of capital depreciation. The agent maximises discounted lifetime utility subject to the capital law of motion, the budget constraint and the time-varying labour wedge (where $\Phi = \Phi_t = \Phi_{t-1}$ represents a fixed exogenous sequence

[4]This simple amplification mechanism captures two key observable features of the US labour market. Firstly, estimates of the labour-wedge exhibit significant counter-cyclicality (Atesagaoglu and Elgin, 2014; Zhang, 2018) Secondly, the amplification in the labour supply decision generated by the counter-cyclical wedge generates an improvement of fit in terms of second moments for US hours worked data.

and τ_t is the time-varying wedge):

$$w_t L_t + r_t K_{t-1} = C_t + I_t + \Phi \tag{3.11}$$

$$K_t = (1-\delta) K_{t-1} + I_t X_t \tag{3.12}$$

$$\Phi = w_t L_t \tau_t \tag{3.13}$$

Substituting (3.12) and (3.13) into (3.11) yields the constraint for the consumer's problem. The labour-leisure condition and the consumption-Euler equation are derived from the maximisation problem:

$$\chi L_t^{\eta} C_t = w_t (1-\tau_t) \tag{3.14}$$

$$\frac{C_{t+1}}{C_t} = \beta \left(X_t r_{t+1} + (1-\delta) \frac{X_t}{X_{t+1}} \right) \tag{3.15}$$

Additionally, $\widetilde{w}_t$ is written to represent the effective-wage rate (analogous to a post-tax wage rate):

$$\widetilde{w}_t = w_t (1-\tau_t) \tag{3.16}$$

The labour wedge τ_t fluctuates optimally over the business cycle in order to meet the exogenous sequence Φ (equivalent to an endogenous labour tax). The following auxiliary maximisation problem is required to ensure optimal consumer decisions with respect to the time-varying wedge. Manipulating equations (3.11) through (3.16) provides the constraints for the auxiliary welfare maximisation problem:

$$\begin{gathered}
\mathcal{L} = \textstyle\sum_{t=0}^{\infty} \beta^t \left[\ln C_t - \chi \frac{L_t^{1+\eta}}{1+\eta} \right] \\
+\lambda_t^1 \left[w_t L_t - \widetilde{w}_t L_t - \Phi \right] \\
+\lambda_t^2 \left[\widetilde{w}_t L_t + r_t K_{t-1} + \tfrac{(1-\delta)}{X_t} - \tfrac{K_t}{X_t} - C_t \right] \\
+\lambda_t^3 \left[\chi L_t^{\eta} C_t - \widetilde{w}_t \right]
\end{gathered}$$

First order conditions are taken with respect to consumption, leisure and the effective-wage rate to ensure the optimal path of the labour wedge τ_t:

$$\frac{\partial \mathcal{L}}{\partial C_t} = \frac{1}{C_t} - \lambda_t^2 + \lambda_t^3 \chi L_t^{\eta} = 0 \tag{3.17}$$

$$\frac{\partial \mathcal{L}}{\partial L_t} = L_t^{\eta} + \lambda_t^1 \left(w_t - \widetilde{w}_t\right) + \lambda_t^2 \widetilde{w}_t + \lambda_t^3 \chi \eta L_t^{\eta-1} C_t = 0 \tag{3.18}$$

$$\frac{\partial \mathcal{L}}{\partial \widetilde{w}_t} = -\lambda_t^1 L_t + \lambda_t^2 L_t - \lambda_t^3 = 0 \tag{3.19}$$

Furthermore, the representative firm faces Cobb-Douglas production technology and operates in a perfectly competitive market.

$$Y_t = K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{3.20}$$

Firms maximise profit by hiring capital and labour. The first order conditions follow trivially:

$$r_t = \alpha \frac{Y_t}{K_{t-1}} \tag{3.21}$$

$$w_t = (1-\alpha)\frac{Y_t}{L_t} \tag{3.22}$$

The model is closed with the autoregressive process where $\rho \in (0,1)$ is persistence of the shock, and where $\varepsilon_t \sim (0, \sigma^{\varepsilon})$ is the zero mean shock process and X_t is the relative price of investment goods:

$$\ln\left(X_t\right) = \rho \ln\left(X_{t-1}\right) + \varepsilon_t \tag{3.23}$$

Equations (3.11) through (3.23) represent the equilibrium conditions for the decentralised economy (shown in the Table 3.5 below):

Steady state allocations be expressed in terms of exogenous parameters. Replacing time subscripts with ss to denote equilibrium, the remaining steady states are obtained accordingly by rearranging and consolidating the equations from the Table 3.5. Due to the production technology being Cobb-Douglas there is no variation in the elasticity of substitution (as shown earlier), therefore output does not need to be normalised to unity. The steady states in Table 3.6 are used for deriving analytical expressions to create boundary conditions which show the parameter subspace for

Table 3.5: Equilibrium Conditions

Model Equations	
Labour Supply	$\chi L_t^{\eta} C_t = w_t \left(1-\tau_t\right)$
Capital Supply	$\frac{C_{t+1}}{C_t} = \beta \left(X_t r_{t+1} + (1-\delta) \frac{X_t}{X_{t+1}} \right)$
Labour Demand	$w_t = (1-\alpha) \frac{Y_t}{L_t}$
Capital Demand	$r_t = \alpha \frac{Y_t}{K_{t-1}}$
Production Function	$Y_t = K_{t-1}^{\alpha} L_t^{1-\alpha}$
Accounting Identity	$Y_t = C_t + I_t + \varPhi$
Law of Motion	$K_t = (1-\delta) K_{t-1} + I_t X_t$
Net-Wage Rate	$w_t (1-\tau_t) = \widetilde{w}_t$
Exogenous Distortion	$\varPhi = w_t L_t \tau_t$
Consumption Optimality	$\frac{1}{C_t} - \lambda_t^2 + \lambda_t^3 \chi L_t^{\eta} = 0$
Labour Optimality	$-\chi L_t^{\eta} + \lambda_t^1 \left(w_t - \widetilde{w}_t\right) + \lambda_t^2 \widetilde{w}_t + \lambda_t^3 \chi \eta L_t^{\eta-1} C_t = 0$
Net-Wage Optimality	$-\lambda_t^1 L_t + \lambda_t^2 L_t - \lambda_t^3 = 0$
Autoregressive Process	$\ln X_t = \rho \ln X_{t-1} + \varepsilon_t$

which positive comovement is generated on impact. Recall, the scaling constant ensures that labour is normalised to unity in the steady state; this feature becomes key in deriving analytical boundary exclusively in terms of parameters as labour is equal to one in the immediate neighborhood of the steady state.

Table 3.6: Steady State Expressions

Steady States	
Price of Investment	$X_{ss} = 1$
Labour	$L_{ss} = 1$
Rental Rate	$r_{ss} = \frac{1-\beta+\beta\delta}{\beta}$
Capital Stock	$K_{ss} = \left(\frac{\alpha\beta}{1-\beta+\beta\delta}\right)^{\frac{1}{1-\alpha}}$
Investment	$I_{ss} = \delta \left(\frac{\alpha\beta}{1-\beta+\beta\delta}\right)^{\frac{1}{1-\alpha}}$
Output	$Y_{ss} = \left(\frac{\alpha\beta}{1-\beta+\beta\delta}\right)^{\frac{\alpha}{1-\alpha}}$
Consumption	$C_{ss} = \left(\frac{\alpha\beta}{1-\beta+\beta\delta}\right)^{\frac{\alpha}{1-\alpha}} - \delta \left(\frac{\alpha\beta}{1-\beta+\beta\delta}\right)^{\frac{1}{1-\alpha}} - \varPhi$
Wage Rate	$w_{ss} = (1-\alpha) \left(\frac{\alpha\beta}{1-\beta+\beta\delta}\right)^{\frac{\alpha}{1-\alpha}}$
Net-Wage Rate	$\widetilde{w}_{ss} = (1-\alpha) \left(\frac{\alpha\beta}{1-\beta+\beta\delta}\right)^{\frac{\alpha}{1-\alpha}} - \varPhi$
Labour Wedge	$\tau_{ss} = \frac{\varPhi}{(1-\alpha)} \left(\frac{1-\beta+\beta\delta}{\alpha\beta}\right)^{\frac{\alpha}{1-\alpha}}$
Scaling Constant	$\chi = \frac{(1-\alpha)\left(\frac{\alpha\beta}{1-\beta+\beta\delta}\right)^{\frac{\alpha}{1-\alpha}} - \varPhi}{\left(\frac{\alpha\beta}{1-\beta+\beta\delta}\right) - \delta\left(\frac{\alpha\beta}{1-\beta+\beta\delta}\right) - \varPhi}$

Model simulations imply that for a subset of the parameter space (low inverse frisch elasticity or high exogenous distortion) consumption rises on impact and exhibits a positive comovement with investment, hours and output. A fall in the relative price of investment does not causes consumption to decrease on impact; instead, it rises modestly. In response to an investment shock, an additional increase in hours worked is induced by a rise in the effective-wage rate as a result of a reduction in the labour wedge (on top of intra-temporal substitution away from leisure towards labour). This is because the value of the exogenous sequence remains unchanged in absolute terms. Therefore, when output rises, the labour wedge (labour tax) falls to ensure the constraint $\Phi = w_t L_t \tau_t$ is satisfied in each period. The labour response on impact is key to achieving positive comovement between consumption and investment; the inverse Frisch elasticity is of crucial importance.

The intuition behind the main result of this appendix can be summarised as follows. Shocks to the marginal efficiency of investment appear only in the capital law of motion. Capital is a state variable; therefore, it is fixed in the current period. Subsequently, investment shocks are not capable of impacting the existing capital stock in the same period (unlike Hicks-neutral productivity shocks which enter directly into the production function). As a result, it is only possible to increase output in the period of the investment shock by increasing labour input. From the accounting identity ($Y_t = C_t + I_t + \Phi$) it can be seen that it is a necessary condition for output to rise if both consumption and investment are to increase contemporaneously with the shock (i.e. generate positive comovement between labour, output, consumption and investment). Therefore, the immediate increase in labour input must be sufficient to provide the necessary slack in the resource constraint such that both consumption and investment can rise together.

Numerical and analytical expressions are derived which demonstrate when this is achieved with respect to a positive investment shock. Starting from the steady state ($t = ss$) assume there is a small perturbation from equilibrium ($t = ss + 1$), the following must hold for comovement to be achieved:

$$\frac{dC_{ss+1}}{dX_{ss+1}} > 0 \quad \frac{dL_{ss+1}}{dX_{ss+1}} > 0 \quad \frac{dY_{ss+1}}{dX_{ss+1}} > 0 \quad \frac{dI_{ss+1}}{dX_{ss+1}} > 0 \tag{3.24}$$

Once the first two conditions are proved (consumption and labour), the second two follow trivially. The conditions will be dealt with in the aforementioned order, starting with consumption. Manipulating the static labour-leisure optimality condition, it can be shown that an explicit relationship exists between current consumption and current labour input

$$\chi L_t^{\eta} C_t = w_t (1 - \tau_t)$$

$$\chi L_t^{\eta+1} C_t = w_t L_t - \Phi$$

$$\chi L_t^{\eta+1} C_t = (1 - \alpha) K_{t-1}^{\alpha} L_t^{1-\alpha} - \Phi$$

$$C_t = \frac{(1 - \alpha) K_{t-1}^{\alpha} L_t^{1-\alpha} - \Phi}{\chi L_t^{\eta+1}}$$

Evaluate this equation assuming some small perturbation from the steady state ($t = ss + 1$):

$$C_{ss+1} = \frac{(1 - \alpha) K_{ss}^{\alpha} L_{ss+1}^{1-\alpha} - \Phi}{\chi L_{ss+1}^{\eta+1}}$$

Since capital is a state variable it is fixed in the period the shock hits, K_{ss} can be substituted in from Table 3.6:

$$C_{ss+1} = \frac{(1 - \alpha) \left(\frac{\beta\alpha}{1-\beta+\beta\delta}\right)^{\frac{\alpha}{1-\alpha}} L_{ss+1}^{1-\alpha} - \Phi}{\chi L_{ss+1}^{\eta+1}}$$

This yields a one-to-one mapping of consumption to labour in the period of the shock hits. Taking the first derivative of this univariated function and evaluating the sign informs us as to whether consumption is increasing or decreasing (with respect to its steady state) when there is a small change in labour input from its steady state value of unity.[5]

$$\frac{dC_{ss+1}}{dL_{ss+1}} = \underbrace{-\frac{(\eta + \alpha)(1 - \alpha)}{\chi L_{ss+1}^{1+\eta+\alpha}} \left(\frac{\beta\alpha}{1 - \beta + \beta\delta}\right)^{\frac{\alpha}{1-\alpha}}}_{ISE} + \underbrace{\frac{(\eta + 1)\Phi}{\chi L_{ss+1}^{\eta+2}}}_{LWE}$$

It can directly be seen that if the exogenous distortionary sequence Φ is set to zero (the model nests a 'plain-vanilla' RBC with no labour wedge) that consumption will be always fall on impact, with respect to a small change in labour input. Whether

[5] Naturally, this expression only holds in the period of the shock, for $t > ss + 1$ onward the state variable is no longer fixed at its equilibrium value.

consumption increases on impact depends on if the intertemporal substitution effect (ISE) or the labour wedge effect (LWE) dominates. Intertemporal substitution is induced by the shock, consumption is reduced in order to invest more today and to increase future income. However, the rise in output on impact also reduces the labour wedge distortion, as a result, hours worked increase such that there is additional slack in the resource constraint so that consumption can rise.

Since labour is normalised to unity in the steady state through the scaling constant χ, evaluating the derivative in the immediate neighborhood of the steady state gives:

$$\frac{dC_{ss+1}}{dL_{ss+1}} \approx -\frac{(\eta+\alpha)(1-\alpha)}{\chi}\left(\frac{\beta\alpha}{1-\beta+\beta\delta}\right)^{\frac{\alpha}{1-\alpha}} + \frac{(\eta+1)\Phi}{\chi} > 0$$

Rearrange the inequality to make Φ the subject:

$$\Phi > \frac{(\eta+\alpha)(1-\alpha)}{(\eta+1)}\left(\frac{\beta\alpha}{1-\beta+\beta\delta}\right)^{\frac{\alpha}{1-\alpha}} \tag{3.25}$$

The exogenous sequence must be greater than the above expression for consumption to be positive on impact (when labour is increasing on impact with respect to the shock). Additionally, to ensure a steady state exists, Φ must also not be too large (this feasibility condition is derived from the resource constraint):

$$\Phi < \left(\frac{\beta\alpha}{1-\beta+\beta\delta}\right)^{\frac{\alpha}{1-\alpha}} - \delta\left(\frac{\beta\alpha}{1-\beta+\beta\delta}\right)^{\frac{1}{1-\alpha}}$$

Applying benchmark values of $\alpha = 0.35$, $\beta = 0.96$, $\eta = 1$ Figure 3.3 displays the δ-Φ subset of the parameter space. This diagram below shows for which parameter values consumption moves positively with respect to a small change in labour input from the steady state (the grey shaded area is where the labour wedge effect dominates the intertemporal substitution effect). As can be seen, if the exogenous sequence is too small, the labour wedge amplification mechanism is too small, and the intertemporal substitution effect dominates the labour wedge effect; consumption falls with respect to a small increase in labour input (the blue shaded area in figure 4). The white area represent the combinations of δ-Φ where no feasible steady state exists (in this region steady state consumption would have to be negative).

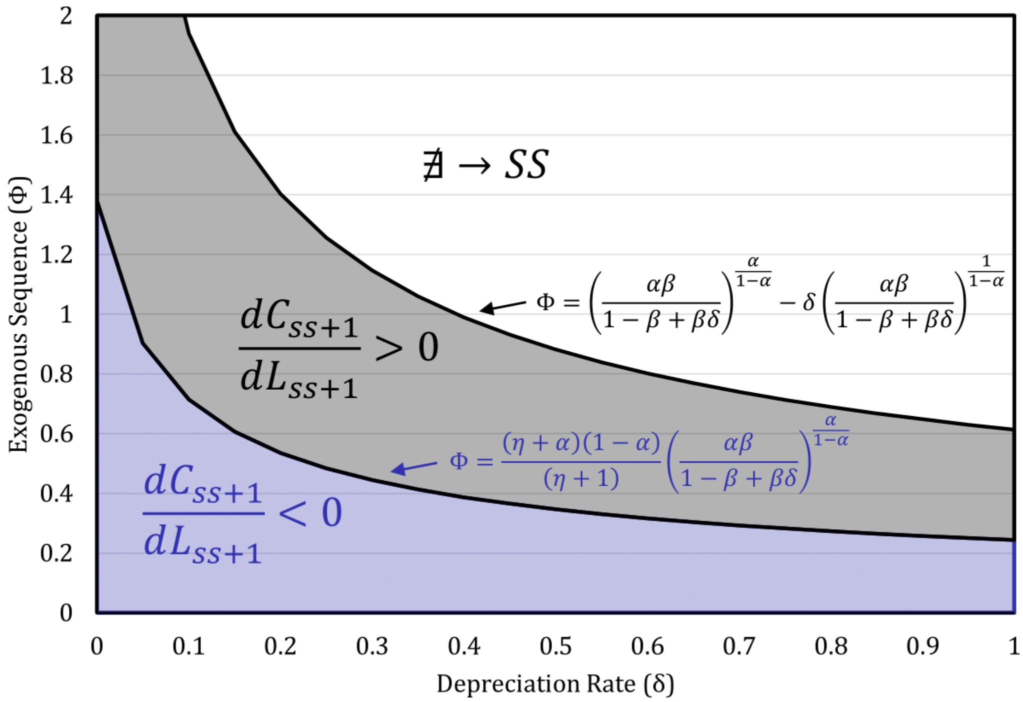

Figure 3.3: Consumption-Leisure static δ-$\varPhi$ parameter space

Therefore, it follows that if $\varPhi$ lies between the following boundaries, consumption will be increasing in labour:

$$\frac{(\eta+\alpha)(1-\alpha)}{(\eta+1)}\left(\frac{\beta\alpha}{1-\beta+\beta\delta}\right)^{\frac{\alpha}{1-\alpha}} < \varPhi < \left(\frac{\beta\alpha}{1-\beta+\beta\delta}\right)^{\frac{\alpha}{1-\alpha}} - \delta\left(\frac{\beta\alpha}{1-\beta+\beta\delta}\right)^{\frac{1}{1-\alpha}} \tag{3.26}$$

However, the main objective is to show that consumption is increasing with respect to the investment shock on impact. Therefore, it must be shown that $\frac{dL_{ss+1}}{dX_{ss+1}} > 0$ also holds, then as a corollary, it proves $\frac{dC_{ss+1}}{dX_{ss+1}} > 0$ since it has already be shown that $\frac{dC_{ss+1}}{dL_{ss+1}} > 0$ is achieved from (3.26).

Given that there is some optimal transition path back to the steady state after an investment shock (and the static consumption-labour relationship has already been utilised) a numerical simulation must provide boundary conditions for when positive intra-temporal substitution towards labour is achieved (shown in Figure 3.4 below).

Investment shocks generally induces intra-temporal substitution towards labour; when investment goods become less costly, leisure is postponed in order to increase income and invest more. Therefore, to show $\frac{dL_{ss+1}}{dX_{ss+1}} > 0$, we simply need δ not to be too large (given reasonable parameter values of α, β, and η). When δ becomes too large it thwarts the intra-temporal substitution channel towards labour as the marginal gain from investment is strictly decreasing with respect to capital depreciation.

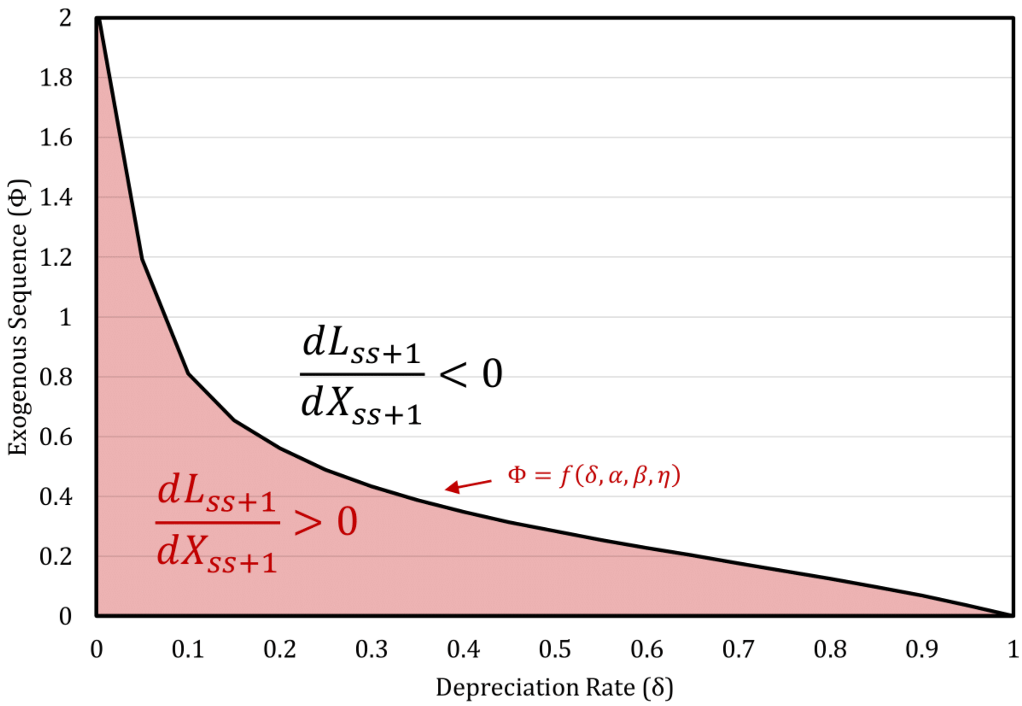

Figure 3.4: Labour-Investment Shock static δ-Φ parameter space

There is no neat analytical expression for this threshold, so a numerical simulation is generated to approximate the boundary condition:

$$\Phi < f(\alpha, \beta, \delta, \eta)$$

Additionally, Φ must be sufficiently small such that it does not induce intra-temporal substitution towards leisure and cause labour to move in the 'wrong'direction on impact of the investment shock. If the exogenous distortion is too large then the labour wedge in the steady state will be large (as shown in Table 3.5). The larger the wedge τ_{ss} the larger the marginal utility of leisure in the neighbourhood of the steady state becomes. This may be such that for an extreme subset of the parameter space a positive investment shock induces a reduction in hours worked.

Examining Figure 3.4 and contrasting it with Figure 3.3, it can be seen that the numerical threshold $\Phi < f(\alpha, \beta, \delta, \eta)$ is strictly below the feasibility constraint threshold $\Phi < \left(\frac{\beta\alpha}{1-\beta+\beta\delta}\right)^{\frac{\alpha}{1-\alpha}} - \delta\left(\frac{\beta\alpha}{1-\beta+\beta\delta}\right)^{\frac{1}{1-\alpha}}$. As this appendix is interested in generating positive comovement on impact such that (3.24) is satisfied; the latter condition becomes superfluous. Laying figure 3.3 and figure 3.4 on top of each other to create a composite image illustrates this argument clearly. Figure 3.5 shows that when the two diagrams are overlaid, there emerges an overlap in the thresholds. In this region of the parameter subspace (the shaded dark red area) positive comovement $\frac{dC_{ss+1}}{dX_{ss+1}} > 0$ and $\frac{dL_{ss+1}}{dX_{ss+1}} > 0$ is achieved.

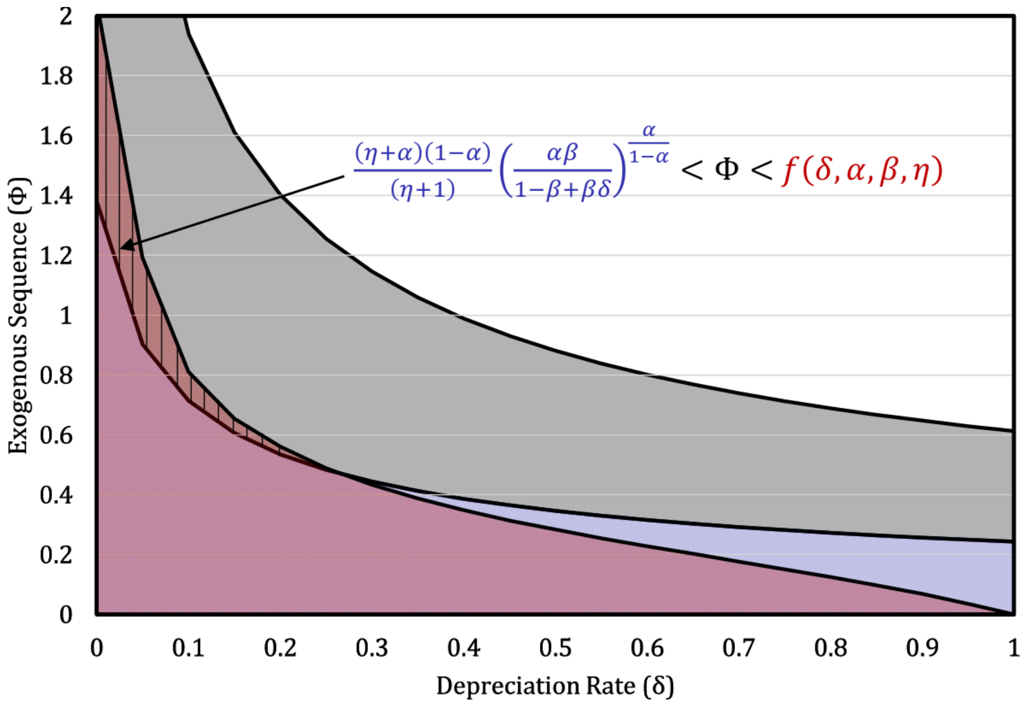

Figure 3.5: Composite of Figure 4 and 5

This composite diagram also shows that, for benchmark values of the discount factor and the labour share, capital depreciation must not exceed approximately 0.26; the point where the two boundaries intersect. Therefore, if $\delta \gtrsim 0.26$, there is no corresponding value for Φ which will generate the amplification in hours worked (as the intra-temporal substitution channel is weakened) such that all cross-correlations on impact in (3.24) are satisfied.

Now it has been shown for what parameter values positive consumption and labour responses are achieved with respect to the shock, it must be shown for the remaining two conditions of output and investment.

Proving $\frac{dY_{ss+1}}{dX_{ss+1}} > 0$ follows trivially from $\frac{dY_{ss+1}}{dL_{ss+1}} > 0$ once numerical simulations ensure that $\frac{dL_{ss+1}}{dX_{ss+1}} > 0$ (as shown). Since capital stock is fixed in the period of the shock, only labour can influence output when the investment shock hits. Additionally, from the Inada condition under Cobb-Douglas we know that the marginal product of capital must always be positive.

$$Y_{ss+1} = K_{ss}^{\alpha} L_{ss+1}^{1-\alpha}$$

$$Y_{ss+1} = \left(\frac{\beta\alpha}{1-\beta+\beta\delta}\right)^{\frac{\alpha}{1-\alpha}} L_{ss+1}^{1-\alpha}$$

Steady state capital is substituted into the production function. Differentiating with respect to labour gives the marginal product of labour on impact of the shock; evaluating in the neighbourhood of the steady state (labour is normalised to unity) yields:

$$\frac{dY_{ss+1}}{dL_{ss+1}} = (1-\alpha)\left(\frac{\beta\alpha}{1-\beta+\beta\delta}\right)^{\frac{\alpha}{1-\alpha}} > 0$$

This proves $\frac{dY_{ss+1}}{dL_{ss+1}} > 0$ and implies $\frac{dY_{ss+1}}{dX_{ss+1}} > 0$. Finally, it must be shown that investment also increases with respect to the shock. This also follows trivially:

$$K_t = (1-\delta)\,K_{t-1} + I_t X_t$$

$$\overbrace{K_{ss+1} - (1-\delta)\left(\frac{\beta\alpha}{1-\beta+\beta\delta}\right)^{\frac{1}{1-\alpha}}}^{Investment} = I_{ss+1}X_{ss+1}$$

It can be observed that on the left hand side of the above equation is investment. Directly it can be seen that $\frac{dI_{ss+1}}{dX_{ss+1}} > 0$. (Unsurprisingly) investment rises on impact relative to its steady state when there is a positive investment shock (capital stock in the next period must rise).

Therefore, it can be concluded that to satisfy all conditions in (24) that Φ must lie between the following threshold (3.27). Given typical values for α, β and η this is achieved provided that δ is not too large.

$$\frac{(\eta+\alpha)(1-\alpha)}{(\eta+1)}\left(\frac{\beta\alpha}{1-\beta+\beta\delta}\right)^{\frac{\alpha}{1-\alpha}} < \Phi < f(\alpha,\beta,\delta,\eta) \tag{3.27}$$

While the inverse Frisch elasticity is set to unity to give logarithmic utility in leisure, allowing η to vary also yields important results. Since the labour response on impact of the shock is the key to resolving the comovement problem in this framework, η plays a very important role. The lower the inverse Frisch elasticity, the stronger the labour response and thus the easier it is to generate the slack needed in the resource constraint for consumption to rise alongside investment in the period of the shock. Figure 3.6 below shows the thresholds generated when the inverse Frisch elasticity is varied. These thresholds are derived from rearranging inequality (25) to make η the subject:

$$\eta < \frac{\alpha\frac{(1-\alpha)}{\Phi}\left(\frac{\beta\alpha}{1-\beta+\beta\delta}\right)^{\frac{\alpha}{1-\alpha}} - 1}{1-\frac{(1-\alpha)}{\Phi}\left(\frac{\beta\alpha}{1-\beta+\beta\delta}\right)^{\frac{\alpha}{1-\alpha}}}$$

With further manipulation, it can be seen that β and δ only enter into the above condition insofar that they determine steady state output $Y_{ss} = \left(\frac{\beta\alpha}{1-\beta+\beta\delta}\right)^{\frac{\alpha}{1-\alpha}}$. Substituting in and rearranging gives a simple and elegant expression where the inverse frisch elasticity is a function of only the capital share α and the exogenous sequence-output ratio Φ/Y_{ss}

$$\eta < \frac{\alpha(1-\alpha)-\frac{\Phi}{Y_{ss}}}{\frac{\Phi}{Y_{ss}}-(1-\alpha)}$$

This states that the inverse frisch elasticity must be below a given threshold to satisfy the conditions in (3.24). The larger the relative distortion to output, the higher the marginal utility of leisure is in the steady state. Therefore, the larger Φ/Y_{ss}, the larger the reduction in the relative size of the labour wedge $\tau_{ss+1}/Y_{ss+1}$and the larger the amplification in the labour response on impact. Unsurprisingly, when η is small, a lower Φ/Y_{ss} is required for positive comovement to occur; since a smaller η directly increases the volatility of the labour response. There exists trade-off between these two parameters in the subset of the parameter space which enables positive comovement (shown in figure 3.6 alongside the labour output elasticity).

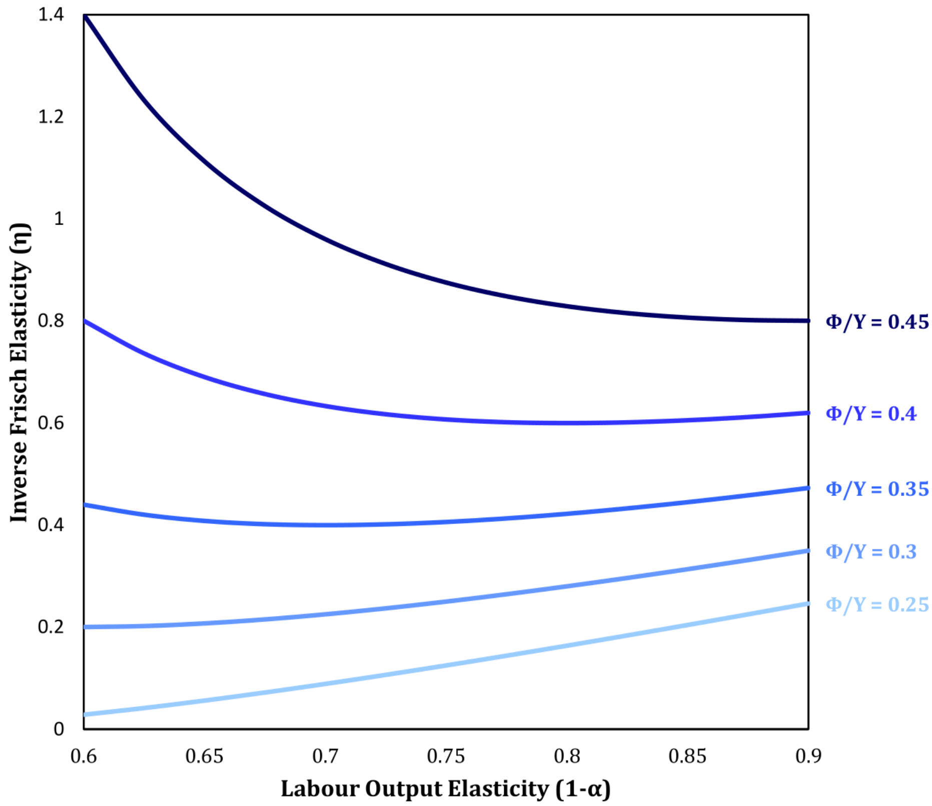

Figure 3.6: Inverse Frisch elasticity thresholds

Therefore, the inclusion of a time-varying labour wedge into a simple RBC framework is capable of generating an empirically recognisable business cycles where consumption, hours, investment and output all rise on impact of an investment shock. Therefore, this framework breaks the "Barro-King Curse", in an extremely simple and tractable framework with analytical proofs; fewer frictions than the New Keynesian model proposed by Ascari, et al (2016)

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