How oviform is the chicken egg? New mathematical insight into the old oomorphological problem

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Abstract

The chicken egg, a major food product, has been, as a model oviform object, the focus of advanced description in mathematical terms, and such a description has practical applications in engineering, industrial and biological disciplines. A precise mathematical description circumscribing major oomorphological characteristics, coupled with their non-destructive measurement, remains, to a certain extent, problematic and has not yet been achieved, hampering their effective control and use. A contour of any chicken egg can be accurately defined with Hügelschäffer's model by means of three main measures: length, L, maximum breadth, B, and a parameter w that corresponds to a distance of shifting the ellipse center to form an egg ovoid. The goal of the study was a comprehensive theoretical evaluation and development of basic geometrical formulae to define egg external traits by using Hügelschäffer's model and employing simulation modelling, digital imaging and image processing. As a result, we deduced novel geometrical formulae for the egg long circumference, C, volume, V, area of a plane curve obtained by the normal/orthogonal projection, A, surface area, S, distance parameter, w, and radius of curvature, R, at any point on the x-axis. For practical use in the poultry industry and food engineering, the proposed formulae can be instrumental in the non-destructive and accurate definition of the external parameters of any chicken egg.

Keywords: Chicken egg quality; Oviform objects; Oomorphology; Egg geometrical parameters; Hügelschäffer's model; Simulation modelling and digital imaging

1 Introduction

A hen's egg is one of the main and traditional food products that is also characterized by the specific and unique shape. Oomorphology implies an accurate and fast estimation of various external parameters of avian eggs. This problem and its solutions are extremely important not only for researchers involved in various studies related to food and poultry science, biology, ecology, etc. (Narushin, 1997), but also for engineers when new specific technologies and equipment are designed for control in incubation of hatching eggs and industrial processing of table eggs (Narushin, Romanov, Lu, Cugley, & Griffin,Narushin, Romanov, Lu, Cugley, & Griffin, 2020 in print). Representing an ideal oviform object, the chicken egg can be basically defined with such geometrical parameters as the egg volume, surface area, long circumference, area of the plane curve obtained by the normal/orthogonal projection, and radius of egg curvature for which a number of mathematical approaches, models and formulae were proposed (e.g., Severa et al., 2013). However, the problem, in our opinion, was that most previous calculation formulae were empirical and based on estimating a particular sample of eggs used in the experiments, or derived from similar geometric shapes, e.g., an ellipse, that impedes their use for controlling egg characteristics.

Recently, we postulated an oviform model that can exclusively and accurately describe the chicken egg (Narushin et al., in print[Instruction: TO DC: Link ref]Narushin, Romanov, et al., 2020). It transpires that a hen's egg can be expressed geometrically in very accurate way using Hügelschäffer's model (as reviewed in Schmidbauer, 1948; Petrovic & Obradovic, 2010; Petrovic et al., 2011; Ferréol, 2017) that is not present so far in standard geometry texts. The model we adopted and improved (Narushin et al., in print[Instruction: TO DC: Link ref Narushin, Romanov, et al., 2020]Narushin, Romanov, et al., 2020) involves direct measurements of egg length, L, maximum breadth, B, and a parameter w meaning a distance between two vertical axes, one of which coincides with B and the other one is crossing the egg at the point of $L/2$:

$$y = \pm\frac{B}{2}\sqrt{\frac{L^2 - 4x^2}{L^2 + 8wx + 4w^2}} \quad (1)$$

Based on this model, the appropriate theoretical formulae for measuring of the egg volume, V, area of the plane curve obtained by the normal/orthogonal projection of a hen's egg, A, and surface area, S, were deduced as follows (Narushin et al., in print[Instruction: TO DC: Link ref]Narushin, Romanov, et al., 2020):

$$V = \frac{\pi B^2}{256w^3}\left(4wL\left(L^2 + 4w^2\right) - \left(L^2 - 4w^2\right)^2 \cdot \ln\left|\frac{L + 2w}{L - 2w}\right|\right) \quad (2)$$

$$A = \frac{BL^2}{9}\left(\frac{1.49}{\sqrt{L^2-2.667wL+4w^2}} + \frac{2}{\sqrt{L^2+4w^2}} + \frac{1.49}{\sqrt{L^2+2.667wL+4w^2}} + \frac{0.943}{\sqrt{L^2-1.333wL+4w^2}} + \frac{0.943}{\sqrt{L^2+1.333wL+4w^2}}\right) \quad (3)$$

$$S = \frac{\pi BL^2}{12}\left(\frac{B}{L^2+4wL+4w^2} - \frac{B}{L^2-4wL+4w^2} + \frac{2\sqrt{3\left(L^2-2wL+4w^2\right)^3+B^2\left(5wL-L^2-4w^2\right)^2}}{\left(L^2-2wL+4w^2\right)^2} + \frac{2\sqrt{3\left(L^2+2wL+4w^2\right)^3+B^2\left(5wL+L^2+4w^2\right)^2}}{\left(L^2+2wL+4w^2\right)^2} + \frac{2\sqrt{\left(L^2+4w^2\right)^3+4B^2L^2w^2}}{\left(L^2+4w^2\right)^2}\right) \quad (4)$$

Although the above equations would be satisfactory for direct computational analysis, they appear to be insufficient and too complicated for further processing and deriving other relevant mathematical formulae. We have faced such disadvantages in Eqns (2)–(4) when (i) recalculating the values of w using the measured data of V, (ii) defining an expression of recalculating S from the values of V, (iii) transforming an egg profile into a well-known geometrical figure with the same value of A, and (iv) taking partial derivatives of the obtained functions for solving applied problems in the study of functions.

There is a further demand pertaining to egg-related studies involving shell strength assessment, egg incubation and processing technology development, etc., which recommends the development of mathematical formulae for estimating the long circumference, C, and radius of egg shape curvature, R. Available equations from the field of classic geometry for defining C and R in curves and the mathematical expression of Hügelschäffer's model would suggest a feasibility of their calculation, but this seems too complicated and tedious for practical use. Maulana et al. (2015) also used Hügelschäffer's model in evaluating the volume of egg-shaped solids and proposed to use numerical calculation of other parameters, e.g. the surface area, due to the difficulties in their analytical definitions. Recently, we developed a geometrical transformation method for non-destructive evaluation of the volume and surface area of avian eggs using 2-D imaging technique and simulation modelling (Narushin et al., 2020Narushin, Lu, Cugley, Romanov, & Griffin, 2020[Instruction: TO DC: Link ref Narushin, Lu, et al., 2020]) that is also applicable in this kind of research.

The objectives of the present study were therefore to (1) extend our previous work calculating Hügelschäffer's model and thence simplify the theoretical equations for evaluating the egg volume, V, area of the plane curve obtained by the normal/orthogonal projection, A, and surface area, S, and define the following functions: $w = f(V)$ and $S = f(V)$, and (2) deduce valid formulae for measuring the long circumference, C, and radius of curvature, R, at any point of the function as can be expressed using Hügelschäffer's model. In the course of the mathematical calculation, coupled with employing simulation modelling, the 2-D digital imaging and image processing, we inferred the desired equations for calculating and controlling the above sought-for external parameters that comprehensively and accurately circumscribe such an oviform object as chicken egg.

2 Methodology

2.1 Long circumference

The formula for estimating the length of any curve including the egg long circumference, C, is known as follows (see, for example, Lambers, 2009):

$$C = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \tag{5}$$

In our case of the egg contours, considering two equal parts of the curve above and below x-axis, the long circumference can be presented with the following formula:

$$C = 2\int_{-\frac{L}{2}}^{\frac{L}{2}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \tag{6}$$

or taking into account Eqn (1):

$$C = 2\int_{-\frac{L}{2}}^{\frac{L}{2}} \sqrt{1 + \frac{4B^2\left(4wx^2 + \left(L^2 + 4w^2\right)x + wL^2\right)^2}{\left(L^2 + 8wx + 4w^2\right)^3\left(L^2 - 4x^2\right)}}dx \tag{7}$$

The integral in Eqn (7) cannot be resolved using classic methods; therefore, a universal formula from Simpson's rule (Recktenwald, 2000) can be used. The subsequent recalculations of the C value (as presented in more detail in Supplementary Data A) were resulted in the following equation:

$$\begin{aligned} C = L\Bigg(& \sqrt{0.01 + \frac{0.19B^2}{L^2-3.6wL+4w^2}} + \sqrt{0.01 + \frac{0.19B^2}{L^2+3.6wL+4w^2}} + \\ & + \frac{3}{20}\sqrt{1 + \frac{4.3B^2\left(L^2-4Lw+4w^2\right)^2}{\left(L^2-3.6Lw+4w^2\right)^3}} + \\ & + \frac{3}{5}\sqrt{1 + \frac{B^2\left(L^2-5.3Lw+4w^2\right)^2}{3.9\left(L^2-1.8Lw+4w^2\right)^3}} + \frac{3}{5}\sqrt{1 + \frac{B^2\left(L^2+5.3Lw+4w^2\right)^2}{3.9\left(L^2+1.8Lw+4w^2\right)^3}} + \\ & + \frac{3}{10}\sqrt{1 + \frac{4B^2L^2w^2}{\left(L^2+4w^2\right)^3}} + \frac{3}{20}\sqrt{1 + \frac{4.3B^2\left(L^2+4Lw+4w^2\right)^2}{\left(L^2+3.6Lw+4w^2\right)^3}}\Bigg) \end{aligned} \tag{8}$$

Because the obtained formula (Eqn (8)) is rather huge, tedious and thus not suitable for practical calculations, the undergoing procedure of its simplification was applied using the method of simulation modelling of egg parameters. In particular, each ratio in Eqn (8) was multiplied and divided by the same value of L in any power, which enabled receiving the following expression (for more detail see Supplementary Data A):

$$\begin{aligned} C = L\Bigg(& \sqrt{0.01 + \frac{0.19SI^2}{1-3.6\frac{w}{L}+4\left(\frac{w}{L}\right)^2}} + \sqrt{0.01 + \frac{0.19SI^2}{1+3.6\frac{w}{L}+4\left(\frac{w}{L}\right)^2}} + \\ & + \frac{3}{20}\sqrt{1 + \frac{4.3SI^2\left(1-4\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^2}{\left(1-3.6\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^3}} + \frac{3}{5}\sqrt{1 + \frac{SI^2\left(1-5.3\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^2}{3.9\left(1-1.8\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^3}} + \\ & + \frac{3}{5}\sqrt{1 + \frac{SI^2\left(1+5.3\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^2}{3.9\left(1+1.8\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^3}} + \frac{3}{10}\sqrt{1 + \frac{4SI^2\left(\frac{w}{L}\right)^2}{\left(1+4\left(\frac{w}{L}\right)^2\right)^3}} + \\ & + \frac{3}{20}\sqrt{1 + \frac{4.3SI^2\left(1+4\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^2}{\left(1+3.6\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^3}}\Bigg) \end{aligned} \tag{9}$$

where *SI* is the egg shape index that equals to B to L ratio.

Eqn (9) can be adduced as

$$C = L \cdot K_C \tag{10}$$

where K_C is a coefficient that in our case equal to:

$$\begin{aligned} K_C = &\sqrt{0.01 + \frac{0.19SI^2}{1-3.6\frac{w}{L}+4\left(\frac{w}{L}\right)^2}} + \sqrt{0.01 + \frac{0.19SI^2}{1+3.6\frac{w}{L}+4\left(\frac{w}{L}\right)^2}} + \\ &+\frac{3}{20}\sqrt{1 + \frac{4.3SI^2\left(1-4\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^2}{\left(1-3.6\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^3}} + \frac{3}{5}\sqrt{1 + \frac{SI^2\left(1-5.3\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^2}{3.9\left(1-1.8\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^3}} + \\ &+\frac{3}{5}\sqrt{1 + \frac{SI^2\left(1+5.3\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^2}{3.9\left(1+1.8\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^3}} + \frac{3}{10}\sqrt{1 + \frac{4SI^2\left(\frac{w}{L}\right)^2}{\left(1+4\left(\frac{w}{L}\right)^2\right)^3}} + \\ &+\frac{3}{20}\sqrt{1 + \frac{4.3SI^2\left(1+4\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^2}{\left(1+3.6\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^3}} \end{aligned} \tag{11}$$

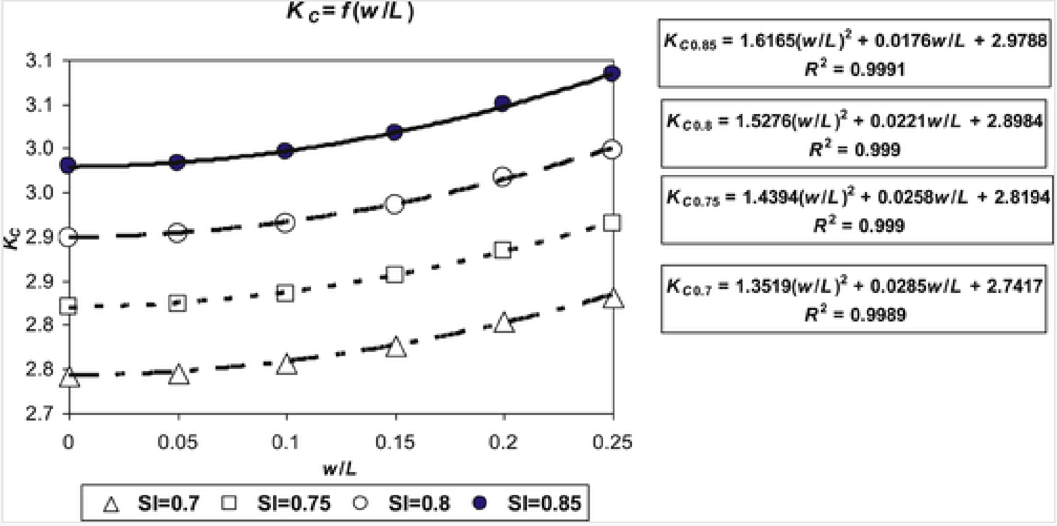
Next, the method of simulation modelling that enabled to take into consideration all the possible variety of sizes of chicken eggs was employed for further simplifying the formulae for both C and other major egg external traits (see sections 2.2 and 2.3).

Studying the geometric variants of Hügelschäffer's model resulted from the parameter w changing in a wide range from 0 to ∞, Obradovic et al. (2013) demonstrated a possible variability of the ovoid and limits to which the resulting ovoids can correspond to the shape of a bird's egg. On the basis of this analysis, we assumed that a range of w/L = [0 ... 0.25] would correspond to any avian egg. Additionally, possible values of the shape index were chosen within a range of SI = [0.70 ... 0.85] that is mostly typical for any hen's egg (e.g., Romanoff & Romanoff, 1949).

Accordingly substituting w/L in Eqn (11) with a series of values 0, 0.05, 0.1, 0.15, 0.2 and 0.25, and SI with 0.7, 0.75, 0.8 and 0.85, the appropriate graphs were plotted by approximating with square polynomials (Fig. 1).

alt-text: Fig. 1

Fig. 1



Graphic dependences of the coefficient K_C from the values of w/L and SI.

All the approximating regressions are of the same type which can generally be expressed as follows:

$$K_C = a\left(\frac{w}{L}\right)^2 + b\frac{w}{L} + c \quad (12)$$

where a, b and c are coefficients shown in Table 1 for each value of SI.

alt-text: Table 1

Table 1

The table layout displayed in this section is not how it will appear in the final version. The representation below is solely purposed for providing corrections to the table. To preview the actual presentation of the table, please view the Proof.

Values of the coefficients from Eqn (12) for each value of SI.

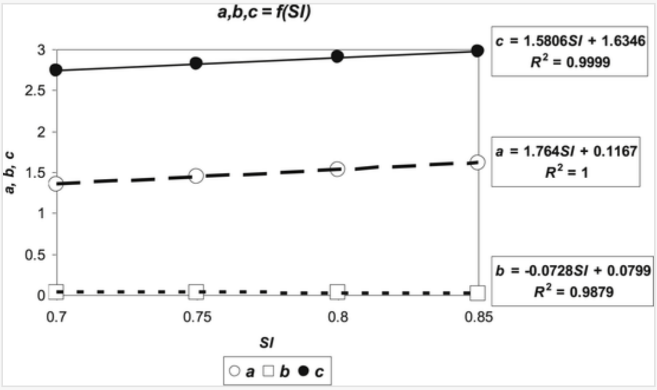
Shape index, *SI*	**Coefficients in Eqn (12)**		
	a	**b**	**c**
0.7	1.3519	0.0285	2.7417
0.75	1.4394	0.0258	2.8194

0.8	1.5276	0.0221	2.8984
0.85	1.6165	0.0176	2.9788
Average	1.484	0.024	2.860

Subsequently, each coefficient was approximated with a dependence $f(SI)$ (Fig. 2).

alt-text: Fig. 2

Fig. 2



Approximations of the coefficients a, b and c from Eqn (12) with corresponding functions $f(SI)$.

Then:

$$K_C = \left(1.764\frac{B}{L} + 0.117\right)\cdot\left(\frac{w}{L}\right)^2 - \left(0.073\frac{B}{L} - 0.08\right)\cdot\frac{w}{L} + 1.581\frac{B}{L} + 1.635 =$$
$$= \frac{(1.764B+0.117L)w^2}{L^3} - \frac{(0.073B-0.08L)w}{L^2} + \frac{1.581B+1.635L}{L} =$$
$$= \frac{(1.764B+0.117L)w^2-(0.073B-0.08L)Lw+(1.581B+1.635L)L^2}{L^3} \quad (13)$$

Whence, taking into account Eqn (10), we finally obtained:

$$C = \frac{(1.764B + 0.117L)\,w^2 - (0.073B - 0.08L)\,Lw + (1.581B + 1.635L)\,L^2}{L^2} \quad (14)$$

Due to rather small values of the coefficient b (Table 1), we can simplify Eqn (14) even further by using the average value $b = 0.024$. Then, the long circumference equation can be written in a simpler form:

$$C = \frac{(1.764B + 0.117L)\,w^2 + 0.024Lw + (1.581B + 1.635L)\,L^2}{L^2} \quad (15)$$

2.2 Simplifying formulae for egg volume, area of its projection and surface area

As was explained in the Introduction, the theoretical formulae for the calculation of the egg volume, V (Eqn (2)), area of the plane curve obtained by the egg normal/orthogonal projection, A (Eqn (3)), and the egg surface area, S (Eqn (4)), are too complicated and not useful in some kind of recalculations and/or applications. To simplify these, we used the same principle as for Eqn (8) for estimating the egg long circumference, C.

After the mathematical transformations presented in more detail in Supplementary Data B, the following equations were obtained:

$$V = \frac{\pi B^2 L}{256} \cdot \frac{w}{L}\left(4 \cdot \frac{L}{w}\left(\left(\frac{L}{w}\right)^2 + 4\right) - \left(\left(\frac{L}{w}\right)^2 - 4\right)^2 \cdot \ln\left|\frac{\frac{L}{w} + 2}{\frac{L}{w} - 2}\right|\right) \tag{16}$$

Previous Version

$$A = \frac{BL}{9}\left(\frac{1.49}{\sqrt{1 - 2.667\frac{w}{L} + 4\left(\frac{w}{L}\right)^2}} + \frac{2}{\sqrt{1 + 4\left(\frac{w}{L}\right)^2}} + \frac{1.49}{\sqrt{1 + 2.667\frac{w}{L} + 4\left(\frac{w}{L}\right)^2}} + + \frac{0.943}{\sqrt{1 - 1.333\frac{w}{L} + 4\left(\frac{w}{L}\right)^2}} + \frac{}{\sqrt{1}}\right.$$

Updated Version

$$A = \frac{BL}{9}\left(\frac{1.49}{\sqrt{1 - 2.667\frac{w}{L} + 4\left(\frac{w}{L}\right)^2}} + \frac{2}{\sqrt{1 + 4\left(\frac{w}{L}\right)^2}} + \frac{1.49}{\sqrt{1 + 2.667\frac{w}{L} + 4\left(\frac{w}{L}\right)^2}} + \frac{0.943}{\sqrt{1 - 1.333\frac{w}{L} + 4\left(\frac{w}{L}\right)^2}} + \frac{}{\sqrt{1 +}}\right. \tag{17}$$

$$S = \frac{\pi B L^2}{12}\left(-\frac{8BLw}{\left(L^2-4w^2\right)^2} + \frac{2\sqrt{3\left(L^2-2wL+4w^2\right)^3+B^2\left(5wL-L^2-4w^2\right)^2}}{\left(L^2-2wL+4w^2\right)^2} + \right.$$
$$\left. + \frac{2\sqrt{3\left(L^2+2wL+4w^2\right)^3+B^2\left(5wL+L^2+4w^2\right)^2}}{\left(L^2+2wL+4w^2\right)^2} + \frac{2\sqrt{\left(L^2+4w^2\right)^3+4B^2L^2w^2}}{\left(L^2+4w^2\right)^2}\right) \tag{18}$$

The obtained formulae (Eqns(16)–(18)) can be reconsidered as follows:

$$V = \frac{\pi B^2 L}{256} \cdot K_V \tag{19}$$

$$A = \frac{BL}{9} \cdot K_A \tag{20}$$

$$S = \frac{\pi BL}{12} \cdot K_S \tag{21}$$

Where the correspondent coefficients are equal to:

$$K_V = \frac{w}{L}\left(4 \cdot \frac{L}{w}\left(\left(\frac{L}{w}\right)^2 + 4\right) - \left(\left(\frac{L}{w}\right)^2 - 4\right)^2 \cdot \ln\left|\frac{\frac{L}{w}+2}{\frac{L}{w}-2}\right|\right) \tag{22}$$

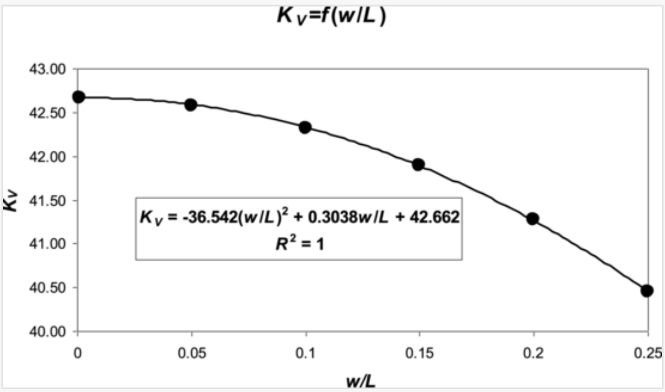
$$K_A = \frac{1.49}{\sqrt{1-2.667\frac{w}{L}+4\left(\frac{w}{L}\right)^2}} + \frac{2}{\sqrt{1+4\left(\frac{w}{L}\right)^2}} + \frac{1.49}{\sqrt{1+2.667\frac{w}{L}+4\left(\frac{w}{L}\right)^2}} + \frac{0.943}{\sqrt{1-1.333\frac{w}{L}+4\left(\frac{w}{L}\right)^2}} + \frac{0.943}{\sqrt{1+1.333\frac{w}{L}+4\left(\frac{w}{L}\right)^2}} \tag{23}$$

$$K_S = \frac{SI}{1+4\frac{w}{L}+4\left(\frac{w}{L}\right)^2} - \frac{SI}{1-4\frac{w}{L}+4\left(\frac{w}{L}\right)^2} + \frac{2\sqrt{3\left(1-2\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^3+SI^2\left(5\frac{w}{L}-1-4\left(\frac{w}{L}\right)^2\right)^2}}{\left(1-2\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^2} + \frac{2\sqrt{3\left(1+2\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^3+SI^2\left(5\frac{w}{L}+1+4\left(\frac{w}{L}\right)^2\right)^2}}{\left(1+2\frac{w}{L}+4\left(\frac{w}{L}\right)^2\right)^2} + \frac{2\sqrt{\left(1+4\left(\frac{w}{L}\right)^2\right)^3+4SI^2\left(\frac{w}{L}\right)^2}}{\left(1+4\left(\frac{w}{L}\right)^2\right)^2} \tag{24}$$

Furthermore, the similar simulation procedure and substitutions of the possible ranges of w/L and SI as we undertook for the simplification of Eqn (11) were performed leading to dependences graphically plotted in the diagrams shown in Figs. 3–5, each approximated with corresponding square polynomials.

alt-text: Fig. 3

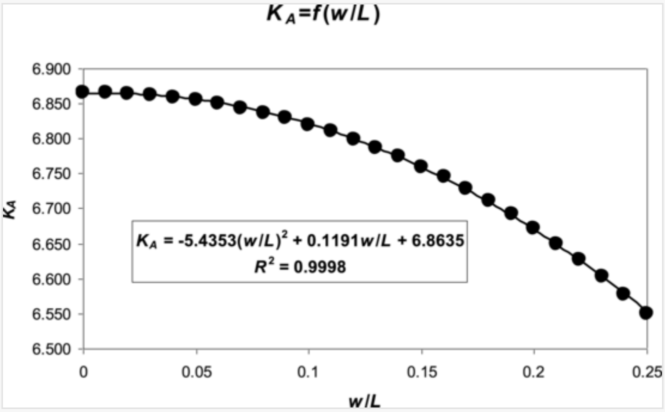
Fig. 3



Graphic dependence of the coefficient K_V from the values of w/L.

alt-text: Fig. 4

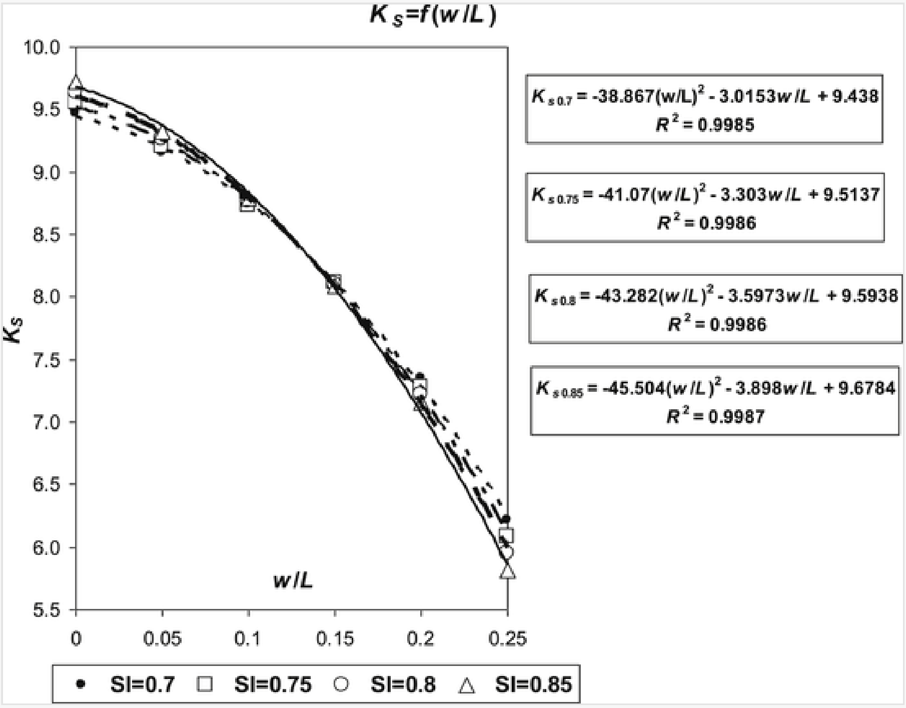
Fig. 4



Graphic dependence of the coefficient K_A from the values of w/L.

alt-text: Fig. 5

Fig. 5



Graphic dependences of the coefficient K_S from the values of w/L and SI.

Ultimately, we can produce the following simpler and proper formulae for computing the egg volume, V, and area of the plane curve obtained by the normal/orthogonal projection, A:

$$V = \frac{\pi B^2 L}{256} \cdot \left(42.662 + 0.3038\frac{w}{L} - 36.542\left(\frac{w}{L}\right)^2\right) = \frac{0.5233B^2\left(L^2 + 0.0071Lw - 0.8565w^2\right)}{L} \tag{25}$$

$$A = \frac{BL}{9} \cdot \left(-5.44\left(\frac{w}{L}\right)^2 + 0.12\frac{w}{L} + 6.86\right) = \frac{0.76B}{L} \cdot \left(L^2 + 0.02wL - 0.8w^2\right) \tag{26}$$

Considering Eqn (24), all the approximating regressions are of the same type that is generally expressed as follows:

$$K_S = a(w/L)^2 + bw/L + c \quad (27)$$

where a, b and c are coefficients shown in Table 2 for each value of SI.

alt-text: Table 2

Table 2

The table layout displayed in this section is not how it will appear in the final version. The representation below is solely purposed for providing corrections to the table. To preview the actual presentation of the table, please view the Proof.

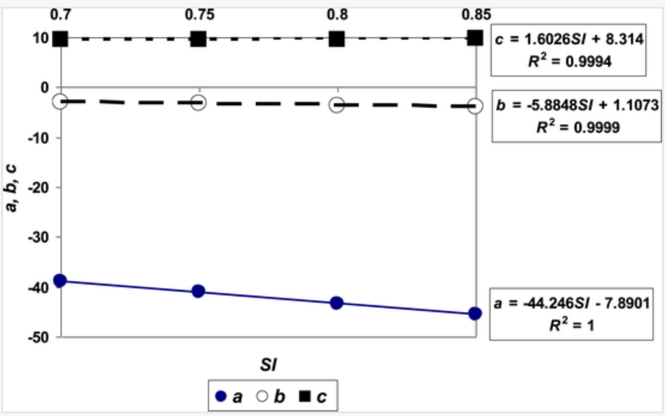
Values of the coefficients from Eqn (27) for each value of SI.

Shape index, *SI*	**Coefficients in Eqn (27)**		
	a	**b**	**c**
0.7	−38.867	−3.0153	9.438
0.75	−41.07	−3.303	9.5137
0.8	−43.282	−3.5973	9.5938
0.85	−45.504	−3.898	9.6784
Average	−42.18	−3.45	9.56

Furthermore, each coefficient in Table 2 was approximated with a dependence $f(SI)$ (Fig. 6).

alt-text: Fig. 6

Fig. 6



Approximations of the coefficients a, b and c from Eqn (27) with corresponding functions $f(SI)$.

Consequently:

$$K_S = \left(-44.25 \cdot \frac{B}{L} - 7.89\right) \cdot \left(\frac{w}{L}\right)^2 - 3.45 \cdot \frac{w}{L} + 9.56 = \\ = \frac{-44.25Bw^2 - 7.89Lw^2 - 3.45L^2w + 9.56L^3}{L^3} \quad (28)$$

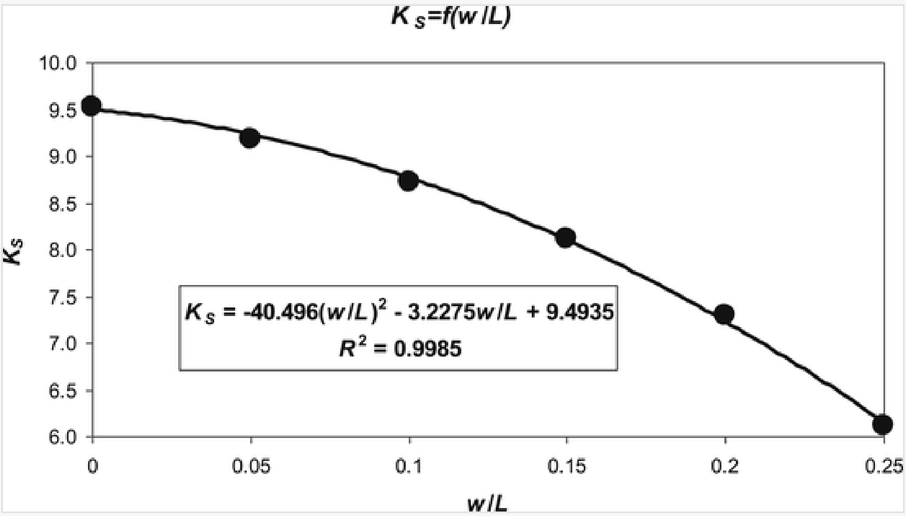
Where from we can finally infer a simpler and accurate equation for computing the egg surface area, S:

$$S = \frac{\pi BL}{12} \cdot \frac{-44.25Bw^2 - 7.89Lw^2 - 3.45L^2w + 9.56L^3}{L^3} = \\ = \frac{2.48B}{L^2} \cdot \left(L^2 (L - 0.36w) - 0.83w^2 (L + 5.61B)\right) \quad (29)$$

We noticed that all the diagrams in Fig. 5 almost coincided with each other. Therefore, we suggested to check an accuracy of recalculating S if we would choose only the average value of SI that, according to Romanoff and Romanoff (1949), equals to 0.737. This was resulted in the mathematical dependence presented in Fig. 7.

alt-text: Fig. 7

Fig. 7



Graphic dependences of the coefficient C_S from the values of w/L when $SI = 0.737$.

Thereafter:

$$S = \frac{\pi BL}{12} \cdot \frac{-40.5w^2 - 3.23Lw + 9.45L^2}{L^2} = \frac{2.48B}{L} \cdot \left(L^2 - 0.34Lw - 4.27w^2\right) \quad (30)$$

Generation of the simplified formulae for V (Eqn (25)) and S (Eqn (29)) made it possible to define the functions $w = f(V)$ and $S = f(V)$, as these definitions were among the goals of this study.

The formula for the recalculation of w from the values of the egg volume was deduced from Eqn (25):

$$w = 0.00415L + \frac{\sqrt{1.1675L^2B^2 - 2.2311LV}}{B} \quad (31)$$

The estimation of the function $S = f(V)$ was grounded on the theoretical basis of Narushin,[Instruction: TO DC: Link ref] Lu, et al. (2020)~~Narushin et al. (2020)~~:

$$S = k \cdot V^{\frac{2}{3}} \quad (32)$$

where k is a function of certain egg parameters.

As from Eqn (32):

$$k = \frac{S}{V^{\frac{2}{3}}} \quad (33)$$

and considering Eqns (25) and (30):

$$k = \frac{\frac{2.5B}{L^2} \cdot \left(L^2\left(L - 0.36w\right) - 0.83w^2\left(L + 5.61B\right)\right)}{\left(\frac{0.5233B^2\left(L^2 + 0.0071Lw - 0.8565w^2\right)}{L}\right)^{\frac{2}{3}}} \quad (34)$$

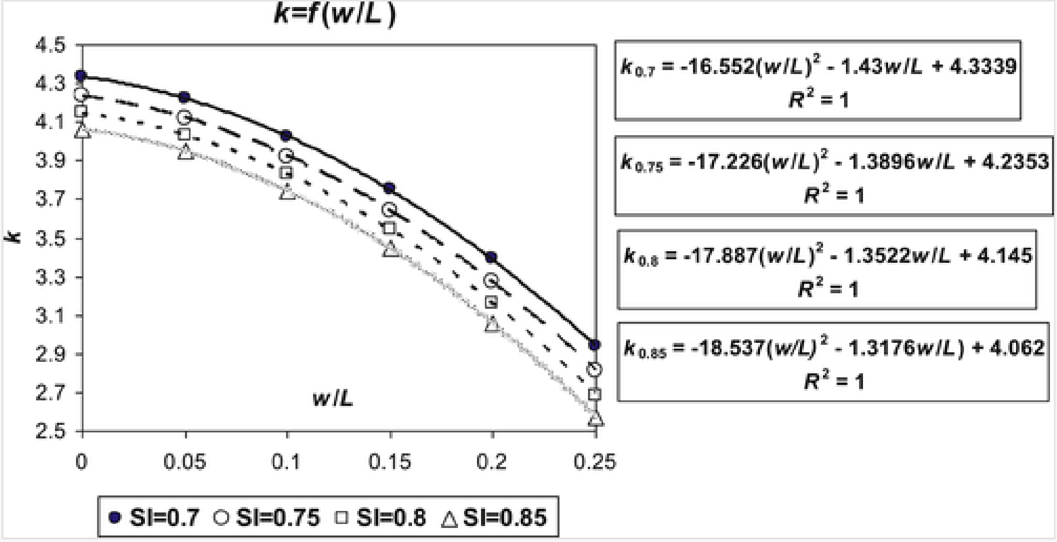
this was transformed into the following expression:

$$k = \frac{2.5SI \cdot \left(\left(1 - 0.36\frac{w}{L}\right) - 0.83\left(\frac{w}{L}\right)^2 \cdot \left(1 + 5.61SI\right)\right)}{\left(0.5233SI^2 \cdot \left(1 + 0.0071\frac{w}{L} - 0.8565\left(\frac{w}{L}\right)^2\right)\right)^{\frac{2}{3}}} \quad (35)$$

The same intervals for w/L and SI were used in the simulation processing of Eqn (35) that resulted in the diagrams and approximating formulae shown in Fig. 8.

alt-text: Fig. 8

Fig. 8



Graphic dependences of the coefficient k from the values of w/L and SI.

The produced approximating regressions were related to the square polynomial types, e.g., Eqn (27), and the values of the coefficients a, b and c were computed for each value of SI as shown in Table 3.

alt-text: Table 3

Table 3

The table layout displayed in this section is not how it will appear in the final version. The representation below is solely purposed for providing corrections to the table. To preview the actual presentation of the table, please view the Proof.

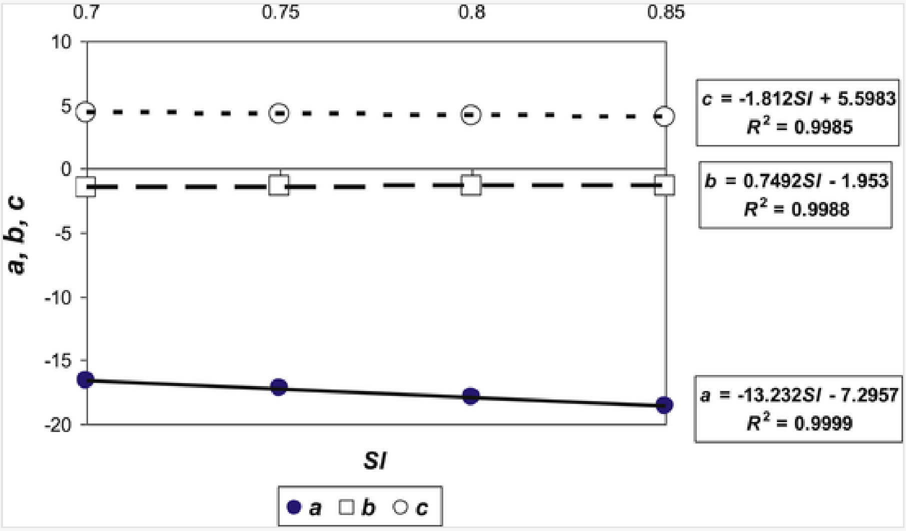
Values of the coefficients a, b and c for each value of SI.

Shape index, *SI*	**Coefficients**		
	a	**b**	**c**
0.7	−16.552	−1.430	4.3339
0.75	−17.226	−1.3896	4.2353
0.8	−17.887	−1.3522	4.145
0.85	−18.537	−1.3176	4.062
Average	−17.551	−1.372	4.194

Then, each coefficient was approximated with a dependence $f(SI)$ (Fig. 9).

alt-text: Fig. 9

Fig. 9



Approximations of the coefficients a, b and c with corresponding functions $f(SI)$.

Thereafter, we can transform Eqn (35) into the following formula:

$$k = -\left(13.232\frac{B}{L} + 7.296\right) \cdot \left(\frac{w}{L}\right)^2 + \left(0.749\frac{B}{L} - 1.953\right) \cdot \frac{w}{L} - 1.812\frac{B}{L} + 5.598 \quad (36)$$

Where from:

$$S = \left(5.598 - 1.812\frac{B}{L} + \left(0.749\frac{B}{L} - 1.953\right) \cdot \frac{w}{L} - \left(13.232\frac{B}{L} + 7.296\right) \cdot \left(\frac{w}{L}\right)^2\right) \cdot V^{\frac{2}{3}} \quad (37)$$

Taking into account that we can use the obtained simplified Eqns (25), (26) and (30) to solve problems of finding the extremum and determining the areas of functions' increase and decrease by taking partial derivatives, we assessed the effect of the parameter w on the egg volume, V, and its surface area, S. This problem may be relevant to poultry selection practice to understand what the relationship of w should be with other parameters included in Eqn (30) to ensure, for example, the maximum volume or minimum surface area of the egg. For this purpose, we applied the partial derivative method using ∂w.

$$\frac{\partial V}{\partial w} = 0.0037B^2 - \frac{0.8964B^2}{L}w \quad (38)$$

$$\frac{\partial S}{\partial w} = -0.84B - \frac{21.18B}{L}w \quad (39)$$

Equating the obtained derivatives (Eqns (38) and (39)) to zero, we have: (a) in the case of V, $w = 0.004L > 0$ for any positive value of L, and (b) with respect to S, $w = -0.04L < 0$ for any positive value of L.

Given that when the first derivative >0, the function increases, and when the first derivative <0, the function decreases (HEC Montréal, 2020), it can be asserted that with increasing w, the values of V lower, while the values of S, on the contrary, grow.

2.3 Radius of egg shape curvature

There is the following formula for estimating the radius, R, of any smooth curve (as, for example, shown in Hobson, 2002):

$$R = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} \quad (40)$$

In our previous study [Instruction: TO DC: Link ref](~~Narushin et al., in print~~Narushin, Romanov, et al., 2020), we deduced the formula of the first derivative of the Hügelschäffer's model as follows:

$$\frac{dy}{dx} = -2B \cdot \frac{4wx^2 + \left(L^2 + 4w^2\right)x + wL^2}{\left(L^2 + 8wx + 4w^2\right)\sqrt{\left(L^2 + 8wx + 4w^2\right)\left(L^2 - 4x^2\right)}} \quad (41)$$

which appeared to be rather difficult for both definition of a second derivative and its usage for further mathematical transformations of Eqn (40). Therefore, the following steps of its simplification by simulation were performed:

$$\frac{dy}{dx} = -2\frac{B}{L} \cdot \frac{4\frac{w}{L} \cdot \left(\frac{x}{L}\right)^2 + \left(1 + 4\left(\frac{w}{L}\right)^2\right) \cdot \frac{x}{L} + \frac{w}{L}}{\left(1 + 8\frac{w}{L} \cdot \frac{x}{L} + 4\left(\frac{w}{L}\right)^2\right)\sqrt{\left(1 + 8\frac{w}{L} \cdot \frac{x}{L} + 4\left(\frac{w}{L}\right)^2\right) \cdot \left(1 - 4\left(\frac{x}{L}\right)^2\right)}} \quad (42)$$

Similar to our previous transformations, Eqn (42) was reconsidered as:

$$\frac{dy}{dx} = -2\frac{B}{L} \cdot K_{dy/dx} \quad (43)$$

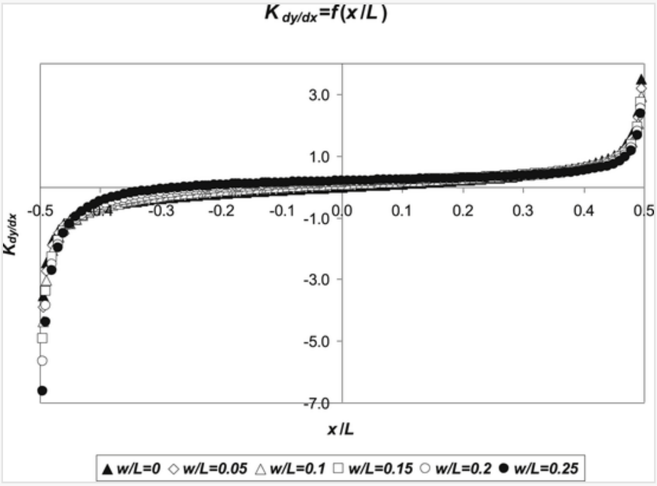
Where:

$$K_{dy/dx} = \frac{4\frac{w}{L} \cdot \left(\frac{x}{L}\right)^2 + \left(1 + 4\left(\frac{w}{L}\right)^2\right) \cdot \frac{x}{L} + \frac{w}{L}}{\left(1 + 8\frac{w}{L} \cdot \frac{x}{L} + 4\left(\frac{w}{L}\right)^2\right)\sqrt{\left(1 + 8\frac{w}{L} \cdot \frac{x}{L} + 4\left(\frac{w}{L}\right)^2\right) \cdot \left(1 - 4\left(\frac{x}{L}\right)^2\right)}} \quad (44)$$

It was shown in [Instruction: TO DC: Link ref Narushin, Romanov, et al. (2020)]Narushin et al., (in print)Narushin, Romanov, et al. (2020) that the egg shape changes over its x-axis from $-L/2$ to $L/2$, wherefrom the possible variations of x/L are [−1/2; 1/2]. Inputting the data of this interval into Eqn (44), the graphs in Fig. 10 were plotted for different values of w/L.

alt-text: Fig. 10

Fig. 10



Graphic dependences of the coefficient $K_{dy/dx}$ from x/L with different values of w/L.

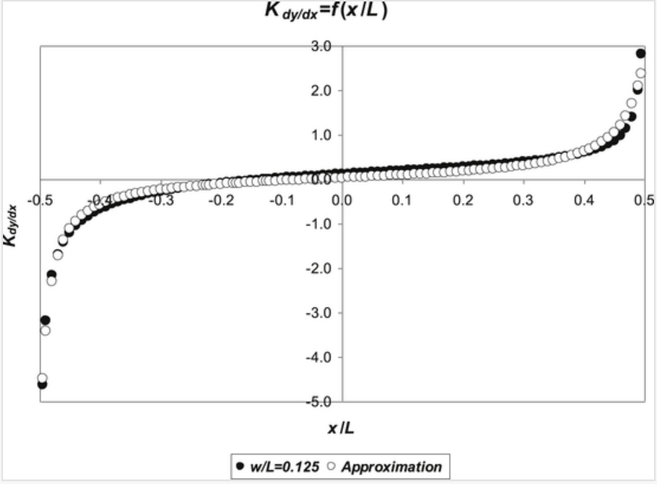
As can be seen in Fig. 10, the diagrams with different values of w/L are located very close to each other and almost coincide. Therefore, we assumed that the value of w does not cause significant effect on Eqn (44) and accept the average data when $w/L = 0.125$. The corresponding diagram is presented in Fig. 11 (black line) that was approximated with a simpler formula:

$$K_{dy/dx} = \frac{0.03 + 0.62\frac{x}{L}}{1 + 0.08\frac{x}{L} - 3.66\frac{x^2}{L^2}} \tag{45}$$

which is also shown in Fig. 11 (pink line).

alt-text: Fig. 11

Fig. 11



Graphic dependences of the coefficient $K_{dy/dx}$ from x/L when $w/L = 0.125$ (black line) and its approximation with Eqn (26) (pink line). (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

Eqn (45) can be easily transformed into the following one:

$$K_{dy/dx} = \frac{L\,(0.03L + 0.62x)}{L^2 + 0.08Lx - 3.66x^2} \quad (46)$$

Where from:

$$\frac{dy}{dx} = -2\frac{B}{L}\cdot\frac{L\,(0.03L + 0.62x)}{L^2 + 0.08Lx - 3.66x^2} = 0.34B\cdot\frac{x + 0.05L}{x^2 - 0.02Lx - 0.27L^2} \quad (47)$$

The second derivative d^2y/dx^2 was estimated from Eqn (47):

$$\frac{d^2y}{dx^2} = 0.34B\cdot\frac{x^2 - 0.02Lx - 0.27L^2 - (2x - 0.02L)(x + 0.05L)}{(x^2 - 0.02Lx - 0.27L^2)^2} = $$
$$= -0.34B\cdot\frac{x^2 + 0.1Lx + 0.27L^2}{(x^2 - 0.02Lx - 0.27L^2)^2} \quad (48)$$

Taking into account Eqns (41) and (47), Eqn (40) was transformed into the following formula to compute the radius of curvature, R:

Previous Version

$$R=\frac{\left(1+\frac{0.12B^{2}(x+0.05L)^{2}}{\left(x^{2}-0.02Lx-0.27L^{2}\right)^{2}}\right)^{\frac{3}{2}}}{-0.34B\cdot\frac{x^{2}+0.1Lx+0.27L^{2}}{\left(x^{2}-0.02Lx-0.27L^{2}\right)^{2}}}=$$

$$=\frac{\sqrt{x^{2}-0.02Lx-0.27L^{2})x+0.05L)(x+0.05L)(x+0.05L)^{2})^{3}}}{0.34B\left(x^{2}+0.1Lx+0.27L^{2}\right)\left(0.27L^{2}+0.02Lx-x^{2}\right)}$$

Updated Version

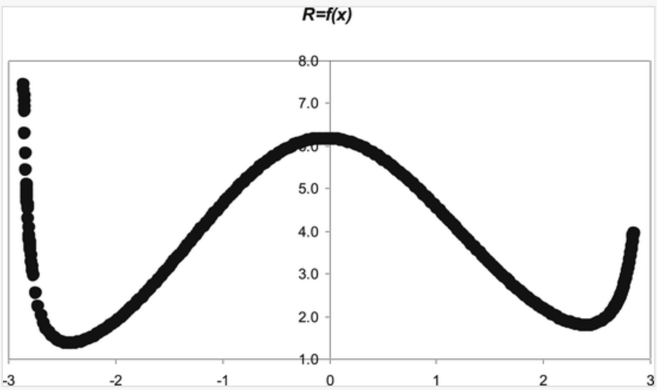
$$R=\frac{\left(1+\frac{0.12B^{2}(x+0.05L)^{2}}{\left(x^{2}-0.02Lx-0.27L^{2}\right)^{2}}\right)^{\frac{3}{2}}}{-0.34B\cdot\frac{x^{2}+0.1Lx+0.27L^{2}}{\left(x^{2}-0.02Lx-0.27L^{2}\right)^{2}}}==\frac{\sqrt{\left(\left(x^{2}-0.02Lx-0.27L^{2}\right)^{2}+0.12B^{2}(x+0.05L)^{2}\right)^{3}}}{0.34B\left(x^{2}+0.1Lx+0.27L^{2}\right)\left(0.27L^{2}+0.02Lx-x^{2}\right)}$$

(49)

The diagram in Fig. 12 shows how the R values are changing over the profile of a typical ("standard") hen's egg (Romanoff & Romanoff, 1949) in which L = 5.7 cm and B = 4.2 cm.

alt-text: Fig. 12

Fig. 12



Graphic dependence on the variation of the radius of egg curvature, R, over the profile of the standard hen's egg, where the values of x are the parts of egg length, L.

Thus, thanks to the above simplification and simulation procedures, we newly produced the more suitable and accurate formulae for computing the following major oomorphological characteristics: long circumference, C (Eqn (15)), volume, V (Eqn (25)), area of the plane curve obtained by the normal/orthogonal projection, A (Eqn (26)), surface area, S (Eqn (29)), distance parameter, w (Eqn (31)), and radius of curvature, R (Eqn (49)). All these equations include three variables: two of them, egg length, L, and maximum breadth, B, can be measured experimentally and directly, while the distance w can be calculated with the appropriate formula (31).

3 Materials and measurements

This study was a part of the GCRF-funded project at the University of Kent, Canterbury, UK, with the materials and methods being previously described in detail in ~~Narushin et al. (2020)~~Narushin, L[Instruction: TO DC: Link ref]u, et al. (2020) and ~~Narushin et al., (in print)~~Narushin, Romanov, [Instruction: TO DC: Link ref]et al. (2020).

Briefly, 40 fresh chicken eggs were used for direct measuring the length, L, and maximum breadth, B, with a Vernier caliper and the volume, V, by Archimedes' principle and, then, examined with the 2-D digital imaging and subsequent image processing techniques [Instruction: TO DC: Link ref](Narushin et al., 2020Narushin, Lu, et al., 2020). This approach allowed producing digital images of each egg. The images of the eggs were then processed using MatLab in order to compute the following egg parameters: egg long circumference, C, egg volume, V, area of the plane curve obtained by the egg normal/orthogonal projection, A, surface area, S, and distance w. The computation was undertaken similar to the procedure as described in more detail in Zlatev et al. (2018).

As we performed the approximation of theoretical equations with simpler simulation models, the accuracy of each formula deduced in the theoretical part of our study was estimated with the correlation coefficient, R, and approximating mean percentage error, ε (e.g., Makridakis et al., 1982):

$$\varepsilon = \frac{1}{n} \cdot \sum_{1}^{n} \left| \frac{v_1 - v_2}{v_1} \right| \cdot 100\% \tag{50}$$

where n is a number of samples in the calculations, and v_1 and v_2 are the values of a corresponding egg parameter defined with a direct measurement and calculated with a theoretical equation. In our case, the approximating mean percentage error, ε, enabled to estimate the results better than the usual statistical error of the calculations. The same estimation procedure was applied when two theoretical formulae were compared.

4 Results

4.1 Egg long circumference

After a sample of the 40 chicken eggs were directly measured to obtain the length, L, maximum breadth, B, and volume, V, they were subjected to the 2-D digital imaging followed by image processing. The results of the theoretical calculations of the egg long circumference using Eqn (8) (a full formula), C_t, and Eqn (15) (a simplified one), C_s, were compared with the computations based on the 2-D images, C_i, as can be seen in Table 4.

alt-text: Table 4

Table 4

The table layout displayed in this section is not how it will appear in the final version. The representation below is solely purposed for providing corrections to the table. To preview the actual presentation of the table, please view the Proof.

Data of different measurements of the egg long circumference (all in cm).

Parameters	C_i	C_t	C_s
Max. value	17.52	17.28	16.95
Min. value	15.43	15.32	15.39
Mean	16.33	16.06	16.07

Standard deviation	0.55	0.52	0.43
Correlation with C_i		0.927	0.967
Approximating mean percentage error, ε, considering C_i (%)		1.55	1.57

As a result, a high correlation was observed between the theoretical calculations and those derived from the 2-D images (0.927 and 0.967, respectively), and the mean percentage error, ε, was low (1.55 and 1.57, respectively; Table 4).

4.2 Egg volume

The data of the direct measurement of the egg volumes, V, results of its computation using the 2-D images, V_i, and calculations by theoretical formula (Eqn (2)), V_t, and simplified formula (Eqn (25)), V_s are provided in Table 5.

alt-text: Table 5

Table 5

The table layout displayed in this section is not how it will appear in the final version. The representation below is solely purposed for providing corrections to the table. To preview the actual presentation of the table, please view the Proof.

Data of different measurements of the egg volume (all in cm^3).

Parameters	V	V_i	V_t	V_s
Max. value	63.63	63.42	64.24	64.24
Min. value	47.94	46.88	49.12	49.12
Mean	55.83	54.72	55.33	55.33
Standard deviation	3.94	4.35	4.32	4.32
Correlation with V		0.967	0.962	0.962
Approximating mean percentage error, ε, considering V (%)		2.21	1.73	1.73

The computed values of egg volume were highly correlated with the actual measurement data, the correlation coefficient being 0.962 to 0.967 and the mean percentage error, ε, being 1.73–2.21% (Table 5).

Furthermore, one of the goals to simplify the formula for estimating the egg volume was a feasibility to recalculate the parameter w. In our previous study [Instruction: TO DC: link ref](~~Narushin et al., in print~~Narushin, Romanov, et al., 2020), w was determined through the 2-D imaging and adjusting theoretically defined contours of the investigated eggs to the actual ones. So, these data of w were compared to the w values calculated by Eqn (31) using the measurements of such egg variables as L, B and V. The correlation coefficient, R, between these two w estimates was equal to 0.994, the approximation error, ε, being 5.76%.

4.3 The area of the plane curve obtained by the normal/orthogonal projection

Similar to the approach used for analyzing the surface area, the data of the theoretical calculations of area of the plane curve obtained by the normal/orthogonal projection using Eqn (3) (a full form), A_t, and Eqn (26) (a simplified formula), A_s, were compared with the results of the computations using the 2-D images, A_i, and are shown in Table 6.

alt-text: Table 6

Table 6

The table layout displayed in this section is not how it will appear in the final version. The representation below is solely purposed for providing corrections to the table. To preview the actual presentation of the table, please view the Proof.

Data of different measurements of area of the egg plane obtained by the curve normal/orthogonal projection (all in cm^2).

Parameters	A_i	A_t	A_s
Max. value	20.90	20.50	20.57
Min. value	17.09	16.92	17.07
Mean	19.01	18.42	18.56
Standard deviation	1.00	1.05	0.97
Correlation with A_i		0.964	0.977
Approximating mean percentage error, ε, considering A_i (%)		3.10	2.43

Values of the calculated A_t and A_s, were highly correlated with the A_i data from the 2-D images (0.964 and 0.977, respectively), with the mean percentage error, ε, being relatively low (3.10 and 2.43, respectively; Table 6).

4.4 Egg surface area

Considering that there is no any accurate direct method for measuring the egg surface area, the conformation of calculations can be proved by examining the computation accuracy of the egg volume because these two parameters are closely related (Narushin, 1997; Narushin, Lu, et al., 2020[Instruction: TO DC: Link ref] Narushin et al., 2020). We showed above (Table 5) that the 2-D imaging appeared to be a useful tool for the accurate calculation of the egg volume and, thus, we can assume that the same would be true when applying the imaging analysis for the egg surface area. Therefore, the results of its theoretical calculations using Eqn (4), S_t, simplified formulae of Eqn (29), S_{s1}, and Eqn (30), S_{s2}, and values $S(V)$ based on the measurements of the egg volume (Eqn (37)) were compared with the data of the computations based on the 2-D images, S_i, and are given in Table 7.

alt-text: Table 7

Table 7

The table layout displayed in this section is not how it will appear in the final version. The representation below is solely purposed for providing corrections to the table. To preview the actual presentation of the table, please view the Proof.

Data of different measurements of the egg surface area (all in cm^2).

Parameters	S_i	S_t	S_{s1}	S_{s2}	$S(V)$
Max. value	65.67	66.87	67.15	66.64	66.554
Min. value	53.69	55.43	55.51	55.10	54.645
Mean	59.71	60.31	60.45	60.00	60.88
Standard deviation	3.15	3.33	2.70	3.28	3.03
Correlation with S_i		0.969	0.974	0.974	0.967
Approximating mean percentage error, ε, considering S_i (%)		1.28	1.37	0.98	1.99

The generated values of S_t, S_{s1}, S_{s2} and $S(V)$ demonstrated a very high correlation with S_i (0.967–0.974) and a lower mean percentage error, ε (0.98–1.99%; Table 7).

5 Discussion

Defining the contours of chicken eggs is of a specific importance for applications related to technological development in engineering, food industry, and poultry science. As we demonstrated in the previous study (~~Narushin et al., in print~~[Instruction: TO DC: Link ref]Narushin, Romanov, et al., 2020), these contours can be fairly accurately described by Hügelschäffer's model. Nevertheless, the derivation of three basic equations for the egg volume, V, area of the plane curve obtained by the normal/orthogonal projection, A, and surface area, S, using Hügelschäffer's model were resulted in unreasonably cumbersome formulae for practical application. Moreover, for a complete description of this geometric figure, the oviform ovoid, the set of equations obtained should be supplemented with the calculation formulae of the long circumference, C, and radius of curvature, R, at any point on the egg contour. As a result of this study, we resolved these oomorphological problems and newly introduced the respective mathematical equations for the oviform objects exemplified by the chicken egg. Simpler mathematical dependencies also open up wide opportunities for further mathematical research including easier differentiation and/or integration of these dependencies, which will enable optimization of geometrical variables and their proportions, thereby making breeders' work more efficiently on selecting for eggs of a given shape.

In addition to deriving the required formulae, we showed that the calculation equations can be greatly simplified if we use the simulation modelling approach as well as such an egg external characteristic as the shape index, *SI*, that typically changes for chicken eggs in the range of [0.70 ... 0.85]. Indeed, this particular type of bird eggs is of maximum interest for solving a number of applied problems in the food industry and commercial poultry farming.

Our studies suggest that the applicability of these theoretical formulae for practical purposes because the approximation error for almost all parameters did not exceed 3%, and, in most cases, was significantly lower, which is sufficient for acceptable accuracy of mathematical calculations.

In a comparative analysis of the accuracy of the derived formulae, we based on the outcome of our previous studies [Instruction: TO DC: link ref](Narushin et al., 2020Narushin, Lu, et al., 2020) in which the external parameters of chicken eggs were measured using the 2-D machine vision technique. To understand how much one could trust the accuracy of egg image processing, we compared the data of direct egg volume measurements and their conversion values from digital pixels to the actual sizes measured by Archimedes' principle. The produced accuracy corresponded to 2.21%, with $R = 0.967$ (Table 5), that enabled us to conclude that results of the 2-D machine vision-assisted analysis can be used for estimating the other egg parameters, since for most of them direct measurement methods are not available. This measurement accuracy is also consistent with the results of other researchers who used machine vision techniques to measure the basic geometric parameters of bird eggs (Chan et al., 2018; Soltani et al., 2015; Zhang et al., 2016; Zhou et al., 2009; Zlatev et al., 2018).

In our previous work [Instruction: TO DC: Link ref](Narushin et al., in printNarushin, Romanov, et al., 2020), we were convinced that the determination of the parameter w by ordinary linear measurements might be difficult. Therefore, sometimes, when there would be no restriction that an egg can be immersed in water to measure the egg volume, V, analytically by Archimedes' principle, the parameter w can be computed using Eqn (31). Although the recalculated values of w appeared to be not too much accurate (>5% error), this seems rather sufficient for rough estimations.

As there are no proper methods for direct measurements of the radius of non-circular curves, we assumed that if all other deduced theoretical formulae are correct and correspond to the true values of the hen's eggs, Eqn (47) might be adequate for the estimation of the radius of Hügelschäffer's ovoid. The evaluation of the curvature and radius of a complex geometrical figure may be difficult due to the complicity of a calculative formula. Even for such typical and well-studied geometrical figure as an ellipse, these equations can be given in a parametric type only. Therefore, our proposed solution to define the radius of egg curvature at any point of its profile would facilitate deeper studies of the shell strength and some other egg physical characteristics.

Ultimately, such an oviform as the chicken egg that can be exclusively described by Hügelschäffer's model is not present yet in geometric reference books. Hoping that this gap can now be filled in, we herein derived the formulae for calculating the major external parameters for this oviform that can be used in a variety of engineering, industrial and biological applications.

6 Conclusion

To conclude, we examined the egg shape figure described with Hügelschäffer's model that can be characterized using the following newly inferred geometrical formulae:

Long circumference, C:

$$C = \frac{(1.764B + 0.117L)\,w^2 + 0.024Lw + (1.581B + 1.635L)\,L^2}{L^2} \tag{51}$$

Egg volume, V:

$$V = \frac{0.5233B^2\left(L^2 + 0.0071Lw - 0.8565w^2\right)}{L} \tag{52}$$

Area of the plane curve obtained by the normal/orthogonal projection of the hen's egg, A:

$$A = \frac{0.76B}{L} \cdot \left(L^2 + 0.02wL - 0.8w^2\right) \tag{53}$$

Surface area, S:

$$S = \frac{2.48B}{L} \cdot \left(L^2 - 0.34Lw - 4.27w^2\right) \tag{54}$$

Distance parameter, w:

$$w = 0.00415L + \frac{\sqrt{1.1675L^2B^2 - 2.2311LV}}{B} \tag{55}$$

Radius of curvature, R, at any point on the x-axis:

Previous Version

$$R = \frac{\sqrt{x^2 - 0.02Lx - 0.27L^2 \Big) x + 0.05L \Big) \left(x + 0.05L\right)(x + 0.05L)^2 \big)^3}}{0.34B\left(x^2 + 0.1Lx + 0.27L^2\right)\left(0.27L^2 + 0.02Lx - x^2\right)}$$

Updated Version

$$R = \frac{\sqrt{\left(\left(x^2 - 0.02Lx - 0.27L^2\right)^2 + 0.12B^2(x + 0.05L)^2\right)^3}}{0.34B\left(x^2 + 0.1Lx + 0.27L^2\right)\left(0.27L^2 + 0.02Lx - x^2\right)} \tag{56}$$

The above formulae can be successfully used for accurate estimating and controlling the parameters of the hen's eggs and be included in reference books for geometric figures in which, along with formulae for calculating such rotation figures as sphere, ellipsoid and others, an egg-shaped ovoid according to the Hügelschäffer's model can be added. In terms of practical use, the proposed formulae can be instrumental in non-destructive and accurate defining and controlling the external parameters of any chicken egg in the research and development relevant to poultry industry and food engineering.

Declaration of competing interest

The authors declare no competing financial interest.

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Appendix A Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.foodcont.2020.107484.

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The corrections made in this section will be reviewed and approved by a journal production editor. The newly added/removed references and its citations will be reordered and rearranged by the production team.

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Highlights

- Egg is a top food product and oviform object model needed for precise description.
- For this purpose, we used Hügelschäffer's model, simulation and digital imaging.
- Novel formulae were deduced for main external egg quality traits.
- These can be instrumental in non-destructive control of external egg parameters.

Appendix A Supplementary data

The following are the Supplementary data to this article:

Multimedia Component 1

Multimedia component 1

alt-text: Multimedia component 1

Multimedia Component 2

Multimedia component 2

alt-text: Multimedia component 2