Digital imaging assisted geometry of chicken eggs using Hügelschäffer's model

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Abstract

Geometrical description of the egg shape is of a great importance in a variety of studies and can be instrumental in predicting quality traits of table and hatching poultry eggs. Importantly, developments of non-destructive oomorphological models can drive novel insights in engineering and physical science and lead to new egg-related technologies and egg sorting systems for poultry industry. We attempted to test the Hügelschäffer's egg model according to which an egg profile curve can be transformed from an ellipse using a specific parameter *w*. For this purpose, two-dimensional digital imaging and follow-up image processing techniques of chicken eggs were employed. The formulae for recalculation of the egg volume and surface area were consequently deduced from the Hügelschäffer's equation. Eventually, we refined the Hügelschäffer's egg model and proved its applicability for defining the contours of hen's eggs. For practical use in poultry industry and food engineering, the proposed non-destructive methodology can be contributory in defining accurately the contour of any avian egg and determining such characteristics of the egg shape as volume, surface area, etc., with an expected potential in designing automated systems in poultry industry and in egg-related applications in biology, physical science, engineering and other areas.

Keywords: Egg quality; Non-destructive measurements; Egg volume and surface area; Hügelschäffer's model; Digital imaging; Image processing

1. Introduction

The basic principles of biosystems engineering are the analysis, design, and control of biologically-based systems (Alocilja, 2013). All these require an accurate and acceptable methodological foundation that would allow the implementation of analytical procedures with complex and variable biological objects. One of these objects is a poultry egg. Despite all the efforts of breeders and geneticists to breed chickens laying highly identical eggs, the variety of their shapes and sizes continues to amaze and create difficulties for scientists involved in egg-related research. These include poultry researchers focused on optimising egg incubation conditions, food scientists involved in egg processing and agricultural engineers that develop optimised technologies and equipment for egg production, incubation and processing. By virtue of an extraordinary biological diversity of the shapes and sizes of bird eggs, a specific term, oomorphology, was introduced.

Oomorphology has been a focal attraction and theme in biological, physical and engineering research due to the following reasons:

1. *Competent scientific description of a biological object*. If we manage to describe each egg with a general mathematical formula, the methodical work of researchers involved in the field of biological systematics, optimisation of technological parameters, egg incubation and selection of poultry will be greatly simplified. In this case, to distinguish one egg from another will be as simple as, say, a sphere from an ellipsoid.
2. *Accurate and simple determination of the physical characteristics of a biological object*. The external properties of the egg are extremely important for researchers and engineers who develop technologies for incubating, processing, storing and sorting eggs. Currently, the main parameter used for these purposes is the egg mass. However, in many instances, there is a need to identify and use egg volume, surface

area, radius of curvature and other indicators that are not difficult to measure if there is a defined mathematical formula for describing the contours of the egg.

3. *Biologically inspired engineering*. In terms of applications in bionics, the egg can be a suitable biological system found in nature to be studied in design of engineering systems and state-of-the-art technologies. It is not without reason that the egg-shaped geometric figure is adopted in architecture and construction as well as in shallow shell and spudcan constructions because it can withstand maximum loads with a minimum consumption of materials (Lazarus et al., 2012; Maulana et al., 2015; Zhang et al., 2017a; Zhang et al., 2017b; Zhang et al., 2019; Guo et al., 2020). Thus, the study of oomorphological parameters positively influences not only agricultural engineering and technology, but also other relevant specialties.

Description of avian eggs and their shapes in mathematical terms (see for review Smart, 1991) has been stirring the minds of many scholars involved not only in poultry science and ornithology, but also in engineering, architecture, construction, decoration, fashion design, vessel manufacturing, etc. If the appropriate equation were to be deduced, it would be rather straightforward to recalculate such parameters of the avian egg as its volume, surface area, curvature, and perimeter. In turn, this could give an opportunity to compare the shapes of different eggs, clutches and species defined with simple mathematical indices, which can be computerised and are easily processed within any analytical investigation. Developments of non-destructive oomorphological models assisted by such modern techniques as, for example, two-dimensional (2-D) digital imaging, have strong potential in research and industrial applications, and particularly in design of technological solutions and automated production systems including egg sorting machines.

2. Theory

2.1. Geometry of an avian egg

Most authors involved in egg geometry research have leant towards a similarity of various avian eggs that can be described as an ellipsoid of a slightly distorted shape. Indeed, if we consider a plane curve obtained by the normal/orthogonal projection of an avian egg, this can be imagined as an ellipse with a shifted vertical axis along the horizontal one. Preston (1953) was one of the first scholars to propose a modification of the ellipse equation to make it closer to the shape of the egg. He introduced a linear formula in the parametric equation of the ellipse. Later, Smart (1967) made this formula a bit more complicated, and Carter (1968) modified it in such a way that a scale of the long axis of the ellipse was pulled out towards one pole and compressed towards the other one.

Reviewing the studies on mathematical transformations of the ellipse equation, Köller (2000) showed that the classic formula of an ellipse, which is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \tag{1}$$

where the parameters a and b are called lengths of axes, easily transforms into the shape of the egg if one uses a function $t(x)$, by means of which each y becomes larger on the right side and smaller on the left side, so the ellipse is transformed into a curve which resembles an egg shape.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}\cdot t(x)=1 \tag{2}$$

Köller (2000) also proposed three simple functions $t(x)$ for the above linear distortion of the ellipse. These functions and the corresponding transformations of the ellipse into the egg contour are shown in Figure 1.

If we consider the main dimensions of the egg contour as the length, L, and the maximum breadth, B, so that the variables a and b from Eq. (1) are equal to

$$a = \frac{L}{2} \text{ and } b = \frac{B}{2},$$

and apply the Köller's functions $t(x)$, the corresponding formulae for the egg contour can be expressed by the following three functions:

1) $\frac{x^2}{(L/2)^2} + \frac{y^2}{(B/2)^2} \cdot (1 + 0.2x) = 1$

or

$$y = \pm \frac{B}{2L} \sqrt{\frac{L^2 - 4x^2}{1 + 0.2x}} \quad (3)$$

2) $\frac{x^2}{(L/2)^2} + \frac{y^2}{(B/2)^2 \cdot (1 - 0.2x)} = 1$

or

$$y = \pm \frac{B}{2L} \sqrt{1 - 0.2x} \cdot \sqrt{L^2 - 4x^2} \quad (4)$$

3) $\frac{x^2}{(L/2)^2} + \frac{y^2}{(B/2)^2} \cdot e^{0.2x} = 1$

or

$$y = \pm \frac{B}{2L} \sqrt{\frac{L^2 - 4x^2}{e^{0.2x}}} \quad (5)$$

If we now represent Köller's (2000) three functions, Eqs. (3)–(5), in a graphic form (Fig. 2), one can notice that the appropriate curves are rather similar and almost coincide between x values of -1 and 1.

The Pearson correlation coefficients r between all the three functions are high: 0.953, between $t_1(x) = 1 + 0.2x$ and $t_2(x) = 1/(1 - 0.2x)$; 0.991, between $t_1(x) = 1 + 0.2x$ and $t_3(x) = e^{0.2x}$; and 0.985, between $t_2(x) = 1/(1 - 0.2x)$ and $t_3(x) = e^{0.2x}$. Taking into account these high correlations and the similarity between the three functions, let us assume that Eqs. (4) and (5) give the same result as Eq. (3) and, therefore, only this function, *i.e*., Eq. (3), may be considered further.

To check if Eq. (3) is valid for an actual avian egg, it should fit the following condition:

$$y_{\max} = \frac{B}{2}, \quad (6)$$

where $y_{\max}$ can be estimated by equating a derivative of Eq. (3) to zero; afterwards, we should input the data of x into the obtained formula. The related mathematical calculations shown in Supplementary Data A suggested that the transformation of the ellipse with the functions $t_1(x)$, $t_2(x)$ and $t_3(x)$ would lead to an incorrect outcome and cannot be applied for description of an actual egg.

Cook (2018) proposed a more universal function of $t(x)$, which is

$$t(x) = 1 + kx, \quad (7)$$

where k is a coefficient that is determined experimentally depending on the shape of an actual egg.

Then, the egg contour can be defined with the following equation:

$$y = \pm \frac{B}{2L}\sqrt{\frac{L^2 - 4x^2}{1 + kx}}. \quad (8)$$

As was also shown by Cook (2018), for any k the width of the egg, B, corresponds to the midpoint of the length at which y equals to 0, and that is not the maximum height of the egg.

In our case, y_0 can be calculated after inputting x=0 into Eq. (8):

$$y_0 = \frac{B}{2L}\sqrt{\frac{L^2 - 0}{1 + 0}} = \frac{BL}{2L} = \frac{B}{2}.$$

Thus, the egg breadth, B, is not the maximum value but appears at the midpoint of L, and, therefore, Cook's formula has the same drawback as the one described by Köller (2000).

2.2. Hügelschäffer's egg model

Petrovic and Obradovic (2010) and Petrovic et al. (2011) drew attention to the scientific heritage of the German mathematician Fritz Hügelschäffer who proposed an egg curve by employing the process of transformation of an ellipse into an egg contour and shifting the minor circle along the *x*-axis from the concentric position to the egg's blunt end by a specific distance designated as the parameter *w* (as reviewed in Schmidbauer, 1948; Ferréol, 2017). Hügelschäffer's transformation model is shown in detail in Figure 3 adopted from Petrovic et al. (2011).

Petrovic and Obradovic (2010) deduced an equation for *t(x)*, which is defined as

$$t(x) = 1 + \frac{2wx + w^2}{a^2}, \tag{9}$$

where *w* is the difference between the distance from the narrowed point to the maximum breadth axis and the value of *a*.

Considering Eq. (9), the final formula for Hügelschäffer's egg contour will be as follows:

$$\frac{x^2}{(L/2)^2} + \frac{y^2}{(B/2)^2} \cdot \left(1 + \frac{2wx + w^2}{(L/2)^2}\right) = 1. \tag{10}$$

This can be rewritten in a more suitable form:

$$y = \pm \frac{B}{2}\sqrt{\frac{L^2 - 4x^2}{L^2 + 8wx + 4w^2}}. \tag{11}$$

To test if the Hügelschäffer's formula has no drawbacks like other examined functions *t(x)* and, therefore, has no limitation to its practical use, the value of $x = -w$, which corresponds to $y_{\max}$ (Fig. 3), was put into Eq. (11):

$$y_{\max} = \pm \frac{B}{2}\sqrt{\frac{L^2 - 4w^2}{L^2 - 8w^2 + 4w^2}} = \pm \frac{B}{2}\sqrt{\frac{L^2 - 4w^2}{L^2 - 4w^2}} = \pm \frac{B}{2}.$$

The obtained condition can be applied to an actual egg, so Eq. (11) is valid and useful for empirical calculations in avian egg studies.

For practical use of Eq. (11), the formulae for measuring the area of the plane curve obtained by the normal/orthogonal projection of a hen's egg, *A*, volume, *V*, and surface area, *S*, of an ovoid resulted from the revolution of the Hügelschäffer's egg contour, can be recalculated as follows:

1. Area of the plane curve:

$$A \approx \frac{BL^2}{9}\left(\frac{1.49}{\sqrt{L^2-2.667wL+4w^2}}+\frac{2}{\sqrt{L^2+4w^2}}+\frac{1.49}{\sqrt{L^2+2.667wL+4w^2}}+\right.$$

$$\left.+\frac{0.943}{\sqrt{L^2-1.333wL+4w^2}}+\frac{0.943}{\sqrt{L^2+1.333wL+4w^2}}\right). \qquad (12)$$

2. Volume:

$$V=\frac{\pi B^2}{256w^3}\left(4wL(L^2+4w^2)-(L^2-4w^2)^2\cdot\ln\left|\frac{L+2w}{L-2w}\right|\right). \qquad (13)$$

Notably, the above formula is very close to the one deduced by Maulana et al. (2015) who evaluated egg-shaped solids obtained by rotating the Hügelschäffer's egg-shaped curve relative to the *x*-axis.

3. Surface area:

$$S \approx \frac{\pi BL^2}{12}\left(-\frac{8BLw}{(L^2-4w^2)^2}+\frac{2\sqrt{3\left(L^2-2wL+4w^2\right)^3+B^2\left(5wL-L^2-4w^2\right)^2}}{\left(L^2-2wL+4w^2\right)^2}+\right.$$

$$\left.+\frac{2\sqrt{3\left(L^2+2wL+4w^2\right)^3+B^2\left(5wL+L^2+4w^2\right)^2}}{\left(L^2+2wL+4w^2\right)^2}+\frac{2\sqrt{\left(L^2+4w^2\right)^3+4B^2L^2w^2}}{\left(L^2+4w^2\right)^2}\right). \qquad (14)$$

Detailed mathematical formulations for deducing the formulae for *A*, *V* and *S*, *i.e*., Eqs. (12)–(14), are shown in Supplementary Data B.

To proceed further, the application of Eqs. (12)–(14) requires determination of the parameter w. The latter can be found directly from an image of the investigated egg by measuring the distance NC_2 (Fig. 3). Then, the distance w would correspond to the difference between NC_2 and the half-length of the egg. However, it is rather difficult to conduct such measurements in practice. In our preliminary study (Narushin et al., 2020) in which the 2-D digital images of egg contours were obtained with a high-resolution camera, the egg maximum breadth corresponded to several points on the egg surface, forming a plateau (Fig. 4). Yet, we were unable to determine the right location of the point C_2 to measure the distance NC_2 correctly as its position varied within the interval 180 to 230 pixels.

Therefore, some theoretical attempts to simplify the procedure for the experimental evaluation of w would be needed. For this purpose, we revised the transformation model shown in Figure 3 implementing the following amendments: the distance O_wG equals the half value of the egg maximum breadth, $B/2$; the distance OD corresponds to the value of y in Eq. (11) when $x = 0$ and is indicated as y_0; and the distance ON is the half length of the egg, $L/2$ (Fig. 5).

Evaluation of y_0 seems to be easier and more accurate than that of the distance O_wN. In this case, the egg length is divided into two, and the egg diameter is measured at that point. The value of y_0 can also be defined from Eq. (11) after inputting $x = 0$:

$$y_0 = \frac{BL}{2\sqrt{L^2 + 4w^2}}, \tag{15}$$

whence

$$w = \frac{L\sqrt{B^2 - 4{y_0}^2}}{4y_0}. \tag{16}$$

Considering possible difficulties and complications in measuring y_0, a more universal formula for calculating w at any point on the x-axis was deduced using Eq. (11). It was defined that for any set of x and y:

$$w_1 = \frac{\sqrt{(4x^2 - L^2)(4y^2 - B^2)}}{4y} - x, \tag{17}$$

$$w_2 = -x - \frac{\sqrt{(4x^2 - L^2)(4y^2 - B^2)}}{4y}. \tag{18}$$

The detailed mathematical transformation of Eq. (11) is shown in Supplementary Data C. When executing this transformation, only the upper half of the egg curve was being considered.

In theory, the values of w calculated using Eqs. (17) and (18) should be valid for the values of x for different parts of the egg curve from the point of inflection, meaning that Eq. (17) defines the values of x at any point on the distance O_wN, while Eq. (18) determines the values of x that correspond to any point on the distance O_wM (Fig. 5). However, this statement needs to be verified further experimentally.

Previously, we suggested a method for recalculation of the egg volume and surface area via geometrical transformation of an actual egg contour into a well-known geometrical figure with a shape that mostly resembles the investigated egg under coequality of some of their parameters (Narushin, 1993, 1997, 2001). Recently, we found that the method of such transformation under the equality of the areas of plane curves of the investigated egg and its geometrical counterpart seems to be mostly easy and accurate (Narushin et al., 2020).

For a validated use of the Hügelschäffer's egg contour model, Eq. (11), the defined formula for computing the area of its projection, A, Eq. (12), should be expressed as $B = f(A)$. To perform this calculation, we considered Eq. (12) as

$$A = \frac{BL^2}{9} \cdot k_A, \tag{19}$$

where

$$k_A = \frac{1.49}{\sqrt{L^2 - 2.667wL + 4w^2}} + \frac{2}{\sqrt{L^2 + 4w^2}} + \frac{1.49}{\sqrt{L^2 + 2.667wL + 4w^2}} +$$

$$+\frac{0.943}{\sqrt{L^2-1.333wL+4w^2}}+\frac{0.943}{\sqrt{L^2+1.333wL+4w^2}}. \quad (20)$$

The above equation can be simplified by simulating the data of L and B to L ratio (*i.e.*, shape index), which would be adequate for the variety of avian eggs, and approximating the obtained data with a simpler dependence. To achieve a greater accuracy, we limited the analysis to the data obtained for chicken eggs only. As the typical length of hens' eggs varies between 5 and 7 cm and the shape index between 0.70 to 0.78 (Narushin, 1994), the simulation of a mathematical equation was accordingly carried out by enumeration of possibilities for the egg length range of 5 to 7 cm with the increment of 0.1 cm and for B/L with the increment of 0.01.

Mathematical approximation resulted in the following formula:

$$k_A=-0.0209\frac{B}{L}\cdot L^2+1.7176=-0.0209BL+1.7176, \quad (21)$$

$R^2 = 0.945$.

Then,

$$A=\frac{BL^2}{9}\cdot(-0.0209BL+1.7176)=-0.0023B^2L^3+0.191BL^2, \quad (22)$$

whence

$$B=\frac{41.52}{L}\left(1-\sqrt{1-\frac{0.252A}{L}}\right). \quad (23)$$

Thus, to identify B, we should determine the area of the investigated egg, A, and egg length, L, by direct measurements. Using these two variables, one can proceed with the geometrical transformation of the investigated egg into the ovoid defined by Hügelschäffer's formula and recalculate the maximum breadth, B_t, of the transformed egg following Eq. (23). Subsequently, estimations of the egg volume and surface area can be done after inputting the value of B_t into Eqs. (13) and (14), instead of B.

The objective of this study was to test the applicability of Hügelschäffer's model for digital imaging assisted estimation of egg volume and surface area in two consequent steps:

(1) the actual measuring of two linear variables for eggs, length and maximum breadth, and determination of the distance w using the following formulae: $w = O_wN - L/2$ and Eqs. (16)–(18); and

(2) comparison of the empirical data with that computed using the geometrical transformation of the egg into the ovoid if its shape is described with Hügelschäffer's formula and if the area of the egg plane curve is estimated using Eq. (23) and Eqs. (13)–(14) accordingly.

The proposed non-destructive methodology would be useful in practice for the poultry industry and food engineering to estimate accurately the contour of any avian egg as well as a number of characteristics of the egg shape like volume, surface area, circumference length, radius of curvature, area of the plane curve, etc., providing new insights in oomorphological research relevant to biology, physical science, engineering and design of automated poultry production systems.

3. Materials and methods

This study was based on the materials and methods as previously described by Narushin et al. (2020). Briefly, 40 fresh chicken eggs were purchased from Woodlands Farm, Canterbury and Staveley's Eggs Ltd, Coppull, UK. Each egg was weighed, and the length and maximum breadth were measured. They were also examined to directly measure their volume, V, surface area, S, length, L, and maximum breadth, B, followed by detection of their scanned plane curves using 2-D digital imaging and subsequent image processing techniques. For this purpose, we exploited a digital camera, a non-reflection enclosure with LED (light emitting diode) lighting facilities, and a personal computer. The camera (UI-2230RE) has a CMOS (Complementary Metal Oxide Semiconductor) RGB (Red, Green and Blue) imaging sensor with a resolution of 1024 (H) × 768 (V) pixels and transmits images to the computer via USB 3.0 data transmission at a frame rate of 25 frames per second. This approach produced digital images of each egg. The images of the eggs were then processed using MatLab in order to compute the geometric parameters of eggs under the plane curve, *i.e.*, A, L, and B. To convert the image dimensions in pixels into an absolute unit (cm), the measured L and B of a reference template were compared with the same variables from their digital counterparts.

Considering that one of the basic parameters in the Hügelschäffer's egg model is w, the following three options for its evaluation were explored:

Option 1. Estimation of the distance O_wN (Fig. 5) where the location of the point O_w on the x-axis was considered to be a middle point of the plateau on the egg curve which corresponded to the maximum value of y. Using the obtained data of the distance O_wN, the parameter w was recalculated as the difference between O_wN and $L/2$ (Fig. 5).

Option 2. Estimation of the distance y_0 with the subsequent recalculation of w by employing Eq. (16).

Option 3. Estimation of different sets of the values of x and y on the obtained digital images with the successive recalculation of w using Eqs. (17) and (18).

4. Results and discussion

The direct measurement of the 40 eggs provided values of their actual volumes V, length L and maximum breadth B, and the appropriate measured egg variables including those based on 2-D digital egg images are given in Table 1. The subsequent generation of the egg images and their processing provided the other data (as analysed in detail in Narushin et al., 2020) needed to examine and compare the validity of the three options for evaluating the parameter w.

Table 1 – Data of measuring the egg variables and determining the parameter *w*.

Parameters	Max. value	Min. value	Mean	Standard deviation
Length[1], L (cm)	6.00	5.27	5.65	0.19
Max breadth[1], B (cm)	4.59	4.16	4.32	0.12
Volume[1], V (cm^3)	63.63	47.94	55.83	3.94
Parameter w (cm)				
Option 1 ($w = O_wN - L/2$)	0.282	0.011	0.165[a]	0.067
Option 2 ($w = f(y_0)$ and Eq. (16)	0.461	0.158	0.280[b]	0.100
Option 3 (Eqs. (17)–(18) and digital egg images)	0.249	0.021	0.120[c]	0.050

[1]Data from Narushin et al. (2020).

[a–c] Mean w values for the three determination options with no common letters differed significantly: a, c with $p < 0.01$; a, b and b, c with $p < 0.001$.

At first, we examined Options 1 and 2 by computing the appropriate w values. As can be seen in Table 1, the first two options for measuring w resulted in significantly different data, and the correlation between these values was 0.130, which is too low and not clearly showing which of the two methods of w determination is valid.

The recalculations of the w values by employing Eqs. (17) and (18) (Option 3) verified our assumption to use two different formulae before and after the inflection point by applying Eq. (17) for the x values in the O_wN interval and Eq. (18) for those in the O_wM interval (Fig. 5), on the other hand. An example of the recalculations of w for each possible measured set of x and y is given in Figure 6.

Examination of the obtained data for the parameter w using the three measurement options demonstrated an essential variability of the w values, especially in the areas close to the egg ends and its middle part. Therefore, the determination of true w values turned out to be critical for conducting all further recalculations in a proper way.

For this purpose, we performed a graphical visualisation using the egg contours plotted for the data measured with the imaging technique and the ones recalculated with Eq. (11) when imputing different w values in the formula. A w value, which shows the full conformity of two images, should correspond to the true w value, and diagrams in Figure 7 (a–c) illustrate how this process was done.

As shown on an example in Figure 7c, the contours of an actual single egg coincided with the recalculated egg shape if the w value for this given egg was equal to 15 px. The same procedure was applied for all the studied eggs that resulted in the following metrics of the true w parameter (Option 3): maximum value, 0.249; minimum value, 0.021; mean, 0.120; and standard deviation, 0.050 (Table 1).

The correlation estimates between the true w value (Option 3) and the measured ones (Table 1) were 0.439 for $w = O_wN - L/2$ (Option 1) and 0.332 for w recalculated from Eq. (16) (Option 2). Both these estimates appeared to be too low to use for practical measurements of w. Taking into account that the way of defining the true w values that we used in this study is too tedious and time-consuming, the following amendments were put forward to address this issue.

It is obvious from Figure 6 that the w values are smoother within the areas near the points of $L/4$ and $-L/4$ on the x-axis. Based on this, the Figure 5 data were revised and modified with several additional measurements of the egg width to check which one can provide the most accurate and meaningful results for recalculating w (Fig. 8).

The appropriate data of egg widths ($y_{-3L/8}$ … $y_{3L/8}$ as shown in Figure 8) were taken from the 2-D measurements based on the digital imaging that, along with the corresponding x values accordingly equal to -3L/8 … 3L/8, were put into Eqs. (17) and (18) for recalculating w. The obtained results are summarised in Table 2.

Table 2 – Data of *w* recalculations in accordance with Figure 8 (in cm).

Parameters	Max. value	Min. value	Mean	Standard deviation	Correlation with w_{true}
w_{true}	0.249	0.021	0.120[a]	0.050	N/A
$w_{3L/8}$	0.334	0.004	0.184	0.072	0.615
$w_{L/4}$	0.296	0.010	0.170	0.062	0.659
$w_{L/8}$	0.328	0.000	0.184	0.061	0.661
$w_{-L/8}$	0.272	-0.012	0.122	0.074	0.648
$w_{-L/4}$	0.302	0.007	0.114	0.065	0.721
$w_{-3L/8}$	0.258	-0.046	0.055	0.063	0.693
Mean $w_{3L/8 \ldots -3L/8}$	0.259	0.038	0.138[a]	0.048	0.915
Mean $w_{L/4} \ldots w_{-L/4}$	0.274	0.022	0.142[a]	0.050	0.881

[a] Mean $w_{3L/8 \ldots -3L/}$ and $w_{L/4} \ldots w_{-L/4}$ values did not differ significantly ($p > 0.05$).

The best w prediction and the appropriate greatest correlations were found if we consider the mean values resulting from all the measurements (Table 2). Overall, the data in Table 2 gave the better estimation of w than the ones from Table 1. However, because w should be greater than 0, the minimum values of $w_{L/8}$, $w_{-L/8}$ and $w_{-3L/8}$ reflected an incorrect condition, and despite their rather high correlation with the true numbers, we discarded these as alternative measurements for the w estimations. Because of that, the values of $w_{L/4}$ and $w_{-L/4}$ seemed to be more realistic having high correlation coefficients with w_{true}. The mean of these values showed a correlation of 0.881 that was high enough for these two measurements to be used in further calculations. Considering the appropriate two pairs of x and y (*i.e.*, x =

$L/4$; $y = y_{L/4}$ and $x = -L/4$; $y = y_{-L/4}$) and inputting these accordingly into Eqs. (17) and (18), we obtained the formulae for the recalculation of $w_{L/4}$ and $w_{-L/4}$:

$$w_{L/4} = L \cdot \frac{\sqrt{3B^2 - 12y_{L/4}^2} - 2y_{L/4}}{8y_{L/4}}, \tag{24}$$

$$w_{-L/4} = L \cdot \frac{2y_{-L/4} - \sqrt{3B^2 - 12y_{-L/4}^2}}{8y_{-L/4}}. \tag{25}$$

Substituting the w values in Eq. (13), we performed the calculation of egg volumes. The egg volume was also estimated as the geometrical transformation of the investigated eggs into the Hügelschäffer's model ovoid under the equality of their areas of the plane curve, as above explained (see subsection *1.2. Hügelschäffer's egg model*). The comparative results of the egg volume estimates are presented in Table 3.

Table 3 – Results of the calculation of the egg volume, *V*.

Parameters	Mean, cm^3	Correlation with V measured	Standard deviation of V values	Error of calculation %
V, actual value	55.83 ^{a}	–	3.94	–
V_1 using the values of w_{true}	55.33 ^{a}	0.962	4.33	0.89
V_2 using the mean of $w_{L/4}$ and $w_{-L/4}$	55.32 ^{a}	0.962	4.32	0.91
V_3 using the estimates of $w = O_wN - L/2$	55.31 ^{a}	0.962	4.32	0.93
V_4 using the data of A and B (Eq. (23))	59.57 ^{b}	0.953	5.71	6.71

$^{a, b}$ Mean V values with no common letters differed significantly ($p < 0.01$).

These results showed the greatest correlation coefficient (0.962) and the least calculation error (< 1%) for the egg volume estimates V_1, V_2 and V_3, meaning that the recalculation accuracy and congruity for the measured parameter w based on estimates of V_1, V_2 and V_3 were better than V_4 evaluated on the basis of the area of the plane curve. That could be explained by the fact that Hügelschäffer's formula describes the egg shape more accurately, thus the geometrical transformation is not needed. While conducting the transformation procedure, calculation and measurement errors are accumulated due to the approximate nature of Eqs. (12) and (23) as well as the conversion of square pixels into square cm, which was shown to be more inaccurate than that for the linear transformation.

Overall, the results for the volume estimations by means of both measuring and/or recalculating of w appeared to be sufficiently accurate and almost the same for all methods. The least calculation error was observed if the values w_{true} were used, however the appropriate mean differences relative to all other w values were insignificant (Table 3). Therefore, we would suggest that the use of any tested method of w estimation is acceptable for both industrial and analytical applications.

The recalculations of the surface area of the investigated eggs using Eq. (14) using the same variables as for the volume estimations are shown in Table 4.

Table 4 – Results of calculation of the egg surface area, *S*.

Parameters	Mean ± SD, cm^2
S_1 using the values of w_{true}	60.68 ± 3.27^a
S_2 using the mean of $w_{L/4}$ and $w_{-L/4}$	60.57 ± 3.27^a
S_3 using the estimates of $w = O_wN - L/2$	60.42 ± 3.24^a
S_4 using the data of A and B (Eq. (23))	62.83 ± 3.69^b

$^{a,\,b}$ Mean S values with no common letters differed significantly ($p < 0.01$).

As there is no direct method for accurately measuring the egg surface area, it is difficult to state which S from Table 4 is the most adequate and true. However, previously we provided theoretical deliberations suggesting that the validity of the computed egg surface area depends on the accuracy of the appropriate formula for estimating the egg volume, *V* (Narushin et al., 2020). The estimated values of *S* have similar variations as those for the calculated *V* (Table 3), therefore we could suggest that any way to determine the surface area would have the same drawbacks and advantages as methods for estimating the egg volume.

In the light of our findings, we can note some new concepts relevant to research and applications in biology, physical science, engineering and poultry industry that can be potentially based on the proposed non-destructive, digital imaging assisted oomorphology model. First, we have developed a theoretical approach to assess the adequacy of mathematical equations for evaluating the avian egg geometry including egg shape. Given the extraordinary number of such equations, with a persistent interest in their creation for almost two centuries, our approach enabled many of them to be rejected, as they do not comply with the principles of stability of the shape of the geometric figure. On the basis of this approach, the Hügelschäffer's model was selected as an equation that completely meets the basic principles and requirements for a comprehensive egg shape description. Using the exact description of the geometric shape of the eggs, the principles of a common engineering method known as Finite Element Analysis (FEA) have been developed and widely used to study the strength properties of the shell (Coucke et al., 1998; Nedomová et al., 2009; Perianu et al., 2010; Sellés et al., 2019).

Second, we have carried out a thorough study using a sophisticated digital imaging hardware that made it possible to develop a technique for measuring the parameter *w* (vertical axis shift) needed for applying Hügelschäffer's model. We described in detail all the problems that can be encountered as a result of this measurement, even when using such a methodological approach as computer scanning of egg contours.

Third, even for well-known geometric figures, e.g., ellipse, it is difficult to derive basic geometric formulas. To date, researchers have not come to an agreement and unified approach for determining the surface area of an ellipsoid. In our work, we demonstrated a new geometric figure, an ovoid built on the basis of the Hügelschäffer's model, for which all the previous attempts to describe it comprehensively were reduced only to inferring the formula for its volume, and even that without direct reference to the egg. In this regard, our successful derivation of the main mathematical formulas for this new geometric object can be certainly considered, in our opinion, not only as an interesting mathematical exercise but rather as an innovative work on a fully-fledged theoretical study of a new geometric body.

We suggest that the theoretical findings and digital imaging assisted tests we have reported here can be further incorporated in developing egg-related non-destructive technologies and automated systems applicable in research and industry.

5. Conclusions

Our analysis has demonstrated the validity of Hügelschäffer's formula for defining digital imaging assisted oomorphology including the contours of the hens' eggs and recalculating their geometrical variables. To tailor Hügelschäffer's model, we would recommend defining the value of the parameter w for each investigated egg. In industrial applications of such a technology, a machine vision technique can be of a great advantage. From a practical viewpoint, the measurement of linear egg parameters is a more straightforward process than, for instance, determination of the egg surface area. For laboratory use, egg images can be easily processed with Photoshop-like software or even less sophisticated programs (*e.g.*, MS Paint). Measurements of the parameter w can also be performed using a modified caliper (Smart, 1991). Even such a simple measurement using a ruler and a printed version of the image can be employed, too. Considering the simplicity and accuracy of the proposed method, it would also be worthwhile to test its applicability and validity for measuring eggs of other avian species, especially those with shapes that differ from the chicken egg profile. If Hügelschäffer's egg model were to be suitable for other species, this would deliver a novel research instrument for ornithological, especially oological studies. The proposed non-destructive methodology is practically ideal in the poultry industry and food engineering areas for an accurate representation of the contour of any avian egg and can also be easily used for the exploration of such characteristics of the egg shape as volume, surface area, circumference length, radius of curvature, area of the plane curve, etc. Based on the proposed digital imaging assisted egg geometry model, we expect it will have great potential in applications for designing automated systems in the poultry industry and in egg-related research in biology, physical science, engineering and other disciplines.

Declaration of Competing Interest

The authors declare no competing financial interest.

Acknowledgements

We acknowledge the financial support of this work via a University of Kent internal research grant sponsored by the Global Challenges Research Fund (GCRF) Partnership Fund.

Supplementary data.

Supplementary data A, B, C to this article can be found online at

References

Alocilja, E. C. (2013). *Principles of Biosystems Engineering*. https://www.egr.msu.edu/~alocilja/Teaching/Principles%20of%20BE%20Book%208-12-2013.pdf. Accessed on 6 April 2020.

Carter, T. C. (1968). The hen's egg: a mathematical model with three parameters. *British Poultry Science*, *9*(2), 165–171.

Cook, J. D. (2018). *Equation to Fit an Egg*. https://www.johndcook.com/blog/2018/04/18/equation-to-fit-an-egg/. Accessed on 6 April 2020.

Coucke, P., Jacobs, G., Sas, P., & De Baerdemaeker, J. (1998). Comparative analysis of the static and dynamic mechanical eggshell behaviour of a chicken egg. In *Proceedings of the 23rd International Conference on Noise and Vibration Engineering* (pp. 1469–1474). Leuven, Belgium.

Ferréol, R. (2017). *Hügelschäffer egg*. http://www.mathcurve.com/courbes2d.gb/oeuf/oeuf.shtml. Accessed on 6 April 2020.

Guo, S., Zhuang, H., Tang, W.-X., Liu, Q., & Wang, Y.-Y. (2020). Design of a bionic spudcan and analysis of penetration and extraction performances for jack-up platform. *China Ocean Engineering*, *34*(1), 80–88.

Köller, J. (2000). *Egg Curves and Ovals*. http://www.mathematische-basteleien.de/eggcurves.htm. Accessed on 6 April 2020.

Lazarus, A., Florijn, H. C. B., & Reis P. M. (2012). Geometry-induced rigidity in nonspherical pressurized elastic shells. *Physical Review Letters*, *109*(14), 144301.

Maulana, A. R., Yunus, M., & Sulistyaningrum, D. R. (2015). The constructions of egg-shaped surface equations using Hugelschaffer's egg-shaped curve. *Indonesian Journal of Physics*, *26*(2), 26–30.

Narushin, V. G. (1993). New indestructive methods of egg parameters and eggshell quality determination. In *5th European Symposium on the Quality of Eggs and Egg Products* (Vol. 2, pp. 217–222). Tours, France.

Narushin, V. G. (1994). Express method for determination of egg morphological parameters. In *Proceedings of 9th European Poultry Conference* (Vol. 1, pp. 385–386). Glasgow, UK.

Narushin, V. G. (1997). The avian egg: geometrical description and calculation of parameters. *Journal of Agricultural Engineering Research*, *68*(3), 201–205.

Narushin, V. G. (2001). AP—Animal Production Technology: Shape geometry of the avian egg. *Journal of Agricultural Engineering Research*, *79*(4), 441–448.

Narushin, V. G., Lu, G., Cugley, J., Romanov, M. N., & Griffin, D. K. (2020). A 2-D imaging-assisted geometrical transformation method for non-destructive evaluation of the volume and surface area of avian eggs. *Food Control*, *112*, 107112. https://doi.org/10.1016/j.foodcont.2020.107112.

Nedomová, Š., Trnka, J., Dvořáková, P., Buchar, J., & Severa, L. (2009). Hen’s eggshell strength under impact loading. *Journal of Food Engineering*, *94*(3–4), 350–357.

Perianu, C., De Ketelaere, B., Pluymers, B., Desmet, W., DeBaerdemaeker, J., & Decuypere, E. (2010). Finite element approach for simulating the dynamic mechanical behaviour of a chicken egg. *Biosystems Engineering*, *106*(1), 79–85.

Petrovic, M., & Obradovic, M. (2010). The complement of the Hugelschaffer's construction of the egg curve. In *2nd International Scientific Conference MonGeometrija 2010* (pp. 520–531). Vlasina, Serbia.

Petrovic, M., Obradovic, M., & Mijailovic, R. (2011). Suitability analysis of Hugelschaffer's egg curve application in architectural and structures’ geometry. *Buletinul Instutului Politehnic din Iasi: Secţia Construcţii de maşini*, *57*(3), 115–122.

Preston, E. W. (1953). The shapes of birds’ eggs. *Auk*, *70*(2), 160–182.

Recktenwald, G. W. (2000). *Numerical Methods with MATLAB: Implementations and Applications*. Upper Saddle River, New Jersey: Prentice Hall.

Schmidbauer, H. (1948). Eine exakte Eierkurvenkonstruktion mit technischen Anwendungen. *Elemente der Mathematik*, *3*(3), 67–68.

Sellés, A. G., Marcè-Nogué, J., Vila, B., Pérez, M. A., Gil, L., Galobart, À., & Fortuny, J. (2019). Computational approach to evaluating the strength of eggs: Implications for laying in organic egg production. *Biosystems Engineering*, *186*, 146–155.

Smart, I. H. M. (1967). The curve of the avian egg. *Journal of Anatomy*, *101*(3), 634–635.

Smart, I. H. M. (1991). Egg-shape in birds. In D. C. Deeming, & M. W. J. Ferguson (Eds.), *Egg incubation: its effects on embryonic development in birds and reptiles* (pp. 101–116). Cambridge, UK: Cambridge University Press.

Zhang, J., Tan, J., Tang, W., Zhao, X., & Zhu, Y. (2019). Experimental and numerical collapse properties of externally pressurized egg-shaped shells under local geometrical imperfections. *International Journal of Pressure Vessels and Piping*, 175, 103893.

Zhang, J., Zhu, B., Wang, F., Tang, W., Wang, W., & Zhang, M. (2017a). Buckling of prolate egg-shaped domes under hydrostatic external pressure. *Thin-Walled Structures*, 119, 296–303.

Zhang, J., Wang, M., Wang, W., Tang, W., & Zhu, Y. (2017b). Investigation on egg-shaped pressure hulls. *Marine Structures*, 52, 50–66.

Figure captions

Fig. 1. – Transformation of an ellipse into an egg contour with the three different functions $t(x)$ (adopted from Köller, 2000).

Fig. 2. – Curves of the three Köller's (2000) egg contour functions: $t_1(x) = 1 + 0.2x$ (green triangles); $t_2(x) = 1/(1 - 0.2x)$ (dark blue diamonds) and $t_3(x) = e^{0.2x}$ (red circles).

Fig. 3. – Hügelschäffer's transformation of an ellipse into an egg contour (from Petrovic et al., 2011).

Fig. 4. – Digital egg contour based on the data of Narushin et al. (2020).

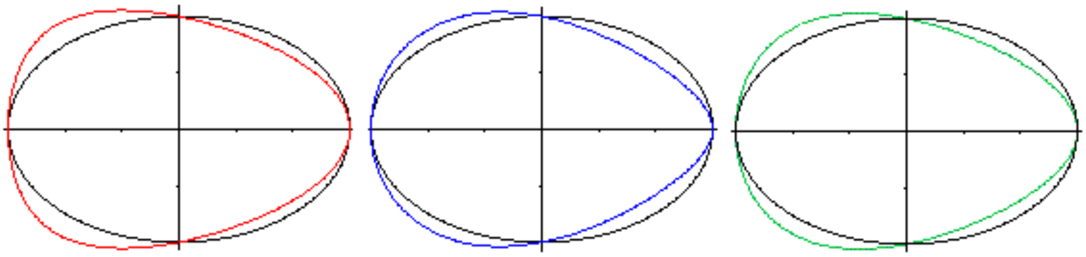
Fig. 5. – Estimation of the parameter w by measuring y_0 on the basis of the amended Hügelschäffer's egg model.

Fig. 6. – Recalculation of w (in pixels) with Eqs. (17) and (18) using the data of scanning the egg images, where the range of x values reflects the egg length, L, and the range of y values the egg maximum breadth, B.

Fig. 7. – An example of graphical visualisation of the egg contours (all dimensions are given in pixels): (a) A diagram plotted for the data of egg imaging. (b) A brown line corresponds to the egg shape calculated with Eq. (11) and w=45 and is plotted relative to the egg contour (blue line) measured using machine vision. (c) The both lines of the actual egg shape (blue) and the recalculated one (brown) coincided when w=15.

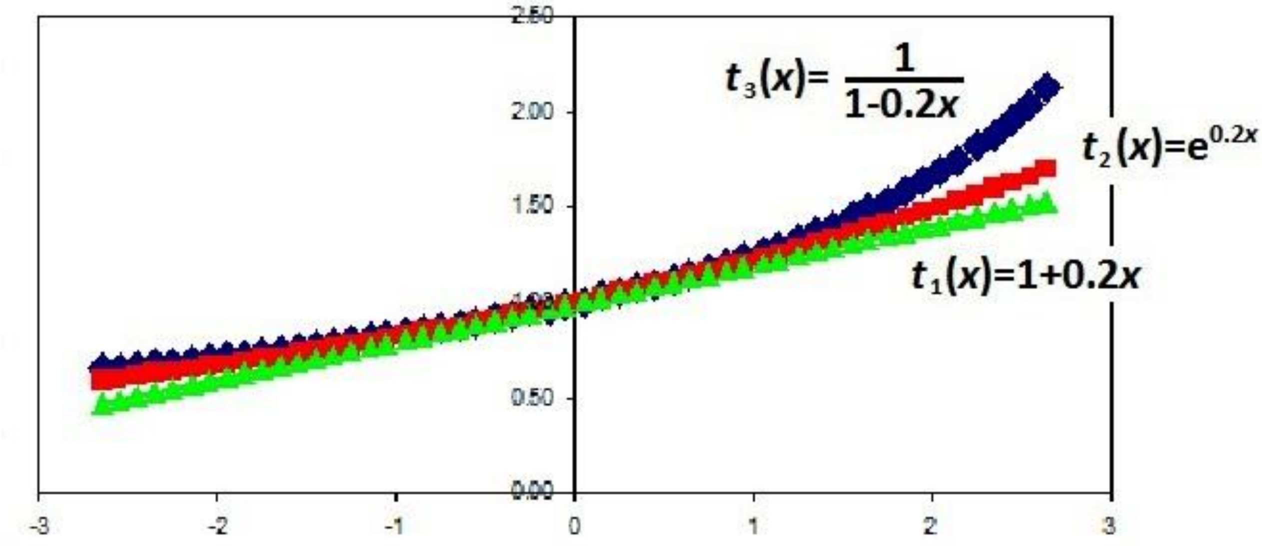
Fig. 8. – Amended measurements of the egg width along its longitudinal axis for recalculation of w.

Figure 1



$t_1(x) = 1 + 0.2x$ $\quad$ $t_2(x) = \dfrac{1}{1-0.2x}$ $\quad$ $t_3(x) = e^{0.2x}$

Figure 2



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$t_2(x)=e^{0.2x}$
$t_1(x)=1+0.2x$

Figure 3

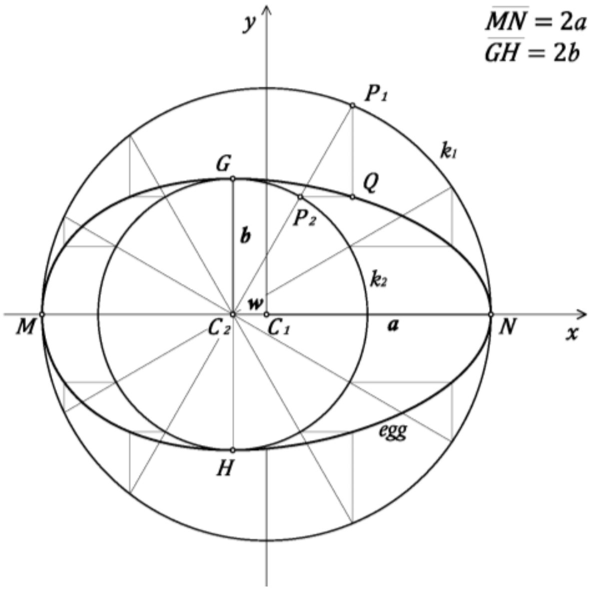

Figure 4

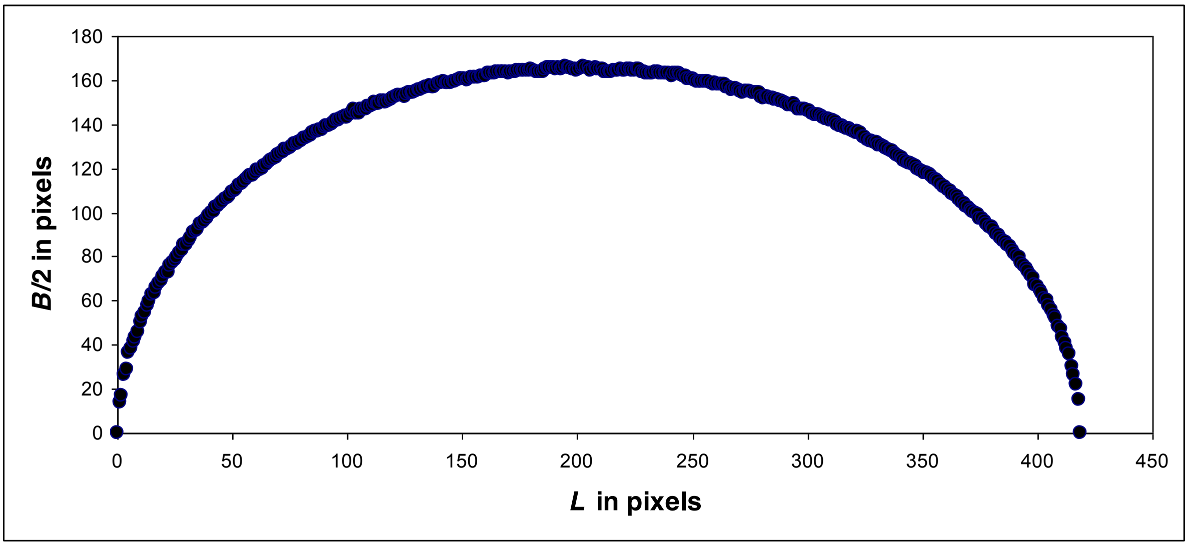

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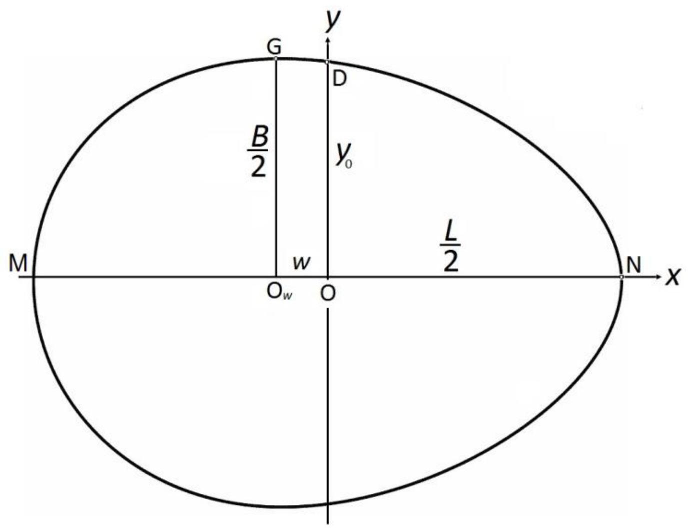
Figure 5



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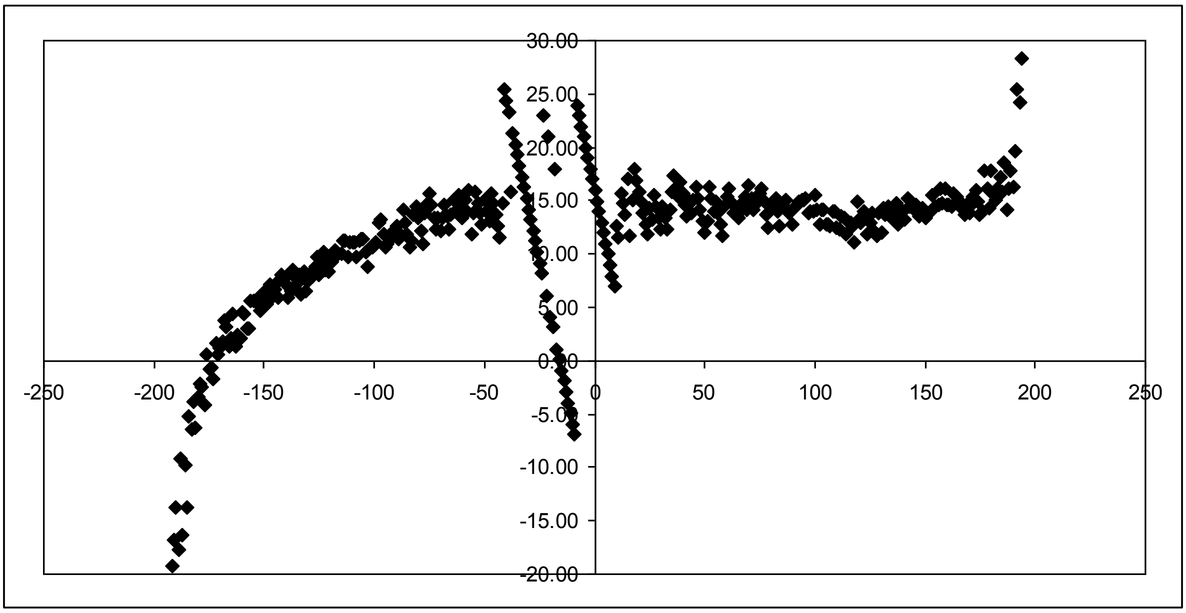

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Figure 7

(a) (b) (c)

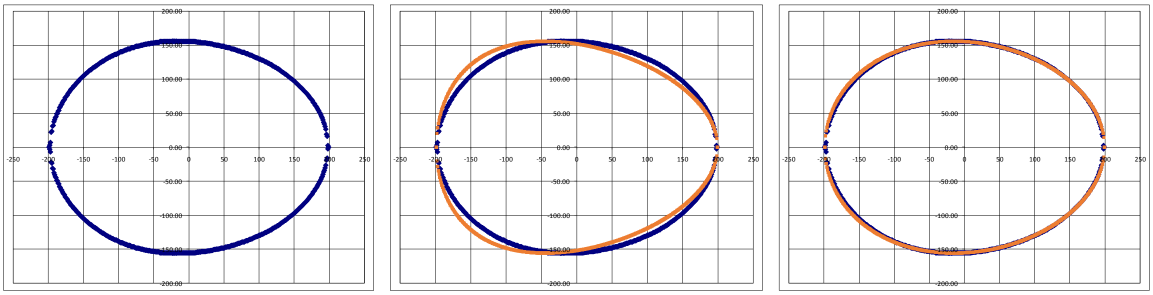

Figure 8