Wu, Di, Gong, Min, Peng, Rui, Yang, Xiangbin and Wu, Shaomin (2020) ***Optimal Product Substitution and Dual Sourcing Strategy considering Reliability of Production Lines.*** **Reliability Engineering and System Safety, 202 . ISSN 0951-8320.**

Optimal Product Substitution and Dual Sourcing Strategy considering Reliability of Production Lines*

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Abstract: Most of the supply chain literature assumes that product substitution is an effective method to mitigate supply chain disruptions and that all production lines either survive or are disrupted together. Such assumptions, however, may not hold in the real world: (1) when there is a shortfall of all products, product substitution may be inadequate unless it is paired with other strategies such as dual sourcing; and (2) production lines do not survive forever and may fail. To relax such assumptions, this paper therefore investigates the situations that the manufacturer may optimize substitution policy and dual sourcing policy to cope with supply chain disruptions. The paper obtains and compares the optimal policies for both deterministic and stochastic demands. A real-world case is also studied to verify the effectiveness of the proposed model.

Keywords: Reliability; dual sourcing; random supply failures; production lines; product substitution; supply chain disruption.

* Suggested citation: Di Wu, Min Gong, Rui Peng, Xiangbin Yan, Shaomin Wu, Optimal Product Substitution and Dual Sourcing Strategy considering Reliability of Production Lines, Reliability Engineering & System Safety, 2020, 107037, DOI: 10.1016/j.ress.2020.107037

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1. Introduction

Supply chain disruptions caused by unforeseen incidents such as natural disasters, labor strikes, terrorism attack, and financial defaults may occur with a high probability and can result in enormous ramifications for a firm [1, 2]. For the example in March 2011, Toyota's production lines were shut down for two weeks when their sole supplier suffered from an earthquake, which caused a supply chain disruption. To cope with supply chain disruption, authors regard flexible supply chains as an effective strategy [3, 4] and have developed methods such as multiple sourcing, product substitution, and flexible product volume to enhance supply chain resilience [5, 6]. For example, during the harsh winter in China's northern provinces in 2010, to combat the disorder in the transportation and production of goods, the Chinese government substituted their imported coal supply for the supply of southern power plants. In addition, the northern grocery market substituted products from southern farms for the supply that was previously provided by the local farms. In this case, product substitution proved valuable [7]. Similarly, the electricity meter subdivision of major oilfield services companies is another good example, where the effective management of changeover and substitution costs enabled those companies to supply radio-frequency-enabled meters in place of the cheaper traditional meters, which leaded to a great decrease in cost and an increase in profit [8]. The most recent example is the Covid-19 pandemic, due to which many countries shut down their nonessential businesses. For example, we witnessed that many supermarkets provided a large quantity of whole milk but did not provided semi-skimmed milk in the UK.

All the existing work on dual sourcing and product substitution is restricted to the assumption that all the production lines of the supplier either survive together or are disrupted together. In practice, some events may disrupt part of a supplier's production lines whereas other production lines are still functioning. For instance, [9] discussed the mixed oxide fuel exploitation and its destruction in power reactors. Among all six reactors, four of them are totally disrupted and the rest still remain functioning. In light of this, this paper considers a supply chain with two separate production lines that are subject to random failures, where the manufacturer decides on the optimal sourcing and substitution strategy.

In the literature, [10] considered the agility and proximity in a supply chain design.[11] solved a similar problem with consideration of distributional uncertainty. Both [3] and [12] assumed that all the production lines of a supplier either survive together or are disrupted together. However, in practice, two separate production lines may produce different goods at the same time, and the supplier may only be partially disrupted.

In this paper, we propose a model where two separate production lines are subject to disruption with different probabilities. Through mathematical derivations and numerical simulation, we obtain some useful results about the joint strategy of product substitution and dual sourcing. Specifically, a supply chain involving a manufacturer and two suppliers (one reliable and one unreliable) is analyzed. The unreliable supplier in this case might only be partially disrupted. The optimal sourcing strategy and the corresponding substitution strategy are solved under different settings of cost and disruption parameters. The solution is obtained under both deterministic and stochastic demands, respectively, and the effects of the interactions between dual-sourcing and substitution are discussed. Besides, a real-world example is proposed to illustrate the application of the proposed model.

It should be noted that the methods used in our paper are different from [13,14] although they considered the mix-flexibility and analyzed the optimal substitution and dual sourcing strategies under uncertainty. [13] considered the game between investment and dual-sourcing whereas this paper analyzes the sourcing-substitution decision from a manufacturer's perspective. [14] utilized a decision tree approach to determine the optimal number of suppliers whereas this paper analytically formulates the optimal sourcing and substitution problem and studies the solution.

This paper makes the following contributions.

- Supply chain authors may benefit from this paper as a theoretical method is employed to obtain the optimal policy under deterministic demand and numerical examples and a real-world case are solved under stochastic demand. The paper also summarizes the interaction between substitution strategy and dual sourcing strategy and illustrates their impacts on optimal managerial decision.
- Practitioners such as supply chain managers may benefit from the paper as it provides a guidance in their decision making on sourcing and substitution strategy for the more realistic case where production lines can be partially disrupted.
- This paper considers the reliability of a production line in supply chain management and has an intension to describe the real-world problem with a more practical manner.

The remainder of this paper is organized as follows. Section 2 reviews previous literature and highlight differences. Section 3 provides the problem description and formulates the supply chain model with deterministic demand and stochastic demand, respectively. Section 4 solves the optimal policy under both cases and provides numerical examples. Section 5 applies the proposed model to a real case study. Section 6 concludes the paper and discusses further research avenues.

2. Literature Review

Literature relevant to this paper includes: unreliable supply chains, product substitution between higher-grade products and lower-grade products, and multiple sourcing in supply chain management.

A supply chain is a network between a manufacturer and its suppliers to produce and distribute a specific product to the end consumer. Unreliable supply chains may confront with internal and external impact. Specifically, some components may be disrupted and the performance of supply chain will degrade. Many papers have been published to investigate various challenges caused by unreliable supply chains [3,15-17]. [18] evaluated the impact of supply disruption risks on the choice between the single and dual sourcing methods in a two-stage supply chain with a non-stationary and price-sensitive demand. They found that the dual sourcing strategy can be employed to increase supply chain efficiency. [19] analyzed the interaction between demand substitution and product changeovers, performing another commonly employed strategy in flexible supply chains. [20] considered the case where a supplier facing the prospect of disruption must decide whether or not to invest in restoration capability. Their discussion of disruption leads to the analysis of an unreliable supply chain. [21] considered risk pooling, risk diversification and supply chain disruption in a multi-location system where the supply is subject to disruptions. Their results show that when the demand is deterministic, the use of a decentralized inventory design can reduce cost variance through the effect of risk diversification. Recently, [22] analyzes the maintenance policy of competing failures under random environment. This paper differs from existing research since it considers both deterministic and stochastic demands, which provides a better depiction of the reality.

Product substitution refers to using other product to substitute an existing product to meet the same needs, which is widely studied under a retail background [5,23]. [24] adopted a stylized two-segment setup with uncertain market sizes under endogenous substitution and illustrated the interplay between risk-pooling and market segmentation. [25] studied a real case in the substitution of cars. Specifically, they considered the demand for two-car households and showed that the car efficiency and substitution are strongly correlated. [26] considered inventory decisions for a finite horizon problem with product substitution options and time varying demand. We conduct a similar research topic as theirs but differ in the partial production line destruction. [27] considered the process flexibility design in heterogeneous and unbalanced networks and employed a stochastic programming approach to solve the optimal substitution strategy. [28] employed substitution strategies in a time allocation model that considers external providers. [29] considered two inventory-based substitutable products in an inventory replenishment system. In this paper, we analogically consider product substitution and introduce the flexible production into concern. The reliable supplier can enlarge its production quantity in case some production lines of unreliable suppliers are disrupted.

A multiple sourcing in a supply chain is outsourcing several of manufacturer's most important operations to several different vendors instead of using a single source [30]. Multiple sourcing, when used in conjunction with product substitution, is another efficient method to mitigate supply disruptions. [31] first proposed a dual sourcing model with random lead time and uncertain demand. [14] connected the mix-flexibility and dual-sourcing literatures by studying unreliable supply chains that produce multiple products. The model was extended by [32], which further considered capacity constraint and flexibility in order quantities. [13] compared the effectiveness of dual sourcing, contingent sourcing and product substitution, and the model proposed by [13] was further extended by [4], which used product substitution as the primary disruption mitigation method while regarding dual sourcing and contingent supply as supporting mechanisms. It is found that the optimal dual sourcing policy is to guarantee the effect of product substitution. [33] examined a double-layered supply chain where a buyer facing the end-users has the option of selecting from a cohort of suppliers that have different yield rates and unit costs in the related field. [34] considered the pricing strategy and coordination in a supply chain with risk-averse retailer taking dual sourcing policy. Nonetheless, little research analyzes the case where only part of the unreliable supplier's production lines is disrupted. This paper therefore bridges the gap by considering this realistic case.

Furthermore, many papers studied the reliability of the production lines and the related optimization problems. For example, [35] and [36] analyzed the optimal maintenance policy for a given product line. Our paper can be also regarded as an optimization problem related to unreliable productions lines and its aim is to make the optimal sourcing and substitution decisions to deal with the possible unsupplied demand caused by disruption of production lines. In order to increase the reliability of a system, methods like redundancy and performance sharing are widely employed [37,38]. [39] studied the optimal replacement policy in terms of the substitution cost. In our paper, the dual sourcing strategy can be regarded as a type of redundancy and the substitution strategy can be taken as a type of performance sharing. On the other hand, research has been done in the analysis of supply chain management from the perspective of reliability. For instance, [40] performed the mathematical definition and the theoretical structure in analyzing the supply chain based on the reliability theory. Specifically, they discussed the structural reliability model and introduced a case study of a supply chain for the personal computer assembly. [41] considered a supply chain where the service provider has limited resources and proposed an emergency supply contract based method to maximize the expected profit. [42] considered both reliability and disruption and analyzed the optimal network design problem for perishable products. Recently, [43] proposed a resilience measure to characterize the interruption in cyber-physical supply chain systems.

3. Problem description and model foundation

Following prior literature [24,28], we assume that product substitution is the primary disruption mitigation technique, and dual sourcing and contingent supply are the supporting mechanisms and that there is a supply chain model in which two products are downward substitutable (i.e. only higher-grade products can be substituted for lower-grade ones) and are sourced from two suppliers.

Notation table

R	perfectly reliable supplier
U	unreliable supplier
$\delta_i(x)$	flexibility function, where δ_i is the flexibility coefficient for P_i
c_s	substitution cost for each unit of product
$c_{Ui}, i=1,2$	sourcing cost for each unit of product P_i ordered from U
$c_{Ri}, i=1,2$	sourcing cost for each unit of product P_i ordered from R
P_1	lower-grade product
P_2	higher-grade product
$q_i, i=1,2$	order quantity of P_i before supply status is observed
r_i	proportion of P_i ordered from supplier R
q_s	substitution policy: number of P_2 used to substitute P_1
$r_i, i=1,2$	sourcing policy: proportion of P_i ordered from supplier R
$b_i, i=1,2$	penalty for P_i when the supply does not meet the demand
π_i	probability that production line i in supplier U is disrupted
(d_1, d_2)	total demand for P_1 and P_2
$C_i(r_1, r_2), i=1,2,3,4$	cost under four diverse cases
$F_d(.)$	cumulative distribution function when demand is random

(ξ_1, ξ_2)	demand realization under stochastic demand

Suppose a manufacturer sells two products: P_1 and P_2. P_2 is a higher grade product than P_1 and can substitute for P_1 if P_1 is unavailable. Note that any unmet demand is lost and each unmet unit of demand for product i incurs a penalty cost b_i. Each unit of P_2 that substitutes for P_1 incurs a substitution cost because P_2 has a higher price than P_1. Therefore, when the product substitution is adopted, it simultaneously results in a revenue loss and a revenue gain. The revenue loss results from substituting a lower-grade product with a high-grade product. The revenue gain results from the avoidance of customer churn. Whether a manufacturer benefits or not depends on whether or not the customer churn may cost more than the gain. There are two suppliers: R, which is perfectly reliable; and U, which is unreliable because its two production lines are subject to random failures. When the production line of P_1 in supplier U is disrupted, P_1, which is ordered from supplier U, is unavailable, and vice versa. Note that each unit of product P_1 ordered from supplier R costs c_{R1} and each unit ordered from supplier U costs c_{U1}. Similarly, each unit of product P_2 ordered from supplier R costs c_{R2} and each unit ordered from supplier U costs c_{U2}. Since P_2 is of a higher quality than P_1, we assume that the cost for P_2 is higher than for P_1. Thus, $c_{R2} \geq c_{R1}, c_{U2} \geq c_{U1}, c_{R2} > c_{U2}$ and $c_{R1} > c_{U1}$. The subscript "$_U$" denotes an unreliable supplier and "$_R$" denotes a reliable supplier.

In reality, it is easy to deduce that supplier R has more flexibility in contingent volume than supplier U. That is, if the demand of any product cannot be satisfied due to the disruption of supplier U, the manufacturer can increase its order from supplier R. Since only P_2 can substitute for P_1, it is easy to know that the manufacturer may increase the order of P_2 due to the disruption of either the production line for P_1 or the production line for P_2 whereas it increases the order of P_1 only due to the disruption of P_1 production line but not P_2 production line. Suppose x units of P_i are ordered from supplier R, the manufacturer can order as many as $\delta_i(x)$ units from the supplier R.

Without loss of generality, we assume that the flexibility function is linear, that is, $\delta_i(x) = \delta_i x$ where $\delta_i > 1$ is the flexibility coefficient and it represents the supplier's ability to supply product units even if disruption occurs. We also assume that both products own the same level of flexibility, say that, $\delta_1 = \delta_2 = \delta$, without loss of generality.

The game proceeds as follows. At the beginning, the manufacturer makes sourcing decisions. Then two production lines of unreliable suppliers break down independently with different probabilities. Each supplier fulfills the sourcing order. The manufacturer makes a substitution decision based on the sourcing situation.

First, the manufacturer determines the quantity of order based on the demand. Assume we need q_i units of P_i ($i = 1,2$). According to the sourcing policy, the manufacturer splits the order between the two suppliers, where the proportion of P_i ordered from supplier R is r_i. Second, the supply status is observed, where supplier U has two production lines that may break down independently from each other with different probabilities. Third, the manufacturer receives the ordered volume from the suppliers. Fourth, the manufacturer allocates the available products to customers by making the substitution decision.

The production line for P_1 may break down with probability π_1, while the production line for P_2 may break down with probability π_2. Generally, the greedy allocation algorithm is still optimal: first, we should satisfy the demand for P_2 as much as possible; second, we should satisfy the demand for P_1 with the available volume of P_1 as much as possible; third, we should consider substituting the remaining demand of P_1 with P_2. Note that the holding of the greedy allocation needs to be supported by the conditions that $c_{R1} < b_1$, $c_{R2} < b_2$, $c_{R1} + c_s < b_1$ and $c_{R1} + c_s + b_2 > c_{R2} + b_1$, where

- $c_{R1} < b_1$ guarantees that obtaining a P_1 from R is cheaper than bearing the penalty for P_1,
- $c_{R2} < b_2$ guarantees that obtaining a P_2 from R is cheaper than bearing the penalty for P_2,
- $c_{R1} + c_s < b_1$ guarantees that obtaining a P_2 from R and substituting for P_1 is better than

bearing the penalty for P_1 but worse than satisfying the demand for P_1 by obtaining a P_1 from R, and

- $c_{R1} + c_s + b_2 > c_{R2} + b_1$ guarantees that P_2 should first satisfy its own demand before substituting P_1.

Without loss of generality, we assume that the higher-grade product has a higher penalty than the lower-grade product. To illustrate the model, we give a numerical example, as shown in Table 1. In this table, the respective demands for P_1 and P_2 are five units and four units, respectively. For the dual sourcing policy, three units of P_1 are ordered from the unreliable supplier U, two units of P_1 are ordered from the reliable supplier R, and two units of P_2 are ordered from both U and R, respectively. The flexibility coefficient is $\delta = 2$. Here we assume that the production line of P_1 from unreliable supplier U is broken.

Table 1. An illustrative example for the case when only one production line is disrupted.

	Demand	From U	From R	Available	Substituted	Satisfied
P_1	5	3(broken)	2	4	1	5
P_2	4	2	2	6	---	4

3.1 Deterministic demand

First, we consider the model under the deterministic demand. Assume the demand for P_1 and P_2 is (d_1, d_2). The working state of supplier U can be classified into the following four cases.

1. Perfect working state

If supplier U is not disrupted, only sourcing cost is incurred. Thus, the cost without disruption is

$$C_1(r_1, r_2) = c_{R1} r_1 q_1 + c_{U1}(1 - r_1) q_1 + c_{R2} r_2 q_2 + c_{U2}(1 - r_2) q_2, \quad (1)$$

where $c_{R1} r_1 q_1$ is the cost of sourcing P_1 from reliable supplier, $c_{U1}(1 - r_1) q_1$ is the cost of sourcing P_1 from unreliable supplier, $c_{R2} r_2 q_2$ is the cost of sourcing P_2 from reliable supplier, and $c_{U2}(1 - r_2) q_2$ is the cost of sourcing P_2 from unreliable supplier.

2. Partial working state: production line for P_1 breaks down

The demand for P_2 can be satisfied. Therefore, we first try to satisfy the demand for P_1 as much as possible with the available volume of P_1. We then use P_2 to satisfy any unmet demand for P_1, as detailed in Table 2.

Table 2. Detailed situation in partial working state.

	Available from U	Demand from R	Available from R	Maximum Available
P_1	0	r_1q_1	δr_1q_1	δr_1q_1
P_2	$(1-r_2)q_2$	r_2q_2	δr_2q_2	$(\delta r_2 - r_2 + 1)q_2$

- If $\delta r_1q_1 \geq d_1$, then no substitution is needed.
- If $\delta r_1q_1 < d_1$ and $d_1 - \delta r_1q_1 < (\delta - 1)r_2q_2$, then $q_s = d_1 - \delta r_1q_1$.
- If $\delta r_1q_1 < d_1$ and $d_1 - \delta r_1q_1 \geq (\delta - 1)r_2q_2$, then $q_s = (\delta - 1)r_2q_2$.

To summarize, the number of P_2 used to substitute P_1 is

$$q_{s1} = \text{Min}([d_1 - \delta r_1q_1]^+, (\delta - 1)r_2q_2). \quad (2)$$

Note that we use $[x]^+$ to represent $\text{Max}[x, 0]$. Thus, the total cost under disruption on production line P_1 is

$$C_2(r_1, r_2) = c_{R1}(\text{Min}(d_1, \delta r_1q_1) + q_{s1}) + c_{R2}r_2q_2 + c_{U2}(1-r_2)q_2 + c_sq_{s1} + b_1(\text{Max}(0, d_1 - \delta r_1q_1 - q_{s1})). \quad (3)$$

where $c_{R1}(\text{Min}(d_1, \delta r_1q_1) + q_{s1})$ is the cost of sourcing P_1 from a reliable supplier, $c_{R2}r_2q_2$ is the cost of sourcing P_2 from the reliable supplier, $c_{U2}(1-r_2)q_2$ is the cost of sourcing P_2 from the unreliable supplier, and c_sq_{s1} is the cost of substitution. The penalty $b_1(\text{Max}(0, d_1 - \delta r_1q_1 - q_{s1}))$ in Eq. (3) corresponds to the unsupplied demand for P_1.

3. Partial working state: P_2 production line breaks down

The demand for P_1 can be satisfied, so substitution is not necessary. We try to satisfy the demand for P_2 as much as possible. The cost will contain both the sourcing cost and the penalty costs incurred for any unmet demand for P_2:

$$C_3(r_1,r_2)=c_{R1}r_1q_1+c_{U1}(1-r_1)q_1+c_{R2}\text{Min}(d_2,\delta r_2q_2)+b_2(\text{Max}[0,d_2-\delta r_2q_2)). \tag{4}$$

where $c_{R1}r_1q_1$ is the cost of sourcing P_1 from reliable supplier, $c_{U1}(1-r_1)q_1$ is the cost of sourcing P_1 from the unreliable supplier, and $c_{R2}\text{Min}(d_2,\delta r_2q_2)$ is the cost of sourcing P_2 from the reliable supplier. Since a lower-grade product cannot substitute for a higher-grade product, there is no substitution cost. The penalty $b_2(\text{Max}(0,d_2-\delta r_2q_2))$ corresponds to the unsupplied demand for P_2.

4. Failure state

Both production lines of supplier U are disrupted. We apply the greedy allocation algorithm.

- If $\delta r_1q_1 \geq d_1$, then no substitution is needed.
- If $\delta r_1q_1 < d_1$ and $\delta r_2q_2 > d_2$ and $d_1-\delta r_1q_1 < \delta r_2q_2-d_2$, then $q_s=d_1-\delta r_1q_1$.
- If $\delta r_1q_1 < d_1$ and $\delta r_2q_2 > d_2$ and $d_1-\delta r_1q_1 \geq \delta r_2q_2-d_2$, then $q_s=\delta r_2q_2-d_2$.
- If $\delta r_2q_2 \leq d_2$, then nothing can be used for substitution.

To summarize, the number of P_2 used to substitute for P_1 is

$$q_{s2}=\text{Min}([d_1-\delta r_1q_1]^+,\text{Max}(0,\delta r_2q_2-d_2)). \tag{5}$$

Thus, the total cost under disruption on production line P_1 is

$$\begin{aligned}C_4(r_1,r_2)&=c_{R1}\text{Min}(d_1,\delta r_1q_1)+c_{R2}(\text{Min}(d_2,\delta r_2q_2)+q_{s2})+c_sq_{s2}\\&+b_1(\text{Max}(0,d_1-q_{s2}-\delta r_1q_1))+b_2(\text{Max}(0,d_2+q_{s2}-\delta r_2q_2)).\end{aligned} \tag{6}$$

where $c_{R1}\text{Min}(d_1,\delta r_1q_1)$ and $c_{R2}(\text{Min}(d_2,\delta r_2q_2)+q_{s2})$ represent the sourcing cost of P_1 from reliable supplier and the sourcing cost of P_2 from the reliable supplier, respectively. Due to production line disruption, c_sq_{s2} is now the substitution cost. The penalty under this case consists of the penalty $b_1(\text{Max}(0,d_1-q_{s2}-\delta r_1q_1))$ corresponding to the unsupplied demand of

P_1 and the penalty $b_2(\text{Max}(0, d_2 + q_{s2} - \delta r_2 q_2))$ corresponding to the unsupplied demand of P_2

Finally, the expected cost $C(r_1, r_2)$ can be expressed by:

$$C(r_1,r_2) = (1-\pi_1)(1-\pi_2)C_1(r_1,r_2) + \pi_1(1-\pi_2)C_2(r_1,r_2) + \pi_2(1-\pi_1)C_3(r_1,r_2) + \pi_1\pi_2 C_4(r_1,r_2). \quad (7)$$

If the demand is deterministic, it is optimal to choose $q_1 = d_1$ and $q_2 = d_2$ since the manufacturer makes the sourcing decision before the failure state of the production lines is observed. Therefore, since the manufacturer is rational and does not predict the future, the optimal order quantity is equal to the demand. Thus, the sourcing problem is to minimize the cost $C(r_1, r_2)$ such that $0 \le r_1, r_2 \le 1$.

3.2 Stochastic demand

Another problem under consideration is stochastic demand. Under this case, the demand (d_1, d_2) is random and has a cumulative distribution function $F_d(.)$. Suppose the manufacturer's order quantity for each product equals to the product's expected demand. This assumption holds since the status of suppliers cannot be observed before a decision is made. Indeed, [44] assumed that the optimal ordering quantity do not equal to the product's expected demand in a newsvendor-type setting. Nonetheless, they studied an investment and production game where the investment decisions are made in advance. In reality, the fluctuation of demand is assumed to be low, say that, the probability that the realized demand is larger than the expected demand multiplied by the flexible coefficient can be neglected. Again, for a given demand realization (ξ_1, ξ_2), we have four cases:

1. Perfect working state. The cost function under this case is the same as $C(r_1, r_2)$, as shown in Eq. (1).
2. Partial working state: production line for P_1 breaks down:

$$q_{s1} = \text{Min}([\xi_1 - \delta r_1 q_1]^+, (\delta - 1) r_2 q_2), \quad (8)$$

$$C_2(r_1, r_2; \xi_1, \xi_2) = \underbrace{c_{R1}(\text{Min}(\xi_1, \delta r_1 q_1) + q_{s1}) + c_{R2} r_2 q_2 + c_{U2}(1 - r_2) q_2}_{\text{sourcing}} + \underbrace{c_s q_{s1}}_{\text{substitution}} + \underbrace{b_1(\text{Max}(0, \xi_1 - \delta r_1 q_1 - q_{s1}))}_{\text{penalty}}. \quad (9)$$

The expected cost is therefore given by $C_2(r_1,r_2)=\int C_2(r_1,r_2;\xi_1,\xi_2)dF_d(\xi_1,\xi_2)$. Specifically, "*sourcing*", "*substitution*", and "*penalty*" in Eq. (9) represent the different parts of total cost.

3. Partial working state: production line for P_2 breaks down

   The demand for P_1 can be satisfied, so substitution is not necessary. We try to satisfy the demand for P_2 as much as possible. The cost will include both the sourcing cost and the penalty costs incurred for any unmet demand for P_2:

$$C_3(r_1,r_2;\xi_1,\xi_2)=\underbrace{c_{R1}r_1q_1+c_{U1}(1-r_1)q_1+c_{R2}\mathrm{Min}(\xi_2,\delta r_2q_2)}_{\text{sourcing}}+\underbrace{b_2(\mathrm{Max}(0,\xi_2-\delta r_2q_2))}_{\text{penalty}}. \quad (10)$$

   Therefore, the expected cost is $C_3(r_1,r_2)=\int C_3(r_1,r_2;\xi_1,\xi_2)dF_d(\xi_1,\xi_2)$.

4. Failure state

$$q_{s2}=\mathrm{Min}([\xi_1-\delta r_1q_1]^+,\mathrm{Max}(0,\delta r_2q_2-\xi_2)), \quad (11)$$

$$\begin{aligned}C_4(r_1,r_2;\xi_1,\xi_2)=&\underbrace{c_{R1}\mathrm{Min}(\xi_1,\delta r_1q_1)+c_{R2}(\mathrm{Min}(\xi_2,\delta r_2q_2)+q_{s2})}_{\text{sourcing}}+\underbrace{c_sq_{s2}}_{\text{substitution}}+\\&\underbrace{b_1(\mathrm{Max}(0,\xi_1-q_{s2}-\delta r_1q_1))+b_2(\mathrm{Max}(0,\xi_2+q_{s2}-\delta r_2q_2))}_{\text{penalty}}.\end{aligned} \quad (12)$$

   Therefore, the expected cost is $C_4(r_1,r_2)=\int C_4(r_1,r_2;\xi_1,\xi_2)dF_d(\xi_1,\xi_2)$.

   Again, the expected cost $C(r_1,r_2)$ can be expressed by Eq. (7). Like the deterministic demand problem, the optimal sourcing policy can be obtained by minimizing the expected cost.

4. The optimal sourcing and substitution policies

In this section, we solve the optimal sourcing policy (r_1^*,r_2^*) and corresponding substitution policy $(q_{s1}{}^*,q_{s2}{}^*)$ for deterministic demand and stochastic demand, respectively. When dealing with the deterministic demand, we employ theoretical analysis and minimize the cost of the manufacturer. The solution of optimization leads to the optimal strategy combination. When dealing with stochastic demand, we employ numerical analysis and assign specific numbers to the parameters in our proposed model. We run the simulation and obtain the numerical solution by similarly minimizing the cost of the manufacturer [46].

4.1 Deterministic demand

For deterministic demand, the total cost is denoted in Eq. (7). Note that $C(r_1, r_2)$ contains several functions. To obtain the optimal $C(r_1, r_2)$, different ranges of r_1 and r_2 should be explored. We consider eighteen cases and only perform the specific derivation and optimality proof for the first case (a) here. See Appendix C for the other seventeen cases (b-r).

(a) $\delta r_2 q_2 \le d_2$, $\delta r_1 q_1 \le d_1$ and $0 \le d_1 - \delta r_2 q_2 \le \delta r_1 q_1 - r_2 q_2$

The total cost can be simplified as

$$\begin{aligned} C(r_1, r_2) = {} & (1-\pi_1)(1-\pi_2)(c_{R1} r_1 q_1 + c_{U1}(1-r_1)q_1 + c_{R2} r_2 q_2 + c_{U2}(1-r_2)q_2) + \\ & \pi_1(1-\pi_2)(c_{R1} d_1 + c_{R2} r_2 q_2 + c_{U2}(1-r_2)q_2 + c_s(d_1 - \delta r_1 q_1)) + \\ & \pi_2(1-\pi_1)(c_{R1} r_1 q_1 + c_{U1}(1-r_1)q_1 + c_{R2}\delta r_2 q_2 + b_2(d_2 - \delta r_2 q_2)) + \\ & \pi_1\pi_2(c_{R1}\delta r_1 q_1 + c_{R2}\delta r_2 q_2 + b_1(d_1 - \delta r_2 q_2) + b_2(d_2 - \delta r_2 q_2)). \end{aligned} \tag{13}$$

Since the expected total cost is a linear function of r_1, r_2, q_1 and q_2, the problem can be translated into linear programming. To find the optimal sourcing policy and related substitution policy, we take (q_1, q_2) as an entirety and use the first-order condition to solve this issue. Let $\partial C(r_1, r_2)/\partial r_1 = 0$ and $\partial C(r_1, r_2)/\partial r_2 = 0$, we obtain

$$\begin{cases} c_{U1}(\pi_1 - 1)(r_1 - 1) + c_s \pi_1(\pi_2 - 1)\delta r_1 + c_{R1} r_1 (1 - \pi_1 + \pi_1\pi_2\delta) = 0 \\ c_{U2}(\pi_2 - 1)(r_2 - 1) - (b_2 + b_1\pi_1)\pi_2\delta r_2 + c_{R2} r_2 (1 - \pi_2 + \pi_2\delta) = 0 \end{cases}. \tag{14}$$

Thus, the optimal sourcing policy can be represented by

$$\begin{cases} r_1^* = \dfrac{c_{U1}(1-\pi_1)}{c_{U1}(1-\pi_1) + c_s\pi_1(1-\pi_2)\delta_1 + c_{R1}(\pi_1\pi_2\delta + \pi_1 - 1)} \\ r_2^* = \dfrac{c_{U2}(1-\pi_2)}{c_{U2}(1-\pi_2) + (b_2 + b_1\pi_1)\pi_2\delta - c_{R2}(1 - \pi_2 + \pi_2\delta)} \end{cases}. \tag{15}$$

Similarly, the optimal substitution strategy can be denoted by

$$\begin{cases} q_{s1}^* = d_1 - \dfrac{\delta q_1 c_{U1}(1-\pi_1)}{c_{U1}(1-\pi_1) + c_s\pi_1(1-\pi_2)\delta_1 + c_{R1}(\pi_1\pi_2\delta + \pi_1 - 1)} \\ q_{s2}^* = 0 \end{cases}. \tag{16}$$

In this case, if the respective demands of the higher-grade and lower-grade products are greater than the flexible quantity (contingently increasable ordering) from the reliable supplier, the optimal sourcing strategy is a function of the disruption probabilities of both production lines, the flexible coefficient, the sourcing cost, and the substitution cost. As for the substitution policy, the best strategy is to substitute some lower-grade products rather than to substitute higher-grade products. This is because the manufacturer would rather retain higher-grade products than substitute them if the demands are greater than the flexible quantity. We conduct sensitivity analysis in Section 3.2 to test the robustness of the proposed model.

Now we prove the optimality of the given sourcing policy and the corresponding substitution policy. Since the total cost is a linear function, the policy is optimal within the boundaries. We illustrate this by comparing the expected total cost between our obtained policy and the boundary. In this case, the boundaries of the sourcing policy are $0 \le r_1 \le \frac{d_1}{\delta q_1}$ and $0 \le r_2 \le \frac{d_2}{\delta q_2}$. In case of the mathematical derivation, we do not substitute the specific value of the optimal sourcing policy in the main body.

Lemma 1. The obtained policy is optimal and possible within the boundaries. The following inequalities are obtained

$$C(r_1^*, r_2^*) < C(0,0), C(r_1^*, r_2^*) < C(\frac{d_1}{\delta q_1}, \frac{d_2}{\delta q_2}).$$

The proof of Lemma 1 can be found in Appendix A.

The other seventeen cases are the same as (a). We will therefore go directly to Proposition 1 (the remaining seventeen derivations are shown in Appendix C, for your reference). Our derivations show there are five different patterns.

1. (a)–(d): The substitution of higher-grade products is equal to zero while the sourcing amounts of lower-grade products slightly changes. From their preconditions, the demands for both products are higher than the flexible quantities.
2. (e)–(g): When the demand for the lower-grade product is higher than the flexible quantity but the demand for the higher-grade product is lower than the flexible quantity and with some other limitations, the optimal substitution strategy follows the same pattern.
3. (h)–(k), (l)–(o) and (p)–(r): These refer to three different patterns. We can further prove that as long as $d_2 \le \delta r_2 q_2$, the pattern remains no matter what relationship between d_1 and $\delta r_1 q_1$ is.

This leads us to Proposition 1.

Proposition 1.

A. Cases (a)–(d) conform to Pattern 1. If the demand for both products are higher than their flexible quantity, the optimal substitution strategy is that any higher-grade product is unsubstituted.
B. Cases (e)–(g) conform to Pattern 2. If the demand for the higher-grade product is higher than its

flexible quantity, the demand for the lower-grade product is not higher than its flexible quantity, and the latter one is larger, the optimal substitution strategy has a similar pattern to $q_{s1}^* = d_1 - \delta r_1^* q_1$ and $q_{s2}^* = \delta r_2^* q_2 - d_2$.

C. Cases (h)–(k) conform to Pattern 3. If the demand for the higher-grade product is higher than its flexible quantity, the demand for the lower-grade product is not higher than its flexible quantity, and the former one is larger, the optimal substitution strategy has a similar pattern as $q_{s1}^* = q_{s2}^* = d_1 - \delta r_1^* q_1$.

D. Cases (l)–(o) conform to Pattern 4. If the demand for the higher-grade product is lower than its flexible quantity and the difference between the demand and the flexible quantity of the higher-grade product is less than the difference of the lower-grade product, then the optimal substitution strategy has a similar pattern to $q_{s1}^* = (\delta - 1) r_2^* q_2$ and $q_{s2}^* = \delta r_2^* q_2 - d_2$.

E. Cases (p)–(r) conform to Pattern 5. If the demand for the higher-grade product is lower than its flexible quantity and the difference between the demand and the flexible quantity of the higher-grade product is more than the difference of the lower-grade product, then the optimal substitution strategy has a similar pattern to $q_{s1}^* = (\delta - 1) r_2^* q_2$ and $q_{s2}^* = d_1 - \delta r_1^* q_1$.

Proposition 1 discusses the patterns for optimal strategies under different scenarios. Pattern A corresponds to the no substitution case where the manufacturer would bear the penalty rather than substituting higher-grade product with lower-grade product. Pattern B-E provides guidance in deciding the optimal substitution amount under different cases. Pattern B is the most intuitive case since the optimal substitutions only depend on their own demand and flexible quantity. Pattern C corresponds to the case where both production lines can be regarded as homogeneous, making the optimal substitution equal to each other. Pattern D corresponds to the case where the substitution is functioning. The higher-grade product is now employed to compensate for the deficiency of the lower-grade product in case the penalty is incurred. Pattern E is a worse version of Pattern D where the normal production line will also be severely influenced by the destruction of unreliable production line.

Lemma 2.

The optimal substitution strategy shares a similar pattern if the precondition that the demand for the higher-grade product is lower than the flexible quantity of the higher-grade product is met.

In general, there are five different expressions of q_{s1}^* and q_{s2}^*, depending on the diverse value of demand and the flexible quantity of both products, as shown in Table 3.

Table 3. Summary of proposition.

Proposition & Lemma	Higher Grade Product	Lower Grade Product	Difference between the demand and flexible quantity	q_{s1}^*	q_{s2}^*
1A	$\delta r_2 q_2 \le d_2$	$\delta r_1 q_1 \le d_1$	----------	----------	0
1B	$\delta r_2 q_2 \le d_2$	$\delta r_1 q_1 \ge d_1$	$\delta r_2 q_2 - d_2 \ge d_1 - \delta r_1 q_1$	$d_1 - \delta r_1^* q_1$	$\delta r_2^* q_2 - d_2$
1C	$\delta r_2 q_2 \le d_2$	$\delta r_1 q_1 \ge d_1$	$\delta r_2 q_2 - d_2 \le d_1 - \delta r_1 q_1$	$d_1 - \delta r_1^* q_1$	$d_1 - \delta r_1^* q_1$
1D	$\delta r_2 q_2 \ge d_2$	$\delta r_1 q_1 \ge d_1$	$\delta r_2 q_2 - d_2 \le d_1 - \delta r_1 q_1$	$(\delta - 1) r_2^* q_2$	$\delta r_2^* q_2 - d_2$
1E	$\delta r_2 q_2 \ge d_2$	$\delta r_1 q_1 \ge d_1$	$\delta r_2 q_2 - d_2 \ge d_1 - \delta r_1 q_1$	$(\delta - 1) r_2^* q_2$	$d_1 - \delta r_1^* q_1$
Lemma 1	$\delta r_2 q_2 \ge d_2$	$\delta r_1 q_1 \le d_1$	$\delta r_2 q_2 - d_2 \ge d_1 - \delta r_1 q_1$	$(\delta - 1) r_2^* q_2$	$\delta r_2^* q_2 - d_2$
Lemma 2	$\delta r_2 q_2 \ge d_2$	$\delta r_1 q_1 \le d_1$	$\delta r_2 q_2 - d_2 \le d_1 - \delta r_1 q_1$	$(\delta - 1) r_2^* q_2$	$d_1 - \delta r_1^* q_1$

From Table 3, we can easily obtain the optimal substitution strategy under deterministic demand. Note that the optimal strategy under both $\delta r_2 q_2 \ge d_2$ and $\delta r_1 q_1 \ge d_1$ is similar to the case when $\delta r_2 q_2 \ge d_2$ and $\delta r_1 q_1 \le d_1$. The supply chain managers can locate their demand and flexible quantity in Table 3 to find out the corresponding optimal sourcing and substitution decision.

4.2 Stochastic demand

Since it is difficult to obtain analytic solutions under the stochastic demand, we now illustrate the model with numerical examples, where both d_1 and d_2 are random and have a joint cumulative distribution function $F_d(d_1, d_2)$. The goal of the company is to minimize the expected total cost, which includes the sourcing cost and substitution cost, and the demand realization is (ξ_1, ξ_2). The backward induction is one of the most commonly used methods to solve such a problem, see [18,20,21], for example. Therefore, we first give the essential parameters that are necessary to obtain the optimal solution based on a real-world case, used in [24]. We then find the optimal substitution strategy when the two production lines may fail with different probabilities. After taking this substitution into account, the optimal sourcing strategy is obtained. Finally, sensitivity analysis is conducted to test the robustness of our model.

As assumed in the traditional inventory control field, the demand in the market follows the Poisson distribution, where $d_1 \sim P(\lambda_1)$ and $d_2 \sim P(\lambda_2)$, respectively [6,20,44]. The Poisson probability function is given by

$$P(X=k)=\frac{\lambda_i^{\,k}}{k!}e^{-\lambda_i}. \tag{17}$$

Therefore, we can calculate the probability of different combinations of the realized demand (ξ_1,ξ_2) and then calculate the expected total cost. Nonetheless, the optimal sourcing and substitution strategy is hard to obtain in this case. Without loss of generality, we assume that $d_1 \sim P(2)$ and $d_2 \sim P(1)$ since P_2 is a higher-grade product and can substitute for P_1. The flexible coefficient is 2. If $c_s=\frac{1}{2}, c_{Ri}=\frac{1}{2}, c_{Ui}=\frac{1}{4}, b_i=\frac{1}{2}$, the expected cost can be represented as

$$\begin{aligned}C(r_1,r_2)=&(1-\pi_1)(1-\pi_2)(2+r_1+r_2)+\pi_1(1-\pi_2)(0.5(\mathrm{Min}[\xi_1,2r_1]+q_{s1})+1+r_2+0.5q_{s1}\\&+0.5[\xi_1-2r_1-q_{s1}]^+)+\pi_2(1-\pi_1)(1+r_1+0.5\mathrm{Min}[\xi_2,2r_2]+0.5[\xi_2-2r_2]^+)\\&+\pi_1\pi_2(0.5\mathrm{Min}[\xi_1,2r_1]+0.5(\mathrm{Min}[\xi_2,2r_2]+q_{s2})+0.5q_{s2}\\&+0.5[\xi_1-q_{s2}-2r_1]^+ +0.5[\xi_2+q_{s2}-2r_2]^+).\end{aligned}$$

$q_{s1}=\mathrm{Min}[[\xi_1-2r_1]^+,2r_2]$ and $q_{s2}=\mathrm{Min}[[\xi_1-2r_1]^+,[2r_2-\xi_2]^+]$ since the number of substituted products must be an integer.

Note that the realized demand follows the Poisson distribution and the probability that each line in d_2 is disrupted is given. We calculate the cost when each realized demand occurs, and the summation of these costs gives us the expected total cost. By minimizing the expected total cost, the optimal sourcing and substitution strategy can be obtained. The expected total cost can be obtained by

$$E[C(r_1,r_2)]=\sum_{\xi_1=\underline{\xi_1}}^{\overline{\xi_1}}\sum_{\xi_2=\underline{\xi_2}}^{\overline{\xi_2}}\Pr(X=\xi_1)\Pr(X=\xi_2)C(r_1,r_2). \tag{18}$$

The goal is to find the minimal expected cost through the optimal sourcing strategy. Therefore, the program is

$$FindMinimum[E(r_1^*,r_2^*)]=\sum_{\xi_1=\underline{\xi_1}}^{\overline{\xi_1}}\sum_{\xi_2=\underline{\xi_2}}^{\overline{\xi_2}}\Pr(X=\xi_1)\Pr(X=\xi_2)C(r_1^*,r_2^*). \tag{19}$$

To better illustrate the optimal strategy under each case, we vary the disruption probability of each production line from 0 to 1 by increments of 0.2. The results are performed in Table 4.

Table 4. The optimal sourcing strategy and corresponding total cost under benchmark.

π_2 \ π_1	0	0.2	0.4	0.6	0.8	1
0	$r_1^* = 0, r_2^* = 0, C^* = 2$					
0.2	$r_1^* = 0$ $r_2^* = 0$ $C^* = 2$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 2.080$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 2.166$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 2.257$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 2.354$	$r_1^* = 1$ $r_2^* = 0.800$ $C^* = 2.457$
0.4	$r_1^* = 0$ $r_2^* = 0$ $C^* = 2$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 2.138$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 2.299$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 2.481$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 2.686$	$r_1^* = 1$ $r_2^* = 0.818$ $C^* = 2.913$
0.6	$r_1^* = 0$ $r_2^* = 0$ $C^* = 2$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 2.174$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 2.398$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 2.672$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 2.996$	$r_1^* = 1$ $r_2^* = 0.824$ $C^* = 3.370$
0.8	$r_1^* = 0$ $r_2^* = 0$ $C^* = 2$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 2.188$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 2.464$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 2.830$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 3.284$	$r_1^* = 1$ $r_2^* = 0.830$ $C^* = 3.827$
1	$r_1^* = 0$ $r_2^* = 0$ $C^* = 2$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 2.179$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 2.497$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 2.954$	$r_1^* = 1$ $r_2^* = 0$ $C^* = 3.549$	$r_1^* = 1$ $r_2^* = 0.728$ $C^* = 4.283$

If the sourcing strategy $r_i^* = 0$, then sourcing from the cheaper supplier is optimal. If $r_i^* = 1$, then sourcing from the expensive and reliable supplier is optimal. From Table 5, we see that if the disruption probabilities of both production lines are equal to zero, the best sourcing strategy is to source all goods from the cheaper supplier. In contrast, if the disruption probability of both production lines is equal to one, the best sourcing strategy is to source all lower-grade product from the expensive and reliable supplier and source over 70% of the higher-grade product from the expensive and reliable supplier. Because the cost of sourcing from the reliable supplier and of substitution is so high that it is preferable to lose part of the sales, products should not be sourced from the reliable supplier when the disruption probability reaches one (i.e. when the production line is certainly disrupted). We should also point out that the cost under this case is still the highest among all possible cases. If the disruption probability of the lower-grade production line is less than one, then the optimal sourcing strategy for higher-grade products remains the same. This is reasonable as only the higher-grade product can substitute for the lower-grade product, so sourcing the higher-grade product from the reliable supplier will always be guaranteed by the manufacturer at first. Moreover, the percentage of higher-grade product from the reliable supplier diminishes when the disruption probability of both production lines changes from $\pi_2 = 0.8$ to $\pi_2 = 1$ while keeping π_1 fixed. This is counterintuitive as it seems normal to source more higher-grade product from the reliable supplier than from the unreliable supplier since the disruption of the higher-grade production line is unavoidable. The manufacturer should first satisfy the demand for each product before considering the substitution since the lower-grade production line will be disrupted.

4.3 Further Explanation of the Proposed Model

We first compare the results obtained from the proposed model under deterministic and stochastic demand. Since the expectation of the Poisson distribution is equal to the variance and the arrival rate, there is no difference between the two cases when the deterministic demand is equal to the arrival rate. In other types of demand distributions, the difference between the two cases depends on the degree of risk aversion of the manufacturer. When the expectation of demand remains the same and the variance is higher (i.e. the demand is more unpredictable), the manufacturer with a high-risk aversion might source more from the reliable supplier to mitigate the destruction of the production lines. The corresponding substitution fraction will decrease and then remain at a very low degree. In contrast, when the manufacturer is risk-seeking, they might source more goods from the cheaper but unreliable supplier. This increases the possible amount of substitution. In this paper, we assume that all parties in the supply chain are risk-neutral; variations on this can be explored in future research.

Figure1 shows the interaction effect of substitution and dual sourcing. The sourcing strategies when there are two suppliers and no substitution are represented by the full lines, and those with substitution are represented by the dotted lines.

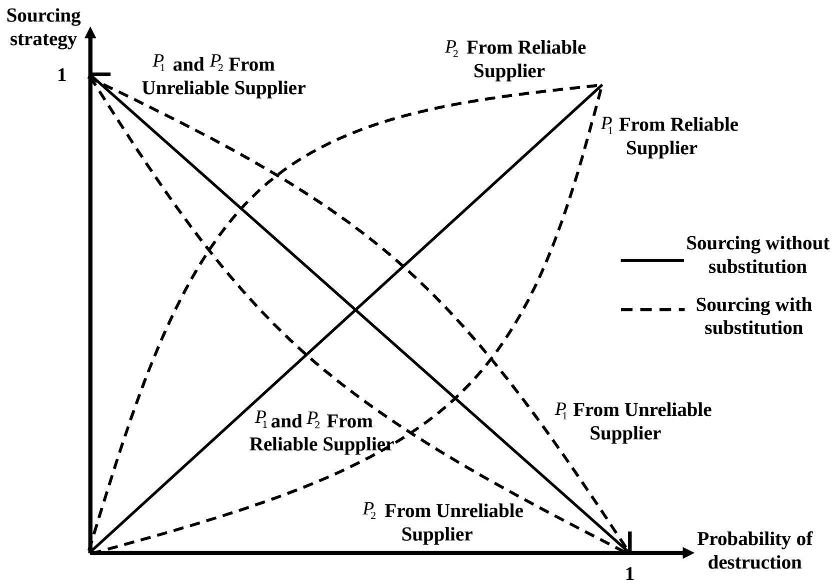

Figure 1. Interaction effect of substitution and dual sourcing.

Fig. 1 can be obtained through Proposition 1 as well as numerical examples. The vertical axis represents the sourcing strategy for the manufacturer range from 0 to 1, where 1 represents that all products are sourced from reliable supplier and 0 represents that all products are sourced from unreliable supplier. The area between the full lines and dotted lines is called the flexible area, which leaves the manufacturer more space to use a substitution strategy to adjust the sourcing strategy. Moreover, we find that the substitution strategy is more effective for the reliable supplier than for the unreliable supplier. If the probability of destruction increases, the sourcing strategy from the reliable supplier increases, leading to the necessity of the flexible area, i.e. substitution. In contrast, the unreliable supplier suffers from the disruption and own less flexibility in substitution than the reliable supplier. Interestingly, from the results obtained in Section 3.2, we find that the substitution effect is at its peak when the probability of destruction is at a middle level. Under this circumstance, the sourcing strategies for the reliable supplier and the unreliable supplier are similar. Additionally, the integrated profit of the supply chain is maximized because of the substitution effect (where both suppliers maximize their flexibility), forming a Pareto area. When the probability of destruction is at a low or high level, the substitution effect is maximized, that is, the manufacturer should adjust their sourcing strategy instead of relying on substitution. This is counter-intuitive since existing literature usually concludes that substitution should be employed as much as possible when production lines may be disrupted. Nonetheless, by relaxing the assumption that both production lines can suffer from disruption, we prove that this conclusion is incorrect. Rather, substitution should be significant when anticipating that the destruction probability is of a middle level. By using the results from Figure 2, a manufacturer can better adjust their substitution and dual-sourcing strategy. In the following section, we introduce a case study to illustrate the effectiveness of our proposed model in reality. Additionally, the previous sensitivity analysis can further perform the alteration of the optimal strategy under different variants.

5. Case Study

We now illustrate the practical application of our model by using real case data collected from a steel product factory in China to analyze the optimal sourcing and substitution strategy. Managerial insights are proposed to help the factory make better decisions when their production line may be disrupted.

First, we test the assumption that the arrival of demand follows a Poisson process. We use the one-sample Kolmogorov-Smirnov test to check the goodness of fit of the Poisson distribution to the data obtained from a downstream firm of the steel product factory from June 01, 2012 to April 06, 2013 (with annual and monthly inspection times removed) [35].The specific data can be found in the online Appendix B. Suppose that the arrival of demands follows a Poisson process with arrival rate λ_1. Through the one-sample Kolmogorov-Smirnov test, we have $\lambda_1 = 0.529$ per day. The hypothesis test summary is shown in Table5.

Table 5. Hypothesis test summary for the lower-grade steel product.

Null hypothesis	Test	Sig.	Decision
The distribution is Poisson with mean 5.29per 10 days.	One-sample Kolmogorov -Smirnov Test	0.938	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is 0.05.

Similarly, we use another product that can substitute for the steel product (higher-grade product) and analyze the data from the same period to obtain the arrival rate. Nonetheless, the higher-grade steel product in this case is a product of constant demand. There is a downstream factory ordering 6 specific goods per 10 days. The demand that the factory is confronted with is a random demand following a Poisson distribution with $\lambda_1 = 0.529$ and a deterministic demand $d_2 = 0.6$. The objective function can now be rewritten as

$$E[C(r_1, r_2)] = \sum_{\xi_1=\underline{\xi_1}}^{\overline{\xi_1}} \Pr(X = \xi_1) C(r_1, r_2). \tag{20}$$

Using our investigation of the steel product factory and the average price of a single mold, we estimate the cost parameters for the practical example. The substitution cost between the different product grades is around $c_s = \$195$. The sourcing cost of the higher-grade product from the reliable supplier is $c_{R1} = \$19.5$/ton while the sourcing cost of the lower-grade product is around $c_{R2} = \$12.5$/ton. Additionally, the sourcing cost of the higher-grade product from the unreliable supplier is $c_{U1} = \$9.5$/ton while the sourcing cost of the lower-grade product is around $c_{U2} = \$6.5$/ton. The penalty cost is 30% of the initial price, which means for the higher-grade product it is $b_1 = \$148$ and for the lower-grade product it is $b_2 = \$206$. The flexible coefficient is still 2. After several interviews and surveys, we found that the disruption probabilities of the lower- and higher-grade products are 80% and 40%, respectively. Using our model to calculate the optimal sourcing strategy and the corresponding total cost, we find that $r_1^* = 0.001,\ r_2^* = 0.667,\ C^* = 124.051$. The optimal data we obtained is very close to the factory's actual practice, where they source none of the lower-grade steel product from the reliable supplier and they source around two-thirds of their higher-grade steel product from the reliable supplier. The corresponding cost minus the fundamental sourcing cost is 10%, which is also close to the cost we obtained. This verifies the usefulness and effectiveness of our proposed model. All possible strategies were performed, and their related total costs under different disruption probabilities are shown in Figures2–4 below.

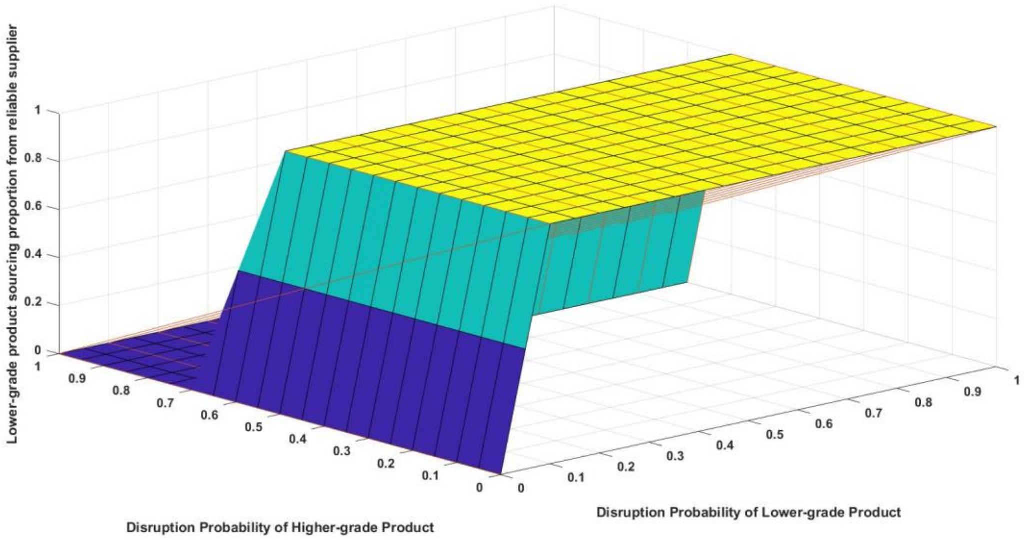

Figure 2.Lower-grade product sourcing proportion from reliable supplier with varying disruption probabilities for both products.

Figure 2 shows that the optimal sourcing strategy for the lower-grade product remains at 1 if the disruption probability of the lower-grade product is less than 0.7, except the case when the disruption probability of the higher-grade product is 0 as well. If the destruction probability of the lower-grade product is high enough, the manufacturer prefers to leave the demand unfulfilled rather than source them from the reliable supplier.

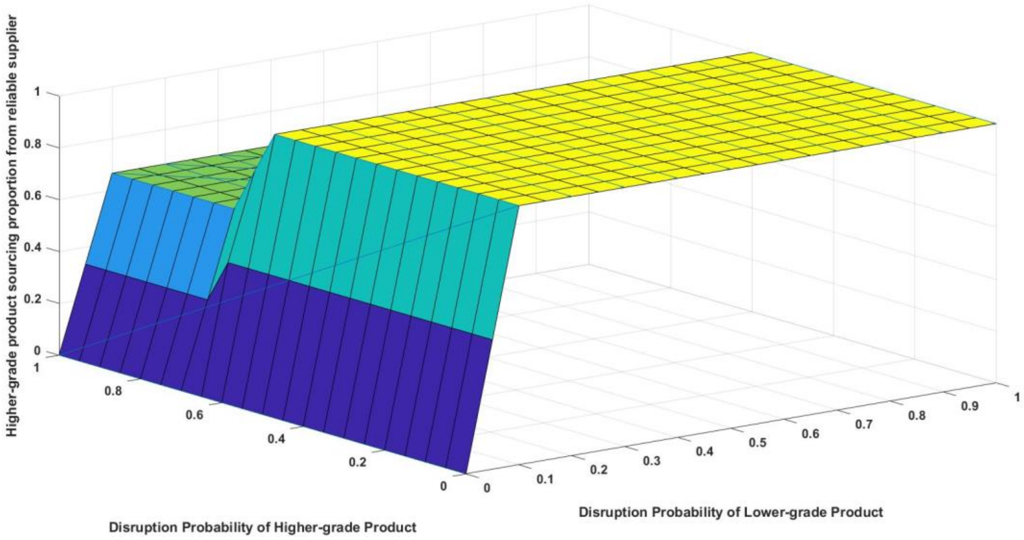

Figure 3.Higher-grade product sourcing proportion from reliable supplier with varying disruption probabilities for both products.

Figure 3 shows that the optimal sourcing strategy for the higher-grade product remains at 1 when the disruption probability of the lower-grade product is less than 0.7, except when the disruption probability of the higher-grade product is 0 as well. When the destruction probability of the lower-grade product is high enough, the manufacturer prefers to source two-thirds of the higher-level product from the reliable supplier. This becomes a dominating strategy.

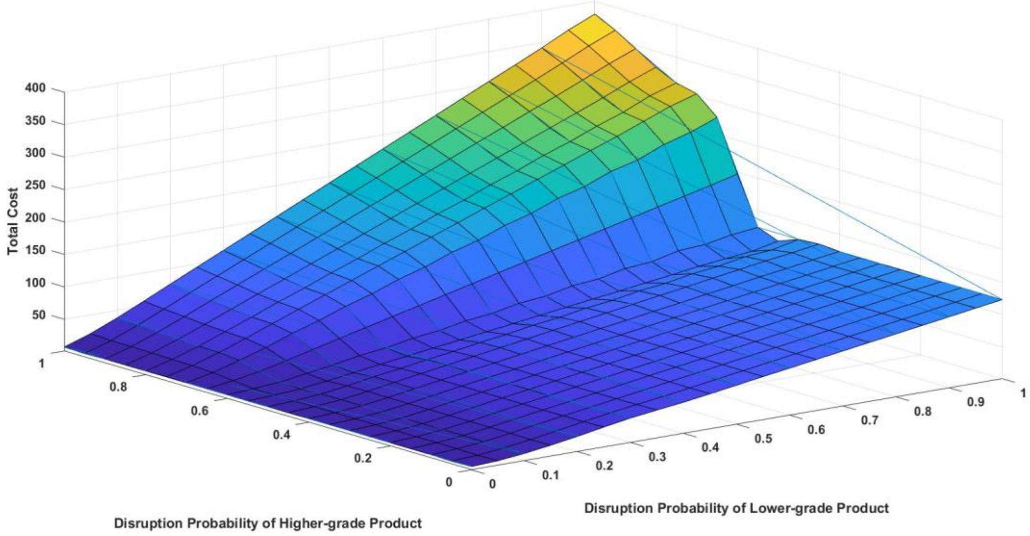

Figure 4. Expected total cost for varying disruption probabilities for both products.

Figure 4 shows that, after the given strategies shown in Figure 3 and Figure 4, the expected cost shows the following trend: before the disruption probability of the lower-grade product reaches 0.7, the expected total cost increases at a normal rate. However, when this destruction probability becomes high enough, the expected cost rises rapidly because of the alteration of the optimal strategies. All results obtained here agree with our major conclusion in the model foundation, illustrating the effectiveness of our proposed model.

We now conduct some sensitivity analysis to discuss what managers should alter in their strategy under different contexts. The probability density function, cost of sourcing, cost of penalty and cost of substitution may vary. For simplification, we only consider the alteration of product 1. In fact, the increase in sourcing cost of product 1 can be regarded as the relative decrease in sourcing cost of product 2. We directly illustrate the results in Figure 5.

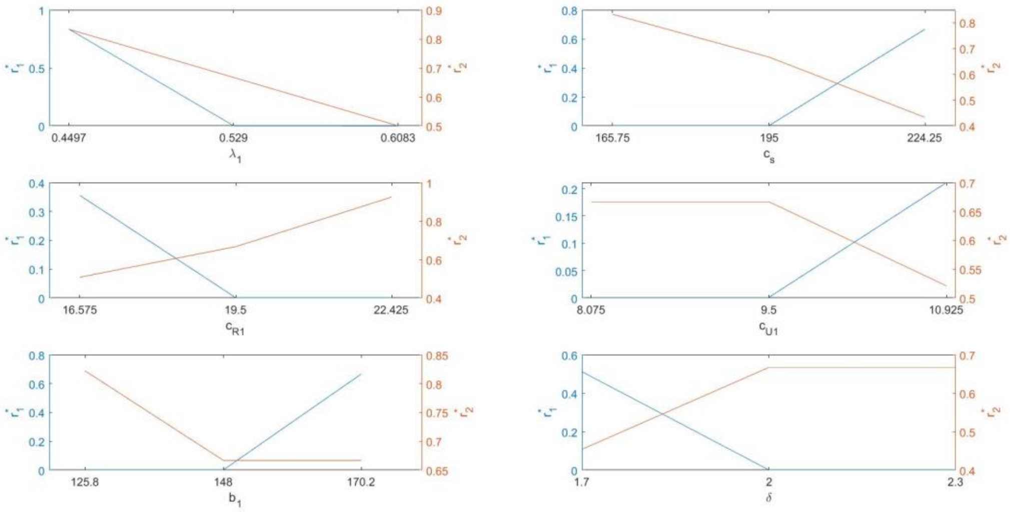

Figure 5 Sensitivity analysis of case study

From Figure. 5, we can see that when the expected value of the Poisson distribution is increasing, the manufacturer should source more products from the reliable suppliers, no matter for high-grade or low-grade products. One explanation for this could be that the best strategy when the parameter is increasing is to keep the product sourced in a steady state rather than taking the risk of penalty. When the substitution cost between two types of product is increasing, the manufacturer would source more low-grade products from a reliable supplier since the substitution is more costly under this case. When the sourcing cost of low-grade products from a reliable supplier is increasing, it is reasonable that the sourcing for low-grade from a reliable supplier is decreasing and the sourcing for high-grade products is increasing since the substitution cost becomes relatively cheaper now. In contrast, when the sourcing cost of low-grade product from unreliable supplier is increasing, the ordering of low-grade products from a reliable supplier becomes relatively cheaper and thus the manufacturer now orders more low-grade products from a reliable supplier and fewer high-grade products. Now we go to the case where there is an augment in substitution cost. Under this case, the manufacturer chooses to order more low-grade product to satisfy the demand instead of relying on substitution. Finally, the increase in flexible capacity of a reliable supplier has no impact on the sourcing strategy of the manufacturer since the initial sourcing amounts from a reliable supplier is less than the maximal value of the flexible capacity. Contrary from that, when the flexible capacity is going down, then the manufacturer has to source more low-grade products in order to satisfy the demand.

Thus, we conclude our managerial insights through theoretical analysis and numerical examples as follows: For a manufacturer, it can decide the optimal substitution and sourcing policy under different scenarios to maximize its profit. Actually, there are five patterns that the manufacturer can find themselves in and take the corresponding strategy combination. The employment of dual sourcing and substitution strategies forms a flexible area, where two types of strategies can compensate with each other. For a supplier, anticipating the sourcing policy of the manufacturer, the supplier can alter its flexible capacity to better coordinate with downstream, leading to a win-win situation.

6. Conclusion and future work

This paper considered a supply chain that utilizes product substitution and dual sourcing. Suppose that products can be ordered from a supplier that may or may not be reliable. A reliable supplier may be able to offer more choices at any time than an unreliable one. Assume that there are two separate production lines, which are subject to random disruptions with different probabilities of occurrence. The manufacturer chooses the optimal substitution policy and the dual sourcing policy to minimize the total cost. Through backward induction, we found that under deterministic demand there are five possible substitution functions, given that different relationships between demand and flexible quantity are held. We analyzed the case of stochastic demand through numerical study, and the different strategies from the manufacturer's perspective were established through sensitivity analysis. The interaction between the substitution and dual-sourcing strategy was performed under a more realistic case. We also employed real world data to gain a better understanding of the practical applicability of our model.

Our future research will aim to improve the proposed model from a variety of aspects. First, in our proposed model, we did not consider the backorder cost, which incurs commonly in supply chain models. Further research could assume that some consumers will backorder the product. It might also be of interest to investigate what types of product can be backordered easily, i.e. higher-grade products or lower-grade products. Second, in our proposed model, we assumed that the products are not perishable. In the real world, some products might perish during transportation, which should be considered by the manufacturer indecision making. Third, in our proposed model, we considered only two products, where one product can substitute for the other. In practice, a supplier might have a great number of different product combinations. This would increase the number of product categories, which is worth investigating in the future. Fourth, the game under asymmetric information and competitive market are also an interesting direction that deserves further analyzing [45]. Finally, in our proposed model, we only considered a one-period game between the manufacturer and the suppliers. The analysis of multi-period game, i.e., newsvendor, is definitely needed. The optimal ordering will thus be influenced by the information disturbance in different stages, making the product's expected demand unequal to the optimal ordering. Besides, our work analyses a supply chain problem through reliability modelling and optimization. Our future work aims to solve other types of management challenges by taking into account practical reliability issues, i.e., redundancy. We believe that the consideration of reliability can lead to more interesting and convincing managerial implications in practice.

Acknowledgment

The research was supported by the NSFC under grant numbers 71671016, 71832011,71901025, and the Beijing Nova Program of Science & Technology under grant Z191100001119100.

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