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Warranty and sustainable improvement of used products through remanufacturing

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Abstract: Currently, a large number of used/second-hand products are being sold with remanufacturing. Remanufacturing is a process of bringing used products to a better functional state and can be applied as a way for (1) controlling the deterioration process, (2) reducing the likelihood of a failure over the warranty period and (3) making the used item effectively younger. Remanufacturing is relatively a new concept and has received very limited attention. In this paper, we develop an important sustainable improvement approach for used items sold with failure free warranty to determine the optimal improvement level. Our model makes a useful contribution to the reliability growth literature, as it captures the uncertainty and suggests improvement in the remanufacturing process. By using this model, the dealers can decide whether and how much to invest in remanufacturing projects.

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1 Introduction

The development of cost/benefit models for investments in sustainable improvement is crucial because it can help the manufacturers in evaluating the effectiveness of the amount of investment they spend and selecting optimal investment opportunities. The investment in sustainable improvement should not be based on faith, and should be analysed by the 'quantified' measures of quality (Lee, 2008). Therefore, what is needed is a way of measuring the impact of quality improvement programmes and a mechanism for predicting the return of an investment in these programmes.

As the competition intensifies in the global market, the product life cycle gets shortened fast and manufacturers try to introduce new products more frequently as one of their key strategic responses. Some products such as computers and mobile phones have a short lifetime and technologies of these products are released to the market every day. As a result, the sale of new products is often tied to a trade-in, resulting in a market for used products (Chattopadhyay and Murthy, 2000).

The used product exchange network is an example of these markets. This network is a growing trend of selling, distributing, buying, marketing and selling of used products and services using the internet as a tool and has launched a new way for sellers to link directly with the buyers. Before this, the sellers could not sell their products directly to buyers. Generally, the activities which carry out in these networks are as follows (see Figure 1):

- the end user goes to the exchange network websites

- the end user posts his/her product's description to the dealer

- the dealer/third party offers different warranty policies and sets the sales price

- the new buyer searches and selects second-hand product in the website.

Furthermore, customers expect excellent quality even for used products. In line with this, the warranty period is extended from half a year to sometimes three years, or even longer. For example, the warranty period for used cars was three months in the early 90s, whereas, currently, it varies from one to three years (The Used Car Market Report, 2007).

Currently, a large number of used items are being sold with remanufacturing. Dealers of used products must ensure that the products they supply are in compliance with the safety requirements and regulation. Research shows that high percent of the used products have high-failure rate just after the purchase by the new buyer. Therefore, dealers should arrange some effective ways to certify the safety of the products and prevent any danger that may arise from the use of the products. These effective ways can be called, in general, as 'remanufacturing'.

In this paper, we develop an important sustainable improvement approach for used items sold with failure free warranty to determine the optimal improvement level. Our model makes a useful contribution to the reliability growth literature, as it captures the uncertainty and suggests improvement in the remanufacturing process. By using this model, the dealers can decide whether and how much to invest in remanufacturing projects.

The outline of this paper is as follows. In Section 2, we carry out a review of the literature on warranty and new product improvement. In Section 3, we give a brief overview of remanufacturing so as to set the background for the main contribution of this paper. In Section 4, we present the model assumptions and notation and use these in Section 5 to formulate our stochastic model and analyse it. We also examine the optimal improvement level for achieving a sensible trade-off between increasing the investment in remanufacturing and reducing the warranty cost and the condition required for remanufacturing projects to be beneficial is derived. In Section 6, we provide a numerical example to verify some of our results. Section 7 concludes this paper and offers some suggested directions for future research.

Figure 1   Activity description in a used product exchange network (see online version for colours)

2   Literature review

Sustainable improvement of new products has also received some attention in the reliability literature. A chronological review of models dealing with warranty and new product improvement is shown in Table 1.

Table 1   A review of the literature linking warranty and new product improvement

Author/s	Year	Abstract
Davis	1952	- Deals with the reliability characteristics of reconditioned bus engines and finds that the reliability improves after each reconditioning
Duane	1964	- Assumes that the development effort leads to a reduction in the failure rate
Crow	1974	- Treats the improvements as occurring according to a non-homogeneous Poisson process (NHPP)
Malik	1979	- Introduces the main concept of 'improvement factor' for modelling repairable systems
Nakagawa	1979	- Assumes that the improvement action is minimal with probability a and perfect with probability $1 - a$

Table 1 A review of the literature linking warranty and new product improvement (continued)

Author/s	Year	Abstract
Murthy and Nguyen	1987	- Develops three different models (Models I–III) for determining optimal reliability improvement taking into account the impact of reliability on the expected warranty cost. Model I is based on the Crow's model (1974). In Model II, it is assumed that the failure rate changes after each design modification and is independent of the development time. The development programme is terminated after k modifications. In Model III, the failure rate of the product changes after each modification, and the amount of reduction in the failure rate is a function of development time
Kijima et al.	1988	- Introduces the main concept of 'virtual age' and calls it the 'Type-I Kijima' where the improvement results in a reduction in the age of the system
Kijima	1989	- Generalises the 'Type-I Kijima' and presents a second virtual age model 'Type-II Kijima'
Chan and Shaw	1993	- Considers the failure rate of an item to be reduced after each repair and this reduction depends on the item age and the number of repairs
Jack and Dagpunar	1994	- Uses a virtual age model to determine the quality and the period between overhauls
Zhang and Jardine	1998	- Proposes a failure rate model, where after an overhaul, the item performance is between as good as before and as good as after the previous overhaul
Djamaludin et al.	2001	- Develops a framework to study preventive policies when the vendor offers an initial period of warranty and pays labour, materials and downtime costs if a failure occurs
Jiang and Ji	2002	- Suggests a repair limit based on the virtual age concept to decide on the type of the maintenance action to apply at the time of a maintenance fault
Jung and Park	2003	- Uses a virtual age model for the failure rate and considers post-warranty preventive maintenance activities
Hussain and Murthy	2003	- Develops a simple stochastic model for failure rate reduction based on Duane (1964)
Doyen and Gaudoin	2004	- Proposes two imperfect repair models. In the first one, repair effect is expressed by a reduction of the failure intensity whereas in the second model it reduces the virtual age of the system
Kim et al.	2004	- Develops a strategy to determine maintenance policies over the warranty duration following Kijima's virtual age model
Dimitrov et al.	2004	- Develops an age-dependent repair model and evaluates the expected warranty costs under different warranty scenarios
Bai and Pham	2005	- Models the imperfect repairs in a single-dimensional warranty context as a quasi-renewal process

Table 1   A review of the literature linking warranty and new product improvement (continued)

Author/s	Year	Abstract
Pascual and Ortega	2006	- Uses a virtual age model to determine optimal intervals between overhauls by minimising global maintenance costs
Chien	2008	- Investigates the impact of an imperfect renewing free-replacement warranty on the age-replacement policy with an increasing failure rate
Yun et al.	2008	- Looks at two new warranty servicing strategies involving minimal and imperfect repairs
Guida et al.	2009	- Proposes a Bayesian procedure to formalise the prior information available about the failure probability of an upgraded automotive component
Samatli-Pac and Taner	2009	- Extends the Bai and Pham's quasi-renewal methodology (2005) to multi-dimensional warranties and adopt the appropriate version in both one- and two-dimensional analyses
Yeo and Yuan	2009	- Develops a model to determine the optimal maintenance period and optimal level of repair based on the structures of the cost function and failure rate function

In contrast, a literature review shows that only few researchers have worked in the area of remanufacturing. The existing papers deal with different issues as illustrated by the following list:

- *Definition and concept*: McCaskey (1994), Andreu (1995), Amezquita et al. (1996), Ijomah (2002), Sundin (2002), Ijomah et al. (2004) and Ijomah et al. (2005).

- *Environmental*: Lund (1996) and Hormozi (1996).

- *Economic*: Shu and Flowers (1995), Ferrer (1996), Hormozi (1996) and Nasr (1996).

- *Engineering*: Haynesworth and Lyons (1987), Ishii (1998), Guide (1999), Ferrer (2001), Ijomah et al. (2007) and Ijomah (2008).

- *Practical experience*: Lund (1984) and Ijomah (1999).

3   Remanufacturing: an overview

3.1   Influence diagram of a used product

Figure 2 shows the influence diagram of a used product that illustrates the interactions among three lifetime stages: past life, remanufacturing duration and warranty period as follows:

The dealer purchases the used product from an end user. Often the dealer has knowledge of the age and usage of the product. These are usually obtained from sources such as registration forms and/or log books (Chattopadhyay and Murthy, 2000). We assume that the item has been maintained or repaired through minimal repairs during its past life all the way until age of x. At age x, the used product is subjected to a remanufacturing process (such as overhaul, reconditioning and upgrade action) with level u, which affects the performance of the item. This improvement can be seen as an investment by the dealer to reduce the risk of early failure. The purpose of this

investment is to reduce the variance and the deviation from the target value of the quality characteristic, and hence, to reduce the expected warranty cost and also to increase the reliability. Since the buyer pays nothing for repairs during the warranty period, from the buyer's perspective, investing in remanufacturing projects is viewed as an unnecessary cost and hard to justify. However, from a life cycle perspective, remanufacturing can have a significant impact on the maintenance cost after the warranty has expired which is borne by the buyer.

After the remanufacturing process, the dealers offer different kinds of warranty policies for their used products. Warranty policy is a statement, in connection with the sale of a product, on the kind (e.g. free repair/replacement, lump sum payment or a pro-rata reimbursement) and the extent (length of period) of compensation offered by the dealer in the event of failure. Offering the warranty implies that the dealer incurs additional costs to service any claims made by the customers. For example, US manufacturers spend over \$25 billion (about 2% of the revenue) annually on warranty service (Manna et al., 2007). These costs are critically dependent on the product reliability, past age and usage, servicing strategy used by the dealer and the terms of the warranty policy. Through effective warranty servicing strategies, the dealer can reduce the warranty costs. The warranty period w is fixed, and starts from the purchase time by the new user. The buyer incurs the full replacement and maintenance cost on failures of the used product after the original warranty period has expired.

Figure 2 Influence diagram of a used product (see online version for colours)
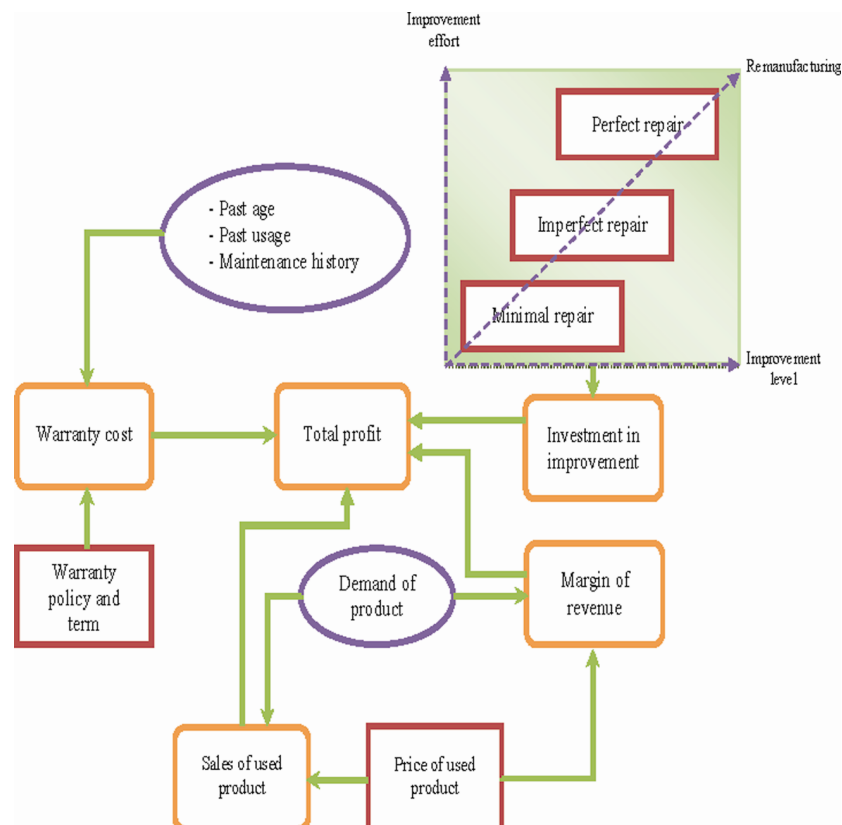
Finally, to stimulate consumers' purchase willingness, dealers must determine the reasonable selling prices for their second-hand items. The price of a second-hand item should not be too high to dissuade consumers from purchasing.

3.2   Remanufacturing concept

Remanufacturing can be defined as actions to

1   restore the used item to a better functional state

2   control the deterioration process

3   reduce the likelihood of a failure

4   make the used item effectively younger.

Remanufacturing typically begins with the arrival of a used product (called a core) at the remanufacturer, where it passes through a series of industrial stages including disassembly, cleaning, part remanufacture and replacing of unremanufacturable parts, reassembly and testing to produce the remanufactured product (Ijomah et al., 2007). This involves research and development (R&D) effort where the used item is subjected to an iterative process of test, analyse, recondition and fix cycles. During this process, an item is tested for a certain period of time or until a failure occurs. Based on the analysis of the test run and failure mode, design and/or engineering modifications are made to improve the reliability. This process is repeated resulting in reliability improvement (or growth) of the item.

The remanufacturing strategies can be classified according to degree of restorability of the used item (see Figure 3).

1   *Replacement with a new item* turns failure rate of the item to zero if replaced with new one (see curve 'a' in Figure 3).

2   *Perfect repair* is a restorative improvement action that enables the used item to be 'as good as new' condition (see curve 'a').

3   *Overhauling or imperfect repair (replacement with a younger one)* restores a substantial portion and the hazard rate falls in between 'as good as new' and 'as bad as old' (see curve 'b'). This strategy is referred as '*reconditioning*' in the literature.

4   *A minimal repair* makes insignificant improvement and the condition after remanufacturing is 'as bad as old' (see curve 'c').

In the context of used-car market, a typical example of minimal repair would be changing the tires, rectifying the ignition or wiring system or any repair of the engine that does not change the overall performance of the car, whereas a typical perfect repair would be a transmission replacement or an engine replacement.

Figure 3 Failure rate with effect of remanufacturing (see online version for colours)
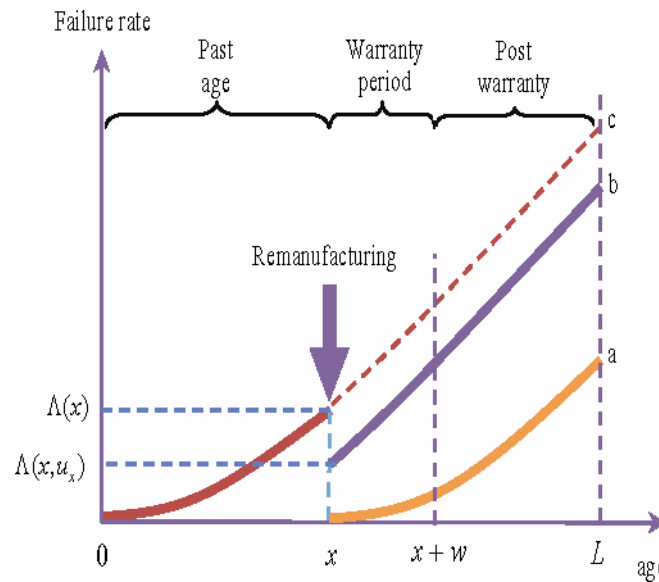
4 Assumptions and notation

In this section, we present the assumptions and notations used in our model formulation.

- the used product is subjected to two types of remanufacturing action: minimal repair and perfect repair; each action has its own associated costs

- the dealer rectifies all failures occurring during the warranty period

- whenever a failure occurs, it results in an immediate claim

- all warranty claims are valid

- the mean time to remanufacture a used item is small in relation to the mean time between failures and it can be ignored

- failures are statistically independent

- the investment in remanufacturing of each used item is comprised a fixed setup cost and a variable cost which depends on the past age and the improvement level.

Nomenclature

x past age of the used product

L expected lifetime of the new product

u_x improvement level (decision variable) $[0 \le u_x \le x]$ $[u_x = 0$ implies no improvement]

λ	scale parameter of product failure distribution
β	shape parameter of product failure distribution
$F(t)$	cumulative failure distribution of the product
$f(t)$	density function associated with $F(t)$
$\Lambda(t)$	intensity function for product failure
θ_1, θ_2	the parameters of the Beta distribution
w	warranty period
$c_u(x)$	investment in remanufacturing process of a used product with past age x and improvement level u_x
$\Lambda(x,u_x)$	failure rate after the remanufacturing
c_s	setup cost of the remanufacturing process
c_u	the remanufacturing cost per unit of time
$E[c_w(x,u_x)]$	the dealer's expected warranty cost
$\bar{c}$	expected cost of each rectification over the warranty period

5   Model formulation

5.1   Investment in remanufacturing

Let u_x be the improvement level of the used item with past age of x. The investment in remanufacturing process is expressed as a function, $c_u(x)$ is a function of the improvement level u_x (> 0) and can be modelled as

$$c_u(x) = c_s + c_u u_x \tag{1}$$

where c_s is the setup cost of the remanufacturing process per unit of the item and c_u is the remanufacturing cost per unit of time (e.g. tools and labour cost). Note that $u_x = 0$ implies minimal repair, $u_x \in (0, x)$ implies imperfect overhaul or replacement the failed item with a younger one and $u_x = x$ implies perfect repair or replacement with a new item.

5.2   Model

Let T be the random variable describing the failure time of the product. We assume cumulative failure distribution of the product is modelled as $F(t)$ with density function:

$$f(t) = \frac{dF(t)}{dt}$$

From Barlow and Hunter (1960), the rate of occurrence of failures (ROCOF) is given by the failure intensity function

$$\Lambda(t) = f(t) / [1 - F(t)]$$

We assume that the remanufacturing process results in a rejuvenation of the used item so that it effectively reduces the failure rate of the item. Let the used item fail after a period t subsequent to the sale and denote, respectively, $f(t,u_x)$, $F(t,u_x)$ and $\Lambda(t,u_x)$ as the failure time probability density function, cumulative distribution function and failure intensity function after the remanufacturing, where

$$\Lambda(t,u_x) = f(t,u_x) / [1 - F(t,u_x)]$$

We use a model based on Nakagawa's probabilistic approach (1979). In this approach, we assume that the used product undergoes 'perfect repair' condition with a probability of p and 'minimal repair' condition with a probability of $1 - p$. Therefore,

$$\Lambda(t,u_x) = p \times \Lambda(t - x) + (1 - p) \times \Lambda(t) \tag{2}$$

or

$$\Lambda(t,u_x) = \Lambda(t) + p \times \left[\Lambda(t - x) - \Lambda(t) \right] \tag{3}$$

where $p \in [0,1]$ denotes the degree of the restorability of the item. The value of $p = 1$ can be achieved only when the remanufacturing programme is perfect repair. We assume p as a random variable distributed according to a Beta distribution with parameters θ_1 and θ_2, which are dependent on u_x. This implies that the density function for p is given

$$B_P(p) = \frac{\Gamma(\theta_1 + \theta_2)}{\Gamma(\theta_1)\Gamma(\theta_2)} p^{\theta_1 - 1} (1 - p)^{\theta_2 - 1} \tag{4}$$

with the expected value of $E[p] = \theta_1 / (\theta_1 + \theta_2)$. The parameters of θ_1 and θ_2 are considered as non-negative random variables which are related to u_x as follows:

$$\theta_1 = a_1 u_x^{b_1} \tag{5}$$

and

$$\theta_2 = a_2 u_x^{b_2} \tag{6}$$

The best way of estimating these parameters is by observing the past history of the product.

5.3   Warranty cost

This section derives the expected warranty cost for a used product with past age of x sold under failure free warranty. Under a failure free policy, the dealer agrees to rectify a failed unit, free of charge to the new buyer, during $[0,w]$ after sale. Second-hand products sold under failure free warranties might include electronics such as large-screen colour TVs, automobiles, refrigerators and household appliances.

Let $E[N_w(x,u_x)]$ be the density number of claims over the warranty period w when the product age is x and the improvement level is u_x at sale. The expected number of claims over the warranty period is given by

$$\begin{aligned}
E\left[N_w(x,u_x)\right] &= \int_x^{x+w} \Lambda(t,u_x)\,dt \\
&= \int_x^{x+w} \int_0^1 p \times \Lambda(t-x) \times B_P(p)\,dp\,dt \\
&\quad + \int_x^{x+w} \int_0^1 (1-p) \times \Lambda(t) \times B_P(p)\,dp\,dt \\
&= E[p] \times \int_x^{x+w} \Lambda(t-x)\,dt + E[1-p] \\
&\quad \times \int_x^{x+w} \Lambda(t)\,dt \\
&= \frac{\theta_1 \int_x^{x+w} \Lambda(t-x) + \theta_2 \int_x^{x+w} \Lambda(t)}{\theta_1 + \theta_2}
\end{aligned} \tag{7}$$

Therefore, the dealer's expected warranty cost, $E\left[c_w(x)\right]$, for a product of age x at sale is given by

$$E\left[c_w(x,u_x)\right] = \bar{c}\,E\left[N_w(x,u_x)\right] \tag{8}$$

where $\bar{c}$ is the expected cost of each rectification action over the warranty period.

5.4 Total mean cost

The total mean cost of the product includes the investment in remanufacturing and the warranty cost. Therefore, the total mean cost is given by

$$c(x,u_x,w) = c_u(x) + E\left[c_w(x,u_x)\right] \tag{9}$$

The optimal improvement level u_x^* can be obtained by minimising, $c(x,u_x,w)$, the total mean cost per item. A higher improvement level is usually costly and adds to the sale price of the used product but leads to a lower warranty cost for the dealer. It is worthwhile for the dealer to carry out remanufacturing only if the reduction in the warranty servicing cost is greater than the extra cost incurred with investment in improvement.

5.5 Weibull distribution

Let $\Lambda(t)$ be given by

$$\Lambda(t) = \lambda\beta(\lambda t)^{\beta-1} \tag{10}$$

Note that $u_x = 0$ implies no improvement and the total expected cost is given by

$$E\left[c_w(x,0)\right] = \bar{c}\lambda^\beta\left[(x+w)^\beta - x^\beta\right] \tag{11}$$

and the expected warranty cost under improvement becomes:

$$E\left[c_w(x,u_x)\right] = \frac{\bar{c}\lambda^\beta}{\theta_1+\theta_2}\left\{\theta_1 w^\beta + \theta_2\left[(x+w)^\beta - x^\beta\right]\right\} \tag{12}$$

The condition required for investment to be beneficial: if

$$c_s + c_u u_x + \frac{\bar{c}\lambda^\beta}{\theta_1 + \theta_2}\left\{\theta_1 w^\beta + \theta_2\left[(x+w)^\beta - x^\beta\right]\right\} < \bar{c}\lambda^\beta\left[(x+w)^\beta - x^\beta\right] \qquad (13)$$

or

$$c_s + c_u u_x < \frac{\theta_1}{\theta_1 + \theta_2}\bar{c}\lambda^\beta\left[(x+w)^\beta - x^\beta - w^\beta\right] \qquad (14)$$

then the minimum reduction in warranty cost is more than the investment in remanufacturing process, and hence, investment in remanufacturing is beneficial.

If investment is beneficial, the optimal improvement level u^* can be obtained by minimising

$$c(x, u_x, w) = c_s + c_u u_x + \frac{\bar{c}\lambda^\beta}{\theta_1 + \theta_2}\left\{\theta_1 w^\beta + \theta_2\left[(x+w)^\beta - x^\beta\right]\right\} \qquad (15)$$

6 Numerical results

In this section, we conduct a numerical example to verify the results established in the previous sections. Table 2 summarises the problem and lists the numerical values.

We consider the case where the failure distribution is Weibull with the following parameter values:

$$\beta = 2, \lambda = 0.22156 / \text{year}$$

This implies that the mean time to first failure is four years.

The warranty length and improvement level are measured in years. As a result, selling a used product with $u = 0.5$ and $w = 2$ implies that the dealer makes the items effectively six months younger and releases them to the market with warranty of 24 months.

Table 3 gives the optimal values u_x^* and the corresponding expected total cost, $c(x, u^*, w)$ for a range of values of w and x.

Also in Table 3, we provide the cost comparison between remanufacturing vs. non-remanufacturing. It shows that remanufacturing decision is worthwhile compared to selling without remanufacturing. From Table 2, it is easy to see that as w increases, the improvement level increases. Also, we observe that the improvement level is higher for higher values of x.

Table 2 Summary of the example

Expected lifetime of the new product (year)	*L = 10*
Warranty policy	FRW
The terms of warranty	$w = 1.0, 1.5, 2.0, 2.5, \bar{c} = \$50$
The parameters of improvement	$c_u = \$10, c_s = \$1$
The coefficient of Beta parameters	$b_2 = 1, a_2 = 10, b_1 = 2, a_1 = 100$

**Table 3** $c(x,u^*,w)$ and u^* for different combinations of w and x

	w = 1.0			w = 1.5			w = 2.0			w = 2.5		
x	c(x,u*,w)	u*	c(x,u = 0,w)	c(x,u*,w)	u*	c(x,u = 0,w)	c(x,u*,w)	u*	c(x,u = 0,w)	c(x,u*,w)	u*	c(x,u = 0,w)
1.0	6.91	0.1	7.36	10.98	0.2	12.89	16.09	0.2	19.64	22.41	0.3	27.61
2.0	8.73	0.2	12.27	13.20	0.3	20.25	18.73	0.3	29.45	25.25	0.4	39.88
3.0	10.14	0.3	17.18	14.94	0.4	27.61	20.71	0.4	39.27	27.48	0.5	52.16
4.0	11.36	0.3	22.09	16.41	0.4	34.98	22.36	0.5	49.09	29.35	0.6	64.43
5.0	12.36	0.4	27.00	17.66	0.5	42.34	23.83	0.6	58.91	31.01	0.7	76.70
6.0	13.35	0.4	31.91	18.83	0.6	49.70	25.18	0.7	68.72	32.52	0.8	88.97
7.0	14.18	0.5	36.82	19.89	0.6	57.07	26.41	0.7	78.54	33.89	0.8	101.25

Total mean cost for different combinations of x and a 2.5-year warranty is shown in Figure 4.

Based on the results of the analysis, the following observations may be summarised:

- The improvement level depends on many factors such as product failure distribution, past age of the used product, warranty length and cost parameters.

- Remanufacturing becomes important when
  - warranty periods are long
  - the initial failure rate of the used product is large
  - the failures during the warranty period are costly.

- Remanufacturing is appropriate where there is a market for the remanufactured product. Thus, fashion-affected products are inappropriate because users may prefer the newer product no matter the quality and cost of the remanufactured alternative.

- From the buyer's perspective, a myopic buyer might decide not to invest in any remanufacturing since the dealer/third party rectifies all failures over the warranty period at no cost to the buyer. However, from a life cycle perspective the total life cycle cost to the buyer is influenced by remanufacturing.

Figure 4   Total mean cost for different combinations of x and 2.5-year warranty (see online version for colours)
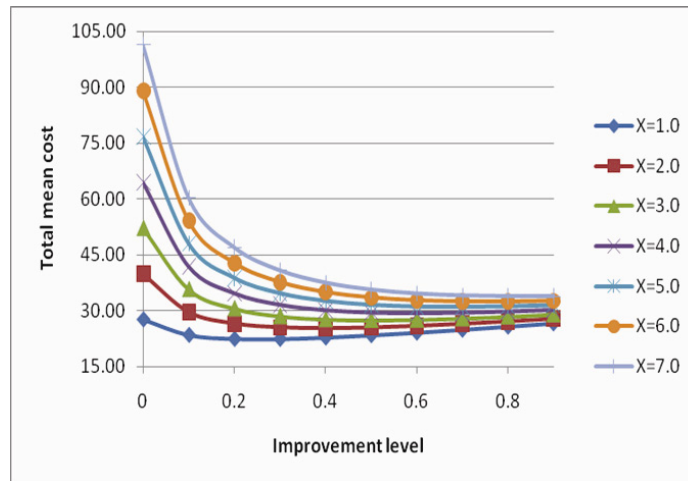
7 Conclusions and topics for future research

In this paper, we established a cost optimisation model to determine the optimal level of remanufacturing process for used items sold with failure free warranty. In the past, most of the studies concentrated on developing deterministic functions of the improvement, whereas in real life, the outcome of any remanufacturing process is uncertain. For solving this problem, we integrated both uncertain reliability improvement and cost models for investments to successfully deal the optimal improvement decision problem.

Possible topics for future research include the following:

1 The method of estimating the model parameters is not discussed in this paper. This is a topic for considerable new research.

2 The validation of the model using real data is yet to be studied.

3 We have confined our analysis to the failure free policy and failed item being always repaired minimally. The analysis of other types of warranty policies, for example, pro-rata, combination is yet to be carried out.

4 This approach can be generalised to work with a mixture of other applicable statistical distributions.

5 Extending the results to build models which incorporate past usage and maintenance strategy.

The authors have obtained some results for 1–5 and these will be reported elsewhere.

References

Amezquita, T., Hammond, R., Salazar, M. and Bras, B. (1996) 'Characterizing the remanufacturability of engineering systems', *Proceedings of ASME advances in design automation conference*, DE-Vol. 82, September 17–20, Boston, MA, USA, pp.271–278.

Andreu, J-J. (1995) *The Remanufacturing Process*. Internal paper from MA, UK: Metropolitan University.

Bai, J. and Pham, H. (2005) 'Repair-limit risk-free warranty policies with imperfect repair', *IEEE Transactions on Systems, Man and Cybernetics, Part A*, Vol. 35, pp.765–772.

Barlow, R.E. and Hunter, L. (1960) 'Optimum preventive maintenance policies', *Operation Research*, Vol. 8, pp.90–100.

Chan, J-K. and Shaw, L. (1993) 'Modeling repairable systems with failure rates that depends on age and maintenance', *IEEE Transaction on Reliability*, Vol. 42, pp.566–571.

Chattopadhyay, G.N. and Murthy, D.N.P. (2000) 'Warranty cost analysis for second-hand products', *Mathematical and Computer Modeling*, Vol. 31, pp.81–88.

Chien, Y.H. (2008) 'Optimal age-replacement policy under an imperfect renewing free replacement warranty', *IEEE Transaction on Reliability*, Vol. 57, pp.125–133.

Crow, L.H. (1974) 'Reliability analysis for complex repairable system', In F. Proschan and R.J. Serfling (Eds.), *Reliability and Biometq*, SIAM, pp.379–410.

Davis, D.J. (1952) 'An analysis of some failure data', *Journal of American statistical Association*, Vol. 47, pp.113–150.

Dimitrov, B., Chukova, S. and Khalil, Z. (2004) 'Warranty costs: an age-dependent failure/repair model', *Naval Research Logistic*, Vol. 51, pp.959–976.

Djamaludin, I., Murthy, D.N.P. and Kim, C.S. (2001) 'Warranty and preventive maintenance', *Int. J. Reliability, Quality and Safety Engineering*, Vol. 8, pp.89–107.

Doyen, L. and Gaudoin, O. (2004) 'Classes of imperfect repair models based on reduction of failure intensity or virtual age', *Reliability Engineering and System Safety*, Vol. 84, pp.45–56.

Duane, J.T. (1964) 'Learning curve approach to reliability monitoring', *IEEE Transaction on Aerospace*, Vol. 2, pp.563–566.

Ferrer, G. (1996) *On the Economics of Remanufacturing a Widget*. France: INSEAD.

Ferrer, G. (2001) 'Theory and methodology on the widget remanufacturing operation', *European Journal of Operational Research*, Vol. 135, pp.373–393.

Guida, M., Pulcini, G. and Vianello, M. (2009) 'Early inference on reliability of upgraded automotive components by using past data and technical information', *Journal of Statistical Planning and Inference*, Vol. 139, pp.1604–1618.

Guide, Jr., V.D. (1999) 'Remanufacturing production planning and control: US industry best practice and research issues', *Second International Working Paper on Re-use*, Eindhoven, pp.115–128.

Haynesworth, H.C. and Lyons, R.T. (1987) 'Remanufacturing by design, the missing link', *Production and inventory management*, Second quarter, pp.24–28.

Hormozi, A. (1996) 'Remanufacturing and its consumer economic and environmental benefits', *APEX Remanufacturing Symposium*, May 20–22, USA, pp.5–7.

Hussain, A.Z.M.O. and Murthy, D.N.P. (2003) 'Warranty and optimal reliability improvement through product development', *Mathematical and Computer Modelling*, Vol. 38, pp.1211–1217.

Ijomah, W. (2002) 'A model-based definition of the generic remanufacturing business process', Doctoral Thesis, University of Plymouth, UK.

Ijomah, W. (2008) 'A tool to improve training and operational effectiveness in remanufacturing', *Int. J. Computer Integrated Manufacturing*, Vol. 21, pp.676–701.

Ijomah, W., Bennett, J. and Pearce, J. (1999) 'Remanufacturing evidence of environmentally conscious business practices in the UK', *EcoDesign '99: First International Symposium on Environmentally Conscious Design and Inverse Manufacturing*, February 1999, Tokyo, Japan.

Ijomah, W., Childe, S. and McMahon, C. (2004) 'Remanufacturing – a key strategy for sustainable development', *Proceedings of the Third International Conference on Design and Manufacture for Sustainable Development*, 1–2 September 2004, Loughborough, UK.

Ijomah, W., Childe S., McMahon, C. and Hammond, G.P. (2005) 'A robust description and tool for remanufacturing: a resource and energy recovery strategy', *Proceedings of EcoDesign 2005: Fourth International Symposium on Environmentally Conscious Design and Inverse Manufacturing*, 12–14 December, Tokyo, Japan.

Ijomah, W., McMahon, C., Hammond, G.P. and Newman, S. (2007) 'Development of design for remanufacturing guidelines to support sustainable manufacturing', *Robotics and Computer Integrated Manufacturing*, Vol. 23, pp.712–719.

Ishii, K. (1998) 'Modularity: a key concept in product life-cycle engineering', In A. Molina and A. Kusiak (Eds.), *Handbook of Life-Cycle Enterprise*. Dordrecht: Kluwer.

Jack, N. and Dagpunar, J.S. (1994) 'An optimal imperfect maintenance policy over a warranty period', *Microelectronics Reliability*, Vol. 34, pp.529–534.

Jiang, R. and Ji, P. (2002) 'Age replacement policy: a multi-attribute value model', *Reliability Engineering and System Safety*, Vol. 76, pp.311–318.

Jung, G.M. and Park, D.H. (2003) 'Optimal maintenance policies during the post-warranty period', *Reliability Engineering and System Safety*, Vol. 82, No. 2, pp.173–185.

Kijima, M. (1989) 'Some results for repairable systems with general repair', *Journal of Applied Probability*, Vol. 26, pp.89–102.

Kijima, M., Morimura, H. and Sujuki, Y. (1988) 'Periodical replacement problem without assuming minimal repair', *European Journal of Operational Research*, Vol. 37, pp.194–203.

Kim, C.S., Djamaludin, I. and Murthy, D.N.P. (2004) 'Warranty and discrete preventive maintenance', *Reliability Engineering and System Safety*, Vol. 84, pp.301–309.

Lee, H-H. (2008) 'The investment model in preventive maintenance in multi-level production systems', *Int. J. Production Economics*, Vol. 112, pp.816–828.

Lund, R.T. (1984) 'Remanufacturing: the experience of the USA and implications for the developing countries', *World Bank Technical Paper No. 3*.

Lund, R.T. (1996) *The Remanufacturing Industry: Hidden Giant*. Boston University.

Malik, M.A.K. (1979) 'Reliable preventive maintenance policy', *AIIE Transactions*, Vol. 11, pp.221–228.

Manna, D.K., Pal, S. and Sinha, S. (2007) 'A use-rate based failure model for two-dimensional warranty', *Computers and Industrial Engineering*, Vol. 52, pp.229–240.

McCaskey, D. (1994) 'Anatomy of adaptable manufacturing in the remanufacturing environment', *APICS Remanufacturing Seminar Proceedings*, USA, pp.42–45.

Murthy, D.N.P. and Nguyen, D.G. (1987) 'Optimal development testing policies for products sold under warranty', *Reliability Engineering*, Vol. 19, pp.113–123.

Nakagawa, T. (1979) 'Optimal policies when preventive maintenance is imperfect', *IEEE Transactions on Reliability*, Vol. 28, pp.331–332.

Nasr, V. (1996) 'Lifecycle analysis and costing in an environmentally conscious manufacturing environment', *APICS Remanufacturing Symposium Proceedings*, USA, pp.44–47.

Pascual, R. and Ortega, J.H. (2006) 'Optimal replacement and overhaul decisions with imperfect maintenance and warranty contracts', *Reliability Engineering and System Safety*, Vol. 91, pp.241–248.

Samatli-Pac, G. and Taner, M.R. (2009) 'The role of repair strategy in warranty cost minimization: an investigation via quasi-renewal processes', *European Journal of Operational Research*, Vol. 197, pp.632–641.

Shu, L. and Flowers, W. (1995) 'Considering remanufacture and other end-of-life options in selection of fastening and joining methods', *Proceedings of the IEEE International Symposium on Electronics and the Environment*, Orlando, FL, USA, May 1–3, pp.75–80.

Sundin, E. (2002) 'Design for remanufacturing from a remanufacturing process perspective', Linko¨ping Studies in Science and Technology, Licentiate Thesis No. 944, LiU-TEK-LIC-2002-17, Department of Mechanical Engineering, Linkö ping Universitet, Sweden, ISBN 91-7373-336-9.

The Used Car Market Report (2007) BCA group; 'Produced in association with Sewells information and research', Available at: http://www.sewells.co.uk. Accessed on 3 November 2008.

Yeo, W.M. and Yuan, X-M. (2009) 'Optimal warranty policies for systems with imperfect repair', *European Journal of Operational Research*, Vol. 199, pp.187–197.

Yun, W-Y., Murthy, D.N.P. and Jack, N. (2008) 'Warranty servicing with imperfect repair', *Int. J. Production Economics*, Vol. 11, pp.159–169.

Zhang, F. and Jardine, A.K.S. (1998) 'Optimal maintenance models with minimal repair, periodic overhaul and complete renewal', *IIE Transactions*, Vol. 30, pp.1109–1119.