Developing a trade-off between upgrade action time and warranty length for second-hand electrical components

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Abstract: One way of improving the reliability and reducing the warranty servicing cost for second-hand items is through actions such as overhaul and upgrade which are carried out by the dealer or a third party. For second-hand electrical components, the improvement involves testing of items for a short time, called upgrade action time, before their release to the market. The items that fail during the test are scrapped or repaired; only items that survive the test are considered to be of good quality and released to the market. In this paper, a cost model is developed to achieve a trade-off between reducing the warranty servicing cost and increasing the upgrade action cost for a second-hand electrical component sold under various warranty policies.

Keywords: expected warranty cost; second-hand product; total mean cost; trade-off; upgrade action.

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1 Introduction

Second-hand items are products that have previously been used by an end-user/consumer. Users change their products even if they are still in good condition because of new functions and technologies coming to the market. For example, products such as computers and mobile phones have a short lifetime and new technologies for these products are released on the market almost on a daily basis. As a result, the sale of new products is often tied to a trade-in, resulting in a growing market for second-hand items (Chattopadhyay and Murthy, 2000).

In France, for example, between 1990 and 2006, the used car unit sales increased from 4.7 million to 5.4 million, and at the same time the new car sales declined from 2.3 million to 2.07 million units (Car Internet Research Program II, 2007).

The demand for warranties for second-hand products has been growing along with the growth of the market for second-hand products. Nowadays, warranties for second-hand products are becoming increasingly more important in consumer and commercial transactions. So, they are widely used and they serve many purposes including: providing protection for both the dealer and the new buyer, being an indicator of second-hand product quality and reliability, a promotional tool to gain reputation and assuring buyers against products that do not perform as promised. In line with this, the warranty period for second-hand products is extended from a half year to sometimes three years, or even longer. For example, in the early 1990s in the used automobile market, the basic warranty coverage was 6 months/6000 miles, which nowadays has increased to as much as 3 years/30,000 miles on the entire vehicle, and 5 years/50,000 miles on the used power trains.

In spite of increasing the share market for second-hand products, often customers of second-hand products encounter the following three problems:

a they are uncertain about durability and performance of these products owing to lack of knowledge related to their past usage and maintenance history b they are uncertain about the accurate pricing of warranties and the post-warranty repair costs

c sometimes, right after the sale, second-hand items may have high failure rate and could be harmful to their new owner.

Owing to these problems, the dealers are currently carrying out actions such as overhaul and upgrade on the second-hand items before their release. These actions are especially important when warranty periods are quite long.

Upgrade action, which is closely related to the concept of warranty, for second-hand products is a relatively new concept and has received very limited attention. Chattopadhyay and Murthy (2004) develop two models to decide on the reliability improvement strategies for used items sold with Free Repair Warranty (FRW) policy. Pongpech et al. (2006) deal with an optimisation model for identifying an upgrade action for used equipment under a fixed period of lease.

For second-hand electrical components, the upgrade action involves testing of items for a short time, called upgrade action time, before their release to the market. The items that fail during the test are scrapped or repaired; only items that survive the test are considered to be of good quality and released to the market. The screening process can be applied at various stages during improvement (e.g., disassembly, cleaning, reconditioning and so on).

Screening test has also received some attention in the reliability improvement literature. Coleman (1990) suggests the use of screening test to improve product reliability. Pohl and Dietrich (1995) deal with the optimal screening duration and develop a three-level stress-screening model for a complex electronic system. Pohl and Dietrich (1999) deal with a model that is an extension of their earlier model.

In this paper, a cost model is developed to achieve a trade-off between reducing the warranty servicing cost and increasing the upgrade action cost for a second-hand electrical component sold under various warranty policies (failure free, rebate warranty and a combination of free replacement and lump sum). In our study, the product lifetime distribution is assumed to be the Weibull distribution with shape parameter β and scale parameter λ. It is well known that, in reliability studies, the Weibull distribution is often used to model the failure times. By an appropriate choice of the parameter values, it provides a flexible model for a distribution with an Increasing Failure Rate (IFR) or Decreasing Failure Rate (DFR).

The rest of this paper is organised as follows. We present the model assumptions and notation in the next section. Section 3 derives the upgrade action cost and warranty cost models and examines upgrade action time and warranty length to achieve a trade-off between reducing the warranty servicing cost and increasing the upgrade action cost. In Section 4, the proposed model is applied to a real-life case study. Section 5 summarises the paper with concluding remarks.

2 Model assumptions and notation

We consider three stages for a second-hand product – past life, upgrade action duration and warranty period (see Figure 1).

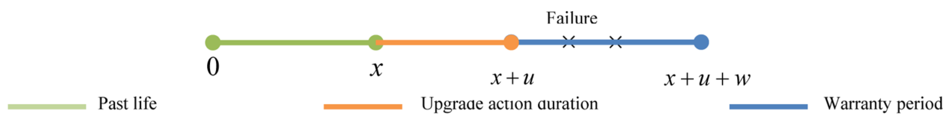
a *Past age:* Second-hand products differ from new ones as the condition varies from item to item. The condition depends on age, usage and maintenance history. The dealer can assess the condition indirectly through registration forms or log books.

b *Upgrade action duration*: The dealers should arrange some effective ways to certify the reliability and safety of the products and prevent any harm that may arise from the use of the products. These effective ways can be called, in general, as '*upgrade action*'. The dealer can employ this action to improve product reliability, minimise warranty servicing costs and control the degradation of the second-hand item.

For electrical components, the upgrade is carried out by operating the products under electrical or thermal conditions for a short time, which is called upgrade action time, before their release. Finding an optimal upgrade action time is an interesting topic of research in optimisation area, in which both the dealer's costs and the dealer's benefits need to be taken into account. On the one hand, providing an upgrade action is usually costly and adds directly to the sale price of the second-hand product. On the other hand, an upgrade action can reduce the warranty cost by improving the performance of the product. Also, it allows the dealer to offer better warranty terms and sell the item at a higher price.

c *Warranty period*: Offering warranty for second-hand items promotes the sales but at the same time, it results in additional cost, which is associated with the warranty repairs. This cost depends on several factors such as the past age or the past usage, the warranty terms, the maintenance strategy and the buyer's risk attitude. The warranty period w is fixed, and starts immediately after the sale to the new buyer. After the expiration of the warranty period, all expenses are borne by the buyer.

Figure 1 Three lifetime stages of a second-hand product (see online version for colours)



We also assume the following:

1 The product is either repairable or non-repairable. The dealer rectifies all failures during the warranty period.

2 Whenever a failure occurs, it results in an immediate claim.

3 All warranty claims are valid.

4 The time to rectify a failed item (either through repair or replacement) is sufficiently small in relation to the mean time between failures and it can be ignored.

5 Since the item is minimally repaired at each failure, rectification action has a negligible impact on its reliability, and hence, the system failure intensity after repair is the same as that before failure.

6 Failures are statistically independent.

7 The dealer sells products that have gone through a screening test.

3 The cost function

The total mean cost of the product includes the initial cost and the warranty servicing cost.

3.1 Initial cost

3.1.1 Purchase price from an end-user

Let the dealer pay c_x to the end-user for purchasing a second-hand product with age x. This price can be estimated from the market feedback data or from the depreciation rate of the product. In the literature, two methods have been applied, Straight Line (SL) method and Double Declining Balance (DDB) method.

In SL method, the dealer's purchase price can be modelled as a function of the past age x, by Chattopadhyay and Murthy (2004)

$$c_x = P_0\left(1 - \frac{x}{L}\right) \tag{1}$$

and in DDB method,

$$c_x = P_0\left(1 - \frac{2}{L}\right)^x \tag{2}$$

where P_0 is the price and L is the expected lifetime of the new product.

3.1.2 Upgrade action cost

Let u be the upgrade action time and T be the random variable describing the failure time of the product with cumulative distribution function $F(t)$ and density function $f(t) = \mathrm{d}F(t)/\mathrm{d}t$.

Case I: Repairable product

The upgrade action cost for a repairable second-hand product comprised a fixed cost (e.g., screening test overheads per unit of item), a variable cost that depends on the upgrade action time (e.g., tool cost or labour cost) and a rectification cost of failed items during upgrade action. By modifying Nguyen and Murthy's general model for repairable products (Nguyen and Murthy, 1982), we can obtain the expected upgrade action cost for a repairable second-hand product with past age x as follows

$$E[c_u(x)] = c_s + c_u u + \overline{c}\int_x^{x+u} \Lambda(t)\,\mathrm{d}t \tag{3}$$

where c_s is the set-up cost of the upgrade action per unit of product, c_u is the upgrade action cost per unit of time and $\overline{c}$ is the expected rectification cost of each failed item. Differentiating equation (3) with respect to u yields

$$\frac{\partial}{\partial u}E[c_u(x)] = c_u + \overline{c} \times \Lambda(x+u) > 0. \tag{4}$$

Since $\Lambda(t)$ is an increasing function of t, the expected upgrade action cost, $E[c_u(x)]$, increases with the upgrade action time. Therefore, the expected initial cost for a repairable second-hand product with past age x and upgrade time u, $c_I(x, u)$, is

$$c_I(x,u) = c_x + E[c_u(x)] \tag{5}$$

where c_x is given by equations (1) or (2) and $E[c_u(x)]$ by equation (3).

Case II: Non-repairable product

The upgrade action cost for a non-repairable second-hand product comprised a fixed cost and a variable cost that depends on the failure time during upgrade. By modifying Sheu and Chien's general model for non-repairable products (Sheu and Chien, 2005), we can obtain the upgrade action cost for a non-repairable second-hand product with past age x as follows

$c_s + c_u(t-x) \quad t \in [x, x+u)$ if the item fails at age t during upgrade

$c_s + c_u u \;\; t \in [x+u, +\infty)$ if the item survives the upgrade action

and the expected upgrade action cost for a non-repairable second-hand product with age x, $E[c_u(x)]$, is (see Appendix I)

$$E[c_u(x)] = c_s\overline{F}(x) + c_u\int_x^{x+u}\overline{F}(t)\,\mathrm{d}t \tag{6}$$

where $\overline{F}(t) = 1 - F(t)$ is the survival function of the item.

Differentiating equation (6) with respect to u yields

$$\frac{\partial}{\partial u}E[c_u(x)] = c_u \times \overline{F}(x+u) > 0. \tag{7}$$

It also shows that the expected upgrade action cost, $E[c_u(x)]$, increases with the upgrade action time. The probability that a second-hand product survives at the upgrade action time is

$$P(t > x+u \mid t > x)) = \frac{\overline{F}(x+u)}{\overline{F}(x)}. \tag{8}$$

Therefore, the expected initial cost for a non-repairable second-hand product with past age x and upgrade action time u, $c_I(x, u)$, is

$$c_I(x,u) = \frac{\overline{F}(x)}{\overline{F}(x+u)}.(c_x + E[c_u(x)]) \tag{9}$$

where c_x is given by equations (1) or (2) and $E[c_u(x)]$ by equation (6).

3.2 Warranty servicing cost

This section derives the expected warranty cost for a second-hand product with age x sold with upgrade action time u and warranty period w for different warranty policies – the failure-free policy, the rebate warranty and the FRW/LSW policy.

Denoted by $F_u(t)$ [$f_u(t)$], the failure time cumulative [density] distribution function of a second-hand item of age x is subjected to an upgrade action time u. Then, the cumulative [density] distribution function can be expressed as

$$F_u(t) = \frac{F(x+u+t) - F(x+u)}{1 - F(x+u)} \tag{10}$$

with density function

$$f_u(t) = \frac{f(x+u+t)}{1 - F(x+u)}. \tag{11}$$

3.2.1 Repairable product

Case I: Free Repair Warranty

When a repairable second-hand product is sold with an FRW policy, the dealer agrees to rectify the faulty item, free of charge to the new buyer, during the warranty period [0, w] after sale. Let $E[N_w(x)]$ be the expected number of claims over the warranty period w when the product age is x at sale. Therefore, the expected number of claims over the warranty period is given by

$$E[N_w(x)] = \int_0^w \Lambda_u(t)\,\mathrm{d}t \tag{12}$$

where

$$\Lambda_u(t) = f_u(t)/[1 - F_u(t)].$$

Then, the dealer's expected warranty costs, $E[c_w(x)]$, for a repairable second-hand item of age x at sale are given by

$$E[c_w(x)] = \overline{c}\int_0^w \Lambda_u(t)\,\mathrm{d}t. \tag{13}$$

3.2.2 Non-repairable product

For non-repairable products, the expected free replacement frequency followed by a renewal process constitutes a renewal equation (Wu et al., 2007). We assume that the dealer is required to provide a new product at no cost to the buyer from the time of purchase. Then, free replacements are provided until a product having a life of at least w is found. By assuming that the failure time of the products is independently and identically distributed, the number $N_w(x)$ of warranty repairs within the warranty is a random variable with a geometric distribution with parameter $(1 - F_u(w))$ and the probability distribution of $N_w(x)$ is

$$P[N_w(x)=n]=\begin{cases}1-F_u(w) & n=0\\ \left[F_u(w)\right]^n\left[1-F_u(w)\right] & n\geq 1\end{cases}. \tag{14}$$

Then, the expected number of replacements for a non-repairable second-hand product, during the time interval [0, w] after sale, is

$$E[N_w(x)]=\frac{F_u(w)}{1-F_u(w)}. \tag{15}$$

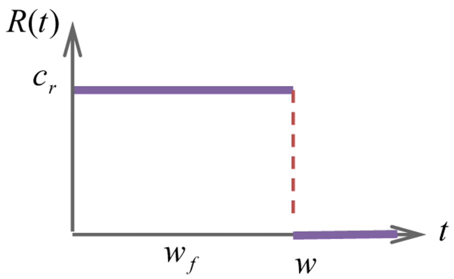
Case I: Free Replacement Warranty

When a non-repairable second-hand product is sold with an FRW policy, a product that fails within its warranty period is replaced by a new one at no cost to the new buyer. The cost function for warranty per replacement under the FRW policy can be expressed as (see Figure 2)

$$R(t)=\begin{cases}c_r & 0\leq t<w\\ 0 & \text{otherwise}\end{cases}, \tag{16}$$

where c_γ is the expected replacement cost per failure during the warranty length.

Figure 2 The refund function for an FRW policy (see online version for colours)



Then, the expected warranty cost per replacement, $E[R(t)]$, is

$$E[R(t)]=\int_0^w c_r f_u(t)\,\mathrm{d}t=c_r F_u(w). \tag{17}$$

Case II: Pro-rata warranty

Under a pro-rata warranty, the dealer agrees to replace the item that fails during the warranty period at a charge to the new buyer that is prorated to the age of the failed item. The PRW is sometimes offered on relatively cheap non-repairable second-hand products such as batteries, tyres and ceramics.

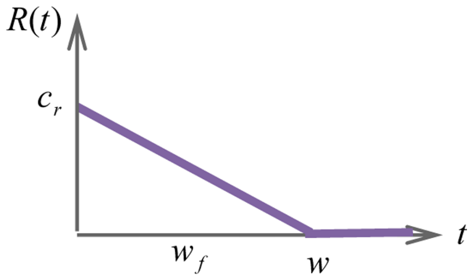
The cost function for warranty per replacement under the PRW policy can be expressed as (see Figure 3)

$$R(t)=\begin{cases}c_r\left[1-\dfrac{t}{w}\right] & 0\leq t<w\\ 0 & \text{otherwise}\end{cases}. \tag{18}$$

Then, the expected warranty cost per replacement, $E[R(t)]$, is

$$E[R(t)] = \int_0^w c_r \left[1 - \frac{t}{w}\right] f_u(t)\,\mathrm{d}t = \frac{c_r}{w}\int_0^w F_u(t)\,\mathrm{d}t \tag{19}$$

Figure 3 The refund function for a PRW policy (see online version for colours)



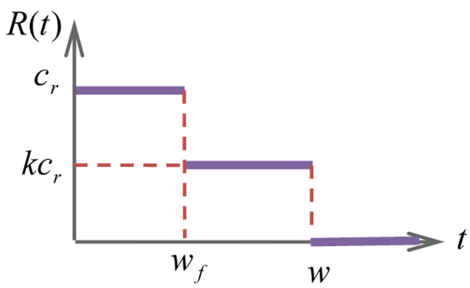
Case III: FRW/LSW

Under this policy, the failed or defective product is replaced free of cost to the new buyer up to a certain time w_f; meanwhile, the dealer refunds a customer some proportion of the replacement cost if the second-hand product fails during the w_f to w after sale. The cost function for warranty per replacement under the FRW/LSW policy can be expressed as (see Figure 4)

$$R(t) = \begin{cases} c_r & 0 \le t < w_f \\ kc_r & w_f \le t < w \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

where $0 \le k \le 1$ is the proportionality coefficient of c_γ and may be established by the dealer based on economic and competitive considerations (Mitra and Patankar, 1998).

Figure 4 The refund function for an FRW/LSW policy (see online version for colours)



Then, the expected warranty cost per replacement, $E[R(t)]$, is

$$E[R(t)] = \int_0^{w_f} c_r f_1(t)\mathrm{d}t + \int_{w_f}^{w} kc_r f_1(t)\mathrm{d}t = c_r[(1-k)F_u(w_f) + kF_u(w)]. \tag{21}$$

According to Wald's renewal equation (Wald, 1944), the expected warranty cost for a non-repairable second-hand product, under upgrade time u and warranty length w, is

$$E[c_w(x)] = E[N_w(x)] \times E[R(t)] \tag{22}$$

where $E[N_w(x)]$ is given by equation (15), and $E[R(t)]$ is given by equations (17), (19) or (21) depending on the offered warranty policy.

3.3 Total mean cost per product

We assume that the dealer of second-hand products sells products of ages within the interval $[m, M]$. The expected warranty costs for older products are higher than those for younger products. If these high costs are built in to sale price, then older products become unattractive to the new buyers. To make the sale price of the older products attractive to the new buyers, the dealer can subsidise older products by charging less than the expected warranty cost and recover the subsidy from younger products by charging more than the expected warranty cost (Chattopadhyay and Murthy, 2000).

We model this by viewing the age X of the second-hand item, as a random variable with support $[m, M]$ and a cumulative [density] distribution function $H(x)$ $[h(x)]$, with $H(m) = 0$ and $H(M) = 1$. On carrying out the expectation over X, we obtain that the total mean cost per product, $c(u, w)$, is given by

$$c(u, w) = \int_m^M [c_I(x, u) + c_w(x)] \times h(x)\, \mathrm{d}x. \tag{23}$$

To stimulate consumers' purchase willingness, dealers must minimise the total mean cost per product and determine the reasonable selling prices for their second-hand products. One possible, mathematically tractable form of $h(x)$ is

$$h(x) = \frac{a\mathrm{e}^{-ax}}{\mathrm{e}^{-am} - \mathrm{e}^{-aM}} \tag{24}$$

with $a > 0$, i.e., the past age X of the second-hand product has a truncated exponential distribution with parameter $a > 0$, and m and M are the lower and upper limits of past age X at statutory base. In real life, distribution of lifetime coverage might not be possible to model using a particular distribution and can be modelled using probability mass function.

3.4 Iso-cost curves

In *fixed* policy, all the customers are offered just one pair of upgrade action time and warranty length. To provide the new buyers a wide range of choices, we can offer several alternative iso-cost curves to every customer to choose one that suits him/her best. In the literature, this type of policy is referred as *flexible*, where alternative curves are iso-cost. These curves are also called indifference curves (Manna et al., 2006).

We can generate iso-cost curves from the following equation

$$I = \{\Omega(u, w) : c(u, w) = c_0\} \tag{25}$$

by solving for the upgrade action time and warranty length (u, w). It must be noticed that there are many iso-cost regions for any specified value of total mean cost.

4 Application to a real-life example

To illustrate the proposed model, we present a real-life case study. Consider a dealer who sells second-hand circuit boards that are used in electrical devices such as plasma. It is non-repairable, and is currently sold under fixed FRW policy without any upgrade action.

Basic data

Statistical analysis was performed on a particular brand of circuit boards. Our data is based on a sample of 314 warranty claims collected by the Electronic repair centre between June 2008 and August 2008. The data records consist of

- warranty claim identification code
- the sale month
- the failure date
- life, etc. (see Table 1).

Table 1 Sale month and claim information (314 claims from June 2008 until August 2008)

Claim no.	*Sale month*	*Failure date*	*Life (in month)*
1	August-06	June-08	22
2	April-08	June-08	3
3	January-07	June-08	15
314	December-07	August-08	9
Total	*314*		

Failure distribution parameters

We assume that the failure distribution is two-parameter Weibull distribution with $\Lambda(t) = \lambda\beta(\lambda t)^{\beta-1}$, where λ is the scale parameter and β is the shape parameter. We used the maximum likelihood approach to estimate the model parameters (for more see Lawless, 1982). The MLEs are given by

$$\hat{\beta} = 2.0 \quad \text{and} \quad \hat{\lambda} = 0.3\,/\,\text{year}.$$

To validate the distribution model, a χ^2 goodness-of-fit test is carried out. The observed χ^2-test value is found to be equal to 8.039 with corresponding p-value = 0.62. Therefore, the data do not provide enough evidence to reject our hypothesis that the underlying failure distribution is the two-parameter Weibull distribution. As a result, after the screening test, the intensity function for product failure is

$$F_u(t) = \frac{\exp[-(0.3(x+u))^2] - \exp[-(0.3(x+u+t))^2]}{\exp[-(0.3(x+u))^2]}. \tag{26}$$

Life parameters

Expected lifetime of a new circuit board is 5 years. After considering the past age data and using Monte Carlo simulation, we estimated the parameters of the truncated exponential distribution $h(x)$ of the past age X as $\rho = 0.2$/year, $m = 1.0$ and $M = 2.5$.

Cost parameters

The dealer of second-hand circuit boards aims to control the warranty costs by providing a screening test. This test includes a series of industrial stages such as component cleaning, reconditioning and electrical test. The upgrade action cost is composed of a fixed set-up cost and a variable cost, which depends on the upgrade action time. On the basis of the available data, the cost parameters of an upgrade action are estimated to be equal to $c_s = \$0.2$ and $c_u = \$5.0$ and the replacement cost with a new item is $c_r = \$10.0$.

Total mean cost per product

Table 2 shows the total mean cost per product based on two methods of depreciation for different combinations of w varying from 1 to 2 years and u varying from 0.05 to 0.2 years for FRW, FRW/LSW and PRW policies.

Table 2 $c(u, w)$ for different combinations of u, w for non-repairable second-hand products sold under FRW, PRW and FRW/LSW policies

		SL				*DDB*			
		$u = 0.05$	$u = 0.1$	$u = 0.15$	$u = 0.2$	$u = 0.05$	$u = 0.1$	$u = 0.15$	$u = 0.2$
FRW	$W = 1.0$	5.36	5.53	5.70	5.88	4.21	4.35	4.50	4.66
	$W = 1.5$	7.57	7.81	8.06	8.31	6.42	6.64	6.87	7.11
	$W = 2.0$	12.50	12.91	13.34	13.78	11.34	11.74	12.15	12.57
	$W = 2.5$	21.14	21.86	22.60	23.36	19.98	20.69	21.41	22.15
FRW/LSW	$W = 1.0$	5.09	5.25	5.41	5.57	3.93	4.07	4.21	4.36
$w_f = 0.5$	$W = 1.5$	6.77	6.98	7.20	7.42	5.62	5.81	6.01	6.22
$k = 0.3$	$W = 2.0$	10.52	10.87	11.23	11.60	9.36	9.70	10.04	10.39
	$W = 2.5$	17.04	17.63	18.23	18.85	15.88	16.45	17.04	17.64
PRW	$W = 1.0$	4.45	4.58	4.72	4.86	3.30	3.41	3.52	3.64
	$W = 1.5$	5.58	5.75	5.92	6.10	4.43	4,58	4.73	4.89
	$W = 2.0$	8.19	8.47	8.75	9.04	7.04	7.30	7.56	7.84
	$W = 2.5$	12.98	13.46	13.94	14.44	11.83	12.29	12.75	13.23

Iso-cost curves

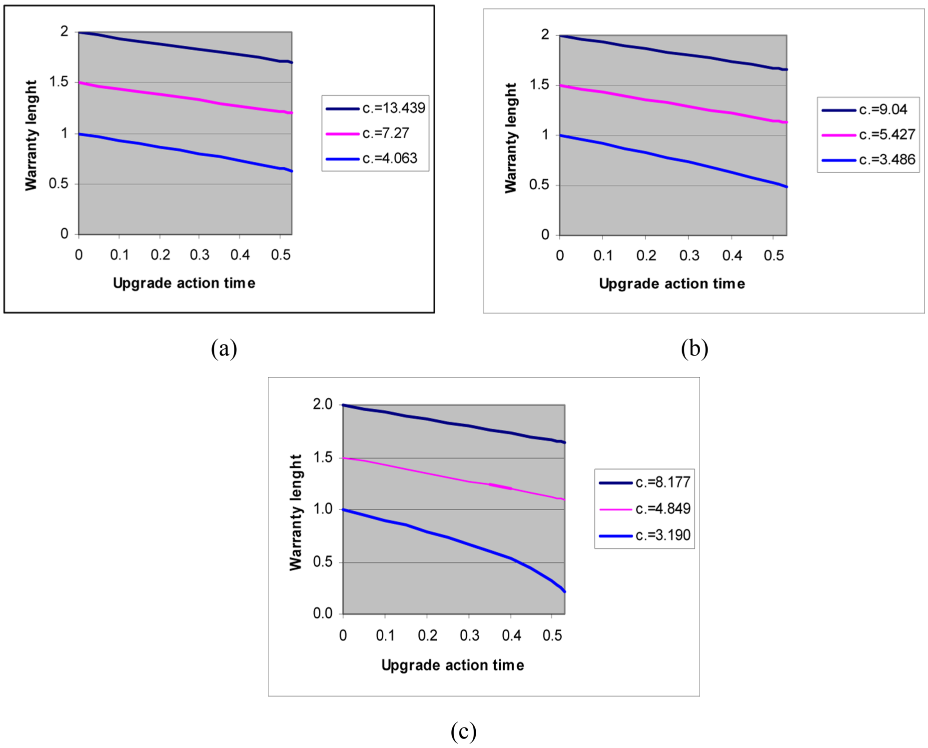
Figure 5 gives three iso-cost curves based on DDB method of depreciation for FRW, FRW/LSW and PRW, drawn to the scale, with three pairs of ($u = 0$ and $w = 1$), ($u = 0$ and $w = 1.5$), ($u = 0$ and $w = 2$). From equations (23) and (25), total mean cost per product for FRW, FRW/LSW and PRW policies are as follows:

FRW: $c_0(u = 0, w = 1) = 4.063$, $c_0(u = 0, w = 1.5) = 7.27$, $c_0(u = 0, w = 2) = 13.439$

FRW/LSW: $c_0(u = 0, w = 1) = 3.486$, $c_0(u = 0, w = 1.5) = 5.427$, $c_0 = (u = 0, w = 2) = 9.04$

PRW: $c_0(u = 0, w = 1) = 3.190$, $c_0(u = 0, w = 1.5) = 4.849$, $c_0(u = 0, w = 2) = 8.177$.

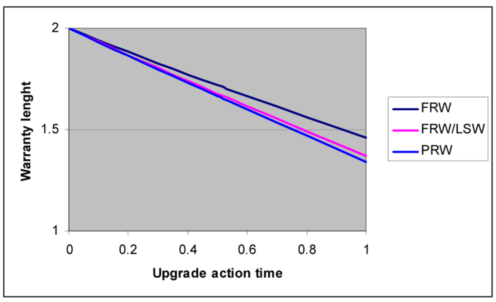
Figure 5 Indifference curves for: (a) FRW; (b) FRW/LSW and (c) PRW (see online version for colours)



The curves are obtained by connecting the points that represent the pairs of upgrade action time and warranty length. Points on an iso-cost curve are obtained as follows: Given the values of all the parameters (λ, β, c_s, c_u, c_r, m, M) and c_0, we can compute u for any specified value of w. This gives a point (u, w) on the iso-cost curve. To draw the curve, we set u at certain pre-specified discrete values – $u_1, u_2, \ldots, u_n$. Let $w_1, w_2, \ldots w_n$ be the corresponding values of w derived from the above-mentioned equation. By connecting the points (u_i, w_i) for $i = 1, 2, \ldots, n$, we get the curve.

The iso-cost curves for FRW, FRW/LSW and PRW policies with $u = 0$ and $w = 2$ are compared in Figure 6.

Figure 6 The comparisons of indifference curves for FRW, FRW/LSW and PRW policies with $u = 0$ and $w = 2$ (see online version for colours)



As shown in Figure 6, the gap between the offered warranty lengths increases as upgrade action time increases.

On the basis of the results of the analysis, the following observations may be summarised:

- The upgrade action time u depends on many factors such as product failure distribution, past age of the second-hand product, warranty length and cost parameters.
- The upgrade action is beneficial if the initial failure rate of the second-hand product is large.
- Failures during the warranty period are costly and providing an upgrade action can reduce these costs.
- We assumed $k = 0.3$ for FRW/LSW policy. We can show that smaller values of k indicate that the dealer will shoulder a lower warranty cost and larger values of k lead to a higher warranty cost for the dealer.
- For a specific upgrade action time and warranty length, the failure-free warranty policy leads to a higher total mean cost than the other warranty policies.
- The proposed strategy can give the new buyers more flexible and equitable choices on the warranty plans suited to their operating propensity and thus may give a firm decisive competitive edge in sales promotion.

5 Conclusion and extensions

In this paper, we develop a cost model to achieve a trade-off between reducing the warranty cost and increasing the upgrade action cost for a second-hand electrical component sold under various warranty policies – failure free, rebate warranty and a combination of free replacement and lump sum.

Determination of the upgrade action time is an optimisation problem in which both the costs and benefits from the dealer's viewpoint should be considered. On the one hand, providing upgrade actions is usually costly and adds directly to the sale price of the product and, on the other hand, upgrade actions can reduce the warranty cost. The model developed in this paper could be utilised for second-hand electric products such as processors, hard drives, monitors, memory components, DVD drives, HDTV, Projection and LCD TVs.

There is huge scope for future research in the area of upgrade action cost modelling for second-hand products. Some extensions are: Modelling the upgrade action cost for a general repairable second-hand product (when the second-hand product fails, minimal repair occurs with probability p and replacement occurs with probability $1 - p$, $0 \le p \le 1$), Determining the optimal upgrade action time for second-hand products to minimise the dealer's total mean cost, Considering the fixed cost and the variable cost per unit time as a random variable that depends on the past age and the past usage of the second-hand product, and build more refined models that incorporate past usage and maintenance history. The authors are currently studying some of the above-mentioned topics.

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Notations

x	Past age of the second-hand product
m	Minimum of x (lower limit)
M	Maximum of x (upper limit)
a	Parameter for the truncated exponential distribution used in the life distribution of products
L	Expected lifetime of the new product
$H(x)$	Distribution function of the past age
$h(x)$	Density function associated with $H(x)$

u	Upgrade action time
$\Lambda(t)$	Intensity function for product failure
P_0	Sale price of the new product
c_x	Purchase price from an end-user when the age is x
w	Warranty length
$F(t)$	Cumulative failure distribution of the product
$f(t)$	Density function associated with $F(t)$
$R(t)$	Refund function

Appendix I

The expected upgrade action cost for a non-repairable second-hand product with age x, $E[c_u(x)]$, is

$$\begin{aligned}
E[c_u(x)] &= \int_x^{x+u} [c_s + c_u(t-x)] f(t)\,\mathrm{d}t + \int_{x+u}^{\infty} (c_s + c_u u) f(t)\,\mathrm{d}t \\
&= c_s \int_x^{x+u} f(t)\,\mathrm{d}t - c_u x[F(x+u) - F(x)] + c_u \int_x^{x+u} tf(t)\,\mathrm{d}t + c_u u[1 - F(x+u)] \\
&= c_s[1 - F(x)] - c_u x[F(x+u) - F(x)] + c_u \left[tF(t)\Big|_x^{x+u} - \int_x^{x+u} F(t)\,\mathrm{d}t \right] \\
&\quad + c_u u[1 - F(x+u)] \\
&= c_s[1 - F(x)] + c_u \int_x^{x+u} [1 - F(t)]\,\mathrm{d}t.
\end{aligned}$$