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The continuous single-source capacitated multi-facility Weber problem with setup costs: Formulation and solution methods

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Abstract The continuous single-source capacitated multi-facility Weber problem (SSCMFWP) where setup cost of opening facilities is taken into account is investigated. The aim is to locate a set of facilities on the plane, to define their respective capacities which can be linked to the configuration of the processing machines used, and to allocate customers to exactly one facility with the objective being the minimisation of the total transportation and setup costs. A new nonlinear mathematical model based on the SSCMFWP is introduced where Rectilinear and Euclidean distances are used. Efficient metaheuristic approaches based on Variable Neighbourhood Search (VNS) and Simulated Annealing (SA) are also developed. The proposed metaheuristics incorporate an exact method and the commonly used Cooper's alternate location-allocation method. We also constructed a new data set to reflect the characteristic of this particular location problem as no data set is available in the literature. Computational experiments show that the proposed metaheuristics generate interesting results for this class of continuous location problems.

Keywords location on the plane · setup cost · single-source · VNS · Simulated Annealing

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1 Introduction

The continuous single-source capacitated multi-facility Weber problem (SSCMFWP) is to seek the location of p facilities on the plane and the allocation of each customer to exactly one of the open facilities so that the sum of the total transportation costs is minimised. This problem is one of the variants of the Multi-facility Weber problem (MFWP) which is introduced by Cooper (1963, 1972) who states that the objective function of MFWP is neither convex nor concave. Megiddo and Supowit (1984) and Sherali and Nordai (1988) reveal that the MFWP is NP-hard. The SSCMFWP is relatively more difficult to solve than its counterpart the MFWP. This is mainly due to the binary nature of the decision variables when allocating each customer to a single open facility. The SSCMFWP has several real life applications. For example, in a telecommunication network design, a user is assigned to a single base transceiver station while in the case of finding the location of oil drill platforms, each oil well has to be allocated to one platform (Devine and Lesso, 1972; Rosing, 1992).

In this paper, the SSCMFWP is studied where the setup cost of establishing an open facility is based on the capacity which is related to the configuration of the machines used. In this setting, a set of machine types is available where each type is defined by its corresponding different capacity and cost. Here, the configuration of the machines relates to the number of each machine type to be installed at each open facility. In the discrete version, Li et al (2014) investigated the two-stage capacitated facility location problem with handling costs. The problem that we study differs from the recent study of Irawan et al (2017) where the authors considered each facility to require one type of machinery only. This kind of applications can be found in engineering and manufacturing where the number of machine types is limited to 3 or4 due to high design cost and also due to the limited usage. We refer to this problem as the SSCMFWP-SC (SSCMFWP with Setup Cost). To tackle such a strategic decision problem, we propose a new mathematical model and efficient VNS- and SA-based metaheuristics.
Our contributions are threefold:

i. A new nonlinear mathematical model for the SSCMFWP-SC with the presence of facility setup cost and its linearization are developed,
ii. Effective metaheuristic approaches based on VNS and SA are designed where an exact method and a Cooper's alternate location-allocation (ALA) method are incorporated.
iii. Newly constructed data sets for the SSCMFWP-SC are produced and interesting and competitive computational results presented.

The paper is organised as follows. In Section 2, a review of the literature is presented followed by Section 3 that contains the mathematical model for the SSCMFWP-SC. The description of the proposed metaheuristic approaches is given in Section 4. The computational results are provided in Section 5. Finally, the conclusions and some highlights of future research are presented in the last section.

2 Literature Review

In this section, a review on recent papers on the SSCMFWP are first presented followed by a brief on related problems such as the capacitated MFWP (CMFWP), the Weber problem in the presence of facility fixed cost and the CMFWP with fixed costs.

2.1 A review on recent papers on the SSCMFWP

Gong et al (1997) study a hybrid evolutionary method based on a Genetic Algorithm (GA) and a Lagrangian relaxation (LR) method. The former is used to determine the location of the facilities while the latter is implemented in the allocation phase. An iterative two phase heuristic approach is proposed by Manzour-al Ajdad et al (2012) where in the first part, an enhanced ALA method based on two assignment rules is applied, while in the second, the generalised assignment problem (GAP) is solved optimally. Manzour et al (2013) also put forward a simpler but less promising version of their earlier method.

Öncan (2013) develop three solution methods for the SSCMFWP with Euclidean and Rectilinear distances. The first one is an enhanced ALA method where the allocation problem is solved optimally, while in the second, the allocation problem is efficiently solved using a very large neighbourhood search procedure. The third method is a discrete approximation technique where a LR is used to find lower and upper bounds. Recently, Irawan et al (2017) propose two methods dealing with the SSCMFWP with the presence of facility fixed cost. The first one is a generalised two stage heuristic scheme whereas the second is based on VNS. The proposed methods were also adapted to the SSCMFWP where these outperform those recently published solution methods.

In this study, the fixed cost of an open facility is defined by a combination of machine types which has led to a modified mathematical formulation. In addition, two efficient solution methods based on VNS and SA are proposed incorporating an exact method and an ALA heuristic. Moreover, an aggregation technique based on a pseudo-random scheme that relies on randomness as well as the already found promising sites is adopted alongside appropriate adaptations of the VNS heuristic.

2.2 A brief on related continuous location problems

Contrarily to the SSCMFWP, for example in the capacitated MFWP (CMFWP) each customer can be served by more than one facility. In this case, when the location of facilities is fixed, the allocation problem reduces to solving the transportation problem (TP) instead of the GAP. Cooper (1972) proposes exact and heuristic methods for the CMFWP. There are a few papers in this area and among the recent ones we have for example Luis et al (2011) and Akyüs et al (2014). The former propose a novel guided reactive greedy randomised adaptive search procedure by incorporating the concept of restricted regions while the latter design two types of branch and bound algorithms with the first one dealing with the allocation space whereas the second for the partition of the location space. For more information and references therein, see Irawan et al (2017).

There is however a shortage of papers which study the Weber problem in the presence of fixed costs. Brimberg et al (2004) are among the first to investigate the multi-source Weber problem with constant fixed cost. They propose a multiphase heuristic to solve the problem where the discrete version of the problem is first solved to obtain the number of facilities and then the location of open facilities is improved by implementing the ALA heuristic. Brimberg and Salhi (2005) consider fixed costs when locating a single facility in the continuous space. The fixed costs are zone-dependent defined as non-overlapping convex polygons. They also propose an efficient method to optimally solve the problem. A discretization method to address the multi facility problem is also presented.

Luis et al (2015) investigate the CMFWP with the presence of facility fixed costs. The authors consider three types of fixed costs which are constant, zone-based, and continuous fixed cost functions. Heuristic approaches that incorporate the concept of restricted regions and a GRASP metaheuristic are applied to solve the problem. Hosseininezhad et al (2015) develop a cross entropy heuristic to solve the CMSWP with a zone-based fixed cost which consists of production and installation costs.

3 Mathematical formulation

The mathematical model of the SSCMFWP-SC is given here. A set of machine types (M) that can be used for each facility is taken into account where each machine type ($m \in M$) is characterized by its capacity (q_m) and its purchasing cost (c_m). The model also considers the number of machines available for each type (u_m) . In addition, we consider that the maximum allowed capacity for each facility j ($j = 1, \ldots, p$) is b_j. The following notations are used.

Set

I : the set of customers with i as its index where $n = |I|$
M : the set of machine types with m as its index.

Parameter

p: the number of facilities to open
$a_i = (a_i^1, a_i^2)$: the location of customer i where $a_i \in \mathbb{R}^2, i \in I$
w_i: the demand of customer i ($i \in I$)
$\hat{c}$: the unit transportation cost per unit demand and per km
b_j : the maximum allowed capacity for facility j, $j = 1, \ldots, p$
q_m : the capacity of machine type $m \in M$
c_m : the purchasing cost of machine type $m \in M$
u_m : the total number of machines available for machine type $m \in M$.

Decision Variable

$$Y_{ij} = \begin{cases} 1 & \text{if customer } i \ (i \in I) \text{ is assigned to facility } j \ (j = 1, \ldots, p), \\ 0 & \text{otherwise} \end{cases}$$

$X_j = (x_j, y_j)$: coordinates of facility j where $X_j \in \mathbb{R}^2$, $j = 1, \ldots, p$
L_{jm} = the number of machines of type $m \in M$ used at facility j, $j = 1, \ldots, p$.

Let $d(X_j, a_i)$ be the distance between facility j and customer i which is defined as follows:

$$d(X_j, a_i) = ||X_j - a_i||_1 = \left(\left|X_j^1 - a_i^1\right| + \left|X_j^2 - a_i^2\right|\right) \text{ for Rectilinear distance}$$
$$d(X_j, a_i) = ||X_j - a_i||_2 = \left(\left(X_j^1 - a_i^1\right)^2 + \left(X_j^2 - a_i^2\right)^2\right)^{1/2} \text{ for Euclidean distance}$$

The mathematical model of the SSCMFWP-SC can be formulated as follows.

Objective function:

$$\min \sum_{j=1}^{p} \sum_{i \in I} (Y_{ij} \cdot d(X_j, a_i) \cdot \hat{c} \cdot w_i) + \sum_{j=1}^{p} \sum_{m \in M} (L_{jm} \cdot c_m) \tag{1}$$

subject to

$$\sum_{j=1}^{p} Y_{ij} = 1, \qquad \forall i \in I \tag{2}$$

$$\sum_{m \in M} (L_{jm} \cdot q_m) \leq \left(b_j + \max_{m \in M} q_m \right), \qquad \forall j = 1, \ldots, p \tag{3}$$

$$\sum_{i \in I} (w_i \cdot Y_{ij}) \leq b_j, \qquad \forall j = 1, \ldots, p \tag{4}$$

$$\sum_{i \in I} (w_i \cdot Y_{ij}) \leq \sum_{m \in M} (L_{jm} \cdot q_m), \qquad \forall j = 1, \ldots, p \tag{5}$$

$$\sum_{j=1}^{p} L_{jm} \leq u_m, \qquad \forall m \in M \tag{6}$$

$$Y_{ij} \in \{0, 1\}, \qquad \forall i \in I; j = 1, \ldots, p \tag{7}$$

$$L_{jm} \geq 0, \text{ integer}, \qquad \forall j = 1, \ldots, p; m \in M \tag{8}$$

$$X_j \in \mathbb{R}^2, \qquad \forall j = 1, \ldots, p \tag{9}$$

The objective function (1) aims to minimise the sum of the total costs. The first part of the objective function (1) is the total transportation cost (z^t) whereas the second defines the total setup cost (z^s). Constraints (2) ensure that each customer is served by one facility.

Motivated by Li et al (2014), Constraints (3) state that the configuration of machines used by a facility must be restricted by its maximum capacity. It looks unlikely that the total capacity based on all machines used by a facility is exactly equal to its maximum capacity. Therefore, a simple upper bound ($\max_{m \in M} q_m$) is added to indicate that in the worst case, the total capacity can be larger than the maximum allowed capacity. Constraints (4) and (5) make sure that capacity constraints of the facilities are satisfied. Here, the capacity constraints are based on the total capacity of all used machines and the maximum allowed capacity. Constraints (6) ensure that the number of machines (for each type of machine) used for all facilities does not exceed the one available in the market. Constraints (7) refer to the binary nature of the variables. Constraints (8) specify that the number of machines (for each type of machine) used for each facility is an integer variable. Constraints (9) indicate the continuous location variables. The above model is nonlinear due to the first part of the objective function. This nonlinear model can be solved by a commercial optimiser such as Lingo. Another way is to linearise the model whenever possible and use powerful commercial solvers such as CPLEX, Lindo, Gurobi, Xpress, etc. In this study, we can linearise the case of rectilinear distances as will be presented next. For the optimiser, we opt for CPLEX due to its availability and ease of implementation.

3.1 Linearization of the model for the case of rectilinear distance

For this particular type of distance, the term of the objective function in the above model can be linearized in a standard way as follows:
Given that

$$Y_{ij} \cdot w_i \cdot \hat{c} \cdot d(X_j, a_i) = w_i \cdot \hat{c} \left(\left| Y_{ij} \cdot X_j^1 - Y_{ij} \cdot a_i^1 \right| + \left| Y_{ij} \cdot X_j^2 - Y_{ij} \cdot a_i^2 \right| \right)$$

Let the new real variables be $V_{ij}^1 = Y_{ij} \cdot X_j^1$ and $V_{ij}^2 = Y_{ij} \cdot X_j^2$.

The objective function in Equation (1) can be rewritten as follow:

$$\min \sum_{j=1}^{p} \sum_{i \in I} \left(\hat{c} \cdot w_i \cdot \left(\left| V_{ij}^1 - Y_{ij} \cdot a_i^1 \right| + \left| V_{ij}^2 - Y_{ij} \cdot a_i^2 \right| \right) \right) + \sum_{j=1}^{p} \sum_{m \in M} (L_{jm} \cdot c_m) \tag{10}$$

We now have a linear model with the following additional constraints:

$$V_{ij}^1 \leq a_{\max}^1 \cdot Y_{ij}, \quad \forall i \in I, j = 1, \dots, p \tag{11}$$

$$V_{ij}^1 \geq a_{\min}^1 \cdot Y_{ij}, \quad \forall i \in I, j = 1, \dots, p \tag{12}$$

$$V_{ij}^1 \leq X_j^1 - a_{\min}^1 \cdot (1 - Y_{ij}), \quad \forall i \in I, j = 1, \dots, p \tag{13}$$

$$V_{ij}^1 \geq X_j^1 - a_{\max}^1 \cdot (1 - Y_{ij}), \quad \forall i \in I, j = 1, \dots, p \tag{14}$$

$$V_{ij}^2 \leq a_{\max}^2 \cdot Y_{ij}, \quad \forall i \in I, j = 1, \dots, p \tag{15}$$

$$V_{ij}^2 \geq a_{\min}^2 \cdot Y_{ij}, \quad \forall i \in I, j = 1, \dots, p \tag{16}$$

$$V_{ij}^2 \leq X_j^2 - a_{\min}^2 \cdot (1 - Y_{ij}), \quad \forall i \in I, j = 1, \dots, p \tag{17}$$

$$V_{ij}^2 \geq X_j^2 - a_{\max}^2 \cdot (1 - Y_{ij}), \quad \forall i \in I, j = 1, \dots, p \tag{18}$$

where $a_{\min}^1 = \min_{i \in I} a_i^1$, $a_{\max}^1 = \max_{i \in I} a_i^1$, $a_{\min}^2 = \min_{i \in I} a_i^2$ and $a_{\max}^2 = \max_{i \in I} a_i^2$.

Note that for the case of rectilinear distance, the linearized model is still hard to solve optimally for $n \geq 25$ as will be shown in the computational results section. This is mainly due to the large number of additional constraints (11)-(18) resulting in $8p|I| = 8pn$ new constraints. For $p = 10$ and $n = 100$ this amounts to 8,000 new constraints and if $p = 50$ and $n = 500$ this number jumps to 200,000.

3.2 Mathematical model of the related discrete problem

For benchmarking purposes, we also present a similar model for the discrete problem namely the capacitated facility location problem with setup cost (CFLP-SC). Let K be a set of potential facility sites where the location of each potential site $(k \in K)$ is known. As the location of potential facility sites and customers is fixed, the distance between a potential site and a customer $(d_{ik}, i \in I, k \in K)$ is also known. The problem is to determine whether a facility is located at a potential site or not. If a potential site is selected, the maximum allowed capacity of this facility must be determined. Moreover, the configuration of machines used for each facility needs also to be optimised. The notation adopted for sets and parameters in the CFLP-SC model is relatively similar to the one of the previous model with the exception of the following additional items.

Set and parameter

K : the set of potential facility sites with k as its index.
d_{ij} : the distance between potential site $k \in K$ and customer $i \in I$.

Decision Variable

$$Y_{ik} = \begin{cases} 1 & \text{if customer } i \in I \text{ is assigned to an open facility located in site } k \in K \\ 0 & \text{otherwise} \end{cases},$$

$$S_{kj} = \begin{cases} 1 & \text{if facility } j \text{ (with maximum capacity } b_j\text{) is located in site } k \in K, \\ 0 & \text{otherwise} \end{cases}$$

L_{km} = the number of machines of type $m \in M$ used at a facility at site $k \in K$.

The CFLP-SC can be expressed as follows:

Objective function:

$$\min \sum_{k \in K} \sum_{i \in I} (Y_{ik} \cdot d_{ik} \cdot \hat{c} \cdot w_i) + \sum_{k \in K} \sum_{m \in M} (L_{km} \cdot c_m) \tag{19}$$

subject to

$$\sum_{k \in K} Y_{ik} = 1, \qquad \forall i \in I \tag{20}$$

$$\sum_{j=1}^{p} S_{kj} \leq 1, \qquad \forall k \in K \tag{21}$$

$$\sum_{k \in K} S_{kj} = 1, \qquad \forall j = 1, \dots, p \tag{22}$$

$$\sum_{m \in M} (L_{km} \cdot q_m) \leq \sum_{j=1}^{p} \left(S_{kj} \cdot \left(b_j + \max_{m \in M} q_m \right) \right), \qquad \forall k \in K \tag{23}$$

$$\sum_{i \in I} (w_i \cdot Y_{ik}) \leq \sum_{j=1}^{p} (b_j \cdot S_{kj}), \qquad \forall k \in K \tag{24}$$

$$\sum_{i \in I} (w_i \cdot Y_{ik}) \leq \sum_{m \in M} (L_{km} \cdot q_m), \qquad \forall k \in K \tag{25}$$

$$\sum_{k \in K} L_{km} \leq u_m, \qquad \forall m \in M \tag{26}$$

$$Y_{ik} \in \{0,1\}, \qquad \forall i \in I; k\ inK \tag{27}$$

$$L_{km} \geq 0, \text{ integer}, \qquad \forall k \in K; m \in M \tag{28}$$

$$S_{kj} \in \{0,1\}, \qquad \forall k \in K; j = 1, \ldots, p \tag{29}$$

The model is linear and has $|K| \cdot (|I| + p)$ binary variables, $|K| \cdot |M|$ integer ones and $p + 4|K| + |I| + |M|$ constraints.

4 Solution methods

In this section, the two proposed solution methods to solve the SSCMFWP-SC are described. These are designed to solve both rectilinear and Euclidean problems. In the first approach, a powerful metaheuristic technique using Variable Neighbourhood Search (VNS) is implemented while the second method is based on Simulated Annealing (SA).

4.1 The proposed VNS approach

In this subsection, the proposed approach incorporates an aggregation technique, an exact method, a Variable Neighbourhood Search (VNS) and the modified Alternate Location-Assignment (ALA) heuristic introduced by Cooper (1964). VNS is a powerful metaheuristic approach that consists of a local search and a neighbourhood search. The local search is to intensify the search whereas the neighbourhood search aims to escape from the local optima. In the neighbourhood search, the next (usually the larger) neighbourhood is systematically used if there is no improvement, otherwise it will revert back to the first (usually the smallest) one (Hansen and Mladenović, 1997). For more information on VNS and its applications and key points, see Hansen et al (2010) and Salhi (2017). The main steps of the proposed VNS are presented in Algorithm 1 which consists of two stages.

4.1.1 Stage 1 of Algorithm 1

Here, a good initial solution is obtained using an aggregation technique, an exact method, a local search and the ALA heuristic. Stage 1 is an iterative process where a set of the reduced discrete problems (CFLP-SC) is solved by the exact method (CPLEX) and a local search that we propose. Each discrete problem consists of a set of selected customer sites to be used as the potential facilities. When n and $|K|$ are large, the CFLP-SC is difficult to solve optimally. Here, an aggregation approach is applied to reduce the number of potential facility sites from n to η sites ($\eta << n$) while all customers are still served. In other words, in each discrete problem, a set of selected customer sites ($\hat{K}$) is pseudo randomly selected and used as the potential facilities location. Note that the η customer

Algorithm 1 The proposed VNS algorithm for the SSCMFWP-SC

Define H, $k_{\max}$ and η. Set $z^* = \infty$.
STAGE 1
repeat
  Choose randomly η customer sites including the promising sites (F^*). These sites are considered as a set of potential facility sites ($\hat{K}$) for the reduced discrete CFLP-SC problem.
  Solve the reduced discrete problem (CFLP-SC) which consists of n customers and η potential facilities using an exact method (CPLEX) within τ' seconds. Let $F = \{F_1, \dots, F_p\}$ be the set of customer sites to locate the p facilities where $C_j(x_j, y_j)$ be the coordinates of facility j. Denote σ_{jm} the number of machines of type m used by facility j and N_j the set of customers to be served by facility j ($j = 1, \dots, p$).
  Apply the proposed local search presented in Algorithm 2 starting from F obtained from the previous step.
  Compute the total setup cost (z^s) and the capacity for each facility κ_j based on σ_{jm} and determine the total transportation cost (z^t) based on F and N_j, see Equation (30).
  Improve (z^t) by applying the ALA heuristic presented in Algorithm 3 using C_j, N_j and κ_j.
  Calculate total cost $z = z^s + z^t$.
  if $z < z^*$) **then**
    Update $z^* = z$ along with $F^* \leftarrow F$, $C_j^* \leftarrow C_j$, $\sigma_{jm}^* \leftarrow \sigma_{jm}$ and $N_j^* \leftarrow N_j$.
  end if
until H times
STAGE 2
Update $z = z^*$ along with $F \leftarrow F^*$, $C_j \leftarrow C_j^*$, $\sigma_{jm} \leftarrow \sigma_{jm}^*$ and $N_j \leftarrow N_j^*$.
Set $k = 1$.
Shaking
Update C_j, σ_{jm} and N_j using Procedure Shaking given in Algorithm 4.
Local Search
Determine z^s, z^t and κ_j based on C_j, σ_{jm} and N_j.
Apply the modified ALA heuristic given in Algorithm 3 to improve the transportation cost (z^t) using κ_j and C_j.
Compute the total cost $z = z^s + z^t$.
Move or Not
if $z < z^*$) **then**
  Update $k = 1$ and $z^* = z$ along with $F^* \leftarrow F$, $C_j^* \leftarrow C_j$, $\sigma_{jm}^* \leftarrow \sigma_{jm}$ and $N_j^* \leftarrow N_j$.
else
  Update $k = k+1$ and $z = z^*$ along with $F \leftarrow F^*$, $C_j \leftarrow C_j^*$, $\sigma_{jm} \leftarrow \sigma_{jm}^*$ and $N_j \leftarrow N_j^*$.
end if

sites include the promising sites (F^*) which are the solution obtained from solving the discrete problem that provides the smallest total cost (initially $F^* = \emptyset$). The remaining potential facility sites ($\eta - p$) are randomly chosen from the customer sites. A similar method has shown to be promising when solving large p-median (Irawan et al, 2014; Irawan and Salhi, 2015b) and p-centre problems (Irawan et al, 2016). Irawan and Salhi (2015a) also produce an interesting review on aggregation techniques for large facility location problems.

The reduced CFLP-SC is solved by CPLEX within τ' seconds to speed up and control the search process. By solving the reduced CFLP-SC, the locations of the p facilities (F), the configuration of the machines that need to be installed for each facility σ_{jm} and the set of allocations N_j are found. The capacity of each open facility κ_j is determined based on the value of σ_{jm}. The solution found (F and σ_{jm}) is then fed into the proposed local search given in Algorithm 2 as the initial solution. The obtained total cost can be divided into the setup cost (z^s) and the transportation cost (z^t). The total setup cost (z^s) and the capacity of each open facility κ_j are calculated based on σ_{jm} while the total transportation cost (z^t) is determined based on N_j and the set of the p open facilities $F = \{F_1, \ldots, F_p\}$ as follows:

$$\min \sum_{j \in F} \sum_{i \in N_j} \left(d_{iF_j} \cdot \hat{c} \cdot w_i \right) \tag{30}$$

The ALA heuristic is then applied to improve the total transportation cost based on the locations of the p facilities and by fixing the capacity of each open facility (κ_j). This process is repeated H times and the solution that yields the smallest objective function value is chosen as the one to be fed into Stage 2 of Algorithm 1 which is the VNS based algorithm.

A Local Search for the Discrete Problem

A local search for the reduced discrete problem (CFLP-SC) is proposed to improve the quality of solution produced by the exact method in Line 5 of Algorithm 1. The input of this local search is the objective function value (z), the location of facilities (F) and the capacity of each facility (σ_{jm}) which are obtained by the exact method. The proposed local search comprises two phases where the first phase uses the best improvement strategy whereas the second one applies the first improvement strategy. In the first phase, the algorithm tries to find the best facility in the current solution to be located in the best location in the potential facility site ($\hat{K}$). In the search, we fix the capacity of each facility (κ_j) which does not change the total setup cost (z^s). Here, the first phase will try to improve the transportation cost. To speed up the search, the approximated approach is used where the quality of solution is evaluated by solving the transportation problem (TP) which is relatively easy to solve by the exact method. Moreover, facility j can be moved to potential site k if a customer located in site k is served by facility j. It means that facility j will not move far from its current location. In the TP, the decision variable x_{ij} is used representing the size of the shipment from facility j to customer i. The TP can be expressed as follows:

$$\min \sum_{j \in F} \sum_{i \in I} (x_{ij} \cdot d_{ij} \cdot \hat{c}) \tag{31}$$

subject to

$$\sum_{j \in F} x_{ij} \geq w_i, \qquad \forall i \in I \tag{32}$$

$$\sum_{i \in I} x_{ij} \leq \kappa_j, \qquad \forall j \in F \tag{33}$$

$$x_{ij} \geq 0, \text{ integer}, \qquad \forall i \in I; j \in F \tag{34}$$

Based on the objective function value obtained by solving the TP, the best pair is then taken. This new facility configuration (F') is evaluated by solving the CFLP-SC using an exact method. This reduced CFLP-SC problem is relatively easy to solve as the reduced model is solved to obtain the assignment problem and the facility capacity configuration (σ_{jm}). If the new configuration gives a better result then this configuration is treated as the incumbent solution. This procedure is repeated until there is no improvement or it reached the time limit.

In Phase 2, the algorithm tries to locate facility j on potential site k if a customer located in site k is served by facility j. This new solution (F') is then assessed by solving the CFLP-SC using an exact method. If the new facility configuration together with its capacity configuration give a better result then this facility configuration is treated as the incumbent solution and it will go back to the beginning of Phase 2. This procedure is repeated until there is no improvement or it reached the time limit.

Algorithm 2 The Local Search for the Discrete Problem

Require: z, F, and σ_{jm} $\forall j = 1, \ldots, p; m \in M$
1: **Phase 1**.
2: Set Imp = true, the time limit (τ^L) and determine κ_j based on σ_{jm}
3: **repeat**
4: Set $z' = \infty$ and Imp = false.
5: **for** $j = 1$ to p **do**
6: **for** $k = 1$ to η **do**
7: **if** The customer in site $k \in \hat{K}$ is allocated to facility j **then**
8: Set $F' \leftarrow F$
9: Move facility F'_j to potential site k, $F'_j = k$. The capacity for each facility (κ_j) does not change.
10: Using the set of customers (I) and the location of facilities (F') together with their capacity (κ_j), solve the transportation problem (TP) where z'' is the obtained objective function value.
11: **if** $z'' < z'$ **then**
12: Set $\hat{j} = j$, $\hat{k} = k$ and $z' = z''$
13: **end if**
14: **end if**
15: **end for**
16: **end for**
17:
18: Set $\hat{F} \leftarrow F$ and $F_{\hat{j}} = \hat{k}$
19: Based on the set of customers I and facilities $\hat{F}$, solve the discrete problem (CFLP-SC) using an exact method. Denote $\hat{z}$ the obtained objective function and $\hat{\sigma}_{jm}$ the facility capacity configuration.
20: **if** $\hat{z} < z$ **then**
21: Update $z = \hat{z}$, $F \leftarrow \hat{F}$, $\sigma_{jm} \leftarrow \hat{\sigma}_{jm}$ and Imp = true.
22: **end if**
23: **until** Imp = false or CPU $>$ time limit (τ^L)
24: **Phase 2**.
25: **for** $j = 1$ to p **do**
26: **for** $k = 1$ to η **do**
27: **if** The customer in site $k \in \hat{K}$ is allocated to facility j **then**
28: **if** CPU $>$ time limit (τ^L) **then**
29: Set $F' \leftarrow F$
30: Move facility F'_j to potential site k, $F'_j = k$.
31: Based on set of customers (I) and the location of facilities (F'), solve the CFLP-SC using an exact method. Denote z' the obtained objective function and σ'_{jm} the facility capacity configuration.
32: **if** $z' < z$ **then**
33: Update $z = z'$, $F \leftarrow F'$ and $\sigma_{jm} \leftarrow \sigma'_{jm}$. Go to Line 25.
34: **end if**
35: **end if**
36: **end if**
37: **end for**
38: **end for**

Cooper's Alternate Location-Assignment (ALA) Heuristic

The modified ALA heuristic incorporates the Weiszfeld's formula to seek a new location for an open facility on the plane. The main steps of this heuristic are given in Algorithm 3. The ALA heuristic requires the location of initial p open facilities (C_j). The Weiszfeld equations (31) are iteratively performed to get the new location of these p facilities ($\hat{C}_j, j = 1, \ldots, p$). Using the new location of these p open facilities, the generalized assignment problem (GAP) is solved using an exact method to find the new allocation of the customers to their respective facilities N_j. The location-allocation problem and the GAP are alternately applied until no improvement in total transportation cost can be attained. Note that in the original ALA, the allocation is performed either by assigning customers to the nearest facility (case of uncapacitated) or by solving the transportation problem (case of capacitated). In other words, the same procedure is relatively slow due to using GAP within Weiszfeld (Weiszfeld, 1937).

Algorithm 3 The ALA Heuristic

Require: z^t, $C_j(x_j, y_j)$, N_j and κ_j $\forall j = 1, \ldots, p$
1: Define ϵ.
2: **repeat**
3:    **for** $j = 1$ to p **do**
4:       Let $d(C_j, a_i)$ is the distance between facility j and customer i
5:       Determine the new coordinate of facility j ($\hat{C}_j(\hat{x}_j, \hat{y}_j)$) using Weiszfeld's equations

$$\hat{x}_j = \frac{\sum_{i \in N_j} \frac{w_i \cdot a_i^1}{d(C_j, a_i)}}{\sum_{i \in N_j} \frac{w_i}{d(C_j, a_i)}}; \ \hat{y}_j = \frac{\sum_{i \in N_j} \frac{w_i \cdot a_i^2}{d(C_j, a_i)}}{\sum_{i \in N_j} \frac{w_i}{d(C_j, a_i)}} \tag{35}$$

6:       **if** $d(C_j, \hat{C}_j) > \epsilon$ **then**
7:          $C_j \leftarrow \hat{C}_j$
8:          Go back to Line 4.
9:       **end if**
10:    **end for**
11:    Solve the GAP using C_j and κ_j. Let $\hat{z}^t$ be its objective function value and update N_j.
12:    Set $dif = z^t - \hat{z}^t$.
13:    **if** $dif > 0$ **then**
14:       Update $z^t = \hat{z}^t$
15:    **end if**
16: **until** $dif < \epsilon$
17: Return z^t, C_j and N_j

4.1.2 Stage 2 of Algorithm 1

In Stage 2 of Algorithm 1, the VNS algorithm is designed. This stage is divided into three parts, namely shaking, local search and move or not. The shaking process is presented in Algorithm 4, which is performed as follows: Firstly, a randomly chosen facility from the current solution configuration is removed, and replaced by a facility located at a customer site which is obtained by solving the discrete CFLP-SC. Here, the new machines configuration for the new facility location is also obtained. Let Γ be the set of open facilities except the removed facility where

$|\Gamma| = p - 1$. Denote $\tilde{j}_k (k \in \Gamma)$ as the index of the open facilities and $\tilde{\sigma}_{km} = \sigma_{\tilde{j}_k m}$ $k \in \Gamma$). The CFLP-SC is solved using all n customers and the set of potential location sites $K = \Gamma \cup I'$ where I' being a subset of customer sites ($I' \subseteq I$). For relatively large problems ($n \geq 300$), I' is populated by the sites of customers served by the removed facility only, otherwise, all customers' sites are included in the set I'.

Algorithm 4 The algorithm for Shaking

Require: k, C_j, N_j and σ_{jm}

1: **repeat**
2: Select randomly a facility, say facility $\hat{j}$, from the set of open facilities.
3: Let Γ be a set of open facilities except facility $\hat{j}$ where $|\Gamma| = p - 1$. Denote $\tilde{j}_k (k \in \Gamma)$ be the index of the open facilities that are not being removed and $\tilde{\sigma}_{km} = \sigma_{\tilde{j}_k m}$ $k \in \Gamma$).
4: Set $K = \Gamma \cup I'$ as potential location sites where I' is a subset of customer sites ($I' \subseteq I$). For relatively large problem ($n \geq 300$), I' is populated by the sites of customers served by the removed facility ($\hat{j}$) only. Otherwise, all customers' sites are included in set I'.
5: Solve the discrete problem (CFLP-SC) which consists of n customers and $|K|$ potential facilities using an exact method (CPLEX) within τ'' seconds.
6: Take the solution of the discrete problem to determine the coordinate of the open facilities (C_j), the configuration of machines installed for each facility (σ_{jm}) and the customers allocation to their facilities (N_j).
7: **until** k times

The discrete problem (CFLP-SC) is solved by an exact method (CPLEX) within τ'' seconds. Note that in the CFLP-SC model, we fixed some decision variables so that the obtained solution will include the incumbent facilities (Γ) with their machine configurations. This is achieved by introducing the following additional constraints:

$$S_{kj} = 1, \quad \forall k \in \Gamma, j = \tilde{j}_k \tag{36}$$

$$L_{km} = \tilde{\sigma}_{km}, \quad \forall k \in \Gamma, m \in M \tag{37}$$

The inclusion of these additional constraints into the model makes the problem relatively easier to solve. Also, for relatively small problems, Constraints (33) may not be included. By solving this problem, a location for a facility at a customer site and its machines configuration are obtained. In the local search, the ALA heuristic presented in Algorithm 3 is applied to find the local minima by improving the total transportation cost. In the move or not move step, a larger neighbourhood is systematically used if there is no improvement (i.e., $k = k + 1$), otherwise the search reverts back to the smallest one (i.e., $k = 1$). The VNS terminates when $k > k_{\max}$.

4.2 The proposed SA approach

Simulated annealing (SA) is a metaheuristic introduced by Kirkpatrick et al (1983) to seek for feasible solutions and converge to a very good if not optimal solution. This method benefits from a technique proposed by Metropolis et al (1953) who simulate the cooling of material in a heat bath. In the method, the cooling process is simulated by gradually decreasing the temperature of the system until it has converged to a steady (freeze) state. This type of process is known as annealing.

For more detailed information on SA and its variants, see Nikolaev and Jacobson (2010), Bertsimas and Nohadani (2010), Dowsland and Thompson (2012), Ferreiro et al (2013), Salhi (2017) and Gerber and Bornn (2017). The proposed SA algorithm for solving SSCMFWP-SC is presented in Algorithm 5.

Algorithm 5 The proposed SA algorithm for the SSCMFWP-SC

Define H, T, $T_{\min}$, α and L. Set $z^* = \infty$.
repeat
  Select randomly p customer sites and locate the p facilities in these locations. Let F be the set of customer sites to locate the p facilities
  Solve optimally the reduced discrete problem (CFLP-SC) using an exact method where the problem consists of n customers and the location of facilities is fixed. The objective function value (z), the machines configuration (σ_{jm}) and the customers allocation (N_j) are obtained.
  if $z < z^*$) **then**
    Update $z^* = z$ along with $F^* \leftarrow F$, $\sigma^*_{jm} \leftarrow \sigma_{jm}$ and $N^*_j \leftarrow N_j$.
  end if
until H times
Determine the coordinates of each facility (C^*_j) based on F^*.
Update $z = z^*$ along with $F \leftarrow F^*$, $C_j \leftarrow C^*_j$, $\sigma_{jm} \leftarrow \sigma^*_{jm}$ and $N_j \leftarrow N^*_j$.
while $T > T_{\min}$ **do**
  repeat
    Select randomly a facility in the current solution (C_j), say facility $\hat{j}$.
    Locate facility $\hat{j}$ to a randomly chosen customer site that is assigned to facility $\hat{j}$ in the current solution. Let $\hat{C}_j$ be the set of new coordinates of open facilities.
    Solve optimally the reduced CFLP-SC using an exact method where the location of facilities ($\hat{C}_j$) is fixed. The new ($\hat{z}$), ($\hat{\sigma}_{jm}$) and ($\hat{N}_j$) are also obtained.
    Compute the total setup cost ($\hat{z}^s$) and the capacity for each facility $\hat{\kappa}_j$ based on $\hat{\sigma}_{jm}$ and determine the total transportation cost ($\hat{z}^t$) based on $\hat{C}_j$ and $\hat{N}_j$.
    Apply the ALA heuristic (Algorithm 3) to improve ($\hat{z}^t$) by using $\hat{C}_j$, $\hat{N}_j$ and $\hat{\kappa}_j$.
    Calculate the new total cost $\hat{z} = \hat{z}^s + \hat{z}^t$.
    Set $\Delta = \hat{z} - z$
    if $\Delta \leq 0$ **then**
      Update $z = \hat{z}$ along with $C_j \leftarrow \hat{C}_j$, $\sigma_{jm} \leftarrow \hat{\sigma}_{jm}$ and $N_j \leftarrow \hat{N}_j$.
      if $z < z^*$ **then**
        Update $z^* = z$ along with $C^*_j \leftarrow C_j$, $\sigma^*_{jm} \leftarrow \sigma_{jm}$ and $N^*_j \leftarrow N_j$.
      end if
    else
      Generate a random number (φ) from $U(0,1)$
      if (φ) $< exp(-\Delta/T)$ **then**
        Update $z = \hat{z}$ along with $C_j \leftarrow \hat{C}_j$, $\sigma_{jm} \leftarrow \hat{\sigma}_{jm}$ and $N_j \leftarrow \hat{N}_j$.
      end if
    end if
  until L times
  Update the temperature by setting $T = \alpha \cdot T$.
end while

The initial solution used in SA is obtained using an iterative procedure where a set of discrete problems (CFLP-SC) is optimally solved using an exact method. In each discrete problem, p facilities are located at randomly chosen customer sites. In other words, the location of facilities is fixed and the CFLP-SC is reduced by replacing the set of potential facility sites (K) by p facility sites. The reduced CFLP-SC is relatively easy to solve using an exact method which results in obtaining the machines configuration (σ_{jm}) and the customers allocation (N_j). The best solution from this iterative process is then taken to be an initial solution for the SA algorithm.

In the SA algorithm, the parameters T, $T_{\min}$, α and L are defined first. T is a temperature parameter dynamically adjusted using the correction parameter α which is the rate of decrease of temperature. The termination condition of the search is determined by $T_{\min}$ and L. In the search, the new solution is generated by moving the location of a randomly chosen facility in the current solution to a randomly selected customer site that is assigned to this facility. Therefore, the new location of the chosen facility is not too far from the original location which will reduce the computational time. The new facility configuration is then evaluated by solving optimally the reduced CFLP-SC using an exact method. The new objective function value, machine and allocation configurations are then obtained. The modified ALA heuristic given in Algorithm 3 is implemented to reduce the transportation cost by improving the location of facilities. If an improvement is found ($\Delta \leq 0$), a new solution is accepted, otherwise, the new solution is accepted if $(\varphi) < exp(-\Delta/T)$ with (φ) being a random number generated in the range $[0, 1]$. In other words, contrary to VNS, a non-improvement (worse) solution is also allowed here.

5 Computational Experiments

As there is no data available in the literature to cater for the characteristics of the SSCMFWP-SC, we constructed a new generated dataset where $n = 50$ to 500 with a step size of 50. The location of customers is also randomly and uniformly generated in the square $(n \times n)$ with integer coordinates values. The demand of each customer is randomly generated between 1 and 10. The number of open facilities (p) is set to $\max(5, 0.1n)$. The number of machine types is set to 3 (i.e., $|M| = 3$). The capacity, purchasing cost and the number of machines available for each type of machine are also estimated based on the total demand of customers, the average distance from one customer to others, and the number of open facilities (p). The unit transportation cost per km and per unit of demand $(\hat{c})$ is set to \$1. Here, the dataset is constructed in such way that in a good solution the total transportation cost obtained is close to the total setup cost.

It is worthwhile noting that a facility may have a different capacity compared to other. For completeness, the coordinates (x, y) of customers' location are also provided along with the demand of each customer. The full dataset can be collected from the authors or downloaded from the Centre for Logistics and Heuristic Optimisation (CLHO) website (https://www.research.kent.ac.uk/clho/datasets/). The implementation was written in C++ .Net 2012 and the mathematical model is solved using the IBM ILOG CPLEX version 12.63 Concert Library and Lingo version 12.0. The experiments were run on a PC with an Intel Core i5 CPU @ 3.20GHz processor and 8.00 GB of RAM.

5.1 Computational results on the SSCMFWP-SC using the exact method

This subsection analyses the performance of the exact method when solving the SSCMFWP-SC problem. The non-linear model of the SSCMFWP-SC problem

(Equations 1–9) are solved using Lingo within 24 hours for both cases of Rectilinear and Euclidean distance. We record the upper bound (UB), the lower bound (LB) and the Gap (%) obtained where the Gap (%) is calculated as follow:

$$\%Gap = \frac{UB - LB}{UB} \times 100 \tag{38}$$

For the case of Rectilinear distance, the linearized model is solved using CPLEX within 2 hours. Our experiments show that the model can be optimally solved using CPLEX when a relatively small problem ($n = 25$ and $p = 5$) is used. For relatively large problems (($n \geq 50$ and $p \geq 5$), CPLEX experienced difficulties to solve the problems. We have tried to run CPLEX for more than three days using a powerful computer, however the Gap (%) is not significanly improved compared to the one obtained within 2 hours.

Table 1 presents the computational results for the SSCMFWP-SC problem solved using the exact method. For the case of Rectilinear distance Lingo was able to obtain the UB and LB within 24 hours for the problems with $n \leq 350$ with a relatively high average %Gap of 81.16. The performance of linearized model solved by CPLEX within 2 hours is much better where it managed to yield the average %Gap of 64.60 for the problem with $n \leq 350$. Moreover, CPLEX was able to obtain the UB and the LB for all instances with the average %Gap of 73.48 which is very high. It is also noted that the %Gap increases with the size of the problem. This indicates that the linearized model is also still very hard to solve. As mentioned in subsection 3.1, this added complexity is mainly due to the large number of additional constraints resulting form the linearisation (i.e., $8pn$ in total).

Table 1 Computational Results for the SSCMFWP-SC using the exact method

Ins	n	p	SSCMFWP-SC (Rectilinear)						SSCMFWP-SC (Euclidean)		
			Linearized Model - CPLEX			Non Linear Model - Lingo			Non Linear Model - Lingo		
			UB	LB	Gap(%)	UB	LB	Gap(%)	UB	LB	Gap(%)
1	50	5	4,813.00	3,203.84	33.43	4,738.11	2,365.00	50.09	4,146.05	2,208.00	46.74
2	100	10	13,319.00	5,502.29	58.69	19,778.20	5,223.30	73.59	11,705.05	5,223.30	55.38
3	150	15	21,835.00	8,645.00	60.41	40,343.00	8,606.00	78.67	41,723.20	8,606.00	79.37
4	200	20	34,250.50	13,035.50	61.94	73,815.50	13,035.50	82.34	53,506.10	13,035.50	75.64
5	250	25	53,520.00	18,096.00	66.19	251,155.00	18,096.00	92.79	NA	NA	NA
6	300	30	121,537.50	25,287.50	79.19	271,111.50	25,287.50	90.67	NA	NA	NA
7	350	35	435,290.00	33,288.00	92.35	354,116.00	0.00	100.00	NA	NA	NA
8	400	40	576,533.00	37,166.50	93.55	NA	NA	NA	NA	NA	NA
9	450	45	833,233.00	50,347.00	93.96	NA	NA	NA	NA	NA	NA
10	500	50	1,185,433.00	58,653.00	95.05	NA	NA	NA	NA	NA	NA
			Average Ins 1-7		64.60	Average Ins 1-7		81.16	Average Ins 1-4		64.28
			Average Ins 1-10		73.48						

NA: Not applicable as the solver (Lingo) was not able to obtain the UB and LB

For the case of Euclidean distance, Lingo was able to obtain the UB and LB for the problems with $n \leq 200$ only. According to Table 1, Lingo produce the average %Gap of 64.28 for the first 4 instances. It seems that the problem with Euclidean distance is harder to solve than the one with Rectilinear. Based on this finding, we therefore opted to use metaheuristic approaches to overcome the limitation of the exact method.

5.2 Computational results on the proposed solution methods

To evaluate the performance of the proposed solution methods (VNS and SA approaches), the solutions for the SSCMFWP-SC found by the exact method which are presented in Table 1 are used for comparison purposes. Moreover, the solutions obtained by solving the discrete CFLP-SC problem using the exact method (CPLEX) are also used. In our experiments, we limit the computing time of CPLEX for solving the discrete CFLP-SC problem to 2 hours. The performance of the proposed methods is measured using the Gap (%) which is defined in Equation 38. Here, minor change is made in the equation where UB is replaced by Z referring to the feasible solution cost obtained by either the exact method (UB) or the proposed methods (VNS or SA approaches).

To assess the consistency of the proposed method, in each instance the proposed methods (VNS and SA approaches) was executed 5 times and the average results as well as the best ones are presented. In this study, we consider both the rectilinear and Euclidean distances. In this subsection, we first present the preliminary experiments for parameters setting used in the proposed methods.

5.2.1 Preliminary Experiments

In this subsection, preliminary experiments are performed to determine the most suitable parameters setting for the proposed approaches. Table 2 shows the alternatives of the parameters setting for each method which classifies as low, medium and high setting values. It is worth noting that higher values of these parameters could be used and may have a greater chance to obtain better solutions but at the expense of more computational time. In the ALA heuristic, the value of ϵ is set to 0.0001. Computational experiments on three instances are conducted to analyse which paramaters setting is used for full experiments on the 10 instances. The three instances are $n = 50$ with $p = 5$, $n = 250$ with $p = 25$ and $n = 500$ with $p = 50$, representing two extreme sizes of instances and one in the middle.

Table 3 reveals the results of the preliminary experiments where the CPU time is given together with the Dev (%) defined as follow:

$$Dev = \frac{Z - Z^b}{Z^b} \times 100 \tag{39}$$

where Z^b refers to the best objective function obtained by the proposed methods (VNS or SA approaches). According to Table 3, in general the quality of solution (%Dev) obtained is better if the values of parameters (high) are used except for the use of medium parameters on the SA. However, it needs long computational time when high parameter is used which is almost double than medium parameter.

Table 2 The alternatives of parameters setting

VNS				SA			
Params.	Low	Medium	High	Params.	Low	Medium	High
η	$\min\{2p, 100\}$	$\min\{4p, 100\}$	$\min\{6p, 100\}$	H for $n < 200$	5	10	15
H	3	5	7	H for $n \geq 200$	3	5	8
$k_{\max}$	3	5	7	T	500	1000	1500
τ'	$1p$	$2p$	$4p$	$T_{\min}$	100	100	100
τ''	$p/2$	$p/2$	$p/2$	α	0.9	0.9	0.9
τ^L	$1p$	$1p$	$1p$	L	$\min\{5, p\}$	$\min\{10, p\}$	$\min\{15, p\}$

Therefore, it is decided to use medium parameter for both methods in the full computational experiments which is presented in the next subsection.

Table 3 The preliminary study results

n	p	Z^b	VNS					
			Low		Medium		High	
			Dev (%)	CPU (s)	Dev (%)	CPU (s)	Dev (%)	CPU (s)
Rectilinear Distance								
50	5	4,675.48	0.1644	12	0.7908	24	0.0000	46
250	10	44,670.46	1.0324	135	0.2810	449	0.0000	1,040
500	50	136,443.05	1.0778	485	0.0000	1,495	0.3780	2,118
Average			0.7582	210	0.3573	656	0.1260	1,068
Euclidean Distance								
50	5	4,145.90	0.7428	18	0.0000	35	0.0036	71
250	10	39,594.57	1.0549	162	0.1527	444	0.0000	948
500	50	120,882.12	0.6236	504	0.0261	1,015	0.0000	2,100
Average			0.8071	228	0.0596	498	0.0012	1,040

n	p	Z^b	SA					
			Low		Medium		High	
			Dev (%)	CPU (s)	Dev (%)	CPU (s)	Dev (%)	CPU (s)
Rectilinear Distance								
50	5	4,673.00	0.7354	21	0.1913	28	0.0000	43
250	10	45,030.20	0.3936	197	0.0000	509	0.6982	1,052
500	50	135,751.62	0.8700	1,387	0.2130	2,073	0.0000	4,497
Average			0.6664	535	0.1348	870	0.2327	1,864
Euclidean Distance								
50	5	4,145.87	0.0000	49	0.0000	44	0.0000	70
250	10	39,573.21	0.6947	307	0.3152	389	0.0000	2,057
500	50	120,127.10	0.0000	1,605	0.6412	2,762	0.1544	3,856
Average			0.2316	654	0.3188	1,065	0.0515	1,994

5.2.2 Experiments on the rectilinear distance

In this scenario, for $n \geq 50$, optimality is not guaranteed as the positive value of %Gap is obtained when solving the non-linear or linearized model by the exact method (Lingo and CPLEX) which is presented in Section 5.1. Here, we only present the results of linearized model as this model performs better than the non-linear one. The discrete CFLP-SC model is relatively easy to solve using the exact method. For the discrete problem, CPLEX was able to obtain the optimal solutions for $n \leq 150$ within 2 hours. The summary results on the Rectilinear distance are shown in Tables 4 where the best objective function value (Z^b), the Gap (*Gap*) obtained by each solution method, and the computational time (CPU) in seconds are provided. The *Gap* is calculated using Equation (38). The bold numbers in the table refer to the best gap found including ties.

Based on Tables 4, it can be noted that the proposed methods (VNS and SA approaches) produce better results when compared to both the exact method on the linearized model and the CFLP-SC. The average %Gap produced by the proposed VNS and SA approaches based on the best results of 5 executions are 55.15% and 55.17% respectively. These are much better than the average %Gap yielded by the exact method on the linearized model (73.48%) and the discrete CFLP-SC (55.39%). According to the results, the proposed VNS outperforms the SA approach as the VNS produce the smallest average %Gap based on the best results and average results of 5 executions. Moreover, the VNS approach runs faster than the SA method. In general, the proposed method is shown to be efficient as it produces good quality solutions within a relatively short computational time. The proposed methods also provide consistent results as the average deviation is not too far from the best one.

We also record the proportion of setup and transportation costs in the solutions produced by the exact method and the proposed approach for the rectilinear distance. It reveals an interesting observation that in a good solution, the share of setup cost is quite close to the one of transportation cost. For instance, based on the best results of the proposed VNS approach, the average share of transportation cost is 56.27%. It is worth noting that the proposed method produces the smallest transportation cost compared to the exact method on the linearized model (72.72%) and on the discrete CFLP-SC (56.58%). It is also worthwhile mentioning that if the problem has a much higher setup cost than the transportation cost, the problem can be considered in general to be relatively easier to solve.

Table 4 Computational Results for the SSCMFWP-SC with Rectilinear Distance

n	p	Z^b	EM-LM (2 hours)		EM (CFLP-SC)		The proposed VNS			The proposed SA		
			LB	Gap (%)	Gap (%)	CPU (s)	Best Gap(%)	Average Gap(%)	Average CPU (s)	Best Gap(%)	Average Gap(%)	Average CPU (s)
50	5	4,659.55	3,203.84	33.43	31.86	12	31.36	31.52	34	31.24	31.52	24
100	10	12,525.61	5,502.29	58.69	56.25	69	56.17	56.26	122	56.07	56.08	63
150	15	20,435.09	8,645.00	60.41	57.88	760	57.70	57.86	375	57.78	58.07	118
200	20	31,172.09	13,035.50	61.94	58.45	7,209	58.18	58.30	599	58.27	58.40	266
250	25	44,694.10	18,096.00	66.19	59.74	7,238	59.51	59.73	943	59.61	59.87	457
300	30	59,986.82	25,287.50	79.19	57.98	7,202	57.84	57.92	668	57.87	58.23	617
350	35	80,563.61	33,288.00	92.35	59.25	7,200	58.68	58.73	878	58.69	58.92	1,458
400	40	88,379.26	37,166.50	93.55	57.98	7,201	57.95	58.10	998	58.26	58.52	924
450	45	117,569.87	50,347.00	93.96	57.57	7,201	57.24	57.43	1,171	57.18	57.25	2,166
500	50	135,598.10	58,653.00	95.05	56.95	7,201	56.85	57.02	1,389	56.74	56.88	2,167
Average				73.48	55.39	5,129	**55.15**	55.29	717	55.17	55.37	826

EM-LM: Exact mehtod (CPLEX) on the linearized model

EM (CFLP-SC): Exact mehtod (CPLEX) on the discrete CFLP-SC model

5.2.3 Experiments on the Euclidean distance

The summary results on the SSCMFWP-SC for this scenario are presented in Tables 5 and 6. In Table 5, the performance of the proposed method is evaluated using %Gap (Equation 38) for the problems with $n \leq 200$. While in Table 6, the %Dev is presented to assess the proposed methods for the problems with $n > 200$ as the exact method (Lingo) did not find the feasible solutions for these problems. The tables show that the proposed methods (VNS and SA approaches) provide better results when compared to the exact method on the SSCMFWP-SC and the discrete CFLP-SC. In contrast to the previous results, for the problem with Euclidean distance the proposed SA method performs better than the VNS approach. Based on the average results of 5 executions, the SA approach yields a better average %Gap and %Dev compared to the VNS approach. However, the SA approach requires relatively more computational time than the VNS method. In brief, the proposed methods are found to run much faster than the exact method while producing better solutions. We also notice that both setup and transportation costs share a similar portion to the total cost, an observation noted earlier for the first scenario. Based on the best results, the average portion of transportation cost is 50.80% to the total cost whereas the exact method on the CFLP-SC only provides 51.06%. Here, it can be shown that the proposed method improves the quality of the solution by reducing the total transportation cost.

6 Conclusions

The continuous single-source capacitated multi-facility Weber problem (SSCMFWP) is studied where setup cost of facilities is considered by a set of capacitated machines. We refer this problem to the SSCMFWP-SC. A new non-linear mathematical model for the SSCMFWP-SC is developed in order to minimise the sum of setup and transportation costs. In case that the SSCMFWP-SC uses rectilinear distance, the new linearized model is introduced. Two metaheuristic frameworks based on Variable Neighbourhood Search and Simulated Annealing are proposed to efficiently solve the problem. The proposed approaches integrate the aggregation technique, the application of an exact method using CPLEX, and the alternate location-allocation method of Cooper. A set of new instances which we constructed for the new model is used to examine the performance of the proposed methods. Very competitive results are attained by the proposed approaches when compared against the exact method on the non-linear problem and the discrete location problem (CFLP-SC).

The following research avenues could be worth exploring. For instance, the neighbourhoods in both VNS and SA could be adaptively adjusted during the search by introducing forbidden regions to avoid exploring already visited areas as successfully exploited by Luis et al (2009) for the capacitated Weber problem. The proposed method can also be extended to incorporate routing effects, see Salhi and Nagy (2009). The fleet can obviously be based on homogeneous or heterogeneous vehicle fleet.

Table 5 Computational Results for the SSCMFWP-SC with Euclidean Distance measured by Gap(%)

n	p	Z^b	EM (Non-Linear) - 24h		EM (CFLP-SC)		The proposed VNS			The proposed SA		
			LB	Gap (%)	Gap (%)	CPU (s)	Best Gap(%)	Average Gap(%)	Average CPU (s)	Best Gap(%)	Average Gap(%)	Average CPU (s)
50	5	4,145.87	2,208.00	46.74	46.82	10	46.74	46.74	32	46.74	46.74	46
100	10	11,005.81	5,223.30	55.38	52.77	23	52.61	52.64	79	52.54	52.54	63
150	15	18,087.01	8,606.00	79.37	52.56	152	52.44	52.46	210	52.42	52.46	151
200	20	27,533.93	13,035.50	75.64	52.95	7,245	52.70	52.73	306	52.66	52.70	390
Average				64.28	51.28	1,857	51.12	51.14	157	**51.09**	51.11	163

Table 6 Computational Results for the SSCMFWP-SC with Euclidean Distance measured by Dev(%)

n	p	Z^b	EM (CFLP-SC)		The proposed VNS			The proposed SA		
			Dev (%)	CPU (s)	Best Dev(%)	Average Dev(%)	Average CPU (s)	Best Dev(%)	Average Dev(%)	Average CPU (s)
250	25	39,655.11	0.3410	7,214	0.0000	0.1284	442	0.1090	0.5207	516
300	30	53,193.37	0.6028	7,202	0.1339	0.1931	585	0.0000	0.7303	927
350	35	70,797.90	1.3518	7,201	0.4610	0.6112	791	0.0000	0.4114	1,563
400	40	78,159.08	0.4679	7,201	0.1912	0.3365	886	0.0000	0.5819	1,400
450	45	103,746.48	0.8820	7,201	0.3222	0.3751	1,386	0.0000	0.2604	2,681
500	50	120,199.45	0.6792	7,201	0.5942	0.6501	1,007	0.0000	0.3612	3,034
Average			0.72089	7,203	0.2837	0.3824	850	**0.0182**	0.47762	1,687

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