Introducing Combined Weights and Centrality Measures To Evaluate Network Topologies

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July, 2017

A thesis submitted to the University of Kent in the subject of Computer Science in fulfilment for the degree of Doctor of Philosophy

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Acknowledgements

I hereby wish to acknowledge my supervisor Prof. Frank Z. Wang, and members of the review panel viz:- Prof. Sally Fincher, Dr. Matteo Migliavacca and Mr Ian Utting. Dr. Fred Yamoah and Dunnhumby, of the United Kingdom (www.dunnhumby.com) are also acknowledged for providing the TESCO datasets that were used for the purpose of this research. My wife and daughters, Mrs. Bukola R. Akanmu, Miss Zainab I. Akanmu and Miss Zafirah L. Akanmu respectively for their understandings all through my absence from home. Most of all I thank God almighty for the gift of life.

List of Publications from the Thesis

PUBLICATIONS AND PRESENTATIONS

JOURNAL

Akanmu, Amidu A.G., Wang, Frank Z. & Yamoah, Fred A. (2014). Weighted Marking, Clique Structure and NodeWeighted Centrality to Predict Distribution Centre?s Location in a Supply Chain Management. International Journal of Advanced Computer Science and Applications(IJACSA), 5(12), 2014. http://dx.doi.org/10.14569/IJACSA.2014.051217

Akanmu, A.A.G., Wang, F.Z & Huankai, C.(2012). Introducing Weighted Nodes To Evaluate The Cloud Computing Topology. Special Issue (Vol. 5 No. 11A, 2012) of JSEA. http:\\www.scirp.org/journal/jsea

CONFERENCE PROCEEDINGS

Akanmu, Amidu A.G. & Wang, Frank Z. (2016). "Newtonian Gravitational Force for Predicting Distribution Centre Location in a Supply Chain Network". Proceedings of the 7th International Conference on Computational Methods (ICCM2016), 1st to 4th August 2016, University of California, Berkeley, ScienTech Publisher, Paper ID (ISSN 2374-3948, online). http://www.sci-entech.com/ICCM/index.php/iccm2016/2016/paper/view/1758

Akanmu, Amidu A.G., Wang, Frank Z. & Yamoah, Fred A. (2014). "Clique Structure and Node-Weighted Centrality Measures for Predicting Distribution Centre Location in the Supply Chain Management". IEEE Technically Co-Sponsored Science and Information Conference (SAI), 2014 , vol., no., pp.100,111, 27-29 Aug. 2014 doi:10.1109/SAI.2014.6918178. IEEE/Science and Information Conference 2014, London, UK. http://ieeexplore.ieee.org/xpl/articleDetails.jsp?arnumber=6918178

Akanmu, A.A.G., Wang, F.Z & Yamoah, A. F.(2013). "Using Weighted Centrality Measures to Predict Distribution Centre Location in Supply Chain Management". ICCCSS 2013 : International Conference on Cloud Computing and Services Science, World Academy of Science and Technology, Dubai, October 22-23, 2013. WASET 82, 2013, pp1235-1242.

ORAL/POSTER PRESENTATION

Akanmu, A.A.G., Wang, F.Z & Yamoah, A. F.(2014). "Clique Structure and Weighted Centrality Measures for Predicting Distribution Centre Location in the Supply Chain Management". International Network for Social Network Analysis' XXXIV Sunbelt Social Networks Conference St. Petersburg, Florida, U.S.A (February 18 to February 23, 2014). (Book of Abstracts).

Akanmu, A. A. G. (2013). Centrality Measure and Location of Structures. Technical Report of the Postgraduate Conference 2013 for the School of Computing, University of Kent, England. https://kar.kent.ac.uk/33889/7/Proceedings2.pdf. pp11-12

Abstract

In this research of network structure analysis, the knowledge of centrality measures is applied to discover or predict a most important actor or node in a network/graph. Problems of energy efficiency and sustainability are considered, and also those of allocation of resources. In order to enable an efficient allocation of energy resources to the right path in a distributed network such as obtained in a data center, authorś network and supply chain network, new measures of centralities are introduced aside from the traditional ones of Closeness, Betweenness, Degree and Eigen-Vector centralities. Mixed-mean centrality, which is based on the generalized degree centrality, was developed as a measure to emphasise the importance of a node in the authorship network and the distributed system of a data centre. Weighted centrality measure when used as against the traditional measures mentioned above was able to make prediction for a Distribution Centre (DC) of a Supply Chain Network with an accuracy of 91.6%. Clique-Structure/Node-weighted centrality measure was able to make a prediction with 66% accuracy, while the Weighted Marking, Clique-Structure/Node-Weighted Centrality made a prediction accuracy of 96.2%. The Top Eigen-Vector Weighted Centrality and Newtonian Gravitational Force were also used to predict the location of distribution centre (DC) in a supply chain network with accuracies of 92.9% and 96.9% respectively.

Chapter 1

Introduction

When mention is being made of weighted networks, much reference and importance is always attached to the weights on the edges in a network. In actual fact what makes up a network is both the nodes and the edges of the network. It is therefore pertinent to investigate the effects and importance of the weights attributed unto the nodes in a network as well as the weights on the links of such networks as they both play important roles in determining the prominence or popularity of the nodes which are also referred to as actors within any particular network.

This thesis used the idea of mixed-mean centrality in solving problem of cloud computing. This concept of mixed-mean centrality combines the main types of centralities (degree centralities and closeness centralities) and takes the mean results, thereby encompassing all the centrality measures in one concept. The idea is used to discover the most central and therefore the most energy consuming nodes in a data centre, this is with an aim of making provision for energy-efficiency, thus minimising costs and saving the environment. This concept of mixed-mean centrality can also be used in locating the performance level of a particular node or edge and thus aiding in decision on which node or edge deserves attention. This can be most especially useful for security and fault-tolerance purposes. Resource allocation is also an applicable area of this centrality measure as it will aid in optimisation of resources, thereby saving costs. This new model was equally applied to the EIES (Electronic Information Exchange System) data set. This is in order to find the most central author (in a system of mail exchange between several nodes (in this case the authors).

The idea of node-weighted centrality measure was also used to predict the location of a Distribution Centre. It shows the most probable node that could serve as a Distribution center out of all other sales outlets of a supply chain network. This was made possible after having considered the centrality values and percentage accuracy of predictions of all the nodes (which are shops in this case).

Subsequently, the concepts of Clique structures of a network, Weighted Marking Methods , Top-Eigen Vector Weighted Centrality measures, and Newtonian Gravitational Forces were employed with node-weighted centrality measures to predict the location of a Distribution Centre of a Supply Chain Management Netwrok.

The remaining parts of the thesis are structured as follow :
Chapter Two deals with the literature review , while chapter three explained the Mixed-Mean Centrality concepts in details. Chapter four explained the use of Weighted Centrality Measure to predict location of nodes or structures. A combined concept of Clique structure and Node-Weighted Centrality was used in Chapter five to predict location of structures and the measures of accuracy was noted. In Chapter six, Weighted Marking, Clique Structure and Node-Weighted Centralities methods are considered, while Chapter seven predicted the location of a Distribution Centre of a Supply Chain Network using the Top-Eigen Vector Weighted Centrality measure. The concept of Newtonian Gravitational Force for predicting a Distribution Centre of Supply Chain Network. Finally, Chapter nine focused on the future studies, Summary and Conclusion.

Chapter 2

Literature Review

2.1 Introduction

Graph theory is a branch of computing and mathematics that enables better understanding of network structures. It concerns points that are referred to as vertices and the links connecting them, these links are also referred to as arcs, edges and lines. According to [Bruce , 2009], a graph is an ordered pair G=(V,E) where V is a set of vertices (nodes) and E is a set of edges (links). He referred to each edge as a pair of vertices, that is, each element of E is a pair of elements of V. Furthermore, [Wasserman and Faust, 1994] noted that graph theory in addition to its utility as a mathematical system, gives a representation of a social network as a model of a social system consisting of a set of actors and the ties between them.

Accordingly any network at all can be viewed from the perspective of graph theory for ease of understanding. A vertex also called node is usually perceived to be a single whole object with its own typical structure depending on what it represents and what the whole graph is all about. For example, a node might be seen as each city of a country if the whole graph is all about say cities, distances between cities and city populations of a particular country. Edges can only be formed when there are vertices to be linked, hence any two vertices forming an edge are called the endpoints of such vertices, and the edges would be said to be incident on the two such vertices. Nodes e1 and e2 are said to be adjacent to each other if there exist an edge e1e2 in the graph of interest.

Graphs are diagrams of vertices and lines, of networks because the vertices are the objects in the network (people, countries, computers, etc.) and the edges are relationships [Bonacich & Lu, 2012].

A graph G(V,E) represents the relationships between vertices $v_i \in$ V whereby E is the set of edges $e_{ij} \in$ E , connecting or linking the vertices i and j , however, the edges can be present or absent. Vertices which can can also be referred to as nodes as said earlier can also be referred to as points or actors while edges can be referred to as links, ties, arcs or lines.

According to [Krackhardt, 1994], a graph (G) is defined as a set of N points P=Pi and a set of unordered pairs of those points L = Pi, Pj, wherein the subscripts i and j are positive integers and they are not greater than N, the total number of nodes or vertices.

[Freeman, 1978] measures were designed for binary networks, hence they are dichotomous from the onset (i.e. it is either there is a tie between nodes or not), hence the only weights that could be assigned to the ties are unitary.

The importance of ties/edges as the conduit pipes of flow cannot be overemphasized as well, as such, [Granovetter, 1973] refers to the strength of a tie as a (probably linear) combination of the amount of time, the emotional intensity, the intimacy (mutual confiding), and the reciprocal services that characterize the tie. From the above, it means, the strength of a tie (also called the link or edge) plays a critical role in the bonding to the nodes/vertices, hence its main focus in the study of centrality.

By dichotomizing the network, much of the information contained in a weighted network datasets is lost, and consequently, the complexity of the network topology cannot be described to the same extent or as richly. As a result, there has been a growing need for network measures that directly account for tie weights.[Opsahl et al., 2010]. Actually ties can have some strengths, for example, in a road network, road distances (now representing ties) between towns or cities can have values other than binary digits, and different widths as well. In fact, [Barrat et al , 2004] concludes in their study of weighted networks that a more complete view of complex networks is provided by the study of the interactions defining the links of these systems and the weights characterizing the various connections which exhibit complex statistical features with highly varying distributions and power-law behaviour. Similarly, [Newman, 2001] also suggested a measure of the strength of collaborative ties which takes account of the number of papers a given pair of scientists have written together, as well as the number of other coauthors with whom they wrote those papers. Using this measure he added weightings to a collaboration networks and used the resulting networks to find which scientists have the shortest average distance to others. While the foci of [Barrat et al., 2004] and [Newman, 2001] are on the tie weights/strengths, [Opsahl, 2010] had a combined focus on both the number of ties and weights of ties.

It is of course pertinent to note that vertices and ties are complementary, hence an edge can only be incident on two nodes and two nodes can only be adjacent if they have a tie linking them. This implies that nodes can similarly have strengths and not only binary in nature, for example, in a road network, if the edges (roads) can have weights/strengths (distances), the nodes (cities) can also have weights/strengths (e.g. populations).

The need to study network flow and centralities is emphasized by [Ghoshal and Bartlett, 1990] when they said that It may be more useful to explore the actual content of strategy in a complex organizational systems like a Network theoretic analysis of internal flows of resources, products, people, and information might be more relevant for developing middle-range theories on resource commitment, decision making, strategic control, normative integration, and creation and diffusion of innovations in such companies. Still on centralities, [Carolan, 2014] explained that conceptually, centrality captures the extent to which a focal actor occupies an important position of prestige and visibility.

Buttressing the above, [Borgatti & Li, 2009] noted that a key concept in social network analysis has been the notion of node centrality, which they defined as the importance of a node due to its structural position in the network as a whole.

Centrality is hereby introduced as a suggested solution to the challenge of predicting the location of a distribution centre to other nodes of a graph/network as required. Specifically, a node-weight modulated centrality which stems from the idea of Generalizing Degree of Centrality and shortest paths of [Opsahl, 2010] and Topological Centrality of [Zhuge, 2010] is hereby proposed.

In an attempt to evaluate centrality, [Brandes, 2001] affirmed that many centrality indices are based on shortest paths linking pairs of actors, thereby considering for example, the measure of the average distance from other actors, or the ratio of shortest paths an actor lies on. Many network-analytic studies rely at least in part on an evaluation of these indices. [Newman, 2001] introduced an algorithm for counting the number of shortest paths between vertices on a graph that pass through each other vertex, and this is used to calculate the betweenness measure of centrality on graphs. Accordingly, [Newman, 2002] said it has been found that the connectivity of many networks (i.e., the existence of paths between pairs of vertices) can be destroyed by the removal of just a few of the highest degree vertices, a result that may have applications in, for example, vaccination strategies. The degree of vertices as being described here is the number of ties connecting the nodes as would shortly be explained in the next section. [Barrat et al, 2004] in their study of weighted networks carried out statistical analysis of complex networks whose edges have assigned a given weight (the flow or the intensity), and such according to them can generally be described in terms of weighted graphs and more so that a more complete view of complex networks is provided by the study of the interactions defining the links of those systems. Furthermore, they confirmed that a more complete view of complex networks is provided by the study of the interactions defining the links of these systems and the weights characterizing the various connections which exhibit complex statistical features with highly varying distributions and power-law behaviour.

[Brandes(2001) , Newman(2001), & Barrat et al(2004)] have all attempted to attach weights to the centralities - Degree, Betweenness and Closeness, but they have only succeeded in attaching weights to the edges and not the nodes.

With this study one can be able to locate which nodes deserve more or less attention because of their location and/or performance.

On ties/edges of nodes, [Borgatti & Halgin, 2011] infer that both state-type ties and event-type ties can be seen as roads or pipes that enable (and constraint) some kind of flow between nodes. Flows are what actually pass between nodes as they interact, such as ideas or goods. Hence two friends (state-type social relation) may talk (event-type interaction) and, in so doing, exchange some news (flow). [Carolan, 2014] is of the opinion that, centrality captures the extent to which a focal actor occupies an important position of prestige and visibility.

Centrality is hereby introduced as a suggested solution to the challenge of allocation of resources or traffic to a node as required. Specifically, a mixed-mean centrality and a node-weight modulated centrality which stems from the idea of Generalizing Degree of Centrality and shortest paths of [Opsahl, Agneessens & Skvoretz, 2010] and Topological Centrality of [Zhuge & Zhang, 2010] is hereby proposed.

There are four standard measures of centrality namely Degree Centrality; EigenVector Centrality; Closeness Centrality and Betweenness Centrality. Three of them Degree, Betweenness and Closeness are all formalised by [Freeman, 1978]. Each of these centralities either concern itself with nodes or edges [Zhuge & Zhang, 2010], however this work concerns itself with nodes and edges while considering the degree, eigenvector, betweenness and closeness centralities.

2.2 Centrality

The formal theory of social network analysis encompasses centrality measures, and these are to be employed in this research that dwells on the mergers of weights (link-weights and node weights) to evaluate network topologies and make a prediction. The strength attached to the nodes also called the node-weights represents a certain attribute of a particular node (e.g. population of a city), and the same goes for the strength attached to edges (e.g. distance between cities). [Akanmu, Wang & Yamoah, 2014].

According to [Barrat et al, 2004], in their study of weighted networks, they carried out statistical analysis of complex networks whose edges have assigned a given weight (the flow or the intensity), and such according to them can generally be described in terms of weighted graphs and more so that a more complete view of complex networks is provided by the study of the interactions defining the links of those systems. Although, [Granoveter, 1973], [Brandes, 2001], [Barrat et al, 2004], [Newman, 2001] have only emphasized on the attachment of weights to the edges and not to the nodes in their various studies, [Opsahl, Agneessens & Skvoretz, 2010] and [Zhuge & Zhang, 2010] have considered both the weights on the edges and also the number of edges attached to a particular node. This work however concerns itself with both nodes and edges while considering the degree, eigenvector, betweenness and closeness centralities.

The importance of location of distribution centre is echoed by [Thai & Grewal, 2005], when they explained that the advantage of an optimal location for distribution centre is not only to reduce transportation costs, but also to improve business performance, increase competitiveness and profitability.

Further investigations can be made into how the objects, nodes or actors are connected within a particular network, of particular interest is the subgroups based on complete mutuality. In fact, [Wasserman and Faust ,1994] defined the clique in a graph to be a maximal complete subgraph of three or more nodes. Another explanation was that a clique is a maximal subset of nodes in which the density is 100%, that means in a clique, everybody has a tie with everybody else. [Borgatti & Li, 2009].

Here, the focus is on the number of ties, weights of ties/nodes and centralities while also investigating the clique structures of the network being considered. With this study one can be able to predict the siting or location of a node that deserves more or less attention because of their location and/or performance.

The idle time of a node and the traffic on its links (edges) has been a concern for the cloud operators. As more energy are being consumed so also the strength and weights of the links to the nodes (data centres) increases, as such, the knowledge of centrality will allow efficient allocation of energy source to the right path.

2.3 Standard Centrality Measures

2.4 Degree Centrality

Degree centrality measure is concerned with the degree of a certain node in a directed graph (or network), that is the number of edges or links or ties that enters a node (wherein referred to as in-degree) or the number of edges that come out of the node (wherein referred to as out-degree), this being applicable to directed graphs.Conversely, in an undirected graph it is the number of ties or edges attached to the node that becomes a concern. Formally, [Newman, M.J., 2008] in the Mathematics of Networks defines the degree K_i of a node i as

$$K_i = C_D(i) = \sum_{j=1}^{n} A_(i,j) \tag{2.1}$$

where n = number of nodes in the network

$$A_(i,j) = \begin{cases} 1 & \text{ifthere is a tie between nodes i and j} \\ 0 & \text{otherwise} \end{cases} \tag{2.2}$$

by tie , we mean an edge and $A_(i,j)$ is an element of the adjacency matrix A, that is, A is an n X n symmetric matrix (implying $A_(i,j)$=$A_(j,i)$). e.g.

$$A_(i,j) = \begin{pmatrix} A_{1,1} & A_{1,2} & \cdots & A_{1,n} \\ A_{2,1} & A_{2,2} & \cdots & A_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{j,1} & A_{j,2} & \cdots & A_{j,j} \end{pmatrix} \tag{2.3}$$

where $A_{1,1} = A_{2,2} = ... = A_{j,j} = 1$

The matrix in (2.3) above implies that the entry $A_{1,2}$ will be equal to $A_{2,1}$ if A is an adjacency matrix. The degree centrality has as part of its advantage that only local structure round the node could be known, for ease of calculation this is acceptable, but it becomes a concern if a node is central and not easily accessible to other nodes for one reason or another. It is described by [Zhuge, 2010] as shown in (2.4) below:

$$C_D(s) = \frac{degree(s)}{n-1} \tag{2.4}$$

where s = node and n = total number of nodes in the graph.

Closeness Centrality

Closeness centrality takes the distance from the focal node to other nodes in a graph into consideration but it has the setback of not taking disconnected nodes (those who are not in the same graph) into consideration. [Zhuge, 2010] formally expresses closeness centrality as

$$C_c(s) = \frac{n-1}{\sum_{r \in S s \notin S} d_G(s,t)} \tag{2.5}$$

where n = number of nodes, dG(s,t) = geodesic distance between s and t.

For example, in Table 2.1, Node B has the highest of the centralities for the degree and closeness centrality measures.

2.5 Generalised Degree Centrality Measure

Each of the three standard degrees mentioned earlier concerns itself with either nodes or edges [Zhuge, 2010]. There have been several attempts to generalise the three node centrality measures but most have solely focused on weights of edges and not number of edges [Opsahl et al, 2010]. Since weights are of importance, the equations in (2.4) to (2.5) above can be

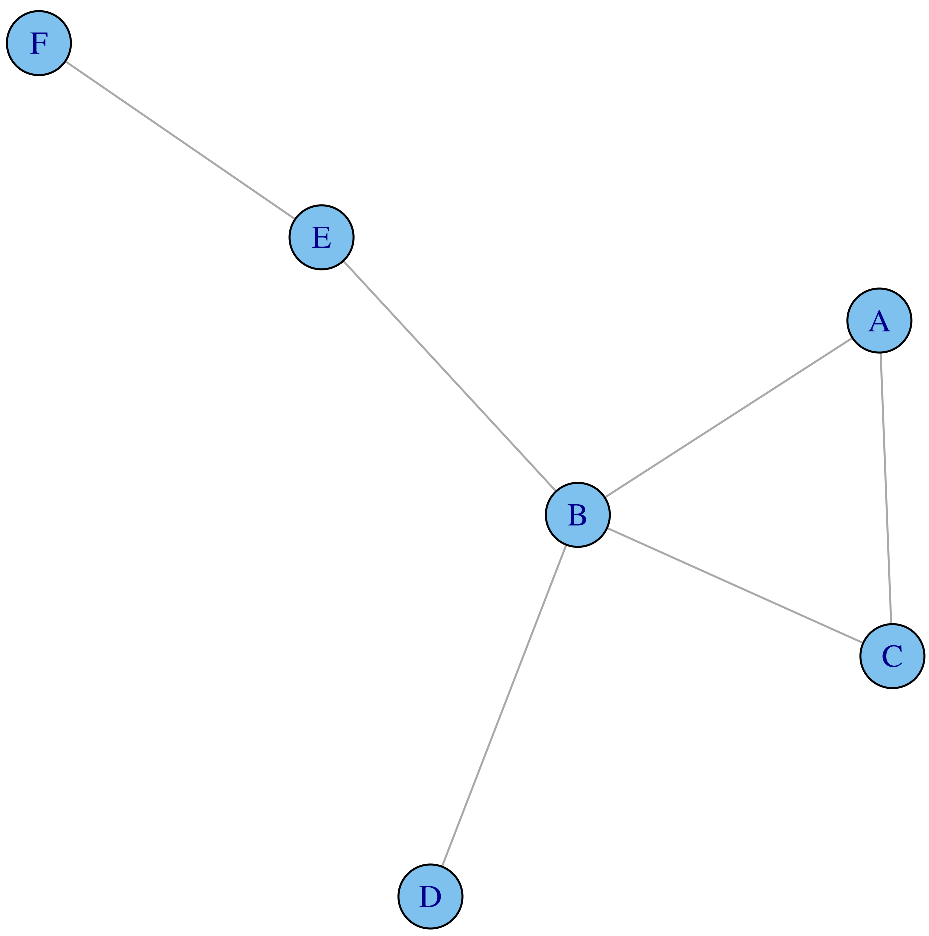

Figure 2.1: (Graph1). A six nodes network, circle represents nodes. (Source: T. Opsahl et al (2010) pg 245)

Table 2.1: The results of the two standard centrality measures according to figure 2.1

Node	**Number of Degrees**	**Degree Centrality (C_D)**	**No of Geodesic Paths from s to t($\sum_d(s,t)$)**	**Closeness Centrality**
A	2	0.4	9	0.555556
B	4	0.8	6	0.833333
C	2	0.4	9	0.555556
D	1	0.2	9	0.555556
E	2	0.4	8	0.625
F	1	0.2	12	0.416667

extended to include weights, therefore, the weighted degree centrality of nodes is hereby represented by

$$C_D^W(s) = \frac{\sum_t^n W_(s,t)}{n-1} \tag{2.6}$$

where $W_(s,t)$ is the sum of the weights of edges connected to the particular source node s and , t represents a particular target node. In the same vein, the weighted closeness centrality $C_c^w(s)$, is also represented by

$$C_c^w(s) = \frac{n-1}{\sum d_w(s,t)} \tag{2.7}$$

which is the weight of geodesic paths between s and t.

In the attempt to incorporate the measures of both degree and strength of edges (i.e. numbers and weights of edges respectively), [Opsahl et al, 2010] considered a graph with 6 nodes (figure 2.2) as shown and also introduced the ideal of generalised degree centrality measure.

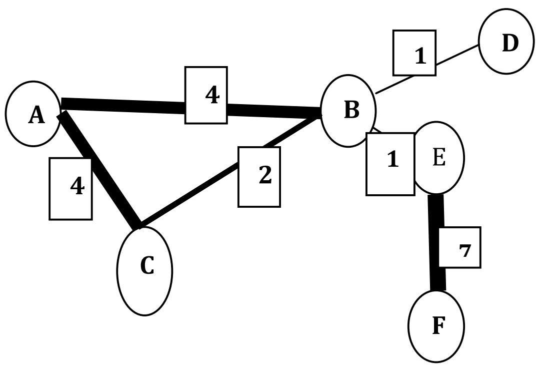

Figure 2.2: (Graph2). A six nodes network, circle represents nodes and square represents weights of edge. e.g. Number of Visits. (Source: T. Opsahl et al (2010) pp 245)

A tuning parameter α was introduced to take care of the weightedness of the degree and strength of the edges, this being the product of degree of a focal node and the average weight to these nodes as adjusted by the introduced tuning parameter. So, for weighted degree centrality at α we have

$$C_d^{w\alpha}(i) = K_i \times (\frac{S_i}{K_i})^{\alpha} = K_i^{1-\alpha} \times S_i^{\alpha} \tag{2.8}$$

where K_i = degree of nodes, $S_i = C_d^w(s)$ as defined in (2.6) above , and α is ≥ 0
For weighted closeness centrality at α we also have

$$C_c^{w\alpha}(i) = K_i \times (\frac{S_i}{K_i})^{\alpha} = K_i^{1-\alpha} \times S_i^{\alpha} \tag{2.9}$$

where K_i = degree of nodes, $S_i = C_c^w(s)$ as defined in (2.7) above , and α is ≥ 0
When applied to figure 2.2, Table 2.2 shows the results for the weighted degree centrality measure and the closeness degree centrality measure.

2.6 Closeness Centrality

Closeness centrality takes the distance from the focal node to other nodes in a graph into consideration but it has the setback of not taking disconnected nodes (those who are not in the same graph) into consideration. [Zhuge, Zhang & Junsheng, 2010] formally expresses closeness centrality as

$$C_c(s) = \frac{n-1}{\sum_{r \in S s \notin S} d_G(s,t)} \tag{2.10}$$

where n = number of nodes, dG(s,t) = geodesic distance between s and t.

2.7 Betweenness Centrality

Assuming node A has two ties to nodes B and C; while B has ties to nodes D and E and node C in turn has ties to nodes F and G as in figure 2.3 below:

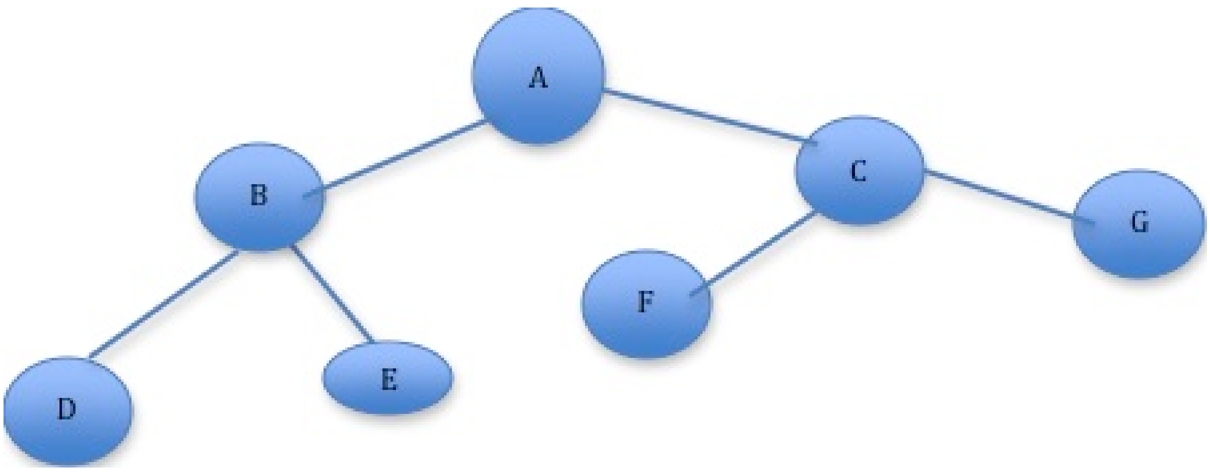

Figure 2.3: Figure showing vertices/nodes A to G and their ties/edges.

Table 2.2: Results for the weighted degree centrality measure and the closeness degree centrality measure with the positive tuning parameters.

Nodes	**No of Degrees**	**Degree Centrality** (C_D)	**No of Geodesic Paths from s to t** ($\sum_d (s,t)$)	**Closeness Centrality**	**CDWI** $\sum_W (s,t)$	C_D^w (S_i)	α **=0**	α= $\frac{1}{2}$	α **=1**	α **=1**$\frac{1}{2}$	**CCW1** ($\sum_d$ (Ws,t))	C_c^W (S_i)	α **=0**	α =$\frac{1}{2}$	α **=1**	α **=1**$\frac{1}{2}$
A	2	0.4	9	0.556	8	1.6	2	1.79	1.6	1.431	30	0.167	2	0.577	0.167	0.048
B	4	0.8	6	0.833	8	1.6	4	2.53	1.6	1.012	15	0.333	4	1.155	0.333	0.096
C	2	0.4	9	0.556	6	1.2	2	1.55	1.2	0.93	18	0.278	2	0.745	0.278	0.104
D	1	0.2	9	0.556	1	0.2	1	0.45	0.2	0.089	20	0.25	1	0.5	0.25	0.125
E	2	0.4	8	0.625	8	1.6	2	1.79	1.6	1.431	18	0.278	2	0.745	0.278	0.104
F	1	0.2	12	0.417	7	1.4	1	1.18	1.4	1.657	46	0.109	1	0.33	0.109	0.036

Node A connects two branches of ties B and C and it also lies on many geodesic paths, hence we say that node A has a high betweenness. Nodes B and C also have betweenness because they lie between node A and their own subordinate nodes. Nodes D, E, F and G however have zero betweenness. Suppose two nodes without direct link needs to have a link, the geodesic path between them can be ′opened′ or ′blocked′ by a brokerage node that sits in between, that is a node that has betweenness relative to them. e.g. Node A is the only brokerage node that serves as link to node B from anyone of nodes C, F and G. One takes note of the fact that resource allocation and distribution is of vital importance in cloud computing. According to [Borgatti & Li , 2009], it is reasonable to deduce that if short paths are important, then the nodes that lie among many short paths between others (a property known as betweenness centrality) are structurally important nodes that are well-positioned to control, filter or colour information flows and possibly become over-burdened bottleneck that slows down the network. [Zhuge, 2010] formally expresses betweenness centrality as

$$C_B(v) = \sum_{s \notin v \notin t \in v; s \neq t} \frac{\sigma_{st}(v)/\sigma_{st}}{(n-1)(n-2)} \tag{2.11}$$

where σ_{st} is the number of the shortest geodesic paths from s to t, and $\sigma_{st}(v)$ is the number of the shortest geodesic paths from s to t that pass through node v.

2.8 Eigen-Vector Centrality

This is a measure that implies that the connections by a source node to more important nodes/vertices that are in turn connected to other important nodes and so on, make the source node to be important. It is in other words the values emanating from the weights that have the value from the highest eigenvector of the adjacency matrix of the graph in question. λ is said to be the eigenvalue of the matrix A if the following holds:

$$\lambda x = Ax \tag{2.12}$$

where A is a square matrix and x is an eigenvector of A.

2.9 Weighted Centrality Measures Due To Weights On The Edges

Each of the four measures of centrality mentioned earlier concerns itself with either nodes or edges [Zhuge & Zhang, 2010]. There have been several attempts to generalise the three node centrality measures (degree, betweenness and closeness) but most have solely focused on weights of edges and not number of edges [Opsahl, Agneessens & Skvoretz, 2010]. Therefore when weights are attached in line with (2.8), the weighted degree centrality of edges incident on a source node p will be defined as:

$$S_p = C_D^W(p) = \frac{\sum_q^n W_{pq}}{n-1} \tag{2.13}$$

where q is the target node and N is the number of nodes considered, w is the weight on the edge that runs from p to q.

In the attempt to incorporate the measures of both degree and strength on edges (i.e. numbers of edges and weights on edges respectively), [Opsahl, 2010] introduced the idea of generalised degree centrality measure.

A tuning parameter α was introduced to determine the relative importance of the number of ties compared to the weights on the ties. The equation (2.14) below thereby represents the product of degree of a focal node and the average weight to these nodes as adjusted by the introduced tuning parameter. So, for weighted degree centrality at α we have:

$$C_d^{w\alpha\beta}(i) = K_i \times (\frac{S_i}{K_i}^{\alpha}) \times (Z_i)^{\beta} = K_i^{1-\alpha} \times S_i^{\alpha} \times (Z_i)^{\beta} \qquad (2.14)$$

where K_p = degree of nodes, $S_p = C_d^w(p)$ as defined in (2.13) above and $\alpha \geq 0$

Different meanings have been adduced to the weighted-ness of a network, so many literature have at instances made references to link-weights as the weights of the entire network, even though any network as described above would at least consist of node(s) and link(s) as the case may be. This therefore implies that there has to be node-weights which are being separated from link-weights and the combination of the two would thereby emerge as weights of any typical network.

For the sake of clarity, in this chapter , (2.14) shall be referred to as linkweighted degree centrality since it deals solely with the weights on the edges and number of edges. Also, from (2.14) it can be deduced trivially that at α=0 the number of degrees would be returned while at α=1 the sum of the weights of the incident edges to a node would be returned. Therefore, our interest shall be on the situations whereby α values are less than 1 or greater than 1 , i.e. $\alpha=\frac{1}{2}$ and $\alpha=1\frac{1}{2}$.

We shall now formally define the weighted centrality of the four measures, i.e. Degree, Closeness, Betweenness and the Eigenvector. The degree centrality of any node S taking cognisance of the strength of the incident edges is herein defined as the weighted degree centrality of node s and is represented in normalised form as:

$$C_D^W(s) = \frac{\sum_t^n W_{st}}{n-1} \qquad (2.15)$$

where W_{st} is the sum of the weights of edges connected to the particular source node s and t represents a particular target node. In the same vein, the weighted closeness centrality, C_c^w is also represented by

$$C_c^w(s) = \frac{n-1}{\sum d_w(s,t)} \qquad (2.16)$$

where $d_w(s,t)$ is the weight of geodesic paths between s and t, while the weighted betweenness centrality is

$$C_B^w(v) = \sum_{s \notin v \notin t \in v; s \neq t} \frac{\sigma_{st}^w(v)/\sigma_{st}^w}{(n-1)(n-2)} \qquad (2.17)$$

where σ_{st} is the number of the shortest geodesic paths from s to t, $\sigma_{st}(v)$ is the number of the shortest geodesic paths from s to t that pass through node v and w is the assigned weights to the ties. Similarly, the weighted eigenvector centrality could be seen as

$$\lambda x = A^w X \tag{2.18}$$

where A^w is a square matrix of the weights on the edges of A and x is an eigenvector of A . So for weighted closeness centrality at α we have

$$C_c^{w\alpha}(i) = K_i \times (\frac{S_i}{K_i})^\alpha = K_i^{(1-\alpha)} \times S_i^\alpha \tag{2.19}$$

where K_i = degree of nodes, $S_i = C_c^w(s)$ as defined in (2.16) above
where α is ≥ 0, and similarly for the degree centrality; betweenness centrality and eigenvector centrality.

Typical data centers house several powerful ICT (Information and Communication Technology) equipment such as servers, storage devices and network equipment that are high-energy consuming. The nature of these high-energy consuming equipment is mostly accountable for the very large quantities of emissions which are harmful and unfriendly to the environment. The costs associated with energy consumption in data centers increases as the need for more computational resources increases, so also the appalling effect of CO2 (Carbon IV Oxide) emissions on the environment from the constituent ICT facilitiesŚervers, Cooling systems, Telecommunication systems, Printers, Local Area Network etc. Energy related costs would traditionally account for some of the total costs of running a typical data center. There is a need to have a good balance between optimization of energy budgets in any data center and fulfillment of the Service Level Agreements (SLAs), as this ensures continuity/profitability of business and customerś satisfaction.

A greener computing from what used to be would not only save/sustain the environment but would also optimize energy and by implication saves costs. This study addresses the challenges of sustainable (or green computing) in the cloud and proffer appropriate, plausible and possible solutions. The idle and uptime of a node and the traffic on its links (edges) has been a concern for the cloud operators because as the strength and weights of the links to the nodes (data centres) increases more energy are also being consumed by and large. It is hereby proposed that the knowledge of centrality can achieve the aim of energy sustainability and efficiency therefore enabling efficient allocation of energy resources to the right path.

2.10 Weights on Nodes

In the supply chain management (SCM), there could be a need to predict where to cite a proposed Distribution Centre (DC). Ordinarily, many factors would be taken into consideration for such an exercise, for example, factors like population density of the area, accessibility by different mode of transportation, the standard of living etc. In this case , application of centraliy measures would be sufficient to predict the citing of the location of the DC, by using the principles of node-weighted and link-weighted centralities. The node-weights could be any of the volume of sales, cost of storage or turnover at a depot/store, while the edges will be the distance between each depot and a proposed distribution centre (DC). TESCO shops of different counties are used as case studies here. For the SCM, since the shops sampled are maximally connected, the advantage of the clique structure was exploited to map out different clique of shops and thereby making the most central node of the chosen clique to be representative of that clique for the purpose of prediction of a proposed DC.

Furthermore, in green computing, the same principles of node and link-weighted centralities can be used to detect the redundant servers (nodes) in a data centre, and then reduce the resources being allocated to such servers (nodes), thereby reducing pollution and saving costs.

Discussed further below are the linkweighted centralities of the four measures, i.e. Link-Weighted Degree, LinkWeighted Closeness, LinkWeighted Betweenness and the LinkWeighted Eigenvector.

2.11 Node-Weight Modified Centrality Measure

As noted from (2.14), when $\alpha = 0$ only the degree of nodes will be measured and if $\alpha = 1$ only the weights on the ties are measured. In view of this only the cases whereby α is less than 1 or greater than 1 shall be considered, specifically cases of $\alpha = 0.25; 0.5; 0.75; 1.25; 1.5$ and 1.75 A tuning parameter β was introduced by [Akanmu, Wang & Yamoah, 2014], [Barrat et al, 2004] & [Granovetter, 1973] to take care of the weightedness on the nodes, although the tuning parameter α was applied to the degree/strength of the edges. The newly evolved equation by way of introduction of a tuning parameter β will now be the product of degree of a focal node, the average weight to these nodes as adjusted by the newly introduced tuning parameter β and the weight accorded to each node. So, for weighted degree centrality at α and β we shall now have

$$C_d^{w\alpha\beta}(i) = K_i \times (\frac{S_i}{K_i}^{\alpha}) \times (Z_i)^{\beta} = K_i^{1-\alpha} \times S_i^{\alpha} \times (Z_i)^{\beta} \qquad (2.20)$$

where K_i = degree of nodes, $S_i = C_d^w(s)$ as defined in (2.14) above

Z_i = weight of nodes, α is ≥ 0 ; $\{\beta \in \mathrm{Z} : -1 \leq \beta \leq 1\}$

The choice of value of β depends on what effect the weight is having on the new centrality measure, if for instance the weight is having a positive effect (e.g. profit) the positive value of β is employed otherwise the negative value(e.g. loss) shall be used in our calculation.

In his work on identifying cohesive subgroups [Frank, 1995] laid emphasis on the link of a graph that the ”definitions based on path length are restrictive in that they specify the nature of the relationship between each pair of actors within a subgroup instead of a general relationship between each actor and all others in the subgroup”, thereby leaving out the actors/nodesśtrength. According to the definition of the Topological Centrality (TC) of an edge, the weights of edges are the sum of the weights of its two end nodes [Zhuge & Zhang, 2010]. Here, the definitions of the weights of edges and weights of nodes are somehow fuzzy, as it is not clear cut what made up the weights of the end nodes. [Opsahl, Agneessens & Skvoretz, 2010] defined a weighted network as that in which ties are not just either present or absent, but have some form of weights attached to them, hence the emphasis of his paper on the trade-off between the weight on the tie and the number of ties. This was however silent on the attributes of the node (which in most cases form the weights on the nodes). This viewpoint was partly shared by [Abbasi, 2013] when he said ”Second category of measures (i.e., h-Degree, a-Degree and g-Degree) takes into account the linksẃeights of a node in a weighted network. Third category of measures (i.e., Hw-Degree, Aw-Degree and Gw-Degree) combines both neighborsd́egree and their linksẃeight.” [Barrat et al, 2004] [Brandes, 2001] [Newman, 2001] have also attempted to generalize the traditional network centrality measures (degree, betweenness and closeness) to weighted networks, but they were only able to implement their generalisations as the link-weighted network, thus not putting the node-weights into consideration.

Another emphasis on link-weighted-ness in terms of duration is that by [Uddin, Hossain & Wigand, 2013] whereby they introduced a time-variant approach to the degree centrality measure, that is, the time scale degree centrality (TSDC), whereby the presence and duration of links between actors are considered while leaving out the node attributes. In their paper on hybrid centrality measures, [Abbasi & Hossain, 2013] in an attempt to define the new hybrid centrality measure reported having considered a network as having the centrality measures of each node as the attribute of the node. They however go further to state thus "To have generalized measures, considering weighted networks which their links have different strengths, we can extend definitions by considering the weight of the links". [Abbasi, Wigand & Hossain, 2014] in their analysis of results for scholars performance and social capital measures also buttressed this view point by submitting that repeated co-authorships are merged by increasing more weight (tie strength) to their link (tie) for each relation, so also [Walker, Fooshee & Davidson, 2015] whereby they referred to weight of undirected graph as the link-weight. However, all these arguments are again centred on link-weights as against the weights of the network that could have considered a combination of node-weights and link-weights. Recently, [Liu, et al, 2015] in their new method to construct co-authorship concluded by saying "We used the times of co-authorship to calculate the distance between each pair of authors, and evaluate the importance of their cooperation to each other with the law of gravity". The mixed-mean centrality measure of [Akanmu, Wang & Chen, 2012] took into consideration, the number of links, link-weights and node-weights in their study of co-authorship network, while [Akanmu, Wang & Yamoah, 2014] used the clique structure and node-weighted centrality to predict the distribution centre location in a supply chain management, thus clarifying what the link-weights and node-weights are in a weighted network. It is still largely unknown how top eigen-vector weighted centrality can be applied to predict location of structures in a network. Thus, it is important to still find out whether the attributes of the nodes in any network is of importance or not; one might also want to know how accurate the mergers of node-weights and link-weights can be in terms of prediction of where to cite structures (for example, where to cite a distribution centre); and finally how accurate would the prediction of the location for a DC become, given a new centrality measure, which takes into consideration, the clique structure of a network combined with the node-weights and link-weights of the network. The nodes of the clique for each of the cities considered are ranked in line with their eigenvectors, and the representative node (the highest ranking node) for that clique becomes the representative node of that city. The centre of mass for the emergent nodes is thereafter taken into consideration. This method is important in that it only takes the node-weights and link-weights into consideration while trying to achieve the results, thereby saving other resources.

Different meanings have been adduced to the weighted-ness of a network, so many literature have at instances made references to link-weights as the weights of the entire network, even though any network as described above would at least consist of node(s) and link(s) as the case may be. This therefore implies that there has to be node-weights as separated from link-weights and the combination of the two would thereby emerge as weights of any typical network. In his work on identifying cohesive subgroups [Frank, 1995] laid emphasis on the link of a graph that the definitions based on path length are restrictive in that they specify the nature of the relationship between each pair of actors within a subgroup instead of a general relationship between each actor and all others in the subgroup, thereby leaving out the actors/nodesśtrength. According to the definition of the Topological Centrality (TC) of an edge, the weights of edges are the sum of the weights of its two end nodes [Zhuge & Zhang, 2010]. Here, the definitions of the weights of edges and weights of nodes are somehow fuzzy, as it is not clear cut what made up the weights of the end nodes. [Opsahl, Agneessens & Skvoretz, 2010] defined a weighted network as that in which ties are not just either present or absent, but have some form of weight attached to them, hence the emphasis of his paper on the trade-off between the weight on the tie and the number of ties. This was however silent on the attributes of the node (which in most cases form the weights on the nodes). This viewpoint was partly shared by [Abbasi, 2013] when he said that a second category of measures (i.e., h-Degree, a-Degree and g-Degree) takes into account the linksẃeights of a node in a weighted network. Third category of measures (i.e., Hw-Degree, Aw-Degree and Gw-Degree) combines both neighborsd́egree and their linksẃeight. [Barrat et al, 2004] [Brandes, 2001] [Newman, 2001] have also attempted to generalize the traditional network centrality measures (degree, betweenness and closeness) to weighted networks, but they were only able to implement their generalisations as the link-weighted network, thus not putting the node-weights into consideration.

Another emphasis on link-weighted-ness in terms of duration is that by [Uddin, Hossain & Wigand, 2013] whereby they introduced a time-variant approach to the degree centrality measure, that is, the time scale degree centrality (TSDC), whereby the presence and duration of links between actors are considered while leaving out the node attributes. On hybrid centrality measures, [Abbasi & Hossain, 2013] reported having considered a network as having the centrality measures of each node as the attribute of the node, while [Abbasi, Wigand & Hossain, 2014] in their analysis of results for scholars performance and social capital measures also buttressed this view point by submitting that repeated co-authorships are merged by increasing more weight(tie strength) to their link(tie) for each relation, so also [Walker, Fooshee & Davidson, 2015] whereby they referred to weight of undirected graph as the link-weight. However, all these arguments are again centred on link-weights as against the weights of the network that could have considered a combination or mergers of node-weights and link-weights. In their new method of constructing co-authorship, [Liu et al, 2015] used the times of co-authorship to calculate the distance between each pair of authors, and to also evaluate the importance of their cooperation to each other with the law of gravity. This relies again on the use of link weights. The mixed-mean centrality measure of [Akanmu, Wang & Chen, 2012] took into consideration, the number of links, link-weights and node-weights in their study of co-authorship network, while [Akanmu, Yang & Yamoah, 2014] used the clique structure and node-weighted centrality to predict the distribution centre location in a supply chain management, thus clarifying what the link-weights and node-weights actually represent in a weighted network. It is still largely unknown how newtonian gravitational force of attraction and the top eigenvector weighted centrality can be applied to predict location of structures in a network. Thus, it is important to still find out whether the attributes of the nodes in any network is of importance or not; one might also want to know how accurate the mergers of node-weights and link-weights can be in terms of prediction of where to cite structures (for example, where to cite a distribution centre); and finally how accurate would the prediction of the location for a DC become, given a new centrality measure, which takes into consideration, the clique structure of a network combined with the node-weights and link-weights of the network. The nodes of the clique for each of the cities considered are ranked in line with their eigenvectors, and the representative node (the highest ranking node) for that clique becomes the representative node of that city. The centre of mass for the emergent nodes is thereafter taken into consideration. This method is important in that it only takes the node-weights and link-weights into consideration while trying to achieve the results, thereby saving other resources. Section II discusses the link-weighted centrality and node-weighted centrality and the third section discusses methods employed and their implementation, while the fourth lays out the output results from the methodology.

2.12 Weighted Centralities

2.12.1 LinkWeighted Centrality

The equation (2.21) below represents the weighted degree centrality with respect to the edges or links.

$$S_p = C_D^W(p) = \frac{\sum_q^N W_{pq}}{n-1} \tag{2.21}$$

Where C_D^W represents the weighted degree centrality; p is the focal node ; q= adjacent node ; w= weight attached to the edge ; and n= total number of nodes in the graph. This reasoning can be extended to the weighted centrality of the Closeness, Betweenness and the Eigenvector. As an example, the weighted eigenvector centrality could be seen as

$$\lambda x = A^w x \tag{2.22}$$

where A^w is a square matrix of the weights on the edges of A and x is an eigenvector of A .

It is to be recalled that a tuning parameter α was introduced to determine the relative importance of the number of ties compared to the weights on the ties by [Opsahl, Agneessens & Skvoretz, 2010]. Equation (2.23) below represents the product of degree of a focal node and the average weight to these nodes as adjusted by the introduced tuning parameter. So, for weighted degree centrality at α we have:

$$C_d^{w\alpha}(i) = K_i \times (\frac{S_i}{K_i})^\alpha = K_i^{(1-\alpha)} \times S_i^\alpha \tag{2.23}$$

where K_i = degree of nodes, $S_i = C_d^w(s)$ as defined in (2.23) above , and α is ≥ 0

This argument could also equally be applied to the closeness centrality; betweenness centrality and eigenvector centrality.

2.12.2 NodeWeighted Centrality

As an extension to equation (2.23), a tuning parameter β was introduced by [Akanmu, Wang & Chen, 2012] to include the weightedness on the nodes, therefore, for weighted degree centrality at α and β we shall now have

$$C_d^{w\alpha\beta}(i) = K_i \times (\frac{S_i}{K_i})^\alpha \times (Z_i)^\beta = K_i^{(1-\alpha)} \times S_i^\alpha \times (Z_i)^\beta \tag{2.24}$$

where K_i = degree of nodes, $S_i = C_d^w(s)$ as defined in (2.24) above

Z_i = weight of nodes, α is ≥ 0 ; $\{\beta \in Z : -1 \leq \beta \leq 1\}$

The value β depends on whether the weight is having positive or negative effect on the centrality measure, if for instance the weight is having a positive effect (e.g. profit) β is +1 else it is -1 (i.e. loss). Values of α ranging from$\frac{1}{4}$ to $1\frac{3}{4}$ is used in order to vary the effect of α, i.e. values less than 1 and those greater than 1.

Mixed-Mean centrality as a new measure of the importance of a node in a graph is introduced, based on the generalized degree centrality. The mixed-mean centrality reflects not only the strengths (weights) and numbers of edges for degree centrality but it combines these features by also applying the closeness centrality measures while it goes further to include the weights of the nodes in the consideration for centrality measures. We illustrate the benefits of this new measure by applying it to cloud computing, which is typically a complex system. Network structure analysis is important in characterizing such complex systems.
Application of graph-theoretic approach to a system or problem can provide a different point of view and make the problems at hand become clearer and much simple, as it provides the appropriate tools (pictorial and quantitative) for solving complications. The basis of centrality measures lies in the formal theory of social network mechanism, and this is largely applied in this research that dwells on the mergers of centrality measures in the study of weightsćombination to evaluate network topologies. In this study, this idea is applied to solve problems of resource allocation via siting of a resource center to augment the existing ones when the need arises. The strengths attached to vertices (weights) differentiate each vertex from another, similar to the strength attached to edges. The evaluation of these weights and the number of edges together contribute effectively to the measure of centrality for an entire graph or network. This novel approach has huge potential for resource optimization in the location of structures in a distributed system of supply chain management. With the use of this method in the prediction of new site for the location of distribution center an accuracy of 91.6 percent was obtained and the more effective measures of centralities were determined.

Much importance is attached to the weights on the edges in a network, but in actual fact what makes up a network is both the nodes and the edges linking up the network. It is therefore pertinent to investigate the effects and importance of the weights attributed unto the nodes in a network as well as the weights on the links of such networks as they both play important roles in determining the prominence or popularity of actors within any particular network. Principles of centrality measures were employed in the supply chain management to show that the weighted-ness of the edges/nodes together with the clique structure that emanates from it can be a pointer to centrality or otherwise of members of a group in the network of a distribution system. As expected, it was affirmed that the nodes belonging to the high clique members have a high percentage of being chosen/predicted as the most likely distribution centre. We examined the cliques of the weighted centrality matrix for the distributed system of a supply chain management network, and from the outcome we are able to predict a location of a new distribution centre in and around a particular area/region with an accuracy of more than 66%. In addition, the distinction between the notion of link-weightedness and node-weightedness were clarified.

Despite the importance attached to the weights or strengths on the edges of a graph, a

graph is only complete if it has both the combinations of nodes and edges. As such, this research brings to bare the fact that the node-weight of a graph is also a critical factor to consider in any graph/networkś evaluation, rather than the link-weight alone as commonly considered. In fact, the combination of the weights on both the nodes and edges as well as the number of ties together contribute effectively to the measure of centrality for an entire graph or network, thereby clearly showing more information. Two methods which take into consideration both the link-weights and node-weights of graphs (the Weighted Marking method of prediction of location and the Clique/Node-Weighted centrality measures) are considered, and the result from the case studies shows that the clique/node-weighted centrality measures gives a more accuracy of 18% than the weighted marking method in the prediction of Distribution Centre location of the Supply Chain Management.
Occasions do arise when researchers and industrialists alike are faced with the decision of where to cite new structures (shops, stores, distribution centers etc) in order to benefit the consumers and the business entity as well. Such decisions might take the importance of vertices and/or edges of a network (e.g. Supply Chain Network) into consideration. In particular, the strength of the vertices and those of the edges play an important role in arriving at such decisions. In this thesis, as against the most common and traditional measures of centralities, that is - Degree, Closeness, Betweenness and Eigen-Vector centralities, a new centrality measure, Top Eigen-Vector Weighted Centrality (TEVWC) which takes into consideration the clique structure of a network and the strengths attached to the vertices/edges of the network, was used to predict the location of a distribution center in a supply chain management. The accuracy of prediction on a sample dataset of supply chain network, using the TEVWC was found to be 92.9%, which is 11.4% higher than the result outcome from the method of min-cut algorithm.

Chapter 3

Mixed-Mean Centrality

3.1 Introduction

Introduced hereby is a second similar graph to the one in figure 2.2 differing only in the weights attached, it is believed that weights of each tie can actually be dependent on two scenarios (in this case, visit by the focal node and number of messages sent by the nodes through the network). The weights in figure 2.2 are the number of visitation by the focal nodes to their neighbors while the weights in figure 3.1 below represents the number of messages sent by the focal node, this was applied to a subset of EIES (Electronic Information Exchange System) dataset (Freeman and Freeman, 1979). This is a dataset which, describes the communication between authors by way of exchange of mails, each author is depicted as a node, and the link represents the number of messages exchanged between two nodes.

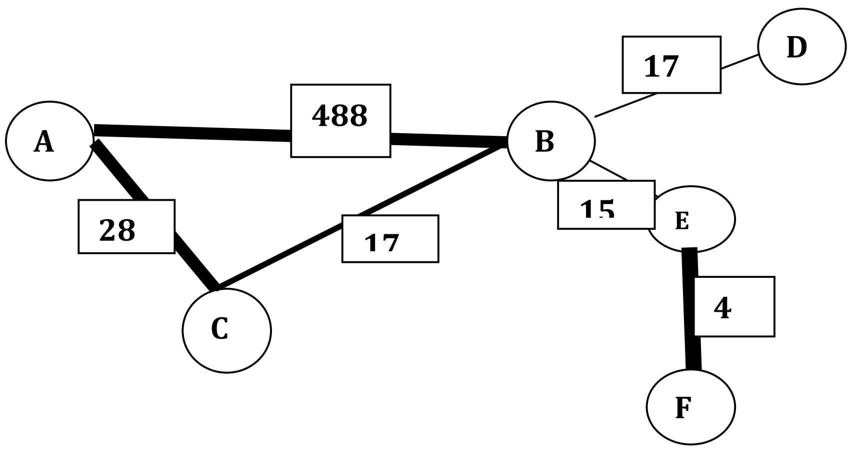

Figure 3.1: Represents a six nodes network, whereby circle represents nodes and square represents weights of edge (Number of Messages)

Figure 3.1 represents a six nodes network, whereby circle represents nodes and square represents weights of edge (Number of Messages), and it is particularly expressed in Table 3.1.

Figure 3.1 shows different ranking values for different graphs/or networks, for instance, in Table 3.1, the first graph (after the application of the tuning parameter to weighted degree

Table 3.1: Table showing the effect of the tuning parameters on the two graphs of discuss

		Data for first Graph									Data for second Graph					
No-des	**No of Deg-rees**	**Deg-ree Centr-ality** (C_D)	**No of Geodesic Paths from s to t** ($\sum_d(s,t)$)	**Close-ness Centr-ality**	**CDWI** $\sum_W(s,t)$	C_D^w (S_i)	α **=0**	α= $\frac{1}{2}$	α **=1**	α **=1**$\frac{1}{2}$	**CDW2** ($\sum_d$ (Ws,t))	C_D^W (S_i)	α **=0**	α =$\frac{1}{2}$	α **=1**	α **=1**$\frac{1}{2}$
A	2	0.4	9	0.556	8	1.6	2	1.79	1.6	1.431	516	103.2	2	14.4	103	741.3
B	4	0.8	6	0.833	8	1.6	4	2.53	1.6	1.012	537	107.4	4	20.7	107	556.5
C	2	0.4	9	0.556	6	1.2	2	1.55	1.2	0.93	45	9	2	4.24	9	19.09
D	1	0.2	9	0.556	1	0.2	1	0.45	0.2	0.089	17	3.4	1	1.84	3.4	6.269
E	2	0.4	8	0.625	8	1.6	2	1.79	1.6	1.431	19	3.8	2	2.76	38	5.238
F	1	0.2	12	0.417	7	1.4	1	1.18	1.4	1.657	4	0.8	1	0.89	0.8	0.716

centrality measure) made node F to have the highest centrality when α=1$\frac{1}{2}$, while the same application to the second graph made node A to have the highest centrality. When α <1 , node B has the highest centrality in both cases.

Our intention is to combine the results for the two graphs so as to have a fairly well-representative result and not only by using weighted degree centrality but also including weighted closeness centrality.

When the weighted closeness centrality is considered for α >1 , node D has the highest centrality in graph1 while node F has the highest in graph2 , but with α <1 , node B retains the highest centrality position in graph 1 and in graph 2. (see table 3.2).

In Table 3.3 , considering the average (i.e. mean) of the weighted degree centralities for the two graphs; when α=1$\frac{1}{2}$ returns node A as the most central while with tuning parameter α=$\frac{1}{2}$, node B is returned as the most central. Whereas the weighted closeness centrality measure when α=1$\frac{1}{2}$ and α=$\frac{1}{2}$ returns nodes D and node B respectively in both cases as having the highest centrality. From the results above when α=$\frac{1}{2}$, node B has higher marginal value of cardinality than any other nodes in both graphs.

Table 3.4 below presents the results of the Mixed-Mean centralities of the two graphs, that is the summation of the mean of the Weighted Degree and Weighted Closeness centralities. That is,

$$Mixed - MeanCentralities = (CDW1 + CDW2 + CCW1 + CCW2)/2 \tag{3.1}$$

From the result of Table 3.4 above, when α=$\frac{1}{2}$ the most central node is B while node A becomes the most central when α=1$\frac{1}{2}$

As shown earlier, in the trivial case when no weights are attached to the nodes (as in figure 1.1 - Graph 1, Table 1.1) , node B is the most central of all nodes as regards degree centrality and closeness centrality. The table 3.5 below shows the summary result of the activity carried out so far:

In the trivial case of Graph 1, and all other cases whereby the tuning parameter α=$\frac{1}{2}$, node B is always the most central. However, the centrality varies when the tuning parameter α=1$\frac{1}{2}$, in fact node B was never the most central in this case, the centrality varies between node D and node F, while for our mixed-mean centrality node A becomes the most central in this case of α=1$\frac{1}{2}$

Application to Data Set:

On applying the same to a subset of the Freeman EIES (Electronic Information Exchange System) dataset as presented by [Opsahl et al, 2010], the results in Table 3.6 were generated.

The ranking positions of the mixed-mean weighted centrality in Table 3.6 shows the ranking according to the mean of the weightedness of closeness and degree centrality measures at different level of α , thus, it can be inferred that Gary Coombs ranked highest in terms of Mixed-Mean centralities for both α=1$\frac{1}{2}$ and α=$\frac{1}{2}$.

However, in terms of the mean of the closeness centralities for both tuning parameters α=1$\frac{1}{2}$ and α=$\frac{1}{2}$, it ranked lowest, thereby indicating that the mixture of the centralities for the two graphs could and actually make a less important or influential node to become the most important and influential when considered on the basis of mixed-mean centrality.

Table 3.2: The weighted closeness centrality results from the two graphs

Nodes	**Number of Degrees** K_i	**CCWI** $\sum_W (s,t)$	C_c^w (S_i)	α **=0**	α $=\frac{1}{2}$	α **=1**	α $=\mathbf{1}\frac{1}{2}$	**CCW2** $(\sum_d$ (Wv,t)	C_c^W (S_i)	α **=0**	α $=\frac{1}{2}$	α **=1**	α $=\mathbf{1}\frac{1}{2}$
A	2	30	0.17	2	0.58	0.17	0.05	598	0.008	2	0.13	0.008	0.0005
B	4	15	0.33	4	1.15	0.33	0.1	87	0.057	4	0.48	0.057	0.0069
C	2	18	0.28	2	0.75	0.28	0.1	121	0.041	2	0.29	0.041	0.0059
D	1	20	0.25	1	0.5	0.25	0.13	155	0.032	1	0.18	0.032	0.0058
E	2	18	0.28	2	0.75	0.28	0.1	117	0.043	2	0.29	0.043	0.0062
F	1	46	0.11	1	0.33	0.11	0.04	123	0.041	1	0.2	0.041	0.0082

Table 3.3: The two weighted centralities measure and their averages for the two graphs. (Mean Weighted Centralities for Degree and Closeness Centralities)

	(CDW1 + CDW2)/2				**(CCW1 + CCW2)/2**			
Nodes	α $=\mathbf{0}$	α $=\frac{1}{2}$	α $=\mathbf{1}$	α $=\mathbf{1}\frac{1}{2}$	α $=\mathbf{0}$	α $=\frac{1}{2}$	α $=\mathbf{1}$	α $=\mathbf{1}\frac{1}{2}$
A	2	8.08	52.4	371	2	0.353	0.0875	0.024
B	4	11.6	54.5	279	4	0.817	0.1954	0.052
C	2	2.9	5.1	10	2	0.516	0.159	0.055
D	1	1.15	1.8	3.18	1	0.34	0.141	0.065
E	2	2.27	2.7	3.33	2	0.519	0.160	0.055
F	1	1.04	1.1	1.19	1	0.266	0.075	0.022

Table 3.4: Mixed-Mean Centralities of the two Graphs

(CDW1 + CDW2)/2 + (CCW1 + CCW2)/2				
Nodes	α $=\mathbf{0}$	α $=\frac{1}{2}$	α $=\mathbf{1}$	α $=\mathbf{1}\frac{1}{2}$
A	2	4.22	26.24	185.70
B	4	6.22	27.35	139.40
C	2	1.71	2.63	5.03
D	1	0.74	0.97	1.62
E	2	1.40	1.43	1.69
F	1	0.65	0.59	0.60

Table 3.5: A summary of the graphs considered and the most central node in each case

Graphs & Tables	Degree Centrality		Closeness Centrality	
	$\alpha = \frac{1}{2}$	$\alpha = 1\frac{1}{2}$	$\alpha = \frac{1}{2}$	$\alpha = 1\frac{1}{2}$
Graph1 with weights on the edges (Table 1.1)	B	F	B	D
Graph2 with different weights on the edges (Table1.2 & Table3.1)	B	A	B	F
MeanWeighted Centralities for Graph1 & Graph2 (Table 3.2)	B	A	B	D

Table 3.6: Mixed-Mean Weighted centrality applied to EIES Dataset

	Number of Degrees		**(CCWI+CCW2)**				**(CDW1+CDW2)**				**(MIXED-MEAN CENTRALITY)**		
Nodes	K_i	α **=0**	α $=\frac{1}{2}$	α **=1**	α $=1\frac{1}{2}$	α **=0**	α $=\frac{1}{2}$	α **=1**	α $=1\frac{1}{2}$	α **=0**	α $=\frac{1}{2}$	α **=1**	α $=1\frac{1}{2}$
JOEL LEVINE	27	27	2.64	0.27	0.03	27	13	6.3	3.04	27	7.84	3.29	1.54
JOHN SONQUIST	28	15	0.33	4	1.15	0.33	0.1	87	0.057	4	0.48	0.057	0.0069
BRIAN FOSTER	29	18	0.28	2	0.75	0.28	0.1	121	0.041	2	0.29	0.041	0.0059
EV ROGERS	30	20	0.25	1	0.5	0.25	0.13	155	0.032	1	0.18	0.032	0.0058
GARY COOMBS	31	18	0.28	2	0.75	0.28	0.1	117	0.043	2	0.29	0.043	0.0062
ED LAUMAN	32	46	0.11	1	0.33	0.11	0.04	123	0.041	1	0.2	0.041	0.0082

3.2 Mixed-Mean Centrality with Nodes' Weights

Consideration has been given to weightedness of edges before now, but the nodes can and do have weights, so the principle discussed above is hereby extended to include the node weights for the two graphs of discourse. This means, consideration will now be given not only to the number of ties and tie weights but also to the weights of the nodes.

Introducing weights to the nodes of figure 3.1 above , we have the new figure 3.2 shown below:

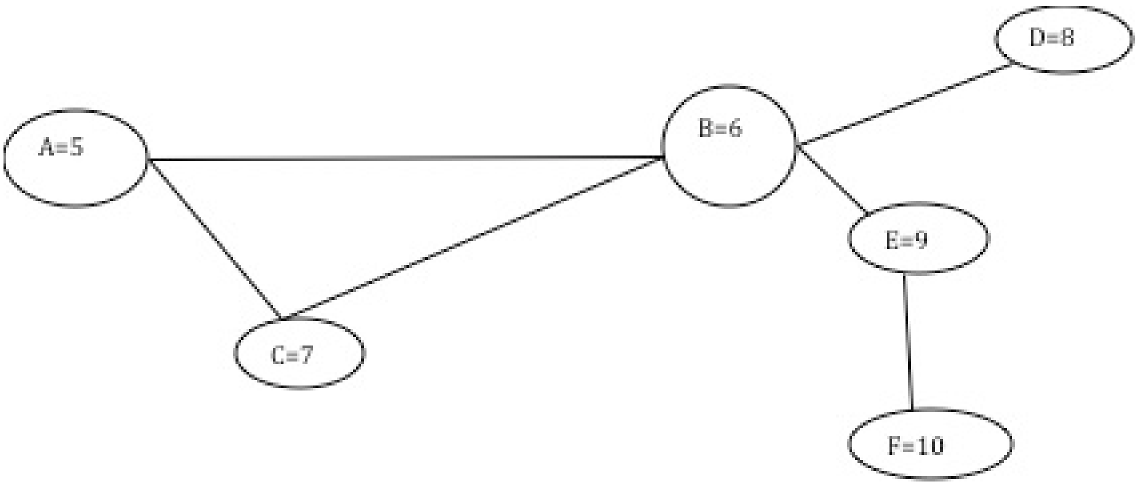

Figure 3.2: A six nodes network, circle represents nodes and their respective weights using the EIES sample data set.

Where A(Joel Levine)= 5; B(Jon Sonquist)=6; C(Brian Foster)= 7; D(Ev Rogers)= 8; E(Gary Coombs) = 9; F(Ed Lauman) = 10 are arbitrary weights assigned to the nodes of the sample EIES data set, these could be the number of resources used up by each of the nodes.

The tuning parameter β was now introduced to take care of the weightedness of the nodes, and degree/strength of the edges, this being the product of degree of a focal node, the average weight to these nodes as adjusted by the introduced tuning parameter and the weight accorded to each node. So, for weighted degree centrality at α and β we shall now have

$$C_d^{w\alpha\beta}(i) = K_i \times (\frac{S_i}{K_i}^{\alpha}) \times (Z_i)^{\beta} = K_i^{1-\alpha} \times S_i^{\alpha} \times (Z_i)^{\beta} \quad (3.2)$$

where K_i = degree of nodes, $S_i = C_d^w(s)$ as defined in (1.6) above

Z_i = weight of nodes, α is ≥ 0 ; $\{\beta \in Z : -1 \leq \beta \leq 1\}$

Applying the new model above to the EIES data set of our discuss will generate the following tables for the new weighted degree centrality (with node weights):

Table 3.7 shows that when weights are attached to the nodes of the EIES data set, Gary Coombs ranked first while Ed Lauman ranked second when $\alpha=\frac{1}{2}$ and $\alpha=1\frac{1}{2}$, whereas in Table 3.8 whereby the nodes were not accorded weights, Gary Coombs though ranked first but Ed Lauman ranked last, even when the $\alpha=1\frac{1}{2}$.

Table 3.7: Table showing the effect of the tuning parameters on the EIES data sets using the Mixed-Mean Centrality after attaching weights to the nodes, β in this case is 1.

	Number of Degrees	**MIXED-MEAN CENTRALITY WITH NODE WEIGHTS**			
Nodes	K_i	α **=0**	$\alpha = \frac{1}{2}$	α **=1**	α **=1** $\frac{1}{2}$
JOEL LEVINE	27	135	43.4	16.4	7.68
JOHN SONQUIST	28	168	45.4	14.3	5.11
BRIAN FOSTER	29	203	47.1	16.5	6.61
EV ROGERS	30	240	57	18.7	7.22
GARY COOMBS	31	279	84.6	43.3	24.9
ED LAUMAN	32	320	67	20.9	7.83

However the above ranking is only for the case whereby the value of β = 1, it is quite of interest to us to see what would happen if β = -1 . Table 3.8 illustrates the result when β = -1.

Table 3.8 also shows that when weights are attached to the nodes of the EIES data set at β = -1, Gary Coombs and Joel Levine together ranked first and second respectively when α=$1\frac{1}{2}$ but when α=$\frac{1}{2}$ Joel Levine ranked first and Jon Sonquist ranked second.

Finally, the rankings for the different situations of beta after weights z have been attached to the nodes of Table 3.8 gives the following result:

From Table 3.9, it can be seen clearly that as β changes value from 0 to 1 and -1 Gary Coombs (though not the one with highest degree or highest weight) maintains the highest ranking when α=$1\frac{1}{2}$. When α=$\frac{1}{2}$, Gary Coombs maintained the lead at β = 1 only to drop to the third position when β=-1.

Having applied the new model above to the EIES data set, one sets out to try this model on an existing electricity consumption data extract of six states in the United States of America as shown below in Table 3.10. Here, the number of consumers in a particular state are stated, with the average monthly bill consumption, any of these parameters can be designated as the node-weights for each of the states , while Table 3.11 showed the distances within the states, these distances can be designated as the link-weights, then the weighted centralities (node-weighted, link-weighted and Mixed-Mean centralities can then be determined, and as such we can deduce the most central of the states and then determine where to locate a central resource such as data centre that can serve the six states optimally.

The geodesic distance in kilometres between the six states is also extracted and tabulated below.

In the above scenario, we want to assume the power plant serving the six states is located in Connecticut, and we then apply the equation of Mixed-Mean Centralities in (3.1) , that is

Mixed-Mean Centralities = (CDW1 + CDW2 + CCW1 + CCW2) / 2

where

CDW1 = Average monthly consumption for graph1

CDW2 = Average retail price for graph 2

CCW1 = Geodesic distance from one state to any other

CCW2 = Geodesic distance from electricity company to each state.

The following results were obtained at different values of beta, i.e. β=-1, 0 and 1.

From Table 3.12, it can be seen that Vermont even though with least number of consumers ranked highest when α=$1\frac{1}{2}$ and it ranked lowest when α=$\frac{1}{2}$.

In the table 3.13, at α=$1\frac{1}{2}$ Vermont still ranked highest as in Table 3.12 while Massachussets ranked highest when α=$\frac{1}{2}$ and it actually has the highest number of consumers.

Table 3.14 indicates that Vermont retains its highest ranking position when α=$1\frac{1}{2}$ but remained lowest when α=$\frac{1}{2}$ while Massachussets retains the highest centrality with α=$\frac{1}{2}$. To make it all clearer, Table 3.15 summarises the effect of beta on the measure of centralities:

3.3 Power Usage Effectiveness (PUE)

It is generally known that even when machines are idle they still consume reasonable energy in that process, but the lower the PUE of a data centre the better and most energy-efficient

Table 3.8: Table showing the effect of the tuning parameters on the EIES data sets using the Mixed-Mean Centrality after attaching weights to the nodes, β in this case is -1.

	Number of Degrees	**MIXED-MEAN CENTRALITY WITH NODE WEIGHTS**			
Nodes	K_i	α $=\mathbf{0}$	α $=\frac{1}{2}$	α $=\mathbf{1}$	α $=\mathbf{1}\frac{1}{2}$
JOEL LEVINE	27	5.4	1.74	0.66	0.3072
JOHN SONQUIST	28	4.67	1.26	0.4	0.1419
BRIAN FOSTER	29	24.14	0.96	0.34	0.135
EV ROGERS	30	3.75	0.89	0.29	0.1128
GARY COOMBS	31	3.44	1.04	0.53	0.3077
ED LAUMAN	32	3.2	0.67	0.21	0.0783

Table 3.9: Table showing different rankings of nodes after weight z have been attached to the nodes of Table 8 with varying values of β

			$\beta = 0$				$(\beta = 1$				$\beta = -1$			
Nodes	**z**	**Deg**	α **=0**	α $=\frac{1}{2}$	α **=1**	α $=1\frac{1}{2}$	α **=0**	α $=\frac{1}{2}$	α **=1**	α $=1\frac{1}{2}$	α **=0**	α $=\frac{1}{2}$	α **=1**	α $=1\frac{1}{2}$
JOEL LEVINE	5	27	27	7.84	3.29	1.54	135	43.38	16.43	7.68	5.40	1.74	0.66	0.3072
JON SON-QUIST	6	28	28	7.57	2.38	0.85	168	45.43	14.30	5.11	4.67	1.26	0.40	0.1419
BRIAN FOS-TER	7	29	29	6.73	2.35	0.94	203	47.08	16.47	6.61	4.14	0.96	0.34	0.1350
EV ROGERS	8	30	30	7.13	2.34	0.90	240	57.00	18.70	7.22	3.75	0.89	0.29	0.1128
GARY COOMBS	9	31	31	9.40	4.81	2.77	279	84.62	43.32	24.92	3.44	1.04	0.53	0.3077
ED LAUMAN	10	32	32	6.70	2.09	0.78	320	66.96	20.90	7.83	3.20	0.67	0.21	0.0782

Table 3.10: Industrial average monthly bill by Census Division, and State 2011 : Source of extract - Table T5.c on http://www.eia.gov/electricity/sales_revenue_price/

Census Divisions	**Number of Consumers**	**Average Monthly Consumption**	**Average Retail Price**	**Average Monthly Bill**
States		**kWh**	**Cents Per KilowattHour**	**Dollar and Cents**
Connecticut	4757	64260	13.24	$8508.16
Maine	2823	89023	8.88	$7908.07
Massachusetts	21021	67288	13.38	$9000.45
New Hampshire	3491	46224	12.27	$5669.84
Rhode Island	1958	38979	11.27	$4394.53
Vermont	221	534353	9.83	$52503.39

Table 3.11: Geodesic Dist in kilometres: Source of extract: http://www.timeanddate.com/worldclock/distance.html

STATE	**CONNEC TICUT**	**MAINE**	**MASSAC HUSSETS**	**NEW HAMPSHIRE**	**RHODE ISLAND**	**VER MONT**
CONNECTICUT	0	370	150	187	105	278
MAINE	370	0	241	188	309	224
MASSACHUSSETS	50	241	0	103	67	245
NEW HAMPSHIRE	187	188	103	0	156	143
RHODE ISLAND	105	309	67	156	0	288
VERMONT	278	224	245	143	288	0

Table 3.12: Mixed-Mean Centrality With Node Weights At $\beta = 0$

Nodes	**Number of Degrees No. of Consumers**	**MIXED-MEAN CENTRALITY WITH NODE WEIGHTS WHEN** $\beta = 0$			
Cities	K_i	α **=0**	$\alpha = \frac{1}{2}$	α **=1**	$\alpha = 1\frac{1}{2}$
CONNECTICUT	4757	4757	18426.12	3214.92	5281.22
MAINE	2823	2823	9291.21	4451.60	11178.50
MASSACHUSSETS	21021	21021	156658.51	3365.10	2691.94
NEW HAMPSHIRE	3491	3491	11759.03	2311.83	3761.09
RHODE ISLAND	1958	1958	5327.76	1949.54	3888.90
VERMONT	221	221	1384.95	26718.15	587531.66

Table 3.13: Mixed-Mean Centrality With Node Weights At β= 1

Nodes	**Number of Degrees No. of Consumers**	**MIXED-MEAN CENTRALITY WITH NODE WEIGHTS WHEN** $\beta = 1$			
Cities	K_i	α **=0**	α $=\frac{1}{2}$	α **=1**	α **=1**$\frac{1}{2}$
CONNECTICUT	4757	40473317.12	12891485645	27353086.97	44933451.21
MAINE	2823	22324481.61	5288192178	35203596.26	88400320.60
MASSACHUSSETS	21021	189198459.5	1.30159E+11	30287378.2	24228704.59
NEW HAMPSHIRE	3491	19793411.44	4411220752	13107719.36	21324755.16
RHODE ISLAND	1958	8604489.74	1266371035	8567329.864	17089890.75
VERMONT	221	11603249.19	2040328623	1402793698	30847404131

Table 3.14: Mixed-Mean Centrality With Node Weights At β= -1

Nodes	**Number of Degrees No. of Consumers**	**MIXED-MEAN CENTRALITY WITH NODE WEIGHTS WHEN** $\beta = 1$			
Cities	K_i	α **=0**	α $=\frac{1}{2}$	α **=1**	α **=1**$\frac{1}{2}$
CONNECTICUT	4757	0.56	0.258	0.378	0.621
MAINE	2823	0.36	0.237	0.563	1.414
MASSACHUSSETS	21021	2.34	0.653	0.374	0.299
NEW HAMPSHIRE	3491	0.62	0.279	0.408	0.663
RHODE ISLAND	1958	0.45	0.241	0.444	0.885
VERMONT	221	0.004	0.023	0.509	11.190

Table 3.15: Summarised Centralities with Different Values of β

	$\beta = -1$				$\beta = 0$ **Values in** 10^3				$\beta = 1$ **Values in** 10^7			
Rank	**Node**	$\alpha = \frac{1}{2}$	**Node**	$\alpha = \mathbf{1}\frac{1}{2}$	**Node**	$\alpha = \frac{1}{2}$	**Node**	$\alpha = \mathbf{1}\frac{1}{2}$	**Node**	$\alpha = \frac{1}{2}$	**Node**	$\alpha = \mathbf{1}\frac{1}{2}$
1	MASSACH	0.653	VERMONT	11.190	MASSACH	156.66	VERMONT	587.53	MASSACH	13000	VERMONT	3084.74
2	NEW HAMP-SHIRE	0.279	MAINE	1.414	CONNEC	18.43	MAINE	11.18	CONNEC	1289.15	MAINE	8.84
3	CONNEC	0.258	RHODE ISLAND	0.885	NEW HAMP-SHIRE	11.76	CONNEC	5.28	MAINE	528.82	CONNEC	4.49
4	RHODE ISLAND	0.241	NEW HAMP-SHIRE	0.663	MAINE	9.29	RHODE ISLAND	3.89	NEW HAMP-SHIRE	441.12	MASSACH	2.42
5	MAINE	0.237	CONNEC	0.621	RHODE ISLAND	5.33	NEW HAMP-SHIRE	3.76	VERMONT	204.03	NEW HAMP-SHIRE	2.13
6	VERMONT	0.023	MASSACH	0.299	VERMONT	1.38	MASSACH	2.69	RHODE ISLAND	126.64	RHODE ISLAND	1.71

conscious data centres will always aim to reduce the PUE to the barest minimum, that is conceivably as close as possible to the unit value. As such, one can say that the PUE has a reasonable impact on the energy-efficiency of a data centre.

[Google, 2012] says that its best site could boast a PUE as low as 1.06 if it uses an interpretation commonly used in the industry. However, it stucks to a higher standard because it's better to measure and optimize everything on its site, not just part of it. Thereby reporting a comprehensive PUE of 1.13 across all its data centers, in all seasons, including all sources of overhead.

An extract of the quarterly PUE from Googleś fourteen (14) data centres spanning the period between 2008 to 2012 is shown in the Table 3.16 below:

Avela,V(2011) defined that PUE is the ratio of two numbers, data center input power over IT load power, even though it appears first to be a problem of simply obtaining two measurements and taking their ratio, it is not that simple in production data centers, but Google (2012) postulates the equation below as being a measure of PUE:

$$PUE = \frac{ESIS + EITS + ETX + ELV + EF}{EITS - ECRAC - EUPS - ELV + ENet1} \tag{3.3}$$

ESIS: Energy consumption for supporting infrastructure power substations feeding the cooling plant, lighting, office space, and some network equipment
EITS: Energy consumption for IT power substations feeding servers, network, storage, and computer room air conditioners (CRACs)
ETX: Medium and high voltage transformer losses
EHV: High voltage cable losses
ELV: Low voltage cable losses
EF: Energy consumption from onsite fuels including natural gas & fuel oils
ECRAC: CRAC energy consumption
EUPS: Energy loss at uninterruptible power supplies (UPSes) which feed servers, network, and storage equipment
ENet1: Network room energy fed from type 1 unit substitution

Non-availability of data from data centres was an hindrance to experiments being performed as regards PUE unlike in the SCM and EIES.

3.4 Traffic Density Ratio

We introduce a Traffic Density Ratio (TDR) for each of the nodes and it can be described as:

$$TDR = \frac{Traffic\ Density\ of\ a\ node\ i}{Sum\ Total\ of\ Traffic\ Density\ of\ all\ the\ nodes\ in\ the\ graph}$$

$$TDR_i = \frac{TD_i}{\sum_i^n TD_i} \tag{3.4}$$

where TD_i is the traffic density at node i , and n= total number of nodes in the graph.

The traffic density at each node i is measured in MB/s (megabytes per second), and the lower the TDR of a node, the lower the traffic on that particular node, but the TDR is directly proportional to the PUE, that is,

Table 3.16: Google's PUE: Source of extract: http://www.google.com/about/datacenters/efficiency/internal/index.html#measuring-efficiency

Year/Quarter	**2008**	**2009**	**2010**	**2011**	**2012**
Q1	N/A	1.15	1.15	1.13	1.11
Q2	N/A	1.20	1.17	1.14	1.12
Q3	1.23	1.22	1.20	1.16	N/A
Q4	1.16	1.16	1.13	1.12	N/A
Average/annum	1.20	1.18	1.16	1.14	1.12

$$PUE \propto TDR \Rightarrow PUE = kTDR_i \tag{3.5}$$

whereby $0 \leq TDR_i \leq 1$

Thus, as TDR of any node approaches zero, it is reasonable to believe that the PUE will also tend to zero, and as such all the traffic on such a node could be diverted towards a more performing and capable node, while the node whose PUE is almost at zero could be shut down, thereby saving much energy.

3.5 Summary

Two graphs were considered and effects of the combined weights on both edges and nodes were evaluated taking the closeness centrality and degree centrality into cognisance. The resulting Mixed-Mean centrality was then applied to the EIES data set while introducing two tuning parameters α and β. This was later applied to real data of electricity consumption.

3.6 Conclusion

The scenario presented here can be applied to cloud computing by using the idea of mixed-mean centrality to discover the most central and therefore the most energy consuming nodes, so as to help in making provision for energy-efficiency, thus minimising costs and saving the environment.

It can be used in locating the performance level of a particular node or edge and thus aiding in decision on which node or edge deserves attention. This can be most especially useful for security and fault-tolerance purposes.

Resource allocation is also an applicable area of this centrality measure as it will aid in optimisation of resources, thereby saving costs.

Chapter 4

Using Weighted Centrality Measures to Predict Location of Structures

4.1 The Need for Introducing Weights on Nodes

So far, consideration has been given only to the weightedness on edges (i.e. the strength on edges) before now, but the weights on the nodes have not been considered whereas nodes can and do also have weights on them, so the principles discussed above shall henceforth be extended to include the weights on the nodes, also referred to as the strengths on the nodes. This means, consideration will now be given not only to the number of edges and strengths on edges but also to the weights on the nodes, however the weights/strength on the nodes can represent different thing for different scenario, for example, in a transport system, weights on nodes might be the daily number of passengers accommodated in a particular transport companyś bus station while the weight on edges might be distances of the companyś bus stations apart. The following reasons are adduced for the study of weights on nodes:-

4.1.1 Clarity of Definitions

Often times, most literature mention node strength while actually making reference to the weights attached to the ties/edges. So, it is pertinent to distinguish between the node strength (herein referred to as the weights of or on the nodes/vertices/actors) and the tie strength (herein referred to as the weights attached to the tie/edge/link/line).

4.1.2 Regaining Loss of Valuess

Sometimes in sociometrics, the exact values (strengths or weights) attached to the nodes are usually left out of various considerations as if they are immaterial, whereas when given the deserved considerations they do affect how centralities are perceived. By not considering the weights/strength of nodes, valuable information are actually lost for any meaningful analysis.

4.1.3 Robust and More Representative Analysis

Networks or graphs are not and cannot be formed by edges alone, they are usually formed by combinations of edges and vertices, as such, an all-encompassing sociometric analysis will lead to a largely representative and more robust analytic outcome.

4.1.4 Specificity

Specificity Apart from studying the interactions or linkages of the nodes, the behaviour or nature of the nodes itself is worth studying (e.g. how a computer works, how sales in a depot/store is affected by the type of links it has, how an actor reacts to changes etc).

The concern here shall be that of a supply chain system whereby, the nodes could be anyone of the number of sales, cost of storage or turnover at a depot/store, while the edges will be the distance between each depot and a proposed distribution centre and the degree will be the number of other depot/store(s) interacted with in terms of supply by the source depot or store.

4.2 Node-Weight Modulated Centrality Measure

It will be recalled that it was earlier mentioned that the tuning parameter α was introduced to determine the relative importance of the number of the ties compared to the weights on the ties. [Akanmu, Wang & Huankai, 2012] introduced a tuning parameter β to take care of the weightedness on the nodes, although the tuning parameter α was applied to the degree/strength of the edges as denoted earlier in equations (2.9) and (2.14) above. The newly evolved equation by way of introduction of a tuning parameter β will now be the product of degree of a focal node, the average weight to these nodes as adjusted by the newly introduced tuning parameter β and the weight accorded to each node. So, for weighted degree centrality at α and β we shall now have (2.14) which shall be recalled here as (4.1)

$$C_d^{w\alpha\beta}(i) = K_i \times (\frac{S_i}{K_i})^{\alpha} \times (Z_i)^{\beta} = K_i^{(1-\alpha)} \times S_i^{\alpha} \times (Z_i)^{\beta} \tag{4.1}$$

where K_i = degree of nodes, $S_i = C_d^w(s)$ as defined in (4.1) above

Z_i = weight of nodes, α is ≥ 0 ; $\{\beta \in Z : -1 \leq \beta \leq 1\}$

The choice of value of β depends on what effect the weight is having on the new centrality measure, if for instance the weight is having a positive effect (e.g. profit) the positive value of β is employed otherwise the negative value(e.g. loss) shall be used in our calculation.

4.3 Evaluation: Node-Weight Modulated Centrality Measures Applied to Supply Chain Management

In this network, an existing distribution centre of an existing chain stores in a particular region were investigated and the store outlets are considered as nodes with the value of sales being the weights on the nodes while distances between nodes are regarded as the weights on the edges. The Table 4.1 and Table 4.2 below show the respective results that were obtained when the Link-Weight Modulated Centrality and Node-Weight Modulated Centrality Measures are applied to a subset of 6 nodes from the supply chain management dataset.

4.3.1 Implementation

The results were obtained using the software UCINET [Borgatti, Everett & Freeman, 2002] and tnet [Opsahl, 2012] to generate the four weighted centrality results of Degree, Eigenvector, Betweenness and Closeness, while Excel spreadsheet was used to carry out the final calculations of the Node-Weight Modulated Centrality measures of each node/vertex. The schematic diagram of how the process was implemented is as shown in figure 4.1 below.

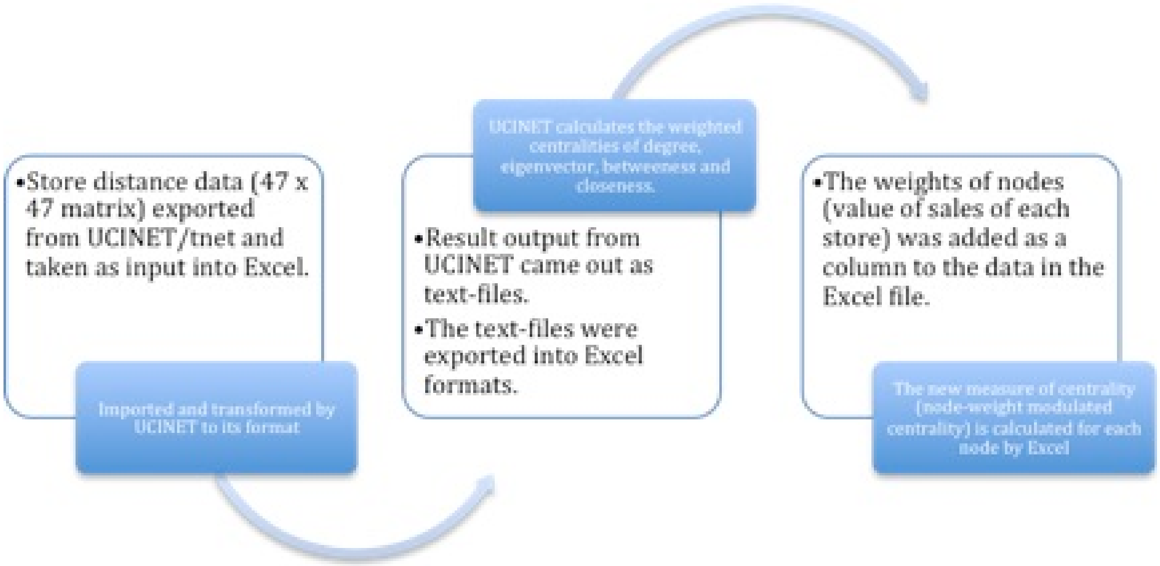

Figure 4.1: Figure showing the implementation of the Node-Weighted Centrality Measure

The initial dataset of the distances between the 47 sales outlets of the considered supply chain system presented as a 47x47 square matrix which was obtained from the UCINET software, saved in Excel format and later imported into UCINET for the purpose of centralities calculations. Table 4.1 depicts the link-weighted centrality measures at different values of alpha for a subset of twelve highest central nodes (in terms of degree, closeness, eigenvector and betweenness) from among the forty-seven nodes considered.

The results came out as text files listing the different columns for each centrality measure, and for the purpose of calculations of the node-weight modulated centrality, the values from the text files were exported into Excel where a column was created for the weights on the nodes. (See Table 4.2).

The sparseness of a sample of 12 stores (nodes A to L) and their linkage to the existing distribution centre (node M) is as shown in figure 4.2 below:

The exact real-life distribution centre (Node M) that supplies all the other nodes (A to L) is 9.3 miles to the finally predicted Distribution centre , which is Node C from the results obtained from the node-weighted centrality measures. The farthest distance apart of any two nodes is that from Node K to Node L, and it is a distance of 143miles. This was used in column 9 of Table 4.3, for the calculation of ratio of distances from any node to that of farthest distance apart. The percentage error of prediction is therefore calculated by

Table 4.1: Table showing the link-weighted measure of centrality

	LINK-WEIGHTED DEGREE CENTRALITY						**LINK-WEIGHTED EIGEN-VECTOR CENTRALITY**						**LINK-WEIGHTED BETWEENNESS CENTRALITY**						**LINK-WEIGHTED CLOSENESS CENTRALITY**				
Node	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = \mathbf{1}$	$\alpha = \mathbf{1}\frac{1}{4}$	$\alpha = \mathbf{1}\frac{1}{2}$	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = \mathbf{1}$	$\alpha = \mathbf{1}\frac{1}{4}$	$\alpha = \mathbf{1}\frac{1}{2}$	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = \mathbf{1}$	$\alpha = \mathbf{1}\frac{1}{4}$	$\alpha = \mathbf{1}\frac{1}{2}$	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = \mathbf{1}\frac{1}{4}$	$\alpha = \mathbf{1}\frac{1}{2}$
A	84.99	157	290.1	990	1830	3380	10.13	2.23	0.49	0.02	0.01	0.0012	9.04	1.78	0.35	0.014	0.003	5E-04	46	46	46	46	46
B	82.29	147	263.3	843	1507	2696	9.93	2.14	0.46	0.02	0	0.001	9.04	1.78	0.35	0.014	0.003	5E-04	46	46	46	46	46
C	87.85	168	320.4	1169	2232	4263	10.22	2.27	0.50	0.02	0.01	0.0012	9.04	1.78	0.35	0.014	0.003	5E-04	46	46	46	46	46
D	91.14	181	357.8	1405	2783	5515	10.42	2.36	0.53	0.03	0.01	0.0014	9.04	1.78	0.35	0.014	0.003	5E-04	46	46	46	46	46
E	86.53	163	306.2	1084	2038	3834	10.20	2.26	0.50	0.02	0.01	0.0012	9.04	1.78	0.35	0.014	0.003	5E-04	46	46	46	46	46
F	90.06	176	345.3	1324	2591	5074	10.37	2.34	0.53	0.03	0.01	0.0014	9.04	1.78	0.35	0.014	0.003	5E-04	46	46	46	46	46
G	82.72	149	267.5	865	1555	2797	9.98	2.17	0.47	0.02	0	0.001	9.04	1.78	0.35	0.014	0.003	5E-04	46	46	46	46	46
H	82.46	148	265.0	851	1526	2736	9.96	2.16	0.47	0.02	0	0.001	9.04	1.78	0.35	0.014	0.003	5E-04	46	46	46	46	46
I	83.06	150	270.8	883	1594	2879	9.98	2.17	0.47	0.02	0	0.001	9.04	1.78	0.35	0.014	0.003	5E-04	46	46	46	46	46
J	83.57	152	275.8	910	1653	3004	10.01	2.18	0.47	0.02	0	0.0011	9.04	1.78	0.35	0.014	0.003	5E-04	46	46	46	46	46
K	97.36	206	436.1	1954	4135	8751	10.80	2.54	0.60	0.03	0.01	0.0018	9.04	1.78	0.35	0.014	0.003	5E-04	46	46	46	46	46
L	151.6	500	1646	17874	58898	2E+05	15.08	4.94	1.62	0.17	0.06	0.0187	9.04	1.78	0.35	0.014	0.003	5E-04	46	46	46	46	46

Table 4.2: Table showing the node-weighted centralities at different alpha and at $\beta = 1$

			NODE-WEIGHTED DEGREE CENTRALITY				**NODE-WEIGHTED EIGEN-VECTOR CENTRALITY**				**NODE-WEIGHTED BETWEENNESS CENTRALITY**				**NODE-WEIGHTED CLOSENESS CENTRALITY**		
	Node	**Sales Value**	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = 1\frac{1}{4}$	$\alpha = 1\frac{1}{2}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = 1\frac{1}{4}$	$\alpha = 1\frac{1}{2}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = 1\frac{1}{4}$	$\alpha = 1\frac{1}{2}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = 1\frac{1}{4}$
A	23539	3.7E+6	6.8E+6	2.3E+7	4.3E+7	5.2E+4	1.16E+4	559.60	123.18	4.2E+4	8	317.96	62.51	1.1E+6	1.1E+6	1.1E+6	1.1E+6
B	19065	2.8E+6	5.0E+6	1.6E+7	2.9E+7	4.1E+4	8.8E+3	411.67	88.89	3.3E+4	6663.35	257.3	50.63	8.8E+5	8.8E+5	8.8E+5	8.8E+5
C	27104	4.5E+6	8.7E+6	3.2E+7	6.1E+7	6.2E+4	1.4E+4	674.32	149.79	4.8E+4	9	366.12	71.98	1.2E+6	1.2E+6	1.2E+6	1.2E+6
D	22112	3.99E+6	7.9E+6	3.1E+7	6.2E+7	5.2E+4	1.2E+4	605.93	137.22	3.9E+4	7	728.30	58.72	1.02E+6	1.02E+6	1.02E+6	1.02E6
E	9	1.5E+3	2.8E+3	9.8E+3	1.8E4	2.0E+1	4.51	0.22	0.05	16.0	3.15	0.12	0.00	414	414	414	414
F	11	1.9E+3	3.8E+3	1.5E+4	2.9E+4	2.6E+1	5.80	0.30	0.07	19.56	3.84	0.15	0.03	506	506	506	506
G	1831	2.7E+5	4.9E+5	1.6E+6	2.8E+6	4.0E+3	8.6E+2	40.53	8.79	3.3E+3	639.95	24.73	4.86	8.4E+4	8.4E+4	8.4E+4	8.4E+4
H	1786	2.6E+5	4.7E+5	1.6E+6	2.7E+6	3.8E+3	8.3E+2	39.05	8.45	3.2E+3	624.22	24.13	4.74	8.2E+4	8.2E+4	8.2E+4	8.2E+4
I	3827	5.7E+5	1.04E+6	3.4E+6	6.1E+6	8.3E+3	1.8E+3	18.38	6.8E+3	1.33	51.69	10.16	1.7E+5	1.7E+5	1.7E+5	1.7E+5	
J	19341	2.9E+6	5.3E+6	1.8E+7	3.2E+7	4.2E+4	9.2E+3	433.35	94.27	3.4E+4	6.75	261.26	51.36	8.9E+5	8.9E+5	8.9E+5	8.9E+5
K	18724	3.9E+6	8.2E+6	3.7E+7	7.7E+7	4.8E+4	1.1E+4	144.61	33.97	6.54	252.92	49.72	8.6 E+5	8.6 E+5	8.6 E+5	8.6 E+5	
L	5106	2.6E+6	8.4E+6	9.1E+7	3.0E+8	2.5E+4	8.3E+3	888.71	291.30	9.1E+3	1.78	68.97	13.56	2.3E+5	2.3E+5	2.3E+5	2.3E+5

multiplying this ratio by 100 and from this emerges the percentage accuracy.

Figure 4.2: Figure showing the road network of existing distribution center (node M) to the other sales outlets. [Google Map of Selected Locations In UK]

4.4 Summary

For the purpose of measures of centralities, the Node-Weighted Betweenness and Node-Weighted Closeness measures returned highest percentage of accuracies, thus presenting as better measures of centrality in this case because how close a node is to all the other nodes is considered more importantly, so also the ability to control the flow between nodes, as opposed to Node-Weighted Degree and Node-Weighted EigenVector which concentrates more on the number of links to a particular node.

4.5 Conclusion

With all the results above one is now in a position to predict the location of a Distribution Centre based on the resulting centrality measures obtained, for example in the case of node-weighted centrality measure, Table 4.3 shows the most probable node that could serve as a Distribution center for all other outlets considering its centrality value and its percentage accuracy of prediction.

This same exercise when carried out for each of the node-weighted centralities and link-weighted centralities yield the results of Table 4.4.

From Table 4.4, it could be seen that when the link-weighted centralities for degree and eigenvectors are considered, node L is the most central but with an accuracy of 23.08 percent while for the link-weighted betweenness and closeness the most central node is node F with percentage accuracy of 95.73 percent. However, with the introduction of weights on the

Table 4.3: Table showing the scores for the node-weighted degree centrality of each node with percentage accuracy of prediction.

	NODE WEIGHTED CENTRALITY AS α VARIES FROM 0.25 TO 1.75								
Node	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = 1\frac{1}{4}$	$\alpha = 1\frac{1}{2}$	$\alpha = 1\frac{3}{4}$	**DIST FROM EXISTING D C**	**RATIO OF DIST FROM THIS NODE TO EXISTING DC & THE FARTHEST SPAN OF NETWORK**	**PERCENT ACCURACY**
A	2000542	3696148	6828905	2.3E+07	4.3E+07	8E+07	20.7	0.178	82.171
B	1568774	2806249	5019864	1.6E+07	2.9E+07	5.1E+07	16.3	0.140	85.960
C	2381166	4547660	8685332	3.2E+07	6.1E+07	1.2E+08	12	0.103	89.664
D	2015385	3993283	7912289	3.1E+07	6.2E+07	1.2E+08	16.9	0.146	85.444
E	778.783	1464.98	2755.81	9.7E+03	18344.1	3.5E+04	16.8	0.145	85.530
F	990.713	1939.75	3797.89	1.5E+04	28505.8	5.6E+04	6.1	0.053	94.746
G	151458	272358	489764	1.6E+06	2847915	5.1E+06	19.1	0.165	83.549
H	147273	264002	473251	1.5E+06	2726107	4.9E+06	18.2	0.157	84.324
I	317873	573973	1036405	3.4E+06	6101587	1.1E+07	15.9	0.137	86.305
J	1616243	2936140	5333922	1.8E+07	3.2E+07	5.8E+07	11	0.095	90.525
K	1822919	3858144	8165624	3.7E+07	7.7E+07	1.6E+08	31.5	0.271	72.868
L	773979	2550466	8404464	9.1E+07	3E+08	9.9E+08	110	0.947	5.254

Table 4.4: Table showing the percentage accuracy of the most central nodes.

	$\alpha = 0.25$		$\alpha = 0.5$		$\alpha = 0.75$		$\alpha = 1.25$		$\alpha = 1.5$		$\alpha = 1.75$	
Centrality Type	**Most entral Node**	**Percentage**	**Most entral Node**	**Percentage**	**Most entral Node**	**Percentage**	**Most entral Node**	**Percentage**	**Most entral Node**	**Percentage**	**Most entral Node**	**Percentage**
Link_Weighted Degree Centrality	Node L	23.08	Node L	23.08	Node L	23.08	Node L	23.08	Node L	23.08	Node L	23.08
Link_Weighted EigenVector Centrality	Node L	23.08	Node L	23.08	Node L	23.08	Node L	23.08	Node L	23.08	Node L	23.08
Link_Weighted Betweenness Centrality	Node F	95.73	Node F	95.73	Node F	95.73	Node F	95.73	Node F	95.73	Node F	95.73
Link_Weighted Closeness Centrality	Node F	95.73	Node F	95.73	Node F	95.73	Node F	95.73	Node F	95.73	Node F	95.73
Node_Weighted Degree Centrality	Node C	91.61	Node C	91.61	Node C	91.61	Node L	23.08	Node L	23.08	Node L	23.08
Node_Weighted EigenVector Centrality	Node C	91.61	Node C	91.61	Node C	91.61	Node L	23.08	Node L	23.08	Node L	23.08
Node_Weighted Betweenness Centrality	Node C	91.61	Node C	91.61	Node C	91.61	Node C	91.61	Node C	91.61	Node C	91.61
Node_Weighted Closeness Centrality	Node C	91.61	Node C	91.61	Node C	91.61	Node C	91.61	Node C	91.61	Node C	91.61

nodes (i.e. the sales values of each store) it could be seen that in all the 24 cases of node-weighted centrality scores, node C has the highest centrality in 18 of the cases with the highest percentage of accuracy of 91.6% except in cases whereby node L has the highest centrality scores but with low accuracy of 23.08% as shown in Table 4.4.

Thus it could be concluded that Node C is the most probable node to be used as the distribution centre because of its high centrality in cases of node-weighted centralities and this is attributed to the corresponding high sales value of 27104 as shown in Table 4.2 . This value of sales (representing the node-weight) is a significant contributory factor in making Node C the most central as opposed to the situations when the linkweightedness are considered.

Chapter 5

Clique Structure and Node-Weighted Centrality Measures to Predict Location of Structures

5.1 Weights on Nodes

The weights/strengths on the nodes can for example in a supply chain management system (which is the focus of this chapter) be the value of sales made by each shop or cost of rentage per square feet while the weight on edges might be the distances linking the shops or sales outlets. There is a need to clarify the distinction between the node strength (herein referred to as the weights of or on the nodes/vertices/actors) and the tie strength (herein referred to as the weights attached to the tie/edge/link/line). This way one can give the deserved considerations to weightedness and how they affect centralities, thus enhancing our analysis. Obviously networks are usually formed through the combinations of edges and vertices, as such, considerations of attributes of both will possibly bring out a largely representative and more robust analytic outcome. In addition to studying the links of the nodes, the attribute of the nodes itself may be worthy of consideration (e.g. how the sales values in a depot/store is affected by the links it has to the supplying unit or distribution centre).

5.2 Modulated Centrality Measure of Node-Weight

A modified centrality measure that combines the weights of nodes, weights/strengths on edges and number of edges with consideration for each of the four basic centrality measures (degree, eigenvector, betweenness and closeness) is hereby considered. It will be recalled that it was earlier mentioned that the tuning parameter α was introduced to determine the relative importance of the number of the ties compared to the weights on the ties. A tuning parameter β was introduced by Akanmu et al(2012, 2013) to take care of the weightedness on the nodes, although the tuning parameter α was applied to the degree/strength of the edges as denoted earlier in equations (2.9) and (2.14) above. The newly evolved equation by way of introduction of a tuning parameter β will now be the product of degree of a focal node, the average weight to these nodes as adjusted by the newly introduced tuning parameter β and the weight accorded to each node. So, for weighted degree centrality at α and β we shall now have equation (2.14) being recalled as (5.1) below:

$$C_d^{w\alpha\beta}(i) = K_i \times (\frac{S_i}{K_i})^{\alpha} \times (Z_i)^{\beta} = K_i^{(1-\alpha)} \times S_i^{\alpha} \times (Z_i)^{\beta} \quad (5.1)$$

where K_i = degree of nodes, $S_i = C_d^w(s)$ as defined in (2.8) above

Z_i = weight of nodes, α is ≥ 0 ; $\{\beta \in Z : -1 \leq \beta \leq 1\}$

The choice of value of β depends on what effect the weight is having on the new centrality measure, if for instance the weight is having a positive effect (e.g. profit) the positive value of β is employed otherwise the negative value(e.g. loss) shall be used in our calculation.

5.3 Case Study: Clique Structure/Node-Weight Modulated Centrality Measures Applied To Supply Chain Management

The network coverage of an existing distribution centre (DC) located at Scotland was investigated and the retail outlets or shops are considered as nodes with the value of sales taken to be the weights on the nodes while distances between nodes are regarded as the weights on the edges. For our sample a 30miles radius coverage of shops from the existing DC was taken and this makes 63nodes all connected by distances (see figure 5.1 below). The nearest DC to this existing one is some 171miles away, so our coverage for this purpose is of 60miles diameter, although this could be extended in future. Out of the community of 63 shops, the Central and Lothian Counties accommodated 33 of these shops while Glasgow city and Edinburgh have 30 of these. The existing DC at Livingston is actually situated in-between these two cities. The clique of shops within Glasgow and Edinburgh were examined and the most central from the two cliques were considered for the prediction of the new DC.

Tables 5.1 to Table 5.4 show the respective results that were obtained when the Link-Weight Modulated Centrality and Node-Weight Modulated Centrality Measures are applied to the 7 nodes of Glasgow and 23 nodes of Edinburgh from the supply chain management dataset.

5.3.1 Implementation

The results were obtained using the software UCINET and tnet to generate the four weighted centrality results of Degree, Eigenvector, Betweenness and Closeness, while Excel spreadsheet was used to carry out the final calculations of the Node-Weight Modulated Centrality measures of each node/vertex. The schematic diagram of how the process was implemented is as shown in figure 5.3 .

The initial dataset of the distances between the 30 sales outlets of Glasgow and Edinburgh presented as a 7x7 square matrix and 23 x 23 square matrix respectively which was obtained from the UCINET software, saved in Excel format and later imported into UCINET for the purpose of centralities calculations. Tables 5.1 & 5.2 depict the link-weighted centrality measures at different values of alpha (in terms of degree, closeness, eigenvector and betweenness).

The results came out as text files listing the different columns for each centrality measure, and for the purpose of calculations of the node-weight modulated centrality, the values from the text files were exported into Excel where a column was created for the weights on the nodes. (See Tables 5.3 & 5.4).

Figure 5.1: Figure showing the coverage of the 30miles radius, Source www.rightmove.co.uk

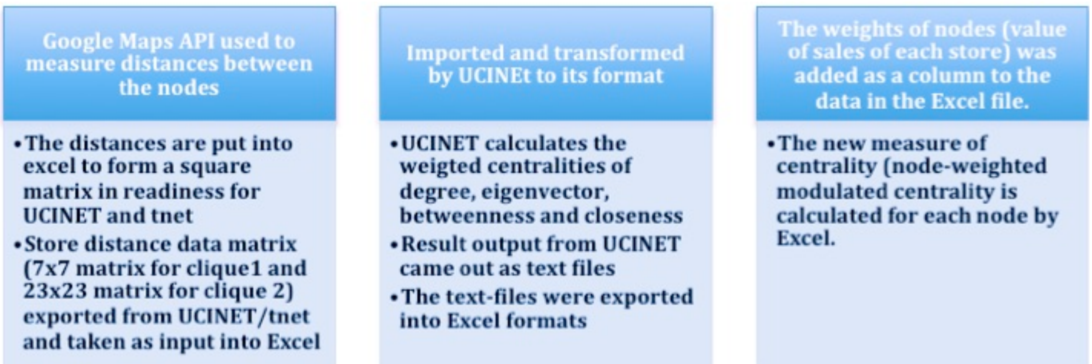

Figure 5.2: Figure showing the implementation of the Node-Weighted Centrality Measure

Table 5.1: Table showing the link-weighted measure of centrality for Glasgow (Clique 1)

			LINK-WEIGHTED DEGREE CENTRALITY						**LINK-WEIGHTED EIGEN-VECTOR CENTRALITY**			**LINK-WEIGHTED BETWEENNESS CENTRALITY**				**LINK-WEIGHTED CLOSENESS CENTRALITY**							
Node	α $=\frac{1}{4}$	α $=\frac{1}{2}$	α $=\frac{3}{4}$	α $=\mathbf{1}\frac{1}{4}$	α $=\mathbf{1}\frac{1}{2}$	α $=\mathbf{1}\frac{3}{4}$	α $=\frac{1}{4}$	α $=\frac{1}{2}$	α $=\frac{3}{4}$	α $=\mathbf{1}\frac{1}{4}$	α $=\mathbf{1}\frac{1}{2}$	α $=\mathbf{1}\frac{3}{4}$	α $=\frac{1}{4}$	α $=\frac{1}{2}$	α $=\frac{3}{4}$	α $=\mathbf{1}\frac{1}{4}$	α $=\mathbf{1}\frac{1}{2}$	α $=\mathbf{1}\frac{3}{4}$	α $=\frac{1}{4}$	α $=\frac{1}{2}$	α $=\frac{3}{4}$	α $=\mathbf{1}$	α $=\mathbf{1}\frac{1}{4}$
1	8.51	12.06	17.09	34.35	48.69	69.03	9.76	15.88	25.84	68.38	111.24	180.98	7.44	9.23	11.45	17.62	21.86	27.12	3.69	2.27	1.40	0.53	0.33
3	8.72	12.67	18.41	38.89	56.52	82.14	9.93	16.42	27.16	74.34	122.99	203.46	7.74	9.99	12.88	21.44	27.66	35.69	3.79	2.40	1.51	0.60	0.38
4	9.47	14.95	23.60	58.80	92.82	146.51	10.67	18.97	33.73	106.62	189.58	337.08	8.60	12.32	17.66	36.26	51.96	74.47	4.02	2.69	1.80	0.81	0.54
5	9.55	15.20	24.20	61.31	97.59	155.34	10.93	19.91	36.27	120.38	219.30	399.50	7.82	10.19	13.28	22.54	29.37	38.28	4.15	2.87	1.98	0.95	0.65
11	9.68	15.48	24.88	64.20	103.14	165.69	10.96	20.01	36.53	121.82	222.45	406.21	8.37	11.66	16.26	31.62	44.08	61.46	4.14	2.85	1.97	0.94	0.65
13	8.54	12.15	17.28	34.99	49.79	70.84	9.78	15.95	26.01	69.16	112.76	183.87	7.21	8.66	10.41	15.03	18.06	21.70	3.70	2.29	1.41	0.54	0.33
15	8.66	12.50	18.04	37.58	54.25	78.30	9.91	16.38	27.07	73.93	122.17	201.88	7.42	9.18	11.36	17.39	21.51	26.62	3.85	2.47	1.58	0.65	0.42

Table 5.2: Table showing the link-weighted measure of centrality for Edinburgh (Clique 2)

	LINK-WEIGHTED DEGREE CENTRALITY						LINK-WEIGHTED EIGEN-VECTOR CENTRALITY				LINK-WEIGHTED BETWEENNESS CENTRALITY				LINK-WEIGHTED CLOSENESS CENTRALITY								
Node	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = 1\frac{1}{4}$	$\alpha = 1\frac{1}{2}$	$\alpha = 1\frac{3}{4}$	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = 1\frac{1}{4}$	$\alpha = 1\frac{1}{2}$	$\alpha = 1\frac{3}{4}$	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = 1\frac{1}{4}$	$\alpha = 1\frac{1}{2}$	$\alpha = 1\frac{3}{4}$	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = 1$	$\alpha = 1\frac{1}{4}$
22	24.18	26.58	29.21	35.29	38.79	42.64	7	27.97	31.54	40.11	45.23	51.00	16.60	12.52	9.44	5.37	4.05	3.06	11.15	5.65	2.86	0.73	0.37
23	20.76	19.59	18.49	16.46	15.53	14.66	21.82	21.63	21.45	21.09	20.91	20.74	13.29	8.03	4.85	1.77	1.07	0.65	9.83	4.39	1.96	0.39	0.18
24	23.68	25.48	27.43	31.77	34.19	36.80	24.40	27.06	30.01	36.91	40.93	45.40	16.24	11.99	8.85	4.83	3.56	2.63	10.82	5.32	2.61	0.63	0.31
25	20.56	19.22	17.97	15.70	14.68	13.72	21.78	21.56	21.34	20.92	20.71	20.50	12.98	7.66	4.52	1.57	0.93	0.55	10.10	4.63	2.13	0.45	0.21
30	21.13	20.29	19.49	17.97	17.26	16.58	22.31	22.62	22.94	23.59	23.92	24.25	13.45	8.22	5.03	1.88	1.15	0.70	10.14	4.68	2.16	0.46	0.21
31	23.38	24.85	26.40	29.82	31.69	33.67	24.12	26.45	29.00	34.87	38.24	41.93	15.79	11.33	8.13	4.19	3.00	2.16	10.52	5.03	2.40	0.55	0.26
32	25.03	28.49	32.42	41.98	47.77	54.36	25.63	29.86	34.79	47.22	55.02	64.10	17.14	13.36	10.41	6.32	4.93	3.84	11.38	5.89	3.05	0.82	0.42
33	22.01	22.02	22.03	22.05	22.05	22.06	23.11	24.28	25.51	28.16	29.59	31.09	14.30	9.29	6.04	2.55	1.66	1.08	10.43	4.94	2.34	0.53	0.25
35	20.77	19.61	18.51	16.49	15.57	14.70	21.95	21.89	21.84	21.73	21.68	21.63	12.90	7.57	4.44	1.53	0.90	0.53	10.13	4.67	2.15	0.46	0.21
36	20.66	19.41	18.23	16.08	15.10	14.19	21.89	21.79	21.68	21.47	21.36	21.26	12.88	7.55	4.42	1.52	0.89	0.52	10.09	4.62	2.12	0.45	0.20
37	24.26	26.76	29.51	35.89	39.58	43.65	25.46	29.46	34.09	45.65	52.82	61.12	15.64	11.12	7.91	4.00	2.84	2.02	11.38	5.89	3.04	0.81	0.42
38	23.94	26.05	28.34	33.56	36.51	39.73	24.49	27.25	30.33	37.57	41.81	46.53	16.60	12.52	9.44	5.37	4.05	3.06	10.99	5.49	2.74	0.68	0.34
39	21.03	20.10	19.21	17.55	16.78	16.04	21.93	21.87	21.80	21.66	21.60	21.53	13.85	8.72	5.49	2.17	1.37	0.86	9.67	4.25	1.87	0.36	0.16
40	24.76	27.88	31.38	39.76	44.76	50.38	25.59	29.76	34.62	46.83	54.46	63.35	16.44	12.28	9.18	5.12	3.83	2.86	11.20	5.70	2.90	0.75	0.38
41	20.80	19.67	18.59	16.62	15.72	14.86	21.94	21.88	21.82	21.71	21.65	21.59	12.83	7.48	4.36	1.48	0.87	0.50	10.12	4.66	2.14	0.45	0.21
42	20.82	19.70	18.64	16.69	15.79	14.94	21.94	21.88	21.83	21.71	21.65	21.59	12.95	7.62	4.48	1.55	0.91	0.54	9.89	4.45	2.00	0.40	0.18
43	25.76	30.17	35.33	48.45	56.73	66.44	26.30	31.44	37.58	53.71	64.21	76.76	17.87	14.52	11.80	7.79	6.33	5.14	11.77	6.30	3.37	0.96	0.52
44	22.19	22.38	22.57	22.96	23.16	23.36	23.31	24.69	26.16	29.35	31.10	32.94	14.37	9.39	6.13	2.62	1.71	1.12	10.49	5.00	2.38	0.54	0.26
45	23.53	25.17	26.92	30.79	32.93	35.22	24.79	27.94	24.79	39.98	45.06	50.78	15.06	10.30	7.05	3.30	2.26	1.55	11.20	5.70	2.90	0.75	0.38
46	21.69	21.39	21.09	20.50	20.22	19.93	22.56	23.14	23.73	24.96	25.60	26.26	14.49	9.54	6.28	2.72	1.79	1.18	10.02	4.57	2.08	0.43	0.20
47	20.34	18.80	17.38	14.85	13.73	12.69	21.49	20.99	20.50	19.56	19.11	18.66	12.75	7.39	4.28	1.44	0.83	0.48	9.78	4.34	1.93	0.38	0.17
48	21.74	21.49	21.23	20.74	20.49	20.25	22.94	23.91	24.93	27.09	28.24	29.44	13.76	8.60	5.38	2.10	1.32	0.82	10.50	5.02	2.39	0.55	0.26
49	20.37	18.86	17.47	14.98	13.87	12.84	21.64	21.28	20.93	20.25	19.92	19.59	12.46	7.06	4.00	1.28	0.73	0.41	10.06	4.60	2.10	0.44	0.20

Table 5.3: Table showing the node-weighted centralities at different alpha and at beta = 1 for Glasgow (Clique1) , with centrality measures values in 000s

	NODE-WEIGHTED DEGREE CENTRALITY						NODE-WEIGHTED EIGEN-VECTOR CENTRALITY						NODE-WEIGHTED BETWEENNESS CENTRALITY						NODE-WEIGHTED CLOSENESS CENTRALITY				
Node	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = \mathbf{1}\frac{1}{4}$	$\alpha = \mathbf{1}\frac{1}{2}$	$\alpha = \mathbf{1}\frac{3}{4}$	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = \mathbf{1}\frac{1}{4}$	$\alpha = \mathbf{1}\frac{1}{2}$	$\alpha = \mathbf{1}\frac{3}{4}$	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = \mathbf{1}\frac{1}{4}$	$\alpha = \mathbf{1}\frac{1}{2}$	$\alpha = \mathbf{1}\frac{3}{4}$	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = \mathbf{1}$	$\alpha = \mathbf{1}\frac{1}{4}$
1	44.8	63.6	90.1	181.2	256.9	364.2	51.5	83.8	136.3	360.7	586.9	954.8	39.3	48.7	60.4	93.0	115.3	143.1	19.5	12.0	7.4	2.8	1.7
3	97.7	142.0	206.3	435.7	633.2	920.2	111.2	183.9	304.3	832.8	1377.8	2279.2	86.7	111.9	144.3	240.2	309.9	399.8	42.5	26.8	17.0	6.8	4.3
4	65.4	103.1	162.9	405.8	640.5	1011.1	73.6	130.9	232.7	735.8	1308.3	2326.2	59.3	85.0	121.9	250.2	358.6	513.9	27.7	18.6	12.4	5.6	3.7
5	126.8	201.8	321.2	813.8	1295.4	2061.9	145.1	264.3	481.5	1597.9	2910.9	5302.9	103.8	135.2	176.2	299.2	389.9	508.1	55.0	38.0	26.3	12.6	8.7
11	36.5	58.7	94.3	243.4	391.1	628.3	41.5	75.9	138.5	461.9	843.5	1540.2	31.7	44.2	61.7	119.9	167.2	233.1	15.7	10.8	7.5	3.6	2.5
13	3.9	5.5	7.9	16.0	22.7	32.3	4.5	7.3	11.9	31.6	51.5	84.0	3.3	4.0	4.8	6.9	8.2	9.9	1.7	1.0	0.6	0.2	0.2
15	3.2	4.6	6.7	13.9	20.0	28.9	3.7	6.1	10.0	27.3	45.1	74.6	2.7	3.4	4.2	6.4	7.9	9.8	1.4	0.9	0.6	0.2	0.2

Table 5.4: Table showing the node-weighted centralities at different alpha and at beta = 1 for Edinburgh (Clique 2) with centrality measures values in 0́000s

	NODE-WEIGHTED DEGREE CENTRALITY						**NODE-WEIGHTED EIGEN-VECTOR CENTRALITY**				**NODE-WEIGHTED BETWEENNESS CENTRALITY**				**NODE-WEIGHTED CLOSENESS CENTRALITY**								
No de	α $=\frac{1}{4}$	α $=\frac{1}{2}$	α $=\frac{3}{4}$	α $=\mathbf{1}\frac{1}{4}$	α $=\mathbf{1}\frac{1}{2}$	α $=\mathbf{1}\frac{3}{4}$	α $=\frac{1}{4}$	α $=\frac{1}{2}$	α $=\frac{3}{4}$	α $=\mathbf{1}\frac{1}{4}$	α $=\mathbf{1}\frac{1}{2}$	α $=\mathbf{1}\frac{3}{4}$	α $=\frac{1}{4}$	α $=\frac{1}{2}$	α $=\frac{3}{4}$	α $=\mathbf{1}\frac{1}{4}$	α $=\mathbf{1}\frac{1}{2}$	α $=\mathbf{1}\frac{3}{4}$	α $=\frac{1}{4}$	α $=\frac{1}{2}$	α $=\frac{3}{4}$	α $=\mathbf{1}$	α $=\mathbf{1}\frac{1}{4}$
22	44.2	48.5	53.4	64.5	70.8	77.9	45.3	51.1	57.6	73.3	82.6	93.2	30.3	22.9	17.2	9.8	7.4	5.6	20.4	10.3	5.2	1.3	0.7
23	3.2	3.0	2.9	2.6	2.4	2.3	3.4	3.4	3.3	3.3	3.2	3.2	2.1	1.2	0.8	0.3	0.2	0.1	1.5	0.7	0.3	0.1	0.2
24	6.1	6.6	7.1	8.2	8.	9.6	6.3	7.0	7.8	9.6	10.6	11.7	4.2	3.1	2.3	1.3	0.9	0.7	2.8	1.4	0.7	0.2	0.1
25	7.4	6.9	6.5	5.7	5.3	5.0	7.9	7.8	7.7	7.5	7.5	7.4	4.7	2.8	1.6	0.6	0.3	0.2	3.6	1.7	7.7	0.2	0.1
30	14.4	13.9	13.3	12.3	11.8	11.4	15.3	15.5	15.7	16.1	16.4	16.6	9.2	5.6	3.4	1.3	0.8	0.5	6.9	3.2	1.5	0.3	0.1
31	31.8	33.8	36.0	40.6	43.2	45.9	32.9	36.0	39.5	47.5	52.1	57.1	21.5	15.4	11.1	5.7	4.1	2.9	14.3	6.8	3.3	0.7	0.4
32	29.0	33.0	37.6	48.7	55.4	63.0	29.7	34.6	40.4	54.8	63.8	74.4	19.9	15.5	12.1	7.3	5.7	4.5	13.2	6.8	3.5	0.9	0.5
33	15.8	15.8	15.8	15.8	15.8	15.8	16.5	17.4	18.3	20.2	21.2	22.3	10.2	6.7	4.3	1.8	1.2	0.8	7.5	3.5	1.7	0.4	0.2
35	0.3	0.3	0.3	0.3	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.3	0.2	0.1	0.1	0.0	0.0	0.0	0.16	0.1	0.0	0.0	0.0
36	2.0	1.9	1.8	1.6	1.5	1.4	2.1	2.1	2.1	2.1	2.1	2.1	1.2	0.7	0.4	0.1	0.1	0.1	1.0	0.4	0.2	0.0	0.0
37	0.5	0.6	0.6	0.7	0.8	0.9	0.5	0.6	0.7	0.9	1.1	1.3	0.3	0.2	0.2	0.1	0.1	0.0	0.2	0.1	0.1	0.0	0.0
38	0.9	1.0	1.0	1.2	1.3	1.5	1.0	1.0	1.1	1.4	1.5	1.7	0.6	0.5	0.3	0.2	0.1	0.1	0.4	0.2	0.1	0.0	0.0
39	0.8	0.7	0.7	0.6	0.6	0.6	0.8	0.8	0.8	0.8	0.8	0.8	0.5	0.3	0.2	0.1	0.0	0.0	0.4	0.2	0.1	0.0	0.0
40	0.4	0.5	0.6	0.7	0.8	0.9	0.5	0.5	0.6	0.8	1.0	1.1	0.3	0.2	0.2	0.1	0.1	0.1	0.2	0.1	0.1	0.0	0.0
41	2.7	2.6	2.4	2.1	2.0	1.9	2.8	2.8	2.8	2.8	2.8	2.8	1.7	1.0	0.6	0.2	0.1	0.1	1.3	0.6	0.3	0.1	0.0
42	1.1	1.0	1.0	0.9	0.8	0.8	1.1	1.1	1.1	1.1	1.1	1.1	0.7	0.4	0.2	0.1	0.0	0.0	0.5	0.2	0.1	0.0	0.0
43	0.2	0.3	0.3	0.4	0.5	0.6	0.2	0.3	0.3	0.5	0.5	0.6	0.1	0.1	0.1	0.1	0.1	0.0	0.1	0.1	0.0	0.0	0.0
44	0.9	0.9	0.9	0.9	0.9	0.9	0.9	1.0	1.1	1.2	1.3	1.3	5.8	3.8	0.2	0.1	0.1	0.0	0.4	0.2	0.1	0.0	0.0
45	0.4	0.5	0.5	0.6	0.6	0.7	0.5	0.5	0.6	0.8	0.9	1.0	0.3	0.2	0.1	0.1	0.0	0.0	0.2	0.1	0.1	0.0	0.0
46	1.7	1.7	1.6	1.6	1.6	1.5	1.8	1.8	1.8	1.9	2.0	2.0	1.1	0.7	0.5	0.2	0.1	0.1	0.8	0.4	0.2	0.0	0.0
47	0.4	0.4	0.3	0.3	0.3	0.2	0.4	0.4	0.4	0.4	0.4	0.4	0.3	0.1	0.1	0.0	0.0	0.0	0.2	0.1	0.0	0.0	0.0
48	2.2	2.1	2.1	2.1	2.0	2.0	2.3	2.4	2.5	2.7	2.8	2.9	1.4	0.9	0.5	0.2	0.1	0.1	1.0	0.5	0.2	0.1	0.0
49	1.7	1.5	1.4	1.2	1.1	1.0	1.8	1.7	1.7	1.7	1.6	1.6	1.0	0.6	0.3	0.1	0.1	0.0	0.8	0.4	0.2	0.0	0.0

The radius of coverage according to figure 5.1 is 30miles, accordingly the farthest possible distance apart of any two nodes will be the diameter of such circle which is 60miles. This was used (in column 9 of Table 5.5) for the calculation of ratio of distances from any node to that of farthest distance apart. The percentage error of prediction is therefore calculated by multiplying this ratio by 100 and from this emerges the percentage accuracy.

5.4 Conclusion and Discussion

From the tables 5.6 and 5.7, the two nodes 5 and 22 are the most central in terms of the node-weightedness thereby representing the cliques of Glasgow and Edinburgh respectively. Considering figure 5.3 ,

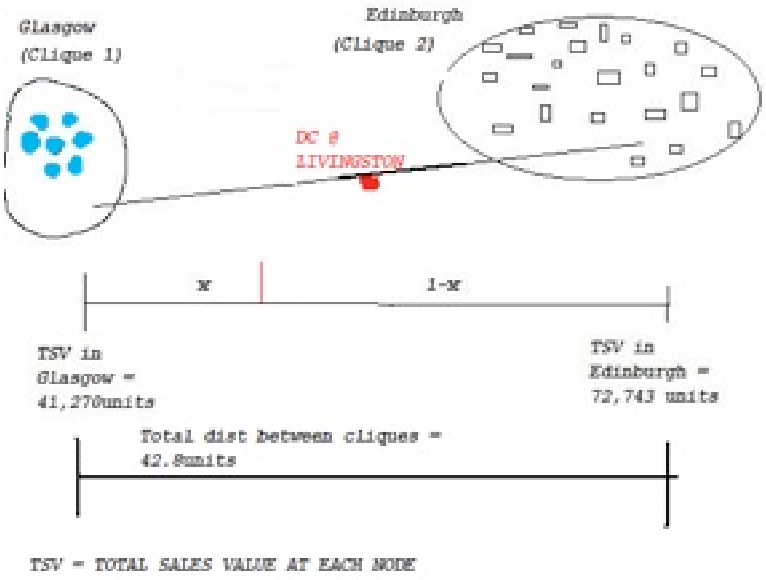

Figure 5.3: The proportional distance of the proposed DC

x is the proportional distance to the proposed Distribution Centre. TSV = Total Sales Values The driving distance between node 5 (representing Glasgow clique) and node 22 (representing Edinburgh clique) is 42.8miles.

1-x/ x = 72743/41270 x = 0.36 (i.e. 36% of 42.8) which is 13.1miles

If x is some 13.1miles away from node 5 (Clique 1), and the existing DC is 27.4 miles away from the same node 5, the difference of the predicted DC will be 14.3miles away from the existing DC, hence The error rate of the predicted DC = (14.3/42.80) x 100 = 33.4% i.e. the percentage accuracy of the prediction = 66.6%

With all the results above one is now in a position to predict the most probable (regions with respect to the nodes) that could serve as a distribution center for all other outlets considering their node-weighted centrality and clique structures going by the percentage accuracy of the prediction.

In Table 4.6, the node-weights are taken as the sales-value and:

NODE 43 (EH12 9BH) is 11.30units of distance to the existing DC. NODE 22 (EH12 7UQ) is 13.10units of distance to the existing DC.

Table 5.5: Table showing the scores for the node-weighted degree centrality of each node with percentage accuracy of prediction for Glasgow.

	NODE WEIGHTED CENTRALITY AS α VARIES FROM 0.25 TO 1.75								
Node	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = 1\frac{1}{4}$	$\alpha = 1\frac{1}{2}$	$\alpha = 1\frac{3}{4}$	**DISTANCES FROM EXISTING D C**	**RATIO OF DIST FROM THIS NODE TO EXISTING DC & THE FARTHEST SPAN OF NETWORK**	**PERCENT ACCURACY**
1	44871 —	63610 —	90174 —	181213 —	256889 —	364167 —	29.60 —	0.493 —	50.7
3	97681 —	141956 —	206300 —	435704 —	633195 —	920203 —	29.10 —	0.485 —	51.5
4	65360 —	103171 —	162856 —	405786 —	640536 —	1011090 —	28.10 —	0.468 —	53.2
5	126772 —	201789 —	321198 —	813810 —	1295383 —	2061929 —	29.70 —	0.495 —	50.5
11	36548 —	58714 —	94324 —	243435 —	391077 —	628263 —	29.60 —	0.493 —	50.7
13	3896 —	5544 —	7888 —	15971 —	22724 —	32334 —	30.60 —	0.510 —	49.0
15	3199 —	4617 —	6664 —	13884 —	20039 —	28924 —	30.10 —	0.502 —	49.8

Table 5.6: Percentage accuracy of prediction with respect to the clique at Glasgow with all shops located within a 30miles radius of the distribution centre at Livingstone (EH54 8QW).

	$\alpha = 0.25$		$\alpha = 0.5$		$\alpha = 0.75$		$\alpha = 1.25$		$\alpha = 1.5$		$\alpha = 1.75$	
Centrality Type	**Most Central Node**	**Percent Accuracy**	**Most Central Node**	**Percent Accuracy**	**Most Central Node**	**Percent Accuracy**	**Most Central Node**	**Percent Accuracy**	**Most Central Node**	**Percent Accuracy**	**Most Central Node**	**Percent Accuracy**
Link_Weighted Degree Centrality	NODE 11	50.7	NODE 11	50.7	NODE 11	50.7	NODE 11	50.7	NODE 11	50.7	NODE 11	50.7
Link_Weighted EigenVector Centrality	NODE 11	50.7	NODE 11	50.7	NODE 11	50.7	NODE 11	50.7	NODE 11	50.7	NODE 11	50.7
Link_Weighted Betweenness Centrality	NODE 4	53.2	NODE 4	53.2	NODE 4	53.2	NODE 4	53.2	NODE 4	53.2	NODE 4	53.2
Link_Weighted Closeness Centrality	NODE 5	50.5	NODE 5	50.5	NODE 5	50.5	NODE 5	50.5	NODE 5	50.5	NODE 5	50.5
Node_Weighted Degree Centrality	NODE 5	50.5	NODE 5	50.5	NODE 5	50.5	NODE 5	50.5	NODE 5	50.5	NODE 5	50.5
Node_Weighted EigenVector Centrality	NODE 5	50.5	NODE 5	50.5	NODE 5	50.5	NODE 5	50.5	NODE 5	50.5	NODE 5	50.5
Node_Weighted Betweenness Centrality	NODE 5	50.5	NODE 5	50.5	NODE 5	50.5	NODE 5	50.5	NODE 5	50.5	NODE 4	53.2
Node_Weighted Closeness Centrality	NODE 5	50.5	NODE 5	50.5	NODE 5	50.5	NODE 5	50.5	NODE 5	50.5	NODE 5	50.5

Chapter 6

Weighted Marking, Clique Structure and Node-Weighted Centrality in Predicting Location of Structure

6.1 Weighted Marking Method and Clique Structure : NodeWeight Modulated Centrality Measures Applied To Supply Chain Management

6.1.1 Weighted Marking Method

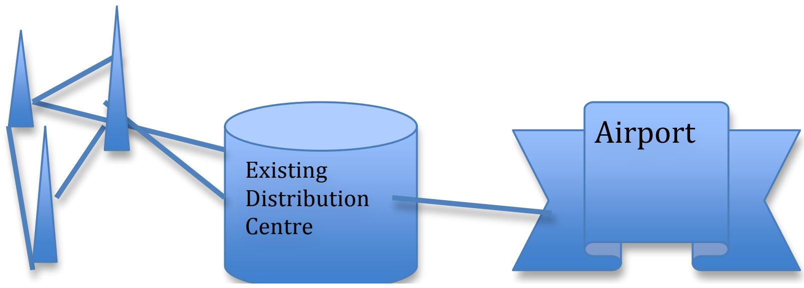

Figure 6.1: Figure showing schematic diagram of Weighted Marking Method, with cones as shops & EDC as existing DC.

Three main stages were proposed by [Thai & Grewal, 2005] in choosing a location for DC using the Weighted Marking Method (WMM):

Stage1 Identification of a general geographical area for DC based on the principle of centre of gravity while considering socio-economic factors. For the Scotland region in our case study, Glasgow and Edinburgh are considered as being the most populated and with tendencies for more economic activities.

Stage2 Identification of alternative locations of DC, these are the shops (cones) as in figure 5.1 whereby EDC is the existing DC. The considered criteria for the cities in stage1 are: Criteria1 - C1 (proximity to customer bases); C2(Expansion capability); C3(percentage of unemployment [to measure availability of labour force]) and C4(Average Income of residents[to measure standard of living]).

Stage3 Selection of specific sites among the alternative locations in Stage2 using quantitative approach after having set a certain treshold (e.g. Composite functions greater than or equal to 5), i.e. the composite point for each node is calculated using the formula below:

$$CompositePoint = \sum_{1}^{4}\{Pointofeachcriteria \times weightingfactorofcriteria\} \qquad (6.1)$$

Thereafter the minimum from the products of Sales Volume and Distance is chosen as in (5.10) below

$$MinVD = min\{Volume\ of\ Sales \times\ Distance\} \qquad (6.2)$$

Applying the technique of [Thai, V.V. & Grewal, D.(2005)], the result of Table 6.1 was obtained: From above, node 79 is the winner and its distance from the existing DC is 14.1 units, therefore the error of prediction is 14.1/60 * 100 = 23.5%, which gives an accuracy of 76.5%.

The network coverage of an existing distribution centre (DC) located at Scotland was investigated and the retail outlets or shops are considered as nodes with the value of sales taken to be the weights on the nodes while distances between nodes are regarded as the weights on the edges. For our sample a 30miles radius coverage of shops from the existing DC was taken and this makes 63nodes all connected by distances (see figure 5.2 below). The nearest DC to this existing one is some 171miles away, so our coverage for this purpose is of 60miles diameter, although this could be extended in future. Out of the community of 63 shops, the Central and Lothian Counties accommodated 33 of these shops while Glasgow city and Edinburgh have 30 of these. The existing DC at Livingston is actually situated in-between these two cities. The clique of shops within Glasgow and Edinburgh were examined and the most central from the two cliques were considered for the prediction of the new DC.

6.2 Clique Structure and Node-Weight Modulated Centrality Measures

The first case study was the region of Scotland and the second was for the region of Northern Ireland. As depicted in figure 5.3 below, the clique of a graph is considered from among which the most central of the nodes is taken to be representative of that clique, which in turn is considered for the prediction test along with the other cliques.

From the Node-Weighted Centrality Measure, the two nodes 5 and 22 are the most central in terms of the nodeweightedness, thereby representing the cliques of Glasgow and Edinburgh respectively.

Tables 6.1 & 6.2 show the respective results that were obtained when the Node-Weight Modulated Centrality Measures are applied to the 7 nodes of Glasgow and 23 nodes of Edinburgh from the supply chain management dataset.

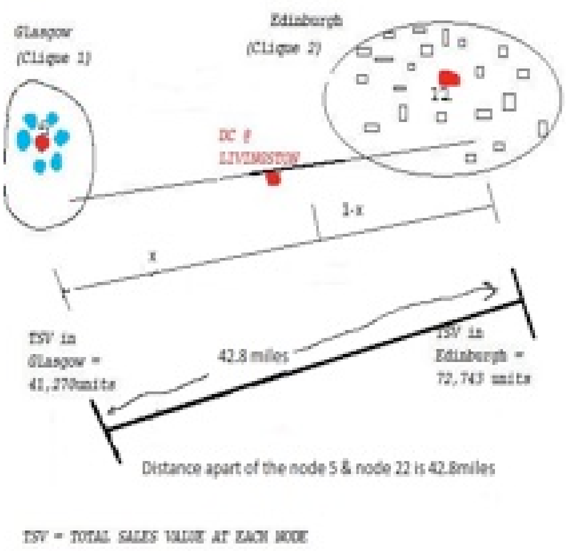

Figure 6.2: Figure showing the coverage of the 30 miles radius of Scotland, cliques at Glasgow & Edinburgh.

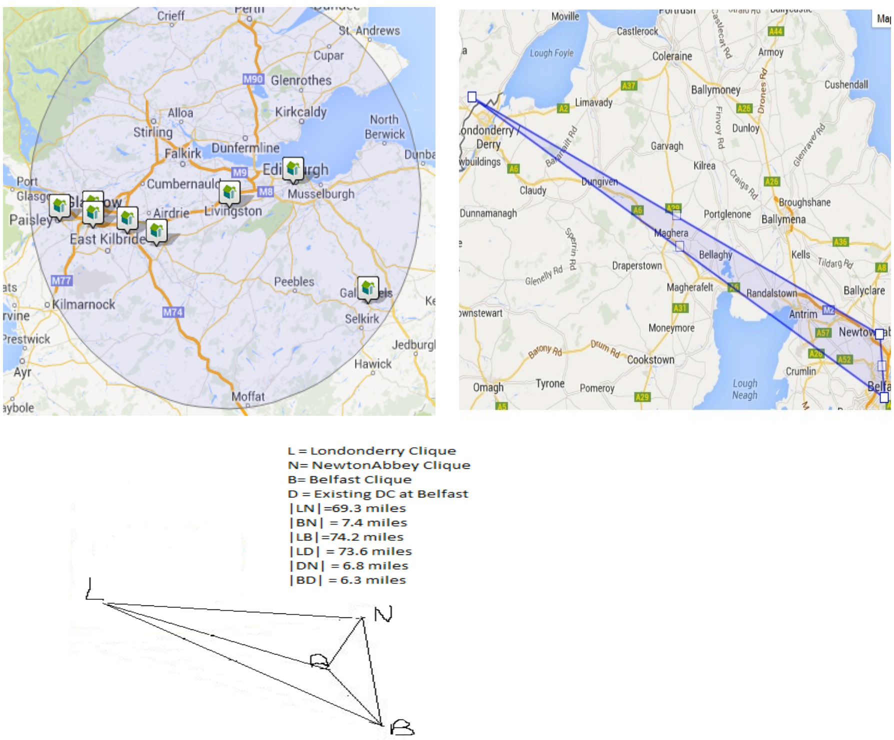

Figure 6.3: Figure showing the cliques at Northern Ireland cities (Londonderry, NewtonAbbey & Belfast)

6.3 Implementation

The initial dataset of the distances between the 30 sales outlets of Glasgow and Edinburghś cliques were presented as a 7x7 square matrix and 23 x 23 square matrix respectively, these were obtained from the UCINET and tnet software, saved in Excel format and later imported into UCINET for the purpose of centralities calculations, see figure 6.2 . The results came out as text files listing the different columns for each centrality measure, and for the purpose of calculations of the node-weight modulated centrality, the values from the text files were exported into Excel where a column was created for the weights on the nodes. Table 6.1 & Table 6.2 depict the node-weighted centrality measures at different values of alpha (in terms of degree, closeness, eigenvector and betweenness).

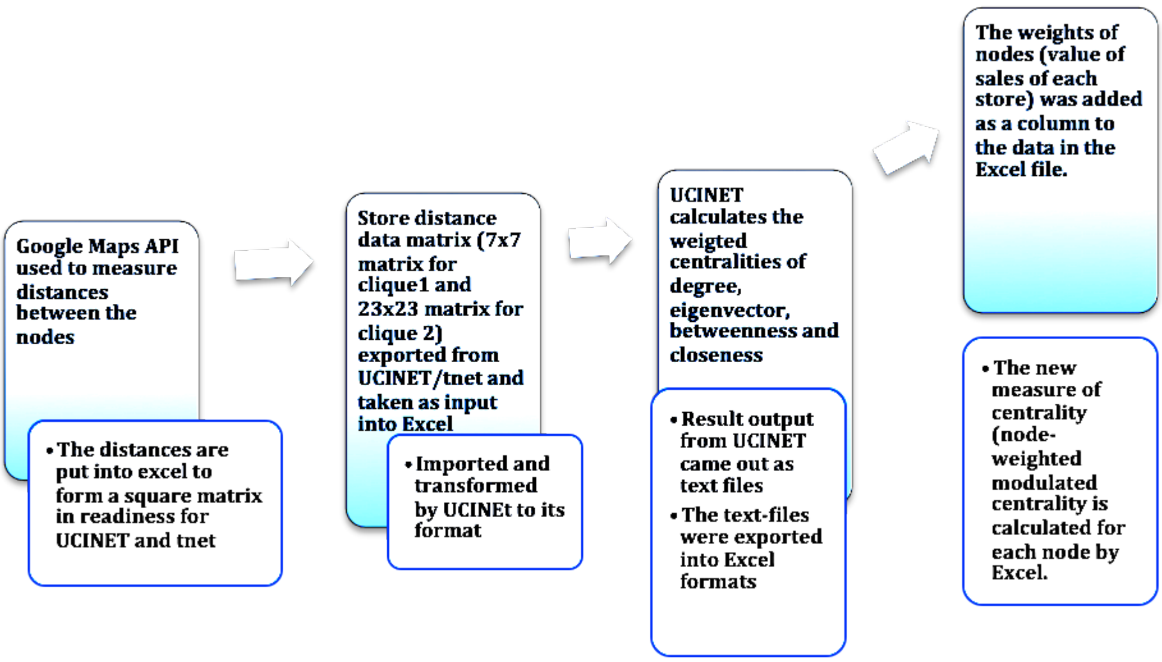

Figure 6.4: Figure showing the implementation of node-weighted centrality measure to the cliques of SCM

The radius of coverage according to figure 6.3 is 30miles, accordingly the farthest possible distance apart of any two nodes will be the diameter of such circle which is 60miles. This was used for the calculation of ratio of distances from any node to that of farthest distance apart. The percentage error of prediction is therefore calculated by multiplying this ratio by 100 and from this emerges the percentage accuracy.

6.4 Clique/Node-Weighted Measure Applied to the Supply Chain Management (SCM)

x is the proportional distance to the proposed Distribution Centre.
TSV = Total Sales Values and the driving distance between node 5 (representing Glasgow clique) and node 22 (representing Edinburgh clique) is 42.8miles.

1-x/x = TSV1 / TSV2

1-x/ x = 41270/72743

x = 0.36 (i.e. 36% of 42.8) which is 13.1miles

If x is some 13.1miles away from the highest sales valued node 22 (Edinburgh) , and the existing DC is 15.4 miles away from the same node 22, the difference of the predicted DC will be 2.3miles away from the existing DC, hence,
the error rate of the predicted DC = (2.3/42.80) x 100 = 5.37%
i.e. the percentage accuracy of the prediction = 94.63%

With all the results above one is now in a position to predict the most probable (regions with respect to the nodes) that could serve as a distribution center for all other outlets considering their node-weighted centrality and clique structures going by the percentage accuracy of the prediction.

Similar argument is also extended to some 51 shops at the Northern Ireland whereby three cliques are considered, that is the cliques at Belfast (14 shops); Londonderry (three shops) and NewtonAbbey (four shops). See figure 6.4 , whereby the centre of mass of the triangle was considered to be the predicted Distribution Centre, while the angles of the nodes are calculated using the cosine rule, see figure 6.5 :

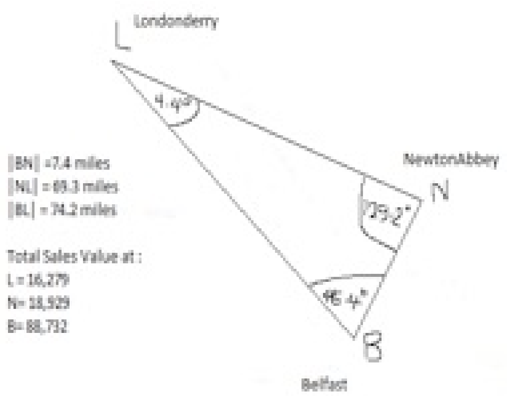

Figure 6.5: Figure showing the angles of triangle BNL, distances apart of the nodes and sales values of each clique.

Since only the distance between the shops are available, from the calculated angles the co-ordinates were arrived at. See figure 6.6 :

From the figure 5.5 above, B(0,0) indicates origin whereby x1= 0 and y1 = 0

a1 = 74.2 Cos46.4 = 51.17

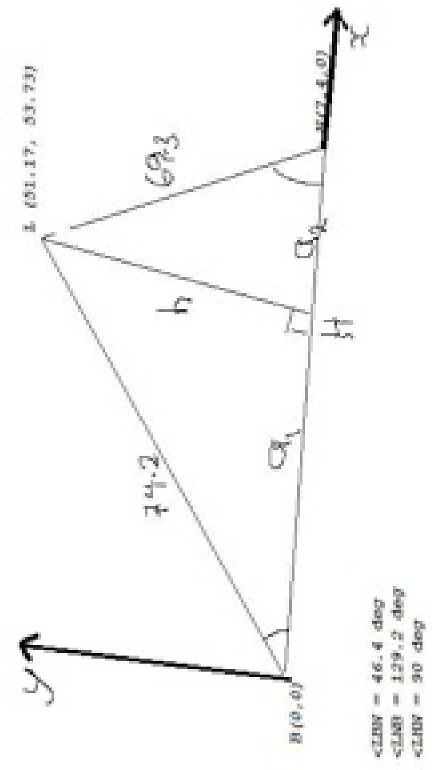

Figure 6.6: Figure showing the co-ordinates of the triangle

h = 74.2 Sin46.4 = 53.73

The co-ordinate of the centre of mass for the triangle BNL is calculated using facts from figure 5.6. above,
whereby x1= 0; x2= 7.4 ; x3=51.17 ; y1 = 0; y2 = 0; and y3 = 53.73
Total sales value at clique B (w1 = 88732) ; at clique N (w2 =18929) and at clique L (w3 = 16279) .
Hence, for the predicted DC (the centre of mass), the co-ordinates are

$$x_{cm} = \frac{1}{n}\frac{\sum_{i=1}^{n} m_i x_i}{M} \tag{6.3}$$

$$y_{cm} = \frac{1}{n}\frac{\sum_{i=1}^{n} m_i y_i}{M} \tag{6.4}$$

where M = $\sum_{i=1}^{n} m_i$ (the total weights on the nodes) and n = number of nodes/vertices.

Substituting in the values from figure 6.6 , equation (6.3) becomes,

x_{cm} =
[0 x 88732]/[88732+18929+16279] + [7.4 x 18929]/[88732+18929+16279] + [51.17 x 16279]/[88732+18929

therefore, x_{cm} =2.62 .

Similarly, y_{cm} = 2.35

So, the predicted DC has co-ordinates (2.62, 2.35), hence the distance from clique B as shown in figure 6.7 below is BP

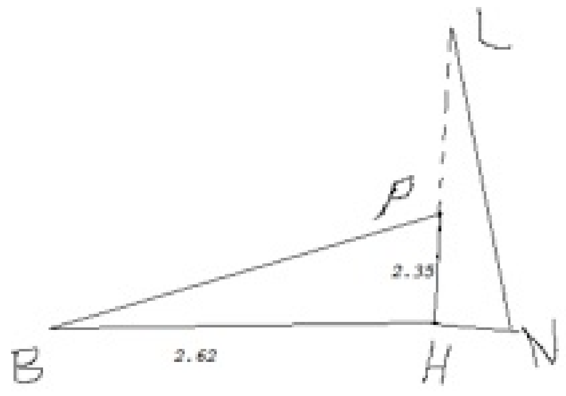

Figure 6.7: Figure showing the carved out portion of triangle BPH from triangle BNL of figure 6.6 above. P is the predicted DC.

BP = $\{2.352^2 + 2.622^2\}^{1/2} = 3.52$

Recall that from figure 6.3, the existing DC is 6.3 miles to the clique at Belfast, so the error is 6.3 - 3.52 = 2.78 , that is, the percentage error is (2.78/73.6 x 100) , the farthest distance apart from the existing DC being 73.6, therefore the percentage accuracy for this prediction is 96.2% .

6.5 Summary and Conclusion

Two case studies were considered in the supply chain management, one considered two cliques with horizontal distance apart (case study of Glasgow clique and Edinburgh cliques in Scotland), while the second case study considered the triangular shaped cliques of (Londonderry, NewtonAbbey and Belfast, in the Northern Ireland).

The results obtained show that the combined weights have an obvious effect on the centralities of the nodes considered as evidenced in the case studies of the Supply Chain Management(SCM). The tuning parameters alpha (whose values range between 0.25 and 1.75) acts as the bounds for the relative importance of number of ties/weight of ties and the tuning parameter beta (whose values are -1 and +1) serves as multiplicative/dividing factors for weights of nodes. Graphs in the SCM were considered and effects of the combined weights on edges (distance between shops) and weights on nodes (sales value for SCM) were evaluated taking the betweenness, closeness, eigenvector and degree centrality into cognisance. The resulting node-weight modulated centrality was then applied to the sales dataset while introducing an additional tuning parameter β thereby making use of two parameters β and α.

Table 6.1: Results Of Weighted Marking Method Applied To Scotland Datasets

				C1		%age of C1		%age of C2	C3		%age of C3	C4		%age of C4		
Node	Units Sold	Store Format	Town/ City	Dist to Existing DC	Evaluation Point	PT* WT Factor	Expansion Capacity	PT* WT Factor	Availabie Workforce	Evaluation Point	PT* WT Factor	Living Standard	Evaluation Point	PT* WT Factor	Composite Pt	Product of Dis to DC· Units Sold
1	4390	Extra	GLAS	29.5	1	0.25	8	2	VH	9	2.25	852	7	1.75	6.25	129505
6	9300	Extra	GLAS	29	2	0.5	8	2	VH	9	2.25	852	7	1.75	6.5	269700
8	10790	Extra	GLAS	30	1	0.25	8	2	FH	7	1.75	860	7	1.75	5.75	323700
55	14780	Extra	EDIN	15	9	2.25	8	2	-	0	0	939	9	2.25	6.5	221700
56	1110	Metro	EDIN	20.3	6	1.5	4	1	VL	3	0.75	867	7	1.75	5	22533
57	1850	Metro	EDIN	17.1	7	1.75	4	1	LST	1	0.25	939	9	2.25	5.25	31635
64	5320	Super	EDIN	20.3	6	1.5	6	1.5	FH	7	1.75	860	7	1.75	6.5	107996
65	10480	Super	EDIN	17.9	7	1.75	6	1.5	FH	7	1.75	860	7	1.75	6.75	187592
66	9400	Super	EDIN	13.1	9	2.25	6	1.5	VH	9	2.25	852	7	1.75	7.75	123140
67	6150	Super	EDIN	21.2	5	1.25	6	1.5	FH	7	1.75	860	7	1.75	6.25	130380
71	130	Express	EDIN	19	7	1.75	2	0.5	FH	7	1.75	810	5	1.25	5.25	2470
72	690	Express	EDIN	19.4	6	1.5	2	0.5	FH	7	1.75	860	7	1.75	5.5	13386
73	200	Express	EDIN	25.8	3	0.75	2	0.5	VH	9	2.25	852	7	1.75	5.25	5160
74	230	Express	EDIN	15.8	8	2	2	0.5	VL	3	0.75	899	7	1.75	5	3634
75	260	Express	EDIN	17.1	7	1.75	2	0.5	FH	7	1.75	860	7	1.75	5.75	4446
77	890	Express	EDIN	22.7	5	1.25	2	0.5	FH	7	1.75	860	7	1.75	5.25	20203
78	390	Express	EDIN	17.7	7	1.75	2	0.5	FH	7	1.75	860	7	1.75	5.75	6903
79	70	Express	EDIN	14.1	9	2.25	2	0.5	VL	3	0.75	860	7	1.75	5.25	987
80	320	Express	EDIN	20.8	6	1.5	2	0.5	FH	7	1.75	860	7	1.75	5.5	6656
82	520	Express	EDIN	16.5	8	2	2	0.5	LO	5	1.25	860	7	1.75	5.5	8580
84	700	Express	EDIN	21.3	5	1.25	2	0.5	FH	7	1.75	860	7	1.75	5.25	14910

Table 6.2: Table Showing The Node-Weighted Centralities at Different alpha and at Beta=1 For Glasgow (Clique1) with centralities values in ́000s

No de	**NODE-WEIGHTED DEGREE CENTRALITY**						**NODE-WEIGHTED EIGEN-VECTOR CENTRALITY**						**NODE-WEIGHTED BETWEENNESS CENTRALITY**						**NODE-WEIGHTED CLOSENESS CENTRALITY**				
	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = 1\frac{1}{4}$	$\alpha = 1\frac{1}{2}$	$\alpha = 1\frac{3}{4}$	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = 1\frac{1}{4}$	$\alpha = 1\frac{1}{2}$	$\alpha = 1\frac{3}{4}$	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = 1\frac{1}{4}$	$\alpha = 1\frac{1}{2}$	$\alpha = 1\frac{3}{4}$	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = 1$	$\alpha = 1\frac{1}{4}$
1	44.9	63.6	90.2	181.2	256.9	364.2	51.5	83.8	136.3	360.7	586.9	954.8	39.3	48.7	60.4	93.0	115.3	143.1	19.5	12.0	7.4	2.8	1.7
3	97.7	142.0	206.3	435.7	633.2	920.2	111.2	183.9	304.3	832.8	1377.8	2279.2	86.7	111.9	144.3	240.2	309.9	399.8	42.5	26.8	17.0	6.8	4.3
4	65.4	103.2	162.9	405.8	640.5	1011.1	73.6	130.9	232.7	735.8	1308.3	2326.2	59.3	85.0	121.9	250.2	358.6	513.9	27.7	18.6	12.4	5.6	3.7
5	126.8	201.8	321.2	813.8	1295.4	2061.9	145.1	264.3	481.5	1597.9	2910.9	5302.9	103.8	135.2	176.2	299.2	389.9	508.1	55.0	38.0	26.3	12.6	8.7
11	36.5	58.7	94.3	243.4	391.1	628.3	41.5	75.9	138.5	461.9	843.5	1540.2	31.7	44.2	61.7	119.9	167.2	233.1	15.7	10.8	7.5	3.6	2.5
13	3.9	5.5	7.9	16.0	22.7	32.3	4.5	7.3	11.9	31.6	51.5	83.9	3.3	4.0	4.8	6.9	8.2	9.9	1.7	1.0	0.1	0.2	0.2
15	3.2	4.7	6.7	13.9	20.0	28.9	3.7	6.1	10.0	27.3	45.1	74.6	2.7	3.4	4.2	6.4	7.9	9.8	1.4	0.9	0.6	0.2	0.2

Table 6.3: Table Showing The Node-Weighted Centralities at Different alpha and at Beta=1 For Edinburgh (Clique 2) with Centralities Values in 0́00s

	NODE-WEIGHTED DEGREE CENTRALITY						NODE-WEIGHTED EIGEN-VECTOR CENTRALITY				NODE-WEIGHTED BETWEENNESS CENTRALITY				NODE-WEIGHTED CLOSENESS CENTRALITY								
Node	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = 1\frac{1}{4}$	$\alpha = 1\frac{1}{2}$	$\alpha = 1\frac{3}{4}$	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = 1\frac{1}{4}$	$\alpha = 1\frac{1}{2}$	$\alpha = 1\frac{3}{4}$	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = 1\frac{1}{4}$	$\alpha = 1\frac{1}{2}$	$\alpha = 1\frac{3}{4}$	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = 1$	$\alpha = 1\frac{1}{4}$
22	441.6	485.4	533.6	644.6	708.5	778.7	453.1	510.9	576.1	732.6	826.1	931.5	303.1	228.7	172.5	98.2	74.0	55.9	203.6	103.1	52.3	13.4	6.8
23	32.2	30.4	28.7	25.6	24.1	22.8	33.9	33.6	33.3	32.7	32.5	32.2	20.6	12.5	7.5	2.7	1.7	1.0	1.5	6.8	3.0	0.6	0.3
24	61.3	66.0	71.0	82.2	88.5	95.3	63.2	70.1	77.7	95.6	106.0	117.5	42.1	31.0	22.9	12.5	9.2	6.8	28.0	13.8	6.8	1.6	0.1
25	74.2	69.4	64.8	56.7	53.0	49.5	78.6	77.8	77.0	75.5	74.7	74.0	46.8	27.6	16.3	5.7	3.3	2.0	3.6	16.7	7.7	1.6	0.1
30	144.7	139.0	133.5	123.1	118.2	113.5	152.8	155.0	157.1	161.6	163.8	166.1	92.1	56.3	34.4	12.9	7.9	4.8	69.5	32.0	14.8	3.1	1.4
31	318.4	338.4	359.6	406.1	431.5	458.6	328.5	360.2	395.0	474.9	520.8	571.0	215.0	154.3	110.7	57.0	40.9	29.4	143.3	685.0	32.8	7.5	3.6
32	290.4	330.4	376.0	487.0	554.1	630.5	297.3	346.4	403.6	547.8	638.2	743.5	198.9	155.0	120.8	73.3	57.1	44.5	132.1	68.3	35.4	9.5	4.9
33	157.6	157.6	157.7	157.8	157.9	156.0	165.5	173.9	182.7	201.6	211.8	222.6	102.4	66.5	43.2	18.3	11.9	7.7	7.5	35.4	16.8	3.8	1.8
35	3.2	3.0	2.9	2.6	2.4	2.3	3.4	3.4	3.4	3.4	3.4	3.4	2.0	1.2	0.1	0.0	0.0	0.0	1.6	0.1	0.0	0.0	0.0
36	19.9	18.7	17.6	15.5	14.6	13.7	21.1	21.0	20.9	20.7	20.6	20.5	12.4	7.3	4.3	1.5	0.1	0.1	9.7	4.5	2.0	0.0	0.0
37	5.0	5.6	6.1	7.5	8.2	9.1	5.3	6.1	7.1	9.5	11.0	12.7	3.2	2.3	1.6	0.1	0.1	0.0	2.4	1.2	0.1	0.0	0.0
38	8.8	9.5	10.4	12.3	13.4	14.6	8.	10.0	11.1	13.8	15.3	17.1	6.1	4.6	3.5	2.0	1.5	1.1	4.0	2.0	1.0	0.0	0.0
39	7.6	7.3	7.0	6.4	6.1	5.8	7.9	7.9	7.9	7.8	7.8	7.8	5.0	3.2	2.0	0.1	0.0	0.0	3.5	1.5	0.1	0.0	0.0
40	4.4	4.9	5.6	7.0	7.9	8.9	4.5	5.3	6.1	8.3	9.6	11.2	2.9	2.2	1.6	0.1	0.1	0.1	2.0	1.0	0.1	0.0	0.0
41	26.9	25.4	24.0	21.5	20.3	19.2	28.3	28.3	28.2	28.0	28.0	27.9	16.6	9.7	5.6	2.0	1.1	0.1	13.1	6.0	2.8	0.1	0.0
42	10.7	10.1	9.6	8.6	8.1	7.7	11.3	11.3	11.2	11.2	11.2	11.1	6.7	3.9	2.3	0.8	0.5	0.3	5.1	2.3	1.0	0.2	0.1
43	2.2	2.5	3.0	4.1	4.7	5.6	2.2	2.6	3.1	4.5	5.4	6.4	1.5	1.2	1.0	0.7	0.5	0.4	1.0	0.5	0.3	0.1	0.0
44	9.0	9.0	9.1	9.3	9.3	9.4	9.4	10.0	10.6	11.8	12.6	13.3	5.8	3.8	2.5	1.1	0.7	0.5	4.2	2.0	1.0	0.2	0.1
45	4.5	4.8	5.1	5.8	6.2	6.7	4.7	5.3	6.0	7.6	8.5	9.6	2.9	2.0	1.3	0.6	0.4	0.3	2.1	1.1	0.6	0.1	0.0
46	16.8	16.6	16.4	15.9	15.7	15.5	17.5	18.0	18.4	19.4	19.9	20.4	11.3	7.4	4.9	2.1	1.4	0.9	7.8	3.5	1.6	0.3	0.2
47	4.0	3.7	3.4	2.9	2.7	2.5	4.2	4.1	4.0	3.8	3.8	3.7	2.5	1.5	0.8	0.3	0.2	0.1	1.9	0.9	0.4	0.0	0.0
48	21.6	21.4	21.1	20.6	20.4	20.2	22.8	23.8	24.8	27.0	28.1	29.3	13.7	8.6	5.4	2.1	1.3	0.8	10.5	5.0	2.4	0.5	0.3
49	16.6	15.4	14.3	12.2	11.3	10.5	17.7	17.4	17.1	16.5	16.3	16.0	10.2	5.8	3.3	1.0	0.6	0.3	8.2	3.8	1.7	3.6	0.2

The resulting predictions in both cases were 94.6% accurate for the Scotland cliques compared with the accuracy of 76.5% obtained with the Weighted Marking Method while 96.2% of accuracy was obtained in the case study involving the Northern Ireland with the clique/node-weighted centrality measure.

Chapter 7

Top Eigen-Vector Weighted Centrality for Predicting Distribution Center Location of a Supply Chain Network

7.1 Top-Eigen Weighted Vector Centrality Applied to a Quadrilateral

The node-weights of the sample used for this study is the sales value while the edges are the driving distances between the shops in the sampled area. The sampled shops here are maximally connected as all of them have road links, hence we take the advantage of the clique structure by making the most central node of the chosen clique to be representative of that clique. By that, we have a representative node each from the four cliques considered for the purpose of the prediction of a proposed DC (see figure 7.1). In the county of Greater Manchester, four cities were chosen for our sample, the city of Manchester (M); Bolton (B); Oldham(L); and Wigan(W). In each of the cities, the ranking of the nodes(i.e. shops) based on eigenvector centrality were considered, tested for all the four centralities (degree, closeness, betweeness and eigenvector), thereafter, the highest ranking node called the top eigenvector weight based was made to be representative of that city (see Table 7.1). The driving distances apart of each of the representative cliques (M, B, L & W) were obtained from google MAPI. UCINET , tnet and Excel software are used for obtaining the centralities and doing the final calculations.

In Table 7.1, Node 18 with postcode BL4 9LS being the highest ranking always, was chosen as the representative of the clique from Bolton when the Top eigen-vector weighted centrality is used. Similar procedure was carried out for the other three nodes of Manchester , Wigan and Oldham, hence the outcome of figure 6.1 In order to find the predicted Distribution Centre (P), the following geometry had to be resolved.

Find $\prec BML$:
$17.9^2 = 1.3^2 + 18.5^2 - 2 * 1.3 * 16.4$ Cos$\prec BML$ (using Cosine rule)
therefore $\prec BML = 60.7^0$
Find $\mid LP \mid$:
$Sin 29.3^o = \mid LP \mid / 18.5$ (using Sine of a right angle)
$\mid LP \mid = 9.1$ units

Table 7.1: Table showing the clique result according to Top Eigen-Vector Weighted Centrality of selection from Bolton City

Centrality Type	$\alpha = \frac{1}{4}$	$\alpha = \frac{1}{2}$	$\alpha = \frac{3}{4}$	$\alpha = 1\frac{1}{4}$	$\alpha = 1\frac{1}{2}$	$\alpha = 1\frac{3}{4}$
Node-Weighted Degree	Node 18	Node 18	Node 18	Node 18	Node 18	Node 18
Node-Weighted Eigen Vector	Node 18	Node 18	Node 18	Node 18	Node 18	Node 18
Node-Weighted Betweenness	Node 18	Node 18	Node 18	Node 18	Node 18	Node 18
Node-Weighted Closeness	Node 18	Node 18	Node 18	Node 18	Node 18	Node 18

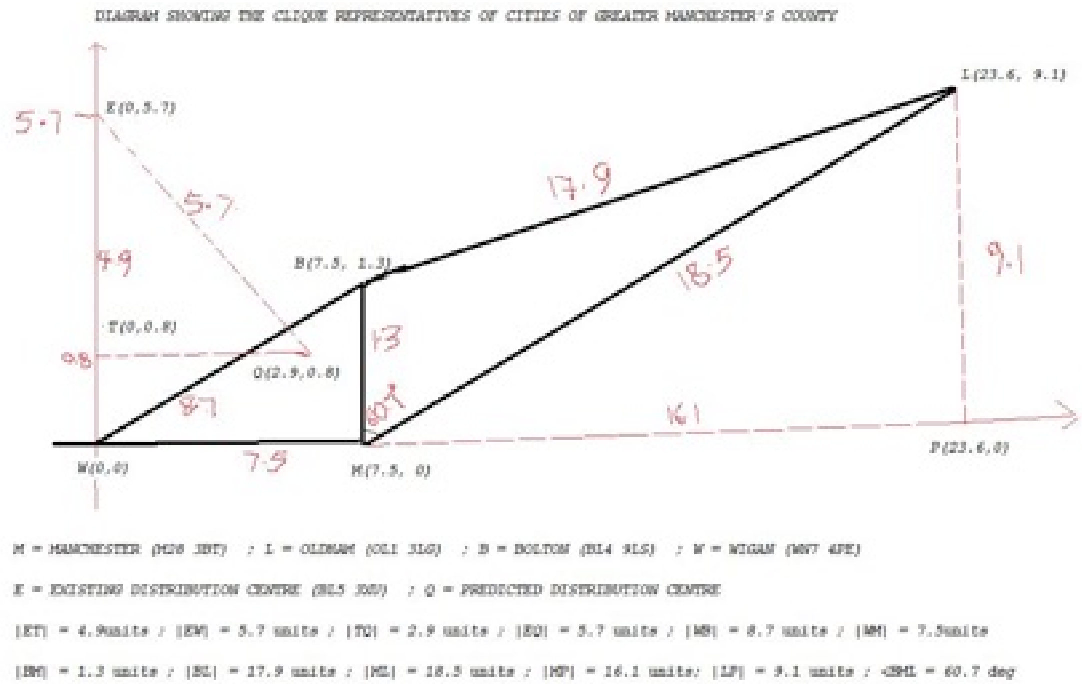

Figure 7.1: Figure showing the clique representative of County of Greater Manchester

Similarly, by using Cosine of a right angle $\mid MP \mid$=16.1units

The mass m_i's (node-weights) of each of the representative node for each clique is as shown below:
Manchester ' M ' = 16,330
Oldham ' L ' = 16,501
Bolton ' B ' = 8,550
Wigan ' W ' = 7,753
Total = 49,134

To find the co-ordinates for the centre of mass (x_c,y_c) which eventually becomes the predicted DC, we use the calculation as shown below:

$$x_c = \frac{1}{n} \frac{\sum m_i x_i}{\sum m} \tag{7.1}$$

= $\frac{1}{4}$ {(16330*7.5 + 16501*23.6 + 8550*7.5 + 7753*0)/49134} =2.93
Similarly,

$$y_{cm} = \frac{1}{n} \frac{\sum m_i y_i}{\sum m} \tag{7.2}$$

= $\frac{1}{4}$ {(16330*0 + 16501*9.1 + 8550*1.3 + 7753*0)/49134} =0.82

Distance from E to Q is then calculated by solving the right angle triangle formed from EQ , which makes $\mid EQ \mid$ = 5.69units.

Therefore, the error rate of prediction is:
$\mid EQ \mid$/(farthest span of any nodes) * 100
= 5.69/80 * 100
=7.11%
Hence the percentage accuracy of prediction = 100% - 7.11% = 92.89%

7.2 Summary/Conclusion

The TEVW centrality provides a more accurate percentage of 92.9% when the Manchester county was introduced. The set of input resources for this method are the nodeweights and link-weights, even though there are other factors to consider in the citing of a distribution centre, this makes this method a cheaper one with high accuracy of prediction. The assumptions in this study is that the driving distances are taken to be a straight line in the model figures, whereas in reality this might not necessarily be so.

It is clear that the node-weights (node attributes) actually count in any network as confirmed in this research whereby it forms the basis of prediction of a distribution centre with a high accuracy while making use of the new centrality measure - Top Eigen-Vector Weighted Centrality (TEVWC).

Chapter 8

Newtonian Gravitational Force for Predicting Distribution Centre Location of a Supply Chain Network

8.1 Top-Eigen Weighted Vector Centrality and Newtonian Gravitational Force

The node-weights of the sample used for this study is the sales value while the edges are the driving distances between the shops in the sampled area. The sampled shops here are maximally connected as all of them have road links. Hence, we take the advantage of the clique structure by making the most central node (the one with highest centrality) from each clique to be a representative of that clique. By that, we have a representative node each from the two cliques considered for the purpose of the prediction of a proposed DC (see figure 8.1 below). In the county of Scotland, two major cities with higher concentration of shops were chosen for our sample, the city of Glasgow and Edinburgh. In each of the cities, the ranking of the nodes(i.e. shops) based on eigen-vector centrality were considered, tested for all the four centralities (degree, closeness, betweeness and eigen-vector), thereafter, the highest ranking node called the top eigen-vector weight was made to be representative of that city (see Table 8.1). The driving distances apart of each of the representative cliques for Glasgow and Edinburgh were obtained from google MAPI. UCINET , tnet and Excel software are used for obtaining the centralities and doing the final calculations (see Figure 8.2).

The newtonian gravitational force was later introduced after the implementation of the Top eigenvector weighted centrality.

8.1.1 Top-Eigen Weighted Vector Centrality

Node 22 with postcode EH12 7UQ being the highest ranking always, was chosen as the representative of the clique from Edinburgh when the Top eigenvector weighted centrality is used. Similar procedure was carried out for Glasgow clique and Node 5 with postcode G21 1YL came out being the representative of that clique.(Figure 8.3 and Figure 8.4)

From Figure 8.4, let x be the proportional distance to the predicted Distribution Centre, and since the driving distance between node 5 (representing Glasgow clique) and node 22

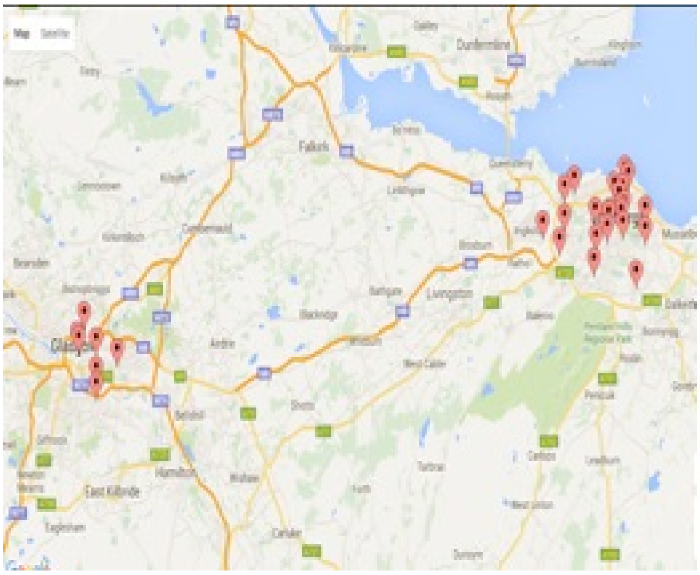

Figure 8.1: Figure showing the two cliques of Scotland shops (Glasgow on the left and Edinburgh on the right)

Figure 8.2: Figure showing the implementation of TopEigen vector weighted centrality measure to the cliques of Scotland

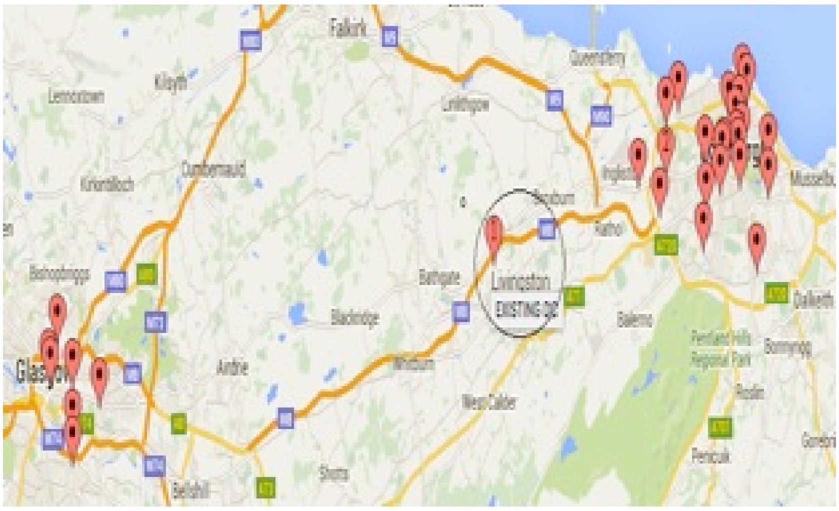

Figure 8.3: Figure showing the Existing DC at Livingston(encircled) and the clique representative node at Edinburgh marked "2".

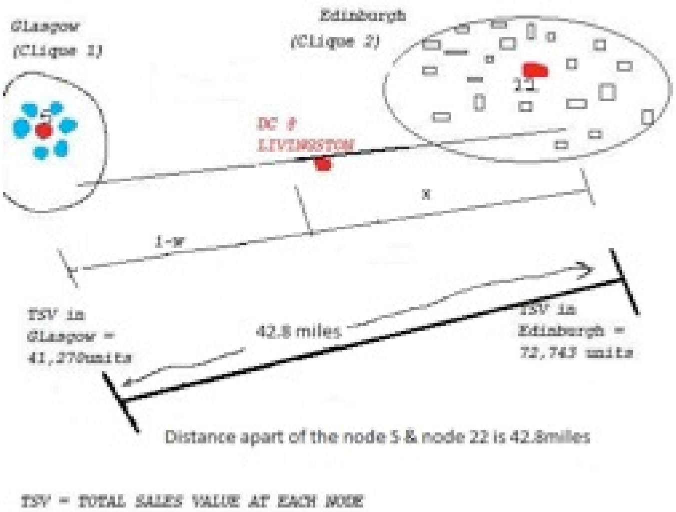

Figure 8.4: Figure showing the representative cliques at Scotland cities of Glasgow and Edinburgh

(representing Edinburgh clique) is 42.8miles, by proportion

1-x / x = 72743/41270, then x = 0.36 (i.e. 36% of 42.8) which is 15.4miles

If x is some 15.4miles away from the Edinburgh clique representative, and the existing DC is 13.1 miles away from node 22, the difference of the predicted DC will be 2.3miles away from the existing DC, hence, the error rate of the predicted DC = (2.3/42.80) x 100 = 5.37% i.e. the percentage accuracy of the prediction = 94.63%

8.2 Newtonian Gravitational Force

This method is fashioned after the Newtonś gravitational law which ascerts that every objectś mass will ascertain some amount of force on any neighboring object, no matter the distance between them. The formula is:

$$F = K \times \frac{m * M}{R^2} \tag{8.1}$$

where
F = Gravitational Force
k = constant
m =the mass of the first object
M=the mass of the second object
R=Distance between the two objects (it can be driving distance or the earth distance)

In case of the objects, which in this case are the 30 shops of Scotland (consisting of seven shops from Glasgow and 23 shops from Edinburgh) as shown in Figure 8.4 . The shops have pull effects on the DC at Livingston, as such the vectorial resultant force F of each node(shop) is calculated using the earth distances apart and the driving distances apart.

8.2.1 Earth Distance with 30 shops/nodes

When the representative clique (EH12 7UQ , i.e. Node 22) was used as origin (leaving 29 shops for consideration) as shown in Figure 8.5 , the total force is 314.53units but when the actual DC for Scotland (EH54 8QW) was used as origin (as in Figure 8.3) for all 30shops the total force was 12.28units.

In the Figure 8.5 above, the point marked "1" is the representative clique (node 22) EH12 7UQ. This node is used as the origin for the other 29nodes in the region of Glasgow and Edinburgh, which is, excluding the existing DC (EH54 8QW) at Livingston.

To make things clearer, the figure 8.6 below shows the existing DC - EH54 8QW (at Livingston) as "1" while "2" represents the predicted DC - EH12 7UQ (at Edinburgh)

8.2.2 Driving Distance with 30 shops/nodes

For the driving distance, the total force for the DC as origin is 1,394,170.15 while the representative clique as origin yielded 29,690,905.18 . The Table 8.2 summarises the findings of the resultant forces when each of the driving distances and earth distances is used in the calculations.

Table 8.1: Table of the clique result according to Top Eigen-Vector Weighted Centrality of selection from Edinburgh & Glasgow

NODE-WEIGHTED EIGEN-VECTOR CENTRALITY FOR EDINBURGH							**NODE-WEIGHTED EIGEN-VECTOR CENTRALITY FOR GLASGOW**						
No de	α $=\frac{1}{4}$	α $=\frac{1}{2}$	α $=\frac{3}{4}$	α $=1\frac{1}{4}$	α $=1\frac{1}{2}$	α $=1\frac{3}{4}$	**No de**	α $=\frac{1}{4}$	α $=\frac{1}{2}$	α $=\frac{3}{4}$	α $=1\frac{1}{4}$	α $=1\frac{1}{2}$	α $=1\frac{3}{4}$
22	453097.5	510928.4	576140.5	732596.9	826101.5	931540.6	1	51495.5	83776.9	136294.6	360734.7	586870.9	954766.6
23	33870.3	33585.7	33303.6	32746.4	32471.3	32198.5	3	111193.7	183949.4	304310.2	832822.6	1377750.4	2279232.3
24	63165.7	70052.7	77690.6	95555.4	105973.8	117528.2	4	73621.3	130901.3	232747.0	735808.2	1308292.6	2326189.8
25	78587.1	77797.5	77015.8	75475.8	74717.4	73966.6	5	145086.8	264307.3	481493.6	1597913.3	2910948.7	5302930.0
30	152807.8	154952.0	157126.2	161566.5	163833.5	166132.3	11	41543.2	75860.3	138525.5	461912.0	843478.6	1540241.6
31	328540.4	360246.8	395013.1	474935.1	520769.5	571027.4	13	4465.1	7280.7	11871.6	31563.2	51465.8	83918.1
32	297308.6	346383.0	403557.8	547777.5	638195.0	743537.0	15	3662.5	6052.2	10001.1	27309.7	45128.5	74573.5
33	165476.9	173857.7	182662.9	201633.7	211845.6	222574.7							
35	3412.6	3404.2	3395.9	3379.3	3371.0	3362.7							
36	21128.7	21025.6	20923.1	20719.5	20618.5	20518.0							
37	5285.0	6115.7	7076.9	9476.3	10965.7	12689.2							
38	8971.3	9984.7	11112.6	13764.9	15319.7	17050.2							
39	7943.9	7919.5	7895.2	7846.8	7822.7	7798.7							
40	4531.7	5270.7	6130.4	8293.1	9645.7	11218.9							
41	28334.9	28259.3	28183.9	28033.6	27958.7	27884.1							
42	11302.1	11272.0	11242.1	11182.4	11152.7	11123.0							
43	2201.3	2631.5	3145.8	4495.6	5374.3	6424.6							
44	9406.4	9964.9	10556.6	11847.4	12550.8	13296.0							
45	4698.1	5294.3	5966.3	7576.8	8538.3	9622.0							
46	17504.4	17952.4	18411.8	19366.2	19861.8	20370.1							
47	4229.0	4130.8	4034.8	3849.6	3760.1	3672.8							
48	22823.0	23793.4	24805.0	26959.2	28105.4	29300.4							
49	17663.5	17373.2	17087.6	16530.6	16258.9	15991.7							

Figure 8.5: Figure showing the representative clique of Edinburgh herein marked "1" with other shops in Scotland

Figure 8.6: Figure showing the Existing DC at Livingston and the representative clique at Edinburgh

Table 8.2: Table showing the total force for Earth and Driving forces for Scotland shops

S/No.	**Type of Distance**	**Existing Distribution CentreC**	**Predicted Distribution Centre**
1	EARTH DISTANCE	1.23 E01	6.0 E01
2	DRIVING DISTANCE	1.39 E06	4.76 E06

8.3 Sample Nodes and Outliers

We consider three additional shops which are outliers , that is, not within Glasgow and Edinburgh but within an increased coverage radius of 36miles against the previous 30miles radius. This means we now consider 33 shops as our sample instead of the previous 30 shops, these newly added shops are at South Queenferry, Hardington and Bathgate. With these additional three shops added from within Scotland but outside Glasgow and Edinburgh, we have the results in Figure 8.7 below:

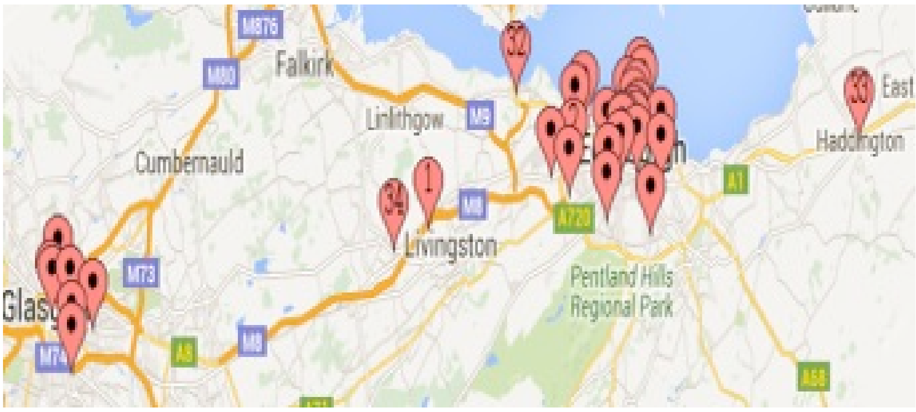

Figure 8.7: Figure showing newly added nodes 32, 33 & 34 outside Glasgow and Edinburgh

The details of the new shops/nodes are as shown in the Table 8.3 :

Table 8.3: Table showing details of the three new nodes added to the existing 30 nodes/shops

Node	**Post Code**	**Distance to Existing Distribution Centre**	**City**	**Sales Value**	**Latitude**	**Longitude**
32	EH30 9QZ	11.9	SOUTH QUEENS-FERRY	7948	55.9828	3.3990
33	EH41 3LZ	36.4	HADDINGTON	9358	55.9571	2.7777
34	EH48 2ES	3.8	BATHGATE	13746	55.8936	3.6215

With the addition of the three new shops and using each one as the origin to the remaining 32 shops, Table 8.4 compares the results with the existing DC and former representative clique node using centrality measures.

Table 8.4: Table showing the total force for Earth and Driving forces for Scotland with additional three shops as new origins

S/No	Type of Distance	Existing DC	Predicted DC	New Shop1 (EH30 9QZ) as Origin	New Shop2 (EH41 3LZ) as Origin	New Shop3 (EH48 2ES) as Origin
1	EARTH DIS- TANCE	1.98 E01	1.3 E02	2.8 E01	6.6 E01	4.4 E01
2	DRIVING DIS- TANCE	6.1 E06	2.3 E07	1.1 E07	1.3 E07	5.9 E06

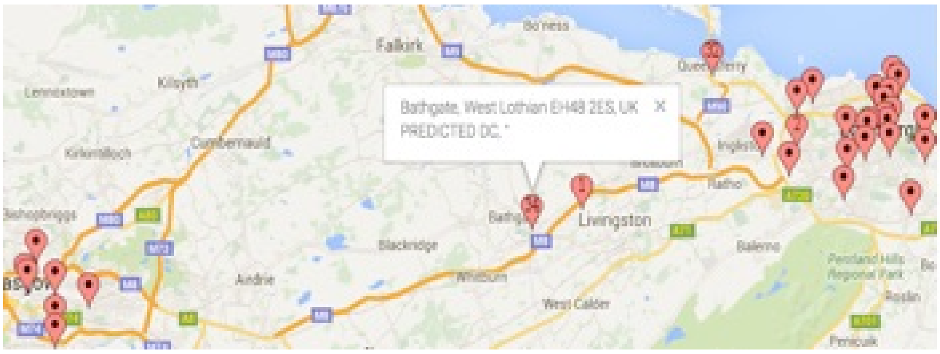

Figure 8.8: Figure shows the newly predicted DC as against the earlier predicted node labeled 2

8.4 Newtonian Gravitational Force with 30 nodes of Glasgow and Edinburgh

Using the Earth distance between the shops and the Existing Distribution Centre (EDC) as origin, we have the results in Table 8.5:

Using the driving distance between the shops and the Existing Distribution Centre (EDC) as origin, we have the results in Table 8.6:

8.5 Newtonian Centrifugal Force with 33 shops of Glasgow and Edinburgh

Using the driving distance between the shops and the Existing Distribution Centre (EDC) as origin, we have the results in Table 8.7 :

Using the earth distance between the shops and the Existing Distribution Centre (EDC) as origin, we have the results in Table 8.8 :

8.6 Summary of Accuracy with the Sales Values Used as Node-Weights

It is hereby summarised in Table 8.9

8.7 Conclusion

The Newtonian Gravitational force provides a more accurate percentage of 4.4% more than when the TEVW centrality was applied. The set of input resources for this method are the node-weights and link-weights, even though there are other factors to consider in the citing of a distribution centre, this makes this method a cheaper one with high accuracy of prediction. The assumptions in this study is that the driving distances are taken to be a straight line in the model figures in this study, whereas in reality this might not necessarily be so.

Table 8.5: Table showing the forces exerted by highest/lowest valued nodes while considering earth distance

	GLASGOW			EDINBURGH			
	Node	**Post Code**	**Value of Force**	**Node**	**Post Code**	**Value of Force**	**Distance Apart of Nodes**
Highest Value Nodes	Node5	G21 1YL	1.5890	Node22	EH12 7UQ	1.7703	42.2
Lowest Value Nodes	Node15	G1 1EJ	0.0447	Node43	EH12 9BH	0.0080	41.3

Table 8.6: Table showing the forces exerted by highest/lowest valued nodes while considering driving distance

	GLASGOW			EDINBURGH			
	Node	**Post Code**	**Value of Force**	**Node**	**Post Code**	**Value of Force**	**Distance Apart of Nodes**
Highest Value Nodes	Node5	G21 1YL	57189.34	Node8	EH12 7UQ	404474.18	2.2
Lowest Value Nodes	Node15	G1 1EJ	1549.52	Node45	EH8 7NG	1081.94	53.7

Table 8.7: Table showing the forces exerted by highest/lowest valued nodes while considering driving distance

	GLASGOW			EDINBURGH			
	Node	**Post Code**	**Value of Force**	**Node**	**Post Code**	**Value of Force**	**Distance Apart of Nodes**
Highest Value Nodes	Node5	G21 1YL	66150.20	Node52A	EH48 2ES	4184638.00	27.7
Lowest Value Nodes	Node15	G1 1EJ	1792.31	Node45	EH8 7NG	1251.47	53.7

Table 8.8: Table showing the forces exerted by highest/lowest valued nodes while considering earth distance

	GLASGOW			EDINBURGH			
	Node	**Post Code**	**Value of Force**	**Node**	**Post Code**	**Value of Force**	**Distance Apart of Nodes**
Highest Value Nodes	Node5	G21 1YL	2.0218	Node22	EH12 7UQ	2.2525	42.2
Lowest Value Nodes	Node15	G1 1EJ	0.0568	Node43	EH12 9BH	0.0102	41.3

Table 8.9: Accuracy of results obtained for both earth/driving distances for 30 shops and 33 shops

	Percentage Accuracy of Highest Force Nodes From Glasgow to Edinburgh	**Percentage Accuracy of Lowest Force Nodes From Glasgow to Edinburgh**
EARTH DISTANCE WITH 30 SHOPS	64.9%	63.2%
EARTH DISTANCE WITH 33 SHOPS	99.1%	99%
DRIVING DISTANCE WITH 30 SHOPS	64.9%	79.9%
DRIVING DISTANCE WITH 33 SHOPS	63.5%	84%

8.8 Summary

It is clear that the node-weights (node attributes) actually count in any network as confirmed in this research whereby it forms the basis of prediction of a distribution centre with a higher accuracy while making use of the newtonian gravitational force as compared with the centrality measure - Top Eigen-Vector Weighted Centrality (TEVWC).

Chapter 9

Future Studies, Summary and Conclusion

9.1 Future Studies

Future Studies:
(i) The concept developed herein shall be applied to real data aggregate from cloud data centres while taking betweenness centrality into consideration.
(ii) The tuning parameters α and β can each be considered as a range of values, thereby introducing dynamism as a measure of centrality, In future, the range of values for α might transcend the range of $\frac{1}{4}$ and $1\frac{3}{4}$ as some interesting outcomes might surface.
(iii) While considering the centrality for the location of the high/low energy consuming nodes/edges, other important factors to take into cognizance are the Power Usage Effectiveness (PUE) and Traffic Density Ratio (TDR).
(iv) So far, the links considered is just one link/edge between two nodes, whereby in this case the weights on the links are distances. Occasion could arise in which case there could be more than one link between two nodes, e.g communication bandwidth within two nodes and physical distances between them could be combined in future, this could be a more complex problem to solve in the future.
(v) This scenario is presently applied to the energy efficiency in the data centres, authorship network (Social network analysis) and supply chain management system. In future one can consider applications on issues arising from the distribution systems of wireless ad-hoc networks, educational system (e.g. siting of new institutions), security issues (siting of the most sensitive and central nodes) in a network and the models could be further extended to other datasets such as in the area of disease control, whereby the model can be used to detect the most central region where epidemic diseases are proned to spread easily or to find the most vulnerable group in the society to an epidemic disease. Here the node weight could be the preponderance of an infectious disease in a particular node and the edge weight will be the distance apart from of highly infected nodes to other nodes in such a graph.
The domain of application could still be further expanded to cover area such as bioinformatics whereby the visualisation and understanding of biology networks will make one to be able to predict the reaction of cells to pharmaceutical drugs due to their positioning in such a network.
Healthcare is another area of consideration, as the study of the connections between hospi-

tals, patients, doctors and healthworkers can help a lot in the prediction of where to cite new hospitals and even how to arrest or prevent epidemics.
In terms of network security, a more central node is protected and given more attention in order to prevent or repel attacks from any form of intrusion.

9.2 Original Contributions

My original contributions are :
(i) Introduction of an additional tuning parameter β (spanning from -1 to +1) thereby making use of two parameters α and β.
(ii) Mixed-Mean Centrality
(iii) Introduction of node-weighted degree centrality as measures in predicting the citing or allocation of resources.
(iv) Node-Weighted Eigen-Vector Centrality
(v) Top Eigen-Vector Weighted Centrality and Newtonian Gravitational Force .

Finally, the workdone in the past and on-going can be described briefly as below:

Table 9.1: Table of the past research on the area of Degree centrality measure

	Centrality Measure	**Author**	**Year**	**Concern**
1	Degree Centrality	Granovetter	1973	Edges (i.e. number of edges connecting focal nodes)
2	Degree Centrality	Freeman	1978	Edges (i.e. number of edges connecting focal nodes)
3	Degree Centrality	Barrat et al	2003	Edges (i.e. number of edges connecting focal nodes)
4	Degree Centrality	Brandes	2001	Edges (i.e. number of edges connecting focal nodes)
5	Degree Centrality	Borgatti	2009	Edges (i.e. number of edges connecting focal nodes)

Table 9.2: Table of the past research on the area of Closeness centrality measure

	Centrality Measure	Author	Year	Concern
6	Closeness Centrality	Granovetter	1973	Edges (i.e. shortest path between the source node and the target node)
7	Closeness Centrality	Freeman	1978	Edges (i.e. shortest path between the source node and the target node)
8	Closeness Centrality	Brandes	2001	Edges (i.e. shortest path between the source node and the target node)
9	Closeness Centrality	Barrat et al	2003	Edges (i.e. shortest path between the source node and the target node)
10	Closeness Centrality	Borgatti	2009	Edges (i.e. shortest path between the source node and the target node)

Table 9.3: Table of the past research on the area of Eigen-Vector centrality measure

	Centrality Measure	Author	Year	Concern	
11	Eigenvector Centrality	Bonacich P.	1972	Edges(Status scores of nodes)	
12	Eigenvector Centrality	Bonacich P.	1987	Edges (Power and Centrality of measures)	
13	Eigenvector Centrality	Costenbader E.	Valente T.W	2003	Edges(Correlations between centralities)
14	Eigenvector Centrality	Valente T.W et al	2008	Edges (Correlations and Regressions among centralities)	

Table 9.4: Table of the past research in the area of Betweenness centrality measure

	Centrality Measure	Author	Year	Concern
15	Betweenness Centrality	Granovetter	1973	Edges and nodes (i.e. number of shortest paths that pass through a particular node)
16	Betweenness Centrality	Freeman	1977to1978	Edges and nodes (i.e. number of shortest paths that pass through a particular node)
17	Betweenness Centrality	Brandes	2001	Edges and nodes (i.e. number of shortest paths that pass through a particular node)
18	Betweenness Centrality	Newman	2001	Edges and nodes (i.e. number of shortest paths that pass through a particular node)
19	Betweenness Centrality	Borgatti	2009	Edges and nodes (i.e. number of shortest paths that pass through a particular node)

Table 9.5: Table of the past and On-going research in the other areas of centrality measure

	Centrality Measure	Author	Year	Concern
20	Generalised Degree Centrality	Opsahl	2010	Edges (i.e. Number of edges and weights of edges)
21	Topological Centrality	Zhuge	2010	Edges and nodes (i.e. number of edges and number of nodes)
22	Node-Weight Modulated Centrality	Akanmuś Thesis	2012/13/14/16	Edges and nodes (i.e. mergers of numbers/weights of edges and nodes)

Table 9.6: Advantages/Disadvantages of the Degree, Closeness and Betweenness Centrality Measures

	Centrality Measure	**Advantage**	**Disadvantage**	**Recommended Scenario**
1	Degree Centrality	It is easier to calculate ,as only the local ties to the focal node is considered, and also, the focal node with the highest centrality has more nodes to interact with.	The global structure of the network is not taken into consideration. There could be many ties from other nodes to the focal node but the distances apart may not be close enough for good interaction.	Where farness is not important.
2	Closeness Centrality	Takes into consideration the shortest distance from the focal node to all other nodes.	It lacks applicability to disconnected components.	Applicable where proximity is of essence.
3	Betweenness Centrality	This determines the control or flow of resources through the shortest path between the nodes.	It can create a bottleneck due to the pressure on the focal node that sits in-between the shortest paths between nodes.	Recommended for a scenario whereby there is a need for censorship and control.

Table 9.7: Advantages/Disadvantages of the Eigen-Vector and Generalised Degree Centrality Measures

	Centrality Measure	**Advantage**	**Disadvantage**	**Recommended Scenario**
4	Eigenvector Centrality	It is global because it takes into consideration the importance of the nodes to which a focal node is connected, therefore it includes the whole network structure.	Nodes that are not attched or connected to more centralised nodes are penalised, even though they might have their own local importance.	Recommended for a scenario whereby greater outreach connectivity is needed.
5	Generalised Degree Centrality	Considered the number of the ties and weights attached to the ties, thereby improving on the idea of dichotomous weightedness.	There are no weights attached to the nodes, only the ties get weights.	Recommended for a scenario whereby it is of no importance to attach weights to the nodes.

Table 9.8: Advantages/Disadvantages of the Topological and Node-Weighted Centrality Measures

	Centrality Measure	**Advantage**	**Disadvantage**	**Recommended Scenario**
6	Topological Centrality	Considered the topological positions of nodes and ties as well as influence between nodes and ties.	The nodes have no weights attached.	Recommended for situations where there is need to measure influence between the ties and nodes.
7	Node-Weight Modulated Centrality	Considered the number of ties, weights on the ties and weights attached to the nodes.	It does not take care of the momentary dynamism of the weights attached to the ties and nodes. Discrete values of α and β were only considered.	Recommended for situation whereby the weights of the nodes, ties and consideration for different centrality measures is important.

9.3 References

Abbasi, A.(2013). H-Type hybrid centrality measures for weighted networks. Scientometrics (2013) 96:633-640.
DOI . 10.1007/s11192-013-0959-y

Abbasi, A. & Hossain, L. (2013). Hybrid centrality measures for binary and weighted netwroks. In Complex networks (pp.1-7). Springer Berlin Heidelberg.

Abbasi, A., Wigand, R. T. & Hossain, L.(2014). Measuring social capital through network analysis and its influence on individual performance. Accessed from `http://works.bepress.com/alireza_abbasi/21` on 03 Mar, 2015.

Akanmu, A.A.G., Wang, F.Z & Yamoah, A. F.(2014a). Clique Structure and Node-Weighted Centrality Measures for Predicting Distribution Centre Location in the Supply Chain Management", IEEE Technically Co-Sponsored Science and Information Conference Aug.27-29, 2014, London UK.

Akanmu, A.A.G., Wang, F. Z. & Yamoah, A. F.(2014b). Weighted Marking, Clique Structure and Node-Weighted Centrality Measures to Predict Distribution Centre?s Location in a Supply Chain Management. International Journal of Advanced Computer Science and Applications. Vol.5, No. 12.

Akanmu, A. A. G., Wang, F. Z. and Yamoah, F. A.(2013). Using Weighted Centrality Measures to Predict Distribution Centre Location in Supply Chain Management. ICCCSS 2013 : International Conference on Cloud Computing and Services Science, World Academy of Science and Technology, Dubai, October 22-23, 2013. WASET 82, 2013, pp1235-1242.

Akanmu, G.A.A., Wang, F. Z. & Chen, H(2012). Introducing weighted nodes to evaluate the cloud computing topology. Journal of Software Engineering and Applications, 2012, 5, 961-969. Special Issue (Vol. 5 No. 11A, 2012) of JSEA. www.scirp.org/journal/jsea

Avela, V. (2011). Guidance for Calculation of Efficiency (PUE) in Data Centres. White Paper 158. Schneider Electric's Data Centre Science Center.

Barrat, A., Barthelemy, M., Pastor-Satorras, R., Vespignani, A., (2004). The Architecture of Complex Weighted Networks. Proceedings of the National Academy of Sciences 101 (11), 3747-3752. arXiv:cond-mat/0311416

Basmadjian, R., De Meer, H., Lent, R. & Giuliani, G. (2012). Cloud Computing and Its Interest in SavingEnergy: the Use Case of a Private Cloud. Journal of Cloud Computing: Advances, Systems and Applications.
Vol.1 No.5. Springer.

Berl, A. et al(2010). Energy-Efficient Cloud Computing. The Computer Journal. Vol.53 No.7. Oxford University Press.pp1045-1051

Bonacich, P. & Lu, P.1(2012). Introduction to Mathematical Sociology. Princeton University Press. USA.

Bolder Technologies (2014). Multiple Distance Calculator. Accessed from http://www.boldertechnologies.com/mdcm/index.jsp on 15 Feb 2014.

Borgatti, S.P., Everett, M.G. and Freeman, L.C. 2002. Ucinet for Windows: Software for Social Network Analysis. Harvard, MA: Analytic Technologies.

Borgatti, S.P. & Halgin, D.S.(2011) . On Network Theory. Organization Science, Articles in Advance, pp.1-14. Informs.

Borgatti, S.P & Li, X(2009). On Social Network Analysis In A Supply Chain Context. Journal Of Supply Chain Management. Volume 45, Issue 2, pp 5-22.

Brandes, U.(2001). A Faster Algorithm for Betweenness Centrality. Journal of Mathematical Sociology 25, 163-177.

Bruce, H. (2009). Introduction to Network Mathematics. Accessed from http://webmathematics.net/\#undirected_directed on 02 Feb 2014.

Carolan,B.V.(2014). Social Network Analysis and Education: Theory, Methods & Applications. SAGE Publications, U.S.A.

Dudin, E. B. & Smetanin, Y. G. (2011). A Review of Cloud Computing. Scientific and Technical Information Processing, Vol.38, No.4, pp.280-284. Allerton Press Inc.

Eriksson, B., Berggren, E. & Froschl, F.(2012). The Cloud-When and Why?. SuccessFactors? Whitepaper, accessed from http://www.cloudtweaks.tradepub.com/free/w_su54/prgm.cgi on 16 Aug 2012

Familypedia. Subdivisions of Scotland. Accessed from http://familypedia.wikia.com/wiki/Subdivisions_of_Scotland on 10 Feb. 2014

Fortunato, S., Latora, V. and Machiori, M. (2004). Method to find community structures based on Information centrality. The American Physical Society. Physical Review E70, 056104 (2004)

Frank, K. A. (1995). Identifying cohesive subgroups. Social Networks 17 (1995) 27 ? 56. N.H. Elsevier.

Freeman, L.C. (1978). Centrality in Social Networks: Conceptual Clarification. Social Networks 1, 215-239. Accessed from http://moreno.ss.uci.edu/27.pdf on 06 June, 2012.

Freeman, S.C. & Freeman, L.C. (1979). The Networkers Network: A study of the impact of a new communications medium on sociometric structure. Social Science Research Reports S46. University of California, Irvine, CA.

Ghoshal, S., & Bartlett, C. A(1990). The Multinational Corporation as an Interorganizational Network. Academy of Management Review,. Vol.15, No.4, 603-625

Google(2012). Efficiency How We Do It. accessed on 01 Oct, 2012 from http://www.google.com/about/datacenters/efficiency/internal/index.html#measuring-efficiency

Google Map JavaScript. (2013). Distance Matrix Service. Accessed from https:/\/developers.google.com/maps/documentation/javascript/examples/distance-matrix on 15 Feb 2014.

Google Map of Selected Locations In UK(n.d). Accessed from http://maps.google.co.uk/maps?hl=en&tab=wl on 02 April, 2013.

Granovetter, M. S. (1973). The Strength of Weak Ties. American Journal of Sociology, University of Chicago Press. pp. 1360-1380.

Green ICT Strategies. Accessed 24 April, 2013 from http://scis.athabascau.ca/graduate/comp635syllabus.php

Hyoungshick, K., John, T., Ross, A., Mascolo, C. (2012). Centrality prediction in dynamic human contact networks, Computer Networks, Volume 56, Issue 3, 23 February 2012, Pages 983-996, ISSN 1389-1286,10.1016/j.comnet.2011.10.022. http://www.sciencedirect.com/science/article/pii/S1389128611003975

Jing, S. Y, Ali, S., She, K. & Zhong, Y. (2011).State-of-the-art Research Study For Green Cloud Computing. Journal Of Supercomputing. Springer

Klaus, J. S. (2012). Data Center Energy: Past, Present and Future. Accessed from http://www.datacenterjournal.com/it/data-center-energy-past-\\present-and-future-part-one/?goback=2Eanb_61513_*2_*1_*1_*1_*1_*1 on 30 Sep 2012.

Koomey Jonathan (n.d). ?Growth in Data Centre Electricity Use 2005 to 2010? Analytic Press, Oakland, CA. Accessed 24 April 2012 from http://www.analyticspress.com/datacenters.html

Krackhardt, D. (1994). ?Graph Theoretical Dimensions of Informal Organizations.? In Kathleen Carley & Michael Prietula (eds.), Computational Organizational Theory. Lawrence Erlbaum Associates, Inc, pp.89-111.

Latora, V. & Marchiori, M. (2004). How the science of complex networks can help developing strategies against terrorism. Chaos, Solitons & Fractals 20(2004) 69-75.

Latora, V. & Marchiori, M. (2007). A measure of centrality based on network efficiency.

New Journal of Physics 9 (2007) 188.

Liu, J. et al (2015). A new method to construct co-author networks. Physica A 419 (2015) 29-39. Elsevier.

Michael, H. et al (2009). Above The Cloud: A Berkeley View of Cloud Computing. Electrical Engineering and Computer Sciences.University of California at Berkeley. Technical Report No. UCB/EECS-2009-28. Accessed from http://www.eecs.berkeley.edu/Pubs/TechRpts/2009/EECS-2009-28.html

Murugesan, S. (2008). "Harnessing Green IT: Principles and Practices," IT Professional, vol. 10, no. 1, pp. 24-33, Jan.-Feb. 2008, doi:10.1109/MITP.2008.10

Newman, M.E.J. (2001). Scientific Collaboration networks. II. Shortest paths, weighted networks, and centrality. The American Physical Society. Physical review E, Volume 64, 016132

Newman, M. E. J., 2002. Assortative mixing in networks. Physical Review Letters 89 (208701).

Newman, M. E. J. (2008). The mathematics of networks. The New Palgrave Encyclopedia of Economics, 2, 2008, pp1-12.

NIST Definition of Cloud Computing v 15. http://csrc.nist.gov/groups/SNS/cloud-computing/cloud-def-v15.doc

Opsahl, T. (2012). Tnet: Software for Analysis of Weighted, Two-mode and Longitudinal networks

Opsahl, T., Agneessens, F. & Skvoretz, J. (2010). Node Centrality in Weighted Networks: Generalizing degree and shortest Paths. Social Networks 32(2010) 245-251. Elsevier B.V.

Secretary Chu Announces $47million to Improve Efficiency in Information Technology and Communications Sectors. Accessed 24 April 2012 from http://www1.eere.energy.gov/recovery/news_detail.html?news_id=15705

Thai, V. V. & Grewal, D. (2005).Selecting the location of Distribution Centre in Logistics Operations: A Conceptual Framework and Case Study. Asia Pacific Journal of Marketing and Logistics, 17(3), 3-24.

The Official Gateway To Scotland. Population of Scotland. Accessed from http://www.scotland.org/about-scotland/the-scottish-people/population-of-scotland on 02 Feb. 2014 .

Uddin, S., Hossain, L. & Wigand, R. T.(2013). New direction in degree-centrality measure: Towards a time- variant approach.. International Journal of Information Technology

& Decision Making. World Scientific Publishing Company.

U.S. Department of Energy. (nd). Secretary Chu Announces $47million to Improve Efficiency in Information Technology and Communications Sectors. Accessed 24 April 2012 from http://www1.eere.energy.gov/recovery/news_detail.html?news_id=15705

Walker, P. B., Fooshee, S. G. & Davidson, I. (2015). Complex interactions in social and event network analysis. In N. Agarwal et al. (Eds.): SBP 2015, LNCS 9021, pp. 440-445. Springer International Publishing Switzerland.

Wasserman, S and Faust, K. (1994). Social Network Analysis: Methods and Applications. Cambridge University Press, USA. 1994.

Zhuge, H. (2011). Semantic Linking Through Spaces for CyberPhysicalSocio Intelligence: A Methodology. Artificial Intelligence 175(2011). pp.988-1019. Elsevier.

Zhuge, H. & Zhang, J. (2010). Topological Centrality and Its e-Science Applications. Wiley Interscience.