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# Two-Beam Multiplexing with Inter-Subarray Coding for Arbitrary Directions Based on Interleaved Subarray Architectures

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**Abstract**—A new method is proposed to achieve millimeter-wave two-beam multiplexing with arbitrary directions based on the interleaved subarray architecture. Beam interference can be mitigated and beam gain augmented by multiplexing multiple beams. Previous techniques can only multiplex two beams whose directions satisfy a specific relationship. By the proposed design and the associated inter-coding technique, two-beam multiplexing for arbitrary directions to serve two users is achieved. Design examples are provided to demonstrate the effectiveness of the proposed method.

## I. INTRODUCTION

For the next-generation (5G) communication systems, two key enabling technologies are massive multiple-input and multiple output (MIMO) and millimeter wave communication [1], and both require the employment of a large number of antennas working at high frequencies with a wide bandwidth. If the traditional beamforming process is implemented completely in the digital domain, the extremely high cost associated with the large number of high-speed analogue to digital converters (ADCs) and the high-level power consumption will render it practically infeasible.

One solution to the problem is to employ some analogue beamforming techniques [2]–[4], in combination with the digital beamforming technique [5], leading to the well-known hybrid beamforming structure [6]–[10], where analogue beamforming is performed first to reduce the number of analogue channels, which are then converted into digital via a reduced number of ADCs, and after that digital beamforming can then be performed.

Various hybrid beamforming structures have been proposed in the past and one of them is the sub-aperture based hybrid beamformer [8], [10]–[13]. There are mainly two types of implementation for the subarray scheme: one is the side-by-side type or localised archi-

tecture and the other one is the interleaved architecture [14], [15]. In the side-by-side structure, all the antenna elements belonging to the same subarray are located within a local region next to each other; as a result, the beam width generated by this scheme is relatively large. For the interleaved structure, the antenna elements of each subarray are distributed over a much larger aperture and the spacing between adjacent subarray elements is much larger than the standard array spacing. So a much narrower beam can be formed by the interleaved structure, which makes it a good candidate for beam multiplexing; however, this narrow beamwidth is achieved at the cost of generating high sidelobes or even grating lobes or spatial aliasing, although this effect can be suppressed at a later stage by digital beamforming to some extent with improved desired beam gain [16]–[18].

Recently, a hybrid beamforming method which involves multiplexing two beams was proposed in [19]. However, as pointed out later in this paper, a limitation of the proposed method is that the directions of the two beams must satisfy a specific relationship and therefore it is not suitable for users located at arbitrary directions. In this work, we focus on the interleaved subarray beamforming scheme, and investigate the inter-subarray coding technique involved in multiplexing two beams with arbitrary directions to serve two users in a user-dense network, which was not considered in [19].

This paper is organized as follows. A review of the interleaved subarray scheme is provided in Sec. II, and the inter-subarray coding scheme for two users with arbitrary directions is proposed in Sec. III. Design examples are provided in Sec. IV and conclusions are drawn in Sec. V.

## II. THE INTERLEAVED SUBARRAY STRUCTURE

The interleaved subarray structure based on an  $N$ -element uniform linear array (ULA) is shown in Fig.

1, where the adjacent antenna spacing is  $d$ . Suppose the  $N$  elements of the ULA are split into  $M$  interleaved subarrays. Then, each subarray consists of  $N_s = N/M$  antennas with an adjacent antenna spacing  $d_m = Md$ . The phase shift between adjacent subarrays is  $e^{j2\pi\frac{d}{\lambda}\sin\theta}$ .

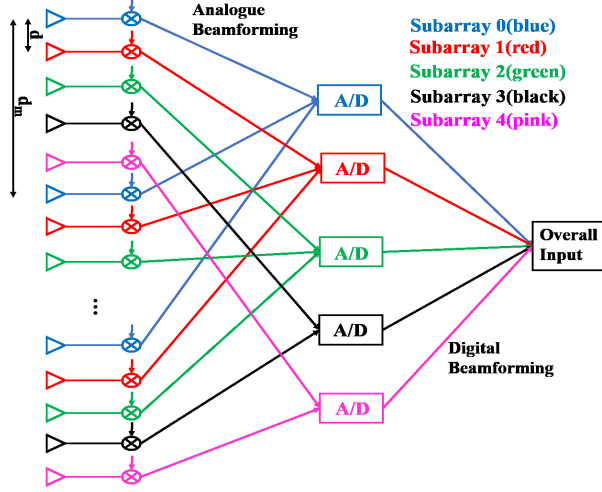


Fig. 1. An interleaved subarray based hybrid beamforming structure

The beam pattern  $P_m(\theta, \phi_m)$ ,  $m = 0, 1, \dots, M-1$ , generated by the  $m$ -th subarray pointing to the direction  $\phi_m$  can be expressed as follows

$$P_m(\theta, \phi_m) = e^{j2\pi p_m \sin\theta} \sum_{n=0}^{N_s-1} w_{A,m,n}(\phi_m) \exp \left[ j2\pi \frac{d_m}{\lambda} n \sin\theta \right], \quad (1)$$

where  $p_m = m\frac{d}{\lambda}$  denotes the starting location of the  $m$ -th subarray in terms of the signal wavelength  $\lambda$ , and  $w_{A,m,n}(\phi_m)$  denotes the analogue weighting factor of the  $n$ -th antenna of the  $m$ -th subarray for the main beam direction pointing to  $\phi_m$ . Through inter-subarray coding, the  $M$ -subarray based hybrid beamforming scheme produces  $M$  beams using the interleaved subarray architecture. The overall beam pattern using the  $M$  subarrays with a main beam in the direction  $\phi_i$  can be expressed as

$$P(\theta, \phi_i) = \sum_{m=0}^{M-1} w_{D,i,m} P_m(\theta, \phi_m) = \sum_{m=0}^{M-1} w_{D,i,m} e^{j2\pi p_m \sin\theta} \sum_{n=0}^{N_s-1} w_{A,m,n}(\phi_m) \exp \left[ j2\pi \frac{d_m}{\lambda} n \sin\theta \right], \quad (2)$$

where  $w_{D,i,m}$  denotes the digital weighting factor for the  $m$ -th subarray.

If we choose  $w_{A,m,n}(\phi_m) = \exp \left[ -j2\pi \frac{d_m}{\lambda} n \sin\phi_m \right]$ , i.e. a uniform weighting compensating only for the phase

difference between the adjacent antenna elements of the  $m$ -th subarray,  $P_m(\theta, \phi_m)$  changes to

$$P_m(\theta, \phi_m) = e^{j2\pi m \frac{d}{\lambda} \sin\theta} \sum_{n=0}^{N_s-1} \exp \left[ j2\pi \frac{Md}{\lambda} n (\sin\theta - \sin\phi_m) \right]. \quad (3)$$

### III. THE PROPOSED SUBARRAY DESIGN AND ASSOCIATED INTER-SUBARRAY CODING SCHEME

In [19], the two-user case is studied, where there are two subarrays in total ( $M=2$ ) and two beams are produced by an inter-subarray coding scheme with a linear combination of analogue weighting factors for two subarrays. With  $d = \frac{\lambda}{2}$ , the antenna analogue weighting factors of the zeroth and first subarrays are given by

$$\begin{aligned} w_{A,0,n}(\phi_0) &= e^{-j2\pi n \sin\phi_0}, \\ w_{A,1,n}(\phi_1) &= e^{-j2\pi(n+\frac{1}{2})\sin\phi_1}, \end{aligned} \quad (4)$$

where  $\phi_0$  and  $\phi_1$  are the two desired directions. This interleaved-subarray beamforming system generates the first user's own beam naturally and the second user's beam in the direction of the first user's grating lobe and vice versa, and as a result, data for the first user is divided in anti-phase and data for the second user is divided in phase. Specifically, as proposed in [16], [19], the digital beamformer coefficient vector  $\mathbf{w}_{D,i}$  can be characterized as follows

$$\mathbf{w}_{D,0} = [w_{D,0,0}, w_{D,0,1}] = [1 \quad -1], \quad (5)$$

$$\mathbf{w}_{D,1} = [w_{D,1,0}, w_{D,1,1}] = [1 \quad 1]. \quad (6)$$

Finally, the two generated beams are

$$P(\theta, \phi_0) = \sum_{n=0}^{N_s-1} e^{j2\pi n (\sin\theta - \sin\phi_0)} - \sum_{n=0}^{N_s-1} e^{j2\pi(n+\frac{1}{2})(\sin\theta - \sin\phi_1)}, \quad (7)$$

$$P(\theta, \phi_1) = \sum_{n=0}^{N_s-1} e^{j2\pi n (\sin\theta - \sin\phi_0)} + \sum_{n=0}^{N_s-1} e^{j2\pi(n+\frac{1}{2})(\sin\theta - \sin\phi_1)}. \quad (8)$$

Ideally (7) should form a beam pointing to direction  $\phi_0$  while (8) forms a beam pointing to direction  $\phi_1$ . Generally, for a ULA of  $2N_s$  antennas with adjacent antenna spacing  $d = \frac{\lambda}{2}$ , to have beam responses with main beam directions in  $\phi_0$  and  $\phi_1$ , respectively, one way is to steer the broadside main beam with uniform weighting to the corresponding directions as follows,

$$P(\theta, \phi_0) = \sum_{n=0}^{2N_s-1} e^{j\pi n (\sin\theta - \sin\phi_0)}, \quad (9)$$

$$P(\theta, \phi_1) = \sum_{n=0}^{2N_s-1} e^{j\pi n(\sin\theta - \sin\phi_1)}. \quad (10)$$

The key is to find some appropriate parameters so that (7) will be transformed into the form of (9), while (8) transformed into the form of (10). To achieve that, we first expand (7) into the following form

$$P(\theta, \phi_0) = \sum_{n=0}^{N_s-1} e^{j2\pi n(\sin\theta - \sin\phi_0)} - \sum_{n=0}^{N_s-1} e^{j\pi(2n+1)(\sin\theta - \sin\phi_0)} e^{j\pi(2n+1)(\sin\phi_0 - \sin\phi_1)}. \quad (11)$$

It can be seen that as long as

$$e^{j\pi(\sin\phi_0 - \sin\phi_1)} = -1, \quad (12)$$

or equivalently

$$|\sin\phi_0 - \sin\phi_1| = 1, \quad (13)$$

(7) and (8) will be transformed into the form of (9) and (10), respectively. Thus, in [19], [20], there is an important limitation to this scheme: the two user directions cannot be arbitrary and have to follow this specific relationship. Although in practice, it is difficult to find two user directions meeting exactly this relationship, it theoretically proves that it is possible to use a simple hybrid analogue and digital beamforming technique to form two beams without spatial aliasing.

Now in the following we try to overcome the limit of the scheme in [19], [20] and design a new scheme with two interleaved subarrays which can form beams in two arbitrary directions. First, given the simple form of the digital beamformer coefficient  $w_{D,i}$  in (5) and (6), they are still adopted in the new scheme. Consider a general uniform linear array with adjacent spacing  $d = \alpha \frac{\lambda}{2}$ , where  $\alpha$  is a coefficient with its value to be determined later. Then, the adjacent antenna spacing for the  $m$ -th subarray is given by

$$d_m = Md = M\alpha \frac{\lambda}{2} = \alpha\lambda, \quad (14)$$

where  $M = 2$  has been used for two subarrays. Moreover, the start position  $p_m$  of the  $m$ -th subarray is

$$p_m = m \frac{d}{\lambda} = m \frac{\alpha}{2}. \quad (15)$$

In order to form two beams to directions  $\phi_0$  and  $\phi_1$  separately using a ULA of  $2N_s$  antennas with adjacent antenna spacing  $d = \alpha \frac{\lambda}{2}$ , similar to (9) and (10), the desired beam responses for the two beams can be achieved by beam steering in combination with uniform weighting as follows

$$P(\theta, \phi_0) = \sum_{n=0}^{2N_s-1} e^{j\alpha\pi n(\sin\theta - \sin\phi_0)}, \quad (16)$$

$$P(\theta, \phi_1) = \sum_{n=0}^{2N_s-1} e^{j\alpha\pi n(\sin\theta - \sin\phi_1)}. \quad (17)$$

On the other hand, based on (2), the new antenna analogue weighting factors for the zeroth and first subarrays can be chosen to compensate for the phase difference corresponding to look directions only with uniform magnitudes as follows

$$\begin{aligned} w_{A,0,n}(\phi_0) &= e^{-j2\alpha\pi n \sin\phi_0}, \\ w_{A,1,n}(\phi_1) &= e^{-j2\alpha\pi(n+\frac{1}{2})\sin\phi_1}. \end{aligned} \quad (18)$$

Then, the desired beam responses for the two beams are

$$P(\theta, \phi_0) = \sum_{n=0}^{N_s-1} e^{j2\alpha\pi n(\sin\theta - \sin\phi_0)} - \sum_{n=0}^{N_s-1} e^{j2\alpha\pi(n+\frac{1}{2})(\sin\theta - \sin\phi_1)}, \quad (19)$$

$$P(\theta, \phi_1) = \sum_{n=0}^{N_s-1} e^{j2\alpha\pi n(\sin\theta - \sin\phi_0)} + \sum_{n=0}^{N_s-1} e^{j2\alpha\pi(n+\frac{1}{2})(\sin\theta - \sin\phi_1)}. \quad (20)$$

To find  $\alpha$ , similar to (11), we modify (19) into

$$P(\theta, \phi_0) = \sum_{n=0}^{N_s-1} e^{j2\alpha\pi n(\sin\theta - \sin\phi_0)} - \sum_{n=0}^{N_s-1} e^{j\alpha\pi(2n+1)(\sin\theta - \sin\phi_0)} e^{j\alpha\pi(2n+1)(\sin\phi_0 - \sin\phi_1)}. \quad (21)$$

For (21) to match with (16), similar to (12), one solution is to have

$$e^{j\alpha\pi(\sin\phi_0 - \sin\phi_1)} = -1. \quad (22)$$

For arbitrary  $\phi_0$  and  $\phi_1$ , the value of  $\alpha$  can then be calculated by

$$\alpha = \frac{1}{|\sin\phi_0 - \sin\phi_1|}. \quad (23)$$

#### IV. DESIGN EXAMPLES

In this section, we provide some design examples for the proposed scheme. Suppose that the number of antenna elements  $N_s$  for each subarray is ten. The beam multiplexing performance between the scheme in [19] and the proposed scheme is compared for two arbitrary-direction beams.

Assume the two desired beam directions are  $\phi_0 = -50^\circ$  and  $\phi_1 = 40^\circ$ . For the scheme in [19], the two-beam multiplexing results of one desired beam pointing to  $-50^\circ$  and one pointing to  $40^\circ$  are provided in Fig. 2 and Fig. 3, respectively. The corresponding analogue weighting factors for the above two cases are listed in Table I and Table II, respectively.

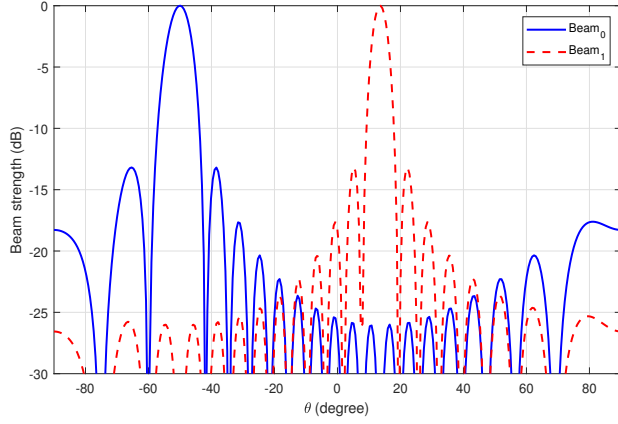


Fig. 2. Beam pattern of two interleaved subarrays when  $\phi_0 = -50^\circ$  with the scheme in [19].

TABLE I  
ANALOGUE WEIGHTING FACTORS WITH ITS BEAM POINTING TO  $-50^\circ$  WITH INTER-CODING SCHEME IN [19].

n \ m	$\phi_0$	$\phi_1$
0	1.0000+0.0000i	0.7418-0.6706i
1	0.1006-0.9949i	-0.5925-0.8056i
2	-0.9797-0.2003i	-0.8611+0.5084i
3	-0.2978+0.9546i	0.4192+0.9079i
4	0.9198+0.3924i	0.9455-0.3257i
5	0.4830-0.8756i	-0.2289-0.9735i
6	-0.8226-0.5686i	-0.9916+0.1297i
7	-0.6485+0.7612i	0.0293+0.9996i
8	0.6920+0.7219i	0.9974+0.0715i
9	0.7878-0.6159i	0.1715-0.9852i

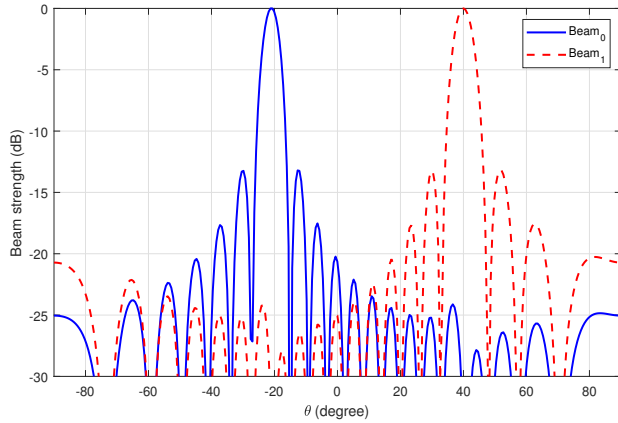


Fig. 3. Beam pattern of two interleaved subarrays when  $\phi_1 = 40^\circ$  with the scheme in [19].

We can clearly observe that in Fig. 2, beam one has pointed to the direction  $13.5^\circ$  instead of  $40^\circ$  as specified by the design, while in Fig. 3, beam zero has pointed to  $-21^\circ$  instead of  $-50^\circ$  as required, highlighting the issue of the design in [19].

As to the proposed inter-subarray coding scheme, from (23),  $\alpha$  can be calculated as 0.71 for  $\phi_0 = -50^\circ$  and  $\phi_1 = 40^\circ$ . Hence, the adjacent antenna spacing for

TABLE II  
ANALOGUE WEIGHTING FACTORS WITH ITS BEAM POINTING TO  $40^\circ$  WITH INTER-CODING SCHEME IN [19].

n \ m	$\phi_0$	$\phi_1$
0	1.0000+0.0000i	-0.4337-0.9011i
1	-0.6295+0.7770i	0.9748+0.2232i
2	-0.2075-0.9782i	-0.7825+0.6226i
3	0.8907+0.4546i	0.0015-1.0000i
4	-0.9139+0.4059i	0.7806+0.6250i
5	0.2599-0.9656i	-0.9755+0.2202i
6	0.5867+0.8098i	0.4364-0.8997i
7	-0.9985-0.0539i	0.4309+0.9024i
8	0.6704-0.7420i	-0.9741-0.2261i
9	0.1545+0.9880i	0.7844-0.6202i

the two subarrays is  $d_0 = d_1 = \alpha\lambda = 0.71\lambda$ . The two resultant beams by the proposed inter-subarray coding scheme are presented in Fig. 4. The corresponding analogue weighting factors for two desired beams with directions of  $\phi_0 = -50^\circ$  and  $\phi_1 = 40^\circ$  are shown in Table III. It can be clearly seen that the two beams are in the desired directions and the proposed scheme is working effectively.

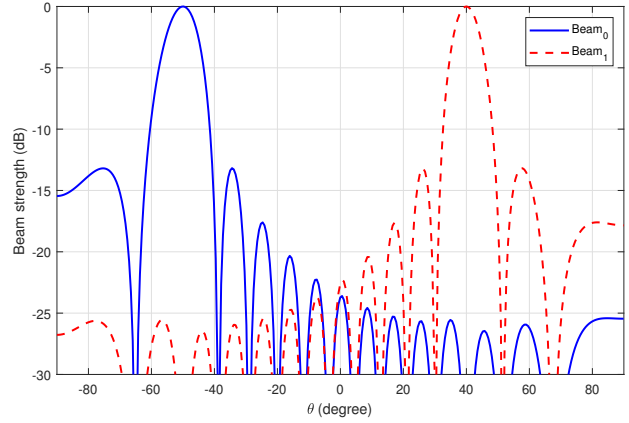


Fig. 4. Beam pattern of two interleaved subarrays when  $\phi_0 = -50^\circ$  and  $\phi_1 = 40^\circ$  with the proposed scheme.

TABLE III  
ANALOGUE WEIGHTING FACTORS FOR TWO BEAMS OF  $\phi_0 = -50^\circ$  AND  $\phi_1 = 40^\circ$  WITH THE PROPOSED SCHEME.

n \ m	$\phi_0$	$\phi_1$
0	1.0000+0.0000i	0.1366-0.9906i
1	-0.9622-0.2723i	-0.3996+0.9167i
2	0.8517+0.5240i	0.6328-0.7743i
3	-0.6768-0.7361i	-0.8188+0.5741i
4	0.4508+0.8926i	0.9436-0.3311i
5	-0.1907-0.9816i	-0.9980+0.0633i
6	-0.0838+0.9965i	0.9779+0.2092i
7	0.3520-0.9360i	-0.8848-0.4660i
8	-0.5935+0.8048i	0.7256+0.6881i
9	0.7902-0.6128i	-0.5123-0.8588i

## V. CONCLUSION

In this paper, a millimeter-wave beam multiplexing scheme has been proposed based on the interleaved subarray architecture. The new scheme overcomes the limitation of an existing one so that the two beam directions can be arbitrary, instead of having a fixed relationship. As demonstrated by design examples, the proposed method works effectively. However, one issue with the proposed scheme is that the antenna spacing is not fixed any more and how to make such a scheme more practical is a new challenge which will be tackled at the next step of our research.

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