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Title: A 2-D Imaging-assisted Geometrical Transformation Method for Non-destructive Evaluation of the Volume and Surface Area of Avian Eggs

Article Type: Research Paper

Keywords: Egg quality; Non-destructive measurements; Egg volume; Egg surface area; Digital imaging; Image processing

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Abstract: Egg volume and surface area are reliable predictors of quality traits for both table and hatching chicken eggs. A new non-destructive technique for the fast and accurate evaluation of these two egg variables is addressed in the present study. The proposed method is based on the geometrical transformation of actual egg contour into a well-known geometrical figure which shape most of all resembles the examined egg. The volume and surface area of an examined egg were recomputed using the formulae appropriate for three figures including sphere, ellipsoid, and egg-shape ovoid. The method of the geometrical transformation includes the measurements of the egg length and the area of the examined eggs. These variables were determined using two-dimensional (2-D) digital imaging and image processing techniques. The geometrical transformation approach is proven to be reliable to turn the studied chicken eggs into the three chosen ovoid models, with the best prediction being shown for the ellipsoid and egg-shape ovoid, whilst the former was slightly more preferable. Depending on the avian species studied, we hypothesise that it would be more suitable to use the sphere model for more round shaped eggs and the egg-shaped ovoid model if the examined eggs are more conical. The choice of the proposed transformation technique would be applicable not only for the needs of poultry industry but also in ornithological, basically zoological studies when handling the varieties of eggs of different shapes. The experimental results show that the method proposed is accurate, reliable, robust and fast when coupled and assisted with the digital imaging and image processing techniques, and can serve as a basis for developing an appropriate instrumental technology and bringing it into the practice of poultry enterprises and hatcheries.

Research Data Related to this Submission

There are no linked research data sets for this submission. The following reason is given:
Data will be made available on request

Highligths

- Egg volume and surface area are valuable predictors of egg quality traits.
- A method of geometrical transformation of an egg contour into a geometrical figure was examined.
- Theoretical dependence between egg volume and surface area was studied.
- 2-D (two-dimensional) digital imaging and image processing techniques were applied.
- The elabourated method showed a correlation coefficient of 0.96 and standard error of 2.14%.

Figure 1

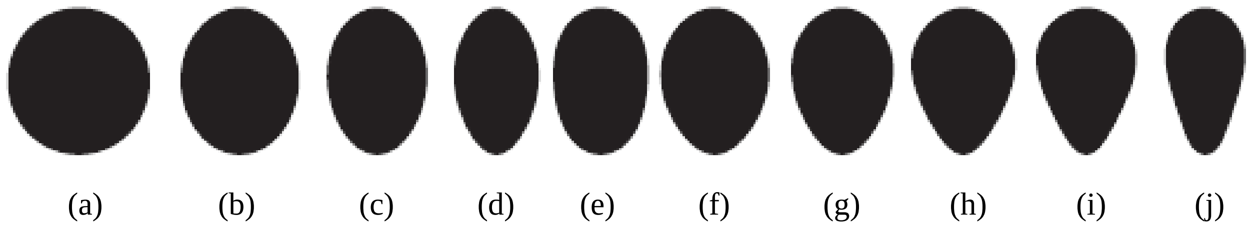
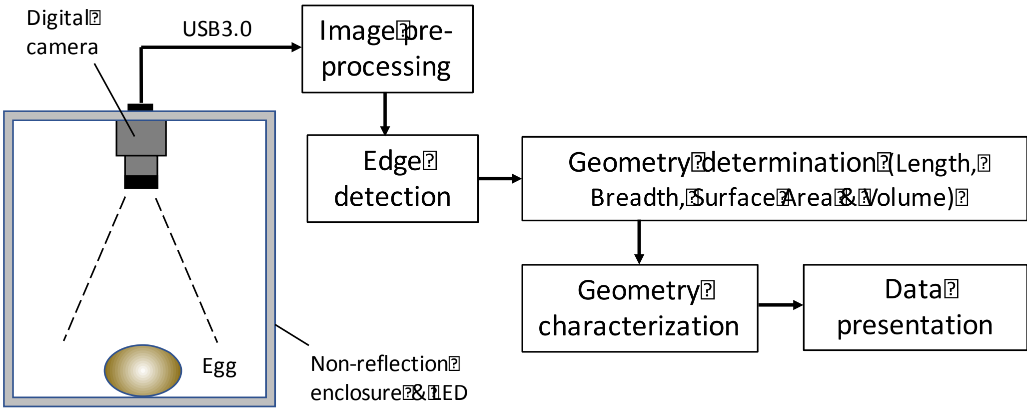


Figure 2



Digital camera
USB3.0
Image pre-processing
Edge detection
Geometry determination (Length, Breadth, Surface Area & Volume)
Geometry characterization
Data presentation
Egg
Non-reflection enclosure & LED

Figure 3

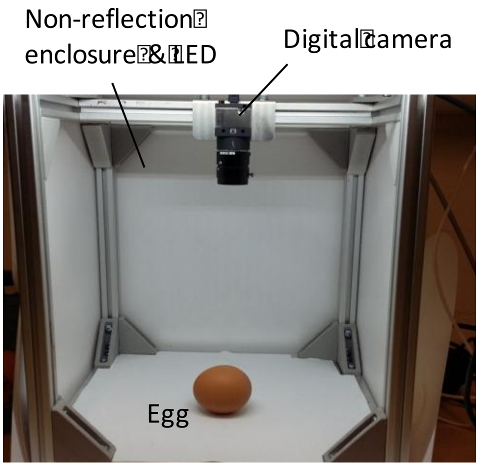


Figure 4

(a) (b)

Figure 5

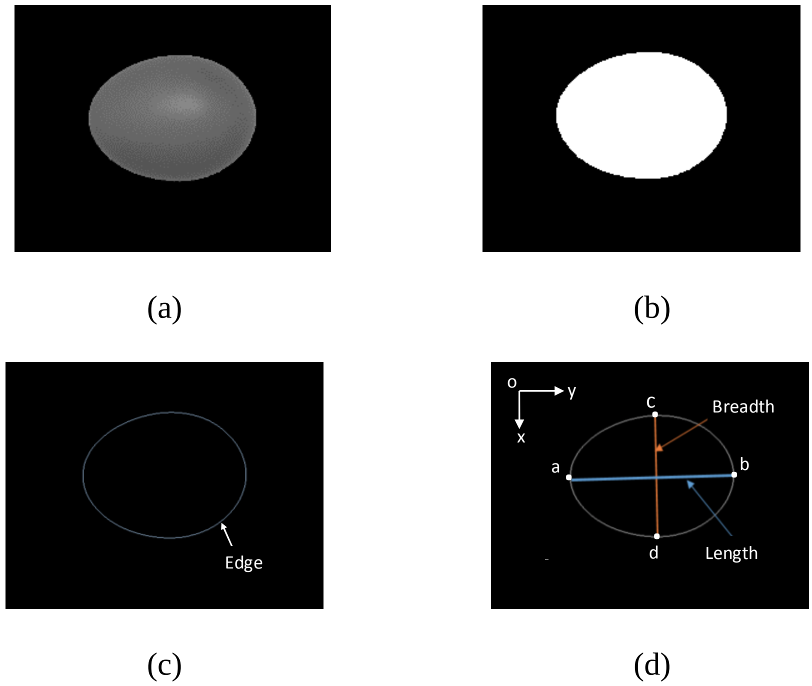
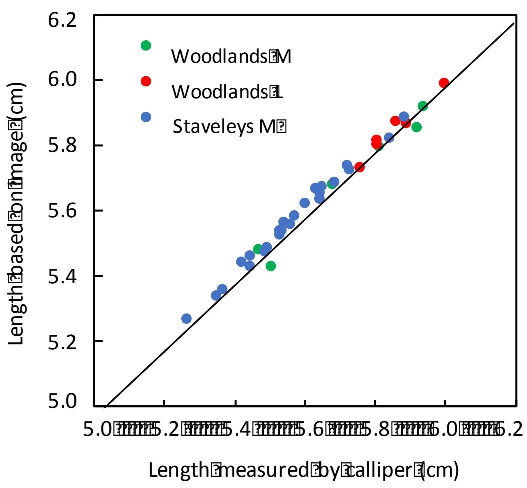


Figure 6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Woodlands L
Staveleys M
Length based on image (cm)
Length measured by calliper (cm)
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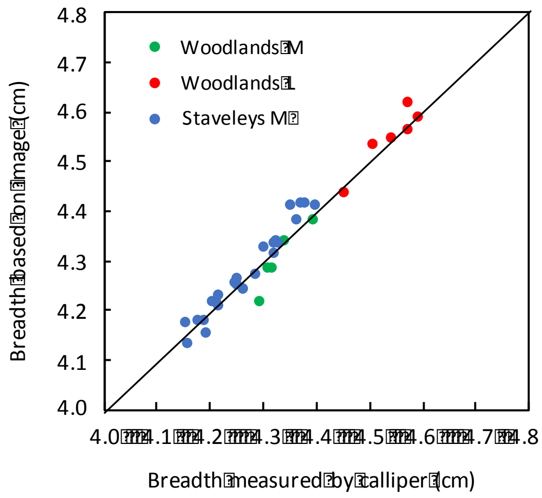
(a)


Woodlands M
Woodlands L
Staveleys M
Breadth based on image (cm)
Breadth measured by calliper (cm)
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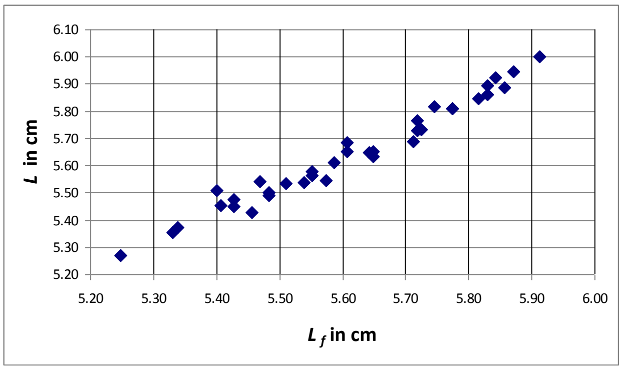
(b)

Figure 7



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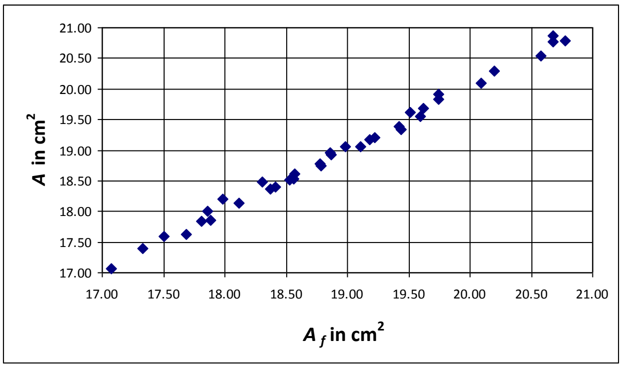

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20.50
21.00
A f in cm2

b

***Conflict of Interest Form**

AUTHOR DECLARATION

We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

We confirm that the manuscript has been read and approved by all named authors and that there are no other persons who satisfied the criteria for authorship but are not listed. We further confirm that the order of authors listed in the manuscript has been approved by all of us.

We confirm that we have given due consideration to the protection of intellectual property associated with this work and that there are no impediments to publication, including the timing of publication, with respect to intellectual property. In so doing we confirm that we have followed the regulations of our institutions concerning intellectual property.

We understand that the Corresponding Author is the sole contact for the Editorial process (including Editorial Manager and direct communications with the office). He/she is responsible for communicating with the other authors about progress, submissions of revisions and final approval of proofs. We confirm that we have provided a current, correct email address which is accessible by the Corresponding Author and which has been configured to accept email from D.K.Griffin@kent.ac.uk

Signed by all authors as follows:

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James Cugley

Michael N. Romanov

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University of Kent

Dear Sir/Madam,

I am submitting a revised version of the manuscript entitled **‘A 2-D Imaging-assisted Geometrical Transformation Method for Non-destructive Evaluation of the Volume and Surface Area of Avian Eggs’** after addressing the following suggestions of Reviewer 1 as follows:

Reviewer notes:
it will be good to include more than one edge detection algorithm. Include it or describe why do you use only one algorithm.
Authors’ response:
Many thanks for your valuable suggestion. According to it, we added the appropriate statement on Lines 225–240 of the revised manuscript.

Reviewer notes:
It will be good to describe more detailed the error sources of measurement.
Authors’ response:
We appreciate this comment and added accordingly a more detailed description of the error sources of measurement on Lines 283-288.

By submitting the updated manuscript, I hope that the Editor will count the above changes as minor revision for acceptance of the paper by your journal.

Thank you very much.

Sincerely,

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A 2-D Imaging-assisted Geometrical Transformation Method for Non-destructive Evaluation of the Volume and Surface Area of Avian Eggs

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Abstract

Egg volume and surface area are reliable predictors of quality traits for both table and hatching chicken eggs. A new non-destructive technique for the fast and accurate evaluation of these two egg variables is addressed in the present study. The proposed method is based on the geometrical transformation of actual egg contour into a well-known geometrical figure which shape most of all resembles the examined egg. The volume and surface area of an examined egg were recomputed using the formulae appropriate for three figures including sphere, ellipsoid, and egg-shape ovoid. The method of the geometrical transformation includes the measurements of the egg length and the area of the examined eggs. These variables were determined using two-dimensional (2-D) digital imaging and image processing techniques. The geometrical transformation approach is proven to be reliable to turn the studied chicken eggs into the three chosen ovoid models, with the best prediction being shown for the ellipsoid and egg-shape ovoid, whilst the former was slightly more preferable. Depending on the avian species studied, we hypothesise that it would be more suitable to use the sphere model for more round shaped eggs and the egg-shaped ovoid model if the examined eggs are more conical. The choice of the proposed transformation technique would be applicable not only for the needs of poultry industry but also in ornithological, basically zoological studies when handling the varieties of eggs of different shapes. The experimental results show that the method proposed is accurate, reliable, robust and fast when coupled and assisted with the digital imaging and image processing techniques, and can serve as a basis for developing an appropriate instrumental technology and bringing it into the practice of poultry enterprises and hatcheries.

Keywords: Egg quality; Non-destructive measurements; Egg volume; Egg surface area; Digital imaging; Image processing

1. Introduction

Such egg variables as the volume and surface area are valuable predictors of quality traits for both table and hatching eggs. Current technical solutions in poultry industry require a non-destructive method for the fast and accurate evaluation of these egg's physical parameters. One of the methodological approaches toward developing this non-invasive technique is to describe the egg shape with a valid mathematical model enabling to evaluate the egg volume and surface area with classic geometrical equations (Narushin, 1997a). Attempts to derive an appropriate formula for description of egg contours were undertaken previously (Narushin, 1997a,b, 2001b; Nishiyama, 2012; Troscianko, 2014; Mytiai and Matsyura, 2017; Biggins *et al.*, 2018). A common prerequisite for these estimations is to increase the quantity of measured points in order to make the egg geometry as close to the original egg as possible. Nevertheless, this approach is still far from being adapted for practical uses.

In our previous research, we focused on the extensive evaluation of the egg volume and surface area (Narushin, 2001a; Narushin and Romanov, 2002a,b; Narushin *et al*., 2002, 2016). In the present study, we revise and lay out a theoretical appraisal that would allow us to figure out an appropriate modus operandi for an optimal solution to compute the egg volume and surface area using mathematical modelling and a minor set of non-destructive instrumental measurements including the application of digital imaging and image processing techniques.

Previously, we proposed a method for computing the egg volume and surface area through the geometrical transformation of an actual egg contour into a well-known geometrical figure which shape mostly resembles the examined egg (Narushin, 1993, 1997b, 2001b). For this purpose, two candidates were suggested for such a geometrical model, i.e., an ellipse (Narushin, 1993), and a theoretically derived egg-shaped contour (Narushin, 2001b) defined by the egg length, L, and the maximum breadth, B, and estimated with the following mathematical formula:

$$y = \pm\sqrt{L^{\frac{2}{n+1}} x^{\frac{2n}{n+1}} - x^2} \qquad (1)$$

where n is a function of the egg shape index, B/L.

It was found that these both the transformation models (i.e., the ellipsoidal and egg-shaped geometrical figures) would give rather similar results when determining the volume, with a slight domination in accuracy of the egg-shaped model (Narushin, 2001b). Narushin *et al.* (1997b) also suggested three possible procedures of the geometrical transformation: (1) the coequality of long circumferences of the actual egg and the geometrical analogues, (2) the coequality of their areas of normal projections, and (3) the coequality of the volumes, and explored the transformation under the first scenario. However, the previously proposed manual measurements of the egg long circumference (Narushin, 1996) were rather tedious and not accurate enough. Recent development of machine vision techniques have made it possible for measuring the area of egg's normal projection in a much simple, fast and accurate way (Zhou *et al.*, 2009; Soltani *et al.*, 2015; Zhang *et al.*, 2016; Dangphonthong and Pinate, 2016; Zlatev, 2018; Chan *et al.*, 2018). In view of this technological development, there is a need in revising the methods for the geometrical transformation of avian eggs to estimate their volumes and surface areas non-invasively.

In this study, we set out an objective to explore a feasibility of using a method of the geometrical transformation of an actual egg into the contours of a known ovoid for estimating the egg volume and surface area based on non-destructive, 2-D (two-dimensional) digital imaging-based measurements of the egg length and area of its normal projection. This approach has been proven to be promising and opening further research avenues toward development of the appropriate instrumental technology for non-invasive assessment of the egg's inner variables that can be used for industrial egg sorting.

2. Methodology

According to Biggins *et al.* (2018), ten types of avian egg shape occur more often in the nature as can be presented schematically in Fig. 1. There are three geometrical figures that can be used as models for the transformation of the contours of an actual examined egg, i.e., a sphere, an ellipsoid, and an egg-shape ovoid. Let us overview the basic calculative formulae for these three egg shape models that can aid in the geometrical transformation and are used to compute the egg area of the normal projection, *A*, the volume, *V*, and the surface area, *S*, as follows.

2.1. Sphere

A normal projection of the sphere is a circle. Then, the length, *L*, and the maximum breadth, *B*, of a projected egg are simply equal to the circle diameter, and the appropriate calculative formula for the projection area, *A*, would be as follows:

$$A = \frac{\pi B^2}{4} \ . \qquad (2)$$

Then, for *V* and *S*, we would have:

$$V = \frac{\pi B^3}{6} \ , \qquad (3)$$

$$S = \pi B^2 . \qquad (4)$$

It is assumed that the 2-D image of the egg reflects the area of the actual egg's normal projection (*A*), the latter should be input in Eq. 2. As a result, the egg can be geometrically transformed into the sphere, the diameter (*B*, or *L*) of this transformed egg, B_t, being determined as follows:

$$B_t = 2\sqrt{\frac{A}{\pi}} = 1.129\sqrt{A} \ . \qquad (5)$$

B_t also means a provisional dimension that corresponds to the empirical diameter of the circle into which the examined egg image is geometrically transformed. Thus, to compute the egg volume and surface area, the value of B_t should be used instead of *B* in Eqs. 3 and 4.

2.2. Ellipsoid

A normal projection of the ellipsoid is an ellipse, the long axis of which corresponds to *L* and the short one to *B*. The projection area of such an ellipse is determined by:

$$A = \frac{\pi LB}{4} \ . \qquad (6)$$

The calculation of V for ellipsoids can then be done by:

$$V = \frac{\pi LB^2}{6} \ . \qquad (7)$$

The formula for computing the surface area of ellipsoid contains several prerequisites and depends on its eccentricity, ε (Tee, 2004). For a prolate ellipsoid that is most similar to the egg shape, we have:

$$S = \frac{\pi B}{2}\left(L \cdot \frac{\arcsin \varepsilon}{\varepsilon} + B \right) , \qquad (8)$$

where

$$\varepsilon = \sqrt{1 - \frac{B^2}{L^2}} \ . \qquad (9)$$

In this case, A and L should be measured instrumentally. Using these two variables, it is possible to perform the geometrical transformation of the examined egg into the ellipsoid computing B_t from Eq. 6:

$$B_t = \frac{4A}{\pi L} = 1.274 \cdot \frac{A}{L} \ . \qquad (10)$$

The computation of A and S can be done after inputting B_t into Eqs. 7–9 instead of B.

2.3. Egg-shaped ovoid

A formula of the egg-shaped curvature (Eq. 1) was deduced by Narushin (2001b) based on a polar equation of a folium (e.g., Kokoska, 2012). This appeared to be a geometrical figure model that resembles the contours of actual eggs in the best way. The variable n in Eq. 1 that reflects a function of the egg shape index, B/L, was previously expressed as a power function (Narushin, 2001b) and, later on, in a form of quadratic dependence (Narushin, 2005), being defined by simulating the B/L data. This approach described adequately a variety of avian eggs in the nature and showed a rather high correlation coefficient of the calculative data. We decided to repeat this simulation trial using a more advanced mathematical apparatus that had been notably improved over the last 15 years since the initial study was carried out. As a result, a more appropriate and precise formula was obtained for n for which the correlation coefficient R^2 would equal to 1:

$$n = 1.466\left(\frac{L}{B}\right)^2 - 0.473. \qquad (11)$$

Our preliminary theoretical findings (Narushin, 1997b, 1998, 2001b, 2005) also suggested derivation of several basic formulae for the egg-shaped ovoid model obtained by revolving the egg-shaped curvature around its long axis. A formula for estimating the volume of the egg-shaped ovoid was composed after the corresponding integration of Eq. 1 (Narushin, 2001b) and resulted in the following:

$$V = \frac{2\pi L^3}{3(3n+1)}. \qquad (12)$$

Substituting Eq. 11 into Eq. 12 and completing some simplifications yielded the following formula for V:

$$V = \frac{5}{10.5 - \frac{B^2}{L^2}} \cdot LB^2. \qquad (13)$$

A detailed mathematical transformation for deriving Eq. 13 is given in Appendix A.

The area of the normal projection, A, is normally estimated with definite integration formulae. Narushin (2001b) found that only approximate methods could assist in resolving such an integral based on the Simpson's rule (Recktenwald, 2000). To improve the accuracy of the computation for any egg which shape can be described with Eq. 1, we performed the computation using actual numbers of the linear variables of a typical hen's egg (Romanoff and Romanoff, 1949). A step-by-step solution of the integral for measuring A (refer to Appendix B) led to:

$$A = 0.118B^2 + 0.637LB + 0.014L^2. \qquad (14)$$

To proceed with the geometrical transformation of the examined egg into the egg-shaped ovoid, B can be derived from Eq. 14 (refer to Appendix C):

$$B = 2.677\sqrt{L^2 + 1.183A} - 2.699L. \qquad (15)$$

The equation for estimating the surface area of the egg-shaped ovoid was proposed by Narushin (2001b), although it was not accurate enough since it was simulated under the data of only four values of coefficient n from Eq. 1. To make the further comparative investigations between the egg volume and surface area simpler, another trial of simulation process for computing S was performed that resulted in a more appropriate and accurate function for which the correlation coefficient R^2 would be equal to 1:

$$S = 1.077B^2 + 1.879BL + 0.08L^2. \qquad (16)$$

To solve Eq. 16, the projection area of the examined egg (A) and the egg length (L) should be measured instrumentally. The instrumental assessment of these two variables makes it possible to get the geometrical transformation of the examined egg into the egg-shaped ovoid recalculating B_t using Eq. 15. Afterwards, we can compute V and S after changing B for B_t in Eqs. 13 and 16.

2.4. Relation between surface area and volume

Considering that there is no any accurate direct method for measuring the egg surface area (Narushin, 1997a), the conformation of calculations can be proved by examining the computation accuracy of the egg volume because these two parameters are closely related. As shown in the past (Romanoff and Romanoff, 1949; Paganelli *et al.*, 1974; Shott and Preston, 1975; Tatum, 1977), the relation between these variables can be written as:

$$S = k_1 V^{\frac{2}{3}} \qquad (17)$$

where k_1 is a dimensionless constant.

Narushin (1997b) also confirmed the validity of Eq. 17 using the dimensional analysis (Schenk, 1979) and compared the theoretical formulae for computing the volume and surface area of the egg-shaped ovoid. Gonzalez *et al.* (1982) explained such dependence as a typical thermogenic process, which corresponds to basal metabolic rate.

To test eventually the correctness of Eq. 17, the above appropriate equations for the calculation of V and S were compared for the three models of the chosen geometrical figures that are most similar to the egg shape as follows:

Sphere. The comparison of Eqs. 3 and 4 leads to:

$$S = 4.835V^{\frac{2}{3}}. \qquad (18)$$

Ellipsoid.

$$S = 2.418\left(\frac{B}{L}\right)^{\frac{2}{3}} \cdot \left(\frac{L}{B} \cdot \frac{\arcsin\sqrt{1-\frac{B^2}{L^2}}}{\sqrt{1-\frac{B^2}{L^2}}} + 1\right) \cdot V^{\frac{2}{3}}, \qquad (19)$$

in which k_1 equals to $2.418\left(\frac{B}{L}\right)^{\frac{2}{3}} \cdot \left(\frac{L}{B} \cdot \frac{\arcsin\sqrt{1-\frac{B^2}{L^2}}}{\sqrt{1-\frac{B^2}{L^2}}} + 1\right)$.

Egg-shaped ovoid.

$$S = \left(1.077\frac{B^2}{L^2} + 1.879\frac{B}{L} + 0.08\right)\left(2.1\frac{L^2}{B^2} - 0.2\right)^{\frac{2}{3}} \cdot V^{\frac{2}{3}} \qquad (20)$$

in which k_1 is $\left(1.077\frac{B^2}{L^2} + 1.879\frac{B}{L} + 0.08\right)\left(2.1\frac{L^2}{B^2} - 0.2\right)^{\frac{2}{3}}$.

The detailed derivation of Eqs. 18–20 is given in Appendix D.

Thus, based on the validity of Eq. 18, it can be stated that the implementation of the calculative method for V using the direct, non-invasive egg measurement can lead to the appropriate computation of S.

3. Materials and Measurements

A total of 40 fresh chicken eggs of medium and large sizes were purchased from Woodlands Farm, Canterbury and Staveleys Eggs Ltd, Coppull, UK. The weight of the eggs was measured using a precision balance (Mettler Toledo PL602E, 620 g capacity, 0.01 g readability). The length (L) and maximum breadth (B) of the eggs were measured with a Vernier calliper (with a 0.01 mm accuracy), and the volume (V) was determined using the Archimedes' method by immersing the eggs into water.

The image system that was used in this study is shown on the block diagram in Fig. 2 whilst Fig. 3 illustrates the physical setup of the system. The system basically consists of a digital camera, a non-reflection enclosure with LED (liquid emitted diode) lighting facilities, and a personal computer. The camera (UI-2230RE) has a CMOS (Complementary Metal Oxide Semiconductor) RGB (Red, Green and Blue) imaging sensor with a resolution of 1024 (H) × 768 (V) pixels transmits images to the computer via USB 3.0 data transmission at a frame rate of 25 frames per second. The LED laminated non-reflection enclosure provides a uniformed and stable illumination environment for the image acquisition. The system acquired 2-D images of the eggs and collected the measurement data for the same 40 eggs. As demonstrated by Chan *et al.* (2018), if the egg is located in a free position on a flat ground or a stage surface, it would be tilted due to its elongated shape and liquid interior. Accordingly, the images of all the eggs were taken under two different conditions: (1) the eggs were free lied on the test bench leading to free projection, and (2) taped on the test bench to ensure that the maximum length was levelled to the test bench providing normal projection. A typical example of the acquired egg images is given in Fig. 4. The images of the eggs were processed using MatLab that allows to compute the geometric parameters of egg including the area (A, normal projection), the length (L), and the maximum breadth (B_t).

(a) Edge detection. The edge detection was performed to determine the outer contour of the egg. This was achieved by firstly converting the RGB images to grey-scale images (Fig. 5a). The choice of the Sobel edge detection technique is because of its simplicity and fast computation in

determining the distinct and low noise spatial gradient in an image such as an egg image (note that the edge of an object is expected to show a great spatial gradient with reference to the image background). In comparison, other edge detection techniques, such as Canny, Roberts and Prewitt edge algorithms (Chandwadkar, 2013), often have greater computational complexity and time consumption. In the edge detection, a pair of 3×3 Sobel operators, as shown in Fig. 6, were applied over the images to estimate the gradient of the image in both the horizontal (G_y) and vertical (G_x) directions. The magnitude (G) and direction (θ) of the gradient at a pixel over the image can then be computed by (Chandwadkar, 2013):

$$G = \sqrt{G_x^2 + G_y^2} \qquad (21)$$

$$\theta = \arctan\left(\frac{G_y}{G_x}\right) \qquad (22)$$

When the gradient vectors (magnitude and direction) of all pixels are computed over the image, the pixels with great magnitudes are regarded to be the edge of the egg, and the its contour can then be drawn. The Sobel edge detection technique (Chandwadkar, 2013) was then applied on the pre-processed image to determine the edge of the egg, i.e., its outer contour. The output of the Sobel edge detection processing is a binary image of the detected edge (Figs. 5b and 5c).

Once the edge of egg is detected, the egg's area (A), length (L) and maximum breadth (B_t) can be determined from the edge-detected image.

(b) Egg area A (cm^2). The egg area was computed by counting the total number of pixels within the egg image region, R, defined by its outer contour (Fig. 5c), as follows:

$$A = k_2 \sum_{i \in R} 1 \qquad (23)$$

where i is a pixel within R, and k_2 is a scale factor, which is used to convert the area from the number of pixels to an absolute unit (cm^2) and can be obtained through the system calibration.

(c) Length (L) and maximum breadth (B_t) (cm). The length and breadth of the eggs were calculated by searching the maximum point-to-point distances along the y-axis for the length, and the x-axis for the breadth over the outer contour of the egg image (Fig. 5d). It is known that the distance between two points in a space is determined based on the Euclidean's distance measurement principal:

$$d(p_1, p_2) = k_3\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad (24)$$

where d is the distance between points $p_1(x_1, y_1)$ and $p_2(x_2, y_2)$. In this case as shown in Fig. 5d, the length (L) is the distance from points a to b, and the breadth (B_t) the distance from points c to d. k_3 is a distance factor, which converts the length from the number of pixels to an absolute unit (cm), and again obtained through the system calibration.

All statistical data and corresponding mathematical approximations were estimated using the computer software package Statistica (StatSoft Inc).

4. Results

The measurement data of the examined eggs based on this direct measurement is summarised in Table 1. The results showed a reasonable variation in physical properties of the eggs. For instance, among the 40 chicken eggs randomly selected and examined, their weight ranged between 51.41 g and 68.72 g, with a mean of 59.19 ± 4.72 g, which can normally be observed for commercial table eggs in the field. Also, the mean egg length, breadth and volume in this experiment were 5.65 ± 0.19 cm, 4.33 ± 0.12 cm and 55.83±3.94 cm^3, respectively.

Table 1

The geometrical properties of examined eggs based on direct measurements.

Parameters	Maximum	Minimum	Mean	Standard deviation
Weight, W (g)	68.72	51.41	59.19	4.72
Length, L (cm)	6.00	5.27	5.65	0.19
Max breadth, B (cm)	4.59	4.16	4.323	0.12
Volume, V (cm^3)	63.63	47.94	55.83	3.94

Based on the digitally acquired egg images after their processing, L, B_t, and A were obtained, which were 405.81 ± 12.59, 312.65 ± 9.22, and 98,984.10 ± 5226.20 pixels, respectively (Table 2). A conversion of the pixels into metric units was done using the initial dataset of the measured egg linear parameters, L and B in centimetres, by which their corresponding values in the numbers of pixels were divided. The conversion coefficient was found to be 72.09 pixels in 1 cm in length (please note that the number of pixels should normally be an integer, however a decimal is used here just for a conversion purpose). Squaring of this value provided the conversion coefficient for A that was equal to 5197.03 pixels in 1 cm^2 (Table 2). Comparing the results obtained by the calliper and the imaging system, respectively (Fig. 6), it was determined that both measurement techniques had a reasonable level of agreement with the averaged relative error being 0.42% and the maximum relative error being 1.88% in linear measurements. There are possible sources which may contribute to the measurement errors. The first is the inherent difference between the working principles of the two measurements. The second may be from the perspective effect along the optical path of the camera which could cause small variations of the length and area conversion coefficients across the 2-D image of the egg considering eggs varies in sizes. However, the level of the errors is small and regarded to be acceptable.

Table 2

The measurement data based on the image system.

Parameter	Maximum	Minimum	Mean	Standard deviation
L (pixels)	429	380	405.81	12.59
Maximum B (pixels)	333	299	312.65	9.22
A (pixels *vs* cm^2)	108,888 / 20.95	89,039 / 17.13	98,984.100 / 19.05	5226.200 / 1.01

As proposed in the theoretical section of this paper, the computation of B was performed using Eqs. 5, 10, 15, and the corresponding evaluation of V and S was done with Eqs. 3, 7, 13 and Eqs. 4, 8, 16, respectively. The data of L was taken from the direct measurements, while B_t was recalculated using the measurements of A through 2-D imaging. The results of this analysis for the three models of ovoids are presented in Table 3.

Table 3

Egg geometrical transformation into three models of ovoids.

Transformation model	Mean B_t (cm)	Mean V_t (cm^{3})	Mean S_t (cm^2)	R^2 between V and V_t	SD for difference $V_t - V$	SE for difference $V_t - V$, %
Sphere	4.93 ± 0.11[a]	62.66 ± 4.01[a]	76.23 ± 3.25	0.945	7.06	12.25
Ellipsoid	4.307 ± 0.10	54.64 ± 3.38	79.55 ± 3.50	0.960	1.70	2.14
Egg-shaped ovoid	4.51 ± 0.10	58.22 ± 3.59	72.26 ± 2.99	0.960	2.75	4.29

[a] $p < 0.01$ as compared to the appropriate, actually measured values of B and V from Table 1; R^2, coefficient of correlation; SD, standard deviation; SE, standard error

Comparing the data of Tables 1 and 3, it was found that actual values of B and V (Table 1) were consistent with the appropriately computed B_t and V_t for the ellipsoid and egg-shaped ovoid models (Table 3). The appropriate differences for the respective values that were actually measured and those computed using the either model were insignificant. If we look at the difference $V_t - V$ depending on the transformation model, the lower values of standard deviation (1.70 *vs* 2.75) and standard error (2.14% *vs* 4.29%) were obtained for the ellipsoid and egg-shaped ovoid models, respectively, with a slight preference toward the ellipsoid. The usage of the transformation equations for the sphere model led to significantly different numbers of the direct measured values, B and V (Table 1), and the computed ones, B_t and V_t ($p < 0.01$; Table 3).

In addition, we compared the computed lengths based on the images of eggs, which were taped and those laid free on the test bench. The tilted position corresponding to free projection could lead to a bias in determining the egg length, L_f, as well as that of normal projection, L. However, the differences appeared to be rather small and insignificant, with the means being L = 5.65 ± 0.19 cm and L_f = 5.62 ± 0.18 cm. Such a negligible difference did also not affect significantly the area A for the normal projection, the means of which being A = 19.05 ± 1.01 cm^2 and A_f = 19.01 ± 1.00 cm^2.

To explore those cases when a certain accuracy of the recomputed egg geometry is needed, the relationships between the respective variables of the normal projection egg images (L and A) and the free projection ones (L_f and A_f) were evaluated and presented in the form of scattergrams (Fig. 7) after their approximating with the following equations for which high correlation coefficients R^2 were also obtained:

$$L = 1.0377L_f - 0.1903, \qquad (25)$$

$$R^2 = 0.969;$$

$$A = 1.0063A_f - 0.0836, \qquad (26)$$

$$R^2 = 0.994.$$

5. Discussion

A combination of the mathematical computation and experimental measurement performed in this study has suggested that the proposed non-destructive, 2-D imaging-based method of geometrical transformation is accurate, reliable, user-friendly, cost effective, and can be easily implemented in both laboratory and industry conditions. The digital camera provides multi-dimension and high-resolution data that is very helpful in re-computing geometrical variables of an examined object, which could not be done using conventional approaches. All the above can lead to a remarkable breakthrough in various related areas including research of egg quality traits and their impact on incubation, poultry breeding, storage conditions, etc., as well as development of industrial applications such as automated egg sorting. For instance, the egg density (sometimes referred to in the egg-related papers as specific gravity) is still one of the basic parameters that can predict egg freshness (e.g., Usturoi *et al.*, 2014; Mezemir *et al.*, 2017), shell thickness (e.g., Nordstrom and Ousterhout, 1982; Sooncharenying and Edwards, 1989), shell strength (e.g., Ahmad *et al.*, 1976; Hamilton *et al.*, 1979; Voisey *et al.*, 1979), hatchability (e.g., Bennett, 1992; Rozempolska-Rucińska *et al.*, 2011), and some variables of its interior (Narushin, 1997c). Taking into account that the egg density is physically determined as the ratio of egg weight and its volume (e.g., Paganelli *et al.*, 1974), these both parameters should be obtained in a fast, accurate and non-invasive manner as we demonstrated in this study. Whilst the procedure of measuring the egg weight is common and easily applicable in poultry industry, determination of the egg volume is still a difficult task, and another similar problem is a solution for non-invasive detection of the egg surface area. Thus, the image processing technique along with the computation formulae examined in this study can be a valuable and high-throughput approach for solving the problems related to the measurement of the egg volume and its surface area.

As theoretically proved in this study, the surface area of the chosen ovoids depends on their volume. It can be suggested further that the validity of the computed egg surface area would depend on the accuracy of the appropriate formula for estimating the egg volume.

We demonstrated here that the method of geometrical transformation is reliable to turn the egg into all three chosen ovoid models, the appropriate correlation coefficient R^2 for the recalculation of the egg volumes being fairly high, around 0.95, for the three ovoids. Judging from the studied sample of the chicken eggs, the ellipsoid and egg-shaped ovoid models seem to be the most plausible geometric figures, with a slight predisposition toward the ellipsoid. However, we would suggest that the proposed computation formulae for these three ovoids would be applicable at examining various eggs depending on their actual shape. Apparently, the chicken eggs in this experiment were of a more ellipsoid shape. We hypothesise that in a variety of avian species it would be more suitable to apply the sphere model for more round shaped eggs and the egg-shaped ovoid model if the examined eggs are more conical. These options enable using the proposed computation technique not only for the needs of poultry industry but also in ornithological, basically zoological studies when researchers handle varieties of eggs of different shapes.

In the long run, we would suggest that a major application of such non-destructive technology would be industrial egg sorting lines that can be easily equipped with a camera and computer system. To simulate the field conditions, we also tested in the present study whether there would be an imaging error for the egg length and projection area if the eggs are located free, in a tilted position and on a flat surface, and found that it would not introduce any error in calculation of these egg parameters.

The simplicity of the proposed technology of the geometrical transformation could also be suitable for measuring the volumes and surface areas of other objects which shapes resemble ovoids, e.g., fruits, nuts, vegetables, grains, etc.

In conclusion, the present study has shown that the 2-D imaging-assisted geometrical transformation of an egg into one of the known ovoids that mostly resemble the egg shape is a worthy, fast and reliable approach for determining the egg volume and surface area. The geometrical transformation tested for a sample of the chicken eggs showed valid results for the ellipsoid and egg-shaped ovoid models. We suggest that the method can be used for practical applications in examining avian eggs and that the digital imaging and image processing techniques coupled with the non-destructive method can serve as a basis for developing the appropriate instrumental technology and bringing it into practice.

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Appendices A–D. Supplementary data

Supplementary data to this article can be found online at

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Appendix A

$$V = \frac{2\pi L^3}{3(3n+1)},$$

$$n = 1.466\left(\frac{L}{B}\right)^2 - 0.473,$$

$$V = \frac{2\pi L^3}{3(4.398\frac{L^2}{B^2} - 1.419 + 1)} = \frac{2\pi L^3}{13.194\frac{L^2}{B^2} - 1.257} = \frac{2\pi L^3 B^2}{13.194L^2 - 1.257B} =$$
$$= \frac{2\pi L^3 B^2}{1.257L^2(10.496 - \frac{B^2}{L^2})} = \frac{5LB^2}{10.5 - \frac{B^2}{L^2}}$$

Appendix B

$$A = 2\int_0^L y \, \mathrm{d}x = 2\int_0^L \sqrt{L^{\frac{2}{n+1}} x^{\frac{2n}{n+1}} - x^2} \, \mathrm{d}x .$$

According to the Simpson's rule (Recktenwald, 2000), the above integral can be resolved using a universal formula:

$$\int_a^b f(x)dx \approx \frac{h}{3}\left(f(x_0) + 4\sum_{i=1}^{n} f(x_{2i-1}) + 2\sum_{i=1}^{n-1} f(x_{2i}) + f(x_{2n}) \right) \qquad \text{(B1)}$$

where

$$h = \frac{b-a}{2n} \qquad \text{(B2)}$$

and n is a number of pivot points.

In our case a = 0, b = L and let's choose n = 3. Then,

$$h = \frac{L}{6},$$

$$f(x_0) = f(0) = \sqrt{L^{\frac{2}{n+1}} \cdot 0 - 0} = 0$$

$$f(x_1) = f\left(\frac{L}{6}\right) = \sqrt{L^{\frac{2}{n+1}} \cdot \left(\frac{L}{6}\right)^{\frac{2n}{n+1}} - \left(\frac{L}{6}\right)^2} = \sqrt{\frac{L^{\frac{2+2n}{n+1}}}{6^{\frac{2n}{n+1}}} - \frac{L^2}{36}} = \sqrt{\frac{L^{\frac{2(n+1)}{n+1}}}{36^{\frac{n}{n+1}}} - \frac{L^2}{36}} = L\sqrt{\frac{1}{36^{\frac{n}{n+1}}} - \frac{1}{36}}$$

$$f(x_2) = f\left(\frac{L}{3}\right) = \sqrt{L^{\frac{2}{n+1}} \cdot \left(\frac{L}{3}\right)^{\frac{2n}{n+1}} - \left(\frac{L}{3}\right)^2} = L\sqrt{\frac{1}{9^{\frac{n}{n+1}}} - \frac{1}{9}}$$

$$f(x_3) = f\left(\frac{L}{2}\right) = \sqrt{L^{\frac{2}{n+1}} \cdot \left(\frac{L}{2}\right)^{\frac{2n}{n+1}} - \left(\frac{L}{2}\right)^2} = L\sqrt{\frac{1}{4^{\frac{n}{n+1}}} - \frac{1}{4}}$$

$$f(x_4) = f\left(\frac{2L}{5}\right) = \sqrt{L^{\frac{2}{n+1}} \cdot \left(\frac{2L}{5}\right)^{\frac{2n}{n+1}} - \left(\frac{2L}{5}\right)^2} = L\sqrt{\left(\frac{4}{25}\right)^{\frac{n}{n+1}} - \frac{4}{25}}$$

$$f(x_5) = f\left(\frac{5L}{6}\right) = \sqrt{L^{\frac{2}{n+1}} \cdot \left(\frac{5L}{6}\right)^{\frac{2n}{n+1}} - \left(\frac{5L}{6}\right)^2} = L\sqrt{\left(\frac{25}{36}\right)^{\frac{n}{n+1}} - \frac{25}{36}}$$

$$f(x_6) = f(L) = \sqrt{L^{\frac{2}{n+1}} \cdot L^{\frac{2n}{n+1}} - L^2} = 0$$

Thus,

$$A = 2 \cdot \frac{L}{6} \cdot \frac{1}{3}\left(0 + 4L \cdot \left(\sqrt{\frac{1}{36^{\frac{n}{n+1}}} - \frac{1}{36}} + \sqrt{\frac{1}{4^{\frac{n}{n+1}}} - \frac{1}{4}} + \sqrt{\left(\frac{25}{36}\right)^{\frac{n}{n+1}} - \frac{25}{36}}\right) + 2L \cdot \left(\sqrt{\frac{1}{9^{\frac{n}{n+1}}} - \frac{1}{9}} + \sqrt{\left(\frac{4}{25}\right)^{\frac{n}{n+1}} - \frac{4}{25}}\right) + 0\right) =$$
$$= \frac{2L^2}{9}\left(2\left(\sqrt{\frac{1}{36^{\frac{n}{n+1}}} - \frac{1}{36}} + \sqrt{\frac{1}{4^{\frac{n}{n+1}}} - \frac{1}{4}} + \sqrt{\left(\frac{25}{36}\right)^{\frac{n}{n+1}} - \frac{25}{36}}\right) + \sqrt{\frac{1}{9^{\frac{n}{n+1}}} - \frac{1}{9}} + \sqrt{\left(\frac{4}{25}\right)^{\frac{n}{n+1}} - \frac{4}{25}}\right)$$

If one considers the latter equation as

$$A = \frac{2L^2}{9} \cdot k_A, \qquad \text{(B3)}$$

then,

$$k_A = 2\left(\sqrt{\frac{1}{36^{\frac{n}{n+1}}} - \frac{1}{36}} + \sqrt{\frac{1}{4^{\frac{n}{n+1}}} - \frac{1}{4}} + \sqrt{\left(\frac{25}{36}\right)^{\frac{n}{n+1}} - \frac{25}{36}}\right) + \sqrt{\frac{1}{9^{\frac{n}{n+1}}} - \frac{1}{9}} + \sqrt{\left(\frac{4}{25}\right)^{\frac{n}{n+1}} - \frac{4}{25}}. \qquad \text{(B4)}$$

The equation (B4) can be simplified by simulating the data of *B*/*L*, being adequate to the variety of avian eggs and approximating of the obtained data with a simpler dependence. The *B* to *L* ratio is a function of *n* in accordance with the Eq. 11.

Mathematical approximation led to the following formula:

$$k_A = 0.53\left(\frac{B}{L}\right)^2 + 2.868\left(\frac{B}{L}\right) + 0.063, \qquad \text{(B5)}$$

$r = 1$.

Then,

$$A = \frac{2L^2}{9} \cdot \left(0.53\left(\frac{B}{L}\right)^2 + 2.868\left(\frac{B}{L}\right) + 0.063\right) = 0.118\frac{B^2 \cdot L^2}{L^2} + 0.637\frac{B \cdot L^2}{L} + 0.014L^2 \cdot$$

Finally,

$$A = 0.118B^2 + 0.637BL + 0.014L^2. \qquad \text{(B6)}$$

Appendix C

$$A = 0.118B^2 + 0.637LB + 0.014L^2$$

The formula can be rewritten as follows:

$$0.118B^2 + 0.637LB + 0.014L^2 - A = 0$$
$$B^2 + 5.398LB + 0.119L^2 - 8.475A = 0$$

The obtained function can be resolved with a general quadratic formula:

$$B_1 = \frac{-5.398L + \sqrt{29.138L^2 - 0.476L^2 + 33.9A}}{2} = -2.699L + \sqrt{7.166L^2 + 8.475A}$$

or

$$B_1 = 2.677\sqrt{L^2 + 1.183A} - 2.699L. \qquad \text{(C1)}$$

Similar to B_1,

$$B_2 = \frac{-5.398L - \sqrt{29.138L^2 - 0.476L^2 + 33.9A}}{2} = -2.699L - \sqrt{7.166L^2 + 8.475A}. \qquad \text{(C2)}$$

It is obvious that Eq. (C2) is negative, and that is impossible for the actual egg breadth, so only Eq. (C1) makes sense.

Appendix D

1. Sphere.

From

$$V = \frac{\pi B^3}{6},$$

$$B = \left(\frac{6V}{\pi}\right)^{\frac{1}{3}}.$$

From

$$S = \pi B^2,$$

$$B = \left(\frac{S}{\pi}\right)^{\frac{1}{2}}.$$

Thus,

$$\left(\frac{S}{\pi}\right)^{\frac{1}{2}} = \left(\frac{6V}{\pi}\right)^{\frac{1}{3}},$$

$$\left(\left(\frac{S}{\pi}\right)^{\frac{1}{2}}\right)^2 = \left(\left(\frac{6V}{\pi}\right)^{\frac{1}{3}}\right)^2,$$

$$S = \frac{6^{\frac{2}{3}}\pi}{\pi^{\frac{2}{3}}} \cdot V^{\frac{2}{3}} = 6^{\frac{2}{3}}\pi^{\frac{1}{3}}V^{\frac{2}{3}} = 4.835V^{\frac{2}{3}}.$$

2. Ellipsoid.

From

$$V = \frac{\pi L B^2}{6} \cdot \frac{B}{B} = \frac{\pi}{6} \cdot \frac{L}{B} \cdot B^3,$$

$$B = \left(\frac{6}{\pi} \cdot \frac{B}{L}\right)^{\frac{1}{3}} \cdot V^{\frac{1}{3}}.$$

Taking into consideration that

$$S=\frac{\pi B}{2}\left(L\cdot\frac{\arcsin\sqrt{1-\frac{B^2}{L^2}}}{\sqrt{1-\frac{B^2}{L^2}}}+B\right)=\frac{\pi B^2}{2}\left(\frac{L}{B}\cdot\frac{\arcsin\sqrt{1-\frac{B^2}{L^2}}}{\sqrt{1-\frac{B^2}{L^2}}}+1\right)$$

and

$$S=k_S\cdot B^2,$$

we can determine

$$k_S=\frac{\pi}{2}\left(\frac{L}{B}\cdot\frac{\arcsin\sqrt{1-\frac{B^2}{L^2}}}{\sqrt{1-\frac{B^2}{L^2}}}+1\right).$$

Then,

$$B=\left(\frac{S}{k_S}\right)^{\frac{1}{2}},$$

$$\left(\frac{S}{k_S}\right)^{\frac{1}{2}}=\left(\frac{6}{\pi}\cdot\frac{B}{L}\right)^{\frac{1}{3}}\cdot V^{\frac{1}{3}},$$

$$\left(\left(\frac{S}{k_S}\right)^{\frac{1}{2}}\right)^2=\left(\left(\frac{6}{\pi}\cdot\frac{B}{L}\right)^{\frac{1}{3}}\cdot V^{\frac{1}{3}}\right)^2,$$

$$S=k_S\cdot\left(\frac{6}{\pi}\right)^{\frac{2}{3}}\cdot\left(\frac{B}{L}\right)^{\frac{2}{3}}\cdot V^{\frac{2}{3}}=2.418\left(\frac{B}{L}\right)^{\frac{2}{3}}\cdot\left(\frac{L}{B}\cdot\frac{\arcsin\sqrt{1-\frac{B^2}{L^2}}}{\sqrt{1-\frac{B^2}{L^2}}}+1\right)\cdot V^{\frac{2}{3}}.$$

3. Egg-shaped ovoid.

$$V=\frac{5}{10.5-\frac{B^2}{L^2}}\cdot LB^2=\frac{5}{10.5-\frac{B^2}{L^2}}\cdot LB^2\cdot\frac{L^2}{L^2}=\frac{5\frac{B^2}{L^2}}{10.5-\frac{B^2}{L^2}}\cdot L^3.$$

If we consider

$$V = k_V \cdot L^3,$$

we can put down

$$k_V = \frac{5\frac{B^2}{L^2}}{10.5 - \frac{B^2}{L^2}},$$

$$L = \left(\frac{V}{k_V}\right)^{\frac{1}{3}},$$

$$S = 1.077B^2 + 1.879BL + 0.08L^2 = \left(1.077\frac{B^2}{L^2} + 1.879\frac{B}{L} + 0.08\right)L^2.$$

If we take into account that

$$S = k_S \cdot L^2,$$

we obtain

$$k_S = 1.077\frac{B^2}{L^2} + 1.879\frac{B}{L} + 0.08.$$

Then,

$$L = \left(\frac{S}{k_S}\right)^{\frac{1}{2}},$$

$$\left(\frac{S}{k_S}\right)^{\frac{1}{2}} = \left(\frac{V}{k_V}\right)^{\frac{1}{3}},$$

$$\left(\left(\frac{S}{k_S}\right)^{\frac{1}{2}}\right)^2 = \left(\left(\frac{V}{k_V}\right)^{\frac{1}{3}}\right)^2,$$

$$S = \frac{k_S}{k_V^{\frac{2}{3}}} \cdot V^{\frac{2}{3}} = \left(1.077\frac{B^2}{L^2} + 1.879\frac{B}{L} + 0.08\right) \cdot \left(\frac{10.5 - \frac{B^2}{L^2}}{5\frac{B^2}{L^2}}\right)^{\frac{2}{3}} \cdot V^{\frac{2}{3}}.$$

Finally,

$$S = \left(1.077\frac{B^2}{L^2} + 1.879\frac{B}{L} + 0.08\right)\left(2.1\frac{L^2}{B^2} - 0.2\right)^{\frac{2}{3}} \cdot V^{\frac{2}{3}}.$$

Figure captions

Fig. 1. Typical shapes of bird eggs (Biggins *et al.*, 2018): (a) White-breasted Kingfisher (*Halcyon smyrnensis*); (b) Adelie Penguin (*Pygoscelis adeliae*); (c) Dalmatian Pelican (*Pelecanus crispus*); (d) Greater Flamingo (*Phoenicopterus roseus*); (e) Southern Brown Kiwi (*Apteryx australis*); (f) Little Grebe (*Tachybaptus ruficollis*); (g) Royal Tern (*Thalasseus maximus*); (h) King Penguin (*Aptenodytes patagonicus*); (i) Pheasant-tailed Jacana (*Hydrophasianus chirurgus*); (j) Common Guillemot (*Uria aalge*).

Fig. 2. Block diagram of the imaging system for egg measurement.

Fig. 3. Physical setup of the imaging system.

Fig. 4. Example images of tested eggs: (a) free position; (b) taped.

Fig. 5. Edge detection of the egg image as shown in Fig. 4b: (a) grey-scale image; (b) binary image; (c) edge of the egg; (d) length and breadth.

Fig. 6. Measurement of length (a) and maximum breadth (b) for the chicken eggs of different origin: Woodlands M, Woodlands Farm medium sized; Woodlands L, Woodlands farm large sized; and Staveleys M, Staveleys Eggs Ltd medium sized.

Fig. 7. Relationship between the actual length (a) and surface area (b) and that of free projection eggs computed based on the digital images.

***Detailed Response to Reviewers**

Dear Sir/Madam,

We are addressing the following suggestions of Reviewer 1 as follows:

Reviewer notes:
it will be good to include more than one edge detection algorithm. Include it or describe why do you use only one algorithm.
Authors' response:
Many thanks for your valuable suggestion. According to it, we added the appropriate statement on Lines 225–240 of the revised manuscript.

Reviewer notes:
It will be good to describe more detailed the error sources of measurement.
Authors' response:
We appreciate this comment and added accordingly a more detailed description of the error sources of measurement on Lines 283-288.

Thank you very much.

Sincerely,

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***Credit Author Statement**

VGN: Conceptualization; Investigation; Methodology; Roles/Writing - original draft; Writing - review & editing.

GL: Data curation; Formal analysis; Investigation; Resources; Software; Visualization; Roles/Writing - original draft; Writing - review & editing.

JC: Data curation; Formal analysis; Visualization; Roles/Writing - original draft.

MNR: Conceptualization; Funding acquisition; Investigation; Roles/Writing - original draft; Writing - review & editing.

DKG: Conceptualization; Funding acquisition; Project administration; Resources; Supervision; Roles/Writing - original draft; Writing - review & editing.