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CHAPTER

# 2

# Adaptive Observer Design for Nonlinear Interconnected Systems With Applications


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## 1 INTRODUCTION

The advancement of modern technologies has produced many complex systems. An important class of such systems, which is frequently called a system of systems, or large-scale system, can be expressed by sets of lower-order ordinary differential equations that are linked through interconnections. Such models are typically called large-scale interconnected systems (see, e.g., [1–4]). In order to achieve the desired performance of the closed-loop system in the presence of uncertainties, robust control methods are needed. In recent decades, much of the literature has focused on designing advanced robust controllers for such systems using $H_\infty$ control [5], backstepping techniques [6], robust adaptive control [7], and sliding mode control [8–10].

Increasing requirements for system performance have resulted in increasing complexity within systems modeling, and thus, it becomes interesting to consider nonlinear, large-scale interconnected systems. Such models are then used for controller design. In order to obtain the required levels of performance from the controllers, it is desirable to




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have knowledge of all the system states for use by the control scheme. This state information may be difficult or expensive to obtain, therefore, it would be advantageous to design an observer to estimate all the system states using only the subset of information available from the measured and known inputs and outputs of the system. If the uncertainties in the system are in the form of unknown parameters, then adaptive techniques can be applied such that these unknown parameters are estimated by designing adaptive observers, which is more challenging [11]. This motivates efforts to use an adaptive scheme to design adaptive observers that simultaneously estimate the unknown parameters, and the unavailable states of a dynamical system.

## 1.1 Interconnected Systems

Large-scale interconnected systems have been studied since the 1960s (see [12] and references therein) due to their relevance in a number of practical application areas, and the availability of pertinent theoretical results. Large-scale interconnected systems widely exist in the real world, for example, power networks, ecological systems, transportation networks, biological systems, and information technology networks [2, 13]. A large-scale system is composed of several subsystems with interconnections, whereby the dynamics interact [14]. The application of centralized control [15] to prescribe stability of an interconnected system, particularly when the system is spread over a wide geographical area, may require additional costs for implementation, and careful consideration of the required information sharing between subsystems. This motivates consideration of the design of decentralized control strategies, whereby each subsystem has a local control based only upon locally available information.

Early work focused on linear systems [16, 17]. However, due to the uncertainties and disturbances present in large-scale interconnected systems, study of the stability of such systems is a very challenging task [18]. Subsequent results used decentralized control frameworks for nonlinear large-scale interconnected systems. The study of such decentralized controllers has stimulated a great deal of literature (e.g., [19–21]), and recently, [22, 23]. In much of this work, however, it is assumed that all the system state variables and the system parameters are available for use by the controller [1, 2, 24, 25]. However, this assumption can limit practical application, as usually only a subset of state variables may be available/measurable [26]. Moreover, many practical systems have unknown parameters. It becomes of interest to establish adaptive observers to estimate the system states and the system parameters simultaneously. It should also be noted that such adaptive observer design has been applied for fault detection and isolation [26–28]. This further motivates the study of adaptive observer's design for nonlinear large-scale interconnected systems.

## 1.2 Adaptive Observer

The concept of an observer was first introduced by Luenberger [29], in which the difference between the output measurements from the actual plant and the output measurements of a corresponding dynamical model were used to develop an injection signal to force the resulting output error to zero. In the 1970s, the problem of designing observers for estimating system states for large-scale interconnected systems was addressed in [16]. Subsequently, many methods have been developed to design observers for large-scale interconnected linear systems [30–33]. In the real world, many practical control systems involve unknown parameters due to the mechanical wear and modeling errors. Therefore, adaptive observers have been developed to estimate the unavailable states and the unknown parameters simultaneously. Over the past few decades, much literature has been devoted to the design of adaptive observers for linear and nonlinear systems. The early results are mainly for linear systems [34, 35]. In the case of nonlinear systems with unknown parameters, many adaptive observers have been developed (see e.g., [36–38]). Adaptive observers for nonlinear systems have been published in [36], based on the fact that the nonlinear systems can be transformed to a particular observable canonical form. The authors in [37, 38] proposed adaptive observers for nonlinear systems that can be transformable by a global state space transformation to other coordinates, with some extra constraints and conditions imposed on the system. These proposed adaptive observers have been extended in [39, 40] to deal with a general class of nonlinear systems. However, the convergence of the parameters' estimation errors depends on persistence of the excitation condition.

More recently, adaptive observers using different techniques have been proposed in, for example, [41–43], where the unknown parameters are limited to be constant. Compared with much existing work in adaptive observer design with unknown constant parameters, the corresponding observation results for unknown time varying parameters (TVPs) are very limited. The authors in [44] proposed a sampled output, high-gain observer for a class of uniformly observable nonlinear systems in which the unknown parameters are bounded. An adaptive estimator is proposed in [45] to estimate TVPs for nonlinear systems. However, all the system states are assumed to be available. The $H_-/H_\infty$ fault detection observer in the finite frequency domain has been designed in [46] for a class of linear-parameter, varying descriptor systems.

Boizot et al. [47] developed an adaptive observer by using an extended Kalman filter to reduce the effect of perturbations. However, in terms of the parameter estimation for nonlinear systems, it is usually very difficult to analyze the stability of the extended Kalman filter. It should be noted that unknown parameters considered in these papers are constant. An adaptive redesign of reduced order nonlinear observers is presented in

[48], in which the solution of a partial differential equation is required, which may not be possible in most cases. In order to improve the quality of the current drawn from the utility grid, an adaptive nonlinear observer is designed in [49] to estimate the inductor current, which is required in the closed-loop control system of power factor correction as an essential part of AC/DC converters. An adaptive observer is designed for a class of MIMO uniformly observable nonlinear systems with linear and nonlinear parameterizations in [50], and the exponential convergence of the error dynamics for both types of parameterization are guaranteed under the persistent excitation condition. Tyukin et al. [51] considered the problem of asymptotic reconstruction of the state and parameter. However, in both [50, 51], it is required that the unknown parameters are constant. The literature in [52] proposed an adaptive state estimator for a class of multiinput and multioutput nonlinear systems with uncertainties in the state and the output equations, in which the systems considered are not interconnected systems. The work in [53] proposed an adaptive observer that expands the extended state observer to nonlinear disturbed systems. However, the adaptive extended state observer is linear, and requires that the error dynamics can be transformed into a canonical form.

An adaptive observer applying sliding mode techniques has been developed in [54] to enhance the performance of the adaptive observer proposed by Yan and Edwards [55]. Adaptive sliding mode, observer-based fault reconstruction for nonlinear systems with parametric uncertainties is considered in [56]. However, the unknown parameters considered in these papers are constant. Many adaptive observers have been developed using sliding mode techniques for particular applications and for particular purposes (see e.g., [57–60]); and thus, corresponding specific conditions need to be imposed on the systems considered. Sliding mode techniques with super twisting algorithms are used in [61] to design adaptive observers for nonlinear systems in which the unknown parameter vector is assumed to be constant.

## 1.3 Contribution

In this chapter, observers are designed for a class of nonlinear interconnected systems with uncertain TVPs, in which both the isolated subsystems and the interconnections are nonlinear. The designed observers are variable structure interconnected systems, but may not result in sliding motion. Under the condition that the difference between the unknown TVPs and the corresponding uncertain nominal values are bounded by constants, adaptive updating laws are proposed to estimate the parameters. The persistence of excitation conditions is not required. A set of sufficient conditions are proposed such that the error dynamics formed by the system states and the designed observers are asymptotically stable, while the parameters' estimation errors are uniformly, ultimately bounded using LaSalle's theorem. The results obtained are applied to a coupled inverted pendulum system, and simulation results are presented to demonstrate the effectiveness and feasibility of the developed results. The main contribution includes: (i) Both the interconnections and isolated subsystems take nonlinear forms. (ii) The unknown parameters considered in the system are time varying, and the corresponding nominal values are not required to be known. (iii) The asymptotic convergence of the observation error between the states of the considered systems and the states of the designed observers is guaranteed; while the estimate errors of the TVPs are uniformly, ultimately bounded.

### 1.4 Notation

For a square matrix $A$, $A > 0$ denotes a symmetric positive definite matrix, and $\lambda_{\min}(A)(\lambda_{\max}(A))$ denotes the minimum (maximum) eigenvalues of $A$. The symbol $I_n$ represents the $n$th-order unit matrix and $R^+$ represent the set of nonnegative real numbers. The set of $n \times m$ real matrices will be denoted by $R^{n\times m}$. The Lipschitz constant of the function $f$ will be written as $\ell_f$. Finally, $\|\cdot\|$ denotes the Euclidean norm, or its induced norm.

## 2 SYSTEM DESCRIPTION AND PROBLEM FORMULATION

Consider a nonlinear interconnected system composed of $N$ subsystems described as follows

$$\dot{x}_i = A_i x_i + f_i(x_i, u_i) + B_i\theta_i(t)\xi_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij}(x_j) \tag{2.1}$$

$$y_i = C_i x_i \tag{2.2}$$

where $x_i \in R^{n_i}$, $u_i \in U_i \in R^{m_i}$ ($U_i$ is the admissible control set), and $y_i \in R$ are the state variables, inputs, and outputs of the $i$th subsystem, respectively. The functions $f_i(\cdot)$ are known to be continuous, the scalars $\theta_i(t) \in R$ are unknown TVPs, and $\xi_i(t) \in R$ are known regressor signals. The matrices $A_i \in R^{n_i\times n_i}$, $B_i \in R^{n_i\times 1}$, and $C_i \in R^{1\times n_i}$ are constants, and $C_i$ is full column rank. The terms

$$\sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij}(x_j)$$

are the known interconnections of the $i$th subsystems for $i = 1, \ldots, N$.

**Assumption 1.** The matrix pairs $(A_i, C_i)$ are observable for $i = 1, \ldots, N$.

From Assumption 1, there exist matrices $L_i$ such that $A_i - L_iC_i$ are Hurwitz stable. This implies that, for any positive-definite matrices $Q_i \in R^{n_i \times n_i}$, the Lyapunov equations

$$(A_i - L_iC_i)^T P_i + P_i(A_i - L_iC_i) = -Q_i \tag{2.3}$$

have unique positive-definite solutions $P_i \in R^{n_i \times n_i}$.

**Assumption 2.** There exist matrices $F_i \in R$ such that solutions $P_i$ to the Lyapunov equations (2.3) satisfy the constraints

$$B_i^T P_i = F_iC_i, \quad i = 1, \ldots, N \tag{2.4}$$

*Remark 1.* To solve the Lyapunov equations (2.3) in the presence of the constraints, Eq. (2.4) is the well-known constrained Lyapunov problem (CLP) [62]. Although there is no general solution available for this problem, associated discussion, and an algorithm, can be found in [63], which may help to solve the CLP for a specific system.

**Assumption 3.** The uncertain TVPs $\theta_i(t)$ satisfy

$$|\theta_i(t) - \theta_{0_i}| \leq \epsilon_{0_i} \tag{2.5}$$

where $\theta_{0_i}$ are unknown constants, and $\epsilon_{0_i}$ are known constants for $i = 1, \ldots, N$.

*Remark 2.* Assumption 3 is to specify a class of uncertainties tolerated in the observer design. The unknown constants $\theta_{0_i}$ given in Eq. (2.5) are called the nominal value of the uncertain TVPs $\theta_i(t)$ throughout this chapter. Different from the existing work (see e.g., [64, 65]), the unknown parameters $\theta_i(t)$ are time varying, and the nominal values $\theta_{0_i}$ are not required to be known.

For further analysis, the terms $B_i\theta_i(t)\xi_i(t)$ in system (2.1) are rewritten as

$$B_i\theta_i(t)\xi_i(t) = B_i[\theta_{0_i} + \epsilon_i(t)]\xi_i(t) \tag{2.6}$$

where the scalers $\epsilon_i(t) = \theta_i(t) - \theta_{0_i}$.

**Assumption 4.** The nonlinear terms $f_i(x_i, u_i)$, with respect to $x_i \in R^{n_i}$, for $u_i \in U_i \in R^{m_i}$ for $i = 1, 2, \ldots, N$ and $H_{ij}(x_j)$ satisfy the Lipschitz condition.

Assumption 4 implies that there exists nonnegative function $\ell_{f_i}$ and constant $\ell_{H_{ij}}$ such that

$$\|f_i(\hat{x}_i, u_i) - f_i(x_i, u_i)\| \leq \ell_{f_i}(u_i)\|\hat{x}_i - x_i\| \tag{2.7}$$

$$\|H_{ij}(\hat{x}_j) - H_{ij}(x_j)\| \leq \ell_{H_{ij}}\|\hat{x}_j - x_j\| \tag{2.8}$$

for $i = 1, 2, \ldots, N$ and $i \neq j$.

*Remark 3.* Assumption 4 is the limitation to the nonlinear terms, and the interconnections that are necessary to achieve the asymptotic stability of the observation error dynamics. It should be noted that in Assumption 4, it is required that $f_i(x_i, u_i)$ satisfies Lipschitz condition, with respect to only the variable $x_i$.

For nonlinear interconnected system (2.1)–(2.2) satisfying Assumptions 1–4, the objective of this chapter is to design an observer with appropriate adaptive laws such that the states of the system (2.1)–(2.2) can be estimated asymptotically, and the estimation errors of the unknown parameters $\theta_i(t)$ in Eq. (2.1) are uniformly bounded.

## 3 ADAPTIVE OBSERVER DESIGN WITH PARAMETERS ESTIMATION

In this section, an asymptotic observer is designed, and the proposed adaptive laws are presented.

From Eq. (2.6), system (2.1) can be rewritten as

$$\dot{x}_i = A_i x_i + f_i(x_i, u_i) + B_i[\theta_{0_i} + \epsilon_i(t)]\xi_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij}(x_j) \tag{2.9}$$

$$y_i = C_i x_i \tag{2.10}$$

For systems (2.9)–(2.10), construct dynamical systems

$$\begin{aligned}\dot{\hat{x}}_i &= A_i\hat{x}_i + f_i(\hat{x}_i, u_i) + L_i(y_i - \hat{y}_i) + B_i\hat{\theta}_i(t)\xi_i(t) - 2P_i^{-1}(F_iC_i)^T|\xi_i(t)|\epsilon_{0_i}\\ &\quad \times \psi_i(\hat{y}_i, y_i) - B_i\hat{\epsilon}_i(t)\xi_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij}(\hat{x}_j)\end{aligned} \tag{2.11}$$

$$\hat{y}_i = C_i\hat{x}_i \tag{2.12}$$

where $P_i$ and $C_i$ satisfy Eqs. (2.3), (2.4),

$$\psi_i(\hat{y}_i, y_i) = \begin{cases} \frac{F_i(\hat{y}_i - y_i)}{\|F_i(\hat{y}_i - y_i)\|}, & F_i(\hat{y}_i - y_i) \neq 0 \\ 0, & F_i(\hat{y}_i - y_i) = 0 \end{cases} \tag{2.13}$$

for $i = 1, 2, \ldots, N$, and $\hat{\theta}_i(t)$ is given by the adaptive law as follows

$$\dot{\hat{\theta}}_i(t) = -2\delta_i(F_i(\hat{y}_i - y_i))^T\xi_i(t) \tag{2.14}$$

where $\delta_i$ is a positive constant that is a design parameter, the known constant $\epsilon_{0_i}$ satisfies the inequality in Assumption 3, and $\hat{\epsilon}_i(t)$ is defined by

$$\hat{\epsilon}_i(t) = -\frac{1}{\delta_i}\hat{\theta}_i(t) \tag{2.15}$$

for $i = 1, 2, \ldots, N$.

Let $e_{x_i} = \hat{x}_i - x_i$. Then, from systems (2.9), (2.10) and (2.11), (2.12), the error dynamical systems can be described by

$$\begin{aligned}\dot{e}_{x_i} &= (A_i - L_iC_i)e_{x_i} + [f_i(\hat{x}_i, u_i) - f_i(x_i, u_i)] \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N}[H_{ij}(\hat{x}_j) - H_{ij}(x_j)] + B_i\tilde{\theta}_i(t)\xi_i(t) \\ &\quad - B_i\hat{\epsilon}_i(t)\xi_i(t) - B_i\epsilon_i(t)\xi_i(t) - 2P_i^{-1}(F_iC_i)^T|\xi_i(t)|\epsilon_{0_i}\psi_i(\hat{y}_i, y_i) \end{aligned} \tag{2.16}$$

where $\tilde{\theta}_i(t)$ is defined by

$$\tilde{\theta}_i(t) = \hat{\theta}_i(t) - \theta_{0_i} \tag{2.17}$$

for $i = 1, 2, \ldots, N$.

For the convenience of further analysis, let

$$\tilde{\epsilon}_i(t) = \hat{\epsilon}_i(t) - \epsilon_{0_i} \tag{2.18}$$

where the known constant $\epsilon_{0_i}$ satisfies the inequality in Assumption 3 and $\hat{\epsilon}_i(t)$ is defined in Eq. (2.15), for $i = 1, 2, \ldots, N$.

## 4 STABILITY OF THE ERROR DYNAMICAL SYSTEMS

The following result is ready to be presented:

**Theorem 1.** *Under Assumptions 1–4, the error dynamical systems (2.16) with adaptive law (2.14) are uniformly ultimately bounded if the matrix $W^T + W$ is positive definite, where the matrix $W = [w_{ij}]_{N\times N}$ and its entries $w_{ij}$ are defined by*

$$w_{ij} = \begin{cases} \lambda_{\min}(Q_i) - 2\ell_{f_i}\|P_i\|, & i = j \\ -2\|P_i\|\ell_{H_{ij}}, & i \neq j \end{cases} \tag{2.19}$$

*where $P_i$ and $Q_i$ satisfy Lyapunov equation in Eq. (2.3) and $\lambda_{\min}(Q_i)$ represents the minimum eigenvalue of the matrix $Q_i$ for $i = 1, 2, \ldots, N$. Further, the errors $e_{x_i}$ given in Eq. (2.16) satisfy*

$$\lim_{t\to\infty}\|e_{x_i}(t)\| = 0, \quad i = 1, 2, \ldots, N \tag{2.20}$$

*Proof* . For systems (2.14), (2.16), consider the candidate Lyapunov function

$$V = \sum_{i=1}^{N} e_{x_i}^T P_i e_{x_i} + \frac{1}{2}\sum_{i=1}^{N}\left(\frac{1}{\delta_i}\tilde{\theta}_i^2(t) + \tilde{\epsilon}_i^2(t)\right) \tag{2.21}$$

where $\delta_i > 0$ is a design parameters given in Eq. (2.14) for $i = 1, 2, \ldots, N$. Note that, in Eq. (2.21), $\tilde{\epsilon}_i(t)$ is dependent on $\tilde{\theta}_i(t)$. From Eqs. (2.15), (2.17), (2.18), it can be seen that the relationship between $\tilde{\epsilon}_i(t)$ and $\tilde{\theta}_i(t)$ is given by

$$
\begin{aligned}
\tilde{\epsilon}_i(t) &= \hat{\epsilon}_i(t) - \epsilon_{0_i} \\
&= -\frac{1}{\delta_i}\hat{\theta}_i(t) - \epsilon_{0_i} \\
&= -\frac{1}{\delta_i}(\tilde{\theta}_i(t) + \theta_{0_i}) - \epsilon_{0_i}
\end{aligned}
$$

Then, from Eq. (2.16)

$$
\begin{aligned}
\dot{V} &= \sum_{i=1}^{N}(\dot{e}_{x_i}^T P_i e_{x_i} + e_{x_i}^T P_i \dot{e}_{x_i}) + \sum_{i=1}^{N}\left(\frac{1}{\delta_i}\tilde{\theta}_i(t)\dot{\tilde{\theta}}_i(t) + \tilde{\epsilon}_i(t)\dot{\tilde{\epsilon}}_i(t)\right) \\
&= \sum_{i=1}^{N}\Bigg\{ e_{x_i}^T[(A_i - L_iC_i)^T P_i + P_i(A_i - L_iC_i)]e_{x_i} + 2e_{x_i}^T P_i[f_i(\hat{x}_i, u_i) - f_i(x_i, u_i)] \\
&\quad + 2e_{x_i}^T P_i \sum_{\substack{j=1 \\ j\neq i}}^{N}[H_{ij}(\hat{x}_j) - H_{ij}(x_j)] + 2e_{x_i}^T P_i B_i \tilde{\theta}_i(t)\xi_i(t) - 2e_{x_i}^T P_i B_i \epsilon_i(t)\xi_i(t) \\
&\quad - 2e_{x_i}^T P_i B_i \hat{\epsilon}_i(t)\xi_i(t) + \frac{1}{\delta_i}\tilde{\theta}_i(t)\dot{\tilde{\theta}}_i(t) + \tilde{\epsilon}_i(t)\dot{\tilde{\epsilon}}_i(t) \\
&\quad - 4e_{x_i}^T P_i P_i^{-1}(F_iC_i)^T|\xi_i(t)|\epsilon_{0_i}\psi_i(\hat{y}_i, y_i)\Bigg\}
\end{aligned} \tag{2.22}
$$

By using condition (2.4) and $C_ie_{x_i} = \hat{y}_i - y_i$,

$$
\begin{aligned}
e_{x_i}^T P_i B_i &= ((P_iB_i)^T e_{x_i})^T = (B_i^T P_i e_{x_i})^T \\
&= (F_iC_ie_{x_i})^T = (F_i(\hat{y}_i - y_i))^T
\end{aligned} \tag{2.23}
$$

Substituting Eq. (2.23) into Eq. (2.22), it follows that

$$
\begin{aligned}
\dot{V} = \sum_{i=1}^{N}\Bigg\{ & e_{x_i}^T[(A_i - L_iC_i)^T P_i + P_i(A_i - L_iC_i)]e_{x_i} + 2e_{x_i}^T P_i[f_i(\hat{x}_i, u_i) - f_i(x_i, u_i)] \\
& + 2e_{x_i}^T P_i \sum_{\substack{j=1 \\ j\neq i}}^{N}[H_{ij}(\hat{x}_j) - H_{ij}(x_j)] + \left[2(F_i(\hat{y}_i - y_i))^T\xi_i(t) + \frac{1}{\delta_i}\dot{\tilde{\theta}}_i(t)\right]\tilde{\theta}_i(t) \\
& - 2(F_i(\hat{y}_i - y_i))^T\epsilon_i(t)\xi_i(t) - 2(F_i(\hat{y}_i - y_i))^T\hat{\epsilon}_i(t)\xi_i(t) \\
& + \tilde{\epsilon}_i(t)\dot{\tilde{\epsilon}}_i(t) - 4(F_i(\hat{y}_i - y_i))^T|\xi_i(t)|\epsilon_{0_i}\psi_i(\hat{y}_i, y_i)\Bigg\}
\end{aligned} \tag{2.24}
$$

From Eq. (2.17), it can be seen that $\dot{\tilde{\theta}}_i(t) = \dot{\hat{\theta}}_i(t)$ because $\theta_{0_i}$ is constant. By substituting Eqs. (2.13), (2.14) into Eq. (2.24) gives

$$\dot{V} = \sum_{i=1}^{N} \left\{ e_{x_i}^T[(A_i - L_iC_i)^T P_i + P_i(A_i - L_iC_i)]e_{x_i} + 2e_{x_i}^T P_i[f_i(\hat{x}_i, u_i) - f_i(x_i, u_i)] \right.$$
$$+ 2e_{x_i}^T P_i \sum_{\substack{j=1 \\ j\neq i}}^{N} [H_{ij}(\hat{x}_j) - H_{ij}(x_j)] - 2(F_i(\hat{y}_i - y_i))^T \epsilon_i(t)\xi_i(t)$$
$$\left. - 2(F_i(\hat{y}_i - y_i))^T \hat{\epsilon}_i(t)\xi_i(t) + \tilde{\epsilon}_i(t)\dot{\tilde{\epsilon}}_i(t) - 4\|F_i(\hat{y}_i - y_i)\| \, |\xi_i(t)|\epsilon_{0_i} \right\}$$

From Eq. (2.18), it can be seen that $\dot{\tilde{\epsilon}}_i(t) = \dot{\hat{\epsilon}}_i(t)$.

$$\dot{V} = \sum_{i=1}^{N} \left\{ e_{x_i}^T[(A_i - L_iC_i)^T P_i + P_i(A_i - L_iC_i)]e_{x_i} + 2e_{x_i}^T P_i[f_i(\hat{x}_i, u_i) - f_i(x_i, u_i)] \right.$$
$$+ 2e_{x_i}^T P_i \sum_{\substack{j=1 \\ j\neq i}}^{N} [H_{ij}(\hat{x}_j) - H_{ij}(x_j)] - 2(F_i(\hat{y}_i - y_i))^T \epsilon_i(t)\xi_i(t)$$
$$\left. - [2(F_i(\hat{y}_i - y_i))^T \xi_i(t) - \dot{\hat{\epsilon}}_i(t)]\hat{\epsilon}_i(t) - \epsilon_{0_i}\dot{\hat{\epsilon}}_i(t) - 4\|F_i(\hat{y}_i - y_i)\| \, |\xi_i(t)|\epsilon_{0_i} \right\} \tag{2.25}$$

Substituting Eq. (2.15) into Eq. (2.25) yields

$$\dot{V} = \sum_{i=1}^{N} \left\{ e_{x_i}^T[(A_i - L_iC_i)^T P_i + P_i(A_i - L_iC_i)]e_{x_i} + 2e_{x_i}^T P_i[f_i(\hat{x}_i, u_i) - f_i(x_i, u_i)] \right.$$
$$+ 2e_{x_i}^T P_i \sum_{\substack{j=1 \\ j\neq i}}^{N} [H_{ij}(\hat{x}_j) - H_{ij}(x_j)] - 2(F_i(\hat{y}_i - y_i))^T \epsilon_i(t)\xi_i(t)$$
$$\left. - 2\epsilon_{0_i}(F_i(\hat{y}_i - y_i))^T \xi_i(t) - 4\|F_i(\hat{y}_i - y_i)\| \, \|\xi_i(t)\|\epsilon_{0_i} \right\}$$

It is clear from Eq. (2.3) that

$$\dot{V} \leq \sum_{i=1}^{N} \Bigg\{ -e_{x_i}^T Q_i e_{x_i} + 2\|e_{x_i}\| \; \|P_i\| [f_i(\hat{x}_i, u_i) - f_i(x_i, u_i)]$$

$$+ 2\|e_{x_i}\| \; \|P_i\| \sum_{\substack{j=1\\ j\neq i}}^{N} [H_{ij}(\hat{x}_j) - H_{ij}(x_j)]$$

$$-2(F_i(\hat{y}_i - y_i))^T \xi_i(t)[\epsilon_i(t) + \epsilon_{0_i}] - 4\|F_i(\hat{y}_i - y_i)\| \; \|\xi_i(t)\|\epsilon_{0_i} \Bigg\}$$

$$\leq \sum_{i=1}^{N} \Bigg\{ -e_{x_i}^T Q_i e_{x_i} + 2\|e_{x_i}\| \; \|P_i\| [f_i(\hat{x}_i, u_i) - f_i(x_i, u_i)] + 2\|e_{x_i}\| \; \|P_i\| \sum_{\substack{j=1\\ j\neq i}}^{N} [H_{ij}(\hat{x}_j)$$

$$-H_{ij}(x_j)] + 4\|F_i(\hat{y}_i - y_i)\| \; \|\xi_i(t)\|\epsilon_{0_i} - 4\|F_i(\hat{y}_i - y_i)\| \; \|\xi_i(t)\|\epsilon_{0_i} \Bigg\}$$

$$\leq \sum_{i=1}^{N} \Bigg\{ -e_{x_i}^T Q_i e_{x_i} + 2\|e_{x_i}\| \; \|P_i\| [\ell_{f_i} \|\hat{x}_i - x_i\|] + 2\|e_{x_i}\| \; \|P_i\| \sum_{\substack{j=1\\ j\neq i}}^{N} [\ell_{H_{ij}} \|\hat{x}_j - x_j\|] \Bigg\}$$

$$\leq -\sum_{i=1}^{N} \Bigg\{ (\lambda_{\min}(Q_i) - 2\|P_i\|\ell_{f_i})\|e_{x_i}\|^2 - \sum_{\substack{j=1\\ j\neq i}}^{N} (2\|P_i\|\ell_{H_{ij}} \|e_{x_i}\| \|e_{x_j}\|) \Bigg\} \tag{2.26}$$

Then, from the definition of the matrix $W$ in Eq. (2.19) and the preceding inequality, it follows that

$$\dot{V} \leq -\frac{1}{2} X^T [W^T + W] X \tag{2.27}$$

where $X = [\|e_{x_1}\|, \|e_{x_2}\|, \ldots, \|e_{x_N}\|]^T$.

From the LaSalle's theorem (see e.g., [66]), all the solutions of Eq. (2.16) are globally, uniformly bounded and satisfy

$$\lim_{t\to\infty} X^T [W^T + W] X = 0 \tag{2.28}$$

Further, from the facts

$$\lambda_{\min}(W^T + W)\|X\|^2 \leq X^T (W^T + W) X$$

and

$$\|X\|^2 = \|e_{x_1}\|^2 + \|e_{x_2}\|^2 + \cdots + \|e_{x_N}\|^2$$

it is straightforward to see from Eq. (2.28) and the condition $W^T + W > 0$ that

$$\lim_{t\to\infty} \|e_{x_i}(t)\| = 0, \quad i = 1, 2, \ldots, N$$

Hence the conclusion follows. □

*Remark 4.* It should be noted that the constructed Lyapunov function (2.21) is a function of variables $e_{x_i}$, $\tilde{\theta}_i$, and $\tilde{\epsilon}_i$ while the right-hand side of inequality (2.27) is a function of variables $e_{x_i}$ only. Therefore, Theorem 1 implies that $\dot{V}$ is semipositive definite instead of positive definite.

*Remark 5.* Theorem 1 shows that the augmented systems formed by Eq. (2.16) and the adaptive law (2.14) are uniformly ultimately bounded. It should be noted that the estimated states $\hat{x}_i$ given by the observer (2.11) converge to the system states $x_i$ in Eq. (2.1) asymptotically, although the estimate error for the parameters may not be asymptotically convergent. As the uncertain parameters $\theta_i$ in system (2.1) are time-varying, the approaches developed in [28, 65] cannot be applied to the systems considered in this chapter.

*Remark 6.* The designed observer is a variable structure interconnected system, but it may not produce a sliding motion, which is different from the work in [65]. In addition, the unknown parameters are considered constants in [65], while in this chapter they are TVPs.

## 5 CASE STUDY EXAMPLES

In order to illustrate the method developed in this chapter, case study examples on a coupled pendulum system and a quarter-car suspension are carried out in this section.

### 5.1 A Coupled Inverted Pendulum

Consider a system formed by two inverted pendulums connected by a spring, as given in Fig. 2.1. There are two balls that are attached at the end of two rigid rods, respectively. The symbols $u_1$ and $u_2$ denote external torques imposed on the two pendulums, respectively, which are the control inputs. The distance $b$ between the two pendulums is assumed to be changeable, with respect to time $t$.

Let $\varphi_1 = x_{11}$, $\varphi_2 = x_{21}$, $\dot{\varphi}_1 = x_{12}$, and $\dot{\varphi}_2 = x_{22}$. The coupled inverted pendulums can be modeled as (see e.g., [66, 67])

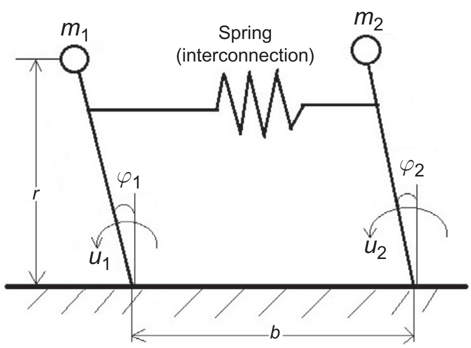


**FIG. 2.1** Coupled inverted pendulums.

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ \left(\frac{m_1 g r}{J_1} - \frac{kr^2}{4J_1}\right) \sin(x_{11}) + \frac{1}{J_1} u_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{kr}{2J_1} \end{bmatrix} (l - b) + \begin{bmatrix} 0 \\ \frac{kr^2}{4J_1} \sin(x_{21}) \end{bmatrix} \tag{2.29}$$

$$y_1 = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} \tag{2.30}$$

$$\dot{x}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} + \begin{bmatrix} 0 \\ \left(\frac{m_2 g r}{J_2} - \frac{kr^2}{4J_2}\right) \sin(x_{21}) + \frac{1}{J_2} u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{kr}{2J_2} \end{bmatrix} (l - b) + \begin{bmatrix} 0 \\ \frac{kr^2}{4J_2} \sin(x_{11}) \end{bmatrix} \tag{2.31}$$

$$y_2 = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} \tag{2.32}$$

The end masses of pendulums are $m_1 = 0.7$ kg and $m_2 = 0.6$ kg, the moments of inertia are $J_1 = 5$ kg and $J_2 = 4$ kg, the constant of connecting spring is $k = 90$ N/m, the pendulum height is $r = 0.25$ m, and the gravitational acceleration is $g = 9.81$ m/s$^2$. In order to illustrate the developed theoretical results, it is assumed that $(l - b(t)) = \theta_1(t) = \theta_2(t)$ is an unknown TVP for $i = 1, 2$; where $l$ is the natural length of the spring, and $b(t)$ is the distance between the two pendulum hinges. The aim is to estimate the unknown TVP $(l - b(t)) = \theta_1(t) = \theta_2(t)$ for $i = 1, 2$, where $l$ is the natural length of the spring, and $b(t)$ is the distance between the two pendulum hinges.

In order to avoid system states going to infinity, and for simulation purposes, the following feedback transformation is introduced

$$u_i = -k_i x_i + v_i, \quad i = 1, 2 \tag{2.33}$$
$$k_1 = \begin{bmatrix} 10 & 15 \end{bmatrix} \tag{2.34}$$
$$k_2 = \begin{bmatrix} 8 & 12 \end{bmatrix} \tag{2.35}$$

Then, with the given preceding parameters, the systems (2.29)–(2.32) can be rewritten as

$$\begin{aligned} \dot{x}_1 &= \underbrace{\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}}_{A_1} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0.06215 \sin(x_{11}) + \frac{1}{5} v_1 \end{bmatrix}}_{f_1(x_1, u_1)} \\ &\quad + \underbrace{\begin{bmatrix} 0 \\ 3 \end{bmatrix}}_{B_1} \underbrace{(l - b(t))}_{\theta_1(t)} + \underbrace{\begin{bmatrix} 0 \\ 0.2813 \sin(x_{21}) \end{bmatrix}}_{H_{12}(x_2)} \end{aligned} \tag{2.36}$$

$$y_1 = \underbrace{\begin{bmatrix} 1 & 1 \end{bmatrix}}_{C_1} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} \tag{2.37}$$

$$\begin{aligned} \dot{x}_2 &= \underbrace{\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}}_{A_2} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0.01632 \sin(x_{21}) + \frac{1}{4} v_2 \end{bmatrix}}_{f_2(x_2, u_2)} \\ &\quad + \underbrace{\begin{bmatrix} 0 \\ 3.75 \end{bmatrix}}_{B_2} \underbrace{(l - b(t))}_{\theta_2(t)} + \underbrace{\begin{bmatrix} 0 \\ 0.352 \sin(x_{11}) \end{bmatrix}}_{H_{21}(x_1)} \end{aligned} \tag{2.38}$$

$$y_2 = \underbrace{\begin{bmatrix} 1 & 1 \end{bmatrix}}_{C_2} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} \tag{2.39}$$

Choose

$$L_i = \begin{bmatrix} 0 & 0 \end{bmatrix} \quad \text{and} \quad Q_i = 4I$$

for $i = 1, 2$. It follows that the Lyapunov equations (2.3) have unique solutions:

$$P_i = \begin{bmatrix} 5 & 1 \\ 1 & 1 \end{bmatrix}, \quad i = 1, 2 \tag{2.40}$$

satisfying the condition (2.4) with

$$F_1 = 3, \quad \text{and} \quad F_2 = 3.75$$

For simplicity, it is assumed that

$$\xi_i(t) = 1, \quad \epsilon_{0_i} = 1, \quad \text{and} \quad \delta_i = 2$$

for $i = 1, 2$.

By direct computation, it follows that the matrix $W^T + W$ is positive definite. Thus, all the conditions of Theorem 1 are satisfied. This implies that the following dynamical systems are the asymptotic observers of the nonlinear interconnected systems (2.36)–(2.39):

$$\begin{aligned}\dot{\hat{x}}_1 = &\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} \hat{x}_{11} \\ \hat{x}_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ 0.06215\sin(\hat{x}_{11}) + \frac{1}{5}v_1 \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ 3 \end{bmatrix}\hat{\theta}_1(t) - \begin{bmatrix} 0 \\ 0.4 \end{bmatrix}\frac{(\hat{y}_1 - y_1)}{\|\hat{y}_1 - y_1\|} \\ &- \begin{bmatrix} 0 \\ 3 \end{bmatrix}\hat{\epsilon}_1(t) + \begin{bmatrix} 0 \\ 0.2813\sin(\hat{x}_{21}) \end{bmatrix}\end{aligned} \tag{2.41}$$

$$\hat{y}_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} \hat{x}_{11} \\ \hat{x}_{12} \end{bmatrix} \tag{2.42}$$

$$\begin{aligned}\dot{\hat{x}}_2 = &\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} \hat{x}_{21} \\ \hat{x}_{22} \end{bmatrix} + \begin{bmatrix} 0 \\ 0.01632\sin(\hat{x}_{21}) + \frac{1}{4}v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 3.75 \end{bmatrix}\hat{\theta}_2(t) \\ &- \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}\frac{(\hat{y}_2 - y_2)}{\|\hat{y}_2 - y_2\|} - \begin{bmatrix} 0 \\ 3.75 \end{bmatrix}\hat{\epsilon}_2(t) + \begin{bmatrix} 0 \\ 0.352\sin(\hat{x}_{11}) \end{bmatrix}\end{aligned} \tag{2.43}$$

$$\hat{y}_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} \hat{x}_{21} \\ \hat{x}_{22} \end{bmatrix} \tag{2.44}$$

The designed adaptive laws are given by

$$\dot{\hat{\theta}}_1(t) = -4(2.25(\hat{y}_1 - y_1))^T \tag{2.45}$$

$$\dot{\hat{\theta}}_2(t) = -4(2.8125(\hat{y}_2 - y_2))^T \tag{2.46}$$

For simulation purposes, the unknown parameters $\theta_{0_i}$ and $\theta_i(t)$ are chosen as 0 and $0.6\sin t$, respectively, for $i = 1, 2$. Simulation in Figs. 2.2 and 2.3 shows that the estimation error between the states of the system (2.29)–(2.32) and the states of the observer (2.41)–(2.44) converges to zero asymptotically. Fig. 2.4 shows that the estimation of the parameters is uniformly bounded with satisfactory accuracy.

## 5.2 A Quarter-Car Suspension

Consider a vehicle (car, bus, etc.) divided into four parts. In the case of the four wheels, each part is a composite mechanical spring-damper system consisting of the quarter part of the mass of the body (together with passengers), and the mass of the wheel. The vertical positions are described by upward directed $x_1$ and $x_2$, see Fig. 2.5. The distance between the road surface and the wheel's contact point is the disturbance $w$, varying

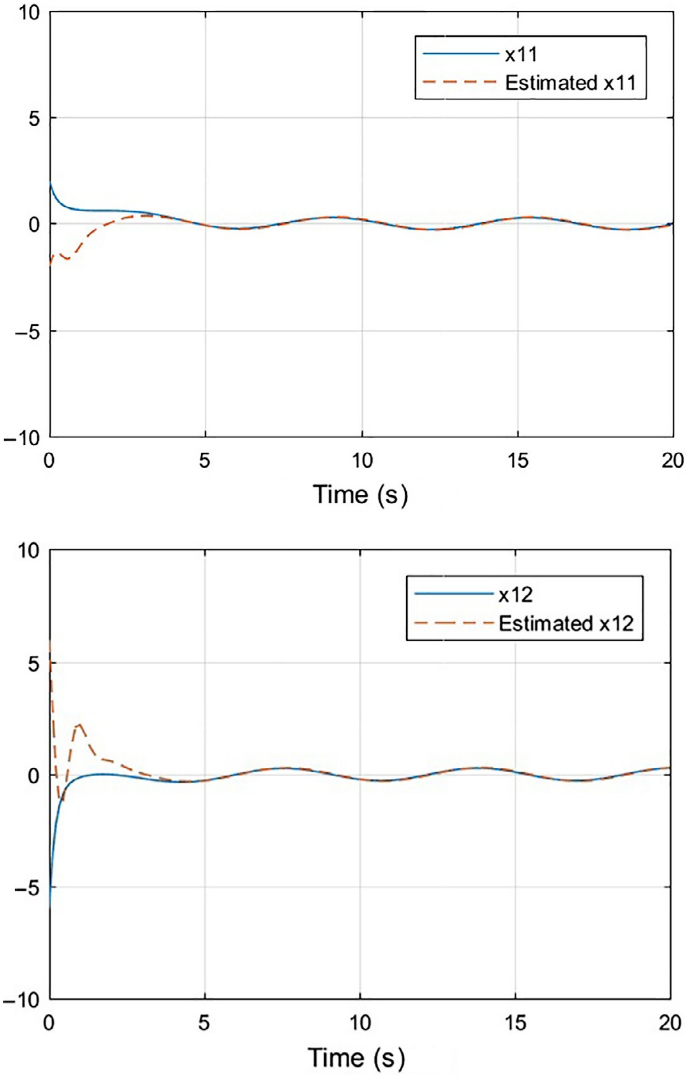


**FIG. 2.2** The time response of the first subsystem states $x_1 = \mathrm{col}(x_{11}, x_{12})$ and their estimation $\hat{x}_1 = \mathrm{col}(\hat{x}_{11}, \hat{x}_{12})$.

together with the road surface. The suspension is active, which means that the actuator produces the force $F$ (control signal).

A good suspension system should have satisfactory road holding stability, while providing good traveling comfort when riding over bumps and holes in the roads.

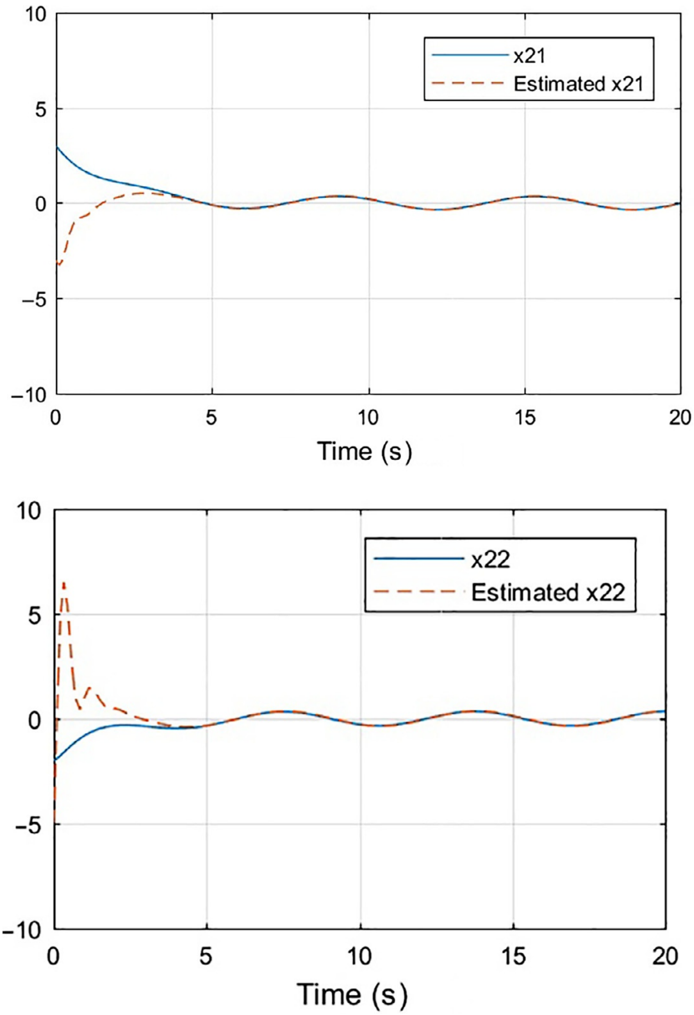


**FIG. 2.3** The time response of the second subsystem states $x_2 = \mathrm{col}(x_{21}, x_{22})$ and their estimation $\hat{x}_2 = \mathrm{col}(\hat{x}_{21}, \hat{x}_{22})$.

Denote $m_1$ and $m_2$ as the mass of the quarter body and the wheel, respectively, while the flexible connections are described by the viscous damping factor $b_1$ and the spring constants $(k_1, k_2)$. The motion equations can be described as follows (see e.g., [68]):

$$m_1\ddot{x}_1 = F - b_1(\dot{x}_1 - \dot{x}_2) - k_1(x_1 - x_2) \tag{2.47}$$

$$m_2\ddot{x}_2 = -F + b_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) - k_2(x_2 - w) \tag{2.48}$$

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 \\ \frac{-k_1}{m_1} & \frac{-b_1}{m_1} \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_1}F \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_1}(k_1x_{21} + b_1x_{22}) \end{bmatrix} \tag{2.49}$$

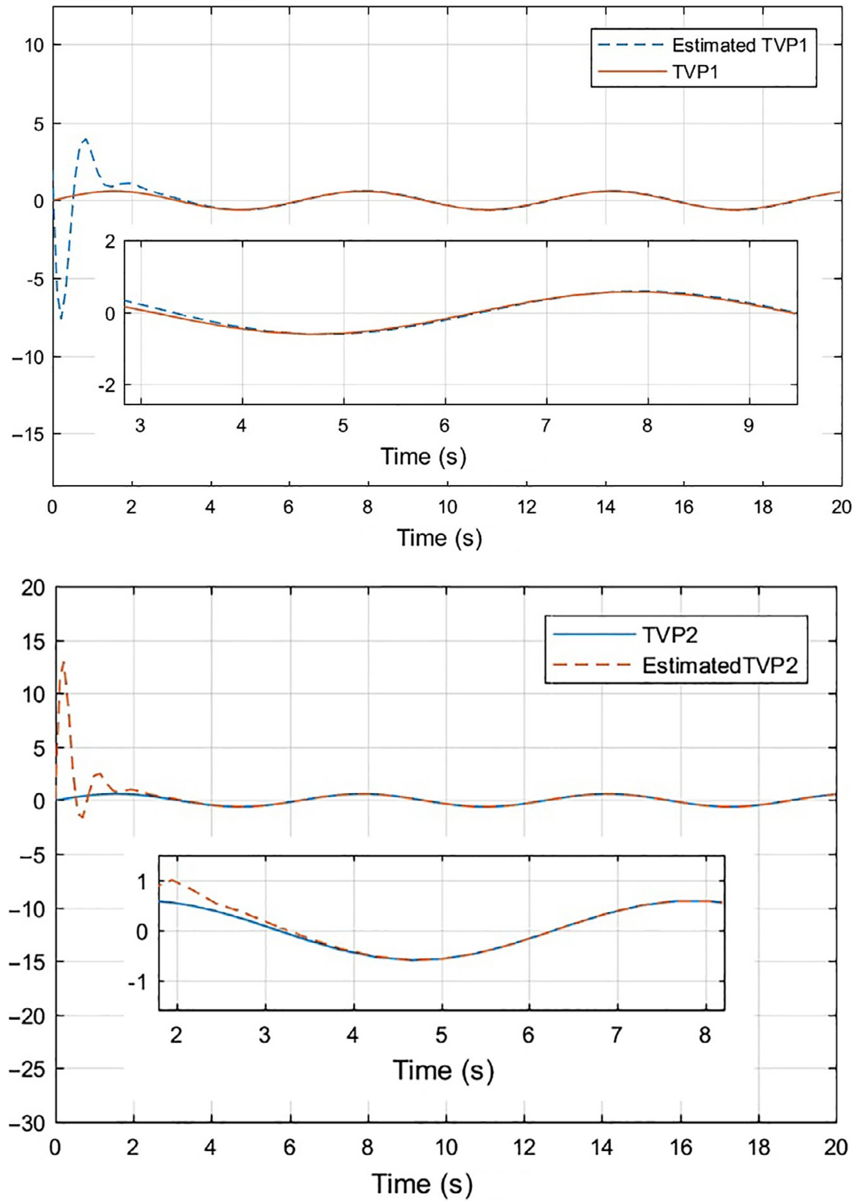


**FIG. 2.4** *Upper*: The time response of $\hat{\theta}_1(t)$ (*dashed line*) and $\theta_1(t)$ (*solid line*); *bottom*: the time response of $\hat{\theta}_2(t)$ (*dashed line*) and $\theta_2(t)$ (*solid line*).

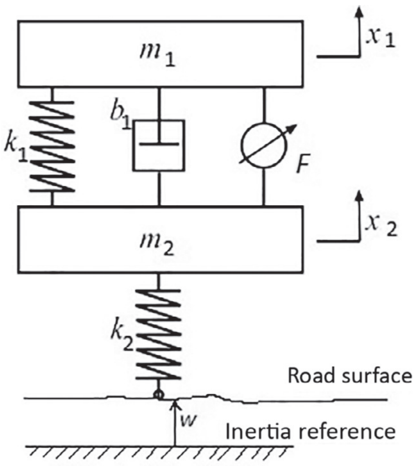


**FIG. 2.5** A quarter-car suspension.

$$y_1 = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} \tag{2.50}$$

$$\dot{x}_2 = \begin{bmatrix} 0 & 1 \\ \frac{-k_2}{m_2} & \frac{-b_1}{m_2} \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-k_1}{m_2} x_{21} - \frac{1}{m_2} F \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{k_2}{m_2} \end{bmatrix} w + \begin{bmatrix} 0 \\ \frac{1}{m_2}(k_1 x_{11} + b_1 x_{12}) \end{bmatrix} \tag{2.51}$$

$$y_2 = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} \tag{2.52}$$

where $m_1 = 500\,\text{kg}$, $m_2 = 300\,\text{kg}$, $b_1 = 900\,\text{N/m/s}$, $k_1 = 900\,\text{N/m}$, and $k_2 = 600\,\text{N/m}$.

In order to avoid system states going to infinity, and for simulation purposes, the following feedback transformation is introduced

$$u_i = -k_i x_i \tag{2.53}$$

$$k_i = \begin{bmatrix} 3.18 & 4.18 \end{bmatrix}, \quad i = 1, 2 \tag{2.54}$$

Then, with the given parameters, the systems (2.49)–(2.52) can be rewritten as

$$\dot{x}_1 = \underbrace{\begin{bmatrix} 0 & 1 \\ -1.8 & -1.8 \end{bmatrix}}_{A_1} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0.002\, u \end{bmatrix}}_{f_1(x_1, u_1)} + \underbrace{\begin{bmatrix} 0 \\ 1.8(x_{21} + x_{22}) \end{bmatrix}}_{H_{12}(x_2)} \tag{2.55}$$

$$y_1 = \underbrace{\begin{bmatrix} 3 & 3 \end{bmatrix}}_{C_1} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} \tag{2.56}$$

$$\dot{x}_2 = \underbrace{\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}}_{A_2} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ -3x_{21} - 0.0033\,u \end{bmatrix}}_{f_2(x_2,u_2)} + \underbrace{\begin{bmatrix} 0 \\ 2 \end{bmatrix}}_{B_2} \underbrace{w}_{\theta(t)} + \underbrace{\begin{bmatrix} 0 \\ 3(x_{11} + x_{12}) \end{bmatrix}}_{H_{21}(x_1)} \tag{2.57}$$

$$y_2 = \underbrace{\begin{bmatrix} 1.8 & 1.8 \end{bmatrix}}_{C_2} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} \tag{2.58}$$

In order to illustrate the developed theoretical results, it is assumed that all the system states are available, and the aim is to estimate the distance between the road surface and the wheel's contact point $w$, which is varying together with the road surface.

Choose

$$L = \begin{bmatrix} 0 & 0 \end{bmatrix} \quad \text{and} \quad Q = 4I$$

It follows that the Lyapunov equations (2.3) have unique solutions:

$$P = \begin{bmatrix} 5 & 1 \\ 1 & 1 \end{bmatrix} \tag{2.59}$$

satisfying the condition (2.4) with

$$F = 0.667$$

For simplicity, it is assumed that

$$\xi(t) = 1, \quad \epsilon_0 = 1, \quad \text{and} \quad \delta = 5$$

The following dynamical systems are the observer of the nonlinear interconnected system (2.55)–(2.58):

$$\dot{\hat{x}}_1 = \begin{bmatrix} 0 & 1 \\ -1.8 & -1.8 \end{bmatrix} \begin{bmatrix} \hat{x}_{11} \\ \hat{x}_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ 0.002\,u \end{bmatrix} + \begin{bmatrix} 0 \\ 1.8(\hat{x}_{21} + \hat{x}_{22}) \end{bmatrix} \tag{2.60}$$

$$\hat{y}_1 = \begin{bmatrix} 3 & 3 \end{bmatrix} \begin{bmatrix} \hat{x}_{11} \\ \hat{x}_{12} \end{bmatrix} \tag{2.61}$$

$$\begin{aligned} \dot{\hat{x}}_2 = {} & \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} \hat{x}_{21} \\ \hat{x}_{22} \end{bmatrix} + \begin{bmatrix} 0 \\ -3\hat{x}_{21} - 0.0033\,u \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} \hat{\theta}(t) - \begin{bmatrix} 0 \\ 1.2 \end{bmatrix} \frac{(\hat{y}_2 - y_2)}{\|\hat{y}_2 - y_2\|} \\ & - \begin{bmatrix} 0 \\ 2 \end{bmatrix} \hat{\epsilon}_2(t) + \begin{bmatrix} 0 \\ 3(\hat{x}_{11} + \hat{x}_{12}) \end{bmatrix} \end{aligned} \tag{2.62}$$

$$\hat{y}_2 = \begin{bmatrix} 1.8 & 1.8 \end{bmatrix} \begin{bmatrix} \hat{x}_{21} \\ \hat{x}_{22} \end{bmatrix} \tag{2.63}$$

The designed adaptive law is given by

$$\dot{\hat{\theta}}(t) = -10(0.667(\hat{y}_2 - y_2))^T \tag{2.64}$$

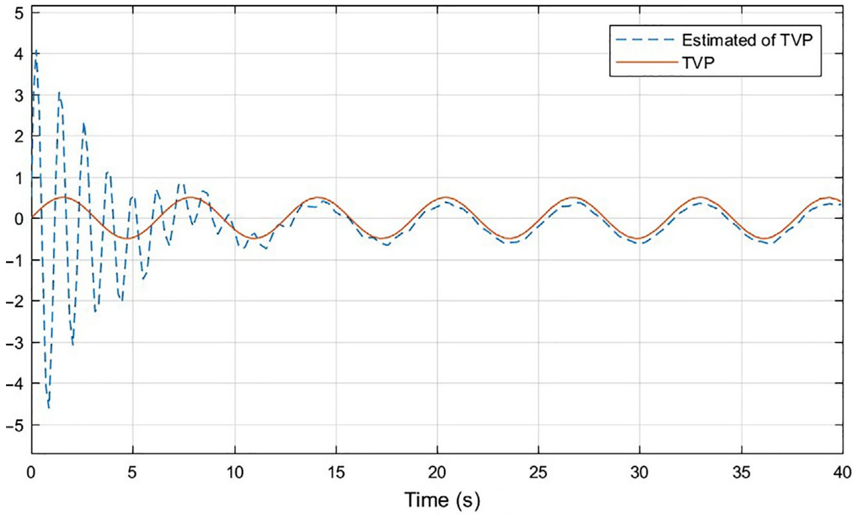


**FIG. 2.6** The time response of $\hat{\theta}(t)$ (*dashed line*) and $\theta(t)$ (*solid line*).

For simulation purposes, the unknown parameters $\theta_0$ and $\theta(t)$ are chosen as 0 and $0.5 \sin t$, respectively. Fig. 2.6 shows that the estimation of the parameter is uniformly, ultimately bounded with satisfactory accuracy.

*Remark 7.* For a real system, the positions and/or the velocities are usually chosen as system outputs. However, sometimes, the linear combination of the position and velocity are taken as system outputs. Physically, such an aggregation of the output might arise in some real systems [69, 70], for example, certain remote-control applications where the number of transmission and receive lines/frequencies are limited [69].

## 6 CONCLUSION

In this chapter, an adaptive observer design for a class of nonlinear, large-scale interconnected systems with unknown TVPs has been proposed based on the Lyapunov direct method. The unknown parameters vary within a given range. A set of sufficient conditions has been developed to guarantee that the observation error system, with the proposed adaptive laws, is globally, uniformly bounded. The states of the designed observer are asymptotically convergent to the original system states. Therefore, from the state estimation point of view, the designed observers are asymptotic observers. Case study examples on a coupled inverted pendulum system and a quarter-car suspension show the practicability of the developed observer scheme for nonlinear interconnected systems.

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