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# **Delivery time dynamics in an assemble-to-order inventory and order based production control system**

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System dynamics play a critical role in influencing supply chain performance. However, the dynamic property of the assemble-to-order (ATO) system remain unexplored. Based on control theory, the inventory and order based production control system (IOBPCS) family, can be utilized as a base framework for assessing system dynamics. However, the underlying assumption in traditional IOBPCS-based analytical studies is that the system is linear and the delivery time to end customers is negligible or backlog is used as a surrogate indicator. Our aim is to incorporate customer delivery lead-time variance as the third assessment measure alongside capacity availability and inventory variance as part of the so-called 'performance triangle'— capacity at the supplier, the customer order decoupling point (CODP) inventory and the delivery lead-time. Using the 'performance triangle' and adopting non-linear control engineering techniques, we assess the dynamic behaviour of an ATO system in the electronics sector. We benchmark the ATO system dynamics model against the IOBPCS family. We exploit frequency response analysis to ensure a robust system design by considering three measures of the 'performance triangle'. The findings suggest delivery LT variance can be minimised by maintaining the ATO system as a true Push-Pull hybrid state with sufficient CODP stock, although increased operational cost driven by bullwhip and CODP variance need to be considered. However, if the hybrid ATO system 'switches' to the pure Push state, the mean and variance of delivery LT can be significantly increased.

*Keywords:* System dynamics, Nonlinear Control theory, Trade-off analysis, Minimum Reasonable Capacity, Minimum Reasonable Lead-time.

## 1. Introduction

The Assemble-to-order system, that is, a combining Make-to-stock (MTS) and Make-to-order (MTO) productions separated by a customer order decoupling point (CODP) (Harrison et al. 2005) in the final assembly plant, has been well-adopted in various industries. e.g. electronic consumer (Chen et al., 2012); semiconductor (Lin et al. 2018), and automobile (Choi et al. 2012). Given the attractiveness of the ATO strategy for companies, including increasing product variety, achieving quick response time and low cost, hence benefiting from the potential risk-pooling effect (Xiao et al. 2010), academics and practitioners have become increasingly interested in analysing ATO systems.

The focus of this paper is to study the dynamic performance of the ATO system by using combined control theory and system dynamics simulation. Systems dynamics plays a critical role in influencing supply chain performance (Spiegler and Naim 2017). Dynamic characteristics, particularly the bullwhip effect (Lee et al., 1997), are considered to be the main sources of disruptions in the business world (Christopher and Peck 2004). System dynamics performance has been extensively studied in pure MTS, or Push-based, systems by assessing order variability (bullwhip) and inventory variance as the two main performance indicators. However, the dynamic behaviour of delivery lead-time, a key performance metric alongside the traditional assessment of bullwhip and inventory variance in hybrid systems, such as ATOs, remains unexplored. This makes it difficult for managers to set control policies for dynamic performance assessment and control, as some of the control rules in traditional MTS systems may break down in the order-driven or hybrid MTO and MTS systems (Akkermans and Vos 2003; Edward et al. 2005). For instance, Goncalves et al. (2005) simulated a complex semiconductor manufacturer and producer supply chain that combines both MTS and MTO manufacturing. They indicated that reducing the level of finished goods inventory can induce greater bullwhip effects in the semiconductor industry. By simplifying and linearizing Goncalves et al. (2005)'s complex hybrid model, Lin et al. (2018) then showed that in such a hybrid model the forecasting policy may result in greater bullwhip than the impact of delays often highlighted in the studies of MTS-based literature.

From an analytical control theory perspective, the well-recognised inventory and order-based production control system (IOBPCS) family of models (Lin et al. 2017), also, ignore the dynamic behaviour of delivery lead-time. This is because the underlying assumption of the IOBPCS family models in most studies is that the system is linear (Lin et al. 2017) and final customer delivery lead-time can be disregarded and set as zero. This has greatly limited the

applicability of published results and has made it difficult to measure end customer delivery lead-time dynamic performance. The customer delivery lead-time measure is especially important in ATO systems, due to capacity constraints, where products cannot always be delivered within the planned lead-time (Wikner et al. 2007).

As a result, motivated by the practical need and academic gap identified for delivery lead-time dynamics assessment, we study the dynamic behaviour of a nonlinear ATO supply chain system within the context of personal computer (PC) sector. This ATO system is well-adopted by the personal computer (PC) industry and its associated semiconductor products (Govindan and Popiuc 2014; Katariya et al. 2014; Lin et al. 2017). However, such systems has suffered severely from capacity unevenness, or the bullwhip effect (Karabuk and Wu 2003; Gonçalves et al. 2005) due to the characteristics of high levels of stochasticity and nonlinearity (Wang and Rivera 2008). For this reason, by incorporating delivery lead-time variance as the third measure as part of the so-called ‘performance triangle’ (delivery lead-time, inventory and capacity) (Klasse and Menor 2007), we aim to analytically assess the delivery lead-time dynamics within the context of the PC ATO system. Overall, we fill the gaps between theoretical predictions and empirical observations, and also advance methodologies that can be used to overcome the difficulties associated with nonlinear systems analysis. Our key contributions in this paper are as follows:

1. Based on nonlinear control theory, we develop a linearisation method to allow for the dynamic analysis of delivery lead-time. To the best of our knowledge, that is the first time that delivery lead-time is analytically assessed as a dynamic metric in combination with capacity (bullwhip) and inventory variability. The so-called the 'performance triangle' (delivery lead-time, inventory and capacity) is analysed within the context of the ATO system dynamics. This offers both practical implications in assessing time-based dynamic behaviour and extends the theoretical understanding from the balanced design of capacity-inventory dynamics to the trade-off consideration of inventory, capacity and delivery lead-time.
2. We develop a nonlinear system dynamics model of the ATO system. Although the model is based on the PC industry, it is a generic ATO representation that can be extended to study other ATO-based systems for different industries or products. Also, the modelling concept, i.e. the Pull and Push parts separated by the CODP stock point, can be exploited for modelling other types of hybrid systems with different locations of

the CODP. For example, in the case of the engineering-to-order systems where the CODP is located fully upstream in the design process (Gosling et al., 2017).

The remainder of this paper is organized as follows. We first model the information and material flow of the PC supply chain in block diagram form based on the descriptions given by Kapuscinski et al. (2004), Huang and Li (2010) and Katariya et al. (2014). Utilizing a Taylor series expansion technique, we linearise the nonlinear representation of delivery lead-time dynamics in the ATO system. Based on the constraints of two stocking points, the nonlinear ATO system can be categorised as three distinctive operational states and the dynamic performance of the 'performance triangle' in responding to sinusoidal demand are analysed separately by using frequency response techniques. Analysing the system dynamics model via the 'filter lens' or sinusoidal input (Towill et al. 2007) allows us to investigate the parameters that impacts on the dynamic properties of the system, including the natural frequency ( $\omega_n$ ) and damping ratio ( $\zeta$ ). The former determines how fast the system's output oscillates during the transient response, while the latter describes how oscillations in the system decay with time. Simulation and numerical studies of the nonlinear ATO system then will be presented for verification and further analysis. We then end with a discussion and conclusion.

## **2. Literature review**

### **2.1 Control theory applications in system dynamics research**

Classic control theory techniques, with feedback thinking and sufficient analytical tools, are advantageous for analysing system dynamics (Lin et al. 2019). The application of classic control theory in a production system can be traced back to Simon (1952). By adopting classic control theory, Towill (1982) translated Coyle's (1977) system dynamics work to represent the Inventory and Order based Production Control System (IOBPCS) in a block diagram form. John et al. (1994) then extended the original model to the automatic pipeline, inventory and order-based production control system (APIOBPCS) by incorporating an automatic work in progress (WIP) feedback loop. These two original models and their variants, i.e. the IOBPCS family, have been recognized as a base framework for production planning and control systems (Lin et al. 2017).

Traditionally, the IOBPCS family represents a typical MTS-based production system in which its service level capabilities are determined by net stock variance and capacity availability. Topics include stability (e.g. Wang et al. 2012), forecasting (e.g. Li et al. 2014)

and supply chain resilience (e.g. Spiegler et al. 2012), to name but a few. However, limited effort has been made to model and analyse the dynamic behaviour of time-oriented production systems, e.g. MTO or ATO system. Also, most of IOBPCS-based studies assume the system is completely linear, although linear assumptions are often criticized for failing to capture the nature of nonlinear attributes of the real supply chain systems with resource and physical constraints (Lin et al. 2017).

Nonlinearities can naturally occur through the existence of physical and economic constraints in supply chain system. For instance, fixed and variable capacity constraints in the manufacturing and shipping processes, variable delays and variable control parameters. Recent IOBPCS-based works analytically studied some forms of nonlinearities in supply chain systems, such as capacity (Jeong et al. 2010; Spiegler et al. 2016a; Spiegler et al. 2016b) and non-negative order constraints (Wang et al. 2012; Wang et al. 2014; Wang et al. 2015) and shipment constraints (Spiegler et al. 2017). Such authors identified the impact of different nonlinearities on the system dynamics such as bullwhip effect in responding to cyclical demand with different mean and frequencies. Also, system structure simplification and linearisation methods are proposed for giving further analytical insights in managing system dynamics in supply chain systems. However, most analyses are limited to a single echelon system and are restricted to the analysis of the different nonlinearities individually. Furthermore, all studies solely explore the dynamic performance of a MTS-based production-inventory control system utilizing bullwhip and inventory variance as performance indicators. To the best of our knowledge, no previous work has analytically assessed the nonlinear ATO system by incorporating the end customer delivery lead-time dynamics.

Beside the IOBPCS family applications, other control approaches / frameworks have also been implemented in studying supply chain dynamics. Specifically, based on optimal control, Ma and Koren (2004) developed a novel method to simultaneously achieve the production target and minimize WIP inventory in large manufacturing systems. Using transfer function and block diagram techniques, Duffie and Shi (2009) presented a method to study the WIP dynamics of a production network. They found that unwanted dynamic variations of the work systems and the production network can be triggered if the structure of order flows between the work systems is omnidirectional and variable. Also, the desired and consistent WIP dynamic behaviour can be produced by sharing order flow structure information.

Jeken et al. (2012) investigated the dynamic interaction of autonomous products and work systems by using hybrid simulation and classic control theory. Such hybrid methods

provide traceable insight into dynamic behaviour of the interacting autonomous products and autonomous work systems. In a multi-product manufacturing environment, Sagawa and Nagano (2015) used Bond Graphs to develop a production control model and study its dynamic performance at the job shop level. Sagawa and Mušič (2019) further revisited a bond graph / mathematical model to depict the dynamics of multi-workstation production systems, and propose closed-loop simulation models with state feedback controllers. Their model provides prescriptive capacity adjustments and can help to define appropriate reference levels for the WIP in the system.

It should be noted that system dynamics and control theory are often criticised as theory-rich but data-poor modelling approach (Pruyt et al. 2014), some attempts have been implemented to integrate other data sources into system dynamics and control theory models. Examples such as combined static value stream mapping and system dynamics models (Agyapong-Kodua et al. 2009; Stadnicka and Litwin 2017), combine big data and control-theoretic models (Hofmann 2017), to name but a few.

## **2.2. ATO system dynamics**

From the system dynamics perspective, existing literature puts the major emphasis on the dynamic modelling and analysis of general customized system such as MTO and hybrid MTS-MTO related systems (Hedenstierna and Ng 2011; Choi et al. 2012; Wikner et al. 2017), while giving little analytical implications specifically for the ATO system. Regarding case study based research, there have been a few studies of the dynamics of ATO systems. Berry and Towill (1992) developed causal loop diagrams to explain the ‘gaming’ that yields bullwhip in the electronics supply chains, including semiconductor production, while Berry et al. (1994) undertook simulation modelling of a generic electronics industry supply chain to highlight the opportunities afforded by different supply chain reengineering strategies to mitigate bullwhip. Gonçalves et al. (2005) developed a system dynamics simulation model to explore how market sales and production decisions interact to create unwanted production and inventory variances in the Intel hybrid ATO supply chain. Lin et al. (2018) then analytically explore Intel’s hybrid ATO model using control engineering. The analytical insights, including the derivation of the stability region as well as the root causes of bullwhip effect, are verified by simulation tests. In recent publication, Lin et al. (2019) developed a PC ATO system dynamics model and explored the impact of capacity and non-negative order nonlinearities on dynamic performance. However, delivery lead-time is not considered in their model due to the assumption that CODP

inventory is always sufficient and thereby all incoming customized orders can be fulfilled by desired lead times.

In terms of analytical/conceptual works, Wikner et al. (2007) developed an MTO system dynamics model and explore its dynamic performance by using the order book feedback control concept. They suggested that managers may be able to control the level of capacity and lead-time flexibility by selecting appropriate forecast smoothing and order book control parameters. Although the model could potentially be extended and used for the dynamic analysis of decoupled systems, it lacks a mechanism for integration between the MTS and MTO elements. Also, Özbayrak, et al. (2007) developed a four-echelon MTO based system dynamics model and analysed some key dynamic metrics such as inventory, WIP levels, backlogged orders and customer satisfaction.

Anderson et al. (2005) assessed the dynamic performance of order-based service supply chains with different degrees of demand variability and information sharing. They developed a capacity management model for a serial chain by presenting related capacity, processing, backlog and service delays at each supply chain stage. By using the system dynamics simulation approach, they characterise the bullwhip phenomenon exist in such supply chain systems. The impact of different levels of information sharing and management strategies on capacity and service delay variability are also studied. Furthermore, using a closed-loop production planning and control system proposed by Duffie and Falu (2002), Knollmann et al. (2014) compared different control strategies (planned lead time and proactive lead time control) to study capacity and lead time dynamics.

Overall, three main limitations are identified. First, most studies do not consider the impact of nonlinearities, such as inventory and capacity constraints, on the dynamics of the ATO system. Second, simulation is the primary choice for most studies and thereby gives little analytical insight or guidance in understanding the system control policies and structures to reduce supply chain dynamics. Finally, although a number of researches investigate the dynamic property of the hybrid MTS-MTO/ATO model, delivery lead time dynamics is largely ignored in literature and most models developed are purely conceptual and thus lead to the difficulty in bridging the gap between theory and real-world observation. We aim to address these gaps by incorporating customer delivery lead-time variance as part of the ‘performance triangle’ within the context of PC ATO supply chains, as found in Lin et al. (2019) as an example.



### **3. Empirical PC model formulation and validation**

#### ***3.1 The PC supply chain description***

There are three manufacturing and one distribution echelons for the PC supply chain: PC part production, sub-assembly, final assembly and distribution (Katariya et al. 2014; Lin et al. 2019). From the material flow perspective, the part and sub-assembly echelons offer what the company called ‘commodities’ required by final PC assembly such as the processor, graphic, motherboard, disk, and software. The corresponding production time is measured in terms of weeks. As the material flows downstream, the process transitions from automated production to highly manual approaches. Final assembly of a PC at the assembly echelon is a largely manual process to allow quick changeover and a high level of flexibility. The corresponding time is measured in terms of days/hours. The final products are shipped either to several company owned distribution centres or directly to final customers.

From the information flow perspective, as illustrated in Figure 1, the hybrid ATO production strategy implements the CODP in the OEMs’ final assembly plants. It should be noted that Figure 1 only show a two-echelon PC ATO supply chain, i.e. the supplier (sub-assembly) and the Original Equipment Manufacturer (OEM) echelons. Specifically, the downstream production of the CODP (final assembly) essentially operates as a MTO in which end customers’ orders pull the available CODP inventory based on their specific PC configurations. However, production upstream of the CODP, i.e. the PC sub-assemble manufacturing, is characterized by MTS: long-term demand forecasting is shared by the OEM and the CODP inventory to determine production rates. It should be noted that although the delayed manufacturing point is located in the OEM’s final assembly plant, there are two CODP inventory stock points due to the adoption of the vendor-managed inventory (VMI) strategy in most of PC supply chains (Huang and Li 2010). Specifically, PC part suppliers are required to manage the finished PC part (CODP inventory) at both their supplier and OEMs sites, by renting or building inventory hubs near the OEMs’ final assembly factories to be pulled by customer orders at a high frequency. This is because of the long geographical distance between OEMs’ final assembly and PC part suppliers’ plants driven by the global supply chain strategy, i.e. longer delay between suppliers and OEMs comparing the short lead-time requirements pulled by customer orders.

As a result, the VMI hub inventory is directly pulled by end customer orders and the inventory at the supplier site is also pulled by the required replenishment of the VMI hub, while the supplier’s part manufacturing push the finished CODP inventory into its stock point. In

return, the OEMs may share important information, e.g. forecasting, real-time backlog, and shipment, to help their suppliers make better CODP inventory replenishment decisions. In other words, the material CODP is incorporated in the final assembly site, while the information CODP is moved to the upstream suppliers' sites to ensure information transparency.

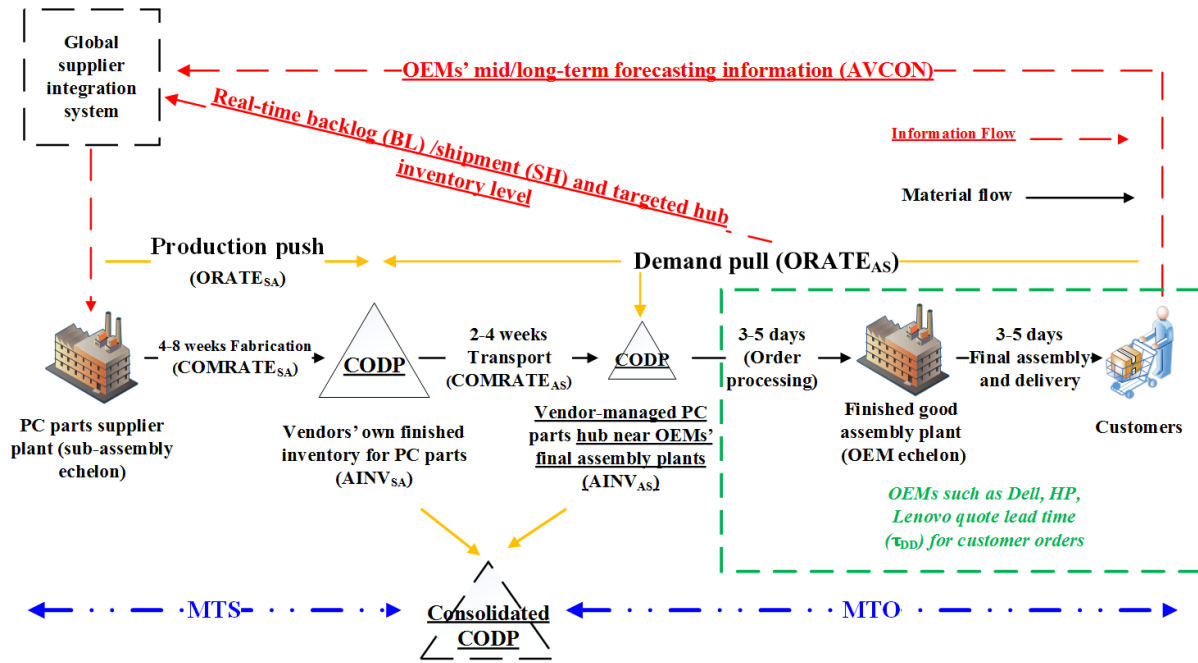


Figure 1. Main material and information flows in PC supply chains, based on Kapuscinski et al. (2004) and Katariya et al. (2014).

### 3.2 Modelling the PC supply chain.

We model the material and information flows of the PC ATO supply chain at an aggregate/single product level. Although in practice, PC and semiconductor companies offer a variety of customized products by a number of commodity parts, the study of the ATO system dynamics based on a single product and a single part setting provide insights of system dynamics at an aggregate level. This assists the long-term strategic planning (e.g. capacity planning, labour expansion, inventory holding) and offers the benchmark of system dynamics performance for subsequent dis-aggregate dynamic modelling and analysis. (Größler and Jörn-Henrik 2008; Lin and Naim 2019) The entire supply chain is modelled as a two-stage system, i.e. *the PC sub-assembly manufacturing supplier and the OEM's final assembly* systems connected by the CODP inventory to represent a typical hybrid ATO structure. The downstream distribution/sales and marketing echelons are not considered in this study, since the orders can be directly transferred to final assembly plant via on-line shopping. Also, the

upstream PC part fabrication echelon is not considered due to the same ordering policy adopted in the sub-assembly supplier echelon, i.e. MRP replenishment rule. All notations used in this paper are presented in Table 1.

$AINV_{AS}$	CODP inventory at VMI hub site near OEM site	$\tau_A$	Time to average consumption
$AINV_{SA}$	CODP inventory at the subassembly supplier site	$\tau_{AS}$	Transport delay for CODP inventory between the supplier site and VMI hub site
$AINV^*_{AS}$	Desired $AINV_{AS}$ level	<b>Pull</b> $ORATE_{AS}$	Desired $ORATE_{AS}$
$AINV^*_{SA}$	Desired $AINV_{SA}$ level	<b>Push</b> $ORATE_{AS}$	Maximum $ORATE_{AS}$
$AVCON$	Averaged consumption rate	<b>SH</b>	Actual shipment rate
<b>BL</b>	Current backlog orders level	<b>SH*</b>	Desired SH rate
<b>BL*</b>	Target backlog orders level	<b>SH<sub>MAX</sub></b>	Maximum shipment rate
<b>BL<sub>ADJ</sub></b>	Backlog adjustment	$\tau_{AINV}$	Time to adjust $AINV_{SA}$ discrepancies
<b>CONS</b>	Customer demand rate	$\tau_{BL}$	Time to adjust backlog discrepancies
<b>COMRATE<sub>AS</sub></b>	VMI inventory (PC parts) arrival rate at the OEM site	$\tau_I$	Time to adjust $AINV_{AS}$ discrepancies
<b>COMRATE<sub>SA</sub></b>	Sub-assembly PC completion rate	$\tau_{WIP}$	Time to adjust WIP discrepancies
<b>WIP</b>	Sub-assembly PC manufacturing work in process level	$\tau_{DD}$	Final assembly operations time (including order processing and final assembly), and transport delay at the OEM site
<b>WIP*</b>	Desired WIP level	$\tau_{SA}$	PC sub-assemble manufacturing delay
<b>WIP<sub>ADJ</sub></b>	WIP adjustment	$s$	$s$ transform operator
<b>ORATE<sub>AS</sub></b>	Order rate for the replenishment of VMI inventory at the OEM site	$a$	Exponential smoothing coefficient
<b>ORATE<sub>SA</sub></b>	Order rate at PC sub-assembly manufacturing site	$b$	First order smoothing coefficient (final assembly)
$\Delta T$	Time interval between samples	<b>LT</b>	Delivery lead-time
<b>IOBPCS</b>	Inventory and Order Based Production Control System		
<b>VIOBPCS</b>	Variable Inventory and Order Based Production Control System		
<b>APIOBPCS</b>	Automatic Pipeline and Inventory and Order Based Production Control System		
<b>APVIOBPCS</b>	Automatic Pipeline and Variable Inventory and Order Based Production Control System		

Table 1. Notations and model descriptors used in modelling PC ATO supply chain system.

Regarding the OEM's final assembly echelon, it focuses on the control of physical final assembly transformation under the pure order-driven strategy. To model this echelon at aggregate level, the relationship between incoming orders and the replenishment of CODP

inventory at the OEM VMI hub site ( $AINV_{AS}$ ) should be captured. Specifically, the supply chain order fulfilment starts by receiving end customers' customized orders at the OEM's final assembly and distribution. If all required  $AINV_{AS}$  are available, they are immediately collected to start the final assembly activities including processing orders, final assembly, test, pack orders and ship them out to the customer by quoted lead times ( $\tau_{DD}$ ). This is reflected by desired shipment rate ( $SH^*$ ). However, if  $AINV_{AS}$  constrains the desired  $SH^*$ , the OEM can only assemble and ship out all PC parts they currently hold on hand, which is represented by  $SH_{MAX}$ .

The first order lag approach (Sarimveis et al. 2008) can be used for aggregately modelling such a process. Specifically, the output of the first order delay, i.e. actual shipment rate,  $SH$ , is determined by:

$$SH(t) = \text{Min}(SH^*(t), SH_{MAX}(t)) \quad (1)$$

If required PC parts in  $AINV_{AS}$  are available for immediate final assembly,  $SH=SH^*$ :

$$SH^*(t) = SH(t) = \frac{BL(t)}{\tau_{DD}} \quad (2)$$

As illustrated in Equation (2), the output, shipment rate ( $SH$ ) under such condition, can be modelled as the desired fraction (determined by  $\tau_{DD}$ ) of current backlog orders ( $BL$ ) (Wikner 2003).  $BL$  refers to cumulative order level determined by the difference between outflow shipment rate  $SH (= SH^*)$  and inflow customer demand rate ( $CONS$ ):

$$BL(t) = BL(t - 1) + CONS(t) - SH(t) \quad (3)$$

Thus,  $BL$  can be treated as a form of 'intangible WIP' in the order driven system, which indicates those customised orders that the OEM has already received but not yet shipped to customers. Moreover,  $\tau_{DD}$  refers to the average physical delay of each received customised order including all final assembly operations (e.g. order processing, final assembly) and transportation delay. As suggested by Atan et al. (2017), a fixed  $\tau_{DD}$  is normally assumed for modelling such delays due to the usually highly reliable final physical assembly operations and delivery times.

As a result, if  $SH=SH^*$ , all incoming customized orders can be fulfilled by the quoted  $\tau_{DD}$  due to sufficient PC parts in  $AINV_{AS}$  available for the immediate final assembly operations and delivery. Hence, customers' waiting time is the sum of the physical final assembly operations and transport lags.

However, if there insufficient  $AINV_{AS}$  constrains  $SH^*$ , the OEM can only ship at  $SH_{MAX}$ , which is represented by a function of on hand  $AINV_{AS}$  and  $\tau_{DD}$ .

$$SH(t) = SH_{MAX} = \frac{AINV_{AS}(t)}{\tau_{DD}} \quad (4)$$

As a result, the average delivery lead-time (LT) may increase due to insufficient, and the further replenishment process of,  $AINV_{AS}$ .  $AINV_{AS}$  is the accumulation of VMI inventory driven by its replenishment, which is given by  $COMRATE_{AS}$  minus its depletion (i.e. SH):

$$AINV_{AS}(t) = AINV_{AS}(t-1) + COMRATE_{AS}(t) - SH(t) \quad (5)$$

In practical terms,  $AINV_{AS}$  is an inbound inventory of parts and subassemblies for the assembling operation, and it contains the same parts and subassemblies as the outbound inventories of the suppliers,  $AINV_{SA}$ . Furthermore, given the dynamical measurement of the delivery LT dynamics is implicit in the ATO model, we incorporate the nonlinear division loop ( $\textcircled{\Pi}$ ) based on Little's Law (Simchi-Levi and Trick, 2011):

$$LT(t) = \frac{BL(t)}{SH(t)} \quad (6)$$

While SH depletes  $AINV_{AS}$ ,  $COMRATE_{AS}$  replenishes it.  $COMRATE_{AS}$  is the delayed order rate ( $ORATE_{AS}$ ) for the replenishment of VMI inventory at the OEM site, due to the transport delay between the supplier and the OEM's final assembly plant. A first order lag can be used to model such a delay (Sipahi and Delice 2010),

$$COMRATE_{AS}(t) = COMRATE_{AS}(t-1) + b \cdot (ORATE_{AS}(t) - COMRATE_{AS}(t-1)) \quad (7)$$

$$\text{where } b = \frac{1}{(1 + \frac{\tau_{AS}}{\Delta T})} \quad (\text{Tohill 1977})$$

$ORATE_{AS}$  is determined by the minimum of the desired Pull  $ORATE_{AS}$  from the final assembly echelon and the feasible Push order rate ( $ORATE_{SA}$ ) from the supplier echelon:

$$ORATE_{AS}(t) = \text{Min}(\text{Pull } ORATE_{AS}(t), \text{Push } ORATE_{SA}(t)) \quad (8)$$

Equation (7) states that customers' orders pull the replenishment of  $AINV_{AS}$  if there are enough finished PC parts in the sub-assembly supplier manufacturing echelon. However, the constrained CODP inventory at the supplier site ( $AINV_{SA}$ ) will lead to the scenario that the upstream supplier plant pushes all feasible  $AINV_{SA}$  ( $\text{Push } ORATE_{SA}(t)$ ) to meet the OEM's demand as soon as possible. By design, Pull  $ORATE_{AS}$  aims to eliminate gaps between targeted inventory ( $AINV_{AS}^*$ ) and  $AINV_{AS}$ , as well as target backlog orders ( $BL^*$ ) and BL adjusted by  $\tau_I$  and  $\tau_{BL}$ . SH, as a more reliable proxy for current demand, is also utilized for deciding Pull  $ORATE_{AS}$  and a non-negativity constraint is given to avoid negative orders being placed on the supplier:

$$\text{Pull } ORATE_{AS}(t) = \text{Max}(0, AINV_{ASadj}(t) + SH(t) + BL_{ADJ}(t)) \quad (9)$$

$AINV_{ASadj}$  is the  $AINV_{AS}$  error control loop adjusted by  $\tau_I$ :

$$AINV_{ASadj}(t) = \frac{1}{\tau_I} \cdot (AINV_{AS}^*(t) - AINV_{AS}(t)), \quad AINV_{AS}^*(t) = SH(t) \cdot \tau_{AS} \quad (10)$$

and  $BL_{ADJ}$  is the backlog error control loop adjusted by  $\tau_{BL}$ :

$$BL_{ADJ}(t) = \frac{1}{\tau_{BL}} \cdot (BL(t) - BL^*(t)), \quad BL^*(t) = CONS(t) \cdot \tau_{DD} \quad (11)$$

$COMRATE_{SA}$  will replenish the depletion of  $AINV_{SA}$ . Due to the long production delay,  $\tau_{SA}$ , usually 4-8 weeks, the supplier echelon is characterized by push production. The APVIOBPCS archetype, i.e. the general case of order-up-to policy (Wang et al. 2014), can be utilized to model such a system. For each replenishment cycle, order rate at PC sub-assembly manufacturing site ( $ORATE_{SA}$ ) is determined by:

$$ORATE_{SA}(t) = \text{Min} \left( \text{Capacity Limit}, AVCON(t) + AINV_{ASadj}(t) + FWIP_{ADJ}(t) \right) \quad (12)$$

where a capacity limit (Min) is utilized to represent the manufacturing plant production resources constraints.  $AVCON(t)$  is a feedforward forecasting policy where the exponential smoothing is adopted (Dejonckheere et al. 2003):

$$AVCON(t) = AVCON(t-1) + a \cdot (CONS(t) - AVCON(t-1)), \quad a = \frac{1}{(1 + \frac{\tau_A}{\Delta T})} \quad (13)$$

$AINV_{SAadj}$  is the CODP inventory error feedback loop adjusted by  $\tau_{AINV}$  and targeted  $AINV_{SA}$  ( $AINV_{SA}^*$ ), where  $AINV_{SA}^*$  for upstream suppliers are based on actual pull  $ORATE_{AS}$  and  $\tau_{SA}$ , although different safety stock policies can be considered:

$$AINV_{SAadj}(t) = \frac{1}{\tau_{AINV}} \cdot (AINV_{SA}^*(t) - AINV_{SA}(t)), \quad AINV_{SA}^*(t) = \tau_{SA} \cdot \text{Pull } ORATE_{AS}(t) \quad (14)$$

$AINV_{SA}$  depends on the accumulation between  $COMRATE_{SA}$  and  $ORATE_{AS}$ :

$$AINV_{SA}(t) = AINV_{SA}(t-1) + COMRATE_{SA}(t) - ORATE_{AS}(t) \quad (15)$$

Furthermore, the dynamic role of WIP inventory at the PC sub-assembly manufacturing site is considered, in line with John et al.'s (1994) standard modelling approach, which a fraction of WIP error is corrected based on the difference between target WIP ( $WIP^*$ ) and WIP:

$$WIP_{ADJ} = \frac{1}{\tau_{WIP}} \cdot (WIP^*(t) - WIP(t)) \quad (16)$$

$WIP^*$  depends on  $AVCON$  and estimated  $\tau_{SA}$  (assume equal to actual  $\tau_{SA}$ , consistent with John et al., 1994), and WIP is an accumulative level between  $COMRATE_{SA}$  and  $ORATE_{SA}$ :

$$WIP^*(t) = \tau_{SA} \cdot AVCON(t) \text{ and } WIP(t) = WIP(t-1) + ORATE_{SA}(t) - COMRATE_{SA}(t) \quad (17)$$

A first order delay is used to model the supplier manufacturing time. This can be interpreted as a production smoothing element representing how slowly the production units adapts to changes in  $ORATE_{AS}$  (Wikner 2003):

$$COMRATE_{SA}(t) = COMRATE_{SA}(t - 1) + c \cdot (ORATE_{SA}(t) - COMRATE_{SA}(t - 1)), \quad c = \frac{I}{(I + \frac{\tau_{SA}}{\Delta T})} \quad (18)$$

Based on Equations (1) - (18), we developed the PC ATO supply chain model in block diagram form, using the continuous time domain, Laplace  $s$ , representation as shown in Figure 2. The rationale for using continuous time domain approach is primarily to examine the stability of the system, although neither continuous or discrete approaches are superior for application in different scenarios (Warburton and Disney 2007). Also, a continuous time approach has the advantage of handling nonlinearities present in the system in an easier way than a discrete time approach (Spiegler et al. 2016b). The entire system consists of a form of Variable Inventory and Order based Production Control System (VIOBPCS) (Edghill and Towill 1990), with the addition of final distribution and BL adjustment loops, for the MTO element and an exact APVIOBPCS for the MTS phase. Also, the two Min functions result in the hybrid ATO system potentially interchanging between three operational states depending on availability of  $AINV_{SA}$  and  $AINV_{AS}$ :

1. Supplier manufacturing Push + final assembly (Pull+ Pull) state, named as the *Push-Pull-Pull state*. The system performs as the desired ATO production if enough  $AINV_{AS}$  and  $AINV_{SA}$  can be guaranteed, all incoming orders thereby fulfilled by  $\tau_{DD}$ .
2. Supplier manufacturing Push + final assembly (Pull+ Push) state, named as the *Push-Pull-Push state*. If  $AINV_{AS}$  is insufficient for incoming orders' pull, the final assembly plant can only ship  $SH_{MAX}$ . The increased backlog and inventory correction signals increase the replenishment rate of  $AINV_{AS}$ , given the condition that customer orders can still pull the  $AINV_{SA}$  at the supplier manufacturing site. The averaged delivery LT is larger than  $\tau_{DD}$ , due to the extra PC part transport acquisition time ( $\tau_{AS}$ ) needed.
3. Supplier manufacturing Push + final assembly (Push+ Push) state, termed as the *Pure Push state*. If pull  $ORATE_{AS}$  is still constrained by  $AINV_{SA}$ , the whole supply chain system will switch to the pure push production, i.e. all  $AINV_{SA}$  and  $AINV_{AS}$  are 'pushed' out as long as they are produced at the supplier site or arrived at the VMI hub. The increase of customer orders cannot be fulfilled for a short time period due to the long supplier manufacturing delay.

Having developed the model, it is important to verify the logic and correctness of the model (Sargent 2013). This verification process is done by simulation on Matlab<sup>TM</sup>. Although we do not show the full verification results, part of the simulation analysis is reported in Table 2. The verification result shows the hybrid ATO model is logical and correct.

Verification test	Details	Verification process	Verification results
<i>Family member and parameters</i>	Behaviour reproduction for cognate system and be consistent with system data and description	1.Regarding the final assembly system, we use the similar Intel supply chain model (Lin et al. 2018) to reproduce its dynamic behaviour by utilizing the same system parameter settings, i.e. $\tau_{AS} = \tau_I = \tau_{BL} = 2\tau_{DD} = 4$ with a unit step increase. 2. For the supplier manufacturing system, OUT settings (Dejonckheere et al. 2003), i.e. $\tau_{SA} = \tau_A/2 = 8$ , $\tau_{AINV} = \tau_{WIP} = 1$ is utilized to check whether the dynamic behaviour is what we expected. That is, the special case of APVIOBPCS archetype.	1. Dynamic behaviour of the final assembly is consistent with the Intel hybrid supply chain model e.g. maximum overshoot/undershoot, rising time and setting time. 2. The dynamic performance of the order-up-to policy can be reproduced.
<i>Boundaries and Structure</i>	Include all important factors and be consistent with system description	Related empirical and conceptual works including Kapuscinski et al. (2004), Katariya et al. (2014) and Wikner et al. (2017) are utilized to check the consistency regarding the system framework and important factors of the PC ATO supply chain.	1. The ATO system dynamic model is consistent with previous descriptions characterized by combined MTO and MTS production, VMI strategy, and material and information decoupling points. 2. All important factors are included for the system dynamics model.
<i>Extremities</i>	Model is logical for extreme values	1. We check whether the dynamic performance of the final assembly system is consistent with the VIOBPCS archetype (Edghill and Towill 1990) if $\tau_{BL} = \tau_{DD} = \infty$ 2. For the supplier manufacturing part, we increase the value of $\tau_{WIP}$ , $\tau_{AINV}$ and $\tau_A$ to extreme conditions to see whether the dynamic performance of the system is still what we expected.	1. The dynamic behaviour of the final assembly system is consistent with corresponding performance in the original VIOBPCS if the backlog and shipment loops are removed. 2. The extreme values of $\tau_A$ , $\tau_{AINV}$ , and $\tau_{WIP}$ will lead to the expected dynamic performance in responding to a step demand increase. For example, the infinite $\tau_{AINV}$ will remove the inventory feedback loop, which result the permanent inventory drift.

Table 2. The verification of the hybrid PC supply chain model.

### 3. Dynamic analysis of the hybrid ATO structure.



### 3.1 Model simplification and delivery LT linearisation

There are multiple nonlinearities in the hybrid ATO system and depending on the rate of change in the output in relation to input, they can be categorized as continuous and discontinuous nonlinearities. To analytically explore the dynamic ‘performance triangle’ of the ATO supply chains, an explanation of the main characteristics of different types of nonlinearities and corresponding simplification / linearisation approaches are reported in Table 3.

Type of nonlinearity in this study	Main characteristics	Simplification/linearisation methods in this study
<p><b>Single-valued discontinuous nonlinearity:</b></p> <p>1) Non-negative order constraint in final assembly plant, i.e. Equation (9).            2) Capacity constraint in the supplier manufacturing plant, i.e. Equation (12).</p>	<p>Sharp changes in output values or gradients in relation to input (e.g. piecewise linear function). Single-valued nonlinearities are also called memory-less, which means that the output value does not depend on the history of the input.</p>	<p>This study focuses on the multi-valued nonlinearities, i.e. shipment and non-negative CODP inventory constraints, since their analyses are crucial for understanding the dynamics of ATO systems. Moreover, single-valued capacity and non-negative order constraints have already been explored in Wang et al. (2012), Wang and Disney (2014) and Spiegler et al. (2016b).</p>
<p><b>Multi-valued discontinuous nonlinearity:</b></p> <p>1) Shipment constraint, i.e. Equation (1)            2) CODP inventory constraint, i.e. Equation (8).</p>	<p>In contrast to the single-value nonlinearity, the output value of multi-valued discontinuous nonlinearity does depend on the history of the input. e.g. changes in capacity constraint subject to the long-term demand forecasting (Spiegler et al. 2016b)</p>	<p>As highlighted previously, two multi-valued nonlinearities (i.e. switches) govern three operational states (Push-Pull-Pull, Push-Pull-Push, and Pure Push) of the hybrid ATO system depending on the feasible <math>AINV_{AS}</math> and <math>AINV_{SA}</math>. We analyse them separately by assuming all discontinuous nonlinearities are not active and temporarily operates as a certain state, that is, the analysis of three operational states</p>
<p><b>Continuous nonlinearity:</b></p> <p>Delivery LT, as shown by Equation (6).</p>	<p>A feature of the outputs in continuous nonlinearity functions is that they are smooth enough to possess convergent expansions at all points and therefore can be linearised.</p>	<p>Taylor series expansion with small perturbation theory will be utilized to linearise the delivery LT variable.</p>

Table 3. Main characteristics of different type of nonlinearities in the ATO system and corresponding simplification/linearisation methods.

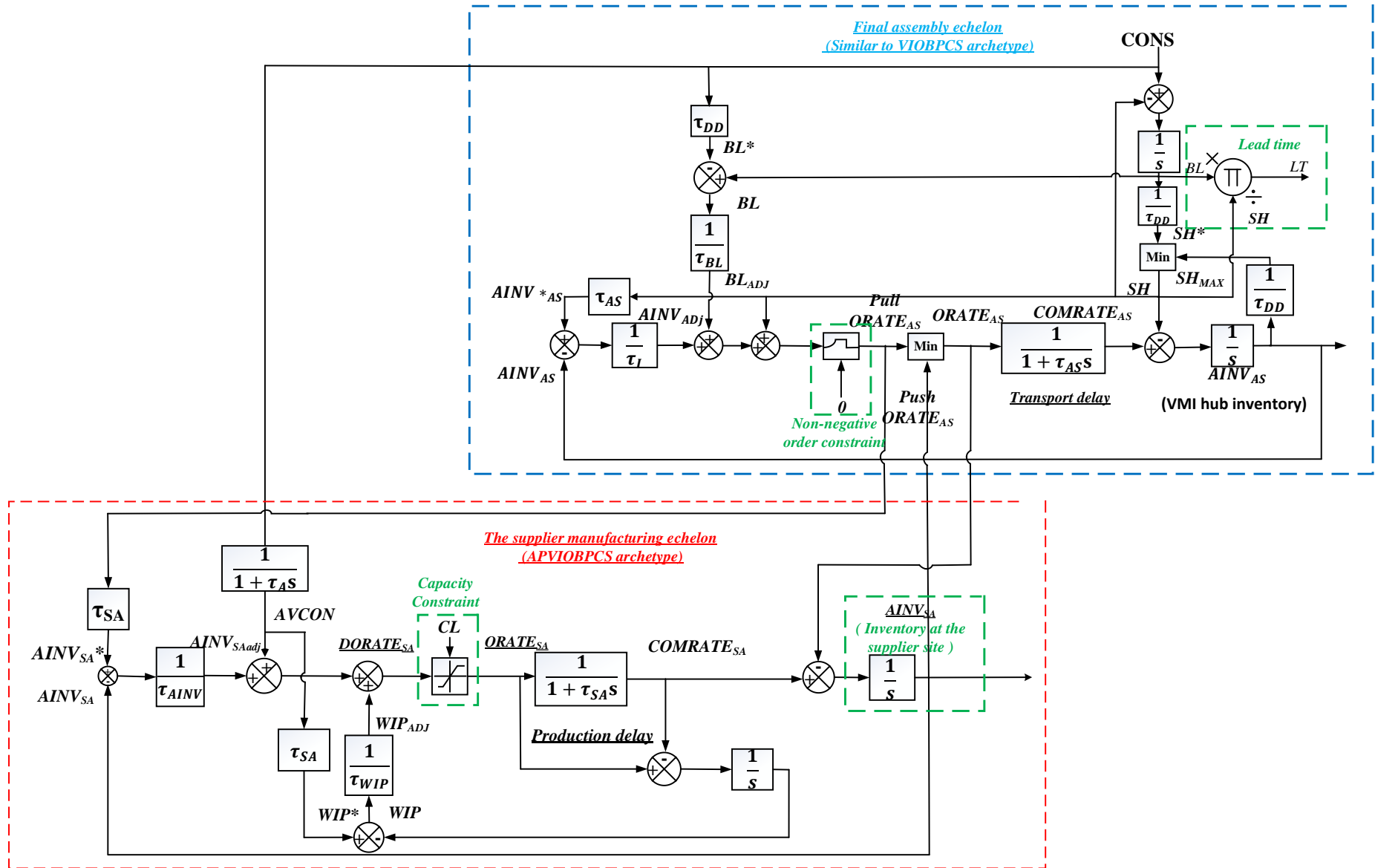


Figure 2. System dynamics model of the PC hybrid ATO supply chain.

Noting that the exclusion of capacity and non-negative order constraints is often criticized for being incapable of representing real supply chain scenarios, the analytical insights derived from the linearised model will be verified by simulation, with the nonlinearities retained, in Section 4. We now linearise the continuous nonlinearity, delivery LT, Equation (6), as shown in Figure 3, by using the Taylor series expansion technique. By temporarily removing all discontinuous nonlinearities, the whole system can be represented by a set of linear differential equations that do not need to be linearised. It should be noted that two multi-valued nonlinearities govern the three-different operational status of the system (Push or Pull), and thereby there are three sets of linear differential equations depending on the specific operational state. e.g. the system will become Push-Pull-Pull state if  $SH^* > SH_{MAX}$  and  $Pull\ ORATE_{AS} < Push\ ORATE_{SA}$ . The only nonlinearity now is the delivery LT, so the problem becomes the linearisation of a nonlinear, continuous function with *one state variable* and *one input variable* only. Let the output delivery  $LT = y$ , input  $BL = u$  and  $SH = x$ , we have

$$LT = \frac{BL}{SH} \rightarrow y = g(x, u) \quad (19)$$

The delivery LT can be linearised about a nominal operating state space  $x^*$  for a given input  $u^*$ , by using small perturbation theory with Taylor series expansion. The first order Taylor series approximation of the nonlinear state derivatives leads to the following linearised function

$$y - y^* = \frac{\partial g}{\partial x} /_{x^*, u^*} (x - x^*) + \frac{\partial g}{\partial u} /_{x^*, u^*} (u - u^*) \quad (20)$$

The equilibrium or resting points  $(x^*, u^*)$  is determined by the final value theorem of a step input demand (CONS) with zero initial condition,  $\frac{\partial g}{\partial x} /_{x^*, u^*}$  (final value of SH in responding a step CONS) and  $\frac{\partial g}{\partial u} /_{x^*, u^*}$  (final value of BL in responding to a step CONS) can be found through the partial derivatives of the output LT equations:

$$\frac{\partial g}{\partial x} /_{x^*, u^*} = -\frac{\tau_{DD}}{CONS}, \quad \frac{\partial g}{\partial u} /_{x^*, u^*} = \frac{1}{CONS}, \quad y^* = \tau_{DD} \quad (21)$$

Thus, delivery LT can be linearised by

$$LT - \tau_{DD} = \left( -\frac{\tau_{DD} \cdot CONS}{CONS^2} (SH - CONS) \right) + \frac{1}{CONS} (BL - \tau_{DD} \cdot CONS) = \frac{BL - \tau_{DD} \cdot SH}{CONS} \quad (22)$$

So

$$LT = \frac{BL - \tau_{DD} \cdot SH}{CONS} + \tau_{DD} \quad (23)$$

Where SH depends on the minimum value of  $SH^*$  and  $SH_{MAX}$ , so if  $SH^*$  can always be satisfied, i.e.  $SH^* = SH$ , then we have

$$BL = SH^* \cdot \tau_{DD} = SH \cdot \tau_{DD} \quad (24)$$

and as a result, delivery LT will become constant:

$$LT = \frac{BL - \tau_{DD} \cdot SH}{CONS} + \tau_{DD} = \tau_{DD} \quad (25)$$

From the customers' perspective, this means that their customized PC products can be received by the 'promised'  $\tau_{DD}$  i.e. 100% customer service. However, if there is insufficient  $AINV_{AS}$  to meet  $SH^*$  i.e.  $SH^* < SH_{MAX}$ , then:

$$SH = \frac{AINV_{AS}}{\tau_{DD}} \quad (26)$$

and therefore, LT is time varying so that

$$LT = \frac{BL - \tau_{DD} \cdot \frac{AINV_{AS}}{\tau_{DD}}}{CONS} + \tau_{DD} = \frac{BL - AINV_{AS}}{CONS} + \tau_{DD} \quad (27)$$

As a result, if Equation (26) holds, LT can be approximated by the summation of  $\tau_{DD}$  and the difference between BL and  $AINV_{AS}$ . Since  $BL - AINV_{AS} > 0$  when  $SH^* < SH_{MAX}$ , the averaged delivery LT now is larger than  $\tau_{DD}$  and this means the time for end customers to wait is longer than the promised  $\tau_{DD}$  and thus lead to a decrease in customer service. Moreover,  $AINV_{AS}$  will be further determined by the CODP inventory constraint between downstream final assembly and the supplier, i.e. the minimum value of Pull  $ORATE_{AS}$  and Push  $ORATE_{SA}$ .

Figures 3a and 3b reported the numerical simulation comparison between original nonlinear LT and linearised LT response in responding to a sinusoidal input with the same Mean (1) but different frequency ( $\omega$ ) and amplitude (A). Note that we deliberately cross-check the different operational states based on the discontinuous nonlinearity switch between Pull  $ORATE_{AS}$  and Push  $ORATE_{SA}$ , that is, compare the original and linearised lead-time response for Push-Pull-Push and pure Push production scenarios. Moreover, two single-valued nonlinearities, capacity and non-negative constraints, are kept in the simulation verification to ensure the system stability, as we will show in the next section, the linear system with Push-Pull-Push state is fundamentally unstable.

Overall, based on Figure 3, the linearised delivery LT response is reasonably accurate by comparing the blue dash line (the original LT response) and the orange solid line (the linearised LT response). Furthermore, comparing Figures 3a and 3b, the linearisation accuracy is increased from Push-Pull-Push to pure Push state and the linearised LT response tends to be more accurate with an increase in demand frequency.

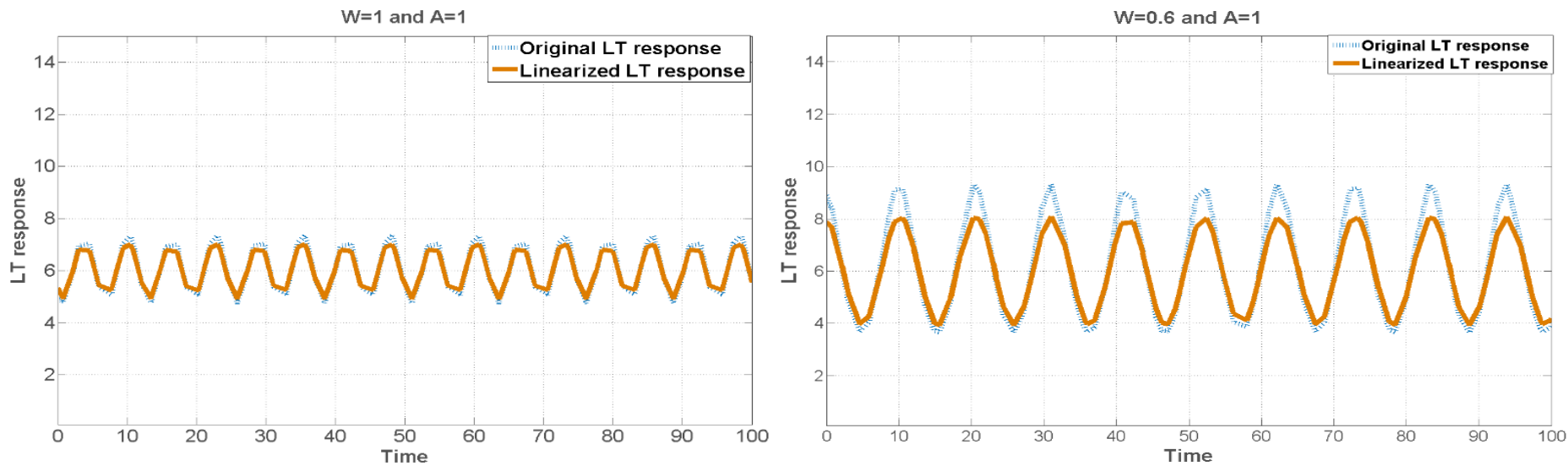


Figure 3a. Comparison between linearised and original LT response for Pull  $ORATE_{AS} < Push ORATE_S$

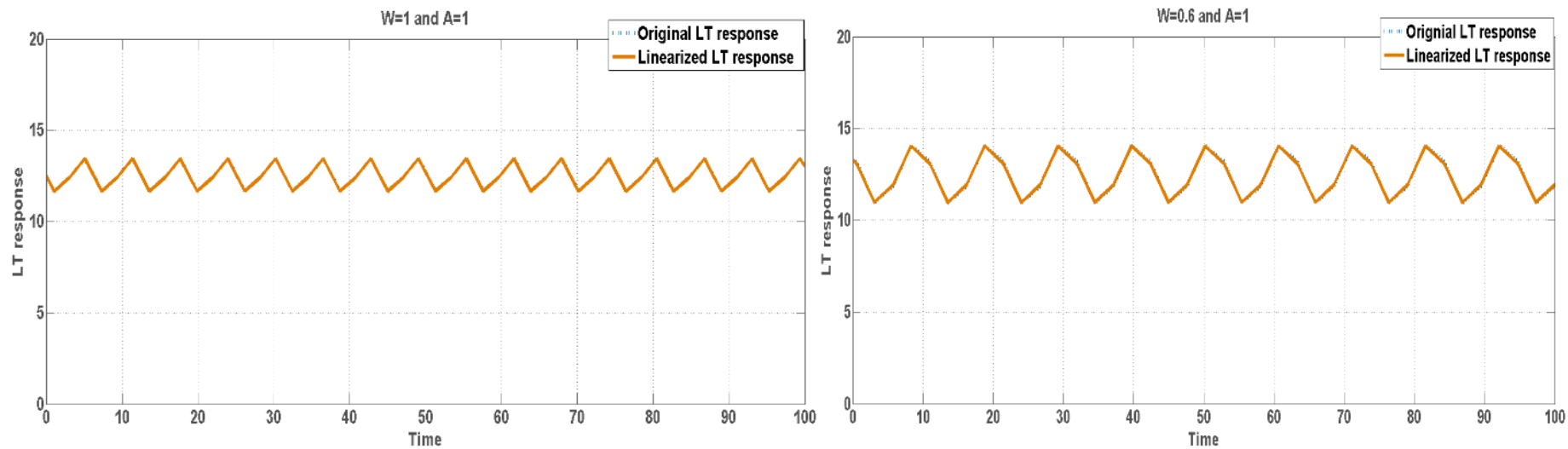


Figure 3b. Comparison between linearised and original LT response for Pull  $ORATE_{AS} > Push ORATE_{SA}$  (Different scales in the y-axis).

### 3.2 The analysis of multi-valued nonlinearities

As summarized in Section 2.2, the PC ATO supply chain can operate between three different states based on two multi-valued nonlinearities, i.e. Equation (1) and (8), which is determined by the availability of  $AINV_{AS}$  and  $AINV_{SA}$ . In the nonlinear environment, different operational states may result during the dynamic response. To understand the impact of such multi-valued nonlinearities on the dynamic performance, that is, the ‘performance triangle’, we analyse the three operational states, *Push-Pull-Pull*, *Push-Pull-Push* and *pure Push* separately by assuming all discontinuous nonlinearities inactive and the whole system temporarily operates in a particular production state.

As a result, for each state the system is completely linear and the corresponding transfer function techniques can be applied. We start by analysing the Characteristics Equations (CE) of the transfer functions for the three operational states. The CE, defined by equating the denominator of overall transfer function to zero, can be used to find poles (roots), which give an initial understanding of the underlying dynamic mechanism of the hybrid ATO system including *system stability* and *unforced system dynamics property* (i.e. natural frequency and damping ratio). Also, the location of roots is a useful indicator for designing system transfer functions to yield ‘good’ system dynamics performance.

#### ***Push-Pull-Pull operational state.***

By design, the entire supply chain operates as the Push-Pull-Pull state in which all incoming end customer order can be satisfied by  $\tau_{DD}$ :

$$\begin{aligned} SH(t) &= SH^*(t) < SH_{MAX}(t) \\ ORATE_{AS}(t) &= Pull\ ORATE_{AS}(t) < Push\ ORATE_{SA}(t) \end{aligned} \quad (28)$$

By maintaining such a state and removing the two Min functions, the corresponding system state in block diagram form can be illustrated as in Figure 4. We derive the CEs for both final assembly and supplier sub-assembly manufacturing echelons, although the full transfer functions can be found in Appendix 1:

$$\begin{aligned} CE_{final\ assembly}: & (1 + \tau_i s + \tau_i \tau_{ASS} s^2)(\tau_{BL} + \tau_{BL} \tau_{DD} s) \quad (29) \\ CE_{supplier\ manufacturing}: & (1 + \tau_i s + \tau_i \tau_{ASS} s^2)(\tau_{BL} + \tau_{BL} \tau_{DD} s)(1 + \tau_{AS} s)(\tau_{WIP} + (\tau_{AINV} \tau_{SA} + \tau_{AINV} \tau_{WIP}) s + \tau_{AINV} \tau_{SA} \tau_{WIP} s^2) \quad (30) \end{aligned}$$

We start the analysis via the Initial Value Theorem (IVT) and Final Value Theorem (FVT). The IVT is a useful tool to mathematically crosscheck the correctness of a transfer function and guide the appropriate initial conditions required for a simulation. The FVT is useful to understand the end steady state value of the dynamic response of a transfer function

and can also help verify any simulation. Since we are interested in the performance triangle, that is, the dynamic performance of the final assembly variable, delivery LT and supplier manufacturing's variables,  $ORATE_{SA}$ ,  $AINV_{SA}$  (the interface between push and pull loops, although  $AINV_{AS}$  is also part of CODP inventory), the initial and final values of  $AINV_{AS}$ ,  $AINV_{SA}$ , and  $ORATE_{SA}$  in responding to a unit step input are obtained. Note that there is no dynamic behaviour for LT for the Push-Pull-Pull state as it is a constant value ( $\tau_{DD}$ ).

$$\begin{aligned}
 \lim_{s \rightarrow \infty} s \frac{AINV_{SA}}{CONS} &= 0 & \lim_{s \rightarrow 0} s \frac{AINV_{SA}}{CONS} &= \tau_{SA} \\
 \lim_{s \rightarrow \infty} s \frac{ORATE_{SA}}{CONS} &= 0 & \lim_{s \rightarrow 0} s \frac{ORATE_{SA}}{CONS} &= 1
 \end{aligned} \tag{31}$$

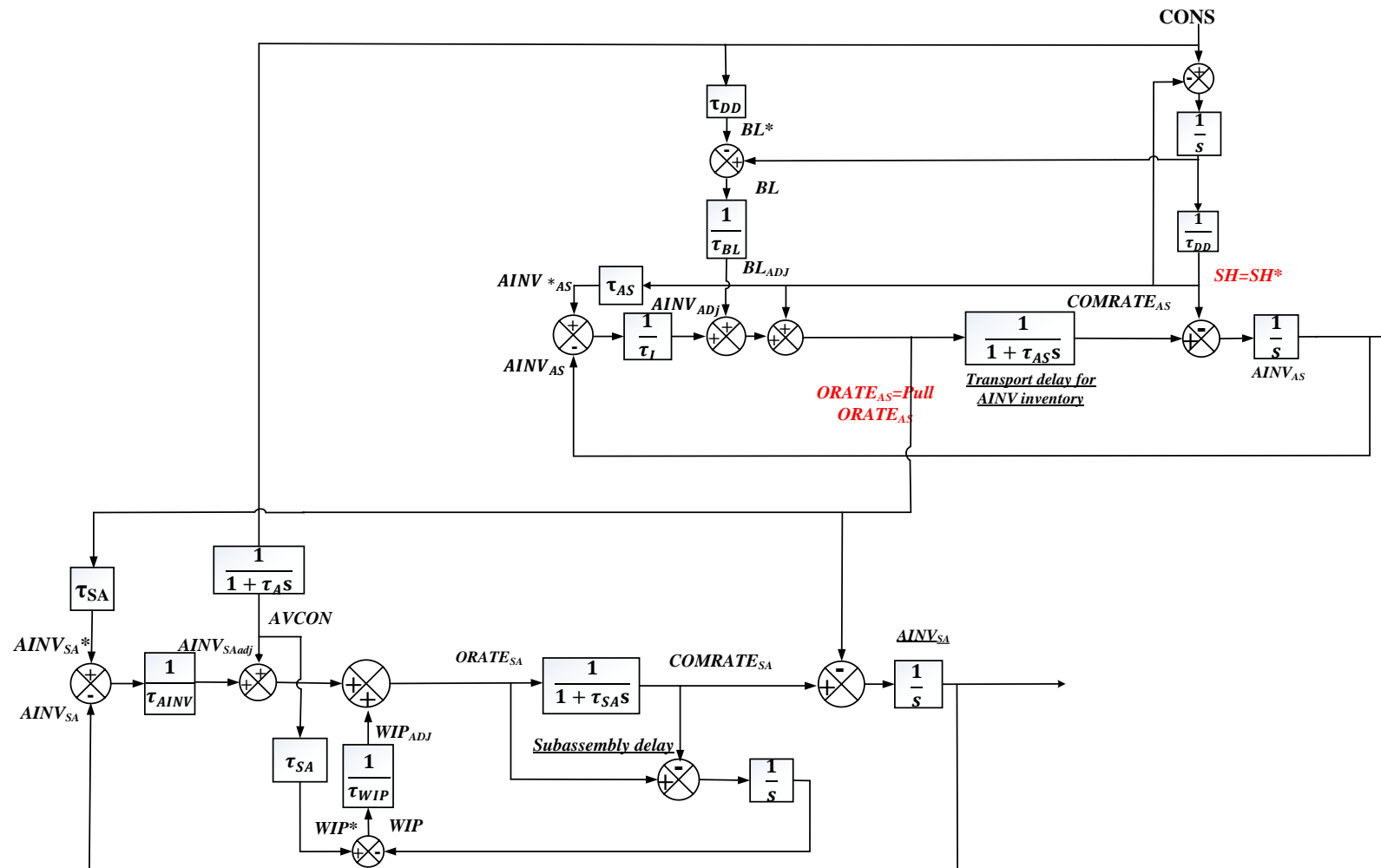


Figure 4. The Push-Pull-Pull state in the block diagram form.



As expected, the initial values of  $AINV_{SA}$  and  $ORATE_{SA}$  are zero, the same as obtained by John et al. (1994). The final value for the  $ORATE_{SA}$ , as expected, equals to the unit demand. The final value of the  $AINV_{SA}$  however, is defined by  $\tau_{SA}$ . By inspecting Equation (29) and (30), a third-order polynomial describes the dynamic nature of the final assembly system, while a sixth order polynomial characterises the dynamic behaviour of the supplier's manufacturing system. Also, there is a third-order polynomial,  $(1 + \tau_i s + \tau_i \tau_{AS} s^2)(\tau_{BL} + \tau_{BL} \tau_{DD} s)$ , in both CEs, which illustrates that, in the Push-Pull-Pull state, the dynamic performance of the supplier sub-assembly system can be partially manipulated by the final assembly control policies, while the dynamic property of the final assembly system is not influenced by the supplier manufacturing system.

By further inspecting Equation (30), we obtain its six roots as follow:

$$R_{1\&2} = -\frac{1}{2\tau_{AS}} \pm \frac{\sqrt{\tau_i^2 - 4\tau_i\tau_{AS}}}{2\tau_i\tau_{AS}}, \quad R_3 = -\frac{1}{\tau_A}, R_4 = -\frac{1}{\tau_{DD}}$$

$$R_{5\&6} = -\frac{1}{2} \left( \frac{1}{\tau_{SA}} + \frac{1}{\tau_{WIP}} \right) \pm \frac{\sqrt{-4\tau_{AINV}\tau_{SA}\tau_{WIP}^2 + (\tau_{AINV}\tau_{SA} + \tau_{AINV}\tau_{WIP})^2}}{2\tau_{AINV}\tau_{SA}\tau_{WIP}} \quad (32)$$

From Equation (32), we can conclude that the hybrid Push-Pull-Pull state is permitted to be stable for possible value of  $\tau_A, \tau_{AINV}, \tau_{WIP}$  and  $\tau_I$ , given  $\tau_{SA}$  and  $\tau_{AS}$  (the system physical delays) are positive. However, the system's response will be continuously oscillatory if  $\tau_{SA} = -\tau_{WIP}$ , that is, the  $R_{5\&6}$  become purely imaginary with no real part. Also, three feedback inventory loops,  $AINV_{AS}, AINV_{SA}$  and WIP adjustment, may generate oscillations of the Push-Pull-Pull state if the square root part of  $R_{1\&2}$  and  $R_{5\&6}$  become negative, i.e.  $\tau_i^2 - 4\tau_i\tau_{AS} < 0$  and  $-4\tau_{AINV}\tau_{WIP}^2\tau_{SA} + (\tau_{AINV}\tau_{WIP} + \tau_{AINV}\tau_{SA})^2 < 0$ . The corresponding CODP inventory-based control policies,  $\tau_I, \tau_{AINV}$  and  $\tau_{WIP}$ , should be carefully adjusted to avoid the possible oscillatory dynamic response.

Furthermore, the inventory proportional control parameters,  $\tau_{AINV}$  and  $\tau_{WIP}$ , and their associated upstream inventory feedback loops, may dominate the dynamic behaviour of the entire Push-Pull-Pull state regarding oscillatory behaviour and recovery speed. This is due to the fact that the real part of  $R_{5\&6}$ ,  $-\frac{1}{2} \left( \frac{1}{\tau_{SA}} + \frac{1}{\tau_{WIP}} \right)$ , is smaller than the real part of  $R_{1\&2}$ ,  $-\frac{1}{2\tau_{AS}}$ , given the sub-assembly manufacturing delay,  $\tau_{SA}$ , and associated  $\tau_{WIP}$  are longer than downstream transport acquisition delay  $\tau_{AS}$ .

***The Push-Pull-Push operational state.***

If  $AINV_{AS}$  continuously depletes and is insufficient for satisfying  $SH^*$ , the company can only assemble and ship what they have on hand,  $SH_{MAX}$ , to customers. As a result, the final assembly and distribution is switched from Pull to Push and if CODP inventory at the supplier site ( $AINV_{SA}$ ) still can be pulled by replenishment of VMI, the system now operates as Push-Pull-Push state, that is:

$$SH(t) = SH_{MAX}(t) < SH^*(t) \quad (33)$$

$$ORATE_{AS}(t) = Pull\ ORATE_{AS}(t) < Push\ ORATE_{SA}(t) \quad (34)$$

Similarly, we can derive the corresponding block diagram to represent such operational state, as illustrated in Figure 5. The only difference between Figure 4 and 5 is that  $SH$  now equals to  $SH_{MAX}$  due to the  $AINV_{AS}$  constraint. We obtain the CEs for both final assembly and sub-assembly manufacturing echelons, although the entire transfer function can be found in Appendix 1 in the supplementary file:

$$CE_{final\ assembly}: \quad s^3\tau_i\tau_{AS}\tau_{BL}\tau_{DD} + s^2(\tau_i\tau_{AS}\tau_{BL} + \tau_i\tau_{BL}\tau_{DD}) - s(\tau_{AS}\tau_{BL} + \tau_{BL}\tau_{DD}) + \tau_i \quad (35)$$

$$CE_{supplier\ manufacturing}: \quad \frac{(1 + s\tau_A) \left( \tau_i + s\tau_{BL} \left( \tau_{DD} - \tau_{AS} + s\tau_i(\tau_{DD} + \tau_{AS}(1 + s\tau_{DD})) \right) \right)}{(\tau_{WIP} + (\tau_{AINV}\tau_{SA} + \tau_{AINV}\tau_{WIP})s + \tau_{AINV}\tau_{SA}\tau_{WIP}s^2)} \quad (36)$$

Equations (35) and (36) illustrate that the upstream part of the Push-Pull-Push state,  $(\tau_{WIP} + (\tau_{AINV}\tau_{SA} + \tau_{AINV}\tau_{WIP})s + \tau_{AINV}\tau_{SA}\tau_{WIP}s^2)$ , remain the same as the Push-Pull-Pull state, due to the assumption that CODP inventory at the sub-assembly supplier site can still be pulled by customer orders. However, the structure of downstream final assembly and distribution echelon changes due to the constraint of  $AINV_{AS}$ . Also, the delivery LT is no longer a constant level and its dynamic property can be characterised by a third order polynomial in non-factorised form including BL and  $AINV_{AS}$  loops, that is, Equation (35). Furthermore, the non-factorised third order polynomial indicates that the independent feedforward  $BL \rightarrow SH^* \rightarrow BL$  loop in the desired Push-Pull-Pull state now has been transformed into part of feedback loop, due to the  $SH_{MAX}$  constraint caused by insufficient  $AINV_{AS}$ , i.e.  $BL \rightarrow ORATE_{AS} \rightarrow AINV_{AS} \rightarrow BL$ . Thereby  $AINV_{AS}$  becomes work-in-process inventory and will be pushed out for final assembly as long as they are arrived in the VMI inventory hub.

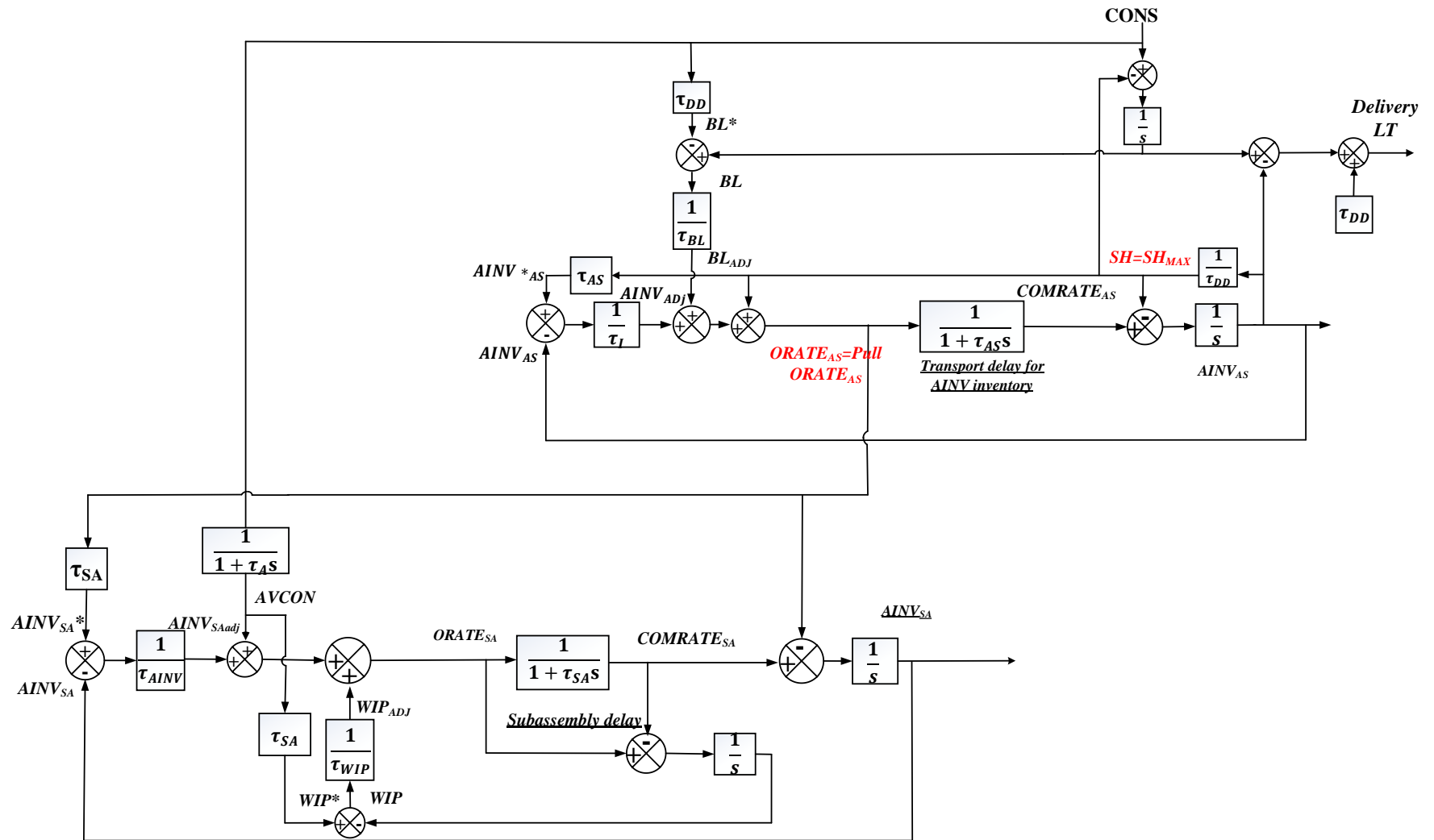


Figure 5. The Push-Pull-Push operational state in the block diagram form.

To access the stability condition of the Push-Pull-Push state, that is, the stability of the non-factorised third-order polynomial, the Routh-Hurwitz stability criterion is utilized. Such method has the advantage of easily and quickly determining the system stability without solving the root of the equations (Disney and Towill 2002). Details of the stability analysis can be found in Appendix 2 in the supplementary file. Based on the Routh-Hurwitz stability analysis, the Push-Pull-Push state characterized by the third order polynomial is unstable. This means the switch from desired Push-Pull-Pull to Push-Pull-Push, resulted by stock out of  $AINV_{AS}$ , not only decreases customer service level due to the increase of delivery LT, but also yields unstable dynamic response (exponentially growing) of CODP inventory and  $ORATE_{SA}$  and thus such a state cannot be maintained for a long period of time.

***The pure Push operational state.***

If CODP inventory at the sub-assembly supplier site,  $AINV_{SA}$ , still constrains the Pull  $ORATE_{AS}$  required by VMI hub replenishment, the system state will switch to a pure Push system with its block diagram representation given as in Figure 6:

$$SH(t) = SH_{MAX}(t) < SH^*(t); ORATE_{AS}(t) = Push ORATE_{SA}(t) < Pull ORATE_{AS}(t) \quad (37)$$

We derived the corresponding CEs as:

$$CE_{final\ assembly} = CE_{supplier\ manufacturing} \left( \begin{array}{c} \left( \tau_i \tau_{SA} + s \tau_{BL} \left( (1 + s \tau_{AS}) (1 + s \tau_{DD}) \tau_i - (\tau_{AS} - \tau_{DD} + \tau_i) \tau_{SA} \right) \right) \\ \tau_{WIP} + s(1 + s) \tau_{AINV} (1 + s \tau_{AS}) \tau_{BL} \\ (1 + s \tau_{DD}) \tau_i (\tau_{WIP} + \tau_{SA} (1 + s \tau_{WIP})) \end{array} \right) \quad (38)$$

Comparing with the Push-Pull-Push and Push-Pull-Pull state, the pure Push state is characterized by a sixth-order polynomial including a first order forecasting loop, and a new fifth-order polynomial in the non-factorised form. This suggests that the final assembly structure independent of the sub-assembly supplier site in the former two states, i.e.  $BL \rightarrow ORATE_{AS} \rightarrow AINV_{AS} \rightarrow BL$ , now is incorporated into the supplier's  $AINV_{SA} \rightarrow ORATE_{SA} \rightarrow AINV_{SA}$  feedback loop, i.e. a fifth order production push loop as in Figure 6. The reduction of independent feedback loops thus may reduce the oscillatory behaviour and contribute to the corresponding decrease of bullwhip and inventory variance.

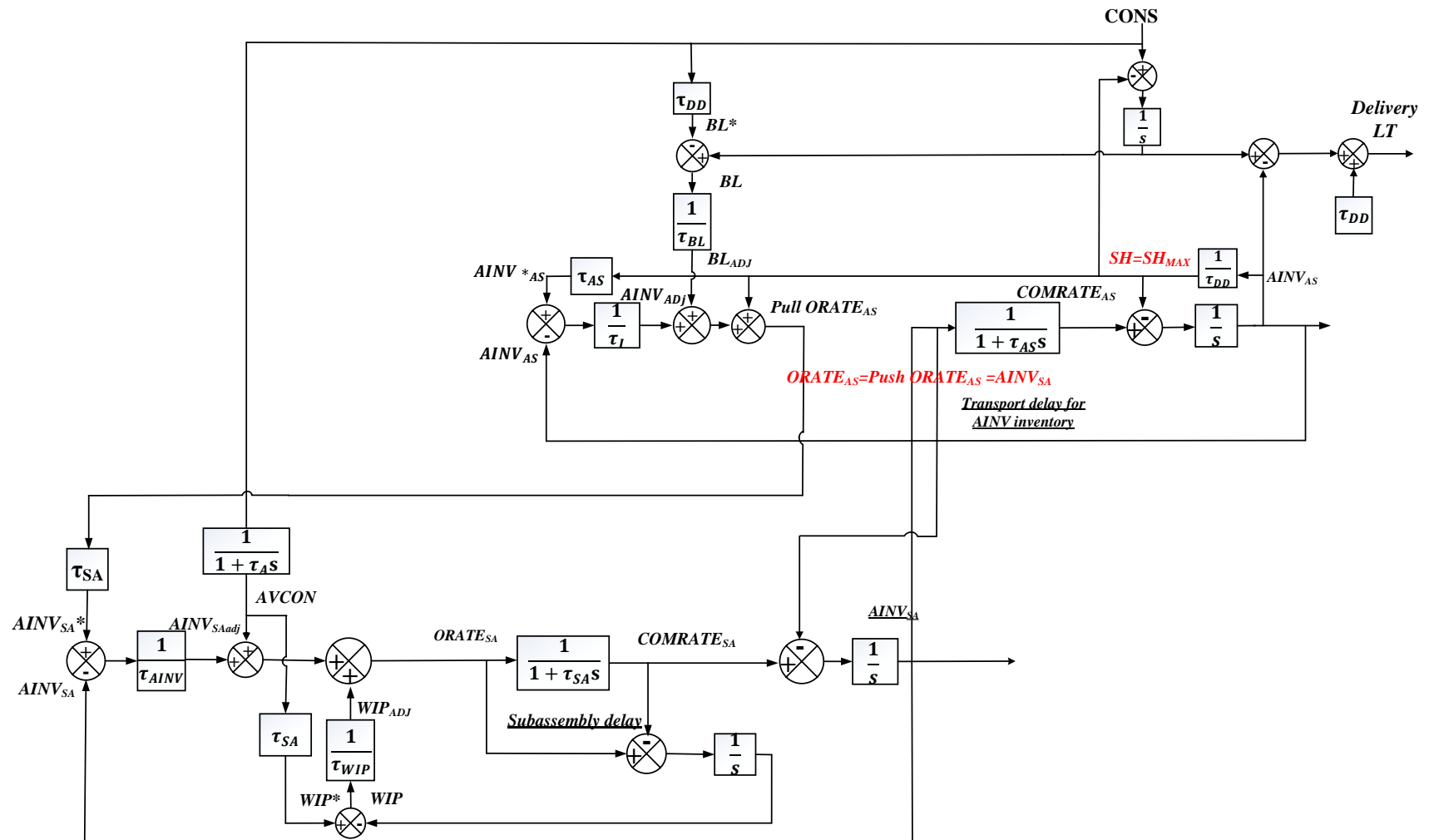


Figure 6. The pure Push state in the block diagram.

The Routh-Hurwitz stability criterion is utilized to examine the stability of Pure Push operational process as given in Appendix 2 in supplementary file. We conclude that depending on the physical delay, the system is partially stable for certain values of control parameters. Specifically, the system can be stable for long time adjustment of two inventory stocks ( $\tau_{AINV}$  and  $\tau_i$ ), although  $\tau_{AINV}$  have a more profound impact on the system stability condition. We obtained the initial value and final value of performance triangle related variables based on the transfer functions of such state, although the full transfer functions can be seen in Appendix 1 in the supplementary file:

$$\begin{aligned}
\lim_{s \rightarrow \infty} s \frac{LT}{CONS} &= 0 & \lim_{s \rightarrow 0} s \frac{LT}{CONS} &= \tau_A + \tau_{DD} + \tau_{BL} \left( \frac{\tau_{DD} - \tau_{AS}}{\tau_i} + \frac{1}{\tau_{SA}} - 1 \right) \\
\lim_{s \rightarrow \infty} s \frac{AINV_{SA}}{CONS} &= 0 & \lim_{s \rightarrow 0} s \frac{AINV_{SA}}{CONS} &= 1 \\
\lim_{s \rightarrow \infty} s \frac{ORATE_{SA}}{CONS} &= 0 & \lim_{s \rightarrow 0} s \frac{ORATE_{SA}}{CONS} &= 1
\end{aligned} \tag{39}$$

The final value of  $AINV_{SA}$  is 1, due to the stock-out condition that  $AINV_{SA}$  becomes WIP inventory, which, similar to the status of  $AINV_{AS}$  in hybrid Push-Pull-Push state, all finished PC parts at the sub-assembly supplier are batched and pushed out as long as they are produced. As a result, the average of  $AINV_{SA}$  will be equal to the average of  $CONS$ . The final value of delivery  $LT$ , as expected, is larger than the desired constant  $\tau_{DD}$  and depends on the combined control parameters for the final assembly and sub-assembly systems. This is due to the increased average of  $BL$  driven by insufficient  $AINV_{AS}$  and  $AINV_{SA}$  as well as transport and manufacturing delays ( $\tau_{AS}$  and  $\tau_{SA}$ ), if the system switches to the pure Push state.

### ***Inter-state comparisons.***

Table 4 summarizes the analytical findings for the three operational states. Depending on the availability of  $AINV_{AS}$  and  $AINV_{SA}$ , the ATO system may switch between the different states. By design, the system operates in the desired Push-Pull-Pull state in which two inventory stocks are pulled by end customer orders. Thus, all customized orders can be fulfilled by the quoted  $\tau_{DD}$ . Such a system state is stable for all positive values of control parameters with two feedback inventory control loops that characterize the oscillatory behaviour, with the control parameter in the supplier manufacturing system, i.e.  $\tau_{AINV}$ , being key in determining the dynamic behaviour and recovery speed.

Operational state	Structure		Initial insights for the dynamic properties	Stability
<i>Push-Pull-Pull</i>	Final assembly	Includes first order BL and second order $AINV_{AS}$ adjustment loops	1. Delivery LT is a constant level, $\tau_{DD}$ . 2. the state is characterised by a two-degree-of-freedom system with independent feedback adjustment loop at final assembly and the sub-assembly supplier sites, which may lead to complex dynamic response, such as two-resonance peak frequencies	The system is stable for all positive values of control parameters
	Sub-assembly supplier manufacturing	Includes first order forecasting and second order $AINV_{SA}$ adjustment loops		
<i>Push-Pull-Push</i>	Final assembly	Characterized by a third order, non-factorized loop, due to the incorporation of BL adjustment loop into feedback $AINV_{AS}$ loop (i.e. stock out of $AINV_{AS}$ )	Not applicable due to the state is unstable	The system is inherently unstable
	Sub-assembly supplier manufacturing	Same structure as the Push-Pull-Pull state		
<i>Pure Push</i>	The whole system is characterised by a first order forecasting loop and a fifth order, non-factorized loop, due to the incorporation of final assembly structure into the sub-assembly supplier loops		1. The average delivery LT larger than, $\tau_{DD}$ and its dynamic performance due to physical delay and system control policies at both final assembly (VMI) and the sub-assembly supplier site. 2. $AINV_{SA}$ becomes WIP inventory (all $AINV_{SA}$ are pushed out as long as they are produced at the sub-assembly site) and the average level equal to the mean of demand. 3. The variance of $ORATE_{SA}$ and $AINV_{SA}$ may be reduced due to the incorporation of final assembly structure.	The system is conditionally stable for positive values of control parameters

Table 4. Summary of three operational states based on two multi-valued nonlinearities.

As the level of  $AINV_{AS}$  falls it will eventually reach a level such that the OEM can no longer pull the required PC parts from  $AINV_{AS}$ . Instead, all available  $AINV_{AS}$  at the VMI hub is pushed into the final assembly plant at the maximum shipment rate,  $SH_{MAX}$ . This leads to the switch from desired Push-Pull-Pull state to Push-Pull-Push state under the condition that  $AINV_{SA}$  are still sufficient to be pulled by the VMI hub replenishment. As a result, the delivery LT is increased driven by a new third-order feedback loop (one real root and two complex roots), which depends on all control and physical parameters in the final assembly echelon and leads to instability. Such an operational state is fundamentally not stable and cannot be maintained due to permanent  $AINV_{AS}$  discrepancies.

If  $AINV_{SA}$  still constrains the pull  $ORATE_{AS}$ , the whole system switches to the pure Push production state. Two inventory stock points,  $AINV_{AS}$  and  $AINV_{SA}$ , become WIP inventory to be pushed out as soon as possible. In other words, there is no ‘finished stock’ at the OEM (VMI inventory hub) and upstream sub-assembly supplier sites. All produced PC parts in  $AINV_{SA}$  at the subassembly echelon are immediately batched and pushed out for shipment to the OEM site, and all PC parts arriving into  $AINV_{AS}$  at the VMI hub are immediately pushed out for final assembly and the subsequently final delivery to meet the incoming customised orders as soon as possible.

As a result, LT is further increased due to the longer upstream sub-assembly supplier manufacturing delay. The whole system is characterized as a first order forecasting loop and a fifth-order push loop. The new non-factorized fifth-order loops may increase the instability but reduce the complex dynamic property contributed by independent feedback loops in the Push-Pull-Pull state. The pure Push system is conditionally stable subject to the choice of control parameter and actual physical lead time ratio, with  $\tau_{AINV}$  being the key parameter on system stability.

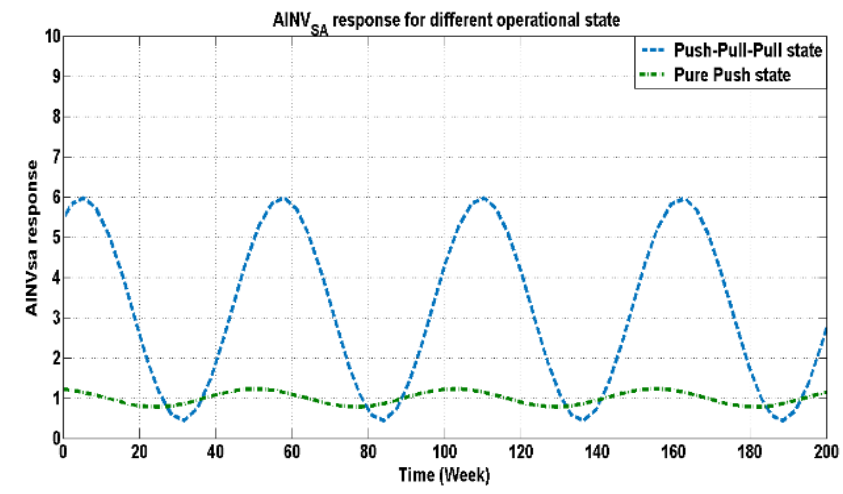
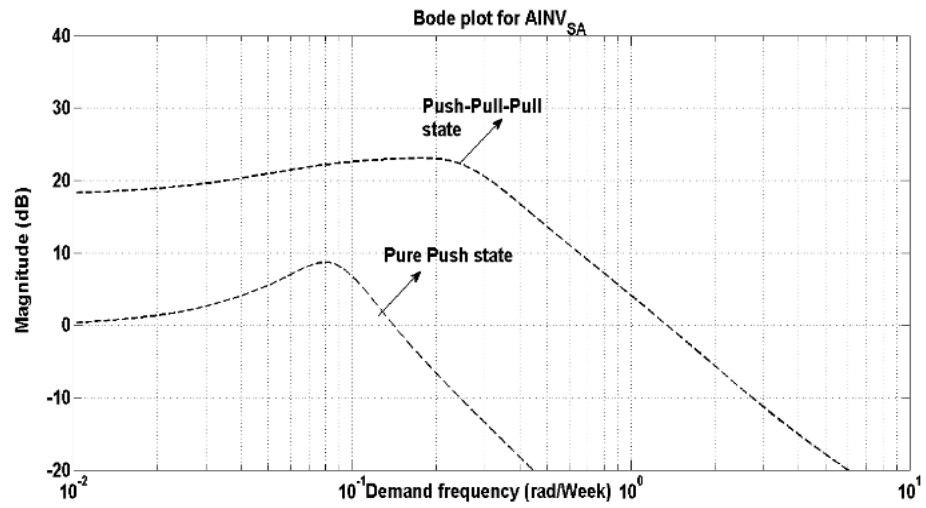
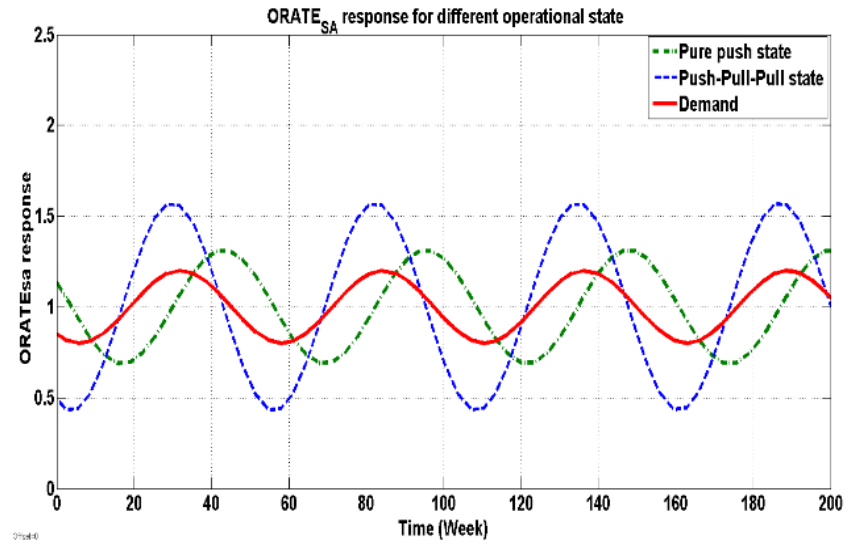
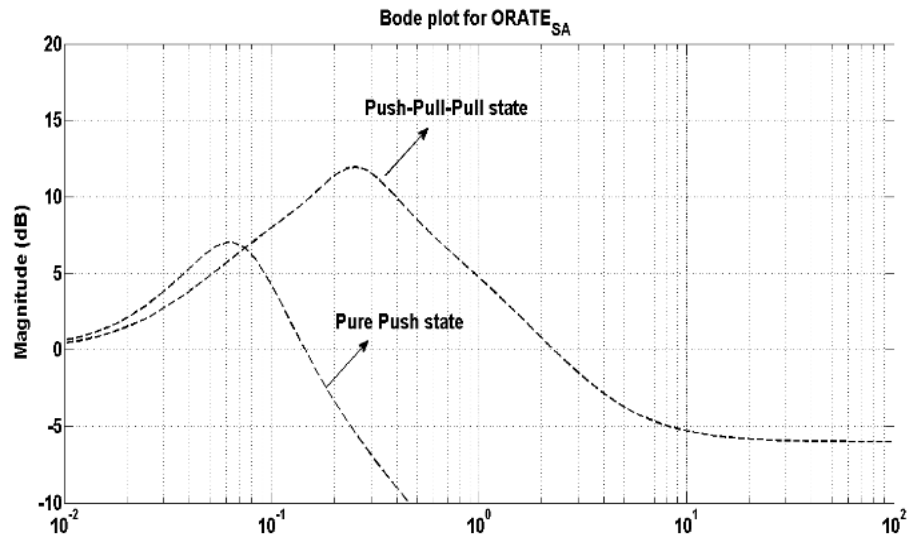
#### **4. Simulation analysis**

To further analyse the dynamic performance of  $ORATE_{SA}$ ,  $AINV_{SA}$  and delivery LT as the ‘performance triangle’ and to consider the hybrid ATO system switch from one state to another, Bode plots and system dynamics simulation are utilized. A Bode Plot is a useful tool to show the gain response of a given linear, time-invariant system for different demand frequencies (Towill et al.,2003; Towill et al. 2007), which we undertake for bullwhip, inventory variance and LT variance in the ATO context.



As shown in Figure 7, we present the Bode plot (Left) and the corresponding simulation results (Right) for the dynamic response of ‘performance triangle’, i.e.  $ORATE_{SA}$ ,  $AINV_{SA}$  and linearised LT under two stable operational states, the Push-Pull-Pull and pure Push states. For both Bode plot and simulation, the control parameters selected are as per the recommended settings of APVIOBPCS (Wang et al. 2014) and VIOBPCS (Edghill and Towill 1990) archetypes with varying values of  $\tau_{AINV}$ , which allows us to deliberately maintain two different states. Regarding the simulation, we select sinusoidal demand input, i.e.  $\omega=0.12\text{rad/week}$ , mean=1 and amplitude=0.2, to represent the cyclical demand pattern evident in the real-world PC industry (Kapuscinski et al. 2004; Katariya et al. 2014). Note that there is no bode plot of delivery LT for the Push-Pull-Pull state due to the constant value of LT ( $\tau_{DD}$ ), i.e. there is no dynamic oscillations (variance) of delivery LT but a constant value.

Overall, the simulation results support the analytical insights derived by the transfer function analysis. As the shift from the Push-Pull-Pull to pure Push state is due to stock-outs in  $AINV_{SA}$  and  $AINV_{AS}$ , the speed of  $ORATE_{SA}$  response slows (the decrease of the cross-over frequency) and the unwanted demand amplification (bullwhip) is significantly decreased for a range of frequencies. This behaviour is due to a change from demand pull to production push, that is, the shift from a two-degrees-of-freedom state with two independent feedback loops to a one-degree-of-freedom Push state. Although the corresponding bullwhip related cost will be decreased, e.g. ramping up / down machines, hiring and firing staff, the mean and variance of delivery LT is significantly increased as the system switches from the desired hybrid Push-Pull-Pull state to the pure Push state. For the desired Push-Pull-Pull state, the delivery LT is a constant value, i.e. the amplification ratio is zero (infinitely small) for all demand frequencies, so that consistent customer service levels can be guaranteed even for highly volatile demand patterns. However, if the desirable state cannot be maintained, the peak magnification and bandwidth of LT response are dramatically increased for low frequencies, which means both the variance and mean of LT are significantly increased due to the influence of CODP inventory shortage and long sub-assemble manufacturing and transport delays. Hence, high customer service cannot be maintained with the increase of demand fulfilment uncertainty.



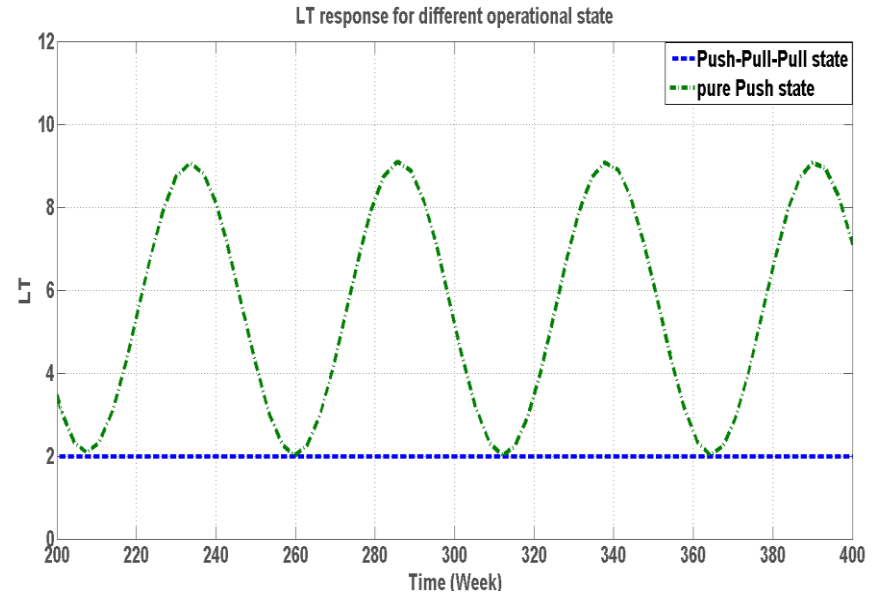
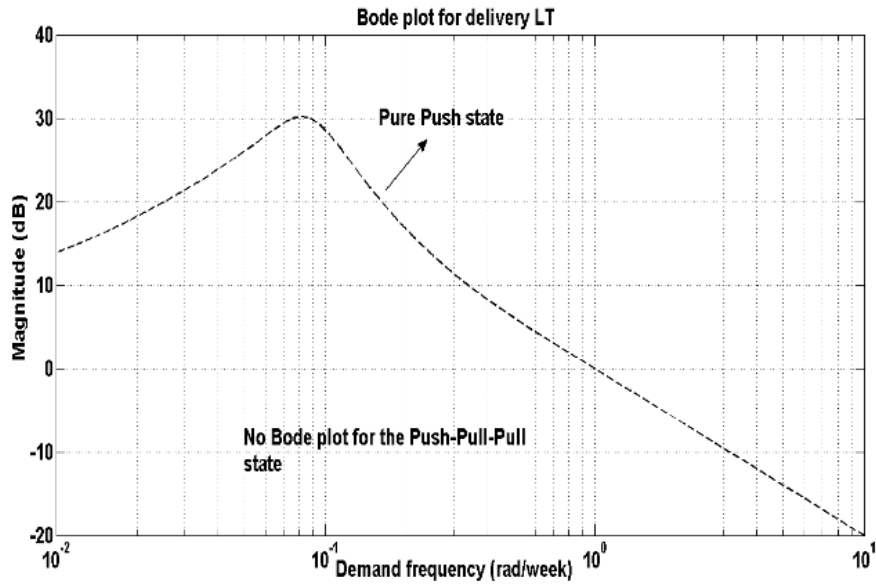


Figure 7. Bode plot of  $ORATE_{SA}$ ,  $AINV_{SA}$  and linearised LT for different operational state

(Demand frequency  $\omega=0.12\text{rad/week}$ ,  $\tau_{SA}=2\tau_{AS} = 2\tau_1 = 8\tau_{DD} = 8$ ,  $\tau_{WIP} = 16$   $\tau_{AINV} = 8$  for the Push-Pull-Pull and  $\tau_{AINV} = 40$  for the Pure Push states)

The frequency response performance of  $AINV_{SA}$ , as expected, is significantly improved from the desired Push-Pull-Pull to the pure Push state. This is because  $AINV_{SA}$  becomes WIP inventory with the change of system structure, i.e.  $AINV_{SA}$  will be pushed out immediately as long as they are produced in the supplier plant. Note that  $AINV_{SA}$  exhibits significant oscillatory behaviour for the desired hybrid state for demand with low frequencies (e.g. between 0.01 rad/week - 0.1 rad/week), suggesting CODP inventory utilized as the buffer will unavoidably experience high variance for maintaining ‘Leagile’ balance (Naylor et al. 1999).

Also, although information sharing is often regarded as a means to radically reducing / eliminating the bullwhip effect (Yang et al. 2011; Hosoda and Disney 2012), our simulation results demonstrate that for both operational states the bullwhip cannot be totally avoided at the low / medium demand frequency range.”

## 5. Conclusion.

In this paper, we study the delivery lead-time dynamics within the context of PC ATO system. We summarize the main results and the corresponding managerial implications in Table 6. Also, we link the ATO analysis results to the general impact of supply chain decoupling point (Gosling et al. 2017; Wikner et al., 2017) on ‘performance triangle’ shown in Figure 8.

The analysis indicates that the hybrid Push-Pull-Pull state can only be maintained if there are sufficient  $AINV_{AS}$  and  $AINV_{SA}$ . In such circumstance, delivery LT is a constant level in which all customer orders can be fulfilled with the scheduled time.

The system will fail to operate in the desired state with a decrease of the CODP inventory at final assembly (VMI hub) and the sub-assembly supplier manufacturing site, leading to the shift from Push-Pull-Pull to pure Push state. Although the CODP inventory variance and bullwhip (the corresponding capacity adjustment) will be significantly decreased, the mean and variance of the delivery LT, however, are dramatically increased due to the stock out issues as well as long physical delays ( $\tau_{SA}$  and  $\tau_{AS}$ ).

	<b>Findings/Outcomes</b>	<b>Managerial implications</b>
<b>Model development</b>	We developed a nonlinear system dynamics model of the PC ATO supply chains, which capture main characteristics of the pull and push parts separated by a CODP point	The hybrid ATO model is helpful for practitioners to design their hybrid supply chain system from the supply chain dynamics perspective. The model also could be a base framework for possible further modification, generalization and analysis in PC or other related industries.
<b>Delivery LT linearisation</b>	Based on Taylor series expansion with small perturbation theory, delivery LT can be linearised by following equations: $LT = \tau_{DD} \quad \text{if } SH_{MAX} > SH^*$ $LT = \tau_{DD} + BL - AINV_{AS} \quad \text{if } SH_{MAX} < SH^*$	Managers may simply calculate the estimated delivery LT by considering difference between current backlog and raw materials inventory level at final assembly plant (VMI hub), under the condition that desired shipment rate cannot be fully satisfied.
<b>Three measures of performance triangle</b>	As the switch from true hybrid Push-Pull to pure Push state, the mean and variance of delivery LT can be significantly increased, although $ORATE_{SA}$ and $AINV_{SA}$ variance can be mitigated due to independent inventory feedback loops at both final assembly and supplier manufacturing sites has been integrated as a fifth-order production push feedback loop.	Due to nonlinear switch between different operational process, maintaining the ‘true’ hybrid ATO operational state is always desirable to ensure customer service level, that is, the reliable LT.
	$\tau_{AINV}$ significantly influence the dynamic behaviour of three measures of performance triangle regarding system oscillatory behaviours and recovery speed.	Managers need to be aware that CODP inventory control policy should be fine-tuned for dynamic performance balance between delivery LT, capacity adjustment and CODP inventory.
	Two peak frequencies can be observed in the bode plot diagram of $ORATE_{SA}$ due to the effect of two natural frequencies driven by two independent feedback loops (two-degree-of-freedom system).	It is important for managers to consider the adoption of collaborative control policy design with their supply chain partners to reduce the influence of supply chain dynamics.

Table 6. Summary of findings and managerial implications.

This is an undesirable condition because of the significant decrease of customer service. In PC supply chains, the upstream suppliers, such as semiconductor manufacturers, may slowly adjust inventory error, i.e. long inventory adjustment time, to maintain the ‘Lean’ production and avoid expensive capacity fluctuation. On the other hand, from the entire ATO supply chain perspective, this may cause an operational shift from desired hybrid structure to a pure Push state driven by frequent stockouts, which significantly influence the downstream OEMs’ customer service, i.e. the long and unreliable delivery LT. Such findings also support the

importance of adopting a collaborative design and planning strategy between suppliers and OEMs to reduce operational cost driven by poor supply chain dynamics.

Overall, this study provides the theoretical foundation of modelling and assessing dynamic performance of the ATO system within the context of the PC sector. In particular, we contribute to the system dynamics analysis with respect to the lead-time dynamics and the corresponding ‘performance triangle’ by means of adequate tools of classical control theory. We demonstrate how competing performance trade-offs that need to be considered with respect to the performance triangle and how it relates the general supply chain decoupling point and upstream / downstream activities. Such a three-way trade-off may be summarised as shown in Figure 8.

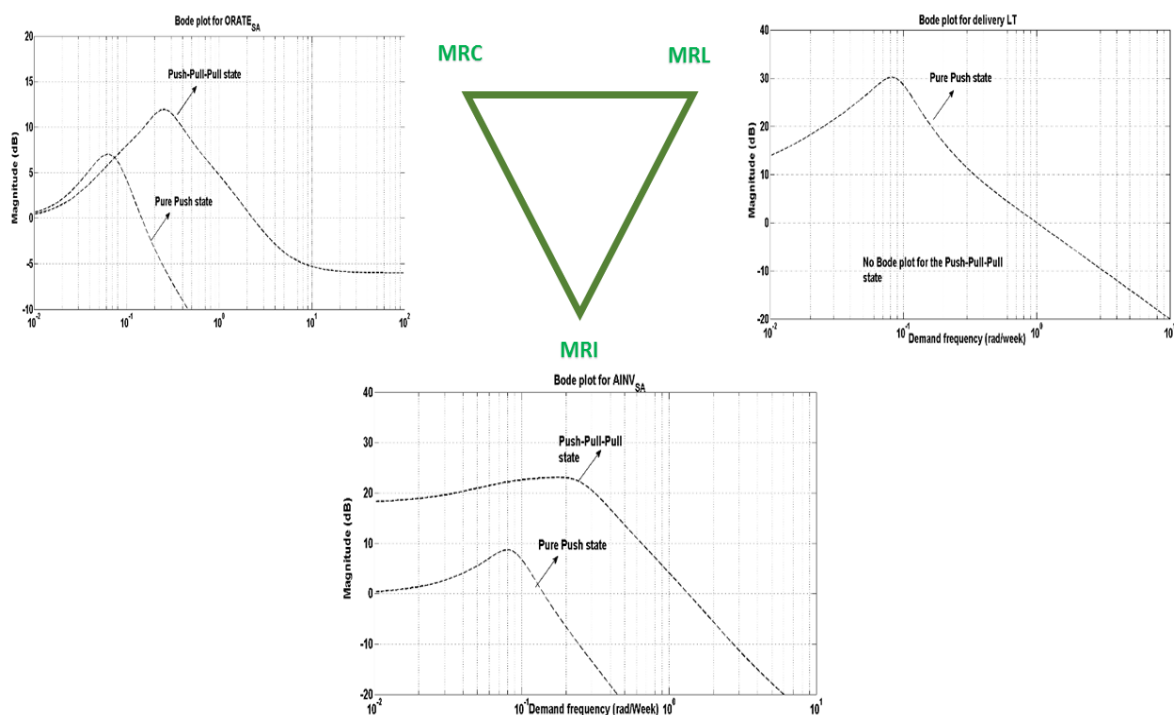


Figure 8. Linking the ATO system dynamics analysis to ‘performance triangle’ trade-off considerations.

Previous control-theoretic research, limited to linear model representations (e.g. Dejonckheere et al., 2003), have only ever considered the trade-off between inventory, such as the CODP, and capacity in satisfying customer service requirements. Such trade-off would allow the establishment of the minimum reasonable inventory (MRI) (Grünwald and Fortuin, 1992), following a ‘level scheduling’ strategy (i.e. minimising bullwhip) or, if following a ‘chase’ strategy, minimising inventory variance, hence the need to determine what we call the minimum reasonable capacity (MRC). The final design, for instance, can be based on the cost assignment for capacity and inventory related factors, such as machine/labour adjustment costs

and inventory holding costs. We can also see from Figure 8 our contribution to the field by the inclusion of a third metric, namely a maximum reasonable lead-time (MRL) i.e. the maximum time a customer is willing to wait for a delivery. Although the performance of MRI and MRC may be improved under a Pure Push state, lead-time now becomes the ‘buffer’ to absorb fluctuating end customer demand and thereby the MRL can no longer be achieved.

The research, however, is limited to the analysis of non-negative inventory constraints without considering capacity limits. Some analytical methods, such as describing functions (Wang et al.2015; Spiegler et al. 2016b), for instance, can be considered for future work. This research can also be extended to the analysis of the impact of demand characteristics (e.g. amplitude and mean of sinusoidal demand) on the occurrence of nonlinearities in the PC ATO system. Furthermore, due to the importance of maintaining hybrid ATO structure to ensure customer service level, further control policy trade-off design between capacity and CODP inventory should be considered to minimize the corresponding operational cost within the context of the PC sector.

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