Feng, Jiehua, Ma, Luning, Zhao, Dongya, Yan, Xinggang and Spurgeon, Sarah K. (2019) *Output Feedback Sliding Mode Control for Continuous Stirred Tank Reactors.* In: Proceedings of the 2019 12th Asian Control Conference (ASCC). IEEE.

Output Feedback Sliding Mode Control for Continuous Stirred Tank Reactors

Jiehua Feng[1] Luning Ma[1] Dongya Zhao[1⋆] Xinggang Yan[2] Sarah K. Spurgeon[3,1]

***Abstract*— The continuous stirred tank reactor (CSTR) is representative of a typical class of chemical equipment where the dynamics is nonlinear. The problematic issues in control of a CSTR are the model uncertainties and external disturbances. Driven by these challenging problems coupled with the need for demanding levels of performance, this paper establishes the dynamic model of CSTR and then proposes an output feedback sliding mode control in light of the established model. The validity of the control algorithm and of the presented model are further verified by MATLAB simulation and experimental trials.**

I. INTRODUCTION

CSTR plays a primary role in many chemical processes [1]. From the control point of view, the CSTR is highly nonlinear. Meanwhile, the difficulty of accurate modeling and the influence of external disturbances make the control of the CSTR challenging [2], [3]. The study of modeling and control for a CSTR will provide a useful reference for other nonlinear processes by reasonable modifications of the modeling and control strategy.

There have been many contributions to the modelling of the CSTR. A model for an immobilized biocatalyst CSTR is established by the transfer function and Laplace method [4], and this can be used to analyze the system's input and output behaviour. However the model does not fully consider the internal mechanisms of the CSTR. A dimensionless dynamic equation of a CSTR has been established in [5], and it is widely cited in the literature [6], [7]. This model describes a first-order, exothermic and irreversible reaction. It should be noted that the model is built with $A \rightarrow B$ reaction as the research object and uses the jacket temperature as the control input. However, the reaction is not common in chemical process control. Moreover, the temperature of the reactor is controlled by the flow of the cooling or heating reagent (mainly water) within the jacket which means that using the temperature as the control input is not appropriate. Motivated by the above analysis, in this study, a mechanism model is built based on the general reaction $A+B \rightarrow C+D$, while the jacket flow is used as the control input.

A lot of work has already considered CSTR controller design. Based on PID control, numerous new control algorithms, such as fuzzy control [8], model predictive control [9] have been applied to the CSTR. Based on full state feedback, a robust control has been presented to achieve disturbance rejection [10]. However, full state feedback control is not always feasible, since some states of the CSTR are not measurable directly online in practice. Some observers [11], [12] have been studied to resolve this issue. High robustness and rapid response are required in today's industry and such observer based control methods may require high control gains which in turn can lead to controller saturation. It is challenging to design a controller which achieves demanding performance levels for the CSTR only using system output information.

Sliding mode control is a widely used control method due to its excellent performance characteristics and strong robustness properties [13], [14]. This control method has been successfully applied to the CSTR. A novel output feedback terminal sliding mode control is proposed to stabilize the temperature tracking error to zero in finite time [15]. A nonlinear adaptive tracking controller is proposed in [16] based on fuzzy sliding mode control and Fourier integral control. It should be noted that the two control methods mentioned above use jacket temperature as the control input, which is hard to achieve in industry [17]. An output feedback sliding mode control is proposed for a class of nonlinear systems in [18]. However, the method did not consider problems that may arise in implementation, such as the effects caused by chattering.

The purpose of this study is to build the mechanism model for a CSTR based on the general reaction and design a sliding mode control. An approximate linear system is obtained through Taylor expansion at the equilibrium point. There after, the model uncertainties and external disturbance are considered. In addition, a dynamic compensator is designed to estimate the unmeasurable state. Finally, an output feedback sliding mode controller is designed based on the contributions in [18].

II. MECHANISM MODEL OF CSTR

Without loss of generality, in this paper, a class of endothermal irreversible reaction shown in (1) is considered for the CSTR.

$$A+B \rightarrow C+D \tag{1}$$

⋆This paper is partially supported by the National Nature Science Foundation of China under Grant (61473312) and the Thousand Person Program for Foreign Expert. Sarah K. Spurgeon gratefully acknowledges the support provided to enable her to visit China University of Petroleum to work with the CSTR experiment.

[1]Jiehua Feng, Luning Ma and Dongya Zhao are with the College of Chemical Engineering, China University of Petroleum, Qingdao 266555, P. R. China `dyzhao@upc.edu.cn; dongyazhao@139.com`

[2]Xinggang Yan is with the School of Engineering & Digital Arts, University of Kent, Canterbury CT2 7NT, United Kingdom `X.Yan@kent.ac.uk`

[3]Sarah K. Spurgeon is with the Department of Electronic & Electrical Engineering, University College London, Torrington Place, London WC1E 7JE, United Kingdom `s.spurgeon@ucl.ac.uk`

TABLE I: Parameter specification

Sign	Physical meaning	Sign	Physical meaning
V	Reactor volume	T_{Bf}	Feed B temperature
C_p	Reactor specific heat capacity	C_{Bf}	Feed B concentration
ρ	Reactor density	q_B	Feed B flow
T	Reactor temperature	C_{PB}	B specific heat capacity
q	Reactor flow	ρ_B	B density
M	Reactor mass	C_B	B concentration
T_{Af}	Feed A temperature	T_{E1}	Jacket inlet temperature
C_{Af}	Feed A concentration	T_{E2}	Jacket outlet temperature
q_A	Feed A flow	Q_E	Jacket flow
C_{PA}	A specific heat capacity	V_E	Jacket volume
ρ_A	A density	ρ_E	Jacket density
C_A	A concentration	C_{pE}	Jacket specific heat capacity
k_0	Index factor	E	Activation energy
R	Gas constant	A	Heat transfer area
ΔH	Reflect the enthalpy change	U	Coefficient of heat transfer

In dt time, the principle of material conservation is applied to the reactants:

$$dn_A = q_A C_{Af} - qC_A dt - V(-r)dt \tag{2}$$

$$dn_B = q_B C_{Bf} - qC_B dt - V(-r)dt \tag{3}$$

where $n_A = VC_A$, $n_B = VC_B$ and $r = -kC_AC_B$, $k = k_0\exp(-E/RT)$. The physical meaning of all parameters in this section is given in Table 1.

The heat balance of the reaction is expressed as [19]:

$$\begin{aligned} MC_p dT =& q_A T_{Af}\rho_A C_{pA} dt + q_B T_{Bf}\rho_B C_{pB} dt \\ &+ V(-\Delta H)kC_AC_B - UA(T - T_{E2})dt \\ &- qT\rho C_p dt \end{aligned} \tag{4}$$

The temperature and heat balance equation in the jacket is:

$$V_E\rho_E C_{pE} dT_{E2} = Q\rho_E C_{pE}(T_{E1} - T)dt + UA(T - T_{E2})dt \tag{5}$$

Since the mass ratio of the reactants is $1:1$, the feed flow, temperature and initial concentration of the reactants A and B are chosen to be the same for convenience; they are denoted as $q/2$, T_{f0}, C_0, respectively.

Integrating (2)-(5) , the model can be expressed as:

$$\begin{aligned} \dot{C}_A &= \frac{q}{2V}(C_0 - 2C_A) - k_0 C_A C_B \exp(-\frac{E}{RT}) \\ \dot{C}_B &= \frac{q}{2V}(C_0 - 2C_B) - k_0 C_A C_B \exp(-\frac{E}{RT}) \\ \dot{T} &= \frac{qT_{f0}(\rho_A C_{pA} + \rho_B C_{pB})}{2\rho V C_p} - \frac{qT}{V} \\ &+ \frac{(-\Delta H)}{\rho C_p} k_0 \exp(-\frac{E}{RT}) C_A C_B + \frac{UA}{\rho V C_p}(T_{E2} - T) \\ \dot{T}_{E2} &= \frac{Q_E}{V_E}(T_{E1} - T) + \frac{UA}{V_E\rho_E C_{pE}}(T - T_{E2}) \end{aligned} \tag{6}$$

Writing (6) in matrix and vector form:

$$\dot{x} = g(x, u)$$

where $x = [\ C_A \quad C_B \quad T \quad T_{E2}\ ]$ is the state vector and $u = Q_E$ is the control input, which represents the water flow in the jacket.

III. Model Linearization and Problem Description

Assume that u_e is a constant input which forces the system (6) to settle into a constant equilibrium state $x_e = [x_{1e}\ x_{2e}\ x_{3e}\ x_{4e}]$. (x_e, u_e) is the system equilibrium point, that is, $g(x_e, u_e) = 0$. The equilibrium point of the system (6) can be obtained because x_3 is well chosen.

The objective is to linearize (6) around the equilibrium point such that the nonlinear control system $\dot{x} = g(x, u)$ can be approximated by a linear control system $\dot{x} = Ax + Bu$.

Let $x = x_e + \Delta x, u = u_e + \Delta u$. From the Taylor's expansion, $\dot{x} = g(x,u) = g(x_e + \Delta x, u_e + \Delta u) = g(x_e, u_e) + \left[\frac{\partial g}{\partial x}\right]_{(x_e,u_e)} + \left[\frac{\partial g}{\partial u}\right]_{(x_e,u_e)} + O(\Delta x, \Delta u)$.

For (6), after neglecting the higher order term, the following linearization can be obtained:

$$\begin{aligned} \dot{x} &= \left[\frac{\partial g}{\partial x}\right]_{(x_e,u_e)} x + \left[\frac{\partial g}{\partial u}\right]_{(x_e,u_e)} u \\ &= Ax + Bu \end{aligned} \tag{7}$$

Remark 1. (7) is an error state equation which represents the deviation from the equilibrium point of each state.

Remark 2. The system state x_4 cannot be measured, so the system output is selected as $y = [\ x_1 \quad x_2 \quad x_3\ ]^T$.

Considering the modeling error and external disturbance, (7) is rewritten as:

$$\begin{aligned} \dot{x} &= Ax + Bu + f(x,t) \\ y &= Cx \end{aligned} \tag{8}$$

where $x \in R^n, u \in R^m, y \in R^p$ are the system state, input and output respectively, and $n = 4, m = 1, p = 3$, the function $f(x,t)$ represents the modeling error and external disturbance.

Assumption 1. The matrix pair (A, C) is observable.

There exists a matrix L such that $A - LC$ is stable. Then given any matrix Q_1, the Lyapunov equation

$$(A - LC)^T P_1 + P_1(A - LC) = -Q_1 \tag{9}$$

has a solution $P_1 > 0$.

Assumption 2. $f(x,t)$ has a structural decomposition:

$$f(x,t) = E\Delta\xi(x,t) \tag{10}$$

where $\|\Delta\xi(x,t)\| \leqslant \zeta(x,t) \leqslant \eta(x,t)\|x\|$, where $\zeta(x,t)$ is Lipschitz with respect to x and K_ζ represents the Lipschitz constant.

Note that not all disturbances will affect the actual system through the control input channel. Let $f(x,t)$ belong to a class of mismatched uncertainty, that is, $E \not\subset span(B)$.

Assumption 3. There exist a matrice F such that $E^T P_1 = FC$ holds.

The aim of this paper is to design a controller to make all the states in (8) converge to zero asymptotically only using the system output information and the estimated state while exhibiting good robustness properties.

IV. Dynamic Compensator Design

Since the system state x_4 cannot be measured, it is necessary to design a compensator. Based on the analysis above, a dynamical compensator, or observer is designed for (8):

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - c\hat{x}) + \phi(\hat{x}, y, t) \tag{11}$$

$$\phi(\hat{x}, y, t) = \begin{cases} E\frac{FCe}{\|FCe\|}\zeta(\hat{x}, t) & FCe \neq 0 \\ 0 & FCe = 0 \end{cases} \tag{12}$$

where $e = x - \hat{x}$.

Combining (8) and (11), it is straightforward to see that:

$$\dot{e} = (A - LC)e + f - \phi(\hat{x}, y, t) \tag{13}$$

Theorem 1. Under Assumptions 1-3, for the system (8) and (11), if $\underline{\lambda}(Q_1) > 2K_\zeta \|FC\|$, e is asymptotically stable.

Proof:

Choose a Lyapunov function candidate $V_1 = e^T P_1 e$:

$$\dot{V}_1 = -e^T Q_1 e + 2e^T P_1 (f - \phi) \tag{14}$$

In the case $FCe = 0$:

$$e^T P_1 (f - \phi) = 0 \leq K_\zeta \|FC\| \|e\|^2 \tag{15}$$

In the case $FCe \neq 0$:

$$\begin{aligned} e^T P_1 (f - \phi) &= (FCe)^T \Delta\xi(x, t) - \frac{(FCe)^T FCe}{\|FCe\|}\zeta(\hat{x}, t) \\ &\leq \|FCe\| \zeta(x, t) - \|FCe\| \zeta(\hat{x}, t) \\ &\leq K_\zeta \|FC\| \|e\|^2 \end{aligned} \tag{16}$$

The following derivation can be obtained:

$$\begin{aligned} \dot{V}_1 &\leqslant -\left(\underline{\lambda}(Q_1) - 2K_\zeta \|FC\|\right) \|e\|^2 \\ &\leqslant -\frac{\underline{\lambda}(Q_1) - 2K_\zeta \|FC\|}{\bar{\lambda}(P_1)} e^T P_1 e \\ &= -2\alpha_2 V_1 \end{aligned} \tag{17}$$

where $\alpha_2 = \frac{\underline{\lambda}(Q_1) - 2K_\zeta \|FC\|}{2\bar{\lambda}(P_1)}$.

Based on the above analysis,

$$\begin{aligned} \underline{\lambda}(P_1) \|e\|^2 \leqslant V_1(t) \leqslant V_1(t_0) \exp(2\alpha_2(t_0 - t)) \\ \|e\| \leqslant \alpha_1 \exp(-\alpha_2 t) \end{aligned} \tag{18}$$

where $\alpha_1 = \sqrt{\frac{V_1(t_0)}{\underline{\lambda}(P_1)}} \exp(\alpha_2 t_0)$.

This means that $\lim_{t \to \infty} e = 0$ and Theorem 1 holds.

V. Output Feedback Sliding Mode Control Design

From (8) and (13), the dynamics in the (x, e) coordinate system can be described as:

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} f \\ f - \phi \end{bmatrix} \tag{19}$$

$$y = Cx \tag{20}$$

The purpose of this section is to design a sliding mode controller based on knowledge of y and $\hat{x}$. The sliding function is defined as:

$$\sigma = S_1 y + S_2 N \hat{x} \tag{21}$$

where $S_1 \in R^{m \times p}$, $S_2 \in R^{m \times (n-p)}$ and $N \in R^{(n-p) \times n}$ are matrices to be designed.

Equation (21) can be further expressed as

$$\sigma = Sx - S_2 N e \tag{22}$$

where $S = S_1 C + S_2 N$.

By making $\dot{\sigma} = 0$, the equivalent control can be obtained:

$$u_{eq} = -(SB)^{-1}(SAx - S_2 N(A - LC)e + S_1 Cf + S_2 N\phi) \tag{23}$$

Meanwhile the sliding mode dynamics can be expressed by:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} &= \begin{bmatrix} A_{eq} & B(SB)^{-1}S_2 N(A - LC) \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} \\ &+ \begin{bmatrix} (I_n - B(SB)^{-1}S_1 C)f - B(SB)^{-1}S_2 N\phi \\ f - \phi \end{bmatrix} \end{aligned} \tag{24}$$

where $A_{eq} = (I_n - B(SB)^{-1}S)A$.

There exist two nonsingular matrices $T_1 \in R^{n \times n}$ and $T_2 \in R^{m \times m}$ such that

$$T_2 S T_1 = \begin{bmatrix} I_m & 0 \end{bmatrix} \tag{25}$$

Introducing the coordinate transformation $z = T_1^{-1}x$, the sliding function (22) becomes

$$\sigma = T_2^{-1} z_1 - S_2 N e \tag{26}$$

where $z = col(z_1, z_2)$ with $z_1 \in R^m$ and $z_2 \in R^{n-m}$.

The sliding surface can be expressed as:

$$z_1 = T_2 S_2 N e \tag{27}$$

Meanwhile the sliding mode dynamics (24) become:

$$\begin{aligned} \begin{bmatrix} \dot{z} \\ \dot{e} \end{bmatrix} &= \begin{bmatrix} T_1^{-1}A_{eq}T_1 & T_1^{-1}B(SB)^{-1}S_2 N(A - LC) \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} z \\ e \end{bmatrix} \\ &+ \begin{bmatrix} T_1^{-1}(I_n - B(SB)^{-1}S_1 C)f - T_1^{-1}B(SB)^{-1}S_2 N\phi \\ f - \phi \end{bmatrix} \end{aligned} \tag{28}$$

where $T_1^{-1}A_{eq}T_1 = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ and $T_1^{-1}B(SB)^{-1}S_2 N(A - LC) = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$, where $A_{11} \in R^{m \times m}$ and $D_1 \in R^{m \times n}$.

(28) can be expressed by:

$$\begin{bmatrix} \dot{z}_2 \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A_{22} & A_{21}T_2 S_2 N + D_2 \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} z_2 \\ e \end{bmatrix} + \begin{bmatrix} f_2 \\ \psi \end{bmatrix} \tag{29}$$

where f_2 is the last $n - m$ components of $\left[T_1^{-1}(I_n - B(SB)^{-1}S_1 C)f - T_1^{-1}B(SB)^{-1}S_2 N\phi\right]_{z_1 = T_2 S_2 N e}$ and $\psi = [f - \phi]_{z_1 = T_2 S_2 N e}$.

Based on Assumption 1 and the compensator design, the following inequality can be obtained:

$$\begin{aligned}&\left\|T_1^{-1}(I_n-B(SB)^{-1}S_1C)f-T_1^{-1}B(SB)^{-1}S_2N\phi\right\|\\&\leqslant\left\|T_1^{-1}(I_n-B(SB)^{-1}S_1C)\right\|\|E\|\eta\|T_1z\|\\&+\left\|T_1^{-1}B(SB)^{-1}S_2N\right\|\|E\|\eta\left(\|T_1z\|+\|e\|\right)\end{aligned}\tag{30}$$

From Theorem 1, it follows that

$$e^TP_1\psi\leqslant K_\zeta\|FC\|\|e\|^2\tag{31}$$

Then, from the inquality

$$\|T_1z\|=\left\|T_1\begin{bmatrix}T_2S_2Ne\\z_2\end{bmatrix}\right\|\leqslant\|T_1\|\left(\|T_2S_2N\|\|e\|+\|z_2\|\right)$$

it follows that there exist χ_1 and χ_2 such that

$$\|f_2\|\leqslant\chi_1\|z_2\|+\chi_2\|e\|\tag{32}$$

where χ_1 and χ_2 are all dependent on η,T_1,T_2,S_1,S_2.

Meanwhile since A_{22} is stable [18], this means that given any matrice $Q_2>0$, the equation

$$A_{22}{}^TP_2+P_2A_{22}=-Q_2\tag{33}$$

has a solution $P_2>0$.

Theorem 2. Under Assumptions 1-3, the sliding mode dynamics (29) are asymptotically stable if $M>0$. where $M=\begin{bmatrix}M_{11}&M_{12}\\M_{21}&M_{22}\end{bmatrix}$ and $M_{11}=\underline{\lambda}(Q_2)-2\bar{\lambda}(P_2)\chi_1$, $M_{12}=M_{21}=-\left(\|P_2(A_{21}T_2S_2N+D_2)\|+\bar{\lambda}(P_2)\chi_2\right)$, $M_{22}=\underline{\lambda}(Q_1)-2\|FC\|K_\zeta$.

Proof: Choose a Lyapunov candidate function:

$$V(e,z_2)=e^TP_1e+z_2^TP_2z_2\tag{34}$$

The time derivative of V is given as:

$$\begin{aligned}\dot V=&-e^TQ_1e-z_2^TQ_2z_2+2z_2^TP_2(A_{21}T_2S_2N+D_2)e\\&+2z_2^TP_2f_2+2e_2^TP_1\psi\end{aligned}\tag{35}$$

Combining (31) and (32), it follows that

$$\begin{aligned}\dot V\leqslant&-\left(\underline{\lambda}(Q_1)-2\|FC\|K_\zeta\right)\|e\|^2\\&-\left(\underline{\lambda}(Q_2)-2\bar{\lambda}(P_2)\chi_1\right)\|z_2\|^2\\&+2\left(\|P_2(A_{21}T_2S_2N+D_2)\|+\bar{\lambda}(P_2)\chi_2\right)\|z_2\|\|e\|\\&=-\begin{bmatrix}\|z_2\|&\|e\|\end{bmatrix}M\begin{bmatrix}\|z_2\|\\\|e\|\end{bmatrix}\end{aligned}$$

Hence Theorem 2 holds.

Based on the analysis above, the following output feedback sliding mode control is designed:

$$u=-(SB)^{-1}\left\{SA\hat x+S_2NL(y-Cx)+\frac{\sigma}{\|\sigma\|}K(\hat x,y,t)\right\}\tag{36}$$

where the control gain

$$\begin{aligned}K(\hat x,y,t)=&\left(\|S_1CE\|+\|S_2NE\|\right)\zeta(\hat x,t)\\&+\alpha_1\left(K_\zeta\|S_1CE\|+\|S_1CA\|\right)e^{-\alpha_2t}+\beta\end{aligned}\tag{37}$$

where β is a positive constant.

Theorem 3. Under Assumptions 1-3 and given Theorems 1 and 2, the control (36) can guarantee that the system (8) reaches the sliding surface and maintains a sliding motion.

Proof:

From Assumption 1 and the compensator design, the following inequality can be obtained:

$$\begin{aligned}S_1Cf&\leqslant\|S_1CE\|\left(\zeta(x,t)-\zeta(\hat x,t)\right)+\|S_1CE\|\zeta(\hat x,t)\\&\leqslant K_\zeta\|S_1CE\|\|e\|+\|S_1CE\|\zeta(\hat x,t)\end{aligned}\tag{38}$$

$$S_2N\Phi\leqslant\|S_2NE\|\zeta(\hat x,t)\tag{39}$$

Based on (6) and (11), the time derivative of the sliding function (22) can be expressed as:

$$\begin{aligned}\dot\sigma(y,\hat x)=&SA\hat x+S_2NLCe+SBu\\&+S_1Cf+S_2N\phi+S_1CAe\end{aligned}\tag{40}$$

By applying the control (36) to (40), it follows that

$$\dot\sigma=-\frac{\sigma}{\|\sigma\|}K(\hat x,y,t)+S_1Cf+S_2N\phi+S_1CAe\tag{41}$$

Further, based on (8), (39) and Theorem 1, the following inequality can be obtained

$$\begin{aligned}\sigma^T\dot\sigma\leqslant&-\|\sigma\|\left\{K(\hat x,y,t)-S_1Cf-S_2N\Phi-S_1CAe\right\}\\\leqslant&-\|\sigma\|\{K(\hat x,y,t)-\left(\|S_1CE\|+\|S_2NE\|\right)\zeta(\hat x,t)\\&-\alpha_1\left(K_\zeta\|S_1CE\|+\|S_1CA\|\right)e^{-\alpha_2t}\}\\\leqslant&-\beta\|\sigma\|\end{aligned}\tag{42}$$

Thus Theorem 3 holds.

Remark 3. As chattering may seriously damage the actuators, a smoothing technique is used in which $\sigma/\|\sigma\|$is replaced by $\sigma/(\|\sigma\|+\delta)$ where δ is a small positive number in the testing.

VI. Simulation and Experimental Verification

In this paper, the saponification process (43) with ethyl acetate and sodium hydroxide as raw materials is selected.

$$CH_3COOC_2H_5 + NaOH \rightarrow CH_3COONa + C_2H_5OH\tag{43}$$

Relevant parameters are given in Table 2. The data are substituted into (6) and the final model can be obtained through model identification as follows:

$$\begin{aligned}\dot x_1&=0.08-0.15128x_1-0.02x_1x_2\exp(-\frac{601.4}{x_3})\\\dot x_2&=0.08-0.15128x_2-0.02x_1x_2\exp(-\frac{601.4}{x_3})\\\dot x_3&=0.0097-0.0048\times\exp(-\frac{601.4}{x_3})x_1x_2\\&+2.1(x_4-x_3)-0.001234x_3\\\dot x_4&=\frac{u}{7510}(319.15-x_3)+0.5421(x_3-x_4)\end{aligned}\tag{44}$$

where the states x_1,x_2,x_3,x_4 represent the concentration of ethyl acetate, the concentration of sodium hydroxide, reactor temperature and jacket outlet temperature respectively; the

TABLE II: Parameter values

Sign	Value	Sign	Value
V	$0.00877m^3$	T_{f0}	$298.15K$
C_p	$7.55e+004J/(kgmol\cdot K)$	C_0	$125mol/m^3$
ρ	$993.924kg/m^3$	U	$1200W/(m^2\cdot K)$
k_0	0.02	C_{PB}	$7.53e+004J/(kgmol\cdot K)$
q	$40L/h$	ρ_B	$989kg/m^3$
E	$50000KJ/kgmol$	ΔH	$158000\ kJ/kmol$
R	$8.314J/(mol\cdot K)$	T_{E1}	$319.15K$
C_{pE}	$7.535e+004J/(kgmol\cdot K)$	A	$0.128m^2$
C_{PA}	$7.57e+004J/(kgmol\cdot K)$	V_E	$0.02m^3$
ρ_A	$982kg/m^3$	ρ_E	$997kg/m^3$

control input u represents the jacket water flow.

Remark 4. Since the mass ratio of the reactants is $1:1$, the responses of x_1 and x_2 are very similar. For reasons of space, this paper only presents the simulation and experimental results corresponding to x_1.

A PID controller is applied to the actual system. Then the same control signal is applied as an open-loop input to the model (44). The corresponding data is compared. The results shown in Fig. 1-2 indicate that the model (44) is reasonable.

The equilibrium point of the system (44) can be obtained since the reactor temperature x_3 is 303K:

$$\begin{aligned}[x_{1e}\ \ x_{2e}\ \ x_{3e}\ \ x_{4e}] &= [0.5238\ \ \ 0.5238\ \ \ 303\ \ \ 303.1735] \\ u_e &= 43.7407\end{aligned} \tag{45}$$

When the equilibrium point is substituted into (7), the following linear model is obtained:

$$A=\begin{bmatrix} -0.1527 & -0.0014 & -4.9391e-06 & 0 \\ -0.0014 & -0.1527 & -4.9391e-06 & 0 \\ 3.4547e-04 & 3.4547e-04 & -2.1012 & 2.1 \\ 0 & 0 & 0.5363 & -0.5421 \end{bmatrix}$$

$$B=\begin{bmatrix} 0 & 0 & 0 & 0.0425 \end{bmatrix}^T$$

From Remark 2 it is obvious that:

$$C=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{46}$$

It is straight forward to verify that (A,C) is observable.

For $Q_1 = I_4$, the Lyapunov equation (9) has a solution :

$$P_1=\begin{bmatrix} 0.1127 & -0.0077 & -0.0269 & -0.0462 \\ -0.0077 & 0.1127 & -0.0269 & -0.0462 \\ -0.0269 & -0.0269 & 0.7421 & -0.3915 \\ -0.0462 & -0.0462 & -0.3915 & 0.5943 \end{bmatrix} \tag{47}$$

Suppose the modeling error and external disturbance $f(x,t) = E\Delta\xi(x,t)$, where $E = \begin{bmatrix} 0.4357 & -0.4357 & 0.1452 & 0.0957 \end{bmatrix}^T$ and $\|\Delta\xi(x,t)\| \leqslant 1/9(\sin^2 x_4 + |x_1|)$.

Meanwhile, choose $F = \begin{bmatrix} 0.0441 & -0.0608 & 0.0703 \end{bmatrix}$ such that $E^T P_1 = FC$ holds. Assumptions 1-3 are guaranteed.

Then let

$$S=\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \tag{48}$$

After direct calculation, it follows that

$$M=\begin{bmatrix} -4.1106 & -3.3459 \\ -3.3459 & -15.1869 \end{bmatrix} \tag{49}$$

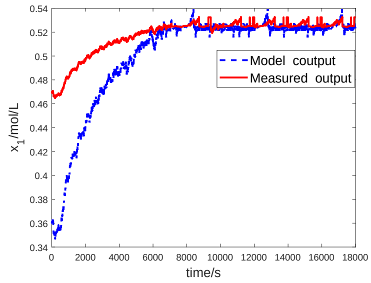

Fig. 1: The test for concentration of ethyl acetate

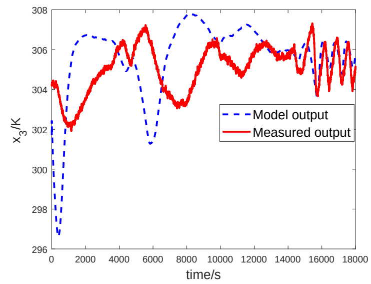

Fig. 2: The test for reactor temperature

Since $M > 0$ and $\underline{\lambda}(Q_1) > 2K_\zeta\|FC\|$, Theorem 1-3 can be guaranteed. From (37), $K(\hat{x},y,t)$ can be chosen as:

$$K(\hat{x},y,t) = 0.0268(\sin^2\hat{x}_4 + |\hat{x}_1|) + \alpha_1 3.0271 e^{-\alpha_2 t} + \beta \tag{50}$$

where α_1 and α_2 are already defined in Theorem 1.

The initial condition is given as :

$$col(x,\hat{x}) = (0.25, 0.25, 1, 1, 0.5, 0.5, 2, 2) \tag{51}$$

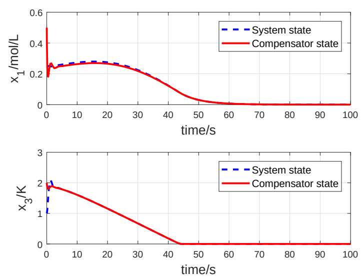

Fig. 3: x_1 and x_3 performance

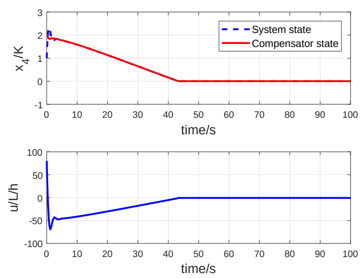

Fig. 4: x_4 and u performance

Fig. 5: The experimental rig

The results in Fig. 3-4 show the effectiveness of the designed controller. The compensator can effectively observe the system states and the system shows good robustness against mismatched disturbances.

The Process Modelling and Control Group from the China University of Petroleum (East China) has developed the experimental rig shown in Fig. 5. The experimental results in Fig. 6-7 show that both the system states and control input can reach the corresponding equilibrium point.

VII. Conclusion

In this paper, the mechanism model for a typical CSTR is developed. The model is linearized for controller design. A compensator is developed to estimate the unmeasurable state and then an output feedback sliding mode control is designed for the system. The simulation and experimental tests have demonstrate the effectiveness of the proposed approach and also validate the model.

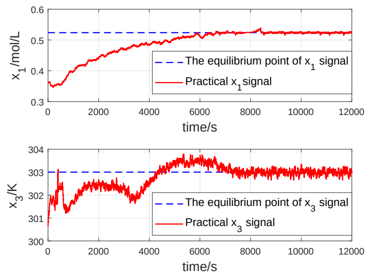

Fig. 6: x_1 and x_3 performance

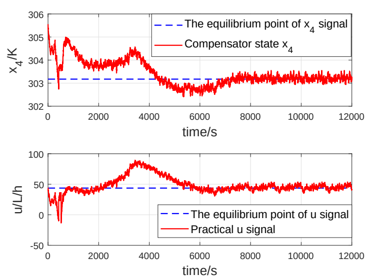

Fig. 7: x_4 and u performance

References

[1] B. W. Bequette, *Process control: modeling, design, and simulation*. Prentice Hall Professional, 2003.

[2] J. Feng, L. Cao, L. Ma, D. Zhao, and S. K. Spurgeon, "Model-free adaptive sliding mode control for continuous stirred tank reactor," in *2018 37th Chinese Control Conference (CCC)*. IEEE, 2018, pp. 3035–3040.

[3] V. Ghaffari, S. V. Naghavi, and A. Safavi, "Robust model predictive control of a class of uncertain nonlinear systems with application to typical cstr problems," *Journal of Process Control*, vol. 23, no. 4, pp. 493–499, 2013.

[4] L. Gao, "Modeling and dynamics analyses of immobilized cstr bioreactor using transfer function model," in *Information Technology in Medicine and Education (ITME), 2012 International Symposium on*, vol. 2. IEEE, 2012, pp. 692–695.

[5] W. H. Ray, *Advanced process control*. McGraw-Hill Companies, 1981.

[6] M. C. Colantonio, A. C. Desages, J. A. Romagnoli, and A. Palazoglu, "Nonlinear control of a cstr: disturbance rejection using sliding mode control," *Industrial & engineering chemistry research*, vol. 34, no. 7, pp. 2383–2392, 1995.

[7] L. Ma, D. Zhao, and S. K. Spurgeon, "Disturbance observer based discrete time sliding mode control for a continuous stirred tank reactor," in *2018 15th International Workshop on Variable Structure Systems (VSS)*. IEEE, 2018, pp. 372–377.

[8] K. Bingi, R. Ibrahim, M. N. Karsiti, and S. M. Hassan, "Fuzzy gain scheduled set-point weighted pid controller for unstable cstr systems," in *Signal and Image Processing Applications (ICSIPA), 2017 IEEE International Conference on*. IEEE, 2017, pp. 289–293.

[9] U. Ratnakumari and M. B. Triven, "Implementation of adaptive model predictive controller and model predictive control for temperature regulation and concentration tracking of cstr," in *Communication and Electronics Systems (ICCES), International Conference on*. IEEE, 2016, pp. 1–6.

[10] C. Kravaris and J. C. Kantor, "Geometric methods for nonlinear process control. 2. controller synthesis," *Industrial and Engineering Chemistry Research*, vol. 29, no. 12, pp. 2295–2310, 2002.

[11] T. Pan, S. Li, and W.-J. Cai, "Lazy learning-based online identification and adaptive pid control: a case study for cstr process," *Industrial & engineering chemistry research*, vol. 46, no. 2, pp. 472–480, 2007.

[12] H. Hoang, F. Couenne, C. Jallut, and Y. Le Gorrec, "Lyapunov-based control of non isothermal continuous stirred tank reactors using irreversible thermodynamics," *Journal of Process Control*, vol. 22, no. 2, pp. 412–422, 2012.

[13] C. Edwards and S. Spurgeon, *Sliding mode control: theory and applications*. CRC Press, 1998.

[14] X.-G. Yan, S. K. Spurgeon, and C. Edwards, "Global decentralised static output feedback slidingmode control for interconnected time-delay systems," *IET control theory & applications*, vol. 6, no. 2, pp. 192–202, 2012.

[15] D. Zhao, Q. Zhu, and J. Dubbeldam, "Terminal sliding mode control for continuous stirred tank reactor," *Chemical engineering research and design*, vol. 94, pp. 266–274, 2015.

[16] Z. Huaguang and L. Cai, "Nonlinear adaptive control using the fourier integral and its application to cstr systems," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 32, no. 3, pp. 367–372, 2002.

[17] X.-G. Yan, S. K. Spurgeon, and C. Edwards, *Variable structure control of complex systems*. Springer, 2017.

[18] X.-G. Yan, C. Edwards, and S. K. Spurgeon, "Output feedback sliding mode control for non-minimum phase systems with non-linear disturbances," *International Journal of control*, vol. 77, no. 15, pp. 1353–1361, 2004.

[19] D.-X. Gao and H. Liu, "Optimal dynamic control for cstr nonlinear system based on feedback linearization," in *Control and Decision Conference (CCDC), 2015 27th Chinese*. IEEE, 2015, pp. 1298–1302.