Essays on Financial Econometrics and Forecasting

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This dissertation is submitted for the degree of

Doctor of Philosophy

July 2019

Word count: approx. 35000 excl. Tables and Figures

Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for any other degree or professional qualification.

The work presented in Chapter 2 was previously published in Journal of Real Estate Finance and Economics as "Detecting Bubbles in the US and UK Real Estate Markets", with F. J. Fabozzi, E. Panopoulou and R. Tunaru.

Abstract

This thesis is comprised of three essays on the topics of financial econometrics and forecasting. In the first essay we examine whether speculative bubbles are present in the US and UK commercial, equity and residential real estate markets. The real estate indices are decomposed to fundamental and non-fundamental components using a wide set of economic indicators and penalized regressions. In order to determine whether the observed deviations of the actual price index from its fundamental value are due to the presence of bubbles, we use two complementary methodologies, the first based on right-side unit root tests for explosive behavior and the second on regime switching models for bubbles. The models using the alternative fundamental specifications are found to exhibit superior out-of-sample performance compared to the stylized alternative models.

In the second essay we set out to evaluate the benefits of integrating return forecasts from a variety of machine learning and forecast combination methods into an out-of-sample asset allocation framework. The performance of the portfolios consisting of stock, bond and commodity indices is evaluated for different levels of risk aversion and investment constraints, around business cycles and for different rebalancing frequencies. The mean-variance allocations are based on several estimates of the covariance matrix, while the effects of the return forecasts are also investigated when using the Conditional Value-at-Risk as an alternative risk measure in optimization. Comparing the multi-asset portfolios incorporating machine learning return forecasts, we find evidence of added economic value relative to the equally-weighted or the historical average benchmark portfolios.

In the final essay we propose a new approach to pairs trading, which takes advantage of the information in the conditional quantiles of the distribution of asset returns. In this framework the pairs are sorted and selected based on cointegration tests and during trading the trading signal is extracted using quantile regression. We apply the strategy to the S&P 100 constituents and evaluate the performance of the pairs trading strategy using a variety of economic and risk-adjusted metrics and under an asset pricing framework, in order to examine whether the profitability of the new strategy can be explained by various risk factors. Our findings suggest that the quantile regression pairs trading strategies based on the lower quantiles tend to outperform all other models.

Acknowledgements

First and foremost I would like to thank my supervisor Professor Ekaterini Panopoulou for her continuous support and advice throughout the past three years. I would also like to thank my second supervisor Professor Radu Tunaru for his support and guidance throughout my PhD.

Moreover, I would like to thank the members of Kent Business School for all the help they provided throughout my PhD.

Finally, I would like to thank my family and especially my parents who have always supported and encouraged me.

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Chapter 1: Introduction

This thesis is comprised of three essays on financial econometrics and forecasting that cover the following three topics: (1) the detection and date stamping of speculative bubbles in real estate markets and forecasting the returns of real estate indices using models that take into account the non-fundamental component of the asset price; (2) the return predictability of stock, bond and commodity indices using a variety of machine learning and forecast combination methods and the benefits of integrating return forecasts into an out-of-sample asset allocation framework and (3) an extension to the cointegration-based pairs trading framework that incorporates quantile regression.

In Chapter 2 we examine whether the prices of the commercial, residential and equity real estate sectors in the US and the UK are driven by market fundamentals or speculative bubbles. The real estate indices are decomposed to fundamental and non-fundamental components using a wide set of economic indicators and penalized regressions. To determine whether the observed deviations of the actual price index from the fundamental value are due to the presence of bubbles two complementary methodologies are used; the first is based on right-side unit root tests for explosive behavior and the second on regime switching models for bubbles. The out-of-sample performance of the bubble model is compared against the historical average and stylized alternative models. The results for all indices indicate that the actual price diverges from the respective fundamental value and the degree of over- or under-valuation could be explained by the presence of rational bubbles. The right-side unit root tests showed significant evidence of the presence of periodically collapsing bubbles in all indices. When comparing the regime switching model for bubbles based on the proposed fundamental specifications to the alternative models, the results showed that for the US and the UK equity and residential real estate the bubble model is preferable to the benchmarks.

Chapter 3 sets out to explore whether portfolios consisting of stock, bond and commodity indices, utilizing forecasts generated from a variety of machine learning and forecast combination methods, can outperform simpler benchmarks, such as the equally-weighed portfolio and portfolios using the historical average forecast. The analysis is conducted for different levels of risk aversion and investment constraints, around business cycles and for monthly or quarterly rebalancing. The mean-variance allocations are based on several different estimates of the covariance matrix, while the effects of the return forecasts are also investigated when using the Conditional Value-at-Risk as an alternative risk measure in optimization. Finally, to assess the value of adding commodities to a traditional portfolio, stock-bond portfolios are constructed and their performance is compared with that of commodity-augmented allocations.

The empirical results for the asset allocation show that the majority of the portfolios, outperform the equally-weighted and historical average portfolio benchmarks. When comparing portfolios across different combinations of weight constraints, the findings indicate that allocations that allow short sales or leverage further improve the performance of portfolios based on machine learning methods. The results persist for mean-variance allocations with different specifications of the covariance matrix and for mean-CVaR portfolios. Additionally, when introducing transaction costs to portfolios with monthly rebalancing the results tend to favor forecast combination techniques, however, reducing the rebalancing frequency to quarterly, leads the portfolios of an aggressive investor that are based on shrinkage and dimensionality reduction methods to generate the highest performance. Finally, when comparing the results of stock-bond portfolios with those that include commodities for the full sample, commodities add value to a traditional portfolio when short-selling is allowed, with portfolios belonging to an aggressive investor benefiting more from the inclusion of commodities.

Pairs trading is a statistical arbitrage strategy which is based on the principle that the prices of two assets co-move with each other. If the spread between the two prices widens, a long-short position can be used to profit from the expected mean-reversion of the spread in the future. The focus of Chapter 4 is to incorporate quantile regression in pairs trading. In this new approach the pairs are formed based on cointegration tests and during trading the trading signal is estimated using quantile regression. The new strategy is applied to a dataset consisting of all stocks in the S&P 100. The performance of the new strategy is assessed using a variety of economic and risk-adjusted metrics and compared against simpler alternatives that are prominent in the pairs trading literature. Additionally, the performance of the pairs trading strategies is evaluated under an asset pricing framework, in order to examine whether the returns of each strategy can be explained by various risk factors. The findings suggest that the quantile regression pairs trading strategies based on the lower quantiles tend to outperform all other models. Finally, Chapter 5 concludes.

Chapter 2: Detecting Bubbles in the US and UK Real Estate Markets

2.1. Introduction

Amongst the earliest bubble detection methods are the variance-bound tests proposed by Shiller (1981) and LeRoy and Porter (1981), who check the validity of the fundamental asset pricing equation by comparing the variance of the observed asset price with an upper bound limit given by the *ex post* rational price. Another method, proposed by Diba and Grossman (1984) and Hamilton and Whiteman (1985), uses stationarity tests to detect bubbles. Furthermore, Campbell and Shiller (1987) apply unit root and cointegration tests to examine the behavior of the fundamental and bubble component of present value models. However, Evans (1991) shows that unit root and cointegration tests have limitations[1] because they are not capable of detecting the explosive patterns of periodically collapsing bubbles.

Although it has been proven that bubbles cannot exist in finite horizon rational expectation models (Tirole, (1982), Santos and Woodford (1997)), bubbles can appear in markets with some particular characteristics that can be also attributed to real estate markets, such as (1) when some particular traders behave myopically (Tirole (1982)), (2) in infinite horizon growing economies with rational traders (Tirole (1985) and Weil (1990)), (3) when there are irrational traders (De Long, Shleifer, Summers, and Waldmann (1990)), (4) in economies where rational traders have differential beliefs and when arbitrageurs cannot synchronize trades (Abreu and Brunnermeier

[1] Recently, a number of econometric methods have been developed that deal with Evans' critique and are capable of distinguishing between pure unit root processes and periodically collapsing bubbles.

(2003)) or (5) when there are short sale/borrowing constraints (Scheinkman and Xiong (2003)). Applying the martingale theory of asset price bubbles in continuous time and continuous trading economies, Jarrow and Porter (2010) demonstrate that in the presence of bubbles, market price indices and fundamental values diverge and lead to serious errors in decision making by investors, financial institutions and regulators.

Debating the idea that the market cannot be efficient because it did not predict the 2008 subprime crisis, John Cochrane stated “crying ‘bubble’ is empty unless you have an operational procedure for identifying bubbles, in real time and not just after the fact, distinguishing them from rationally low-risk premiums, telling a ‘bubble’ from a justified ‘boom,’ and crying wolf too many years in a row”, see Buckner (2017). In this study we offer a procedure that can be used to timely detect bubbles in the real estate markets and we highlight the usefulness of our approach using an extended out-of-sample period 2009-2015.

The subprime mortgage crisis of 2007-2009 had its roots in a real estate bubble of gigantic proportions. There were clear signals (Case and Shiller (2004), Belke and Wiedmann (2005) and Zhou and Sornette (2006)) that something was wrong with the residential real estate prices in the United States. There was evidence of real estate bubbles in the United Kingdom as well at the beginning of the 2000s (Zhou and Sornette (2003), Black, Fraser and Hoesli (2006), Fraser, Hoesli and McAlevey (2008)). Nneji, Brooks, and Ward (2013a, 2013b) examined the residential market in the United States between 1960 and 2011 and found evidence of an intrinsic bubble pre-2000 and, based on a regime-switching model, evidence of periodically rational bubbles in the post-2000 market. Even in real estate investment trusts (REITS) that behave more like an equity asset class, there was evidence of speculative bubbles (Brooks, Katsaris, McGough and Tsolacos (2001), Payne and Waters (2005, 2007) and Jirasakuldech, Campbell and Knight (2006)). It is therefore highly desirable to have a mechanism for signalling the emergence of a bubble in the most valuable asset class of all, real estate.

In this study, real estate price indices are decomposed into a fundamental and a non-fundamental component using a rich dataset of 19 variables covering financial indicators, price indicators, national income and business activity indicators, and employment and labour market indicators. Our study tries to cover exhaustively the real estate markets in the United States and the United Kingdom going back from the end of 2015 to the beginning of historical available data for real estate indices and their drivers in commercial, residential and REIT markets. We employ several subset selection and shrinkage procedures (stepwise regression, ridge regression, lasso, bridge regression and the elastic net along with the commonly employed least squares regression). In order to avoid model selection risk in extracting the fundamental value component of the real estate indices, we propose averaging the fundamental components of all models employed. Our findings suggest the existence of significant periods of *overvaluation* in real estate markets, particularly in residential real estate, as well as economically significant periods of *undervaluation*, particularly in equity real estate markets. The evolution of specific real estate indices in the United States is like the evolution of the corresponding indices in the United Kingdom.

In order to determine whether the observed deviations of the actual prices from their fundamental values are due to the presence of speculative bubbles, we use two complementary methodologies, both taking into account the information contained in the non-fundamental component of the asset price. To verify whether the deviation of the asset price from the fundamental value is due to the presence of speculative bubbles we employ the right-side augmented Dickey-Fuller test for explosive behavior developed by Phillips, Wu and Yu (2011) and Phillips, Shi and Yu (2015) and the Van Norden and Schaller (1993, 1996) two-state regime switching model. The first methodology can also be used to date-stamp the periods of explosiveness in the real estate sectors. The second methodology is based on regime-switching models with two regimes: one where the bubble survives and continues to grow and the other where the bubble collapses. The findings from both methodologies provide significant in-sample evidence that the observed deviations of the actual price from the fundamental value were due to the presence of speculative bubbles. More importantly, our out-of-sample results show that in most cases the proposed regime-switching model for bubbles (averaged across all models employed) outperforms the historical average benchmark and the stylized alternative models.

The chapter is organised as follows. Section 2.2 provides a description of the econometric methodology that we follow and Section 2.3 presents the data that are used. In Section 2.4 we present the in-sample bubble detection results, while in Section 2.5 we discuss the out-of-sample empirical results. Last section concludes the chapter.

2.2. How to Detect Bubbles in Asset Markets?

Starting from Campbell, Lo and McKinlay (1997) and Cochrane (2005), the fundamental price of an asset is derived[2] as

$$P_t = E_t\left[\sum_{i=1}^{T}\left(\frac{1}{1+R}\right)^{i} D_{t+i}\right] + E_t\left[\left(\frac{1}{1+R}\right)^{T} P_{t+T}\right] \qquad (2.1)$$

where the first term of the right-hand side of equation (2.1) represents the fundamental component, which is the expectation of all discounted cash flows, and the second term is the expectation of the

[2] Lai and van Order (2017) investigate US house prices between 1980 and 2012 across 45 metropolitan areas, employing a version of the Gordon dividend discount model.

discounted asset price T periods from time t, and P_t is the asset price at time t and D_{t+1} is the next period's cash flow.

In the case of real estate markets, expected cash flow payments are not directly available. One proxy widely used in the literature is the rent income stemming from holding the property, which is also not available for the majority of indices. To this end, we develop alternative models for the estimation of the fundamental component and consequently the bubble component of the real estate price indices. Specifically, we propose extracting it using subset selection and shrinkage procedures, such as stepwise regression, ridge regression, lasso, bridge regression and elastic net.[3] This is the first time these techniques have been employed in this context. The subsequent description of these methods is largely based on Hastie, Tibshirani, and Friedman (2009).

2.2.1. Model Selection Procedures for the Fundamentals

The *benchmark model* in our study is the classic normal linear regression model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon},$$

where $\mathbf{X} = (\mathbf{x}_1, \dots, \mathbf{x}_p)$ is the $T \times p$ matrix of predictors, $\boldsymbol{\beta} = (\beta_1, \dots, \beta_p)'$ is the coefficient vector and $\boldsymbol{\epsilon} \sim N(\mathbf{0}, \sigma^2 \mathbf{I}_n)$ is the error vector. The ordinary least squares (OLS) estimator $\widehat{\boldsymbol{\beta}}_{OLS} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$ typically has poor predictive accuracy with low bias and high variance.

[3] In a recent paper, Shi (2017) employs a vector autoregressive (VAR) model and variables reflecting aggregate macroeconomic conditions in order to predict fundamental prices.

Ridge regression is a regression method estimating the coefficients subject to the l_2 penalty:

$$\underset{\boldsymbol{\beta}}{\operatorname{argmin}}[\|\mathbf{y}-\mathbf{X}\boldsymbol{\beta}\|^2+\lambda\|\boldsymbol{\beta}\|^2] \quad (2.2)$$

where $\lambda \geq 0$ is a parameter for the amount of shrinkage. The second term of the equation is called the shrinkage penalty and in the case of the ridge regression it is based on l_2 regularization, where $\lambda\|\boldsymbol{\beta}\|^2 = \lambda\sum_{j=1}^{p}\beta_j^2$ and is small when $\beta_1, \dots, \beta_p$ are close to zero and has the effect of shrinking the coefficient estimates towards zero. When $\lambda = 0$ the penalty term has no effect and ridge regression will produce similar estimates to OLS. However, as $\lambda \to \infty$ the impact of the ridge penalty grows and the coefficient estimates will approach zero[4].

The *least absolute shrinkage and selection operator* (lasso) has a penalty term based on the l_1 norm, capable of yielding sparse models. The lasso coefficient estimates are obtained by solving:

$$\underset{\boldsymbol{\beta}}{\operatorname{argmin}}[\|\mathbf{y}-\mathbf{X}\boldsymbol{\beta}\|^2+\lambda\|\boldsymbol{\beta}\|_1] \quad (2.3)$$

where $\lambda \geq 0$ is the lasso tuning parameter. The regression penalty for the lasso is $\lambda\|\boldsymbol{\beta}\|_1 = \lambda\sum_{j=1}^{p}|\beta_j|$. The difference between this and ridge regression is that the lasso method imposes a penalty based on the l_1 norm instead of the l_2 norm, allowing for both shrinkage and variable selection, by setting some of the coefficients equal to zero.

[4] A disadvantage of ridge regression is that the penalty $\lambda\|\boldsymbol{\beta}\|^2$ will shrink all the coefficients towards zero, but it will never set them to zero. Having a model which uses all p predictors can be a problem for model interpretation.

Bridge regression has a penalty term which is based on the l_γ norm and the coefficients are estimated by minimizing:

$$\underset{\boldsymbol{\beta}}{\operatorname{argmin}}\left[\|\mathbf{y}-\mathbf{X}\boldsymbol{\beta}\|^2+\lambda\|\boldsymbol{\beta}\|_\gamma^\gamma\right] \tag{2.4}$$

subject to the constraint $\lambda \geq 0$ and $\gamma > 0$ are the two tuning parameters. The penalty term in the case of bridge regression is $\lambda\|\boldsymbol{\beta}\|_\gamma^\gamma = \lambda\sum_{j=1}^{p}\left|\beta_j\right|^\gamma$ and it is a generalization of the lasso ($\gamma = 1$) and ridge regression ($\gamma = 2$). The bridge regression ($1 < \gamma < 2$) performs shrinkage by keeping all predictors, similarly to ridge regression.

Finally, the *elastic net* (EN) method combines both l_1 and l_2 terms in the penalty, thus simultaneously performing continuous shrinkage and automatic variable selection, but it can also select groups of correlated variables. The elastic net coefficients are estimated by minimizing the following penalized residual sum of squares function:

$$\underset{\boldsymbol{\beta}}{\operatorname{argmin}}\left[\|\mathbf{y}-\mathbf{X}\boldsymbol{\beta}\|^2+\lambda\big((1-\alpha)\|\boldsymbol{\beta}\|_1+\alpha\|\boldsymbol{\beta}\|^2\big)\right] \tag{2.5}$$

where λ is the tuning parameter, $\|\boldsymbol{\beta}\|_1 = \sum_{j=1}^{p}\left|\beta_j\right|$ and $\|\boldsymbol{\beta}\|^2 = \sum_{j=1}^{p}\beta_j^2$. The term $(1-\alpha)\|\boldsymbol{\beta}\|_1 + \alpha\|\boldsymbol{\beta}\|^2$ with $\alpha \in [0,1]$ is called the elastic net penalty, which is a combination of the ridge regression and the lasso penalties. When $\alpha = 1$, the elastic net becomes a ridge regression; if $\alpha = 0$ it is the lasso, while if $\alpha \in (0,1)$ it has the properties of both methods.

2.2.2 Right-Side Unit Root Tests and Date Stamping Procedure

The tests for speculative bubbles we employ in this study are based on right-side unit root tests implemented repeatedly on a forward expanding sample sequence to search for mildly explosive behavior in the data. Those are the supremum augmented Dickey-Fuller (SADF) test and the generalized SADF (GSADF) test developed by Phillips, Wu and Yu (2011, PWY)[5] and Phillips, Shi and Yu (2015, PSY) respectively. The GSADF test has the advantage that it has an increased capacity to detect multiple bubbles in the data.

The PWY and PSY tests are based on the assumption that asset prices follow a random walk process with an asymptotically negligible drift:

$$y_t = dT^{-\eta} + \theta y_{t-1} + e_t, \qquad e_t \sim iid\ N(0, \sigma^2) \tag{2.6}$$

where d is a constant, T is the sample size, $\eta > 1/2$ is a localizing coefficient that controls for the magnitude of the drift as the sample size approaches infinity and e_t is the error term. PWY sets $\eta \to \infty$ and PSY set d, η and θ to unity. Strong upward departures from fundamental values lead the asset price time series to follow an explosive process.

The econometric implementation is based on the ADF test and the use of recursive regressions with variable window widths. This test is applied to each time series y_t to test for a unit root against the alternative of an explosive root. By defining the window's start and end points as r_1 and r_2 respectively, the empirical regression model is specified as:

$$\Delta y_t = a_{r_1,r_2} + \beta_{r_1,r_2} y_{t-1} + \sum_{i=1}^{k} \psi^i_{r_1,r_2} \Delta y_{t-i} + \varepsilon_t \tag{2.7}$$

where y_t can be either a price-to-income ratio or the non-fundamental component, a_{r_1,r_2} is the intercept, k is the maximum number of lags and $\varepsilon_t \sim iid\ N(0, \sigma^2_{r_1,r_2})$. The sample interval is [0,1]

[5]Astill, Harvey, Leybourne and Taylor (2016), propose tests that improve upon the detection of an end-of-sample asset price bubble of finite length and show that their tests detect several well-documented periods of exuberance earlier than existing methods. Fabozzi and Xiao (2019) propose a new recursive algorithm to deal with the inconsistency encountered when estimating the timeline of a bubble based on different samples. This method improves upon the PWY procedure by identifying more consistent starting points and by implementing a two-direction searching process for initialization.

after normalizing the original sample by T and the number of observations in each recursive regression is $T_w = \lfloor Tr_w \rfloor$, where $r_w = r_2 - r_1$ is the fractional window size of the regression. The ADF t-statistic that is used is: $\mathrm{ADF}_{r_1}^{r_2} = \frac{\beta_{r_1,r_2}}{SE(\beta_{r_1,r_2})}$, where β_{r_1,r_2} and $\mathrm{ADF}_{r_1}^{r_2}$ are the regression coefficient and its corresponding ADF t-statistic over the sample $[r_1, r_2]$, respectively.

The SADF test is based on calculating the ADF statistic in each recursive regression performed on a forward expanding sample window. The starting point r_1 of the estimation window remains fixed for all recursive regressions and is the first observation of the sample. The end point r_2 of the first estimation window is set according to some choice of minimum window size r_0 required for the adequate initial estimation of equation (2.7). Therefore, the first regression involves $T_0 = \lfloor Tr_0 \rfloor$ observations for a minimum fraction, r_0, of the total sample. Each subsequent regression increments the initial fraction of the sample by one observation, giving a forward expanding window size $r_2 \in [r_0, 1]$. The ADF statistic is calculated for each recursive regression and is denoted by ADF_{r_2}. The SADF test statistic is defined as the supremum value of ADF_{r_2} for $r_2 \in [r_0, 1]$:

$$\mathrm{SADF}(r_0) = \sup_{r_2 \in [r_0, 1]} \left\{ \mathrm{ADF}_{r_2} \right\} \tag{2.8}$$

The GSADF test generalizes the SADF test by having more flexible estimation window widths and by allowing the starting point r_1 to change within the range $[0, r_2 - r_0]$ for each regression. The GSADF test statistic is defined as the supremum value of $\mathrm{ADF}_{r_1}^{r_2}$ for $r_1 \in [0, r_2 - r_0]$ and $r_2 \in [r_0, 1]$:

$$\mathrm{GSADF}(r_0) = \sup_{\substack{r_2 \in [r_0, 1] \\ r_1 \in [0, r_2 - r_0]}} \left\{ \mathrm{ADF}_{r_1}^{r_2} \right\} \tag{2.9}$$

The SADF and GSADF tests can also be used to date stamp the origination and collapse of the bubbles in a time series. The date stamping procedure of the SADF test compares each ADF_{r_2} statistic to each respective right-side critical value of the standard ADF statistic to identify whether a bubble exists at time Tr_2. The origination date of a bubble, Tr_e, where r_e is the fractional estimate of the beginning of the bubble period, is determined as the time point when the ADF_{r_2} sequence crosses its respective critical value sequence from below. The collapse date of the bubble, Tr_f, where r_f is the fractional estimate of the end of the bubble period, is marked when the ADF_{r_2} sequence crosses its respective critical value sequence from above. The fractional origin and collapse points of the bubble for the SADF test are denoted as:

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \left\{ r_2 \colon \mathrm{ADF}_{r_2} > cv_{r_2}^{\beta_T} \right\}, \tag{2.10}$$

$$\hat{r}_f = \inf_{r_2 \in [\hat{r}_e, 1]} \left\{ r_2 \colon \mathrm{ADF}_{r_2} < cv_{r_2}^{\beta_T} \right\}$$

where $cv_{r_2}^{\beta_T}$ is the $100(1-\beta_T)\%$ critical value of the limit distribution of the standard ADF statistic based on $\lfloor Tr_2 \rfloor$ sample observations and β_T is the size of the one sided test.

The date stamping procedure for the GSADF test is based on calculating a sup ADF statistic on backward expanding samples, with fixed ending points at r_2 and varying starting points $r_1 = [0, r_2 - r_0]$. The backward SADF statistic is defined as:

$$\mathrm{BSADF}_{r_2}(r_0) = \sup_{r_1 = [0, r_2 - r_0]} \left\{ \mathrm{ADF}_{r_1}^{r_2} \right\} \tag{2.11}$$

Similarly to the SADF date stamping procedure, the fractional origin and collapse points of the bubble for the GSADF test are denoted as:

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \left\{ r_2 \colon \mathrm{BSADF}_{r_2}(r_0) > cv_{r_2}^{\beta_T} \right\}, \tag{2.12}$$

$$\hat{r}_f = \inf_{r_2 \in [\hat{r}_e, 1]} \left\{ r_2 \colon \mathrm{BSADF}_{r_2}(r_0) < cv_{r_2}^{\beta_T} \right\}$$

where $cv_{r_2}^{\beta_T}$ is the $100(1-\beta_T)\%$ critical value of the limit distribution of the standard ADF statistic based on $\lfloor Tr_2 \rfloor$ sample observations.

2.2.3. The Regime-Switching Bubble Model

Blanchard (1979) and Blanchard and Watson (1982) suggested a model for rational bubbles with two possible bubble states; one state is that the bubble survives and the other state is that the bubble collapses. The bubble process is then defined by:

$$B_{t+1}|S = \left(\frac{1+R}{q}\right) B_t + u_{t+1}, \text{ with probability } q \tag{2.13}$$

and

$$B_{t+1}|C = u_{t+1}, \text{ with probability } 1-q$$

A rational bubble that has the above form obeys the restriction: $B_t = E_t\left[\frac{B_{t+1}}{1+R}\right]$, as long as the shock u_{t+1} satisfies $E(u_{t+1}) = 0$. Then

$$E_t(B_{t+1}|S) = \frac{(1+R)}{q} B_t, \text{ with probability } q \tag{2.14}$$

and

$$E_t(B_{t+1}|C) = 0, \text{ with probability } 1-q$$

where S indicates the state that the bubble survives and C the state that it collapses. If the bubble survives in period $t+1$, it will grow at a rate $\left(\frac{1+R}{q}\right) - 1$, which is faster than R, in order to compensate the investors for the risk they take for the probability of a crash.

The Blanchard and Watson model was generalized[6] by van Norden and Schaller (1993, 1999) in two ways. First, they allow the probability of the bubble being in the surviving state q to depend on the relative size of the bubble $q = q(b_t)$ where $b_t = B_t/P_t$ is the relative size of the bubble, which is the ratio of the non-fundamental component B_t to the actual price P_t. The absolute value of b_t is used since there can be positive or negative bubbles.

The second generalization allows for partial collapses, by permitting the expected value of the bubble conditional on the collapsing state being non-zero. Van Norden and Schaller (1993, 1999) defined the expected size of a bubble in state C as $u_t P_t$ and assumed that it depends on the relative size of the bubble in a previous period:

$$E_t(B_{t+1}|C) = u(b_t)P_t \tag{2.15}$$

where $u(\cdot)$ is a continuous and differentiable function such that $u(0) = 0$ and $0 \leq \frac{du(b_t)}{db_t} \leq 1$. The condition ensures that a collapsing bubble is smaller than the bubble in the previous period.

The two generalizations made by van Norden and Schaller lead to the following modified bubble model:

$$E_t(B_{t+1}|S) = \frac{(1+R)}{q(b_t)}B_t - \frac{1-q(b_t)}{q(b_t)}\,u(b_t)P_t, \text{ with probability } q(b_t) \tag{2.16}$$

and

$$E_t(B_{t+1}|C) = u(b_t)P_t, \text{ with probability } 1 - q(b_t)$$

The expected gross returns R^* for each regime are:

[6] Van Norden and Schaller (1993, 1999) and Brooks and Katsaris (2005a, 2005b) criticised the Blanchard and Watson (1982) model because of the lack of theoretical support and empirical evidence.

$$E_t(R^*_{t+1}|S) = (1+R) + \frac{1-q(b_t)}{q(b_t)}[(1+R)b_t - u(b_t)], \text{ with probability } q(b_t) \qquad (2.17)$$

and

$$E_t(R^*_{t+1}|C) = (1+R)(1-b_t) + u(b_t), \text{ with probability } 1-q(b_t)$$

Thus, the returns in time $t+1$ depend on the regime of the previous period t. To estimate the model, the first-order Taylor series approximations of $E_t(R^*_{t+1}|S)$ and $E_t(R^*_{t+1}|C)$ with respect to b_t around some arbitrary value b_0 are taken, giving the linear regime switching model:

$$E_t(R^*_{t+1}|S) = \beta_{S0} + \beta_{S1}b_t, \qquad (2.18)$$

$$E_t(R^*_{t+1}|C) = \beta_{C0} + \beta_{C1}b_t$$

where

$$\beta_{S1} = \left(-\frac{1}{q(b_0)^2}\frac{dq(b_0)}{db_t}[(1+R)b_0 - u(b_0)] + \frac{1-q(b_0)}{q(b_0)}\left[1+R-\frac{du(b_0)}{db_t}\right]\right) \qquad (2.19)$$

and

$$\beta_{C1} = \left(\frac{du(b_0)}{db_t} - (1+R)\right).$$

The regime switching model can be rewritten as:

$$R^*_{S,t+1} = \beta_{S0} + \beta_{S1}b_t + \varepsilon_{S,t+1}, \qquad \varepsilon_{S,t+1} \sim N(0,\sigma_S^2) \qquad (2.20)$$

and

$$R^*_{C,t+1} = \beta_{C0} + \beta_{C1}b_t + \varepsilon_{C,t+1}, \qquad \varepsilon_{C,t+1} \sim N(0,\sigma_C^2)$$

where σ_S, σ_C are the standard deviations of the error terms of $\varepsilon_{S,t+1}$ and $\varepsilon_{C,t+1}$ respectively. The parameters β_{S0} and β_{C0} represent the mean returns for the surviving and the collapsing state respectively and the coefficients β_{S1} and β_{C1} show how changes in the relative size of the bubble affect the returns in each state.

For the functional form of $q(b_t)$, van Norden and Schaller use a probit model:

$$P(R^*_{t+1}|S) = q(b_t) = \Phi\big(\beta_{q0} + \beta_{q1}|b_t|\big) \tag{2.21}$$

and

$$P(R^*_{t+1}|C) = 1 - q(b_t) = 1 - \Phi\big(\beta_{q0} + \beta_{q1}|b_t|\big)$$

where Φ is the standard normal cumulative density function and β_{q1} describes the effect that the absolute value of the relative size of the bubble has on the probability of being in the surviving state. Van Norden (1996) uses both b_t and b_t^2 instead of $|b_t|$, van Norden and Vigfusson (1998) employ b_t, while Schaller and van Norden (2002) use b_t^2. The model considers the restriction $\beta_{q1} < 0$, since as the deviation from the fundamentals grows, so does the probability of collapse. Furthermore, assuming $R > 0$, then $\beta_{C1} < 0$, because as the relative size of the bubble grows, it leads to greater capital losses when the bubble collapses and $\beta_{S1} > \beta_{C1}$, since a large relative size of the bubble means that the difference between the returns of the surviving and collapsing state will be greater.

The parameters estimates are found by maximizing the log-likelihood function:

$$\sum_{t=1}^{T} \ln\left[q(b_t)\varphi\left(\frac{R^*_{S,t+1} - \beta_{S0} + \beta_{S1}b_t}{\sigma_S}\right)\sigma_S^{-1} + \big(1 - q(b_t)\big)\varphi\left(\frac{R^*_{C,t+1} - \beta_{C0} + \beta_{C1}b_t}{\sigma_C}\right)\sigma_C^{-1}\right] \tag{2.22}$$

where φ is the standard normal probability density function and the parameters to be estimated are $\beta_{S0}, \beta_{S1}, \beta_{C0}, \beta_{C1}, \beta_{q0}, \beta_{q1}, \sigma_S$ and σ_C. The probability of being in regime $i = S, C$ in period $t + 1$ depends on the relative size of the bubble b_t and is given by the formula: $\Phi\left(l(i)\big(\beta_{q0} + \beta_{q1}|b_t|\big)\right)$, where $l(i) = 1$ in the surviving state and $l(i) = -1$ in the collapsing state.

2.3. Data Description

2.3.1 Real Estate Data

We analyse the main real estate indices in each real estate market from each country. For commercial, residential and equity real estate sectors, the indices for the United States are NCREIF, S&P/Case-Shiller and US FTSE EPRA/NAREIT, while for the United Kingdom the indices in this study are IPD UK All Property, UK House Price Index and UK FTSE EPRA/NAREIT.

All real estate indices price levels are retrieved from Bloomberg. For the US, data on quarterly frequency is available for the NCREIF from the fourth quarter of 1977 to the fourth quarter of 2015, while commercial real estate data for the UK are available on a monthly frequency for the IPD index for the period from December 1986 to December 2015, providing a total of 153 quarterly and 349 monthly observations respectively. For equity real estate monthly data on the transactions-based FTSE EPRA/NAREIT indices for the US and the UK are available for the period from December 1989 to December 2015, with a total number of 313 monthly observations for each index. Finally, for the residential real estate market, monthly data is available on the S&P/Case-Shiller home price index for the US from January 1987 to December 2015 and on the UK House Price Index from January 1995 to December 2015, totalling 348 and 252 monthly observations for each time series respectively. The real estate indices are adjusted for inflation using the Consumer Price Index (CPI) for the US and the Retail Price Index (RPI) for the UK. The augmented Dickey-Fuller tests (ADF tests) for all indices indicate[7] that the level series are non-stationary.

2.3.2. Economic Data

We are guided by the extant literature in selecting the economic variables employed to construct the fundamental value models for the real estate indices. Ghysels, Plazzi, Valkanov and Torous (2013) provide an extensive review of the literature on real estate forecasting based on the type of predictive information used. We extracted the potential drivers of the fundamental value of real estate markets from previous studies.[8]

For both economies, we employ a set of 19 explanatory variables, which are classified into four broad categories: financial indicators, price indicators, national income and business activity indicators, and employment and labour market indicators. Specifically, the financial variables are a stock price index, the US/UK exchange rate, the money supply M2, the central bank rate, the 5-year and 10-year government bond yields and a mortgage rate. The price indicators include the inflation rate, gold price, oil price and the rent price index. The national income and business activity indicators are the real GDP, real personal disposable income, industrial production and housing starts. Finally, the labour market indicators are the unemployment rate, labour cost and labour productivity. Variable definitions are presented in Table 2.1, while data sources are outlined in Table A2.1 in the Appendix of this chapter.

[7] All tests for stationarity are presented in Table A2.1 in the Appendix of Chapter 2.

[8] See Case and Shiller (1990), Dobson and Goddard (1992), Liu and Mei (1992), Mei and Liu (1994), Ling and Naranjo (1997), Ling, Naranjo and Ryngaert (2000), De Wit and Van Dijk (2003), Himmelberg, Mayer and Sinai (2005), Clayton, Ling and Naranjo (2009), MacKinnon and Al Zaman (2009) and Plazzi, Torous and Valkanov (2010). The list is by no means exhaustive and there is a very long list of articles in this area.

[Insert Table 2.1 Here]

2.4. In-Sample Empirical Analysis

2.4.1. Results for the Fundamental Value

In order to apply the right-side unit root tests and the regime switching model on the real estate indices, the fundamental and bubble components must first be retrieved. This is usually done by constructing a supply and demand model, through which the price index is regressed on various economic variables using OLS. The fitted value of the regression model represents the fundamental value of the index, which is determined by the economic variables. The error term of the model is the part of the index that is not explained by the model predictors and represents the non-fundamental or bubble component of the index price.

Due to the large number of predictors, we employ several shrinkage and model selection procedures along with OLS to create alternative measures for the fundamental and bubble component. The SADF and GSADF tests are applied to the non-fundamental component. Furthermore, in order to estimate the regime switching model the relative size of the bubble is required, which is constructed using the actual price and fundamental price. Specifically, to extract the fundamental price from the regressions the following formula is used:

$$p_t^f = \left(1 + r_t^f\right) p_{t-1}^f, \text{ where } p_0^f = p_0 \quad (2.23)$$

where p_t^f is the fundamental price at time t, r_t^f is the fitted value of the regression of the index returns on the stationary predictors[9] at time t and p_0 is the actual price of the index at $t = 0$. Subsequently, the relative size of the bubble is computed using the following formula:

$$b_t = \frac{p_t - p_t^f}{p_t} \tag{2.24}$$

where b_t is the relative size of the bubble and p_t is the actual price of the index at time t.

Figure 2.1 plots the actual price, the average fundamental value and the average relative size of the bubble, for all six real estate indices. The average fundamental value or bubble size is simply computed by taking the average of the fundamental value or relative bubble size of all fitting procedures for each market. In this way, we overcome the model risk associated with the employment of one particular model for bubble estimation. The left-hand scale of Figure 2.1 plots the actual index price against the average fundamental price and on the right-hand scale the extent of under- or overvaluation is depicted. There have been periods of overvaluation and undervaluation in all six markets across our sample.

For the US, the commercial real estate as reflected by the NCREIF index was often undervalued, from the end of the 1980s right to the eruption of the subprime crisis in 2007. There were short periods of overvaluation between 1982 and 1986 and between 2007 and 2008. A similar picture is portrayed for the US FTSE EPRA/NAREIT Index with long periods of undervaluation around the dot.com crisis of 2000-2002 and in the aftermath of the subprime crisis.

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[9] The predictors are the 19 economic variables listed in Table 2.1. The OLS, stepwise regression, ridge regression, the lasso, bridge regression and the elastic net were applied and the tuning parameters were selected using tenfold cross validation. For the lambda tuning parameters, a grid of 100 values between 10^{-2} and 10^{2} was chosen. The bridge regression tuning parameter, gamma, is given a grid of values between 1.1 and 1.9 with step 0.1, while for the elastic net alpha tuning parameter a grid of values between 0 and 1 with step 0.1 is chosen.

[Insert Figure 2.1 Here]

The residential real estate evolution in the United States paints a different picture, with a long undervaluation period between 1991 and 2002, followed by an economically significant overvaluation period ending in 2009 and followed by undervaluation that peaked in 2012.

The IPD index in the UK seems to be closer to the fundamental value. There are short periods of overvaluation, the most notable one being the period before the start of the subprime crisis, and likewise short periods of undervaluation, the only economically significant one being the period 2009-2015. Similar to the US, the equity index for the UK indicates that this market was generally characterised by undervaluation. Mei and Saunders (1997) found evidence of a trend-chasing strategy of buying high and selling low followed by commercial banks and thrifts on their real estate investments. Their conclusion is in line with our results on REITS markets in the US and the UK reaching an overall judgement that undervaluation was omnipresent.

The residential real estate in the United Kingdom had a similar evolution with the residential real estate in the United States, with the only difference being the period 2002-2003 indicating the start of a bubble in the United Kingdom that ended in 2012. In both countries there has been a long period of significant overvaluation of house prices that started after 2002 and ended in 2009 in the United States and in 2011 in the United Kingdom. Holly, Pesaran and Yamagata (2011) argued that there is a direct link between London house prices and New York house prices and also suggested that economic shocks to the metropolis prices propagated contemporaneously and spatially to other regions in the same country. Their argument may explain our evidence on the similarity of overvaluation and undervaluation periods in the two countries.

2.4.2. Results of the Right-Side ADF Tests for Explosive Behavior

Table 2.2 summarizes the results for the SADF and GSADF tests on the real estate indices for the US and the UK. In the interest of saving space, we report the tests based on the average of the non-fundamental components derived from the alternative proposed models described in Section 2.2. Following the rule suggested by PSY, the minimum window size is set to $0.01 + 1.8/\sqrt{T}$ of the total sample size for each index. The finite sample critical value sequences are obtained by Monte Carlo simulation with 2000 replications, while the ADF lag is chosen to minimize the Schwarz Information Criterion.

[Insert Table 2.2 Here]

Overall, the SADF and GSADF tests provide evidence of bubble formation for all real estate indices. Specifically, both tests find evidence of explosive behavior for all US real estate indices at the 1% significance level. According to the SADF test, all UK real estate indices exhibit explosive behavior at a 1% significance level, with the exception of the UK equity real estate index, where the null hypothesis that there is a unit root is rejected at a 10% significance level. The results of the GSADF tests for the UK and the US reveal evidence that multiple bubbles are present in the commercial, equity and residential real estate indices of both countries.

Our tests point to strong evidence of exuberance in all real estate indices and we employ the BSADF test in order to identify the origin and collapse date of the bubble periods for each index. Similarly to the GSADF test, the minimum window is set to $0.01 + 1.8/\sqrt{T}$ of the total sample observations and the ADF lag is chosen to minimize the Schwarz Information Criterion.

Figure 2.2 illustrates that for the NCREIF Property Index the two major bubble periods occur in the late 80s to early 90s and from 2005 to 2008, while the bubble period with the greatest duration for the IPD UK Property Index is from 2005 to 2008, with shorter periods appearing in the late 90s and in 2013-2014. For the US and the UK real estate indices the bubbles with the longer duration occur in the late 90s and early 2000s, with shorter bubble periods appearing between 2006 and 2007. For the S&P/Case-Shiller Index the two major bubbles are observed for the period 1990-1998 and another one for the period 2000 to 2007, while for the UK residential real estate the bubble with the longest duration is between 2001 and 2007, with smaller bubble periods after 2009.

[Insert Figure 2.2 Here]

2.4.3. Results for the Regime Switching Models for Bubbles

To determine whether the deviations of the actual prices from their fundamentals were due to the presence of periodically collapsing bubbles, we apply the van Norden and Schaller (vNS) regime switching model to the returns of the real estate indices. Tables 2.3-2.5 present the results of the regime switching model based on both the average bubble size and the model specific ones for the commercial, equity and residential real estate markets, respectively. The regime switching model we apply has two regimes. In the first regime the bubble survives and continues to grow yielding a positive return, while in the second regime the bubble collapses and prices fall. According to the bubble theory, realised returns should be higher in the surviving regime, while volatility should be higher in the collapsing regime. We first focus on the findings with respect to the average bubble size and then we compare it to the individual model ones.

The coefficient of the bubble term for the surviving regime (β_{S1}) is statistically significant at the 5% level only for the S&P/Case-Shiller Home Price Index. In this case, all individual bubble models provide positive statistically significant results. For the IPD UK Index, β_{S1} is positive and statistically significant for the average bubble size at the 10% level, while the results of individual models is mixed with only bridge and elastic net pointing in the same direction. Furthermore, the coefficient of the bubble term when the bubble collapses, β_{C1}, is statistically significant for all indices with the exception of the UK commercial and equity index. For these indices, only bridge supports the theoretical negative coefficient. Overall, the coefficients in the surviving regime are greater than those in the collapsing regime, which suggests that the bubble in the collapsing regime leads to more negative returns than in the surviving regime.

The coefficient β_{q1} is negative, in the case of the US residential and the UK equity and residential indices, which indicates that the larger the bubble size, the higher the probability of the bubble collapsing in the next period. The estimates for β_{q1} are statistically significant at the 5% level for the US and the UK equity and the UK residential real estate indices. For the equity indices, both OLS and stepwise point to non-statistically significant coefficients, while for the UK residential index, all models agree.

The estimates for the mean returns in the surviving regime are 1.75%, 0.40%, 0.44%, 0.70%, -0.29% and 0.57%, while in the collapsing regime they are -6.10%, -18.98%, -0.44%, -0.20%, -25.93% and -0.06% for the commercial, equity and residential real estate markets for the US and the UK respectively. These represent the expected yields when there is no bubble and are quite similar across models.

[Insert Tables 2.3-2.5 Here]

Turning to coefficient restriction tests and the results based on the average bubble, we note that the restriction $\beta_{S0} \neq \beta_{C0}$ holds for all sectors (at the 10% level) except for the UK equity real estate sectors (marginally), while the restriction $\beta_{S1} \neq \beta_{C1}$ holds for all indices except for the IPD UK Property Index and the UK FTSE EPRA/NAREIT index. It is interesting to note, though, that we observe considerable heterogeneity among individual bubble specifications. More in detail, for the NCREIF index, both restrictions are rejected when the bridge bubble is employed and for IPD UK, the restriction $\beta_{S1} \neq \beta_{C1}$ holds for the bridge and elastic net specification. In a similar vein, OLS rejects both restrictions and stepwise only the second one for the US equity real estate index. On the other hand, both restrictions hold based on the bridge bubble specification and the UK FTSE index.

Finally, we perform likelihood ratio tests to determine whether the vNS bubble model can explain returns better than alternative models such as volatility regimes, fads and mixture-normal models. Our results, based on the average bubble specification, indicate that the vNS model is more efficient in capturing return dynamics for all indices, except for the two commercial real estate indices. For the NPI the volatility regimes and the mixture-normal models outperform the bubble model, while for the IPD the mixture-normal model is better at describing the returns. For these indices, all bubble specifications point to the same direction with the exception of the bridge bubble that points to superiority of the vNS model over the mixture-normal model. With respect to US FTSE index, contrary to the average bubble and the majority of fundamental models, stepwise and OLS reject the superiority of the vNS models versus all alternative stylised models (OLS at the 5% level for the fads model). On the other hand, for the UK FTSE index, only lasso and bridge (along with the average) are in favor of the vNS model. Similarly to the coefficient restriction tests, all fundamental model specifications agree on the superiority of the vNS model for the US and UK residential indices.

Figure 2.3 illustrates the evolution of the probability of collapse for each specific real estate sector (based on the average bubble size) in both the US and the UK. The only indication of a possible crash in the commercial real estate market in the US is for 1992-1993 and 2009. The equity market in the US was close to a crash in 2004, 2009 and 2012. For the residential real estate in the US as reflected by the Case-Shiller index, clear problems related to the collapse of the market were in 1990-1991, 2006-2011, 2014 and 2015. The situation in the United Kingdom was slightly different. The probability of collapse attached to the IPD index was very high between 1990-1994 and 2007-2010. The equity market in the United Kingdom was only ever close to a crash around 2009. The residential market as represented by the UK House Price index was close to a collapse between 2008 and 2009 and the probability of collapse even reached zero in the period 2002-2008.

[Insert Figure 2.3 Here]

In the next section, we assess the out-of-sample forecasting ability of the vNS regime switching model relative to the stylised bubble models and the historical average model (random walk with drift). We also scrutinise the forecasting ability of the proposed fundamental models employed for the relative bubble calculation and check whether employing the average relative bubble offers a hedge against model uncertainty.

2.5. Out-of-Sample Empirical Analysis

This section examines whether the van Norden-Schaller regime-switching model can be used to generate reliable out-of-sample forecasts. We consider 1-month, 3-month and 6-month forecasting horizons (the analysis for the NCREIF Property Index is for only 1-quarter and 2-quarters ahead). Given the total number of T observations of each index, the sample is split into an out-of-sample part, Q and an in-sample part, $P = T - Q$. In our experiment, the out-of-sample window is set to eight years for all indices (32 observations for the NCREIF Property Index and 96 observations for the rest of the indices). In this respect, the out-of-sample period starts at 2008 and coincides with the global financial crisis, creating considerable challenges for our forecasting experiment. The h-period ahead forecasts ($h = 1$, 3 and 6 months) of the regime switching model are generated by estimating the van Norden-Schaller model recursively, increasing the initial window, P, by one observation at a time. The average relative and individual fundamental bubble sizes, which are used as an input in the model, are also constructed recursively from the estimates of all the fundamental models at each iteration.

The forecasting performance of the van Norden-Schaller model and the alternative nested regime switching specifications are evaluated using the mean square forecast error (MSFE) criterion, which is given by:

$$MSFE_i = \frac{1}{Q}\sum_{t=1}^{Q}\left(r_{P+t} - \hat{r}_{i,P+t}\right)^2 \quad (2.25)$$

where $\hat{r}_{i,P+t}$ denotes the forecast from model i. In order to evaluate the forecasting accuracy of the regime switching models, we compare them with the historical average benchmark model (random walk with drift). We compute the MSFE ratios of the regime switching models relative to the benchmark and alternative nested regime specifications. A ratio below unity implies that the regime switching model forecast is more accurate than the benchmark and alternative models in terms of MSFE. Additionally, to test whether the improvement in MSFE for the regime switching models against the historical average (and the nested regime switching specifications) is statistically significant, we employ the Clark and West (2007) test that utilises the MSFE-adjusted statistic, which is approximately normally distributed when comparing forecasts from nested models. The MSFE-adjusted statistic is computed by first defining:

$$f_{i,t} = (r_{P+t} - \bar{r}_{P+t})^2 - \left(r_{P+t} - \hat{r}_{i,P+t}\right)^2 + \left(\bar{r}_{P+t} - \hat{r}_{i,P+t}\right)^2 \quad (2.26)$$

where $\bar{r}_{P+t}$, is the forecast of r_{P+t}, using the historical average benchmark. The Clark-West t-statistic is compared to the critical value of 1.282 corresponding to the 10% significance level. The null-hypothesis is that the MSFE of the benchmark is less or equal to the MSFE of model i, while the alternative is that MSFE of the benchmark is greater than the MSFE of model i.

2.5.1. One-Month ahead Forecasts

Tables 2.6, 2.7 and 2.8 detail the MSFE ratios of the various models relative to the benchmark for the 1-month ahead horizon, while the Clark-West t-statistics are reported below in parenthesis. Overall, our 1-month out-of-sample findings suggest that the van Norden and Schaller model is more accurate than the benchmark in all the indices considered while it beats the alternative regime switching models in four of the indices under consideration.

More in detail, the top panel of Table 2.6 compares the performance of the forecasts with the historical average for the UK commercial index, while the bottom panel compares the out-of-sample performance of the van Norden and Schaller model with each of the stylized alternative models. Our findings suggest that both the normal-mixture model and the bubble model have statistically significant better out-of-sample performance compared to the historical average. The fundamental bubble calculated via the bridge regression attains the lowest MSFE (0.7898) among the alternative fundamental models and the average bubble. Comparing the performance of the vNS model to the stylized alternative models, we note that the vNS model beats both the volatility regimes and the fads model (but not the normal-mixture one).

Turning to equity real estate indices, our findings, reported in Table 2.7, suggest that for the US, the forecasts generated by the vNS model and the average bubble are the most accurate (with an MSFE of 0.9069) albeit non-significant. However, stepwise vNS model forecasts are statistically significantly lower that the historical average benchmark. We should also note that the elastic net, the bridge and average bubble fads model attain superior forecasts. For the UK equity index, all normal mixture and vNS models (with the exception of lasso) achieve lower forecast errors than the historical average benchmark. Stepwise vNS delivers more accurate forecasts among the alternative bubble models followed by the average bubble. With respect to the residential real estate indices, the vNS model achieves superior forecasting performance irrespective of the fundamental bubble employed for both the US and UK markets (Table 2.8). Specifically, for the S&P/Case-Shiller Home Price Index, the average bubble delivers the lowest MSFE (0.6079) followed by ridge (0.6176) and lasso (0.6354). As expected, the vNS model outperforms all stylised nested specifications (Panel B) by a wide margin. Similar findings pertain for the UK House Price Index. In this case, the lowest MSFE is achieved by lasso vNS (0.6852) followed by the average bubble vNS (0.6958). As expected, Panel B of Table 2.8 verifies the forecasting superiority of vNS relative to the volatility regimes, the fads and the mixture of normal model.

[Insert Tables 2.6-2.8 Here]

To gain a visual understanding of the accuracy of our models, the cumulative difference between forecast errors for the historical average against each of the alternative bubble vNS models for all real estate markets are plotted in Figure 2.4. These graphs can be used to assess whether the alternative models consistently outperform the historical average benchmark for any particular out-of-sample period. To determine this, the height of the curve at the beginning and end points of the period of interest are compared. If the curve is higher at the end of the segment compared to the beginning, then the forecast based on the regime switching model has a lower MSFE than the historical average benchmark during that period. For a model to always outperform the historical average, the slope should be positive for the whole out-of-sample period.

Overall, the path of the cumulative forecast error differences are quite diverse for the indices considered. More specifically, for the IPD, we observe all fundamental bubble models along with the average being in the positive territory for the whole out-of-sample period, experiencing small losses in the aftermath of the financial crisis. They then stabilise and retain their ranking position up to the end of the sample period. For the US FTSE real estate index, the financial crisis period is marked with losses for all the models followed by a quick recovery in 2009. Beyond 2009, all models move similarly with the average ranking higher and on the other hand, the bridge model deteriorating to rank lowest at the end of 2015. Turning to the UK FTSE real estate index, all models behave similarly during the financial crisis showing divergent patterns in the aftermath. Specifically, stepwise vNS followed by the average quickly gain ground and retain their superiority up to the end of the sample, while lasso vNS is for the majority of the out-of-sample period in negative territory showing worse forecasting performance than the historical average. The superior forecasting performance of all fundamental models is apparent in the case of the US house price index, as all models exhibit quick gains during the financial crisis which they manage to retain and increase (upward sloping curve) up to the end of the sample period. Although all models move close together, the average bubble ranks first while the bridge one takes the lowest position. Finally, the UK house price index paints a different picture. Similarly, to the

US house price index, the financial turmoil benefits all specifications, but soon after our fundamental models form three groups. In the best performing one, associated with consistent forecasting gains over the out-of-sample period, we see lasso and the average bubble vNs model, while elastic net and bridge form the group of worst performing models.

[Insert Figure 2.4 Here]

2.5.2. Longer Forecasting Horizons

Since most investors except portfolio investors need more lead time than one month, we also consider 3-month and 6-month forecasting horizons. Table 2.9 reports the related findings for all the indices at hand. Overall, the majority of alternative vNS bubble models are superior to the random walk in all cases. More in detail, for the commercial real estate indices, all fundamental vNS models achieve superior forecasting ability relative to the historical average with the bridge vNS achieving the lowest MSFE (0.7181 and 0.6347, for the US and UK respectively). This performance is closely followed by the average fundamental vNS model, which ranks second for the NCREIF index. The normal mixture model also outperforms the historical average, but is associated with inferior forecasts relative to the vNS. Turning to equity real estate indices, we note that the best model in terms of MSFE is the elastic net fads model for the US (0.9349) and the bridge fads model for the UK (0.8378). The vNS model ranks second with the average bubble and bridge bubble model performing best for the US and UK, respectively. Finally, for the residential real estate indices, all vNS specifications rank first and succeed in reducing the random walk MSFE by almost half both for the US and UK. For example, for the Case-Shiller index, the best forecasting model is the elastic net vNS that achieves an MSFE of 0.5989 closely followed by all models with the average just a little over 0.6077. For the UK house price index, the best performing model is the stepwise vNS model (with an MSFE of 0.6175) followed by ridge, lasso and the average fundamental vNS model.

[Insert Table 2.9 Here]

Figure 2.5 plots the cumulative difference between forecast errors for the historical average against each of the alternative bubble vNS models for all real estate markets and the 3-month horizon. In both the US and UK commercial real estate indices, all vNS models appear successful in improving forecasts in the aftermath of the financial crisis. However, for NCREIF all specifications underperform in 2008 followed by sharp gains after 2009 and small losses afterwards. These movements are rather muted for the elastic net bubble model. The best performance is attained by the bridge vNS model followed by the average bubble one. For the IPD, all specifications move quite similarly, experiencing sharp gains during the financial crisis followed by stabilisation in the aftermath. In contrast, performance of the equity real estate indices is quite diverse among fundamental specifications. Specifically, for the US only the average and elastic net manage to retain gains at the end of the out-of-sample period with bridge showing the worst performance. However, bridge and the average are consistently superior and rank first for the UK FTSE index. Finally, all fundamental models move closely together in the case of the US residential index, while for the UK fundamental models form two groups, with elastic net and bridge belonging to the worst performing one.

[Insert Figure 2.5 Here]

Turning to the 6-month horizon, our findings reported in Table 2.10, suggest that the best forecasting performance is attained for the Case-Shiller Index followed by the IPD UK index. For the Case Shiller index, all fads, normal mixture and vNs models appear significantly more accurate than the random walk, with the vNS ranking first. In this set of models, the one with the lowest MSFE is the stepwise (0.4951), while the average bubble model also proved accurate with a statistically significant MSFE of 0.5488. In the case of the IPD, the most accurate model is the bridge vNS (0.5689) followed by the elastic net (0.7132). The average bubble vNS fares well with a statistically significant MSFE of 0.7658. On the other hand, all vNS specifications fail to improve upon the historical average model for both the US and UK FTSE indices. In these cases, the fads model beats the historical average model with the average bubble and the ridge bubble fads model ranking first for the US and UK, respectively. Finally, the NCREIF and the UK house price index provide mixed evidence. For the NCREIF, the bridge and elastic net fads model are the best followed by stepwise OLS. For this index, all fundamental bubble vNS models (with the exception of the OLS) offer improvements over the historical average as judged by the Clark-West test.

[Insert Table 2.13 Here]

The pattern of the forecasting ability of the various bubble vNS models for the 6-month horizon is graphically shown in Figure 2.6. For this forecasting horizon, overall we get diminished forecasting power for the majority of indices and more divergent behavior across specifications with the exception of the US residential index. For the US commercial index, all models experience some gains in 2009, followed by sharp losses in 2010, which for the stepwise and lasso they are smaller and lead to significant improvements over the out-of-sample period. For the UK IPD index, bridge and average are the models benefiting more from the financial crisis compared to the remaining specifications. Finally, two groups of forecasting models can be identified for the UK residential index. In the group of best performing specifications are OLS, stepwise, ridge and lasso.

This group experiences gains in 2008 that are mostly retained in the out-of-sample period, while the worst performing group quickly loses any benefits and continues to underperform the historical average model until the end of the out-of-sample period.

[Insert Figure 2.6 Here]

2.6. Conclusion

In this research we confirm the existence of bubbles in real estate markets in the United States and the United Kingdom using shrinkage and variable selection models to extract the fundamental component underpinning these markets. To investigate the bubble dynamics in real estate markets, fundamental models were constructed using several fitting procedures and a wide range of economic variables. The fundamental value underpinning commercial, residential and equity real estate markets was extracted using stepwise regression, ridge regression, lasso, bridge regression, elastic net and an average of those models. In all real estate markets, the actual price diverges from the respective fundamental value. The right-side unit root tests showed significant evidence of the presence of periodically collapsing bubbles in all indices. The regime switching model for bubbles was compared to alternative models and the results showed that for the United States and the United Kingdom equity and residential real estate, the bubble model is preferable to the alternatives. The out-of-sample analysis reveals that for one period ahead, the van Norden and Schaller model exhibits superior forecasting performance for residential real estate markets in the United Kingdom and the United States.

Chapter 2 Tables

Table 2.1: Economic Predictors

	United States	United Kingdom
Financial indicators	• S&P 500 Index • US/UK exchange rate • M2 • Effective federal funds rate • 3-month Treasury bill: (secondary market rate) • 5-year Treasury constant maturity rate • 10-year Treasury constant maturity rate • 30-year fixed rate mortgage average	• FTSE All-Share Index • US/UK exchange rate • Retail M4 (or M2) • Official bank rate • 3-month Treasury bill • Generic government 5-year yield • Generic government 10-year yield • Mortgage rate
Price indicators	• Inflation rate (CPI) • London Bullion Market Association (LBMA) gold price • WTI crude oil price • US rent price index	• Inflation rate (RPI) • London Bullion Market Association (LBMA) gold price • IMF Brent crude oil price • UK rent price index
National income and business activity indicators	• Real GDP • Dallas Fed US real personal disposable income index • Industrial production • Housing starts	• Real GDP • Dallas Fed UK real personal disposable income index • Industrial production • Housing starts
Employment and labour market indicators	• Unemployment rate • OECD Labour cost • OECD Labour productivity	• Unemployment rate • OECD Labour cost • OECD Labour productivity

Notes: This table reports the set of predictors used to construct the fundamental models for the US and UK real estate indices.

Table 2.2: The SADF and GSADF Test Results on the Non-fundamental Component

	NCREIF Property Index		US EPRA/NAREIT Index		S&P/Case-Shiller Index	
	SADF	GSADF	SADF	GSADF	SADF	GSADF
Test statistic	5.3090	6.4302	2.9105	2.9986	2.7291	6.9542
90% Critical Value	**1.0845**	**1.7792**	**1.1439**	**1.9219**	**1.1442**	**1.9424**
95% Critical Value	**1.3843**	**2.0663**	**1.4257**	**2.1340**	**1.4350**	**2.1843**
99% Critical Value	**1.9300**	**2.7919**	**1.9585**	**2.6837**	**1.9417**	**2.8751**
	IPD UK Property Index		UK EPRA/NAREIT Index		UK House Price Index	
	SADF	GSADF	SADF	GSADF	SADF	GSADF
Test statistic	3.1161	4.5725	1.2414	2.9815	3.9483	7.0706
90% Critical Value	**1.1471**	**1.9431**	**1.1416**	**1.9179**	**1.1664**	**1.9113**
95% Critical Value	**1.4620**	**2.1877**	1.4027	**2.1460**	**1.4760**	**2.1935**
99% Critical Value	**2.0303**	**2.7542**	2.0189	**2.8789**	**2.0308**	**2.8802**

Notes: The null hypothesis is that there is a unit root and the alternative that there is explosive behavior. Figures in bold indicate the rejection of the null hypothesis at the respective significance level. The critical values for the SADF and GSADF tests were computed from Monte Carlo simulations with 2000 replications, with the minimum window set to $0.01+1.8/\sqrt{T}$ of the total sample observations. The ADF lag is chosen to minimize the Schwarz Information Criterion with the maximum lag length set to 4 quarters for the NCREIF Property Index and to 12 months for the remaining indices. Sample size: 151 for the NCREIF Property Index, 311 for the US FTSE EPRA/NAREIT Index, 346 for the S&P/Case-Shiller Home Price Index, 348 for the IPD UK Property Index, 312 for the UK FTSE EPRA/NAREIT Index and 251 for the UK House Price Index.

Table 2.3: Results from the van Norden and Schaller Speculative Bubble Model for the US and the UK Commercial Real Estate Indices

	NCREIF Property Index							IPD UK Property Index						
Parameters	OLS	Stepwise	Ridge	Lasso	Bridge	Elastic Net	Average	OLS	Stepwise	Ridge	Lasso	Bridge	Elastic Net	Average
β_{S0}	1.0188	1.0185	1.0181	1.0182	1.0167	1.0165	1.0175	1.0069	1.0070	1.0069	1.0069	1.0068	1.0072	1.0070
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_{S1}	0.0089	0.0070	0.0027	0.0051	-0.0056	-0.0110	-0.0003	-0.0026	-0.0029	0.0031	0.0052	0.0121	0.0148	0.0097
	(0.3844)	(0.4695)	(0.7632)	(0.5839)	(0.4589)	(0.3842)	(0.9800)	(0.7355)	(0.6625)	(0.6648)	(0.4548)	(0.0002)	(0.0043)	(0.0962)
β_{C0}	0.9459	0.9449	0.9393	0.9433	0.9962	0.9340	0.9390	0.9982	0.9973	0.9981	0.9981	0.9986	0.9977	0.9980
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_{C1}	-0.1604	-0.1622	-0.1611	-0.1586	0.0510	-0.1746	-0.1668	0.0321	0.0411	0.0136	0.0135	-0.0375	-0.0288	-0.0127
	(0.0005)	(0.0003)	(0.0000)	(0.0002)	(0.5702)	(0.0000)	(0.0000)	(0.2424)	(0.1734)	(0.5998)	(0.6000)	(0.0145)	(0.1523)	(0.5815)
β_{q0}	-2.8408	-2.8196	-2.9355	-2.8555	-1.0180	-3.4508	-3.0083	-1.4803	1.1517	-1.4112	1.3888	1.2678	-1.2781	-1.2584
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.2767)	(0.0001)	(0.0000)	(0.0003)	(0.0051)	(0.0004)	(0.0006)	(0.0017)	(0.0007)	(0.0012)
β_{q1}	3.4307	2.9801	3.8308	2.8063	-5.4348	6.5097	3.5989	7.2884	-0.9526	6.1666	-5.4300	-1.8742	3.1424	3.0181
	(0.4451)	(0.4902)	(0.2954)	(0.5233)	(0.2249)	(0.2046)	(0.4573)	(0.0842)	(0.8222)	(0.1166)	(0.1553)	(0.4339)	(0.3184)	(0.3781)
σ_S	0.0150	0.0151	0.0151	0.0151	0.0140	0.0154	0.0152	0.0062	0.0061	0.0061	0.0061	0.0058	0.0059	0.0060
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
σ_C	0.0202	0.0196	0.0184	0.0194	0.0360	0.0124	0.0174	0.0173	0.0171	0.0174	0.0174	0.0162	0.0169	0.0173
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Tests of coefficient restrictions														
$\beta_{S0}\neq\beta_{C0}$	37.0124	41.8808	51.0395	44.0316	1.8871	125.0816	56.3834	10.6412	11.6538	10.5023	10.3259	10.4075	12.3070	10.7871
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.1695)	(0.0000)	(0.0000)	(0.0011)	(0.0006)	(0.0012)	(0.0013)	(0.0013)	(0.0005)	(0.0010)
$\beta_{S1}\neq\beta_{C1}$	12.5608	13.4290	20.0220	13.3886	0.3955	45.5019	19.1911	1.3235	1.8596	0.1340	0.0856	10.1999	4.3316	0.8217
	(0.0004)	(0.0002)	(0.0000)	(0.0003)	(0.5294)	(0.0000)	(0.0000)	(0.2500)	(0.1727)	(0.7143)	(0.7698)	(0.0014)	(0.0374)	(0.3647)
Bubble model specification test against alternative models														
Volatility regime	6.0245	5.8360	5.9050	5.3145	3.4929	5.9956	5.1383	13.1066	10.1463	11.1716	11.1679	28.2891	18.4964	11.3828
	(0.1973)	(0.2117)	(0.2064)	(0.2565)	(0.4790)	(0.1995)	(0.2734)	(0.0108)	(0.0380)	(0.0247)	(0.0247)	(0.0000)	(0.0010)	(0.0226)
Fads	10.2856	10.4791	11.1973	10.2752	8.0119	11.3421	10.6333	20.6783	17.6434	18.4485	17.7409	29.6868	21.4947	17.0178
	(0.0163)	(0.0149)	(0.0107)	(0.0164)	(0.0458)	(0.0100)	(0.0139)	(0.0001)	(0.0005)	(0.0004)	(0.0005)	(0.0000)	(0.0001)	(0.0007)
Mixture-normal	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	12.8417	3.0491	0.0100
	(0.9997)	(0.9997)	(0.9997)	(0.9997)	(0.9997)	(0.9997)	(0.9997)	(0.9997)	(0.9997)	(0.9997)	(0.9997)	(0.0050)	(0.3841)	(0.9997)

Notes: This table reports the in-sample results of the van Norden and Schaller model for each fundamental specification. The first panel reports the coefficient estimates of the bubble model along with the respective p-values in parenthesis that are derived by taking the inverse of the Hessian matrix. The second panel reports the results of the likelihood ratio tests of the two restrictions implied by the bubble model, while the third panel presents the results from tests that examine whether stylized alternative models can better explain the returns than the regime-switching model for bubbles.

Table 2.4: Results from the van Norden and Schaller Speculative Bubble Model for the US and the UK Equity Real Estate Indices

	US FTSE EPRA/NAREIT Index							UK FTSE EPRA/NAREIT Index						
Parameters	OLS	Stepwise	Ridge	Lasso	Bridge	Elastic Net	Average	OLS	Stepwise	Ridge	Lasso	Bridge	Elastic Net	Average
β_{S0}	1.0064	1.0073	1.0051	1.0039	1.0026	1.0034	1.0040	0.9997	0.9998	0.9983	0.9969	0.9896	0.9971	0.9971
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_{S1}	-0.0049	-0.0018	-0.0073	-0.0114	-0.0177	-0.0128	-0.0115	-0.0048	-0.0048	-0.0064	-0.0081	-0.0191	-0.0083	-0.0085
	(0.6552)	(0.8518)	(0.4771)	(0.2826)	(0.0467)	(0.2264)	(0.2977)	(0.5504)	(0.5405)	(0.4163)	(0.3437)	(0.0222)	(0.3264)	(0.3287)
β_{C0}	0.8892	0.9129	0.8002	0.7968	0.8100	0.7900	0.8102	0.8582	0.8887	0.7326	0.7954	0.7540	0.7782	0.7407
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0014)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_{C1}	-0.2197	-0.2032	-0.2710	-0.2910	-0.1464	-0.2864	-0.2705	-0.1708	-0.1462	-0.2616	-0.1990	-0.1418	-0.2202	-0.2430
	(0.1113)	(0.1414)	(0.0333)	(0.0407)	(0.0411)	(0.0240)	(0.0283)	(0.4181)	(0.3583)	(0.2344)	(0.1370)	(0.0078)	(0.1518)	(0.0984)
β_{q0}	3.0088	2.5812	-4.2995	-4.3159	-6.6387	-4.6459	-4.4448	4.0527	3.5410	5.2037	4.5707	-7.2487	4.7347	5.2640
	(0.0000)	(0.0000)	(0.0000)	(0.0001)	(0.0009)	(0.0001)	(0.0000)	(0.0201)	(0.0112)	(0.0014)	(0.0010)	(0.0000)	(0.0011)	(0.0002)
β_{q1}	-1.8062	-0.5839	3.6171	3.7348	6.0346	4.2569	4.1916	-2.1116	-1.5229	-3.0491	-2.6715	4.3614	-2.7937	-3.3972
	(0.1128)	(0.5938)	(0.0038)	(0.0077)	(0.0095)	(0.0047)	(0.0050)	(0.1663)	(0.2075)	(0.0473)	(0.0341)	(0.0006)	(0.0367)	(0.0163)
σ_S	0.0407	0.0407	0.0418	0.0420	0.0427	0.0421	0.0419	0.0503	0.0499	0.0506	0.0493	0.0477	0.0498	0.0497
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
σ_C	0.1247	0.1274	0.1196	0.1184	0.1355	0.1189	0.1199	0.1097	0.1091	0.1012	0.0934	0.0874	0.0956	0.0916
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Tests of coefficient restrictions														
$\beta_{S0}\neq\beta_{C0}$	2.6195	2.8041	3.9578	3.2508	3.1973	3.9697	3.7989	0.6008	0.8302	1.3514	2.1014	5.8611	1.9121	2.5970
	(0.1056)	(0.0940)	(0.0467)	(0.0714)	(0.0738)	(0.0463)	(0.0513)	(0.4383)	(0.3622)	(0.2450)	(0.1472)	(0.0155)	(0.1667)	(0.1071)
$\beta_{S1}\neq\beta_{C1}$	2.4327	2.1340	4.3270	3.9251	3.2147	4.7053	4.4589	0.6213	0.7927	1.3455	2.0333	5.0551	1.9103	2.5266
	(0.1188)	(0.1441)	(0.0375)	(0.0476)	(0.0730)	(0.0301)	(0.0347)	(0.4306)	(0.3733)	(0.2461)	(0.1539)	(0.0246)	(0.1669)	(0.1119)
Bubble model specification test against alternative models														
Volatility regime	4.5484	1.8080	11.6828	12.5982	30.1942	15.3703	14.7110	3.2372	2.9189	5.8300	8.4368	25.6241	7.6889	10.1248
	(0.3368)	(0.7710)	(0.0199)	(0.0134)	(0.0000)	(0.0040)	(0.0053)	(0.5189)	(0.5715)	(0.2122)	(0.0768)	(0.0000)	(0.1037)	(0.0384)
Fads	7.1879	5.0067	13.5061	13.6537	26.5998	15.8503	15.4786	4.0729	3.6249	6.2864	7.8439	22.9221	7.3379	9.8032
	(0.0661)	(0.1713)	(0.0037)	(0.0034)	(0.0000)	(0.0012)	(0.0015)	(0.2537)	(0.3049)	(0.0985)	(0.0494)	(0.0000)	(0.0619)	(0.0203)
Mixture-normal	3.5658	0.8254	10.7002	11.6156	29.2116	14.3877	13.7284	1.4639	1.1456	4.0567	6.6635	23.8508	5.9156	8.3515
	(0.3123)	(0.8434)	(0.0135)	(0.0088)	(0.0000)	(0.0024)	(0.0033)	(0.6906)	(0.7661)	(0.2554)	(0.0834)	(0.0000)	(0.1158)	(0.0393)

Notes: This table reports the in-sample results of the van Norden and Schaller model for each fundamental specification. The first panel reports the coefficient estimates of the bubble model along with the respective p-values in parenthesis that are derived by taking the inverse of the Hessian matrix. The second panel reports the results of the likelihood ratio tests of the two restrictions implied by the bubble model, while the third panel presents the results from tests that examine whether stylized alternative models can better explain the returns than the regime-switching model for bubbles.

Table 2.5: Results from the van Norden and Schaller Speculative Bubble Model for the US and the UK Residential Real Estate Indices

	S&P/Case-Shiller Home Price Index							UK House Price Index						
Parameters	OLS	Stepwise	Ridge	Lasso	Bridge	Elastic Net	Average	OLS	Stepwise	Ridge	Lasso	Bridge	Elastic Net	Average
β_{S0}	1.0042	1.0042	1.0042	1.0042	1.0038	1.0039	1.0044	1.0051	1.0055	1.0052	1.0053	1.0059	1.0056	1.0057
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_{S1}	0.0106	0.0113	0.0118	0.0112	0.0166	0.0149	0.0139	-0.0132	-0.0125	-0.0077	-0.0085	-0.0035	-0.0046	-0.0078
	(0.0021)	(0.0013)	(0.0003)	(0.0009)	(0.0000)	(0.0000)	(0.0000)	(0.0379)	(0.0686)	(0.2019)	(0.1813)	(0.4841)	(0.4002)	(0.2403)
β_{C0}	0.9942	0.9942	0.9949	0.9944	0.9967	0.9960	0.9956	0.9991	0.9991	0.9996	0.9996	0.9975	0.9985	0.9994
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_{C1}	-0.0346	-0.0349	-0.0318	-0.0322	-0.0228	-0.0243	-0.0275	-0.1268	-0.1295	-0.1294	-0.1338	-0.0773	-0.1013	-0.1140
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_{q0}	-0.5549	-0.6088	-0.3381	-0.5343	-0.4052	-0.4632	-0.0426	-0.7556	-0.1297	0.0777	0.0652	0.2084	0.0178	0.2309
	(0.2096)	(0.1710)	(0.4430)	(0.2403)	(0.4049)	(0.3317)	(0.9331)	(0.2728)	(0.8636)	(0.9043)	(0.9276)	(0.7623)	(0.9793)	(0.7339)
β_{q1}	2.6717	2.7929	-0.1604	1.6158	0.0758	0.1247	-2.3939	-12.1367	-14.9164	14.9327	15.2631	9.2006	12.1565	-14.7843
	(0.5244)	(0.5138)	(0.9655)	(0.6969)	(0.9775)	(0.9651)	(0.5039)	(0.0152)	(0.0051)	(0.0011)	(0.0015)	(0.0036)	(0.0017)	(0.0005)
σ_S	0.0042	0.0042	0.0041	0.0042	0.0039	0.0040	0.0040	0.0091	0.0088	0.0089	0.0089	0.0088	0.0089	0.0090
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
σ_C	0.0042	0.0044	0.0046	0.0045	0.0056	0.0054	0.0050	0.0027	0.0045	0.0042	0.0042	0.0047	0.0049	0.0049
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Tests of coefficient restrictions														
$\beta_{S0} \neq \beta_{C0}$	218.7708	190.0803	156.4087	170.2617	48.2574	64.2196	92.9463	21.0044	13.4726	10.7906	10.1537	16.5139	12.3667	9.6589
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0002)	(0.0010)	(0.0014)	(0.0000)	(0.0004)	(0.0019)
$\beta_{S1} \neq \beta_{C1}$	76.9262	68.2720	64.0140	64.0555	50.8936	49.7649	53.3688	75.9723	25.7643	31.7482	26.4138	31.5721	29.3276	22.4570
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Bubble model specification test against alternative models														
Volatility regime	51.8753	47.9361	48.5529	45.4679	46.3767	44.5277	45.4770	21.7529	25.6001	25.1875	25.8914	18.5668	21.3688	25.3115
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0002)	(0.0000)	(0.0000)	(0.0000)	(0.0010)	(0.0003)	(0.0000)
Fads	58.6704	55.5599	56.1470	53.1805	60.1836	53.0791	53.7911	17.6257	21.5197	24.6443	22.5199	23.8871	25.3749	26.8535
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0005)	(0.0001)	(0.0000)	(0.0001)	(0.0000)	(0.0000)	(0.0000)
Mixture-normal	38.0458	34.1067	34.7235	31.6385	32.5473	30.6983	31.6476	10.7870	14.6343	14.2216	14.9256	7.6009	10.4029	14.3457
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0129)	(0.0022)	(0.0026)	(0.0019)	(0.0550)	(0.0154)	(0.0025)

Notes: This table reports the in-sample results of the van Norden and Schaller model for each fundamental specification. The first panel reports the coefficient estimates of the bubble model along with the respective p-values in parenthesis that are derived by taking the inverse of the Hessian matrix. The second panel reports the results of the likelihood ratio tests of the two restrictions implied by the bubble model, while the third panel presents the results from tests that examine whether stylized alternative models can better explain the returns than the regime-switching model for bubbles.

Table 2.6: MSFE Ratios and Clark and West (2007) t-statistics for the IPD UK Property Index: 1 Month Horizon

A. Historical average set as the benchmark.

	IPD UK Property Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
Historical Average	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
Volatility regimes model	1.0462	1.0462	1.0462	1.0462	1.0462	1.0462	1.0462
	(-1.2571)	(-1.2571)	(-1.2571)	(-1.2571)	(-1.2571)	(-1.2571)	(-1.2571)
Fads model	1.0321	0.9992	1.0324	1.0202	1.0493	1.0698	1.0319
	(-1.7195)	(0.5254)	(-1.9392)	(-1.5285)	(-2.6045)	(-3.2751)	(-2.4887)
Normal-mixture model	**0.7606**	**0.7606**	**0.7606**	**0.7606**	**0.7606**	**0.7606**	**0.7606**
	(4.1167)	(4.1167)	(4.1167)	(4.1167)	(4.1167)	(4.1167)	(4.1167)
vNS bubble model	**0.7984**	**0.7991**	**0.8382**	**0.8173**	**0.8401**	**0.7898**	**0.8672**
	(3.6898)	(3.5236)	(3.5104)	(3.4117)	(3.9675)	(4.2887)	(3.6808)

B. Van Norden and Schaller model set as the benchmark.

	IPD UK Property Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
vNS bubble model	0.0001	0.0001	0.0002	0.0002	0.0002	0.0001	0.0002
Volatility regimes model	**0.7632**	**0.7638**	**0.8012**	**0.7812**	**0.8031**	**0.7549**	**0.8289**
	(3.4830)	(3.4269)	(3.2986)	(3.3190)	(3.5447)	(3.9338)	(3.3448)
Fads model	**0.7736**	**0.7997**	**0.8119**	**0.8010**	**0.8007**	**0.7383**	**0.8403**
	(3.5773)	(3.4211)	(3.4100)	(3.3577)	(3.7697)	(4.0934)	(3.5798)
Normal-mixture model	1.0497	1.0506	1.1020	1.0744	1.1045	1.0383	1.1401
	(-0.7878)	(-0.7175)	(-2.3504)	(-1.2532)	(-2.2685)	(-0.1573)	(-2.8420)

Notes: The first panel reports the MSFE ratios between the historical average benchmark and the volatility regimes, fads, mixture-normal and the vNS bubble models respectively. A below unity ratio indicates that the respective model outperforms the historical average. The second panel reports the MSFE ratios between the van Norden and Schaller model and the alternative regime switching models. A below unity ratio indicates that the bubble model outperforms the respective regime switching model. The figures in parenthesis are the t-statistics from the Clark and West (2007) test. Figures in bold indicate the rejection of the null hypothesis at the 10% significance level. The out-of-sample period is set to eight years.

Table 2.7: MSFE Ratios and Clark and West (2007) t-statistics for the Equity Real Estate Indices: 1 Month Horizon

A. Historical average set as the benchmark.

	US FTSE EPRA/NAREIT Index							UK FTSE EPRA/NAREIT Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
Historical Average	0.0062	0.0062	0.0062	0.0062	0.0062	0.0062	0.0062	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044
Volatility regimes model	1.0014	1.0014	1.0014	1.0014	1.0014	1.0014	1.0014	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
	(-0.0590)	(-0.0590)	(-0.0590)	(-0.0590)	(-0.0590)	(-0.0590)	(-0.0590)	(0.1402)	(0.1402)	(0.1402)	(0.1402)	(0.1402)	(0.1402)	(0.1402)
Fads model	0.9969	0.9953	0.9872	0.9879	**0.9846**	**0.9557**	**0.9815**	1.0015	0.9992	1.0032	**0.9862**	**0.9883**	**0.9495**	**0.9917**
	(0.5345)	(0.6754)	(1.2187)	(1.2597)	(1.3823)	(1.9051)	(1.5898)	(-0.1741)	(0.3484)	(-0.5220)	(2.0726)	(1.6339)	(2.8627)	(2.0140)
Normal-mixture model	1.0239	1.0239	1.0239	1.0239	1.0239	1.0239	1.0239	**0.9178**	**0.9178**	**0.9178**	**0.9178**	**0.9178**	**0.9178**	**0.9178**
	(0.0182)	(0.0182)	(0.0182)	(0.0182)	(0.0182)	(0.0182)	(0.0182)	(1.3715)	(1.3715)	(1.3715)	(1.3715)	(1.3715)	(1.3715)	(1.3715)
vNS bubble model	0.9317	**0.9176**	0.9249	0.9291	0.9169	0.9426	0.9069	**0.9825**	**0.9390**	**0.9814**	1.0022	**0.9931**	**0.9751**	**0.9669**
	(1.2486)	(1.2919)	(1.0732)	(1.0534)	(1.1150)	(1.2132)	(1.1902)	(1.5844)	(1.4917)	(1.6810)	(0.2171)	(2.0022)	(2.0128)	(1.4041)

B. Van Norden and Schaller model set as the benchmark.

	US FTSE EPRA/NAREIT Index							UK FTSE EPRA/NAREIT Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
vNS bubble model	0.0058	0.0057	0.0058	0.0058	0.0057	0.0059	0.0056	0.0044	0.0042	0.0043	0.0044	0.0044	0.0043	0.0043
Volatility regimes model	0.9305	**0.9164**	0.9236	0.9278	0.9156	0.9413	0.9057	**0.9827**	**0.9392**	**0.9816**	1.0024	**0.9933**	**0.9753**	**0.9671**
	(1.2705)	(1.3299)	(1.1204)	(1.0976)	(1.1547)	(1.2088)	(1.2334)	(1.4787)	(1.4559)	(1.6237)	(0.2033)	(1.7987)	(2.0614)	(1.3528)
Fads model	0.9346	**0.9220**	0.9369	0.9404	0.9312	0.9864	0.9241	**0.9809**	**0.9397**	**0.9782**	1.0163	1.0049	1.0269	0.9750
	(1.2320)	(1.2828)	(1.0307)	(0.9984)	(1.0619)	(0.8839)	(1.1217)	(1.4027)	(1.4737)	(1.7131)	(-1.5630)	(0.1118)	(-1.5260)	(1.1540)
Normal-mixture model	0.9100	0.8963	0.9033	0.9074	0.8955	0.9206	0.8858	1.0705	1.0231	1.0693	1.0920	1.0821	**1.0624**	1.0535
	(1.1370)	(1.1449)	(0.9934)	(0.9948)	(1.0476)	(1.1165)	(1.0820)	(1.1895)	(1.2049)	(1.2184)	(1.1077)	(1.1574)	(1.2953)	(1.1594)

Notes: The first panel reports the MSFE ratios between the historical average benchmark and the volatility regimes, fads, mixture-normal and the vNS bubble models respectively. A below unity ratio indicates that the respective model outperforms the historical average. The second panel reports the MSFE ratios between the van Norden and Schaller model and the alternative regime switching models. A below unity ratio indicates that the bubble model outperforms the respective regime switching model. The figures in parenthesis are the t-statistics from the Clark and West (2007) test. Figures in bold indicate the rejection of the null hypothesis at the 10% significance level. The out-of-sample period is set to eight years.

Table 2.8: MSFE Ratios and Clark and West (2007) t-statistics for the Residential Real Estate Indices: 1 Month Horizon

A. Historical average set as the benchmark.

	S&P/Case-Shiller Home Price Index							UK House Price Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
Historical Average	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Volatility regimes model	1.0205	1.0205	1.0205	1.0205	1.0205	1.0205	1.0205	**0.9961**	**0.9961**	**0.9961**	**0.9961**	**0.9961**	**0.9961**	**0.9961**
	(-2.2443)	(-2.2443)	(-2.2443)	(-2.2443)	(-2.2443)	(-2.2443)	(-2.2443)	(2.2354)	(2.2354)	(2.2354)	(2.2354)	(2.2354)	(2.2354)	(2.2354)
Fads model	1.0294	1.0215	1.0184	1.0184	1.0445	1.0569	1.0328	1.0156	1.0268	1.0311	1.0347	1.0129	1.0172	1.0387
	(-3.4020)	(-2.5539)	(-2.4455)	(-2.4782)	(-3.1169)	(-3.4699)	(-3.0841)	(-0.5857)	(-1.2351)	(-1.9521)	(-2.2302)	(-1.2342)	(-1.7342)	(-2.4942)
Normal-mixture model	**0.8184**	**0.8184**	**0.8184**	**0.8184**	**0.8184**	**0.8184**	**0.8184**	1.0050	1.0050	1.0050	1.0050	1.0050	1.0050	1.0050
	(3.3062)	(3.3062)	(3.3062)	(3.3062)	(3.3062)	(3.3062)	(3.3062)	(-0.9123)	(-0.9123)	(-0.9123)	(-0.9123)	(-0.9123)	(-0.9123)	(-0.9123)
vNS bubble model	**0.6424**	**0.6409**	**0.6176**	**0.6354**	**0.6423**	**0.6626**	**0.6079**	**0.7416**	**0.7503**	**0.7312**	**0.6852**	**0.8540**	**0.8351**	**0.6958**
	(5.3701)	(5.4697)	(5.3216)	(5.3637)	(4.9644)	(4.6450)	(5.4104)	(4.2838)	(3.8842)	(4.0720)	(4.5234)	(2.8027)	(2.6092)	(3.8015)

B. Van Norden and Schaller model set as the benchmark.

	S&P/Case-Shiller Home Price Index							UK House Price Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
vNS bubble model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Volatility regimes model	**0.6295**	**0.6280**	**0.6052**	**0.6226**	**0.6294**	**0.6493**	**0.5957**	**0.7445**	**0.7532**	**0.7341**	**0.6879**	**0.8574**	**0.8384**	**0.6986**
	(5.4441)	(5.5419)	(5.3984)	(5.4359)	(5.0473)	(4.7386)	(5.4824)	(4.2610)	(3.8628)	(4.0459)	(4.5029)	(2.7834)	(2.5843)	(3.7871)
Fads model	**0.6241**	**0.6274**	**0.6065**	**0.6239**	**0.6149**	**0.6269**	**0.5885**	**0.7302**	**0.7306**	**0.7091**	**0.6623**	**0.8431**	**0.8210**	**0.6699**
	(5.3565)	(5.3800)	(5.3239)	(5.3571)	(5.1148)	(4.9509)	(5.4607)	(4.4499)	(4.2550)	(4.2805)	(4.7128)	(2.9297)	(2.7303)	(3.9995)
Normal-mixture model	**0.7850**	**0.7831**	**0.7547**	**0.7764**	**0.7848**	**0.8097**	**0.7428**	**0.7378**	**0.7465**	**0.7275**	**0.6818**	**0.8498**	**0.8309**	**0.6923**
	(4.6070)	(4.6536)	(4.6515)	(4.6372)	(4.0624)	(3.7924)	(4.5175)	(4.2777)	(3.8755)	(4.0480)	(4.4926)	(2.8539)	(2.6678)	(3.8103)

Notes: The first panel reports the MSFE ratios between the historical average benchmark and the volatility regimes, fads, mixture-normal and the vNS bubble models respectively. A below unity ratio indicates that the respective model outperforms the historical average. The second panel reports the MSFE ratios between the van Norden and Schaller model and the alternative regime switching models. A below unity ratio indicates that the bubble model outperforms the respective regime switching model. The figures in parenthesis are the t-statistics from the Clark and West (2007) test. Figures in bold indicate the rejection of the null hypothesis at the 10% significance level. The out-of-sample period is set to eight years.

Table 2.9: MSFE Ratios and Clark and West (2007) t-statistics, with the Historical Average set as the Benchmark, for all Real Estate Indices: 3 Month Horizon

	NCREIF Property Index							IPD UK Property Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
Historical Average	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014
Volatility regimes	0.9930	0.9930	0.9930	0.9930	0.9930	0.9930	0.9930	1.0635	1.0635	1.0635	1.0635	1.0635	1.0635	1.0635
model	(0.5526)	(0.5526)	(0.5526)	(0.5526)	(0.5526)	(0.5526)	(0.5526)	(-1.2038)	(-1.2038)	(-1.2038)	(-1.2038)	(-1.2038)	(-1.2038)	(-1.2038)
Fads model	1.0264	1.0165	0.9966	1.0059	0.9845	0.9811	0.9993	1.0641	1.0488	1.0614	1.0581	1.0955	1.1330	1.0701
	(-0.3252)	(-0.1479)	(0.3920)	(0.0616)	(1.1374)	(1.2366)	(0.2823)	(-1.3750)	(-1.5190)	(-1.7616)	(-1.8475)	(-2.8762)	(-3.4383)	(-2.6131)
Normal-mixture	**0.9175**	**0.9175**	**0.9175**	**0.9175**	**0.9175**	**0.9175**	**0.9175**	**0.7675**	**0.7675**	**0.7675**	**0.7675**	**0.7675**	**0.7675**	**0.7675**
model	(1.5778)	(1.5778)	(1.5778)	(1.5778)	(1.5778)	(1.5778)	(1.5778)	(3.4933)	(3.4933)	(3.4933)	(3.4933)	(3.4933)	(3.4933)	(3.4933)
vNS bubble model	**0.7414**	**0.7797**	**0.7674**	**0.7657**	**0.9209**	**0.7181**	**0.7259**	**0.6733**	**0.7420**	**0.7569**	**0.6983**	**0.7132**	**0.6347**	**0.7354**
	(1.7668)	(1.7229)	(1.7421)	(1.7539)	(1.6654)	(1.7507)	(1.8234)	(4.1215)	(4.4968)	(4.1575)	(3.7781)	(4.1421)	(4.3549)	(4.0974)

	US FTSE EPRA/NAREIT Index							UK FTSE EPRA/NAREIT Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
Historical Average	0.0160	0.0160	0.0160	0.0160	0.0160	0.0160	0.0160	0.0139	0.0139	0.0139	0.0139	0.0139	0.0139	0.0139
Volatility regimes	0.9974	0.9974	0.9974	0.9974	0.9974	0.9974	0.9974	0.9927	0.9927	0.9927	0.9927	0.9927	0.9927	0.9927
model	(0.4594)	(0.4594)	(0.4594)	(0.4594)	(0.4594)	(0.4594)	(0.4594)	(1.2321)	(1.2321)	(1.2321)	(1.2321)	(1.2321)	(1.2321)	(1.2321)
Fads model	**0.9834**	**0.9766**	**0.9605**	**0.9472**	**0.9349**	**0.9476**	**0.9424**	**0.9597**	**0.9514**	**0.9648**	**0.9436**	**0.9428**	**0.8378**	**0.9683**
	(1.4476)	(1.9775)	(2.3340)	(2.8474)	(2.7515)	(2.6859)	(2.6620)	(2.8172)	(2.9397)	(2.0129)	(2.8863)	(2.7143)	(4.0156)	(1.8745)
Normal-mixture	1.1394	1.1394	1.1394	1.1394	1.1394	1.1394	1.1394	**0.9802**	**0.9802**	**0.9802**	**0.9802**	**0.9802**	**0.9802**	**0.9802**
model	(-0.3977)	(-0.3977)	(-0.3977)	(-0.3977)	(-0.3977)	(-0.3977)	(-0.3977)	(3.8694)	(3.8694)	(3.8694)	(3.8694)	(3.8694)	(3.8694)	(3.8694)
vNS bubble model	**1.0052**	1.0573	1.0244	1.0126	0.9886	1.2666	0.9820	**0.9806**	1.0320	1.1126	**0.9104**	**0.9365**	**0.8758**	**0.9023**
	(1.2847)	(1.0491)	(1.2778)	(1.2560)	(1.4228)	(1.1984)	(1.4384)	(1.4512)	(1.0737)	(0.4425)	(2.0172)	(3.9340)	(2.6308)	(3.2333)

	S&P/Case-Shiller Home Price Index							UK House Price Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
Historical Average	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008
Volatility regimes	1.0343	1.0343	1.0343	1.0343	1.0343	1.0343	1.0343	1.0319	1.0319	1.0319	1.0319	1.0319	1.0319	1.0319
model	(-3.1533)	(-3.1533)	(-3.1533)	(-3.1533)	(-3.1533)	(-3.1533)	(-3.1533)	(-3.3564)	(-3.3564)	(-3.3564)	(-3.3564)	(-3.3564)	(-3.3564)	(-3.3564)
Fads model	1.0303	1.0241	1.0156	1.0135	1.0566	1.0884	1.0307	1.0239	1.0363	1.0253	1.0407	1.0219	1.0193	1.0224
	(-1.8896)	(-1.4370)	(-1.1528)	(-0.8687)	(-3.0618)	(-3.6629)	(-1.9311)	(-1.4583)	(-2.1629)	(-1.6776)	(-2.9089)	(-1.7182)	(-1.9625)	(-1.5865)
Normal-mixture	**0.7397**	**0.7397**	**0.7397**	**0.7397**	**0.7397**	**0.7397**	**0.7397**	1.0046	1.0046	1.0046	1.0046	1.0046	1.0046	1.0046
model	(4.1302)	(4.1302)	(4.1302)	(4.1302)	(4.1302)	(4.1302)	(4.1302)	(-2.0022)	(-2.0022)	(-2.0022)	(-2.0022)	(-2.0022)	(-2.0022)	(-2.0022)
vNS bubble model	**0.6025**	**0.6087**	**0.6048**	**0.6020**	**0.5989**	**0.6150**	**0.6077**	**0.7482**	**0.6175**	**0.6575**	**0.6761**	**0.9284**	**0.9996**	**0.7439**
	(5.6693)	(5.5950)	(5.3509)	(5.4391)	(5.2564)	(5.2443)	(5.3781)	(4.0465)	(4.4968)	(4.7331)	(4.6012)	(3.1015)	(2.4182)	(4.0023)

Notes: This table reports the MSFE ratios between the historical average benchmark and the volatility regimes, fads, mixture-normal and the vNS bubble models respectively. A below unity ratio indicates that the respective model outperforms the historical average. The figures in parenthesis are the t-statistics from the Clark and West (2007) test. Figures in bold indicate the rejection of the null hypothesis at the 10% significance level. The out-of-sample period is set to eight years.

Table 2.10: MSFE Ratios and Clark and West (2007) t-statistics, with the Historical Average set as the Benchmark, for all Real Estate Indices: 6 Month Horizon

	NCREIF Property Index							IPD UK Property Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
Historical Average	0.0033	0.0033	0.0033	0.0033	0.0033	0.0033	0.0033	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044
Volatility regimes	0.9927	0.9927	0.9927	0.9927	0.9927	0.9927	0.9927	1.0551	1.0551	1.0551	1.0551	1.0551	1.0551	1.0551
model	(0.5630)	(0.5630)	(0.5630)	(0.5630)	(0.5630)	(0.5630)	(0.5630)	(-0.7260)	(-0.7260)	(-0.7260)	(-0.7260)	(-0.7260)	(-0.7260)	(-0.7260)
Fads model	1.0268	1.0124	0.9989	1.0061	**0.9422**	**0.9374**	1.0000	1.0548	1.0414	1.0257	1.0452	1.0379	1.1543	1.0029
	(-0.0856)	(0.1530)	(0.4131)	(0.2148)	(2.2199)	(2.6104)	(0.3692)	(-0.0457)	(-0.2217)	(0.1048)	(-0.6090)	(-0.3326)	(-2.7337)	(1.0107)
Normal-mixture	**0.9609**	**0.9609**	**0.9609**	**0.9609**	**0.9609**	**0.9609**	**0.9609**	**0.9120**	**0.9120**	**0.9120**	**0.9120**	**0.9120**	**0.9120**	**0.9120**
model	(1.3390)	(1.3390)	(1.3390)	(1.3390)	(1.3390)	(1.3390)	(1.3390)	(2.2890)	(2.2890)	(2.2890)	(2.2890)	(2.2890)	(2.2890)	(2.2890)
vNS bubble model	1.0643	**0.9550**	**1.0985**	**0.9805**	**1.1076**	**1.1186**	**1.0502**	**0.7840**	**0.7815**	**0.7530**	**0.7546**	**0.7132**	**0.5689**	**0.7658**
	(1.0718)	(1.5859)	(1.5097)	(1.4848)	(1.7334)	(1.6191)	(1.6353)	(2.9586)	(3.5745)	(3.8847)	(4.0102)	(3.5106)	(3.9689)	(4.5953)

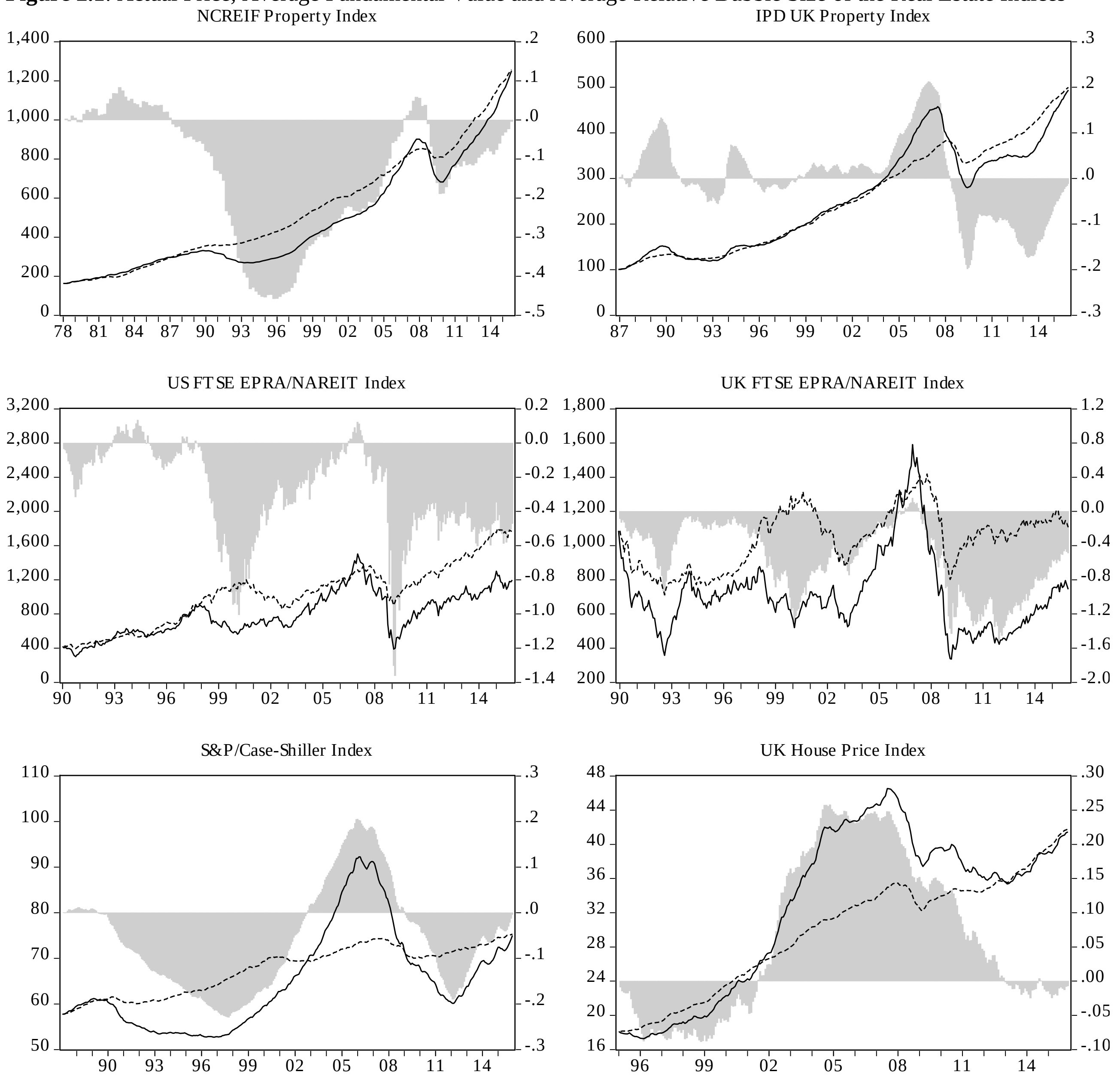
	US FTSE EPRA/NAREIT Index							UK FTSE EPRA/NAREIT Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
Historical Average	0.0372	0.0372	0.0372	0.0372	0.0372	0.0372	0.0372	0.0297	0.0297	0.0297	0.0297	0.0297	0.0297	0.0297
Volatility regimes	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003	1.0011	1.0011	1.0011	1.0011	1.0011	1.0011	1.0011
model	(0.0882)	(0.0882)	(0.0882)	(0.0882)	(0.0882)	(0.0882)	(0.0882)	(-0.1547)	(-0.1547)	(-0.1547)	(-0.1547)	(-0.1547)	(-0.1547)	(-0.1547)
Fads model	0.9907	**0.9885**	**0.9762**	**0.9724**	**0.9709**	**1.1553**	**0.9660**	**0.9722**	**0.9731**	**0.9719**	**0.9817**	**0.9677**	**0.9490**	**0.9896**
	(1.2491)	(1.5471)	(2.1842)	(2.3441)	(2.3337)	(2.7924)	(2.4995)	(2.5971)	(2.6878)	(2.4150)	(1.5209)	(2.3381)	(2.2633)	(1.8501)
Normal-mixture	1.3093	1.3093	1.3093	1.3093	1.3093	1.3093	1.3093	**0.9907**	**0.9907**	**0.9907**	**0.9907**	**0.9907**	**0.9907**	**0.9907**
model	(-1.5283)	(-1.5283)	(-1.5283)	(-1.5283)	(-1.5283)	(-1.5283)	(-1.5283)	(2.0716)	(2.0716)	(2.0716)	(2.0716)	(2.0716)	(2.0716)	(2.0716)
vNS bubble model	1.8717	1.7965	1.8768	1.7959	1.7922	1.8334	1.7534	1.0630	1.1848	1.0416	1.0188	1.0247	1.2123	1.1353
	(-1.1075)	(-0.9840)	(-0.9747)	(-0.6848)	(-0.8389)	(-0.5541)	(-0.8372)	(0.6969)	(0.5675)	(0.9367)	(0.6848)	(0.4449)	(0.5747)	(-0.7401)

	S&P/Case-Shiller Home Price Index							UK House Price Index						
	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average	OLS	Stepwise	Ridge	Lasso	Elastic Net	Bridge	Average
Historical Average	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020
Volatility regimes	1.0034	1.0034	1.0034	1.0034	1.0034	1.0034	1.0034	1.0366	1.0366	1.0366	1.0366	1.0366	1.0366	1.0366
model	(0.2716)	(0.2716)	(0.2716)	(0.2716)	(0.2716)	(0.2716)	(0.2716)	(-5.3407)	(-5.3407)	(-5.3407)	(-5.3407)	(-5.3407)	(-5.3407)	(-5.3407)
Fads model	**0.9180**	**0.9101**	**0.8980**	**0.9096**	**0.9059**	**0.9683**	**0.9039**	**0.9866**	0.9982	1.0084	1.0003	1.0172	1.0183	1.0124
	(5.4353)	(5.4666)	(7.1082)	(5.5138)	(4.6663)	(2.1240)	(5.5595)	(1.7551)	(0.6764)	(-0.2153)	(0.4340)	(-0.7367)	(-1.0012)	(-0.3375)
Normal-mixture	**0.7381**	**0.7381**	**0.7381**	**0.7381**	**0.7381**	**0.7381**	**0.7381**	1.3690	1.3690	1.3690	1.3690	1.3690	1.3690	1.3690
model	(4.2411)	(4.2411)	(4.2411)	(4.2411)	(4.2411)	(4.2411)	(4.2411)	(1.2160)	(1.2160)	(1.2160)	(1.2160)	(1.2160)	(1.2160)	(1.2160)
vNS bubble model	**0.5036**	**0.4951**	**0.5388**	**0.5136**	**0.5506**	**0.5545**	**0.5488**	**0.5977**	**0.7013**	**0.7736**	**0.8164**	**1.3968**	**1.3352**	**1.1736**
	(7.5880)	(7.1996)	(7.0621)	(7.2244)	(7.2499)	(7.2342)	(7.0663)	(4.2701)	(3.9288)	(3.5832)	(3.6796)	(2.1236)	(2.2183)	(2.9174)

Notes: This table reports the MSFE ratios between the historical average benchmark and the volatility regimes, fads, mixture-normal and the vNS bubble models respectively. A below unity ratio indicates that the respective model outperforms the historical average. The figures in parenthesis are the t-statistics from the Clark and West (2007) test. Figures in bold indicate the rejection of the null hypothesis at the 10% significance level. The out-of-sample period is set to eight years.

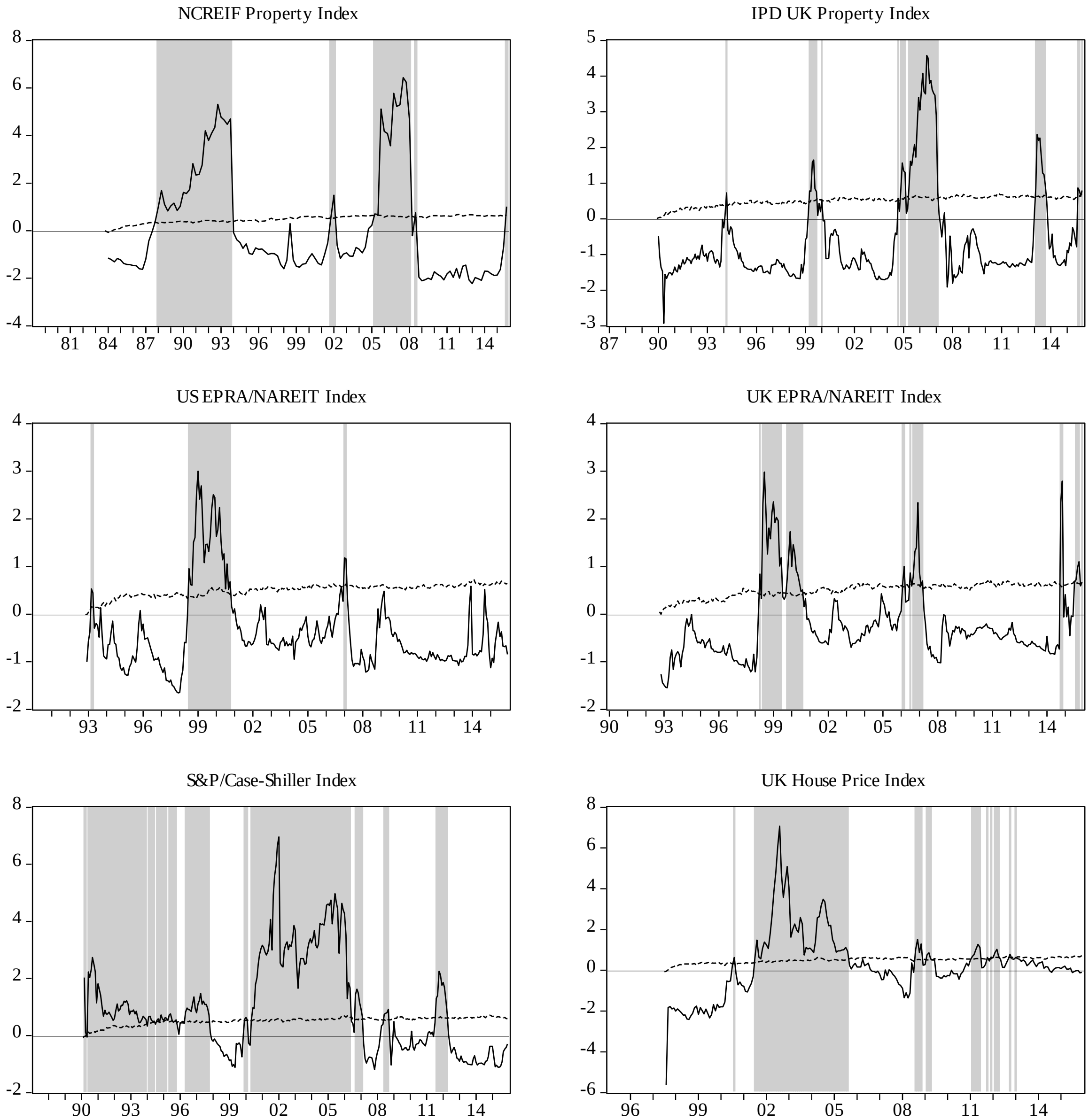
Chapter 2 Figures

Figure 2.1: Actual Price, Average Fundamental Value and Average Relative Bubble Size of the Real Estate Indices



Notes: The shaded areas indicate the periods of under- or overvaluation, the dashed line is the fundamental price Q7 and the full line is the actual price.

Figure 2.2: Date Stamping the Periods of Explosiveness in the Non-fundamental Component of the Real Estate indices



Notes: The shaded areas indicate the bubble periods, the dashed line is the 95% critical value sequence and the full line is the BSADF test statistic sequence. The bubble periods are identified using the BSADF test based on Monte Carlo simulations with 2000 replications, with the minimum window set to $0.01+1.8/\sqrt{T}$ of the total sample observations. Sample size: 151 for the NCREIF Property Index, 311 for the US FTSE EPRA/NAREIT Index, 346 for the S&P/Case-Shiller Home Price Index, 348 for the IPD UK Property Index, 312 for the UK FTSE EPRA/NAREIT Index and 251 for the UK House Price Index.

Figure 2.3: Estimated Probability of Collapse for all Real Estate Sectors based on the Average Bubble Size

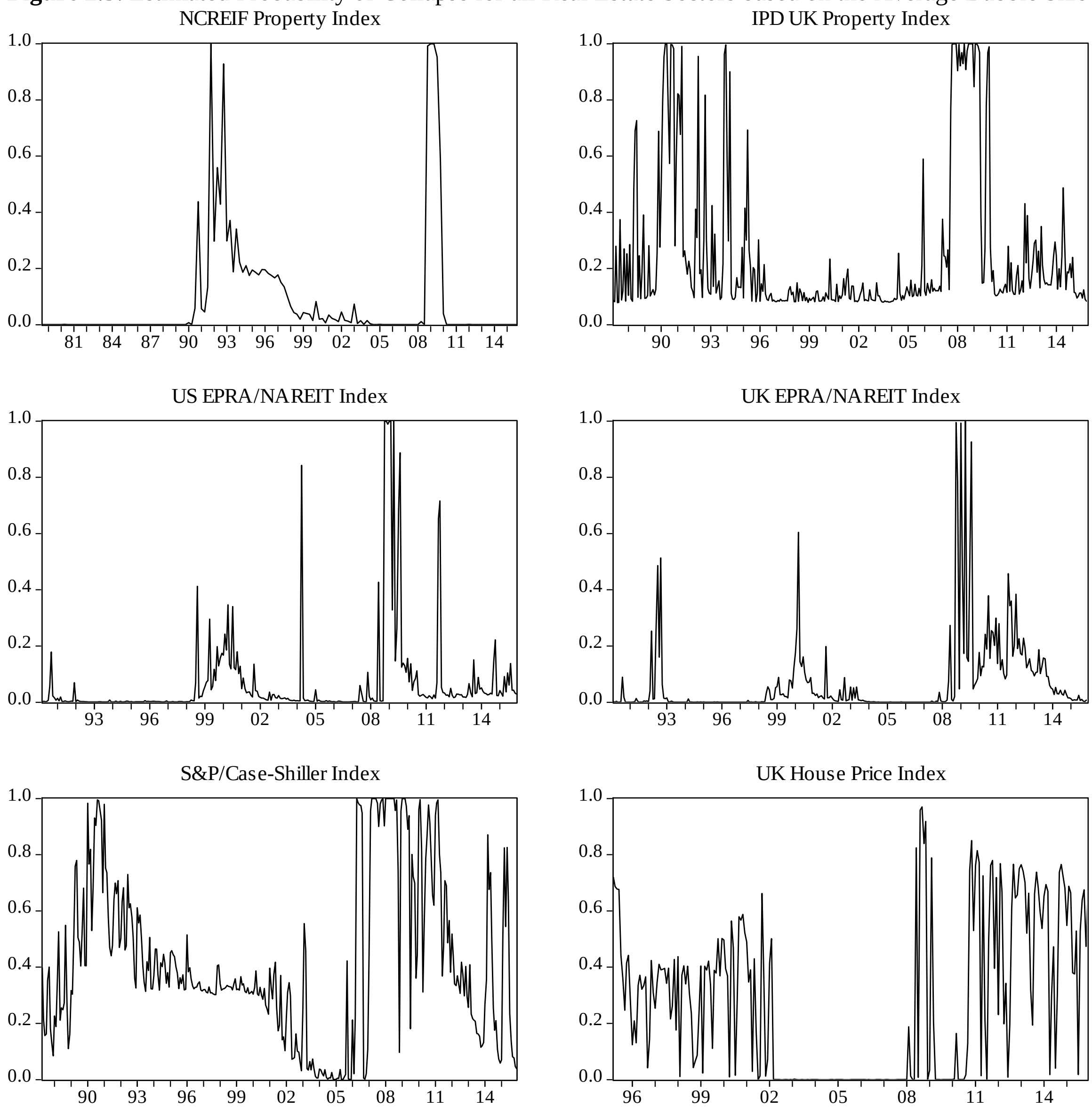


Figure 2.4: The Cumulative Difference between Forecast Errors for the Historical Average against the vNS Regime Switching Model based on Different Fundamental Specifications: 1 Month Horizon

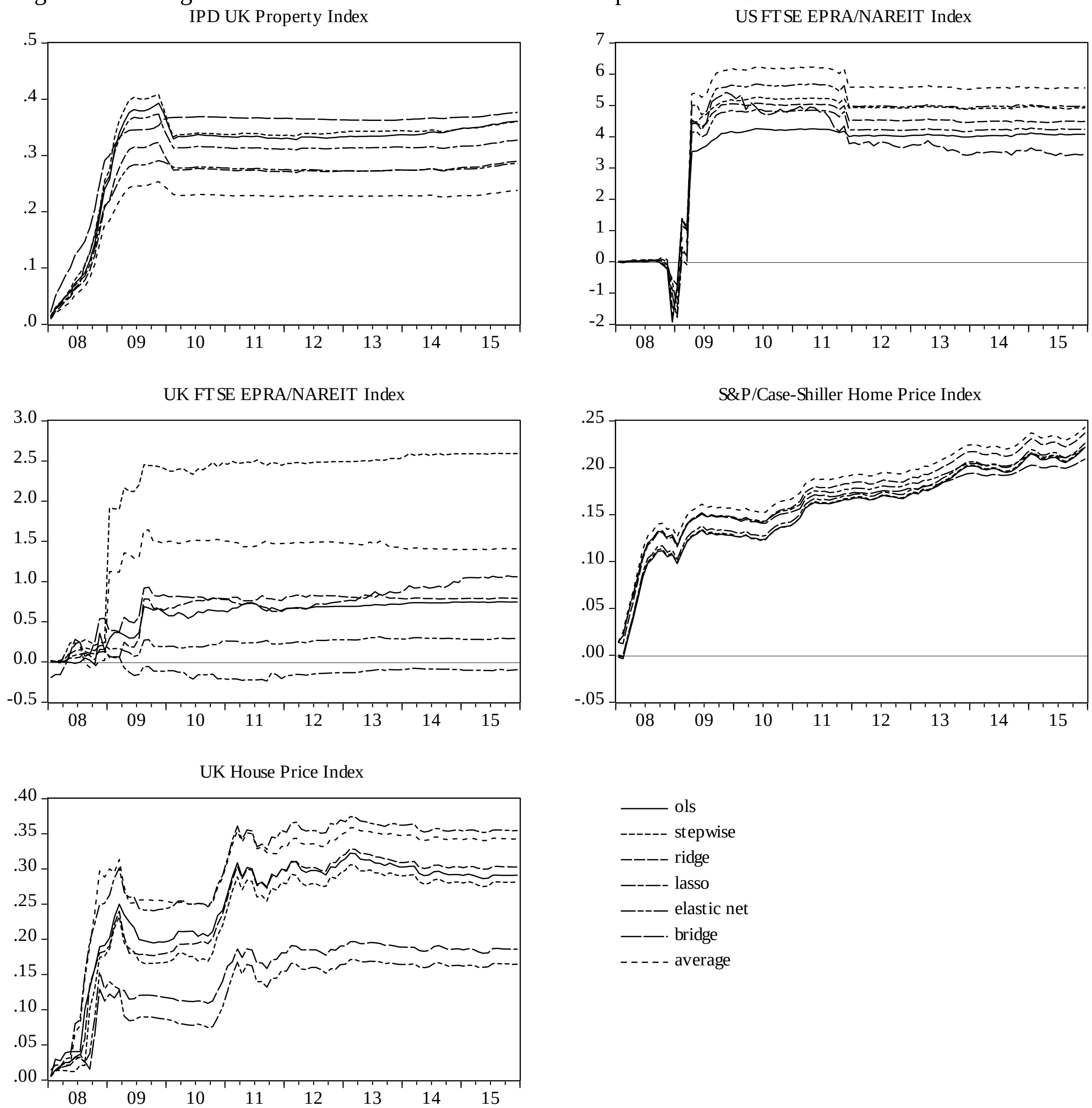


Figure 2.5: The Cumulative Difference between Forecast Errors for the Historical Average against the vNS Regime Switching Model based on Different Fundamental Specifications: 3 months horizon

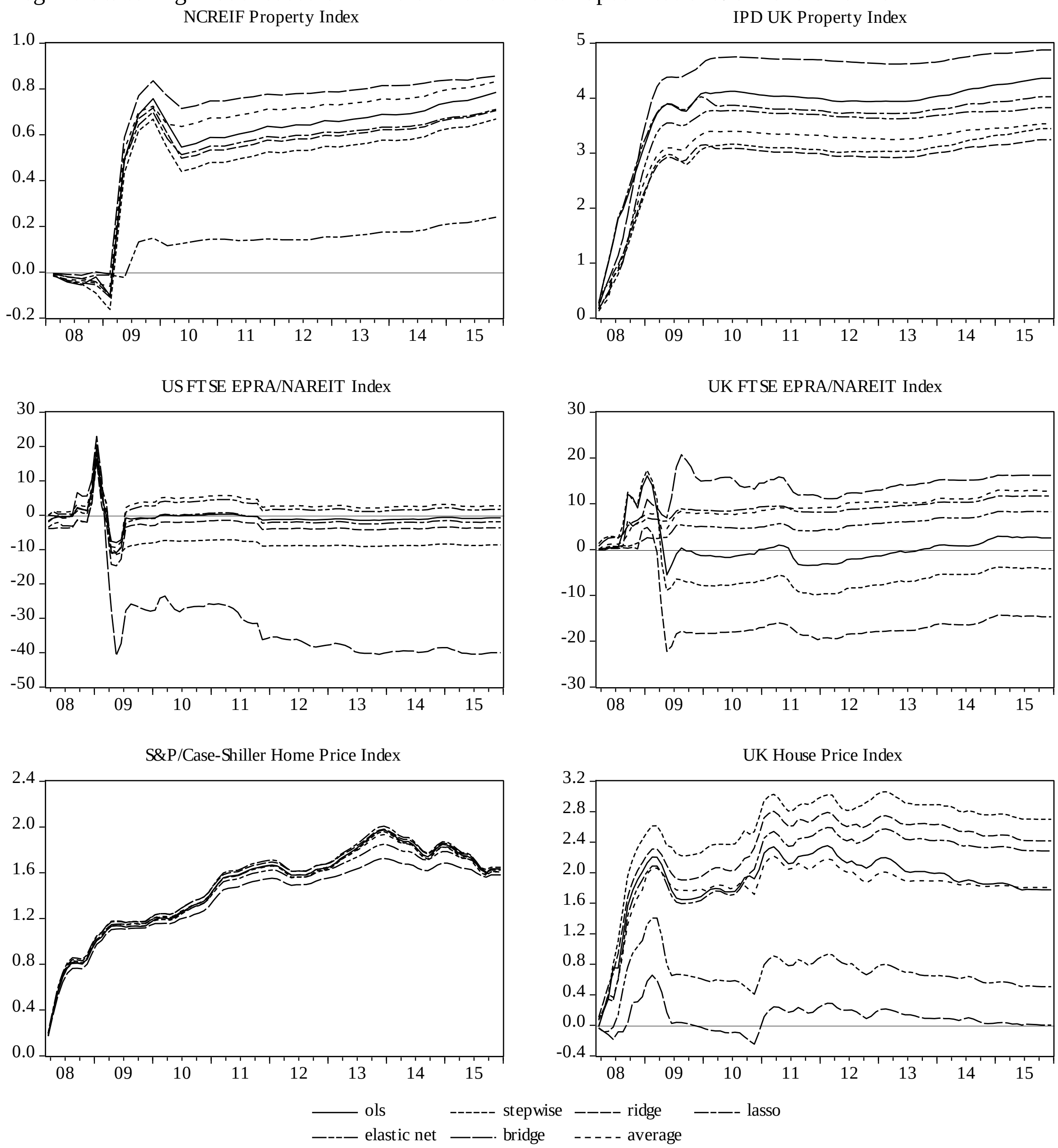
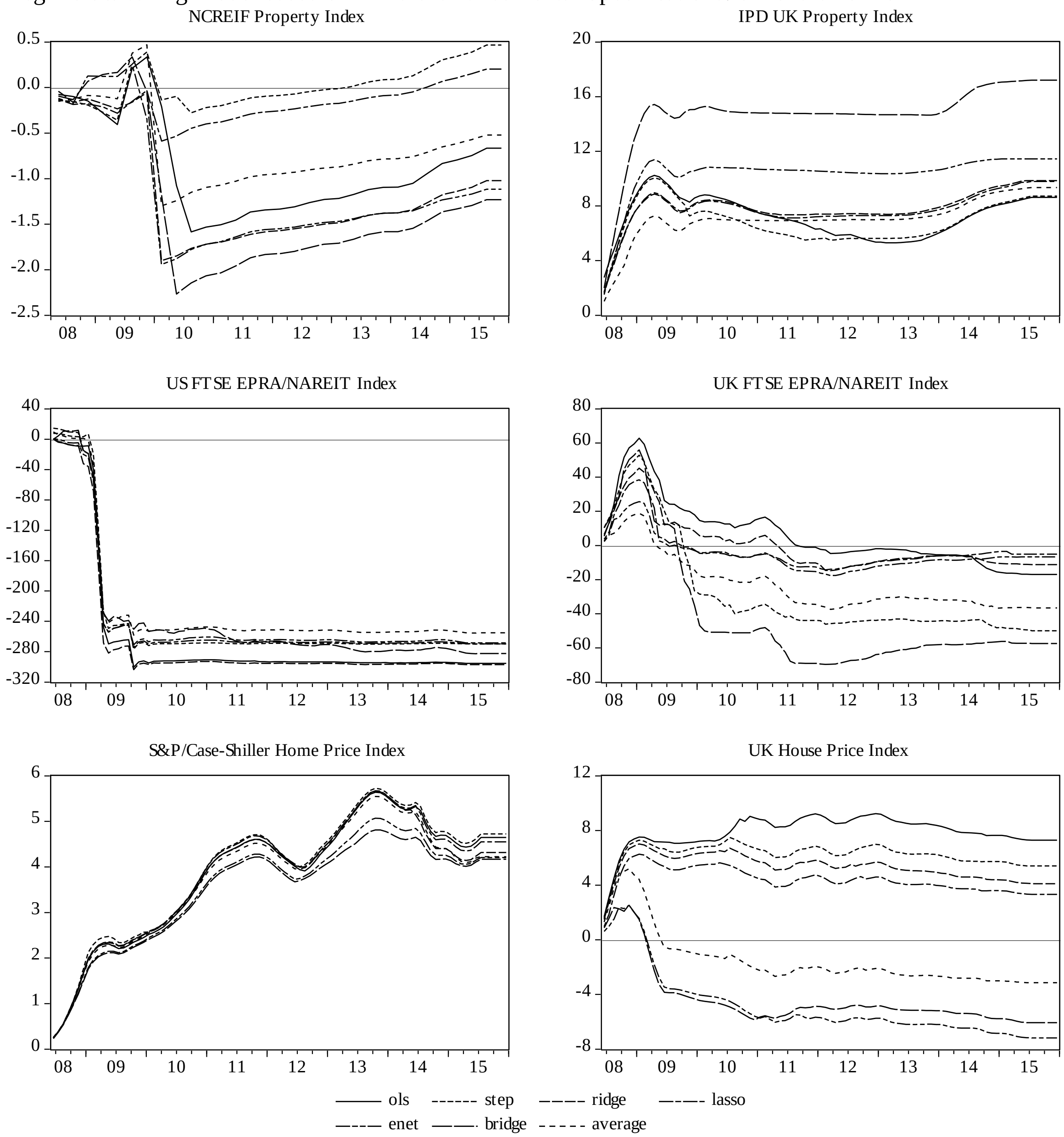


Figure 2.6: The Cumulative Difference between Forecast Errors for the Historical Average against the vNS Regime Switching Model based on Different Fundamental Specifications: 6 months horizon



Chapter 2 Appendix

Table A2.1: The ADF Test Results for the log Real Price of the Real Estate Indices

	NCREIF Property Index	US EPRA/NAREIT Index	S&P/Case-Shiller Home Price Index
ADF statistic	-0.0072	-2.2872	-0.8251
90% Critical Value	**-2.5771**	**-2.5717**	**-2.5712**
95% Critical Value	**-2.8807**	**-2.8707**	**-2.8697**
99% Critical Value	**-3.4743**	**-3.4514**	**-3.4491**
	IPD UK Property Index	UK EPRA/NAREIT Index	UK House Price Index
ADF statistic	-0.9260	-2.3257	-2.8640
90% Critical Value	**-2.5712**	**-2.5717**	**-2.5732**
95% Critical Value	**-2.8697**	**-2.8706**	**-2.8734**
99% Critical Value	**-3.4491**	**-3.4512**	**-3.4576**

Notes: The null hypothesis is that there is a unit root and the alternative that the series are stationary. Figures in bold indicate the rejection of the null hypothesis at the respective significance level. The ADF lag is chosen to minimize the Schwarz Information Criterion with the maximum lag length set to 4 quarters for the NCREIF Property Index and to 12 months for the US FTSE EPRA/NAREIT Index, the S&P/Case-Shiller Home Price Index, the IPD UK Property Index, the UK FTSE EPRA/NAREIT Index and the UK House Price Index.

Table A2.2: Source of the Potential Determinant Variables

Variable name	US variables	UK variables
Stock market index	Bloomberg (SPX Index)	Bloomberg (ASX Index)
Exchange rate	FRED (EXUSUK)	BoE (XUMAGBD)
Money supply	FRED (M2SL)	BoE (LPMVQWK)
Central bank rate	FRED (FEDFUNDS)	BoE (IUMABEDR)
3-month Treasury Bill	FRED (TB3MS)	BoE (IUMAAJNB)
5-year Govt bond yield	FRED (GS5)	BoE (IUMASNZC)
10-year Govt bond yield	FRED (GS10)	BoE (IUMAMNZC)
Mortgage rate	FRED (MORTGAGE30US)	BoE (CFMHSDE) and Three centuries of data version 2.3 dataset.
Inflation rate	Bloomberg (CPURNSA% Index)	Bloomberg (UKRPMOM Index)
Gold price	Bloomberg (GOLDLNAM Index)	Bloomberg (GOLDBPAM Index)
Oil price	FRED (OILPRICE and POILWTIUSDM)	Bloomberg (WRCOBREN Index)
Rent price index	OECD (2016), "Prices: Analytical house price indicators".	OECD (2016), "Prices: Analytical house price indicators".
GDP	Bloomberg (GDP CHWG Index)	Bloomberg (UKGRABMI Index)
Disposable income	Bloomberg (DDIRUS Index)	Bloomberg (DDIRGB Index)
Industrial production	Bloomberg (IP Index)	Bloomberg (UKIPI Index)
Housing starts	FRED (HOUST)	Department for Communities and Local Government
Unemployment rate	Bloomberg (USURTOT Index)	Bloomberg (UKUEILOR Index)
Labor cost	Bloomberg (EOUSU001 Index)	Bloomberg (EOUKU001 Index)
Labor productivity	Bloomberg (EOUSD007 Index)	Bloomberg (EOUKD007 Index)

Notes: This table reports the name and sources of the predictors used in this study.

Chapter 3: Does Model Complexity add Value to Asset Allocation? Evidence from Machine Learning Forecasting Models

3.1. Introduction

The allocation of wealth among risky assets is one of the most important problems faced by investors. The problem of constructing optimal portfolios depends on the objective of the investor, the constraints and the estimation of expected returns. Since forecasting returns is quite challenging, the historical average is often used as an input in portfolio optimization. However, existing literature shows that out-of-sample return predictability adds economic value in asset allocation. This study sets out to examine whether return forecasts generated by shrinkage, variable selection and dimensionality reduction methods from the machine learning literature benefit portfolios consisting of stock, bond and commodity indices, when compared to forecast combination, the equal-weighted portfolio or portfolios based on the historical average.

Our study contributes primarily in three strands of literature. First, it contributes to the growing literature that uses machine learning methodologies to forecast economic and financial variables. The methodologies used in this study have mainly been applied in the context of macroeconomic forecasting using a large number of predictors. Notable studies include Bai and Ng (2008), De Mol, Giannone and Reichlin (2008) and Stock and Watson (2012), who use factor models in conjunction with shrinkage methods to examine the predictability of key macroeconomic indicators. The advantages of machine learning in the context of return predictability and forecasting the equity premium have been explored among others by Rapach, Strauss and Zhou (2013), Kelly and Pruitt (2015), Kelly, Pruitt and Su (2018) and Rapach, Strauss, Tu and Zhou (2018). A comprehensive review of the predictive accuracy of machine learning methodologies has been performed by Gu, Kelly and Xiu (2018) in the context of forecasting the equity premium, by Bianchi, Büchner and Tamoni (2018) who compare the ability of various methods to forecast bond risk premia and by Kim and Swanson (2014) who use a large number of models to predict key macroeconomic variables. Our contribution to this literature stems from exploring the ability of a wide range of machine learning methods to predict stock, bond and commodity returns using a different set of predictors for each index.

Second, our study adds to the literature of asset allocation and portfolio formation that exploits the predictability of asset returns. There exists a rich literature in finance, such as DeMiguel, Garlappi and Uppal (2009), Duchin and Levy (2009), Kritzman, Page and Turkington (2010), Kirby and Ostdiek (2012), Bianchi and Guidolin (2014) and Gao and Nardari (2018), who evaluate the out-of-sample performance of asset portfolios relative to simple benchmarks such as the equal-weighted portfolio. Our contribution to this strand of literature arises from investigating the benefits of integrating return forecasts from machine learning methodologies into an out-of-sample asset allocation framework, by comparing the alternative portfolios to the widely used benchmarks of the equal-weighted portfolio and portfolios based on the historical average forecast.

Third, it extends the literature of commodity return predictability and that of asset allocation exercises that include commodities. Prominent studies that investigate the out-of-sample predictability of commodities include Bessembinder and Chan (1992), Chen, Rogoff, and Rossi (2010), Hong and Yogo (2012) and Gargano and Timmermann (2014). Asset allocation studies that cover commodities include Erb and Harvey (2006), Jensen, Johnson and Mercer (2000), Daskalaki and Skiadopoulos (2011), Belousova and Dorfleitner (2012), You and Daigler (2013), Bessler and Wolff (2015). In a more recent study, Gao and Nardari (2018) assess the value of incorporating commodities in portfolios that exploit the predictability of asset return moments. Our contribution to this literature stems from examining whether traditional portfolios would benefit by the inclusion of commodities, when using forecasts generated by a wide range of machine learning methodologies.

In the empirical analysis, we employ a variety of linear machine learning methods along with forecast combination schemes to generate the return forecasts for each of the stock, bond and commodity indices. In particular, we consider shrinkage and variable selection methods with a wide range of convex and non-convex penalties, along with dimensionality reduction techniques and methods that combine forecasts of single predictor models. The out-of-sample performance of these methods is initially evaluated for each index separately against the historical average benchmark. We conduct the statistical and economic evaluation of the forecasts for the full sample and around NBER-dated recessions and expansions. To explore the potential benefits of using the machine learning methods in an asset allocation setting, stock-bond-commodity portfolios are constructed, each based on the return forecasts generated from a different multivariate prediction model. We compare the performance of the portfolios with that of the equal-weighed portfolio and a portfolio using the historical average forecast. The analysis is conducted for a conservative and an aggressive investor and for different combinations of short-sale and leverage constraints. Furthermore, we examine the performance of the portfolios for the full sample and around business cycles incorporating transaction costs for monthly or quarterly rebalancing. We employ several models for the covariance matrix in a mean-variance allocation framework along with employing Conditional Value-at-Risk (CVaR) as an alternative risk measure in optimization. Finally, to assess the value of adding commodities to a traditional portfolio, mean-variance stock-bond portfolios are constructed and their performance is compared with that of commodity-augmented allocations.

Overall, our study shows that using machine learning techniques can be beneficial for the out-of-sample performance of multiasset portfolios. When, examining the predictive accuracy of the alternative models to forecast the returns of each index individually, the majority of the multivariate prediction models outperform the historical average benchmark and the bivariate predictive regressions. In particular, shrinkage and variable selection methods generate the highest performance for the stock and bond indices, while for the commodity index the results favor dimensionality reduction methods. For the stock and commodity indices the models perform better during recessions, while for the bond index most of the models show increased predictability during expansions, with the exception of shrinkage and variable selection methods that exhibit high performance during recessions.

Our asset allocation results show that the majority of the portfolios outperform the equal-weighted and historical average portfolio benchmarks. When comparing portfolios across different combinations of weight constraints, our findings indicate that allocations that allow short sales or leverage further improve the performance of portfolios based on machine learning methods. Overall, the commodity-augmented portfolios of an aggressive investor outperform those of a conservative investor. Additionally, when introducing transaction costs to portfolios with monthly rebalancing the results tend to favor forecast combination techniques, however, reducing the rebalancing frequency to quarterly leads the portfolios of an aggressive investor based on shrinkage and dimensionality reduction methods to generate the highest performance. Mean-variance portfolios across different specifications of the covariance matrix perform similarly.

Using CVaR as an alternative measure in optimization, results in the vast majority of the mean-CVaR portfolios outperforming the equal-weighted and historical average portfolios.

Our findings for the performance of the stock-bond-commodity portfolios during recessions are mixed. The majority of the long-only allocations that yield positive values are based on variable selection, shrinkage and dimensionality reduction methods. However, when short selling is introduced the return and Sharpe ratio become positive for the majority of the models. During expansions all portfolios result in positive returns and Sharpe ratios and the performance between the proposed models varies less than in recessions. In recessionary periods, all portfolios based on multivariate regression models outperform the equal weighted portfolios or those based on the historical average forecast. In expansionary periods, portfolios with leverage or short selling tend to yield higher performance.

Finally, when comparing the results of stock-bond portfolios with those that include commodities for the full sample, commodities add value to a traditional portfolio when short selling is allowed, with aggressive investors benefiting more from the inclusion of commodities. During recessions, the majority of the commodity-augmented portfolios outperform the traditional portfolios across all weight constraints. In expansions, the long-only traditional portfolios outperform those that include commodities. However, the difference in performance between stock-bond and stock-bond-commodity allocations is greater in recessions, where commodity-augmented portfolios perform best.

The remainder of this chapter is organized as follows. Section 3.2 presents the framework and methods used to generate the out-of-sample return forecasts. Section 3.3 provides details on the data and descriptive statistics, while Section 3.4 discusses the framework and results for the statistical and economic evaluation. Section 3.5 describes the portfolio optimization framework.

Finally, Section 3.6 presents the results of the optimal asset allocation strategies and Section 3.7 concludes.

3.2. Return Prediction Models

In this section, we discuss the general predictive regression model framework and the collection of alternative models that we employ to generate asset returns forecasts.

3.2.1. Bivariate and Kitchen Sink Models

Let $\mathbf{r}$ be the $T \times 1$ vector of asset returns, where $\mathbf{r} = (r_2, r_3, \dots, r_T)'$. We denote by $\mathbf{X}$ the $T \times p$ matrix of p predictors, with elements $x_{i,t}$, where $\mathbf{X} = (\mathbf{x}_1', \mathbf{x}_2', \dots \mathbf{x}_{T-1}')'$ denotes the p-dimentional cross section of the predictors at time t and $\mathbf{X} = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_p)$ denotes the T-dimentional time series of the ith predictor.

The general approach we employ is based on the classic normal linear regression model:

$$\mathbf{r} = \alpha + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{3.1}$$

where α is the intercept, $\boldsymbol{\beta} = (\beta_1, \beta_2, \dots, \beta_p)'$ is the coefficient vector and $\boldsymbol{\varepsilon}$ is the vector of residuals. The most common method to fit the model is by ordinary least squares (OLS), where the estimates of the parameters $\boldsymbol{\theta} = (\alpha, \boldsymbol{\beta})$ are obtained by minimizing the residual sum of squares:

$$\underset{\boldsymbol{\theta}}{\operatorname{argmin}}\, \mathcal{L}(\boldsymbol{\theta}) = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \|\mathbf{r} - (\alpha + \mathbf{X}\boldsymbol{\beta})\|^2 \tag{3.2}$$

where $\mathcal{L}(\cdot)$ indicates the least squares loss. Following the studies of stock return predictability (see for example Goyal and Welch, 2008 and Rapach, Strauss and Zhou, 2010), we consider two approaches based on OLS. The first approach considers the simple bivariate prediction models of asset returns, where each model is based on a single predictor $\mathbf{x}_i$, for $i = 1, 2, \dots, p$. The individual forecast, for $t + 1$ using the ith predictor is given by:

$$\hat{r}_{i,t+1} = \hat{a}_i + x_{i,t}\hat{\beta}_i, \text{ for } i = 1, 2, \dots, p. \tag{3.3}$$

This type of models is often used to examine the predictive power of individual predictors or as the preliminary step to generate forecast combinations based on bivariate prediction models. The second is the kitchen sink (KS) approach, which is a multivariate prediction model utilizing all p predictors. The forecast of a regression with p predictors is given by:

$$\hat{r}_{t+1} = \hat{\alpha} + \mathbf{x}_t\widehat{\boldsymbol{\beta}} \tag{3.4}$$

It is well known that this model, namely the kitchen sink, has poor forecasting performance, as the estimated parameters have low bias but high variance. This problem becomes more acute as the number of predictors increases. To this end, we consider alternative models that belong to the families of forecast combination, shrinkage and dimensionality reduction methods.

3.2.2. Sample Splitting and Cross-Validation

Prior to describing the alternative models for forecasting returns, we discuss how we split our total sample to in- and out-of-sample periods, the forecasting scheme and provide an overview of the concept of cross-validation.

We generate out-of-sample forecasts of asset returns by employing a recursive forecasting scheme. The total sample, T, is divided into the in-sample part, R and the out-of-sample part, $Q = T - R$. The expanding window is updated recursively, by increasing the estimation window by one observation at each step, with the parameters of each model being re-estimated at each iteration. Proceeding this way through the end of the out-of-sample period, a series of Q forecasts are generated for the asset returns. The first q_0 forecasts of the out-of-sample period, Q, serve as the hold-out period for the forecast combination methods that require it, leaving a total of $Q - q_0$ return forecasts for statistical and economic evaluation.

All of the shrinkage procedures and some of the dimensionality reduction methods discussed below rely on hyperparameter tuning. The choice of hyperparameters controls the amount of model complexity and is critical for the performance of the model. We use K-fold cross-validation to select the hyperparameters. Cross-validation is performed in each iteration of the recursive scheme, by using data only of the respective iteration's in-sample period. We split the in-sample data of each iteration into K blocks, with each block containing roughly the same number of observations. The observations assigned to each block are randomly selected. For the kth block we fit the model on the remaining $K - 1$ blocks and calculate the prediction error of the fitted model when predicting the kth block of the data. After repeating this for $k = 1,2, \dots, K$, the K estimates of the prediction error are combined. This procedure is performed for each set of hyperparamer values of the model for $K = 10$ folds. The optimal set of hyperparameters is the one that minimizes the prediction error. After the optimal set of hyperparameters is chosen the model is refitted using all data from the in-sample period and the estimates of the model parameters are kept to construct the forecasts. For a detailed description of cross-validation see Friedman, Hastie and Tibshirani (2009).

For the models based on principal component analysis and independent component analysis the tuning parameters are chosen to minimize the Bayesian Information Criterion (BIC).

3.2.3. Forecast Combination Methods

The forecast combination approach was originally proposed by Bates and Granger (1969) and can be used as an alternative approach to individual forecasting methods (see Timmermann, 2006 for a comprehensive review). Forecast combinations may be preferred over using forecasts based on individual models, since the latter could suffer from model uncertainty and instability, while combining different models can increase accuracy by including valuable information from each model. Following, among others, Rapach, Strauss, and Zhou (2010), forecast combination methods, using forecasts based on individual predictors, are employed to construct one-period ahead expected return estimates.

The forecast combinations, denoted by $\hat{r}^{C}_{t+1}$, are the weighted averages of the p individual forecasts and can be expressed as:

$$\hat{r}^{C}_{t+1} = \hat{\mathbf{r}}_{t+1}\boldsymbol{\omega}_t \tag{3.5}$$

where $\hat{\mathbf{r}}_{t+1} = \left(\hat{r}_{1,t+1}, \hat{r}_{2,t+1}, \dots, \hat{r}_{p,t+1}\right)$ is the vector of p individual forecasts, based on bivariate predictive regressions and $\boldsymbol{\omega}_t = \left(\omega_{1,t}, \omega_{2,t}, \dots, \omega_{p,t}\right)'$ are the combining weights of the individual forecasts at time t. Several combining methods are considered and they all differ in the way that $\boldsymbol{\omega}_t$ is computed. The forecast combination methods used in this study include the mean, median, trimmed mean, model rank based on mean squared forecast error (MSFE), discounted mean squared forecast error and cluster. Some of the forecast combination methods require a holdout period to estimate the weights. The first q_0 observations from the out-of-sample period Q are used as the holdout period.

The first type of forecast combination methods are based on simple averaging schemes and include the mean, median and trimmed mean. The *mean combination* (MC*)* sets the weight $\omega_{i,t} = 1/p$ for $i = 1, 2, \dots, p$ and the *median combination* (MDC) is the median of $\hat{\mathbf{r}}_{t+1}$. The *trimmed mean combination* (TMC) sets $\omega_{i,t} = 0$ for the forecasts with the lowest and highest values and $\omega_{i,t} = 1/p - 2$ for the remaining forecasts. These simple forecast averaging schemes do not require a holdout period.

For the second type of forecasting methods, the combining weights are computed based on the historical forecasting performance of the individual models over the holdout period. Aiolfi and Timmermann (2006) consider a method based on the rank of each model according to the MSFE (Rank). This weighing scheme lets the weights be inversely proportional to the forecast models' rank, RANK_i:

$$\omega_{i,t} = \frac{\mathrm{RANK}_{i,t}^{-1}}{\sum_{i=1}^{p} \mathrm{RANK}_{i,t}^{-1}} \tag{3.6}$$

where the model with the lowest MSFE value gets a rank of 1, the model with the second lowest MSFE value gets a rank of 2 and so forth. Aiolfi and Timmermann (2006) also consider a clustering approach to combine forecasts. The algorithm used is the $\mathrm{C}(L, \mathrm{PB})$. Specifically, the forecasts from the individual models are grouped into L equal-sized clusters based on their past MSFE performance, with the first cluster containing the models with the lowest MSFE. Each combination forecast is the average of the individual forecasts contained in the first cluster. This procedure starts with the initial holdout period q_0 and then goes through the end of the available OOS period using a rolling window. We consider forecast combinations with two ($\mathrm{C}(2, \mathrm{PB})$) and three ($\mathrm{C}(3, \mathrm{PB})$) clusters.

The third type of combining methods considered is also based on past performance of the individual models and uses time-varying combination weights. Stock and Watson (2004), proposed the *discounted mean square forecast error* combining method, which uses the following weights:

$$\omega_{i,t} = m_{i,t}^{-1} \Big/ \sum_{i=1}^{p} m_{i,t}^{-1}, \tag{3.7}$$

where $$m_{i,t} = \sum_{s=R}^{t-1} \psi^{t-1-q_0} \left(r_{s+1} - \hat{r}_{i,s+1} \right)^2, \text{ for } t = R + q_0, \dots, T$$

and ψ is a discount factor, with $0 < \psi \leq 1$. In the case of $\psi < 1$ this method assigns greater weights to recent individual predictive regression forecasts. When $\psi = 1$, then there is no discounting and the equation above produces the optimal combination forecast derived by Bates and Granger (1969) for the case where the individual forecasts are uncorrelated. The values for ψ considered are 1 and 0.9 (D (1) and D (0.9)).

3.2.4. Shrinkage Methods

In general, shrinkage methods regularize the coefficient estimates and involve fitting the model in all p predictors. These procedures shrink the coefficients towards zero relative to the OLS estimates and aim at significantly reducing the respective coefficient variances. Shrinkage methods can also perform variable selection, since depending on the type of regularization, some coefficients may actually be zero.

A shrinkage method is similar to the simple linear model, in that it considers only the baseline, untransformed predictors, however, it modifies the least squares problem by adding one additional term in the loss function. In the most general form, a shrinkage method includes a penalty term in the loss function:

$$\underset{\boldsymbol{\theta}}{\operatorname{argmin}}[\mathcal{L}(\boldsymbol{\theta}) + P(\boldsymbol{\beta}; \cdot)] \tag{3.8}$$

There are several choices for the penalty function $P(\cdot)$[1]. The inputs are standardized to have zero mean and unit variance. We consider the shrinkage methods with the following penalties: ridge, lasso, elastic net, adaptive lasso, bridge, smoothly clipped absolute deviation, minimax concave penalty and smooth integration of counting and absolute deviation.

Ridge regression was introduced by Hoerl and Kennard (1970) and is the classical penalized regression method. Coefficients are estimated by minimizing the residual sum of squares subject to the l_2 penalty:

$$P(\boldsymbol{\beta}; \lambda) = \lambda ||\boldsymbol{\beta}||^2 \tag{3.9}$$

where $\lambda \geq 0$ is a tuning parameter, which is determined separately and controls the amount of shrinkage. The penalty in the case of the ridge regression, is based on l_2 regularization, where $\lambda ||\boldsymbol{\beta}||^2 = \lambda \sum_{i=1}^{p} \beta_i^2$. When $\lambda = 0$, the penalty term has no effect and ridge regression produces similar estimates to OLS. However, as $\lambda \rightarrow \infty$ the impact of the ridge penalty grows and the coefficient estimates will approach zero. A disadvantage of the ridge regression is that while the penalty $\lambda ||\boldsymbol{\beta}||^2$ shrinks all the coefficients towards zero, it never sets them to zero.

[1] Note that the intercept, α, is not included in the penalty term. The penalty is applied to the coefficient vector $\boldsymbol{\beta}$ that measures the association of each predictor with the asset returns and not the intercept, which is a measure of the mean value of the asset returns when, $\mathbf{x}_1 = \mathbf{x}_2 = \cdots = \mathbf{x}_p = 0$. Penalization on the intercept is not typically considered, since it would make the optimization procedure dependent on the origin chosen for the asset returns, $\mathbf{r}$; i.e. adding a constant to each observation of the asset returns, r_t,would not simply result in a shift of the predictions by the same amount (see Friedman, Hastie and Tibshirani (2009)).

The *least absolute shrinkage and selection operator* (lasso) was introduced by Tibshirani (1996) and has a penalty term based on the l_1 norm, which allows it to yield sparse models, the penalty term for the lasso is:

$$P(\boldsymbol{\beta}; \lambda) = \lambda \|\boldsymbol{\beta}\|_1 \tag{3.10}$$

where $\lambda \geq 0$ is the lasso tuning parameter and $\lambda\|\boldsymbol{\beta}\|_1 = \lambda \sum_{i=1}^{p} |\beta_i|$. The difference with ridge regression is that lasso imposes a penalty based on the l_1 norm instead of the l_2 norm, which allows for both shrinkage and variable selection, by setting some of the coefficients equal to zero. One of the problems that lasso faces is that if there is a group of highly correlated variables, then lasso will select arbitrarily only one of the variables in the group.

The *elastic net* (EN) was proposed by Zou and Hastie (2005) which combines both l_1 and l_2 terms in the penalty, thus simultaneously performing continuous shrinkage, automatic variable selection and can also select groups of correlated variables. The elastic net penalty is:

$$P(\boldsymbol{\beta}; \lambda; \alpha) = \lambda\big((1-\alpha)\|\boldsymbol{\beta}\|_1 + \alpha\|\boldsymbol{\beta}\|^2\big) \tag{3.11}$$

where λ is the tuning parameter and $\alpha \in [0,1]$. When $\alpha = 1$ the elastic net becomes ridge regression, if $\alpha = 0$ it is the lasso, while if $\alpha \in (0,1)$ it has the properties of both methods.

The *adaptive lasso* (Alasso) was developed by Zou (2006) and solves the drawback of the original lasso, which is that it does not necessarily satisfy the oracle properties (Fan and Li, 2001). This is achieved by modifying the lasso to include adaptive weights that are used to penalize different coefficients in the l_1 penalty. The adaptive lasso penalty is given by

$$P(\boldsymbol{\beta}; \lambda; \gamma) = \lambda \sum_{i=1}^{p} \widehat{w}_i |\beta_i|, \text{ with } \widehat{w}_i = \frac{1}{|\hat{\beta}_i|^{\gamma}}, \text{ for } \gamma > 0 \tag{3.12}$$

where $\hat{w}_i$ is the weight corresponding to coefficient $|\beta_i|$, $\hat{\beta}_i$ is the OLS estimate and γ is a hyperparameter which controls the strength of the weight. This leads to the adaptive lasso penalizing individual coefficients less severely.

Bridge regression developed by Frank and Friedman (1993) and Friedman (2012), has a penalty term based on the l_γ norm and is given by

$$P(\boldsymbol{\beta};\lambda;\gamma) = \lambda\|\boldsymbol{\beta}\|_\gamma^\gamma \qquad (3.13)$$

where $\lambda \geq 0$ and $\gamma > 0$ are the two tuning parameters and $\lambda\|\boldsymbol{\beta}\|_\gamma^\gamma = \lambda\sum_{i=1}^{p}|\beta_i|^\gamma$. The bridge penalty term for $0 \leq \gamma \leq 2$ represents all the penalties between ridge regression and best subsets. When using the squared error loss it includes ridge regression ($\gamma = 2$), the lasso ($\gamma = 1$) and best-subsets regression ($\gamma = 0$). Ridge regression produces dense solutions, while shrinking the coefficient absolute values, while best-subsets regression produces the sparsest solutions by forcing many coefficients to be equal to zero and applies no shrinkage to the non-zero coefficients, with a large number of λ producing fewer non-zero coefficients. For $\gamma > 1$ all coefficients are strictly non-zero and all penalties in the power family are convex, while for $\gamma < 1$ the penalties are non-convex.

The *smoothly clipped absolute deviation* (SCAD) is a non-convex penalty function, which was proposed by Fan and Li (2001). The SCAD penalty is given by

$$P(\boldsymbol{\beta};\lambda;\gamma) = \sum_{i=1}^{p} P(\beta_i;\lambda;\gamma)\,, \qquad (3.14)$$

$$\text{where} \qquad P(\beta_i;\lambda;\gamma) = \begin{cases} \lambda|\beta_i|, & \text{if } |\beta_i| \le \lambda \\ \dfrac{2\gamma\lambda|\beta_i| - |\beta_i|^2 - \lambda^2}{2(\gamma-1)}, & \text{if } \lambda < |\beta_i| \le \gamma\lambda \\ \dfrac{\lambda^2(\gamma+1)}{2}, & \text{if } |\beta_i| > \gamma\lambda \end{cases}$$

for $\gamma > 2$. SCAD coincides with the lasso until $|\beta_i| = \lambda$, then smoothly transitions to a quadratic function until $|\beta_i| = \gamma\lambda$ and then it remains constant for all $|\beta_i| > \gamma\lambda$. For small coefficients, the SCAD penalty has similar penalization rate as the lasso, but leaves large coefficients not excessively penalized.

The *minimax concave penalty* (MCP) developed by Zhang (2010) is another non-convex penalty function. The MCP is defined by

$$P(\boldsymbol{\beta};\lambda;\gamma) = \sum_{i=1}^{p} P(\beta_i;\lambda;\gamma)\,, \tag{3.15}$$

$$\text{where} \qquad P(\beta_i;\lambda;\gamma) = \begin{cases} \lambda|\beta_i| - \dfrac{|\beta_i|^2}{2\gamma}, & \text{if } |\beta_i| \le \lambda\gamma \\ \dfrac{\gamma\lambda^2}{2}, & \text{if } |\beta_i| > \lambda\gamma \end{cases}$$

for each value of $\lambda > 0$ and $\gamma > 1$, there is a continuum of penalties and threshold operators varying from hard thresholding ($\gamma \to 1+$) to soft thresholding ($\gamma \to \infty$). MCP starts with the same rate of penalization as the lasso but smoothly relaxes the penalization rate to zero as the absolute value of the coefficient increases. Furthermore, MCP relaxes the penalization rate immediately, compared to SCAD, where the rate remains flat for a while before decreasing.

The *smooth integration of counting and absolute deviation* (SICA) penalty (Lv and Fan, 2009) takes the form

$$P(\boldsymbol{\beta};\lambda;\gamma) = \sum_{i=1}^{p} P(\beta_i;\lambda;\gamma)\,, \tag{3.16}$$

where $$P(\beta_i;\lambda;\gamma) = \lambda\frac{(\gamma+1)|\beta_i|}{\gamma+|\beta_i|}$$

ith $\lambda > 0$ and a small shape parameter $\gamma > 0$, such as 10^{-2} or 10^{-4}. SICA is another non-convex regularization method which is a combination between the l_0 and l_1 penalties and therefore gives sparse solutions. For smaller values of γ, SICA yields results closer to the best-subsets regression, while for larger values of γ it is closer to the lasso.

3.2.5. Dimensionality Reduction Methods

The methods described in the previous section used shrinkage and variable selection to reduce the dimensions of the predictors by forcing the coefficients to be close or equal to zero. The next set of models incorporates the information of a large set of economic variables in a predictive regression framework using latent factors, which are estimated either in a supervised way (using information in both **r** and **X**) or an unsupervised way (using information only in **X**).

Partial least squares (PLS), introduced by Wold (1966), identifies the features in a supervised way, by constructing linear combinations based on both **r** and **X**. Specifically, PLS decomposes the matrix of standardized predictors **X** and the zero-mean vector of asset returns **r** into the form: $\mathbf{X} = \mathbf{ZP}' + \mathbf{E}$ and $\mathbf{r} = \mathbf{Zq}' + \mathbf{e}$, where $\mathbf{Z}$ is a matrix that produces k linear combinations or scores, the matrix **P** and the vector **q** are the loadings, while **E** and **e** are the residuals. The score matrix is given by $\mathbf{Z} = \mathbf{XA}$. In order to find the matrix **Z**, the columns of $\mathbf{A} =$

$(\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \dots, \boldsymbol{\alpha}_k)$, where $k < p$, need to be obtained through successive optimization problems. The criterion to find the jth estimated direction vector $\boldsymbol{\alpha}_j$ is:

$$\underset{\boldsymbol{\alpha}}{\operatorname{argmax}} \operatorname{cor}^2(\mathbf{r}, \mathbf{X}\boldsymbol{\alpha})\operatorname{var}(\mathbf{X}\boldsymbol{\alpha}), \text{ s.t. } \boldsymbol{\alpha}'\boldsymbol{\alpha} = 1, \ \ \boldsymbol{\alpha}'\boldsymbol{\Sigma}_{XX}\boldsymbol{\alpha}_j = 0, \text{ for } j = 1, \dots, k-1 \tag{3.17}$$

where $\boldsymbol{\Sigma}_{XX}$ is the covariance of $\mathbf{X}$. PLS can be expressed as a multiple regression model: $\mathbf{r}_{t+1} = \boldsymbol{X}_t\boldsymbol{\beta} + \boldsymbol{\eta}_{t+1}$, where the PLS-regression coefficients can be written as $\boldsymbol{\beta} = \mathbf{A}\mathbf{q}'$ and $\boldsymbol{\eta}$ is the residual vector. The version of PLS used is SIMPLS proposed by de Jong (1993). If $k = p$ then PLS would give a solution equivalent to the OLS estimates.

Kelly and Pruitt (2015), propose the *three-pass regression filter* (3PRF) which is a generalization to PLS to include forecast proxies $\mathbf{V}$. The first pass runs p time-series regressions:

$$x_{i,t} = \varphi_{0,i} + \mathbf{v}'\boldsymbol{\varphi}_i + \varepsilon_{i,t}, \text{ for } i = 1, 2, \dots, p \tag{3.18}$$

The second pass runs T cross-sectional regressions:

$$x_{i,t} = \varphi_{0,t} + \hat{\boldsymbol{\varphi}}'\mathbf{F}_t + e_{i,t}, \text{ for } t = 1, 2, \dots, T \tag{3.19}$$

where $\hat{\boldsymbol{\varphi}}$ is the coefficient estimate from the time-series regression from the first pass. In the third pass a single time-series predictive regression is run:

$$r_{t+1} = \beta_0 + \hat{\mathbf{F}}'\boldsymbol{\beta} + \eta_{i,t}, \tag{3.20}$$

where $\hat{\mathbf{F}}$ are the estimated predictive factors from the second pass. All regressions are estimated using OLS. To estimate 3PRF, the proxies do not necessarily need to be specified, instead we use the automatic proxy selection algorithm, found in Kelly and Pruitt (2015), which constructs the proxies using $\mathbf{X}$ and $\mathbf{r}$. The 3PRF with automatic proxies becomes identical to PLS when the predictors $\mathbf{X}$ have been standardized and the regressions in the first two passes do not include a constant.

Sparse partial least squares (SPLS) is an extension of PLS that imposes the l_1 penalty to promote sparsity onto a surrogate direction vector **c** instead of the original direction vector $\boldsymbol{\alpha}$, while keeping $\boldsymbol{\alpha}$ and **c** close to each other (Chun and Keles, 2010). The first SPLS direction vector solves:

$$\underset{\boldsymbol{\alpha},\mathbf{c}}{\operatorname{argmin}} -\kappa\boldsymbol{\alpha}'\mathbf{M}\boldsymbol{\alpha} + (1-\kappa)(\mathbf{c}-\boldsymbol{\alpha})'\mathbf{M}(\mathbf{c}-\boldsymbol{\alpha}) + \lambda_1\|\mathbf{c}\|_1 + \lambda_2\|\mathbf{c}\|^2, \text{ s.t. } \boldsymbol{\alpha}'\boldsymbol{\alpha} = 1 \tag{3.21}$$

where $\mathbf{M} = \mathbf{X}'\mathbf{r}\mathbf{r}'\mathbf{X}$, λ_1 and λ_2 are non-negative tuning parameters and $0 < \kappa < 1$, is a tuning parameter to control the effect of the concavity of the objective function. To solve SPLS a large λ_2 value is usually required and setting $\lambda_2 = \infty$ yields a solution that has the form of the soft threshold estimator by Zou and Hastie (2005). Furthermore, since we use PLS to predict a univariate response (the vector of asset returns **r**) the solution does not depend on the parameter κ. This reduces the number of tuning parameters to two, the tuning parameter λ_1 and the number of hidden components k.

In the dimensionality reduction methods described above the directions that best represent the predictors **X** are derived in a supervised way since the vector of asset returns, **r**, is used to determine the component directions. The next set of models derives the latent factors in an unsupervised way, before using them in a predictive regression. The regression takes the following form:

$$\mathbf{r}_{t+1} = \alpha + \hat{\mathbf{Z}}_t\boldsymbol{\beta} + \boldsymbol{\varepsilon}_{t+1} \tag{3.22}$$

where $\mathbf{Z} = (\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_k)$, with $k < p$, is the vector of latent factors or components that are estimated through principal component analysis, sparse principal component analysis, independent component analysis or reconstruction independent component analysis. The number of components used to estimate the model varies for each iteration of the recursive scheme, with the optimal number of components chosen to minimize the Bayesian Information Criterion (BIC).

Principal component analysis (PCA) is the most widely used method to obtain estimates of the latent factors called principal components. Principal components are a sequence of projections of the data mutually uncorrelated and ordered in variance. The first principal component captures the maximum variation among all linear combinations of predictors, the second principal component has the highest variation among all linear combinations in the remaining orthogonal subspace, and so on, with the last principal component having minimum variation.

PCA can be viewed as a regression-type problem where the goal is to find the first k principal component loading vectors by minimizing:

$$\underset{\mathbf{A}}{\operatorname{argmin}} \|\mathbf{X} - \mathbf{X}\mathbf{A}\mathbf{A}'\|^2, \quad \text{s.t.} \quad \mathbf{A}'\mathbf{A} = \mathbf{I}_k \tag{3.23}$$

where $\mathbf{A}$ is a $p \times k$ matrix. The solution to this problem is most often obtained via singular value decomposition: $\mathbf{X} = \mathbf{U}\mathbf{D}\mathbf{V}'$, by setting $\mathbf{A} = \mathbf{V}$. The columns of $\mathbf{V} = (\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k)$ are the principal components loadings. Each $\mathbf{v}_j$ is used to derive the jth principal component, $\mathbf{z}_j = \mathbf{X}\mathbf{v}_j$, thus, $\mathbf{Z}\mathbf{V}$ is the dimension reduced version of the original predictors. The derived variable $\mathbf{z}_1$ is the first principal component of $\mathbf{X}$ and has the largest sample variance amongst all normalized linear combinations of the columns of $\mathbf{X}$.

Sparse principal component analysis (SPCA), developed by Zou, Hastie and Tibshirani (2006), is similar to PCA as it is designed to uncover the linear combination of the original predictors in a way that the derived variables capture the maximum variance. However, it produces principal components with sparse loadings. PCA has the drawback that each principal component is a linear combination of all the original predictors and the loadings are typically non-zero, which leads to difficulties in the interpretability of the results. This issue is addressed by SPCA that produces modified principal components with sparse loadings, such that each principal component is a linear combination of only a few of the original predictors.

The approach followed by Zou, Hastie and Tibshirani (2006) is based on the regression/reconstruction property of PCA. They show how PCA can be viewed in terms of a ridge regression problem and by adding the l_1 penalty, they convert it to an elastic net regression, which allows for the estimation of sparse principal components. The following regression criterion is proposed to derive the sparse principal component loadings:

$$\underset{\mathbf{A},\mathbf{C}}{\operatorname{argmin}} \sum_{i=1}^{T} \|\mathbf{x}_i - \mathbf{A}\mathbf{C}'\mathbf{x}_i\|^2 + \sum_{j=1}^{k} \lambda_{1,j} \|c_j\|_1 + \lambda_2 \sum_{j=1}^{k} \|c_j\|^2, \quad \text{s.t.} \quad \mathbf{A}'\mathbf{A} = \mathbf{I}_k \tag{3.24}$$

where $\mathbf{A}$ and $\mathbf{C}$ are both $p \times k$. If $\lambda_1 = \lambda_2 = 0$, $T > p$ and restrict $\mathbf{C} = \mathbf{A}$, then the minimizer of the objective function is exactly the first k loading vectors of ordinary PCA. When $p \gg T$, in order to obtain a unique solution, $\lambda_2 > 0$ is required. The l_1 penalty on c_j induces sparseness of the loadings, with larger values of λ_1 leading to sparser solutions. The algorithm by Zou and Hastie (2005) is used to compute the sparse approximations of each principal component.

Independent component analysis (ICA), developed by Comon (1994), aims at finding a linear representation of non-Gaussian data so that the components are statistically independent. The ICA objective is:

$$\underset{\mathbf{A}}{\operatorname{argmin}} \sum_{i=1}^{T} \|\mathbf{A}'\mathbf{x}_i\|_1, \quad \text{s.t.} \quad \mathbf{A}'\mathbf{A} = \mathbf{I}_k \tag{3.25}$$

Solving the ICA problem amounts to finding an orthogonal $\mathbf{A}$ such that the components of the vector random variable $\mathbf{Z} = \mathbf{X}\mathbf{A}$ are independent and non-Gaussian. More in detail, the independent components are estimated by iterative estimation of the matrix $\mathbf{A}$, systematically increasing the degree of independence of the components. However, since there is no direct measure of independence, non-Gaussianity is used instead. Popular approaches for measuring independence or non-Gaussianity in ICA are based on entropy. We use the FastICA algorithm developed by Hyvärinen and Oja (2000), which uses negentropy as a measure of Non-Gaussianity.

Ordinary ICA has two drawbacks; it requires constrained optimization which can become difficult in high dimensional settings and it is sensitive to whitening, a preprocessing step that decorrelates the input data, which cannot always be computed exactly when $p \gg T$. Le, Karpenko, Ngiam and Ng (2011), propose *reconstruction independent component analysis* (RICA), which overcomes the drawbacks of ICA, by replacing ICA's orthonormality constraint with a reconstruction penalty. This produces the unconstrained problem:

$$\underset{\mathbf{A}}{\operatorname{argmin}} \sum_{i=1}^{T} \|\mathbf{A}'\mathbf{x}_i\|_1 + \lambda \sum_{i=1}^{T} \|\mathbf{A}\mathbf{A}'\mathbf{x}_i - \mathbf{x}_i\|^2 \tag{3.26}$$

where $\lambda > 0$ is a regularization parameter. RICA is equivalent to ICA when $k < p$, data is whitened and λ approaches infinity.

3.3. Data and Descriptive Statistics

Our dataset consists of monthly closing prices of stock, bond and commodity total return indices, denominated in US dollars. Stocks are proxied by the S&P 500 Total Return Index, bonds are measured by the Bloomberg Barclays US Aggregate Bond Index and as a proxy for the commodity class the S&P Goldman Sachs Commodity Total Return Index (GSCI) is used. A different set of predictors is used to forecast the returns of each index. Details on the sources of the series used in this study and the construction of the predictors are given in the appendix of this chapter (Table

A3.1). Our sample period is from January 1977 to December 2016 for a total of 480 observations. The initial estimation period, including the hold-out period, is from January 1977 to December 1996 (240 observations), while the out-of-sample period is from January 1997 to December 2016 (240 observations).

The majority of the predictor variables for the S&P 500 index are from Goyal and Welch (2008). These are the dividend-price ratio (DP), dividend yield (DY), earnings-price ratio (EP), dividend-payout ratio (DE), stock variance (SVAR), book-to-market ratio (BM), net equity expansion (NTIS), Treasury bill rate (TBL), long-term yield (LTY), long-term return (LTR), term spread (TMS), default yield spread (DFY), default return spread (DFR) and inflation (INFL) based on the Consumers Price Index. Following Rapach, Wohar and Rangvid (2005), the industrial production index (IP), the money stock M1 (M1) and the unemployment rate (UR) are also included. Finally, we consider three additional variables, namely the Chicago Board Options Exchange Volatility Index (VXO) and the macroeconomic and financial uncertainty indices (Umacro and Ufin respectively), proposed by Jurado, Ludvigson and Ng (2015) and Ludvigson, Ma and Ng (2015).

Following Ludvigson and Ng (2009), Lin, Wu and Zhou (2017) and Gao and Nardari (2018), candidate predictors for the bond index returns are divided into three sets: interest rate factors, stock market factors and other economic factors. Interest rate factors include the Treasury bill rate, long-term yield, long-term return, term spread, default yield spread, default return spread and the spread between the 6-month Treasury bill and 1- to 5-year government bonds (SP1 and SP5 respectively). The stock market factors include the S&P 500 return, the dividend yield and the VXO. The remaining variables are the Producers Price Index (PPI), capacity utilization (manufacturing, CAP), the inventories-sales ratio (IS), the money stock M1, the unemployment rate and the two uncertainty indices.

The variables used to forecast the returns of the GSCI are primarily based on Gargano and Timmermann (2014) and can be categorized into general economic factors and commodity specific factors. The general factors include the dividend yield, Treasury bill rate, long-term return, term spread, default return spread, inflation, the industrial production index, the money stock M1, the unemployment rate, Kilian's (2009) real economic activity index (REA), the Chicago Fed National Activity Index (CFNAI), and the macroeconomic and financial uncertainty indices. The commodity specific factors are the price of crude oil (WTI) and four commodity currencies (Chen, Rogoff and Rossi (2010)), the Australian dollar-US dollar (USDAUD), the Canadian dollar-US dollar (USDCAD), the Indian rupee-US dollar (USDIND) and the New Zealand dollar-US dollar (USDNZD).

Table 3.1 presents the descriptive statistics for the monthly returns of the three indices. The stock index has the highest mean return (0.98%), a standard deviation of 4.3%, exhibits the most negative skewness (-0.58) and has a kurtosis of 5.10. The bond index has a mean return of 0.61%, the lowest standard deviation (1.57%), exhibits a positive skewness of 0.76 and has the highest kurtosis (10). The commodity index has the lowest mean return (0.56%), the highest standard deviation (5.59%), a skewness of -0.24 and the lowest kurtosis (5.06). The stock index is weakly positively correlated with the bond and commodity indices (0.21 and 0.18 respectively), while the bond and commodity indices are uncorrelated (-0.02). Figure 3.1 plots the prices and returns of the three indices under consideration.

[Insert Table 3.1 Here]

[Insert Figure 3.1 Here]

3.4. Out-of-Sample Performance

3.4.1. Statistical Evaluation

The benchmark against which the alternative forecasting models are compared is the historical average forecast, given by: $\hat{r}_{0,t+1} = (1/t)\sum_{s=1}^{t} r_s$. As a measure of forecast accuracy we use the Cambell and Thomson (2008) out-of-sample R^2 (R^2_{OS}). R^2_{OS} measures the proportional reduction in the mean squared forecast error (MSFE) of the individual model forecast relative to the historical average. The R^2_{OS} for the ith model is given by:

$$R^2_{OS} = 1 - \mathrm{MSFE_i}/\mathrm{MSFE_0}\text{, where MSFE} = \frac{1}{Q - q_0 - 1}\sum_{t=1}^{Q-q_0-1}(r_{t+1} - \hat{r}_{t+1})^2 \qquad (3.27)$$

A positive R^2_{OS} implies that the alternative model outperforms the historical average in terms of MSFE. The statistical significance of the R^2_{OS} is assessed by the Clark and West (2007) MSFE-adjusted statistic. The statistic tests the null hypothesis that the MSFE of the historical average benchmark is equal or less than the MSFE of the alternative model, against the one-sided alternative hypothesis that the historical average MSFE is greater than the MSFE of the alternative model. Clark and West (2007) adjust the MSFE in the following way:

$$\mathrm{MSFE_{adj}} = \frac{1}{Q - q_0 - 1}\sum_{t=1}^{Q-q_0-1}(r_{t+1} - \hat{r}_{t+1})^2 + \frac{1}{Q - q_0 - 1}\sum_{t=1}^{Q-q_0-1}\left(\hat{r}_{0,t+1} - \hat{r}_{t+1}\right)^2 \qquad (3.28)$$

The MSFE-adjusted statistic is equivalent to the t-statistic for the constant, obtained by regressing:

$$f_{t+1} = \left(r_{t+1} - \hat{r}_{0,t+1}\right)^2 - (r_{t+1} - \hat{r}_{t+1})^2 + \left(\hat{r}_{0,t+1} - \hat{r}_{t+1}\right)^2 \quad (3.29)$$

on a constant. The null hypothesis of equal predictive ability is rejected when the t-statistic is greater than 1.282, 1.645 and 2.326, for a one-sided 0.1, 0.05, and 0.01 test respectively.

3.4.2. Economic Evaluation

Following Campbell and Thompson (2008) and Ferreira and Santa-Clara (2011), the economic value of the return forecasts is measured for an investor with moderate risk preferences. The strategy involves a portfolio, with monthly rebalancing, consisting of a risky asset (the stock, bond or commodity index) and a risk-free asset (Treasury bill). The optimal weight of the risky asset, $w_{i,t}$, based on the return forecast of model i, under a mean-variance framework with a one-month ahead horizon is:

$$w_{i,t} = \frac{\hat{r}_{i,t+1} - r_{f,t+1}}{\gamma \hat{\sigma}^2_{t+1}} \quad (3.30)$$

where $\hat{r}_{i,t+1}$ is the return forecast based on model i, $r_{f,t+1}$ is the risk-free rate of return, γ is the coefficient of relative risk aversion and $\hat{\sigma}^2_{t+1}$ is the forecast of the variance. The one-month ahead portfolio return is given by:

$$r_{P,t+1} = w_t r_{t+1} + (1 - w_t) r_{f,t+1} \quad (3.31)$$

The forecast of the variance, $\hat{\sigma}^2_{t+1}$, is derived using a similar approach to Cambell and Thomson (2008), where $\hat{\sigma}^2_{t+1}$ is estimated as the rolling average of the variance of past monthly returns. The length of the rolling window is set to ten years (120 observations). Following Neely, Rapach, Tu and Zhou (2014) γ is set equal to five.

The portfolio performance is evaluated using the certainty equivalent return (CER), the Sharpe Ratio (SR) and portfolio turnover. The certainty equivalent return is defined as:

$$\text{CER} = \bar{r}_P - 0.5\gamma\bar{\sigma}_P^2 \tag{3.32}$$

where $\bar{r}_P$ and $\bar{\sigma}_P^2$ are the mean and variance of the portfolio returns over the out-of-sample period. The CER can be interpreted as the risk-free return that a mean-variance investor with coefficient of relative risk aversion γ is willing to accept instead of investing in the risky portfolio. The difference in CER is reported (ΔCER), which is equivalent to the CER generated by the portfolio utilizing the forecasts minus the portfolio based on the historical average benchmark. ΔCER can be interpreted as the performance fee that the investor would be willing to pay to use the information of each alternative model instead of the benchmark. The Sharpe ratio is defined as the average excess return of the portfolio divided by the standard deviation of the portfolio.

The performance based on the R_{OS}^2 is presented in Table 3.2 for the bivariate prediction models and in Table 3.3 for the multivariate prediction models. We also report the R_{OS}^2 during NBER-dated recessions and expansions. Table 3.3 also includes the performance based the annualized ΔCER and SR measures. All models are compared against the historical average forecast.

From the first panel of Table 3.2, we observe that, even though six models yield positive R_{OS}^2 for the stock index, the MSFEs of all variables are significantly less than the historical average MSFE at conventional levels of significance according to the MSFE_{adj} statistic. Overall, the bivariate prediction models for the stock index perform better during recessions than in expansions, in terms of R_{OS}^2, however, the only case with a statistically significant R_{OS}^2 according to the Clark-West test is the BM variable during recessions. The results for the bond index, as reported in the second panel, show eight variables with positive R_{OS}^2, of those five have significant MSFE_{adj} statistic (PPI, IS and SP500 at the 5% level, while SP1 and SP5 at the 1% level). Additionally, the MSFE_{adj} statistic for DY indicates that the MSFE of the alternative model is significantly less than the MSFE of the historical average despite having $R_{OS}^2 < 0$. This result is possible when comparing forecasts of nested models. The results during business cycles for the bond index, according to the Clarke-West test, favor expansionary periods. Finally, for the commodity index, there are twelve models with positive R_{OS}^2, but only those associated with DFR, CFNAI and WTI are statistically significant. The majority of the models perform better during recessionary periods.

[Insert Table 3.2 Here]

Table 3.3 reports the results of the forecasting performance based on multivariate models for the stock, bond and commodity indices. For the stock index there are 18 models with positive R_{OS}^2 (from 0.05% to 4.05%), with nine models having significant R_{OS}^2 statistics. The best performing models among them are; SCAD, followed by the lasso, MCP, elastic net and ridge regression. Surprisingly, the model estimated by OLS (KS) also has a positive R_{OS}^2 statistic, which, along with SICA, is significant at the 5% level. However, this result is not so strange since several of the original 14 predictors by Goyal and Welch (2008) have been adjusted to stationarity by taking first differences (see the Appendix of Chapter 3) and there are six additional predictors included in the dataset. In terms of R_{OS}^2, the models perform better in recessions, however, only the KS and ICA models have significant MSFE_{adj} statistics. During expansions the KS and penalized regression methods generate statistically significant but lower R_{OS}^2, compared to the recession subperiod. This is consistent with studies such as Rapach, Strauss and Zhou (2010), which report that predictability of stock returns is concentrated in economic recessions. According to the economic evaluation, 18 models generate CER values higher than the historical average benchmark (from 0.03% to 4.63%), while the Sharpe ratios are between 0.26 and 0.70. The highest ΔCER and SR values belong to the portfolio based on the KS model, followed by those based on forecasts generated by SICA, adaptive lasso, 3PRF and lasso models. Overall, for the stock index penalized regressions tend to outperform the majority of the models based on dimensionality reduction and forecast combination methods, while the forecasts generated by the KS model are both statistically significant and yield higher economic value than the other models.

[Insert Table 3.3 Here]

Our findings for the bond index indicate that there are 21 models with significant $\mathrm{MSFE}_{\mathrm{adj}}$ statistics even though there are 15 models with a positive R_{OS}^2 (from 1.45% to 10.62%). The best performing model is the elastic net, followed by ridge regression, the lasso, MCP and bridge regression. During expansionary periods, all models except ICA and RICA have statistically significant R_{OS}^2 while in recessions the lasso, SCAD and MCP are the only models with significant R_{OS}^2. The finding that the majority of the models for the bond index generate higher and statistically significant R_{OS}^2 during expansions is the opposite from recent studies, such as Gargano, Pettenuzzo and Timmermann (2017), which find that R_{OS}^2 values are generally higher during recessions. The ΔCER values for the bond index are relatively low, with seven models yielding positive CER gains, between 0.02 and 0.62. The models with positive ΔCER include ridge regression, the elastic net, PLS, SPLS and KS, while the majority of the portfolios based on forecast combination methods show similar performance to the HA benchmark. The Sharpe ratios are between 0.83 and 1.16, with the highest ratio belonging to the KS model and ridge regression, elastic net, PLS and SPLS having similar performance. Overall, the majority of penalized regressions outperform forecast combination methods, while the performance of all dimensionality reduction methods fails to surpass that of the historical average benchmark in terms of MSFE. The economic evaluation indicates moderate out-of-sample results, especially according to ΔCER, with the portfolios based on forecast combinations having similar performance to the historical average.

Turning to the results for the commodity index, all models except KS and ICA deliver a positive R_{OS}^2 statistic, while there are 18 models with significant MSFE_{adj} statistics. The positive R_{OS}^2 range from 0.14% to 4.36%, with the highest value belonging to PCR, followed by SPCA, RICA, PLS and the 3PRF. In recessionary periods, most of the models generate positive and statistically significant R_{OS}^2, compared to expansionary periods where only the lasso produces statistically significant results. These results are consistent with Gargano and Timmermann (2014), who find that the predictive accuracy of commodity return forecasts tends to be higher during recessions than expansions. The CER gains are positive for 19 of the portfolios based on multivariate prediction models, with values between 1.24% and 3.01%. PLS generates the highest ΔCER, followed by adaptive lasso, lasso and both cluster combinations. The Sharpe ratios of the commodity portfolios are low, with RICA generating the highest ratio, followed by SPLS, PCA, adaptive lasso and lasso. The results for the commodity index favor dimensionality reduction methods, with penalized regressions and forecast combinations yielding similar performance.

3.5. Optimal Asset Allocation

Consider an investor who allocates her wealth among N individual assets with portfolio weight vector: $\mathbf{w} = (w_1, w_2, \dots, w_N)$. The initial wealth is normalized to 1. The benchmark strategy is the naive diversification rule of an equal-weighted portfolio, where $w_j = 1/N$, for $j = 1, 2, \dots, N$. The objective of the main framework is to optimize the trade-off between risk and return. The optimization problem is:

$$\min_{\mathbf{w}}\left[\gamma \Phi_P(\mathbf{w}) - \mathbf{w}'\widehat{\mathbf{R}}\right] \tag{3.33}$$

where Φ_P is the portfolio risk function, $\widehat{\mathbf{R}} = (\hat{\mathbf{r}}_{1,t+1}, \hat{\mathbf{r}}_{2,t+1}, \dots, \hat{\mathbf{r}}_{N,t+1})$ is the matrix of return forecasts for each asset and γ is the coefficient of relative risk aversion. As an alternative to the $1/N$ benchmark, portfolios using historical average forecasts are considered. The two benchmarks are compared to portfolios based on forecasts generated by multivariate prediction models.

All portfolio models include short-selling and leverage constraints to avoid implausible positions. The first constraint sets an upper bound to the sum of the portfolio weights, $\mathbf{w}'\mathbf{I}_N = h$, where $\mathbf{I}_N$ is an N-vector of ones and h denotes the maximum leverage, for example $h = 1$ ensures that the portfolio weights sum up to one, while $h = 1.5$ indicates that the investor cannot borrow more than 50% of total wealth. The second constraint, sets a lower bound to the weight of each asset, $w_j \geq l$, with $j = 1, \dots, N$, where l is the lower bound for each weight, w_j. When $l = 0$, then all weights are positive and the resulting portfolios are long-only, while $l = -0.5$ restricts short sales to 50% of wealth. The portfolio return at $t + 1$ can then be computed as:

$$r_{P,t+1} = \widehat{\mathbf{w}}_t'\mathbf{r}_{t+1} + (1 - \widehat{\mathbf{w}}_t'\mathbf{1}_N)r_{f,t+1} \tag{3.34}$$

where $\mathbf{r}$ is an N-vector of risky asset returns. In the case of $h = 1$, the portfolio return is equivalent to $r_{P,t+1} = \widehat{\mathbf{w}}_t'\mathbf{r}_{t+1}$. As a basic measure of portfolio risk the standard deviation of the portfolio, Markowitz (1952), is used. To construct mean-variance (MV) optimization framework, the risk function of the portfolio, $\Phi_P(\mathbf{w})$, is set to

$$\sigma(\mathbf{w}) = \sqrt{\mathbf{w}'\mathbf{\Sigma}\mathbf{w}} \tag{3.35}$$

where $\mathbf{\Sigma}$ is the $N \times N$ covariance matrix.

3.5.1. Covariance Matrix Estimation

For the mean-variance optimization framework, the forecast of the covariance matrix, $\boldsymbol{\Sigma}_{t+1}$, is estimated using four different approaches. In the first approach $\boldsymbol{\Sigma}_{t+1}$ for assets $i = 1, \ldots, N$ is estimated based on the sample covariance matrix: $\mathbf{S}_{t+1} = \frac{1}{t}\sum_{s=1}^{t}(r_{i,s} - \bar{r}_i)(r_{i,s} - \bar{r}_i)'$, using information up to time t.

The second method used to obtain the estimate of the covariance matrix is the dynamic conditional correlation GARCH model, proposed by Engle (2002). The one-period ahead covariance based on the DCC GARCH model evolves according to:

$$\boldsymbol{\Sigma}_{t+1} = \mathbf{D}_{t+1}\mathbf{R}_{t+1}\mathbf{D}_{t+1} \tag{3.36}$$

where $\mathbf{D}_{t+1}$ is an $N \times N$ diagonal matrix with conditional standard deviation $\hat{\sigma}_{i,t+1}$ on the ith diagonal element and $\mathbf{R}_{t+1}$ is the $N \times N$ correlation matrix, with ones on the diagonal and conditional correlations in the off-diagonal. The estimation of the DCC GARCH has two steps. The first step involves estimating the diagonal elements of the conditional standard deviation matrix, $\mathbf{D}_{t+1}$, where the conditional standard deviation, $\hat{\sigma}_{i,t+1}$, of the ith asset is usually estimated using a GARCH(1,1) model. The second step involves the estimation of the conditional correlation matrix, $\mathbf{R}_{t+1}$. Removing the conditional mean from the N series of asset returns yields the residuals, $\boldsymbol{\varepsilon}_{t+1}$ and the standardized residuals, $\mathbf{u}_{t+1}$, can be obtained using the conditional standard deviation matrix, $\mathbf{D}_{t+1}$: $\mathbf{u}_{t+1} = \mathbf{D}_{t+1}^{-1}\boldsymbol{\varepsilon}_{t+1}$. The conditional correlation structure then is:

$$\mathbf{Q}_{t+1} = (1 - a - b)\bar{\mathbf{Q}} + a\mathbf{u}_t\mathbf{u}_t' + b\mathbf{Q}_t \tag{3.37}$$

$$\mathbf{R}_{t+1} = \mathbf{Q}_{t+1}^{*^{-1}}\mathbf{Q}_{t+1}\mathbf{Q}_{t+1}^{*^{-1}}$$

where $\bar{\mathbf{Q}}$ is the unconditional covariance of the standardized residuals and $\mathbf{Q}_{t+1}^{*}$ is a diagonal matrix composed of the square root of the diagonal elements of $\mathbf{Q}_{t+1}$.

The third approach is based on the shrinkage estimator of the covariance matrix proposed by Ledoit and Wolf (2004), which shrinks the sample covariance matrix towards a one-parameter matrix, where all the variances are the same and all covariances are zero. The shrinkage estimator of the covariance matrix, $\mathbf{\Sigma}^{*}$, can be written as:

$$\mathbf{\Sigma}^{*} = \delta\mathbf{F} + (1-\delta)\mathbf{S}, \tag{3.38}$$

where $\mathbf{S}$ is the sample covariance matrix with entries $s_{i,j}$, $\mathbf{F}$ is the shrinkage target with entries $f_{i,j}$ and δ is a shrinkage constant between 0 and 1. The shrinkage target in this case is set to:

$$\mathbf{F} = \nu\mathbf{I}_N, \text{ with } \nu = \operatorname{tr}(\mathbf{S}\mathbf{I}_N)/N \tag{3.39}$$

where $\mathbf{I}_N$ is the $N \times N$ identity matrix. The optimal shrinkage constant (see Ledoit and Wolf (2003) for details) is:

$$\delta^{*} = \max\left\{0, \min\left\{\frac{\kappa}{T}, 1\right\}\right\}, \text{ with } \kappa = \frac{\pi}{\gamma} \tag{3.40}$$

where π denotes the sum of the asymptotic variances of the entries of the sample covariance matrix:

$$\pi = \sum_{i=1}^{N}\sum_{j=1}^{N}\pi_{i,j}, \text{ with } \pi_{i,j} = \frac{1}{T}\sum_{t=1}^{T}\left[\left(r_{i,t}-\bar{r}\right)\left(r_{j,t}-\bar{r}\right)-s_{i,j}\right]^2 \tag{3.41}$$

and γ measures the misspecification of the shrinkage target, where $\gamma = \sum_{i=1}^{N}\sum_{j=1}^{N}\left(f_{i,j}-s_{i,j}\right)^2$.

The final approach to estimate the covariance matrix is the graphical lasso algorithm, proposed by Friedman, Hastie and Tibshirani (2008), which estimates the sparse precision matrix (inverse of the covariance matrix), using the l_1 (lasso) penalty to enforce sparsity. The graphical lasso problem is to maximize the following penalized log likelihood:

$$\log(\det\mathbf{\Theta}) - \mathrm{tr}(\mathbf{S\Theta}) - \rho\|\mathbf{\Theta}\|_1 \tag{3.42}$$

where $\rho \geq 0$ is a tuning parameter controlling the amount of regularization. Here, $\mathbf{\Theta} = \mathbf{\Sigma}^{-1}$, with entries $\theta_{i,j}$, is the $N \times N$ inverse of the covariance matrix and $\|\mathbf{\Theta}\|_1$ is the l_1 norm of $\mathbf{\Theta}$ – the sum of the absolute value of the elements $\theta_{i,j}$. The penalty parameter ρ is chosen by 10-fold cross validation, to make the value of $-\log(det\mathbf{\Sigma}_1) - tr(\mathbf{\Sigma}_2\mathbf{\Sigma}_1^{-1})$ large, where $\mathbf{\Sigma}_1$ is the covariance matrix estimated using the training set and $\mathbf{\Sigma}_2$ is the covariance estimated over the validation set.

3.6. Portfolio Performance

In this section, we assess the economic value of using return forecasts to construct portfolios and whether commodities add value to a traditional portfolio consisting of bonds and stocks. The evaluation period is the same as the one used for the statistical and economic evaluation of the forecasts. The portfolios are constructed recursively using the related return and covariance forecasts in each iteration, starting in January 1997. The buy-and-hold portfolio returns are calculated for the period of one month and the portfolio is rebalanced monthly until the end of the evaluation period (December 2016). Each portfolio is computed for different combination of weight constraints: unleveraged long-only portfolios ($0 \leq w_j \leq 1$), leverage restricted to 50% of wealth ($0 \leq w_j \leq 1.5$), short selling restricted to 50% of wealth ($-0.5 \leq w_j \leq 1$) and portfolios with both leverage and short selling restricted to 50% of wealth ($-0.5 \leq w_j \leq 1.5$). Two types of investors are considered based on different values of the coefficient of risk aversion, $\gamma = 2$ for an aggressive investor and $\gamma = 10$ for a conservative investor.

The performance of the portfolios is evaluated over the out-of-sample period using the certainty equivalent return of the portfolio and the out-of-sample Sharpe ratio. The Sharpe ratio (SR) is calculated as the fraction of the out-of-sample excess return (average realized return less the risk-free rate) divided by the standard deviation of the out-of-sample portfolio returns:

$$\mathrm{SR} = \frac{\left(\bar{r}_P - r_f\right)}{\hat{\sigma}_P^2} \tag{3.43}$$

where $\bar{r}_P = 1/(Q - q_0)\sum_{t=1}^{Q-q_0} r_{P,t}$ is the average realized return of the portfolio over the out-of-sample period, r_f is the risk free rate and $\hat{\sigma}_P$ is the standard deviation of the portfolio excess returns over the out-of-sample period.

3.6.1. Performance of Stock-Bond-Commodity Portfolios

Tables 3.4 and 3.5 report the performance of stock-bond-commodity portfolios for the full out-of-sample period, based on the certainty equivalent return and Sharpe ratio respectively. Panels A through D present the results based on one of the four different approaches to estimate the covariance matrix. Within each panel the results for the two different types of investors and for different weight constraints are compared.

The first row of Table 3.4 gives the certainty equivalent return of the $1/N$ portfolio, which is 3.87% for an aggressive investor and -0.27% for a conservative investor across all panels, since derivation of the weights for this strategy does not involve any optimization or estimation and ignores the data. The second row of the table gives the certainty equivalent return of the mean-variance portfolio based on the historical average, which varies based on the estimation approach of the covariance matrix, the type of investor and weight constraints. For the sample covariance matrix, the certainty equivalent return of the HA portfolio for an aggressive investor ($\gamma = 2$), is 5.65% for $w_j \in [0,1]$, 5.34% for $w_j \in [0,1.5]$, 5.29% for $w_j \in [-0.5,1]$ and when both leverage and short selling is introduced ($w_j \in [-0.5,1.5]$) the certainty equivalent return increases to 5.85%. On the other hand, a conservative investor ($\gamma = 10$), for weight constraints $0 \leq w_j \leq 1$ the certainty equivalent return is 3.71%, for $0 \leq w_j \leq 1.5$ it is 4.54%, when $-0.5 \leq w_j \leq 1$ it is 3.93% and when weight constraints are set to $-0.5 \leq w_j \leq 1.5$ the portfolio return is 4.71%.

[Insert Table 3.4 Here]

The certainty equivalent return of the mean-variance portfolios, for an aggressive investor, based on the sample covariance matrix and return forecasts generated by multivariate predictive regressions, is between 3.70% and 8.84% for the case when no short sales or leverage is allowed, 5.27% to 10.87% for a 50% leverage constraint, 4.28% to 13.07% for a 50% short-sales constraint, while portfolios with both short sales and leverage allowed generate CER from 5.16% to 15.56%. For leveraged portfolios with short selling all models except ICA yield higher return than the HA, while the models that exhibit the highest performance across all weight constraints are the adaptive lasso, SICA and SPLS. In the case of a conservative mean-variance investor, for weight constraints $0 \leq w_j \leq 1$ the certainty equivalent return is between 3.24% and 6.16%, when leverage is 50% of wealth ($0 \leq w_j \leq 1.5$) it is from 2.46% to 6.11%, when short selling is allowed ($-0.5 \leq w_j \leq 1$) it is from 2.07% to 6.31%, while the return for a less conservative investor with portfolio weights $-0.5 \leq w_j \leq 1.5$, has a range from 1.38% to 7.19%, depending on the model used to construct the return forecasts. For an investor with $\gamma = 10$ the majority of the models outperform the portfolio based on the historical average forecast across the combinations of weight constraints, with models based on PCA and DMSFE combination yielding the highest performance. The differences in CER, when comparing the results for the mean-variance optimization framework across alternative covariance matrix estimates, are not as substantial as those between risk preferences or combinations of weight constraints. For an aggressive investor, portfolios with either short selling or leverage generate higher certainty equivalent return than the unleveraged long-only allocations, while portfolios with both leverage and short selling allowed yield the highest CER. Overall, the observation that can be made is that out-of-sample; the majority of the models utilizing return (and covariance) forecasts outperform the $1/N$ benchmark in terms of certainty equivalent returns.

[Insert Table 3.5 Here]

Table 3.5 reports the annualized Sharpe ratio for the full out-of-sample period. The findings indicate that the majority of the models utilizing mean and covariance forecasts outperform the equal-weighted portfolio benchmark with a ratio of 0.27 (the exception being SPCA for an aggressive investor with no leverage or short selling). The Sharpe ratio for the HA portfolio with $\gamma = 2$ does not change drastically for different weight constraints, the ratio has a range between 0.36 when short selling or leverage is set to 50% of wealth and 0.39 for either $w_j \in [0,1]$ or $w_j \in [-0.5,1.5]$. For a conservative investor the Sharpe ratio is between 0.56 ($w_j \in [0,1]$) and 0.72 ($w_j \in [-0.5,1.5]$). The Sharpe ratio for mean variance portfolios with relative risk aversion of 2, is between 0.25 and 0.6 for unleveraged and long only portfolios, 0.36 to 0.64 for a 50% leverage constraint, 0.31 to 0.69 for a 50% short-sales constraint and 0.36 to 0.75 when both leverage and short selling are restricted to 50%. Some of the portfolios with the highest performance are those with return forecasts generated by the adaptive lasso, SPLS and rank combination scheme. Based on different weight constraints, the Sharpe ratios for a conservative investor are higher compared to those of a more aggressive investor, with values from 0.57 to 0.9 when $w_j \in [0,1]$, 0.65 to 0.92

for $w_j \in [0,1.5]$, 0.61 to 0.92 for $w_j \in [-0.5,1]$ and 0.69 to 1.01 when $w_j \in [-0.5,1.5]$. Adaptive lasso and PCA are among the models that yield the highest ratios across all weight constraints. Overall, when leverage and short sales are allowed, the mean-variance portfolios yield higher Sharpe ratios. Similarly to CER, there are no major changes when comparing the respective Sharpe ratios across different specifications of the covariance matrix. Our findings differ from DeMiguel, Garlappi and Uppal (2009), since we observe that portfolios based on alternative forecasting models for the returns and the covariance matrix consistently outperform the $1/N$ portfolio for different investment constraints and levels or risk aversion. However, in their study the portfolios consist of a larger number of assets and also use sample moments, which may be the reasons for this discrepancy. Additionally, the results are consistent with the recent study by Gao and Nardari (2018) who construct stock-bond-commodity portfolios and find that strategies that employ forecasts of asset return moments outperform strategies with a fixed weighing scheme.

Performance of Stock-Bond-Commodity Portfolios during Business Cycles

To examine the contribution of return forecasts to stock-bond-commodity portfolios during business cycles, the full out-of-sample period is divided into recessionary and expansionary subperiods. Tables 3.6 and 3.7 present the portfolio performance during NBER-dated recessions and expansions, based on the certainty equivalent return and the Sharpe ratio respectively. The portfolios are based on the sample covariance matrix.

[Insert Table 3.6 Here]

[Insert Table 3.7 Here]

The CER and Sharpe ratio of the $1/N$ portfolio are negative during recessions (-17.02% and -0.92 respectively) and positive during expansions (6.43% and 0.57 respectively). The HA portfolio yields positive CER and Sharpe ratios during expansionary periods, with portfolios with $\gamma = 2$ generating higher returns but lower ratios than portfolios with $\gamma = 10$. In recessionary periods the returns and Sharpe ratios are negative, with the portfolios of the aggressive investor significantly underperforming those of the conservative investor in terms of both measures.

For a mean-variance investor with relative risk aversion parameter of 2, the majority of the long-only portfolios produce negative certainty equivalent returns and Sharpe ratios during recessions, with the exception of some models, such as adaptive lasso, SICA, PLS, 3PRF and SPLS, that generate positive values. The CER (Sharpe ratio) of those models is between 4.05% and 12.28% (0.32-0.72) for $w_j \in [0,1]$ and from 5.94% to 14.45% (0.45-0.73) when $w_j \in [0,1.5]$. Other models with positive returns include RICA and rank combination. On the other hand, when short selling is allowed the certainty equivalent return and Sharpe ratio become positive for all portfolios, except for those based on the median combination. For example, when a short selling constraint of 50% is imposed, adaptive lasso, PLS, the 3PRF, SPLS and RICA generate returns between 25.84% and 44.37% and ratios from 1.03 to 1.56. When both short selling and leverage is allowed, the five portfolios with the highest performance yield CER from 23.01% to 45.23% and Sharpe ratios from 0.97 to 1.49. During expansions all mean-variance portfolios with $\gamma = 2$ generate positive CER and Sharpe ratios, with leverage and short selling having a greater (positive) effect on the return of the portfolios. Specifically, the certainty equivalent returns range from 5.14% to 8.81% for unleveraged long-only portfolios, from 7.22% to 11.17% when $0 \leq w_j \leq 1.5$, from 5.09% to 10.65% when $-0.5 \leq w_j \leq 1$ and from 5.83% to 13.20% when both leverage and short sales are restricted to 50% of wealth. On the other hand, the Sharpe ratio does not vary greatly among different sets of weight constraints during expansionary subperiods, with ratios ranging between 0.35 and 0.68, across all models and weight combinations. In recessionary subperiods all portfolios based on multivariate regression models outperform the equal-weighted portfolios or those based on the historical average forecast, in terms of both measures. In expansionary subperiods the majority of the portfolios outperform the $1/N$ benchmark in terms of CER, while the results based on the Sharpe ratio appear mixed. There are no portfolios that outperform the HA when $w_j \in [0,1]$ and for the remaining weight combinations the results are not consistent.

For a conservative investor the pattern of the results during recessions, based on the certainty equivalent return, is similar to that of an aggressive investor. Most of the models generate positive CER when short-selling is allowed, while for long-only allocations the results are mixed. Short sales again have a considerable impact on portfolios, with models based on adaptive lasso, PLS, the 3PRF, rank and cluster combinations leading to a certainty equivalent return between 9.23% and 13.10% for $-0.5 \leq w_j \leq 1$ and from 9.10% to 13.92% for weight constraint $-0.5 \leq w_j \leq 1.5$. Our findings for portfolios with $\gamma = 2$ based on the Sharpe ratio (Table 3.7) paint a similar picture to that based on the CER, with all models generating higher ratios when short selling is allowed. For example, for a 50% short-sale constraint the ratios are from 0.05 to 1.61 and when a 50% constraint is imposed to both short sales and leverage the ratios range from 0.12 to 1.66. During expansionary periods all portfolios result in positive CER and Sharpe ratios, yielding returns from 1.31% to 7.05% and ratios from 0.57 to 1.02, varying based on the weight constraints, with leveraged portfolios exhibiting higher values in both measures. The models with consistent high performance across all weight combinations are the elastic net for the certainty equivalent return and PCA for the Sharpe ratio. In recessions all models, except KS, outperform both the equal-weighed and the HA portfolios, however, in expansions the results appear mixed, with the exception that when short selling is allowed the majority of the models outperform the naïve portfolio.

3.6.2. Performance of Stock-Bond Portfolios

In this section, we focus on the performance of traditional portfolios consisting of only stocks and bonds and compare it with that of the respective stock-bond-commodity portfolios, in order to determine whether including commodities in the portfolio adds economic value. The analysis is conducted for the full out-of-sample period and for the NBER-dated recession and expansion subperiods.

[Insert Table 3.8 Here]

Table 3.8 presents the certainty equivalent return and Sharpe ratio for the stock-bond portfolios for the full sample. For traditional portfolios that utilize forecasts from multivariate predictive regressions, the results indicate that the majority of the models outperform the HA portfolio in terms of both performance measures. When comparing the alternative portfolios against the equal-weighted allocation, in terms of CER, most of the portfolios outperform the benchmark, however, in terms of Sharpe ratio, the majority of the models for an aggressive investor fail to produce higher ratios than the equal-weighted benchmark, but most of the portfolios with relative risk aversion of 10 outperform the $1/N$ portfolio.

According to the $1/N$ strategy, reported in the first row of the table, an investor would be reluctant to include commodities in a portfolio of stocks and bonds, based on either performance measure, which is consistent with the findings of Gao and Nardari (2018) that a fixed weight allocation favors traditional portfolios. For a mean-variance allocation based on the historical average forecast, commodities would add value to traditional portfolio only for an aggressive investor and for weight combinations that allow short selling. This result is consistent with the findings in Daskalaki and Skiadopoulos (2011) and Gao and Nardari (2018), who find that using sample moments to form the long-only portfolios leads to traditional strategies dominating the commodity augmented allocations. When comparing mean-variance portfolios based on alternative forecasts, our findings are not as conclusive and vary depending on the degree of risk aversion, the weight constraints and the models used to generate the return forecasts. For an aggressive investor, when no short selling or leverage is allowed 21 out of the 24 stock-bond portfolios outperform those that include commodities, the exceptions being the portfolios based on the 3PRF, SPLS and RICA, while with 50% leverage only SPLS yields better results when commodities are included. For both weight combinations with a 50% short-sales constraint the results are reversed with 19 and 18 portfolios that include commodities outperforming their stock-bond counterparts, for $w_j \in [-0.5,1]$ and $w_j \in [-0.5,1.5]$ respectively. The results based on the Sharpe ratio follow a similar pattern, with most of the stock-bond portfolios producing higher ratios when no short selling is allowed (23 models for both sets of weight constraints) and when a 50% short-sales constraint is imposed, the results favor the stock-bond-commodity portfolios, with 16 and 17 commodity-augmented portfolios having higher ratios for $-0.5 \leq w_j \leq 1$ and $-0.5 \leq w_j \leq 1.5$ respectively. For a risk averse investor, the majority of the traditional portfolios, for $w_j \in [0,1]$ and $w_j \in [0,1.5]$ respectively, outperform the portfolios that include commodities in terms of certainty equivalent return. When short selling is allowed, there are some portfolios that favor a commodity-augmented allocation.

Performance of Stock-Bond Portfolios during Business Cycles

Tables 3.9 and 3.10 report the performance, based on certainty equivalent return and Sharpe ratio respectively, for the stock-bond portfolios during business cycles. For both benchmark portfolios, the CER and Sharpe ratio are positive during expansions and negative in recessions. For the mean-variance allocations utilizing forecasts from multivariate predictive regressions, there are models that exhibit negative returns and Sharpe ratios during recessionary periods, while in expansionary periods all models yield positive returns.

[Insert Table 3.9 Here]

[Insert Table 3.10 Here]

During recessions all commodity-augmented portfolios of an aggressive investor, except most of those based on KS and ICA, median and $C(3, PB)$ forecasts, outperform the traditional portfolios across all weight constraints, according to both performance measures. For a conservative investor, commodities add value to a portfolio when short-selling is allowed. In expansionary subperiods, commodity augmented portfolios outperform traditional portfolios in the case of an aggressive investor that utilizes machine learning forecasts and for weight combinations with 50% short selling. However, the difference in return values between stock-bond and stock-bond-commodity allocations is not as great in expansions as in recessions, where the commodity-augmented allocations have better performance. In terms of Sharpe ratios, the majority of the traditional portfolios perform better, with the exception of those belonging to an aggressive investor based on forecast combinations that allow short selling. Overall, commodities benefit a traditional portfolio, when short selling is allowed and during NBER-dated recessions. The findings that commodities add greater value to a stock-bond portfolio when short selling is allowed and particularly for an aggressive investor are consistent with Bessler and Wolff (2015).

3.6.3. The Effect of Transaction Costs on Stock-Bond-Commodity Portfolios

To examine the effect of transaction costs on the asset allocation strategies, we compute the turnover as a measure of the amount of trading required to implement a particular strategy. Following DeMiguel, Garlappi and Uppal (2009), the portfolio turnover is defined as the average absolute change of the portfolio weights over the $Q - q_0$ rebalancing periods across the N assets, given as follows:

$$\mathrm{PT_P} = \frac{1}{Q - q_0 - 1} \sum_{t=1}^{Q-q_0-1} \sum_{j=1}^{N} \left| w_{j,t+1} - w_{j,t} \right| \tag{3.44}$$

where $w_{j,t+1}$ is the weight in asset j at time $t + 1$ and $w_{j,t}$ is the weight in asset j at time before rebalancing at $t + 1$. The transaction costs are set to 50 bps for each asset. When a portfolio is rebalanced at $t + 1$, $\left|w_{j,t+1} - w_{j,t}\right|$ denotes the magnitude of trading asset j. Given a transaction cost of c, the trading cost of the entire portfolio is $c \sum_{j=1}^{N} \left|w_{j,t+1} - w_{j,t}\right|$. The return of the portfolio after transaction costs is as follows:

$$r_{P,t+1}^{TC} = \left(1 + r_{P,t+1}\right)\left(c \sum_{j=1}^{N} \left| w_{j,t+1} - w_{j,t} \right| \right) - 1 \tag{3.45}$$

We report results for portfolios including transaction costs for two rebalancing frequencies, namely monthly and quarterly, in Tables 3.11 and 3.12, respectively.

Quite interestingly, our results for monthly-rebalanced portfolios favor forecast combination schemes to more sophisticated methods as these lead to portfolios with high turnover. However, when the rebalancing frequency is reduced to quarterly, the certainty equivalent return and Sharpe ratio of the majority of the portfolios improves, with portfolios of an aggressive investor based on machine learning techniques yielding superior performance to the forecast combination methods with monthly rebalancing. The result that a decrease in rebalancing frequency can lead to better performance in a dynamic asset allocation setting with transaction costs is also supported by Almadi, Rapach and Suri (2014).

[Insert Table 3.11 Here]

[Insert Table 3.12 Here]

3.6.4. Conditional Value-at-Risk Portfolios

As a robustness check, we consider an alternative measure of portfolio risk, namely the Conditional Value-at-Risk (CVaR) of the portfolio. CVaR is defined as the conditionally expected value losses greater or equal to the Value-at-Risk (VaR) at a specific confidence interval. Following Rockafellar and Uryasev (2000, 2002), CVaR is estimated based on the following approximation:

$$\mathrm{CVaR}_a(\mathbf{w}, \mathrm{VaR}_a) = \mathrm{VaR}_a + \frac{1}{J(1-a)} \sum_{j=1}^{J} \left[-\mathbf{w}'\mathbf{r}_j - VaR_a \right]^+ \tag{3.46}$$

where $[t]^+ = \max(0, t)$, a is a probability level, J is the number of scenarios and $\mathbf{r}_j$ is the vector of asset returns in the jth scenario. To estimate CVaR of the portfolio, 5000 scenarios are generated using Monte Carlo simulation based on the multivariate normal distribution with mean varying according to the return forecast and sample variance-covariance matrix. For the mean-CVaR (MCVaR) optimization framework, we set $\Phi_P(\mathbf{w}) = \mathrm{CVaR}_a(\mathbf{w}, \mathrm{VaR}_a)$.

In order to investigate the ability of the proposed models to assess tail risk, we calculate the CVaR measure of all portfolio strategies, in addition to the certainty equivalent return and Sharpe ratios. The a Conditional Value-at-Risk of a portfolio is given by:

$$\text{CVaR}_a = (1-a)^{-1} f\big(F^{-1}(1-a)\big)\hat{\sigma} - \bar{r}_P \tag{3.47}$$

where F is the cumulative standard normal distribution function and f is the probability density function of the standard normal distribution. The CVaR is calculated at the 95% confidence level.

Table 3.13 reports the certainty equivalent return and Sharpe ratio for the mean-CVaR optimization framework for the full sample. The mean-CVaR portfolios based on forecasts from multivariate regression models outperform the equal-weighted and HA portfolios, in terms of certainty equivalent return. Furthermore, our results indicate that the Sharpe ratio of all models, except ICA and PLS, is higher than the $1/N$ and HA models. Weight combinations with a 50% leverage constraint appear to lead to higher CER, however, Sharpe ratios do not vary greatly across different weight constraints. Comparing the results of the CVaR portfolios for the CER and Sharpe ratio (Table 3.13) with those of the mean-variance portfolios (Panel A of Tables 3.4 and 3.5), we note that while mean-CVaR portfolios generate considerably lower returns than mean-variance portfolios, the Sharpe ratios produced by the former optimization framework are higher.

[Insert Table 3.13 Here]

Table 3.14 compares the CVaR at the 95% confidence level measure of the portfolios that use variance or CVaR as a risk measure. Overall, the $CVaR_{95}$ values of the mean-CVaR portfolios are lower than those of the mean-variance portfolios and the degree of relative risk aversion has a greater impact on mean-variance portfolios than on mean-CVaR portfolios, with the majority of the portfolios of a conservative investor generating lower $CVaR_{95}$ values than those of an aggressive investor. The mean-CVaR allocations consistently outperform the equal-weighted portfolio for both types of investors, while the majority of the long-only mean-CVaR portfolios tend to produce lower $CVaR_{95}$ than the HA portfolio.

[Insert Table 3.14 Here]

3.7. Conclusion

This study examines whether return forecasts generated by shrinkage, variable selection and dimensionality reduction methods from the machine learning literature add value in portfolios consisting of stock, bond and commodity indices. We first examine the benefits of forecasting the returns for each individual index. Our results indicate that the majority of the proposed prediction models outperform the historical average benchmark, with shrinkage and variable selection methods yielding the highest performance for the stock and bond indices, while for the commodity index the dimensionality reduction methods achieve superior performance. For the stock and commodity indices, the proposed models perform better during recessions, while the results for the bond index are mixed.

To examine whether return forecasts provide any benefits in an asset allocation setting, stock-bond-commodity portfolios are constructed based on the proposed models and their performance is compared to that of the equal-weighted portfolio and a mean-variance portfolio based on the historical average. For commodity-augmented portfolios, the majority of the models utilizing return forecasts outperform the $1/N$ benchmark in terms of certainty equivalent returns. The models that tend to outperform the HA benchmark are those based on shrinkage and dimensionality reduction for an aggressive investor, while portfolios of a conservative investor favor forecast combination methods. In terms of Sharpe ratios, the majority of the models outperform the equal-weighted and HA portfolio benchmarks. Portfolios with either short selling or leverage generate higher CER than the unleveraged long-only allocations, while portfolios with both leverage and short selling yield the highest return. Sharpe ratios are higher for a conservative investor and when leverage and short sales are allowed the ratios tend to increase. There are no major changes when comparing either performance measure across different specifications of the covariance matrix. When transaction costs are taken into account, the results for monthly-rebalanced portfolios favor forecast combination methods, instead of methods that combine information due to the latter methods leading to portfolios with higher turnover. When the rebalancing frequency is reduced to quarterly, the models with the best performance for an aggressive investor are those based on shrinkage and dimensionality reduction methods. When CVaR is used as a risk measure, the vast majority of the mean-CVaR portfolios based on forecasts from multivariate regression models outperform the equal-weighted and HA portfolios.

For an aggressive investor during recessionary periods, most of the long-only allocations that yield positive values are based on variable selection, shrinkage and dimensionality reduction methods. However, when short selling is allowed, CER and Sharpe ratios become positive for the majority of the models. All portfolios based on multivariate regression models outperform both benchmarks. During expansionary periods, all mean-variance portfolios generate positive certainty equivalent returns and Sharpe ratios, with leverage and short-selling having a greater (positive) effect on the return of the portfolios. The majority of the portfolios outperform the $1/N$ benchmark in terms of CER, while the results based on the Sharpe ratio appear mixed. When comparing the results to the HA average portfolio, only in combinations with leverage or short sales can we find models with higher performance. For a conservative investor, the CER for the majority of the models is positive during recessions, with short sales having a considerable impact on the returns and Sharpe ratios of all portfolios. During expansions, all portfolios yield positive returns and Sharpe ratios and the performance between the different models varies less than in recessions. In recessions, all models based on alternative forecasts outperform both the equal-weighed and the HA portfolios, however, in expansions the results appear mixed, except when short selling is allowed in which case all models outperform the naive portfolio.

To examine whether commodities add value to a stock bond portfolio, our analysis is replicated for traditional portfolios. For stock-bond portfolios that utilize forecasts from multivariate predictive regressions, our results indicate that the majority of the models outperform the HA portfolio. The portfolios that outperform the naive portfolio in terms of CER are those of an aggressive investor, while the portfolios that yield better results than the $1/N$ portfolio in terms of Sharpe ratio belong to a conservative investor. During expansions, stock-bond portfolios exhibit positive returns and Sharpe ratios, however, in recessions the majority of the portfolios for $\gamma = 2$ generate negative values in both measures, for $\gamma = 10$ most returns and Sharpe ratios are positive and improve as the weight constrains allow leverage or short selling. When comparing the results of stock-bond portfolios with those that include commodities for the full sample, commodities add value to a traditional portfolio when short selling is allowed, with portfolios for $\gamma = 2$ benefiting more from the inclusion of commodities. During recessions, the majority of the commodity-augmented portfolios outperform the traditional portfolios across all weight constraints, according to both performance measures. In expansions, the long-only traditional portfolios outperform those that include commodities, while short selling provides a greater benefit to commodity-augmented portfolios. However, the difference in return values between stock-bond and stock-bond-commodity allocations is greater in recessions, where the commodity-augmented allocations have better performance, than in expansions.

Overall, the return forecasts from the majority of alternative multivariate prediction methods outperform the historical average benchmark. When the forecasts are used to construct optimal portfolios, most of the models outperform the $1/N$ and HA portfolio benchmarks, with allocations that allow short sales or leverage further improving the performance of portfolios based on machine learning methods. When introducing transaction costs to portfolios with monthly rebalancing the results tend to favor forecast combination techniques, however, reducing the rebalancing frequency to quarterly leads the portfolios of an aggressive investor based on shrinkage and dimensionality reduction methods to generate the highest performance. Finally, our findings indicate that commodities would benefit a traditional portfolio when short selling is allowed and during recessionary periods.

Chapter 3 Tables

Table 3.1: Descriptive Statistics

Index	N	Mean	Median	Min.	Max.	Std. Dev.	Skew.	Kurt.
Stock	480	0.98	1.29	-21.58	13.52	4.30	-0.58	5.10
Bond	480	0.61	0.60	-6.08	11.34	1.57	0.76	10.00
Commodity	480	0.56	0.64	-28.20	22.94	5.59	-0.24	5.06

Correlation matrix

	Stock	Bond	Commodity
Stock	1.00		
Bond	0.21	1.00	
Commodity	0.18	-0.02	1.00

Notes: The table reports the summary statistics for the returns of the three indices; the S&P 500 Total Return Index (Stock), the Bloomberg Barclays US Aggregate Bond Index (Bond) and the S&P Goldman Sachs Commodity Total Return Index (Commodity) and their sample correlation matrix. The sample period is from January 1977 to December 2016. The mean, median, minimum, maximum and standard deviation of returns are reported as percentages.

Table 3.2: Out-of-Sample Forecasting Performance: Univariate Prediction Models

Stock				Bond				Commodity			
Model	R^2_{OS}	R^2_{OS} REC	R^2_{OS} EXP	Model	R^2_{OS}	R^2_{OS} REC	R^2_{OS} EXP	Model	R^2_{OS}	R^2_{OS} REC	R^2_{OS} EXP
DP	-0.18	1.47	-0.79	DY	-0.72*	3.15	-1.67	DY	0.06	2.28**	-0.75
DY	-0.16	0.4	-0.37	TBL	-0.01	-0.27	0.05	TBL	0.01	0.36**	-0.12
EP	-0.76	-2.49	-0.12	LTY	-0.5	0.32	-0.7	LTR	0.85	4.32*	-0.43
DE	-1.7	-4.49	-0.66	LTR	-1.44	-1.26	-1.49	TMS	-0.19	-0.33	-0.13
SVAR	1.69	6.84	-0.24	TMS	-0.72	-0.55	-0.76	DFR	2.75*	10.55	-0.11
BM	0.63	3.58*	-0.46	DFY	-0.27	-11.39	2.45***	INFL	-0.93	-0.5	-1.09
NTIS	-0.55	0.75	-1.04	DFR	-1.09	-4.48	-0.26	IP	1.46	7.3	-0.68
TBL	-0.64	-1.08	-0.47	PPI	1.25**	6.09	0.07**	M1	0.65	3.49	-0.39
LTY	-0.12	0.27	-0.27	CAP	-4.38	-20.64	-0.41	UR	-0.2	0.93	-0.62
LTR	-0.21	0.21	-0.36	IS	3.61**	11.37	1.72**	WTI	1.92**	5.04**	0.78
TMS	-0.39	-0.38	-0.39	M1	-4.89	-21.49	-0.83	USDAUD	0.59	1.29	0.33
DFY	-1.34	-3.93	-0.37	UR	-2.13	-4	-1.68	USDCAD	0.51	0.85	0.38
DFR	-1.97	-3.37	-1.45	SP1	1.41***	-3.41	2.59***	USDINR	0.64	3.5	-0.42
INFL	0.15	0.86	-0.12	SP5	1.77***	1.1	1.93***	USDNZD	-0.34	1.98	-1.18
IP	0.63	5.63	-1.23	SP500	1.74**	3.3	1.36***	REA	-0.56	-1.03	-0.39
M1	-0.31	3.74	-1.82	VXO	0.19	-1.84	0.69**	CFNAI	2.34*	9.34**	-0.23
UR	-0.99	-1.95	-0.63	Umacro	0.42	-6.09	2.01***	Umacro	0.52	5.09	-1.16
VXO	-1.26	-3.22	-0.53	Ufin	0.26	-5.48	1.66**	Ufin	-0.53	0.8	-1.01
Umacro	0.28	2.23	-0.45								
Ufin	1.63	7.53	-0.57								

Notes: The table reports the R^2_{OS} as a percentage, for the alternative model against the historical average benchmark. Positive values indicate that the univariate model outperforms the benchmark. The alternative model is a univariate return prediction model that includes a constant and the predictor variable listed in each row. The out-of-sample period is from January 1997 to December 2016. The R^2_{OS} is reported for the NBER-dated recessions and expansions. The hypothesis of equal predictive ability is measured based on the Clark and West (2007) test. *, ** and *** indicate significance at the 10%, 5% and 1% levels.

Table 3.3: Out-of-Sample Forecasting Performance: Multivariate Prediction Models

	Stock					Bond					Commodity				
Model	R^2_{OS}	R^2_{OS} REC	R^2_{OS} EXP	ΔCER	SR	R^2_{OS}	R^2_{OS} REC	R^2_{OS} EXP	ΔCER	SR	R^2_{OS}	R^2_{OS} REC	R^2_{OS} EXP	ΔCER	SR
KS	2.23**	15.14*	-2.58**	4.63	0.70	-6.95***	-16.44	-4.63***	0.15	1.16	-1.72*	7.61*	-5.15	-3.66	0.00
MC	0.32	1.72	-0.21	0.16	0.41	3.34***	1.36	3.82***	0.00	0.90	1.11**	3.99**	0.06	2.25	0.03
MDC	-0.08	-0.04	-0.10	-0.72	0.37	1.68**	-1.53	2.47***	0.00	0.90	0.47*	1.75**	0.00	1.57	-0.03
TMC	0.16	1.20	-0.23	-0.13	0.40	2.91***	1.45	3.26***	0.00	0.90	0.93**	3.36**	0.04	2.02	0.01
Rank	0.29	3.31	-0.83	0.90	0.46	1.45	1.71	1.39***	0.00	0.90	0.99	4.43*	-0.27	2.07	0.02
C(2,PB)	0.05	2.74	-0.95	0.03	0.41	2.42**	1.26	2.70***	0.02	0.91	1.53**	5.58**	0.04	2.40	0.05
C(3,PB)	-0.11	3.37	-1.41	0.04	0.41	3.45***	4.72	3.14***	-0.05	0.90	2.07**	7.28**	0.15	2.47	0.07
D(1)	0.31	1.73	-0.22	0.16	0.41	3.29***	1.35	3.77***	0.00	0.90	1.12**	4.00**	0.06	2.25	0.03
D(0.9)	0.32	1.78	-0.23	0.19	0.41	3.34***	0.99	3.91***	0.00	0.90	1.11**	3.99**	0.06	2.26	0.03
Ridge	3.46*	11.99	0.28**	1.84	0.52	10.40***	6.09	11.46***	0.62	1.15	1.73**	5.33**	0.40	2.37	0.06
Lasso	3.96*	12.17	0.91**	2.35	0.54	8.79***	10.66*	8.34***	-0.34	0.96	0.57*	-0.97	1.14*	2.56	0.10
EN	3.62*	10.76	0.96**	2.26	0.54	10.62***	9.93	10.78***	0.21	1.07	0.14	0.34	0.07	1.97	0.03
Alasso	0.32*	8.77	-2.83*	3.37	0.61	3.68***	7.49	2.75***	-0.57	0.93	1.93**	9.34**	-0.79	2.71	0.14
Bridge	1.35	9.44	-1.66*	0.91	0.45	8.06***	11.25	7.29***	-0.27	0.96	1.39*	5.79**	-0.22	2.25	0.05
SCAD	4.05*	14.57	0.13**	1.97	0.53	6.81***	11.81*	5.59***	-0.46	0.96	0.17	-1.01	0.60	1.98	0.04
MCP	3.87*	12.12	0.79**	2.06	0.52	8.31***	10.67*	7.74***	-0.19	1.01	0.16	-1.01	0.59	1.97	0.04
SICA	1.49**	12.80	-2.72**	4.04	0.66	-4.55***	-18.82	-1.06***	-0.84	0.87	0.13*	6.34**	-2.15	-0.64	0.09
PLS	0.92	16.48	-4.89	2.02	0.52	-1.13***	-23.26	4.27***	0.28	1.13	3.87**	21.74**	-2.69	-0.68	0.05
3PRF	2.88*	22.76	-4.53	2.59	0.56	-1.57***	-11.28	0.79***	0.13	1.08	3.12*	20.78**	-3.36	-0.46	0.01
SPLS	0.64	12.12	-3.64	2.29	0.54	-6.97***	-41.62	1.48***	0.21	1.14	1.32*	9.24**	-1.59	2.16	0.19
PCA	-3.61	-10.17	-1.16	-2.88	0.26	-0.27**	-24.11	5.55***	0.01	0.92	4.36**	16.94**	-0.27	3.01	0.14
SPCA	-2.50	-5.74	-1.29	-2.63	0.26	-6.39**	-41.90	2.27***	-0.08	0.92	4.04**	16.83**	-0.66	1.24	0.00
ICA	-4.51	-7.65*	-3.33	-0.63	0.39	-21.10	-74.91	-7.96	-0.41	0.83	-0.67	3.47*	-2.18	-0.87	-0.14
RICA	-0.09	1.38	-0.64	-0.52	0.38	-16.04	-29.40	-12.78	-0.38	0.86	3.24**	11.63**	0.16	2.36	0.22

Notes: The table reports the R^2_{OS} as a percentage, for the alternative model against the historical average benchmark. Positive values indicate that the alternative model outperforms the benchmark. The alternative models are based on a range of multivariate estimation methods, using a different set of predictors for each index. The out-of-sample period is from January 1997 to December 2016. The R^2_{OS} is also reported for the NBER-dated recessions and expansions. The hypothesis of equal predictive ability is measured based on the Clark and West (2007) test. *, ** and *** indicate significance at the 10%, 5% and 1% levels.
Additionally, the table reports the performance for mean-variance portfolios, with monthly rebalancing, for an investor with risk aversion coefficient of five. The ΔCER is the gain in the percentage annualized certainty equivalent return (CER) and SR is the annualized Sharpe Ratio. A 0.00 indicates a number less than 0.005 in absolute value.

Table 3.4: Mean-Variance Portfolio Performance based on Certainty Equivalent Return: Stock-Bond-Commodity Portfolios

	Sample Covariance								DCC-GARCH Covariance							
	γ=2				γ=10				γ=2				γ=10			
	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5
EW	3.87	3.87	3.87	3.87	-0.27	-0.27	-0.27	-0.27	3.87	3.87	3.87	3.87	-0.27	-0.27	-0.27	-0.27
HA	5.65	5.34	5.29	5.85	3.71	4.54	3.93	4.71	5.28	5.67	5.88	6.46	4.36	5.18	4.52	5.32
KS	6.45	7.44	6.87	7.36	3.24	2.46	2.17	1.68	6.22	6.80	6.32	6.61	2.76	1.85	1.98	1.45
MC	7.22	8.39	10.62	11.31	4.39	5.37	**5.17**	**6.03**	6.78	8.30	10.61	11.70	4.98	**6.01**	5.75	6.63
MDC	6.46	7.02	7.96	8.73	4.14	5.01	4.59	5.38	5.67	7.24	8.03	8.67	4.63	5.53	5.03	5.83
TMC	7.03	8.06	9.75	10.41	4.33	5.28	5.02	5.86	6.61	8.13	9.82	10.77	4.94	5.90	5.57	6.42
Rank	7.10	**9.45**	**11.62**	**12.79**	4.45	5.37	**5.11**	**6.08**	6.82	8.90	**11.52**	**13.07**	5.08	**6.21**	**6.18**	**7.19**
C(2,PB)	6.53	7.49	9.85	11.06	3.99	5.02	5.10	6.01	6.27	7.53	9.92	11.03	4.60	5.67	**5.78**	**6.74**
C(3,PB)	6.38	7.81	9.78	10.91	3.74	4.50	4.58	5.54	6.36	7.97	10.13	11.33	4.36	5.46	5.65	**6.71**
D(1)	7.19	8.37	10.61	11.32	4.40	5.37	**5.18**	**6.04**	6.77	8.32	**10.62**	11.73	4.99	**6.02**	**5.76**	6.64
D(0.9)	7.09	8.25	10.53	11.30	4.45	**5.43**	**5.24**	**6.10**	6.69	8.36	**10.65**	**11.83**	5.03	**6.06**	**5.80**	**6.68**
Ridge	5.58	7.29	7.71	9.33	4.57	4.87	4.57	5.50	4.86	6.65	7.72	9.70	4.68	5.32	4.87	5.54
Lasso	6.18	8.87	9.54	**11.56**	**5.08**	5.23	4.50	5.23	6.93	9.26	9.08	11.49	**5.38**	5.90	4.89	5.57
EN	6.47	8.85	9.44	11.12	**4.99**	**5.42**	4.84	5.83	6.41	8.74	9.06	11.32	**5.37**	5.97	4.91	5.58
Alasso	**8.68**	**10.76**	**12.82**	**15.56**	**6.16**	**5.99**	4.99	5.07	**8.51**	**10.85**	**12.84**	**16.46**	**5.99**	5.54	5.11	5.11
Bridge	4.35	6.79	7.16	9.40	4.80	5.18	4.10	4.74	4.54	5.96	6.67	8.55	4.80	5.38	4.10	4.76
SCAD	5.73	8.03	9.15	10.70	4.85	4.90	3.93	4.88	5.86	7.38	7.50	9.39	4.83	5.79	4.75	5.41
MCP	5.79	8.50	9.28	11.37	**5.22**	**5.61**	4.73	5.68	5.94	8.02	8.15	10.76	**5.44**	**6.48**	5.32	6.39
SICA	**7.76**	**9.88**	**10.72**	**12.31**	4.79	3.33	2.37	2.01	**7.93**	**9.65**	10.25	**12.38**	4.45	3.91	3.38	3.23
PLS	**7.64**	9.29	**11.08**	11.55	4.41	4.69	3.91	4.10	**7.78**	**9.42**	10.39	11.55	4.68	4.43	3.52	3.51
3PRF	**8.84**	**9.77**	10.01	11.09	4.40	4.69	3.44	3.60	**8.26**	**9.53**	9.79	10.29	4.43	4.12	3.01	2.73
SPLS	**8.55**	**10.87**	**13.07**	**14.90**	4.60	3.24	2.07	1.38	**8.25**	**10.88**	**13.08**	**14.49**	4.73	3.84	2.64	2.41
PCA	5.00	7.74	8.97	11.38	**5.20**	**6.11**	**6.31**	**7.19**	5.35	6.84	7.83	9.36	**5.12**	5.81	**5.94**	**6.73**
SPCA	3.70	5.55	6.73	8.96	4.46	5.08	4.93	5.95	4.83	6.36	6.61	7.98	4.28	5.16	4.98	5.88
ICA	5.07	5.27	4.28	5.16	3.80	4.13	3.80	4.23	5.71	6.76	5.29	5.34	3.46	3.92	3.98	4.59
RICA	6.63	8.35	9.24	10.52	4.19	4.74	4.46	5.02	7.01	8.57	10.19	11.79	5.09	5.94	5.48	6.32

Table 3.4 (continued): Mean-Variance Portfolio Performance based on Certainty Equivalent Return: Stock-Bond-Commodity Portfolios

	Ledoit-Wolf Shrinkage Covariance								Graphical Lasso Covariance							
	γ=2				γ=10				γ=2				γ=10			
	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5
EW	3.87	3.87	3.87	3.87	-0.27	-0.27	-0.27	-0.27	3.87	3.87	3.87	3.87	-0.27	-0.27	-0.27	-0.27
HA	5.65	5.45	5.11	5.69	3.60	4.10	3.73	4.17	5.38	5.51	4.96	5.65	4.10	5.00	4.23	5.10
KS	6.44	7.39	6.78	6.88	2.85	1.75	2.11	1.27	6.43	7.75	7.42	8.30	4.02	4.00	3.90	3.52
MC	7.27	8.57	10.45	11.22	4.44	5.13	5.06	5.62	7.29	8.49	10.25	10.90	4.80	5.85	5.41	6.34
MDC	6.49	7.13	7.84	8.59	4.09	4.64	4.43	4.90	6.64	7.19	7.70	8.47	4.53	5.48	4.83	5.70
TMC	7.08	8.21	9.52	10.24	4.37	5.04	4.90	5.44	7.18	8.05	9.29	9.97	4.77	5.78	5.26	6.18
Rank	7.08	**9.41**	**11.38**	**12.63**	4.57	**5.32**	**5.35**	**6.06**	7.39	**9.57**	**11.31**	**12.47**	4.81	**5.93**	**5.56**	**6.59**
C(2,PB)	6.57	7.55	9.76	11.01	4.08	4.87	**5.10**	**5.73**	6.62	8.01	10.13	10.96	4.51	5.57	5.43	6.41
C(3,PB)	6.38	7.74	9.66	10.82	3.84	4.47	4.80	5.48	6.52	8.15	9.93	10.99	4.17	5.28	5.17	6.19
D(1)	7.26	8.55	10.45	**11.24**	4.45	**5.14**	5.07	5.64	7.27	8.50	10.27	10.93	4.81	5.86	5.42	6.36
D(0.9)	7.16	8.42	10.39	11.23	4.50	**5.21**	**5.13**	**5.70**	7.19	8.46	10.27	10.93	4.86	5.92	5.48	6.41
Ridge	5.44	7.14	7.61	9.21	4.45	4.48	4.76	5.35	5.70	7.92	8.28	10.02	4.90	5.61	5.35	6.33
Lasso	6.01	8.46	9.16	11.02	**4.76**	4.60	4.54	4.91	6.39	**9.38**	10.09	**12.28**	**5.29**	5.92	5.30	6.28
EN	6.29	8.60	9.16	10.73	**4.73**	4.83	4.79	5.37	6.68	9.28	9.78	11.58	**5.39**	**6.08**	**5.53**	**6.59**
Alasso	**8.53**	**10.54**	**12.34**	**15.11**	**5.70**	**5.22**	4.74	4.63	**8.55**	**11.16**	**13.31**	**16.69**	**6.47**	**6.58**	**5.81**	6.18
Bridge	4.10	6.23	6.74	8.87	4.44	4.46	3.98	4.44	4.92	7.53	7.87	10.05	5.05	5.68	4.69	5.64
SCAD	5.58	7.58	8.79	10.29	4.54	4.28	3.99	4.46	5.92	8.85	9.58	11.20	5.14	5.76	4.90	5.92
MCP	5.62	8.08	8.98	10.95	**4.90**	4.96	4.81	5.29	6.01	9.17	9.95	11.91	**5.46**	**6.44**	**5.67**	**6.66**
SICA	**7.69**	**9.55**	**10.56**	**11.86**	4.42	2.82	2.38	1.71	**7.94**	**10.23**	**11.05**	**13.40**	4.98	4.48	4.08	4.11
PLS	**7.68**	9.22	**10.77**	11.24	4.11	4.07	3.53	3.40	**7.65**	9.35	**10.84**	11.80	4.86	5.24	4.48	4.95
3PRF	**8.89**	**9.80**	9.77	10.66	4.05	3.96	2.99	2.88	**8.51**	9.26	9.76	11.03	4.75	5.04	4.05	4.38
SPLS	**8.51**	**10.73**	**12.75**	**14.61**	4.45	2.93	1.98	1.47	**8.58**	**10.99**	**13.11**	**14.62**	4.63	3.73	2.69	3.06
PCA	4.93	7.63	8.88	11.18	**5.09**	**5.82**	**6.31**	**7.05**	5.55	8.31	9.47	11.65	**5.43**	**6.44**	**6.38**	**7.38**
SPCA	3.53	5.25	6.43	8.61	4.35	4.86	**5.13**	**5.94**	4.18	6.28	7.24	9.33	4.73	5.79	5.45	**6.53**
ICA	4.97	5.03	3.88	4.72	3.50	3.66	3.69	3.95	5.12	5.68	4.87	5.94	3.93	4.46	4.06	4.65
RICA	6.56	8.19	9.04	10.38	3.95	4.18	4.57	4.99	6.76	8.57	9.11	10.40	4.62	5.28	5.01	5.72

Notes: This table reports the certainty equivalent return of the stock-bond-commodity mean-variance portfolios with monthly rebalancing. The out-of-sample period is from January 1997 to December 2016. The benchmarks are the equally-weighted portfolio and the mean-variance portfolio based on the historical average forecast. The alternative mean-variance allocations utilize return forecasts generated by multivariate prediction models. Each panel reports the certainty equivalent return of portfolios relying on different estimates of the covariance matrix. The portfolio performance is reported for different levels of risk aversion ($\gamma = 2, 10$) and portfolio weight constraints ($w_j \in [0,1]$, $w_j \in [0,1.5]$, $w_j \in [-0.5,1]$ and $w_j \in [-0.5,1.5]$). Figures in bold indicate the five models with the best performance.

Table 3.5: Mean-Variance Portfolio Performance based on Sharpe Ratio: Stock-Bond-Commodity Portfolios

	Sample Covariance								DCC-GARCH Covariance							
	γ=2				γ=10				γ=2				γ=10			
	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5
EW	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
HA	0.39	0.36	0.36	0.39	0.56	0.69	0.60	0.72	0.37	0.38	0.39	0.42	0.68	0.79	0.71	0.81
KS	0.43	0.46	0.45	0.50	0.66	0.72	0.76	0.81	0.41	0.43	0.43	0.47	0.61	0.65	0.72	0.78
MC	**0.53**	0.54	**0.64**	0.65	0.68	0.82	0.80	**0.90**	0.50	0.55	0.66	0.68	0.79	**0.92**	**0.92**	**1.00**
MDC	0.46	0.46	0.50	0.53	0.64	0.76	0.71	0.81	0.40	0.47	0.52	0.53	0.73	0.84	0.80	0.88
TMC	0.51	0.52	0.60	0.61	0.67	0.80	0.78	0.88	0.49	0.54	0.62	0.64	0.79	0.90	**0.89**	0.98
Rank	**0.54**	**0.63**	**0.69**	**0.72**	0.68	0.81	0.77	0.89	0.53	**0.59**	**0.70**	**0.73**	0.80	**0.93**	**0.94**	**1.05**
C(2,PB)	0.48	0.48	0.59	0.63	0.61	0.76	0.77	0.88	0.46	0.49	0.61	0.63	0.71	0.86	0.89	**0.99**
C(3,PB)	0.46	0.50	0.58	0.62	0.57	0.69	0.70	0.82	0.47	0.52	0.61	0.64	0.67	0.82	0.84	0.97
D(1)	0.53	0.54	**0.64**	**0.65**	0.68	0.82	**0.80**	**0.90**	0.50	0.55	**0.66**	**0.68**	0.80	**0.92**	**0.92**	**1.00**
D(0.9)	0.52	0.53	0.64	**0.65**	0.69	**0.83**	**0.81**	**0.91**	0.50	0.55	**0.66**	**0.69**	**0.80**	**0.93**	**0.93**	**1.01**
Ridge	0.39	0.46	0.47	0.54	0.70	0.75	0.74	0.84	0.34	0.43	0.48	0.55	0.71	0.80	0.76	0.84
Lasso	0.43	0.54	0.55	0.62	**0.77**	0.81	0.75	0.84	0.49	**0.56**	0.54	0.62	**0.80**	0.87	0.77	0.86
EN	0.45	0.54	0.55	0.61	0.75	**0.82**	0.77	0.87	0.45	0.54	0.54	0.62	**0.80**	0.87	0.76	0.85
Alasso	**0.60**	**0.63**	**0.67**	**0.75**	**0.90**	**0.92**	**0.89**	**0.93**	**0.59**	**0.63**	**0.67**	**0.78**	**0.88**	0.86	0.85	0.89
Bridge	0.30	0.43	0.45	0.54	0.73	0.79	0.70	0.78	0.31	0.39	0.43	0.51	0.73	0.82	0.69	0.79
SCAD	0.39	0.49	0.53	0.59	0.75	0.79	0.71	0.81	0.40	0.46	0.46	0.54	0.74	0.86	0.77	0.86
MCP	0.40	0.52	0.54	0.61	**0.78**	**0.85**	0.77	0.87	0.41	0.50	0.49	0.59	**0.81**	**0.93**	0.81	0.93
SICA	0.52	**0.57**	0.59	0.64	**0.78**	0.77	0.77	0.81	**0.53**	0.56	0.57	0.64	0.75	0.79	0.80	0.85
PLS	0.52	0.55	0.60	0.61	0.71	0.80	**0.81**	0.86	**0.54**	0.56	0.58	0.61	0.74	0.77	0.78	0.82
3PRF	**0.60**	**0.57**	0.56	0.60	0.71	0.79	0.77	0.82	**0.56**	**0.56**	0.55	0.57	0.70	0.73	0.73	0.77
SPLS	**0.60**	**0.64**	**0.69**	**0.73**	0.74	0.71	0.68	0.70	**0.58**	**0.64**	**0.70**	**0.72**	0.75	0.75	0.70	0.74
PCA	0.36	0.51	0.55	0.65	**0.82**	**0.92**	**0.92**	**1.01**	0.39	0.46	0.49	0.55	0.80	0.87	0.87	0.96
SPCA	0.25	0.37	0.43	0.53	0.68	0.77	0.77	0.88	0.35	0.42	0.43	0.49	0.66	0.78	0.77	0.87
ICA	0.35	0.36	0.31	0.36	0.59	0.65	0.61	0.69	0.40	0.44	0.36	0.37	0.54	0.63	0.63	0.72
RICA	0.46	0.52	0.54	0.58	0.65	0.73	0.71	0.78	0.51	0.54	0.60	0.64	0.77	0.87	0.81	0.91

Table 3.5 (continued): Mean-Variance Portfolio Performance based on Sharpe Ratio: Stock-Bond-Commodity Portfolios

	Ledoit-Wolf Shrinkage Covariance								Graphical Lasso Covariance							
	γ=2				γ=10				γ=2				γ=10			
	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5
EW	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
HA	0.39	0.36	0.35	0.38	0.55	0.65	0.57	0.66	0.38	0.37	0.34	0.38	0.64	0.77	0.67	0.78
KS	0.43	0.46	0.45	0.48	0.63	0.70	0.76	0.80	0.43	0.47	0.47	0.52	0.70	0.78	0.81	0.85
MC	**0.53**	0.55	**0.63**	0.65	0.68	0.77	0.77	0.83	**0.54**	0.56	0.64	0.65	0.79	0.92	**0.88**	**0.98**
MDC	0.46	0.46	0.50	0.52	0.62	0.71	0.68	0.74	0.48	0.48	0.50	0.53	0.73	0.85	0.78	0.88
TMC	0.51	0.53	0.59	0.60	0.67	0.76	0.75	0.81	0.53	0.53	0.60	0.61	0.79	0.91	0.86	0.96
Rank	**0.54**	**0.62**	**0.68**	**0.71**	0.70	**0.80**	**0.80**	**0.88**	**0.57**	**0.65**	**0.70**	**0.72**	0.77	**0.92**	0.85	**0.97**
C(2,PB)	0.48	0.49	0.59	0.63	0.62	0.74	0.77	0.84	0.49	0.52	0.62	0.64	0.71	0.86	0.84	0.96
C(3,PB)	0.46	0.49	0.57	0.61	0.59	0.70	0.73	0.82	0.48	0.53	0.60	0.63	0.64	0.80	0.78	0.90
D(1)	0.53	0.55	**0.63**	**0.65**	0.68	0.77	0.77	0.83	0.54	0.56	**0.64**	0.65	0.79	0.92	**0.88**	**0.98**
D(0.9)	0.52	0.54	0.63	**0.65**	0.69	**0.78**	0.78	0.84	0.53	0.56	**0.65**	0.65	**0.80**	**0.93**	**0.89**	**0.99**
Ridge	0.37	0.45	0.47	0.53	0.69	0.73	0.76	0.84	0.40	0.50	0.51	0.57	0.74	0.83	0.80	0.91
Lasso	0.42	0.52	0.54	0.60	**0.74**	0.77	0.75	0.82	0.45	**0.57**	0.58	0.65	0.79	0.87	0.80	0.91
EN	0.44	0.53	0.54	0.59	0.73	0.78	0.76	0.84	0.47	0.57	0.58	0.63	**0.80**	0.88	0.82	0.94
Alasso	**0.58**	**0.61**	**0.65**	**0.73**	**0.86**	**0.87**	**0.87**	**0.91**	**0.59**	**0.65**	**0.70**	**0.80**	**0.93**	**0.95**	**0.91**	0.96
Bridge	0.28	0.40	0.43	0.52	0.69	0.74	0.69	0.77	0.34	0.48	0.48	0.57	0.76	0.84	0.73	0.84
SCAD	0.38	0.47	0.52	0.57	0.72	0.75	0.71	0.79	0.41	0.54	0.56	0.61	0.77	0.85	0.76	0.87
MCP	0.39	0.50	0.53	0.60	**0.75**	**0.80**	0.77	**0.85**	0.42	0.56	0.58	0.64	**0.81**	**0.92**	0.84	0.94
SICA	0.51	**0.55**	0.58	0.63	**0.76**	0.75	0.77	0.80	0.53	**0.59**	0.60	**0.67**	0.78	0.80	0.81	0.86
PLS	0.52	0.55	0.59	0.60	0.69	0.77	**0.79**	0.83	0.53	0.56	0.60	0.62	0.75	0.81	0.81	0.87
3PRF	**0.60**	**0.57**	0.55	0.58	0.69	0.76	0.75	0.79	**0.59**	0.55	0.55	0.60	0.74	0.80	0.77	0.83
SPLS	**0.59**	**0.63**	**0.68**	**0.72**	0.73	0.70	0.67	0.71	**0.60**	**0.65**	**0.70**	**0.73**	0.73	0.71	0.66	0.73
PCA	0.35	0.50	0.54	0.63	**0.78**	**0.86**	**0.91**	**0.99**	0.41	0.55	0.59	**0.67**	**0.89**	**0.98**	**0.93**	**1.03**
SPCA	0.24	0.35	0.41	0.51	0.66	0.75	**0.79**	**0.88**	0.29	0.42	0.46	0.55	0.73	0.87	0.81	0.93
ICA	0.34	0.34	0.29	0.35	0.56	0.62	0.60	0.67	0.36	0.38	0.33	0.39	0.60	0.69	0.63	0.72
RICA	0.46	0.51	0.53	0.58	0.63	0.69	0.72	0.79	0.48	0.54	0.54	0.58	0.70	0.79	0.75	0.84

Notes: This table reports the Sharpe Ratio of the stock-bond-commodity mean-variance portfolios with monthly rebalancing. The out-of-sample period is from January 1997 to December 2016. The benchmarks are the equal-weighted portfolio and the mean-variance portfolio based on the historical average forecast. The alternative mean-variance allocations utilize return forecasts generated by multivariate prediction models. Each panel reports the Sharpe Ratio of portfolios relying on different estimates of the covariance matrix. The portfolio performance is reported for different levels of risk aversion ($\gamma = 2, 10$) and portfolio weight constraints ($w_j \in [0,1]$, $w_j \in [0,1.5]$, $w_j \in [-0.5,1]$ and $w_j \in [-0.5,1.5]$). Figures in bold indicate the five models with the best performance.

Table 3.6: Mean-Variance Portfolio Performance based on Certainty Equivalent Return: Stock-Bond-Commodity Portfolios (Business Cycles)

	γ=2								γ=10							
	Recession				Expansion				Recession				Expansion			
	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5
EW	-17.02	-17.02	-17.02	-17.02	6.43	6.43	6.43	6.43	-28.84	-28.84	-28.84	-28.84	3.34	3.34	3.34	3.34
HA	-22.11	-31.34	-31.71	-34.81	9.08	9.87	9.88	10.90	-6.03	-7.32	-6.03	-7.32	4.91	6.00	5.16	6.19
KS	-6.73	-12.47	4.20	-1.47	8.05	9.84	7.12	8.32	-6.27	-9.09	2.93	-0.27	4.35	3.79	2.16	1.95
MC	-2.43	-5.40	12.19	10.92	**8.39**	10.07	**10.40**	11.32	1.01	0.84	4.72	3.95	4.80	5.91	5.21	6.26
MDC	-12.71	-17.67	-10.32	-11.61	**8.81**	10.05	10.19	11.21	-2.47	-3.45	-1.08	-2.38	4.95	6.04	**5.27**	**6.31**
TMC	-3.60	-7.02	5.97	4.64	8.32	9.90	10.18	11.09	0.27	-0.11	3.22	2.30	4.83	5.93	**5.23**	**6.27**
Rank	0.75	2.51	20.75	20.55	7.87	**10.29**	**10.49**	**11.83**	**2.98**	**3.51**	**9.23**	**9.10**	4.62	5.58	4.62	5.71
C(2,PB)	-4.66	-4.76	14.01	15.13	7.89	8.98	9.32	10.53	**1.74**	**2.04**	8.23	7.80	4.26	5.37	4.71	5.78
C(3,PB)	-5.19	-5.08	15.25	16.00	7.80	9.38	9.09	10.27	**1.80**	**2.05**	**9.29**	**9.16**	3.97	4.79	4.03	5.12
D(1)	-2.45	-5.41	12.28	11.25	8.36	10.05	**10.38**	11.30	1.06	0.91	4.83	4.06	4.80	5.91	5.21	6.26
D(0.9)	-2.64	-6.00	11.96	11.45	8.27	9.98	10.33	11.25	1.13	1.00	4.92	4.15	4.85	5.96	**5.27**	**6.32**
Ridge	-4.59	-9.25	6.97	2.91	6.81	9.29	7.76	10.05	-4.49	-6.71	1.38	0.93	5.69	6.29	4.92	6.00
Lasso	-0.50	-3.24	8.47	4.57	6.98	**10.32**	9.63	**12.35**	-3.66	-6.22	-1.18	-0.73	**6.15**	**6.64**	5.15	5.90
EN	1.94	-1.47	12.58	8.29	7.01	10.08	9.03	11.41	-3.19	-5.80	0.65	0.97	**6.00**	**6.80**	**5.31**	**6.37**
Alasso	**7.85**	**7.07**	**32.93**	**35.07**	**8.76**	**11.17**	**10.40**	**13.20**	**2.83**	1.75	**10.21**	**10.83**	**6.54**	**6.47**	4.45	4.45
Bridge	-4.82	-5.86	10.84	9.55	5.45	8.29	6.68	9.33	-3.94	-4.43	3.13	2.31	5.87	6.35	4.18	4.99
SCAD	-2.09	-5.00	5.26	1.81	6.67	9.60	9.58	11.73	-4.18	-7.28	-3.80	-3.89	**5.96**	6.40	4.83	5.89
MCP	-0.57	-2.98	9.47	6.55	6.54	9.87	9.22	**11.90**	-1.10	-1.52	2.69	2.63	**5.98**	**6.47**	4.94	6.01
SICA	**4.05**	**5.94**	24.02	22.53	8.19	**10.31**	9.09	11.03	-0.95	-3.84	7.98	6.96	5.44	4.14	1.76	1.47
PLS	**10.51**	**14.45**	**44.37**	**45.23**	7.28	8.64	7.15	7.57	**2.88**	**1.82**	**13.10**	**13.92**	4.57	5.01	3.04	3.16
3PRF	**12.28**	**11.49**	**39.48**	**40.91**	**8.40**	9.53	6.51	7.55	1.66	-0.03	**10.30**	**10.94**	4.71	5.23	2.78	2.89
SPLS	**7.62**	**9.75**	**33.16**	**33.20**	**8.65**	**10.98**	**10.65**	**12.68**	1.18	-2.33	4.41	1.52	4.99	3.88	1.75	1.31
PCA	-7.79	-7.91	6.41	7.69	6.57	9.66	9.25	11.79	1.02	0.78	8.60	8.50	5.71	**6.75**	**6.07**	**7.05**
SPCA	-8.06	-8.09	8.00	10.23	5.14	7.22	6.55	8.77	1.62	**1.91**	8.33	9.00	4.81	5.45	4.59	5.63
ICA	-12.54	-18.18	-2.63	-0.63	7.21	8.13	5.09	5.83	-1.66	-2.64	5.57	3.71	4.46	4.95	3.57	4.27
RICA	3.88	3.28	**25.84**	**23.01**	6.95	8.93	7.23	9.00	-0.51	-1.54	5.89	4.37	4.72	5.45	4.34	5.13

Notes: This table reports the certainty equivalent return of the stock-bond-commodity mean-variance portfolios with monthly rebalancing during NBER-dated recessions and expansions. The out-of-sample period is from January 1997 to December 2016. The benchmarks are the equally-weighted portfolio and the mean-variance portfolio based on the historical average forecast. The alternative mean-variance allocations utilize return forecasts generated by multivariate prediction models. All mean-variance portfolios are constructed based on the sample covariance matrix. The portfolio performance is reported for different levels of risk aversion ($\gamma = 2, 10$) and portfolio weight constraints ($w_j \in [0,1]$, $w_j \in [0,1.5]$, $w_j \in [-0.5,1]$ and $w_j \in [-0.5,1.5]$). Figures in bold indicate the five models with the best performance.

Table 3.7: Mean-Variance Portfolio Performance based on Sharpe Ratio: Stock-Bond-Commodity Portfolios (Business Cycles)

	γ=2								γ=10							
	Recession				Expansion				Recession				Expansion			
	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5
EW	-0.92	-0.92	-0.92	-0.92	0.57	0.57	0.57	0.57	-0.92	-0.92	-0.92	-0.92	0.57	0.57	0.57	0.57
HA	-0.90	-0.91	-0.98	-0.94	0.68	0.64	0.63	0.65	-0.53	-0.34	-0.53	-0.34	0.78	0.92	0.82	0.95
KS	-0.19	-0.15	0.48	0.44	0.54	0.57	0.45	0.51	0.25	0.38	1.26	1.21	0.73	0.78	0.68	0.74
MC	-0.14	-0.15	0.65	0.60	**0.62**	**0.65**	**0.65**	**0.67**	0.16	0.28	0.75	0.71	0.75	0.90	**0.81**	**0.95**
MDC	-0.61	-0.64	-0.24	-0.25	**0.65**	**0.65**	**0.64**	**0.66**	-0.25	-0.09	0.05	0.12	0.78	**0.93**	**0.84**	**0.96**
TMC	-0.23	-0.25	0.41	0.36	**0.62**	0.64	0.64	0.66	0.05	0.17	0.57	0.54	0.76	0.91	**0.82**	**0.95**
Rank	0.04	0.20	0.98	**0.97**	0.61	**0.68**	**0.65**	**0.68**	0.47	**0.58**	1.20	1.20	0.70	0.83	0.70	0.85
C(2,PB)	-0.31	-0.15	0.72	0.75	0.59	0.57	0.57	0.61	0.26	0.41	1.12	1.09	0.65	0.81	0.71	0.86
C(3,PB)	-0.34	-0.16	0.76	0.78	0.58	0.59	0.55	0.59	0.28	0.41	1.21	1.21	0.60	0.72	0.61	0.76
D(1)	-0.15	-0.15	0.65	0.61	**0.62**	0.65	**0.65**	0.66	0.17	0.29	0.76	0.72	0.75	0.90	0.81	0.95
D(0.9)	-0.16	-0.20	0.64	0.62	0.62	0.64	**0.65**	0.66	0.17	0.29	0.77	0.73	0.76	0.91	**0.82**	**0.96**
Ridge	-0.09	-0.15	0.47	0.38	0.49	0.58	0.48	0.57	-0.21	-0.13	0.82	0.85	0.84	0.90	0.75	0.87
Lasso	0.09	0.09	0.52	0.45	0.50	0.63	0.56	0.66	-0.05	-0.06	0.49	0.56	**0.89**	**0.94**	0.80	0.88
EN	0.20	0.12	0.66	0.53	0.50	0.61	0.54	0.62	-0.09	-0.07	0.62	0.69	**0.87**	**0.96**	0.80	0.91
Alasso	**0.50**	**0.46**	**1.23**	**1.23**	**0.62**	**0.66**	0.58	**0.67**	**0.63**	**0.66**	**1.44**	**1.48**	**0.94**	**0.95**	0.80	0.84
Bridge	-0.07	0.06	0.60	0.58	0.38	0.52	0.43	0.54	-0.11	0.00	0.88	0.89	0.86	0.91	0.68	0.78
SCAD	-0.01	-0.01	0.42	0.36	0.46	0.58	0.55	0.63	-0.10	-0.12	0.34	0.40	0.87	0.92	0.78	0.89
MCP	0.09	0.09	0.55	0.49	0.46	0.60	0.55	0.64	0.16	0.24	0.71	0.75	**0.87**	0.93	0.78	0.89
SICA	**0.32**	**0.45**	0.95	0.91	0.56	0.60	0.53	0.60	**0.52**	**0.52**	**1.36**	**1.36**	0.83	0.80	0.67	0.72
PLS	**0.65**	**0.73**	**1.56**	**1.49**	0.50	0.52	0.45	0.48	**0.65**	**0.64**	**1.61**	**1.66**	0.72	0.82	0.67	0.72
3PRF	**0.72**	**0.63**	**1.43**	**1.39**	0.58	0.57	0.42	0.48	**0.51**	0.47	**1.48**	**1.52**	0.74	0.84	0.65	0.71
SPLS	**0.49**	**0.56**	**1.27**	**1.18**	0.61	**0.65**	0.60	0.66	0.48	0.36	1.00	0.92	0.77	0.75	0.63	0.67
PCA	-0.63	-0.43	0.44	0.49	0.48	0.62	0.58	**0.67**	0.17	0.29	1.23	1.25	**0.90**	**1.01**	**0.92**	**1.02**
SPCA	-0.66	-0.45	0.50	0.58	0.36	0.47	0.42	0.52	0.26	0.40	**1.26**	**1.31**	0.73	0.81	0.70	0.83
ICA	-0.47	-0.45	0.09	0.20	0.52	0.52	0.35	0.39	-0.05	0.01	0.88	0.79	0.68	0.75	0.57	0.68
RICA	0.30	0.33	**1.03**	0.92	0.50	0.56	0.45	0.53	**0.55**	**0.63**	1.14	1.11	0.72	0.81	0.66	0.76

Notes: This table reports the Sharpe Ratio of the stock-bond-commodity mean-variance portfolios with monthly rebalancing during NBER-dated recessions and expansions. The out-of-sample period is from January 1997 to December 2016. The benchmarks are the equal-weighted portfolio and the mean-variance portfolio based on the historical average forecast. The alternative mean-variance allocations utilize return forecasts generated by multivariate prediction models. All mean-variance portfolios are constructed based on the sample covariance matrix. The portfolio performance is reported for different levels of risk aversion ($\gamma = 2, 10$) and portfolio weight constraints ($w_j \in [0,1]$, $w_j \in [0,1.5]$, $w_j \in [-0.5,1]$ and $w_j \in [-0.5,1.5]$). Figures in bold indicate the five models with the best performance.

Table 3.8: Mean-Variance Portfolio Performance based on Certainty Equivalent Return and Sharpe Ratio: Stock-Bond Portfolios

	Certainty Equivalent Return								Sharpe Ratio							
	γ=2				γ=10				γ=2				γ=10			
	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5
EW	6.39	6.39	6.39	6.39	4.00	4.00	4.00	4.00	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63
HA	5.86	6.36	4.85	5.70	4.47	5.44	4.47	5.44	0.41	0.42	0.33	0.38	0.71	0.84	0.71	0.84
KS	**8.60**	**11.67**	**11.07**	**14.40**	**6.16**	6.08	**5.37**	5.25	**0.65**	**0.72**	**0.67**	**0.76**	**0.90**	0.93	**0.88**	0.92
MC	7.77	8.62	7.58	8.42	5.03	6.09	5.07	6.10	0.57	0.55	0.49	0.52	0.82	**0.96**	0.83	0.96
MDC	6.81	7.54	6.03	7.07	4.78	5.78	4.75	5.75	0.49	0.49	0.40	0.45	0.78	0.90	0.77	0.90
TMC	7.53	8.33	6.96	7.80	4.96	5.99	4.97	5.99	0.55	0.54	0.46	0.49	0.82	0.94	0.82	0.94
Rank	7.97	10.06	8.98	10.22	4.98	6.12	5.16	6.26	0.61	0.66	0.58	0.61	0.79	0.94	0.80	0.95
C(2,PB)	7.21	8.29	7.26	8.22	4.55	5.67	4.74	5.79	0.53	0.53	0.47	0.50	0.71	0.87	0.74	0.88
C(3,PB)	7.08	8.50	7.22	8.53	4.13	5.26	4.37	5.45	0.52	0.54	0.46	0.52	0.63	0.80	0.67	0.82
D(1)	7.75	8.62	7.59	8.44	5.03	6.09	5.07	6.10	0.57	0.55	0.49	0.52	0.82	**0.96**	0.83	0.96
D(0.9)	7.68	8.59	7.57	8.52	5.06	6.13	5.11	6.14	0.56	0.55	0.49	0.52	0.83	**0.96**	**0.84**	**0.96**
Ridge	7.80	9.32	8.38	9.67	5.47	**6.31**	**5.59**	**6.78**	0.58	0.58	0.52	0.56	0.82	0.91	0.83	**0.96**
Lasso	7.71	9.91	8.54	10.10	5.44	6.07	5.13	**6.40**	0.57	0.61	0.53	0.57	0.81	0.89	0.79	0.92
EN	7.36	9.30	7.98	9.37	5.55	**6.50**	**5.65**	**6.88**	0.54	0.57	0.50	0.54	0.82	0.93	**0.84**	**0.97**
Alasso	**10.15**	**13.20**	**11.73**	**14.41**	**6.50**	6.29	4.93	5.25	**0.82**	**0.84**	**0.73**	**0.78**	**0.93**	0.94	0.82	0.89
Bridge	7.46	9.32	7.56	8.32	5.37	**6.43**	5.08	6.26	0.55	0.58	0.48	0.50	0.80	0.92	0.77	0.90
SCAD	7.75	9.45	8.05	9.56	5.23	6.13	4.90	6.13	0.57	0.58	0.50	0.55	0.79	0.90	0.77	0.90
MCP	7.66	9.85	8.41	10.23	**5.66**	**6.97**	**5.82**	**7.01**	0.57	0.60	0.52	0.58	0.84	**0.98**	**0.86**	**0.98**
SICA	**8.95**	**11.90**	**10.65**	**13.15**	**5.90**	5.41	4.23	4.16	**0.69**	**0.73**	**0.65**	**0.71**	**0.87**	0.88	0.80	0.86
PLS	**8.66**	**11.22**	**10.58**	**12.34**	**5.75**	6.21	**5.44**	5.99	**0.67**	**0.71**	**0.66**	**0.68**	**0.85**	0.90	0.82	0.89
3PRF	**8.62**	**10.50**	**10.10**	**11.74**	5.31	5.67	4.86	5.27	**0.68**	**0.66**	**0.63**	**0.65**	0.79	0.85	0.77	0.83
SPLS	7.70	9.66	8.26	9.66	4.04	4.41	2.95	3.23	0.59	0.61	0.52	0.56	0.66	0.74	0.62	0.70
PCA	6.30	8.24	6.42	8.84	5.43	**6.44**	5.30	**6.31**	0.47	0.54	0.43	0.54	**0.91**	**1.01**	**0.85**	**0.97**
SPCA	5.78	7.20	5.17	7.08	4.92	6.07	4.63	5.75	0.42	0.47	0.35	0.45	0.77	0.93	0.71	0.86
ICA	7.55	9.23	7.90	9.83	5.16	5.98	4.83	5.67	0.57	0.59	0.51	0.58	0.80	0.89	0.74	0.85
RICA	6.38	8.51	6.00	7.80	5.12	5.85	3.93	4.83	0.48	0.56	0.40	0.49	0.82	0.88	0.60	0.73

Notes: This table reports the certainty equivalent return and Sharpe Ratio of the stock-bond mean-variance portfolios with monthly rebalancing. The out-of-sample period is from January 1997 to December 2016. The benchmarks are the equally-weighted portfolio and the mean-variance portfolio based on the historical average forecast. The alternative mean-variance allocations utilize return forecasts generated by multivariate prediction models. All mean-variance portfolios are constructed based on the sample covariance matrix. The portfolio performance is reported for different levels of risk aversion ($\gamma = 2, 10$) and portfolio weight constraints ($w_j \in [0,1]$, $w_j \in [0,1.5]$, $w_j \in [-0.5,1]$ and $w_j \in [-0.5,1.5]$). Figures in bold indicate the five models with the best performance.

Table 3.9: Mean-Variance Portfolio Performance based on Certainty Equivalent Return: Stock-Bond Portfolios (Business Cycles)

	γ=2								γ=10							
	Recession				Expansion				Recession				Expansion			
	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5
EW	-6.92	-6.92	-6.92	-6.92	8.02	8.02	8.02	8.02	-12.88	-12.88	-12.88	-12.88	6.11	6.11	6.11	6.11
HA	-20.12	-22.83	-23.58	-23.51	9.06	9.96	8.36	9.31	-3.00	-3.71	-3.00	-3.71	5.39	6.56	5.39	6.56
KS	**0.72**	**-0.35**	**6.53**	**6.72**	**9.55**	**13.12**	**11.61**	**15.30**	-0.44	-1.97	3.33	4.88	**6.96**	7.04	5.57	5.24
MC	-5.26	-7.88	-4.32	-3.44	**9.36**	10.63	9.02	9.86	1.39	1.41	1.75	1.52	5.47	6.65	5.47	6.65
MDC	-12.11	-14.76	-15.52	-14.70	9.13	10.29	8.68	9.75	-1.03	-1.49	-1.26	-1.74	5.49	6.66	5.49	6.66
TMC	-5.88	-7.95	-7.46	-6.63	9.17	10.32	8.72	9.56	0.85	0.62	0.91	0.65	5.46	6.64	5.46	6.64
Rank	-1.10	**0.83**	**5.65**	**6.62**	9.07	11.18	9.37	10.65	**3.29**	**3.80**	**5.46**	**5.64**	5.18	6.39	5.11	6.31
C(2,PB)	-5.68	-5.23	-1.23	1.11	8.78	9.94	8.28	9.06	**2.13**	**2.55**	3.80	3.65	4.84	6.04	4.84	6.04
C(3,PB)	-5.52	-4.54	-1.29	1.81	8.62	10.09	8.25	9.33	**2.51**	**2.89**	4.71	4.70	4.32	5.53	4.31	5.52
D(1)	-5.25	-7.74	-4.11	-3.21	9.34	10.62	9.00	9.85	1.43	1.46	1.82	1.58	5.46	6.64	5.46	6.64
D(0.9)	-5.23	-7.48	-3.72	-2.64	9.25	10.56	8.94	9.88	1.50	1.56	1.92	1.68	5.50	6.67	5.50	6.67
Ridge	-5.20	-11.72	-6.30	-10.04	**9.39**	11.91	**10.17**	**12.08**	-3.32	-4.97	-0.50	0.51	**6.57**	**7.72**	**6.31**	**7.51**
Lasso	-2.45	-6.57	-1.16	-4.39	8.95	**11.91**	9.71	11.84	-2.72	-4.76	-0.49	0.80	6.45	7.41	5.78	7.05
EN	-3.42	-9.54	-4.09	-9.73	8.67	11.61	9.44	11.70	-3.57	-5.61	-1.22	-0.13	**6.70**	**8.02**	**6.46**	**7.70**
Alasso	**-0.30**	-2.73	2.93	1.56	**11.43**	**15.14**	**12.78**	**15.95**	0.37	0.90	**5.29**	**6.90**	**7.23**	6.93	4.85	5.01
Bridge	-3.19	-7.68	-2.32	-7.62	8.76	11.39	8.74	10.25	-2.89	-2.21	0.73	1.53	6.40	**7.50**	5.58	6.81
SCAD	-4.04	-7.86	-3.88	-6.87	9.19	11.56	9.49	11.54	-3.46	-5.95	-3.13	-2.32	6.32	**7.64**	**5.86**	**7.13**
MCP	-4.04	-7.49	-3.30	-4.96	9.09	**11.96**	9.82	**12.05**	-0.30	0.11	3.25	3.97	6.40	**7.82**	**6.11**	**7.35**
SICA	**0.26**	**-1.79**	**5.06**	2.94	**10.01**	**13.56**	**11.31**	**14.36**	-0.05	-1.27	4.19	2.72	**6.61**	6.18	4.19	4.28
PLS	**4.19**	**7.12**	**15.53**	**17.86**	9.19	11.71	9.97	11.65	**3.35**	**2.79**	**8.71**	**10.12**	6.03	6.61	5.03	5.47
3PRF	**3.23**	**2.99**	**10.97**	**10.71**	9.27	11.41	**9.97**	11.84	1.52	0.25	**5.20**	**6.44**	5.76	6.33	4.80	5.10
SPLS	-2.75	-3.20	0.23	0.22	8.98	11.24	9.23	10.80	-0.71	-3.17	-3.06	-4.70	4.63	5.34	3.65	4.17
PCA	-8.08	-8.84	-10.35	-9.23	8.07	10.33	8.47	11.05	0.80	0.54	-1.24	-1.62	6.00	7.15	**6.09**	**7.26**
SPCA	-8.23	-8.75	-7.65	-6.05	7.50	9.15	6.73	8.68	1.35	1.63	1.73	2.16	5.35	6.60	4.97	6.17
ICA	-2.22	-2.45	3.57	**7.26**	8.73	10.64	8.40	10.12	**2.83**	**2.82**	**7.38**	**7.19**	5.44	6.35	4.51	5.47
RICA	-3.74	-4.82	-2.99	-4.41	7.61	10.14	7.08	9.27	1.16	0.40	-2.24	-3.49	5.60	6.49	4.64	5.80

Notes: This table reports the certainty equivalent return of the stock-bond mean-variance portfolios with monthly rebalancing during NBER-dated recessions and expansions. The out-of-sample period is from January 1997 to December 2016. The benchmarks are the equally-weighted portfolio and the mean-variance portfolio based on the historical average forecast. The alternative mean-variance allocations utilize return forecasts generated by multivariate prediction models. All mean-variance portfolios are constructed based on the sample covariance matrix. The portfolio performance is reported for different levels of risk aversion ($\gamma = 2, 10$) and portfolio weight constraints ($w_j \in [0,1]$, $w_j \in [0,1.5]$, $w_j \in [-0.5,1]$ and $w_j \in [-0.5,1.5]$). Figures in bold are the five models with the best performance.

Table 3.10: Mean-Variance Portfolio Performance based on Sharpe Ratio: Stock-Bond Portfolios (Business Cycles)

	γ=2								γ=10							
	Recession				Expansion				Recession				Expansion			
	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5
EW	-0.59	-0.59	-0.59	-0.59	0.91	0.91	0.91	0.91	-0.59	-0.59	-0.59	-0.59	0.91	0.91	0.91	0.91
HA	-0.90	-0.78	-0.86	-0.78	0.67	0.64	0.54	0.56	-0.31	-0.11	-0.31	-0.11	0.91	1.05	0.91	1.05
KS	**0.05**	**0.09**	**0.43**	**0.45**	**0.73**	**0.80**	**0.71**	**0.81**	0.30	0.39	0.85	0.96	**0.98**	1.01	0.89	0.91
MC	-0.36	-0.32	-0.10	-0.05	0.70	0.68	0.58	0.59	0.19	0.33	0.29	0.36	0.91	1.06	0.91	**1.06**
MDC	-0.71	-0.67	-0.72	-0.64	0.68	0.66	0.56	0.58	-0.15	0.03	-0.20	-0.01	0.93	**1.07**	**0.93**	**1.07**
TMC	-0.40	-0.35	-0.30	-0.24	0.68	0.66	0.56	0.57	0.11	0.24	0.13	0.24	0.92	1.07	**0.92**	**1.07**
Rank	-0.11	**0.09**	**0.39**	**0.45**	0.71	0.72	0.61	0.63	**0.54**	**0.63**	0.84	0.87	0.82	0.98	0.80	0.97
C(2,PB)	-0.38	-0.19	0.05	0.18	0.66	0.63	0.53	0.54	0.33	0.47	0.64	0.66	0.76	0.93	0.76	0.93
C(3,PB)	-0.37	-0.15	0.05	0.22	0.64	0.63	0.52	0.55	**0.40**	**0.52**	0.75	0.77	0.66	0.83	0.66	0.83
D(1)	-0.36	-0.31	-0.09	-0.04	0.70	0.68	0.58	0.59	0.20	0.33	0.30	0.37	0.91	1.06	0.91	1.06
D(0.9)	-0.36	-0.33	-0.09	-0.03	0.69	0.67	0.58	0.59	0.21	0.34	0.32	0.39	0.92	1.07	0.92	**1.07**
Ridge	-0.40	-0.54	-0.19	-0.28	0.71	0.72	0.62	**0.66**	-0.34	-0.19	0.38	0.51	**0.97**	**1.08**	**0.92**	1.05
Lasso	-0.14	-0.17	0.11	0.07	0.68	0.72	0.59	0.65	-0.15	-0.09	0.43	0.56	0.93	1.02	0.85	0.98
EN	-0.23	-0.36	-0.06	-0.21	0.65	0.71	0.58	0.64	-0.34	-0.21	0.35	0.47	**0.97**	**1.10**	**0.93**	1.06
Alasso	**-0.01**	-0.03	0.27	0.25	**0.95**	**0.97**	**0.81**	**0.86**	0.34	**0.51**	**0.93**	**1.05**	**1.01**	0.99	0.81	0.87
Bridge	-0.21	-0.23	0.05	-0.09	0.67	0.70	0.54	0.58	-0.24	0.00	0.44	0.55	0.93	1.04	0.82	0.96
SCAD	-0.24	-0.21	0.00	0.00	0.69	0.70	0.58	0.63	-0.22	-0.17	0.24	0.35	0.91	1.05	0.86	0.99
MCP	-0.24	-0.20	0.02	0.05	0.69	**0.73**	0.60	0.65	0.06	0.24	0.65	0.75	0.92	**1.07**	0.88	1.01
SICA	**0.01**	**0.01**	**0.36**	0.31	**0.79**	**0.83**	**0.69**	**0.76**	**0.36**	0.51	**0.95**	**0.98**	0.94	0.94	0.78	0.84
PLS	**0.32**	**0.49**	**0.96**	**0.98**	**0.72**	**0.73**	**0.62**	0.65	**0.58**	**0.58**	**1.16**	**1.28**	0.88	0.94	0.77	0.84
3PRF	**0.24**	**0.24**	**0.66**	**0.61**	**0.73**	0.72	**0.62**	**0.66**	0.33	0.33	**0.86**	**0.98**	0.85	0.91	0.76	0.81
SPLS	-0.37	-0.25	0.05	0.09	0.68	0.69	0.57	0.60	-0.02	0.00	0.25	0.26	0.72	0.83	0.68	0.77
PCA	-0.65	-0.49	-0.50	-0.34	0.62	0.67	0.55	0.66	0.13	0.26	0.12	0.22	**1.03**	**1.16**	**1.03**	**1.17**
SPCA	-0.67	-0.49	-0.34	-0.18	0.57	0.59	0.44	0.53	0.20	0.36	0.42	0.51	0.84	1.01	0.76	0.93
ICA	-0.10	0.00	0.30	**0.47**	0.68	0.69	0.55	0.60	**0.49**	**0.55**	**1.06**	**1.03**	0.85	0.94	0.69	0.82
RICA	-0.35	-0.23	-0.07	-0.05	0.59	0.67	0.47	0.57	0.31	0.44	0.26	0.28	0.93	1.01	0.73	0.90

Notes: This table reports the Sharpe Ratio of the stock-bond mean-variance portfolios with monthly rebalancing during NBER-dated recessions and expansions. The out-of-sample period is from January 1997 to December 2016. The benchmarks are the equal-weighted portfolio and the mean-variance portfolio based on the historical average forecast. The alternative mean-variance allocations utilize return forecasts generated by multivariate prediction models. All mean-variance portfolios are constructed based on the sample covariance matrix. The portfolio performance is reported for different levels of risk aversion ($\gamma = 2, 10$) and portfolio weight constraints ($w_j \in [0,1]$, $w_j \in [0,1.5]$, $w_j \in [-0.5,1]$ and $w_j \in [-0.5,1.5]$). Figures in bold are the five models with the best performance.

Table 3.11: Mean-Variance Portfolio Performance based on Certainty Equivalent Return and Sharpe Ratio: Stock-Bond-Commodity Portfolios (Transaction Costs - Monthly Rebalancing)

	Certainty Equivalent Return								Sharpe Ratio							
	γ=2				γ=10				γ=2				γ=10			
	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5
EW	3.87	3.87	3.87	3.87	-0.27	-0.27	-0.27	-0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
HA	5.46	4.94	4.84	5.38	3.54	4.37	3.76	4.53	0.38	0.34	0.33	0.36	0.53	0.67	0.57	0.69
KS	1.48	-0.11	-5.34	-7.22	-1.43	-3.78	-6.39	-7.80	0.12	0.13	0.03	0.07	0.24	0.27	0.21	0.25
MC	**5.97**	**6.15**	**7.61**	**7.84**	**3.53**	**4.47**	**4.14**	**5.00**	**0.43**	**0.41**	**0.48**	**0.48**	**0.53**	**0.68**	**0.63**	**0.75**
MDC	**5.78**	5.73	6.14	6.74	**3.63**	**4.49**	**3.98**	**4.76**	**0.41**	0.38	0.40	0.43	**0.55**	**0.68**	**0.60**	**0.72**
TMC	**5.89**	**6.02**	**7.05**	**7.36**	**3.57**	**4.50**	**4.12**	**4.95**	**0.42**	**0.40**	**0.45**	**0.46**	**0.53**	**0.68**	**0.63**	**0.75**
Rank	4.78	**5.97**	**6.59**	**6.93**	2.97	3.76	3.09	4.05	0.34	**0.40**	**0.42**	**0.44**	0.44	0.58	0.50	0.64
C(2,PB)	4.74	4.52	5.68	6.22	2.80	3.76	3.55	4.47	0.33	0.31	0.38	0.40	0.42	0.58	0.55	0.68
C(3,PB)	4.16	4.33	4.72	4.92	2.18	2.76	2.42	3.37	0.29	0.30	0.33	0.35	0.34	0.46	0.44	0.58
D(1)	**5.94**	**6.11**	**7.58**	**7.82**	**3.53**	**4.47**	**4.14**	**5.00**	**0.43**	**0.40**	**0.48**	**0.48**	**0.53**	**0.68**	**0.63**	**0.75**
D(0.9)	**5.81**	**5.96**	**7.47**	**7.75**	**3.57**	**4.52**	**4.18**	**5.04**	**0.42**	**0.40**	**0.47**	**0.48**	**0.54**	**0.69**	**0.64**	**0.76**
Ridge	1.71	1.46	-1.05	-1.37	1.21	0.84	-0.68	0.15	0.11	0.16	0.07	0.11	0.28	0.33	0.23	0.35
Lasso	2.58	3.52	1.26	1.67	1.85	1.20	-0.83	-0.32	0.17	0.27	0.19	0.25	0.40	0.43	0.27	0.37
EN	2.69	3.19	0.88	0.64	1.53	1.16	-0.76	0.04	0.18	0.25	0.16	0.20	0.34	0.39	0.24	0.35
Alasso	3.89	3.78	2.17	3.07	2.52	1.43	-1.29	-1.70	0.27	0.29	0.26	0.33	0.52	0.53	0.42	0.46
Bridge	0.37	0.85	-2.08	-1.45	1.24	0.77	-1.83	-1.47	0.02	0.13	0.04	0.12	0.30	0.34	0.16	0.25
SCAD	1.48	1.64	-0.80	-1.01	1.06	0.09	-2.29	-1.54	0.10	0.18	0.12	0.17	0.33	0.35	0.18	0.29
MCP	2.48	3.56	1.61	2.19	2.09	1.74	-0.53	0.22	0.17	0.27	0.20	0.26	0.42	0.47	0.30	0.41
SICA	2.03	1.29	-2.37	-2.84	0.38	-2.43	-5.69	-6.68	0.15	0.18	0.13	0.18	0.37	0.34	0.24	0.28
PLS	1.57	0.38	-2.82	-4.67	-0.49	-1.37	-4.76	-4.99	0.11	0.13	0.08	0.10	0.21	0.27	0.22	0.27
3PRF	2.69	0.77	-3.95	-5.34	-0.65	-1.65	-5.59	-5.95	0.19	0.15	0.04	0.07	0.19	0.25	0.15	0.20
SPLS	2.17	1.51	-1.18	-1.94	-0.64	-3.53	-7.28	-8.65	0.14	0.17	0.12	0.16	0.21	0.18	0.05	0.10
PCA	0.92	1.82	0.98	2.04	2.92	3.74	3.42	4.29	0.03	0.15	0.14	0.21	0.43	0.58	0.57	0.69
SPCA	-0.61	-0.71	-1.86	-1.23	1.88	2.25	1.38	2.34	-0.09	0.00	0.00	0.07	0.29	0.41	0.40	0.52
ICA	-0.34	-2.96	-7.75	-8.96	0.20	0.14	-1.56	-1.22	-0.02	-0.07	-0.26	-0.24	0.06	0.16	-0.05	0.09
RICA	0.55	-0.71	-4.06	-4.89	0.38	0.49	-1.05	-0.62	0.02	0.03	-0.07	-0.04	0.14	0.24	0.08	0.19

Notes: This table reports the certainty equivalent return and Sharpe Ratio of the stock-bond-commodity mean-variance portfolios with monthly rebalancing and transaction costs set to 50 bps for each asset. The out-of-sample period is from January 1997 to December 2016. The benchmarks are the equally-weighted portfolio and the mean-variance portfolio based on the historical average forecast. The alternative mean-variance allocations utilize return forecasts generated by multivariate prediction models. All mean-variance portfolios are constructed based on the sample covariance matrix. The portfolio performance is reported for different levels of risk aversion ($\gamma = 2, 10$) and portfolio weight constraints ($w_j \in [0,1]$, $w_j \in [0,1.5]$, $w_j \in [-0.5,1]$ and $w_j \in [-0.5,1.5]$). Figures in bold indicate the five models with the best performance.

Table 3.12: Mean-Variance Portfolio Performance based on Certainty Equivalent Return and Sharpe Ratio: Stock-Bond-Commodity Portfolios (Transaction Costs - Quarterly Rebalancing)

	Certainty Equivalent Return								Sharpe Ratio							
	γ=2				γ=10				γ=2				γ=10			
	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5
EW	3.87	3.87	3.87	3.87	-0.27	-0.27	-0.27	-0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
HA	4.73	4.51	3.80	4.39	3.92	4.84	4.04	4.93	0.35	0.30	0.29	0.32	0.51	0.64	0.54	0.66
KS	**7.12**	8.99	**9.45**	**10.40**	3.64	3.32	2.85	2.08	0.46	0.51	**0.54**	**0.57**	0.64	0.69	**0.72**	0.74
MC	6.61	7.34	7.53	8.34	**4.63**	**5.67**	**4.95**	**5.94**	**0.48**	0.49	0.50	0.52	0.65	**0.78**	**0.72**	**0.84**
MDC	5.12	5.43	4.32	5.21	4.24	5.18	4.36	5.30	0.35	0.36	0.32	0.35	0.58	0.71	0.61	0.73
TMC	6.52	7.26	6.66	7.58	**4.60**	**5.60**	**4.81**	**5.79**	0.46	0.48	0.46	0.49	0.64	**0.77**	0.70	**0.81**
Rank	5.70	6.87	5.97	6.36	4.17	5.19	4.33	5.39	0.40	0.45	0.41	0.42	0.58	0.72	0.60	0.73
C(2,PB)	5.61	6.42	6.59	7.24	4.21	5.26	**4.69**	**5.72**	0.39	0.42	0.43	0.46	0.57	0.71	0.66	**0.78**
C(3,PB)	5.04	6.10	6.00	6.33	3.97	5.06	4.62	5.68	0.34	0.39	0.39	0.41	0.52	0.68	0.64	0.76
D(1)	6.59	7.34	7.54	8.34	**4.62**	**5.67**	**4.95**	**5.94**	0.48	0.49	0.50	0.52	0.65	**0.78**	**0.72**	**0.84**
D(0.9)	6.52	7.37	7.59	8.36	**4.69**	**5.74**	**5.01**	**6.00**	0.47	0.49	0.50	0.52	0.67	**0.80**	**0.73**	**0.85**
Ridge	5.34	7.22	5.32	7.17	4.12	4.63	3.73	4.87	0.35	0.42	0.31	0.41	0.59	0.63	0.53	0.65
Lasso	6.21	8.99	8.24	**10.10**	4.37	4.88	3.96	5.08	0.42	0.52	0.47	0.54	**0.67**	0.71	0.61	0.72
EN	6.26	8.96	7.53	8.83	4.39	4.95	4.01	5.14	0.44	0.53	0.42	0.50	0.65	0.70	0.61	0.72
Alasso	**10.37**	**13.16**	**12.88**	**15.12**	**5.79**	**5.56**	4.03	4.30	**0.75**	**0.79**	**0.70**	**0.73**	**0.84**	**0.84**	**0.75**	0.78
Bridge	4.29	6.24	3.60	5.28	3.57	3.95	2.93	4.01	0.24	0.37	0.21	0.32	0.55	0.58	0.48	0.59
SCAD	6.28	**9.47**	8.15	9.69	4.28	4.84	3.71	4.78	0.42	**0.54**	0.47	0.53	0.66	0.70	0.60	0.71
MCP	5.32	8.28	7.47	9.30	4.59	5.44	4.19	5.21	0.35	0.48	0.42	0.50	**0.69**	0.75	0.64	0.74
SICA	**7.64**	**9.85**	**10.27**	**11.65**	4.30	4.01	1.78	1.97	**0.52**	**0.59**	**0.57**	**0.61**	**0.69**	0.70	0.61	0.66
PLS	**8.35**	**10.48**	**9.76**	**11.31**	4.09	3.86	2.13	2.49	**0.62**	**0.66**	**0.55**	**0.60**	0.66	0.64	0.62	0.65
3PRF	**8.68**	**10.72**	**8.76**	9.97	3.86	3.38	1.49	1.94	**0.64**	**0.67**	**0.53**	**0.57**	0.62	0.61	0.57	0.63
SPLS	6.14	8.38	6.41	8.47	4.13	3.39	1.56	2.24	0.42	0.52	0.40	0.48	**0.67**	0.61	0.52	0.58
PCA	4.24	5.34	4.76	5.70	3.67	4.76	3.80	4.89	0.27	0.34	0.31	0.37	0.52	0.68	0.60	0.72
SPCA	3.20	4.24	2.93	3.99	3.49	4.41	2.51	3.65	0.18	0.26	0.22	0.28	0.48	0.63	0.50	0.61
ICA	4.07	4.82	4.76	6.18	4.08	4.78	4.52	5.36	0.27	0.31	0.31	0.38	0.57	0.63	0.61	0.71
RICA	5.33	5.37	6.50	6.72	2.59	2.43	2.01	2.60	0.40	0.37	0.42	0.44	0.41	0.46	0.46	0.49

Notes: This table reports the certainty equivalent return and Sharpe Ratio of the stock-bond-commodity mean-variance portfolios with quarterly rebalancing and transaction costs set to 50 bps for each asset. The out-of-sample period is from January 1997 to December 2016. The benchmarks are the equally-weighted portfolio and the mean-variance portfolio based on the historical average forecast. The alternative mean-variance allocations utilize return forecasts generated by multivariate prediction models. All mean-variance portfolios are constructed based on the sample covariance matrix. The portfolio performance is reported for different levels of risk aversion ($\gamma = 2, 10$) and portfolio weight constraints ($w_j \in [0,1]$, $w_j \in [0,1.5]$, $w_j \in [-0.5,1]$ and $w_j \in [-0.5,1.5]$). Figures in bold indicate the five models with the best performance.

Table 3.13: Mean-CVaR Portfolio Performance based on Certainty Equivalent Return and Sharpe Ratio: Stock-Bond-Commodity Portfolios

	Certainty Equivalent Return								Sharpe Ratio							
	γ=2				γ=10				γ=2				γ=10			
	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5
EW	3.87	3.87	3.87	3.87	-0.27	-0.27	-0.27	-0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
HA	4.96	6.29	4.96	6.29	4.49	5.23	4.47	5.19	0.86	0.86	0.85	0.85	0.86	0.86	0.85	0.85
KS	**5.55**	**7.14**	**6.85**	**8.55**	**4.90**	**5.83**	**5.54**	**6.43**	0.87	0.87	0.88	0.93	0.96	0.96	0.93	0.98
MC	5.02	6.38	5.04	6.42	4.54	5.31	4.53	5.29	0.88	0.88	0.88	0.88	0.88	0.88	0.87	0.87
MDC	4.98	6.32	4.99	6.34	4.51	5.26	4.49	5.24	0.87	0.87	0.86	0.86	0.86	0.86	0.86	0.86
TMC	5.01	6.36	5.03	6.39	4.53	5.30	4.52	5.27	0.88	0.88	0.88	0.88	0.87	0.87	0.86	0.86
Rank	5.09	6.48	5.12	6.53	4.59	5.40	4.59	5.38	0.90	0.90	0.90	0.90	0.89	0.89	0.89	0.89
C(2,PB)	5.03	6.39	5.07	6.45	4.56	5.35	4.57	5.36	0.88	0.88	0.89	0.89	0.88	0.88	0.88	0.88
C(3,PB)	5.05	6.43	5.11	6.51	4.56	5.35	4.56	5.35	0.89	0.89	0.90	0.90	0.88	0.88	0.88	0.88
D(1)	5.02	6.38	5.04	6.42	4.54	5.31	4.53	5.30	0.88	0.88	0.88	0.88	0.88	0.88	0.87	0.87
D(0.9)	5.02	6.39	5.05	6.42	4.54	5.32	4.53	5.30	0.88	0.88	0.88	0.88	0.88	0.88	0.87	0.87
Ridge	5.27	6.76	5.80	7.26	4.76	5.66	5.12	6.05	0.96	0.96	0.93	0.95	0.95	0.95	1.01	1.00
Lasso	**5.42**	**6.98**	5.75	7.36	**4.87**	**5.82**	5.13	6.07	**1.00**	**1.00**	**0.99**	**1.02**	**0.98**	**0.98**	**1.03**	**1.02**
EN	5.37	6.90	5.68	7.23	4.82	5.75	5.11	6.04	**0.99**	**0.99**	0.95	**0.99**	0.97	0.97	**1.02**	**1.02**
Alasso	**5.44**	**7.00**	**6.13**	**7.81**	**4.89**	**5.86**	5.14	6.19	0.98	0.98	0.95	0.99	**0.99**	**0.99**	0.98	**1.02**
Bridge	5.25	6.73	5.71	7.26	4.73	5.62	5.06	5.97	0.95	0.95	0.91	0.95	0.94	0.94	1.00	0.99
SCAD	5.40	6.95	5.90	7.46	4.83	5.77	**5.20**	**6.19**	**0.99**	**0.99**	**1.04**	**1.04**	0.97	0.97	**1.06**	**1.05**
MCP	**5.43**	**6.99**	5.75	7.31	4.85	5.79	5.12	6.05	**1.00**	**1.00**	0.96	**1.00**	**0.97**	**0.97**	**1.02**	1.01
SICA	**5.68**	**7.36**	**6.16**	**8.08**	**5.04**	**6.07**	**5.33**	**6.37**	**1.00**	**1.00**	**0.98**	**1.06**	**1.02**	**1.02**	**1.06**	**1.07**
PLS	5.41	6.97	**6.58**	**8.22**	**4.86**	**5.81**	**5.36**	**6.28**	0.98	0.98	0.85	0.94	**0.97**	**0.97**	0.90	0.97
3PRF	5.32	6.83	**6.45**	**8.11**	4.82	5.75	**5.35**	**6.25**	0.96	0.96	0.86	0.94	0.97	0.97	0.91	0.97
SPLS	5.25	6.72	5.94	7.37	4.79	5.69	5.14	6.05	0.90	0.90	0.86	0.90	0.95	0.95	1.00	1.00
PCA	5.18	6.62	5.50	7.09	4.64	5.48	4.78	5.65	0.93	0.93	**0.96**	0.96	0.91	0.91	0.93	0.93
SPCA	5.21	6.67	5.77	7.32	4.70	5.56	4.84	5.75	0.94	0.94	**0.98**	0.99	0.93	0.93	0.95	0.95
ICA	5.04	6.40	5.55	6.96	4.50	5.24	4.77	5.54	0.83	0.83	0.85	0.85	0.86	0.86	0.91	0.90
RICA	5.12	6.53	5.20	6.63	4.61	5.42	4.59	5.35	0.90	0.90	0.86	0.86	0.89	0.89	0.87	0.87

Notes: This table reports the certainty equivalent return and Sharpe Ratio of the stock-bond-commodity mean-CVaR portfolios with monthly rebalancing. The out-of-sample period is from January 1997 to December 2016. The benchmarks are the equally-weighted portfolio and the mean-CVaR portfolio based on the historical average forecast. The alternative mean-variance allocations utilize return forecasts generated by multivariate prediction models. The CVaR of all portfolios is estimated by generating 5000 scenarios using Monte Carlo simulation based on the multivariate normal distribution, with mean varying according to the return forecast and sample variance-covariance matrix. The portfolio performance is reported for different levels of risk aversion ($\gamma = 2, 10$) and portfolio weight constraints ($w_j \in [0,1]$, $w_j \in [0,1.5]$, $w_j \in [-0.5,1]$ and $w_j \in [-0.5,1.5]$). Figures in bold indicate the five models with the best performance.

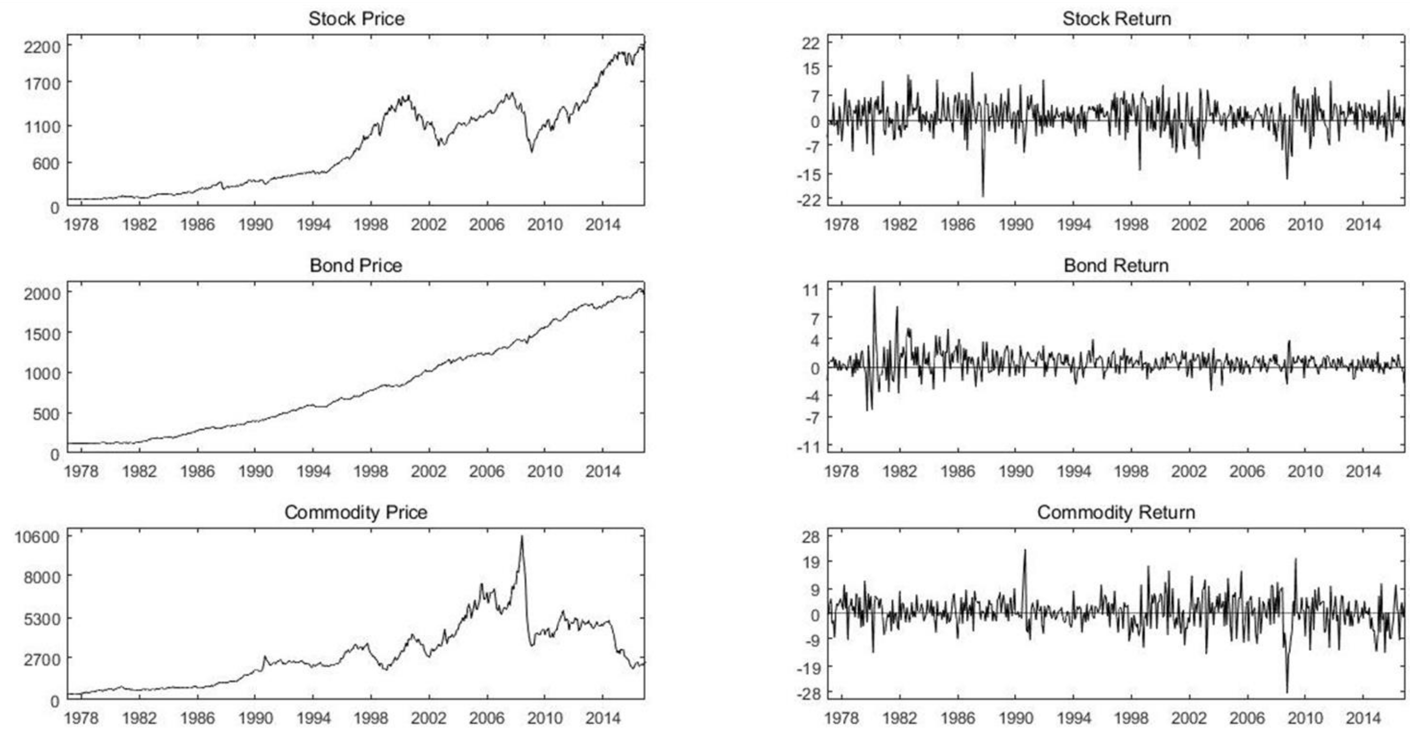
Table 3.14: Mean-Variance and Mean-CVaR Portfolio Performance based on CVaR: Stock-Bond-Commodity Portfolios

	γ=2								γ=10							
	Mean-Variance				Mean-CVaR				Mean-Variance				Mean-CVaR			
	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5	0,1	0,1.5	-0.5,1	-0.5,1.5
EW	5.65	5.65	5.65	5.65	5.65	5.65	5.65	5.65	5.65	5.65	5.65	5.65	5.65	5.65	5.65	5.65
HA	7.62	9.90	10.17	11.40	1.63	2.52	1.64	2.53	2.99	3.50	3.01	3.51	1.63	2.52	1.64	2.54
KS	8.73	12.62	16.02	18.90	1.97	3.05	2.77	3.65	5.86	7.29	7.92	8.65	1.63	2.53	2.36	3.11
MC	6.80	8.98	**9.83**	10.90	1.60	2.48	**1.61**	**2.50**	**2.88**	**3.31**	**3.03**	**3.44**	1.61	2.49	1.62	2.52
MDC	7.24	9.26	**9.69**	**10.81**	1.62	2.51	1.63	2.52	**2.89**	**3.37**	**2.95**	**3.45**	1.62	2.51	1.64	2.53
TMC	6.81	**8.94**	**9.59**	**10.66**	1.61	2.49	1.62	2.51	**2.82**	**3.27**	**2.94**	**3.39**	1.61	2.49	1.63	2.52
Rank	**6.23**	**8.25**	9.84	**10.85**	1.59	2.47	**1.60**	**2.49**	3.13	3.56	3.79	4.05	1.60	2.47	**1.61**	2.50
C(2,PB)	**6.79**	9.24	10.33	11.39	1.60	2.48	**1.61**	**2.50**	3.18	3.55	3.54	3.85	1.60	2.48	1.62	2.51
C(3,PB)	6.84	9.40	10.73	11.88	1.60	2.48	**1.60**	**2.49**	3.44	3.90	4.17	4.39	1.60	2.48	1.62	2.51
D(1)	**6.79**	**8.98**	**9.83**	**10.89**	1.60	2.48	**1.61**	**2.50**	**2.88**	**3.30**	**3.03**	**3.44**	1.61	2.49	1.62	2.51
D(0.9)	**6.79**	**8.96**	**9.81**	**10.84**	1.60	2.48	1.61	2.50	**2.86**	**3.29**	**3.02**	**3.43**	1.61	2.49	1.62	2.51
Ridge	7.65	10.34	11.44	13.26	**1.57**	**2.43**	1.96	2.77	4.01	4.72	5.14	5.39	1.56	2.43	1.63	2.49
Lasso	7.61	10.52	12.07	14.16	**1.55**	**2.41**	1.77	2.56	4.39	5.26	5.49	5.80	**1.55**	**2.40**	**1.59**	**2.41**
EN	7.60	10.37	11.54	13.51	**1.55**	**2.41**	1.81	2.59	4.19	4.95	5.16	5.40	**1.55**	**2.41**	**1.60**	**2.42**
Alasso	7.71	10.97	13.58	15.63	1.62	2.51	2.10	2.93	4.82	6.02	6.86	7.35	**1.54**	**2.39**	1.75	2.54
Bridge	7.92	10.77	12.07	13.73	1.57	2.44	1.95	2.79	4.17	4.86	5.39	5.70	1.56	2.42	1.63	2.49
SCAD	8.00	10.98	12.75	14.86	**1.56**	**2.42**	1.72	2.54	4.52	5.43	5.75	5.96	1.56	2.42	**1.58**	**2.39**
MCP	7.67	10.46	11.88	13.96	**1.57**	**2.44**	1.85	2.61	4.33	5.19	5.48	5.76	1.56	2.43	1.63	**2.47**
SICA	8.34	12.03	15.49	18.11	1.69	2.62	2.01	2.82	5.53	7.04	7.84	8.47	**1.55**	**2.41**	1.64	**2.44**
PLS	7.86	11.33	14.16	16.70	1.59	2.47	2.75	3.42	4.96	5.91	7.06	7.44	1.56	2.42	2.35	3.06
3PRF	7.89	11.22	14.22	16.41	1.60	2.48	2.58	3.32	4.92	5.89	7.06	7.46	**1.55**	**2.40**	2.27	3.01
SPLS	7.49	10.68	13.04	15.49	1.70	2.63	2.24	3.05	4.98	6.47	7.19	7.94	1.58	2.46	1.67	2.54
PCA	**6.79**	**8.92**	10.23	11.40	1.59	2.46	1.69	2.62	2.91	3.47	4.13	4.48	1.59	2.46	1.64	2.55
SPCA	6.91	9.17	10.68	11.93	1.58	2.45	1.80	2.65	3.31	4.02	4.90	5.13	1.58	2.45	**1.61**	2.50
ICA	7.92	10.57	11.61	12.94	1.75	2.71	2.03	3.02	3.54	4.22	4.21	4.77	1.64	2.54	1.70	2.61
RICA	7.53	10.13	11.75	13.52	1.61	2.50	1.77	2.74	3.88	4.49	4.72	5.10	1.61	2.49	1.68	2.60

Notes: This table reports the CVaR calculated at the 95% confidence level of the stock-bond-commodity mean-variance and mean-CVaR portfolios with monthly rebalancing. The out-of-sample period is from January 1997 to December 2016. The benchmarks are the equal-weighted portfolio and the mean-variance and mean-CVaR portfolios based on the historical average forecast. The alternative mean-variance allocations utilize return forecasts generated by multivariate prediction models. All mean-variance portfolios are constructed based on the sample covariance matrix. The CVaR of all portfolios is estimated by generating 5000 scenarios using Monte Carlo simulation based on the multivariate normal distribution, with mean varying according to the return forecast and sample variance-covariance matrix. The portfolio performance is reported for different levels of risk aversion ($\gamma = 2, 10$) and portfolio weight constraints ($w_j \in [0,1]$, $w_j \in [0,1.5]$, $w_j \in [-0.5,1]$ and $w_j \in [-0.5,1.5]$). Figures in bold indicate the five models with the best performance.

Chapter 3 Figures

Figure 3.1: Index Prices and Returns



Notes: The figure plots the monthly prices and returns of the stock, bond and commodity indices. The indices are denominated in US dollars. The sample period is from 1977 to 2016.

Chapter 3 Appendix

This section provides a table with the sources of the series used in this study and detailed description on the construction of the predictors.

Table A3.1 provides the sources of the time series used to construct the variables in this study.

[Insert Table A3.1 Here]

The predictor variables used to forecast the three indices were constructed in the following way:

1. DP: Dividend-price ratio the difference between the log of dividends paid on the S&P 500 index and the log of the S&P 500 index price. The dividends are measured using a 12-month moving sum.
2. DY: Dividend yield is the difference between the log of dividends and the log of lagged stock prices.
3. EP: Earnings-price ratio is the difference between the log of earnings on the S&P 500 Index minus the log of stock prices. The earnings are measured using a 12-month moving sum.
4. DE: Dividend-payout ratio is the difference between the log of dividends and the log of earnings.
5. SVAR: Stock variance is the sum of squared daily returns on the S&P 500 Index.
6. BM: The book-to-market ratio for the Dow Jones Industrial Average.
7. NTIS: Net equity expansion is the ratio of a 12-month moving sum of net equity issues by NYSE-listed stocks to the total end-of-year market capitalization of NYSE stocks.
8. TBL: Treasury bill rate is the interest rate on a three-month Treasury bill.
9. LTY: Long-term yield is the long-term government bond yield.
10. LTR: Long-term return is the return on long-term government bonds.

11. TMS: Term spread is the difference between the long-term yield and the Treasury bill rate.
12. DFY: Default yield spread, difference between Moody's BAA- and AAA-rated corporate bond yields.
13. DFR: Default return spread is the difference between the long-term corporate bond return and the long-term government bond return.
14. INFL: Inflation based on the CPI. To account for the delay in CPI releases, one period lagged values of inflation are used.
15. IP: The first difference in the log-levels of the industrial production index.
16. M1: The first difference in the log-levels of the money stock M1.
17. UR: The change in civilian unemployment rate.
18. VXO: The Chicago Board Options Exchange S&P 100 Volatility Index.
19. Umacro: Macroeconomic uncertainty. An aggregate measure of macroeconomic uncertainty proposed by Jurado, Ludvigson and Ng (2015) and Ludvigson, Ma and Ng (2015), based on a large set of macroeconomic indicators.
20. Ufin: Financial Uncertainty. An aggregate measure of financial uncertainty proposed by Jurado, Ludvigson and Ng (2015) and Ludvigson, Ma and Ng (2015), based on a large set of financial indicators.
21. SP1: The yield spread between a 1-year government bond and the 6-month Treasury bill
22. SP5: The yield spread between a 5-year government bond and the 6-month Treasury bill.
23. PPI: The second difference in the log-levels of the Producers Price Index (Finished Goods).
24. CAP: The change in capacity utilization-manufacturing.
25. IS: The change in inventories-sales ratio (Total Business).
26. REA: The real economic activity index proposed by Kilian (2009).

27. CFNAI: The Chicago Fed national activity index.
28. WTI: The first difference in the log-levels of the spot crude oil price: West Texas Intermediate.
29. USDAUD: The first difference in the log-levels of the Australian dollar-US dollar.
30. USDCAD: The first difference in the log-levels of the Canadian dollar-US dollar.
31. USDIND: The first difference in the log-levels of the Indian rupee-US dollar.
32. USDNZD: The first difference in the log-levels of the New Zealand dollar-US dollar.

The predictors DP, DY, EP, BM, NTIS, TBL, LTY and INFL are non-stationary based on augmented Dickey-Fuller (ADF) tests and are adjusted to stationarity by taking first differences.

Table A3.1: Data Source

Series	Source
S&P 500 Total Return Index	Amit Goyal's Website
Bloomberg Barclays US Aggregate Bond Index	Bloomberg
S&P Goldman Sachs Commodity Total Return Index	Bloomberg
12-Month Moving Sum of Dividends	Amit Goyal's Website
12-Month Moving Sum of Earnings	Amit Goyal's Website
AAA-Rated Corporate Bond Yield	Amit Goyal's Website
BAA-Rated Corporate Bond Yield	Amit Goyal's Website
Long-Term Corporate Bond Return	Amit Goyal's Website
Stock Market Volatility	Amit Goyal's Website
Book-to-Market Value Ratio	Amit Goyal's Website
Total Net Issues of NYSE	Amit Goyal's Website
3-Month Treasury Bill yield	Amit Goyal's Website
Long-Term Government Bond Return	Amit Goyal's Website
Long-Term Government Bond Yield	Amit Goyal's Website
Inflation Rate	Amit Goyal's Website
6-Month Treasury Bill yield	St. Louis Fed’s FRED database
1-Year Treasury Rate	St. Louis Fed’s FRED database
5-Year Treasury Rate	St. Louis Fed’s FRED database
Chicago Board Options Exchange S&P 100 Volatility Index	St. Louis Fed’s FRED database
Total Business: Inventories to Sales Ratio	St. Louis Fed’s FRED database
Capacity Utilization: Manufacturing	St. Louis Fed’s FRED database
Producer Price Index: Finished Goods	St. Louis Fed’s FRED database
M1 Money Stock	St. Louis Fed’s FRED database
Industrial Production Index	St. Louis Fed’s FRED database
Civilian Unemployment Rate	St. Louis Fed’s FRED database
Chicago Fed National Activity Index	St. Louis Fed’s FRED database
Spot Crude Oil Price: West Texas Intermediate	St. Louis Fed’s FRED database
Australian Dollar-US Dollar	St. Louis Fed’s FRED database
Canadian Dollar-US Dollar	St. Louis Fed’s FRED database
Indian Rupee-US Dollar	St. Louis Fed’s FRED database
New Zealand Dollar-US Dollar	St. Louis Fed’s FRED database
Macroeconomic Uncertainty	Sydney C. Ludvigson's Website
Financial Uncertainty	Sydney C. Ludvigson's Website
Real Economic Activity Index	Lutz Kilian's Website

Notes: This table reports the name and sources of the series used in this study.

Chapter 4: Pairs Trading using Quantile Regression

4.1. Introduction

Pairs trading is a statistical arbitrage strategy which is based on the principle that the prices of two assets co-move with each other. Pairs trading is a market-neutral strategy that matches a long position with a short position through a pair of co-moving assets and it is also a mean-reverting strategy that assumes that the spread will revert to its historical mean leading to profits through relatively low risk positions. According to Gatev, Goetzmann and Rouwenhorst (2006), the concept of pairs trading is comprised of two stages. In the formation period, a pair of assets whose prices have moved together historically is identified. In the trading period, the spread between the two asset prices is monitored and if the prices diverge and the spread widens, the higher priced asset is sold and the lower priced asset is bought. If the two assets follow an equilibrium relationship, the prices of the two assets will converge and the spread will revert to its historical mean, resulting in profit.

There are many pairs trading methods in the literature, which, according to Krauss (2017), can be categorized into five groups: the distance approach, the cointegration approach, the time-series approach, the stochastic control approach, with the fifth group, entitled "other approaches", containing pairs trading methods unrelated to the aforementioned approaches and with limited supporting literature. In the distance method (DM), introduced by Gatev, Goetzmann and Rouwenhorst (2006), distance metrics are used to identify co-moving assets, while nonparametric threshold rules are used as triggers to open or close a pair position. In contrast to the distance method, the cointegration method is a model based approach that assumes a cointegrating relationship between the price series of two assets. A theoretical framework for pairs trading is outlined by Vidyamurthy (2004), based on the error correction model representation of cointegrated time series by Engle and Granger (1987). In the time series approach, the focus is on the trading period and on generating optimal trading signals using different methods. The most cited study using the time-series approach is by Elliott, Van Der Hoek and Malcolm (2005), who model the mean reversion of the spread, defined as the difference between the two prices, in a continuous time setting, by using a state-space representation of the spread estimated by Kalman filter. Finally, the primary focus in the stochastic control approach is finding the optimal portfolio holdings in the two legs of the pair when other assets are available. Jurek and Yang (2007) develop a model which allows non-myopic arbitrageurs to allocate capital between a mean-reverting spread and a risk-free asset, while Liu and Timmermann (2013) derive optimal portfolio strategies for convergence trades under recurring and nonrecurring arbitrage opportunities for an investor who maximizes the expected value of a power utility function defined over terminal wealth.

We extend the literature of pairs trading by examining the performance of the cointegration method, when the spread during the trading period is computed using quantile regressions. We use daily data of the stocks in the S&P 100 to conduct a robust analysis of the new pairs trading strategy, which takes advantage of the information in the conditional quantiles of the distribution of asset returns, against the distance method and original cointegration method benchmarks.

The study by Gatev, Goetzmann and Rouwenhorst (2006) is the earliest comprehensive study to examine pairs trading. In the formation period they rank each possible combination of pairs according to the sum of squared differences (SSD) on normalized price series and trading is triggered when the spread diverges more than two historical standard deviations (estimated during the formation period) and closed upon mean reversion or at the end of the trading period, independent of the occurrence of any price convergence. Do and Faff (2010) examine the profitability of the DM and find that it yields the highest performance during the 1970s and the 1980s, with declining performance in the 1990s. The exceptions to the declining performance of the DM are the periods of the dot-com bubble and global financial crisis. Additionally, Do and Faff (2012) examine the effects of transaction costs on the distance method, by accounting for commissions, market impact, and short-selling fees. They find that the strategy becomes largely unprofitable, with only refined portfolios, based on additional selection metrics, achieving a positive excess return. More recently Rad, Low and Faff (2016), perform an extensive evaluation of the DM and find that it generates positive returns and is less affected by transaction costs compared to other methods. While the aforementioned studies focus on the US stock market, the profitability of the distance method has also been examined in international stock markets. Other applications of the distance method include Perlin (2009), who applies the DM to the Brazilian stock market, Broussard and Vaihekoski (2012), who examine the profitability of pairs trading under different weighting schemes and trade initiation conditions in the Finnish stock market and Jacobs and Weber (2015), who analyse the performance of the DM for 34 countries and find that the returns of the strategy depend on the investors' reaction to news.

The framework for pairs trading based on cointegration was proposed by Vidyamurthy (2004). During the formation period the tradability of the pairs is assessed using cointegration tests. The most commonly used approach is the Engle-Granger test for cointegration that is based on the error-correction representation of the relationship between two asset prices. In the trading period entry and exit signals are generated using simple threshold rules, based on the normalized spread between the prices of the two cointegrated assets. In a large scale application using US data, Rad, Low and Faff (2016), compare the performance of three methods; the distance, cointegration and copula methods. They find that the cointegration performs similarly to the DM, with the former generating better results than the other strategies before transaction costs are taken into account. In a recent study, Farago and Hjalmarsson (2018) use data from the Stockholm stock exchange, which contains “ordinary stocks” that are issued by different companies and dual-class firms that issue A- and B-shares, whose prices are closely related and can act as a control group for series that are likely cointegrated. They find that before transaction costs A-B pairs yield similar Sharpe ratios to the theoretical model they developed, which are higher than those of the ordinary pairs. Their findings suggest that cointegration is not the likely reason for the profitability of pairs trading strategies based on ordinary stocks, since they do not satisfy the cointegrating restrictions, compared to the A-B pairs. Other applications include those by Caldeira and Moura (2013), who use a cointegration-based trading strategy on the Brazilian stock exchange and find statistically significant excess returns after accounting for transaction costs, and by Huck and Afawubo (2015) who develop pairs trading strategies using stocks listed on the S&P 500 and find that the cointegration approach significantly outperforms the distance approach.

The contributions of this study are threefold. First, we combine aspects of the cointegration method with quantile regression to produce a new approach to pairs trading. In the formation period stock pairs are sorted and selected similarly to the cointegration method, while in the trading period the trading signal is generated based on the spread of the stock prices in the pair, which has been estimated by quantile regression. Our second contribution, stems from the extensive evaluation of the new method along with the distance method and the original cointegration method, using a dataset consisting of daily observations of all stocks in the S&P 100 from 2000 to 2017. Additionally, we use a variety of economic and risk-adjusted measures to evaluate the performance of the new method, estimated for multiple quantiles, and compare it with the simpler alternatives. Our final contribution arises from evaluating the performance of the pairs trading strategies under an asset pricing framework, in order to examine whether the returns of each strategy can be explained by various risk factors.

The remainder of this chapter proceeds as follows; Section 4.2 provides a description of the data, Section 4.3 describes the pairs trading strategies and Section 4.4 presents the results. Section 4.5 concludes.

4.2. Data

For the empirical application we focus on the S&P 100. The primary reasons being market efficiency and computational feasibility. The S&P 100 consists of the 100 major, blue chip companies across multiple industry sectors. This highly liquid subset of the US stock market serves as a proving ground for any trading strategy, since investor scrutiny and analyst coverage is especially high for these large capitalization stocks. Additionally, handling approximately 100 stocks per iteration of the backtest, renders even the most sophisticated strategies computationally feasible, making the S&P 100 the ideal choice for the application of our strategy. First, we obtain the month end constituent list for the S&P 100 from Bloomberg from December 2000 to December 2017[1]. Then, following Krauss and Stübinger (2015), we consolidate those lists into a binary matrix, with one indicating that the stock is a constituent of the index and zero otherwise. Furthermore, we acquire the daily total return index of all stocks that were included in the S&P

[1] The choice of sample period is due to data availability constraints, since information on the constituents list of the S&P 100 starts on December 2000.

100 for the same period. This leads to a dataset with 4296 daily observations (205 months), for a total of 177 stocks.

4.3. Methodology

Pairs trading is a mean-reverting strategy that assumes a relationship between the prices of two securities. Modelling this relationship could potentially allow us to take advantage of any short-term deviations from the mean, by simultaneously buying the undervalued security and selling short the overvalued security. When the prices revert to their mean, we close the position and realize the profit. To examine the performance of the three pair trading strategies, based on daily data of S&P 100 stocks from December 2000 to December 2017, we use a similar setting for our backtest as Rad, Low and Faff (2016). The performance of the strategies is evaluated using a rolling window of 18 (calendar) months that is updated by moving forward by one month in each iteration. The rolling window in each iteration is divided in a formation period of 12 months, where the pairs are selected and a trading period of six months, where the strategy is executed. Since we do not wait six months for the current trading period to end, we end up with six overlapping portfolios of pairs, with each portfolio having a different starting period, since it belongs to a different iteration. There are a total of 188 backtest iterations (cycles of formation/trading periods) and the number of pairs considered in each are approximately 4950.

The distance method, described in Section 4.3.1, is a popular pairs trading strategy that we use as a benchmark to evaluate the cointegration and quantile regression methods. In the DM, during the formation period, the pairs of stocks are sorted based on the sum of squared differences (SSD) in their normalized prices and throughout the trading period their spreads are monitored for any deviations beyond a certain threshold that would trigger a long and short position. The cointegration method for pairs trading (Section 4.3.2) is based on the property of cointegrated time series, that they excibit a long-term equilibrium and any deviations from that equilibrium will be corrected through time, when the series will mean-revert. In this approach the pairs are selected based on the two-step Engle-Granger method (Engle and Granger (1987)) during the formation period and in the course of the trading period, long and short positions are opened when there are temporary deviations from the estimated stationary spread. The third approach combines the cointegration method and quantile regression. Pairs of stocks are sorted and selected in the same way as in the cointegration method, however, the spread is estimated for a range of quantiles using quantile regression. We provide details of this approach in Section 4.3.3.

4.3.1. Distance Method

The first strategy is based on the distance method proposed by Gatev, Goetzmann and Rouwenhorst (2006), and is similar to its in implementation as in Rad, Low and Faff (2016) and Do and Faff (2010, 2012). In the DM potential pairs are sorted according the sum of squared differences in their normalized prices during the formation period. Let $p_{1,t}$ and $p_{2,t}$ denote the normalized price series of two stocks calculated by dividing the price series $P_{1,t}$ and $P_{2,t}$ with their respective first observation, so that they are scaled to \$1 at the beginning of the formation period. The SSD of a pair is then computed using the following formula:

$$\mathrm{SSD}_{1,2} = \frac{1}{T_f} \sum_{t=1}^{T_f} \left(p_{1,t} - p_{2,t}\right)^2 \tag{4.1}$$

where T_f denotes the number of daily observations in the formation period. During the 12-month formation period the spread between the normalized prices of all possible pair combinations is calculated. Then we select the 20 pairs with the least SSD that are going to be traded in the subsequent 6-month trading period. The mean and standard deviation of the spread is kept since it will be used to generate the trading signal. At the beginning of the trading period, the prices are rescaled to begin at $1 and the spread of the 20 selected pairs is recalculated and monitored. When the spread diverges by two or more historical standard deviations, a long and a short position is simultaneously opened in the pair on the direction of the divergence.

4.3.2. Cointegration Method

The theoretical framework for the cointegration method for pairs trading was developed by Vidyamurthy (2004). In the formation period the pairs are selected using the Engle-Granger two-step approach (Engle and Granger (1987)). Let $P_{1,t}$ and $P_{2,t}$ denote the $I(1)$-nonstationary price processes of the two stocks. If a linear combination of the two series exists that is $I(0)$-stationary then the two price series are said to be cointegrated. If the two price series are cointegrated then there exists a non-zero real number β such that:

$$u_{ij,t} = P_{i,t} - \beta P_{j,t}, \text{ for } i,j = 1,2 \text{ and } i \neq j, \tag{4.2}$$

where β is the cointegration coefficient and the spread $u_{ij,t}$ is a $I(0)$-stationary series known as cointegrating errors. To test whether the spread is cointegrated, the Engle and Granger (1987) Error Correction Model (ECM) framework can be used. According to the ECM representation the cointegrating series exhibit long-run equilibrium, even though short-run deviations from this equilibrium may occur. The ECM representation of the cointegration relationship between price series $P_{1,t}$ and $P_{2,t}$ is:

$$\Delta u_{ij,t} = \gamma u_{ij,t-1} + \delta \Delta u_{ij,t-1} + \varepsilon_t, \text{ for } i,j = 1,2 \text{ and } i \neq j, \tag{4.3}$$

where $\Delta u_{ij,t}$ denotes the first difference of the spread $u_{ij,t}$. The null hypothesis is that the spread is a unit-root process ($H_0: \gamma = 1$), therefore the two price series are not cointegrated and the alternative that the spread is stationary ($H_0: \gamma \neq 1$), which means that the two series are cointegrated. This is equivalent to an Augmented Dickey-Fuller (ADF) test, for a model specification without intercept, linear trend and with a lag of order one. It should be noted that in the Engle-Granger procedure, the usual ADF tabulated critical values cannot be used, since we test for the stationarity on a derived variable - the spread of the two price series estimated by OLS. However, the selection of the pairs in the case of the cointegration method is done based on the t-statistic. Specifically, in the formation period we select the 20 pairs with the lowest t-statistics for the estimate of γ. Cointegrating series exhibit mean-reverting behavior and by going short (long) $P_{i,t}$ and long (short) $P_{j,t}$ when the spread is positive (negative), then the strategy should generate profits once the spread returns back to its long-term equilibrium.

Similar to the DM, the mean μ_u and standard deviation σ_u of the spread $u_{ij,t}$ are computed using data in the formation period. During the trading period the spread is calculated based on a rolling window of 120 observations and is used to form the normalized spread:

$$Z_t = \frac{u_{ij,t} - \mu_u}{\sigma_u} \tag{4.4}$$

where $u_{ij,t} = P_{i,t} - \beta_t P_{j,t}$ and β_t is the is the cointegration coefficient estimated by OLS in a rolling window of 120 observations. To estimate the normalized spread, observations from the formation period need to be used. However, when estimating the trading entry and exit points of the strategy and the return of the pairs, the values of β_t and Z_t used, correspond to the dates of the trading period. Similarly to the DM method we simultaneously open and close long and short positions when the normalized spread diverges beyond 2 and close all positions when the spread returns to zero, which is equivalent to the spread returning to its long-run equilibrium.

4.3.3. Modelling the Spread using Quantile Regression

In the quantile regression method the pairs are selected in the same way as in the cointegration method, during the formation period. However, the cointegrating coefficient, β, is estimated throughout the trading period using quantile regression. The quantile regression estimator for each quantile $\tau \in \mathcal{T}$ is obtained through the following optimization problem:

$$\hat{\beta}(\tau) = \underset{\beta \in \mathbb{R}}{\operatorname{argmin}} \sum_{t=1}^{T} \rho_\tau\left(P_{i,t} - \beta(\tau) P_{j,t}\right), \text{ for } i, j = 1, 2 \text{ and } i \neq j, \tag{4.5}$$

where $\rho_\tau(u) = u\big(\tau - H(u < 0)\big)$ is the asymmetric weights function as in Koenker and Bassett (1978) and $H(\cdot)$ is a Heaviside step function. In the special case where $\tau = 0.5$ quantile regression is equivalent to the Least Absolute Deviation (LAD) estimation of $\beta(\tau)$. The spread between the prices of stocks i and j, for the τth quantile, is then computed as $u_{\tau,t} = P_{i,t} - \hat{\beta}(\tau) P_{j,t}$ which is used to form the normalized spread and open simultaneously long and short positions in the same way as in the cointegration method, where β was estimated using OLS.

4.3.4. Return Calculation

The return of a nominated pair, $r_{p,t}$, within each iteration of the backtest is calculated using the following formula:

$$r_{p,t} = I_{p,t-1}\left(r_{i,t} - \beta_{t-1} r_{j,t}\right) \tag{4.6}$$

where $r_{i,t}$ and $r_{j,t}$ are the percentage returns of stocks i and j respectively and $I_{p,t}$ is a dummy variable which takes the value of 1 for a long position in the spread, value -1 for a short position in the spread and 0 otherwise. The lagged values are used for both the trading signal dummy variable, $I_{p,t}$, and the cointegration coefficient, β_t, when calculating the returns of each nominated pair. In the case of the quantile regression method the return of a pair is calculated based on the lagged estimates $\beta(\tau)$, for each value of τ:

$$r_{p,t}(\tau) = I_{p,t-1}\left(r_{i,t} - \beta_{t-1}(\tau) r_{j,t}\right) \tag{4.7}$$

and for the DM long and short positions are valued equally, therefore the returns of each pair are estimated by the following formula:

$$r_{p,t} = I_{p,t-1}\left(r_{i,t} - r_{j,t}\right) \tag{4.8}$$

Following Gatev, Goetzmann and Rouwenhorst (2006) and Do and Faff (2010), the returns of each strategy are calculated in two ways: return on employed capital and return on committed capital.

$$r_t^{EC} = \frac{\sum_{p=1}^{n} r_{p,t}}{n} \tag{4.9}$$

$$r_t^{CC} = \frac{\sum_{p=1}^{n} r_{p,t}}{20} \tag{4.10}$$

Return on employed capital for day t, r_t^{EC}, is calculated as the sum of the daily returns of all pairs divided by the number of pairs that have traded during that day. Return on committed capital for day t, r_t^{CC}, is calculated as the sum of daily returns of all pairs divided by the number of pairs that were nominated to trade in that day (20), regardless of whether they actually traded or not. As we do not wait for the six-month trading period of the iteration of the backtest to complete, each month we have six overlapping portfolios. The return of each strategy is computed as the equally-weighted average of the returns of these six portfolios. Furthermore, since trades do not necessarily open at the beginning of the trading period or there are days when no trading has occurred and interest is not accrued to the capital when it is not involved in a trade, the performance of the strategies is underestimated.

4.4. Empirical Results

The performance of the pairs trading strategies, based on the return on employed capital and return on committed capital, is estimated using various performance measures.

4.4.1. Pairs Trading Performance

The three simple measures used to evaluate the performance of the pairs trading strategies are the average excess return (AV), the standard deviation (SD) and the end of period value (EPV), which is the value at the end of the period of the backtest for a portfolio with starting wealth of 1 unit at the beginning of the period of the backtest and is based on the cumulative sum of returns.

When measuring portfolio performance the Sharpe ratio (SR) is the most popular metric and is calculated as the fraction of the excess return (average realized return less the risk-free rate) divided by the standard deviation of the excess returns.

$$\mathrm{SR} = \frac{r_i - r_f}{\sigma_i}, \tag{4.11}$$

where r_i is the average realized return of a pairs trading strategy, r_f is the risk free rate and σ_i is the standard deviation of the strategy's excess returns.

To investigate the capacity of the different strategies to assess tail risk we compute the Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) measures for each strategy. VaR describes the possible loss of an investment that is not exceeded with a given probability level of $1 - a$ in a certain period. The a Value-at-Risk of a portfolio is computed as:

$$\mathrm{VaR}_a = -F^{-1}(1-a)\sigma_i - (r_i - r_f), \tag{4.12}$$

where F is the cumulative standard normal distribution function. The a Conditional Value-at-Risk of a portfolio is given by:

$$\mathrm{CVaR}_a = (1-a)^{-1} f\left(F^{-1}(1-a)\right)\sigma_i - (r_i - r_f), \tag{4.13}$$

where f is the probability density function of the standard normal distribution. The VaR and CVaR are calculated at the 95% confidence interval.

[Insert Table 4.1 Here]

Table 4.1 reports the main results for the performance of the three strategies, based on the end-of-period value, average return, standard deviation, Sharpe ratio, VaR and CVaR metrics, for the period from December 2001 to December 2017. The first panel reports the results before transaction costs and the results after transaction costs are reported in the second panel. The table presents the results on return on committed capital and employed capital. The average excess return and Sharpe ratio before transaction costs, based on the employed capital method, are positive for all strategies except those based on quantile regression for $\tau = 0.8$ and 0.9. The positive average returns range from 0.61% to 7.06%, while the ratios range from 0.056 to 0.1970, with the strategies with $\tau = 0.1$ to 0.5 outperforming the DM and original cointegration method benchmarks, in terms of both measures. The lowest standard deviation is produced by the distance method, while the strategy with $\tau = 0.1$ and 0.2 yields the second lowest standard deviations. These are also the two pairs trading strategies with the lowest VaR and CVaR values. After taking into account transaction costs of 3 bps per share, the average return and Sharpe ratio for the distance method and the strategies based on quantile regression for $\tau = 0.6$ and 0.7 become negative. For the remaining strategies the average return has a range from 0.01% to 4.03% and the Sharpe ratio is from 0.0001 to 0.1124, with all quantile regression methods outperforming the cointegration method. Turning to the results for the return on committed capital before transaction costs the rankings are similar to those based on return on employed capital, with strategies based on quantile regression with $\tau = 0.1$ to 0.5 outperforming both benchmarks. Overall, the average excess return and Sharpe ratios are lower, since this is a more conservative way of computing returns. After transaction costs are taken into account the model with $\tau = 0.1$ is the only one generating positive returns and Sharpe ratio.

[Insert Figures 4.1 and 4.2 Here]

Figures 4.1 and 4.2 present the cumulative excess return of all strategies based on the return on committed capital and return on employed capital respectively. It can be seen that the pairs trading strategies based on quantile regression with $\tau = 0.1$ to 0.4 outperform the two benchmarks throughout the full sample period, while the strategy with $\tau = 0.5$ has similar performance with that of the cointegration method. The methods for $\tau = 0.6$ to 0.9 perform poorly in terms of cumulative excess return. The cumulative return series of all strategies fluctuate greatly, experiencing downward trend around 2004 and 2011. After taking into account transaction costs of 3 bps the (Figures 4.3 and 4.4) the performance of all strategies deteriorates, with the quantile regression methods for $\tau = 0.1$ to 0.5 being the best performing models.

[Insert Figures 4.3 and 4.4 Here]

We further analyze the performance of the pairs trading strategies using measures that take into account downside risk. The remaining performance measures can be divided into two categories, according to whether they quantify risk based on lower partial moments or drawdown.

Lower partial moment measures take into account only the negative deviations of returns from a minimal acceptable excess return. The four lower partial moment measures are the Omega (OR, Shadwick and Keating (2002)), Sortino (SOR, Sortino and van der Meer (1991)), Kappa 3 (K3, Kaplan and Knowles (2004)) and Upside Potential (UP, Sortino, van der Meer and Plantinga (1999)) ratios, which are based on the lower partial moments (LPM) and upper partial moments (UPM), given by

$$\mathrm{LPM}_n = \frac{1}{T}\sum_{t=1}^{T} \max\left(0, r_{f,t} - r_{i,t}\right)^n \text{ and } \mathrm{UPM}_n = \frac{1}{T}\sum_{t=1}^{T} \max\left(0, r_{i,t} - r_{f,t}\right)^n. \tag{4.14}$$

The choice of n determines the extent to which the deviation from the minimal acceptable return is weighted. The ratios can then be defined as:

$$\mathrm{OR} = \frac{r_i - r_f}{\mathrm{LPM}_1} + 1, \mathrm{SOR} = \frac{r_i - r_f}{\sqrt{\mathrm{LPM}_2}}, \mathrm{K3} = \frac{r_i - r_f}{\sqrt[3]{\mathrm{LPM}_3}} \text{ and } \mathrm{UP} = \frac{\mathrm{UPM}_1}{\sqrt{\mathrm{LPM}_2}}.$$

Note that while OR, SOR and K3 measure excess return as the difference between average return and the risk-free rate, the UP ratio measures return using an upper partial moment, which measures positive deviations from the minimal acceptable return. Because LPMs consider only negative deviations of returns from a minimal acceptable return, they are more appropriate measures of downside risk than the standard deviation, which considers equally both negative and positive deviations from portfolio returns.

Drawdown performance metrics measure the magnitude of losses of an investment over a certain period. The drawdown (DD) at time t, is given by

$$\mathrm{DD}_t = \min\left(\frac{p_t - p_{\max}}{p_{\max}}, 0\right), \text{ where } p_{\max} = \max_{1\le j\le t} p_j \tag{4.15}$$

and p_t is the current value of the portfolio. The most commonly used drawdown is the maximum value of the DDs over a period of time. The maximum drawdown (MDD) broadly reflects the maximum cumulative loss from a peak to a following bottom. The MDD of a portfolio within the period studied is calculated as $\mathrm{MDD} = \max_{1\le t\le T} \mathrm{DD}_t$.

The drawdown metrics are ratios of the excess return divided by risk measures based on drawdown. The Calmar ratio (CR, Young (1991)), Sterling ratio (STE, Kestner (1996)), and Burke ratio (BR, Burke (1994)) use the maximum drawdown, an average above the K largest drawdowns and the square root of the sum of squares of the K largest drawdowns as risk measures:

$$\mathrm{CR} = \frac{r_i - r_f}{\mathrm{MDD}}, \mathrm{STE} = \frac{r_i - r_f}{K^{-1}\sum_{k=1}^{K}\mathrm{DD}_k} \text{ and } \mathrm{BR} = \frac{r_i - r_f}{\sqrt{\sum_{k=1}^{K}\mathrm{DD}_k^2}}.$$

Following the literature, the $K = 5$ largest drawdowns are used, when computing the Sterling and Burke ratios.

[Insert Table 4.2 Here]

The performance of the pairs trading strategies based on downside measures is reported in Table 4.2. The results for the returns on employed capital before transaction costs show that the best performing strategy according to the Omega, Sortino and Kappa 3 ratios is the one based on quantile regression for $\tau = 0.2$, while according to the Upside Potential measure the distance method outperforms the rest. It is interesting to note that the model with $\tau = 0.6$ exhibits the best performance based on the drawdown measures (Calmar ratio, Sterling ratio and Burke ratio). After taking into account transaction costs, the four metrics using lower partial moments as a measure for risk, select the quantile regression strategy with $\tau = 0.2$, while drawdown measures swift to the one with $\tau = 0.4$. The worst performing pairs trading strategies in both cases are those with $\tau = 0.8$ and 0.9. Turning to the results for the returns on committed capital, the best performing model according to all measures except UP is based on quantile regression with $\tau = 0.1$. The upside potential selects the distance method prior to transaction costs and the strategy with $\tau = 0.2$ after costs.

As both return measures achieve similar rankings for all strategies we use the return on employed capital, referred to as return from this point forward, to conduct the analysis described in the following section.

4.4.2. Risk Characteristics of Pairs Trading Strategies

To further investigate the performance of the pairs trading strategies we employ four factor models that are widely used in the cross-sectional asset pricing literature. The first factor model is based on the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965) and is designed to adjust the excess returns of a portfolio for the effect of the overall market return. The one-factor market model is

$$r_{i,t} - r_{f,t} = a_i + b_i \mathrm{MKT}_t + e_{i,t}, \tag{4.16}$$

where $r_{i,t} - r_{f,t}$ is the excess return of the ith pairs trading strategy for period t, MKT_t is the excess return on the market factor for period t and $e_{i,t}$ is the component of the return of strategy i during period t that is not due to exposure to the factors included in the model.

The second model was originally proposed by Fama and French (1993) and incorporates two additional risk factors to the first model as return proxies associated with size and value. The size effect refers to the fact that stocks with small market capitalizations have outperformed stocks with large market capitalizations, while the value effect refers to the fact that stocks with high book-to-market ratios have historically outperformed stocks with low book-to-market ratios. The Fama and French three-factor model is given by

$$r_{i,t} - r_{f,t} = a_i + b_i \mathrm{MKT}_t + s_i \mathrm{SMB}_t + h_i \mathrm{HML}_t + e_{i,t}, \tag{4.17}$$

where SMB_t is the "small minus big" size factor and HML_t denotes the "high minus low" value factor.

The third model we employ, proposed by Carhart (1997), augments the Fama and French three-factor model with one additional factor that accounts for the momentum phenomenon. The momentum factor, represents the returns of a portfolio that is long in stocks with the highest recent performance and short in stocks with the lowest recent performance. The Carhart four-factor model can be written as

$$r_{i,t} - r_{f,t} = a_i + b_i \mathrm{MKT}_t + s_i \mathrm{SMB}_t + h_i \mathrm{HML}_t + m_i \mathrm{MOM}_t + e_{i,t}, \tag{4.18}$$

where MOM_t denotes the momentum factor.

The fourth model we use was proposed by Fama and French (2015), who augment the Fama and French three-factor model by adding a profitability and an investment factor. The profitability factor is the difference between the returns on portfolios of stocks with robust and weak profitability, while the investment factor is the difference between the returns on portfolios of the stocks of low and high investment firms. The Fama and French five-factor model is given by

$$r_{i,t} - r_{f,t} = a_i + b_i \mathrm{MKT}_t + s_i \mathrm{SMB}_t + h_i \mathrm{HML}_t + r_i \mathrm{RMW}_t + c_i \mathrm{CMA}_t + e_{i,t}, \qquad (4.19)$$

where RMW_t is the "robust minus weak" profitability factor and CMA_t is the "conservative minus aggressive" investment factor.

The intercept, a_i, or Jensen (1968)'s alpha, can be interpreted as the average excess return of the ith pairs trading strategy that is not due to the sensitivity to any of the factors included in the regression. If the exposures to the combination of factors, b_i, s_i, h_i, r_i, c_i or m_i depending on the chosen model, capture all variation in expected excess returns, then the intercept is zero for the ith strategy. To examine whether a strategy generates statistically significant average abnormal returns, we use the p-value associated with the intercept coefficient. Each slope coefficient is an estimate of the strategy's sensitivity to the corresponding factor. We use the coefficients and their respective p-values to determine whether a factor is related to the returns of a pairs trading strategy. Since all factor models are time-series regressions they may exhibit autocorrelation and/or heteroscedasticity in their error terms. To overcome this the Newey and West (1987) estimator is used.

Tables 4.3 and 4.4 present the estimated alphas and factor sensitivities for each of the pair trading strategies, before and after transaction costs of 3 bps per share respectively. When the returns are adjusted using the CAPM risk model (Panel A of Table 4.3), all strategies except those based on $\tau = 0.8$ and 0.9, generate positive excess returns. However, the results indicate that only the strategies based on quantile regression for $\tau = 0.1, 0.2$ and 0.3 generate abnormal returns that are statistically indistinguishable from zero. This indicates that the excess returns of the remaining strategies are due to the portfolio exposure to the market factor, therefore after controlling for the effects of the market factor, the average abnormal return of each of the strategies is statistically insignificant. Furthermore, the results indicate that all strategies except those based on quantile regression for $\tau = 0.7$ have statistically significant sensitivity to the market portfolio, with all strategies except those for $\tau = 0.7, 0.8$ and 0.9 having a positive exposure to the market. When transaction costs are taken into account (Panel A of Table 4.4), the only strategies with statistically significant, but negative, abnormal excess returns are the DM and those based on quantile regression for $\tau = 0.8$ and 0.9, while the sensitivity to the market factor remains significant for the majority of the strategies.

Next, the results from Panel B of Table 4.3, show that the positive and statistically significant alphas, after adjusting for the MKT, SMB and HML risk factors, belong to the strategies based quantile regression for $\tau = 0.1, 0.2$ and 0.3. Moreover, for those three strategies the market and value factor are both statistically significant, with MKT being positively and HML negatively correlated with the excess returns. The remaining strategies do not yield statistically significant abnormal excess returns according to the Fama and French three-factor model, with the majority of the strategies exhibiting statistically significant sensitivity to the market portfolio and the value factor. The DM is the only pairs trading strategy with significant exposure to the size factor. After transaction costs (Panel B of Table 4.4) the DM and strategies based on quantile regression for $\tau = 0.8$ and 0.9 generate significant negative alphas, while none of the other methods yield significant average excess return. The exposure to the market and value factors remains statistically significant for most of the strategies.

Turning to the results from the Carhart four-factor model (Table 4.3, Panel C), all models except those based on quantile regression for $\tau = 0.8$ and 0.9 generate a positive alpha, while the strategies that yield statistically significant results are those with $\tau = 0.1, 0.2, 0.3$ and 0.4. For these strategies the market factor has a significant positive impact, while the exposures to the value and momentum factors are negative and statistically significant, with the size factor being positive and statistically significant for the strategy with $\tau = 0.1$. The results for the majority of remaining pairs trading strategies show that the excess returns are affected by the market and value factors, while the slope estimate of the momentum factor is negative and significant for all models. After transaction costs there are no strategies that yield statistically significant and positive alphas. The results for the four factors are similar to those before transaction costs, with the momentum factor being statistically significant and negative across all strategies.

Finally, the results of the Fama and French five factor model are reported in Panel D of Tables 4.3 and 4.4. The models that generate significant positive excess returns belong to the pairs trading strategies based on quantile regression with $\tau = 0.1, 0.2, 0.3$ and 0.4. These strategies have positive and significant exposures to the market replicating portfolio and negative and significant exposures to the value and profitability factors, with the size and investment factors being statistically insignificant in the case of $\tau = 0.4$. The results for the rest of the models appear mixed, with the exception of the DM where all factors are statistically significant. After transaction costs are taken into account none of the pair trading strategies generate positive and significant average excess returns, while the market, value, profitability and investment factors are statistically significant for most strategies.

4.5. Conclusion

In this study we propose a new approach to pairs trading that combines the cointegration method and quantile regression. Using a sample consisting of daily observations of all stocks in the S&P 100 from 2000 to 2017, we evaluate the performance of the new strategy, along with the distance method and original cointegration method, using a wide range of performance metrics and examine the sensitivity of pairs trading returns to various risk factors. The results indicate that the quantile regression pairs trading strategies based on the lower quantiles tend to outperform all other models.

Chapter 4 Tables

Table 4.1: Pairs Trading Strategies Performance

Panel A: Before transaction costs												
	Return on committed capital						Return on employed capital					
	EPV	AR	SD	SR	VaR	CVaR	EPV	AR	SD	SR	VaR	CVaR
DM	1.2782	0.0017	0.0794	0.0216	0.0260	0.0327	1.9806	0.0061	0.1137	0.0533	0.0371	0.0466
mean	3.9213	0.0180	0.1946	0.0927	0.0633	0.0796	5.8749	0.0301	0.3952	0.0762	0.1288	0.1618
q0.1	5.8991	0.0303	0.1640	0.1846	0.0527	0.0664	11.2477	0.0633	0.3514	0.1801	0.1131	0.1424
q0.2	5.6343	0.0286	0.1720	0.1665	0.0554	0.0698	12.4328	0.0706	0.3585	0.1970	0.1151	0.1451
q0.3	5.5505	0.0281	0.1797	0.1564	0.0580	0.0730	11.1337	0.0626	0.3883	0.1612	0.1252	0.1577
q0.4	5.0144	0.0248	0.1894	0.1310	0.0613	0.0771	10.2198	0.0570	0.3930	0.1449	0.1270	0.1598
q0.5	4.1735	0.0196	0.1915	0.1023	0.0622	0.0782	7.0241	0.0372	0.3801	0.0979	0.1236	0.1553
q0.6	2.5700	0.0097	0.1835	0.0529	0.0600	0.0753	4.1055	0.0192	0.3971	0.0483	0.1299	0.1630
q0.7	1.5743	0.0035	0.1834	0.0193	0.0602	0.0755	1.3682	0.0023	0.4073	0.0056	0.1339	0.1679
q0.8	-0.5614	-0.0096	0.1826	-0.0528	0.0605	0.0757	-4.2022	-0.0321	0.3801	-0.0845	0.1263	0.1581
q0.9	-0.2749	-0.0079	0.1966	-0.0401	0.0650	0.0814	-4.5580	-0.0343	0.3880	-0.0885	0.1290	0.1614
Panel B: After transaction costs												
	Return on committed capital						Return on employed capital					
	EPV	AR	SD	SR	VaR	CVaR	EPV	AR	SD	SR	VaR	CVaR
DM	-2.7760	-0.0233	0.0795	-0.2935	0.0271	0.0337	-2.0736	-0.0190	0.1138	-0.1669	0.0382	0.0477
mean	-0.9453	-0.0120	0.1945	-0.0618	0.0645	0.0807	1.0083	0.0001	0.3952	0.0001	0.1300	0.1630
q0.1	1.2749	0.0017	0.1640	0.0104	0.0539	0.0676	6.6235	0.0347	0.3514	0.0989	0.1142	0.1436
q0.2	0.7233	-0.0017	0.1719	-0.0099	0.0566	0.0710	7.5218	0.0403	0.3585	0.1124	0.1163	0.1463
q0.3	0.5609	-0.0027	0.1797	-0.0151	0.0592	0.0743	6.1441	0.0318	0.3883	0.0818	0.1265	0.1589
q0.4	0.0314	-0.0060	0.1893	-0.0316	0.0625	0.0784	5.2368	0.0262	0.3931	0.0666	0.1283	0.1611
q0.5	-0.8089	-0.0112	0.1914	-0.0584	0.0634	0.0794	2.0417	0.0064	0.3800	0.0169	0.1248	0.1565
q0.6	-2.4418	-0.0213	0.1833	-0.1160	0.0612	0.0765	-0.9063	-0.0118	0.3970	-0.0297	0.1311	0.1643
q0.7	-3.5149	-0.0279	0.1833	-0.1521	0.0614	0.0768	-3.7210	-0.0292	0.4073	-0.0716	0.1351	0.1692
q0.8	-5.6080	-0.0408	0.1825	-0.2237	0.0617	0.0769	-9.2488	-0.0633	0.3801	-0.1666	0.1276	0.1593
q0.9	-5.2135	-0.0384	0.1965	-0.1954	0.0662	0.0826	-9.4966	-0.0648	0.3879	-0.1672	0.1302	0.1626

Notes: This table reports the end-of-period value, average return, standard deviation, Sharpe ratio, Value-at-Risk and Conditional Value-at-Risk, for the three pairs trading strategies. The strategy based on quantile regression has been computed for $\tau \in [0.1,0.9]$. All calculations are based on the excess returns of each strategy, with the 3-month Treasury bill used as the risk-free asset. The formation period is set to 12 months and the trading period to 6 months. The period considered is from December 2001 to December 2017.

Table 4.2: Pairs Trading Strategies Performance based on Downside Measures

Panel A: Before transaction costs

	Return on committed capital							Return on employed capital						
	OR	SOR	K3	UP	CR	STE	BR	OR	SOR	K3	UP	CR	STE	BR
DM	1.0132	0.0063	0.0040	0.4817	0.0001	0.0001	0.0000	1.0315	0.0156	0.0103	0.5089	0.0003	0.0003	0.0001
mean	1.0661	0.0279	0.0155	0.4497	0.0007	0.0007	0.0003	1.0559	0.0228	0.0117	0.4306	0.0002	0.0002	0.0001
q0.1	1.1348	0.0568	0.0318	0.4780	0.0014	0.0014	0.0006	1.1448	0.0527	0.0228	0.4166	0.0001	0.0001	0.0001
q0.2	1.1214	0.0520	0.0290	0.4798	0.0013	0.0013	0.0006	1.1527	0.0644	0.0321	0.4859	0.0001	0.0001	0.0000
q0.3	1.1141	0.0488	0.0270	0.4768	0.0012	0.0012	0.0005	1.1256	0.0502	0.0236	0.4501	0.0002	0.0002	0.0001
q0.4	1.0956	0.0405	0.0224	0.4640	0.0010	0.0010	0.0005	1.1102	0.0458	0.0243	0.4619	0.0004	0.0005	0.0002
q0.5	1.0728	0.0310	0.0172	0.4570	0.0008	0.0008	0.0004	1.0698	0.0296	0.0158	0.4534	0.0006	0.0006	0.0003
q0.6	1.0351	0.0155	0.0093	0.4578	0.0004	0.0004	0.0002	1.0337	0.0143	0.0079	0.4383	0.0008	0.0008	0.0003
q0.7	1.0127	0.0056	0.0034	0.4501	0.0001	0.0001	0.0001	1.0039	0.0016	0.0009	0.4222	0.0001	0.0001	0.0000
q0.8	0.9667	-0.0150	-0.0090	0.4352	-0.0004	-0.0004	-0.0002	0.9457	-0.0240	-0.0142	0.4192	-0.0013	-0.0013	-0.0006
q0.9	0.9741	-0.0113	-0.0066	0.4256	-0.0003	-0.0003	-0.0001	0.9423	-0.0246	-0.0144	0.4014	-0.0014	-0.0014	-0.0006

Panel B: After transaction costs

	Return on committed capital							Return on employed capital						
	OR	SOR	K3	UP	CR	STE	BR	OR	SOR	K3	UP	CR	STE	BR
DM	0.8382	-0.0807	-0.0524	0.4181	-0.0009	-0.0009	-0.0004	0.9077	-0.0469	-0.0315	0.4614	-0.0008	-0.0008	-0.0003
mean	0.9583	-0.0181	-0.0102	0.4172	-0.0005	-0.0005	-0.0002	1.0001	0.0000	0.0000	0.4149	0.0000	0.0000	0.0000
q0.1	1.0071	0.0031	0.0018	0.4391	0.0001	0.0001	0.0000	1.0770	0.0286	0.0125	0.4003	0.0001	0.0001	0.0001
q0.2	0.9932	-0.0030	-0.0017	0.4404	-0.0001	-0.0001	0.0000	1.0843	0.0362	0.0182	0.4659	0.0001	0.0001	0.0000
q0.3	0.9896	-0.0046	-0.0026	0.4381	-0.0001	-0.0001	0.0000	1.0619	0.0252	0.0119	0.4326	0.0001	0.0001	0.0001
q0.4	0.9783	-0.0095	-0.0053	0.4283	-0.0002	-0.0002	-0.0001	1.0491	0.0208	0.0111	0.4442	0.0002	0.0002	0.0001
q0.5	0.9608	-0.0173	-0.0097	0.4233	-0.0004	-0.0004	-0.0002	1.0117	0.0051	0.0027	0.4364	0.0001	0.0001	0.0001
q0.6	0.9272	-0.0332	-0.0202	0.4232	-0.0009	-0.0009	-0.0004	0.9799	-0.0087	-0.0048	0.4226	-0.0005	-0.0005	-0.0002
q0.7	0.9058	-0.0432	-0.0260	0.4152	-0.0011	-0.0011	-0.0005	0.9512	-0.0209	-0.0115	0.4069	-0.0012	-0.0012	-0.0005
q0.8	0.8666	-0.0619	-0.0377	0.4020	-0.0016	-0.0016	-0.0007	0.8960	-0.0469	-0.0279	0.4036	-0.0025	-0.0025	-0.0011
q0.9	0.8802	-0.0539	-0.0320	0.3962	-0.0015	-0.0015	-0.0007	0.8939	-0.0460	-0.0270	0.3871	-0.0026	-0.0026	-0.0012

Notes: This table reports the performance metrics using lower partial moments and drawdown as measures of risk, for the three pairs trading strategies. The strategy based on quantile regression has been computed for $\tau \in [0.1,0.9]$. All calculations are based on the excess returns of each strategy, with the 3-month Treasury bill used as the risk-free asset. The formation period is set to 12 months and the trading period to 6 months. The period considered is from December 2001 to December 2017.

Table 4.3: Pairs Trading Strategies Risk Profile, before Transaction Costs

	DM	mean	q0.1	q0.2	q0.3	q0.4	q0.5	q0.6	q0.7	q0.8	q0.9
Panel A: One-factor market model											
Alpha	0.0030	0.0995	0.2067*	0.2428**	0.2124*	0.1994	0.1280	0.0687	0.0117	-0.1055	-0.0919
MKT	0.6219***	0.6145***	1.3615***	1.1640***	1.1142***	0.8329***	0.6094***	0.2363**	-0.0765	-0.6761***	-1.3315***
Panel B: Fama and French three-factor model											
Alpha	0.0041	0.0982	0.2102**	0.2443**	0.2132*	0.1982	0.1270	0.0655	0.0090	-0.1091	-0.0936
MKT	0.5998***	0.5660***	1.5981***	1.3257***	1.2147***	0.8534***	0.5637***	0.1560	-0.1793	-0.8809***	-1.5461***
SMB	-0.1664***	0.1049	-0.1779	-0.0120	0.0059	0.1717	0.0798	0.3044	0.2305	0.2338	-0.0156
HML	0.2626***	0.2707	-1.5391***	-1.1232***	-0.7066***	-0.2551	0.2682	0.3642*	0.5695***	1.2806***	1.5102***
Panel C: Carhart four-factor model											
Alpha	0.0191	0.1279	0.2226**	0.2660**	0.2402**	0.2272*	0.1561	0.1008	0.0452	-0.0820	-0.0685
MKT	0.4018***	0.1722	1.4335***	1.0377***	0.8569***	0.4684***	0.1774	-0.3115***	-0.6596***	-1.2403***	-1.8787***
SMB	-0.0550	0.3265	-0.0853	0.1500	0.2073	0.3883*	0.2972	0.5674**	0.5007**	0.4360**	0.1715
HML	-0.2549***	-0.7584***	-1.9691***	-1.8756***	-1.6416***	-1.2612***	-0.7412***	-0.8572***	-0.6855***	0.3415	0.6410***
MOM	-0.8923***	-1.7744***	-0.7414***	-1.2974***	-1.6122***	-1.7347***	-1.7404***	-2.1059***	-2.1638***	-1.6192***	-1.4985***
Panel D: Fama and French five-factor model											
Alpha	0.0159	0.1432	0.2410**	0.2855***	0.2587**	0.2471**	0.1734	0.1166	0.0609	-0.0854	-0.0730
MKT	0.5267***	0.3128***	1.4683***	1.1272***	0.9855***	0.5916***	0.3031***	-0.1520	-0.4988***	-1.0694***	-1.7181***
SMB	-0.2043***	-0.1662	-0.5780***	-0.4289**	-0.4039*	-0.1928	-0.2045	0.0968	0.0513	0.3636*	0.1398
HML	0.2898***	0.1030	-2.1130***	-1.6366***	-1.1657***	-0.5859***	0.0859	0.3885*	0.6626***	1.7758***	2.0311***
RMW	-0.4083***	-2.0041***	-2.1289***	-2.4332***	-2.5079***	-2.4273***	-2.0847***	-1.9224***	-1.8416***	-0.0927	0.0709
CMA	-0.5190***	-0.5272	2.0946***	1.4378***	1.0122**	0.2177	-0.4953	-1.7476***	-2.1419***	-3.3970***	-3.4348***

Notes: This table reports the risk-adjusted alphas and factor sensitivities for each of the pairs trading strategies, using the one-factor market model (Panel A), Fama and French three-factor model (Panel B), Carhart four-factor model (Panel C) and Fama and French five-factor model (Panel D). The statistical significance of each parameter is based on t-statistics calculated using Newey-West standard errors.

*, ** and *** indicate significance at the 10%, 5% and 1% levels.

Table 4.4: Pairs Trading Strategies Risk Profile, after Transaction Costs

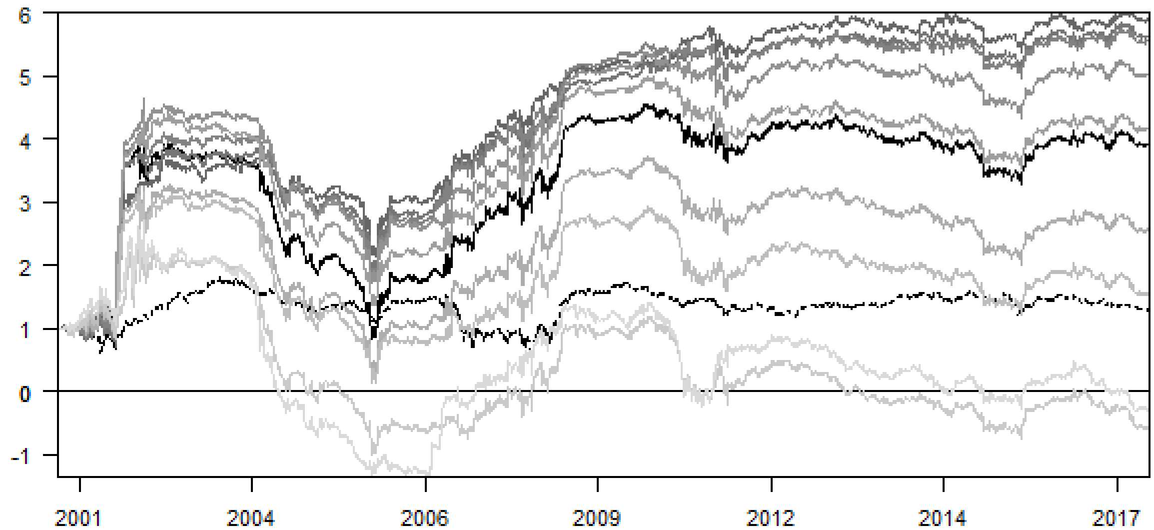
	DM	mean	q0.1	q0.2	q0.3	q0.4	q0.5	q0.6	q0.7	q0.8	q0.9
Panel A: One-factor market model											
Alpha	-0.0973***	-0.0208	0.0924	0.1214	0.089	0.0762	0.0049	-0.0552	-0.1141	-0.2302*	-0.2140*
MKT	0.6247***	0.6145***	1.3631***	1.1654***	1.1158***	0.8338***	0.6090***	0.2379**	-0.0762	-0.6757***	-1.3301***
Panel B: Fama and French three-factor model											
Alpha	-0.0961***	-0.0220	0.0959	0.1228	0.0899	0.0750	0.0039	-0.0584	-0.1168	-0.2338**	-0.2157*
MKT	0.6042***	0.5694***	1.6016***	1.3290***	1.2189***	0.8565***	0.5663***	0.1604	-0.1760	-0.8785***	-1.5433***
SMB	-0.1716***	0.1015	-0.1728	-0.0071	0.0040	0.1743	0.0797	0.3055	0.2304	0.2361	-0.0124
HML	0.2551***	0.2492	-1.5555***	-1.1393***	-0.7235***	-0.2724	0.2468	0.3432	0.5482**	1.2649***	1.4989***
Panel C: Carhart four-factor model											
Alpha	-0.0813**	0.0074	0.1080	0.1442	0.1165	0.1037	0.0327	-0.0234	-0.0809	-0.2070*	-0.1909
MKT	0.4081***	0.1802	1.4415***	1.0453***	0.8659***	0.4758***	0.1850*	-0.3023***	-0.6519***	-1.2341***	-1.8723***
SMB	-0.0613	0.3205	-0.0827	0.1525	0.2026	0.3885*	0.2942	0.5659**	0.4982**	0.4362**	0.1727
HML	-0.2573***	-0.7679***	-1.9740***	-1.8807***	-1.6460***	-1.2672***	-0.7496***	-0.8659***	-0.6952***	0.3357	0.6392***
MOM	-0.8835***	-1.7537***	-0.7215***	-1.2782***	-1.5907***	-1.7153***	-1.7180***	-2.0847***	-2.1440***	-1.6020***	-1.4824***
Panel D: Fama and French five-factor model											
Alpha	-0.0841**	0.0230	0.1266	0.1641	0.1354	0.1240	0.0502	-0.0074	-0.0649	-0.2103*	-0.1952*
MKT	0.5299***	0.3161***	1.4718***	1.1307***	0.9899***	0.5946***	0.3067***	-0.1468	-0.4952***	-1.0663***	-1.7150***
SMB	-0.2105***	-0.1687	-0.5724***	-0.4237**	-0.4048*	-0.1902	-0.2038	0.0995	0.0528	0.3669*	0.1437
HML	0.2817***	0.0833	-2.1286***	-1.6524***	-1.1809***	-0.6031***	0.0648	0.3696*	0.6437***	1.7609***	2.0209***
RMW	-0.4166***	-2.0008***	-2.1269***	-2.4312***	-2.5027***	-2.4280***	-2.0783***	-1.9131***	-1.8340***	-0.0864	0.0754
CMA	-0.5223***	-0.5368	2.0899***	1.4370***	1.0045**	0.2167	-0.4922	-1.7537***	-2.1518***	-3.3978***	-3.4385***

Notes: This table reports the risk-adjusted alphas and factor sensitivities for each of the pairs trading strategies, using the one-factor market model (Panel A), Fama and French three-factor model (Panel B), Carhart four-factor model (Panel C) and Fama and French five-factor model (Panel D). The statistical significance of each parameter is based on t-statistics calculated using Newey-West standard errors.

*, ** and *** indicate significance at the 10%, 5% and 1% levels.

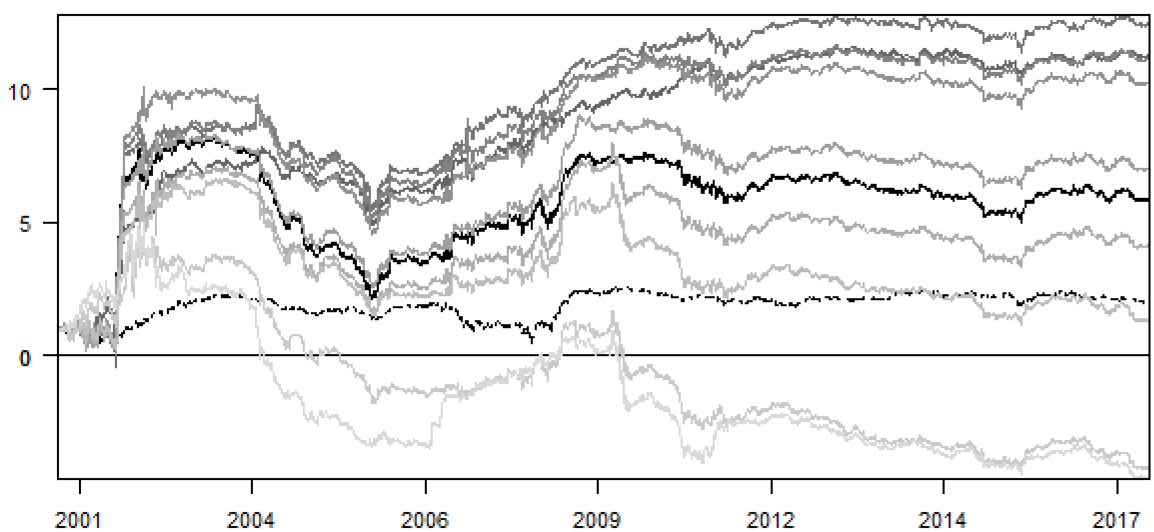
Chapter 4 Figures

Figure 4.1: Cumulative Excess Return on Committed Capital, before Transaction Costs.



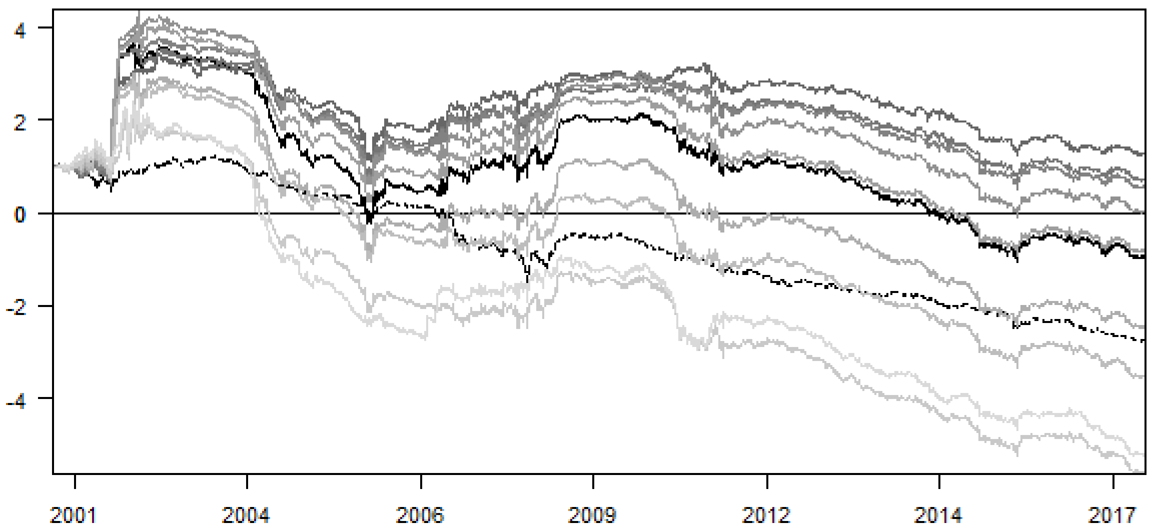
Notes: This figure plots the cumulative return for the DM (dashed line), the cointegration method (solid line) and the quantile regression method (grey lines), based on the return on committed capital.

Figure 4.2: Cumulative Excess Return on Employed Capital, before Transaction Costs.



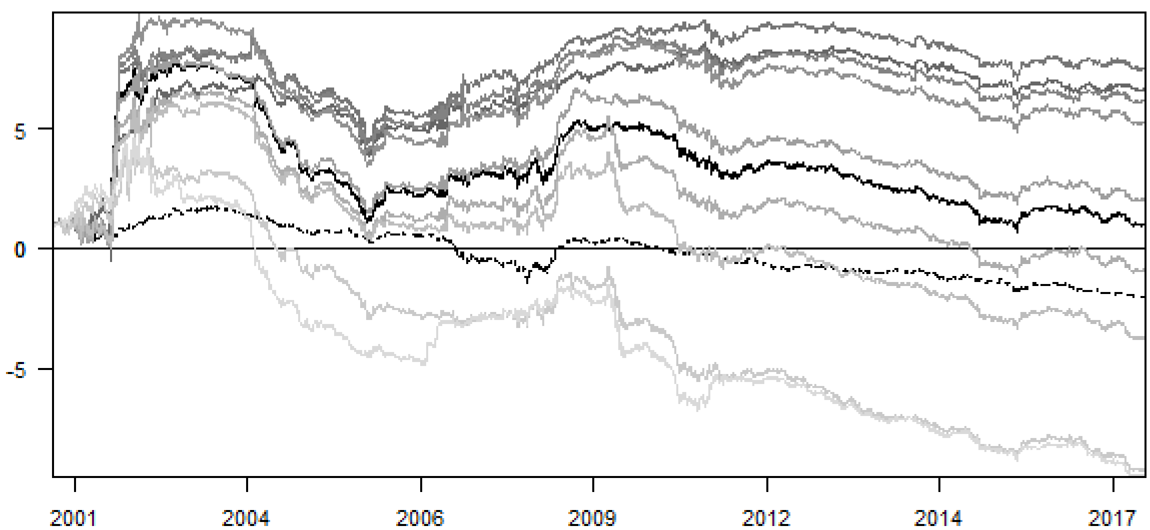
Notes: This figure plots the cumulative return for the DM (dashed line), the cointegration method (solid line) and the quantile regression method (grey lines), based on the return on employed capital.

Figure 4.3: Cumulative Excess Return on Committed Capital, after Transaction Costs.



Notes: This figure plots the cumulative return for the DM (dashed line), the cointegration method (solid line) and the quantile regression method (grey lines), based on the return on committed capital.

Figure 4.4: Cumulative Excess Return on Employed Capital, after Transaction Costs.



Notes: This figure plots the cumulative return for the DM (dashed line), the cointegration method (solid line) and the quantile regression method (grey lines), based on the return on employed capital.

Chapter 5: Concluding Remarks and Future Research

This thesis is comprised of three essays with the following topics: (1) detection of speculative bubbles in real estate and forecasting the returns of real estate indices using models that take into account the bubble component of the asset price; (2) the benefits of integrating return forecasts from machine learning and forecast combination methods to an out-of-sample asset allocation framework and (3) the evaluation of a new approach to pairs trading that incorporates quantile regression.

In the first essay of this thesis we examined whether speculative bubbles are present in the US and UK commercial, equity and residential real estate sectors. First, the real estate price indices are decomposed into a fundamental and a non-fundamental component using a wide range of predictors and the models are estimated using penalized regressions. Our findings suggest the existence of significant periods of overvaluation in real estate markets, particularly in residential real estate, as well as economically significant periods of undervaluation, especially in equity real estate markets. In order to determine whether the observed deviations of the actual prices from their fundamental values are due to the presence of speculative bubbles, we use two complementary methodologies that utilize the information contained in the non-fundamental component of the asset price. The first is based on right-side augmented Dickey-Fuller tests for explosive behavior and the second on a two-state regime switching model for bubbles. The findings from both methodologies provide significant in-sample evidence that the observed deviations of the actual price from the fundamental value were due to the presence of speculative bubbles. The out-of-sample results show that in most cases the proposed regime-switching model for bubbles outperforms the historical average benchmark and the stylized alternative models.

In the second essay we evaluate the benefits of integrating return forecasts from a variety of machine learning and forecast combination methods into an out-of-sample asset allocation framework. When examining the benefits of forecasting the returns for each individual index, the results indicate that the majority of the proposed prediction models outperform the historical average benchmark, with shrinkage and variable selection methods yielding the highest performance for the stock and bond indices, while for the commodity index the dimensionality reduction methods achieve superior performance. To examine whether return forecasts provide any benefits in an asset allocation setting, we construct stock-bond-commodity portfolios and compare their performance to that of the equally-weighted portfolio and a mean-variance portfolio based on the historical average. For commodity-augmented portfolios, the majority of the models outperform the two benchmarks, while the models with the highest performance are those based on shrinkage and variable selection methods or PLS-type methods.

The performance of the portfolios is further evaluated for different levels of risk aversion and investment constraints, around business cycles and for monthly or quarterly rebalancing. Overall, the commodity-augmented portfolios of an aggressive investor outperform those of a conservative investor. Portfolios with either short selling or leverage generate higher certainty equivalent return than the unleveraged long-only allocations, while portfolios with both leverage and short selling yield the highest return. When transaction costs are taken into account, the results for monthly-rebalanced portfolios favor forecast combination methods, instead of methods that combine information due to the latter methods leading to portfolios with higher turnover. When the rebalancing frequency is reduced to quarterly, the models with the best performance for an aggressive investor are those based on shrinkage and dimensionality reduction methods. In recessionary periods, all portfolios based on multivariate regression models outperform the equal weighted portfolios or those based on the historical average forecast, while in expansionary periods, portfolios with leverage or short selling tend to yield higher performance. When CVaR is used as a risk measure, the vast majority of the mean-CVaR portfolios based on forecasts from multivariate regression models outperform the equally-weighted and HA portfolios. Finally, when comparing the results of stock-bond portfolios with those that include commodities for the full sample, commodities add value to a traditional portfolio when short selling is allowed, with aggressive investors benefiting more from the inclusion of commodities.

In the final essay we propose a new approach to pairs trading, which takes advantage of the information in the conditional quantiles of the distribution of asset returns. In the formation period stock pairs are sorted and selected using cointegration tests, while in the trading period the trading signal is generated based on the spread of the stock prices in the pair, which has been estimated by quantile regression. We conduct an extensive evaluation of the new strategy by applying it to the S&P 100 index constituents. The performance of the new strategy is compared to the distance method and cointegration method benchmarks using a variety of economic and risk-adjusted measures and under an asset pricing framework, in order to examine whether the returns of each strategy can be explained by various risk factors. We find that pairs trading strategies based on the lower quantiles generate the highest performance.

The contributions of this thesis can be expanded in several ways. Additional research directions regarding Chapter 2 include the implementation of a greater variety of models to derive the non-fundamental component of the asset price, modifying the states and probabilities of the regime-switching model, investigating the cross-sectional migration of speculative bubbles, economic evaluation of the models and the development of trading strategies that take into account bubble dynamics. Based on Chapter 3, further research could be conducted in order to improve the way that the forecasts are integrated into the objective function, so that the impact of transaction costs on portfolio returns would be reduced. Furthermore, a detailed investigation of the benefits of using alternative estimates of the covariance matrix could be performed, especially for large dimensional portfolios with cardinality constraints. Finally, additional research in regard to Chapter 4 could further explore the source of the profitability of the pairs trading strategy based on quantile regression, incorporate quantile regression to the formation stage of pairs trading and extend the new strategy to a multivariate framework.

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