PHYSICAL REVIEW A **92**, 051602(R) (2015)

Baryon squishing in synthetic dimensions by effective SU(M) gauge fields

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(Received 27 April 2015; published 30 November 2015)

The "synthetic dimension" proposal [A. Celi *et al.*, Phys. Rev. Lett. **112**, 043001 (2014)] uses atoms with M internal states ("flavors") in a one-dimensional (1D) optical lattice, to realize a hopping Hamiltonian equivalent to the Hofstadter model (tight-binding model with a given magnetic flux per plaquette) on an M-sites-wide square lattice strip. We investigate the physics of SU(M) symmetric interactions in the synthetic dimension system. We show that this system is equivalent to particles [with SU(M) symmetric interactions] experiencing an SU(M) Zeeman field at each lattice site *and* a non-Abelian SU(M) gauge potential that affects their hopping. This equivalence brings out the possibility of generating *nonlocal* interactions between particles at different sites of the optical lattice. In addition, the gauge field induces a *flavor-orbital coupling*, which mitigates the "baryon breaking" effect of the Zeeman field. For M particles, concomitantly, the SU(M) singlet baryon which is site localized in the usual 1D optical lattice, is deformed to a nonlocal object ("squished baryon"). We conclusively demonstrate this effect by analytical arguments and exact (numerical) diagonalization studies. Our study promises a rich many-body phase diagram for this system. It also uncovers the possibility of using the synthetic dimension system to laboratory realize condensed-matter models such as the SU(M) random flux model, inconceivable in conventional experimental systems.

DOI: 10.1103/PhysRevA.92.051602 PACS number(s): 67.85.−d, 21.45.−v, 37.10.Jk

Emulation of quantum systems of interest to areas from condensed matter to high-energy physics is made possible with cold atoms [1], as evidenced by recent developments [2–8]. Adding to the soaring interest is the realization of systems with novel physics such as those with SU(M) ($M > 2$) symmetries, and yet are exceedingly difficult to realize by conventional experiments. SU(M) symmetric spin models have interesting phases and phase transitions [9–16], as do Hubbard models with SU(M) symmetry [17–20]. Several theoretical [21–28] and experimental [29–35] works have explored systems with SU(M) symmetry working with ^{6}Li ($M = 4$) [36], ^{173}Yb ($M = 6$) [32,35], and ^{87}Sr ($M = 10$) [30,31,34].

Celi *et al.* [37] proposed, using atoms with M-internal states, to realize a finite strip of the Hofstadter model [38]. This was dubbed as a "synthetic dimension" (SD) since the internal state mimicked the coordinate along an additional spatial dimension. Atoms with M internal states ($\gamma = 1,\ldots,M$) hop in a one-dimensional (1D) optical lattice whose γ-independent hopping t from a site j ($x_j = jd$,d, lattice spacing) to its neighbor preserves their internal state. The states are coherently coupled by light of wave number k_ℓ such that an atom in state γ at j can "hop" to $\gamma + 1$ at j with an amplitude $\Omega_\gamma^j = \Omega_\gamma e^{-ik_\ell x_j}$. An atom picks up a phase factor $e^{-ik_\ell d}$ upon hopping around a plaquette $[(j,\gamma) \to (j+1,\gamma) \to (j+1,\gamma+1) \to (j,\gamma+1) \to (j,\gamma)]$, simulating an enclosed magnetic flux. Choosing $k_\ell d = 2\pi\frac{p}{q}$, where p and q are relative prime integers, thus provides an alternate realization of the Hofstadter model (compare with Refs. [6,7]) with a p/q flux per plaquette. Recent experimental realization [39,40] bolsters this research direction.

The physics of SU(M) symmetric interactions in the SD system is an unexplored area. Previous studies [23,25–27] of fermions with attractive SU(M) interactions in the usual 1D lattice (no flux, i.e., $\frac{p}{q} = 0$, $\Omega_\gamma = 0$) shows SU(M) singlet "baryons" and their quasi-long-range color superfluidity [41]. Viewed from the SD perspective, the SU(M) interaction manifests as "infinite ranged" (distance independent) along the SD. For example, two atoms at site j with $\gamma = 1$ and 2 will interact with the same strength as $\gamma = 1$ and M. This aspect, taken together with the flux p/q, raises several intriguing open questions: What is the fate of the baryons? How is the color superfluidity affected? Are there different many-body states and interesting physics in this system? This Rapid Communication addresses these questions, and points to a plethora of possibilities of this system that would be of wide interest.

We show that the SD system [37] can be mapped to a system of M-flavor particles with SU(M) interactions hopping on the lattice with an on-site SU(M) Zeeman potential (due to Ω_γ) along with an SU(M) gauge field (due to flux p/q, and Ω_γ) that controls their hopping. Further analysis reveals, *inter alia*, the gauge field induces (i) a *flavor-orbital coupling* which mitigates the "baryon breaking" effects of the Zeeman field, and (ii) a *nonlocal* interaction, i.e., interaction between particles at *different* j sites. A crucial outcome is that under favorable circumstances, the SU(M) singlet baryon ($\Omega_\gamma = 0$), which is an object localized at a site j but extended along the synthetic dimension, is transformed into an M-body bound state that is extended in real space (along j), which we dub as the "squished baryon." This is demonstrated by analytical arguments and detailed exact diagonalization calculations. These results point to different many-body phases of these systems. Our mapping further suggests opportunities of using the SD system to simulate interesting models as the SU(M) random flux model [42].

Model and mapping. Denoting the operator that creates a fermion [43] at site j with hyperfine flavor γ as $C^\dagger_{j,\gamma}$, the

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Hamiltonian is $\mathcal{H} = H_t + H_\Omega + H_U$, with

$$H_t = -t \sum_j \sum_{\gamma=1}^{M} (C^\dagger_{j+1,\gamma} C_{j,\gamma} + \text{H.c.}), \tag{1}$$

$$H_\Omega = \sum_j \sum_{\gamma=1}^{M-1} (\Omega^j_\gamma C^\dagger_{j,\gamma+1} C_{j,\gamma} + \text{H.c.}), \tag{2}$$

$$H_U = -\frac{U}{2} \sum_{j,\gamma,\gamma'} C^\dagger_{j,\gamma} C^\dagger_{j,\gamma'} C_{j,\gamma'} C_{j,\gamma}, \tag{3}$$

where t is the intersite hopping amplitude, and U is the strength of the attractive SU(M) interaction. The couplings $\Omega^j_\gamma = \Omega_\gamma e^{-ik_\ell x_j}$, where Ω_γ's depend on the details [37,39,40] of the system.

A mapping gains further insights into the effects of interaction. Towards this end, we introduce the notation $\hat{C}_j = (C_{j,1}, C_{j,2}, \ldots, C_{j,M})^T$. We introduce a local unitary transformation $\hat{C}_j = \hat{W}_j \hat{b}_j$, where $\hat{W}_j = \text{Diag}\{e^{-ik_{\ell\gamma} x_j}\}$, $\gamma = 1, \ldots, M$, with $k_{\ell\gamma} = (\gamma - 1)k_\ell$, and $\hat{b}_j = (b_{j,1}, \ldots, b_{j,M})^T$ is another set of fermionic operators. This results in $H_\Omega = \sum_j \hat{b}^\dagger_j \mathbf{\Omega} \hat{b}_j$, where $\mathbf{\Omega}$ is a site-independent Hermitian matrix,

$$\mathbf{\Omega} = \begin{pmatrix} 0 & \Omega_1^* & 0 & \ldots & 0 \\ \Omega_1 & 0 & \Omega_2^* & \ldots & 0 \\ 0 & \Omega_2 & 0 & \ldots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \Omega^*_{M-1} \\ 0 & 0 & \ldots & \Omega_{M-1} & 0 \end{pmatrix}. \tag{4}$$

On diagonalization, $\mathbf{\Omega} = \mathbf{S}\boldsymbol{\omega}\mathbf{S}^\dagger$, where $\boldsymbol{\omega} = \text{Diag}\{\omega_\zeta\}$ ($\zeta = 1, \ldots, M$) is the diagonal matrix with eigenvalues ω_ζ, and $\mathbf{S}$ is a unitary matrix. Then, $H_\Omega = \sum_j \hat{a}^\dagger_j \boldsymbol{\omega} \hat{a}_j$, where $\mathbf{S}^\dagger \hat{b}_j = \hat{a}_j = \{a_{j,\zeta}\}^T$ is a new set of fermionic operators. Clearly, $\hat{C}_j = \mathbf{U}_j \hat{a}_j$, where $\mathbf{U}_j = \hat{W}_j \mathbf{S}$ is a unitary matrix. We now have

$$H = -t \sum_j (\hat{a}^\dagger_{j+1} \mathbf{U}^\dagger_{j+1} \mathbf{U}_j \hat{a}_j + \text{H.c.}) + \sum_j \hat{a}^\dagger_j \boldsymbol{\omega} \hat{a}_j + H_U, \tag{5}$$

where H_U is the operator defined in Eq. (3) rewritten in terms of $a_{j,\zeta}$ owing to its SU(M) invariance. We immediately see that in terms of the transformed states $\hat{a}_j$, the Hamiltonian can be interpreted as that of particles in a flavor- (ζ-) dependent potential ω_ζ [which is a SU(M) Zeeman field], and whose hopping is influenced by a non-Abelian gauge field $\mathbf{U}^\dagger_{j+1}\mathbf{U}_j = \mathbf{S}^\dagger \Phi \mathbf{S}$ ($\Phi = \text{Diag}\{e^{ik_{\ell\zeta} d}\}$) that produces *flavor-orbital coupling*. The Zeeman field depends solely on Ω_γ, while the gauge field has a crucial additional dependence on the flux [44]. The SD system is thus equivalent to SU(M)-interacting fermions experiencing SU(M) Zeeman and gauge fields (flavor-orbital coupling) [45].

Induced interactions. We now discuss the key outcome of the mapping. Consider the $M = 2$ system with $\frac{1}{2}$ flux. A rather unnatural limit of vanishing hopping $t \to 0$ reveals the main idea. The Zeeman field is $\boldsymbol{\omega} = \text{Diag}\{-\Omega, \Omega\}$. When $\Omega \ll U$, the ground state is an $M = 2$ baryon with two particles at the same site (Fig. 1, top left). The "baryon breaking" effect of the Zeeman field occurs when Ω exceeds $\Omega_c = \frac{U}{2}$ (Fig. 1, top right). The broken baryon has both particles with $\zeta = 1$, located at two distinct sites. Now, $t > 0$ with $\frac{1}{2}$ flux produces hopping, indicated by arrows in Fig. 1 (bottom left) that does not conserve the ζ flavor—flavor-orbital coupling (gauge field). The degeneracy of the broken baryon states is lifted by the flavor-orbit-coupled hopping—two particles with $\zeta = 1$ on neighboring sites can gain energy by hybridizing with the degenerate baryon state (bound along the SD). This induces a *nonlocal* attractive interaction between particles with $\zeta = 1$ located on two neighboring sites. The outcome is a "squished baryon" state that generically has a bound state character along the synthetic and real dimension. In Fig. 1, this state is a bound state of two particles that "resonates" between the vertical and horizontal bonds (Fig. 1, bottom left), hopping on the "dual lattice" indicated by crosses in Fig. 1 (bottom right). As $\Omega \gg \Omega_c$, the bound state is primarily made of particles with $\zeta = 1$—a "fully" squished baryon, a result of the attractive interaction between near-neighbor $\zeta = 1$ states proportional to $\frac{t^2}{2\Omega - U} - \frac{t^2}{2\Omega} \sim \frac{t^2 U}{\Omega^2}$. Indeed, longer-range interactions are also similarly generated. A repulsive U results in an induced nonlocal repulsion.

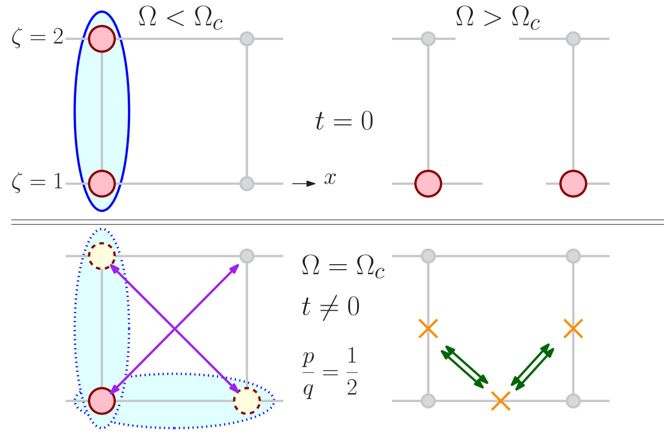

FIG. 1. (Color online) Nonlocal interaction: The top panel shows the state of two fermions when $t = 0$ with $M = 2$, which has $\omega_{\zeta=1} = -\Omega$ and $\omega_{\zeta=2} = \Omega$. The arrows in the left bottom panel show the hopping pattern when $t \neq 0$ in the presence of a $\frac{1}{2}$ flux. If the two particles are in the neighboring sites as shown, then this baryon can effectively hop on a dual lattice shown by crosses (bottom right) by hybridizing with the degenerate baryon (vertical shaded bond), gaining kinetic energy. This produces a net attractive interaction between particles at neighboring sites with $\zeta = 1$.

A similar physics applies to generic M. The key point is that the scale Ω_c and the resulting "broken baryon" state depend on the details of Ω_γ. For a given M and Ω_γ, there are special fluxes that most effectively produce nonlocal binding and baryon squishing.

Exact diagonalization. We have investigated few-body physics using numerical exact diagonalization. We consider a system with N_q lattice sites ($d = 1$) with periodic boundary conditions. For a system with M internal states, this provides $N_q M$ one-particle states. So, for N_p particles, the dimension of the resulting Hilbert space is $\binom{N_q M}{N_p}$. We use translational symmetry, with Q, the center-of-mass momentum, as a good quantum number. If the ground state (GS) has $Q = Q_g$, then the binding energy is $E_b = E_g(Q_g, U = 0) - E_g(Q_g, U)$, where $E_g(Q_g, U = 0)$ is the GS energy of the same system with $U = 0$. We also study the properties of GS by computing the moment of inertia along the x direction, $I_{xx} = \frac{1}{d^2\binom{N_p}{2}} \langle \sum_{i_1 > i_2} (\Delta x_{i_1,i_2})^2 \rangle$, and an average value for the synthetic coordinate $\langle \zeta \rangle = \frac{1}{N_p} \langle \sum_i \zeta_i \rangle$, where i's run over the particle labels and $\Delta x_{i_1,i_2} = (x_{i_1} - x_{i_2})$. We use the following two criteria to detect an N_p-particle bound state. First, the binding energy should be positive. Second, the I_{xx} should be finite and insensitive to the spatial size of the system (N_q) [46]. The quantity $\langle \zeta \rangle$ provides a measure of squishing. For eaxmple, with $N_p = M$, $\langle \zeta \rangle = \frac{(M+1)}{2}$ indicates the usual SU(M)-singlet baryon, while squishing is deduced from a value of $\langle \zeta \rangle < \frac{(M+1)}{2}$.

Results. While we choose the simplest case $\Omega_\gamma = \Omega$ to illustrate the physical ideas, our calculations can be adapted to specific systems. Figure 2 shows the results for $M = 2$. In the absence of a flux $p/q \to 0$, the critical Zeeman field to break the baryon is $\Omega_c = \frac{1}{2}(\sqrt{U^2 + 16t^2} - 4t)$. The "phase diagram" in the p/q-Ω plane, shown in Figs. 2(a) and 2(b), shows that this indeed occurs at $p/q = 0$. For larger Ω, there is no bound state at $p/q = 0$. For $p/q = \frac{1}{2}$ ($\frac{1}{2}$ flux), the situation is entirely different. I_{xx} remains finite with the increase in Ω, and $\langle \zeta \rangle$ goes to unity. The baryon evolves to the squished baryon (see the inset). We have investigated the $\frac{1}{2}$-flux case in greater depth. Figures 2(c)–2(e) clearly demonstrate that for the $\frac{1}{2}$ flux a bound state always exists (except when $t = 0$) *irrespective* of a large Zeeman field—a vivid example of the flavor-orbital coupling mitigating the baryon breaking effects of the Zeeman field. Figures 2(f) and 2(g) further demonstrate the squishing of the baryon by the flavor-orbital coupling. Finally, Figs. 2(h) and 2(i) discuss the case $\Omega = \Omega_c$. From analytic considerations, the binding energy of the squished baryon when $t \ll U$ is $\approx 2t$, $I_{xx} \approx \frac{1}{2}$, and $\langle \zeta \rangle \approx \frac{5}{4}$. The numerical binding energy at small t is indeed in agreement, as are I_{xx} and $\langle \zeta \rangle$ [Figs. 2(h) and 2(i)].

$M = 3$. Here, when $t = 0$, $\Omega_c = \frac{U}{\sqrt{2}}$ with a peculiar feature. Three distinct states are degenerate at Ω_c. These are the usual $M = 3$ baryons [27], a completely broken baryon with three particles at different sites ("$1 + 1 + 1$"), and partially broken "$2 + 1$" baryon which has two particles at a given site with $\zeta = 1$ and 2 and the third particle at a different site with $\zeta = 1$. Figures 3(a) and 3(b) show the phase diagram in the p/q-Ω/U plane. Again, the squishing effect is clearly seen. Figures 3(c) and 3(d) are for the case with a $\frac{1}{2}$ flux ($t/U = 0.1$), which show the squishing of the baryon continuously (most rapidly near Ω_c) with an increase of Ω. The process does not go on forever, and at a value of Ω somewhat larger than Ω_c, the baryon completely breaks up. Therefore, the gauge field produced by the $\frac{1}{2}$ flux is unable to entirely prevent the pair breaking effect. Most interestingly, the situation changes completely if one introduces a $\frac{1}{3}$ flux. Squishing occurs smoothly [Figs. 3(e) and 3(f)], and in fact, we believe that there is a bound state for all Ω (we cannot verify this as I_{xx} becomes large at large Ω). Further, at Ω_c, E_b for small t can be analytically inferred to be proportional to t. This is due to the hybridization between the "$2 + 1$" baryon hybridizing with a "$1 + 1 + 1$" aided by the 1/3-gauge field (flavor-orbital coupling). This is, again, in excellent agreement with our numerical result (not shown).

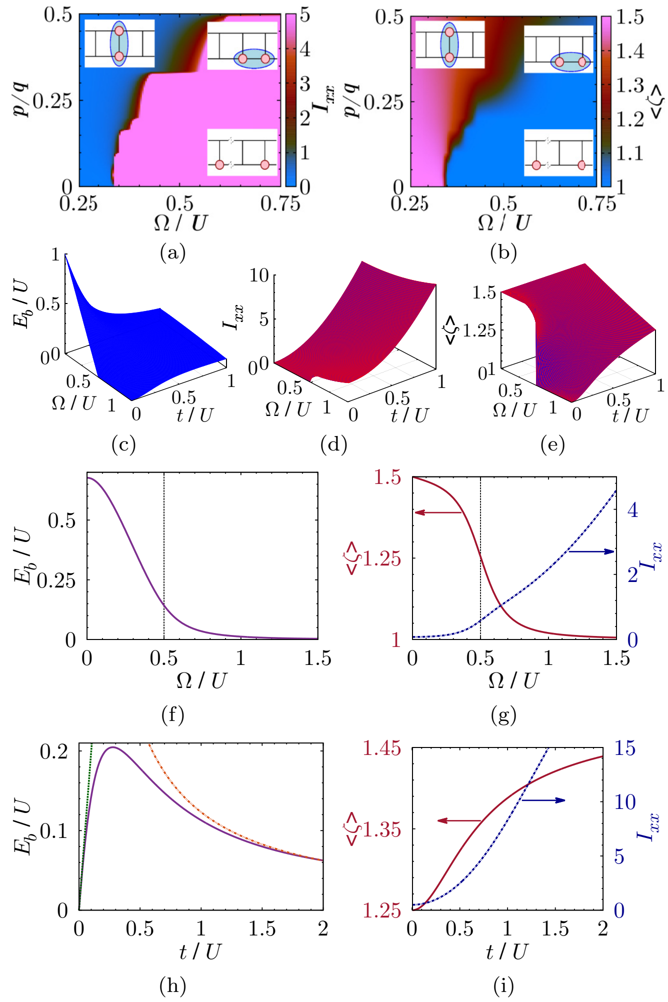

FIG. 2. (Color online) $M = 2$. (a) and (b): "Phase diagram" of two particles showing dependence of (a) I_{xx} and (b) $\langle \zeta \rangle$ on flux p/q and Ω/U for $t/U = 0.1$. The insets show the type of bound state stabilized. (c)–(i), $p/q = \frac{1}{2}$: E_b, I_{xx}, and $\langle \zeta \rangle$ are respectively shown in (c)–(e). (f) shows the dependence of E_b on Ω/U for $t/U = 0.1$, while (g) shows I_{xx} and $\langle \zeta \rangle$ for the same case. The vertical black dotted lines show $\Omega = \Omega_c$. The dependence of E_b in (h) and $I_{xx}, \langle \zeta \rangle$ in (i) on t/U at $\Omega = \Omega_c = U/2$ are shown. The dashed lines in (h) are results of analytical considerations at small and large t/U.

$M = 4$. The distinct aspect here is the presence of two critical Zeeman fields Ω_{c1} and Ω_{c2}. When $t = 0$, the usual 4-baryon is destabilized to a state with two 2-baryons (each of which can be located at any site) at $\Omega_{c1} = \frac{2U}{\sqrt{5}}$. At $\Omega_{c2} = U$, this state is again broken into a $1 + 1 + 1 + 1$ state where each particle can be at any site distinct from others with $\zeta = 1$. Figures 4(a) and 4(b) show the phase diagram in the p/q-Ω/U plane. A $\frac{1}{2}$ flux has a smooth change from the usual 4-baryon to a $2 + 2$ baryon (bound state of 2-baryons)—another good example of squishing. However, the $\frac{1}{2}$ flux is not able to mitigate the effects of the Zeeman field; near Ω_{c2} the squished $2 + 2$ baryon is broken up [Figs. 4(c) and 4(d)]. Remarkably, for a flux of $\frac{1}{4}$, this transition is prevented [Figs. 4(e) and 4(f)], and our calculations suggest a bound state for any Ω (checking this requires larger computational resources). At Ω_{c1}, it can be shown that the binding energy is proportional to t^2 (in order to

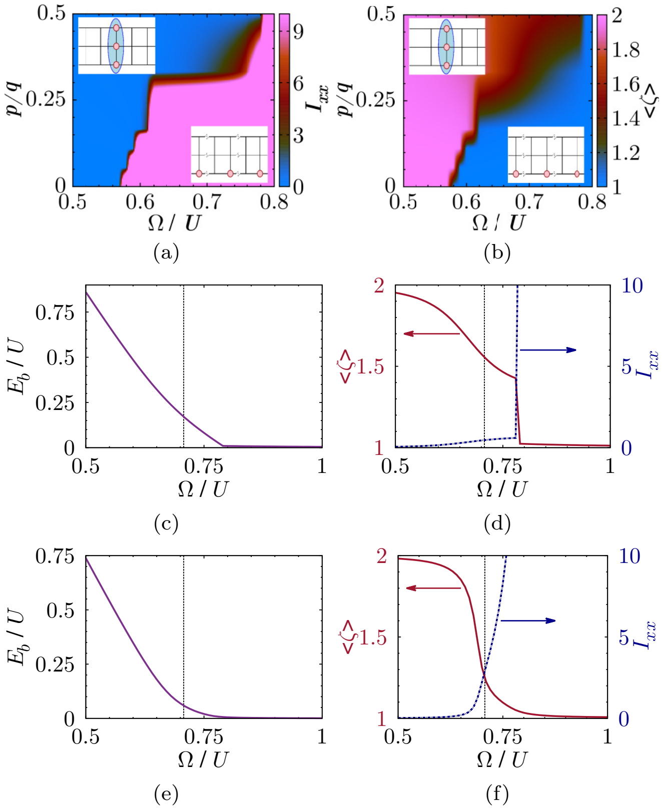

FIG. 3. (Color online) $M = 3$. Phase diagram, (a) and (b): Three-particle phase diagram showing the dependence of (a) I_{xx} and (b) $\langle\zeta\rangle$ on flux p/q and Ω/U for $t/U = 0.1$ ($N_q = 18$). Insets: Type of bound state. (c) and (d), $p/q = \frac{1}{2}$: (c) and (d) show the binding energy, and I_{xx} and $\langle\zeta\rangle$ vs Ω/U with $t/U = 0.1$. (e) and (f), $p/q = \frac{1}{3}$: (e) and (f) show same quantities as (c) and (d) for $\frac{1}{3}$ flux. The vertical black dotted lines show $\Omega = \Omega_c$.

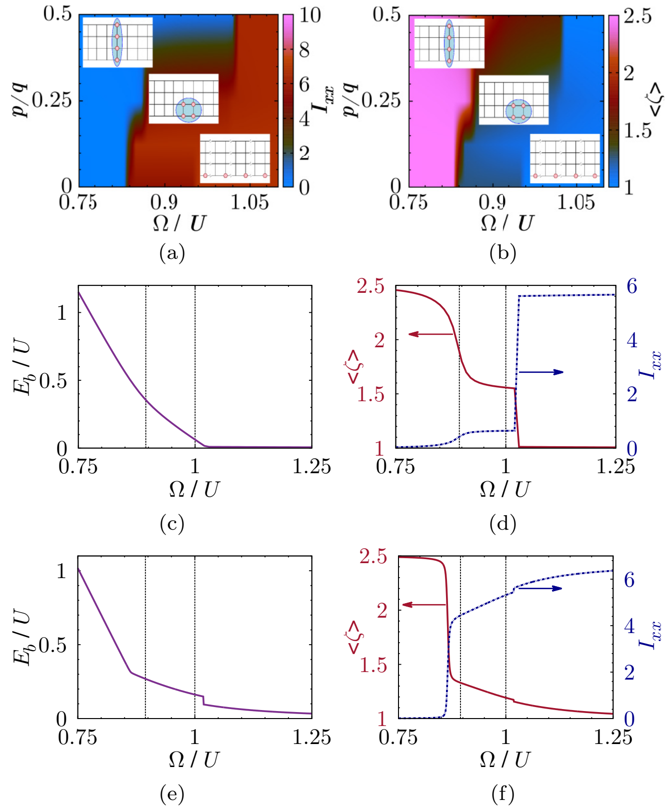

FIG. 4. (Color online) $M = 4$. Phase diagram, (a) and (b): Four-particle phase diagram showing the dependence of (a) I_{xx} and (b) $\langle\zeta\rangle$ on flux p/q and Ω/U for $t/U = 0.1$ obtained with $N_q = 8$. The insets show the type of bound state stabilized. (c) and (d), $p/q = \frac{1}{2}$: (c) and (d) show the dependence of the binding energy, and I_{xx} and $\langle\zeta\rangle$ on Ω/U with $t/U = 0.1$. (e) and (f), $p/q = \frac{1}{4}$: (e) and (f) show same quantities as (c) and (d) for the $\frac{1}{4}$-flux case. The vertical black dotted lines show $\Omega = \Omega_{c1}$ and $\Omega = \Omega_{c2}$ ($\Omega_{c1} < \Omega_{c2}$).

hybridize the 4-baryon and the 2 + 2 baryon); our numerical calculations have borne this out.

What are the general criteria required to produce squishing? To produce squishing, the flavor-orbital coupling induced by the flux must be able to hybridize the degenerate states that occur at the critical Zeeman fields. For example, for $M = 4$, the flavor-orbital coupling with a $\frac{1}{4}$ flux does hybridize the 2 + 2 state with the 1 + 1 + 1 + 1 state, and hence the baryon is squished (unlike the $\frac{1}{2}$ flux). For a given Ω_γ, an appropriate flux can be chosen to achieve this.

In the many-body setting, there is clearly a rich collection of states and crossovers to be explored. For attractive interactions, many-body states with squished baryons are likely to hold interesting physics. The repulsive nonlocal interactions for repulsive U should sustain density waves [47]. The results developed here can be used as a guide for such studies, particularly in the dilute limit.

We conclude this Rapid Communication by pointing out an interesting possibility to use the SD system to create a class of Hamiltonians called "random flux" models [42]. The idea is to introduce some randomness in Ω_γ^j, which in turn will make the gauge fields [Eq. (5)] also random. For a two-dimensional square optical lattice, the Hamiltonian of the type [Eq. (5)] realized will be similar to a "random flux" model.

S.K.G. and U.K.Y. acknowledge support from CSIR, India via a SRF grant and UGC, India via a DSKPDF grant, respectively, and V.B.S. is grateful to DST, India and DAE, India (SRC grant). We thank Sambuddha Sanyal for discussions regarding the random flux model, and Adhip Agarwala for comments on the manuscript.

[1] I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008).

[2] Y.-J. Lin, R. L. Compton, K. Jimenez-Garcia, J. V. Porto, and I. B. Spielman, Nature (London) **462**, 628 (2009).

[3] Y.-J. Lin, R. L. Compton, A. R. Perry, W. D. Phillips, J. V. Porto, and I. B. Spielman, Phys. Rev. Lett. **102**, 130401 (2009).

[4] P. Wang, Z.-Q. Yu, Z. Fu, J. Miao, L. Huang, S. Chai, H. Zhai, and J. Zhang, Phys. Rev. Lett. **109**, 095301 (2012).

[5] L. W. Cheuk, A. T. Sommer, Z. Hadzibabic, T. Yefsah, W. S. Bakr, and M. W. Zwierlein, Phys. Rev. Lett. **109**, 095302 (2012).
[6] H. Miyake, G. A. Siviloglou, C. J. Kennedy, W. C. Burton, and W. Ketterle, Phys. Rev. Lett. **111**, 185302 (2013).
[7] M. Aidelsburger, M. Atala, M. Lohse, J. T. Barreiro, B. Paredes, and I. Bloch, Phys. Rev. Lett. **111**, 185301 (2013).
[8] G. Jotzu, M. Messer, R. Desbuquois, M. Lebrat, T. Uehlinger, D. Greif, and T. Esslinger, Nature (London) **515**, 237 (2014).
[9] N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).
[10] K. Harada, N. Kawashima, and M. Troyer, Phys. Rev. Lett. **90**, 117203 (2003).
[11] F. F. Assaad, Phys. Rev. B **71**, 075103 (2005).
[12] P. Corboz, A. M. Läuchli, K. Penc, M. Troyer, and F. Mila, Phys. Rev. Lett. **107**, 215301 (2011).
[13] C. Xu, Phys. Rev. B **81**, 144431 (2010).
[14] A. Paramekanti and J. B. Marston, J. Phys.: Condens. Matter **19**, 125215 (2007).
[15] M. Hermele and V. Gurarie, Phys. Rev. B **84**, 174441 (2011).
[16] M. Greiter and S. Rachel, Phys. Rev. B **75**, 184441 (2007).
[17] K. Buchta, O. Legeza, E. Szirmai, and J. Sólyom, Phys. Rev. B **75**, 155108 (2007).
[18] S. R. Manmana, K. R. A. Hazzard, G. Chen, A. E. Feiguin, and A. M. Rey, Phys. Rev. A **84**, 043601 (2011).
[19] Z. Cai, H.-H. Hung, L. Wang, and C. Wu, Phys. Rev. B **88**, 125108 (2013).
[20] Z. Zhou, Z. Cai, C. Wu, and Y. Wang, Phys. Rev. B **90**, 235139 (2014).
[21] W. Hofstetter, J. I. Cirac, P. Zoller, E. Demler, and M. D. Lukin, Phys. Rev. Lett. **89**, 220407 (2002).
[22] C. Honerkamp and W. Hofstetter, Phys. Rev. Lett. **92**, 170403 (2004).
[23] G. Klingschat and C. Honerkamp, Phys. Rev. B **82**, 094521 (2010).
[24] A. Rapp, W. Hofstetter, and G. Zaránd, Phys. Rev. B **77**, 144520 (2008).
[25] S. Capponi, G. Roux, P. Lecheminant, P. Azaria, E. Boulat, and S. R. White, Phys. Rev. A **77**, 013624 (2008).
[26] A. Rapp, G. Zaránd, C. Honerkamp, and W. Hofstetter, Phys. Rev. Lett. **98**, 160405 (2007).
[27] J. Pohlmann, A. Privitera, I. Titvinidze, and W. Hofstetter, Phys. Rev. A **87**, 023617 (2013).
[28] T. Graß, R. W. Chhajlany, C. A. Muschik, and M. Lewenstein, Phys. Rev. B **90**, 195127 (2014).
[29] M. A. Cazalilla, A. F. Ho, and M. Ueda, New J. Phys. **11**, 103033 (2009).
[30] M. A. Cazalilla and A. M. Rey, Rep. Prog. Phys. **77**, 124401 (2014).
[31] B. J. DeSalvo, M. Yan, P. G. Mickelson, Y. N. Martinez de Escobar, and T. C. Killian, Phys. Rev. Lett. **105**, 030402 (2010).
[32] T. Fukuhara, Y. Takasu, M. Kumakura, and Y. Takahashi, Phys. Rev. Lett. **98**, 030401 (2007).
[33] M. Prevedelli, F. S. Cataliotti, E. A. Cornell, J. R. Ensher, C. Fort, L. Ricci, G. M. Tino, and M. Inguscio, Phys. Rev. A **59**, 886 (1999).
[34] A. V. Gorshkov, M. Hermele, V. Gurarie, C. Xu, P. S. Julienne, J. Ye, P. Zoller, E. Demler, M. D. Lukin, and A. M. Rey, Nat. Phys. **6**, 289 (2010).
[35] S. Taie, R. Yamazaki, S. Sugawa, and Y. Takahashi, Nat. Phys. **8**, 825 (2012).
[36] J. K. Chin, D. E. Miller, Y. Liu, C. Stan, W. Setiawan, C. Sanner, K. Xu, and W. Ketterle, Nature (London) **443**, 961 (2006).
[37] A. Celi, P. Massignan, J. Ruseckas, N. Goldman, I. B. Spielman, G. Juzeliunas, and M. Lewenstein, Phys. Rev. Lett. **112**, 043001 (2014).
[38] J. K. Jain, *Composite Fermions* (Cambridge University Press, Cambridge, U.K., 2007).
[39] M. Mancini, G. Pagano, G. Cappellini, L. Livi, M. Rider, J. Catani, C. Sias, P. Zoller, M. Inguscio, M. Dalmonte, and L. Fallani, Science **349**, 1510 (2015).
[40] B. K. Stuhl, H.-I. Lu, L. M. Aycock, D. Genkina, and I. B. Spielman, Science **349**, 1514 (2015).
[41] X.-W. Guan, M. T. Batchelor, and C. Lee, Rev. Mod. Phys. **85**, 1633 (2013).
[42] A. Altland and B. Simons, Nucl. Phys. B **562**, 445 (1999).
[43] While we discuss fermionic physics in this Rapid Communication, many of our conclusions will be applicable also to bosonic systems.
[44] The mapping holds for any flux $k_\ell d = 2\pi\varphi$.
[45] See also T. Graß, A. Celi, and M. Lewenstein, Phys. Rev. A **90**, 043628 (2014). We thank the referee for bringing this work to our attention.
[46] In a completely unbound state, such as that obtained with $U = 0$, $I_{xx} \sim N_q^2$.
[47] Indeed, very recent postings [S. Barbarino, L. Taddia, D. Rossini, L. Mazza, and R. Fazio, Nat. Commun. **6**, 8134 (2015); T.-S. Zeng, C. Wang, and H. Zhai, Phys. Rev. Lett. **115**, 095302 (2015)] do find results consistent with this.