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Sliding Mode Control for Complex Time Delay Systems with Application to Power Systems

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By

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ABSTRACT

This thesis is concerned with the stability and control of complex time delay systems with power systems application using sliding mode control (SMC). It is assumed that all the system state is accessible for design and robustness is improved by taking advantage of the system structure and disturbance bounds. Lyapunov-Razumikhin technique is used as analytical tool for the time delay system throughout this thesis. The main contributions in this thesis are as follows

- A systematic technique for obtaining the design parameters was developed using basic linear matrix inequality (LMI) in combination with Lyapunov Razumikhin approach in order to improve the requirements on bounds of the delay. Matched and mismatched uncertainties are considered which involves time delay. Compared with general non-linear uncertainties, the considered non-linear uncertainties involve time delay. This design is more robust for large complex systems network.

- Turbines with reheat unit often neglect delays in load frequency control (LFC) studies. For stability purposes, a simple model used to represent this delays is formulated. A multiple delayed non-linear term is used to describe the disturbance. The idea is to treat the overall effects of delays and disturbances especially on the input side as general non-linearity may be considered to have large time varying delay. It also considers the stability of the load disturbance, such disturbances are generated by purely resistive loads and are non-varnishing around the system origin. Further analysis on this has been extended to decentralised interconnected LFC system with more focus on robust and closed loop systems performance.

- The effect of communication delay in wide area LFC network is addressed using the Lyapunov-Razumikhin function method. The system is tested for large time delay and its robustness to change rate of the time delay which may be increased due to multi-areas, under step and random load disturbances. It further modifies the existing works by treating the interconnection as a non-linear, unknown term. Thereby, making the system more robust to larger class of systems. Decentralised sliding mode LFC is designed for effective closed-loop system performance.

In order to validate the theoretical results obtained, four test examples was conducted using Matlab/Simulink tool. The stabilisation of time delay system with non-linear delayed disturbance was tested using an automatic voltage regulator (AVR) system and a numerical example. An isolated reheat turbine LFC system is used to verify the effectiveness of the modified turbine system model. Two area LFC system have been used to demonstrate the effectiveness of the developed decentralised scheme.

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CHAPTER 1

INTRODUCTION

The eminent challenges of modern control theory brought by technological, environmental and societal processes in the broad areas of ecology, power systems, economic and spacecraft engineering, rely on overcoming the size and complexity of useful models and systems. As systems become larger and complex, great effort is required in its analysis which may either be impossible or uneconomical to deal with irrespective of modern computing machines. One way of ciphering this problem for the purpose of system analysis and control design is through decomposition into a number of interconnected subsystems, in order to consider these subsystems independently while also dealing with interconnection constraints that may arise in the process. More so, it is necessary to consider the causes and effects of behavioural complexity of the systems and how it influences the whole system especially during large interconnection.

Time delay is one of the main causes of this complexity in the systems. It's a phenomenon which occurs in many physical and biological systems whereby the present state of the system depends on the previous states. Time delay is also a source of instability and poor control performance and has been largely considered in various literature [23, 106, 17, 28, 119]. In addition, in the presence of perturbations, the system complex-

ity may be much more pronounced, wherefore increasing complexity may bring about reduced accuracy of the otherwise stable system [78]. Thus, this thesis will be focusing on the analysis and control of complex time delay systems in the field of power systems network.

1.1. BACKGROUND AND MOTIVATION

The importance of power system in the practical world cannot be overemphasised which explains it complex nature due to increased demand for electrical energy. Power systems are modelled as large scale dynamical equations composed of a number of interconnected subsystems and are often distributed in a wide area network through transmission lines. However, structural perturbation, a phenomenon whereby subsystems groups are disconnected and connected again to each other in various ways during operation, high dimensionality, information transfer among individual subsystems, communication time delay and high cost of implementation all account for the complex nature of power systems [78, 102, 105], and strong motivation for investigating this studies.

In power systems, one very important index of power quality is frequency stability. Frequency fluctuations and tie-line power deviation can occur due to sudden load disturbances. Thus, LFC is needed to ensure power quality. LFC aims to return frequency to pre-specified value as well as minimise deviation in tie-line power flow between interconnected areas. As previously mentioned, ceaseless growth and complexity of modern power system may result in wide area system oscillation due to wide area network [11]. Decentralised control strategy has been applied due to its ease of implementation and cost. The concept of decentralisation was first used in stabilising a linear dynamic system in [55], and is such that the system information pattern and control structure are limited such that individual subsystem is controlled by its own input only as seen in Fig 1.1.

Many advanced decentralised control approaches have been applied to LFC, and are an effective control technique for the control of large scale interconnected systems, [105, 57, 8, 31] for review on decentralised control techniques. In practise, LFC is characterised by non-linearities existing in various form in power systems. Typical non-linearities imposed

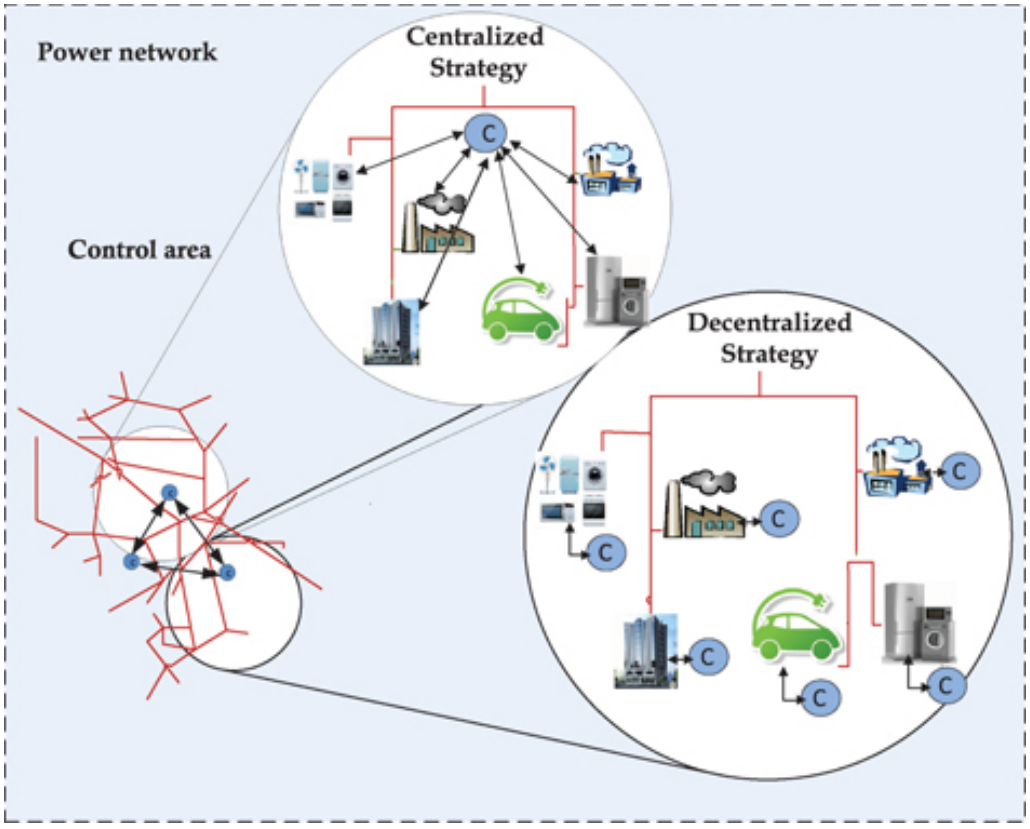

Figure 1.1: Power Network [10]

by physical system dynamics in LFC network include generation rate constraint (GRC), governor dead band (GDB), time delay and uncertainties due to continuous variations in load/dynamics and operating conditions. GRC is a phenomenon associated with the turbine performance as it affects the rate of change in the generating unit. Techniques for modelling this effects have been explored in various literatures [40, 25]. Also, the effect of GDB in power system performance has been considered in a few literatures [3, 85, 75]. Although this non-linearities can degrade the system performance, they are not the main focus of this thesis.

Time delays have often been known to cause instability and performance degradation in many networked and interconnected systems and their effects have been thoroughly investigated in many engineering applications [34, 104, 106]. In power systems, time delays are generally ignored and few works have been carried out in analysing the effects of time delay in power systems. This may be due to explicit mathematical solutions associated

with power system and time delay system analysis. The impact of time delays in power system stability has been studied in [59], it focuses on delays due to terminal voltage measurement and transducers. An effective power systems operation requires an open communication infrastructure to support the growing decentralised property of control such as LFC [33]. This open communication channel is often characterized by constant and time-varying delays in the LFC scheme. Delay in LFC often arises due to several causes involving signal processing, crossover elements within turbines and communication delays [3, 41]. Recent findings also show that the integration of electric vehicles (EV) into LFC network may introduce time-varying delays [39]. More so, reheat turbines due to their more complex turbine structure and multiple control valves often increase the chances of time delay in the systems structure [67, 41]. Following the rapid infrastructural development in power system networks, communication delay has become a major challenge in LFC and has been largely researched in the past decade [63, 62, 116].

Time domain methods have been applied in the stability and control of time delayed power system. This method helps to facilitate robust control design (variable structure control, H∞ control) and analyzing time-varying delays and uncertainties. The Lyapunov Krasovskii theorem and Razumikhin theorem are largely used in the analysis of time delay systems. With regards to LFC of power systems, frequencies and tie-line power deviate from their pre-defined values due to large communication delays. Various control and stability methods have been combined in trying to circumvent this problem. In [117, 39, 33], Lyapunov Krasovskii method was used extensively with LMI to deal with these effects while employing proportional integral (PI) LFC to guarantee its closed-loop performance. However, PI LFC can be suitable when dealing with conventional LFC problem as compared to practical LFC problem which is highly non-linear due to uncertainties, constant load disturbances and renewable energy fluctuations [58]. Delay dependent stability control of power systems with multiple time delays was considered in [47] using H∞ control performance technique to guarantee its closed loop design. Model predictive control (MPC), a type of robust control methods has been applied severally to power system LFC to act against uncertainties and calculate optimal control input. Robust multivariable predictive based LFC considering generation rate constraint have been

considered by [76]. Model predictive based LFC design concerning wind turbines have also been considered in [61]. However, when delays are involved in LFC, there are rarely any of such papers in literature.

More recently, sliding mode control (SMC), a nonlinear type of control method, due to its strong robustness property and good transient response to parameter uncertainties and system disturbances, has been applied to LFC of power system. Prior to this, SMC has recorded huge success in its application to complex systems in many literature [2, 5, 9, 13, 18, 93]. Especially when time delay is involved [65, 66, 106]. Power system is highly nonlinear and characterized by large system disturbances. However, delays are unavoidable due to communications links. Sliding mode load-frequency controller (SMLFC) design for dynamic stability enhancement of large-scale interconnected power systems was investigated in [115], which was used to handle large parameter variations. In [58], SMLFC is proposed for multi-area time delay power system with wind power integration. A delay dependent stability condition is derived using the Lyapunov Krasovskii functional method for dealing with constant and time-varying communication delays in [68]. Although, there are only limited literature in delayed LFC systems, fewer literatures have applied SMC. The use of Lyapunov Krasovskii method can be limited when system involves time delay. Its limitation to the change rate of the time delay often makes it less robust to systems with fast change rate. It has been shown in practice that power system has a fast frequency response, and is insensitive to parameter variations and load disturbances [3]. Therefore, this thesis will focus on employing Lyapunov Razumikhin method for achieving stability for large constant and time-varying delay which may exist in the continuously expanding network systems. It will also seek to highlight the delays associated with the turbine system model which is often ignored in the study of LFC. Above all, studies have shown that LFC performance declines due to increasing delay. Thus, a delay dependent SMC will be designed for improved systems performance.

1.2. CONTRIBUTIONS AND ORGANISATION

This thesis considers a class of complex control systems with time delays, disturbances, non-linearities and possibly interconnections between subsystems. It focuses on developing rigorous theoretical but practical results which are able to be applied to electrical power systems. The main contribution of the thesis is summarised as follows:

- Development of simplified technique for obtaining the design parameters of non-linear delayed disturbances: A systematic technique for obtaining the design parameters was developed using basic LMI in combination with Lyapunov Razumikhin approach in order to improve the requirements on bounds of the delay. Matched and mismatched uncertainties are considered which also involves time delay, and the known delay is utilized in the control design. Compared with general non-linear uncertainties, the considered non-linear uncertainties involve time delay. This design is more robust for large network complexities and has also been extended to real power system application.

- Control of reheat turbine system involving multiple delays: Large turbine units especially turbines with reheat unit often neglect delays in LFC studies. For stability purposes, a simple model used to represent these delays is formulated. A multiple delayed non-linear term is used to describe the disturbance. The idea is to treat the overall effects of delays and disturbances especially on the input side as general non-linearity which may be considered to have a large time varying delay. It also considers the stability of the load disturbance, such disturbances are generated by purely resistive loads and are non-varnishing around the system origin. Further analysis on this has been extended to decentralised interconnected LFC system with more focus on robust and closed loop system performance.

- Decentralised SMC for LFC applications has been considered. Non-reheat turbine cases are considered for interconnected systems with time delay. However, communication delays are eminent due to wide area network. Thus, the system is tested for large time delay and its robustness to change rate of the time delay which may

be increased due to multi-areas. It further modifies the existing works by treating the interconnection as a non-linear, unknown term. Thereby, making the system more robust to larger class of multi-areas LFC models. Decentralised SMC is used to enhance systems performance.

The rest of this thesis is structured as follows:

Chapter 2 introduce the basic mathematical background used in this thesis. It defines the Lyapunov theorems and necessary definitions as used in this text. However, not much details are given as most are standard and can be found in the within references.

Chapter 3 looks at basic examples of state-space representation and controllability of linear systems. It explores the concept of sliding mode control with simulated example and further discusses in brief, the methods of time delay systems as used used in the thesis. An overview of power systems LFC for single and multi area LFC is also considered.

Chapter 4 studies the stabilisation of time delay systems based on sliding mode. It analyses the disturbance effect of the system using basic LMI technique. Lyapunov Razumikhin method is used to deal with the delay and further designs a delay dependent sliding mode control with application to power systems.

Chapter 5 focuses on developing stability result for multiple delay in reheat turbine system. It discusses the effect of multiple delays and non-frequency dependent disturbance in LFC. Stability is analysed based on Razumikhin method and SMC is designed to reduce chattering effects. Application to an isolated reheat turbine system is studied with simulation.

Chapter 6 deals with communication time delay for interconnected power system. Analysis of non-linear interconnection with delayed uncertainty is considered. Stability of the system is analysed and decentralised SMC is designed for systems efficiency. Two area LFC system application is considered.

Chapter 7 discusses the summary and conclusion of the research work based on overall analysis and simulation of the system dynamics, as well as future work.

CHAPTER 2

MATHEMATICAL PRELIMINARIES

This chapter comprises mainly of some background and standard materials already obtainable from many books and literatures, but are used throughout this thesis. More details about understanding their concepts and relationships can be explored in further references [23, 36, 51].

2.1. NORMS

Definition 2.1.1 *[51]* A normed vector space is a pair $(X, \|\cdot\|)$ consisting of a vector space X and a norm $\|\cdot\| : X \to \Re$ such that

- $\|x\| = 0$ if and only if $x = 0$
- $\|\lambda x\| = |\lambda|\|x\| \quad \forall \lambda \in \Re, \forall x \in X$
- $\|x + y\| \leq \|x\| + \|y\| \quad \forall x, y \in X$

For $x = \Re^n$, let

$$x = (x_1 \;\; x_2 \;\; \cdots \;\; x_n)$$

The three most commonly used norms are

$\|x\|_1 = |x_1| + \cdots + |x_n|$= $\sum_{i=1}^{n} |x_i|$

$\|x\|_2 = \sqrt{|x_1|^2 + \cdots + |x_n|^2} = \sqrt{\sum_{i-1}^{n} |x|^2}$ (Euclidean norm)

$\|x\|_\infty = \max |x_i|$

In this thesis, the Euclidean norm will be used.

2.2. LYAPUNOV STABILITY

The Lyapunov stability theorem is a representation of a systems stability in terms of energy, a great advantage which eliminates the need to obtain explicit solutions in order to determine the stability of a system.

2.2.1. BASIC STABILITY THEOREM

Consider an autonomous system

$$\dot{x} = f(x) \tag{2.1}$$

where $x \in \mathcal{D} \subset \Re^n$, and $f : \mathcal{D} \to \Re^n$ is a locally Lipschitz map.

Definition 2.2.1 (Stability and asymptotic stability [83]). An equilibrium point $x = 0$ of system (2.1) is said to be

- stable, for any $t_0 \geq 0$ and $\epsilon > 0$, $\exists \delta = \delta(t_0, \epsilon) > 0$ such that

$$\|x(0)\| < \delta(t_0, \epsilon) \Rightarrow \|x(t)\| < \epsilon \qquad \forall t \geq t_0, \tag{2.2}$$

- unstable if it is not stable.

- uniformly stable if, for each $\epsilon > 0$, there exits $\delta = \delta(\epsilon) > 0$, independent of t_0, such that (2.2) is satisfied.

- asymptotically stable if it is stable and there is a positive constant $\delta_2 = \delta_2(t_0) > 0$ such that

$$\|x(0)\| < \delta_2(t_0) \Rightarrow \lim_{t\to\infty} x(t) = 0 \tag{2.3}$$

Lemma 2.2.1 (Lyapunov stability theorem) [83]
Consider system (2.1). Let $x = 0$ be an equilibrium point for system (2.1) and $\mathcal{D} \subset \Re^n$ be a domain containing $x = 0$. Let $V : \mathcal{D} \to \Re$ be a continuously differentiable function such that

$$V(0) = 0 \quad and \quad V(x) > 0 \quad in \quad \mathcal{D} - \{0\} \tag{2.4}$$

$$\dot{V}(x) \leq 0 \quad in \quad \mathcal{D} \tag{2.5}$$

Then, $x = 0$ is stable. Moreover, if

$$\dot{V}(x) < 0 \quad in \quad \mathcal{D} - \{0\} \tag{2.6}$$

then $x = 0$ is asymptotically stable.

For a non-autonomous system

$$\dot{x} = f(t, x) \tag{2.7}$$

where $\mathcal{D} \subset \Re^n$, contains the origin $x_e = 0$ and $f : \Re^+ \times \mathcal{D} \to \Re^n$ is piecewise continuous in t and locally Lipschitsz in x on $\Re^+ \times \mathcal{D}$.

Definition 2.2.2 (Class $\mathcal{K}$ functions [83]). A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. It is said to belong to class $\mathcal{K}_\infty$ if $a = \infty$ and $\alpha(r) \to$ as $r \to \infty$.

Definition 2.2.3 (Class $\mathcal{KL}$ functions [83]). A continuous function $\beta : [0, a) \times [0, \infty) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. It is said to belong to class $\mathcal{KL}_\infty$ if, for each fixed s, the mapping $\beta(r, s)$ belongs to class $\mathcal{K}$ with respect to r and, for each fixed r, the mapping $\beta(r, s)$ is increasing with respective to s and $\beta(t, s) \to \infty$ as $s \to \infty$.

Lemma 2.2.2 (Expanded Lyapunov stability theorem)[36]: Let $x = 0$ be an equilibrium point for (2.7) and $\mathcal{D} \subset \Re^n$ be a domain containing $x = 0$. Let $V : [0, \infty) \times \mathcal{D} \to \Re$ be a continuously differentiable function such that

$$W_1(x) \leq V(t, x) \leq W_2(x) \tag{2.8}$$

$$\frac{\delta V}{\delta t} + \frac{\delta V}{\delta x} f(t, x) \leq 0 \tag{2.9}$$

$\forall t \leq 0$ and $\forall x \in \mathcal{D}$, where $W_1(x)$ and $W_2(x)$ are continuous positive definite functions on $\mathcal{D}$. Then, $x = 0$ is uniformly stable. $\diamond$

Lemma 2.2.3 (Expanded Lyapunov asymptotic stability theorem) [36]: Suppose the assumptions of Lemma 2.2.2 are satisfied with inequality (2.9) strengthened to

$$\frac{\delta V}{\delta t} + \frac{\delta V}{\delta x} f(t, x) \leq -W_3(x) \tag{2.10}$$

$\forall\ t \geq 0$ and $\forall\ x \in \mathcal{D}$, where $W_3(x)$ is a continuous positive definite function on $\mathcal{D}$. Then, $x = 0$ is uniformly asymptotically stable. Moreover, if r and c are chosen such that $B_r = \{\|x\| \leq r\} \subset \mathcal{D}$ and $c < min_{\|x\|=r} W_1(x)$, then every trajectory starting in $\{x \in B_r \mid W_2(x) \leq c\}$ satisfies

$$\|x(t)\| \leq \beta(\|x(t_o)\|, t - t_0), \quad \forall t \geq t_o \geq 0 \tag{2.11}$$

for some class $\mathcal{KL}$ function β. Finally, if $\mathcal{D} = R^n$ and $W_1(x)$ is radially unbounded, then $x = 0$ is globally uniformly asymptotically stable.

2.2.2. FUNCTIONAL DIFFERENTIAL EQUATION

Consider the following retarded differential equation (RDE)

$$\dot{x}(t) = f(t, x_t), \qquad t_0 \leq t, \tag{2.12}$$

where $x(t) \in \Re^n$ and $f : \Re \times C \to \Re^n$ is continuous in both arguments and is locally Lipschitz continuous in the second argument. Assume that $f(t, 0) = 0$ which guarantees that (2.12) possesses a trivial solution $x(t) \equiv 0$.

Definition 2.2.4 *[16]* (uniform stability, uniform asymptotic stability, global uniform asymptotic stability) The trivial solution of (2.12) is

- uniformly stable if $\forall t_0 \in \Re$ and $\forall \epsilon > 0$, $\exists \delta = \delta(\epsilon) > 0$ such that $\|x_{t_0}\|_c < \delta(\epsilon)$ $\|x(t)\| < \epsilon \quad \forall\, t \geq t_0$.

- uniform asymptotically stable if it is uniformly stable and $\exists \delta_a > 0$ such that for any $\eta > 0$ $\exists T(\delta_a, \eta)$ such that $\|x_{t_0}\|_c < \delta_a$ implies |x(t)|< η for all $t \geq t_0 + T(\delta_a, \eta)$ and $t_0 \in \Re$.

- globally uniformly asymptotically stable if δ_a can be an arbitrary large, finite number.

Lemma 2.2.4 (Lyapunov-Krasovskii stability theorem, [23])
Suppose that $f : \Re \times C \to \Re^n$ in (2.12) maps $\Re\times$ (bounded sets in C) into bounded sets in $\Re^n$, and that u, v, w: $\Re_+ \to \Re_+$ are continuous non-decreasing functions, where $u(\tau)$ and $v(\tau)$ are positive for $\tau > 0$ and $u(0) = v(0) = 0$.

- If there exists a continuous differentiable functional $V : \Re \times C \to \Re$ such that $u(\|\phi(0)\|) \leq V(t, \phi) \leq v(\|\phi\|_c)$ and $\dot{V}(t, \phi) \leq -w(\|\phi(0)\|)$, then the trivial solution of (2.12) is uniformly stable.

- If the trivial solution of (2.12) is uniformly stable, and $w(\tau) > 0$ for $\tau > 0$, then the trivial solution of (2.12) is uniformly asymptotically stable.

- If the trivial solution of (2.12) is uniformly asymptotically stable and if $\lim_{\tau\to\infty} u(\tau) = \infty$, then the trivial solution of (2.12) is globally uniformly asymptotically stable.

Lemma 2.2.5 (Lyapunov-Razumikhin stability theorem, [23])
If there exist class $\mathcal{K}_\infty$ functions $\gamma_i(\cdot)$ with $i = 1, 2$, a class $\mathcal{K}$ function $\gamma_3(\cdot)$ and a continuous function $V_1(\cdot) : [-\overline{d}, \infty] \times \Re^n \mapsto \Re^+$ satisfying

$$\gamma_1(\|x\|) \leq V_1(t, x) \leq \gamma_2(\|x\|), \quad t \in [-\overline{d}, \infty],\ x \in \Re^n \tag{2.13}$$

such that the time derivative of V_1 along the solution of system (2.15) satisfies

$$\dot{V}_1(t, x) \leq -\gamma_3(\|x\|) \tag{2.14}$$

whenever

$$V_1(t + \theta, x(t + \theta)) \leq V_1(t, x(t)) \tag{2.15}$$

for any $\theta \in [-\bar{d}, 0]$, then the system (2.15) is uniformly stable. If, in addition, $\gamma_3(\tau) > 0$ for $\tau > 0$, and there exists a continuous non-decreasing function $\gamma_4(\cdot)$ which satisfies $\gamma_4(\tau) > \tau$ for $\tau > 0$ such that inequality (2.15) is strengthened to

$$\dot{V}_1(t, x) \leq -\gamma_3(\|x\|) \tag{2.16}$$

whenever

$$V_1(t + \theta, x(t + \theta)) \leq \gamma_4(V_1(t, x(t))) \tag{2.17}$$

for any $\theta \in [-\bar{d}, 0]$, then system (2.15) is uniformly asymptotically stable.

2.2.3. ANALYSIS OF LINEAR TIME INVARIANT SYSTEMS

Consider the autonomous linear time-invariant system given by

$$\dot{x} = Ax, \qquad A \in \Re^{n \times n} \tag{2.18}$$

and let $V(\cdot)$ be defined as follows

$$V(x) = x^T P x \tag{2.19}$$

where $P \in \Re^{n \times n}$ is (i) symmetric and (ii) positive definite. With these assumptions, $V(\cdot)$ is positive definite. Then

$$\dot{V}(x) = \dot{x}^T P x + x^T P \dot{x} \tag{2.20}$$

Thus the time derivative of $V(\cdot)$ along the system (2.18) is given by

$$\begin{aligned} \dot{V}(x) &= x^T A^T P x + x^T P A x & (2.21) \\ &= x^T (A^T P + P A) x & (2.22) \end{aligned}$$

or

$$\dot{V}(x) = -x^T Q x, \qquad A^T P + P A = -Q. \tag{2.23}$$

If Q is chosen as symmetric positive definite, then $\dot{V}$ is negative definite and the origin is globally asymptotically stable. Equation (2.23) is called Lyapunov equation.

2.3. OTHERS

Lemma 2.3.1 *[106]* Let the matrix $N_1 \in \Re^{m\times n}$ and vectors $x \in \Re^m$ and $y \in \Re^n$. Then, the inequality

$$x^T N_1 y \leq \frac{1}{2\epsilon} x^T N_1 N_2^{-1} N_1^T x + \frac{\epsilon}{2} y^T N_2 y \tag{2.24}$$

holds for any symmetric positive-definite matrix $N_2 \in \Re^{n\times n}$ and any positive constant ϵ.

Lemma 2.3.2 *[6, 99]* For any $z, y \in \Re^n$ and any positive definite matrix $X \in \Re^{n\times n}$

$$-2z^T y \leq z^T X^{-1} z + y^T X y \tag{2.25}$$

CHAPTER 3

BASIC SYSTEM DYNAMICS AND CONTROL

3.1. LINEAR SYSTEM DYNAMICS

This chapter gives a brief overview of the state-space methods for linear systems, to provide a context for the focus of this thesis. Back in early twenties, linear systems were analysed based on frequency domain approaches with the aim of circumventing issues in communications system such as bandwidth and noise. However, they presented a few drawbacks as systems became larger and more complex involving multiple inputs and outputs, greater uncertainties and non-linearities which may be interrelated in a complicated manner. This led to the adoption of modern control technique usually a time domain approach which involves state space models of control systems in addressing this issues.

3.1.1. STATE-SPACE REPRESENTATION

State space approach is characterized by its ability to represent the internal state of the system, as well as provide the optimal channel during control design.

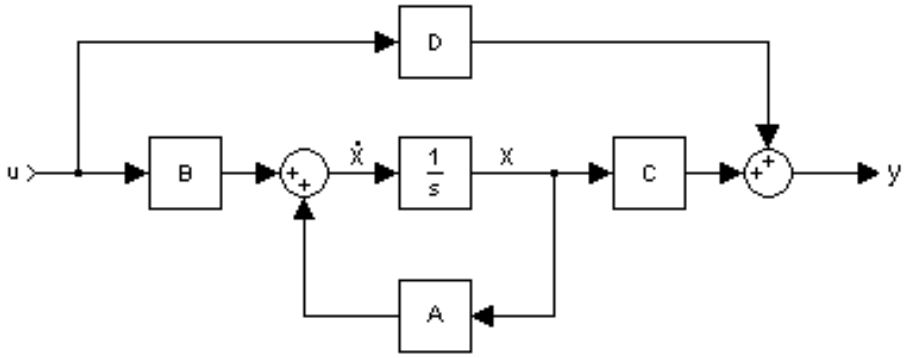

Figure 3.1: Block diagram of the state space representation

A state-space representation for a linear time-invariant system as shown in Fig 3.1 has the general form

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{3.1}$$

$$y(t) = Cx(t) + Du(t) \tag{3.2}$$

$x(t_0) = x_0$ for a specified initial time t_0.

where

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}, \quad u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix}, \quad y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_p(t) \end{bmatrix} \tag{3.3}$$

and A, B, C and D are real constant matrices of appropriate dimension. $x(t)$ is the state vector (n-vector), $u(t)$ is the control vector (m-vector), and $y(t)$ is the output vector (p-vector) respectively. The dynamic response of the system is determined by equation (3.1). Equation (3.2) is called the measurable quantity. It gives the desired output as a linear combination of the states and inputs.

Example 3.1.1 In order to introduce the state space modelling of linear dynamic systems, consider the DC motor drive system shown in Fig. 3.2. As illustrated in [93], the dynamics

of the motor are governed by two coupled first-order equations with respect to the shaft speed and armature current.

$$J\dot{\varpi} = k_t i - \tau_l \tag{3.4}$$

$$L\dot{i} = u - Ri - \lambda_o \varpi \tag{3.5}$$

where L is the armature inductance, R is armature resistance, J is the motor rotor and load inertia, i is the armature current, ϖ is the shaft speed, u is the terminal voltage, λ is the constant back electromotive force (EMF), k_t is torque constant, and τ_l is load torque. For

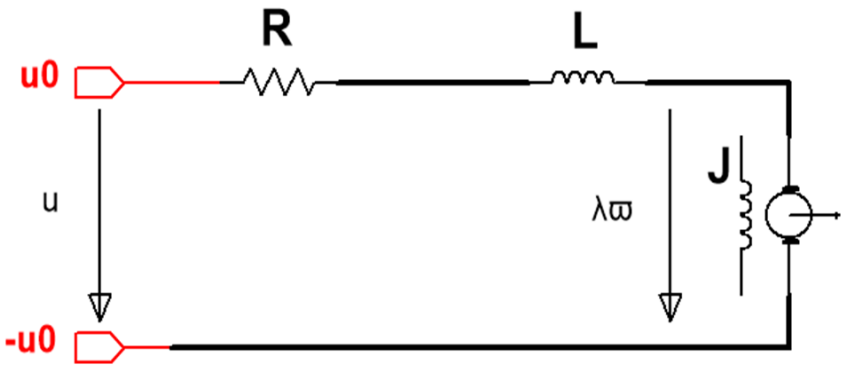

Figure 3.2: Model of a DC motor

the purpose of this work and ease of implementation, we focus on the integrated structure approach for speed control of the DC motor.

From Fig 3.3, taking the reference shaft speed as $\tilde{\varpi}$, $e = \tilde{\varpi} - \varpi$ as the speed tracking error and defining the state variables $x_1 = e$, and $x_2 = \dot{e}$. The following state space realization for the motion equation of a DC motor can be described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{k_t\lambda}{JL} & \frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{k_t}{JL} \end{bmatrix} u + \begin{bmatrix} 0 \\ g(t) \end{bmatrix} \tag{3.6}$$

where $g(t) = \ddot{\tilde{\varpi}} + \frac{R}{L}\dot{\tilde{\varpi}} + \frac{k_t\lambda}{JL}\dot{\tilde{\varpi}} + \frac{R\tau_l}{JL} + \frac{\dot{\tau}_l}{J}$, constitute the disturbance depending on the desired speed and load torque.

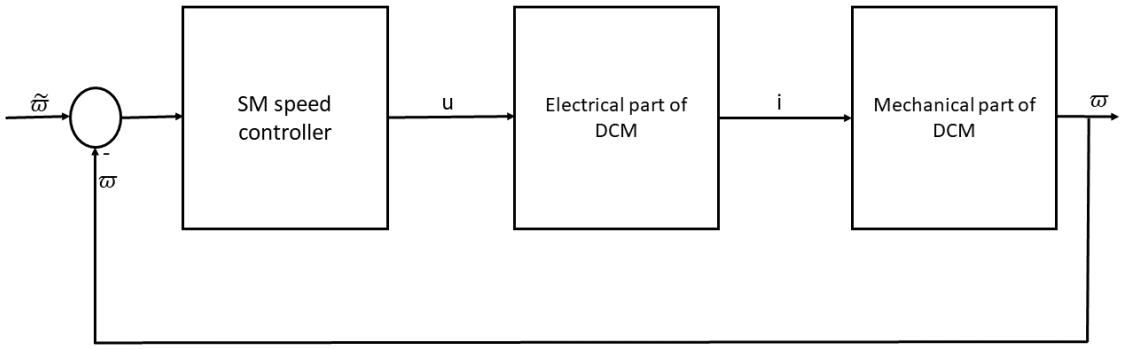

Figure 3.3: Speed control schematic of a DC motor

3.1.2. CONTROLLABILITY

Prior to introducing the state feedback design methods, it is paramount that necessary conditions and tests for controllability and observability are presented. One essential objective of state variable control is the design of systems with excellent performance. This is often based on the choice of specific performance criterion. A successful linear control system is governed by the controllability and observability properties of the system. This section seeks to explore the characteristic relationship between the input and state of an n-dimensional linear time invariant state equation. It specifically looks at the extent to which the state trajectory can be influenced by the input signal. Thus, implying that based on a certain derived condition, the state trajectory can be driven from an initial point to a final point by a suitable input signal. Therefore, we will define the basic idea of controllability by analysing the matrix pair (A,B) of the dynamic systems and illustrate with simple examples.

Consider the system in (3.1)-(3.2),

Definition 3.1.1 [98], For a given initial time t_0, the system is said to be completely state controllable if there exist a final time $t_f > t_0$ and an input function $u(\cdot)$ defined on a finite

interval $[t_0, t_f]$ such that

$$\begin{aligned} x(t_f) &= e^{A(t_f-t_0)}x + \int_{t_0}^{t_f} e^{A(t_f-\tau)}Bu(\tau)d\tau \\ &= 0 \in \Re^n \end{aligned} \tag{3.7}$$

Based on this definition, we arrive at the following prove.

Theorem 3.1.1 The linear state equation (3.1) is controllable if and only if

- (A, B) is controllable
- rank $([B \quad AB \quad A^2B \quad \cdots \quad A^{n-1}B]) = n$

3.1.3. LINEAR FEEDBACK CONTROL

The goal of control engineering is to guide the regulated system to operate in a required manner regardless of unpredictable influence of the environment. In the design of every practical control problem, deviations between the actual plant and the mathematical model used for its control design is unavoidable. These variations or mismatches are called un-modelled dynamics and can be considered as external disturbances. However, as systems become even more complex, this uncertainties abound, thereby making it more challenging to design adequate control laws that would provide the desired performance to the systems. The quest of proffering solution to this re-occurring phenomenon has led to the development of robust controllers aimed at solving this problems. This section takes a short look at the fundamentals of feedback control system in which a set of closed-loop eigenvalues is assigned by state feedback. For system (3.1)-(3.2), the state feedback control law as illustrated in Fig 3.4 is

$$u = -Kx \tag{3.8}$$

where K is the state feedback gain matrix with dimension $m \times n$. By combining Eqs. (3.1) and (3.8) the closed-loop system is defined by

$$\dot{x} = (A - BK)x \tag{3.9}$$

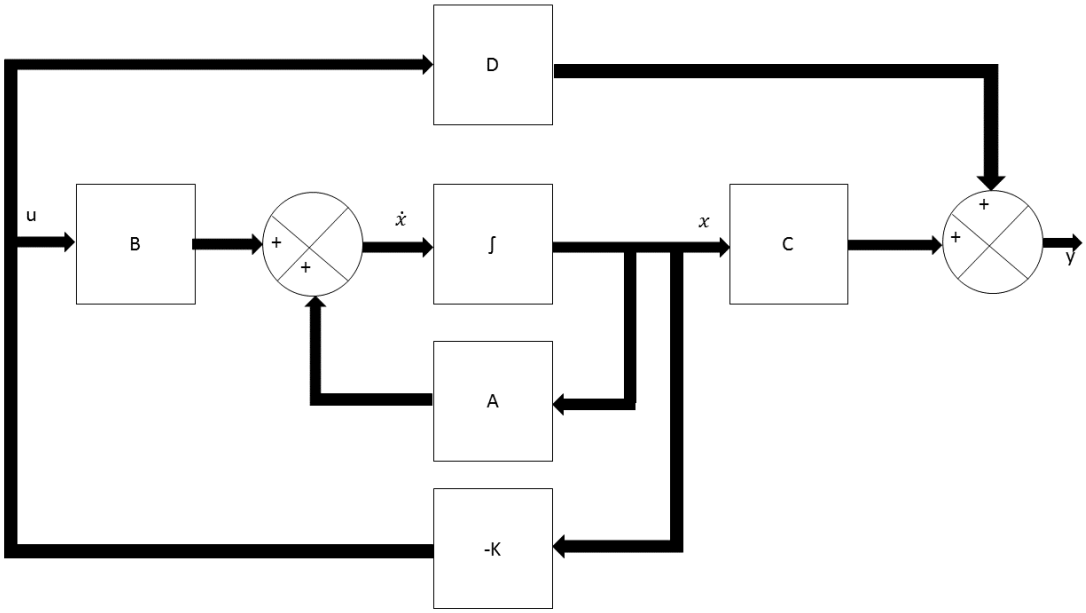

Figure 3.4: State feedback control system

It follows that the closed-loop system matrix produced by the state feedback is

$$A_k = A - BK \tag{3.10}$$

Thereby, resulting in the closed-loop characteristics equation

$$T(\lambda) = |\lambda I - A_k| = |\lambda I - A - BK| = 0 \tag{3.11}$$

Equation (3.11) implies that the closed-loop eigenvalue can be assigned by an appropriate selection of the state feedback matrix K. A necessary and sufficient condition for the selection of K is that the plant is completely state controllable.

3.2. SLIDING MODE CONTROL

Sliding mode control (SMC) [9, 13, 18, 93] is one particular robust control technique, which may be well suitable for achieving the desired performance. Sliding mode was first initiated in the 1950s by Emelyanov and his colleagues in the Soviet Union and was later popularized by Utkin in his published paper [92]. This was motivated by the need to design robust control laws, fuelled by the needs of military aeronautics to achieve

parametric insensitivity, due to lack of adequate compensation in linear networks. Since this period, there has been tremendous works carried out on this topic by vast majority of researchers in the control engineering field and various learned journals [22, 29, 24, 80, 118], aimed at improving dynamic system performances based on variable structure control (VSC) concept.

Sliding mode control (SMC), due to its many applications is currently one of the vital topics within the control engineering domain. It is a special type of variable structure control (VSC) techniques which obeys a particular switching logic and is characterized by discontinuous control action necessary to alter the system structure upon reaching the sliding surface. This controller drives the system to the surface, once the system reaches the surface, it stays on it thereafter. Thereby, sliding along the surface to the origin. This method is illustrated in Example (3.2.1).

Example 3.2.1 An introductory example
Consider the DC motor system in Fig 3.3.

$$\dot{x}_1 = x_2 \tag{3.12}$$

$$\dot{x}_2 = a_1 x_1 + a_2 x_2 + bu + g(t) \tag{3.13}$$

Apply the controller, where the sliding surface is define as

$$s(x) = S_1 x_1 + x_2 \tag{3.14}$$

and

$$u = -b^{-1}\Big\{a_1 x_1 + (S_1 + a_2)x_2 + u_o \text{sign}(s)\Big\} \tag{3.15}$$

where $S_1 = 3$, $a_1 = 0.008$, $a_2 = 0.5$, $b = -8$ and $|g(t)| \leq \bar{g}$ is disturbance associated with the desired speed and load torque.

From Fig 3.6, it can be observed that both signal x_1 and x_2 reaches 0 after about 4 seconds. Also, from the x_1 plot, it can be observed that the trajectory reaches the switching surface when the time is approximately t_s = 3.0 seconds.

However, the systems control signal has the drawback that the control signal chatters when the system trajectory is moving on the switching surface (refers to Fig. 3.7). This

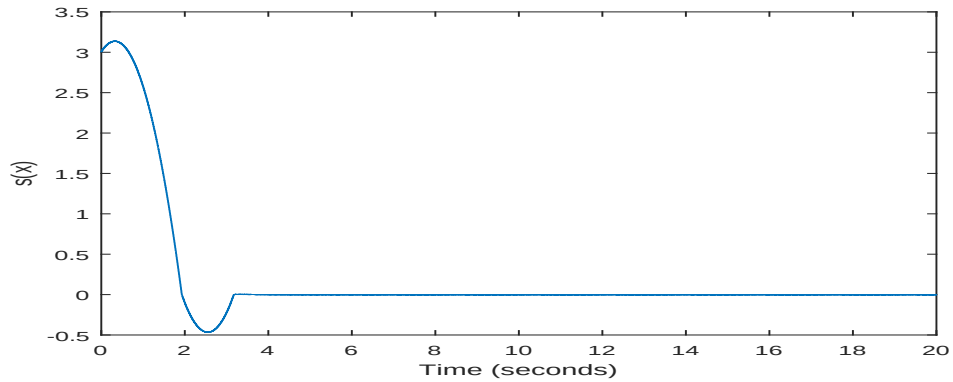

Figure 3.5: Time response of sliding variable

is due to the use of a sign(s) function in the control law, i.e.

$$u = -0.001x_1 - 0.44x_2 - 0.15\text{sign}(s) \tag{3.16}$$

It is obvious from the control law that when the system reaches the sliding surface $s(x) = 0$, it switches between positive and negative small numbers and will cause the control signal, u(t), to fluctuate along the envelope of the signal, with the fluctuation amplitude of 0.17 units, as confirmed from the Fig 3.7.

3.2.1. SIMULATION WITH OTHER INITIAL CONDITIONS

Simulation is done to the same controller with different initial conditions x_{10}, $x_{20} \leq 2$ as shown in Figs 3.8 and 3.9. It can be observed that regardless of the initial position, the controller manage to force the state x to 0 after some time. More so, it is clear that the further the initial condition is from the switching surface, the longest it takes to reach the surface. According to Figs 3.8 and 3.9, chattering of control signal occurs after the phase plane trajectory reaches the switching surface for all the different initial conditions investigated. The control signal chattering drawback can be improved or eliminated by simple modification of the control law. They will be discussed in further sections.

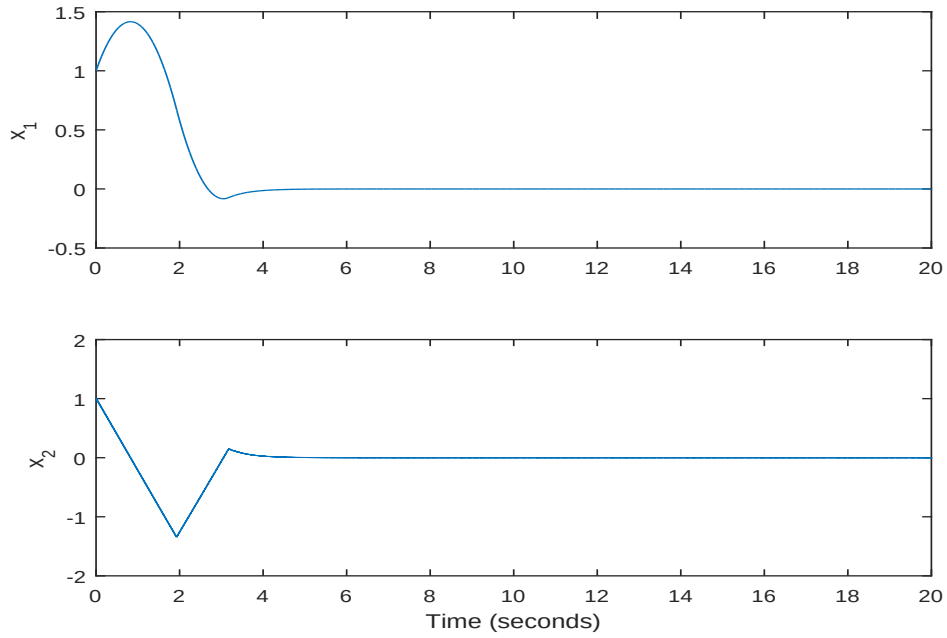

Figure 3.6: Time response of the states x_1 and x_2 for initial conditions $x_{10} = x_{20} = 1$

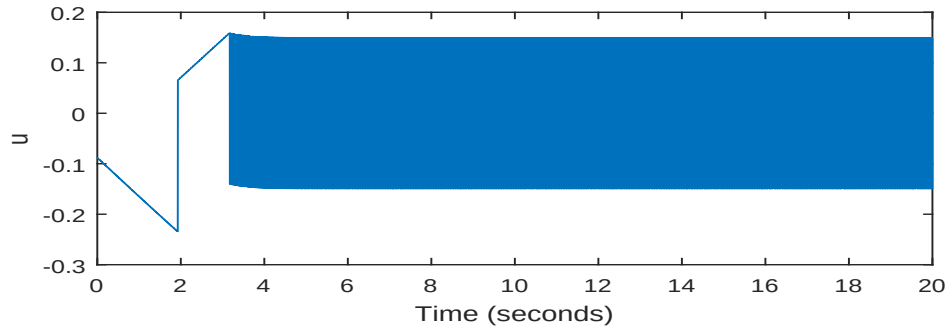

Figure 3.7: Time response of the control signal u for initial conditions $x_{10} = x_{20} = 1$

3.2.2. EXISTENCE PROBLEM

Consider the linear state system

$$\dot{x} = Ax(t) + Bu(t) \tag{3.17}$$

where $x \in \Re^n$ is state vector, $u(t) \in \Re^m$ is control vector. $A \in \Re^{n\times n}$ and $B \in \Re^{n\times m}$ are matrices of appropriate dimensions with B being full column rank. The control input is composed of two components a continuous and discontinuous component u_{eq} and u_n

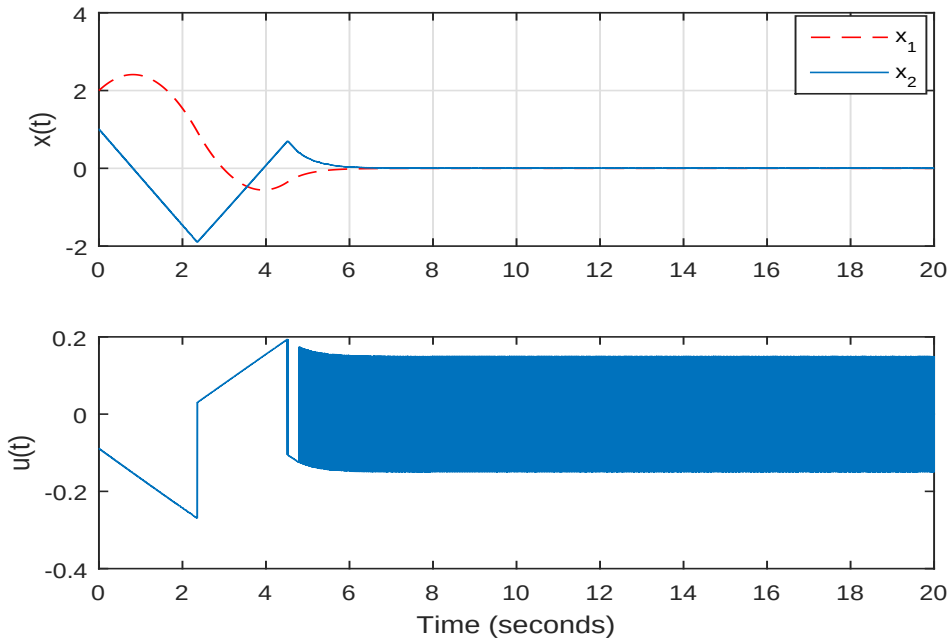

Figure 3.8: Time response of the states $x_1(t)$, $x_2(t)$ and control signal $u(t)$ for initial conditions $x_0 = \text{col}(2, 1)$.

respectively. The switching control u_n has the form

$$u_n = \begin{cases} u^+(t,x) & if\ s>0 \\ u^-(t,x) & if\ s<0 \end{cases} \tag{3.18}$$

where the linear switching function is defined as

$$s(x) = Sx \tag{3.19}$$

where $s(x) = 0$, is the switching surface, $S \in \Re^{m\times n}$ is of full rank. u_n is discontinuous and drives the system states to the sliding surface. The equivalent control term u_{eq} guarantees that the system stays on the sliding surface based on the condition $\dot{s} = 0$ and is only effective when the trajectory hits the sliding surface. The equivalent control term will be discussed later.

Sliding mode control involves mainly two design stages. The first phase concerns designing a discontinuous sliding surface such that when the trajectories of the system is confined to this surface, dynamic behaviour of the overall system will be stable and would exhibit some desired properties. The second phase is the reachability problem, this involves designing variable structure control laws such that the states are forced onto and

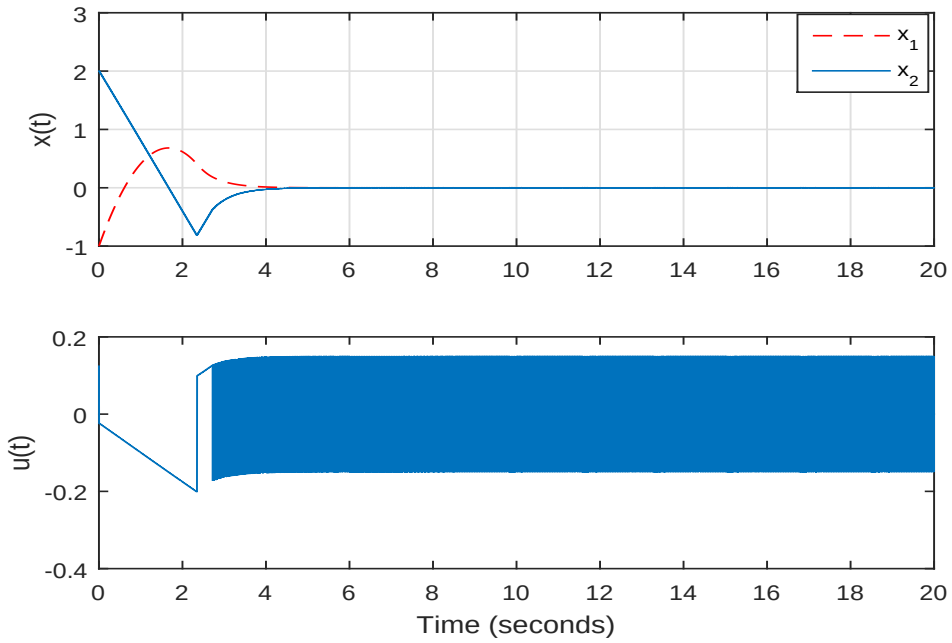

Figure 3.9: Time response of the states $x_1(t)$, $x_2(t)$ and control signal $u(t)$ for initial conditions $x_0 = \text{col}(-1, 2)$.

subsequently remains on the hyperplane $s(x)$ in finite time. In the first phase, conventional control theory methods such as stabilization and eigenvalue placement may be applied. Sliding mode existence principle requires the stability of the state trajectory on the sliding surface $s(x) = 0$ in the neighbourhood of $\{x|s(x) = 0\}$. Thus, the time derivative of the state vector, in the region of attraction which is the largest such domain as illustrated in Fig 3.10, must point towards the sliding surface $s(x) = 0$ [9, 13, 92]. Suppose u encounters discontinuities on the $s(x) = 0$ plane and suppose the trajectories of the state vectors are directed towards the plane. Sliding mode will therefore exist on the plane if the conditions (sufficient)

$$\lim_{s\to-0}\dot{s} > 0 \;\; and \;\; \lim_{s\to+0}\dot{s} < 0 \tag{3.20}$$

holds. The velocity of the state trajectory should be directed towards the sliding surface in its small vicinity. However, this type of generalised stability problem can often be analysed using the classical Lyapunov function $V(t, x)$ which is positive definite and in the region of attraction, has a negative derivative. Further literatures on existence problem can be obtained in [13, 77]. Details of Lyapunov methods and applications also exist in

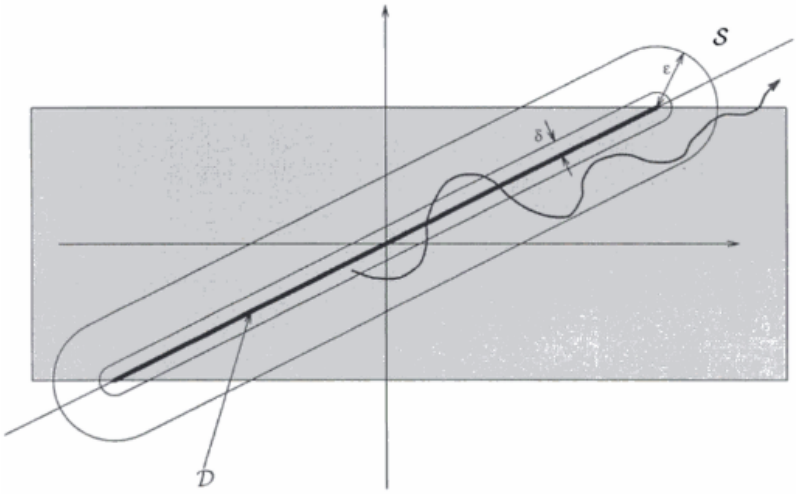

Figure 3.10: Domain of sliding mode

[36, 52]. As mentioned earlier in this section, there are two phases involved in the design of SMC. The existence problem which we have just explored is mainly associated with the design of the sliding surface. In the first phase of sliding mode design, one of two methods namely, 'equivalent control' method or 'regular form' may be used.

- Equivalent Control:

  The equivalent control technique proposed by Utkin [92], is more or less a conventional technique used in determining the systems motion as confined to the sliding surface $s(x) = 0$. In order to achieve this, assume that sliding occurs on the sliding surface $s(x) = 0$ at time $t = t_0$. Thus, based on the existence condition, it follows from (3.17) that

  $$S\dot{x} = SAx(t) + SBu_{eq} = 0 \tag{3.21}$$

  where u_{eq} is the equivalent control. Assuming SB is nonsingular, it follows that the equivalent control can be calculated as

  $$u_{eq} = -(SB)^{-1}SAx(t) \tag{3.22}$$

  Thus, by substituting u_{eq} back into (3.17), the dynamics of the system on the switching surface for $t \geq t_0$ is given by

  $$\dot{x} = I_n - B(SB)^{-1}SAx(t) \tag{3.23}$$

  which represents the ideal sliding motion, independent of the control action.

- Regular Form:

  In this work, converting the systems equations to regular form for the sliding mode design will be performed as a preliminary step procedures. Consider systems (3.17) with rank $(B) = m$. Matrix B is partitioned as

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \tag{3.24}$$

  where $B_1 \in \Re^{(n-m)\times m}$, $B_2 \in \Re^{m\times m}$ with det $B_2 \neq 0$. The non-singular co-ordinate transformation $z = Tx$, where

$$T = \begin{bmatrix} I_{n-m} & -B_1 B_2^{-1} \\ 0 & B_2^{-1} \end{bmatrix} \tag{3.25}$$

  transfer the system equation (3.17) to

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 \tag{3.26}$$

$$\dot{x}_2 = A_{21}x_1 + A_{22}x_2 + B_2 u \tag{3.27}$$

  where $x_1 \in \Re^{n-m}$ and $x_2 \in \Re^m$, and B_2 is non-singular.

  In order to achieve the reduced order sliding mode dynamics, the sliding surface (3.19) is partitioned such that

$$s(x) = \begin{bmatrix} S_1 & S_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \tag{3.28}$$

  Based on assumption, S_2 is non-singular since B_2 is non-singular, it follows that

$$x_2 = -S_2^{-1} S_1 x_1 \tag{3.29}$$

  substituting (3.29) back into (3.26), the reduced order dynamics is

$$\dot{x}_1 = (A_{11} - A_{12}S_2^{-1}S_1)x_1 \tag{3.30}$$

3.2.3. REACHABILITY PROBLEM

From the previous subsection (existence problem), it has been acknowledged that the sliding motion is independent of the control. Thus, the second phase involves discontinuous

control law design which will drive the system state to the designed sliding surface. The attractiveness of the sliding surface can be expressed by the condition

$$s\dot{s} < 0 \tag{3.31}$$

which is referred to as reachability condition. The conditions which ensures that the trajectories converge to the manifold $s(x) = 0$, and for existence of sliding mode in this manifold may be derived based on a Lyapunov function candidate

$$V = \frac{1}{2}s^T s > 0 \tag{3.32}$$

However, in order to achieve finite time convergence. The condition (3.31) is often replaced with the η reachability condition described as

$$s\dot{s} \leq -\eta\|s\| \tag{3.33}$$

which guarantees finite time convergence to $s = 0$. For purpose of illustration, we revisit the DC motor dynamics in (3.12)-(3.13) with sliding function

$$\begin{aligned} s(x) &= S_1 x_1 + x_2 \\ \dot{s} &= -a_1 x_1 - (S_1 + a_2)x_2 + bu + g(t) \end{aligned} \tag{3.34}$$

using the control law $u = u_0 \text{sgn}(s), u_0 > 0$ and based on the condition $s\dot{s} < 0$,

$$s\dot{s} < |s|(|-a_1 x_1 - (S_1 + a_2)x_2 + g(t)|) - b|s|u_0 \tag{3.35}$$

$$< |s|(|-a_1 x_1 - (S_1 + a_2)x_2 + g(t)| - bu_0) \tag{3.36}$$

Thus, the reachability is satisfied in the domain

$$\Omega = \left\{x : b^{-1}|-a_1 x_1 - (S_1 + a_2)x_2 + g(t)| < u_0\right\} \tag{3.37}$$

SMC is known for its strong robust advantages including fast and good transient response. It is very insensitive to matched uncertainties. Due to its computational efficiency, it can be applied to a wide range of possibly time-varying and non-linear plants. Despite these advantages, it is affected by chattering, a phenomenon caused by high frequency oscillation of the sliding variable around the sliding manifold [13, 113]. Chattering is a "zig-zag" motion around the sliding manifold caused by high frequency switching and it

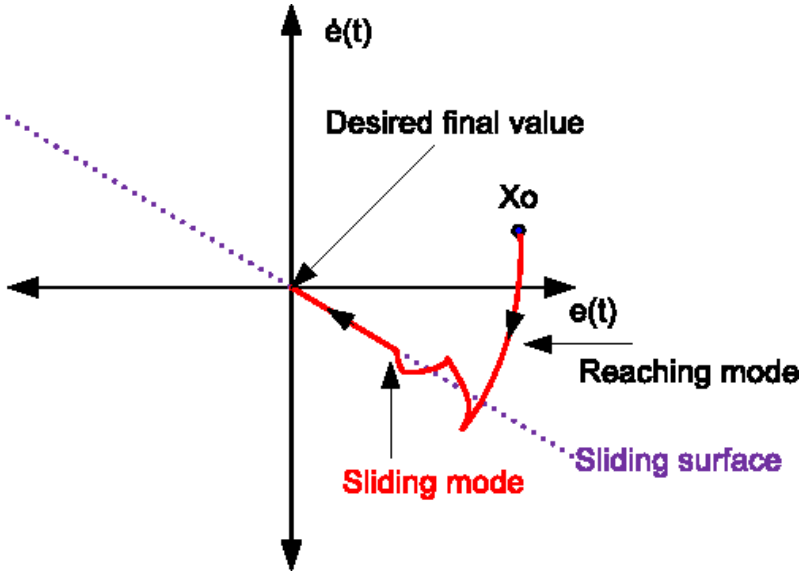

Figure 3.11: Chattering effect

constitutes a serious drawback which obviously hinders their practical applications. The effects of chattering could range from low control accuracy, failure of actuator, high heat losses in electrical power circuits, large control signal, high wear of electrical mechanical parts [93, 36], all of which can degrade the system performance and cause instability. In order to circumvent this effects, various techniques have been applied and are still been considered in literatures [44, 45, 13, 92]. Although, one of the most common approach to eliminate chattering is by using the function

$$\text{sat(s)} = \begin{cases} -1, & \text{for } s < -\rho. \\ \frac{1}{\rho}s, & \text{for } |s| \leq \rho \\ 1, & \text{for } s > \rho \end{cases} \tag{3.38}$$

known as the boundary layer controller [81], where ρ is a positive constant, used in place the $\text{sgn}(s)$ function in the discontinuous control term. The replacement of the $\text{sgn}(s)$ function makes the control term continuous and the switching variable does not converge to zero but to a the close interval $[-\rho, \rho]$ as illustrated in Fig 3.12. Alternatively, a non-linear approximation of the discontinuous control term may be applied which can be defined as

$$\frac{s}{\rho + |s|} \tag{3.39}$$

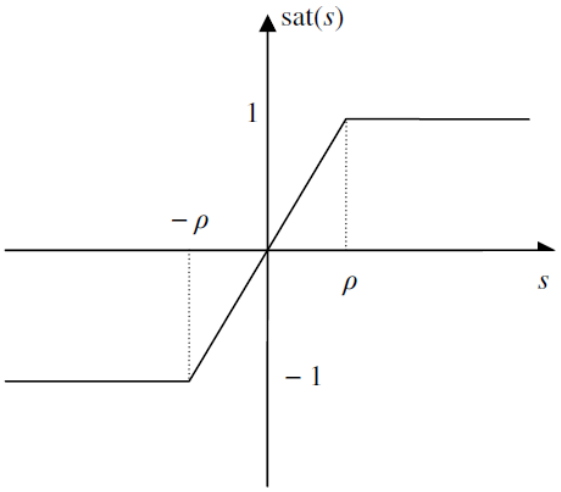

Figure 3.12: Boundary layer

3.3. OVERVIEW OF TIME DELAY SYSTEMS

Time delay often occurs in practical systems, and are mostly described using functional differential equations (FDE). Although, ordinary differential equations (ODE) in the form

$$\dot{x}(t) = f(t, x(t)) \qquad t_0 \leq t < \infty \tag{3.40}$$

have been generally used to model dynamical system. This describes the fact that the future evolution of the system is determined completely by the current value of the state variable $x(t)$. However, this assumption is not satisfied for all systems, since not all dynamical systems in practise can be modelled by an ODE. In some cases, the systems behaviour is required to not only include information about the current states $x(t)$, but also includes the previous states $x(\zeta), t_0 - d \leq \zeta \leq t_0$. This type of systems are referred to as time delay systems. This section will present a short overview of time delay systems, its basic definitions and methods of stability with respect to dependent and independent delays. For the purpose of illustration, examples will be given in later chapters to help clarify its applicability.

3.3.1. EXAMPLES OF TIME DELAY SYSTEMS

Example 3.3.1 Shower System

Consider the shower model in Fig 3.13 [122]: The actual temperature often overshoots

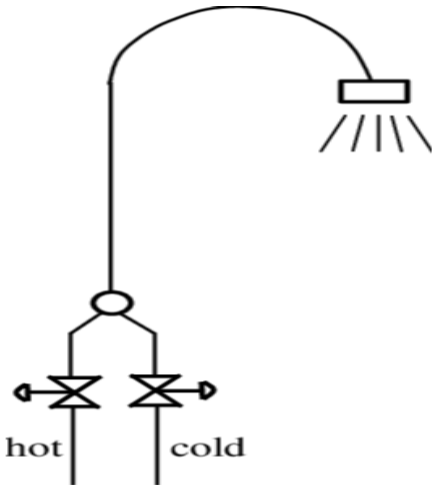

Figure 3.13: Diagram of a shower system

the desired temperature when the shower is turned on. Thus, the water is either too cold or too warm and may take a while to readjust to the actual temperature. Depending on the pressure and pipe length, there is a delay in flow of hot/cold water from the tap to the shower head. Changing the tap position does not immediately change the water temperature, as the water temperature has to wait for the delay to elapse. However, temperature may fluctuate due to constant varying of the tap in line with the perceived current temperature. Assume an incompressible and stationary flow of water.

Based on poiseuille law, water flow rate is

$$F = \frac{\pi R^4}{8\mu L}\Delta P \tag{3.41}$$

where the kinematic viscosity of water $\mu = 0.01$, L is the pipe length, R is the pipe radius and ΔP is the difference in pressure between the two pipe ends. The time delay h can be expressed as

$$h = \frac{\pi R^2 L}{F}\Delta P = \frac{8\mu}{\Delta P}(\frac{1}{R})^2 \tag{3.42}$$

Example 3.3.2 Continuous Stirred Tank Reactor (CSTR)

Consider the cascaded CSTR system shown in Fig 3.14 [27]. In order for recycling to commence, the yield and the input to be recycled must be separated prior to the separation operation, before finally travelling through pipes. This entire process introduces delay in the recycle system. The compositions C_A and C_B of the produce streams from reactor A and reactor B, represents the system states which are to be controlled. The output of one

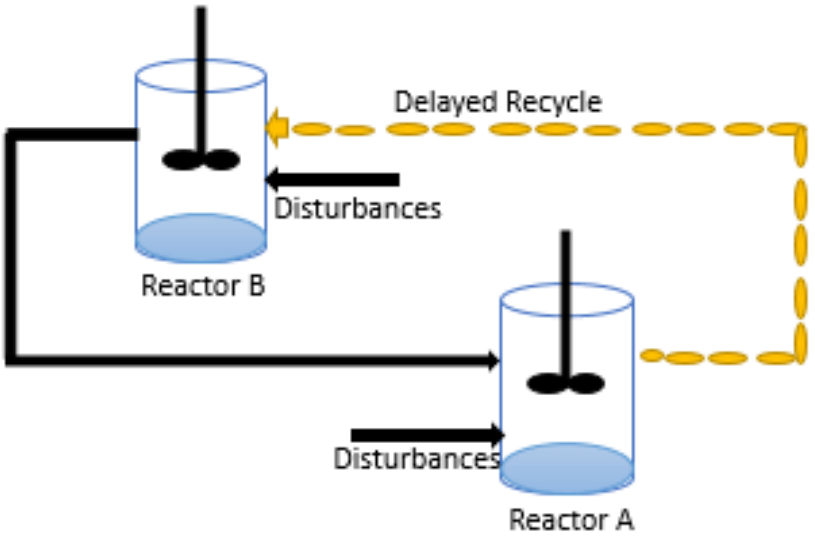

Figure 3.14: Cascade chemical reactor system.

reactor CSTR (Reactor A) determines the flow rate into the second reactor (Reactor B) and the external disturbances. A time delay is added between the output of one reactor and the input (flow rate) of the other reactor such that at a certain time, the state of one reactor is determined by the state of the other reactor at a previous time $t - d(t)$. The entire system as described in (3.43)

$$\begin{aligned} \dot{C}_A &= -k_A C_A - \frac{1}{\theta_A} H_A(C_A, C_A(t-d_A)) + \frac{1-R_B}{V_A} C_B + \delta_A(t, C_A(t-d_A)) \\ \dot{C}_B &= -k_B C_B - \frac{1}{\theta_B} H_B(C_B, C_B(t-d_B)) + \frac{R_A}{V_B} C_B(t-d_B) + \frac{F}{V_B} u(t) \\ &\quad +\delta_B(t, C_B(t-d_B)) \end{aligned} \tag{3.43}$$

where d_A, d_B are delays in the system, k_i are the reaction constants, θ_i are the reactor residence times, δ_i are non-linear functions for describing the external disturbances and system uncertainties, R_i are the recycle flow rates, V_i are reactor volumes, F is the feed rete and H_i are non-linear functions representing the systems complex behavior.

Example 3.3.3 Communication Networks

In recent years, Communication networks constitute one of the rapidly growing and developing areas in engineering and the need to control systems over the networks has grown increasingly [50, 74]. Thanks to high speed networks which has motivated the availability of control-over-internet. From the control point of view, these systems are frequently modeled as time delayed systems due to propagation delays which are inherent

[53]. These delays are crucial to the systems stability and the quality of service. A single connection between a source controlled by an access regulator and a distant destination node served with a constant transmission capacity μ is given as an example here. The fluid model shown in Fig. 3.15 is used to describe this idea [30, 53]. The access regulator controls the input rate $u(t)$, at the source node, according to the congestion status of the destination node. The congestion status $y(t)$ is defined as the difference between the

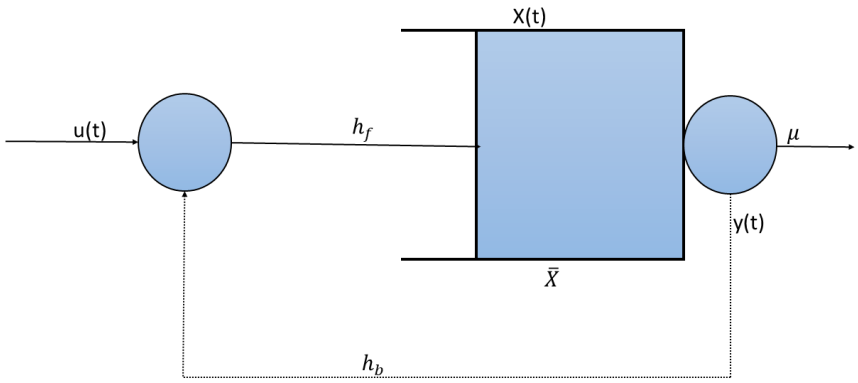

Figure 3.15: Fluid model

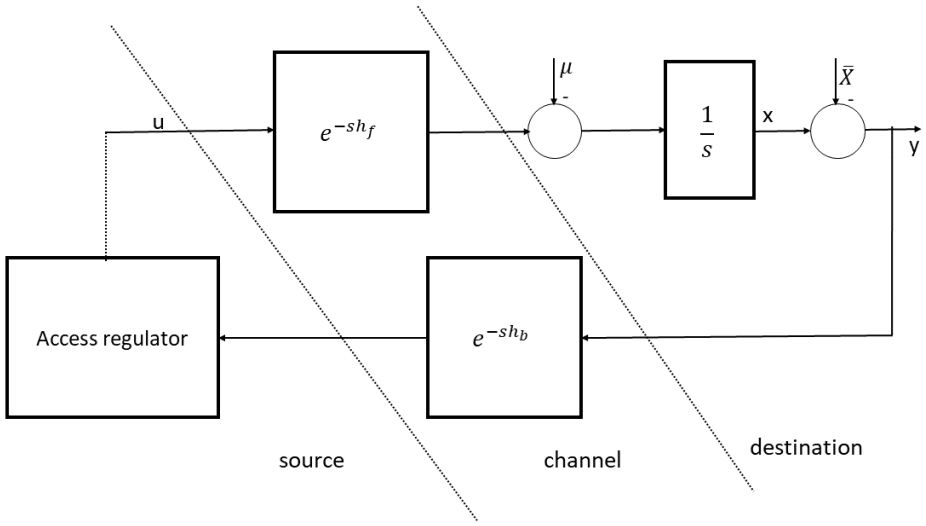

Figure 3.16: Communication networks: A single connection

current buffer contents $x(t)$ and the target value $\bar{X}$, i.e.,

$$y(t) = x(t) - \bar{X}$$

As a result of the propagation delay from the destination node to the source node which is referred to as backward delay h_b, this arrives at the source node (the access regulator) only after this delay period. Also, within the period for the package to arrive at the destination node from the source node, there is a forward delay h_f. The arrived packages are then stored in the buffer and sent with a constant transmission capacity μ. The control objective is to adapt $u(t)$ to μ dynamically while maintaining the buffer $x(t)$ at an acceptable level. The block diagram is shown in Fig 3.16. The communication networks in reality, which are built from single connections, are much more complicated. The delays are often time-varying and stochastic. The information transmitted via communication networks is quantised and there exist package losses as well.

3.3.2. BASIC DEFINITIONS

This section introduce some classes of time delay systems and basic definitions. The main focus is centered on linear time-invariant systems. An example of such system is the single delay system of the form

$$\dot{x} = Ax(t) + A_d x(t-d), \tag{3.44}$$

where A and A_d are constant $n \times n$ matrices, d is a non-negative delay. In order to define a particular solution for the system, it is important to first set initial conditions which includes a time instant t_0 and an initial vector function $\phi(\cdot)$

$$x(t_0 + \theta) = \phi(\theta), \quad \forall \theta \in [-d, 0]. \tag{3.45}$$

Here $\phi(\cdot)$ is an element of the banach space $\mathcal{C}([-d, 0], \Re^n)$ of continuous vector functions mapping the delay interval $[-d, 0]$ into $\Re^n$ with the standard uniform norm

$$\|\phi(\cdot)\| = \max_{\theta \in [-d,0]} \{\|(\theta)\|\} \tag{3.46}$$

For a given initial condition of the form (3.45), the system (3.44) admits the unique solution $x(t) = x(t, t_0, \phi(\cdot))$, which is defined on $[t_0 - d, \infty]$ [38].

An extension of the class of single delay systems is the class of systems with multiple delays of the form

$$\dot{x} = Ax(t) + \sum_{i=1}^{N} A_d x(t - d_i), \tag{3.47}$$

where all delays are non-negative real values. For such class of systems (3.47), in order to define a particular solution one has to specify an initial vector function $\phi(\cdot)$ on the segment $[-d, 0]$, where d= $\max_{1 \leq i \leq N}\{d_i\}$.

The distributed delay is a more general class of time delay of the form

$$\dot{x} = \int_{-d}^{0} [dF(\theta)]x(t - \theta) \tag{3.48}$$

All elements of the matrix valued function $F(\theta)$ are assumed to have bounded variations on $[-d, 0]$

$$\dot{x} = \int_{-d} \|dF(\theta)\| \leq \infty \tag{3.49}$$

The single and multiple delay cases in (3.44) and (3.47) respectively are special cases of the distributed system (3.48) and can be obtained from it with a special choice of piecewise constant system matrix $F(\cdot)$. For a particular solution of the system (3.48), the initial function $\phi(\cdot)$ must be specified on the whole integration segment $[-d, 0]$. In this thesis, the single and multiple delay cases of time delay system are main areas of focus.

3.3.3. ROBUST STABILITY OF TIME DELAY SYSTEM

Here we take a look at the two methods of stabilising continuous time delay systems. Our question bothers on the extent to which the time delay systems are vulnerable under the influence of unknown delay factors. There are two main places where uncertainty may affect a time delay system. First, as in the case of delay free systems, uncertainty may appear in the description of the system matrices, $A, A_1, ..., A_N$ for the systems (3.44) and (3.47). The second source of uncertainty which does not exist in the case of delay free systems is a description of delay elements, scalar d in (3.44) or delays $d_1, d_2, ..., d_N$ in (3.47). Much attention has been given to the robust stability of uncertain time delay systems. A more general approach to stability analysis of time delay systems is based on the Lyapunov's direct method. Two main techniques in time domain, Lyapunov Krasovskii

functional [15, 37, 43], and Lyapunov Razumikhin functions [35, 64, 120] have been considered in literature for developing sufficient conditions. Two such condition are delay dependent and delay independent conditions. While the former makes use of information on the length of the delay, the latter is independent of the length of the delay. Although our approach is mainly focused on using Lyapunov Razumikhin method for delay independent sufficient condition. We will briefly review both cases with simple examples for illustration purposes.

Consider the linear time delay system (3.44)

- Krasovskii methods of Lyapunov functionals

  Lyapunov Krasovskii method is known to extend the classical lyapunov stability analysis for ODE to an infinite dimensional problem. It makes use of functional to map the function $V(t, x_t)$ in the interval $[t - d, t]$ i.e x_t and thus, should measure the deviation of x_t from the trivial solution 0. The Lyapunov-Krasovskii functional is generally written as

  $$V(x_t) = x^T(t)Px(t) + \int_{t-h}^{t} x^T(s)Qx(t)ds \tag{3.50}$$

  where the Lyapunov matrices P and Q are to be determined. Let $V : \Re \times C[-d, 0] \rightarrow \Re$ be a continuous functional, and let $x_\tau(t, \phi)$ be the solution of (3.44) at time $\tau \geq t$ with the initial condition $x_t = \phi$. Thus $\dot{V}(x_t)$ along the trajectory of (3.44) can be defined as follows:

  $$\dot{V}(x_t) = \lim \sup_{\Delta t \rightarrow 0^+} \frac{1}{\Delta t}[V(t + \Delta t, x_{t+\Delta t}(t, \phi))] \tag{3.51}$$

  which yields the delay independent condition

  $$\begin{bmatrix} PA + A^T P + Q & PA_d \\ * & -Q \end{bmatrix} < 0 \tag{3.52}$$

  Based on this, a non-positive $\dot{V}(t, x_t)$ ensures that x_t does not grow with t, implying that the considered system is stable [16, 99]. The delay independent condition expressed in (3.52) is for constant delay case. It has the flexibility for achieving large constant delay and can be reduced to the feasibility of an LMI. However, for unknown time-varying delay factor h satisfying

  $$0 \leq d(t) \leq d_o, \qquad 0 \leq \dot{d}(t) \leq \bar{d} \leq 1 \tag{3.53}$$

where d_o, $\bar{d}$ are known bounds. It follows that system (3.44) is uniform asymptotically stable as long as $d(t)$ satisfies (3.53). Thus, implying a limit on the change rate of the time delay.

- Razumikhin methods of Lyapunov functions

  Compared with Lyapunov-Krasovskii functional involving mainly the use of functional which may seem difficult to manipulate. Lyapunov-Razumikhin theorem is based on classical Lyapunov functions and uses a special estimation procedure which allows exclusion of the delay states in the derivative of the Lyapunov functions. Apparently, the main concept of this approach focuses on the Lyapunov function , $P > 0$.

$$V(x) = x(t)^T P x(t)$$

  Thus, the fact that $V(x)$ represents the size of $x(t)$ suggests that

$$V(x_t) = x^T(t+\theta)Px(t+\theta) \qquad \theta \in [-d, 0] \tag{3.54}$$

  The time-derivative of V along the trajectories solutions of system (3.44) is given by

$$\dot{V}x(t) = \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix}^T \begin{bmatrix} PA + A^T P & PA_d \\ A_d^T P & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix} \tag{3.55}$$

  Applying the Lyapunov-Razumikhin Theorem in Lemma 2.2.5, $\dot{V}x(t)$ must be negative whenever $V(x(t+\theta)) < \gamma_4 V(x(t))$ for some $\gamma_4 > 1$ and for all $\theta \in [-d, 0]$. Since the latter inequality holds for all $\theta \in [-d, 0]$ then we have $V(x(t+d)) < \gamma_4 V(x(t))$ and by application of the S-procedure (see [23]) we get

$$\begin{bmatrix} PA + A^T P + \tau\gamma_4 P & PA_d \\ A_d^T P & -\tau P \end{bmatrix} < 0 \tag{3.56}$$

  with $\tau > 0$. Finally let $\gamma_4 = 1 + \delta$, for a small $\delta > 0$, we get the following result based on Lyapunov theorem, system (3.44) is asymptotically stable independent of delay if there exists $P = P^T > 0$ and a scalar $\tau > 0$ such that

$$\begin{bmatrix} PA + A^T P + \tau P & PA_d \\ A_d^T P & -\tau P \end{bmatrix} < 0 \tag{3.57}$$

Note that the feasibility of matrix inequality (3.57) implies the feasibility of matrix inequality (3.56). It is clear that the latter inequality provides a delay-independent stability test since the matrix inequality does not depend on the delay. Moreover, it can be noted that (3.57) is not a LMI due to bi-linear term τP but fall into the framework of generalized eigenvalues problem [12, 4, 23, 96].

It is important to note that the structure in (3.52) reflects the matrix inequality obtained from the application of the Lyapunov-Razumikhin Theorem, but is LMI in the case of Lyapunov Krasovskii. Moreover, (3.52) seems less conservative than (3.57) since matrix Q is free and independent of P in the Lyapunov-Krasovskii test while the matrix is τP is the Lyapunov-Razumikhin test and strongly correlated to P. As a conclusion the Lyapunov-Krasovskii based test includes the Lyapunov-Razumikhin test as a particular case $Q = \tau P$.

3.4. OVERVIEW OF INTERCONNECTED SYSTEMS

Many systems in the real world can be modelled as dynamical equations composed of interconnections existing between a collection of lower dimensional subsystems. This kind of systems are often referred to as large scale interconnected systems [114], and they exist widely in such diverse fields as economics, ecology, power systems and space flight. Interconnected systems as shown in Fig. 3.17, quite commonly do not stay coupled for all time due to structural perturbations which can affect the entire subsystems performance once one of the systems is affected. In order to avoid information transfer between different subsystems, decentralised control is applied. Decentralised control strategy is in such a way that the controller of each subsystem is allowed to use its own information. This strategy was used to resolve a problem of stabilising a linear dynamic system, and was first studied by Mcfadden in [56], after which it has been applied generally in many control literature. More recently, decentralised control schemes have been combined with modern control approaches such as adaptive [48], back-stepping [88, 123] etc. to large scale interconnected systems. Its reliability and robustness in accommodating inexact knowledge of interactions are additional features of decentralised scheme which must be

taken into account whenever a question of centralized and decentralised strategy appears in controlling of large scale systems.

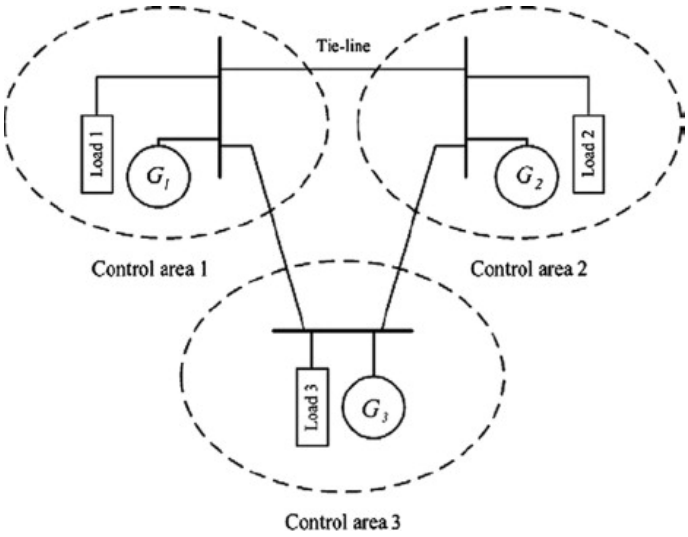

Figure 3.17: Three Area interconnected power system

3.4.1. FUNDAMENTAL CONCEPT

From the mathematical point of view, a large scale interconnected system composed of N n_i^{th} order subsystems can be described as

$$\dot{x}_i = A_i x_i + B_i(u_i + G_i(t, x_i)) + \Delta F_i(t, x_i) + \sum_{\substack{j=1 \\ j \neq i}}^{N} \zeta_{ij}(t, x_j) \tag{3.58}$$

$$y_i = C_i x_i, \qquad i = 1, 2, ..., N \tag{3.59}$$

where $x_i \in \Omega_i \subseteq \Re^{n_i}$ (Ω_i is a neighborhood of the origin) and $u_i \in \Re^{m_i}$, $y_i \in \Re^{p_i}$ are the state, inputs and outputs of the i^{th} subsystem respectively for $i = 1, 2, ...N$. All the constant matrices $A_i \in \Re^{n_i \times n_i}$, $B_i \in \Re^{n_i \times m_i}$ and $C_i \in \Re^{m_i \times n_i}$ are known. The terms $G_i(\cdot)$ and $F_i(\cdot)$ represents the matched and mismatched uncertainties respectively. The term $\sum_{\substack{j=1 \\ j \neq i}}^{N} \zeta_{ij}(t, x_j)$ represents the interconnection of the i^{th} subsystem. It is assumed that all the nonlinear functions are smooth enough such that the unforced systems have unique solutions.

The following basic definitions are introduced for the system (3.58).

Definition 3.4.1 Consider system (3.58). The system

$$\dot{x}_i = A_i x_i + B_i(u_i + G_i(t, x_i)) + \Delta F_i(t, x_i) \tag{3.60}$$

$$y_i = C_i x_i, \qquad i = 1, 2, ..., N \tag{3.61}$$

is called the i^{th} isolated subsystem (3.58), and the system

$$\dot{x}_i = A_i x_i + B_i u_i \tag{3.62}$$

$$y_i = C_i x_i, \qquad i = 1, 2, ..., N \tag{3.63}$$

is called the i^{th} nominal subsystem (3.58). Although, one problem associated with interconnected system is in establishing under what conditions the interconnected system (3.58) exhibits the desired performance if the isolated systems (3.60) and (3.62) are already stable. Thus, it all follows that the main concerns with decentralised control is the ability to manage the interconnections among the various subsystems.

Definition 3.4.2 Consider system (3.58). If there exist controllers for each subsystems depending on the state of the subsystems only, i.e.,

$$u_i = u_i(t, x_i), \qquad i = 1, 2, ..., N \tag{3.64}$$

such that by applying the control u_i in (3.64) to system (3.58), a closed-loop system is formed which has the desired performance. Thus, control (3.64) forms a decentralised state feedback control scheme [78, 114]. Alternatively, if the controllers in (3.64) takes the form

$$u_i = u_i(t, y_i), \qquad i = 1, 2, ..., N \tag{3.65}$$

implying that each local controller depends upon the output of the subsystem only, then they are referred to as decentralised output feedback controllers.

For the purpose of clarity, this thesis is focused on static state feedback controllers.

3.5. OVERVIEW OF POWER SYSTEMS

Power systems network is mainly a combination of generation, transmission, distribution and loads (see Fig. 3.18). Changes in load have major influences on bus voltages and

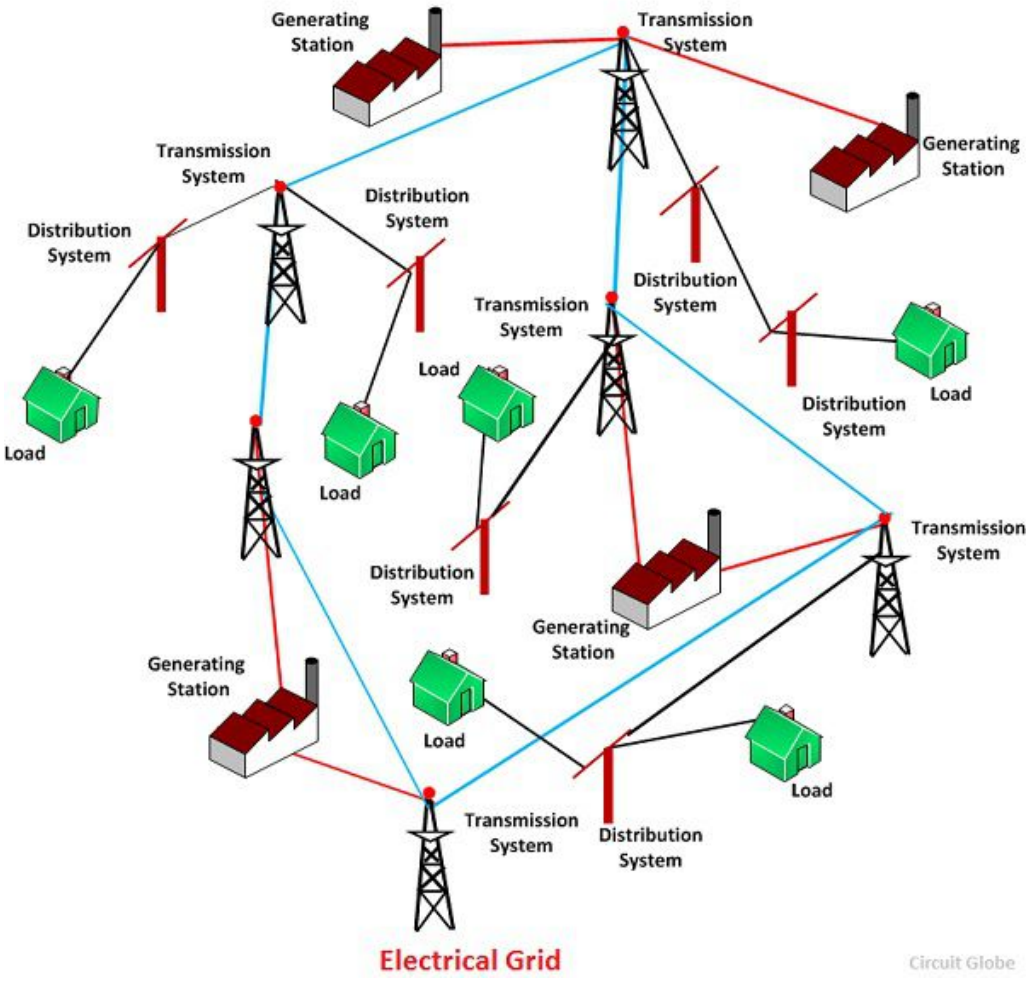

Figure 3.18: Power systems network

frequencies in the systems. Although slight changes in load would normally cause the frequency deviation to be decoupled from the voltage deviation, various circumstances such as voltage instability, rotor instability, and frequency instability can lead to power instability, thereby making power system stability and control an essential measure that must be applied, so that the system can attain a high level of efficiency and reliability. Power system stability is the "ability of an electric power system for a given initial condition, to regain a state of operating equilibrium after being subjected to physical disturbance, with most system variables bounded so that practically the entire system remains intact" [3, 42]. In power system plants, various levels of control involving complex array of devices may be applied such as prime mover controls and excitation controls. Whereas the former mainly deals with speed regulations, the latter is more focused on regulating

the governor voltage and reactive power output. Real and reactive power demand varies for different loads and frequency is affected by change in real power demand. Keeping this changes/deviations to voltage and frequency constant is the main challenges of power systems. Details of this and excitation controls can be obtained in various literatures [41, 49, 87, 103, 105]. As previously stated, power systems control problems are grouped into frequency control and voltage control and can be dealt with independently. For more specific illustration, this work is focused on load frequency stability and control applications. Since the system is affected by constant load change, there is a need for automatic generation control (AGC) at generating stations. LFC is part of AGC, its objective is to minimize these deviations in frequency and tie-line, thereby ensuring a zero steady-state error.

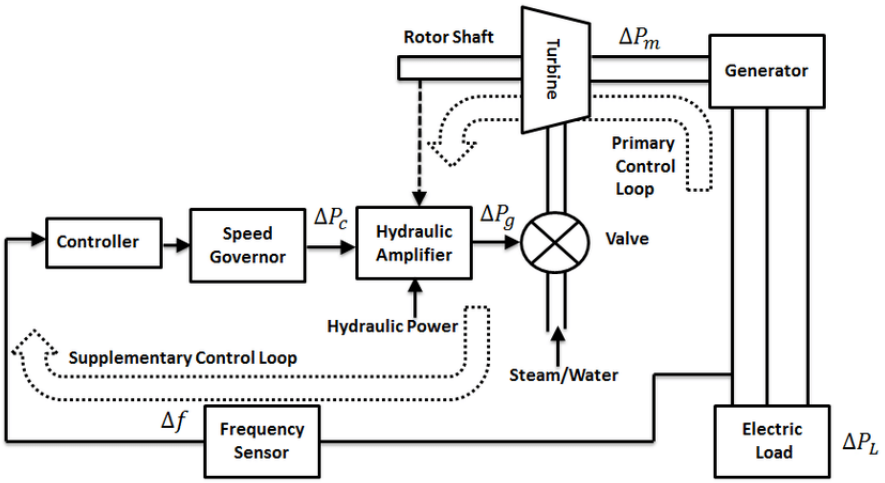

Figure 3.19: Block diagram of synchronous generator with frequency control

From Fig 3.19 above, primary control is undertaken by the speed governor. The speed governor provides primary speed control action by providing instant action to sudden change in load depending on the governor droop characteristics and frequency sensitivity of the load [41]. Although, this is made possible by causing the valve to open/close to increase/decrease water/steam input to the prime mover.

A well designed and operated power system must be robust to variations in load and system disturbances, and should provide a sustainable level of power quality while maintaining voltage and frequency within acceptable bounds. Due to continually changing load, a

secondary control which acts as supplementary control is used in adjusting the output of selected generators through a load reference setting. Thus, restoring system frequency to its normal value and maintaining interchange power between control areas.

3.5.1. SINGLE AREA LFC POWER SYSTEM

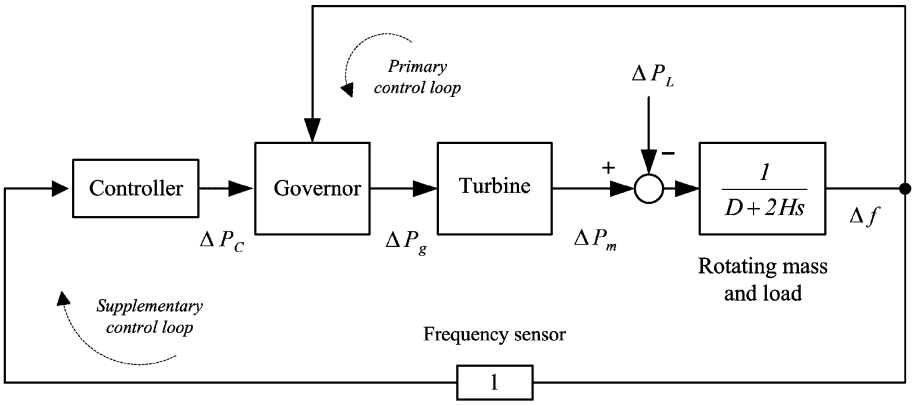

Figure 3.20: Single area LFC model

Most large synchronous generators are equipped with a supplementary frequency control loop as illustrated in Fig. 3.20. The single area LFC system consists mainly of a governor, turbine, generator (Rotating mass and load) with regulation feedback constant and a step load change in the generator input. It is important to note that a single control area may consist of various sources of power generation such as steam, nuclear, wind etc. Therefore, in order to deal with the constant load changes, adjustments are made to contributions of generations from various sources in the area. Hence, the LFC objectives at this stage are to keep the system frequency close to nominal value and to divide the load between generator units. Also, the supplementary control loop is administered to the controller which will usually be a simple integral or proportional-integral control. The resulting error signal is used to regulate the frequency of the system. LFC model uses simple first order transfer function for modelling the power system dynamics. However, our focus in this work is mainly based on applying non-linear control theory specifically variable structure control for control of the system frequency. This type of control may be more suitable compared with conventional linear control theory due to power system complexity such as its non-linear load characteristics and variable operating points.

3.5.2. MULTI-AREA LFC POWER SYSTEM

A multi-area power system is made up of areas, interconnected by high voltage transmission lines or tie-lines. In multi-area power system, LFC is usually more pronounced since primary control action is usually not sufficient to restore the system frequency. The LFC objectives in addition to the objectives stated in the single area are to control the tie-line interchange power. Each control area is dedicated to measuring its own frequency and tie-line power at the control centers.

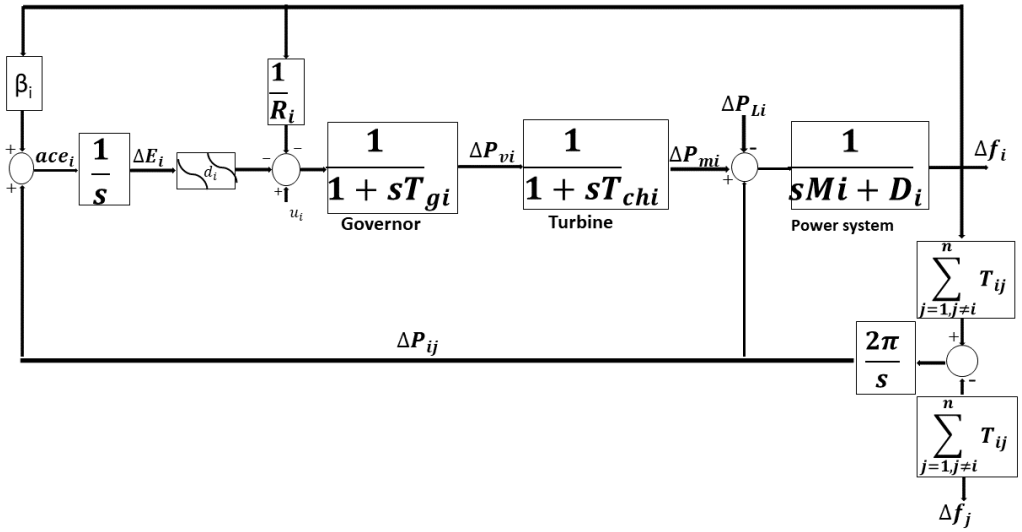

Figure 3.21: Multi area LFC model

In order to employ the LFC, the area control error (ACE) which is a linear combination of the tie-line and error frequency are calculated and administered to the dynamic controller Fig 3.21, whereas the resulted control signal is applied to the turbine-governor unit. However, two input signal effects such as load changes and multi-area interface are carefully considered at this stage. In a two area LFC, increased load disturbance ΔP_L in one area, will amount to frequency reduction in both areas and a tie-line flow of ΔP_{12}. Deviation in tie-line flow shows the contribution of regulation characteristic $\frac{1}{R} + D$ of one area to another. More so, supplementary control for each areas are often responsible for load changes in their local areas. This can be extended to much larger control area Fig 3.17. However, scheduled interchange transfer may seen slight different. Details of this can be

found in [41, 3].

CHAPTER 4

STABILISATION OF TIME DELAY SYSTEMS WITH NONLINEAR DISTURBANCES USING SLIDING MODE CONTROL

This chapter focuses on a class of control systems with delayed states and nonlinear disturbances using sliding mode techniques. Both matched and mismatched uncertainties are considered which are assumed to be bounded by known nonlinear functions. The bounds are used in the control design and analysis to reduce conservatism. A sliding function is designed and a set of sufficient conditions is derived to guarantee the asymptotic stability of the corresponding sliding motion by using the Lyapunov-Razumikhin approach which allows large time varying delay with fast changing rate. A delay dependent sliding mode control is synthesized to drive the system to the sliding surface in finite time and maintain a sliding motion thereafter. Effectiveness of the proposed method is demonstrated via two case studies on a automatic voltage regulator (AVR) system and a numerical example.

4.1. INTRODUCTION

Over the past decades, time delay systems as often referred to as after-effect systems have been an active area of research in a wide range of natural and social sciences. They belong to a class of functional differential equations, existing widely in the practical world, and are mostly encountered in numerous engineering systems such as electrical networks, chemical reactors, and hydraulic, pneumatic and manufacturing processes to mention but a few [23, 73, 121]. Time delay is usually a source of instability and performance degradation in control systems which needs to be considered seriously in design, and as such, has received considerable attention over the past years, see, e.g. [73, 111] and references therein.

Motivated by recent development in large scale system design, multiple or large time delay arise in systems due to vast network complexity. As a result, various techniques have been developed in trying to circumvent the effect of uncertain dynamical time delay systems [7, 73, 111]. LMI techniques are applied in [72, 97], but requires that the considered systems are linear and the non-linear uncertainty satisfy linear growth condition. Lyapunov-Razumikhin technique is applied in this chapter which focuses on developing a robust system with large time delay and fast change rate, which has improved the requirement of the bounds on the delays compared with recent results [20, 69, 32]. In addition, the design parameters can be obtained via LMI techniques systematically. It should be noted that one method which has proved very effective in dealing with uncertainties in the system is the sliding mode control due to its strong robustness properties against parametric uncertainties and external disturbances in the input channel, as well as its attractive features such as fast and good transient response [13].

Due to its high robustness, sliding mode control has been extended to time delay systems with disturbances, and most of the existing results are in combination with other techniques such as LMI [100], adaptive control [2], decentralised control [114], where the common goal is to present less conservative conditions to guarantee high performance of systems considered. It should be noted that sliding mode control techniques can also be used to deal with mismatched uncertainties [21, 113]. The problem of sliding mode

control for uncertain time delay systems has been a continuous area of interest and development. Recent work carried out in this area involved different technique when compared with traditional sliding mode control, e.g. integral sliding surface where the reaching phase is eliminated [5, 95], are different from the usual or conventional sliding surface which has a reaching phase in [13] or that it only considered matched uncertainty [62], while in [26, 104] it is required that the bounds on the uncertainties satisfy the linear growth condition. Moreover, two main techniques based on Lyapunov-Krasovskii functional and Lyapunov-Razumikhin function have been largely used to deal with time delay. It should be noted that sliding mode control for time delay with non-linear disturbances has been studied in [108] where static output feedback was considered, which has strong limitation on the system including the bounds on the uncertainties. [5] proposed the robust sliding mode control of non-linear uncertain systems by analyzing the lump estimated disturbances via a disturbance observer. Although static output feedback or observer based output feedback controllers have certain advantages in real implementation, strong limitations is unavoidably required. In reality, sometimes, all the system states for example, the position and speed of a mechanical system, may be available and thus state feedback will be possible, which will largely reduce the limitation of the considered system. Moreover, the bounds on uncertainty can be much relaxed when compared with output feedback design scheme. Optimal guaranteed cost sliding mode control of interval type-2 fuzzy time-delay systems was proposed in [46] where Lyapunov Krasovskii technique was used to analyze the delay bound. However, due to evolving complex systems, developing large time delay bound may be necessary. It should be noted that the Lypunov-Krasovskii method usually requires that the time varying delay is differentiable and there is limitations on the rate of change of time delay. Hence, Lyapunov Razumikhin method which is used for large constant and time varying delay is used to circumvent this effect.

This chapter proposes a sliding mode control scheme for a class of time delay control systems with non-linear delayed disturbances. The assumptions for non-linear terms are imposed on the transformed systems to avoid unnecessary conservatism caused by coordinate transformation in theoretical analysis. Lyapunov- Razumikhin approach is used to derive a set of conditions to guarantee that the derived sliding motion is asymptotically stable in the presence of time delay. Then under assumption that all the system states are

accessible, sliding mode control is synthesized such that the controlled system is driven to the sliding surface in finite time and maintains sliding motion thereafter. Case study of a automatic voltage regulator (AVR) system is provided to show the feasibility of the developed results and the effectiveness of the proposed method. The main contribution of this chapter is summarized as follows:

i The known bounds on the uncertainties are fully applied in the controller design to reject the effects of the uncertainties on system performance.

ii Compared with associated existing work, the proposed approach not only allows the bounds on the uncertainties have more general non-linear form but all the design parameters relating to the sliding motion can be obtained using LMI techniques.

iii Both matched and mismatched uncertainties are considered and the bounds on matched and mismatched uncertainties involve time varying delay.

iv There is no limitation to the change rate of the time-varying delay which allows fast changing rate of the time varying delay.

4.2. PRELIMINARIES

First, recall some basic linear system theory. Consider a linear system

$$\dot{x} = Ax + Bu \tag{4.1}$$

where $x \in \Re^n, u \in \Re^m$ are states and inputs, respectively, with $m < n$. The matrix pair (A, B) is of appropriate dimensions whereas B is of full rank. Assume that the matrix pair (A, B) is controllable. Then from basic matrix theory, it can be shown that a coordinate transformation $z = Tx$ exists such that the matrix pair (A, B) in the new coordinates z has the following structure [13, 113]:

$$\tilde{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}, \tag{4.2}$$

where $A_{11} \in \Re^{(n-m)\times(n-m)}$, and $B_2 \in \Re^{m\times m}$ is non-singular. It should be noted that such a transformation can be obtained systematically using matrix theory. Further, from [13],

the fact that (A, B) is controllable implies that (A_{11}, A_{12}) is controllable, and thus there exists a matrix $M \in \Re^{m\times(n-m)}$ such that the matrix $A_{11} - A_{12}M$ is Hurwitz stable.

Consider a time-delay system

$$\dot{x}(t) = f(t, x(t - d(t))) \tag{4.3}$$

with an initial condition

$$x(t) = \phi(t), \qquad t \in [-\bar{d}, 0]$$

where $f : \Re^+ \times \mathcal{C}_{[-\bar{d},0]} \mapsto \Re^n$ takes $\Re\times$(bounded sets of $\mathcal{C}_{[-\bar{d},0]}$) into bounded sets in $\Re^n$; $d(t)$ is the time-varying delay and $\bar{d} := \sup_{t\in\Re^+}\{d(t)\} < \infty$.

4.3. PROBLEM FORMULATION

Consider a time varying delay system with delayed disturbance described by

$$\dot{x} \;=\; Ax + A_d x_d + B(u + \bar{G}(t, x, x_d)) + \bar{F}(t, x, x_d) \tag{4.4}$$

where $x \in \Omega \subset \Re^n$ (Ω is an neighborhood of the origin), $u \in \Re^m$ are the states and inputs respectively; A, $A_d \in \Re^{n\times n}$ and $B \in \Re^{n\times m}$ ($m < n$) are constant matrices with B being of full rank; The vectors $\bar{G}(\cdot)$ and $\bar{F}(\cdot)$ represent the matched and mismatched disturbances affecting the system respectively. The symbol $x_d := x(t - d)$ represents the delayed state where $d := d(t)$ is the time varying delay which is assumed to be known, continuous, non-negative and bounded in $\Re^+ := \{t \mid t \geq 0\}$, that is

$$\bar{d} := \sup_{t\in\Re^+}\{d(t)\} < \infty$$

The initial condition related to the time delay is given by

$$x(t) = \Phi(t), \qquad t \in [-\bar{d}, 0] \tag{4.5}$$

where $\Phi(\cdot)$ is continuous in $[-\bar{d}, 0]$. It is assumed that all the non-linear functions are smooth enough for the subsequent analysis, which guarantees that the unforced system has unique continuous solutions.

Remark 4.3.1 In general, (4.4) is a multi input multi output system (MIMO) system with non-linear delayed disturbance. However, in this thesis, single input single output (SISO) case is considered.

In this chapter, the objective is to design a sliding mode control for the system (4.4) under the assumption that all the system states and time delay are available for the design, such that the corresponding closed loop system is asymptotically stable in the presence of time delay and uncertainties, with focus on disturbance tolerability but of convenient parameter design methodology.

4.4. SYSTEM ANALYSIS AND BASIC ASSUMPTIONS

In this section, a sliding surface will be designed for system (4.4) and the stability of corresponding sliding motion will be analyzed. First, it is necessary to impose the following fundamental assumptions on the system (4.4).

Assumption 4.4.1 The matrix pair (A, B) is controllable.

From Section 4.2, it follows that under Assumption 4.4.1, there exists new coordinates

$$z = Tx \tag{4.6}$$

such that in the new coordinates z, the system (4.4) can be described by

$$\begin{aligned} \dot{z}_1 &= A_{11}z_1 + A_{d11}z_1(t-d) + A_{12}z_2 + A_{d12}z_2(t-d) \\ &\quad + F_1(t, z, z_d) \qquad (4.7) \\ \dot{z}_2 &= A_{21}z_1 + A_{d21}z_1(t-d) + A_{22}z_2 + A_{d22}z_2(t-d) \\ &\quad + B_2u(t) + B_2G(t, z, z_d) + F_2(t, z, z_d) \qquad (4.8) \end{aligned}$$

where $z(t) = col(z_1, z_2)$ with $z_1 \in \Re^{n-m}$ and $z_2 \in \Re^m$, $z_d = col(z_1(t-d),\, z_2(t-d))$

with $z_1(t-d) \in \Re^{n-m}$ and $z_2(t-d) \in \Re^m$, and

$$TAT^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{4.9}$$

$$TA_dT^{-1} = \begin{bmatrix} A_{d11} & A_{d12} \\ A_{d21} & A_{d22} \end{bmatrix}, \quad TB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \tag{4.10}$$

where $A_{11}, A_{d11} \in \Re^{(n-m)\times(n-m)}$, $A_{12}, A_{d12} \in \Re^{(n-m)\times m}$, $A_{21}, A_{d21} \in \Re^{m\times(n-m)}$, $A_{22}, A_{d22} \in \Re^{m\times m}$, $B_2 \in \Re^{m\times m}$ is nonsingular, and

$$G(t, z(t), z_d(t)) = \bar{G}(t, x, x_d)|_{x=T^{-1}z} \tag{4.11}$$

$$\begin{bmatrix} F_1(t, z(t), z_d) \\ F_2(t, z(t), z_d) \end{bmatrix} := T\bar{F}(t, x(t), x_d)|_{x=T^{-1}z} \tag{4.12}$$

where $F_1(\cdot) \in \Re^{n-m}$ and $F_2(\cdot) \in \Re^m$. Specifically, the matrix pair (A_{11}, A_{12}) is controllable and thus there exists matrix $M \in \Re^{m\times(n-m)}$ such that

$$A_{11} - A_{12}M$$

is asymptotically stable. Therefore, there exists $P > 0$ such that

$$(A_{11} - A_{12}M)^T P + P(A_{11} - A_{12}M) < 0 \tag{4.13}$$

Assumption 4.4.2 The uncertain terms $G(\cdot)$, $F_1(\cdot)$ and $F_2(\cdot)$ in (4.8) satisfy:

$$\|G(t, z(t), z_d)\| \leq \phi(t, z(t), z_d) \tag{4.14}$$

$$\|F_1(t, z(t), z_d)\| \leq \rho_1(t, z(t), z_d) \tag{4.15}$$

$$\|F_2(t, z(t), z_d)\| \leq \rho_2(t, z(t), z_d) \tag{4.16}$$

where $\phi(\cdot)$, $\rho_1(\cdot)$ and $\rho_2(\cdot)$ are known nonnegative continuous functions.

Remark 4.4.1 Assumption 4.4.2 holds if the uncertainties $\bar{G}(\cdot)$ and $\bar{F}(\cdot)$ in (4.4) are bounded by known non-negative continuous functions. In this chapter, the bounds on $G(\cdot)$ and $F_2(\cdot)$ are assumed to be known which will be used in control design in order to enhance the robustness against the corresponding uncertainties.

4.5. SLIDING MODE ANALYSIS AND CONTROL DESIGN

This section describe the existence of sliding mode and its reachability such that the control law is determined, which drives the system from an initial state to the equilibrium point in finite time. Based on the assumptions in Section 4.4, the main aim of this section is to achieve robust stability of system (4.7)-(4.8) in the presence of disturbances and delay using sliding mode control which guarantees a sliding motion occurs in finite time. From Section 4.2, it follows that under Assumption 4.4.1, the sliding function is defined as

$$\sigma(z) = Mz_1(t) + z_2(t) \tag{4.17}$$

where $M \in \Re^{m\times(n-m)}$ is a designed matrix which satisfies (4.13). When the system is limited to the sliding surface

$$\sigma(z) = 0, \tag{4.18}$$

it follows that $z_2 = -Mz_1$.

From the structure of system (4.7)-(4.8), the sliding motion of system (4.4) associated with the sliding surface (4.18) is dominated by system (4.7). When dynamic (4.7) is limited to the sliding surface (4.18), it can be described by

$$\dot{z}_1 = (A_{11} - A_{12}M)z_1 + (A_{d11} - A_{d12}M)z_1(t-d) + F_{1\delta}(t, z_1(t), z_{1d}) \tag{4.19}$$

where

$$F_{1\delta}(t, z_1, z_{1d}) = F_1(t, z, z_d)|_{z_2=-Mz_1} \tag{4.20}$$

with $z = col(z_1, z_2)$ and $F_1(\cdot)$ defined in (4.7).

Remark 4.5.1 System (4.19) is the sliding mode of system (4.7)-(4.8) corresponding to the sliding surface (4.18). It should be noted that the mismatched uncertainty $F_{1\delta}(\cdot)$ is the uncertainty $F_1(\cdot)$ when it is limited to the sliding surface (4.18).

From equation (4.19) it is clear to see that the mismatched uncertainty $F_{1\delta}(\cdot)$ can affect the sliding mode dynamics and as such it is necessary to impose some constraint on it in

order to guarantee asymptotic stability of the sliding motion.

Assumption 4.5.1 There exist known function $\mu(\cdot)$ such that the uncertainty $F_{1\delta}(\cdot)$ in (4.19) satisfies

$$F_{1\delta}(t, z, z_d) \leq \mu(t, z)\|z_d\| \tag{4.21}$$

The following results is ready to be presented.

4.5.1. STABILITY ANALYSIS OF SLIDING MOTION

Theorem 4.5.1 Under Assumptions 4.4.2 and 4.5.1, the sliding motion of system (4.7)-(4.8) associated with the sliding surface (4.18), governed by system (4.19) is uniformly asymptotically stable if there exist a scalar $\epsilon > 0$ and a real positive definite matrix P such that the inequality

$$\begin{aligned} &\lambda_{min}(A_o^T P + P A_o) - k + \lambda_{max}(P A_1 P^{-1} A_1^T P) \\ &+2\sqrt{k}\mu(\cdot)\lambda_{max}(P) > 0 \end{aligned} \tag{4.22}$$

holds, where $k = \frac{(1+\epsilon)\lambda_{max}(P)}{\lambda_{min}(P)}$, $\epsilon > 0$ and $\mu(\cdot)$ is a known non-negative function.

Proof: For sliding mode (4.19), consider the candidate Lyapunov function

$$V_1 = z_1^T(t) P z_1(t) \tag{4.23}$$

Then the time derivative of V_1 along the trajectory of the system (4.19), is given by

$$\begin{aligned} \dot{V}_1|_{(4.19)} &= z_1^T(t)(A_o^T P + P A_o) z_1(t) + 2 z_1^T P A_1 z_1(t-d) \\ &\quad +2 z_1^T(t) P F_{1\delta}(t, z_1(t), z_1(t-d)) \end{aligned} \tag{4.24}$$

where $A_o = A_{11} - A_{12}M$ and $F_{1\delta}$ is defined in (4.20).

From Lemma 2.3.1, it follows that

$$2 z_1^T(t) P A_1 z_1(t-d) \leq z_1^T(t-d) P z_1(t-d) + z_1^T(t) P A_1 P^{-1} A_1^T P z_1(t) \tag{4.25}$$

From (4.24) and (4.25) it is observed that the derivative V_1 along the trajectory of system (4.19) can be described by

$$\begin{aligned}\dot{V}_1|_{(4.19)} &= z_1^T(t)[A_o^T P + PA_o]z_1(t) + z_1^T(t-d)Pz_1(t-d) \\ &\quad + z_1^T(t)PA_1P^{-1}A_1^T Pz_1(t) + 2z_1^T PF_{1\delta}(t, z_1(t), z_1(t-d)) \quad (4.26)\end{aligned}$$

Applying the Razumikhin condition (see Lemma 2.2.5), for some positive constant $q = (1+\epsilon)$ with $\epsilon > 0$, the following inequality holds:

$$z_1^T(t-d)Pz_1(t-d) \leq (1+\epsilon)z_1^T(t)Pz_1(t) \quad (4.27)$$

From (4.27), it follows that

$$\lambda_{min}(P)\|z_1(t-d)\|^2 \leq z_1^T(t-d)Pz_1(t-d) \leq (1+\epsilon)\lambda_{max}(P)\|z_1\|^2 \quad (4.28)$$

Thus

$$\|z_1(t-d)\|^2 \leq k\|z_1\|^2 \quad (4.29)$$

where k is defined in (4.22). From (4.21), (4.26) and (4.29),

$$\begin{aligned}\dot{V}_1|_{(4.19)} &\leq -\lambda_{min}(Q)\|z_1\|^2 + k\|z_1\|^2 + \lambda_{max}(PA_1P^{-1}A_1^TP)\|z_1\|^2 \\ &\quad + 2\sqrt{k}\mu\lambda_{max}(P)\|z_1\|^2 \\ &= -\Big(\lambda_{min}(Q) - k - \lambda_{max}(PA_1P^{-1}A_1^TP) - 2\sqrt{k}\mu(\cdot)\lambda_{max}(P)\Big)\|z_1\|^2\end{aligned} \quad (4.30)$$

From (4.22), it follows that $\dot{V}$ is negative definite. Hence the result follows.

Remark 4.5.2 Theorem 4.5.1 gives a set of sufficient conditions which guarantee the asymptotic stability of the designed sliding motion. However, the left hand side of the inequality (4.22) is a function due to $\mu(\cdot)$, and thus it is difficult to obtain the design parameters to complete sliding mode design. In order to make the parameters more accessible, the LMI technique is used based on the following assumption.

Assumption 4.5.2 There exist known constants ϖ_1 and ϖ_2 such that the uncertainty $F_1(\cdot)$ in (4.7) satisfies

$$F_1^T(t, z(t), z_d)F_1(t, z(t), z_d) \leq \varpi_1^2 z^T(t)z(t) + \varpi_2^2 z_d^T z_d \quad (4.31)$$

Remark 4.5.3 Assumptions 4.4.2 and 4.5.1 are limitations to the non-linear uncertainties. The bounds on $G(\cdot)$ and $F_2(\cdot)$ in (4.14) and (4.16) have general non-linear form and involve time delay. In order to use LMI techniques to obtain the design parameters for sliding surface design, Assumption 4.5.1 is imposed on the mismatched uncertainty $F_1(\cdot)$ to facilitate the sliding motion analysis. However, it is not required that the bounds on uncertainty $\bar{F}$ in (4.4) satisfies linear growth condition. This is in comparison with many existing work [20, 26, 69, 104] where sliding mode techniques are employed and [72, 97] where LMI is used.

From (4.31) and (4.20), it follows that

$$\begin{aligned}
F_{1\delta}^T(\cdot)F_{1\delta}(\cdot) \leq\ & \varpi_1^2 \left[\begin{array}{cc} z_1^T & -(Mz_1)^T \end{array} \right] \left[\begin{array}{c} z_1 \\ -Mz_1 \end{array} \right] \\
& +\varpi_2^2 \left(\left[\begin{array}{cc} z_1^T(t-d) & -(Mz_1(t-d))^T \end{array} \right] \left[\begin{array}{c} z_1(t-d) \\ -Mz_1(t-d) \end{array} \right] \right) \\
=\ & \varpi_1^2[z_1^T z_1 + z_1^T(M^TM)z_1] + \varpi_2^2[z_1^T(t-d)z_1(t-d) \\
& +z_1(t-d)(M^TM)z_1(t-d)] \\
\leq\ & \varpi_1^2(1+\lambda_{max}(M^TM))z_1^T z_1 + \varpi_2^2(1+\lambda_{max}(M^TM))z_1^T(t-d)z_1(t-d) \\
\leq\ & \psi_1 z_1^T z_1 + \psi_2 z_1^T(t-d)z_1(t-d) \qquad (4.32)
\end{aligned}$$

where

$$\psi_1 = \varpi_1^2[1+\lambda_{max}(M^TM)] \qquad (4.33)$$

$$\psi_2 = \varpi_2^2[1+\lambda_{max}(M^TM)] \qquad (4.34)$$

where ϖ_1 and ϖ_2 are constants satisfying (4.31).

Theorem 4.5.2 Under Assumptions 4.4.1 and 4.5.1, the sliding motion of system (4.7)-(4.8) associated with the sliding surface (4.18), governed by system (4.19) is uniformly asymptotically stable if there exist scalars $\alpha > 0$, $\epsilon > 0$ and a real positive definite matrix P such that the following LMI holds

$$\left[\begin{array}{cc} W_o & P \\ P & -\alpha I \end{array} \right] < 0 \qquad (4.35)$$

where

$$\begin{aligned} W_o &= A_o^T P + P A_o + (1+\epsilon)P + P A_1 P^{-1} A_1^T P + \beta\alpha I \\ A_o &= A_{11} - A_{12} M \\ A_1 &= A_{d11} - A_{d12} M \\ \beta &= \psi_1 + \psi_2 k \end{aligned}$$

where k, ψ_1 and ψ_2 are defined in (4.22), (4.33) and (4.34) respectively, and M is defined in (4.17).

Proof: Using Lyapunov function (4.23), the time derivative of V_1 along the trajectory of the system (4.19), is given by (4.24).

Then from (4.32) and (4.29), it follows that

$$\begin{aligned} & F_{1\delta}^T(\cdot) F_{1\delta}(\cdot) \\ &\leq \psi_1 z_1^T(t) z_1(t) + \psi_2 \|z_1(t-d)\|^2 \\ &\leq \psi_1 z_1^T(t) z_1(t) + \psi_2 k z_1^T(t) z_1(t) \\ &= \Big(\psi_1 + \psi_2 k\Big) z_1^T(t) z_1(t) \end{aligned} \tag{4.36}$$

where ψ_1 and ψ_2 are defined in (4.33) and (4.34) respectively.
Using (4.36), by similar analysis as in Theorem 4.5.1, the proof of the time derivative of $V_1(\cdot)$ is given by

$$\begin{aligned} \dot{V}_1|_{(4.19)} &= z_1^T(t)\Big[A_o^T P + P A_o + (1+\epsilon)P + P A_1 P^{-1} A_1^T P\Big] z_1(t) \\ &\quad + 2 z_1^T P F_{1\delta}[t, z_1(t), z_1(t-d)] \\ &= \begin{bmatrix} z_1(t) \\ F_{1\delta} \end{bmatrix}^T \begin{bmatrix} W & P \\ P & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ F_{1\delta} \end{bmatrix} \end{aligned} \tag{4.37}$$

where

$$W = A_o^T P + P A_o + (1+\epsilon)P + P A_1 P^{-1} A_1^T P, \quad \epsilon > 0 \tag{4.38}$$

The inequality (4.36) can be rewritten as

$$\begin{bmatrix} z_1(t) \\ F_{1\delta} \end{bmatrix}^T \begin{bmatrix} \beta I & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} z_1 \\ F_{1\delta} \end{bmatrix} \geq 0 \tag{4.39}$$

where $\beta = \psi_1 + \psi_2 k$.

It can be seen from (4.37) and (4.39) that,

$$\begin{aligned}
\dot{V}_1|_{(4.19)} &\leq \begin{bmatrix} z_1(t) \\ F_{1\delta} \end{bmatrix}^T \left(\begin{bmatrix} W & P \\ P & 0 \end{bmatrix} + \alpha \begin{bmatrix} \beta I & 0 \\ 0 & -I \end{bmatrix} \right) \begin{bmatrix} z_1(t) \\ F_{1\delta} \end{bmatrix} \\
&= \begin{bmatrix} z_1(t) \\ F_{1\delta} \end{bmatrix}^T \begin{bmatrix} W + \beta\alpha I & P \\ P & -\alpha I \end{bmatrix} \begin{bmatrix} z_1 \\ F_{1\delta} \end{bmatrix} \\
&= \begin{bmatrix} z_1(t) \\ F_{1\delta} \end{bmatrix}^T \begin{bmatrix} W_o & P \\ P & -\alpha I \end{bmatrix} \begin{bmatrix} z_1 \\ F_{1\delta} \end{bmatrix}
\end{aligned} \tag{4.40}$$

where α is a positive constant, W_o and W are defined by (4.35) and (4.38) respectively By applying inequality (4.35) to (4.40), it follows that $\dot{V}$ is negative definite. Hence the result follows.

Remark 4.5.4 It should be noted that the inequality in (4.22) in Theorem 4.5.1 involves function $\mu(\cdot)$ which makes it difficult to determine the design parameters, although Theorem 4.5.1 is less conservative. In connection with this, a set of conditions has been expressed in LMI to guarantee the stability of sliding motion and thus, the associated design parameters can be obtained systematically using LMI techniques. This is in comparison with the work [109].

Remark 4.5.5 From the proof of Theorem 4.5.2, it follows that it is unnecessary to assume that the bound on the uncertainty $F_1(t, z(t), z_d(t))$ has the special form in (4.31). Actually, it is only required that the bound on $F_{1\delta}(\cdot)$ defined in (4.32) has the special form in (4.31). Therefore, in this chapter, the requirement on the bound on mismatched uncertainty is relaxed which is allowed to have more general form.

4.5.2. SLIDING MODE CONTROL DESIGN

The objective now is to design a state feedback sliding mode control law such that the system state is driven to the sliding surface (4.18) in finite time. The following control

is proposed:

$$\begin{aligned} u(t) &= -B_2^{-1}(\Gamma + \{\|B_2\|\phi_1(t, z(t), z_d) + \|M\|\rho_1(\cdot) \\ &\quad +\rho_2(\cdot) + \eta\}\text{sgn}(\sigma(z))) \end{aligned} \tag{4.41}$$

where

$$\begin{aligned} \Gamma(\cdot) &= (MA_{11} + A_{21})z_1(t) + (MA_{d11} + A_{d21})z_1(t-d) \\ &\quad +(MA_{12} + A_{22})z_2(t) + (MA_{d21} + A_{d22})z_2(t-d) \end{aligned} \tag{4.42}$$

$\phi(\cdot)$, $\rho_1(\cdot)$ and $\rho_2(\cdot)$in (4.14)-(4.16) are defined respectively and $\eta > 0$ is the reachability constant. The following result is ready to be presented.

Theorem 4.5.3 Consider the system (4.7)-(4.8). The control (4.41) drives the system (4.7)-(4.8) to the sliding surface (4.18) in finite time and maintains a sliding motion on it thereafter.

Proof:

From (4.17) and (4.7)-(4.8), it can be verified that

$$\begin{aligned} \dot{\sigma}(z) &= M(A_{11}z_1 + A_{d11}z_1(t-d) + A_{12}z_2 + A_{d12}z_2(t-d) \\ &\quad +F_1(t, z(t), z_d)) + (A_{21}z_1 + A_{d21}z_1(t-d) + A_{22}z_2 \\ &\quad +A_{d22}z_2(t-d)) + Bu(t) + BG(t, z(t), z_d) + F_2(t, z(t), z_d)) \\ &= \Gamma(\cdot) + Bu(t) + BG(t, z(t), z_d) + MF_1(t, z(t), z_d) + F_2(t, z(t), z_d) \end{aligned} \tag{4.43}$$

where $\Gamma(\cdot)$ is defined in (4.42).

Applying the control u in (4.41) to (4.43), it follows from (4.14) and (4.16) that

$$\begin{aligned}
\sigma^\tau\dot{\sigma} &= \sigma^\tau(z)\Gamma - \sigma^\tau(z)MF_1(t,z(t),z_d) - \sigma^\tau(z)[\Gamma + \{\|M\|\rho_1(t,z(t),z_d) \\
&\quad +\|B\|\phi(t,z(t),z_d) + \rho_2(t,z(t),z_d) + \eta\}\text{sgn}(\sigma)] + BG(t,z(t),z_d) \\
&\quad +F_2(t,z(t),z_d) \\
&= \sigma^\tau(z)MF_1(t,z(t),z_d) - \|\sigma(z)\|\|M\|\rho_1(t,z(t),z_d) \\
&\quad +\sigma^\tau(z)BG(t,z(t),z_d) - \|\sigma\|\|B\|\phi(t,z(t),z_d) \\
&\quad +\sigma^\tau(z)F_2(t,z(t),z_d) - \|\sigma\|\rho(t,z(t),z_d) - \eta\|\sigma(z)\| \\
&\leq \|\sigma^\tau(z)MF_1(t,z(t),z_d)\| - \|\sigma(z)\|\|M\|\rho_1(t,z(t),z_d) \\
&\quad +\|\sigma^\tau(z)BG(t,z(t),z_d)\| - \|\sigma\|\|B\|\phi(t,z(t),z_d) \\
&\quad +\|\sigma^\tau(z)F_2(t,z(t),z_d)\| - \|\sigma\|\rho(t,z(t),z_d) \\
&\leq -\eta\|\sigma(z)\| \qquad (4.44)
\end{aligned}$$

where the fact that $\sigma^\tau(z)\text{sgn}(\sigma(z)) \geq \|\sigma(z)\|$ (see [109]) is used to obtain the inequality above.

This shows that the reachability condition holds and hence the conclusion follows.

Theorems 4.5.1 and 4.5.2 together show that the corresponding closed-loop system is uniformly asymptotically stable.

Remark 4.5.6 It should be noted that the designed controller (4.41) is expressed in z co-ordinates. The corresponding controller in x co-ordinates is easy to obtain using the transformation $z = Tx$ in (4.6) which can be obtained using basic matrix theory [13]. The reachability analysis above is carried out directly in z co-ordinates, which may reduce the conservatism. It should be noted that there is no limitation to the change rate of time varying delay $d(t)$. This is in comparison with many existing work [32].

4.5.3. STEP BY STEP APPROACH TO SMC DESIGN

The following basic step by step procedure is used to design SMC.

- Obtain the system dynamics from the state-space model.

- Check for controllability of the state and input matrix.
- Transform the system into controller canonical form (This separates the sliding mode and the reaching mode).
- Define the sliding surface s(x)=0.
- Derive the control law based on Lyapunov stability approach.
- Tune the switching gain (η).
- Run the simulation.

4.6. APPLICATION AND SIMULATION EXAMPLES

In this section, the effect of time delay is tested on the designed sliding mode controller. Two illustrative examples (one numerical and one practical example) are used to demonstrate the superiority of the obtained result.

Example 4.6.1 Consider the automatic voltage regulator (AVR) system, also known as excitation control system in Fig 4.1 [1, 41], which is used to illustrate the effectiveness of the developed method in this chapter. When there is an increase in power load demand, especially in reactive power load demand, it can be observed that the generator terminal voltage ($|v_a|$) drops. The magnitude of the voltage is sensed by the phasor measurement unit (PMU) via a potential transformer. The measured voltage is rectified and compared with a reference DC voltage. The regulator (PI controller and amplifier) produces an analogue signal which controls the firing of the controlled rectifier, and further controls and increases the exciter field and terminal voltage respectively. The increase in the generator field current is due to an increase in the exciter terminal voltage. Such increase in the field current results in an increase in the generator electromotive force (emf). This allows the reactive power generation to increase to a new equilibrium point, thereby increasing the generator terminal voltage to the desired value. The PMUs are used to measure power system dynamic datas such as frequency, voltage, current and angle using the Discrete Fourier Transform (DFT), and are susceptible to time delays due to

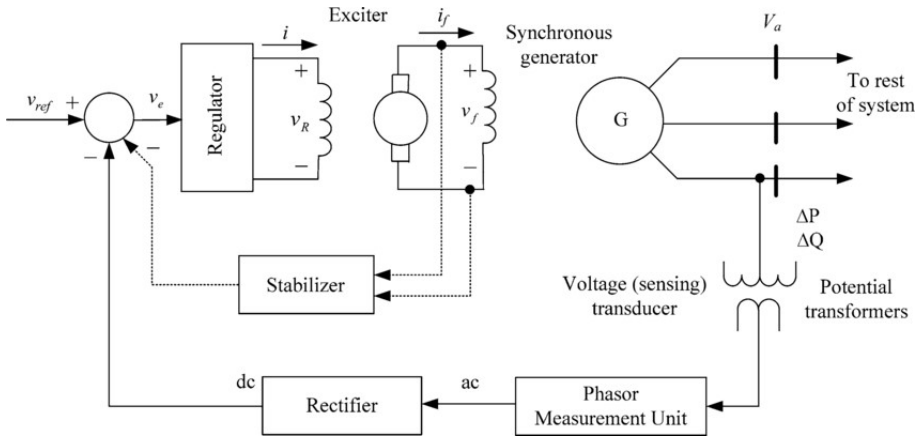

Figure 4.1: Generator excitation system

open and distributed communication network for transferring measured signals and date to the controller. Refer to [1, 41, 71] for more information on AVR. In order to describe the mathematical model of the AVR system [71], the state vector as illustrated in Fig 4.1 chosen as $x(t) = [\Delta V_R(t) \;\; \Delta V_F(t) \;\; \Delta V_T(t) \;\; \Delta V_S(t) \;\; \int \Delta V_S(t)dt$ $\int \Delta V_{FS}(t)dt]^T$. Using the state vector, the state space model of the generator-excitation system is described as:

$$\dot{x} = Ax(t) + A_d x(t-d) + B(u + \bar{G}(t, x, x_d)) + \bar{F}(t, x, x_d) \tag{4.45}$$

where

$$A = \begin{bmatrix} -\frac{1}{T_A} & \frac{K_P K_A K_F}{T_A T_F} & 0 & 0 & 0 & \frac{K_P K_A}{T_A T_F} - \frac{K_I K_A}{T_A} \\ \frac{K_E}{T_E} & -\frac{1}{T_E} & 0 & 0 & 0 & 0 \\ 0 & \frac{K_G}{T_G} & -\frac{1}{T_G} & 0 & 0 & 0 \\ 0 & 0 & \frac{K_R}{T_R} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{K_F}{T_F} & 0 & 0 & 0 & \frac{1}{T_F} \end{bmatrix} \tag{4.46}$$

$$A_d = \begin{bmatrix} 0 & 0 & 0 & -\frac{K_P K_A}{T_A} & \frac{K_I K_A}{T_A} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{T_A} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.47}$$

By choosing the parameters as in [71], and based on the assumption 4.4.1 and (4.6), the AVR system in the new coordinates $z = Tx$ can be described as

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{13} & A_{14} \end{bmatrix} = \left[\begin{array}{ccccc|c} -3 & 0 & 0 & 0 & 0 & 3 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 20 & -20 & 0 & 0 & 0 \\ 0 & 0 & 11 & 0 & 0 & 0 \\ 0 & 0 & 0 & -25 & 0 & 0 \\ \hline -1750 & 0 & 0 & 0 & 835 & -10 \end{array}\right] \tag{4.48}$$

$$\begin{bmatrix} A_{d11} & A_{d12} \\ A_{d13} & A_{d14} \end{bmatrix} = \left[\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & -35 & 40 & 0 & 0 \end{array}\right], \quad B = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline -10 \end{array}\right] \tag{4.49}$$

It should be noted that based on Assumption 4.4.2 in (4.14)-(4.16), the uncertainties $G(\cdot)$, $F_1(\cdot)$ and $F_2(\cdot)$ are added to the system (4.48)-(4.49) specifically to illustrate the theoretical results which are assume to satisfy

$$\|G(t, z(t), z_d)\| \leq \underbrace{0.5|\sin(t)||z_2(t-d)|}_{\phi(t,z(t),z_d)} \tag{4.50}$$

$$\|F_2(t, z(t), z_d)\| \leq \underbrace{0.5|\cos(z_2(t))|}_{\rho(t,z(t),z_d)} \tag{4.51}$$

and $F_1(\cdot)$ satisfies

$$F_1^T F_1 \leq \underbrace{0.58^2}_{\varpi_1} z^T z_d + \underbrace{0.35^2}_{\varpi_2} z_d^T z_d \tag{4.52}$$

By solving the LMI in (4.40), $\alpha = 0.05$, and $\beta = 3.2781$. Thus the matrix (4.35) is symmetric negative definite. From the result above, it can be verified that all the conditions in Theorem 4.5.1 are satisfied. Thus from Theorem 4.5.2, the sliding motion associated with the sliding surface is asymptotically stable.

From Theorem 4.5.2 and by direct calculation, the sliding mode control law (4.41) given

by

$$\begin{aligned} u &= -\Gamma(t,z) - \Big(0.3778\|z(t)\|^2 + 0.845\|z_{d(t)}\|^2 + 0.5\cos(t)|z_{d2}| \\ &\quad +0.5\sin(t)|z_1(t)| + 2\Big)\text{sgn}(\sigma(z)) \end{aligned}$$

stabilises the system (4.48)-(4.52), where

$$\Gamma(t,z) = -2z_1(t) - 0.833z_2(t) + 3z_2(t-d)$$

For simulation purposes, the initial condition relating to the time delay is chosen as $z(t) = \text{col}(\sin(t), \text{e}^t)$ and the time delay is $d(t) = 2 - 5\text{sin}t$. The time responses of the generator output and exciter system output without delay and non-linearities are shown in Figs. 4.2 and 4.3 and the responses with delay and non-linearities are shown in Figs. 4.4 and 4.5 respectively.

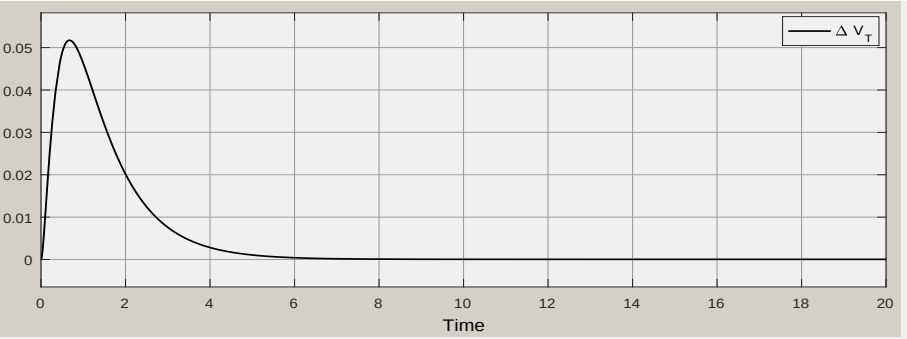

Figure 4.2: Time response of the Generator output $\Delta V_T(t)$ without delay and non-linearities

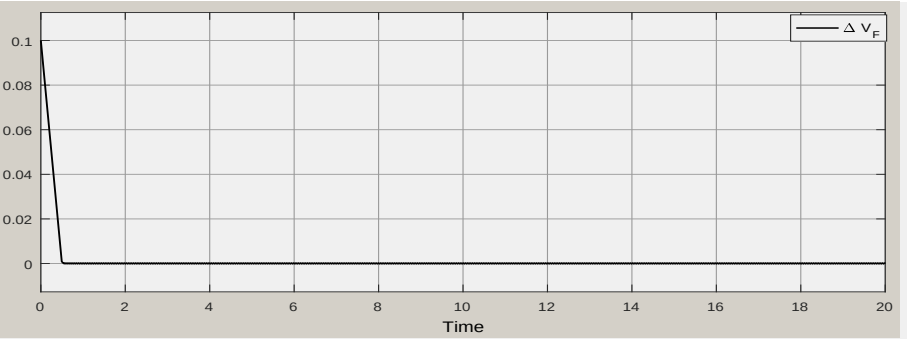

Figure 4.3: Time response of the exciter system output $\Delta V_F(t)$ without delay and non-linearities

Clearly, it can be observed from Figs 4.2 and 4.3 that the deviation signals under normal load and delay free conditions attain steady-state. This physically implies that the perturbed generator terminal voltage (V_T) attains the desired value within a settling time of about $t_s = 6secs$. When delay is applied to the system, it can be observed from Fig 4.4 that generator terminal voltage returns to equilibrium condition after a prolonged transient at approximately $t_s = 9secs$. The generator-excitation system with feedback loop delay is assumed to be stable for all positive delay as shown in exciter system output and the controller in Fig 4.5 and Fig 4.6 respectively. Owing to the presence of delay in the feedback loop, this simulation validates the theoretical result obtained and further demonstrates that the proposed approach is effective.

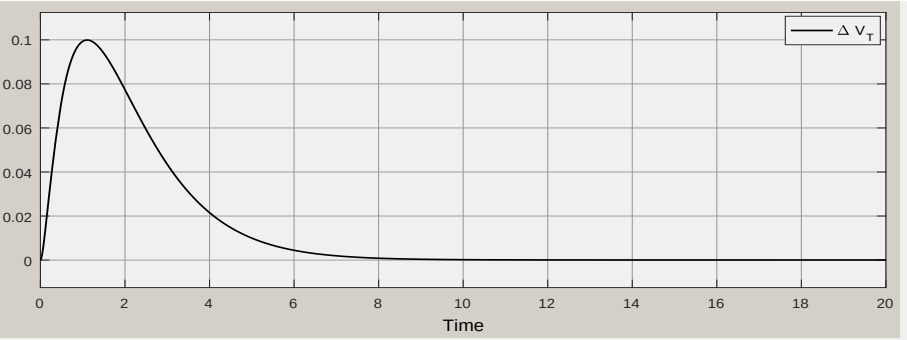

Figure 4.4: Time response of the Generator output $\Delta V_T(t)$ with delay and non-linearities

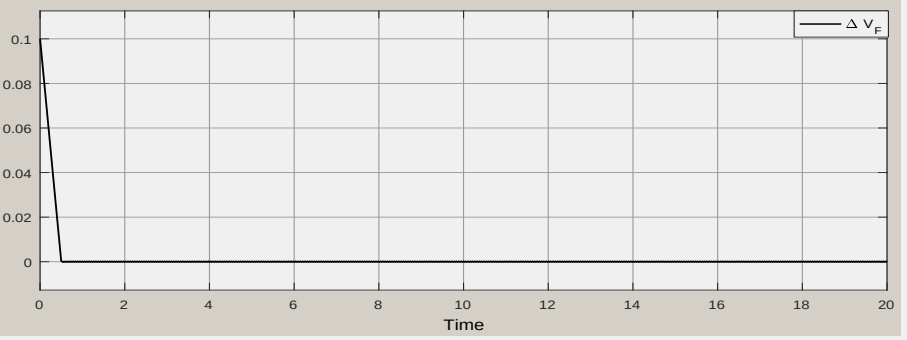

Figure 4.5: Time response of the exciter system output $\Delta V_F(t)$ with delay and non-linearities

Example 4.6.2 Consider the uncertain time delay systems with matched and mismatched

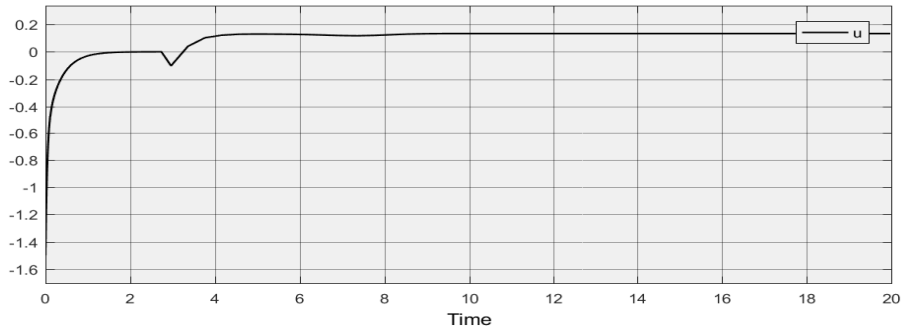

Figure 4.6: Time response of controller u(t)

delayed uncertainties as shown below

$$\dot{x} = \begin{bmatrix} -1 & 0.7 \\ 0.3 & 1 \end{bmatrix} x + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix} x_d + \begin{bmatrix} 1 \\ 1 \end{bmatrix} (u + \bar{G}(\cdot)) + \bar{F}(\cdot) \tag{4.53}$$

It follows from Assumption 4.4.1 that in the new coordinate, the system (4.53) can be described as

$$\dot{z} = \begin{bmatrix} -0.5 & -0.8 \\ -1.2 & 0.5 \end{bmatrix} z + \begin{bmatrix} 0.15 & -0.05 \\ -0.05 & 0.15 \end{bmatrix} z_d + \begin{bmatrix} 0 \\ -1.41 \end{bmatrix} (u + G(\cdot)) + F(\cdot)$$

where $z_d = z(t-d(t))$, $d(t) = 1-0.5\sin t$ and the uncertainties $G(\cdot)$ and $F(\cdot)$ are assumed to be bounded by $\phi(\cdot) = 0.5sin^2(z_1)$ and $\rho = 1.8(|z_1 z_2|)^{\frac{1}{2}}$. Choose the sliding function $\sigma(z) = [\underbrace{0.3}_{M} \quad 1]z$. By direct calculation, $A_o = -0.26$,. With $Q = I$ and $P = 1.92$, the parameters $\alpha = 0.1$, and $\beta = 9.23$ are obtained such that

$$\begin{bmatrix} W_o & P \\ P & -\alpha I \end{bmatrix} = \begin{bmatrix} 1 & 1.92 \\ 1.92 & -0.1 \end{bmatrix} < 0 \tag{4.54}$$

Thus the matrix (4.35) is symmetric negative definite.

From the result above, it can be verified that all the developed conditions are satisfied. Thus, the sliding motion associated with the sliding surface is asymptotically stable.

In this example, the simulation is carried out using Matlab/Simulink in order to observe the role of the matrix A_d and the delay element $d(t)$ on the stability of the time delay systems. The stable response of the delay free system ($d(t) = 0$) is first obtained, in order to be able to investigate and show how increasing time delay makes the system

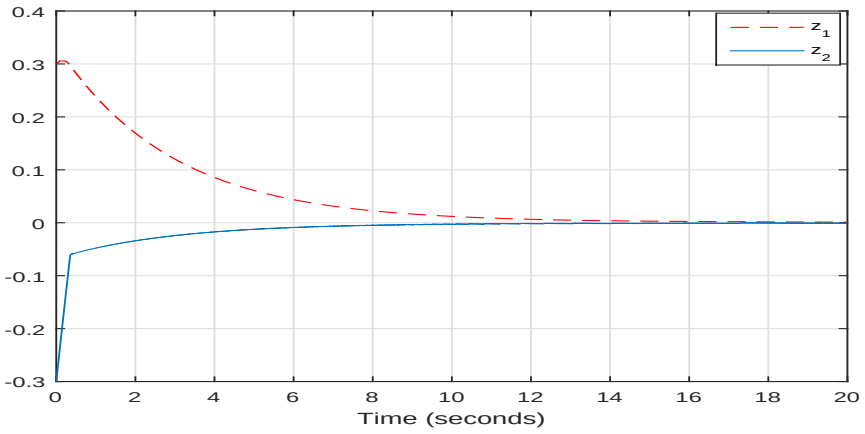

Figure 4.7: Time response of system without delay

unstable. Figs 4.7 and 4.8 show the responses of the system without delay and with delay respectively. It can be observed from Fig 4.7 that the system steadily achieves steady state. By applying delays to the system while keeping all other parameters constant, showed insignificant changes in the system response as in Fig 4.8. This may be due to the effectiveness of the designed controller to counteract the much effect of the delay or that the matrix A_d is rather small. The matrix A_d reflects the strength of the delayed state on the system dynamics. Since the original system is not much affected by delay $d(t)$, it is important to investigate the effect of varying the delayed state matrix A_d.

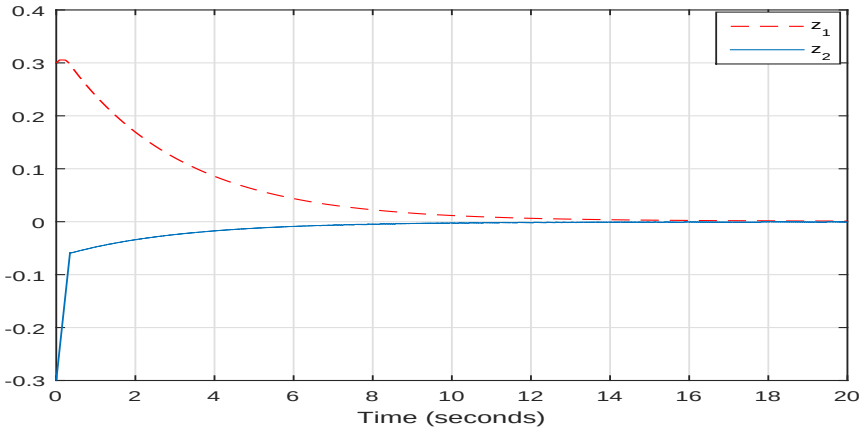

Figure 4.8: Time response of system with delay $d(t) = 3 + 5\sin t$

Now suppose that

$$A_d = \begin{bmatrix} -1.15 & -0.05 \\ -0.25 & 0.55 \end{bmatrix} \tag{4.55}$$

It can be observed from Fig 4.9 that the system experiences overshoot with reduced transient before approaching steady-state. The delayed state matrix when further increased such that

$$A_d = \begin{bmatrix} -2.15 & -0.05 \\ -0.35 & 0.55 \end{bmatrix} \tag{4.56}$$

Further increased overshoot in the system can be observed as shown in Fig 4.10 before the system reaches steady-state. This goes to show that in some cases, the delayed state matrix can yield de-stabilising effects. However, by adjusting the gain of the controller, the system response in Fig 4.10 is improved which is shown in Fig 4.11. This shows that the designed controller works effectively.

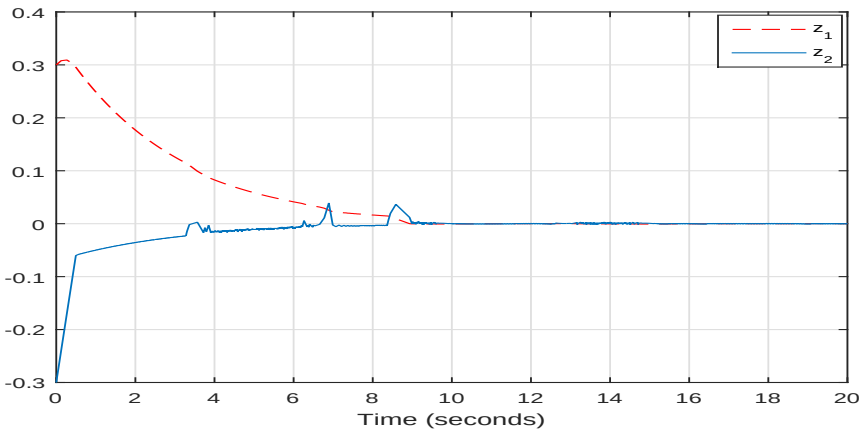

Figure 4.9: Time response of the system (z_1, z_2) for increased delayed state matrix (4.55)

4.7. SUMMARY

In this chapter, state feedback sliding mode control for a class of time delay systems has been considered, where time delay exists in both system states and disturbances. Conservatism is reduced by fully using the property that sliding mode dynamics is of reduced

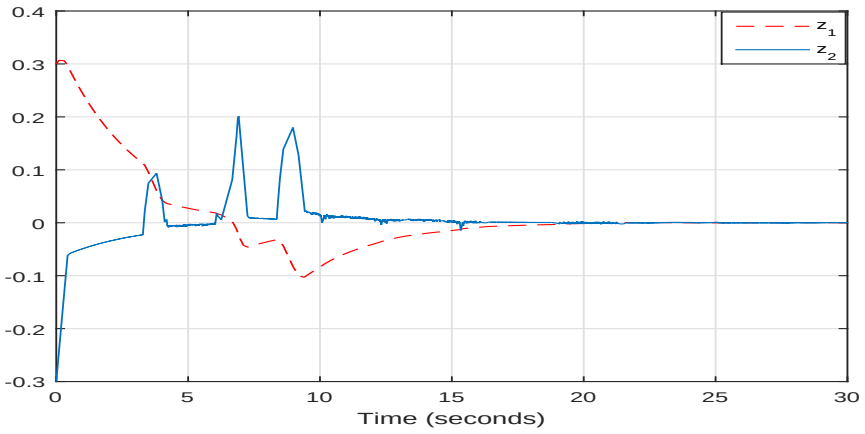

Figure 4.10: Time response of the system (z_1, z_2) when delayed state matrix is further increased (4.56)

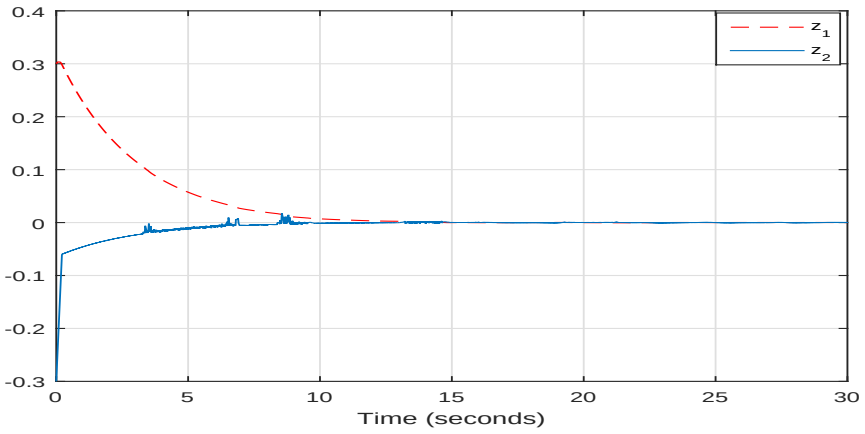

Figure 4.11: Time response of the system for delayed state matrix (4.56) with increased control gain

order, and using the Razumikhin approach, the developed results can accommodate the large time delays. Also, it has been shown that though the uncertainty bounds are non-linear, they can be obtained using LMI technique or may be analysed based on generalised eigenvalue problem. Sliding mode control has been designed to guarantee the systems reachability to the sliding surface, and the non-linear bounds on uncertainties have been fully employed in control design. The results of the simulation verify the theoretical analysis and further illustrate the feasibility of the proposed methodology. Two simulation examples have been used to test the effectiveness of the proposed approach.

CHAPTER 5

REHEAT TURBINE LFC STABILITY WITH MULTIPLE DELAYED NON-LINEARITIES BASED ON SLIDING MODE

This chapter deals with sliding mode control with multiple delayed non-linear terms used to describe the disturbances. Its an extension of the model considered in chapter 4 but with a closer representation of a real case power system model. LFC is considered with multiple delayed non-linear term used to describe random disturbances in power systems including step load disturbances have been considered as well. Therefore, the effects of delays in the reheat turbine of an isolated power system with multiple delayed non-linearities will be considered. A model representation for reheat turbine delay is developed for stability purposes so as to improve the operating condition of the system and avoid the destabilising effects of time delay. Then a multiple delayed non-linear term is used to describe disturbances. This creates an advantage for minimizing the effect of both

delays and non-linearities while avoiding the complications of input delay. On this basis, an admissible upper bound is provided based on Lyapunov Razumikhin theorem and an ultimate bound is calculated for the load disturbance. Finally, an improved load frequency sliding mode control (SMC) is synthesized, which reduces chattering effects and restore power system efficiency. Effectiveness of the proposed method is tested via an isolated power system supplying a service load.

5.1. INTRODUCTION

In real applications, many physical systems are subject to time delay. This delays affect system stability and may degrade system performance. In power systems measurement and controls, delays cannot be avoided [11]. However, they are ignored in traditional power systems controllers which are built based on local information. LFC is one major challenge due to transmission channels, and its studies are aimed at re-adjusting frequency to nominal value after a sustained disturbance. Results of this investigation [94, 3, 85] have shown that uncertainties in power systems arise due to multiple factors ranging from parameter variations, un-modelled dynamics, rate limits etc. In addition, time delays are natural issues which cannot be neglected especially with regards to improving system performance.

Many results have been produced for the stability problem of time delay systems (see, e.g. [17, 23, 65]), and have more recently been extended to multiple state delays and input delay. Lyapunov Krasovskii and Razumikhin techniques are most commonly used to study stability of time delay systems. State and parameter estimation for non-linear delay systems using sliding mode techniques was consider in [110], where Razumikhin technique was used to deal with delays. In [47], delay-dependent stability control for power systems with multiple time-delays has been considered using Krasovskii theorem, but the rate of change of the time delay is bounded and may not be large enough to accommodate power system delays which may be large and fast varying. It should be noted that delays in power systems may be classified into various forms as communications delay, reheaters delay (for reheat turbines), crossover delay etc., all of which can degrade the

systems performance.

Recently, much attention has been paid to the design problem of robust state feedback controllers for linear dynamical systems stabilisation. Sliding mode control due to its robustness to matched parameter perturbations and external disturbances have been applied. However, in power systems which are highly characterized by multiple non-linearities associated with delays, un-modelled and fast varying parameters, various sliding mode techniques have been considered. Decentralised sliding mode LFC for multi-area system was proposed in [57], but did not consider time delay. In [68, 63], sliding mode LFC with state delay and load disturbances has also been considered. Integration of electric vehicles for load frequency output feedback $H\infty$ control of smart grids was proposed in [67], where both state and control input delays were considered. However, the input region of the LFC power system can be characterized by multiple non-linearities such as GDB, delays, all of which can degrade the performance of the system. Therefore, instead of dealing with multiple input delays which may sometimes lead to increased oscillation and conservativeness of the system, it may be less restrictive to assume that the input region is affected by multiple delayed non-linearities. This takes into consideration GDB non-linearities and delays both emanating from governor backlashes and communication signals respectively.

This chapter is to propose a load frequency SMC for a reheat turbine power system with load disturbances and delayed non-linearities. Assumptions for turbine-end delays and multiple delayed uncertainties in the input region are considered. Since the load disturbance is non-vanishing, a sizeable ultimate bound is calculated based on the Lyapunov Razumikhin theorem, which is used as a main analysis tool to deal with the delay. Under the assumption that all states are accessible, a set of conditions is developed to guarantee that sliding motion of the considered reheat turbine power system is ultimately bounded in finite time. A delay dependent sliding mode control which reduces chattering effects is synthesized such that the power system remains uniformly ultimately bounded in the presence of load disturbance and non-linearities. Compared with previous works, the mapped non-linearity associated with the valve position is flexible to adapt both input delay effect and possible non-linear growth and there is no limitation to the size of the delay. Robustness is enhanced by fully using the bounds on the uncertainties in this chapter. A

single area LFC model is simulated to show the feasibility of the developed results and the effectiveness of the proposed method.

5.2. POWER SYSTEM COMPONENTS AND DYNAMICS

This section describes the dynamic model of an isolated power system with reheat turbine as characterized by multiple time delays. The objective of LFC is to restore system frequency (f) to the nominal value by implementing adequate supplementary control action which adjusts the load reference set point. Depending on the turbine configuration, reheat turbines experience changes in high pressure (hp) turbine steam flow which are characterized by time delay, due to crossover piping, re-heaters and turbine valve position adjustment [41, 14]. Therefore, it is suffice to say that multiple delays may abound as turbines grow larger. However, power system is highly nonlinear, and thus, more uncertainties arise due to parameter variations and un-modeled dynamics etc. In order to fully represent this outcomes, the modified dynamics of the reheat turbine system is given as follows (see, e.g. [67]).

5.2.1. GOVERNOR MODEL DYNAMICS

The dynamic model used to describe the reheat turbine system is given as follows

$$\begin{aligned} \Delta\dot{P}_v(t) &= -\frac{1}{RT_g}\Delta f - \frac{1}{T_g}\Delta P_v - \frac{1}{T_g}\Delta E(t) - \frac{1}{T_g}E(t-d_3) \\ &\quad + \frac{1}{T_g}u(t) \end{aligned} \tag{5.1}$$

The governor model for a typical steam unit is shown in Fig 5.1 [94], where $\Delta P_v, \Delta f$, ΔE, R, T_g, u represents governor valve position, frequency deviation, integral control, governor speed regulation, governor time constant and supplementary control respectively. The control valves are naturally discontinuous relays. Due to adjustments in control valves position, delay is introduced into the system. In addition, speed gover-

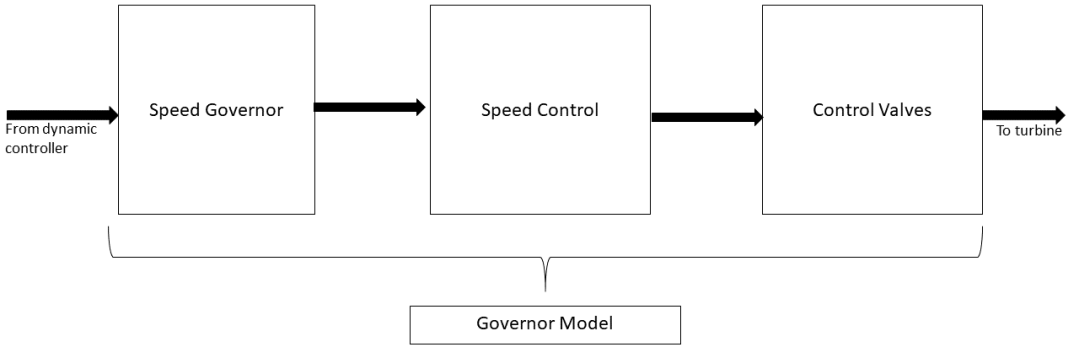

Figure 5.1: Governor block of a steam unit

nors are susceptible to time delay of about 3 to 5 secs and frequency control has been shown to decline due to increase in time delay [41, 14, 3]. Thus, it is necessary to take into account the effects of the delay given in the system (5.1).

5.2.2. TURBINE MODEL

Consider the turbine modelled by

$$\Delta\dot{P}_r(t) = -\frac{1}{T_t}\Delta P_r + \frac{1}{T_t}\Delta P_v - \frac{1}{T_t}\Delta P_r(t-d_1) \tag{5.2}$$

$$\Delta\dot{P}_m(t) = -\frac{1}{T_t}\Delta P_m + \frac{T_t - K_r}{T_t T_r}\Delta P_r + \frac{K_r}{T_t T_r}\Delta P_v + \frac{K_p}{T_p}\Delta P_m(t-d_2) \tag{5.3}$$

where $\Delta P_m, \Delta P_r, T_t, T_p, T_r, K_p, K_r$ represents the generator mechanical power deviation, intermediate power output deviation, turbine time constant, plant model time constant and reheat time constant, plant gain, re-heater gain respectively. The turbine model is characterized by multiple delays depending on its capacity. However, this delays are not represented in most studies. Delays can be introduced via various stop valves within

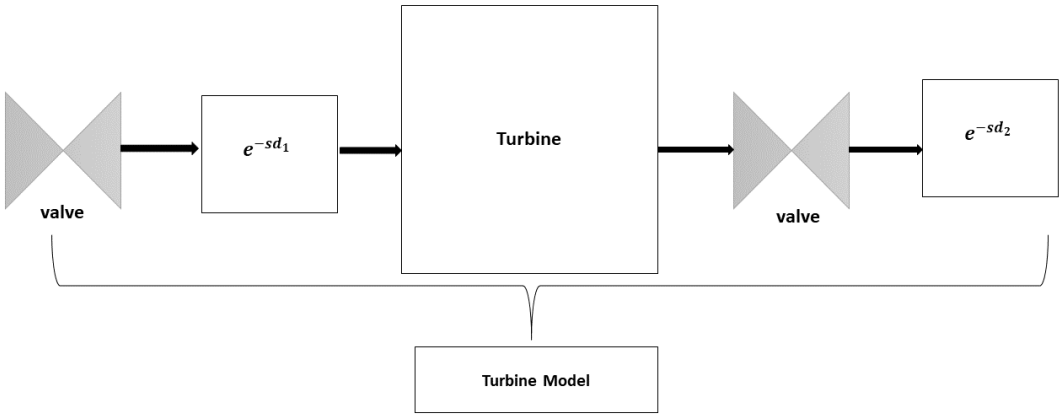

Figure 5.2: Block of a reheat turbine unit with end delays

the steam chest and inlet piping, re-heaters and crossover pipings [14, 94, 67]. In this chapter, these are represented as turbine-end delays for the purpose of stability studies.

5.2.3. FREQUENCY & INTEGRAL CONTROL DYNAMICS

Consider the generator modelled by

$$\Delta\dot{f}(t) = -\frac{1}{T_p}\Delta f + \frac{K_p}{T_p}\Delta P_m - \frac{K_p}{T_p}\Delta P_L(t) \tag{5.4}$$

$$\Delta\dot{E}(t) = \beta\Delta f \tag{5.5}$$

where $\Delta P_L(t) = \Delta(t)$ is the time varying load deviation. β is the frequency bias factor.

Within the overall aim of designing a load frequency sliding mode control for the closed-loop system, the main objectives of this chapter are

- To obtain a stability result for dealing with a class of turbine time delay in power systems.
- To determine a suitable ultimate bound for step/random load disturbances as well as ensure a sufficient allowable bound for the multiple delayed non-linearities.

- Design a load frequency sliding mode control to improve close-loop system performance and reduce the effect of chattering which may increase due to switching relays.

For convenience, the LFC dynamics system is described in the following form

$$\begin{aligned}\dot{x}(t) &= \bar{A}x(t) + \bar{E}_1 x(t-d_1) + \bar{E}_2 x(t-d_2) + \bar{B}u(t) \\ &\quad + \bar{H}_u(t, x, x(t-d_1), x(t-d_2), x(t-d_3)) \\ &\quad + \bar{H}\Delta(t) \end{aligned} \tag{5.6}$$

where $x(t) = [\Delta f\ \Delta P_m\ \Delta P_r\ \Delta P_v\ \Delta E]^T \in \Omega \subset \Re^5$, is the state vector, $u \in \Re$ is the control input at the governor terminal. The matrices $\bar{A}$, $\bar{E} \in \Re^{5\times 5}$, $\bar{B} \in \Re^{5\times 1}$ and $\bar{H} \in \Re^5$ in (5.6) are constant matrices of appropriate dimensions given by

$$\underbrace{\begin{bmatrix} -\frac{D}{M} & \frac{1}{M} & 0 & 0 & 0 \\ 0 & -\frac{1}{T_t} & \frac{T_t - K_r}{T_t T_r} & \frac{K_r}{T_t T_r} & 0 \\ 0 & 0 & -\frac{1}{T_t} & \frac{1}{T_t} & 0 \\ -\frac{1}{RT_g} & 0 & 0 & -\frac{1}{T_g} & -\frac{1}{T_g} \\ \beta & 0 & 0 & 0 & 0 \end{bmatrix}}_{\bar{A}}, \underbrace{\begin{bmatrix} 0_{2\times 1} \\ \frac{1}{T_g} \\ 0_{2\times 1} \end{bmatrix}}_{\bar{B}} \tag{5.7}$$

$$\bar{E}_1,\ \bar{E}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{M} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{T_t} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{T_t} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},\ \bar{H} = \begin{bmatrix} -\frac{1}{M} \\ 0_{4\times 1} \end{bmatrix} \tag{5.8}$$

with $\bar{B}$ being of full rank; $D = \frac{1}{K_p}$ is the damping coefficient, $M = \frac{T_p}{K_p}$ is the equivalent initial. The unknown function $\bar{H}_u(\cdot) : \Re^5$ represents the non-linear characteristics relating to un-modelled parameters including perturbations which affects the system dynamics and performance. The time varying delay $d := d(t)$ for $i = 1, 2, 3$ which are assumed to be known, non-negative and bounded in $\Re^+ := \{t \mid t \geq 0\}$, satisfy

$$\bar{d} = \max\Big\{ \sup_{t\in\Re^+}\{d_1(t)\}, \sup_{t\in\Re^+}\{d_2(t)\}, \sup_{t\in\Re^+}\{d_3(t)\} \Big\} < \infty$$

The initial condition related to the delay is given by

$$x(t) = \phi(t), \quad t \in [-\bar{d}, 0] \tag{5.9}$$

where $\phi(\cdot)$ is continuous in $[-\bar{d}, 0]$. It is assumed that all the non-linear functions are smooth enough such that the unforced system has a unique continuous solution.

Remark 5.2.1 Although this chapter mainly explores the physical limitations such as delays and non-linearities associated with a reheat turbine system, it should be emphasized that other physical constraints such as GRC and GDB non-linearities exist in practice. Results of the impact of these non-linearities are reported in [85, 91, 40]. In this chapter, these non-linearities are generalized in the lumped perturbation term $\bar{H}_u(\cdot)$ in (5.6).

5.3. BASIC ASSUMPTIONS AND LEMMA

In order to facilitate subsequent analysis, the following lemma is required.

Lemma 5.3.1 *(see [36]).* For the system,

$$\dot{x} = f(t, x) \tag{5.10}$$

where $x \in \Re^n$. Let $D \subset \Re^n$ be a domain that contains the origin and $V : [0, \infty) \times D \to R$ be a continuously differentiable function such that

$$\gamma_1(\|x\|) \leq V(t, x) \leq \gamma_2(\|x\|) \tag{5.11}$$

$$\frac{\delta V}{\delta t} + \frac{\delta V}{\delta x} f(t, x) \quad \leq \quad -W_3(x), \ \forall \ \|x\| \geq \varpi > 0 \tag{5.12}$$

$\forall\, t \geq 0$ and $\forall\, x \in D = \{x \in \Re^n : \|x\| < r\}$, where $\gamma_1(\cdot)$ and $\gamma_2(\cdot)$ are class $\mathcal{K}$ functions and $W_3(x)$ is a continuous positive definite function. Take $r > 0$ such that $Br \subset D$ and suppose that

$$\varpi < \gamma_2^{-1}(\gamma_1(r)) \tag{5.13}$$

Then, there exist a class $\mathcal{KL}$ function $\beta(\cdot)$ and for every initial state $x(t_0)$, satisfying $\|x(t)\| \leq \gamma_2^{-1}(\gamma_1(r))$, there is $T \geq 0$ (dependent on $x(t_0)$ and ϖ) such that the solution of (5.10) satisfies

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t - t_0), \quad \forall \quad t_0 \leq t \leq t_0 + T \tag{5.14}$$

$$\|x(t)\| \leq \varpi < \gamma_2^{-1}(\gamma_1(r)) \quad \forall \ t \leq t_0 + T \tag{5.15}$$

Moreover, if $D = \Re^n$ and $\gamma_1(\cdot)$ belongs to class $\mathcal{K}_\infty$, then (5.14) and (5.15) holds for any initial state $x(t_0)$, with no restriction on how large ϖ is.

Assumption 5.3.1 The matrix pair $(\bar{A}, \bar{B})$ defined in (5.7) is controllable.

It should be noted that for a LFC controlled system, $T_g \neq 0$ and matrix $\bar{B}$ is full rank. Thus, from [13] there exists a coordinate transformation $\hat{x}$= $\hat{T}x$ such that the matrix triple $(\bar{A}, \bar{E}, \bar{B})$ in system (5.6) in the new coordinate, has the structure

$$\hat{A} = \begin{bmatrix} \hat{A}_1 & \hat{A}_2 \\ \hat{A}_3 & \hat{A}_4 \end{bmatrix}, \quad \hat{E}_1 = \begin{bmatrix} \hat{E}_{11} & \hat{E}_{12} \\ \hat{E}_{13} & \hat{E}_{14} \end{bmatrix} \tag{5.16}$$

$$\hat{B} = \begin{bmatrix} 0 \\ \hat{B}_2 \end{bmatrix}, \quad \hat{E}_2 = \begin{bmatrix} \hat{E}_{21} & \hat{E}_{22} \\ \hat{E}_{23} & \hat{E}_{24} \end{bmatrix}, \quad \hat{H} = \begin{bmatrix} \hat{H}_1 \\ \hat{H}_2 \end{bmatrix} \tag{5.17}$$

$$\begin{bmatrix} \hat{H}_{u1}(t, \hat{x}, \hat{x}_d) \\ \hat{H}_{u2}(t, \hat{x}, \hat{x}_d) \end{bmatrix} := \hat{T}\bar{H}_u(t, \hat{T}^{-1}x, \hat{T}^{-1}x_d) \tag{5.18}$$

where $\hat{A}_1 \in \Re^{4\times4}$, and $\hat{B}_2 \in \Re$ is positive, $x_d := \text{col}(x(t - d_1), x(t - d_2), x(t - d_3))$. It should be noted that such a transformation can be obtained systematically using matrix theory. Further from [13], since $(\bar{A}, \bar{B})$ is controllable implies that $(\hat{A}_1, \hat{A}_2)$ is controllable, and thus there exists a matrix $M \in \Re^{1\times4}$ such that the matrix $\hat{A}_1 - \hat{A}_2M$ is Hurwitz stable. Further, consider the transformation matrix $z = \tilde{T}\hat{x}$, with $\tilde{T}$ defined by

$$\tilde{T} = \begin{bmatrix} I_4 & 0 \\ M & I \end{bmatrix} \tag{5.19}$$

It follows from the analysis above that in the new coordinate $z = \tilde{T}\hat{x}$, the matrix triple

$(\bar{A}, \bar{E}, \bar{B})$ in system (5.6) has the following form

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \quad E_1 = \begin{bmatrix} E_{11} & E_{12} \\ E_{13} & E_{14} \end{bmatrix} \tag{5.20}$$

$$B = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}, \quad E_2 = \begin{bmatrix} E_{21} & E_{22} \\ E_{23} & E_{24} \end{bmatrix}, \quad H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \tag{5.21}$$

$$\begin{bmatrix} \Theta_1(t, z, z_d) \\ \Theta_2(t, z, z_d) \end{bmatrix} := \tilde{T}\hat{H}_u(t, \tilde{T}^{-1}\hat{x}, \tilde{T}^{-1}\hat{x}_d) \tag{5.22}$$

where $A_1 = \hat{A}_1 - \hat{A}_2 M$ is Hurwitz stable and $B_2 = \tilde{T}\hat{B}_2$ is a positive scalar.

Remark 5.3.1 The analysis above shows the matrix structure (5.20)-(5.22) can be obtained using a linear nonsingular transformation $z = \tilde{T}\hat{x}$. This is called the regular form, which facilitates the sliding surface design and sliding mode control design.

5.4. ANALYSIS AND DESIGN OF SLIDING MODE CONTROL

Section 5.3 shows that there exists new coordinates $z = \tilde{T}\hat{x}$ such that in the new coordinate $z = \text{col}(z_1, z_2)$, the system (5.6) can be described in (5.20)-(5.22). In this section, a sliding surface will be designed and stability of the corresponding sliding motion will be analysed. The matrix structure (5.20)-(5.22) can be rewritten by

$$\begin{aligned} \dot{z}_1 = {} & A_1 z_1 + A_2 z_2 + E_{11} z_1(t - d_1) + E_{12} z_2(t - d_1) \\ & + E_{21} z_1(t - d_2) + E_{22} z_2(t - d_2) + H_1 \Delta(t) \\ & + \Theta_1(t, z, z_d) \end{aligned} \tag{5.23}$$

$$\begin{aligned} \dot{z}_2 = {} & A_3 z_1 + A_4 z_2 + E_{13} z_1(t - d_1) + E_{14} z_2(t - d_1) \\ & + E_{23} z_1(t - d_2) + E_{24} z_2(t - d_2) + B_2 u + H_2 \Delta(t) \\ & + \Theta_2(t, z, z_d) \end{aligned} \tag{5.24}$$

where $z := \text{col}(z_1, z_2)$ with $z_1 := \text{col}(z_{11}, z_{12}, z_{13}, z_{14})$ and $z_2 \in \Re$, A_1 is Hurwitz stable, and B_2 is a positive scalar.

Assumption 5.4.1 There exist known continuous non-negative bounds such that

$$\|\Delta(t)\| \leq \delta \tag{5.25}$$

$$\|\Theta_1(t, z(t), z_d)\| \leq \kappa\rho_1(t, z(t))\|z_d\| \tag{5.26}$$

$$\|\Theta_2(t, z(t), z_d)\| \leq \rho_2(t, z(t), z_d) \tag{5.27}$$

where $\delta > 0, \kappa > 0$ are positive constants, and the functions $\rho_1(\cdot)$ and $\rho_2(\cdot)$ are non-decreasing.

In this chapter, it is assumed that all of the bounds on $\Delta(t)$, $\Theta_1(\cdot)$ and $\Theta_2(\cdot)$ are known which are to be used in the following analysis and design.

5.4.1. STABILITY OF SLIDING MOTION

The main aim of this section is to design a sliding surface for the system (5.23)-(5.24) and then study the stability of the corresponding sliding motion in the presence of disturbances and delays. From the structure of system (5.23) and (5.24), define a switching function of the form

$$\sigma(z) = z_2 \tag{5.28}$$

Therefore, the sliding surface for the isolated system (5.23)-(5.24) is described as

$$\sigma(z) = z_2 = 0 \tag{5.29}$$

Since A_1 in (5.23) is stable, for any $Q > 0$, the following Lyapunov equation has a unique solution $P > 0$

$$A_1^T P + P A_1 = -Q \tag{5.30}$$

From the structure of system (5.23)-(5.24), the sliding motion of system (5.6) associated with the sliding surface (5.29) is dominated by system (5.23). When dynamic system

(5.23) is limited to the sliding surface (5.29), $z_2 = 0$. Thus, the reduced order dynamics can be described as

$$\begin{aligned}\dot{z}_1 &= A_1 z_1 + E_{11} z_1(t-d_1) + E_{21} z_1(t-d_2) \\ &\quad + H_1 \Delta(t) + \bar{\Theta}_1(t, z_1, z_{1d})\end{aligned} \tag{5.31}$$

where $\bar{\Theta}_1(t, z_1(t), z_{1d}) := \Theta_1(t, z(t), z_d)|_{z_2=0}$.

The following theorem is ready to be presented.

Theorem 5.4.1 Under Assumptions 5.3.1, the sliding motion of system (5.23)-(5.24) associated with the sliding surface (5.29), governed by (5.31) is uniformly ultimately bounded with the ultimate bound

$$\bar{\delta} = \sqrt{\frac{\lambda_m(P)}{\lambda_M(P)}}\varpi \tag{5.32}$$

where λ_m and λ_M are the minimum and maximum eigenvalue of matrix P if

$$\begin{aligned}k &= \Big(\lambda_m(Q) - \alpha_1^{-1}\lambda_M(PE_{11}P^{-1}E_{11}^T P) \\ &\quad -\alpha_2^{-1}\lambda_M(PE_{21}P^{-1}E_{21}^T P) - q\bar{\alpha}\lambda_M(P)(\alpha_1+\alpha_2) \\ &\quad -\kappa\rho_1(t,z_1)\lambda_M(P)(q\bar{\alpha}\epsilon^{-1}+\epsilon)\Big) > 0\end{aligned} \tag{5.33}$$

where $\delta, \kappa, \rho_1(\cdot)$ are given in assumption 5.4.1 and

$$\delta < \frac{k\Xi}{2\lambda_M(P)\|H_1\|}\sqrt{\frac{\lambda_m(P)}{\lambda_M(P)}}r \tag{5.34}$$

where $r > 0$, Ξ is a positive constant satisfying $0 < \Xi < 1$.

Proof. For system (5.31), consider a Lyapunov function candidate

$$V(z_1) = z_1(t)^T P z_1(t) \tag{5.35}$$

It follows from (5.30) that the time derivative of V along the trajectories of system (5.31) is given as

$$\begin{aligned}\dot{V}(z_1(t)) &= -z_1^T(t)Qz_1(t) + 2z_1^T(t)PE_{11}z_1(t-d_1) \\ &\quad +2z_1^T(t)PE_{21}z_1(t-d_2) + 2z_1^T(t)PH_1\Delta(t) \\ &\quad +2z_1^T(t)P\bar{\Theta}_1(t, z_1, z_{1d})\end{aligned} \tag{5.36}$$

It follows from inequality $2a^Tb \leq a^TN^{-1}a + b^TNb$, for any $a, b \in \Re^n$ that

$$\begin{aligned}\dot{V}(z_1(t)) &= -z_1^T(t)Qz_1(t) + z_1^T(t)PE_{11}N_1^{-1}E_{11}^TPz_1(t) \\ &\quad +z_1^T(t)PE_{21}N_2^{-1}E_{21}^TPz_1(t) + z_1^T(t-d_1) \\ &\quad \times N_1z_1(t-d_1) + z_1^T(t-d_2)N_2z_1(t-d_2) \\ &\quad +2z_1^T(t)P\bar{\Theta}_1(t,z_1,z_{1d}) + 2z_1^T(t)PH_1\Delta(t)\end{aligned}$$

$$\begin{aligned}\leq\ & -\lambda_m(Q)\|z_1(t)\|^2 + \alpha_1^{-1}\lambda_M(PE_{11}P^{-1}E_{11}^TP)\|z_1(t)\|^2 \\ & +\alpha_2^{-1}\lambda_M(PE_{21}P^{-1}E_{21}^TP)\|z_1(t)\|^2 + \alpha_1\lambda_M(P) \\ & \times\|z_1(t-d_1)\|^2 + \alpha_2\lambda_M(P)\|z_1(t-d_2)\|^2 + 2\kappa\rho_1(t,z_1) \\ & \times\lambda_M(P)\|z_{1d}\|\|z_1(t)\| + 2\delta\lambda_M(P)\|H_1\|\|z_1(t)\| \qquad (5.37)\end{aligned}$$

where $N_1 = \alpha_1P, N_2 = \alpha_2P$, $\alpha_1, \alpha_2, \kappa > 0$ and P is a positive definite matrix. Applying the inequality: $2ab \leq \epsilon^{-1}a^2 + \epsilon b^2$, it follows from (5.37) that

$$\begin{aligned}& 2\kappa\rho_1(t,z_1)\lambda_M(P)\|z_{1d}\|\|z_1(t)\| \\ & \leq \epsilon^{-1}\kappa\rho_1(t,z_1)\lambda_M(P)\|z_{1d}\|^2 + \epsilon\kappa\rho_1(t,z_1)\lambda_M(P)\|z_1(t)\|^2 \qquad (5.38)\end{aligned}$$

where $\epsilon > 0$ is arbitrary constant. Following Razumikhin theorem [23], assume that there exist constant $q > 1$ and $\bar{\alpha} > 0$, such that

$$\|z_{id}\|^2 \leq q\bar{\alpha}\|z_1(t)\|^2, \qquad (5.39)$$

Therefore,

$$\begin{aligned}\dot{V}(z_1(t)) &\leq -\Big(\lambda_m(Q) - \alpha_1^{-1}\lambda_M(PE_{11}P^{-1}E_{11}^TP) \\ &\quad -\alpha_2^{-1}\lambda_M(PE_{21}P^{-1}E_{21}^TP) - q\bar{\alpha}\lambda_M(P)(\alpha_1+\alpha_2) \\ &\quad -\kappa\rho_1(t,z_1)\lambda_M(P)(q\bar{\alpha}\epsilon^{-1}+\epsilon)\Big)\|z_1(t)\|^2 \\ &\quad +2\delta\lambda_M(P)\|H_1\|\|z_1(t)\| \\ &= -(1-\Xi)k\|z_1(t)\|^2 \\ &\quad -(k\Xi\|z_1\| - 2\delta\lambda_M(P)\|H_1\|)\|z_1(t)\| \\ &\quad \forall\ \|z_1(t)\| \geq \frac{2\delta\lambda_M(P)\|H_1\|}{k\Xi} = \varpi \qquad (5.40)\end{aligned}$$

where Ξ and k are defined in (5.34) and (5.33) respectively. Therefore, it follows from Lemma 5.3.1 that defining γ_1, γ_2: $[0, \infty) \rightarrow \Re^+$, by

$$\gamma_1(\|z_1\|) = \lambda_m(P)\|z_1\|^2, \;\; \gamma_2(\|z_1\|) = \lambda_M(P)\|z_1\|^2$$
$$\|z_1(t)\| \leq \bar{\delta} \tag{5.41}$$

where $\bar{\delta}$ is defined in (5.32). Thus, the sliding motion is globally uniformly ultimately bounded. Hence the result follows.

5.4.2. SLIDING MODE CONTROL DESIGN

The objective now is to design a state feedback sliding mode control law such that the system state is driven to the sliding surface (5.29) in finite time. The following control is proposed:

$$u(t) \;\; = \;\; -B_2^{-1}\Big(\Gamma(z) + (\|H_2\|\delta + \rho_2(\cdot) + \eta)\text{sgn}(\sigma(z))\Big) - \mu_l\sigma \tag{5.42}$$

where

$$\begin{aligned}\Gamma(z) \;\; = \;\; & A_3z_1 + A_4z_2 + E_{13}z_1(t-d_1) + E_{14}z_2(t-d_1) \\ & + E_{23}z_1(t-d_2) + E_{24}z_2(t-d_2)\end{aligned} \tag{5.43}$$

where δ and $\rho_2(\cdot)$ are defined in (5.25) and (5.27) respectively, μ_l is a positive design scaler and $\eta > 0$ is the reachability constant. The following result is ready to be presented.

Theorem 5.4.2 Consider the system (5.23)-(5.24). The control (5.42) drives the system (5.23)-(5.24) to the sliding surface (5.29) in finite time and maintains a sliding motion on it thereafter.

Proof: From (5.28) and (5.24), it can be verified that

$$\begin{aligned}\dot{\sigma}(z) = & A_3z_1 + A_4z_2 + E_{13}z_1(t-d_1) + E_{14}z_2(t-d_1) \\ & + E_{23}z_1(t-d_2) + E_{24}z_2(t-d_2) + Bu(t) \\ & + \bar{\Theta}_2(t, z, z_d))\end{aligned} \tag{5.44}$$

Applying the control u in (5.42) to (5.44), it follows from (5.27) that,

$$\begin{aligned}
\sigma^\tau\dot{\sigma} &= \sigma^\tau(z)\Big(A_3z_1 + A_4z_2 + E_{13}z_1(t-d_1) + E_{14}z_2(t-d_1) \\
&\quad + E_{23}z_1(t-d_2) + E_{24}z_2(t-d_2)\Big) - \sigma^\tau(z)\Bigg(\Gamma(z) \\
&\quad + \Big(\|H_2\|\delta + \rho_2(t,z(t),z_d) + \eta\Big)\text{sgn}(\sigma(z)) - B_2\mu_l\sigma\Bigg) \\
&\quad + \sigma^\tau(z)H_2\Delta(t) + \sigma^\tau(z)\bar{\Theta}_2(t,z(t),z_d) \\
&= \sigma^\tau(z)\bar{\Theta}_2(t,z(t),z_d) - \rho_2(t,z(t),z_d)\sigma^\tau(z)\text{sgn}(\sigma(z)) \\
&\quad \sigma^\tau(z)H_2\Delta(t) - \delta\|H_2\|\sigma^\tau(z)\text{sgn}(\sigma(z)) - \sigma^T\sigma B_2\mu_l \\
&\quad -\sigma^\tau(z)\eta\text{sgn}(\sigma(z)) \\
&\leq \|\sigma(z)\|\|\bar{\Theta}_2(t,z(t),z_d)\| - \|\sigma(z)\|\rho_2(t,z(t),z_d) \\
&\quad \|\sigma(z)\|\|H_2\|\|\Delta(t)\| - \|\sigma(z)\|\|H_2\|\delta - \mu_l\|B_2\|\|\sigma\| \\
&\quad -\eta\|\sigma\| \\
&\leq -\mu_l\|B_2\|\|\sigma\| - \eta\|\sigma(z)\| \qquad (5.45)
\end{aligned}$$

Thus it follows from [13] that since $\mu_l\|B_2\|\|\sigma\| > 0$, the η reachability condition holds and hence the conclusion follows. Theorems 5.4.1 and 5.4.2 together show that the corresponding closed-loop system is uniformly asymptotically stable.

Remark 5.4.1 It should be noted that by increasing the term η appearing in the control (5.42), chattering may be induced [110]. In order to revert this effect, μ_l in (5.42) can be chosen arbitrarily in order to increase the rate at which sliding is attained. Thus, creating the possibility for η to be chosen small enough to reduce the switching amplitude.

5.5. SIMULATION EXAMPLE

For simulation purposes, the following parameters $M = 10, T_t = 0.3s, T_g = 0.2s, D = 1.0, T_r = 7.0s, R = 0.05, K_r = 5$ as obtained from [41, 67] are used to obtain the simulation results. Consider the reheat steam turbine generating unit in Fig 5.3, feeding an isolated load. By applying the algorithm in [13], the isolated LFC power system (5.20)-(5.22) in the new coordinates $z = \text{col}(z_1, z_2)$ can be described by

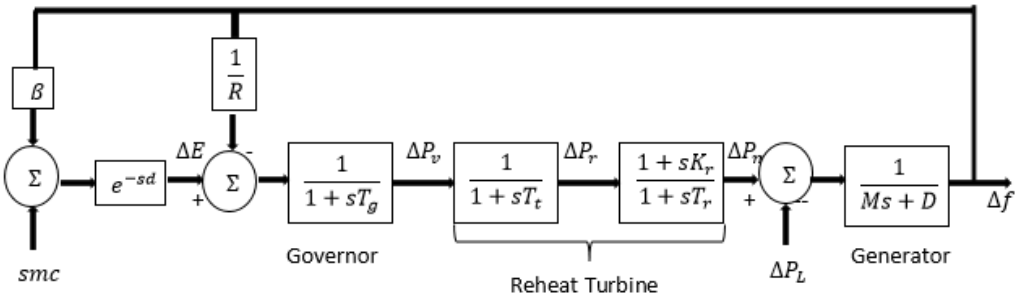

Figure 5.3: Single area reheat generating unit

$$\underbrace{\left[\begin{array}{cccc|c} -0.761 & 132.147 & 5.265 & -112.941 & -0.952 \\ 0.10 & -0.10 & 0 & 0 & 0 \\ -13.507 & -593.773 & -15.139 & 592.941 & 5 \\ 0 & -0.425 & 0 & 0 & 0 \\ \hline 38.10 & 186.70 & 38.40 & -186.6 & -15.9 \end{array}\right]}_{A}, \underbrace{\left[\begin{array}{c} 0_{4\times 1} \\ -5 \end{array}\right]}_{B}$$

$$\underbrace{\left[\begin{array}{cccc|c} 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline -9.27 & -462.51 & -10.09 & 380.15 & 3.33 \end{array}\right]}_{E_1, E_2}, \underbrace{\left[\begin{array}{c} 0 \\ 0.1 \\ 0_{2\times 1} \\ \hline -13.87 \end{array}\right]}_{H}$$

The load deviation $\Delta(t) = 0.02$ and the non-linear delayed perturbation

$$\Theta(\cdot) = \left[\begin{array}{c} 0.2|z_{11}(t)|^{1/2} + \sin(z_{14}(t)) \\ 0 \\ 0 \\ 0.3\sin(|z_{11}(t-d_1)z_{14}(t-d_3)|) \\ 0.1\cos(|z_2(t)z_{14}(t-d_3)) \end{array}\right] \tag{5.46}$$

satisfy the conditions

$$\|\Delta(t)\| \leq \underbrace{0.05}_{\delta} \tag{5.47}$$

$$\|\bar{\Theta}_1(t, z_1, z_{1d})\| \leq \underbrace{2.2\text{sin}^2(z_1) + 0.5\text{cos}^2\|z(t-d)\|}_{\rho_1(\cdot)} \tag{5.48}$$

$$\|\bar{\Theta}_2(t, z_1, z_{1d})\| \leq \underbrace{1.5\text{sin}^2(z_1)\|z_1\| + 0.3\|z(t-d)\|}_{\rho_2(\cdot)} \tag{5.49}$$

Therefore, on the sliding surface (5.29), when the sliding motion takes place, since A_1 in (5.20) is stable, it follows that choosing $Q = I_4$, the Lyapunov equation (5.30) has a unique solution

$$P = \begin{bmatrix} 0 & -5 & 0 & 1.177 \\ 0 & 0 & 0 & 1.177 \\ -0.210 & -7 & -0.040 & -7.765 \\ -0.005 & -0.293 & -0.002 & 1.007 \end{bmatrix} \tag{5.50}$$

By direct calculation, choosing $\alpha_1 = \alpha_2 = 20, \epsilon = 0.1$ and $\bar{\alpha} = 0.05$, it follows from Theorem 5.4.1 that for $\kappa < 1$, $\rho_1(\cdot) \leq 3.6\text{sin}^2\|z_1\|$. Hence, k defined in (5.33) is positive definite.

Furthermore, calculating $\lambda_m = 0.05, \lambda_M = 0.5$ and choosing $r = 0.81, \Xi = 0.6$. The ultimate bound $\bar{\delta} = 0.05$. It is easy to verify that the conditions in Theorem 5.4.1 are satisfied. Thus, the sliding motion associated with the sliding surface is uniformly ultimately bounded. From theorem 5.4.2, the control law

$$\begin{aligned} u = -\Big\{\Gamma(z) + \Big(&2\text{sin}^2(z)\|z\| + 0.3\|z_d\| \\ &+0.44\Big)\text{sgn}(\sigma(z))\Big\} - 2\sigma(z) \end{aligned} \tag{5.51}$$

where $\Gamma(z)$ is defined in (5.43). Thus, the time taken to achieve sliding can be improved by increasing (μ_l) in (5.42). Chattering effects can be reduced by reducing the amplitude of the reachability constant (η) in (5.42).

In order to verify the validity of the proposed LFC design. It is essential to observe the system response under different load disturbance condition and delay based on above systems analysis.

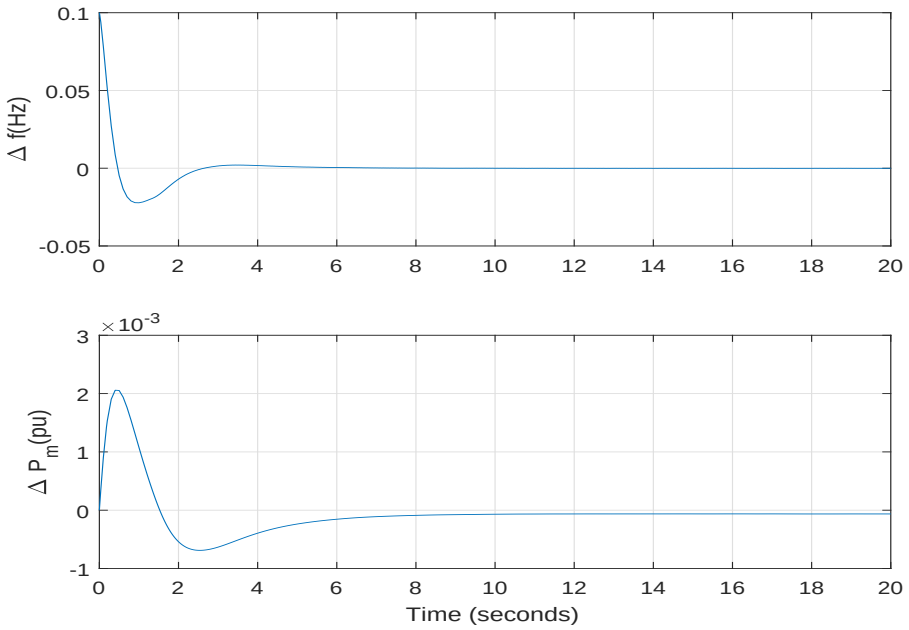

Figure 5.4: Frequency deviation $\Delta f(t)$ (upper) and mechanical power deviation (lower) at $0.02pu$ step load disturbance $\Delta(t)$.

5.5.1. EFFECTS OF LOAD DISTURBANCE

For illustration purposes and based on the calculated ultimate bound, Figs 5.4 and 5.5 shows the step load disturbances applied to the system at 2% pu and 4% pu respectively. Results obtained in Fig 5.4 shows the steady state response of the frequency deviation Δf and mechanical power deviation ΔP_m at 0.02pu. By increasing the size of the load change, Fig 5.5 shows equal transient in frequency deviation due to LFC and a significant change in the mechanical power deviation when subjected to step load change of 0.04 pu, which satisfy the condition given in (5.47). Fig 5.6 shows the systems response when subjected to random load change under same operating conditions.

5.5.2. EFFECT OF NON-LINEARITIES WITH MULTIPLE DELAY

Fig 5.7 shows the responses of the system when subjected to multiple delays in the systems dynamics. Under same operating condition i.e. 0.04 pu load change, by applying constant and time-varying delay, a significant transient and small overshoot can be ob-

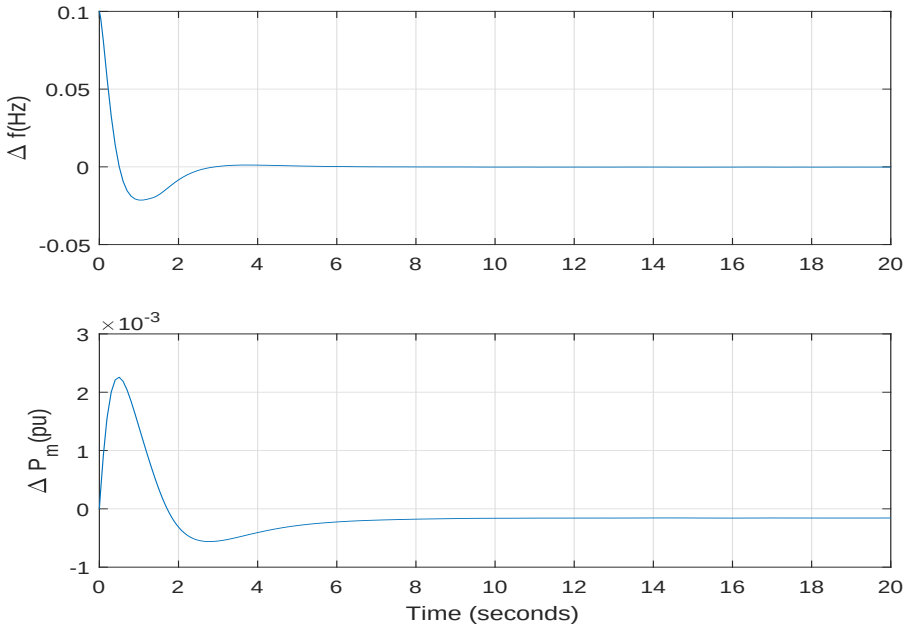

Figure 5.5: Responses of frequency deviation $\Delta f(t)$ and mechanical power deviation $\Delta P_m(t)$ at 0.04 pu step load disturbance

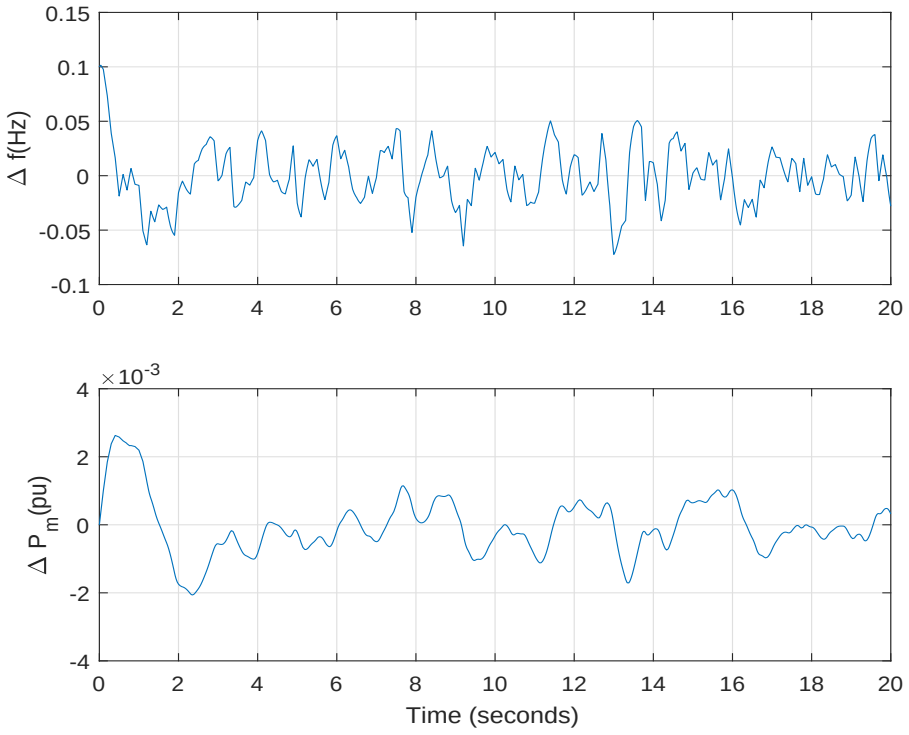

Figure 5.6: Responses of frequency deviation Δf and mechanical power deviation ΔP_m at 0.04 pu random load disturbance.

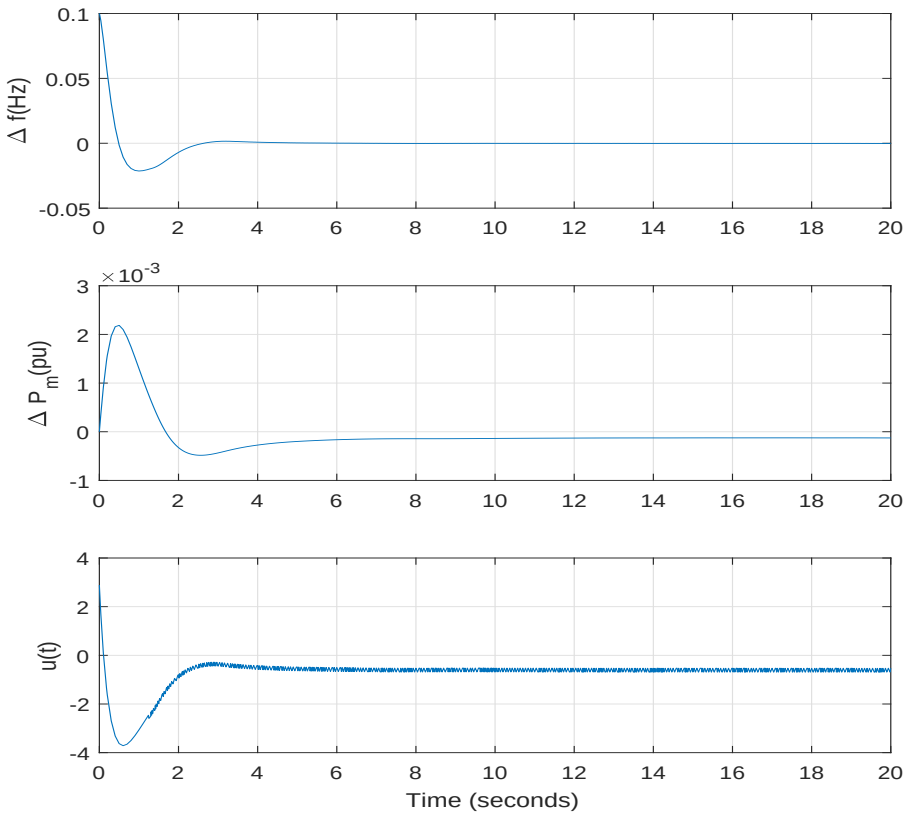

Figure 5.7: Responses of frequency deviation Δf and mechanical power deviation ΔP_m and control $u(t)$ at 0.04 pu random load disturbance and delays $d_1 = 1 + 3\sin t$, $d_2 = 0.8$, $d_3 = 5\sin t$.

served before the system finally achieves steady state. This illustrates the idea that turbine mechanical power (P_m) is a function of the gate signal (P_v), which adjusts the valve to bring the frequency (f) to normalcy [41].

5.6. SUMMARY

A delay dependent state feedback sliding mode control based on the Lyapunov-Razumikhin theorem has been proposed for non-linear isolated power systems with multiple delayed non-linearities. By employing the delay and exploiting the bounds on the uncertainties in both the sliding motion analysis and the control design, conservatism is reduced and the robustness is enhanced by reducing chattering effects. The simulation results on a

single area power systems have demonstrated the effectiveness of the obtained results and further illustrate the feasibility of the proposed methodology.

CHAPTER 6

DECENTRALISED LFC FOR NON-LINEAR INTERCONNECTED POWER SYSTEMS WITH TIME DELAY

In chapter 5, the method has been developed and applied to LFC of a single area. This chapter is an extension of the chapter 5 for a power system from single to multi-area system, with more pronounced communication delays due to long distance interconnection, and may be constant or time-varying. This chapter considers a decentralised sliding mode LFC for multi-area power system with time-varying delays and non-linear uncertainties. Since delays can exert a destabilising effect on the overall system, it is necessary to design a control system to accommodate this delays which may be large so as to regularize the deviation in frequency and tie-line. Robustness is improved by taking advantage of the system structure and uncertainty bounds. A sliding surface is designed, and stability of the corresponding sliding motion is analysed based on Lyapunov-Razumikhin function. A delay dependent decentralised sliding mode control is synthesized to drive the system to the sliding surface and maintain a sliding motion afterwards. The obtained results are

applied to a two-area interconnected power system to demonstrate the effectiveness of the proposed method.

6.1. INTRODUCTION

LFC, being one of the main problems in power system network, has largely been investigated and various methods have been used to enhance power system stability [57, 63, 68]. One reason for the stability of LFC system is to restore systems balancing between total generation and total load demand.

With the advancement of scientific technology, power systems are becoming larger and more complex. The decentralised control for stabilisation of interconnected systems has become a major design goal [8, 90, 106]. Moreover, the integration of renewable sources has made the system more complicated, exchange of information between subsystems becomes more difficult. Thus, centralized control technique may be difficult to apply due to high complexity and cost [8, 106]. Since communication technology constitutes the main backbone for power system data transmission, it is characterized by open communication infrastructure which are unreliable due to time delay, packet loss, and communication failure. As a direct consequence, constant and time varying delay must be considered in order to enhance the performance of LFC systems [63, 68, 79].

It is widely known that time delay is usually a source of instability and performance degradation in control systems. Huge number of results have been dedicated to the development and stability of time delay systems (see, e.g. [17, 73]). Two main techniques based on Lyapunov-Krasovskii functional and Lyapunov-Razumikhin function have been largely used to deal with time delay. The Lyapunov-Razumikhin method, when compared with the Lyapunov-Krasovskii method, can be used to deal with large constant delay and time varying delay with fast change rate [54, 112]. It should be noted that, various control methods have been applied to time delay LFC such as proportional & integral (PI) control [33, 82], robust control [63], Model predictive control [60] and sliding mode control [68, 89, 58] etc.

Sliding mode control (SMC) technique due to its strong robustness properties, fast and

good transient response has proved very effective in dealing with system uncertainties even if delays are involved ([106, 19, 109]). Recent work carried out in LFC with time delay has recorded remarkable improvement amidst differences in techniques. Within the context of sliding mode, some did not consider time delay in the design procedure (see e.g. [57], [79]). Whereas in some others, the interconnection (tie-line) was dealt with as part of the model uncertainty (see [68], [117], [70]). [33] investigates the delay-dependent stability of LFC with constant and time-varying delays, Even so, PI controller was employed and there is limitation to the change rate of the time delay. The effect of electric vehicles (EV) aggregators with time-varying delays on the LFC system was considered in [39]. However, it did not consider uncertainties associated with the system model and EV integration. In addition, multiple input delays were considered in the generic system model, which may result in increased oscillations.

It is important to note that so far when dealing with LFC with time delay using sliding mode techniques, only Lyapunov-Krasovskii method has been used [68]. It should be noted that the Lypunov-Krasovskii method usually requires that the time varying delay is differentiable and the rate of change of time delay is limited/small [68, 17]. However, the delay in multi area power system may be large or time varying with fast change rate. In connection with this, Lyapunov Razumikhin approach is employed to deal with the LFC for the multi area power system.

This work proposes a decentralised sliding mode LFC scheme for the multi area LFC power system with time-varying delays and non-linear delayed disturbances. The assumptions for non-linear terms are imposed on the transformed systems to avoid unnecessary conservatism. The Lyapunov-Razumikhin theorem is used as a main analysis tool to deal with the delay, thereby deriving a set of conditions which guarantee that the derived sliding motion is stable. Then under assumption that each subsystem states are accessible, a delay dependent decentralised sliding mode control is synthesized such that the power system remains uniformly asymptotically stable in the presence of uncertainties and time delays. Compared with existing literatures [68, 33, 39], the main contribution of this chapter can be summarized as follows

i the interconnection is dealt with as a separate non-linear term.

ii Matched and mismatched uncertainties are considered as a measure to improve robustness against non-linearities.

iii Based on Lyapunov-Razumikhin approach, there is no limitation to the size of the delay. In addition, the system allows the bounds on the uncertainties not only have more general non-linear form but also involve time delay.

iv Robustness is enhanced by fully using the bounds on the uncertainties in this chapter.

A two area LFC model is simulated to show the feasibility of the developed results and the effectiveness of the proposed method.

6.2. DYNAMIC MODEL OF POWER SYSTEM

This section describes the dynamic model of a multi-area LFC system with time delay as illustrated in Fig 6.1.

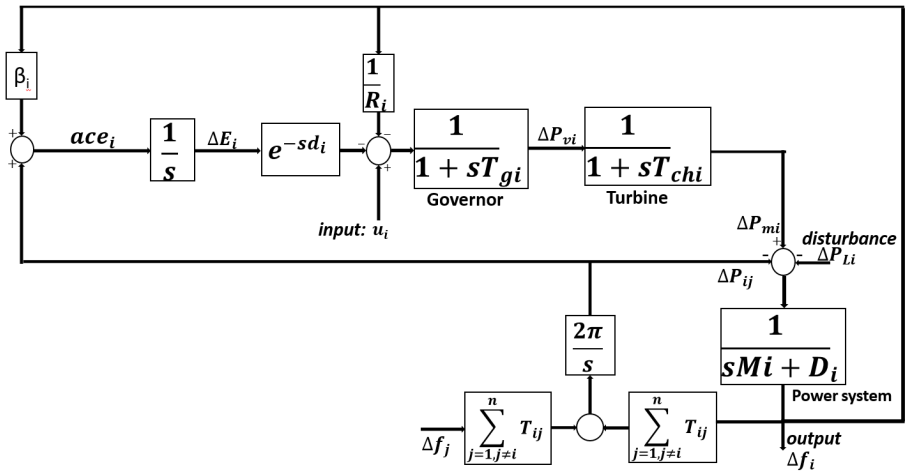

Figure 6.1: Block diagram of the ith-area LFC system

The main objective of LFC is to regulate frequency (f_i) against any load variations and maintain tie-line power (P_{tie_i}) between control areas at the scheduled values. Further, due to control signal data transmission via an open-loop communication channel, delays are introduced into the control loop, and are represented as exponential block $e^{-sd_i(t)}$ ($d_i(t)$ is time delay). In order to fully represent the system model, the conventional LFC model

[3] is modified to include the lumped delay. The dynamics of the N multiarea LFC power system are described by (see, [68])

$$\Delta\dot{f}_i(t) = -\frac{1}{T_{pi}}\Delta f_i + \frac{K_{pi}}{T_{pi}}\Delta P_{mi} - \frac{K_{pi}}{T_{pi}}\Delta P_{tie_i} - \frac{K_{pi}}{T_{pi}}\Delta P_{Li}(t) \tag{6.1}$$

$$\Delta\dot{P}_{mi}(t) = -\frac{1}{T_{chi}}\Delta P_{mi} + \frac{1}{T_{chi}}\Delta P_{vi} + \frac{K_{pi}}{T_{pi}}\Delta P_{mi}(t-d_i(t)) \tag{6.2}$$

$$\Delta\dot{P}_{vi}(t) = -\frac{1}{R_i T_{gi}}\Delta f_i - \frac{1}{T_{gi}}\Delta P_{vi} - \frac{1}{T_{gi}}\Delta E_i(t-d_i(t)) + \frac{1}{T_{gi}}u_i(t) \tag{6.3}$$

$$\Delta\dot{E}_i(t) = \beta_i\Delta f_i + \Delta P_{tie_i} \tag{6.4}$$

$$\Delta\dot{P}_{tie_i}(t) = 2\pi\cdot\sum_{\substack{j=1\\ j\neq i}}^{N} T_{ij}(\Delta f_i(t) - \Delta f_j(t)) \tag{6.5}$$

for $i = 1, 2, ..., N$ where the state variables $\Delta f_i, \Delta P_{mi}, \Delta P_{vi}, \Delta E_i$ and ΔP_{tie_i} represent the frequency deviation, generator mechanical power deviation, governor valve position, integral control, tie-line power deviation between two areas respectively. $D_i = \frac{1}{K_{pi}}$ is the damping coefficient of the i-th area, $M_i = \frac{T_{pi}}{K_{pi}}$ is the equivalent inertia. ΔP_{Li} is the load deviation of the i-th unit. T_{chi}, T_{gi}, T_{pi} are the turbine time constant, governor time constant and plant model time constant respectively. T_{ij} is the synchronizing co-efficient or interconnection between areas i and j $(i \neq j)$. K_{pi} is the plant gain, β_i is the frequency bias factor and R_i is the governor speed regulation.

The objective of this chapter is to design a decentralised sliding mode controller

$$u_i = \varphi_i(\Delta f_i, \Delta P_{mi}, \Delta P_{vi}, \Delta E_i, \Delta P_{tie_i}), \quad i = 1, 2, ..., N$$

such that the closed loop system under the assumption that all the states are available is asymptotically stable in the presence of disturbances and time varying delay. The model (1)-(5) has been widely used to study LFC.

For convenience, describe the LFC power system (6.1)-(6.5) in the following form

$$\dot{x}_i = A_i x_i + E_i x_{id_i} + B_i(u_i + G_i(t, x_i, x_{id_i})) + F_i(t, x_i, x_{id_i}) + \sum_{\substack{j=1\\ j\neq i}}^{N} \Upsilon_{ij}(t, x_j, x_{jd_j}) \tag{6.6}$$

where $x_i = [\Delta f_i \ \Delta P_{mi} \ \Delta P_{vi} \ \Delta E_i \ \Delta P_{tie_i}]^T \in \Omega_i \subset \Re^5$ is the state vector, $u_i \in \Re$ is the control input at the governor terminal of the i-th subsystem for $i = 1, 2, ..., N$. The system matrices in (6.6) are of the dimensions $A_i \in \Re^{5\times5}$, $E_i \in \Re^{5\times5}$, $B_i \in \Re^{5\times1}$ which are given by

$$A_i = \begin{bmatrix} -\frac{D_i}{M_i} & \frac{1}{M_i} & 0 & 0 & -\frac{1}{M_i} \\ 0 & -\frac{1}{T_{chi}} & \frac{1}{T_{chi}} & 0 & 0 \\ -\frac{1}{RT_{gi}} & 0 & -\frac{1}{T_{gi}} & 0 & 0 \\ \beta_i & 0 & 0 & 0 & 1 \\ 2\pi \sum_{\substack{j=1 \\ j\neq i}}^{N} T_{ij} & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{6.7}$$

$$E_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{K_{Pi}}{T_{Pi}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{T_{gi}} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{6.8}$$

$$B_i = \begin{bmatrix} 0_{2\times1} \\ \frac{1}{T_{g_i}} \\ 0_{\times1} \end{bmatrix}, \quad F_i = \begin{bmatrix} \frac{-1}{M_i} \\ 0_{4\times1} \end{bmatrix} \Delta P_{Li} \tag{6.9}$$

The uncertainties $G_i(\cdot)$ and $F_i(\cdot)$ are used to model matched and mismatched non-linear perturbations respectively, with appropriate dimensions; where the terms $\sum_{\substack{j=1 \\ j\neq i}}^{N} \Upsilon_{ij}(\cdot)$ describe the uncertain interconnection of the i-th subsystem; $x_{id_i} := x_i(t - d_i)$ represent the delayed state where $d_i := d_i(t)$ is the time varying delay which is assumed to be known, non-negative and bounded in $\Re^+ := \{t \mid t \geq 0\}$, that is

$$\bar{d_i} := \sup_{t\in\Re^+} \{d_i(t)\} < \infty$$

The initial condition related to the delay is given by

$$x_i(t) = \phi_i(t), \qquad t \in [-\bar{d_i}, 0] \tag{6.10}$$

where $\phi_i(\cdot)$ is continuous in $[-\bar{d}_i, 0]$. It is assumed that all the nonlinear functions are smooth enough such that the unforced interconnected system has a unique continuous solution.

Remark 6.2.1 For the interconnected system (1)-(5), the interconnection term is described in equation (6.5). Consider the disturbance between different areas, the more general interconnections which is allowed to be non-linear and unknown are considered in this chapter. Moreover, in a LFC system, turbines and generators may be affected due to component ageing and non-linearities associated with the governor or stop valve. Thus, as part of modifying the conventional LFC model in [3], this non-linearities are represented as $G_i(\cdot)$ in (6.6) which are matched uncertainties. This is in comparison with most recent works [57, 63, 68, 79], where parameter uncertainties has been considered and non-linear uncertainty is not considered in the interconnection.

6.3. POWER SYSTEM ANALYSIS AND BASIC ASSUMPTIONS

Consider the matrix pair (A_i, B_i) defined in (6.7) and (6.8) for $i = 1, 2, ..., N$. By direct calculation, the controllability matrix

$$[B_i \; A_iB_i \; ... \; A_i^4B_i] = \begin{bmatrix} 0 & 0 & \frac{1}{M_i \cdot T_{chi} \cdot T_{g_i}} & * & * \\ 0 & \frac{1}{T_{chi} \cdot T_{g_i}} & l_i & * & * \\ \frac{1}{T_{g_i}} & \frac{1}{T_{g_i}^2} & \frac{1}{T_{g_i}^3} & * & * \\ 0 & 0 & 0 & \tilde{v}_{i1} & \tilde{v}_{i2} \\ 0 & 0 & 0 & \tilde{v}_{i3} & \tilde{v}_{i4} \end{bmatrix} \tag{6.11}$$

where $*$ denotes the entries which are not necessary to specify and

$$l_i = -\frac{\frac{1}{T_{chi}^2} + \frac{1}{T_{chi} \cdot T_{g_i}}}{T_{g_i}} \tag{6.12}$$

$$\tilde{v}_{i1} = -\frac{\beta_i}{M_i \cdot T_{chi} \cdot T_{g_i}}, \quad \tilde{v}_{i3} = -\frac{2\pi \sum_{\substack{j=1 \\ j\neq i}}^{N} T_{ij}}{M_i \cdot T_{chi} \cdot T_{g_i}} \tag{6.13}$$

$$\tilde{v}_{i2} = \frac{\beta_i (T_{g_i} + T_{chi})}{M_i T_{chi}^2 T_{g_i}^2} - \frac{2\pi M_i \sum_{\substack{j=1 \\ j\neq i}}^{N} T_{ij} - D_i \beta_i}{M_i^2 T_{chi} T_{g_i}} \tag{6.14}$$

$$\tilde{v}_{i4} = \frac{2\pi \sum_{\substack{j=1 \\ j\neq i}}^{N} T_{ij}}{M_i T_{g_i} T_{chi}} \left(\frac{1}{T_{chi}} + \frac{1}{T_{g_i}} + \frac{D_i}{M_i T_{chi} T_{g_i}} \right) \tag{6.15}$$

It follows that the determinant

$$\begin{bmatrix} \tilde{v}_{i1} & \tilde{v}_{i2} \\ \tilde{v}_{i3} & \tilde{v}_{i4} \end{bmatrix} = 2\pi M_i \sum_{\substack{j=1 \\ j\neq i}}^{N} T_{ij} \tag{6.16}$$

If $\sum_{\substack{j=1 \\ j\neq i}}^{N} T_{ij} = 0$, then there is no power exchange between any two different areas i and j $(i \neq j)$ due to $T_{ij} \geq 0$. In this case, the i-th power system areas have no interconnection with any other power areas. It can be modeled individually by the 1st four equations of system (6.6) and thus it is controllable by direct verification. If $\sum_{\substack{j=1 \\ j\neq i}}^{N} T_{ij} \neq 0$, then from $M_i \neq 0$, it is straight forward to see that the matrix pair (A_i, B_i) is controllable for $i = 1, 2...N$.

It should be noted that for a LFC controlled system, $T_{gi} \neq 0$. Matrices B_i are full rank for $i = 1, 2, .., N$. It follows from [13] that there exists a coordinate transformation $\hat{x}_i = \hat{T}_i x_i$ such that the matrix triple (A_i, E_i, B_i) in system (6.6) in the new coordinate, has the structure

$$\hat{A}_i = \begin{bmatrix} \hat{A}_{i1} & \hat{A}_{i2} \\ \hat{A}_{i3} & \hat{A}_{i4} \end{bmatrix}, \quad \hat{E}_i = \begin{bmatrix} \hat{E}_{i1} & \hat{E}_{i2} \\ \hat{E}_{i3} & \hat{E}_{i4} \end{bmatrix}, \quad \hat{B}_i = \begin{bmatrix} 0 \\ \hat{B}_{i2} \end{bmatrix} \tag{6.17}$$

where $\hat{A}_{i1} \in \Re^{4\times4}$, $\hat{E}_{i1} \in \Re^{4\times4}$, and specifically $\hat{B}_{i2} \in \Re$ is a positive scalar for $i = 1, 2, ..., N$. Further, the fact that (A_i, B_i) is controllable implies that $(\hat{A}_{i1}, \hat{A}_{i2})$ is controllable (see, [13]), and thus there exists a matrix $K_i \in \Re^{1\times4}$ such that $\hat{A}_{i1} - \hat{A}_{i2} K_i$

is Hurwitz stable. Further, consider the transformation matrix $z_i = \tilde{T}_i\hat{x}_i$, with $\tilde{T}_i$ defined by

$$\tilde{T}_i = \begin{bmatrix} I_4 & 0 \\ K_i & 1 \end{bmatrix} \tag{6.18}$$

Let $T_i = \tilde{T}_i\hat{T}_i$. It follows from the analysis above that in the new coordinate $z_i = T_i x_i$, system (6.6) has the following form

$$\begin{aligned} \dot{z}_i &= \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix} z_i + \begin{bmatrix} E_{i1} & E_{i2} \\ E_{i3} & E_{i4} \end{bmatrix} z_{id_i} + \begin{bmatrix} 0 \\ B_{i2} \end{bmatrix} \\ &\quad \times \big(u_i + G_i(t, T_i^{-1}z_i, T_i^{-1}z_{id_i})\big) + F_i(t, T_i^{-1}z_i, T_i^{-1}z_{id_i}) \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N} \Upsilon_{ij}(t, T_i^{-1}z_j, T_i^{-1}z_{jd_j}) \end{aligned} \tag{6.19}$$

where

$$\begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix} = T_i A_i T_i^{-1} \tag{6.20}$$

$$\begin{bmatrix} E_{i1} & E_{i2} \\ E_{i3} & E_{i4} \end{bmatrix} = T_i E_i T_i^{-1}, \quad T_i B_i = \begin{bmatrix} 0 \\ B_{i2} \end{bmatrix} \tag{6.21}$$

where $A_{i1} = \hat{A}_{i1} - \hat{A}_{i2}K_i$ is Hurwitz stable and $B_{i2} = \hat{B}_{i2}$ is a positive scalar for $i = 1, 2, ..., N$.

Remark 6.3.1 The analysis above shows that system (6.6) can be transformed to system (6.19) using a linear nonsingular transformation $z_i = T_i x_i$ for $i = 1, 2, ...N$. It should be noted that the linear part of system (6.19) is in regular form, which facilitates the sliding surface design and sliding mode control

Assumption 6.3.1 There exist known continuous non-negative functions $\zeta_i(\cdot)$, $\kappa_i(\cdot)$, and $\alpha_i(\cdot)$ such that

$$\|G_i(\cdot)\| \leq \zeta_i(t, z_i(t), \|z_{id_i}\|) \tag{6.22}$$

$$\begin{aligned} \|F_i^b(\cdot)\| &\leq \alpha_i^a(t, z_i(t))\|z_i\| \\ &\quad +\alpha_i^b(t, \|z_{id_i}\|)\|z_{id_i}\| \qquad (6.23) \\ \|\Upsilon_{ij}^b(\cdot)\| &\leq \kappa_{ij}^a(t, z_j)\|z_j\| \\ &\quad +\kappa_{ij}^b(t, \|z_{jd_j}\|)\|z_{jd_j}\| \quad (i \neq j) \qquad (6.24) \end{aligned}$$

for $i, j = 1, 2, ..., N$.

Remark 6.3.2 From Assumption 6.3.1, it is straightforward to see that the bounds on the uncertainties are nonlinear and involve time delays. Assumption 6.3.1 shows that bounds on all uncertainties are required to be known, which will be employed in LFC design to reject the effects of the uncertainties.

6.4. SLIDING MODE CONTROL ANALYSIS AND DESIGN

Section 6.3 shows that there exists new coordinates $z_i = T_i x_i$ such that in the new coordinate $z = \text{col}(z_1, z_2 ... z_N)$, the system (6.6) can be described in (6.19). In this section, a sliding surface will be designed for system (6.19) and stability of the corresponding sliding motion will be analysed. System (6.19) can be rewritten by

$$\begin{aligned} \dot{z}_i^a &= A_{i1}z_i^a + A_{i2}z_i^b + E_{i1}z_{id_i}^a + E_{i2}z_{id_i}^b \\ &\quad +F_i^a(t, z_i, z_{id_i}) + \sum_{\substack{j=1 \\ j\neq i}}^{N} \Upsilon_{ij}^a(t, z_j, z_{jdj}) \qquad (6.25) \\ \dot{z}_i^b &= A_{i3}z_i^a + A_{i4}z_i^b + E_{i3}z_{id_i}^a + E_{i4}z_{id_i}^b \\ &\quad +B_{i2}(u_i + G_i(t, z_i, z_{id_i})) + F_i^b(t, z_i, z_{id_i}) \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N} \Upsilon_{ij}^b(t, z_j, z_{jdj}) \qquad (6.26) \end{aligned}$$

where $z_i := \text{col}(z_i^a, z_i^b)$ with $z_i^a \in \Re^4$ and $z_i^b \in \Re$, A_{i1} is Hurwitz stable, and B_{i2} is a positive scalar. Thus

$$G_i(t, z_i, z_{id_i}) := G_i(t, T_i^{-1} z_i, T_i^{-1} z_{id_i}) \tag{6.27}$$

$$\begin{bmatrix} F_i^a(t, z_i, z_{id_i}) \\ F_i^b(t, z_i, z_{id_i}) \end{bmatrix} := T_i F_i(t, T_i^{-1} z_i, T_i^{-1} z_{id_i}) \tag{6.28}$$

$$\begin{bmatrix} \Upsilon_{ij}^a(t, z_j, z_{jdj}) \\ \Upsilon_{ij}^b(t, z_j, z_{jdj}) \end{bmatrix} := T_i \Upsilon_{ij}(t, T_i^{-1} z_j, T^{-1} z_{jdj}) \tag{6.29}$$

where $\Upsilon_{ij}^a(\cdot) \in \Re^{4\times 1}$, $F_i^a(\cdot) \in \Re^{4\times 1}$; $\Upsilon_{ij}^b(\cdot) \in \Re$, $F_i^b(\cdot) \in \Re$ for $i \neq j$ and $i, j = 1, 2, ..., N$.

It is well known that sliding mode control design consists of two steps:

i To design a sliding surface such that the corresponding sliding motion has desired performance when the system is limited to the sliding surface.

ii To design a sliding mode control such that the system can be driven to the sliding surface by the designed control.

The following study will reflect this two steps.

6.4.1. STABILITY OF SLIDING MOTION

Based on the above assumption, the main aim of this section is to design a sliding surface for the system (6.25)-(6.26) and then study the stability of the corresponding sliding motion in the presence of disturbances and delay. From the structure of system (6.25) and (6.26), define a switching function of the form

$$\sigma_i(z_i) = S_{i2} z_i^b, \qquad i = 1, 2, ..., N \tag{6.30}$$

where $S_{i2} \neq 0$. Therefore the local sliding surface for the interconnected system (6.19) is described as

$$\sigma_i(z_i) = S_{i2} z_i^b = 0, \qquad i = 1, 2, ..., N \tag{6.31}$$

Since A_{i1} in (6.25) is stable, for any $Q_i > 0$, the following Lyapunov equation has a unique solution $P_i > 0$

$$A_{i1}^T P_i + P_i A_{i1} = -Q_i, \qquad i = 1, 2, ..., N. \tag{6.32}$$

From the structure of system (6.25)-(6.26), the sliding motion of system (6.6) associated with the sliding surface (6.31) is dominated by system (6.25). When dynamic system (6.25) is limited to the sliding surface (6.31), $z_i^b = 0$. The reduced order dynamics can be described as

$$\begin{aligned} \dot{z}_i^a = A_{i1} z_i^a \quad + \quad & E_{i1} z_{id_i}^a + \Psi_i(t, z_i^a, z_{id_i}^a) \\ & + \sum_{\substack{j=1 \\ j \neq i}}^{N} \tilde{\Upsilon}_{ij}^a(t, z_j^a, z_{jdj}^a) \end{aligned} \tag{6.33}$$

where $z_i^a = \text{col}\,(z_{i1}^a, z_{i2}^a, ..., z_i^a)$, $z_i^b = \text{col}\left(z_{i1}^b, z_{i2}^b, .., z_i^b\right)$, and

$$\Psi_i(t, z_i^a, z_{id_i}^a) \;:=\; F_i^a(t, z_i, z_{id_i})|_{z_i^b = 0} \tag{6.34}$$

$$\tilde{\Upsilon}_{ij}(t, z_j^a, z_{jd_j}^a) \;:=\; \Upsilon_{ij}^a(t, z_j, z_{jd_j})|_{z_j^b = 0} \tag{6.35}$$

where $F_i^a(\cdot)$ and $\Upsilon_{ij}^a(\cdot)$ are defined in (6.28)-(6.29) respectively.

From Assumption 6.3.1, it is clear to see that $F_i^a(\cdot)$ and $\Upsilon_{ij}^a(\cdot)$ are bounded by a known continuous function. Therefore, on the sliding surface, it follows from (6.23) and (6.24) that

$$\begin{aligned} \|P_i^{1/2} \Psi_i(t, z_i^a, z_{idi}^a)\| \;\leq\; & \beta_i^a(t, z_i^a) \|P_i^{1/2} z_i^a\| \\ & + \beta_i^b(t, z_i^a) \|P_i^{1/2} z_{id_i}^a\| \end{aligned} \tag{6.36}$$

$$\begin{aligned} \|P_j^{1/2} \tilde{\Upsilon}_{ij}(t, z_j^a, z_{jdj}^a)\| \;\leq\; & \chi_{ij}^a(t, z_j^a) \|P_j^{1/2} z_j^a\| \\ & + \chi_{ij}^b(t, z_j^a) \|P_j^{1/2} z_{jd_j}^a\| \end{aligned} \tag{6.37}$$

where the functions $\beta_i^a(\cdot)$, $\beta_i^b(\cdot)$, $\chi_{ij}^a(\cdot)$, and $\chi_{ij}^b(\cdot)$ are continuous.
The following theorem is ready to be presented.

Theorem 6.4.1 Under Assumption 6.3.1, the sliding motion of system (6.25)-(6.26) associated with the sliding surface (6.31), governed by (6.33) is uniformly asymptotically

stable if $W^T + W > 0$, where the matrix

$$W = \begin{bmatrix} w^a & w^b \\ w^c & w^d \end{bmatrix}_{2N \times 2N} \tag{6.38}$$

Here $w^a = (w^a_{ij})_{N\times N}$, $w^b = (w^b_{ij})_{N\times N}$, $w^c = (w^c_{ij})_{N\times N}$,
$w^d = (w^d_{ij})_{N\times N}$ and

$$w^a_{ij} = \begin{cases} \lambda_{min}(P_i^{-1/2} Q_i P_i^{-1/2}) - 2\beta^a_i(\cdot) - q_i, & \text{if } i = j \\ -\lambda_{max}(P_i^{1/2} P_j^{-1/2})\chi^a_{ij}(\cdot), & \text{if } i \neq j \end{cases}$$

$$w^b_{ij} = \begin{cases} \|P_i^{1/2} E_{i1} P_i^{-1/2}\| + \beta^b_i(\cdot), & \text{if } i = j \\ -\lambda_{max}(P_i^{1/2} P_j^{-1/2})\chi^b_{ij}(\cdot), & \text{if } i \neq j \end{cases}$$

$$w^d_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

where $w^c_{ij} = w^b_{ji}$ for some $q_i > 1$ and $i, j = 1, 2, ..., N$.

Proof. For system (6.33), consider a Lyapunov function candidate

$$V(z^a_1, z^a_2, .., z^a_N) = \sum_{i=1}^{N} (z^a_i)^T P_i z^a_i \tag{6.39}$$

It follows from (6.32) that the time derivative of V along the trajectories of system (6.33) is given as

$$\begin{aligned} \dot{V}(z^a_1, z^a_2, .., z^a_N)|_{(6.33)} = &- \sum_{i=1}^{N} (z^a_i)^T Q_i z^a_i + 2\sum_{i=1}^{N} (z^a_i)^T P_i E_{i1} z^a_{id_i} \\ &+ 2\sum_{i=1}^{N} (z^a_i)^T P_i \Psi_i(\cdot) + 2\sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N} (z^a_i)^T P_i \tilde{\Upsilon}^a_{ij}(\cdot) \end{aligned} \tag{6.40}$$

It is straight forward to see that

$$\begin{aligned} (z^a_i)^T Q_i z^a_i &= (z^a_i)^T P_i^{1/2} P_i^{-1/2} Q_i P_i^{-1/2} P_i^{1/2} z^a_i \\ &\leq \lambda_{min}(P_i^{-1/2} Q_i P_i^{-1/2}) \|P_i^{1/2} z^a_i\|^2 \end{aligned} \tag{6.41}$$

and

$$
\begin{aligned}
&2\sum_{i=1}^{N}(z_i^a)^T P_i E_{i1} z_{id_i}^a \\
&= 2\sum_{i=1}^{N}(z_i^a)^T P_i^{1/2} P_i^{1/2} E_{i1} P_i^{-1/2} P_i^{1/2} z_{id_i}^a \\
&\leq 2\sum_{i=1}^{N}\|P_i^{1/2} z_i^a\|\|P_i^{1/2} E_{i1} P_i^{-1/2}\|\|P_i^{1/2} z_{id_i}^a\| \qquad (6.42)
\end{aligned}
$$

From (6.36), it follows that

$$
\begin{aligned}
&2\sum_{i=1}^{N}(z_i^a)^T P_i \Psi_i(\cdot) = 2\sum_{i=1}^{N}(z_i^a)^T P_i^{1/2} P_i^{1/2} \Psi_i(\cdot) \\
\leq\ & 2\sum_{i=1}^{N}\|P_i^{1/2} z_i^a\|\left(\beta_i^a(\cdot)\|P_i^{1/2} z_i^a\| + \beta_i^b(\cdot)\|P_i^{1/2} z_{id_i}^a\|\right) \\
=\ & 2\sum_{i=1}^{N}\left(\beta_i^a(\cdot)\|P_i^{1/2} z_i^a\|^2 + \beta_i^b(\cdot)\|P_i^{1/2} z_i^a\|\|P_i^{1/2} z_{id_i}^a\|\right)
\end{aligned}
$$

From (6.37)

$$
\begin{aligned}
&2\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}(z_i^a)^T P_i \tilde{\Upsilon}_{ij}^a(\cdot) = 2\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}(z_i^a)^T P_i^{1/2} P_i^{1/2} P_j^{-1/2} P_j^{1/2} \tilde{\Upsilon}_{ij}^a(\cdot) \\
\leq\ & 2\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\lambda_{max}(P_i^{1/2} P_j^{-1/2})\|P_i^{1/2} z_i^a\|\left(\chi_{ij}^a(\cdot)\|P_j^{1/2} z_j^a\| + \chi_{ij}^b(\cdot)\|P_j^{1/2} z_{jd_j}^a\|\right)
\end{aligned}
$$

Substituting (6.41), (6.42), (6.43) and (6.43) into (6.40) yields

$$
\begin{aligned}
&\dot{V}(z_1^a, z_2^a, .., z_N^a)|_{(6.33)} \leq -\sum_{i=1}^{N}\lambda_{min}(P_i^{-1/2} Q_i P_i^{-1/2}) \\
&\times\|P_i^{1/2} z_i^a\|^2 + 2\sum_{i=1}^{n}\|P_i^{1/2} z_i^a(t)\|\|P_i^{1/2} E_{i1} P_i^{-1/2}\| \\
&\times\|P_i^{1/2} z_{id_i}^a\| + 2\sum_{i=1}^{N}\Big(\beta_i^a(t, z_i^a)\|P_i^{1/2} z_i^a\|^2 + \beta_i^b(t, z_i^a) \\
&\times\|P_i^{1/2} z_i^a\|\|P_i^{1/2} z_{id_i}^a\|\Big) + 2\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\lambda_{max}(P_i^{1/2} P_j^{-1/2}) \\
&\times\Bigg(\chi_{ij}^a(\cdot)\|P_i^{1/2} z_i^a\|\|P_j^{1/2} z_j^a\| \\
&\qquad +\chi_{ij}^b(\cdot)\|P_i^{1/2} z_i^a\|\|P_j^{1/2} z_{jd_j}^a\|\Bigg) \qquad (6.43)
\end{aligned}
$$

For any constant $q_i > 1$, assume that

$$V(z^a_{1d_1}, z^a_{2d_i}, ..., z^a_{Nd_N}) \leq q_i V(z^a_1, z^a_2, ..., z^a_N) \tag{6.44}$$

which from (6.39) implies that

$$\sum_{i=1}^{N} \|P_i^{1/2} z^a_{id_i}\|^2 \leq q_i \sum_{i=1}^{N} \|P_i^{1/2} z^a_i\|^2$$

$$q_i \sum_{i=1}^{N} \|P_i^{1/2} z^a_i\|^2 - \sum_{i=1}^{N} \|P_i^{1/2} z^a_{id_i}\|^2 \geq 0 \tag{6.45}$$

Further from (6.43) and (6.45), it follows that

$$\begin{aligned}
&\dot{V}(z^a_1, z^a_2, .., z^a_N)|_{(6.33)} \leq \\
&-\Bigg\{ \sum_{i=1}^{N} \Big(\lambda_{min}(P_i^{-1/2} Q_i P_i^{-1/2}) - 2\beta^a_i(\cdot) - q_i \Big) \|P_i^{1/2} z^a_i\|^2 \\
&+ \sum_{i=1}^{N} \|P_i^{1/2} z^a_{id_i}\|^2 - 2 \sum_{i=1}^{N} \Big(\|P_i^{1/2} E_{i1} P_i^{-1/2}\| + \beta^b_i(\cdot) \Big) \\
&\times \|P_i^{1/2} z^a_i(t)\| \|P_i^{1/2} z^a_{id_i}\| - 2 \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \lambda_{max}(P_i^{1/2} P_j^{-1/2}) \chi^a_{ij}(\cdot) \\
&\times \|P_i^{1/2} z^a_i\| \|P_j^{1/2} z^a_j\| - \lambda_{max}(P_i^{1/2} P_j^{-1/2}) \chi^b_{ij}(\cdot) \\
&\times \|P_i^{1/2} z^a_i\| \|P_j^{1/2} z^a_{jd_j}\| \Bigg\} \\
&\leq -Z^T (W + W^T) Z
\end{aligned} \tag{6.46}$$

where

$$Z =: \Big(\|P_1^{1/2} z^a_1\|, ..., \|P_N^{1/2} z^a_N\|, \|P_1^{1/2} z^a_{1d_1}\|, \cdots, \|P_N^{1/2} z^a_{Nd_N}\| \Big)^T$$

Hence from Lemma 2.2.5, the conclusion follows based on the condition that $W^T + W > 0$.

Remark 6.4.1 It should be noted that the matrix $P^{1/2}$ is introduced in this chapter. The bounds on uncertainties can be calculated in (6.36) and (6.37) which are used for analysis to reduce the conservatism. Similar method is used in [109] where only centralized system is considered while interconnected systems are considered in this chapter.

6.4.2. DECENTRALISED CONTROL DESIGN

Next, it is necessary to design a decentralised sliding mode control law for the interconnected systems such that the systems state is driven to the sliding surface (6.31). A well known reachability condition for the interconnected systems (6.25)-(6.26) is described by [106]

$$\sum_{i=1}^{N} \frac{\sigma_i^T(z_i)\dot{\sigma}_i(z_i)}{\|\sigma_i(z_i)\|} < 0 \tag{6.47}$$

Consider the following decentralised control.

$$u_i = -(S_{i2}B_i)^{-1}\Big\{S_{i2}H_i + \Big(\Xi_i + \eta_i\Big)\text{sgn}(\sigma_i(z_i))\Big\} \tag{6.48}$$

where

$$H_i = A_{i3}z_i^a + A_{i4}z_i^b + E_{i3}z_{id_i}^a + E_{i4}z_{id_i}^b \tag{6.49}$$

is the linear component,

$$\begin{aligned}\Xi_i \;=\; & \|S_{i2}B_i\|\zeta_i(t, z_i, \|z_{id_i}\|) + \|S_{i2}\|\Big(\kappa_{ij}^a(t, z_j)\|z_j\| \\ & +\kappa_{ij}^b(t, \|z_{jd_j}\|)\|z_{jd_j}\|\Big) + \|S_{i2}\|\Big(\alpha_i^a(t, z_i)\|z_i\| \\ & +\alpha_i^b(t, \|z_{id_i}\|)\|z_{id_i}\|\Big)\end{aligned} \tag{6.50}$$

where the scalar $S_{i2} \neq 0$ and η_i is the reachability constant. The functions $\zeta_i(\cdot)$, $\kappa_i^a(\cdot)$, $\kappa_i^b(\cdot)$, $\alpha_i^a(\cdot)$, and $\alpha_i^b(\cdot)$ are given in Assumption 6.3.1.

Theorem 6.4.2 Consider the system (6.25)-(6.26). Under Assumption 6.3.1, a delay dependent, decentralised control law in (6.48) with Ξ_i defined by (6.50) drives the system (6.25)-(6.26) to the sliding surface (6.31) and maintain a sliding motion on it thereafter.

Proof. From (6.30) and (6.26), it is easy to see that

$$\begin{aligned}\sum_{i=1}^{N} \frac{\sigma_i^T(z_i)\dot{\sigma}_i(z_i)}{\|\sigma_i(z_i)\|} = \sum_{i=1}^{N} \frac{\sigma_i^T(z_i)}{\|\sigma_i(z_i)\|}\Big\{S_{i2}H_i + S_{i2}B_iu_i \\ +S_{i2}B_iG_i(\cdot) + S_{i2}F_i^b(\cdot) + S_{i2}\sum_{\substack{j=1\\ j\neq i}}^{N}\Upsilon_{ij}^b(\cdot)\Big\}\end{aligned} \tag{6.51}$$

Substituting u_i in (6.48) into (6.51),

$$\sum_{i=1}^{N} \frac{\sigma_i^T(z_i)\dot{\sigma}_i(z_i)}{\|\sigma_i(z_i)\|} = \sum_{i=1}^{N} \frac{\sigma_i^T(z_i)}{\|\sigma_i(z_i)\|} \Bigg\{ S_{i2}H_i + S_{i2}B_i \Bigg(-(S_{i2}B_i)^{-1}\Big\{ S_{i2}H_i + \Big(\Xi_i + \eta_i\Big)\text{sgn}(\sigma_i(z_i))\Big\}\Bigg) + S_{i2}B_iG_i(\cdot) + S_{i2}F_i^b(\cdot) + S_{i2}\sum_{\substack{j=1 \\ j\neq i}}^{N} \Upsilon_{ij}^b(\cdot) \Bigg\} \tag{6.52}$$

From (6.50), (6.52) and Assumption 6.3.1, it follows that

$$\begin{aligned}
\sum_{i=1}^{N} \frac{\sigma_i^T(z_i)\dot{\sigma}_i(z_i)}{\|\sigma_i(z_i)\|} \leq & \Bigg(-\sum_{i=1}^{N} \frac{\sigma_i^T(z_i)\text{sgn}(\sigma_i(z_i))}{\|\sigma_i(z_i)\|}\|S_{i2}B_i\|\zeta_i(\cdot) \\
& + \|S_{i2}B_i\|\|G_i(\cdot)\|\Bigg) - \Bigg(\sum_{i=1}^{N} \frac{\sigma_i^T(z_i)\text{sgn}(\sigma_i(z_i))}{\|\sigma_i(z_i)\|}\|S_{i2}\| \\
& \Big(\alpha_i^a(t,z_i)\|z_i\| + \alpha_i^b(t,\|z_{id_i}\|)\|z_{id_i}\|\Big) - \|S_{i2}\|\|F_i(\cdot)\|\Bigg) \\
& - \Bigg(\sum_{i=1}^{N} \frac{\sigma_i^T(z_i)\text{sgn}(\sigma_i(z_i))}{\|\sigma_i(z_i)\|}\|S_{i2}\|\Big(\kappa_{ij}^a(t,z_j)\|z_j\| \\
& + \kappa_{ij}^b(t,\|z_{jd_j}\|)\|z_{jd_j}\|\Big) - \sum_{\substack{j=1 \\ j\neq i}}^{N} \|S_{i2}\|\|\Upsilon_{ij}^b(\cdot)\|\Bigg) \\
& - \sum_{i=1}^{N} \frac{\sigma_i^T(z_i)\text{sgn}(\sigma_i(z_i))}{\|\sigma_i(z_i)\|}\eta_i \leq -\sum_{i=1}^{N} \eta_i < 0
\end{aligned} \tag{6.53}$$

where the fact that $\|\sigma_i(z_i)\| \leq \sigma_i^T(z_i)\text{sgn}(\sigma_i(z_i))$ (see Lemma 1 in [107]) was used above. This shows that the reachability condition holds. Hence the conclusion follows.

From sliding mode control theory, Theorems 6.4.1 and 6.4.2 together guarantee that the closed-loop systems formed by applying controllers (6.48)–(6.50) to the system (6.25)-(6.26) are asymptotically stable.

Remark 6.4.2 From (6.48)-(6.50), it is clear to see that the bounds on the uncertainties have been employed in the controller design, and thus it is required that the bounds on the uncertainties are known. An adaptive technique [101], can be combined together with the method used in this chapter if bounds on uncertainties are unknown. Moreover, if there

is any unknown parameter in power system, the parameter can be estimated/identified by combining adaptive control technique.

6.5. SIMULATION EXAMPLE

In order to illustrate the obtained results, the theoretical result obtained here is applicable to N multi-area systems. However, for simplicity, we only consider 2-area systems. For a two-area LFC power system with time varying network delays, Table (6.1) shows the parameters and data used in obtaining the simulation results as in [68, 79]. The areas are

Table 6.1: Parameters of Two-Area LFC Scheme

Area	T_{pi}	K_{pi}	β_i	R_i	T_{gi}	T_{chi}	T_{ij}
Area 1	8	0.66	41.5	0.05	0.4	0.17	0.05
Area 2	8	0.55	61.8	0.05	0.35	0.2	0

interconnected via a tie-line. Thus, as the system load varies, turbine speed also varies which in turn changes the frequency $f(t)$ and the tie-line power $P_{tie}(t)$ of the generating unit. This incremental variation is sensed by the sensing devices which generates the area control error $ace_i(t)$, and transmitted to the integral controller $E_i(t)$ via an open channel.

By using the algorithm in [13], the coordinate transformation $z_i = T_i x_i$ for $i = 1, 2$ can be obtained with T_i defined by

$$T_1 = T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 30 & 62 & 42.5 & 11.50 & 1 \end{bmatrix} \tag{6.54}$$

In the new coordinates $z = \text{col}(z_1, z_2)$ the two area LFC power system can be described by system (6.19) with $N = 2$ as follows

Area 1:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{13} & A_{14} \end{bmatrix} = \left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -30 & -62 & -42.5 & -11.50 & 1 \\ \hline -89 & -156 & -43 & -8 & 3 \end{array}\right] \tag{6.55}$$

$$\begin{bmatrix} E_{11} & E_{12} \\ E_{13} & E_{14} \end{bmatrix} = \left[\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.01 & 0.08 & 0 \\ \hline 0.38 & 50.36 & 0.06 & 0.45 & 0 \end{array}\right]$$

$$B_1 = \begin{bmatrix} 0_{4\times 1} \\ 1 \end{bmatrix}, \quad F_1 = \begin{bmatrix} -0.08(z^a_{1d_1})^2 \\ 0_{3\times 1} \\ -2.48(z^b_1)^2 \end{bmatrix}, \quad \Upsilon_{12} = \begin{bmatrix} 0_{4\times 1} \\ -0.31\text{sin}(z^b_2) \end{bmatrix}$$

Area 2:

$$\begin{bmatrix} A_{21} & A_{22} \\ A_{23} & A_{24} \end{bmatrix} = \left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -30 & -62 & -42.5 & -11.5 & 1 \\ \hline -105 & -188 & -69 & -13 & 4 \end{array}\right] \tag{6.56}$$

$$\begin{bmatrix} E_{21} & E_{22} \\ E_{23} & E_{24} \end{bmatrix} = \left[\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.009 & 0.069 & 0 \\ \hline 0.001 & 40.76 & 0.055 & 0.438 & 0 \end{array}\right]$$

$$B_2(\cdot) = \begin{bmatrix} 0_{4\times 1} \\ 1 \end{bmatrix}, \quad F_2(\cdot) = \begin{bmatrix} -0.07(z^a_{1d_1})^2 \\ 0_{3\times 1} \\ -2.06(z^b_1)^2 \end{bmatrix} \tag{6.57}$$

Based on Assumption 6.3.1. The unknown matched uncertainty $G_i(\cdot)$ and $F_i^b(\cdot)$ is assumed to satisfy

$$\begin{aligned}\|G_1(\cdot)\| &\leq \underbrace{3|z_{1d_1}^b|\sin^2(z_1)}_{\zeta_1(\cdot)}\\ \|G_2(\cdot)\| &\leq \underbrace{2\sin^2 z_2^a\|z_2\|}_{\zeta_2(\cdot)}\end{aligned}$$

$$\begin{aligned}\|F_1^b(\cdot)\| &\leq \underbrace{\sin^2(z_1)\|z_1\| + 0.8\|z_{1d_1}\|}_{\alpha_1(\cdot)}\\ \|F_2^b(\cdot)\| &\leq \underbrace{2\cos^2(z_2)\|z_2\| + 0.5\|z_{2d_2}\|}_{\alpha_2(\cdot)}\\ \|\Upsilon_{12}^b(\cdot)\| &\leq \underbrace{0.5\cos^2(z_2)\|z_2\|}_{\kappa_{12}(\cdot)}\end{aligned}$$

Therefore, on the sliding surface (6.31), when the sliding motion takes place. Since A_{11} and A_{21} in (6.55)-(6.56) are stable, it follows that choosing $Q_1 = Q_2 = I_2$ and solving the Lyapunov equation (6.32) has a unique solution

$$P_1 = P_2 = \begin{bmatrix} 2.0559 & -0.5 & -0.8979 & 0.5 \\ -0.5 & 0.8979 & -0.5 & -1.7320 \\ -0.8979 & -0.5 & 1.7320 & -0.5 \\ 0.5 & -1.7320 & -0.5 & 9.9248 \end{bmatrix} \tag{6.58}$$

Further, from (6.36)-(6.37),

$$\begin{aligned}\beta_1^a(\cdot) = 3\sin^2(z_1^a), \quad \beta_1^b(\cdot) = 2\sin^2(t)\\ \beta_2^a(\cdot) = 2|z_2^a|^4, \quad \beta_2^b(\cdot) = |z_{2d_2}^a|\\ \chi_{12}^a(\cdot) = 3\sin^2(z_2^a), \quad \chi_{12}^b(\cdot) = 0\end{aligned}$$

By direct calculation, W defined in (6.38) is positive definite and it is easy to verify that the conditions in Theorem 6.4.1 are satisfied. Thus the sliding motion associated with the sliding surface is uniformly asymptotically stable. From theorem 6.4.2 and the theoretical

anlysis in Section 6.4, the decentralised control law

$$u_1 = -\Big\{H_1 + \Big(3|z_{1d_1}^b|\sin^2(z_1) + (\sin^2(z_1)\|z_1\| + 0.8\|z_{1d_1}\|) + 0.5\cos^2(z_2)\|z_2\| + 0.8\Big)\mathrm{sgn}(\sigma_i(z_i))\Big\} \quad (6.59)$$

$$u_2 = -\Big\{H_2 + \Big(2\sin^2 z_2^a\|z_2\| + 2\cos^2(z_2\|z_2\| + 0.5\|z_{2d_2}\| + 0.5\Big)\mathrm{sgn}(\sigma_i(z_i))\Big\} \quad (6.60)$$

where H_i is defined in (6.49), can stabilise the two area power system uniformly asymptotically.

Due to effects of load variation and wide area communication time delays which may affect LFC power network. Larger time delays have been considered in this LFC design to forestall future growing network complexity. This is illustrated by introducing delays into the power system as discussed in this chapter. Studies have shown that ace signal processing could span about 2s or more [3, 116], so in a wider area network, large delays may arise which can hinder the controller from neutralizing the oscillations caused by the delay. Thereby, re-emphasizing the importance of considering delays in the design of LFC.

6.5.1. SIMULATION RESULTS AND ANALYSIS

The continued growth in wide area network (WAN) and consumers will often pose challenges to power systems due to increasing communication delays and constant load variations. The fast expansion and longer transmission in power systems has the tendency to introduce longer delays due to signal processing and filtering. Considering the effect of time delay in LFC, two cases are considered here. a) The case of constant and time-varying delay during step load disturbance. b) The case of constant and time-varying delay at random load disturbance. In order to validate the analytical results, comparison is made with non-linear sliding mode controller (NSMC) [68], for similar plant parameters, load disturbance, but larger constant and time-varying delay bound which is achieved using Lyapunov Razumikhin approach as compared with the maximum upper bound obtained in [68], which has limitation to the change rate of the time delay.

- Case 1: Step and Random Load Disturbances with Constant Delay

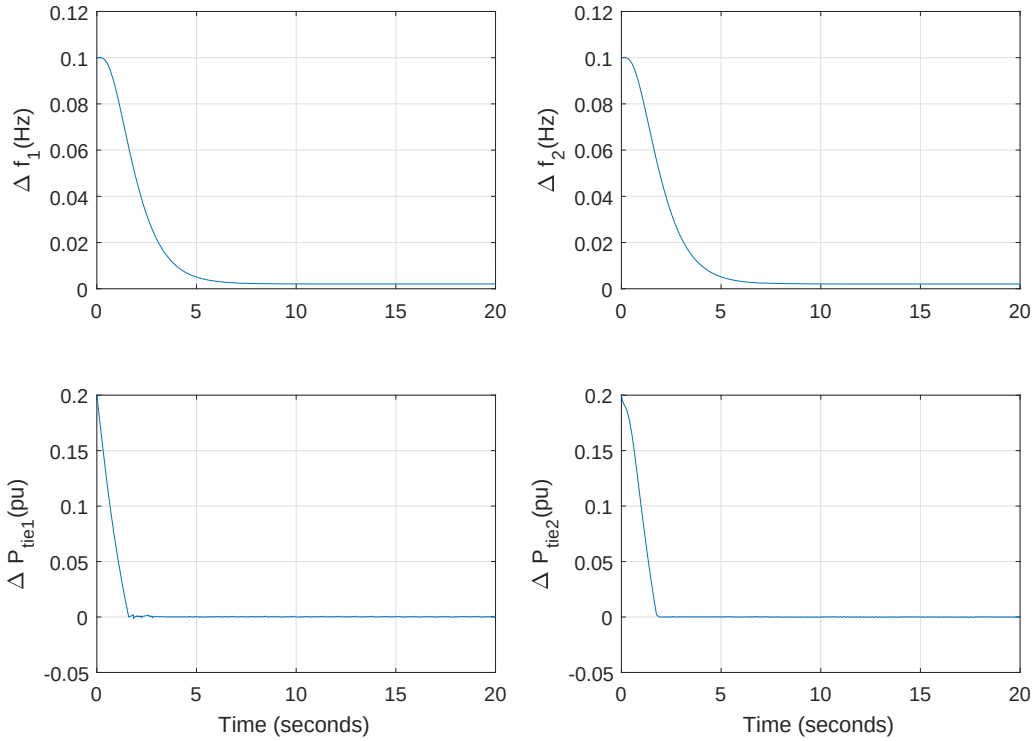

Figure 6.2: Time response of Δf_i and ΔP_{tie_i} for step load disturbance with constant delays $d_1 = 5.5s,\ d_2 = 6.5s$.

The result of the proposed LFC scheme is compared with the maximum delay bounds 1.8s and 2.5s for both areas 1 & 2 respectively as obtained in [68]. Figs 6.2 shows the response of the frequency deviations Δf_i, and tie-line power deviation ΔP_{tie_i} at large constant delays 5.5s and 6.5s for illustration in the proposed LFC scheme. The obtained results shows that the frequency deviation is suppressed significantly for step load disturbance and the interchange power deviation is totally minimized. The effects of random load disturbances is observed in Fig 6.3 for same value of large constant delay. The same result is observed as the frequency deviation is maintained within specified limits ($\pm 0.02Hz$), which verifies the performance of the sliding mode LFC to force the steady state frequency deviation to zero.

- Case 2: Step and Random Load Disturbances with Time-varying Delay

  Here we consider large time-varying communication delay for step and random load disturbances. Due to high fluctuations in renewable integration, change rate of

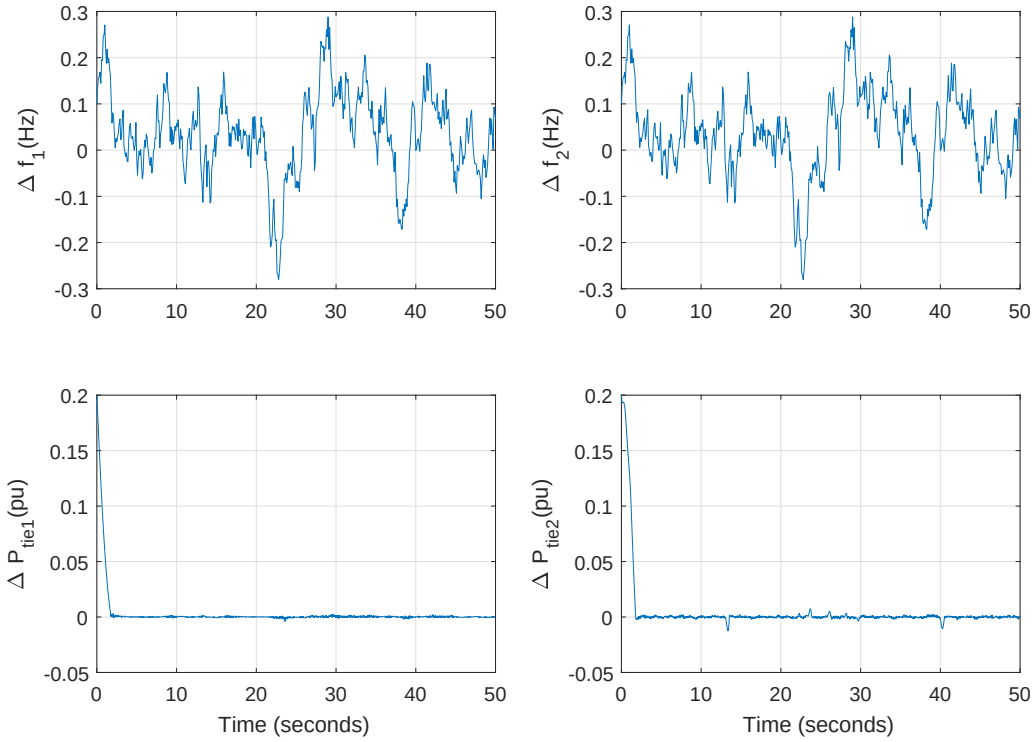

Figure 6.3: Time response of Δf_i and ΔP_{tie_i} for random load disturbance with constant delays $d_1 = 5.5s,\ d_2 = 6.5s$.

communication delays may be large. Compared with the maximum upper bounds for time-varying delays, 320 and 350ms obtained in [68], actually, time varying delay $d_1(t) = 3 + 0.5\text{sin}t$ and $d_i(t) = 4 + 0.5\text{cos}t$ were also used for simulation in [58] which has low change rate as $\dot{d}_i(t) = 0.5\text{cos}t$. Fig 6.4 shows the results of the time-varying delay with constant load change. There are no limitations to the change rate of the time delays and the response of frequency deviation, tie-line and control signal are shown. The frequency deviation approaches zero asymptotically and the control signal drives the interchange power to zero after a slight overshoot. Fig 6.6 shows similar response of time-varying delay at random load change which shows consistency in steady state frequency deviation. It has been shown that the developed control scheme can handle large time delay with fast change rate, giving it more flexibility compared with many existing works, where the time delay is either constant or there is limitation to the change rate of the time delay. However, chattering is observed in the control signal, but can be minimized to avoid wear-out

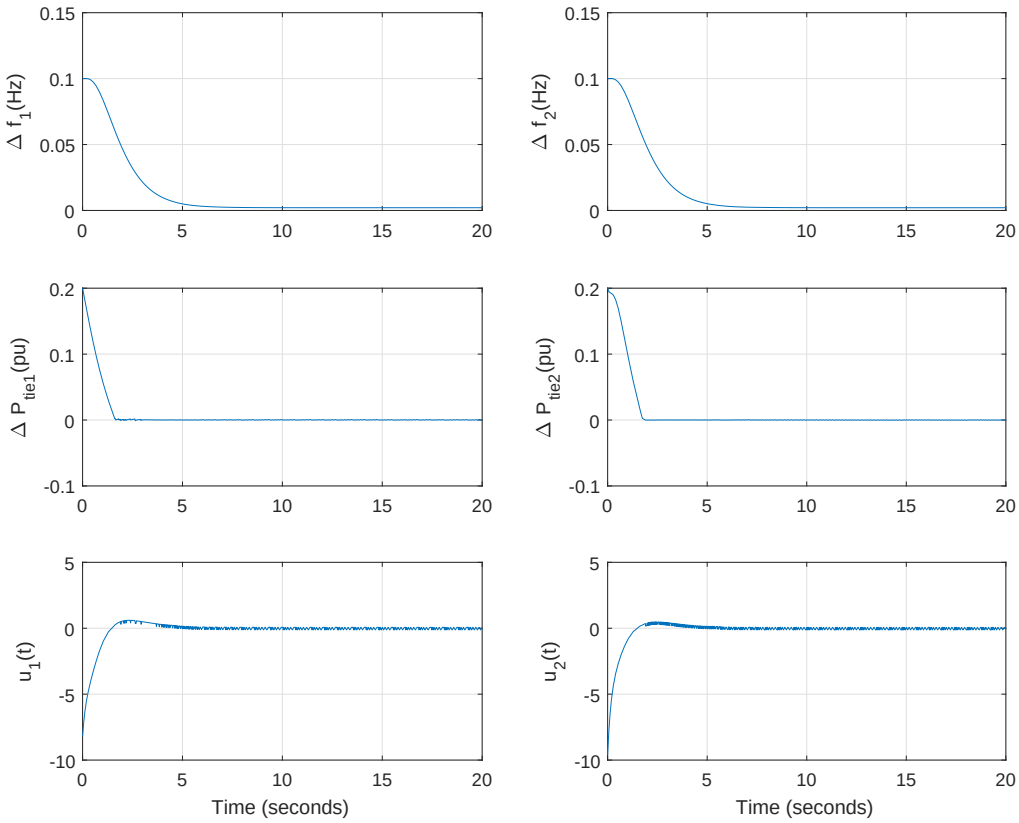

Figure 6.4: Time response of Δf_i, ΔP_{tie_i} and u_i for step load disturbance with time-varying delays $d_1 = 5 + 4\text{sin}t,\ d_2 = 6 + 10\text{sin}t$.

of the actuator valve.

6.5.2. SYSTEM RESPONSE WITH NON-LINEARITIES

Power system is associated with numerous uncertainties which can be due to un-modelled dynamics, continuous parameter variations and inexact measurements, all of which may result in generator-load mismatch [3, 86]. In power system, there are many non-linearities which affect system performance. In this chapter, these are modelled as matched and mismatched uncertainties, and thus compared with existing literatures [68, 84], the non-linearities are bounded by a known function which can be employed in the design to reduce conservativeness.

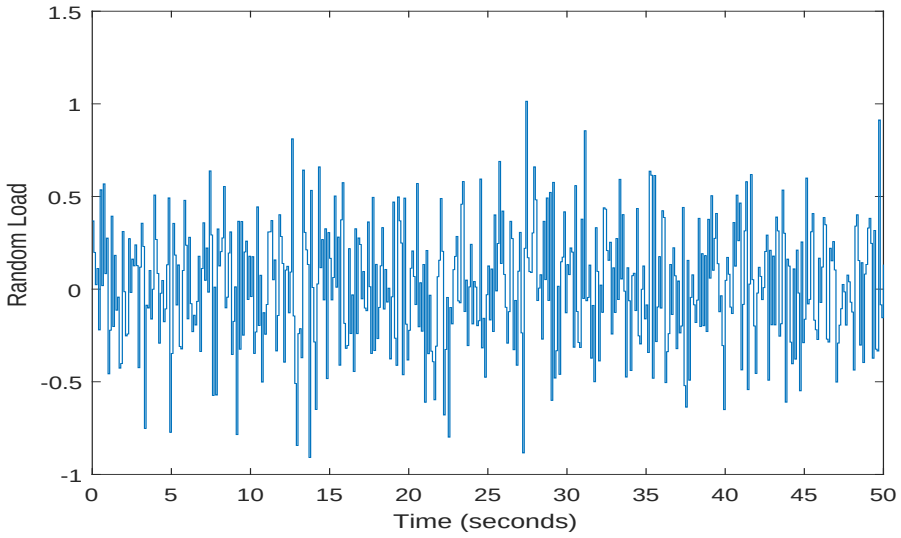

Figure 6.5: Time response of random load ΔP_L.

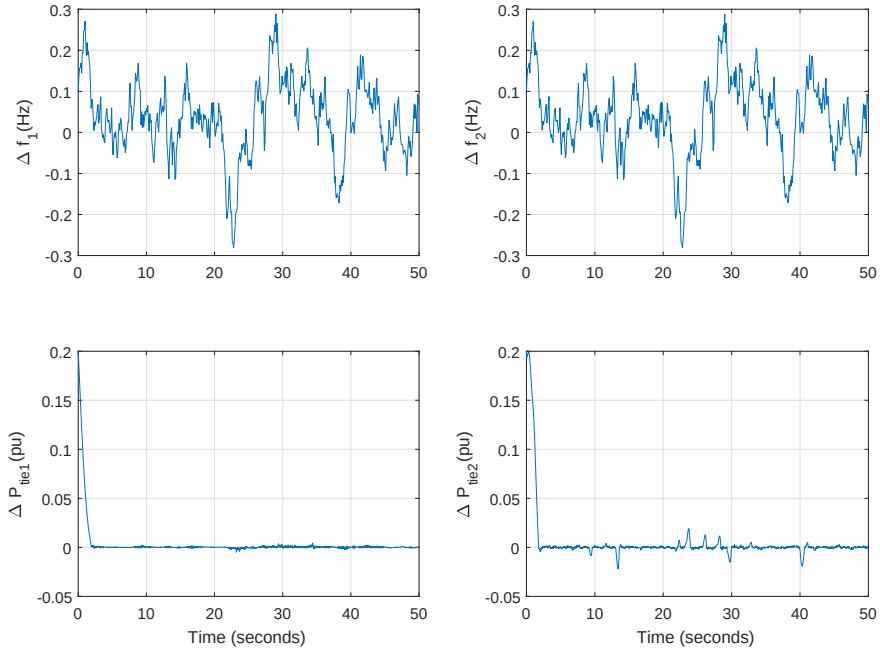

Figure 6.6: Time response of Δf_i, ΔP_{tie_i} for random load disturbance with time-varying delays $d_1 = 5 + 4\text{sin}t,\ d_2 = 6 + 10\text{sin}t$.

6.6. SUMMARY

A decentralised state feedback sliding mode control based on the Lyapunov-Razumikhin theorem has been proposed for non-linear interconnected power systems with time-varying delays and uncertainties. A more systematic approach for dealing with large constant and time-varying communication delays and uncertain non-linear interconnection in the multi-area power system have been presented. By employing the delay and exploiting the bounds on the uncertainties in both the sliding motion analysis and the control design, conservatism is reduced and the robustness is enhanced. The simulation results on a 2-area interconnected power systems have demonstrated the effectiveness of the obtained results and further illustrate the feasibility of the proposed methodology.

CHAPTER 7

CONCLUSION AND FUTURE WORK

7.1. CONCLUSIONS

In this thesis, a survey of the fundamentals and history of SMC and time delay systems with application to power systems have been presented. The systems structure with non-linear delayed disturbances were fully considered and it's applications have been tested on power system models. Although the issue of communication delays in wide area network remains an open problem for multi-area power systems integration, the results presented in this thesis has improved several results in this domain. This thesis has been presented in seven chapters.

In the first and second chapters of this thesis, an introductory knowledge and literature reviews have been presented alongside main contributions of the thesis, followed by the mathematical preliminaries which have been used. The third chapter presents a general introductory knowledge and concepts of controllability, linear feedback control system, SMC, time delay systems, interconnected systems and power systems with simplified illustrative examples have been presented.

In the fourth chapter, a systematic approach for dealing with a class of time delay systems with non-linear disturbances is proposed. The objective of the designed delay dependent smc controller is to force the system position to track a desired trajectory. The systems stability is analysed based on Lyapunov Razumikhin method combined with basic LMI technique to achieve a less conservative result such that the developed uncertainty bounds can be fully applied in the controller design for rejecting the effect of uncertainty in the system. In order to make the system more robust to uncertainties, much focus has been placed on disturbance tolerability, and matched and mismatched uncertainty is also considered which involve time-varying delay. The simulation result indicate that the proposed controller works well for the AVR system and the numerical example. More so, the simulation studies indicate that the proposed control scheme is robust to bounded uncertainties.

A load frequency SMC for reheat turbine system with turbine-end delays and multiple delayed non-linear term is proposed in the fifth chapter. The objective is to introduce turbine-end delays which will account for unnecessary delays in the turbine region, and also consider multiple delay around the input which may result in increased oscillation. Various Lyapunov stability methods have been used to achieve sufficient results such that the multiple delays and non-linearities within the modified time delay system are stable within bounds and a sufficient ultimate bound is achieved for step load disturbances. Load frequency SMC with reduced chattering effects have been developed, and simulation is conducted using matlab/simulink tool for a single area reheat turbine generating unit, such that the closed-loop performance of the considered system is ensured. Result obtained from simulation shows a robust and effective designed controller. This technique is useful for protecting the turbine further in real application, and also reduced oscillations caused by input delays.

The sixth chapter considers the multi-area power systems. It presents a systematic approach for dealing with large constant and time-varying communication delays and uncertain non-linear interconnection in the multi-area power system. Robustness has been achieve by dealing with the interconnection as a separate non-linear uncertain term. The parameter uncertainties are assumed to be matched and based on Lyapunov-Razumikhin

approach. There is no limitation to the size of the delay. In addition, the system allows the bounds on the uncertainties not only have more general non-linear form but also involve time delay. A two area LFC model is simulated to show the feasibility of the developed results and the effectiveness of the proposed method. Furthermore, the technique developed here is robust and sufficient to adapt future growing network delays and uncertain interconnection.

7.2. FUTURE WORK

Future research interest is shaped by the emerging trend towards the increase in size and complexity of interconnected systems due to renewable energy integration. Many challenging issues associated with time delay such as stabilisation, robust performance analysis etc., are still open areas in almost every engineering application and cannot be ignored. Therefore, it is meaningful to explore further ways to improve systems structure and applications.

Although, this thesis have only considered single input single output (SISO) systems, multi input multi output (MIMO) systems will be considered in the future. In order to show the reliability and validity of the proposed method, further comparative studies will be conducted in future work. As wide area network communication time delay increases due to large-scale industrial processes. It is important to understand that certain systems may be at risk of unknown delays and parameters, thereby increasing its likelihood to risk of uncertainties. Therefore, it will be interesting to extend my research to delay independent control cases, a phenomenon of unknown delays as compared with known delay cases in previous works. Also, the use of Lyapunov Krasovskii methods of time delay stability will be studied as compared with Razumikhin in this thesis. More so, if unknown parameters exists in considered system, adaptive technique may be employed in the future.

From application perspective, a much more integrated approach to SMC design will be studied and the developed results will be applied to broad areas of power systems network and other energy related applications.

APPENDIX A

LIST OF PUBLICATIONS

- Onyeka, A.E., Yan, X.G., Mao, Z. and Mu, J., 2019. Stabilisation of time delay systems with nonlinear disturbances using sliding mode control. *International Journal of Modelling, Identification and Control*, 31(3), pp.259-267.

- Onyeka, A.E., Yan, X.G., Mao, Z. and Zhang, Q., 2018, June. Decentralised Sliding Mode LFC for Nonlinear Interconnected Power System with Time Delay. *Annual American Control Conference (ACC)* Milwaukee, USA, 27-29 June, 2018.

- Onyeka, A.E., Yan, X.G. and Mu, J., 2017. Sliding Mode Control of Time-Delay Systems with Delayed Nonlinear Uncertainties. *20th World Congress of the International Federation of Automatic Control (IFAC)*, 9-14 July 2017, Toulouse, France.

- Onyeka, A.E., Yan, X.G., Mao, Z. and Jiang, B., 2019, June. Reheat Turbine LFC of Power Systems with Multiple Delays Based on Sliding Mode Techniques. *12th Asian Control Conference (ASCC)*, 9-12 June 2019, Kitakyushu, Fukuoka, Japan.

- Onyeka, A.E., Yan, X.G., Mao, Z. and Jiang, B. Robust Decentralised LFC for Interconnected Time Delay Power Systems using Sliding Mode Techniques.

IET Generation, Transmission & Distribution, 2019 (Submitted).

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