Denoising The Equity Premium

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Introduction

- The issue of forecasting equity returns is one of the most widely discussed topics in the finance literature
  - mainly due to its central role in asset pricing, portfolio allocation and evaluation of investment managers
- The in-sample predictive ability of a quite exhaustive list of potential predictors was the focus of the earlier studies
  - that typically contains valuation ratios, various interest rates and spreads, distress indicators, inflation rates along with other macroeconomic variables, indicators of corporate activity, etc.
- Lately, interest has turned to the out-of-sample performance of the candidate variables

Literature Review

- Goyal and Welch (2008) show that their long list of predictors can not deliver consistently superior out-of-sample performance
  - variety of predictive regression models ranging from single variable ones to their 'kitchen sink' model

- Campbell and Thompson (2008) show that when imposing simple restrictions, suggested by economic theory, on predictive regressions' coefficients, the out-of-sample performance improves and market timing strategies can deliver profits to investors
  - (see also Ferreira and Santa-Clara (2011))

- Ludvigson and Ng (2007) and Neely, Rapach, Tu and Zhou (2011) adopt a diffusion index approach
  - which can conveniently track the key movements in a large set of predictors, and they find evidence of improved equity premium forecasting ability

Literature Review

- Rapach, Strauss and Zhou (2010) employ forecast combinations of individual single variable predictive regression forecasts,
  - which help reduce model uncertainty/parameter instability and find evidence of improved equity premium predictability

- Meligkotsidou, Panopoulou, Vrontos and Vrontos (2013) incorporate the forecast combination methodology in a quantile regression setting
  - Robust and accurate equity premium forecasts are produced by combining a set of predictive quantile regressions in either a fixed or time-varying manner (see also Lima and Meng, 2017)

- Elliott, Gargano and Timmermann (2013) propose a novel forecast combination method based on complete subset regressions
  - i.e. combining forecasts from all possible linear regression models that keep the number of predictors fixed
  - Their empirical application on equity premium predictability shows that subset combinations of up to four predictors generates superior forecast accuracy

Motivation

- Previous studies have shown that a variety of economic variables fails to deliver consistently accurate out-of-sample forecasts for the equity premium

- Literature suggests that:
  - Quantile Regression outperforms alternative methods

  - Wavelet Analysis is a powerful tool for financial time-series analysis

Methodology

- In this study we propose a wavelet decomposition/denoising framework in the context of equity premium forecasting

1. We decompose the time-series using wavelet analysis

2. We remove the noise in different frequencies by restricting the wavelet coefficients

3. Finally, we apply a quantile combination approach in the decomposed/denoised time-series in order to forecast the equity premium

Univariate Models

- The standard univariate predictive regressions estimated by OLS

$$r_{t+1} = \alpha_i + \beta_i x_{i,t} + \varepsilon_{t+1}, \quad i = 1,\ldots,N$$

- where $x_{i,t}$ is a variable whose predictive ability is of interest;
- $\varepsilon_{i,t+1}$ is an i.i.d. error term
- α_i and β_i are respectively the intercept and slope coefficients specific to model $i = 1,\ldots,N$.
- Each univariate model i yields its own forecast $f^{i}_{t+1,t} = \hat{\alpha}_i + \hat{\beta}_i x_{i,t}$

Quantile Combination Forecast

- The quantile regression model is given by:

$$r_{t+1} = \alpha_i^{(\tau)} + \beta_i^{(\tau)} x_{it} + \varepsilon_{t+1}, \quad i = 1, \ldots, N$$

- where $\tau \in (0,1)$ and the errors ε_{t+1} are assumed independent from an error distribution $g_\tau(\varepsilon)$ with τ^{th} quantile equal to 0, i.e. $\int_{-\infty}^{\tau} g_\tau(\varepsilon) d\varepsilon = \tau$

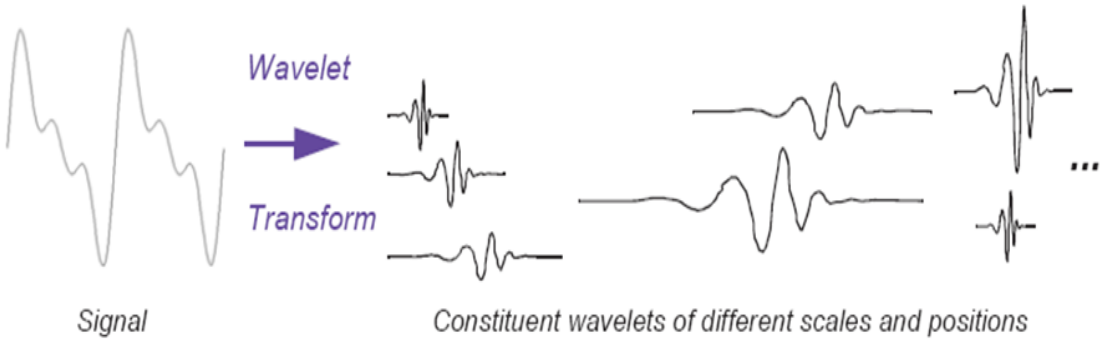
- We consider a discrete grid of quantiles $\tau \in (\tau_1, \tau_2, \ldots, \tau_J)$

- We follow Lima and Meng (2017) for a quantile combination scheme

$$QC = \frac{1}{3} f_{t+1,t}^{0.3} + \frac{1}{3} f_{t+1,t}^{0.5} + \frac{1}{3} f_{t+1,t}^{0.7}$$

Wavelet Analysis

- Wavelet analysis provides a powerful tool to decompose time-series data into orthogonal components with different frequencies
  - the method can accommodate structural change, discontinuity and regime shifts
  - common characteristics of financial time series



Wavelet Analysis

- The Maximal Overlap Discrete Wavelet Transform

$$f(t) = \sum_k s_{j,k}\varphi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + \cdots + \sum_k d_{1,k}\psi_{j,k}(t)$$

- where

$$s_{j,k} = \int_{-\infty}^{+\infty} f(t)\varphi_{j,k}(t)dt, \qquad k, j \in \mathbb{Z}$$

$$d_{j,k} = \int_{-\infty}^{+\infty} f(t)\psi_{j,k}(t)dt, \qquad k, j \in \mathbb{Z}$$

Wavelet Analysis

- By setting

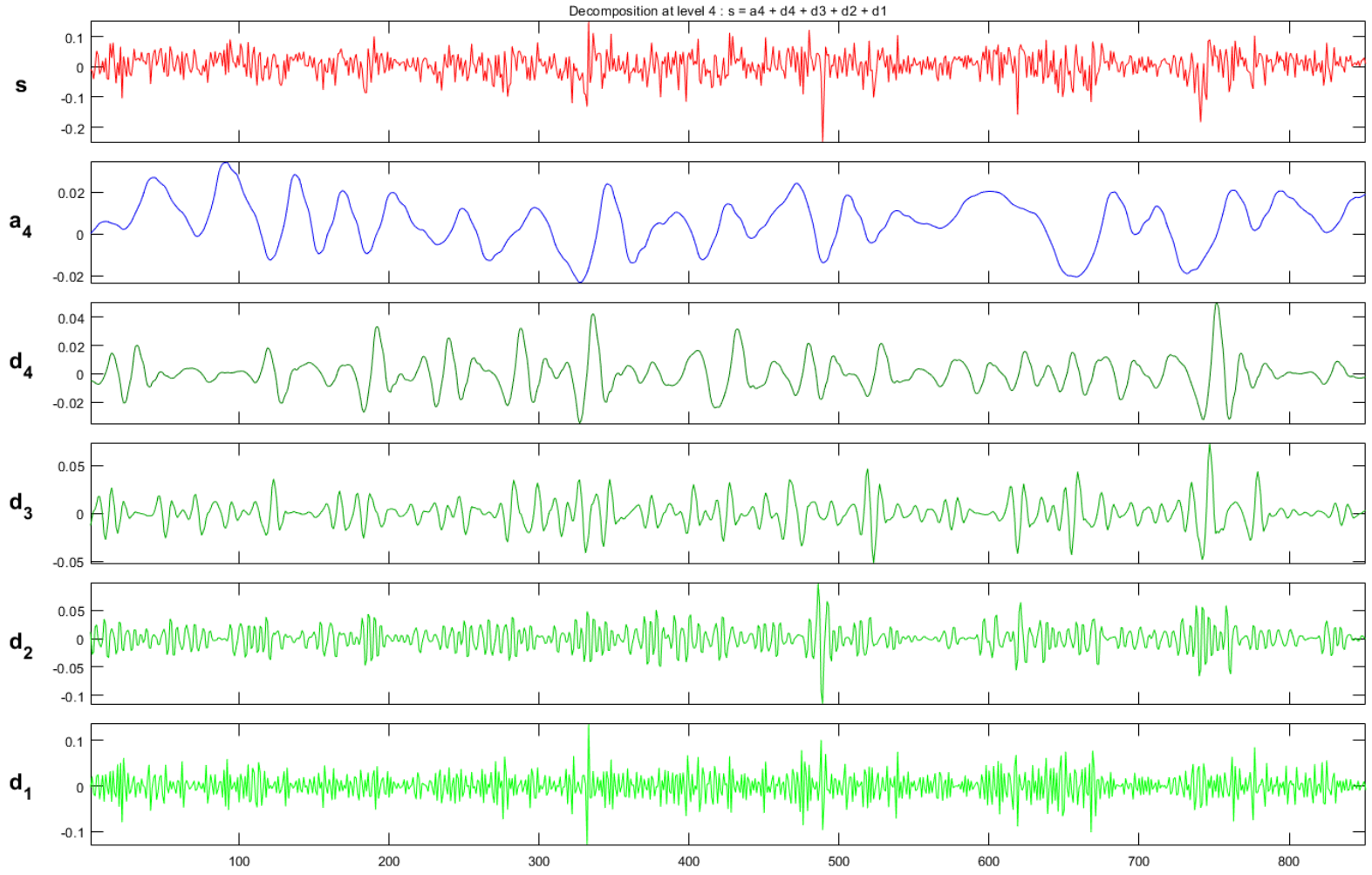
$$S_j(t) = \sum_k s_{j,k}(t)\varphi_{j,k}$$

$$D_j(t) = \sum_k d_{j,k}(t)\psi_{j,k}$$

- Multiresolution Analysis (reconstruction of the original time-series)

$$f(t) \approx \sum_{k \in Z} s_{J,k}\varphi_{J,k}(t) + \sum_{j \in Z}\sum_{k \in Z} d_{j,k}\psi_{j,k}(t) = S_J(t) + D_J(t) + D_{J-1}(t) + \cdots + D_1(t)$$

Wavelet Decomposition of Returns



Wavelet Decomposition and Denoising

- Wavelet X Y
- Wavelet X
- Wavelet Y
- Wavelet Y last
- Wavelet Denoise (Donoho and Johnstone's universal threshold and level-dependent thresholding)
  - XY
  - X
  - Y
- Wavelet Denoise Soft
  - X
  - XY
  - Y

Data

- The data we employ is the updated Goyal and Welch (2008) dataset
  - The equity premium is calculated as the difference of the continuously compounded S&P500 returns, including dividends, and the Treasury Bill rate

- Out-of-sample forecasts of the equity premium are generated by continuously updating the estimation window
  - i.e. following a recursive (expanding) window

Data

- Our forecasting experiment is conducted on a annual, quarterly and monthly basis
- Data span 1947:1 to 2017:12
- Initial holdout in sample period: 1947:1 to 1966:12 (20 years)
- Out-of-sample forecast evaluation period covers the period 1967:1-2017:12

Predictors

- DP = log dividend-price ratio,
- DY = log dividend yield,
- EP = log earnings-price ratio,
- DE = log dividend-payout ratio,
- SVAR = equity risk premium volatility,
- BM = book-to-market ratio,
- NTIS = net equity expansion,
- TBL = Treasury bill rate (annual %),
- LTY = long-term bond yield (annual %),
- LTR = long-term bond return (%),
- TMS = term spread (annual %),
- DFY = default yield spread (annual %),
- DFR = default return spread (%),
- INFL = inflation rate (%).
- IK = Investment to capital

Forecasting Evaluation

- The natural benchmark forecasting model is the historical mean (PM)
  - according to which the forecast of the equity premium coincides with the constant in the linear regression model when no predictor is included, i.e. $k=0$

- As a measure of forecast accuracy, we employ the out-of-sample R^2 computed as

$$R^2_{OS} = 1 - \frac{MSFE_i}{MSFE_{PM}}$$

  - $MSFE_i$ is the Mean Square Forecast Error associated with each of our competing models and specifications
  - $MSFE_{PM}$ is the respective value for the PM model, both computed over the out-of-sample period.

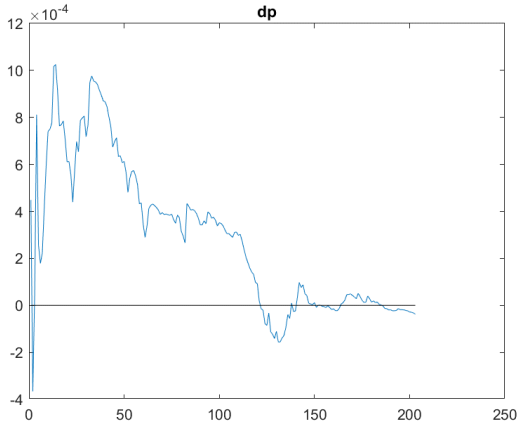
Quarterly – Out-of-Sample R^2

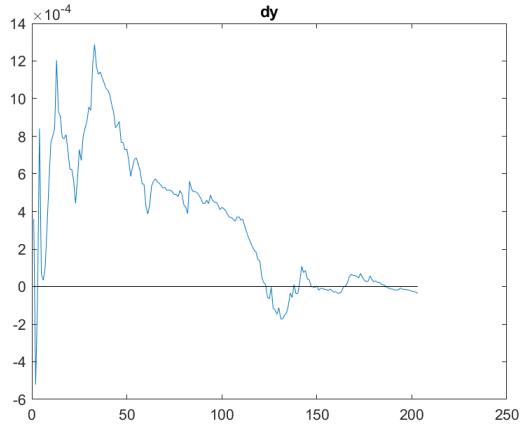
	dp	dy	ep	de	svar	bm	ntis	tbl	lty	ltr	tms	dfy	dfr	infl	ik	Mean
Linear	-0.56	-0.52	-1.52	-1.96	-11.15	-2.66	-2.17	-2.00	-2.53	-1.38	-2.32	-3.17	**0.34**	**0.49**	**2.60****	**2.53*****
Quantile	**1.29***	**2.31****	-1.33	-0.26	-17.92	-3.69	-4.46	-1.14	-1.34	-5.56	-5.55	-5.87	-0.16	**0.13**	**2.96*****	**2.55*****
Wavelet Y Last	**1.16***	**1.61****	-1.06	-2.66	-3.00	-1.83	-1.70	-0.22	-1.97	**0.53**	**0.83***	-3.36	**0.27**	**1.39**	**2.82****	**1.08***
Wavelet Y Den	**0.68***	**0.81***	-1.00	-2.67	-0.78	-1.69	-1.98	-0.70	-2.08	-0.11	**0.01***	-2.35	**1.22**	**1.52**	**2.71****	**1.65****
W Y Den s	**0.98***	**1.35****	-0.80	-2.23	-0.89	-1.48	-1.60	**0.06***	-1.59	**0.37**	**0.50**	-2.69	**0.72**	**1.63**	**2.75****	**1.28****
WQ Y	**1.55****	**1.79****	-0.92	**0.96**	-11.84	-2.77	-3.30	**0.50****	-0.16	-1.29	-3.52	-6.62	**0.21**	**0.10**	**3.64*****	**2.50*****
WQ Y Last	**1.71****	**2.36****	-0.14	-0.33	-0.88	-0.80	-0.55	**0.78****	-0.77	**0.81****	**1.62****	-1.60	**1.53****	**1.91***	**3.40*****	**1.91*****
WQ Y Den	**1.54****	**2.24****	-0.09	-0.23	-1.09	-1.85	-1.01	-0.37	-1.05	-0.18	**0.51***	-1.69	**1.31***	**2.50***	**3.64*****	**2.15*****
WQ Y Den s	**1.73****	**2.36****	-0.16	-0.42	0.41	-1.21	-0.43	**0.78***	-0.43	**0.63**	**1.48***	-1.39	**1.49***	**2.21***	**3.21*****	**1.93*****

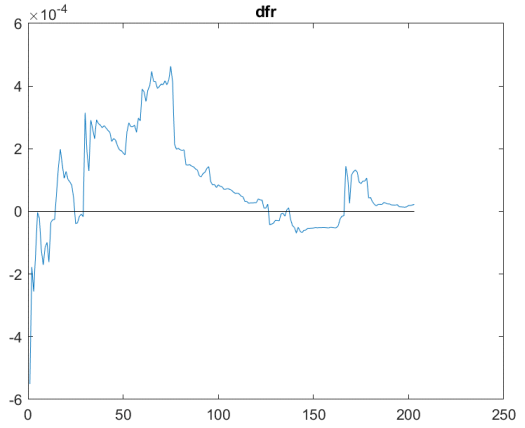
Quarterly – IPOCID

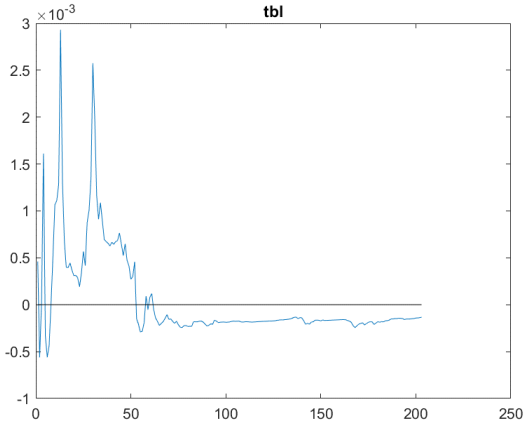
Benchmark	29.7															
	dp	dy	ep	de	svar	bm	ntis	tbl	lty	ltr	tms	dfy	dfr	infl	ik	Mean
Linear	65.84	48.02	62.87	42.08	52.97	50.99	41.58	41.09	**43.56**	49.50	46.04	41.09	48.02	44.06	47.03	44.06
Quantile	63.37	47.03	**63.86**	43.07	**57.43**	52.97	**47.52**	43.07	40.10	48.02	**48.02**	45.05	**49.01**	43.07	47.03	45.54
Wavelet Y Last	63.37	45.54	56.44	38.61	41.09	32.67	39.11	39.60	41.58	49.01	43.56	38.61	42.08	45.05	43.56	35.15
Wavelet Y Den	**67.33**	**50.00**	60.40	43.56	45.05	45.05	44.55	40.10	43.07	49.50	45.54	30.69	47.52	47.03	47.03	42.57
Wavelet Y Den s	66.34	49.01	59.41	41.09	34.65	34.65	40.10	40.10	**43.56**	49.01	45.05	35.15	46.53	46.04	46.53	37.62
WaveletQuant Y	65.84	48.51	61.88	**47.03**	56.44	**54.46**	44.55	**45.05**	42.08	**50.99**	45.54	**52.48**	48.51	43.07	47.03	**48.02**
WaveletQuant Y Last	60.40	47.03	51.49	40.59	36.63	41.58	42.08	41.09	43.07	50.00	46.53	39.60	47.03	47.52	46.53	41.09
WaveletQuant Y Den	**67.33**	49.01	53.96	43.07	45.05	44.06	43.56	41.58	40.10	48.51	47.03	35.15	48.51	**48.02**	**48.51**	41.09
WaveletQuant Y Den s	64.85	47.03	48.51	36.14	38.61	35.64	38.61	43.07	42.08	50.00	44.06	34.65	45.05	46.53	42.08	40.10

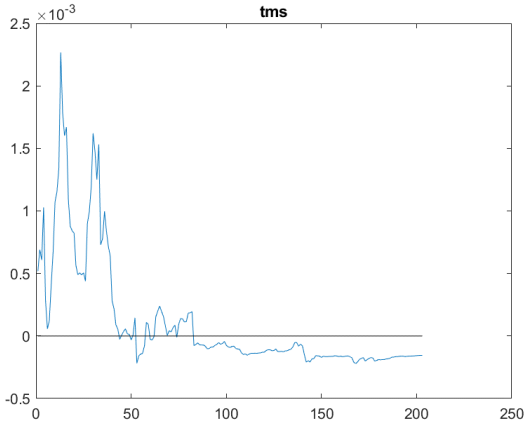
Linear Model – Cumulative Squared Prediction Error Minus the Benchmark

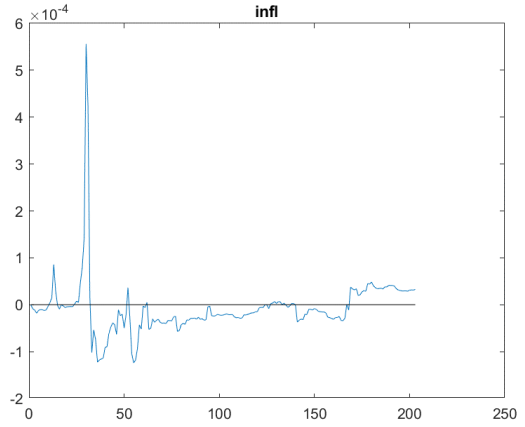


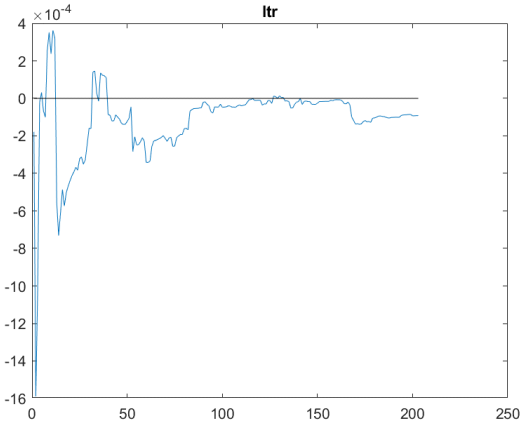


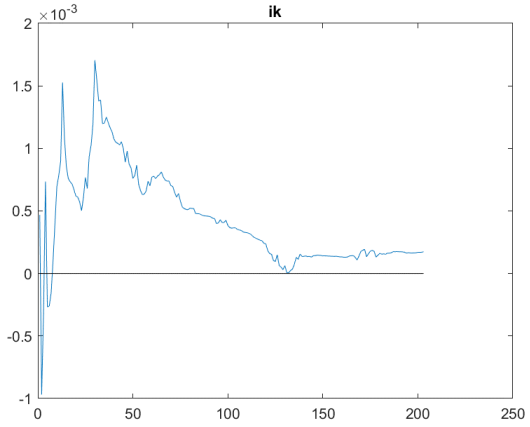




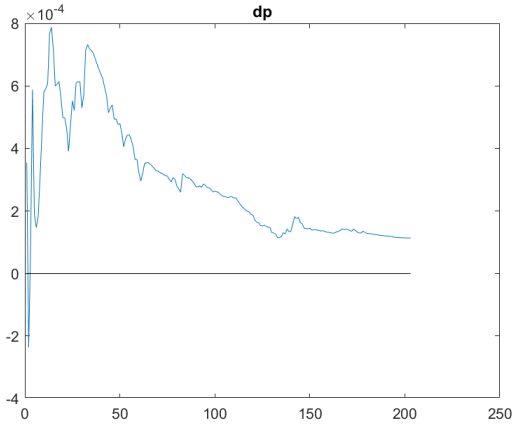


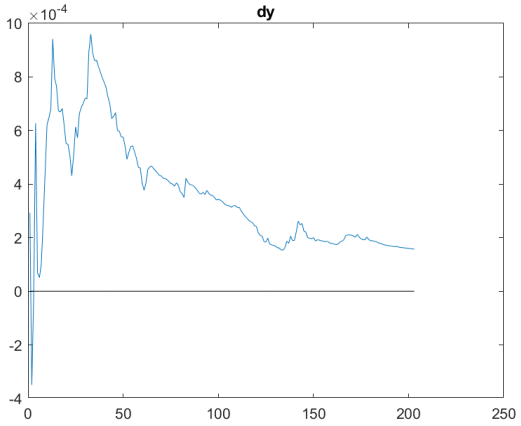


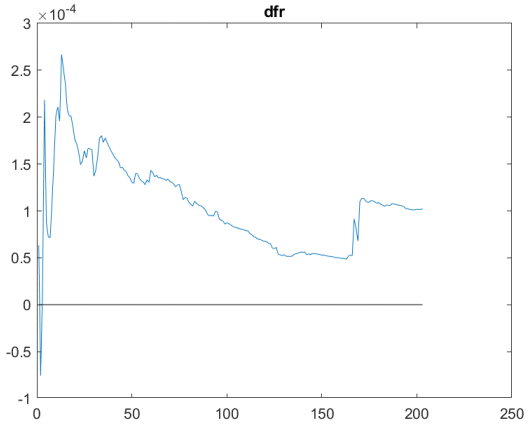


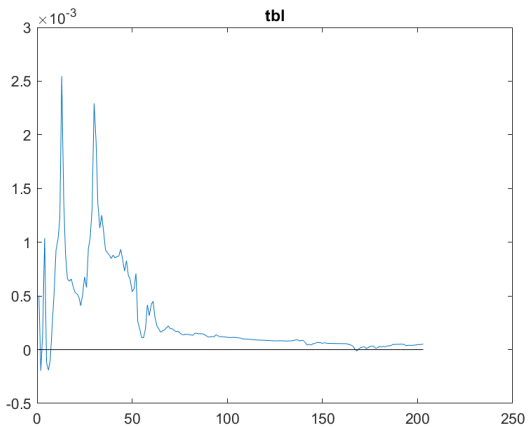


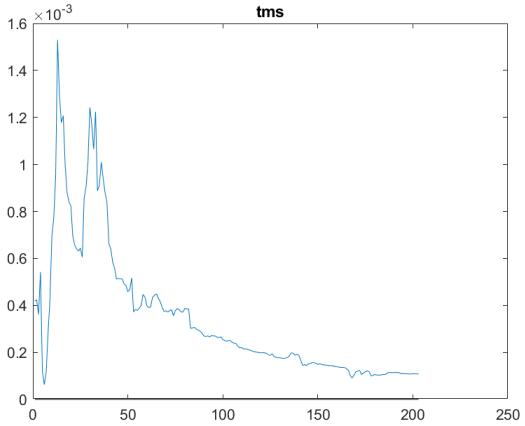
Wavelet Quantiles – Cumulative Squared Prediction Error Minus the Benchmark

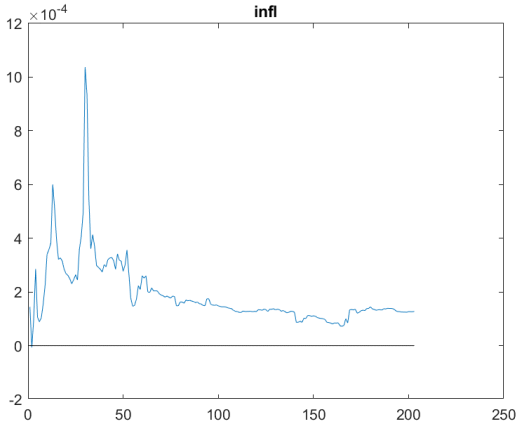


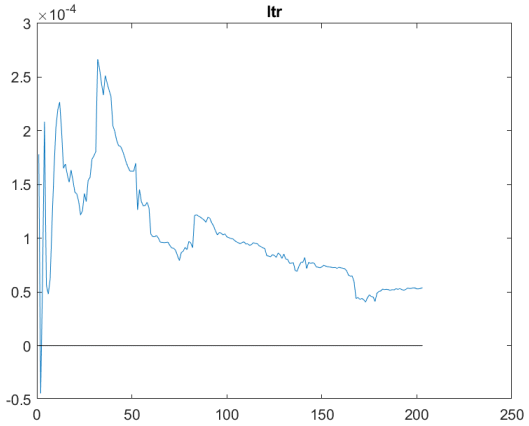


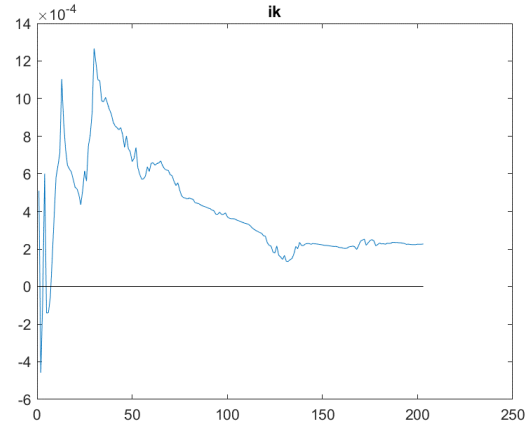












Economic Evaluation

- We compute the certainty equivalent return (CER) for a mean-variance investor who optimally allocates across equities and risk-free bills using various equity risk premium forecasts

- In a mean-variance framework, the solution to the maximization problem of the investor yields the following weight (w_t) on the risky asset

$$w_t = \frac{\hat{r}_{t+1}}{\gamma \hat{\sigma}^2_{t+1}}$$

- The share1-w_t is allocated to risk-free bills, and the month-(t+1) portfolio return is given by

$$r_{p,t+1} = w_t r_{t+1} + r_{f,t+1}$$

Quarterly – Certainty Equivalent Return Gain

Benchmark 2.77

	dp	dy	ep	de	svar	bm	ntis	tbl	lty	ltr	tms	dfy	dfr	infl	ik	Mean
Linear	1.00	1.86	**1.63**	-0.17	-1.60	-1.01	-0.82	2.11	1.96	-0.06	1.49	-1.13	1.70	1.68	2.29	**2.35**
Quantile	0.20	1.39	0.91	0.03	-2.21	-2.01	-1.62	2.23	1.92	-0.31	1.54	-1.46	1.23	0.18	2.59	1.37
Wavelet Y Last	**1.03**	1.71	0.72	-0.54	0.95	-1.19	**0.42**	2.11	1.99	**0.18**	1.48	**0.96**	0.51	2.26	2.34	1.74
Wavelet Y Den	0.61	1.46	0.98	-0.58	-0.76	**-1.00**	-0.78	2.16	2.01	-0.27	1.48	-0.62	**1.77**	2.44	2.24	1.79
Wavelet Y Den s	0.78	1.33	0.70	-0.68	0.81	-1.16	-0.35	2.16	2.05	0.04	1.24	0.43	1.02	2.44	2.24	1.60
WaveletQuant Y	0.68	1.57	0.92	**0.20**	-1.59	-1.43	-1.39	2.23	1.93	0.01	1.53	-1.69	1.47	0.01	2.56	1.07
WaveletQuant Y Last	1.02	**2.08**	-0.01	-0.10	**1.08**	-1.48	-0.42	2.22	**2.10**	0.09	**1.90**	0.47	1.44	2.24	2.74	1.73
WaveletQuant Y Den	0.69	1.72	0.35	0.04	-0.78	-1.81	-0.32	2.26	2.04	-0.28	1.62	-0.38	1.37	2.39	2.74	1.57
WaveletQuant Y Den s	0.93	1.94	0.27	-0.17	0.48	-1.83	-0.40	**2.31**	2.04	-0.06	1.68	0.04	1.38	**2.47**	**2.75**	1.60

Quarterly – Certainty Equivalent Return Gain with TC

Benchmark 0.66

	dp	dy	ep	de	svar	bm	ntis	tbl	lty	ltr	tms	dfy	dfr	infl	ik	Mean
Linear	0.26	0.47	0.40	-0.07	-0.45	**-0.26**	-0.23	0.53	0.50	-0.18	0.31	-0.30	0.25	0.36	0.56	0.57
Quantile	0.14	1.31	0.88	-0.02	-2.34	-2.06	-1.75	**2.23**	1.92	-1.11	1.34	-1.54	0.71	0.05	2.51	1.29
Wavelet Y Last	**1.01**	1.66	0.70	-0.62	0.88	-1.23	**0.37**	2.10	2.01	**0.08**	1.30	**0.90**	0.45	2.02	2.27	1.70
Wavelet Y Den	0.61	1.45	**0.96**	-0.66	-0.88	-1.03	-0.85	2.16	2.05	-0.54	1.27	-0.67	1.34	2.17	2.18	**1.76**
Wavelet Y Den s	0.77	1.30	0.67	-0.74	0.76	-1.19	-0.39	2.15	2.08	-0.11	1.06	0.38	0.87	**2.20**	2.18	1.58
WaveletQuant Y	0.65	1.53	0.87	**0.13**	-1.75	-1.49	-1.52	2.22	1.94	-0.63	1.33	-1.76	0.85	-0.09	2.46	1.01
WaveletQuant Y Last	0.97	**2.03**	-0.05	-0.15	**1.02**	-1.51	-0.48	2.21	**2.13**	0.01	**1.76**	0.42	**1.37**	2.04	**2.69**	1.70
WaveletQuant Y Den	0.66	1.67	0.32	-0.01	-0.89	-1.85	-0.37	2.26	2.05	-0.47	1.43	-0.43	1.09	2.17	2.66	1.55
WaveletQuant Y Den s	0.89	1.90	0.24	-0.22	0.44	-1.86	-0.44	2.30	2.06	-0.18	1.54	-0.01	1.24	2.28	2.71	1.58

Quarterly – Sharpe Ratios

Benchmark 0.08

	dp	dy	ep	de	svar	bm	ntis	tbl	lty	ltr	tms	dfy	dfr	infl	ik	Mean
Linear	0.06	0.10	0.11	0.08	0.08	0.06	0.09	0.15	0.13	0.10	0.18	0.08	0.14	0.14	0.15	0.15
Quantile	0.08	0.11	**0.13**	0.12	0.10	**0.09**	**0.12**	**0.17**	**0.15**	**0.13**	**0.20**	0.11	0.15	0.12	**0.18**	**0.15**
Wavelet Y Last	0.10	0.10	0.09	0.06	0.10	0.06	0.09	0.15	0.13	0.10	0.16	0.09	0.10	0.16	0.14	0.12
Wavelet Y Den	0.05	0.08	0.10	0.06	0.08	0.07	0.07	0.15	0.13	0.09	0.17	0.05	0.14	0.16	0.14	0.13
Wavelet Y Den s	0.08	0.08	0.09	0.06	0.10	0.07	0.07	0.15	0.14	0.10	0.16	0.07	0.11	**0.16**	0.14	0.12
WaveletQuant Y	0.08	0.11	0.12	**0.12**	**0.11**	0.09	0.12	0.16	0.15	0.13	0.19	**0.12**	**0.15**	0.11	0.17	0.14
WaveletQuant Y Last	**0.11**	**0.13**	0.08	0.08	0.10	0.07	0.08	0.15	0.14	0.09	0.16	0.08	0.12	0.16	0.16	0.13
WaveletQuant Y Den	0.10	0.12	0.10	0.10	0.10	0.08	0.10	0.16	0.14	0.10	0.18	0.08	0.14	0.16	0.17	0.13
WaveletQuant Y Den s	0.11	0.12	0.09	0.08	0.09	0.07	0.09	0.16	0.14	0.10	0.16	0.07	0.13	0.17	0.16	0.13

Conclusions

- Wavelet decomposition and wavelet denoising improve the forecasting ability of all predictors
- Wavelet Quantiles outperform alternative methods
- Economic Evaluations
  - Higher utility gains
  - Better SR (together with simple quantile combinations)
- IK in general outperforms all other predictors across methods and frequency
- Robust results in annual, quarterly and monthly returns

Future Work

- 'Kitchen Sink'
- Variable Selection at each time step
- Different Quantile Combinations
- Combinations
- Robustness checks
  - ~~Frequency~~
  - Different periods
  - Different γ

Monthly– Out-of-Sample R^2

	dp	dy	ep	de	svar	bm	ntis	tbl	lty	ltr	tms	dfy	dfr	infl	Mean
Linear	-0.32	-0.28	-0.46	-0.72	-0.11	-1.17	-0.69	-0.48	-0.65	-0.06	-0.75	-0.83	-0.69	-0.27	**0.93*** **
Quantile	-0.01	**0.04**	-0.78	-0.82	-2.09	-1.94	-1.27	-0.86	-1.03	-1.00	-0.83	-2.28	-1.80	-1.15	**0.33***
W Y Last	**0.26**	**0.27***	-0.32	-0.82	**1.06***	-0.69	-0.76	-0.37	-0.51	**0.30***	-0.75	-0.56	**0.22**	-0.41	**0.83****
W Y Den	**0.21**	**0.06***	-0.24	-0.72	**0.94***	-0.63	-0.70	-0.47	-0.59	**0.49***	-0.79	-0.74	-0.03	-0.47	**1.00*** **
W Y Den s	**0.26***	**0.22***	-0.31	-0.84	**1.11***	-0.65	-0.77	-0.38	-0.58	**0.37***	-0.69	-0.77	**0.16**	-0.48	**0.87*** **
WQ Y	**0.52***	**0.40***	-0.11	-0.13	-1.29	-1.02	-0.53	**0.29****	-0.10	**0.01****	-0.11	-1.25	-1.54	-0.27	**0.81*** **
WQ Y Last	**0.26**	**0.54****	-0.19	-0.37	**1.35***	-0.55	-0.13	**0.31****	-0.14	**0.40****	**0.52****	-0.51	**0.11**	-0.27	**0.74*** **
WQ Y Den	**0.45***	**0.57****	-0.30	-0.22	**0.98***	-0.98	-0.69	-0.99	-0.77	**0.47***	-0.83	-0.83	-0.12	-0.51	**0.99*** **
WQ Y Den s	**0.58****	**0.68****	-0.09	**0.02**	**1.15***	-0.46	-0.37	**0.06****	-0.08	**0.50****	**0.14****	-0.54	**0.05**	-0.28	**0.83*** **

Monthly – IPOCID

Benchmark	24.6														
	dp	dy	ep	de	svar	bm	ntis	tbl	lty	ltr	tms	dfy	dfr	infl	Mean
Linear	**69.18**	38.85	**61.48**	34.10	39.18	45.57	36.72	40.33	40.49	47.38	49.34	39.84	44.43	42.79	42.62
Quantile	64.92	40.82	49.84	36.56	40.98	41.15	36.89	40.98	39.34	47.05	47.05	41.48	40.66	42.62	41.15
Wavelet Y Last	65.41	38.20	47.87	34.43	43.61	32.13	38.52	40.98	40.16	47.38	49.51	35.57	45.08	43.44	35.90
Wavelet Y Den	**69.18**	39.67	55.25	37.21	44.10	33.93	39.67	42.62	41.97	47.05	48.85	38.36	45.74	42.95	41.48
Wavelet Y Den s	**69.18**	39.51	52.95	36.72	43.61	34.59	39.67	42.79	41.97	47.38	49.18	39.18	45.90	42.95	38.03
WaveletQuant Y	58.85	**43.93**	54.92	**40.98**	42.13	**51.48**	41.97	43.11	41.48	47.05	46.89	**42.62**	**49.51**	46.23	**44.43**
WaveletQuant Y Last	56.56	43.11	46.39	37.21	43.61	39.51	42.46	41.97	40.33	47.38	48.52	37.38	39.84	43.61	37.21
WaveletQuant Y Den	60.16	39.18	44.26	39.84	**45.57**	44.10	**45.25**	**44.10**	**43.44**	**47.54**	48.20	40.82	41.80	**47.54**	40.33
WaveletQuant Y Den s	58.69	39.67	43.77	40.33	42.95	40.00	44.10	40.16	38.52	46.89	**49.67**	35.25	38.69	42.30	35.57

Monthly – Certainty Equivalent Return Gain

Benchmark 3.97

	dp	dy	ep	de	svar	bm	ntis	tbl	lty	ltr	tms	dfy	dfr	infl	Mean
Linear	0.18	0.52	**0.85**	-0.53	0.42	-1.48	**-0.26**	1.45	1.38	0.84	1.12	-0.95	0.08	**0.15**	1.79
Quantile	-0.93	-0.38	-0.81	-0.58	-1.29	-2.34	-1.20	**1.55**	1.44	0.76	1.09	-1.73	-1.36	-0.44	0.53
Wavelet Y Last	0.24	0.45	0.22	-0.52	1.09	-1.04	-0.30	1.46	1.40	0.46	1.21	-0.40	**0.79**	0.03	1.67
Wavelet Y Den	0.17	0.47	0.49	-0.59	**1.17**	**-1.02**	-0.41	1.45	1.38	0.74	1.17	-0.88	0.61	0.08	**1.91**
Wavelet Y Den s	0.27	0.44	0.33	-0.69	1.08	-1.04	-0.48	1.47	1.39	0.57	1.19	-0.75	0.73	0.01	1.74
WaveletQuant Y	**0.46**	0.22	0.50	**-0.17**	-1.19	-2.09	-0.51	1.45	1.38	**0.94**	1.26	-1.55	-1.47	-0.05	1.19
WaveletQuant Y Last	0.09	**0.58**	-0.40	-0.36	1.07	-1.36	-0.42	1.53	1.35	0.48	**1.40**	**-0.34**	0.39	-0.11	1.27
WaveletQuant Y Den	-0.13	0.27	-1.02	-0.47	0.82	-1.90	-0.68	1.51	**1.57**	0.75	1.16	-1.03	0.22	0.05	1.55
WaveletQuant Y Den s	0.37	0.64	-0.31	-0.04	1.09	-1.37	-0.39	1.54	1.45	0.57	1.22	-0.90	0.19	-0.06	1.34

Monthly – Certainty Equivalent Return Gain with TC

Benchmark 0.32

	dp	dy	ep	de	svar	bm	ntis	tbl	lty	ltr	tms	dfy	dfr	infl	Mean
Linear	0.01	0.03	0.07	**-0.06**	0.00	**-0.13**	**-0.04**	0.12	0.11	-0.17	0.06	**-0.09**	-0.09	**-0.06**	0.12
Quantile	-1.12	-0.68	-0.92	-0.76	-1.50	-2.52	-1.46	1.46	1.35	-1.72	0.73	-1.86	-2.42	-1.45	0.18
Wavelet Y Last	0.17	0.25	0.17	-0.63	0.60	-1.13	-0.58	1.42	1.38	-0.35	0.79	-0.54	0.10	-0.86	1.48
Wavelet Y Den	0.10	0.34	**0.46**	-0.69	0.49	-1.10	-0.68	1.41	1.36	-1.04	0.76	-1.03	-0.19	-0.85	**1.68**
Wavelet Y Den s	0.20	0.30	0.29	-0.78	0.48	-1.11	-0.75	1.43	1.37	-0.43	0.78	-0.87	-0.06	-0.89	1.55
WaveletQuant Y	0.19	-0.15	0.30	-0.40	-1.52	-2.38	-0.77	1.37	1.31	-1.97	0.83	-1.72	-2.77	-0.77	0.75
WaveletQuant Y Last	-0.07	**0.38**	-0.48	-0.47	**0.71**	-1.50	-0.66	**1.48**	1.29	**-0.06**	**1.02**	-0.46	**0.21**	-0.81	1.13
WaveletQuant Y Den	-0.24	0.08	-1.10	-0.58	0.28	-2.05	-0.95	1.46	**1.54**	-0.87	0.79	-1.15	-0.25	-0.86	1.35
WaveletQuant Y Den s	**0.28**	0.47	-0.37	-0.15	0.61	-1.48	-0.63	1.50	1.41	-0.21	0.83	-1.01	-0.04	-0.68	1.20

Monthly – Sharpe Ratios

Benchmark 0.07

	dp	dy	ep	de	svar	bm	ntis	tbl	lty	ltr	tms	dfy	dfr	infl	Mean
Linear	0.04	0.05	0.07	0.05	0.07	0.04	0.08	0.10	0.09	0.10	0.11	0.06	0.07	0.07	0.10
Quantile	0.06	0.06	0.07	**0.07**	0.06	**0.06**	0.07	**0.11**	**0.10**	**0.11**	0.12	0.07	0.06	0.08	0.09
Wavelet Y Last	0.05	0.05	0.07	0.05	0.08	0.05	0.08	0.10	0.09	0.08	0.12	0.06	**0.08**	0.07	0.10
Wavelet Y Den	0.05	0.05	0.07	0.04	0.08	0.05	0.08	0.10	0.09	0.09	0.12	0.06	0.08	0.07	0.10
Wavelet Y Den s	0.05	0.05	0.07	0.04	0.08	0.05	0.08	0.10	0.09	0.08	0.12	0.05	0.08	0.07	0.10
WaveletQuant Y	0.06	0.06	**0.07**	0.07	0.06	0.05	0.09	0.10	0.10	0.11	0.12	**0.07**	0.05	0.08	0.10
WaveletQuant Y Last	0.07	0.07	0.06	0.05	0.08	0.06	0.08	0.10	0.09	0.08	0.12	0.06	0.08	0.07	0.09
WaveletQuant Y Den	0.08	0.07	0.06	0.07	0.08	0.06	0.09	0.10	0.10	0.10	**0.12**	0.07	0.08	**0.08**	**0.11**
WaveletQuant Y Den s	**0.08**	**0.08**	0.07	0.07	**0.09**	0.06	**0.09**	0.10	0.10	0.09	0.12	0.06	0.08	0.07	0.10

Annual – Out-of-Sample R^2

	dp	dy	ep	de	svar	bm	ntis	tbl	lty	ltr	tms	dfy	dfr	infl	ik	Mean
Linear	-3.98	-1.71	-5.39	-6.45	-8.87	-14.12	-11.02	-8.18	-12.49	-16.87	-4.28	-9.37	-9.14	-1.17	**8.69****	**5.45****
Quantile	**1.50***	**2.30**	-5.35	-10.77	-16.46	-12.22	-12.15	-9.99	-10.89	-18.12	-7.33	-14.30	-8.87	-1.42	**9.03****	**6.44*****
W Y Last	**3.63***	**2.21**	**0.00**	-2.79	-2.15	-2.65	-7.06	-1.72	-8.40	-0.93	**3.40****	-11.06	**0.76**	**0.26**	**6.65****	**2.75***
W Y Den	-0.73	-3.24	-1.14	-3.97	-3.47	-3.55	-6.75	-0.60	-3.03	-0.31	-0.21	-4.28	-1.13	-1.13	**5.25****	**1.68**
W Y Den s	-1.85	-2.95	-1.09	-2.66	-1.40	-3.38	-2.01	**0.12**	-1.16	-0.16	**0.47**	-2.40	-0.37	-0.26	**1.54**	-0.04
WQ Y	-3.24	-1.54	-2.24	-0.08	-7.30	-20.65	-13.16	-10.86	-20.39	-19.35	-6.04	-14.52	-19.28	-2.79	**14.22*****	**6.95****
WQ Y Last	**0.33**	-0.68	-1.57	-2.64	-5.38	-4.43	-9.43	-4.38	-15.83	-4.48	**3.92****	-16.51	-0.18	-1.19	**8.98****	**2.58**
WQ Y Den	**2.80***	**1.96**	**1.33**	-5.45	-4.41	-2.83	-11.09	-3.59	-9.61	-0.51	**2.90**	-8.08	**0.69**	-1.14	**8.74****	**3.16***
WQ Y Den s	-0.77	-0.68	**0.11**	-5.34	-5.03	-4.14	-11.19	-2.14	-9.92	-3.57	**0.20**	-11.33	-0.75	-1.34	**7.73****	**1.18**

Annual – IPOCID

Benchmark	20.4															
	dp	dy	ep	de	svar	bm	ntis	tbl	lty	ltr	tms	dfy	dfr	infl	ik	Mean
Linear	71.43	44.90	**63.27**	32.65	48.98	51.02	42.86	32.65	28.57	46.94	51.02	48.98	48.98	20.41	48.98	30.61
Quantile	**75.51**	44.90	**63.27**	36.73	**59.18**	**57.14**	46.94	34.69	36.73	44.90	51.02	**55.10**	**51.02**	36.73	42.86	**36.73**
Wavelet Y Last	61.22	40.82	51.02	34.69	30.61	28.57	22.45	34.69	34.69	**51.02**	46.94	30.61	42.86	26.53	51.02	28.57
Wavelet Y Den	67.35	42.86	55.10	36.73	26.53	28.57	24.49	40.82	34.69	38.78	51.02	28.57	48.98	24.49	48.98	28.57
Wavelet Y Den s	71.43	36.73	53.06	36.73	20.41	28.57	24.49	34.69	32.65	30.61	46.94	30.61	42.86	26.53	48.98	22.45
WaveletQuant Y	67.35	**59.18**	59.18	**40.82**	44.90	55.10	**53.06**	36.73	**38.78**	46.94	**53.06**	53.06	**51.02**	38.78	**53.06**	**36.73**
WaveletQuant Y Last	63.27	48.98	53.06	30.61	32.65	40.82	40.82	**46.94**	36.73	32.65	48.98	32.65	46.94	34.69	**53.06**	24.49
WaveletQuant Y Den	**75.51**	48.98	53.06	30.61	28.57	42.86	22.45	36.73	28.57	38.78	**53.06**	30.61	**51.02**	**42.86**	**53.06**	26.53
WaveletQuant Y Den s	71.43	55.10	53.06	32.65	32.65	32.65	24.49	40.82	36.73	34.69	44.90	30.61	48.98	**42.86**	**53.06**	26.53

Annual – Certainty Equivalent Return Gain

Benchmark -0.26

	dp	dy	ep	de	svar	bm	ntis	tbl	lty	ltr	tms	dfy	dfr	infl	ik	Mean
Linear	**5.09**	4.54	2.10	0.35	-1.32	**-0.37**	-0.85	2.68	**2.37**	**0.55**	1.44	-0.09	0.35	-0.18	**3.41**	1.41
Quantile	4.79	4.08	1.33	-0.55	-1.23	-1.46	-0.83	2.57	2.01	0.48	1.56	**0.17**	**0.49**	-0.35	2.93	1.33
Wavelet Y Last	1.56	2.09	-0.27	-0.71	-0.96	-1.51	-0.56	2.25	2.13	-0.42	0.85	-0.46	0.13	**0.06**	1.60	0.64
Wavelet Y Den	1.18	2.16	-0.11	0.03	-0.43	-0.96	-0.59	2.19	1.87	0.10	0.82	-0.29	-0.04	-0.13	1.58	0.52
Wavelet Y Den s	-0.33	-0.44	-0.42	0.05	-0.24	-0.83	**-0.12**	0.61	0.30	-0.03	0.30	-0.29	0.00	-0.01	0.26	0.17
WaveletQuant Y	4.28	**4.85**	**2.43**	**0.46**	-0.59	-0.86	-1.43	**2.80**	2.16	0.39	**1.89**	-0.13	0.48	0.01	3.10	**1.77**
WaveletQuant Y Last	0.68	1.04	-0.45	-0.20	-1.16	-1.96	-0.55	2.31	2.04	-0.81	1.09	-0.31	0.05	-0.05	2.29	0.79
WaveletQuant Y Den	2.25	4.12	0.07	-0.59	**-0.09**	-1.16	-0.36	2.20	1.93	-0.05	1.13	-0.29	-0.02	-0.21	2.61	0.84
WaveletQuant Y Den s	1.16	3.04	-0.23	-0.59	-0.37	-1.22	-0.23	1.96	1.97	-0.42	0.40	-0.47	-0.11	-0.01	1.96	0.48

Annual – Certainty Equivalent Return Gain with TC

Benchmark -0.41

	dp	dy	ep	de	svar	bm	ntis	tbl	lty	ltr	tms	dfy	dfr	infl	ik	Mean
Linear	**5.18**	4.56	2.10	0.33	-1.34	**-0.35**	-0.86	2.69	**2.40**	**0.43**	1.37	-0.07	0.21	-0.19	**3.37**	1.41
Quantile	4.88	4.07	1.32	-0.58	-1.21	-1.47	-0.85	2.58	2.05	0.41	1.51	**0.20**	**0.39**	-0.35	2.87	1.33
Wavelet Y Last	1.58	2.06	-0.27	-0.72	-0.97	-1.49	-0.60	2.25	2.15	-0.45	0.81	-0.50	0.14	**0.06**	1.56	0.64
Wavelet Y Den	1.21	2.14	-0.12	0.01	-0.44	-0.96	-0.63	2.18	1.88	0.08	0.78	-0.31	-0.04	-0.14	1.54	0.51
Wavelet Y Den s	-0.33	-0.48	-0.43	0.04	-0.24	-0.83	**-0.13**	0.60	0.29	-0.03	0.29	-0.32	0.00	-0.01	0.22	0.17
WaveletQuant Y	4.31	**4.87**	**2.42**	**0.45**	-0.64	-0.87	-1.46	**2.82**	2.20	0.29	**1.82**	-0.13	0.34	0.00	3.04	**1.76**
WaveletQuant Y Last	0.69	1.01	-0.45	-0.20	-1.20	-1.94	-0.59	2.32	2.07	-0.84	1.05	-0.34	0.05	-0.06	2.24	0.79
WaveletQuant Y Den	2.30	4.15	0.07	-0.60	**-0.09**	-1.16	-0.41	2.21	1.96	-0.07	1.10	-0.31	-0.03	-0.23	2.59	0.85
WaveletQuant Y Den s	1.20	3.06	-0.23	-0.61	-0.38	-1.21	-0.26	1.95	1.99	-0.44	0.38	-0.50	-0.13	-0.02	1.92	0.47

Annual – Sharpe Ratios

Benchmark 0.18

	dp	dy	ep	de	svar	bm	ntis	tbl	lty	ltr	tms	dfy	dfr	infl	ik	Mean
Linear	**0.29**	0.23	0.17	0.15	0.17	0.12	0.18	0.23	0.20	0.22	0.29	0.21	0.21	0.17	0.24	0.20
Quantile	0.28	0.21	0.20	0.19	**0.23**	0.14	**0.22**	**0.27**	**0.24**	**0.25**	0.30	**0.24**	**0.27**	**0.20**	0.25	**0.24**
Wavelet Y Last	0.18	0.17	0.16	0.14	0.16	0.18	0.12	0.22	0.18	0.16	0.23	0.14	0.19	0.19	0.19	0.17
Wavelet Y Den	0.12	0.11	0.15	0.13	0.16	0.18	0.12	0.22	0.18	0.20	0.25	0.15	0.18	0.17	0.17	0.17
Wavelet Y Den s	0.13	0.10	0.16	0.15	0.17	**0.20**	0.16	0.17	0.13	0.18	0.21	0.16	0.18	0.19	0.14	0.17
WaveletQuant Y	0.24	**0.28**	**0.21**	**0.22**	0.18	0.14	0.19	0.26	0.22	0.22	**0.31**	0.20	0.26	0.19	**0.27**	0.23
WaveletQuant Y Last	0.15	0.12	0.17	0.14	0.14	0.18	0.13	0.23	0.20	0.14	0.24	0.16	0.20	0.18	0.21	0.18
WaveletQuant Y Den	0.20	0.25	0.21	0.16	0.19	0.19	0.15	0.25	0.22	0.20	0.26	0.17	0.19	0.18	0.23	0.20
WaveletQuant Y Den s	0.13	0.18	0.17	0.13	0.17	0.19	0.15	0.22	0.20	0.16	0.21	0.16	0.18	0.18	0.20	0.17