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Endogenous Time-Varying Volatility and Emerging Market Business Cycles

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Endogenous Time-Varying Volatility and Emerging Market Business Cycles

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Abstract

Time-varying volatility plays a crucial role in understanding business cycles in emerging market economies. However, the literature treats volatility as an exogenous process. This paper endogenizes time-varying volatility in the debt premium and total factor productivity into a standard small open economy model and assesses the quality of the model by comparing it to emerging market data. An additional volatility channel that operates through the debt premium on the interest rate faced by a small open economy can generate countercyclical net exports and excess volatility in consumption as observed in data on emerging market business cycles.

Keywords: Endogenous Volatility, DSGE, Emerging Markets

JEL Classification: E32, F41, F44

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Non-Technical Summary

Time-varying volatility plays a crucial role in understanding business cycles in emerging market economies. There is now plentiful empirical evidence that volatility as measured by the standard deviation in macroeconomic data is time-varying and strongly countercyclical. Volatility increases during an economic recession and becomes lower during an economic boom.

In addition, we observe that standard open-economy macroeconomic models widely used for business cycle analysis fail to address the specific characteristics of many emerging market economies. In emerging market economies the net export to output ratio is typically negatively correlated with output i.e. it is countercyclical. However, standard models predict a near perfect positive correlation between the two. Besides that, emerging market economies show a higher fluctuation in consumption data than in data on output. Standard models, however, predict a higher fluctuation in output than consumption. These models are therefore overemphasizing the role of consumption smoothing.

This work is motivated by the above empirical observations. We augment a standard small-open economy model and introduce time-varying volatility to the interest rate and total factor productivity. In our model the interest rate and total factor productivity automatically turn more volatile when the economy becomes more indebted or when output declines in response to a negative total factor productivity shock. Once we introduce time-varying volatility into a standard open-economy model, the model becomes able to significantly better match emerging market economy data. After the introduction of a time-varying interest rate and total factor productivity we are able to present a model where net exports are strongly negatively correlated with output and consumption shows a higher variation than output as observed in the data. By choosing different parameter values for the time-varying volatility the model is able to characterize both, emerging market and developed economies, or an economy that is in transition to a developed economy.

Although we are not the first to address the problems of macroeconomic models for emerging market economies, our approach is especially simple and does not rely on shocks to the permanent component of total factor productivity or to shocks in the level of the interest rate. Compared to other research in this area our approach only requires one source of external variation, the widely used shock to total factor productivity. From a policy point of view our model can be especially useful for economists and policy makers in emerging market economies as our model now better fits the economic cycle in those countries.

1 Introduction

Time-varying volatility plays a crucial role in understanding business cycles. The literature has so far established a link between high levels of volatility and decreases in output and consumption e.g. as in Fernandez-Villaverde et al. (2011). However, the standard literature considers time-varying volatility to be an exogenously driven process. This is in stark contrast to some empirical observations that argue that volatility itself is caused by changes in macroeconomic variables like output or consumption.¹ In this work we propose a model that endogenizes time-varying volatility which is then able to match emerging market business cycle facts.

We motivate our work by two empirical facts. First, emerging market economies (EME) behave differently than developed economies. Net exports are strongly countercyclical and consumption volatility exceeds output volatility as shown in Table 1. Second, EME business cycle data contains a large amount of time-varying volatility. This time-varying volatility is especially present in the debt premium on the interest rate and total factor productivity (TFP). It can also empirically be observed that this time-variation in volatility is stronger for emerging markets than for developed economies.

TABLE 1
Empirical Business Cycle Features

	Mexico				Canada			
	1995-1999	2000-2004	2005-2009	2010-2014	1995-1999	2000-2004	2005-2009	2010-2014
$\rho_{Y,NX}$	-0.81	-0.35	0.05	-0.22	-0.03	0.64	0.74	0.76
σ_C/σ_Y	0.82	1.02	1.25	1.51	0.73	0.61	0.70	0.41
σ_{r-us}	11.46	2.30	1.72	0.49	1.21	0.91	0.58	0.26

Note: Table 1 shows the correlation of net exports and output as well as the relative standard deviation of consumption to output and the standard deviation of the debt premium relative to the US for Mexico and Canada. All data is quarterly from the Federal Reserve Bank of St Louis.

We quantify these observations by constructing a small open economy model with endogenous time-varying volatility in the debt premium on the interest rate and TFP that can explain EME business cycle features like strongly countercyclical net exports and excess volatility in consumption. Characteristics that standard models often fail to replicate. We include in our model a reduced form process where high debt to output levels trigger endogenously time-varying volatility in the debt premium and TFP. In the case of the debt premium this reduced form process can be interpreted as a situation where high levels of debt relative to output decrease the trading volume of the debt which will increase the variability of returns and hence the debt premium. In the case of TFP the reduced form process can be more precisely interpreted as a case where high debt levels increase a firm's probability of default which subsequently causes misallocations in factor inputs. This misallocation hence leads to temporarily higher variability in total factor productivity. In our model the joint occurrence of debt premium and TFP volatility caused by a negative transitory TFP shock that leads to above than steady state debt

¹See Bachmann et al. (2013) and Ludvigson et al. (2015).

levels reinforce each other and lead to effects comparable to a negative trend shock in TFP. Specifically, a negative TFP shock will decrease output and increase debt which will lead to an above steady state debt to output level. This will increase debt premium volatility and hence higher volatility in the debt price and the amount of debt itself. As TFP volatility is driven by the same fundamental process that depends on the debt to output ratio, higher debt to output will increase volatility in TFP. The increase in TFP volatility will in turn increase volatility in output and hence the volatility of the debt to output ratio. This increase in the volatility of the debt to output ratio caused by both, debt premium and TFP volatility, will lead to a self-reinforcing cycle. The result is that transitory TFP shocks that simultaneously cause endogenous time-varying volatility in the debt premium and TFP can have long lasting effects on output, consumption, and investment similar to a trend shock in TFP. Depending on the degree and persistence of endogenous time-varying volatility we are able to produce countercyclical net exports and excess consumption volatility.

This work is based on two strands of the literature, the emerging market economies business cycle literature and the literature on time-varying volatility in macroeconomic models. Within the EME business cycle literature this work is related to Aguiar and Gopinath (2007) who construct a small open economy model with cycle and trend shocks to TFP and conclude that such a model can fit the characteristics of emerging market economies as well as of developed countries by choosing the correct relative size of cycle and trend shocks. A related work is Boz et al. (2011) who explain emerging market business cycle features with a learning process about cycle and trend TFP shocks. They find that when agents are imperfectly informed about the trend and cycle components of TFP, a learning process using a Kalman filter can greatly improve the performance of a standard real business cycle model to match EME business cycles. Within the EME business cycle literature many authors stress the importance of financial frictions. Among them Neumeyer and Perri (2005) who use a small open economy model to study the effect of interest rates on EME business cycles. They find that exogenous shocks to the level of the interest rate can explain business cycle facts for five EME very well. Boz et al. (2015) use labor market frictions to explain the countercyclical behavior of EME business cycles. Garcia-Cicco et al. (2010) construct a RBC model with financial frictions and level shocks to the debt premium and show that such a model can generate EME business cycle features. It also relates to Chang and Fernandez (2013) who build a model with trend shocks, interest rate shocks, and financial frictions and conclude that financial frictions are the main source of fluctuations in emerging markets. Further Alvarez-Parra et al. (2013) build a small open economy model that includes durable and non-durable goods and shocks to trend TFP and the country risk premium. In line with other papers that stress the importance of financial frictions they find that financial frictions in the form of a countercyclical risk premium are more important than trend shocks. This is because in their model trend shocks would make durable consumption to volatile which therefore imposes an upper limit on the size of the trend shock. Other current work that studies

the case of EME business cycles in a small open economy model include Li (2011) who addresses the high wage volatility in emerging markets and Fernandez and Meza (2015) who build a small open economy model with formal and informal labor markets to match the main business cycle moments.

Within the literature on time-varying volatility in macroeconomic models this work is further related to papers like Fernandez-Villaverde et al. (2011) who look at interest rate volatility in a small open economy framework and analyze the effect of interest rate volatility on output, consumption, and investment in emerging market economies. They find that exogenous volatility shocks to the interest rate have negative effects on output, consumption, and investment. However, exogenous volatility shocks to the interest rate cannot explain the countercyclicality of net exports in emerging markets. Justiniano and Primiceri (2008) build a closed economy model with time-varying volatility in TFP and the investment efficiency and find that these two are major sources of macroeconomic fluctuations. Further papers on volatility include Gourio (2012) who introduces time-varying disaster risk into a standard real business cycle model and Christiano et al. (2014) who combine a Christiano et al. (2005) type model with Bernanke et al. (1999) and time-variation to the productivity shock to find that this risk shock can explain a large share of variation in macroeconomic variables. Seoane (2017) further studies the effect of volatility shocks on markups in a small open economy model using an exogenous shock to the level of productivity and volatility shocks to the risk premium. However, all these papers treat volatility as an exogenous process rather than an equilibrium outcome and show that exogenously driven volatility can cause recessions. We will go one step further and argue that TFP driven business cycles will endogenously produce volatility which then can produce EME business cycle facts.

This paper is different from the previous literature on time-varying volatility in that volatility in the debt premium as well as TFP emerges endogenously as the debt to output ratio diverges too much from its steady state. It is therefore close to Saijo (2017) who constructs a closed economy New-Keynesian model that endogenizes time-varying volatility through a learning process. In his work endogenous time-varying volatility increases the response of output and other variables to technology and monetary shocks. Our paper however is different from Saijo (2017) as it uses a small open economy model that includes time-varying volatility to the debt premium and TFP. This paper therefore builds on the assumption that high levels of debt relative to output might increase uncertainty about firms' profitability and hence trigger an increase in volatility. It hence features a simple reduced form implementation of countercyclical volatility.²

In our model positive deviations of the debt to output ratio from its steady state will trigger higher volatility in the debt premium and TFP faced by a small open economy. Whereas below steady state levels will trigger periods of lower volatility in these variables. Since the debt to output ratio moves slowly, persistent recessions and booms arise endogenously and are similar to a trend shock in TFP. The model contributes to the re-

²See Bloom (2014) for empirical evidence on the countercyclicality of volatility.

cent emerging market business cycle literature pioneered by Aguiar and Gopinath (2007). It can generate negative correlations between output and net exports as well as higher volatility in consumption than in output. Characteristic features that are often found in data on emerging market economies. Introducing endogenous time-varying volatility into an interest rate debt premium faced by the small open economy in addition to endogenous time-varying volatility in transitory TFP allows for these countercyclical net exports and excess consumption volatility. In contrast to Aguiar and Gopinath (2007) this countercyclical behavior of net exports even occurs when cycle TFP shocks are more important than trend TFP shocks, or as in our model, when trend shocks are not present at all. In contrast to Neumeyer and Perri (2005) we introduce endogenous time-varying volatility to the debt premium instead of an exogenous shock to the level of the debt premium. The appealing novelty of our model with endogenous time-varying volatility is the fact that we only require one exogenous shock. Namely a level shock to transitory TFP opposed to other papers that require an additional exogenous interest rate shock or a shock to trend TFP to match EME business cycle moments.³ In addition, by endogenizing the volatility process we address the fact that volatility is not only a source of aggregate dynamics but also a response to it so that time-varying volatility is negatively correlated with the business cycle.

We parameterize the model with standard parameters for a small open economy like Mexico and find that a simple small open economy model augmented for endogenous time-varying volatility is able to match different second moments of the data. By choosing different elasticities for the debt premium and the TFP volatility using SMM we are able to match both, EME and developed economies', business cycle features. By using a Monte Carlo approach we are able to confirm that the countercyclicality of net exports and the excess consumption volatility are indeed caused by different degrees of endogenous time-varying volatility rather than changes in the standard model parameters like e.g. the labor elasticity, discount factor or the consumption share in the utility function.

The rest of the paper is organized as follows. The second part provides some empirical evidence of the relationship of debt premium volatility and TFP with the business cycles in an emerging market economy like Mexico and compares it to a developed economy like Canada. The third part presents a real business cycle model with endogenous time-varying volatility in the interest rate debt premium as well as TFP. The fourth part compares the generated second moments of the model with real data from Mexico and analyzes the results. The fifth part will conclude and point to possible future research.

2 Some Stylized Facts

It is a well known characteristic of emerging market economies business cycles that net exports are strongly countercyclical and consumption volatility exceeds output volatil-

³To ease reading we will refer to the TFP level shock simply as a TFP shock. This is the only exogenous shock in our model.

ity.⁴ We therefore aim in this section not to show EME data moments, but rather how the debt premium volatility as well as TFP volatility interact with the correlation of net exports and output and with the relative standard deviations of consumption to output.

2.1 Data

To establish an empirical relationship between the debt premium and TFP volatility with key features of business cycles in Mexico and Canada we require data on the debt premium as well as on TFP. We further require data on output, consumption, capital, and net exports. For our analysis we choose Mexico as an example for an emerging market economy and Canada as a developed country because both countries can be considered as small open economies with a high degree of trade openness. For this reason Mexico and Canada often stand for the prototypical small open economy countries and are widely used in the literature. The bulk of the data used in the empirical part is quarterly data from the FRED database of the Federal Reserve Bank of St Louis and ranges from the first quarter 1993 to the fourth quarter 2014. In addition, the interest rate data shown in the next section is monthly data from January 1978 to March 2017. Net exports relative to output are constructed of exports minus import relative to output. TFP data is constructed using a Cobb-Douglas production function in logarithmic terms so that

$$y_t = a_t + (1 - \alpha) k_t + \alpha l_t \quad (1)$$

where y_t denotes log-output, k_t denotes log-capital and l_t denotes the logarithm of total hours. α denotes the elasticity of labor in the Cobb-Douglas production function which we assume to be the labor share of the economy and set it to 0.68. In this way the productivity term a_t can be easily calculated given capital, output, total labor, and the labor share of the economy.⁵ All data series are HP-Filtered as in Hodrick and Prescott (1997) with a filter weight of 1600 for quarterly data to obtain the business cycle component.⁶

The volatility of TFP is then constructed as the moving standard deviation for a time period of $k + 1$ quarters centered around the period t . So that the volatility in period t is the standard deviation of the series from period $t - \frac{k}{2}$ to period $t + \frac{k}{2}$ where $k + 1$ is the window size. We provide different estimates of the results to verify that our results are not significantly driven by the value of the window size $k + 1$. However, it should be noted that the choice of k highlights different aspects of the data i.e. short term versus longer term frequencies.

The debt premium for Mexico and Canada is calculated as the difference of the Mex-

⁴See e.g. Neumeyer and Perri (2005), Aguiar and Gopinath (2007) and Fernandez and Gulan (2015) for some empirical evidence on the differences between EME and developed economies business cycles.

⁵We decide to calculate TFP from the production function as TFP estimates are hardly available for an emerging market economy like Mexico at a quarterly frequency. We use spline interpolation to convert the yearly capital stock data to a quarterly frequency.

⁶For the ease of reading we will refer to the cycle component of TFP simply as TFP. However, in the following data section we always consider the HP-Filtered cycle component of TFP.

ican and Canadian interest rates and the US interest rate that acts as the world interest rate. Approximating the world interest rate by the US interest rate seems to be justified as both Mexico and Canada have high trade volumes with the US and US monetary policy has strong effects on the world interest rate. The Mexican, Canadian, and US quarterly interest rates are the 90-day rate on Mexican treasury securities and the 90-day rate on Canadian and US interbank rates. The debt premium volatility is then constructed as the moving volatility analogously to TFP volatility. Since interest rate data is available at higher frequency than aggregate macroeconomic data we can also construct the standard deviations of interest rates for every year. For this we use the monthly interest rate on government securities and treasury bills for Mexico, Canada, and the US, respectively. Table 8 in the Appendix provides an overview about all data sources.

2.2 Debt Premium Volatility

Our working hypothesis is that, besides volatility in TFP, volatility in the debt premium plays a crucial role in driving the business cycle. We therefore start by showing some observations regarding the debt premium for Mexico and Canada. Figure 1 shows the debt premium for Mexico and Canada in percent relative to the US in the upper panel and the calculated volatility in the middle panel for the period January 1978 to March 2017 using monthly data. The blue line in the middle panel shows the moving volatility in standard deviations and the red asterisks denote the standard deviation of the debt premium on the interest rate for every year. We plot the lower and upper estimates of the moving standard deviation when k , the parameter that governs the window size, is set between 6 and 20 as the shaded area. It turns out that for reasonable values of k the standard deviation of the debt premium moves within a relatively close band. One striking fact is that Mexico as an emerging market economy shows a much higher variability in its interest rate debt premium compared to Canada. The debt premium for Mexico also shows a high degree of time-varying volatility, a key feature in the data that is less pronounced for a developed economy like Canada. Especially during the 1980's and mid 1990's Mexico experienced high levels of debt premium volatility that decreased significantly during the 2000's. The pattern for Canada is similar with high levels of volatility in the 1980's and a decline in volatility from the early to mid 1990's. Similar patterns, with slightly different timings, can be observed in many countries and are generally referred to as the Great Moderation especially when output volatility is concerned.

We want to go beyond a pure visual inspection of the data and estimate a stochastic process for the debt premium in both countries for the period January 1978 to March 2017 using monthly data. For this we use the algorithm by Fernandez-Villaverde et al. (2011).⁷ Their algorithm is a particle based Metropolis-Hastings algorithm that allows to estimate the size of stochastic volatility shocks and their persistence. In contrast to

⁷We choose the same prior for both countries. We then run 20000 replications of the model with 2000 particles each and discard the first 5000 runs as a burn-in.

a GARCH algorithm their Metropolis-Hastings algorithm allows for a clear distinction between level shocks and volatility shocks. Figure 1 shows in the lower panel the fitted probability density functions for the persistence of volatility shocks in the left graph and the size of volatility shocks in the right graph. The estimates for Mexico are in blue and for Canada in red with the dashed vertical lines indicating the median estimate. The median estimates for the size of stochastic volatility are 0.27 for Mexico and 0.22 for Canada, respectively. Besides a higher persistence and larger size of stochastic volatility shocks, Mexico also faced a higher mean volatility. The estimates of the Bayesian estimation of the debt premium confirm that Mexico experienced larger volatility shocks and that these volatility shocks are more persistent.

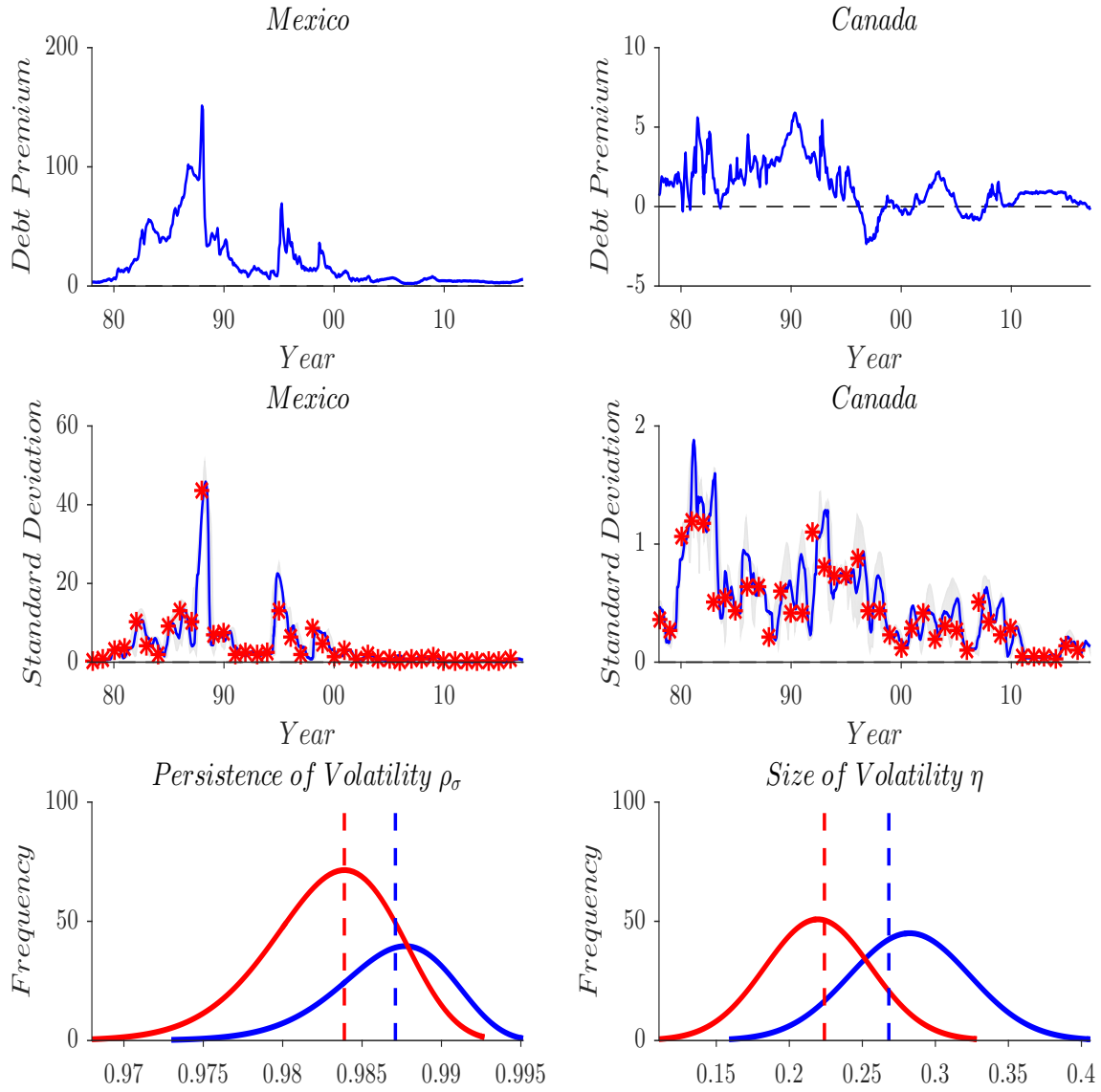
These observations hence allow us to conclude that, (1) there is a significant amount of time-variation in the debt premium on interest rates, (2) this time-variation is stronger for a typical emerging market economy like Mexico than for a developed economy like Canada, (3) high periods of volatility seem to coincide with high levels of the debt premium.

Table 2 shows the contemporaneous correlations for Mexican and Canadian volatility in the debt premium as well as TFP volatility and TFP in levels with the correlation of net exports with output using quarterly data.⁸ For this we construct the moving correlation of net exports and output in a similar way as the moving volatility of a variable. This is the correlation in period t is the correlation of both series from period $t - \frac{k}{2}$ to period $t + \frac{k}{2}$ where $k + 1$ is the window size. The time series of the moving correlation of net exports with output is strongly and negatively correlated with the time series of the moving debt premium volatility for Mexico and Canada. Looking at the correlation of the moving correlation of net exports with TFP in levels the data reveals a low but statistically insignificant correlation for Canada. Further, the debt premium volatility and the volatility of TFP are highly positively correlated for Mexico and slightly negatively for Canada. Whereas the correlation of the debt premium volatility with the TFP level is positive but insignificant for both countries. As k increases it can generally be observed that correlations become stronger hence indicating that correlations between the debt premium volatility and the net export to output ratio become stronger in the long run.

These observations let us conclude that, (1) the correlation of net exports and output is negatively correlated with the debt premium volatility, (2) debt premium volatility and TFP volatility are highly positively correlated in emerging market economies, (3) there is a strong positive correlation of the debt premium volatility and the debt premium in levels for an EME economy but less so for a developed economy.

⁸We revert to quarterly data as data on TFP, net exports, output, and consumption is not available at a monthly frequency.

FIGURE 1
Empirical Debt Premium



Note: Figure 1 shows the debt premium for Mexico and Canada relative to the US in the upper panel and the volatility of the debt premium as the moving standard deviation in the middle panel. Asterisks denote the standard deviation of the interest rate debt premium for every year. The lower panel shows the fitted PDF of the Bayesian estimates. Mexico is shown in blue and Canada in red. Vertical dashed lines indicate median estimates. All data is monthly.

TABLE 2
Empirical Correlations

Correlation	Mexico			Canada		
	12	16	20	12	16	20
Correlation Y/NX - Debt Premium Volatility	-0.64	-0.74	-0.85	-0.23	-0.35	-0.47
Correlation Y/NX - TFP Volatility	-0.53	-0.52	-0.66	0.29	0.36	0.36
Correlation Y/NX - TFP Level	0.00	-0.05	0.03	-0.07	-0.08	-0.03
Debt Premium Volatility - TFP Volatility	0.80	0.86	0.87	-0.21	-0.25	-0.31
Debt Premium Volatility - TFP Level	0.11	0.04	-0.01	0.08	0.04	0.03
Debt Premium Volatility - Debt Premium Level	0.75	0.74	0.72	-0.19	-0.18	-0.18

Note: Table 2 shows the contemporaneous correlations of the moving correlation of net exports and output with the debt premium volatility, TFP volatility and TFP in levels for Mexico and Canada for different values of the window size parameter k. Bold faced values are significant at the 5 percent level. All data is quarterly.

3 A Small Open Economy Model

We construct a small open economy model with endogenous time-varying debt premium and TFP volatility to replicate the dynamics of developing economies i.e. negative correlations of net exports and output and a consumption volatility that exceeds output volatility.

3.1 Model

The model is a small open economy model as used by Aguiar and Gopinath (2007). However, our model only features one kind of TFP process, namely a transitory process. The model is a model with incomplete asset markets as in Mendoza (1991), Neumeyer and Perri (2005), and Uribe and Yue (2006). We include into the model endogenous time-varying volatility in the debt premium on the world interest rate as well as in the transitory TFP process that arise as the debt to output ratio increases above its steady state. Agents can invest in physical capital and an internationally traded, one-period, and un-contingent bond.⁹ The preferences of the representative household are given by the life-time utility function

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{[C_t^\gamma (1 - L_t)^{1-\gamma}]^{1-\sigma}}{1 - \sigma} \right] \quad (2)$$

where C_t is consumption at period t and L_t is labor supply by the households. σ denotes the risk aversion of the agents. And $\gamma \in (0, 1)$ denotes the consumption share in the utility function. Agents discount future utility with the discount factor $\beta \in (0, 1)$.

3.1.1 Technology and Capital Accumulation

The production function is of standard Cobb-Douglas form

$$Y_t = e^{z_t} K_t^{1-\alpha} L_t^\alpha \quad (3)$$

where z_t is a productivity process with transitory effect. K_t denotes the capital stock at time t and L_t denotes labor. Output is denoted by Y_t and $\alpha \in (0, 1)$ denotes the elasticity of labor. The law of motion for capital is given by

$$K_{t+1} = (1 - \delta) K_t + I_t - \frac{\phi}{2} \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 K_t \quad (4)$$

with $\delta \in (0, 1)$ being capital depreciation. I_t is investment and ϕ denote a capital adjustment cost parameter to avoid excess volatility in investment.

⁹Fernandez-Villaverde et al. (2011) argue that a one-period, uncontinend bond reflects well the limited ability of many emerging market economies to borrow in international financial markets at long horizons.

3.1.2 Budget Constraint and Debt Premium

The budget constraint of the economy is

$$C_t + K_{t+1} = Y_t + (1 - \delta) K_t - \frac{\phi}{2} \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 K_t - B_t + q_t B_{t+1} \quad (5)$$

where q_t is the price of debt B_t that depends on the debt level relative to output as in Schmitt-Grohe and Uribe (2003)

$$\frac{1}{q_t} = 1 + r_t = 1 + r^* + p_t \quad (6)$$

where p_t is a premium on the time invariant net world interest rate r^* faced by the small open economy.¹⁰ This premium depends on the deviation of the country's debt to output ratio from the steady state and shows time-varying volatility that emerges endogenously

$$p_t = \psi e^{\sigma_{p_t}} \left(e^{\tilde{B}_t} - 1 \right) \quad (7)$$

where $\psi > 0$ is the elasticity of the debt premium and \tilde{B}_t is the deviation of the debt to output ratio from its steady state value

$$\tilde{B}_t = \frac{B_t}{Y_t} - \frac{B}{Y} \quad (8)$$

The variable σ_{p_t} in Equation (7) follows a first-order autoregressive process that again is driven by deviations in the debt to output ratio from its steady state \tilde{B}_t and an elasticity parameter η_p that governs the response of σ_{p_t} to deviations of the debt to output ratio from its steady state

$$\sigma_{p_t} = \rho_{\sigma_p} \sigma_{p_{t-1}} + \eta_p \tilde{B}_t \quad (9)$$

so that larger deviations from the steady state have a level effect on the debt premium as well as a volatility effect.¹¹ By modeling the volatility term in the above way we closely follow Fernandez-Villaverde et al. (2011) when they introduce stochastic volatility to the debt premium. However, we decide to make the volatility depend on the level of debt relative to output. This reduced form process is motivated by the observation that debt premiums are not only countercyclical and increase in the debt to output level but also their volatility is strongly countercyclical. As it is standard in small open economy models with a debt elastic interest rate, agents do neither internalize the effect on the debt premium level nor on the debt premium volatility when choosing the optimal debt level B_t .

¹⁰The world interest rate is set such that $\beta(1 + r^*) = 1$.

¹¹In a stochastic process this is tantamount to saying that level shocks and volatility shocks are perfectly correlated.

3.1.3 Net Exports and Output Growth

We define the ratio of net exports to output as¹²

$$NX_t = \frac{B_t - q_t B_{t+1}}{Y_t} \quad (10)$$

where B_t denotes the amount of debt so that higher debt in the next period is associated with negative net exports. The growth rate of output is defined as

$$\Delta Y_t = \log(Y_t) - \log(Y_{t-1}). \quad (11)$$

3.1.4 Recursive Problem and Equilibrium

In recursive representation the agent's problem becomes

$$V(K, B, z) = \max_{(C, L, K', B')} \left\{ \frac{[C^\gamma (1-L)^{1-\gamma}]^{1-\sigma}}{1-\sigma} + \beta E[V(K', B', z')] \right\} \quad (12)$$

subject to the budget constraint

$$C + K' = Y + (1 - \delta)K - \frac{\phi}{2} \left(\frac{K'}{K} - 1 \right)^2 K - B + qB'. \quad (13)$$

Given an initial capital stock K_0 and debt level B_0 , the equilibrium of the economy is characterized by the first order conditions of the problem in Equation (12), technology in Equation (3) and budget constraint in Equation (13), and the transversality condition. Where the capital law of motion is given by

$$K' = (1 - \delta)K + I - \frac{\phi}{2} \left(\frac{K'}{K} - 1 \right)^2 K. \quad (14)$$

We provide the full set of equilibrium conditions in the Appendix.

3.2 TFP Shocks

The model includes endogenous time-varying volatility in TFP besides the endogenous time-varying volatility in the debt premium. This choice is motivated by two facts. First, time-varying volatility in TFP emerges as a natural extension of Aguiar and Gopinath (2007) and the EME business cycle literature that includes shocks to TFP. Second, exogenous volatility shocks to TFP are known to be a major source of macroeconomic fluctuations as shown by Justiniano and Primiceri (2008), Fernandez-Villaverde et al. (2011) and others. We have previously shown that debt premium volatility and TFP volatility are

¹²We will call the net export to output ratio simply as net exports in what follows.

highly correlated with each other in the data. We implement the time-varying volatility in TFP as an endogenous process since empirical research suggests volatility shocks to have endogenous components as shown by Bachmann et al. (2013) and Ludvigson et al. (2015). We see it as an additional advantage to endogenize the volatility process as this leaves us with a single exogenous shock, a transitory TFP shock as used in the very standard macroeconomic DSGE models.

The time-varying volatility in TFP is similarly structured as in Fernandez-Villaverde et al. (2011). However, the component causing stochastic volatility in the original model is drawn from a random normal distribution. In our paper the driving process is instead assumed to be the deviation of the debt to output ratio from its steady state, which is the same assumption we used for the debt premium process. In this fashion a larger positive deviation of the debt to output ratio from its steady state induces higher volatility. We choose deviations of the debt to output ratio from its steady state as the driving process for the endogenous time-varying volatility as this allows us as to get a convenient reduced form implementation of countercyclical volatility. Since time-varying volatility in our model is not only driven by output but also by debt, it harmonizes well with the narrative that a higher debt burden induces financial instability. High levels of debt to output increase the uncertainty about the profitability of future investment projects as it increases the probability of firm defaults which subsequently causes misallocations in factor inputs. This hence leads to temporarily higher variability in total factor productivity. Since the debt to output ratio is a slowly moving process, persistent periods of low and high volatility emerge in the model. Once the persistence of the volatility process is high enough and debt premium and TFP volatility are both present, agents will react to such changes in volatility in a similar way as they would react to a permanent TFP shock. The TFP process is then structured as follows

$$z_t = \rho_z z_{t-1} + e^{\sigma_{z_t}} u_{z_t} \quad (15)$$

where $\rho_z < 1$ is the persistence of the TFP process and u_{z_t} is a normally distributed random variable with mean zero and variance $\sigma_{u_z}^2$ that can be considered as a shock in transitory TFP

$$u_{z_t} \sim \mathcal{N}(0, \sigma_{u_z}^2). \quad (16)$$

The variable σ_{z_t} is not assumed to be constant but instead follows a first-order autoregressive process so that the volatility part then follows as

$$\sigma_{z_t} = \rho_{\sigma_z} \sigma_{z_{t-1}} + \eta_z \tilde{B}_t. \quad (17)$$

The parameter η_z in Equation (17) affects the elasticity of endogenous volatility in TFP with respect to deviations of the debt to output ratio \tilde{B}_t from its steady state.¹³ A high

¹³In a stochastic setting η_z would be the degree of stochastic volatility. In an endogenous setting it should

η_z implies a high elasticity of endogenous volatility in the process i.e. volatility reacts strongly to deviations of the debt to output ratio from its steady state. Further ρ_{σ_z} denotes the persistence of the TFP volatility process. By modeling TFP in this way the only driving exogenous shock to the system is a shock in transitory TFP.

3.3 Parameters

The parameterization of the main model parameters follows Aguiar and Gopinath (2007) to ensure comparability of the results. Table 3 reports all parameters of the model. In our baseline case the labor elasticity α is set to 0.68 and the discount factor β is assumed to be 0.98 to fit quarterly data. The capital depreciation rate δ and the capital adjustment cost ϕ are set to conventional levels of 0.05 and 4.00, respectively. The risk aversion parameter σ is set to 2.00 in accordance with the literature in international macroeconomics and the value used by Aguiar and Gopinath (2007). The consumption exponent γ is set to 0.36 and the debt premium elasticity ψ is assumed to be 0.001 in line with the values used by Schmitt-Grohe and Uribe (2003) and Neumeyer and Perri (2005). The steady state debt to output ratio B/Y is finally set to 0.10. We further fix the persistence of TFP shocks ρ_z to 0.95 as used by Aguiar and Gopinath (2007) and various other papers.

To verify the results we choose in a second step a Monte Carlo prior for the parameter values from a uniform distribution centered around the baseline value. For the labor elasticity α we choose values between 0.50 and 0.86 and for the discount factor β we choose values between 0.97 and 0.99. The capital depreciation rate δ and the capital adjustment cost ϕ are set between 0.03 and 0.07 and between 2.00 and 6.00, respectively. We allow for some variation in the risk aversion σ by choosing values between 1.50 and 2.50. The consumption exponent γ is set between 0.20 and 0.52. Finally, the steady state debt to output ratio B/Y is set to values between 0 and 0.20, respectively. We fix the debt premium elasticity ψ to the baseline value of 0.001 as this parameter directly influences the size of the debt premium and therefore the effect of the debt premium volatility term on interest rates. Having additional variation in the debt premium elasticity would make it difficult to pin down the effect of the endogenous volatility parameter η_p . Since the time-varying volatility emerges endogenously in the model, it is not straightforward to estimate the parameters of the volatility process as in Fernandez-Villaverde et al. (2011) who assume normally distributed innovations to the stochastic volatility process. We therefore validate the model for a range of parameter values for the volatility elasticities η_p and η_z .

3.4 Solving the Model

Since we are explicitly interested in the effect of the endogenous volatility terms on macroeconomic dynamics, we solve the model using a third-order approximation to let

rather be called the elasticity of volatility with respect to the deviation of the debt to output ratio from its steady state. We might use both terms interchangeably.

TABLE 3
Model Parameters

Parameter	Description	Baseline	Monte Carlo
α	Labor Elasticity	0.68	[0.50,0.86]
β	Discount Factor	0.98	[0.97,0.99]
δ	Capital Depreciation Rate	0.05	[0.03,0.07]
ϕ	Capital Adjustment Cost	4.00	[2.00,6.00]
σ	Risk Aversion	2.00	[1.50,2.50]
γ	Consumption Exponent	0.36	[0.20,0.52]
ψ	Debt Premium Elasticity	0.001	0.001
B/Y	Steady State Debt to Output	0.10	[0.00,0.20]
ρ_z	TFP Persistence	0.95	0.95
σ_{u_z}	TFP Shock Size	0.01	0.01
ρ_{σ_p}	Debt Premium Volatility Persistence	0.90	0.90
ρ_{σ_z}	TFP Volatility Persistence	0.90	0.90

Note: Table 3 shows the parameter values in the DSGE model for the baseline case and the parameter range of the Monte Carlo prior.

the endogenous volatility terms have an independent effect from the TFP shocks. It is well known that a first-order approximation would imply certainty equivalence and volatility does have no impact on agents decision. When using a second-order approximation of the policy function all effects of volatility would only appear through the effect on the TFP shock. Only with a third-order approximation the volatility terms have direct effects on agents decision making. We provide the analytical solution for the steady state together with a numerical example in the Appendix.

4 Results

The fit of the model is now compared to Mexican and Canadian data for different sizes of the endogenous volatility elasticities η_p and η_z . The theoretical benchmark for our analysis is the model without time-varying volatility by Aguiar and Gopinath (2007) that uses larger trend than cycle shocks to generate countercyclical net exports and excess volatility in consumption.

We compare the fit of the different model specifications using the second moments. The simulated data of the model is HP-Filtered as in Hodrick and Prescott (1997) with a filter weight of 1600 for quarterly data. Each model is simulated for 288 periods with the baseline parameters in Table 3. We then drop the first 200 observations to get rid of initial conditions so that we are left with 88 observations, the same size as the empirical data. We replicate the model 1000 times with a different sequence of exogenous shocks and report the medians and the 5th to 95th percent confidence bands of the moments.

4.1 Simulated Method of Moments Estimates

We are now using a Simulated Method of Moments (SMM) approach to estimate the parameters for the elasticities of the debt premium and TFP volatility, η_p and η_z , by match-

ing the model moments to the empirically observed moments.¹⁴ In a first specification we estimate the elasticities of the debt premium and TFP volatility, η_p and η_z , to match the standard deviation of output and the relative standard deviation of consumption to output. In a second specification we additionally include the standard deviation of the exogenous TFP shock σ_{u_z} that gives us some flexibility to match the standard deviation of output in our model. We further include in this second specification the persistence parameter for the debt premium and TFP volatility process ρ_{σ_p} and ρ_{σ_z} and the capital adjustment cost parameter ϕ . In this specification we try to match all ten moments of the model. In both specifications we try to minimize the sum of squared percentage deviations of the model moments from the targeted empirical moments.¹⁵

Table 4 shows the estimated parameter values and the second moments when we use Mexico and Canada as the target countries together with the results of the model by Aguiar and Gopinath (2007) for comparison.¹⁶ For data comparison we use the data moments calculated by Aguiar and Gopinath (2007) as the benchmark value. These are calculated using quarterly data from 1980Q1 to 2003Q1. As Fernandez and Gulan (2015) mention, they cannot find any change in EME moments when data for the last recession is included. We therefore conclude that the data sample calculated by Aguiar and Gopinath (2007) is still representative for many emerging markets. Most informative for our purpose are the relative standard deviations of consumption to output σ_C/σ_Y , the relative standard deviation of net exports to output σ_{NX}/σ_Y , as well as the correlation between net exports and output $\rho_{Y,NX}$ and the correlation between consumption and output $\rho_{Y,C}$. These moments are most informative in our context as especially the negative correlation of net exports and output as well as the excess volatility in consumption are defining features of emerging market business cycles. It is well known that standard business cycle models usually fail to produce countercyclical net exports and excess volatility in consumption. We therefore take these second moments as a natural benchmark to evaluate our model.

By matching the Mexican standard deviation of output and the relative standard deviation of consumption we obtain estimated elasticities of about 0.475 and 0.149 for η_p and η_z , respectively. Matching the Canadian standard deviation of output and the relative standard deviation of consumption to output on the other hand implies elasticities

¹⁴We use SMM for two reasons. First, since we are using a third-order approximation of our model, theoretical moments are difficult to calculate. Only recently Andreasen et al. (2018) provide an approach using a pruned state space approximation. Second, we are comparing our model moments to empirically observed moments obtained from a finite sample period. We therefore aim to compare like with like and calculate the model moments from a simulation with the same sample length as its empirical counterpart.

¹⁵We minimize the sum of squared percentage deviations rather than the sum of squared residuals as our targeted moments are in different units and sizes. The main results stay the same when we minimize the sum of squared residuals.

¹⁶We show the results of Aguiar and Gopinath (2007) when they estimate the standard deviation of the cycle TFP shock and the standard deviation of the trend TFP shock as parameters as comparison for our first specification. For the second specification we compare our results with the results by Aguiar and Gopinath (2007) when they estimate the standard deviation of cycle and trend shocks, the persistence of cycle and trend shocks, and the growth rate of trend TFP as well as the capital adjustment cost.

of 0.194 and 0.154 for η_p and η_z . The estimated parameter results when only η_p and η_z are estimated seem to indicate that Mexican business cycles are mainly driven by a higher degree of endogenous time-varying volatility in the debt premium rather than differences in the degree of time-varying TFP volatility.

When we match all moments using the six parameters described above we can substantially improve the fit of the model for Mexico. We obtain parameter estimates of 0.438 and 0.162 for η_p and η_z , respectively. Those estimates are of similar size as when we only target the standard deviation of output and the relative standard deviation of consumption to output. We also estimate the persistence of endogenous volatility as 0.966 and 0.935 for ρ_{σ_p} and ρ_{σ_z} . In addition we estimate the standard deviation of the TFP shock σ_{u_z} as 0.39 percent and the capital adjustment cost ϕ as 0.90. The lower standard deviation of the TFP shocks hence makes up for the higher values of the persistence parameters ρ_{σ_p} and ρ_{σ_z} when compared to the values used in our baseline example. Both the estimated standard deviation of TFP shocks as well as the capital adjustment cost are similar in size to what Aguiar and Gopinath (2007) find by matching all moments. Compared to Aguiar and Gopinath (2007) we only have one exogenous shock in the model. We nevertheless require a comparable size of the transitory TFP shock as the TFP shock in our model gets amplified by the presence of the endogenous time-varying volatility.¹⁷ By comparing the sum of squared percentage deviations (SSPD) of the targeted empirical moments from the model moments it turns out that we can slightly increase the fit of the model for Mexico compared to the moments obtained by Aguiar and Gopinath (2007) when they match all moments with six parameters. Turning to the estimates for Canada it turns out that once we target all moments using six parameters we get similar estimates for the elasticities η_p and η_z compared to the values for Mexico. However we get significantly different estimates for the persistence parameters ρ_{σ_p} and ρ_{σ_z} . The persistence parameters are much lower in the case of Canada than their Mexican counterparts. The estimated capital adjustment cost ϕ is higher than the corresponding value for Mexico as Canada shows much less mean volatility in investment.

Table 5 shows in addition the second moments when we match all ten moments of the model to the average moments of 13 emerging market economies and 13 developed economies, respectively.¹⁸ It becomes clear that our model does a good job in replicating both EME and developed economies business cycle moments. Not only are we able to produce excess consumption volatility and countercyclical net exports in emerging market economies, we get in addition close to the empirically observed correlation of consumption with output $\rho_{Y,C}$ and investment with output $\rho_{Y,I}$. Our model also does a remarkably good job in creating the right level of persistence in output ρ_Y and output growth $\rho_{\Delta Y}$. However, in both specifications we slightly underestimate the mean

¹⁷We shed more light on this fact in the next section.

¹⁸Averages are calculated by Aguiar and Gopinath (2007) and are unweighted averages of Argentina, Brazil, Ecuador, Israel, South Korea, Malaysia, Mexico, Peru, Philippines, Slovakia, South Africa, Thailand, and Turkey as emerging market economies and Australia, Austria, Belgium, Canada, Denmark, Finland, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, and Switzerland as developed economies.

TABLE 4
Simulated Method of Moments I

Mexico					
		$\sigma_Y, \sigma_C / \sigma_Y$ Used		All Moments Used	
	Data	AG2007	Model	AG2007	Model
η_p			0.475		0.438
η_z			0.149		0.162
σ_{uz}					0.0039
ρ_{σ_p}					0.966
ρ_{σ_z}					0.935
ϕ					0.900
σ_Y	2.40	2.40	2.40	2.13	2.11
$\sigma_{\Delta Y}$	1.52	1.73	1.86	1.42	1.74
σ_C / σ_Y	1.26	1.26	1.26	1.10	1.35
σ_I / σ_Y	4.15	2.60	1.89	3.83	2.84
σ_{NX} / σ_Y	0.90	0.71	0.50	0.95	0.75
ρ_Y	0.83	0.78	0.72	0.82	0.75
$\rho_{\Delta Y}$	0.27	0.13	0.03	0.18	0.25
$\rho_{Y,NX}$	-0.75	-0.66	-0.64	-0.50	-0.64
$\rho_{Y,C}$	0.92	0.94	0.93	0.91	0.92
$\rho_{Y,I}$	0.91	0.92	0.97	0.80	0.78
SSPD		0.00	0.00	0.28	0.22

Canada					
		$\sigma_Y, \sigma_C / \sigma_Y$ Used		All Moments Used	
	Data	AG2007	Model	AG2007	Model
η_p			0.194		0.451
η_z			0.154		0.113
σ_{uz}					0.0089
ρ_{σ_p}					0.810
ρ_{σ_z}					0.914
ϕ					3.130
σ_Y	1.55	1.55	1.55	1.24	0.95
$\sigma_{\Delta Y}$	0.80	1.14	1.24	0.82	0.73
σ_C / σ_Y	0.74	0.74	0.74	0.76	0.85
σ_I / σ_Y	2.67	1.99	1.96	3.14	2.28
σ_{NX} / σ_Y	0.57	0.41	0.20	0.65	0.42
ρ_Y	0.93	0.75	0.71	0.81	0.72
$\rho_{\Delta Y}$	0.55	0.04	-0.01	0.17	0.02
$\rho_{Y,NX}$	-0.12	0.18	0.02	-0.15	-0.13
$\rho_{Y,C}$	0.87	0.87	0.97	0.87	0.87
$\rho_{Y,I}$	0.74	0.94	0.98	0.82	0.96
SSPD		0.00	0.00	0.66	1.32

Note: Table 4 shows the model moments using estimated parameter values obtained from SMM for Mexico and Canada. σ denotes standard deviations of a variable and ρ denotes the correlation between two variables. Moments are the median of 1000 replications of the model. Data moments are as calculated in Aguiar and Gopinath (2007). If not estimated, parameters are as reported in Table 3. SSPD denotes the sum of squared percentage deviations of the targeted moments from the data moments.

volatility of net exports and investment. The SMM parameter estimates again indicate that differences in EME and developed economies are mainly driven by differences in the persistence of volatility rather than differences in the size of the elasticities once we estimate the full set of parameters.

TABLE 5
Simulated Method of Moments II

	Emerging Markets		Developed Economies	
	Data	Model	Data	Model
η_p		0.426		0.378
η_z		0.099		0.105
σ_{u_z}		0.0071		0.0116
ρ_{σ_p}		0.958		0.850
ρ_{σ_z}		0.938		0.897
ϕ		0.908		1.375
σ_Y	2.74	2.54	1.34	1.34
$\sigma_{\Delta Y}$	1.87	1.94	0.95	0.97
σ_C/σ_Y	1.45	1.74	0.94	0.95
σ_I/σ_Y	3.91	2.91	3.41	3.36
σ_{NX}/σ_Y	1.18	0.98	0.76	0.63
ρ_Y	0.76	0.78	0.75	0.76
$\rho_{\Delta Y}$	0.23	0.20	0.09	0.09
$\rho_{Y,NX}$	-0.51	-0.48	-0.17	-0.18
$\rho_{Y,C}$	0.72	0.81	0.66	0.74
$\rho_{Y,I}$	0.77	0.64	0.67	0.84
SSPD		0.21		0.11

Note: Table 5 shows the model moments using estimated parameter values obtained from SMM for the average of EME and developed economies. σ denotes standard deviations of a variable and ρ denotes the correlation between two variables. Moments are the median of 1000 replications of the model. Data moments are as calculated in Aguiar and Gopinath (2007). If not estimated, parameters are as reported in Table 3. SSPD denotes the sum of squared percentage deviations of the targeted moments from the data moments.

4.2 Baseline Model

We have shown that we can generate EME business cycle moments for a certain combination of values for the elasticities of the debt premium and TFP η_p and η_z , respectively. We now analyze the behavior of the model when only time-varying debt premium or TFP volatility are present. The benchmark model by Aguiar and Gopinath (2007) is again compared to our model with time-varying volatility in the debt premium p_t and to the model with time-varying volatility in TFP z_t . We then continue and analyze the behavior of the model when time-varying volatility in TFP and the debt premium is jointly present. It turns out that only a model that features both, endogenous time-varying debt premium volatility and time-varying TFP volatility, is able to replicate EME business cycle moments.

The second column in Table 6 shows the results by Aguiar and Gopinath (2007) without time-varying volatility. In this model TFP cycle shocks are set to a standard deviation of 0.48 percent and trend shocks to TFP have a standard deviation of 2.81 percent. With such a specification net exports become countercyclical and consumption volatility is higher than output volatility.¹⁹ Model (1) in Table 6 presents the results for the small open

¹⁹Note that Aguiar and Gopinath (2007) match the standard deviation of output and the relative standard deviation of consumption to output by construction.

economy model with a single transitory TFP shock but without time-varying volatility. As mentioned before such a model fails to reproduce the main emerging market business cycle facts.

TABLE 6
Second Moments Baseline Model

	Data	AG2007	(1) No Volatility	(2) Premium Volatility	(3) TFP Volatility	(4) Premium and TFP Volatility
η_p			0.00	0.60	0.00	0.60
η_z			0.00	0.00	0.15	0.15
σ_Y	2.40	2.40	1.67 (1.31;2.18)	4.02 (2.60;8.46)	1.67 (0.40;8.38)	3.12 (0.86;11.14)
$\sigma_{\Delta Y}$	1.52	1.73	1.31 (1.14;1.49)	3.15 (2.15;6.68)	1.28 (0.32;6.20)	2.45 (0.70;8.68)
σ_C/σ_Y	1.26	1.26	0.49 (0.41;0.56)	0.44 (0.18;0.76)	0.54 (0.46;0.62)	1.49 (1.00;2.70)
σ_I/σ_Y	4.15	2.60	1.88 (1.78;1.99)	1.97 (1.71;2.23)	1.90 (1.76;2.02)	1.85 (1.44;2.27)
σ_{NX}/σ_Y	0.90	0.71	0.23 (0.16;0.32)	0.95 (0.72;1.21)	0.19 (0.12;0.29)	0.65 (0.24;1.84)
ρ_Y	0.83	0.78	0.72 (0.56;0.82)	0.74 (0.59;0.84)	0.72 (0.51;0.84)	0.72 (0.54;0.84)
$\rho_{\Delta Y}$	0.27	0.13	0.00 (-0.18;0.19)	0.10 (-0.13;0.34)	0.02 (-0.21;0.28)	0.04 (-0.18;0.31)
$\rho_{Y,NX}$	-0.75	-0.66	0.91 (0.81;0.95)	0.98 (0.97;0.99)	0.84 (0.50;0.92)	-0.69 (-0.90;0.05)
$\rho_{Y,C}$	0.92	0.94	1.00 (0.99;1.00)	-0.97 (-0.99;-0.89)	1.00 (0.97;1.00)	0.92 (0.29;0.98)
$\rho_{Y,I}$	0.91	0.92	0.97 (0.95;0.98)	0.91 (0.82;0.95)	0.97 (0.95;0.98)	0.97 (0.71;0.99)
σ_p	0.25		0.02 (0.01;0.06)	0.61 (0.13;2.45)	0.04 (0.01;0.23)	0.61 (0.13;2.30)
σ_z	3.01		2.40 (1.59;3.73)	2.43 (1.55;3.83)	2.48 (0.61;12.77)	6.85 (1.91;24.22)
$\rho_{Y,p}$	-0.40		0.05 (-0.11;0.28)	-0.06 (-0.30;0.12)	-0.02 (-0.20;0.28)	0.04 (-0.21;0.33)
$\rho_{Y,z}$	0.53		0.69 (0.45;0.89)	0.70 (0.42;0.89)	0.67 (0.30;0.90)	0.62 (0.26;0.87)

Note: Table 6 shows the second moments of the Mexican data and the different DSGE models. σ denotes standard deviations of a variable and ρ denotes the correlation between two variables. Moments are the median of 1000 replications of the model. The 5th to 95th percentile confidence bands are in parenthesis. Data moments are as calculated in Aguiar and Gopinath (2007).

4.2.1 Endogenous Debt Premium Volatility

Model (2) in Table 6 only contains time-varying debt premium volatility i.e. η_p is set to 0.60 and η_z is set to 0. The standard deviation of TFP shocks is set to 1 percent. A model that only features endogenous volatility to the debt premium is not able to generate countercyclical net exports or excess volatility in consumption. The correlation of net exports with output is clearly positive with 0.98 and the relative volatility of consumption to output is far below unity with 0.44. These observations are in line with the findings by Fernandez-Villaverde et al. (2011) who introduce exogenous stochastic volatility to the interest rate. Most striking in our findings is that endogenous debt premium volatility that is caused by the debt to output ratio induces a strongly countercyclical consumption behavior in the model with a correlation of output and consumption of -0.97. This is a

result of the specification of the debt premium process which implies a non-monotonic behavior with respect to the debt to output ratio. After a positive TFP shock the debt to output ratio becomes negative and the debt premium hence turns negative. However, given the functional form, further decreases in the debt to output ratio lead to a convergence back to a zero debt premium. Given the parameter value for the debt premium elasticity ψ this results in a debt premium increase by up to 10 basis points in the long run and therefore negative consumption growth through the Euler equation. The result is then a strongly negative correlation between output and consumption. As a remedy we test an alternative functional form for the debt premium

$$p_t = \psi \left(e^{\sigma p_t} e^{\tilde{B}_t} - 1 \right). \quad (18)$$

Using this functional form the debt premium becomes strictly monotonic and the correlation between output and consumption remains close to unity. Additionally, this functional form is able to generate countercyclical net exports when only endogenous time-varying debt premium volatility is present. However, it is not able to generate the desired excess consumption volatility. We can therefore rule out with certainty that a model with only time-variation in the debt premium volatility is suitable to describe business cycles in EME with both countercyclical net exports and excess consumption volatility.

4.2.2 Endogenous TFP Volatility

Model (3) in Table 6 contains only time-variation in the volatility of TFP i.e. η_p is set to 0 and η_z is set to 0.15. Time-varying volatility in TFP is again not able to generate countercyclical net exports and excess consumption volatility. Correlations of net exports and output are lower than in the previous case but still positive with a value of 0.84 and the relative volatility of consumption to output is again below unity with 0.54. However, compared to the model with only endogenous debt premium volatility, consumption is now highly correlated with output.

4.2.3 Endogenous Debt Premium and TFP Volatility

Finally, model (4) in Table 6 contains endogenous time-varying volatility in the debt premium and TFP simultaneously i.e. η_p is set to 0.60 and η_z is set to 0.15. Both parameter values are therefore set on the upper bound of the SMM estimates for illustrative purpose. Introducing volatility to the debt premium and TFP simultaneously allows for countercyclical net exports and excess volatility in consumption. Net exports become strongly countercyclical with a correlation of -0.69 and the ratio of consumption to output volatility is above unity with 1.49 which is even higher than the empirical value for Mexico with 1.26. The correlation of output and consumption matches now exactly the value for Mexico with 0.92. Similar to Aguiar and Gopinath (2007) our model underpredicts the autocorrelation of output growth and the relative volatility of investment to output and

net exports to output. However, in general our values closely match the empirical data and the moments obtained by Aguiar and Gopinath (2007). In addition this specification is able to generate a substantial mean volatility in the debt premium comparable to levels observed in emerging market economies. However the model falls short of delivering a strongly countercyclical debt premium as observed in the data.

4.2.4 Grid Space of Endogenous Debt Premium and TFP Volatility

Figure 2 then shows the most important moments for different combinations of η_p and η_z within the range of 0 to 0.80 for η_p and 0 to 0.20 for η_z that govern the elasticity of the endogenous time-varying volatility in the debt premium and TFP with respect to the deviation of the debt to output ratio from its steady state. It shows in addition to the standard moments the standard deviations for output, consumption, investment, and net exports for the same grid of η_p and η_z . We classify a model as being able to generate realistic EME business cycle moments when it satisfies certain assumption i.e. when the second moments are within a certain range. These conditions are, (1) NX are countercyclical i.e. correlations with output are less than zero, (2) consumption volatility exceeds output volatility but is less than 3, (3) NX volatility relative to output volatility is between 0.30 and 1.00, (4) the correlation of consumption to output is larger than 0.70 and (5), the autocorrelation of output is larger than 0.50. We additionally impose some restrictions on the absolute volatility of output, consumption, investment, and net exports as shown in Table 7. We require that output has a standard deviation of less than 5 and consumption of less than 10. For the standard deviation of investment and net exports we require the values to be below 20 and 5, respectively. All the restrictions we impose on the moments fit well the empirically observed moments for Mexico. Moments that satisfy our restrictions are colored blue, whereas those that do not satisfy the restrictions are shown in red. Even after imposing these restrictions it becomes clear that there are combinations of η_p and η_z that can endogenously generate emerging market business cycle characteristics in the generated data and that satisfy all imposed restrictions.

It is striking that neither time-varying volatility in the debt premium nor time-varying volatility in TFP can generate emerging market business cycles alone as shown in Table 6 and as visible in Figure 2. Only when time-varying volatility in the debt premium governed by η_p and in TFP as governed by η_z are present simultaneously, a negative correlation between net exports and output and excess volatility in consumption arises. The previously observed negative correlation between consumption and output after introducing debt premium volatility vanishes and turns strongly positive when TFP volatility is added to the model.

4.2.5 Autocorrelation of Net Exports

Garcia-Cicco et al. (2010) argue that standard real business cycle models produce an autocorrelation for net exports that is flat and close to unity so that net exports essentially

TABLE 7
Emerging Market Conditions

Moment	Description	Data	Min	Max
$\rho_{Y,NX}$	Correlation Output and Net Exports	-0.75	-1.00	0.00
σ_C/σ_Y	Relative Standard Deviation Consumption to Output	1.26	1.00	3.00
σ_{NX}/σ_Y	Relative Standard Deviation Net Exports to Output	0.90	0.30	1.00
$\rho_{Y,C}$	Correlation Output and Consumption	0.92	0.70	1.00
ρ_Y	Autocorrelation of Output	0.83	0.50	1.00
σ_Y	Standard Deviation of Output	2.40	1.00	5.00
σ_C	Standard Deviation of Consumption	3.02	1.00	10.00
σ_I	Standard Deviation of Investment	9.96	1.00	20.00
σ_{NX}	Standard Deviation of Net Exports	2.16	0.50	5.00

Note: Table 7 shows the conditions imposed on the moments to classify as a DSGE model that is able to generate EME business cycles. Data moments shown are for Mexico.

follow a random walk. In the data however, autocorrelations are significantly less than unity. Figure 3 shows the autocorrelation of net exports for different lags. It becomes clear that our model does a good job in producing autocorrelations of net exports that are significantly below unity and most of the time within the two standard deviations reported by Garcia-Cicco et al. (2010) as indicated by the blue bars.²⁰ However, it becomes also clear that even in the absence of time-varying volatility in the debt premium and TFP the model generates autocorrelations in line with the empirical data. Nevertheless, time-variation in the volatility of the debt premium and TFP will however lower the autocorrelation even further as the actual debt premium increases as it is amplified by the time-varying volatility.

4.3 Monte Carlo Prior

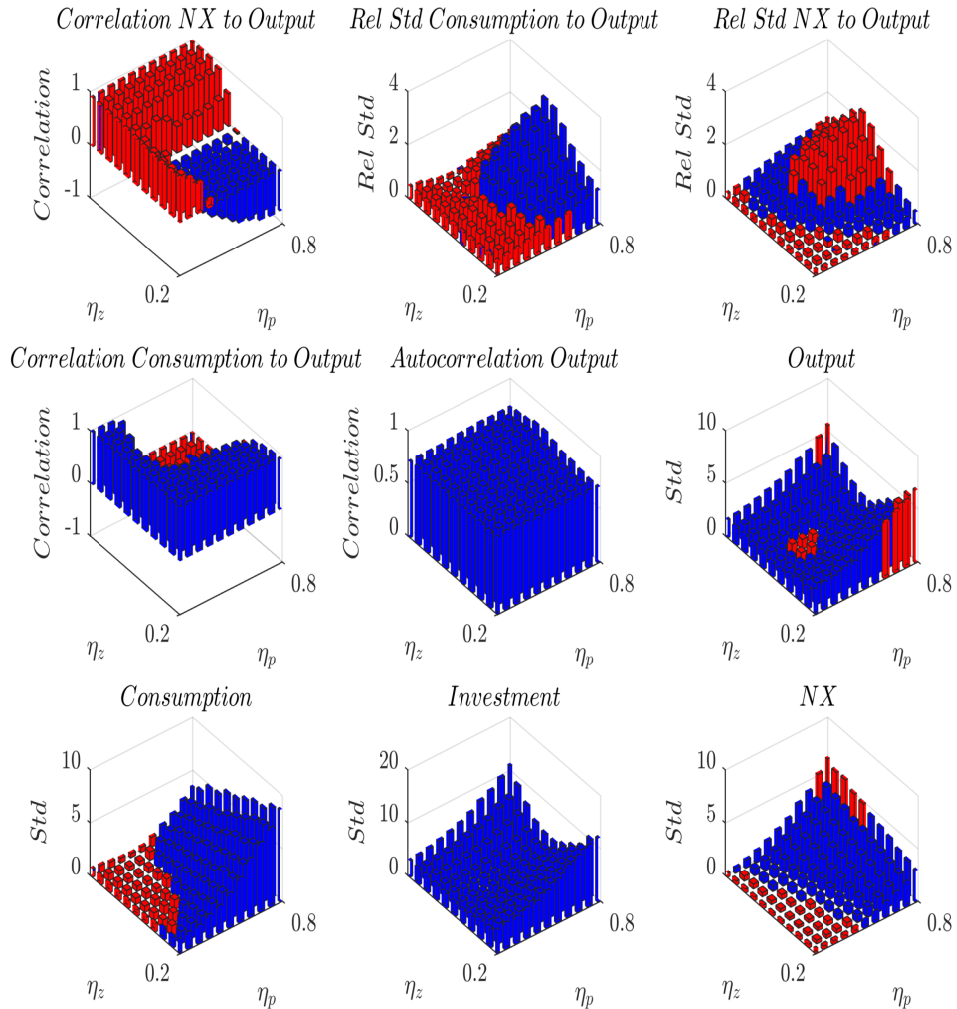
We now want to test how robust our results are to certain variations in the model parameters. For this purpose we use a Monte Carlo prior for the main parameters of the DSGE model as show in Table 3. The simulation of the model with different parameter combinations allows us to verify our findings and to test whether a subset of η_p and η_z exists for each of the draws of the DSGE parameters that can generate EME business cycle features.

Figures 4 shows the moments of the model for the 5th and 95th percentile of the Monte Carlo draws.²¹ Although we have chosen a fairly uninformative and wide prior for the model parameters we get relatively tight results i.e. a small distance between the 5th and 95th percentile of the Monte Carlo draws indicated by the height of the bars. Variations in the importance of endogenous time-varying volatility η_p and η_z seem to have a much larger impact in moving the moments of the model than the variation in the model parameters drawn from the uniform distribution. As previously, blue bars indicate

²⁰We set the acceptable boundaries on the autocorrelation to 0.40 to 0.80 for the first lag, to 0.10 to 0.50 for the second lag and to 0 to 0.40 for the third and fourth lag, respectively. This is in line with the empirical data observed by Garcia-Cicco et al. (2010).

²¹We draw 50 independent parameter combinations for the DSGE parameters in Table 3 where each Monte Carlo draw is the median of 25 replications of the model with a different sequence of exogenous shocks.

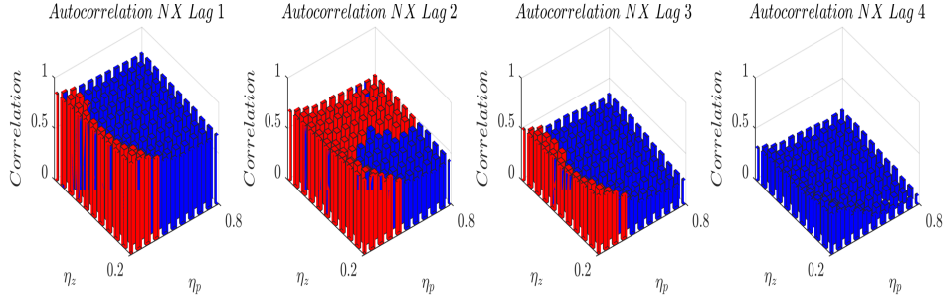
FIGURE 2
Second Moments Baseline Model



Note: Figure 2 shows the moments of the model for the baseline specification of parameters. Blue bars indicate parameter combinations of η_p and η_z that satisfy the characteristics of emerging market data, whereas red bars do not. Moments are the median of 500 replications of the model.

parameter combinations for η_p and η_z that satisfy the EME conditions shown in Table 7 entirely between the 5th and 95th percentile of the Monte Carlo draws. Whereas red bars do not fully satisfy these restrictions. Especially for the relative standard deviation of consumption to output, the Monte Carlo draws are able to produce excess volatility between the 5th and 95 percentile for a large set of combinations of η_p and η_z . However, we find that for the correlation of net exports and output the set of possible combinations of η_p and η_z shrinks somewhat when we require the 5th to 95th percentile of the Monte Carlo draws to be negative. However, requiring only the 10th to 90th percentile to be

FIGURE 3
Autocorrelation Net Exports Baseline Model



Note: Figure 3 shows the autocorrelation of net exports for different lags for the baseline specification of parameters. Blue bars indicate parameter combinations of η_p and η_z that satisfy the characteristics of emerging market data, whereas red bars do not. Moments are the median of 500 replications of the model.

negative creates many more parameter combinations that can generate countercyclical net exports. One can clearly observe that the confidence intervals for most moments increase when the volatility in the debt premium increases as this increases the overall variability in the model.

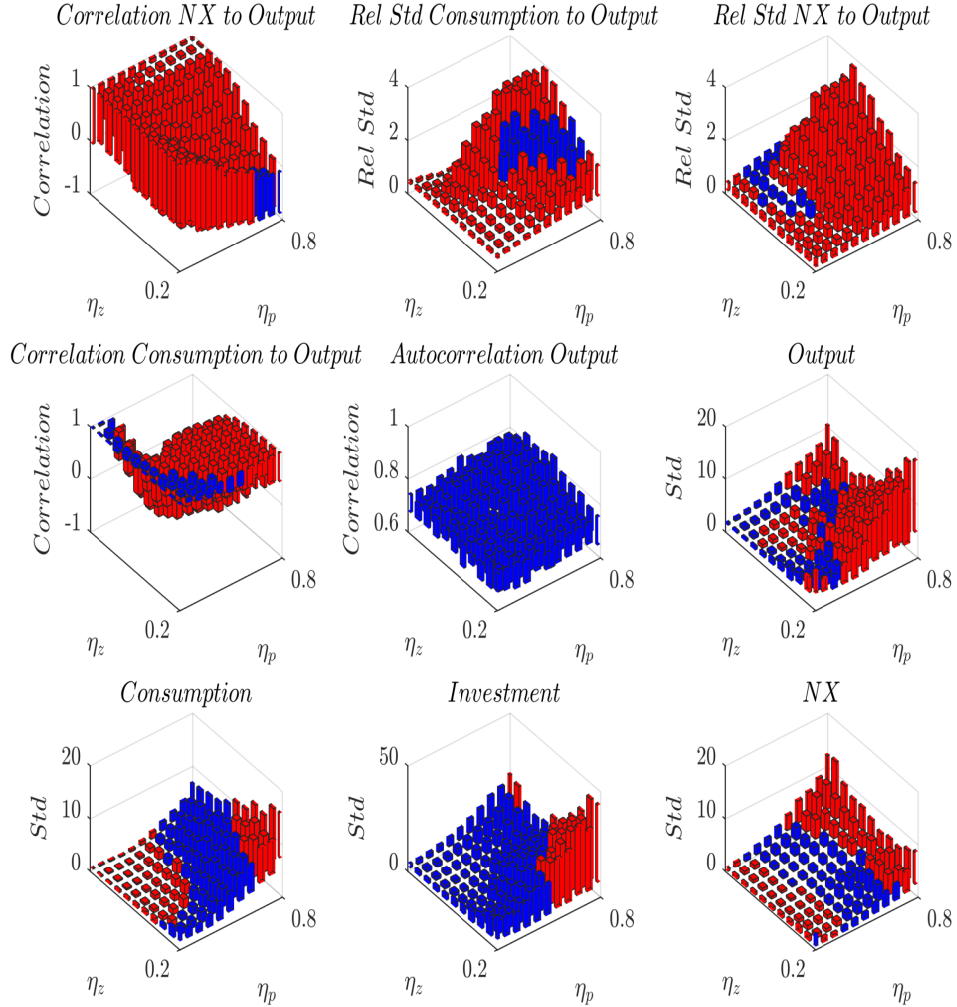
Figure 5 shows the Monte Carlo results for the autocorrelation of net exports. Especially for the first, third, and fourth lag of the autocorrelation of net exports the model is able to generate results where the 5th and 95th percentile of the Monte Carlo draws are mostly within the range of empirical results by Garcia-Cicco et al. (2010). Here again, similar to the baseline case, introducing time-varying volatility seems to slightly lower the autocorrelation of net exports. However, one has to say that our results are still on the upper range of the empirical observations.

Figure 6 finally shows the proportion of Monte Carlo draws that are able to generate EME business cycles for the combinations of η_p and η_z . The maximum frequency occurs for a value of 0.60 for the debt premium volatility parameter η_p and 0.15 for the TFP volatility parameter η_z . With this parameterization about 60 percent of the Monte Carlo draws generate EME business cycles under the set of restrictions in Table 7.

4.4 Exogenous Volatility Shocks

We use exogenous volatility processes to show the difference to a case when the volatility process for the debt premium and TFP are endogenous. We replace the endogenous driving process of volatility \tilde{B}_t for the debt premium and TFP volatility process in Equation (9)

FIGURE 4
Second Moments Monte Carlo Prior



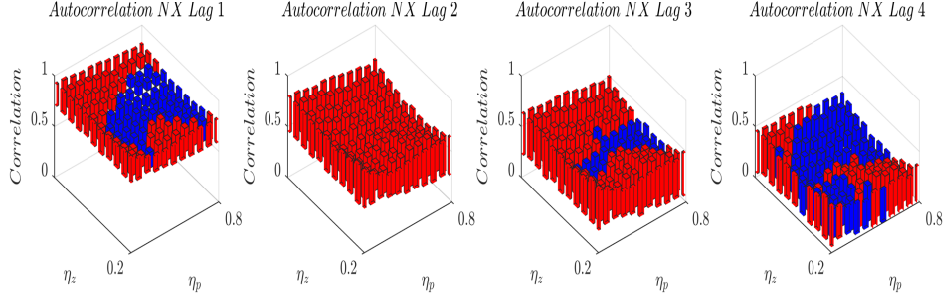
Note: Figure 4 shows the moments of the model for different combinations of η_p and η_z for the 5th and 95th percentile of the Monte Carlo draws. Blue bars indicate parameter combinations for η_p and η_z that satisfy EME conditions between the 5th and 95th percentile of the Monte Carlo draws. Whereas red bars do not fully satisfy these restrictions. Each Monte Carlo draw is the median of 25 replications of the model.

and (17) by exogenous shocks v_{p_t} and v_{z_t} , respectively.²² The exogenous volatility shocks are drawn from a normally distributed random variable with mean zero and variance $\sigma_{v_p}^2$ and $\sigma_{v_z}^2$ of 1 percent

$$v_{p_t} \sim \mathcal{N}\left(0, \sigma_{v_p}^2\right) \quad (19)$$

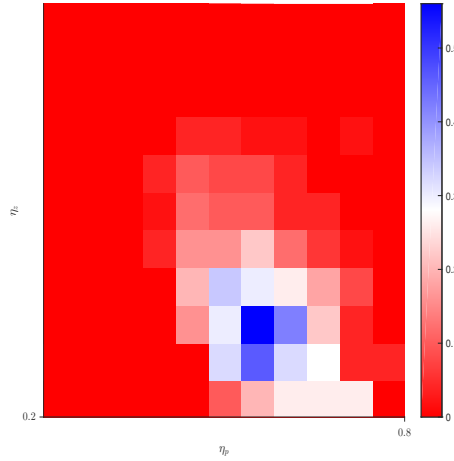
²²Note that this setup is still different from Neumeier and Perri (2005) who use exogenous level shocks to the debt premium.

FIGURE 5
Autocorrelation Net Exports Monte Carlo Prior



Note: Figure 5 shows the autocorrelation of net exports for different combinations of η_p and η_z for the 5th and 95th percentile of the Monte Carlo draws. Blue bars indicate parameter combinations for η_p and η_z that satisfy EME conditions between the 5th and 95th percentile of the Monte Carlo draws. Whereas red bars do not fully satisfy these restrictions. Each Monte Carlo draw is the median of 25 replications of the model.

FIGURE 6
Relative Frequency with Monte Carlo Prior



Note: Figure 6 shows the proportion of Monte Carlo draws that satisfy all EME conditions for different values of the parameters η_p and η_z . Each Monte Carlo draw is the median of 25 replications of the model.

$$v_{z_t} \sim \mathcal{N}(0, \sigma_{v_z}^2). \quad (20)$$

We control the actual size of the standard deviation of the stochastic volatility process by the parameters η_p and η_z that now control the size of the stochastic volatility in the debt premium and TFP, respectively. We choose different values for η_p and η_z in the range

of 1 to 100 so that the actual size of stochastic volatility is comparable to Fernandez-Villaverde et al. (2011).²³ It turns out that once we use exogenous volatility shocks to the debt premium and TFP, the model is not able to generate the key EME business cycle moments anymore. That is the correlation of net exports with output remains positive and the relative volatility of consumption to output stays below unity. If we increase the stochastic volatility beyond the given range the mean volatility of output, consumption, investment, and net exports increases beyond reasonable values without having a sizable effect on the correlation of net exports with output. It is hence crucial to endogenize the volatility processes for the debt premium and TFP in order to get EME business cycle characteristics. This observation is again caused by the fact our endogenous volatility is driven by a slowly moving process, namely the debt to output ratio. Only a slowly moving process can generate persistent increases and decreases in volatility that are then perceived as permanent by risk averse agents. Whenever volatility shocks become exogenous this persistence vanishes and net exports turn procyclical.

4.4.1 Time-Varying Volatility Increases Exogenous Shocks

Every non-zero value for the elasticity parameter η_z will increase the mean standard deviation of the corresponding TFP shocks u_{z_t} in a setup as in Fernandez-Villaverde et al. (2011) because of the non-linear nature of the exponential function.²⁴ In a setup with exogenous volatility shocks it is possible to control for these amplifying effects as the standard deviation of the innovations and hence the volatility term is known apriori. However, when volatility is endogenous, as in our work, the variance of the driving process is not known apriori and it is hence not possible to control for these amplifying effects.

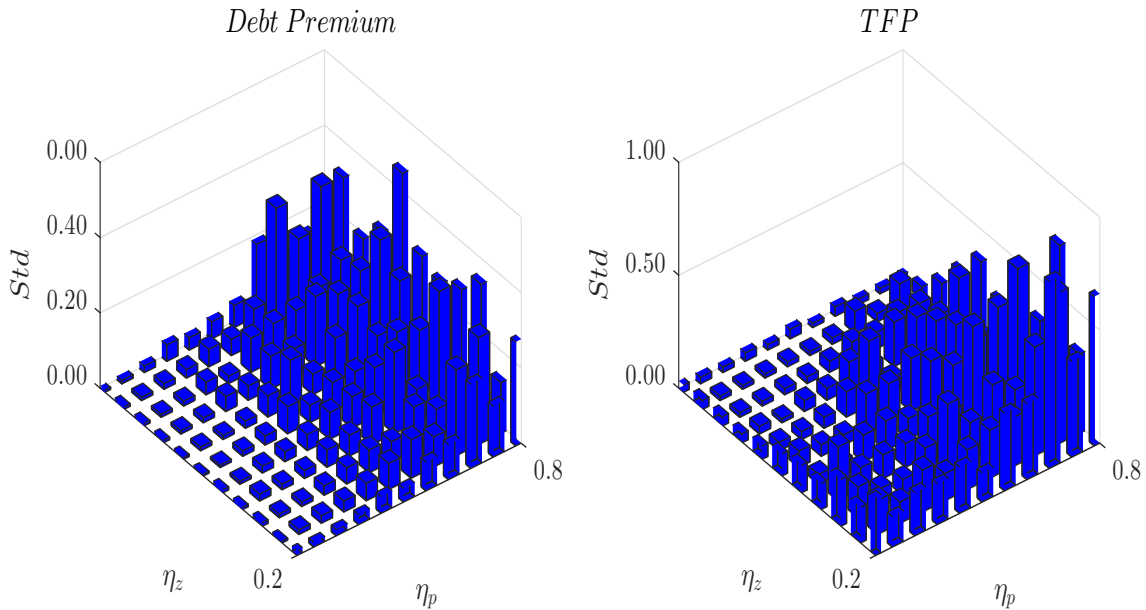
Figure 7 shows the standard deviations of the debt premium and TFP process after amplification through the volatility process for different combinations of the elasticities η_p and η_z when volatility is endogenous. The standard deviation of the debt premium in the left panel increases strongly in the elasticity of premium volatility η_p and increases only slightly as TFP becomes more volatile when η_z increases. Higher volatility in TFP has only a minor effect on the size of debt premium as the transmission is only indirectly through higher volatility in the production function and hence output. The standard deviation of TFP in the right panel increases naturally in the degree of TFP volatility η_z but also strongly in the degree of debt premium volatility η_p through the endogenous deviation of the debt to output ratio from its steady state. This endogenous increase in the mean volatility of the debt premium and TFP is the main driver of the increasing mean volatility in output, consumption, and investment that can be observed in Figure

²³Fernandez-Villaverde et al. (2011) find values for η between 0.28 for Brazil and 0.46 for Argentina in the case of interest rate volatility when a unit variance is used. This is equivalent to values of η of 28 and 46 when a 1 percent standard deviation is used.

²⁴For every normal random variable $Z \sim \mathcal{N}(0, \sigma^2)$ the mean of the exponential of Z is equivalent to $E[e^Z] = e^{\frac{\sigma^2}{2}}$.

2. It is however important to stress that the pure increase in the standard deviation of TFP is not able to create countercyclical net exports or excess consumption volatility. The high correlation between standard deviations of the TFP process and the correlation of net exports with output is rather a side effect of the endogenous volatility process that drives both.

FIGURE 7
Standard Deviation of the Debt Premium and TFP Baseline Model



Note: Figure 7 shows the standard deviations of the debt premium and TFP process after amplification through the volatility process for different combinations of η_p and η_z in the baseline model. Moments are the median of 500 replications of the model.

4.4.2 Shock Size Adjusted Exogenous Volatility Shocks

Once the volatility process is exogenous it becomes possible to resize the level shocks so that the level shocks get not augmented by the presence of time-varying volatility. This allows us to increase the stochastic volatility further without reaching too high values of mean volatility. The process for TFP then takes the form

$$z_t = \rho_z z_{t-1} + \Gamma e^{\sigma_{z_t}} u_{z_t} \quad (21)$$

where Γ is a time invariant scaling factor and u_{z_t} is a normally distributed random variable with mean zero and variance $\sigma_{u_z}^2$ and can be considered as a shock in levels to TFP

$$u_{z_t} \sim \mathcal{N}(0, \sigma_{u_z}^2). \quad (22)$$

As in the case of endogenous volatility, the variable σ_{z_t} is not assumed to be constant but instead follows an AR(1) process so that the volatility process then follows as

$$\sigma_{z_t} = \rho_{\sigma_z} \sigma_{z_{t-1}} + \eta_z w_{z_t} \quad (23)$$

where w_{z_t} is now a normally distributed random variable with mean zero and a a priori known variance $\sigma_{w_z}^2$ that can be regarded as a shock in TFP volatility

$$w_{z_t} \sim \mathcal{N}(0, \sigma_{w_z}^2). \quad (24)$$

By replacing the endogenous volatility process with an exogenous one we can now calculate the variance of the AR(1) process in Equation (23) as²⁵

$$\Sigma_{\sigma_z}^2 = \frac{\eta_z \sigma_{w_z}^2}{1 - \rho_{\sigma_z}^2}. \quad (25)$$

This allows us to resize the level shock u_{z_t} in Equation (21) accordingly by the factor

$$\Gamma = e^{-\frac{\eta_z \sigma_{w_z}^2}{2(1 - \rho_{\sigma_z}^2)}}. \quad (26)$$

The process for the debt premium follows the same pattern. The result is that the mean standard deviation of the level shocks u_{z_t} is not augmented anymore by increases in the volatility parameters η_z and η_p . However, similarly to the case without the resizing of the level shocks, exogenous time-varying volatility is not able to generate EME business cycles. Resizing the level shocks helps to control the mean volatility of output, consumption, investment, and net exports. It does however little to create countercyclical net exports and excess consumption volatility as again the exogenous volatility shocks are not persistent enough.

4.5 Transmission Channels

We finally want to show how the endogenous time-varying volatility propagates through the model after an exogenous shock in TFP and how it affects the correlation of net exports and output.

Figure 8 shows the transmission channels of endogenous volatility after a positive shock in TFP and the effect on net exports. We identify four channels through which net exports, and therefore the correlation of net exports with output, might potentially change. One direct effect in blue and an additional effect through the level of the debt premium in green. These two effects are also present in standard models without volatility that feature a debt elastic interest rate as in Schmitt-Grohe and Uribe (2003). The other two channels are the effect through debt premium volatility in red and TFP volatility in

²⁵We use $\Sigma_{\sigma_z}^2$ to denote the variance of the AR(1) process in Equation 23 and $\sigma_{w_z}^2$ to denote the variance of the exogenous error term of the AR(1) process.

yellow.

The direct effect shown in blue increases both output and consumption. However as agents perceive the shock as transitory they save in anticipation of lower future income. Hence the debt decreases and net exports turn positive resulting in procyclical net exports.

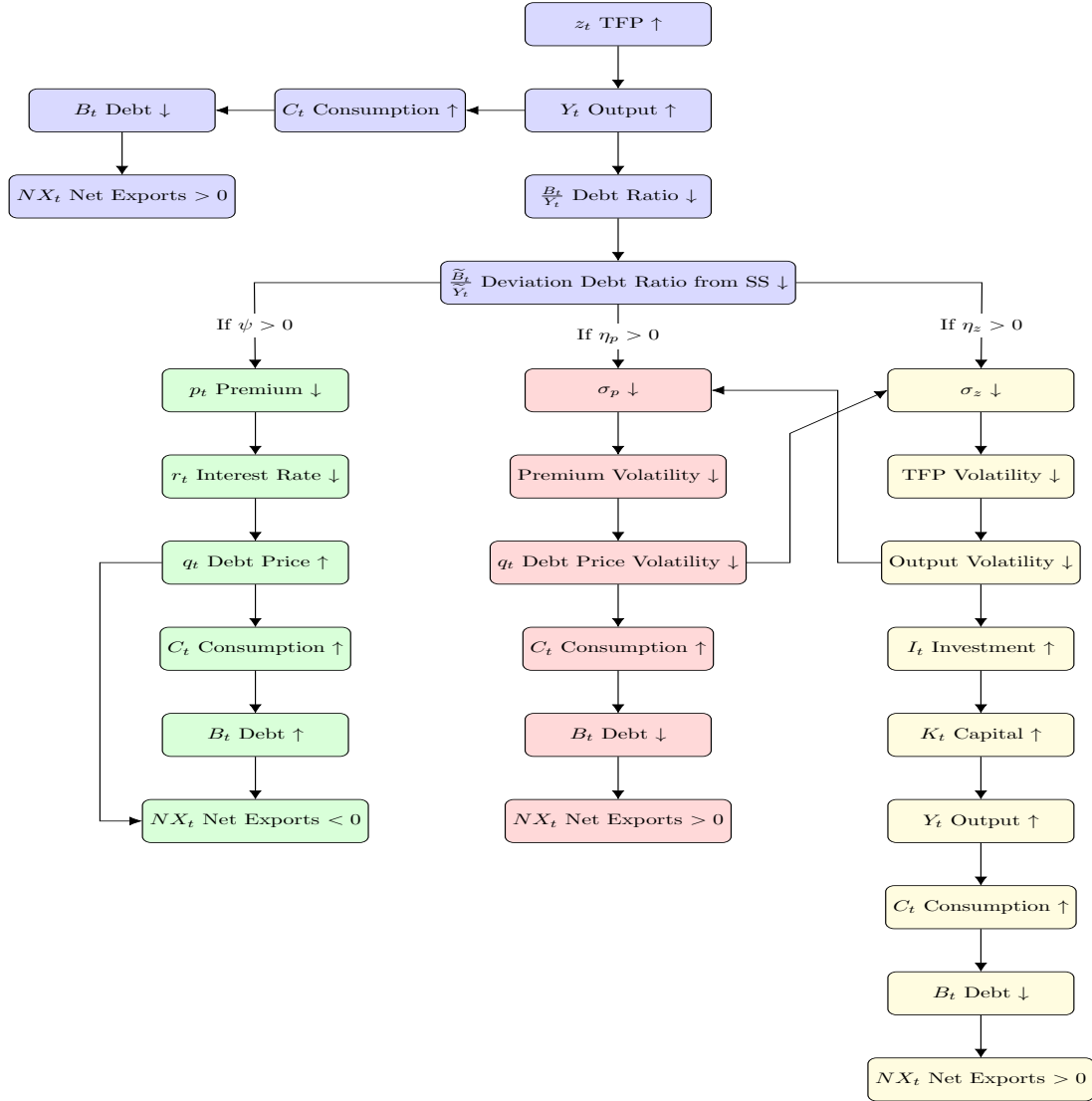
The effect through the level of the debt premium shown in green reduces the debt premium after a positive TFP shock leading to lower interest rates and higher debt prices. Higher debt prices and larger debt will lead to negative net exports and hence countercyclical net exports. For reasonable values of the debt premium elasticity ψ this channel has a rather limited effect on the overall behavior of net exports. So that standard small open economy models with an endogenous debt premium still have procyclical net exports.

In the debt premium volatility channel shown in red a positive TFP shock increases output and hence the volatility of the debt premium decreases as the debt to output ratio is below its steady state. Lower debt premium volatility lowers the debt price volatility and increases consumption and lowers the debt. As the change in volatility originates from a transitory shock in TFP, agents perceive the lower volatility to be of temporary nature. Therefore leading to positive net exports and hence procyclical net exports.

Finally, in the TFP volatility channel shown in yellow a positive TFP shock reduces the volatility of TFP and hence the volatility of output. This in turn leads to higher investment and a higher capital stock which results in higher output and consumption and lower debt and positive net exports. Again, as the lower volatility originates from a transitory TFP shock the effects on output are perceived as temporary and agents react by increased savings, or reduced debt, to prepare for future bad times resulting in procyclical net exports.

The debt premium and TFP volatility channels are connected to each other as lower debt price volatility will also reduce TFP volatility and lower output volatility will reduce the debt premium volatility. The following reduced output volatility will then have a dampening effect on the volatility of the deviation of the debt to output ratio from its steady state. This reduction in volatility then leads to a long lasting reduction in the premium volatility which in turn leads to an increase in consumption and previously positive net exports turn negative and thus became countercyclical. The connection between debt premium volatility and TFP volatility is hence leading to the observed effect that debt premium and TFP volatility need to be jointly present in order to generate countercyclical net exports.

FIGURE 8
Transmission Channels



Note: Figure 8 shows the different transmission channels when endogenous time-varying volatility is present in a model with a single exogenous TFP shock.

5 Conclusion

This paper introduced endogenous time-varying volatility into a standard small open economy model with transitory shocks to total factor productivity. A simplified version of the nowadays standard Aguiar and Gopinath (2007) model is augmented for additional time-varying volatility in the debt premium faced by a small open economy and by time-varying volatility in TFP. The time-varying volatility is driven by deviations of the debt to output ratio from its steady state. Introducing endogenous time-varying volatility into the debt premium and TFP can generate business cycle moments that are in line

with emerging market economy business cycle data even when trend shocks in TFP are not more important than cycle shocks as required by Aguiar and Gopinath (2007) or in a case where they are not present at all. Our work is therefore in-line with the findings by Garcia-Cicco et al. (2010) who emphasize the importance of financial frictions to generate EME business cycles. However, our paper does not require any exogenous shocks on the debt premium.

By using a simulated method of moments approach we can find parameter values for the elasticities of the volatility process and its persistence that allow us to closely match business cycle moments for Mexico and Canada as well as the average of EME and developed economies. To verify our results we ran a Monte Carlo like prior for the main DSGE model parameters and find that the right combination of endogenous time-varying debt premium and TFP volatility can generate EME business cycle characteristics for most of the Monte Carlo draws. We find that to maximize the frequency of the Monte Carlo draws that satisfy the EME restrictions we impose, the debt premium volatility elasticity needs to be about four times larger than the TFP volatility elasticity.

We have deliberately chosen a rather simple reduced form process to generate endogenously time-variation in the volatility of our model, the deviation of the debt to output ratio from its steady state, which allowed us to focus more on the mechanics of the model. Future work might therefore consider a more sophisticated process to endogenize volatility.

A Appendix

A.1 Data Sources

A.2 Steady State

Since our model is a standard small open economy model we can provide analytical solutions for the steady state. Since we assumed that $\beta(1+r) = 1$ and $\frac{1}{q} = 1+r$, as the debt premium is solely dependent on the deviation of the debt to output ratio from its steady state, we know that in the steady state

$$q = \beta \tag{A1}$$

and the output to capital ratio can be derived using the fact that the marginal product of capital equals $r + \delta$ as

$$\frac{Y}{K} = \frac{\left(\frac{1}{q} - (1 - \delta)\right)}{(1 - \alpha)}. \tag{A2}$$

TABLE 8
Data Sources

Variable	Series	FRED Identifier
Mexico GDP	GDP by Expenditure in Constant Prices: Total GDP for Mexico	NAEXKP01MXQ657S
Mexico Consumption	GDP by Expenditure in Constant Prices: Private Final Consumption	NAEXKP02MXQ657S
Mexico Exports	Exports of Goods and Services in Mexico	MEXEXPORTQDSMEI
Mexico Imports	Imports of Goods and Services in Mexico	MEXIMPORTQDSMEI
Mexico Capital	Capital Stock at Constant National Prices for Mexico	RKNANPMXA666NRUG
Mexico Labor	Number of Persons Engaged for Mexico	EMPENGMXA148NRUG
Mexico Hours	Monthly Hours Worked: Manufacturing for Mexico	HOHWMN03MXQ661N
Mexico Exchange Rate	Mexico / U.S. Foreign Exchange Rate	EXMXUS
Mexico Interest Rate 1	Interest Rates, Government Securities, Treasury Bills for Mexico	INTGSTMXM193N
Mexico Interest Rate 2	3-Month or 90-day Rates and Yields: Treasury Securities for Mexico	IR3TTS01MXQ156N
Canada GDP	GDP by Expenditure in Constant Prices: Total GDP for Canada	NAEXKP01CAQ189S
Canada Consumption	GDP by Expenditure in Constant Prices: Private Final Consumption	NAEXKP02CAQ189S
Canada Exports	Exports of Goods and Services in Canada	CANEXPORTQDSMEI
Canada Imports	Imports of Goods and Services in Canada	CANIMPORTQDSMEI
Canada Capital	Capital Stock at Constant National Prices for Canada	RKNANPCAA666NRUG
Canada Labor	Number of Persons Engaged for Canada	EMPENGCAN148NRUG
Canada Hours	Weekly Hours Worked: Manufacturing for Canada	HOHWMN02CAQ065N
Canada Exchange Rate	Canada / U.S. Foreign Exchange Rate	EXCAUS
Canada Interest Rate 1	Interest Rates, Government Securities, Treasury Bills for Canada	INTGSTCAM193N
Canada Interest Rate 2	3-Month or 90-day Rates and Yields: Interbank Rates for Canada	IR3TIB01CAQ156N
US Interest Rate 1	Interest Rates, Government Securities, Treasury Bills for United States	INTGSTUSM193N
US Interest Rate 2	3-Month or 90-day Rates and Yields: Interbank Rates for United States	IR3TIB01USQ156N

Note: Table 8 shows the used data series and their FRED database identifier.

The steady state consumption to output ratio follows from the budget constraint as

$$\frac{C}{Y} = 1 - \delta \frac{K}{Y} - (1 - q) \frac{B}{Y} \quad (\text{A3})$$

where $\frac{B}{Y}$ is the exogenous debt to output ratio and $\frac{K}{Y}$ is the inverse of the output to capital ratio in Equation (A2). Steady state labor can be derived using the first-order condition for labor in Equation (A17) and the marginal utilities for consumption and labor in Equation (A12) and (A13) as

$$L = \frac{\alpha \gamma}{\frac{C}{Y} - \gamma \frac{C}{Y} + \alpha \gamma} \quad (\text{A4})$$

where $\frac{C}{Y}$ is the consumption to output ratio from Equation (A3). Capital follows from the Cobb-Douglas production function as

$$K = \left[\frac{L^\alpha}{\frac{Y}{K}} \right]^{\frac{1}{\alpha}} \quad (\text{A5})$$

where $\frac{Y}{K}$ is the output to capital ratio from Equation (A2). Steady state output follows then from the Cobb-Douglas production function as

$$Y = K^{1-\alpha} L^\alpha. \quad (\text{A6})$$

Consumption follows from the consumption to output ratio in Equation (A3) and output in Equation (A6) as

$$C = \frac{C}{Y}Y. \quad (\text{A7})$$

Steady state investment follows from the capital law of motion as

$$I = \delta K \quad (\text{A8})$$

and net exports to output in steady state are defined as

$$NX = \frac{Y - C - I}{Y}. \quad (\text{A9})$$

The debt level is derived from the exogenous debt to output ratio $\frac{B}{Y}$ and actual output in Equation (A6) as

$$B = \frac{B}{Y}Y. \quad (\text{A10})$$

Finally in steady state the debt premium p and TFP z as well as the debt premium and TFP volatility σ_p and σ_z are all zero. Table 9 provides a numerical example for the steady state using the baseline parameters from Table 3.

TABLE 9
Steady States Baseline Model

Variable	Description	Steady State
Y	Output	0.67
C	Consumption	0.52
I	Investment	0.15
K	Capital	3.07
L	Labor	0.33
B	Debt	0.07
NX	Net Exports	0.002
Y/K	Output to Capital Ratio	0.22
C/Y	Consumption to Output Ratio	0.77
I/Y	Investment to Output Ratio	0.227
q	Debt Price	0.98
p	Debt Premium	0.00
z	TFP	0.00
σ_p	Debt Premium Volatility	0.00
σ_z	TFP Volatility	0.00

Note: Table 9 shows the steady states of the model using the baseline parameter values.

A.3 Equilibrium Conditions

We here give the full set of equilibrium conditions for the model. Utility is given by

$$U_t = \frac{\left[C_t^\gamma (1 - L_t)^{1-\gamma} \right]^{1-\sigma}}{1 - \sigma} \quad (\text{A11})$$

The marginal utility for consumption U_{C_t} follows as

$$U_{C_t} = \frac{\gamma(1-\sigma)U_t}{C_t} \quad (\text{A12})$$

The marginal disutility for labor U_{L_t} follows as

$$U_{L_t} = -\frac{(1-\gamma)(1-\sigma)U_t}{(1-L_t)} \quad (\text{A13})$$

Budget constraint

$$C_t + K_{t+1} = Y_t + (1-\delta)K_t - \frac{\phi}{2} \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 K_t - B_t + q_t B_{t+1} \quad (\text{A14})$$

Capital law of motion including capital adjustment costs

$$K_{t+1} = (1-\delta)K_t + I_t - \frac{\phi}{2} \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 K_t \quad (\text{A15})$$

The first-order condition for capital follows as

$$U_{C_t} \left[1 + \phi \left(\frac{K_{t+1}}{K_t} - 1 \right) \right] = \beta U_{C_{t+1}} \left[(1-\delta) + (1-\alpha) \frac{Y_{t+1}}{K_{t+1}} - \frac{\phi}{2} \left(-2 \left(\frac{K_{t+2}}{K_{t+1}} - 1 \right) \frac{K_{t+2}}{K_{t+1}} + \left(\frac{K_{t+2}}{K_{t+1}} - 1 \right)^2 \right) \right] \quad (\text{A16})$$

The first-order condition for labor is

$$-U_{L_t} = U_{C_t} \alpha \frac{Y_t}{L_t} \quad (\text{A17})$$

The Euler equation for the debt follows as

$$\frac{U_{C_{t+1}}}{U_{C_t}} = \frac{q_t}{\beta} \quad (\text{A18})$$

Production is of standard Cobb-Douglas form

$$Y_t = e^{z_t} K_t^{1-\alpha} L_t^\alpha \quad (\text{A19})$$

Price of debt

$$\frac{1}{q_t} = 1 + r^* + p_t \quad (\text{A20})$$

Debt premium process and the endogenous volatility term are given by

$$p_t = \psi e^{\sigma p_t} \left(e^{\tilde{p}_t} - 1 \right) \quad (\text{A21})$$

$$\sigma_{p_t} = \rho_{\sigma_p} \sigma_{p_{t-1}} + \eta_p \tilde{B}_t \quad (\text{A22})$$

TFP process and the endogenous volatility term are given by

$$z_t = \rho_z z_{t-1} + e^{\sigma_{z_t}} u_{z_t} \quad (\text{A23})$$

$$\sigma_{z_t} = \rho_{\sigma_z} \sigma_{z_{t-1}} + \eta_z \tilde{B}_t \quad (\text{A24})$$

Deviation of the debt to output ratio from steady state

$$\tilde{B}_t = \frac{B_t}{Y_t} - \frac{B}{Y} \quad (\text{A25})$$

Net exports

$$NX_t = \frac{B_t - q_t B_{t+1}}{Y_t} \quad (\text{A26})$$

Output growth is defined as

$$\Delta Y_t = \log(Y_t) - \log(Y_{t-1}) \quad (\text{A27})$$

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