Essays on International Business Cycles

Jan-Philipp Dueber

Thesis for the Degree of Doctor of Philosophy in Economics

School of Economics
University of Kent
Canterbury
CT2 7NP
United Kingdom

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Abstract

This PhD thesis analyzes in three chapters topics in international macroeconomics with a focus on time-varying volatility and international capital flows in emerging market economies.

In the first chapter which is called *"Volatility Driven Capital Flows in Emerging Market Economies"* I construct a two-country open economy model with exogenous volatility shocks to investment efficiency (IE) and neutral total factor productivity (TFP) in each country. The model shows that these volatility shocks can lead to significant cross-country gross capital flows. I show that the theoretical findings of increased capital outflows of domestic agents out of a country and a reduction in the inflows of capital by foreign agents into the country are consistent with empirical observations for the US and Mexico. For this purpose I am using a structural vector autoregression (SVAR) with sign restrictions for the identification of the exogenous volatility shocks. This paper contributes therefore to possible explanations for capital flows during financial crises by arguing that increased volatility is a main cause of international capital flows.

The second chapter with the title *"Endogenous Time-Varying Volatility and Emerging Market Business Cycles"* analyzes the effect of time-variation in the volatility of the interest rate and total factor productivity (TFP) on business cycles in emerging markets using a small open economy model. In this model the volatility of the interest rate and TFP increases endogenously as the debt to output ratio increases in response to a negative TFP shock. The paper finds that when time-varying volatility in the interest rate and TFP are jointly present, a standard small open economy model becomes able to significantly improve the match of the data moments observed in emerging market economies. This finding is particularly important as standard small open economy models fail to address the fact that net exports are countercyclical and consumption volatility exceeds output volatility in emerging markets data. Augmenting the small open economy model with time-varying volatility in interest rates and TFP allows to successfully address these shortcomings. By choosing different degrees of time-varying volatility the model is able to successfully match data moments for emerging markets and developed economies as shown with the example of Mexico and Canada. This paper is different from previous attempts to modify the small open economy model to fit emerging market economies in that I only require one exogenous shock. A standard transitory TFP shock is sufficient to explain the dynamics of emerging markets and developed economies.

In the third chapter titled *"Countercyclical Risk Aversion and International Business Cycles"* I construct a two-country open economy model where the risk aversion faced by agents is endogenous and time-varying. Introducing a time-varying risk aversion into the model is motivated by the fact that empirical evidence suggests that the risk aversion is countercyclical. Thus in this model the risk aversion is decreasing when the economy is in a boom scenario and increasing when the economy is in a recession. After the introduction of a countercyclical risk aversion the model becomes able to successfully address the fact that real exchange rates and relative consumption across countries shows a near zero or even negative correlation in the data for the US and the rest of the world. However, standard models with a constant risk aversion show a near perfect correlation. An anomaly which is usually known as the Backus-Smith puzzle. The paper also finds that investment and labor supply become more volatile after the introduction of a countercyclical risk aversion so that investment and labor volatility now closely match those observed in the data. Finally, the introduction of a countercyclical risk aversion increases the correlation of investment and labor across countries and thereby partially addresses the International Comovement Puzzle.

Acknowledgments

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Chapter 1

Volatility Driven Capital Flows in Emerging Market Economies

1.1 Introduction

Emerging market economies are characterized by high volatility in capital inflows and outflows. Among the countries that are particularly affected by this are the major Latin American economies of Mexico, Brazil, and Argentina for instance. Large gross positions in capital flows pose a potential threat to these countries and the international financial system as these flows might be subject to sudden stops or reversals. Even when net capital flows are small, gross capital flows can be large and they can have important consequences for the stability of the international financial system. Gross capital flows are hence a better measure of exposure to international risk.

At the same time, macroeconomic volatility is subject to large variations in most of these emerging market economies. Fluctuations in output, consumption, investment, or total factor productivity (TFP) are much higher than in developed countries. In an environment of high economic uncertainty, i.e. high perceived relative volatility of macroeconomic variables, risk averse domestic agents might increase their holdings of foreign assets which might then be reflected in increased capital outflows by domestic agents in an open economy. In addition to this mechanism, foreign investors might care about relative volatility when to decide to withdraw capital from emerging markets when idiosyncratic volatility increases. Hence leading to reduced capital inflows by foreigners from developed economies.

The aim of this paper is to bridge the gap between economic uncertainty in the form of time-varying volatility in TFP and investment efficiency (IE) on the one hand and cross-border gross capital flows on the other.[1] For this purpose we construct a two-country dynamic stochastic general equilibrium (DSGE) model with portfolio holdings of stocks that is augmented for stochastic volatility shocks to derive impulse responses of gross capital flows after a volatility shock in TFP and investment efficiency. Once TFP or investment efficiency volatility increases exogenously, output and hence firm dividends become more volatile. This will lead to idiosyncratic volatility increases in asset returns of the home country to which agents react by holding less of the home asset and more of the foreign asset which will hence result in capital outflows of the home country.

We further not only address the change in net capital flows as most previous research in this area, but we widen the view and focus on gross capital flows instead. This becomes especially important as for many countries net capital flows reverted to an almost balanced level in the last years whereas gross inflows and outflows continue to grow and exhibit a significant volatility.

Finally, we contribute by using a structural vector autoregression (SVAR) which allows us to account for potential reverse causality between gross capital flows and idiosyncratic volatility in economic variables that can potentially arise in panel data settings that were used in previous studies. The structural VAR applies combined sign and

[1] We refer to investment efficiency as the relative price of investment to consumption goods. This is often referred to as investment specific technology (IST) in the literature.

zero long run restrictions for the identification of the TFP and IE volatility shocks. The vector autoregressive approach will then allow us to obtain impulse responses to volatility shocks in idiosyncratic TFP and investment efficiency on capital inflows and outflows and their dynamic impact and compare them to the previously obtained results from the DSGE model. Using a structural VAR comes with the advantage of a less restrictive approach that is more data driven and relies less on a specific form of a model.

Our model is able to produce dynamic responses of gross capital flows after a shock in home and foreign stochastic volatility of TFP and the investment efficiency. The structural VAR with sign and zero restrictions is then applied to gross capital flow data and is able to reproduce the predictions from the theoretical model.

In general, this paper fits into the broad literature of stochastic volatility in macroeconomic models like Justiniano and Primiceri (2008) who construct a closed economy model with exogenous shocks in TFP and the investment efficiency. They find that TFP and IE volatility shocks are major sources of macroeconomic fluctuations. It further relates to Fernandez-Villaverde et al. (2011) who show that time-varying volatility in the real interest rate increased in the emerging market economies of Argentina, Brazil, Ecuador, and Venezuela. They can subsequently show that increases in volatility of the real interest rate can decrease output, consumption, investment, hours, and debt in a standard small open economy business cycle model. For this purpose they estimate the stochastic process of the real interest rate using a particle filter that can then be integrated into a DSGE model. Kollmann (2016) considers the effect of output volatility shocks in a two-country model on consumption, trade flows, and the real exchange rate. Further recent research with regard to time-varying volatility is Seoane (2017) who looks at the propagation of TFP and interest rate volatility shocks through mark-ups in a small open economy model. Mumtaz and Theodoridis (2017) use a factor model to decompose the time-varying variance of macroeconomic and financial variables into country-specific and common factors. A DSGE model then allows them to conclude that increased globalization and trade openness are the driving force behind the increased cross-country correlation that can be observed in different volatility measures. When volatility is considered as a major source of uncertainty, this paper also relates to the pioneering work by Bloom (2014) on macroeconomic uncertainty. He finds that uncertainty is countercyclical and stronger in emerging markets than in developed economies. Major recent contributions to the study of capital flows in DSGE models are Tille and van Wincoop (2010) who use a DSGE model to analyze net and gross portfolio asset flows in a two-country framework. Devereux and Sutherland (2009) build a DSGE model with an emerging market economy and a developed economy. Their model can account for the large holdings of foreign assets by emerging markets and at the same time it allows for large inflows of foreign direct investment into the emerging market economy. The empirical part using a structural VAR with sign restrictions is also closely related to work by Uhlig (2005) and Mountford and Uhlig (2009) who use a structural VAR with sign restrictions to study monetary and fiscal policy effects. Scholl and Uhlig (2008) use sign restrictions to study the effect of US monetary policy on exchange rates between the US and Germany. However, there is so far no application of this identification scheme to international capital flows and stochastic volatility. It also relates to work by Fogli and Perri (2015) who find a strong relationship between uncertainty as measured by the variance of a country's GDP and the accumulation of net foreign assets in a panel of OECD countries. In addition, Elgin and Kuzubas (2013) find an association in a larger set of countries in a panel between current account deficits and high output volatility. However, most of the current literature on capital flows does not look at gross capital flows and instead only looks at net capital flows. An exception is Broner et al. (2013) who show that differentiating and disentangling capital inflows and capital outflows proves important as gross flows might significantly contribute to global imbalances. Further work is done by Schmidt and Zwick (2015) who further analyze gross capital flows in the European Monetary Union under uncertainty. Gourio et al. (2016) use stock market volatility to establish a causal relationship between volatility and gross capital flows within a panel of 26 emerging countries. By distinguishing between global volatility and country specific stock market volatility they are then able to reconcile their empirical results in a portfolio choice model.

This paper is different from the above literature in that we are applying stochastic volatility shocks to TFP and the investment efficiency in a two-country model with stock holdings. As shown by Justiniano and Primiceri (2008) volatility shocks in TFP and IE can have a major impact on business cycles. In contrast to many previous papers we look at the disentangled capital inflows and capital outflows separately.

The rest of this paper is structured as follows. The second part will introduce the data and discuss some volatility measures. In the third part a two-country DSGE model with stochastic volatility in TFP and IE is constructed and the parameter of the stochastic process are estimated for Mexico and the US using a Bayesian approach. The fourth part will show the empirical observations for the case of Mexico using a SVAR with combined sign and zero restrictions derived from the DSGE model. The fifth part will then compare the results from the DSGE model and the structural VAR with restrictions derived from the DSGE model and part six will finally conclude and guide to future avenues of research.

1.2 Data

In the following we need data to estimate the stochastic process of TFP and investment efficiency for Mexico and the US that is used in the DSGE model. We further need data for the structural analysis applied to Mexico that consists of a seven variable SVAR that includes the Mexican-US interest rate differential, an idiosyncratic volatility measure for TFP and investment efficiency, capital inflows and outflows as well as the differential between Mexican and US GDP growth and inflation differentials. All data is of quarterly frequency and ranges from the first quarter 1980 to the fourth quarter of 2014. Mexican and US variables used to estimate the structural VAR and to estimate stochastic processes for the DSGE model are from the FRED database of the Federal Reserve Bank of St Louis.

Mexico is chosen in this paper as a representative of a large emerging market economy since the Mexican economy is less susceptible to sudden capital flow movements caused by changes in commodity prices and demand like Argentina or Brazil for instance. It further exhibits strong interlinkages with the US as the representative of the rest of the world. A detailed overview about all data sources is available in Table 1.A1 in the Appendix 1.A.1.

1.2.1 SVAR Data

As we are interested in the behavior of gross capital flows, it is important to clearly disentangle capital inflows from outflows. Hence, capital inflows by foreign agents are defined as the sum of portfolio investment liabilities and other investment liabilities which include bank flows, other public and private loans, and trade credit for Mexico from the financial account of the balance of payments. Capital outflows by domestic agents are defined as the corresponding asset components of the financial account, namely portfolio investment assets and other investment assets for Mexico.[2] [3] A positive value for capital inflows therefore indicates a capital inflow by foreign agents into Mexico whereas a negative value indicates a withdrawal of capital from Mexico by foreign agents. A positive value for capital outflows indicates an increase in the holdings of foreign assets by Mexican agents whereas a negative value indicates a repatriation of capital by Mexican agents back to Mexico.[4] Figure 1.1 shows a diagram of capital flows between Mexico and the US. The blue lines indicate capital inflows by US agents into Mexico, whereas the dashed-dotted red lines indicate capital outflows by Mexican agents.

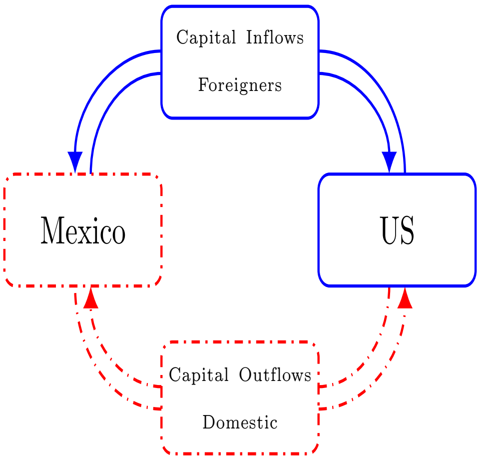
Inflows and *Outflows* are then constructed relative to GDP to normalize the variables by country size. US interest rates used in the SVAR are the effective Federal Funds Rate and Mexican interest rates are approximated by 3-month Mexican treasury securities. The inflation data for Mexico and the US is the growth rate with respect to the previous period of the consumer price index and includes all items. GDP growth rates are the growth rates with respect to the previous period of constant prices GDP and are seasonally adjusted. Differentials of interest rates (*Interest Rate Diff*), GDP growth ($\Delta GDP\ Diff$), and CPI growth ($\Delta CPI\ Diff$) are then constructed by subtracting the US value from the Mexican data. Where data is not available in constant US Dollar terms, data series are transformed using the quarterly GDP deflator for Mexico and then converted to constant US Dollar values.

[2]This approach is hence similar to Broner et al. (2013) who additionally include direct foreign investment liabilities and assets to their definition of capital inflows by foreigners and capital outflows by domestic agents. We drop FDI flows due to limited data availability for Mexico.

[3]For the ease of reading we will refer to capital outflows by domestic agents simply as capital outflows and to capital inflows by foreign agents as capital inflows.

[4]This is the gross term refers to capital flows being disentangle into flows by foreign agents and domestic agents, respectively. However, the paper uses net capital flows for each group of agents. This allows for a one to one mapping in the model later on.

FIGURE 1.1
Capital Flows



Note: Figure 1.1 shows a diagram of capital flows. Capital outflows domestic refers to the net flow caused by Mexican agents investing in the US, whereas capital inflows foreigners refers to the net flow caused by US agents investing in Mexico.

1.2.2 TFP and Investment Efficiency Data

The TFP data used to estimate the stochastic volatility process is estimated using a log-linearized Cobb-Douglas production function of the form

$$\widehat{y}_t = \widehat{a}_t + \alpha \widehat{k}_t + (1-\alpha)\,\widehat{l}_t \tag{1.1}$$

where $\widehat{y}_t$ is the log-deviation of output from the quadratic trend, $\widehat{a}_t$ is the log-deviation of TFP, $\widehat{k}_t$ is the log-deviation of the capital stock, and $\widehat{l}_t$ is the log-deviation of hours worked.[5] The parameter α denotes the capital elasticity and is set to the same value of 0.40 which we later use for the theoretical model. The data used to construct TFP is first detrended using a quadratic trend as in Mendoza (1991) so that e.g. $\widehat{y}_t$ corresponds to the log-deviation of output from its quadratic trend. Data on investment efficiency for Mexico and the US is constructed following Basu and Thoenissen (2011) so that the relative price of investment goods to consumption goods can be approximated around it's steady state using a log-linear approximation as

$$\widehat{\frac{P_t^I}{P_t^C}} = (\theta_C - \theta_I)\,\widehat{T}_t - \widehat{\chi}_t \tag{1.2}$$

[5]Since capital stock data is only available at an annual frequency, the data is converted to a quarterly frequency using spline interpolation.

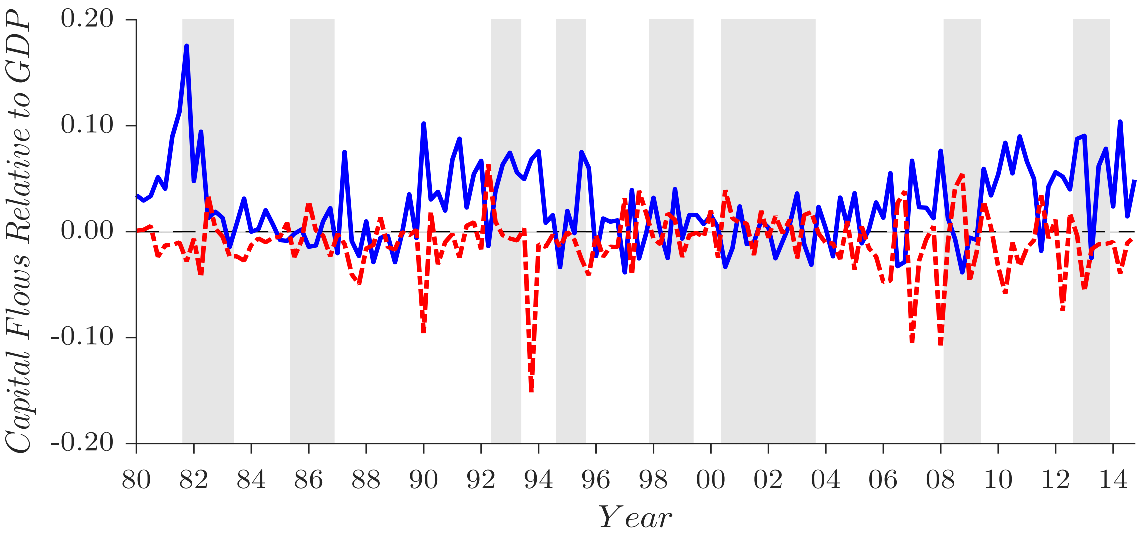
where P_t^I is the price of investment, P_t^C is the price of consumption, and T_t denotes the terms of trade expressed in import prices over export prices. θ_C and θ_I denote the share of domestic goods in consumption and investment and χ_t denotes the investment efficiency term. A hat denotes log-deviations from the quadratic trend of the respective variable. Data on investment and consumption prices is then detrended using a quadratic trend as in the case of TFP. Equation (1.2) shows that the investment efficiency χ_t can be expressed as the relative prices of investment and consumption as long as the share of domestic goods in consumption is the same as in investment.[6] Calculating TFP and investment efficiency data using the production function provides the advantage that the estimates for TFP and the investment efficiency are grounded in economic theory and have a one to one mapping in the DSGE model that we use. It further comes with the advantage that comparable data series can be constructed for both Mexico and the US at a quarterly frequency.

1.2.3 Capital Flows

Figure 1.2 shows capital inflows and capital outflows relative to GDP for Mexico. As previously noted in the text and in accordance with Broner et al. (2013) the term gross flows refers to the distinction in the financial account between domestic agents and foreign agents. However, they are net in the sense that capital inflows and capital outflows by each group are shown as net flows. Capital flows are not trending since each time series shows the net flow for each group i.e. a net change in the Mexican asset and liability position. We can observe that especially in the last decade capital inflows and capital outflows increased both substantially.

[6] The investment efficiency χ_t will be constructed assuming that $\theta_C = \theta_I$. Although θ_C and θ_I will later be allowed to differ in the DSGE model. This, however, would require a costly re-estimation of the stochastic process for any given pair of θ_C and θ_I.

FIGURE 1.2
Mexican Gross Capital Flows



Note: Figure 1.2 shows the Mexican capital flows relative to GDP from 1980 to 2014. Capital inflows are shown in blue and capital outflows are shown as the dashed-dotted red line, respectively. Shaded periods indicate recessions of the Mexican economy as defined by the OECD. The data is quarterly and ranges from 1980Q1 to 2014Q4.

1.2.4 Volatility Measures

In general four broad categories of uncertainty measures can be characterized as described by Bloom (2014) who provides a comprehensive review of uncertainty measures. First, volatility in macroeconomic variables like output, government spending, interest rates, or TFP which can easily be estimated using econometric methods. Second, indicators of stock market volatility like the widely used VIX index of stock option prices for the US that measures volatility in financial markets. Third, micro based indicators based on firm level data or the spread in business forecasts. And finally, measures of political uncertainty as described in Baker et al. (2016). The latter two are difficult to obtain for a large set of countries over long time horizons, whereas the former two are more easily available for a broad set of countries.

All available uncertainty measures come with some apparent advantages and disadvantages. Changes in general macroeconomic variables like output or government expenditure are easily available and might closely mirror agents perceived uncertainty. Exchange rate volatility might be important in the decision of optimal portfolio allocations but it comes most likely with the biggest problem of reverse causality. Sudden inflows or outflows of capital might have a non-neglectable impact on nominal exchange rates.

In the following analysis, two different measures of volatility are used. The first indicator used is the variability of domestic TFP. Changes in total factor productivity are known to be a major driver of macroeconomic fluctuations and hence volatility in TFP might constitute a major factor in the allocation of assets. The second measure is the volatility in investment efficiency, as it will determine uncertainty in the returns to physical capital investment as described by Justiniano and Primiceri (2008) and Justiniano et al. (2010). It will reflect uncertainty about the relative price of investment over consumption, and hence the cost at which units of consumption can be transformed into productive investment. It is important to note that what ultimately matters for capital allocations is idiosyncratic volatility that is caused by country specific factors and not global volatility that affects all countries in the same way. As global shocks are uninsurable, and hence bilateral flows cannot be used as an insurance mechanism through a reoptimization of portfolios. That is, as volatility increases globally, investors cannot benefit from shifting assets to other countries which is quite similar to a shock in the global rate of return on assets which would have no impact on the portfolio allocation. Therefore the approach suggested by Gourio et al. (2016) and applied by them to stock market returns is used to obtain idiosyncratic volatility measures in this paper. This procedure is similar to the approach in the Capital Asset Pricing Model and assumes that world variables measure aggregate, systematic risk. In the case of TFP volatility, the squared Mexican TFP is regressed on the squared US TFP as an approximation for global volatility in TFP[7]

$$\left(\widehat{a}_t^{MX}\right)^2 = \alpha^{MX} + \beta^{MX}\left(\widehat{a}_t^{US}\right)^2 + \zeta_t^{MX} \tag{1.3}$$

where $\widehat{a}_t^{MX}$ is the log-deviation from the quadratic trend of Mexican TFP at time t and $\widehat{a}_t^{US}$ is the log-deviation from the quadratic trend of US TFP at time t. The OLS regression coefficients are denoted by α^{MX} and β^{MX}, respectively. Then the idiosyncratic component of Mexican volatility can be recovered as the OLS constant α^{MX} and the sum of error terms ζ_t^{MX} as

$$\sigma_{t^*}^{MX} = \frac{1}{1+\tau}\sum_{t=t^*-\tau}^{t^*}\left(\alpha^{MX} + \zeta_t^{MX}\right) \tag{1.4}$$

and the US volatility component as

$$\sigma_{t^*}^{US} = \frac{1}{1+\tau}\sum_{t=t^*-\tau}^{t^*}\beta^{MX}\left(\widehat{a}_t^{US}\right)^2 \tag{1.5}$$

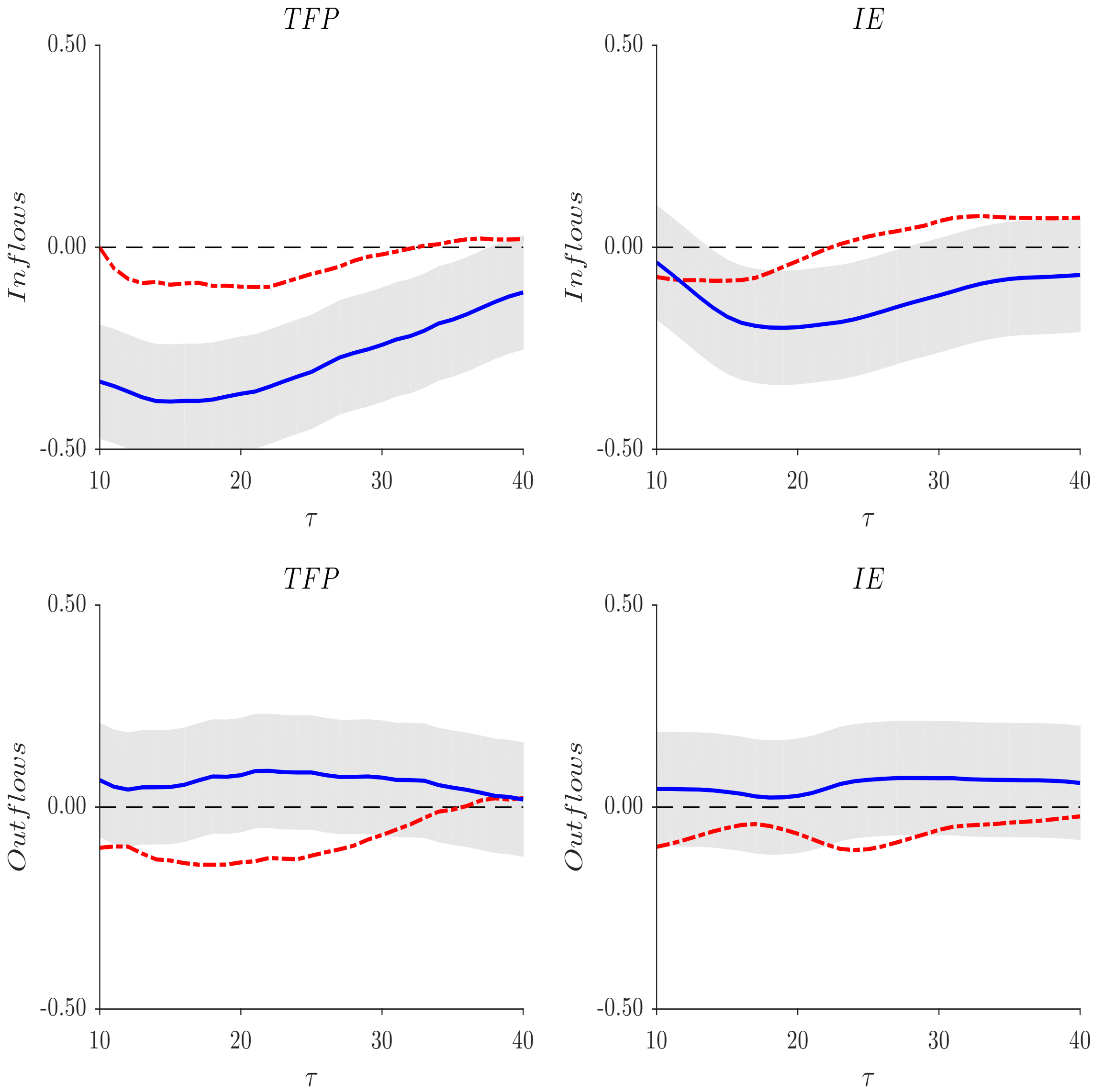
where t^* denotes the last period of the rolling time window and τ corresponds to the length of the rolling time window, which is set to a period of 20 quarters. The idiosyncratic volatility component for Mexico is then set in relation to the US volatility by calculating the difference between both volatility series. The same procedure is used to construct the idiosyncratic volatility of the investment efficiency series.

[7] Approximating global variables by the US is not only justified as the US is the largest world economy, but also that the US is by far the largest trading partner of Mexico.

1.2.5 Empirical Regularities of Volatility and Capital Flows in Mexico

We now document some empirical regularities of TFP and investment efficiency volatility and capital flows in Mexico. Figure 1.3 shows the contemporaneous correlations of idiosyncratic TFP and investment efficiency volatility with capital inflows and outflows for different values of τ. Capital inflows are strongly negatively correlated with TFP and investment efficiency volatility for all values of τ. For capital outflows the correlation is positive as expected but much lower. For comparison the dashed-dotted red line shows the correlation of a simple moving variance estimate for the same values of τ constructed as the difference between Mexican and US variance over a moving window. Clearly, the difference in the moving variance is characterized by a relatively low correlation of volatility and capital flows as it does not clearly distinguish between Mexican and US volatility. For capital outflows the moving variance might even indicate a negative relationship, further showing the need for a decomposition of volatility into country specific and global components. Again, the simple measure of a moving volatility is not able to produce any significant cross-correlations.

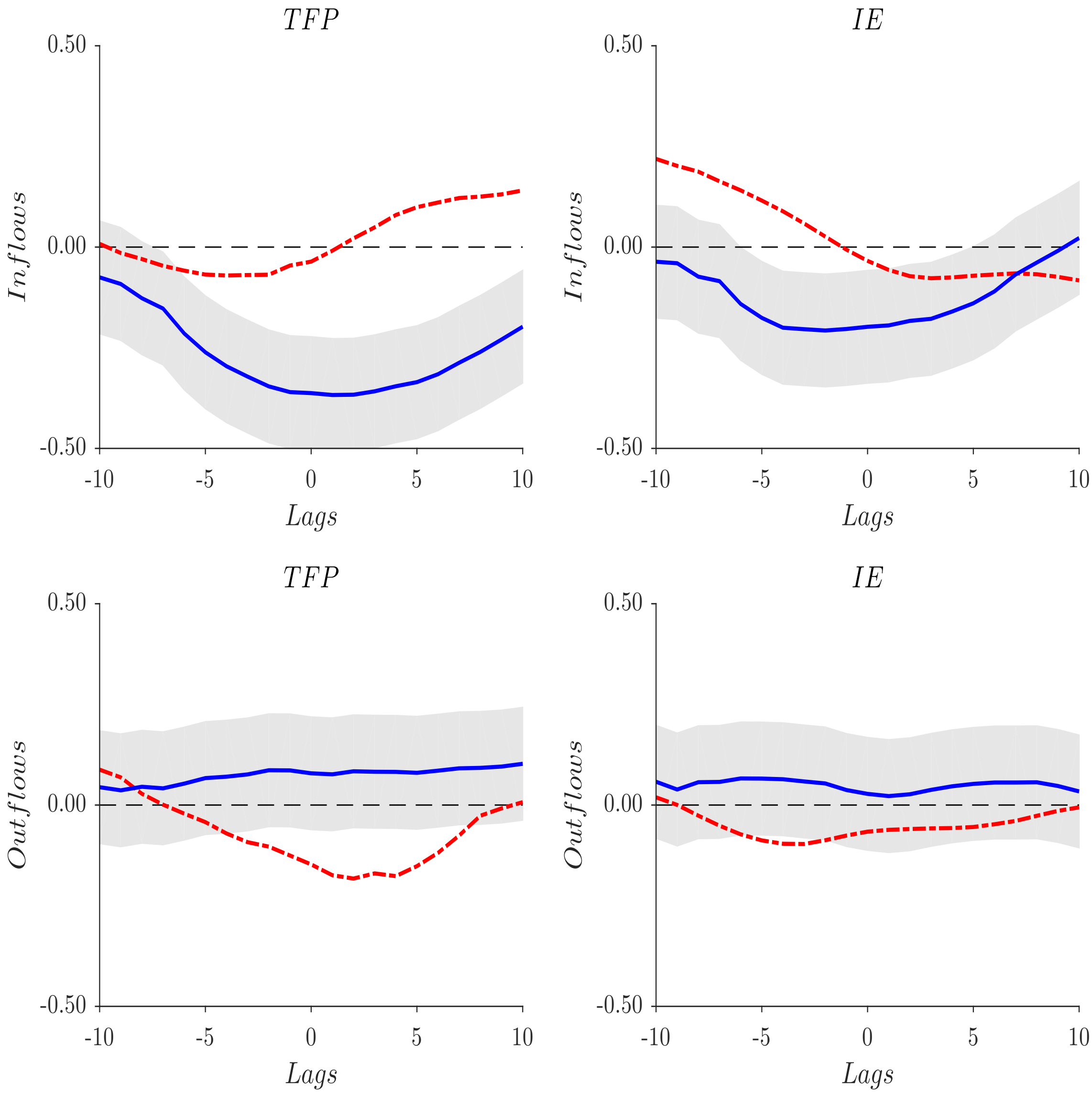
FIGURE 1.3
Correlations of Mexican Gross Capital Flows and Idiosyncratic Volatility



Note: Figure 1.3 shows the contemporaneous correlation of Mexican capital inflows and the volatility of TFP and IE for different values of τ in the upper panel. The lower panel shows the contemporaneous correlation of Mexican capital outflows and the volatility of TFP and IE for different values of τ. The moving volatility is shown as the dashed-dotted red line. The 5 and 95 percent confidence bands of the correlation are the shaded area. The data is quarterly and ranges from 1980Q1 to 2014Q4.

Figure 1.4 shows the cross-correlation of capital inflows with idiosyncratic TFP and investment efficiency volatility for different leads and lags in the upper panel and for capital outflows in the lower panel. The idiosyncratic TFP and investment efficiency volatility is negatively correlated with capital inflows and positively correlated with capital outflows at lag zero. For the correlation of TFP and investment efficiency with inflows it becomes apparent that correlations vary strongly for different lags or leads. As the overall correlation of TFP and investment efficiency is much lower for outflows, no clear pattern emerges for different lags or leads. We show again in addition the correlation with a simple moving variance of TFP and IE as the dashed-dotted red line that does also include global volatility components. Such a series shows no significant correlation with capital inflows and outflows.

FIGURE 1.4
Cross-Correlations of Mexican Gross Capital Flows and Idiosyncratic Volatility



Note: Figure 1.4 shows the cross-correlation of Mexican capital inflows and the volatility of TFP and IE for different lags in the upper panel. The lower panel shows the correlation of Mexican capital outflows and the volatility of TFP and IE for different lags. τ is set to 20. The moving volatility is shown as the dashed-dotted red line. The 5 and 95 percent confidence bands of the cross-correlation are the shaded area. The data is quarterly and ranges from 1980Q1 to 2014Q4.

Due to the change of key characteristics of the data which is mainly illustrated by a decline in volatility, it is self-evident to split the sample into two subperiods and analyze both separately. The first sample period will consist of quarterly data from 1980 to 1999 that includes major episodes of volatility for the Mexican economy like the debt crises of the 1980's and the 1994 Peso crisis. The second sample will include the period 2000 to 2014, which is characterized by a relatively stable behavior of macroeconomic variables. Table 1.1 shows the standard deviations of the main macroeconomic variables that are used in the structural estimation. The differential of the Mexican and US data series is characterized by high amounts of volatility in output and inflation in the 1980's and a subsequent turn to more moderate inflation and growth figures especially after the implementation of economic reforms including the introduction of the New Mexican Peso in the 1990's. These patterns are especially pronounced in the differential between Mexican and US interest rates. For the period 1980 to 1999 the standard deviation is high with 16.13 and falls sharply to 0.96 for the period 2000 to 2014. The idiosyncratic TFP and IE volatility series relative to the US show a similar pattern, where the second period shows less variation then the sample comprising 1980 to 1999. Both the standard deviation of capital inflows as well as capital outflows show a decrease in the second sample period. Variability in the differential of the consumer price index as measured by the standard deviation decreased from 7.79 for the period 1980 to 1999 to only 1.21 for the period 2000 to 2014.

TABLE 1.1
Standard Deviations of Macroeconomic Variables

	1980-2014	1980-1999	2000-2014
Interest Rate Diff	12.22	16.13	0.96
Volatility TFP	1.33	1.39	1.24
Volatility IE	2.01	2.17	1.65
Inflows	14.73	17.51	9.41
Outflows	11.34	12.88	8.83
ΔGDP Diff	1.29	1.53	0.79
ΔCPI Diff	7.03	7.79	1.21

Note: Table 1.1 shows standard deviations of the data for Mexico for the whole sample and the two subperiods.

Table 1.2 shows the correlations of the major economic variables of interest. Striking is the strong negative correlation between capital inflows and capital outflows of -0.46. That is in periods of high capital inflows, domestic agents repatriate assets from the foreign country back home so that capital outflows turn negative. Also as expected, capital inflows show a positive correlation with the differential in output growth rates of 0.25 and a negative correlation with the inflation differential between Mexico and the US.

TABLE 1.2
Correlations of Macroeconomic Variables

	Interest Rate Diff	Volatility TFP	Volatility IE	Inflows	Outflows	ΔGDP Diff	ΔCPI Diff
Interest Rate Diff	1.00						
Volatility TFP	-0.03	1.00					
Volatility IE	-0.01	0.16	1.00				
Inflows	0.04	**-0.36**	**-0.20**	1.00			
Outflows	-0.02	0.08	0.03	**-0.46**	1.00		
ΔGDP Diff	-0.07	0.08	0.15	**0.25**	-0.02	1.00	
ΔCPI Diff	0.15	-0.17	-0.30	-0.15	-0.01	**-0.42**	1.00

Note: Table 1.2 shows the correlations of the main variables used in the structural estimation for Mexico for the whole sample period from 1980 to 2014. Bold faced values are significant at the 5 percent level.

Table 1.3 shows the contemporaneous correlations of capital inflows and capital outflows with idiosyncratic TFP and investment efficiency volatility of Mexico relative to the US for different time periods. Both TFP and investment efficiency volatility seem to have a higher correlation with capital inflows than with capital outflows as is also visible in Figure 1.4. Correlations with capital inflows range from -0.31 for TFP volatility in the period 2000 to 2014 to -0.51 for TFP volatility in the period 1980 to 1999. Correlations of capital inflows with investment efficiency volatility is slightly lower with -0.40 for the period 1980 to 1999 and even slightly, but non-significantly positive for the period 2000-2014. For capital outflows, correlations range from 0.06 for the period 1980 to 1999 to 0.20 for the period 2000 to 2014 in the case of TFP volatility. For capital outflows and investment efficiency volatility only the period 2000 to 2014 is noteworthy to mention with a correlation of 0.14.

TABLE 1.3
Correlations of Capital Flows and Volatility

	1980-2014	1980-1999	2000-2014
Inflows - Volatility TFP	**-0.36**	**-0.51**	**-0.31**
Inflows - Volatility IE	**-0.20**	**-0.40**	0.22
Inflows - Outflows	**-0.46**	**-0.37**	**-0.58**
Outflows - Volatility TFP	0.08	0.06	0.20
Outflows - Volatility IE	0.03	-0.01	0.14
Volatility TFP - Volatility IE	0.16	0.15	0.00

Note: Table 1.3 shows the contemporaneous correlations of capital inflows and outflows with idiosyncratic TFP and investment efficiency volatility for Mexico. τ is set to 20. Bold faced values are significant at the 5 percent level.

1.3 A Two-Country DSGE Model

The correlations obtained in the previous sections are unconditional and hence cannot identify the role of volatility shocks on gross flows. To identify the effect and contribution of TFP and IE volatility shocks, we use a structural VAR. The SVAR allows us to identify shocks without imposing very strong restrictions on the data. However, identification needs to be disciplined by a theoretical model of capital flows and volatility shocks. The model is used to derive robust restrictions for the SVAR and allows us to develop the economic intuition behind the mechanisms at work.

1.3.1 Model

The model is a two-country international real business cycle model with equity holdings as described in Coeurdacier and Rey (2013) which is similar to Coeurdacier et al. (2010). The model is only extended to include stochastic volatility processes as in Fernandez-Villaverde et al. (2011) for the exogenous variables which are total factor productivity A_t and an investment efficiency term χ_t. We use a two-country model to explicitly model not only the capital allocations and flows of the home country but also of the foreign country. This is necessary to get a complete look at capital inflows and outflows for the home country as capital inflows to the home country consist of repatriation of capital held by home country residents in foreign stocks and investment by foreign agents in home country stocks. As the home and foreign economy are identically structured we will only present the home economy. We follow the convention and indicate foreign variables by a *.[8] Representative households maximize life-time utility of the form

$$\max_{C_t, L_t, K_{t+1}} \qquad \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\omega}}{1+\omega} \right) \right] \tag{1.6}$$

where ω is the inverse of the Frisch elasticity of labor supply with $\omega > 0$ and σ is the relative risk aversion parameter of the households. Further, C_t is consumption and L_t is labor supply by the households, respectively. Consumption C_t is a composite good consisting of home and foreign goods

$$C_t = \left[\theta_C^{\frac{1}{\phi_C}} C_{H,t}^{\frac{\phi_C - 1}{\phi_C}} + (1-\theta_C)^{\frac{1}{\phi_C}} C_{F,t}^{\frac{\phi_C - 1}{\phi_C}} \right]^{\frac{\phi_C}{\phi_C - 1}} \tag{1.7}$$

where $C_{F,t}$ is the home country's consumption of the good produced in the foreign country and $C_{H,t}$ is the consumption of the home produced intermediate good. $\theta_C \in (0,1)$ is the share of the home consumption good in composite consumption when their relative prices are unity. Finally, $\phi_C > 0$ is the elasticity of substitution between home and foreign consumption goods. The corresponding consumption price index is then

$$P_t^C = \left[\theta_C P_{H,t}^{1-\phi_C} + (1-\theta_C) P_{F,t}^{1-\phi_C} \right]^{\frac{1}{1-\phi_C}} \tag{1.8}$$

where $P_{H,t}$ and $P_{F,t}$ are the prices of the home and foreign goods, respectively.

[8]For the ease of reading parameters are not indexed with country subscripts in what follows. The baseline version of the model assumes that all parameters are equal across countries except for the domestic share in the input function θ_C and θ_I. However, later to verify robustness all of the parameters are allowed to vary between countries.

Technologies and Capital Accumulation

The production function follows the standard Cobb-Douglas form with the capital elasticity $\alpha \in (0,1)$ and A_t represents total factor productivity which follows an AR(1) process with exogenous shocks and is subject to a stochastic volatility shock so that output is

$$Y_t = e^{A_t} K_t^{\alpha} L_t^{1-\alpha}. \tag{1.9}$$

Capital K_t follows the law of motion

$$K_{t+1} = (1-\delta) K_t + e^{\chi_t} I_t \tag{1.10}$$

where $\delta \in (0,1)$ is the depreciation rate of capital and I_t is gross investment. χ_t denotes the investment efficiency term that follows an AR(1) process which is subject to stochastic volatility similar to the case of TFP. Gross investment I_t is generated using home and foreign inputs so that

$$I_t = \left[\theta_I^{\frac{1}{\phi_I}} I_{H,t}^{\frac{\phi_I - 1}{\phi_I}} + (1-\theta_I)^{\frac{1}{\phi_I}} I_{F,t}^{\frac{\phi_I - 1}{\phi_I}} \right]^{\frac{\phi_I}{\phi_I - 1}} \tag{1.11}$$

where $I_{H,t}$ and $I_{F,t}$ are the amounts of the home and foreign investment goods used for composite investment. $\theta_I \in (0,1)$ is the share of domestic components in investment spending and $\phi_I > 0$ is the substitution elasticity between home and foreign investment goods.[9] The corresponding investment price index P_t^I is then

$$P_t^I = \left[\theta_I P_{H,t}^{1-\phi_I} + (1-\theta_I) P_{F,t}^{1-\phi_I} \right]^{\frac{1}{1-\phi_I}} \tag{1.12}$$

where $P_{H,t}$ and $P_{F,t}$ are the prices of the home and foreign goods, respectively.

Firms' Decision

As the production function is of standard Cobb-Douglas form, workers receive a share $1-\alpha$ of output in each country as a wage W_t

$$W_t L_t = (1-\alpha) P_{H,t} Y_t \tag{1.13}$$

and shareholder receive a share α of output net of physical investment as a dividend D_t so that

$$D_t = \alpha P_{H,t} Y_t - P_t^I I_t. \tag{1.14}$$

[9]For simplicity $\theta_C = \theta_I$ and $\phi_C = \phi_I$ in the following. However, one can easily allow for different input shares and substitution elasticities between consumption and investment.

Then follow the first-order conditions as

$$P_t^I = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t^C}{P_{t+1}^C} \right) e^{\chi_t} \left[P_{H,t+1} \alpha e^{A_{t+1}} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + (1-\delta) \frac{P_{t+1}^I}{e^{\chi_{t+1}}} \right] \right]. \tag{1.15}$$

The intratemporal allocations for intermediate investment goods follow from the firm's profit maximization problem

$$\max_{I_t \geq 0, I_{H,t} \geq 0, I_{F,t} \geq 0} \qquad P_t^I I_t - P_{H,t} I_{H,t} - P_{F,t} I_{F,t} \tag{1.16}$$

subject to the above investment production function in Equation (1.11) so that

$$I_{H,t} = \theta_I \left(\frac{P_{H,t}}{P_t^I} \right)^{-\phi_I} I_t \tag{1.17}$$

$$I_{F,t} = (1-\theta_I) \left(\frac{P_{F,t}}{P_t^I} \right)^{-\phi_I} I_t. \tag{1.18}$$

Financial Markets and Instantaneous Budget Constraint

International trade occurs in stocks besides the trade in intermediate consumption and investment goods. Stocks issued by a firm grant a right to the dividends D_t i.e. a share of output. The budget constraint at date t becomes

$$P_t^C C_t + P_{H,t}^S S_{H,t+1} + P_{F,t}^S S_{F,t+1} = W_t L_t + \left(D_t + P_{H,t}^S \right) S_{H,t} + \left(D_t^* + P_{F,t}^S \right) S_{F,t} \tag{1.19}$$

where $P_{H,t}^S$ is the price of the home stock and $P_{F,t}^S$ is the price of the foreign stock, respectively. $S_{H,t}$ denotes the holdings of the home stocks and $S_{F,t}$ denotes the holdings of the foreign stocks at time t.

Household Decisions and Market Clearing Conditions

Households in each country choose consumption and labor allocations to maximize lifetime utility. The first-order conditions for the optimal allocation of consumption spending across home and foreign goods derive from the consumers maximization problem

$$\max_{C_t \geq 0, C_{H,t} \geq 0, C_{F,t} \geq 0} \qquad P_t^C C_t - P_{H,t} C_{H,t} - P_{F,t} C_{F,t} \tag{1.20}$$

subject to the above consumption production function in Equation (1.7) so that

$$C_{H,t} = \theta_C \left(\frac{P_{H,t}}{P_t^C} \right)^{-\phi_C} C_t \tag{1.21}$$

$$C_{F,t} = (1-\theta_C) \left(\frac{P_{F,t}}{P_t^C} \right)^{-\phi_C} C_t. \tag{1.22}$$

The labor supply decision of households depends on the inverse of the Frisch elasticity of labor supply ω

$$L_t^{\omega} = \left(\frac{W_t}{P_t^C}\right) C_t^{-\sigma}. \tag{1.23}$$

The consumption Euler equations with respect to home and foreign stocks are then

$$1 = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t^C}{P_{t+1}^C} R_{F,t+1}^S \right] \tag{1.24}$$

where

$$R_{H,t+1}^S = \frac{P_{H,t+1}^S + D_{t+1}}{P_{H,t}^S} \tag{1.25}$$

$$R_{F,t+1}^S = \frac{P_{F,t+1}^S + D_{t+1}^*}{P_{F,t}^S} \tag{1.26}$$

are the gross returns of home and foreign stocks between period t and $t+1$. Market clearing in goods markets requires

$$C_{H,t} + C_{H,t}^* + I_{H,t} + I_{H,t}^* = Y_t \tag{1.27}$$

$$C_{F,t} + C_{F,t}^* + I_{F,t} + I_{F,t}^* = Y_t^* \tag{1.28}$$

and market clearing in stock markets requires that home stocks are either held by the home or by the foreign country. In the same way foreign stocks are either held by the home or the foreign country so that

$$S_{H,t} + S_{H,t}^* = 1 \tag{1.29}$$

$$S_{F,t} + S_{F,t}^* = 1 \tag{1.30}$$

where stocks are normalized to one.

1.3.2 Portfolio Allocations

A major feature of the model is the dynamic allocation of assets and the resulting gross capital flows after a stochastic volatility shock in TFP and investment efficiency. Given that the return on the home and foreign asset equalizes one requires an approach that takes into consideration the variance of the asset returns. Since the agents in our model are risk averse, they do not only care about the levels of returns but also about the second moments.

Tille and van Wincoop (2010) provide a method of solving the portfolio allocation problem in a two-country economy using an iterative numerical algorithm. In a series of papers Devereux and Sutherland (2009), Devereux and Sutherland (2010), and Devereux and Sutherland (2011) develop an approach that allows for the allocation of any number of arbitrary assets in an open economy model with complete or incomplete markets.[10] The calculated steady states and portfolio dynamics by Devereux and Sutherland (2010) and Devereux and Sutherland (2011) can be shown to be identical to those obtained by Tille and van Wincoop (2010) for any given model. However, their approach allows for the fast analytical computation of impulse responses in domestic and foreign stock holdings for the home country after an arbitrary shock. The main advantage of the Devereux and Sutherland (2011) approach is in the wide applicability to any class of DSGE models.[11] The algorithm uses a second-order approximation of the portfolio optimality conditions to determine the steady state and a third-order approximation to determine the first-order asset dynamics.[12]

To model the asset structure of the economy we use a domestic and a foreign stock that pay dividends in a two-country model. The gross returns on home and foreign stocks are given by $R^S_{H,t+1}$ and $R^S_{F,t+1}$ as determined in Equation (1.25) and (1.26).

Capital market clearing conditions are used to determine the asset allocations in the foreign country and the resulting capital flows after a stochastic volatility shock in TFP or the investment efficiency. Capital inflows and capital outflows can then be defined as

$$Inflows_t = \Delta S^*_{H,t} \tag{1.31}$$

$$Outflows_t = \Delta S_{F,t} \tag{1.32}$$

where $\Delta S^*_{H,t}$ denotes the change in the home stock held by the foreign country at time t that is flowing into the home country. Such that a positive value for $Inflows_t$ indicates a flow of capital from the foreign to the home country by foreign agents whereas a negative value indicates a flow from the home country back to the foreign country. In the same way a positive value for $Outflows_t$ indicates a flow of capital from home to foreign by domestic agents and a negative value in turn indicates a flow from the foreign country back to the home country. The definition of capital inflows and capital outflows is therefore the same in the DSGE model and the empirical part allowing for an easy comparison of the results.

1.3.3 Shock Structure

Finally, the shocks in the model are structured to allow for stochastic volatility in total factor productivity and the investment efficiency in a similar fashion as in Fernandez-Villaverde et al. (2011). Shocks to the volatility of total factor productivity and the investment efficiency might a priori be seen as a major force in agents' decision of the allocation

[10] See the Appendix 1.A.2 and 1.A.3 for a short explanation of the zero-order asset portfolio and the first-order asset dynamics.

[11] See Rabitsch et al. (2015) for a critical review of the performance of the Devereux and Sutherland (2011) algorithm.

[12] Their approximation is therefore in line with Samuelson (1970) who states that for n-th order accuracy a $n+2$ order approximation of the portfolio problem is required.

of assets. Additionally, shocks in the volatility of the investment efficiency might make it harder for households to make informed decisions about future relative prices of investment and consumption and thus lead to an outflow of capital. For both, the TFP and investment efficiency, we assume transitory shocks to allow for a comparison with the data where we constructed detrended TFP and IE series.

The AR(1) processes for total factor productivity is then as follows

$$A_t = \rho_A A_{t-1} + e^{\sigma_{A,t}} u_{A_t} \tag{1.33}$$

where ρ_A is a persistence parameter and u_{A_t} is a normally distributed random variable with mean zero and unit variance and can be considered as a shock in levels to TFP

$$u_{A_t} \sim \mathcal{N}(0,1). \tag{1.34}$$

The variable σ_{A_t} is not assumed to be constant but instead follows an AR(1) process so that the volatility part then follows as

$$\sigma_{A_t} = (1-\rho_{\sigma_A})\,\sigma_A + \rho_{\sigma_A}\sigma_{A_{t-1}} + \eta_A u_{\sigma_A,t} \tag{1.35}$$

where again ρ_{σ_A} is a persistence parameter and $u_{\sigma_A,t}$ is again a normally distributed random variable with mean zero and unit variance and causes exogenous changes in the volatility of TFP

$$u_{\sigma_A,t} \sim \mathcal{N}(0,1). \tag{1.36}$$

The parameters σ_A and η_A in Equation (1.35) affect the degree of mean volatility and stochastic volatility in total factor productivity. A high σ_A causes a high degree of mean volatility and a high η_A causes a high degree of stochastic volatility in the process.[13]

The remaining processes for investment efficiency χ_t can then be analogously written as

$$\chi_t = \rho_\chi \chi_{t-1} + e^{\sigma_{\chi t}} u_{\chi_t} \tag{1.37}$$

and the exogenous shocks u_{χ_t} follow a normal distribution with mean zero and unit variance

$$u_{\chi_t} \sim \mathcal{N}(0,1). \tag{1.38}$$

Again the AR(1) process for σ_{χ_t} follows as

$$\sigma_{\chi t} = \left(1-\rho_{\sigma_\chi}\right)\sigma_\chi + \rho_{\sigma_\chi}\sigma_{\chi t-1} + \eta_\chi u_{\sigma_\chi,t} \tag{1.39}$$

[13]It is for simplicity assumed that the error terms u_t and u_{σ_t} are uncorrelated. However, Fernandez-Villaverde et al. (2011) argue that innovations to levels and volatility can be highly correlated.

with a normally distributed error term with mean zero and unit variance

$$u_{\sigma_\chi,t} \sim \mathcal{N}(0,1). \tag{1.40}$$

The parameterization of the shocks poses a major burden as the stochastic processes in Equation (1.33) and Equation (1.35) as well as in Equation (1.37) and Equation (1.39) are driven by two innovations, one innovation to levels and one innovation to volatility. The parameters are hence estimated using Bayesian estimation in the form of a particle filter i.e. a sequential Monte Carlo algorithm as used by Fernandez-Villaverde et al. (2011). Their approach proves to be convenient as it provides parameter estimates for a stochastic process which can then be used in a DSGE model to generate stochastic volatility. It is therefore superior to a GARCH approach which does not clearly distinguish between innovations in levels and innovations in volatility.

Table 1.4 shows the used priors of the parameters of the shocks. The priors for ρ and ρ_σ are assumed to be beta distributed with a mean of 0.75 and standard deviation of 0.02 and 0.05, respectively. This ensures that the prior values are bounded between zero and one.[14] The chosen mean is reflecting the fact that the underlying data is quarterly data and that the data shows some persistence in the shocks. The priors of σ and η follow a normal and truncated normal distribution to ensure a positive posterior for η. The prior for η is set to a conservative level of 0.50 implying an amplification of the stochastic shock by around 1.65. The standard deviation of 0.30 ensures some flexibility for the estimation of the posteriors. The priors for the parameter σ that controls for mean volatility is derived from Mexican and US data from total factor productivity and investment efficiency. The value of -6.46 for Mexican TFP reflects the fact that Mexico as the home country shows about twice the degree of mean volatility in the data than the US as the foreign country with a value of -7.40. A similar pattern is observable for investment efficiency with a value of -1.88 for Mexico and -8.37 for the US.[15] The subsequently used Metropolis-Hastings algorithm is run for 20000 iterations with 2000 particles or simulations per iteration with the first 5000 iterations being discarded as a burn-in period until equilibrium is reached.[16]

[14] This implies for the parameters of the beta distribution using the mean μ and variance σ^2 following $\alpha = \left(\frac{1-\mu}{\sigma^2}\right)\mu^2 - \mu$ and $\beta = \frac{\alpha}{\mu} - \alpha$ a value of $\alpha = 350.81$ and $\beta = 116.94$ for $\sigma^2 = 0.02$ and $\alpha = 55.50$ and $\beta = 18.50$ for $\sigma^2 = 0.05$, respectively.

[15] In the model Mexican and US volatility shocks are assumed to be uncorrelated as what matters for the allocation of assets is the relative volatility.

[16] The low dimensionality guarantees that the equilibrium is reached rather quickly and that 20000 iterations are sufficient to be confident to be in equilibrium.

TABLE 1.4
Priors of Shock Parameters

		TFP		IE	
Parameter	Description	Mexico	US	Mexico	US
ρ	Level Persistence	$\mathcal{B}\,(0.75, 0.02)$	$\mathcal{B}\,(0.75, 0.02)$	$\mathcal{B}\,(0.75, 0.02)$	$\mathcal{B}\,(0.75, 0.02)$
σ	Mean Volatility	$\mathcal{N}\,(-6.46, 0.40)$	$\mathcal{N}\,(-7.40, 0.40)$	$\mathcal{N}\,(-1.88, 0.40)$	$\mathcal{N}\,(-8.37, 0.40)$
ρ_σ	Volatility Persistence	$\mathcal{B}\,(0.75, 0.05)$	$\mathcal{B}\,(0.75, 0.05)$	$\mathcal{B}\,(0.75, 0.05)$	$\mathcal{B}\,(0.75, 0.05)$
η	Stochastic Volatility	$\mathcal{N}^+\,(0.50, 0.30)$	$\mathcal{N}^+\,(0.50, 0.30)$	$\mathcal{N}^+\,(0.50, 0.30)$	$\mathcal{N}^+\,(0.50, 0.30)$

Note: Table 1.4 shows the assumed priors of the parameter values of the stochastic shocks. Where the mean and standard deviation are in parentheses.

Table 1.5 shows the posterior distribution of the Bayesian estimation with the 2.5 and 97.5 percent confidence sets in parenthesis for the TFP and investment efficiency series. The posterior estimate for σ reflects the fact that the US has a significantly lower mean volatility in both TFP and investment efficiency.

TABLE 1.5
Posteriors of Shock Parameters

		TFP		IE	
Parameter	Description	Mexico	US	Mexico	US
ρ	Level Persistence	0.78	0.83	0.89	0.81
		(0.75;0.81)	(0.80;0.86)	(0.87;0.91)	(0.78;0.84)
σ	Mean Volatility	-4.12	-5.03	-3.30	-5.44
		(-4.25;-3.97)	(-5.24;-4.81)	(-3.64;-2.99)	(-5.74;-5.00)
ρ_σ	Volatility Persistence	0.74	0.77	0.81	0.77
		(0.62;0.83)	(0.66;0.85)	(0.73;0.88)	(0.68;0.85)
η	Stochastic Volatility	0.09	0.25	0.33	0.40
		(0.01;0.25)	(0.16;0.38)	(0.24;0.46)	(0.29;0.58)

Note: Table 1.5 shows the median of the posterior distribution of the parameter values of the stochastic shocks including the 2.5 and 97.5 percent confidence sets in parenthesis.

1.3.4 Equilibrium and Equilibrium Conditions

An equilibrium of this economy is characterized by a set of allocations for the consumer in each country consisting of consumption C_t, labor L_t, capital K_t, investment I_t, and home and foreign stocks $S_{H,t}$ and $S_{F,t}$. The allocations for home and foreign intermediate consumption goods producers $C_{H,t}$, $C^*_{H,t}$, $C_{F,t}$, and $C^*_{F,t}$. The allocations for home and foreign intermediate investment goods producers $I_{H,t}$, $I^*_{H,t}$, $I_{F,t}$, and $I^*_{F,t}$. The allocations for both the home and foreign goods producers Y_t and Y^*_t and prices of the intermediate goods $P_{H,t}$, $P^*_{H,t}$, $P_{F,t}$, and $P^*_{F,t}$ as well as final prices for consumption P^C_t, P^{C*}_t and investment P^I_t, P^{I*}_t as well as home and foreign stocks $P^S_{H,t}$, $P^S_{F,t}$. Finally, we require the price of labor and stock returns W_t and R^S_t for each country such that (1) Households allocations solve the households' problem. (2) Intermediate goods producers' allocations solve the intermediate goods producers' problem. (3) Final good producers' allocations solve the final goods producers' problem. (4) All markets clear.

1.3.5 Solution Techniques

As we are interested in the effects of shocks to stochastic volatility, a higher order approximation is required than the usual approach of approximating the policy function up to second-order. In this case a third-order approximation of the policy functions is needed in order for the exogenous volatility shocks to have an independent effect on the endogenous variables. Otherwise volatility shocks would have only an indirect effect through the levels shock. Higher order perturbation methods e.g. Taylor series expansions around the steady state can suffer from explosive sample paths. Andreasen et al. (2018) provide an approach for third-order perturbation method approximations to avoid generating these explosive sample paths. This is done using the pruned state space technique. All generated impulse responses in this paper therefore use the pruned state space technique to calculate third-order approximations. The model Equations (1.7) to (1.30) and the shocks structure in Equations (1.33) to (1.39) solve the model.

1.3.6 Parameters

The model is parameterized in tradition of the standard international business cycle literature as in Backus et al. (1994) and in particular Coeurdacier et al. (2010). Table 1.6 shows the parameter values for Mexico as the home country and the US as the foreign country. As the model is used as a foundation for a structural VAR that uses quarterly data, the discount factor β is set to 0.99, thus corresponding to a yearly value of 0.96. The risk aversion σ is fixed at 2.00 following standard literature in international macroeconomics. The capital elasticity in the production function α is set at the conventional level of 0.40 for both countries. The inverse of the Frisch elasticity of labor supply ω is set to 0.40 for Mexico and the US which is in the middle of estimates by Chetty (2012). The substitutability between domestic and foreign consumption goods in the CES production function ϕ_C is assumed to be 2.00 for both countries. The model assumes a domestic share θ_C in the input function of 0.60 for Mexico as the home country and 0.85 for the US as the foreign country which is in line with empirical observations for Mexico and the US.

We use the above parameters to gain intuition into the model. However, to verify the robustness of the results with respect to different parameters and to derive robust sign restrictions for the SVAR, we draw random parameters from a uniform distribution similar to Canova and Paustian (2011). A uniform distribution is therefore chosen to best model the uninformative prior. Table 1.6 shows the support of all parameter draws. The mean of the range is always chosen to coincide with the baseline parameters in Table 1.6. The capital elasticity α is between 0.35 and 0.45 and the discount factor β is between 0.98 and 0.999. For the inverse of the Frisch labor supply elasticity ω we choose values between 0.30 and 0.50 and for the capital depreciation rate δ we pick the parameters from between 0.02 and 0.03. We further allow for some variation in the risk aversion parameter σ in the range 1.50 to 2.50. The domestic share in the consumption and investment function θ_C and θ_I is between 0.50 and 0.70 for Mexico and between 0.75 and 0.95 for the US. Finally, the substitutability between home and foreign goods ϕ_C and ϕ_I is chosen from between 1.50 and 2.50 in both the consumption and investment case. The estimated posterior distribution for the stochastic processes in Table 1.5 is then used to draw parameter values for the TFP and investment efficiency processes. ρ and ρ_σ are both drawn from a beta distribution to obtain bounded values between 0 and 1. Further σ is drawn from a normal distribution and η is drawn from a truncated normal distribution to ensure positive values. This allows us to incorporate the uncertainty of the parameter estimate into the DSGE model.

TABLE 1.6
Model Parameters

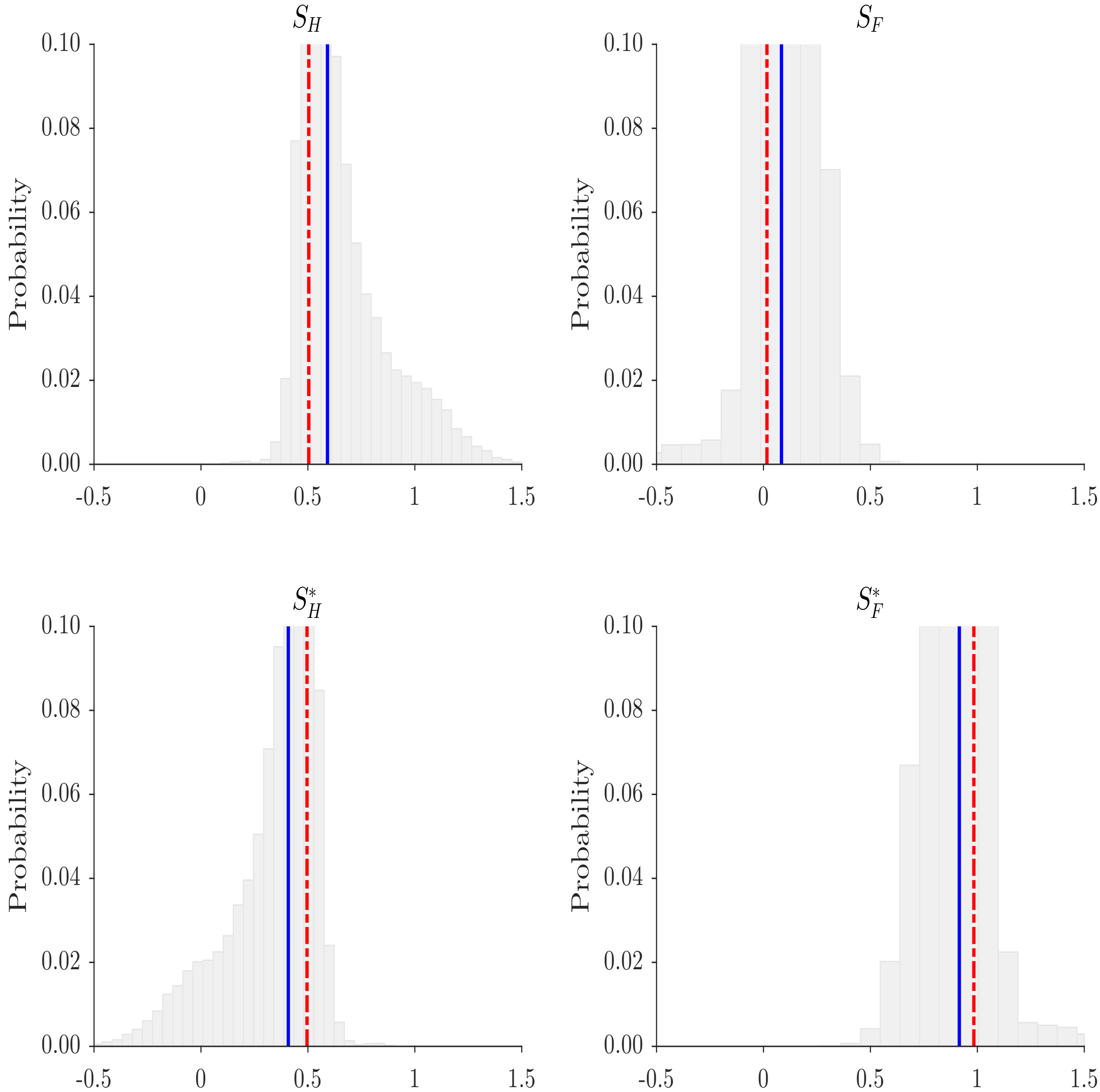
		Baseline		Monte Carlo	
Parameter	Description	Mexico	US	Mexico	US
α	Capital Elasticity	0.40	0.40	$\mathcal{U}\{0.35, 0.45\}$	$\mathcal{U}\{0.35, 0.45\}$
β	Discount Factor	0.99	0.99	$\mathcal{U}\{0.98, 0.999\}$	$\mathcal{U}\{0.98, 0.999\}$
ω	Inverse of Frisch Elasticity	0.40	0.40	$\mathcal{U}\{0.30, 0.50\}$	$\mathcal{U}\{0.30, 0.50\}$
δ	Capital Depreciation Rate	0.025	0.025	$\mathcal{U}\{0.02, 0.03\}$	$\mathcal{U}\{0.02, 0.03\}$
σ	Risk Aversion	2.00	2.00	$\mathcal{U}\{1.50, 2.50\}$	$\mathcal{U}\{1.50, 2.50\}$
θ_C	Domestic Share in Input Function - Consumption	0.60	0.85	$\mathcal{U}\{0.50, 0.70\}$	$\mathcal{U}\{0.75, 0.95\}$
θ_I	Domestic Share in Input Function - Investment	0.60	0.85	$\mathcal{U}\{0.50, 0.70\}$	$\mathcal{U}\{0.75, 0.95\}$
ϕ_C	Substitutability between Goods - Consumption	2.00	2.00	$\mathcal{U}\{1.50, 2.50\}$	$\mathcal{U}\{1.50, 2.50\}$
ϕ_I	Substitutability between Goods - Investment	2.00	2.00	$\mathcal{U}\{1.50, 2.50\}$	$\mathcal{U}\{1.50, 2.50\}$

Note: Table 1.6 shows the parameter values in the DSGE model for Mexico as the home country and the US as the foreign country.

1.3.7 Monte Carlo Zero-Order Asset Portfolios

To verify the robustness of the steady state portfolio allocation simulated using the Devereux and Sutherland (2011) algorithm, 2500 simulations of the model are performed with uniform random parameter draws. Figure 1.5 shows the histogram of simulated steady state estimates of asset portfolios with the median estimate of the Monte Carlo draws indicated by the blue line and the estimate of the baseline estimation shown by the dashed-dotted red line. Due to the structure of the error terms the zero-order asset portfolios show some skewness in their distribution. We can observe that the stocks of the home country S_H are almost evenly split between the home country and the foreign country. However, for the foreign stocks S_F we observe that these are almost entirely owned by the foreign economy due to the higher share of domestic goods in the production of the final consumption and investment good.

FIGURE 1.5
Monte Carlo Zero-Order Asset Portfolios



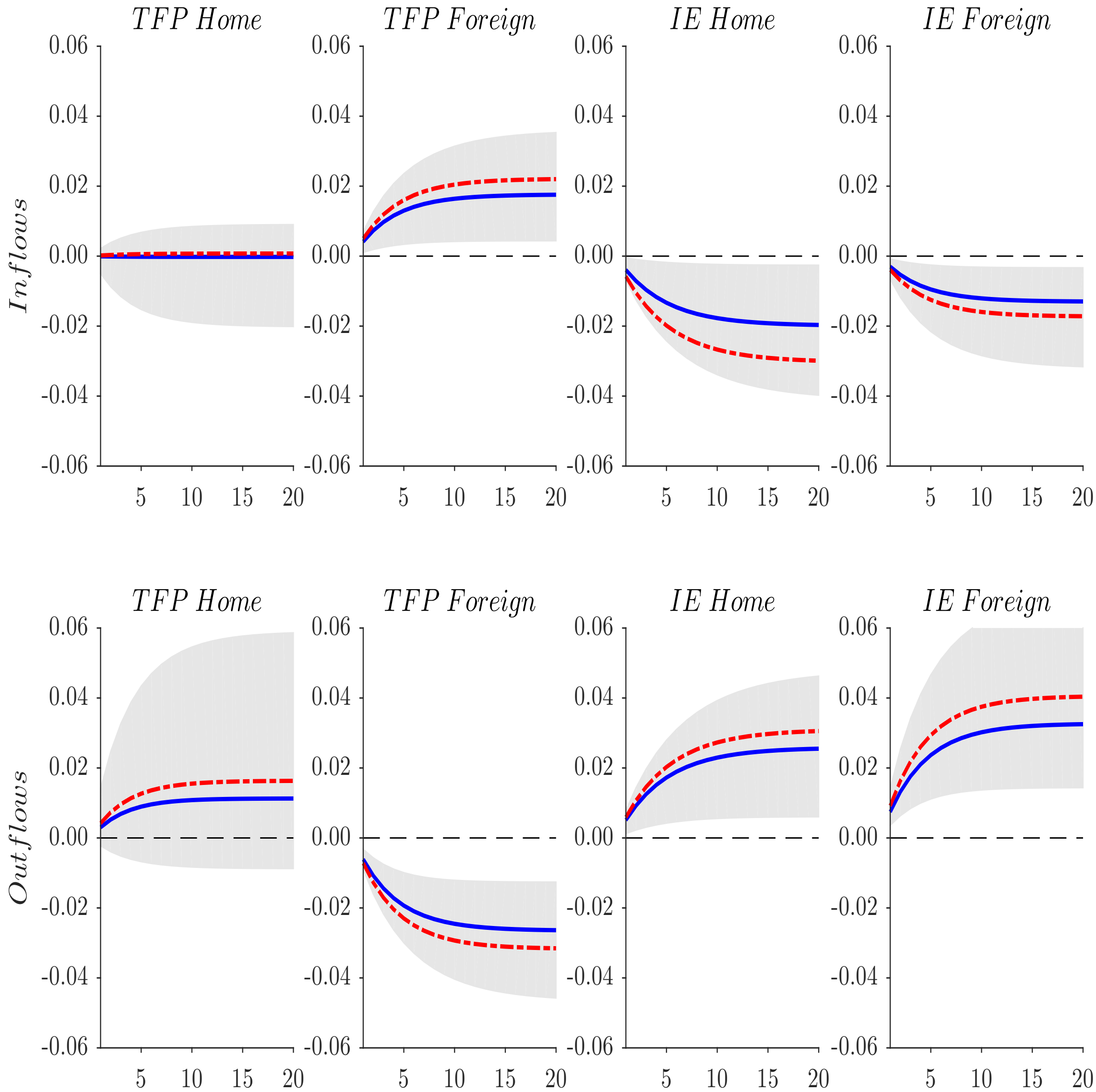
Note: Figure 1.5 shows the histogram of zero-order asset portfolios after 2500 simulations of the model. The median estimate is in blue. The dashed-dotted red line denotes the results from the baseline version.

1.3.8 Monte Carlo First-Order Asset Dynamics

A stochastic shock to TFP volatility will have a direct effect on output volatility and therefore on dividend payments to shareholders and returns on equity. Volatility shocks to the investment efficiency will affect the volatility of relative prices between investment and consumption goods and hence will affect capital accumulation. This will then affect output and hence dividends and the volatility of returns on equity.

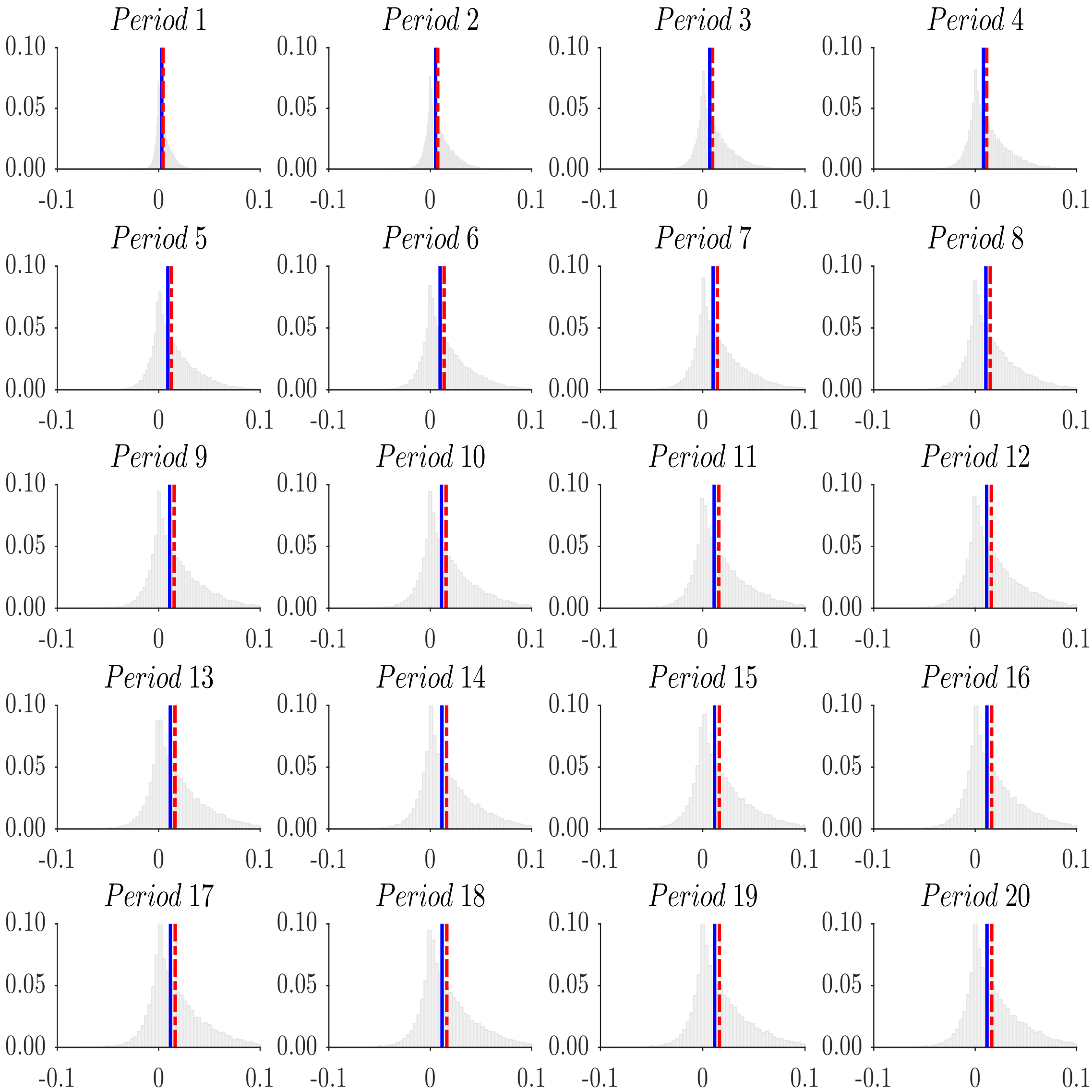
Figure 1.6 shows the impulse responses of TFP and IE volatility shocks on the home country capital inflows and capital outflows with the 10th and 90th percentile confidence bands. The median and the confidence bands are obtained from 1000 independent parameter draws. The displayed flows are deviations from their steady state value relative to output and are cumulated over the displayed time horizon. The impulse responses after a TFP volatility shock in each country on capital inflows and capital outflows have the expected sign and magnitude. However, only investment efficiency volatility shocks in the home country have the expected sign. Investment efficiency shocks originating in the foreign country behave against the expectation. Figure 1.7 shows a cross-section of the impulse response for a home country TFP volatility shock on cumulated capital outflows. Like the zero-order asset portfolios, the impulse responses exhibit some skewness, but are otherwise near normally distributed. Although the results of the Monte Carlo draws are not perfectly normal distributed it still allows us to construct confidence bands for the impulse responses.

FIGURE 1.6
Impulse Responses DSGE - Capital Flows with Monte Carlo Priors



Note: Figure 1.6 shows the median impulse responses on capital inflows (upper panel) and capital outflows (lower panel) for the home country in the DSGE model in blue and the 10th and 90th percentile confidence bands as the shaded area. The dashed-dotted red line denotes the results from the baseline version.

FIGURE 1.7
Impulse Responses DSGE - Cross-Section with Monte Carlo Priors



Note: Figure 1.7 shows the histogram of Monte Carlo draws including the median impulse responses in the DSGE model in blue for a home country TFP volatility shock on cumulated capital outflows. The dashed-dotted red line denotes the results from the baseline version.

1.4 Structural Estimation with Sign and Zero Restrictions

We now apply a structural VAR with combined sign and zero restrictions for the identification of the structural shocks to Mexican capital flow data. The aim of this exercise is to show whether a structural VAR with theoretically derived restrictions is able to reproduce the results from a standard two-country DSGE model. The approach of combined sign and zero restrictions for potentially underidentified models is described by Binning (2013) who applies the derived algorithm to replicate the results by Smets and Wouters (2007). Sign restrictions come with the advantage of providing more flexibility when identifying the shocks as the identifying restrictions are less strict than the traditional way of imposing zero restrictions for the short run or the long run. All what is needed is an assumption of the initial sign of the impulse response either on impact or for a certain time horizon e.g. a positive response of capital outflows to an increase in idiosyncratic investment efficiency volatility for the first four quarters. However, in certain scenarios the long run effect is obvious and can hence serve as an additional identifying restriction. It might therefore be justified to assume that a monetary policy shock has no long run impact on GDP growth rates as it is predominantly done in the literature.

The work by Binning (2013) builds on previous work by Rubio-Ramirez et al. (2010) who develop an algorithm that allows for sign and zero restrictions in exactly identified models. The Binning (2013) approach can therefore be considered as a generalization of the Rubio-Ramirez et al. (2010) approach to allow for underidentified models to be estimated.

1.4.1 Identifying Restrictions

We use the following identifying restrictions consisting of sign restrictions on impact and zero restrictions for the short and the long run.[17] A positive monetary policy shock ϵ^{MP} increases the interest rate differential between Mexico and the US and has a negative short run impact on GDP growth rate differentials as well as on inflation differentials. It is further assumed that the monetary shock has no long run impact on GDP growth following neutrality of money theory so that it is justified to impose a zero long run restriction as in Blanchard and Quah (1989) and Christiano et al. (2006). The paper imposes no a priori assumptions of the effect of monetary policy on capital inflows or capital outflows.

The restrictions for volatility shocks on capital outflows and capital inflows are derived from the DSGE model with stochastic volatility shocks for TFP and the investment efficiency. This part thus imposes a positive effect on impact of an idiosyncratic volatility shock on capital outflows and a negative effect on impact for capital inflows for both the TFP and investment efficiency volatility shock denoted as ϵ^{VolTFP} and ϵ^{VolIE}, respectively. Let us assume that the TFP volatility shock ϵ^{VolTFP} leads to an increase in TFP volatility

[17] See Fry and Pagan (2011) for a review on sign restrictions in structural VARs.

and that the investment efficiency volatility shocks leads to an increase in investment efficiency volatility.[18] We believe that these identifying restrictions make sense as a positive volatility shock in TFP and the investment efficiency will increase capital outflows by domestic agents that try to benefit from lower uncertainty in the US and therefore transfer assets from Mexico to the US. In addition, US agents will repatriate assets from Mexico to the US in response to increased Mexican volatility hence leading to negative capital inflows by foreigners. [19] To have unique identifying restrictions for the TFP and the investment efficiency shock it is further assumed that the TFP volatility shock has no short run impact on investment efficiency volatility and vice versa.[20] This allows us to impose a zero short run restriction. Both volatility shocks are assumed to have no effect on the GDP growth differential between Mexico and the US so that a zero long run restriction can additionally be imposed.

A capital inflow shock ϵ^{In} as well as a capital outflow shock ϵ^{Out} will both increase TFP and investment efficiency volatility on impact. Capital flow shocks are assumed to have only a short run impact on volatility so that it is possible to impose zero long run restrictions for both types of capital flow shocks on volatility. It seems obvious that excessive capital inflows or capital outflows might temporarily increase idiosyncratic volatility. This might be most obvious when investment efficiency volatility is considered that can react significantly to capital inflows and capital outflows. However, it might be more in line with economic theory that a capital inflow or outflow shock has no long run impact on volatility as volatility might eventually return to its steady state value. The structural estimation does further not impose any a priori restrictions on the cross impact between capital inflow and capital outflow shocks on each other. However, the data might indicate a negative correlation between capital inflows and capital outflows as seen in Table 1.3.

For completeness one can further assume that an aggregate demand shock ϵ^{AD} has a positive short run impact on the interest rate differential and differential of GDP growth rates as well as inflation differentials between Mexico and the US. It is further assumed that the long run effect on the differential in the GDP growth rate is zero for the aggregate demand shocks which is in line with the common assumptions for long run identification as e.g. in Blanchard and Quah (1989) and Christiano et al. (2006).

Finally, the unidentified shock ϵ^{Res} is left without any restrictions and is ordered last to catch any unexplained variation. The identifying restrictions of the SVAR are summarized in Table 1.7 where a + (-) indicates a positive (negative) impact upon the structural shocks ϵ. Whereas 0 indicates no impact and blanks indicate no imposed restriction.

[18] Leduc and Liu (2016) find that uncertainty shocks are similar to demand shocks in their data and a DSGE model.

[19] Coeurdacier et al. (2019) find that capital in many cases flows to developed countries as these countries face lower uncertainty.

[20] This assumption is justified by the low correlation between Mexican TFP and investment efficiency volatility. It comes with the huge advantage of unique identification of both shocks due to different restrictions.

TABLE 1.7
Identifying Restrictions SVAR - Baseline

	Short Run Restrictions						
	ϵ^{MP}	ϵ^{VolTFP}	ϵ^{VolIE}	ϵ^{In}	ϵ^{Out}	ϵ^{AD}	ϵ^{Res}
Interest Rate Diff	+					+	
Volatility TFP		+	0	+	+		
Volatility IE		0	+	+	+		
Inflows		-	-	+			
Outflows		+	+		+		
ΔGDP Diff	-					+	
ΔCPI Diff	-					+	

	Long Run Restrictions						
	ϵ^{MP}	ϵ^{VolTFP}	ϵ^{VolIE}	ϵ^{In}	ϵ^{Out}	ϵ^{AD}	ϵ^{Res}
Interest Rate Diff							
Volatility TFP				0	0		
Volatility IE				0	0		
Inflows							
Outflows							
ΔGDP Diff	0	0	0			0	
ΔCPI Diff							

Note: Table 1.7 shows all the identifying restrictions for the short and the long run in the SVAR. A + (-) indicates a positive (negative) impact on the structural shocks ϵ. Whereas 0 indicates no impact.

Using standard VAR notation allows to write the VAR using the lag operator as

$$Y_{t+1} = B\left(L\right) Y_t + u_{t+1} \tag{1.41}$$

and

$$\mathbb{E} u_t u_t' = \Sigma \tag{1.42}$$

where Σ is the variance-covariance matrix of the forecast errors and L is the lag operator so that

$$B\left(L\right) = B_1 + B_2 L + \ldots + B_p L^{p-1} \tag{1.43}$$

where p is the number of lags of the structural VAR. Further imposing some structure on the error terms implies

$$u_t = Z\epsilon_t \tag{1.44}$$

$$ZZ' = \Sigma \tag{1.45}$$

where the Z matrix maps the structural shocks ϵ_t into the reduced form shocks u_t. As Z is not unique there are infinitely many solutions that satisfy Equation (1.45). Some additional economy theory is therefore used to rule out unwanted matrices Z. The structural shocks ϵ_t are assumed to be orthogonal so that

$$\epsilon_t \epsilon_t' = I. \tag{1.46}$$

Using the algorithm provided by Binning (2013) and following the same notation gives for the short run impact matrix L_0

$$L_0 = \begin{bmatrix} + & X & X & X & X & + & X \\ X & + & 0 & + & + & X & X \\ X & 0 & + & + & + & X & X \\ X & - & - & + & X & X & X \\ X & + & + & X & + & X & X \\ - & X & X & X & X & + & X \\ - & X & X & X & X & + & X \end{bmatrix} \tag{1.47}$$

where a $+$ indicates a positive impact and a $-$ indicates a negative impact, respectively. Unclear impacts are denoted by X and are not restricted so that they can be positive, negative, or zero. In this seven variable VAR the ordering is as follows. Interest rate differentials are ordered first, followed by the measure of idiosyncratic TFP volatility and the volatility of the investment efficiency. Capital inflows relative to GDP, and capital outflows relative to GDP follow afterwards as these flows are assumed to be contemporaneously affected by monetary policy and volatility shocks. This is completed by the differential of GDP growth and the differential of CPI changes that are ordered last so that they will react after each of the variables. The corresponding shocks are a monetary policy shock denoted ϵ^{MP}, an idiosyncratic TFP volatility shock ϵ^{VolTFP}, an idiosyncratic investment efficiency volatility shock ϵ^{VolIE}, and capital inflow and capital outflow shocks denoted ϵ^{In} and ϵ^{Out} as well as an aggregate demand shock ϵ^{AD} and an unidentified shock ϵ^{Res} that will account for all unidentified shocks. For the long run matrix including zero restrictions one then gets after imposing the restrictions

$$L_\infty = \begin{bmatrix} X & X & X & X & X & X & X \\ X & X & X & 0 & 0 & X & X \\ X & X & X & 0 & 0 & X & X \\ X & X & X & X & X & X & X \\ X & X & X & X & X & X & X \\ 0 & 0 & 0 & X & X & 0 & X \\ X & X & X & X & X & X & X \end{bmatrix} \tag{1.48}$$

where again X denotes no restriction. The short run and the long run matrix in Equation (1.47) and (1.48) can then be combined to obtain

$$f(Z, B) = \begin{bmatrix} L_0 \\ L_\infty \end{bmatrix} \tag{1.49}$$

where L_0 is the $k \times k$ short run impact matrix and L_∞ is the $k \times k$ long run impact matrix with k being the number of variables in the SVAR. A $k \times 2k$ restriction matrix Q can then be derived such that

$$Q_j f(Z, B) e_j = 0 \tag{1.50}$$

where e_j is the j-th column of a $k \times k$ identity matrix. The initial impact matrix is obtained as

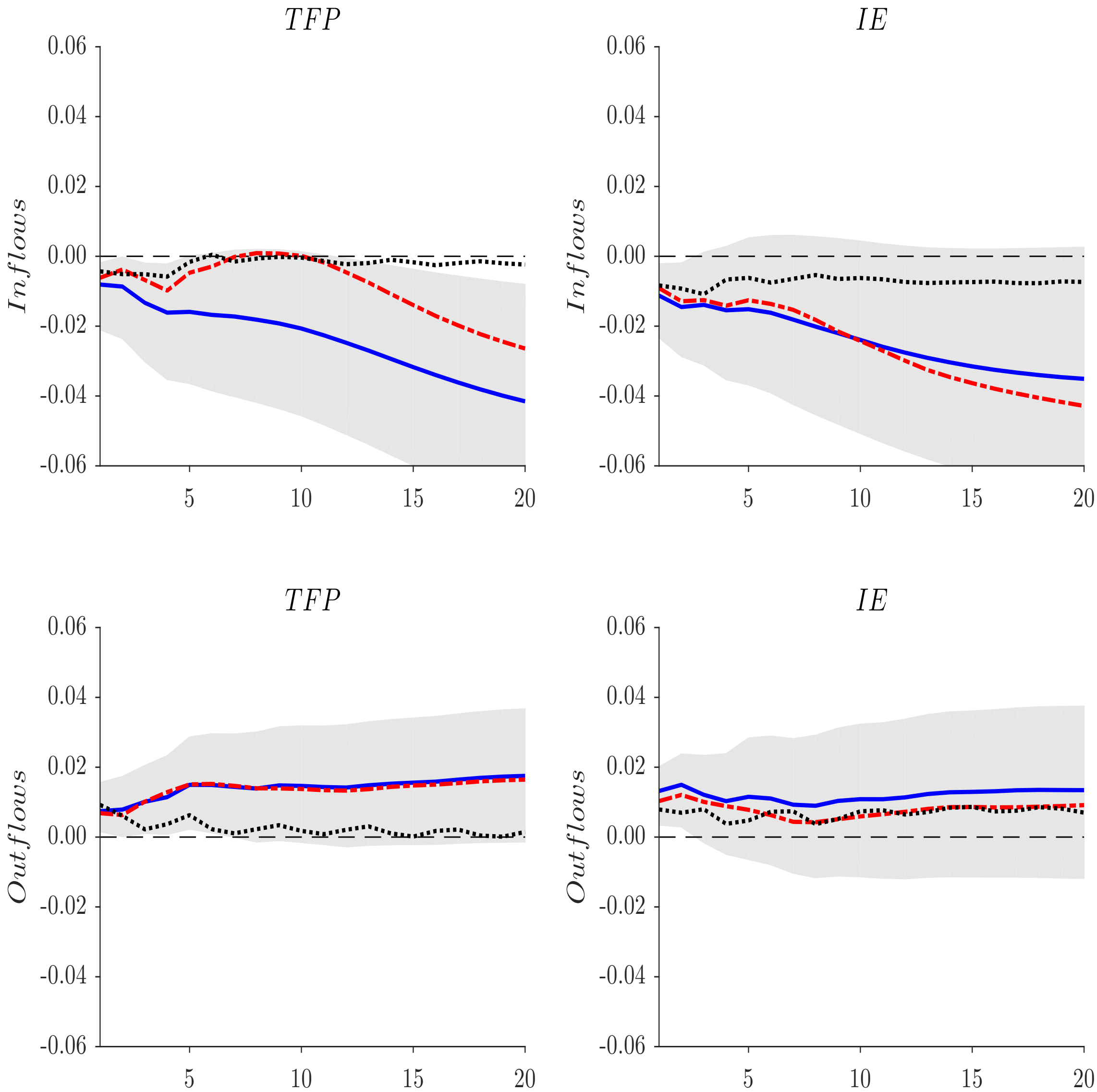
$$Z = CQ^* \tag{1.51}$$

where C is the lower Cholesky decomposition of the variance-covariance matrix Σ so that $CC' = \Sigma$ and Q^* is a randomly drawn orthogonal matrix from a random normal distribution. For each additional draw the Cholesky decomposition C is then again multiplied by a randomly drawn orthogonal matrix Q^*.

1.4.2 Impulse Responses

The structural VAR using the whole sample period and the two subperiods is estimated with four lags to minimize the Bayesian Information Criterion (BIC) and 500 draws of the impact matrix Z that satisfy the sign restrictions of the impulse responses as imposed in Table 1.7. Figure 1.8 shows the impulse responses of capital inflows and capital outflows to a unity shock in idiosyncratic TFP and investment efficiency shocks in the structural VAR. The estimated system is underidentified as there are less than the $k(k-1)/2$ i.e. 21 required identifying restrictions. Hence the Z matrix is not unique so that $ZZ' = \Sigma$ can be satisfied by different Z matrices. This in turn leads to a set of possible impulse responses being calculated for each structural shock that is not uniquely identified. The impulse responses in Figure 1.8 show the median estimate of all 500 draws and the 10th and 90th percentile confidence bands as proposed by Fry and Pagan (2011). By construction, increases in TFP and investment efficiency volatility have a negative short run impact on capital inflows and a positive impact on capital outflows. The impact of a idiosyncratic TFP volatility shock on capital inflows comes with a wider confidence band but still allows to conclude that cumulated changes in capital outflows remain negative for most of the 20 periods. The estimated confidence bands for a TFP volatility shock on capital outflows are relatively tight and allow to conclude that the cumulated response remains positive for the whole plotted period of 20 quarters. Shocks to the volatility investment efficiency decrease capital inflows and increase capital outflows. In the case of investment efficiency shocks, the confidence bands allow to conclude that the impulse responses remain negative for capital inflows and positive for capital outflows throughout the 20 quarters.

FIGURE 1.8
Impulse Responses SVAR - Baseline Restrictions



Note: Figure 1.8 shows impulse responses of capital inflows and outflows to a unity shock in the structural VAR with sign and zero restrictions for Mexico. The data is quarterly from 1980Q1 to 2014Q4. The median impulse response is shown in blue and the 10th and 90th percentile confidence bands as the shaded area. The sample consisting of data from 1980 to 1999 is shown as the dashed-dotted red line and the sample from 2000 to 2014 is shown as the dotted black line.

1.4.3 Forecast Error Variance Decomposition

The forecast error variance decomposition of the structural VAR at an infinite horizon can be seen in Table 1.8 for the whole sample period and the two subperiods. Using the forecast error variance decomposition allows us to judge the relative importance of TFP volatility shocks and investment efficiency volatility shocks on capital inflows and capital outflows. The FEVD confirms that the SVAR is doing a relatively good job when the two subperiods are considered separately. For the period 1980 to 1999 the shock in idiosyncratic TFP volatility can explain about 3 percent of the variation in capital inflows. For the same period, shocks to idiosyncratic investment efficiency volatility can account for about 6 percent in capital inflows and about 4 percent in capital outflows. For the period 2000 to 2014, however, TFP volatility can explain 6 percent of capital inflow variability. Looking at capital outflows, 3 percent in the variability can be attributed to investment efficiency. Looking at the whole sample the FEVD suggests a tiny effect of TFP and investment efficiency volatility on capital inflows and outflows of between 1 and 3 percent. Overall, this suggests that between 3 to 9 percent in the total variation of capital inflows and capital outflows can be attributed to shocks in volatility, either TFP or investment efficiency.

TABLE 1.8
Forecast Error Variance Decomposition SVAR

	1980-2014						
	ϵ^{MP}	ϵ^{VolTFP}	ϵ^{VolIE}	ϵ^{In}	ϵ^{Out}	ϵ^{AD}	ϵ^{Res}
Interest Rates Diff	1.00						
Volatility TFP	0.00	1.00					
Volatility IE	0.03	0.03	0.94				
Inflows	0.00	0.01	0.02	0.97			
Outflows	0.00	0.00	0.03	0.30	0.67		
ΔGDP Diff	0.04	0.00	0.01	0.05	0.01	0.90	
ΔCPI Diff	0.44	0.01	0.04	0.00	0.02	0.00	0.48
	1980-1999						
	ϵ^{MP}	ϵ^{VolTFP}	ϵ^{VolIE}	ϵ^{In}	ϵ^{Out}	ϵ^{AD}	ϵ^{Res}
Interest Rates Diff	1.00						
Volatility TFP	0.00	1.00					
Volatility IE	0.06	0.03	0.91				
Inflows	0.01	0.03	0.06	0.90			
Outflows	0.00	0.00	0.04	0.18	0.77		
ΔGDP Diff	0.05	0.01	0.02	0.03	0.02	0.87	
ΔCPI Diff	0.48	0.02	0.04	0.00	0.01	0.01	0.44
	2000-2014						
	ϵ^{MP}	ϵ^{VolTFP}	ϵ^{VolIE}	ϵ^{In}	ϵ^{Out}	ϵ^{AD}	ϵ^{Res}
Interest Rates Diff	1.00						
Volatility TFP	0.00	1.00					
Volatility IE	0.03	0.00	0.96				
Inflows	0.01	0.06	0.00	0.92			
Outflows	0.02	0.00	0.03	0.45	0.50		
ΔGDP Diff	0.14	0.01	0.01	0.08	0.01	0.75	
ΔCPI Diff	0.00	0.08	0.00	0.07	0.00	0.01	0.83

Note: Table 1.8 shows the Forecast Error Variance Decomposition of the SVAR for the three sample periods at an infinite horizon. The data is quarterly data.

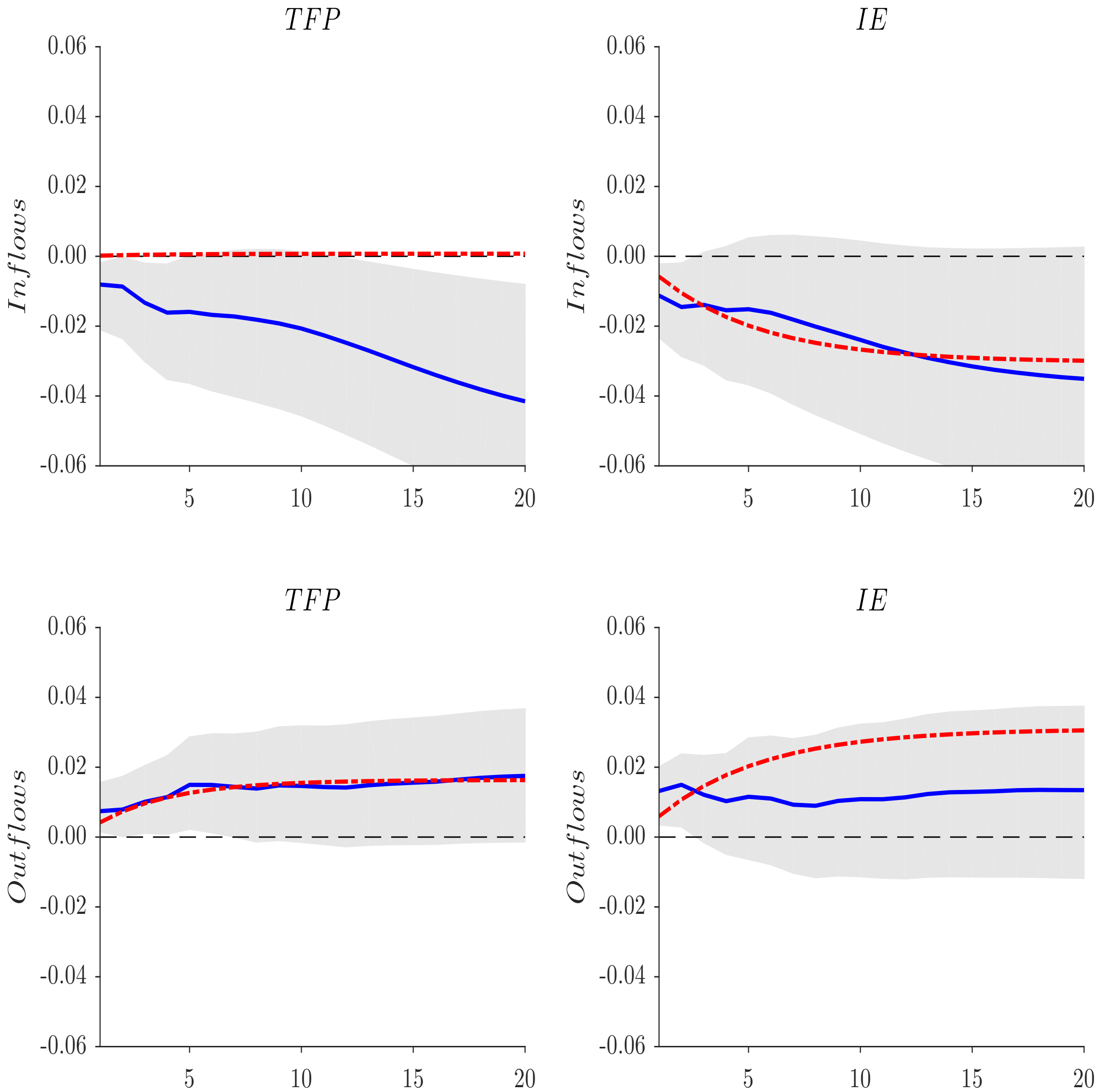
1.5 Comparison of the DSGE and SVAR Results

We now turn to a comparison of the impulse responses of the DSGE model incorporating stochastic volatility shocks to TFP and investment efficiency and the structural VAR with the same set of shocks.

1.5.1 Baseline Sign Restrictions

Figure 1.9 shows the impulse responses from the DSGE model and the SVAR for capital inflows and capital outflows after a one standard deviation shock in domestic volatility. The upper half depicts the cumulated impulse responses of capital inflows to an idiosyncratic TFP and investment efficiency volatility shock in the DSGE model and to shocks in idiosyncratic TFP and investment efficiency volatility in the SVAR with sign restrictions as in Table 1.7. The lower half then shows impulse responses for the same shocks on capital outflows estimated by the DSGE model and the SVAR. The slightly negative impact of a TFP volatility shock on capital inflows generated by the DSGE model is well within the confidence bands of the SVAR similar to the positive impulse response of a TFP volatility shock on capital outflows.

FIGURE 1.9
Impulse Responses Comparison - Baseline Restrictions



Note: Figure 1.9 shows the impulse responses from the DSGE model and the SVAR to a volatility shock in TFP and the investment efficiency on capital inflows in the upper half and capital outflows in the lower half, respectively. The SVAR impulse responses are the median of all draws for the whole sample period using the baseline restrictions and are shown in blue. The 10th and 90th percentile confidence bands of the SVAR are shown as the shaded area. Results for the baseline DSGE model are the dashed-dotted red line. Shown are the deviations of capital flows relative to GDP from the steady state.

1.5.2 Alternative Sign Restrictions

Table 1.9 shows an alternative set of identifying restrictions that allows for both volatility shocks to have either positive or negative impact on capital inflows and capital outflows.[21] Figure 1.10 shows the impulse responses for the alternative set of sign restrictions. A TFP volatility shock in the SVAR again produces a slightly negative median impulse response of capital inflows and a slightly positive median impulse response for capital outflows. In addition, the investment efficiency shock has a small but negative median impact on capital inflows and a small but positive median impact on capital outflows. That implies even without imposing sign restrictions on the impact of a TFP and investment efficiency shock the SVAR is able to produce the expected signs. However, confidence bands are now much larger and well cover a possible positive or negative response of TFP and investment efficiency shocks.

TABLE 1.9
Identifying Restrictions SVAR - Alternative

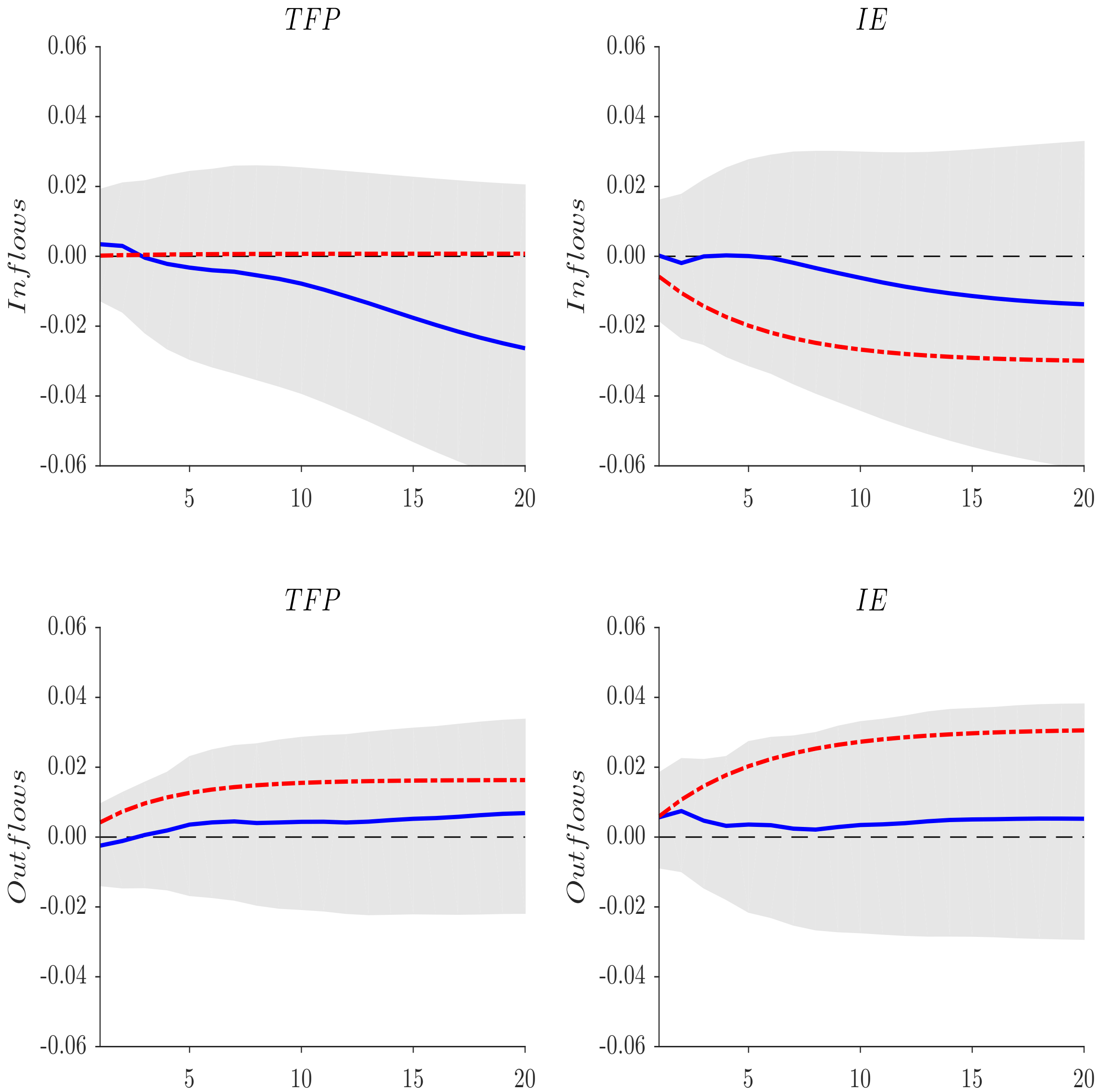
	Short Run Restrictions						
	ϵ^{MP}	ϵ^{VolTFP}	ϵ^{VolIE}	ϵ^{In}	ϵ^{Out}	ϵ^{AD}	ϵ^{Res}
Interest Rate Diff	+					+	
Volatility TFP		+	0	+	+		
Volatility IE		0	+	+	+		
Inflows				+			
Outflows					+		
ΔGDP Diff	-					+	
ΔCPI Diff	-					+	

	Long Run Restrictions						
	ϵ^{MP}	ϵ^{VolTFP}	ϵ^{VolIE}	ϵ^{In}	ϵ^{Out}	ϵ^{AD}	ϵ^{Res}
Interest Rate Diff							
Volatility TFP				0	0		
Volatility IE				0	0		
Inflows							
Outflows							
ΔGDP Diff	0	0	0			0	
ΔCPI Diff							

Note: Table 1.9 shows all the identifying restrictions for the short and the long run in the SVAR. A + (-) indicates a positive (negative) impact on the structural shocks ϵ. Whereas 0 indicates no impact.

[21] Note that the remaining restrictions are still sufficient to correctly identify the TFP and investment efficiency shock.

FIGURE 1.10
Impulse Responses Comparison - Alternative Restrictions



Note: Figure 1.10 shows the impulse responses from the DSGE model and the SVAR to a volatility shock in TFP and the investment efficiency on capital inflows in the upper half and capital outflows in the lower half, respectively. The SVAR impulse responses are the median of all draws for the whole sample period using the alternative restrictions and are shown in blue. The 10th and 90th percentile confidence bands of the SVAR are shown as the shaded area. Results for the baseline DSGE model are the dashed-dotted red line. Shown are the deviations of capital flows relative to GDP from the steady state.

1.6 Conclusion

In this paper we examined the impact of stochastic volatility shocks to TFP and investment efficiency on gross capital flows in an emerging market economy using a DSGE model parameterized to Mexican and US data. The DSGE model with stochastic volatility shows that shocks to the volatility of TFP and the investment efficiency can have a major impact on asset holdings and first-order asset dynamics i.e. gross capital flows between economies. After a stochastic volatility shock in foreign TFP, foreign output and hence foreign dividends become more volatile and hence the volatility of asset returns of the foreign country increases. Since the agents in this model are risk averse, gross capital inflows into the home country increase and capital outflows from the home country decrease in response. To rule out any effect of misparameterization of the model, we used a Monte Carlo prior for the parameters of the model and the stochastic processes. Even after allowing for some variation in the parameters, the model is still able to generate the anticipated responses on gross capital flows.

We then derived identifying restrictions for a structural VAR from the impulse responses of the theoretical DSGE model. The structural VAR for Mexican data was then applied using the sign restrictions from the DSGE model and additional zero long run restrictions. The SVAR confirms the significant negative impact of idiosyncratic volatility shocks in TFP and investment efficiency on capital inflows, and a significant positive impact on cumulated changes in capital outflows in an emerging market economy like Mexico. It is important to note that nothing in the setup of the DSGE model restricts its use to developing countries. The general setup can be applied to any country pair being it emerging market economies or developed countries.

1.A Appendix

1.A.1 Data Sources

TABLE 1.A1
Data Sources

Variable	Series	FRED Identifier
Capital Inflows Part 1	Financial Account: Portfolio Investment Liabilities for Mexico	BPFAPI03MXQ637N
Capital Inflows Part 2	Financial Account: Other Investment Liabilities for Mexico	BPFAOI03MXQ637N
Capital Outflows Part 1	Financial Account: Portfolio Investment Assets for Mexico	BPFAPI02MXQ637N
Capital Outflows Part 2	Financial Account: Other Investment Assets for Mexico	BPFAOI02MXQ637N
Mexico Recession Indicator	OECD based Recession Indicators for Mexico	MEXREC
Mexico GDP	GDP by Expenditure in Constant Prices: Total GDP for Mexico	NAEXKP01MXQ661S
Mexico GDP Growth	GDP by Expenditure in Constant Prices: Total GDP for Mexico	NAEXKP01MXQ657S
Mexico CPI Growth	Consumer Price Index: Total All Items for Mexico	CPALTT01MXQ657N
Mexico Interest Rate	3-Month or 90-Day Rates and Yields: Treasury Securities for Mexico	IR3TTS01MXQ156N
Mexico Consumption Prices	Consumer Price Index: All Items for Mexico	MEXCPIALLQINMEI
Mexico Investment Prices	Price Level of Capital Formation for Mexico	PLICPPMXA670NRUG
Mexico Capital Stock	Capital Stock at Constant National Prices for Mexico	RKNANPMXA666NRUG
Mexico Labor Force	Number of Persons Engaged for Mexico	EMPENGMXA148NRUG
Mexico Hours	Monthly Hours Worked: Manufacturing for Mexico	HOHWMN03MXQ661N
US GDP	Real Gross Domestic Product	GDPC1
US GDP Growth	Real Gross Domestic Product	A191RL1Q225SBEA
US CPI Growth	Consumer Price Index: Total All Items for the United States	CPALTT01USQ657N
US Interest Rate	Effective Federal Funds Rate	FEDFUNDS
US Consumption Prices	Consumption Deflator	CONSDEF
US Investment Prices	Investment Deflator	INVDEF
US Capital Stock	Capital Stock at Constant National Prices for United States	RKNANPUSA666NRUG
US Labor Force	Active Population: Aged 25-54: All Persons for the United States	LFAC25TTUSQ647S
US Hours	Weekly Hours Worked: Manufacturing for the United States	HOHWMN02USM065S

Note: Table 1.A1 shows the used data series and their FRED database identifier.

1.A.2 Zero-Order Asset Portfolios

The zero-order or steady state vector of asset holdings $\tilde{\alpha}$ in the n asset case can be written as

$$\tilde{\alpha} = \left[R_2 \Sigma D_2' R_1' - D_1 R_2 \Sigma R_2'\right]^{-1} R_2 \Sigma D_2' + \mathcal{O}(\epsilon) \tag{1.A1}$$

where Σ is the $k \times k$ covariance matrix of the k exogenous shocks. D_1, D_2, R_1, and R_2 can be obtained from the state space solution. D_1 is in general a scalar containing the first-order decision rule of the wealth shock on consumption differences. D_2 is in general a $1 \times k$ vector containing the first-order decision rules of the k exogenous shocks on consumption differences. Further, R_1 is a $n \times 1$ vector of the first-order decision rules of the wealth shock on the n assets. Finally, R_2 is a $n \times k$ matrix of first-order decision rules of the k exogenous shocks on the n assets. The decision rules can be derived from a first-order approximation of the form

$$y_t = y^s + A y_{t-1}^h + B u_t \tag{1.A2}$$

where y^s is the steady state value of y and $y_t^h = y_t - y^s$. The matrices D_1, D_2, R_1, and R_2 can then be formed of the correct rows and columns of B. It is noteworthy to mention, that as Devereux and Sutherland (2011) describe, a second-order approximation of the underlying approximation equations can be derived using only first-order approximations. This arises because the underlying optimality conditions only contain products, and second-order accurate solutions for products can be obtained from first-order accurate solutions for individual variables. This is, the zero-order asset portfolios can be characterized by a first-order approximation. In general, it can be noted that for $n = 2$ a unique solution exists. For $n > 2$ multiple solutions may exist.

1.A.3 First-Order Asset Dynamics

The first-order dynamics of the asset holdings can be described as

$$\gamma' = -\left(D_1 R_2 \Sigma R_2'\right)^{-1} \left(R_2 \Sigma D_5' + D_2 \Sigma R_5'\right) + \mathcal{O}(\epsilon) \tag{1.A3}$$

where Σ is the $k \times k$ covariance matrix of the k exogenous shocks. D_1, D_2, D_5, R_2, and R_5 can be obtained from the state space solution. D_1 is in general a scalar containing the first-order decision rule of the wealth shock on consumption differences. D_2 is in general a $1 \times k$ vector containing the first-order decision rules of the k exogenous shocks on consumption differences. R_2 is a $n \times k$ matrix of first-order decision rules of the k exogenous shocks on the n assets as in the case of the zero-order asset portfolios. In addition, D_5 is in general a $z \times k$ matrix containing the second-order decision rules of the k exogenous shocks on consumption differences. Where z is the number of predetermined variables in the system. Finally, R_5 is in general a $z \times k$ matrix containing the second-order decision rules of the k exogenous shocks on assets. The decision rules can be derived from a second-order approximation of the form

$$y_t = y^s + \frac{1}{2}\Delta^2 + A y_{t-1}^h + B u_t + \frac{1}{2} C \left(y_{t-1}^h \otimes y_{t-1}^h\right) + \frac{1}{2} D \left(u_t \otimes u_t\right) + E \left(y_{t-1}^h \otimes u_t\right) \tag{1.A4}$$

where again y^s is the steady state value of y and $y_t^h = y_t - y^s$. Further, Δ^2 is the shift effect of the variance of future shocks. The matrices D_1, D_2, and R_2 can then be formed of the correct rows and columns of B in the same way as in the zero-order asset portfolio case. D_5 and R_5 can subsequently be formed of the correct rows and columns of E. Similar to the zero-order asset portfolios, a second-order approximation is all what is required to pin down the first-order accurate behavior of asset dynamics as noted by Devereux and Sutherland (2011).

Chapter 2

Endogenous Time-Varying Volatility and Emerging Market Business Cycles

2.1 Introduction

Time-varying volatility plays a crucial role in understanding business cycles. The literature has so far established a link between high levels of volatility and decreases in output and consumption e.g. as in Fernandez-Villaverde et al. (2011). However, the standard literature considers time-varying volatility to be an exogenously driven process. This is in stark contrast to some empirical observations that argue that volatility itself is caused by changes in macroeconomic variables like output or consumption.[1] In this work we propose a model that endogenizes time-varying volatility which is then able to match emerging market business cycle facts.

We motivate our work by two empirical facts. First, emerging market economies (EME) behave differently than developed economies. Net exports are strongly countercyclical and consumption volatility exceeds output volatility as shown in Table 2.1. Second, EME business cycle data contains a large amount of time-varying volatility. This time-varying volatility is especially present in the debt premium on the interest rate and total factor productivity (TFP). It can also empirically be observed that this time-variation in volatility is stronger for emerging markets than for developed economies.

TABLE 2.1
Empirical Business Cycle Features

	Mexico				Canada			
	1995-1999	2000-2004	2005-2009	2010-2014	1995-1999	2000-2004	2005-2009	2010-2014
$\rho_{Y,NX}$	-0.81	-0.35	0.05	-0.22	-0.03	0.64	0.74	0.76
σ_C/σ_Y	0.82	1.02	1.25	1.51	0.73	0.61	0.70	0.41
$\sigma_{r-r^{US}}$	11.46	2.30	1.72	0.49	1.21	0.91	0.58	0.26

Note: Table 2.1 shows the correlation of output and net exports as well as the relative standard deviation of consumption to output and the standard deviation of the debt premium relative to the US for Mexico and Canada. All data is quarterly from the Federal Reserve Bank of St Louis.

We quantify these observations by constructing a small open economy model with endogenous time-varying volatility in the debt premium on the interest rate and TFP that can explain EME business cycle features like strongly countercyclical net exports and excess volatility in consumption. Characteristics that standard models often fail to replicate. We include in our model a reduced form process where high debt to output levels trigger endogenously time-varying volatility in the debt premium and TFP. In the case of the debt premium this reduced form process can be interpreted as a situation where high levels of debt relative to output decrease the trading volume of the debt which will increase the variability of returns and hence the debt premium. In the case of TFP the reduced form process can be more precisely interpreted as a case where high debt levels increase a firm's probability of default which subsequently causes misallocations in factor inputs. This misallocation hence leads to temporarily higher variability in total factor productivity. In our model the joint occurrence of debt premium and TFP volatility caused by a negative transitory TFP shock that leads to above than steady state debt

[1] See Bachmann et al. (2013) and Ludvigson et al. (2015).

levels reinforce each other and lead to effects comparable to a negative trend shock in TFP. Specifically, a negative TFP shock will decrease output and increase debt which will lead to an above steady state debt to output level. This will increase debt premium volatility and hence higher volatility in the debt price and the amount of debt itself. As TFP volatility is driven by the same fundamental process that depends on the debt to output ratio, higher debt to output will increase volatility in TFP. The increase in TFP volatility will in turn increase volatility in output and hence the volatility of the debt to output ratio. This increase in the volatility of the debt to output ratio caused by both, debt premium and TFP volatility, will lead to a self-reinforcing cycle. The result is that transitory TFP shocks that simultaneously cause endogenous time-varying volatility in the debt premium and TFP can have long lasting effects on output, consumption, and investment similar to a trend shock in TFP. Depending on the degree and persistence of endogenous time-varying volatility we are able to produce countercyclical net exports and excess consumption volatility.

This work is based on two strands of the literature, the emerging market economies business cycle literature and the literature on time-varying volatility in macroeconomic models. Within the EME business cycle literature this work is related to Aguiar and Gopinath (2007) who construct a small open economy model with cycle and trend shocks to TFP and conclude that such a model can fit the characteristics of emerging market economies as well as of developed countries by choosing the correct relative size of cycle and trend shocks. A related work is Boz et al. (2011) who explain emerging market business cycle features with a learning process about cycle and trend TFP shocks. They find that when agents are imperfectly informed about the trend and cycle components of TFP, a learning process using a Kalman filter can greatly improve the performance of a standard real business cycle model to match EME business cycles. Within the EME business cycle literature many authors stress the importance of financial frictions. Among them Neumeyer and Perri (2005) who use a small open economy model to study the effect of interest rates on EME business cycles. They find that exogenous shocks to the level of the interest rate can explain business cycle facts for five EME very well. Boz et al. (2015) use labor market frictions to explain the countercyclical behavior of EME business cycles. Garcia-Cicco et al. (2010) construct a RBC model with financial frictions and level shocks to the debt premium and show that such a model can generate EME business cycle features. It also relates to Chang and Fernandez (2013) who build a model with trend shocks, interest rate shocks, and financial frictions and conclude that financial frictions are the main source of fluctuations in emerging markets. Further Alvarez-Parra et al. (2013) build a small open economy model that includes durable and non-durable goods and shocks to trend TFP and the country risk premium. In line with other papers that stress the importance of financial frictions they find that financial frictions in the form of a countercyclical risk premium are more important than trend shocks. This is because in their model trend shocks would make durable consumption to volatile which therefore imposes an upper limit on the size of the trend shock. Other current work that studies the case of EME business cycles in a small open economy model include Li (2011) who addresses the high wage volatility in emerging markets and Fernandez and Meza (2015) who build a small open economy model with formal and informal labor markets to match the main business cycle moments.

Within the literature on time-varying volatility in macroeconomic models this work is further related to papers like Fernandez-Villaverde et al. (2011) who look at interest rate volatility in a small open economy framework and analyze the effect of interest rate volatility on output, consumption, and investment in emerging market economies. They find that exogenous volatility shocks to the interest rate have negative effects on output, consumption, and investment. However, exogenous volatility shocks to the interest rate cannot explain the countercyclicality of net exports in emerging markets. Justiniano and Primiceri (2008) build a closed economy model with time-varying volatility in TFP and the investment efficiency and find that these two are major sources of macroeconomic fluctuations. Further papers on volatility include Gourio (2012) who introduces time-varying disaster risk into a standard real business cycle model and Christiano et al. (2014) who combine a Christiano et al. (2005) type model with Bernanke et al. (1999) and time-variation to the productivity shock to find that this risk shock can explain a large share of variation in macroeconomic variables. Seoane (2017) further studies the effect of volatility shocks on markups in a small open economy model using an exogenous shock to the level of productivity and volatility shocks to the risk premium. However, all these papers treat volatility as an exogenous process rather than an equilibrium outcome and show that exogenously driven volatility can cause recessions. We will go one step further and argue that TFP driven business cycles will endogenously produce volatility which then can produce EME business cycle facts.

This paper is different from the previous literature on time-varying volatility in that volatility in the debt premium as well as TFP emerges endogenously as the debt to output ratio diverges too much from its steady state. It is therefore close to Saijo (2017) who constructs a closed economy New Keynesian model that endogenizes time-varying volatility through a learning process. In his work endogenous time-varying volatility increases the response of output and other variables to technology and monetary shocks. Our paper however is different from Saijo (2017) as it uses a small open economy model that includes time-varying volatility to the debt premium and TFP. This paper therefore builds on the assumption that high levels of debt relative to output might increase uncertainty about firms' profitability and hence trigger an increase in volatility. It hence features a simple reduced form implementation of countercyclical volatility.[2]

In our model positive deviations of the debt to output ratio from its steady state will trigger higher volatility in the debt premium and TFP faced by a small open economy. Whereas below steady state levels will trigger periods of lower volatility in these variables. Since the debt to output ratio moves slowly, persistent recessions and booms arise endogenously and are similar to a trend shock in TFP. The model contributes to the re-

[2]See Bloom (2014) for empirical evidence on the countercyclicality of volatility.

cent emerging market business cycle literature pioneered by Aguiar and Gopinath (2007). It can generate negative correlations between output and net exports as well as higher volatility in consumption than in output. Characteristic features that are often found in data on emerging market economies. Introducing endogenous time-varying volatility into an interest rate debt premium faced by the small open economy in addition to endogenous time-varying volatility in transitory TFP allows for these countercyclical net exports and excess consumption volatility. In contrast to Aguiar and Gopinath (2007) this countercyclical behavior of net exports even occurs when cycle TFP shocks are more important than trend TFP shocks, or as in our model, when trend shocks are not present at all. In contrast to Neumeyer and Perri (2005) we introduce endogenous time-varying volatility to the debt premium instead of an exogenous shock to the level of the debt premium. The appealing novelty of our model with endogenous time-varying volatility is the fact that we only require one exogenous shock. Namely a level shock to transitory TFP opposed to other papers that require an additional exogenous interest rate shock or a shock to trend TFP to match EME business cycle moments.[3] In addition, by endogenizing the volatility process we address the fact that volatility is not only a source of aggregate dynamics but also a response to it so that time-varying volatility is negatively correlated with the business cycle.

We parameterize the model with standard parameters for a small open economy like Mexico and find that a simple small open economy model augmented for endogenous time-varying volatility is able to match different second moments of the data. By choosing different elasticities for the debt premium and the TFP volatility using SMM we are able to match both, EME and developed economies', business cycle features. By using a Monte Carlo approach we are able to confirm that the countercyclicality of net exports and the excess consumption volatility are indeed caused by different degrees of endogenous time-varying volatility rather than changes in the standard model parameters like e.g. the labor elasticity, the discount factor, or the consumption share in the utility function.

The rest of the paper is organized as follows. The second part provides some empirical evidence of the relationship of debt premium volatility and TFP with the business cycles in an emerging market economy like Mexico and compares it to a developed economy like Canada. The third part presents a real business cycle model with endogenous time-varying volatility in the interest rate debt premium as well as TFP. The fourth part compares the generated second moments of the model with real data from Mexico and analyzes the results. The fifth part will conclude and point to possible future research.

[3] To ease reading we will refer to the TFP level shock simply as a TFP shock. This is the only exogenous shock in our model.

2.2 Some Stylized Facts

It is a well known characteristic of emerging market economies business cycles that net exports are strongly countercyclical and consumption volatility exceeds output volatility.[4] We therefore aim in this section not to show EME data moments, but rather how the debt premium volatility as well as TFP volatility interact with the correlation of net exports and output and with the relative standard deviations of consumption to output.

2.2.1 Data

To establish an empirical relationship between the debt premium and TFP volatility with key features of business cycles in Mexico and Canada we require data on the debt premium as well as on TFP. We further require data on output, consumption, capital, and net exports. For our analysis we choose Mexico as an example for an emerging market economy and Canada as a developed country because both countries can be considered as small open economies with a high degree of trade openness. For this reason Mexico and Canada often stand for the prototypical small open economy countries and are widely used in the literature. The bulk of the data used in the empirical part is quarterly data from the FRED database of the Federal Reserve Bank of St Louis and ranges from the first quarter 1993 to the fourth quarter 2014. In addition, the interest rate data shown in the next section is monthly data from January 1978 to March 2017. Net exports relative to output are constructed of exports minus imports relative to output. TFP data is constructed using a Cobb-Douglas production function in logarithmic terms so that

$$y_t = a_t + (1 - \alpha)\, k_t + \alpha l_t \tag{2.1}$$

where y_t denotes log-output, k_t denotes log-capital and l_t denotes the logarithm of total hours. α denotes the elasticity of labor in the Cobb-Douglas production function which we assume to be the labor share of the economy and set it to 0.68. In this way the productivity term a_t can be easily calculated given capital, output, total labor, and the labor share of the economy.[5] All data series are HP-Filtered as in Hodrick and Prescott (1997) with a filter weight of 1600 for quarterly data to obtain the business cycle component.[6]

The volatility of TFP is then constructed as the moving standard deviation for a time period of $k + 1$ quarters centered around the period t. So that the volatility in period t is the standard deviation of the series from period $t - \frac{k}{2}$ to period $t + \frac{k}{2}$ where $k + 1$ is the window size. We provide different estimates of the results to verify that our results are not significantly driven by the value of the window size $k + 1$. However, it should be

[4]See e.g. Neumeyer and Perri (2005), Aguiar and Gopinath (2007), and Fernandez and Gulan (2015) for some empirical evidence on the differences between EME and developed economies business cycles.

[5]We decide to calculate TFP from the production function as TFP estimates are hardly available for an emerging market economy like Mexico at a quarterly frequency. We use spline interpolation to convert the yearly capital stock data to a quarterly frequency.

[6]For the ease of reading we will refer to the cycle component of TFP simply as TFP. However, in the following data section we always consider the HP-Filtered cycle component of TFP.

noted that the choice of k highlights different aspects of the data i.e. short term versus longer term frequencies.

The debt premium for Mexico and Canada is calculated as the difference of the Mexican and Canadian interest rates and the US interest rate that acts as the world interest rate. Approximating the world interest rate by the US interest rate seems to be justified as both Mexico and Canada have high trade volumes with the US and US monetary policy has strong effects on the world interest rate. The Mexican, Canadian, and US quarterly interest rates are the 90-day rate on Mexican treasury securities and the 90-day rate on Canadian and US interbank rates. The debt premium volatility is then constructed as the moving volatility analogously to TFP volatility. Since interest rate data is available at higher frequency than aggregate macroeconomic data we can also construct the standard deviations of interest rates for every year. For this we use the monthly interest rate on government securities and treasury bills for Mexico, Canada, and the US, respectively. Table 2.A1 in the Appendix 2.A.1 provides an overview about all data sources.

2.2.2 Debt Premium Volatility

Our working hypothesis is that, besides volatility in TFP, volatility in the debt premium plays a crucial role in driving the business cycle. We therefore start by showing some observations regarding the debt premium for Mexico and Canada. Figure 2.1 shows the debt premium for Mexico and Canada in percent relative to the US in the upper panel and the calculated volatility in the middle panel for the period January 1978 to March 2017 using monthly data. The blue line in the middle panel shows the moving volatility in standard deviations and the red asterisks denote the standard deviation of the debt premium on the interest rate for every year. We plot the lower and upper estimates of the moving standard deviation when k, the parameter that governs the window size, is set between 6 and 20 as the shaded area. It turns out that for reasonable values of k the standard deviation of the debt premium moves within a relatively close band. One striking fact is that Mexico as an emerging market economy shows a much higher variability in its interest rate debt premium compared to Canada. The debt premium for Mexico also shows a high degree of time-varying volatility, a key feature in the data that is less pronounced for a developed economy like Canada. Especially during the 1980's and mid 1990's Mexico experienced high levels of debt premium volatility that decreased significantly during the 2000's. The pattern for Canada is similar with high levels of volatility in the 1980's and a decline in volatility from the early to mid 1990's. Similar patterns, with slightly different timings, can be observed in many countries and are generally referred to as the Great Moderation especially when output volatility is concerned.

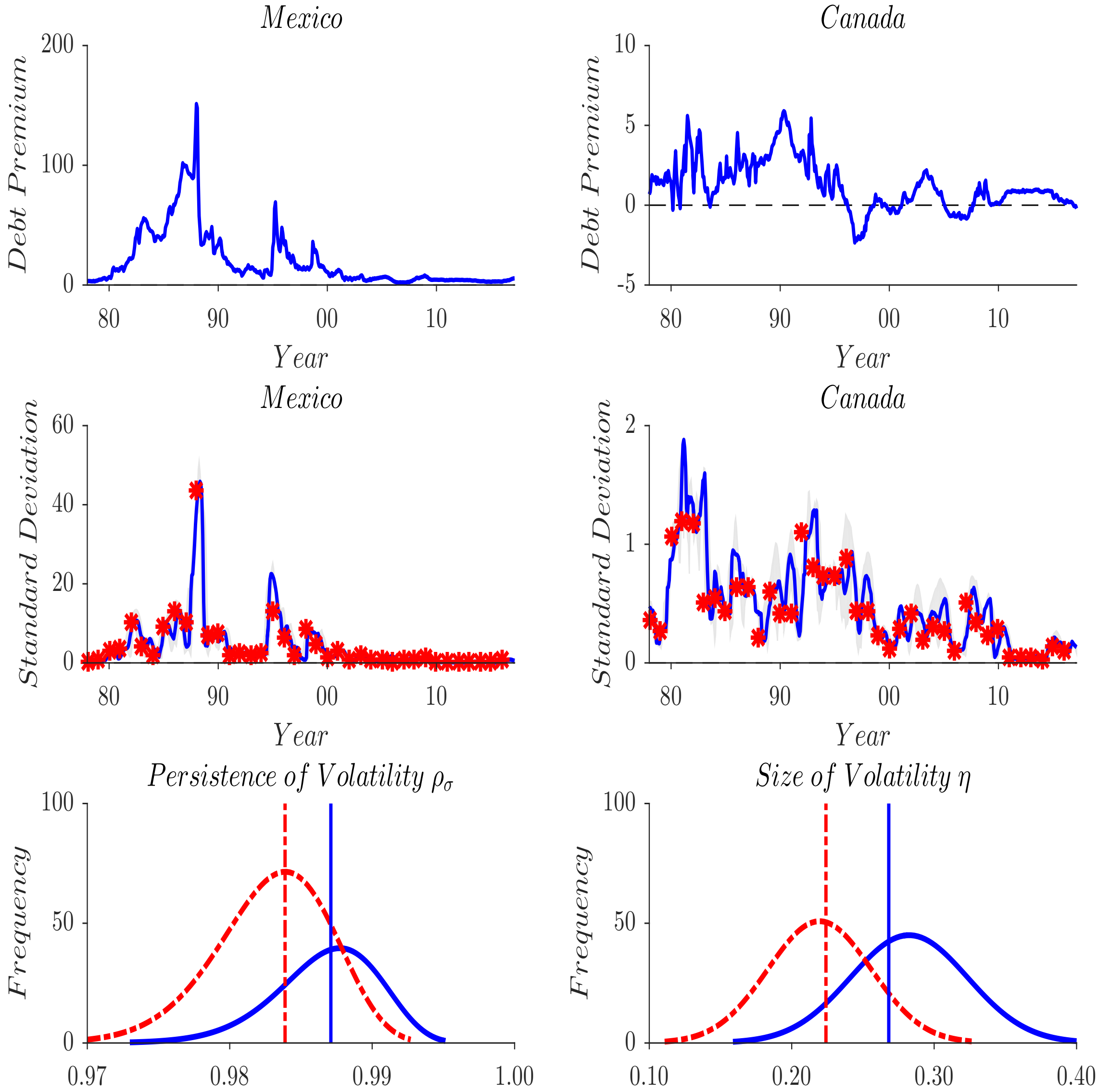
We want to go beyond a pure visual inspection of the data and estimate a stochastic process for the debt premium in both countries for the period January 1978 to March 2017 using monthly data. For this we use the algorithm by Fernandez-Villaverde et al.

(2011).[7] Their algorithm is a particle based Metropolis-Hastings algorithm that allows to estimate the size of stochastic volatility shocks and their persistence. In contrast to a GARCH algorithm their Metropolis-Hastings algorithm allows for a clear distinction between level shocks and volatility shocks. Figure 2.1 shows in the lower panel the fitted probability density functions for the persistence of volatility shocks in the left graph and the size of volatility shocks in the right graph. The estimates for Mexico are shown as the blue line and for Canada as the dashed-dotted red line with the vertical lines indicating the median estimate. The median estimates for the size of stochastic volatility are 0.27 for Mexico and 0.22 for Canada, respectively. Besides a higher persistence and larger size of stochastic volatility shocks, Mexico also faced a higher mean volatility. The estimates of the Bayesian estimation of the debt premium confirm that Mexico experienced larger volatility shocks and that these volatility shocks are more persistent.

These observations hence allow us to conclude that, (1) there is a significant amount of time-variation in the debt premium on interest rates, (2) this time-variation is stronger for a typical emerging market economy like Mexico than for a developed economy like Canada, (3) high periods of volatility seem to coincide with high levels of the debt premium.

[7] We choose the same prior for both countries. We then run 20000 replications of the model with 2000 particles each and discard the first 5000 runs as a burn-in.

FIGURE 2.1
Empirical Debt Premium



Note: Figure 2.1 shows the debt premium for Mexico and Canada relative to the US in the upper panel and the volatility of the debt premium as the moving standard deviation in the middle panel. Asterisks denote the standard deviation of the interest rate debt premium for every year. The lower panel shows the fitted PDF of the Bayesian estimates. Mexico is shown as the blue line and Canada as the dashed-dotted red line. Vertical lines indicate median estimates. All data is monthly.

Table 2.2 shows the contemporaneous correlations for Mexican and Canadian volatility in the debt premium as well as TFP volatility and TFP in levels with the correlation of net exports with output using quarterly data.[8] For this we construct the moving correlation of output and net exports in a similar way as the moving volatility of a variable.

[8]We revert to quarterly data as data on TFP, net exports, output, and consumption is not available at a monthly frequency.

This is the correlation in period t is the correlation of both series from period $t - \frac{k}{2}$ to period $t + \frac{k}{2}$ where $k + 1$ is the window size. The time series of the moving correlation of net exports with output is strongly and negatively correlated with the time series of the moving debt premium volatility for Mexico and Canada. Looking at the correlation of the moving correlation of net exports with TFP in levels the data reveals a low but statistically insignificant correlation for Canada. Further, the debt premium volatility and the volatility of TFP are highly positively correlated for Mexico and slightly negatively for Canada. Whereas the correlation of the debt premium volatility with the TFP level is positive but insignificant for both countries. As k increases it can generally be observed that correlations become stronger hence indicating that correlations between the debt premium volatility and the net export to output ratio become stronger in the long run.

These observations let us conclude that, (1) the correlation of output and net exports is negatively correlated with the debt premium volatility, (2) debt premium volatility and TFP volatility are highly positively correlated in emerging market economies, (3) there is a strong positive correlation of the debt premium volatility and the debt premium in levels for an EME economy but less so for a developed economy.

TABLE 2.2
Empirical Correlations

	Mexico			Canada		
	k			k		
Correlation	12	16	20	12	16	20
Correlation Y/NX - Debt Premium Volatility	**-0.64**	**-0.74**	**-0.85**	**-0.23**	**-0.35**	**-0.47**
Correlation Y/NX - TFP Volatility	**-0.53**	**-0.52**	**-0.66**	**0.29**	**0.36**	**0.36**
Correlation Y/NX - TFP Level	0.00	-0.05	0.03	-0.07	-0.08	-0.03
Debt Premium Volatility - TFP Volatility	**0.80**	**0.86**	**0.87**	**-0.21**	**-0.25**	**-0.31**
Debt Premium Volatility - TFP Level	0.11	0.04	-0.01	0.08	0.04	0.03
Debt Premium Volatility - Debt Premium Level	**0.75**	**0.74**	**0.72**	-0.19	-0.18	-0.18

Note: Table 2.2 shows the contemporaneous correlations of the moving correlation of output and net exports with the debt premium volatility, TFP volatility, and TFP in levels for Mexico and Canada for different values of the window size parameter k. Bold faced values are significant at the 5 percent level. All data is quarterly.

2.3 A Small Open Economy Model

We construct a small open economy model with endogenous time-varying debt premium and TFP volatility to replicate the dynamics of developing economies i.e. negative correlations of output and net exports and a consumption volatility that exceeds output volatility.

2.3.1 Model

The model is a small open economy model as used by Aguiar and Gopinath (2007). However, our model only features one kind of TFP process, namely a transitory process. The model is a model with incomplete asset markets as in Mendoza (1991), Neumeyer and

Perri (2005), and Uribe and Yue (2006). We include into the model endogenous time-varying volatility in the debt premium on the world interest rate as well as in the transitory TFP process that arise as the debt to output ratio increases above its steady state. Agents can invest in physical capital and an internationally traded, one-period, and uncontingent bond.[9] The preferences of the representative household are given by the lifetime utility function

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{\left[C_t^{\gamma} \left(1 - L_t\right)^{1-\gamma} \right]^{1-\sigma}}{1-\sigma} \right] \tag{2.2}$$

where C_t is consumption at period t and L_t is labor supply by the households. σ denotes the risk aversion of the agents. And $\gamma \in (0,1)$ denotes the consumption share in the utility function. Agents discount future utility with the discount factor $\beta \in (0,1)$.

Technology and Capital Accumulation

The production function is of standard Cobb-Douglas form

$$Y_t = e^{z_t} K_t^{1-\alpha} L_t^{\alpha} \tag{2.3}$$

where z_t is a productivity process with transitory effect. K_t denotes the capital stock at time t and L_t denotes labor. Output is denoted by Y_t and $\alpha \in (0,1)$ denotes the elasticity of labor. The law of motion for capital is given by

$$K_{t+1} = (1-\delta) K_t + I_t - \frac{\phi}{2} \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 K_t \tag{2.4}$$

with $\delta \in (0,1)$ being capital depreciation. I_t is investment and ϕ denote a capital adjustment cost parameter to avoid excess volatility in investment.

Budget Constraint and Debt Premium

The budget constraint of the economy is

$$C_t + K_{t+1} = Y_t + (1-\delta) K_t - \frac{\phi}{2} \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 K_t - B_t + q_t B_{t+1} \tag{2.5}$$

where q_t is the price of debt B_t that depends on the debt level relative to output as in Schmitt-Grohe and Uribe (2003)

$$\frac{1}{q_t} = 1 + r_t = 1 + r^* + p_t \tag{2.6}$$

[9] Fernandez-Villaverde et al. (2011) argue that a one-period, uncontingent bond reflects well the limited ability of many emerging market economies to borrow in international financial markets at long horizons.

where p_t is a premium on the time invariant net world interest rate r^* faced by the small open economy.[10] This premium depends on the deviation of the country's debt to output ratio from the steady state and shows time-varying volatility that emerges endogenously

$$p_t = \psi e^{\sigma_{p_t}} \left(e^{\widetilde{B}_t} - 1 \right) \tag{2.7}$$

where $\psi > 0$ is the elasticity of the debt premium and $\widetilde{B}_t$ is the deviation of the debt to output ratio from its steady state value

$$\widetilde{B}_t = \frac{B_t}{Y_t} - \frac{B}{Y}. \tag{2.8}$$

The variable σ_{p_t} in Equation (2.7) follows a first-order autoregressive process that again is driven by deviations in the debt to output ratio from its steady state $\widetilde{B}_t$ and an elasticity parameter η_p that governs the response of σ_{p_t} to deviations of the debt to output ratio from its steady state

$$\sigma_{p_t} = \rho_{\sigma_p} \sigma_{p_{t-1}} + \eta_p \widetilde{B}_t \tag{2.9}$$

so that larger deviations from the steady state have a level effect on the debt premium as well as a volatility effect.[11] By modeling the volatility term in the above way we closely follow Fernandez-Villaverde et al. (2011) when they introduce stochastic volatility to the debt premium. However, we decide to make the volatility depend on the level of debt relative to output. This reduced form process is motivated by the observation that debt premiums are not only countercyclical and increase in the debt to output level but also their volatility is strongly countercyclical. As it is standard in small open economy models with a debt elastic interest rate, agents do neither internalize the effect on the debt premium level nor on the debt premium volatility when choosing the optimal debt level B_t.

Net Exports and Output Growth

We define the ratio of net exports to output as[12]

$$NX_t = \frac{B_t - q_t B_{t+1}}{Y_t} \tag{2.10}$$

where B_t denotes the amount of debt so that higher debt in the next period is associated with negative net exports. The growth rate of output is defined as

$$\Delta Y_t = \log\left(Y_t\right) - \log\left(Y_{t-1}\right). \tag{2.11}$$

[10] The world interest rate is set such that $\beta\left(1 + r^*\right) = 1$.

[11] In a stochastic process this is tantamount to saying that level shocks and volatility shocks are perfectly correlated.

[12] We will call the net exports to output ratio simply as net exports in what follows.

Recursive Problem and Equilibrium

In recursive representation the agent's problem becomes

$$V(K,B,z) = \max_{(C,L,K',B')} \left\{ \frac{\left[C^{\gamma}(1-L)^{1-\gamma}\right]^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}\left[V\left(K',B',z'\right)\right] \right\} \tag{2.12}$$

subject to the budget constraint

$$C + K' = Y + (1-\delta)K - \frac{\phi}{2}\left(\frac{K'}{K} - 1\right)^2 K - B + qB'. \tag{2.13}$$

Given an initial capital stock K_0 and debt level B_0, the equilibrium of the economy is characterized by the first-order conditions of the problem in Equation (2.12), technology in Equation (2.3) and budget constraint in Equation (2.13), and the transversality condition. Where the capital law of motion is given by

$$K' = (1-\delta)K + I - \frac{\phi}{2}\left(\frac{K'}{K} - 1\right)^2 K. \tag{2.14}$$

We provide the full set of equilibrium conditions in the Appendix 2.A.4.

2.3.2 TFP Shocks

The model includes endogenous time-varying volatility in TFP besides the endogenous time-varying volatility in the debt premium. This choice is motivated by two facts. First, time-varying volatility in TFP emerges as a natural extension of Aguiar and Gopinath (2007) and the EME business cycle literature that includes shocks to TFP. Second, exogenous volatility shocks to TFP are known to be a major source of macroeconomic fluctuations as shown by Justiniano and Primiceri (2008), Fernandez-Villaverde et al. (2011), and others. We have previously shown that debt premium volatility and TFP volatility are highly correlated with each other in the data. We implement the time-varying volatility in TFP as an endogenous process since empirical research suggests volatility shocks to have endogenous components as shown by Bachmann et al. (2013) and Ludvigson et al. (2015). We see it as an additional advantage to endogenize the volatility process as this leaves us with a single exogenous shock, a transitory TFP shock as used in the very standard macroeconomic DSGE models.

The time-varying volatility in TFP is similarly structured as in Fernandez-Villaverde et al. (2011). However, the component causing stochastic volatility in the original model is drawn from a random normal distribution. In our paper the driving process is instead assumed to be the deviation of the debt to output ratio from its steady state, which is the same assumption we used for the debt premium process. In this fashion a larger positive deviation of the debt to output ratio from its steady state induces higher volatility.

We choose deviations of the debt to output ratio from its steady state as the driving process for the endogenous time-varying volatility as this allows us as to get a convenient reduced form implementation of countercyclical volatility. Since time-varying volatility in our model is not only driven by output but also by debt, it harmonizes well with the narrative that a higher debt burden induces financial instability. High levels of debt to output increase the uncertainty about the profitability of future investment projects as it increases the probability of firm defaults which subsequently causes misallocations in factor inputs. This hence leads to temporarily higher variability in total factor productivity. Since the debt to output ratio is a slowly moving process, persistent periods of low and high volatility emerge in the model. Once the persistence of the driving process $\widetilde{B}_t$ is high enough and debt premium and TFP volatility are both present, agents will react to such changes in volatility in a similar way as they would react to a permanent TFP shock. The TFP process is then structured as follows

$$z_t = \rho_z z_{t-1} + e^{\sigma_{z_t}} u_{z_t} \tag{2.15}$$

where $\rho_z < 1$ is the persistence of the TFP process and u_{z_t} is a normally distributed random variable with mean zero and variance $\sigma^2_{u_z}$ that can be considered as a shock in transitory TFP

$$u_{z_t} \sim \mathcal{N}\left(0, \sigma^2_{u_z}\right). \tag{2.16}$$

The variable σ_{z_t} is not assumed to be constant but instead follows a first-order autoregressive process so that the volatility part then follows as

$$\sigma_{z_t} = \rho_{\sigma_z} \sigma_{z_{t-1}} + \eta_z \widetilde{B}_t. \tag{2.17}$$

The parameter η_z in Equation (2.17) affects the elasticity of endogenous volatility in TFP with respect to deviations of the debt to output ratio $\widetilde{B}_t$ from its steady state.[13] A high η_z implies a high elasticity of endogenous volatility in the process i.e. volatility reacts strongly to deviations of the debt to output ratio from its steady state. Further ρ_{σ_z} denotes the persistence of the TFP volatility process. By modeling TFP in this way the only driving exogenous shock to the system is a shock in transitory TFP.

2.3.3 Parameters

The parameterization of the main model parameters follows Aguiar and Gopinath (2007) to ensure comparability of the results. Table 2.3 reports all parameters of the model. In our baseline case the labor elasticity α is set to 0.68 and the discount factor β is assumed to be 0.98 to fit quarterly data. The capital depreciation rate δ and the capital adjustment cost

[13] In a stochastic setting η_z would be the degree of stochastic volatility. In an endogenous setting it should rather be called the elasticity of volatility with respect to the deviation of the debt to output ratio from its steady state. We might use both terms interchangeably.

ϕ are set to conventional levels of 0.05 and 4.00, respectively. The risk aversion parameter σ is set to 2.00 in accordance with the literature in international macroeconomics and the value used by Aguiar and Gopinath (2007). The consumption exponent γ is set to 0.36 and the debt premium elasticity ψ is assumed to be 0.001 in line with the values used by Schmitt-Grohe and Uribe (2003) and Neumeyer and Perri (2005). The steady state debt to output ratio B/Y is finally set to 0.10. We further fix the persistence of TFP shocks ρ_z to 0.95 as used by Aguiar and Gopinath (2007) and various other papers.

To verify the results we choose in a second step a Monte Carlo prior for the parameter values from a uniform distribution centered around the baseline value. For the labor elasticity α we choose values between 0.50 and 0.86 and for the discount factor β we choose values between 0.97 and 0.99. The capital depreciation rate δ and the capital adjustment cost ϕ are set between 0.03 and 0.07 and between 2.00 and 6.00, respectively. We allow for some variation in the risk aversion σ by choosing values between 1.50 and 2.50. The consumption exponent γ is set between 0.20 and 0.52. Finally, the steady state debt to output ratio B/Y is set to values between 0 and 0.20, respectively. We fix the debt premium elasticity ψ to the baseline value of 0.001 as this parameter directly influences the size of the debt premium and therefore the effect of the debt premium volatility term on interest rates. Having additional variation in the debt premium elasticity would make it difficult to pin down the effect of the endogenous volatility parameter η_p.

Since the time-varying volatility emerges endogenously in the model, it is not straightforward to estimate the parameters of the volatility process as in Fernandez-Villaverde et al. (2011) who assume normally distributed innovations to the stochastic volatility process. We therefore validate the model for a range of parameter values for the volatility elasticities η_p and η_z. For the persistence parameters ρ_{σ_p} and ρ_{σ_z} we assume values of 0.90 when we do not use them to match the moments of the model.[14]

TABLE 2.3
Model Parameters

Parameter	Description	Baseline	Monte Carlo
α	Labor Elasticity	0.68	$\mathcal{U}\{0.50, 0.86\}$
β	Discount Factor	0.98	$\mathcal{U}\{0.97, 0.99\}$
δ	Capital Depreciation Rate	0.05	$\mathcal{U}\{0.03, 0.07\}$
ϕ	Capital Adjustment Cost	4.00	$\mathcal{U}\{2.00, 6.00\}$
σ	Risk Aversion	2.00	$\mathcal{U}\{1.50, 2.50\}$
γ	Consumption Exponent	0.36	$\mathcal{U}\{0.20, 0.52\}$
ψ	Debt Premium Elasticity	0.001	0.001
B/Y	Steady State Debt to Output	0.10	$\mathcal{U}\{0.00, 0.20\}$
ρ_z	TFP Persistence	0.95	0.95
σ_{u_z}	TFP Shock Size	0.01	0.01
ρ_{σ_p}	Debt Premium Volatility Persistence	0.90	0.90
ρ_{σ_z}	TFP Volatility Persistence	0.90	0.90

Note: Table 2.3 shows the parameter values in the DSGE model for the baseline case and the parameter range of the Monte Carlo prior.

[14]We provide robustness results where we set these two persistence parameters to zero.

2.3.4 Solving the Model

Since we are explicitly interested in the effect of the endogenous volatility terms on macroeconomic dynamics, we solve the model using a third-order approximation to let the endogenous volatility terms have an independent effect from the TFP shocks. It is well known that a first-order approximation would imply certainty equivalence and volatility does have no impact on agents' decision. When using a second-order approximation of the policy function all effects of volatility would only appear through the effect on the TFP shock. Only with a third-order approximation the volatility terms have direct effects on agents' decision making. We provide the analytical solution for the steady state together with a numerical example in the Appendix 2.A.3.

2.4 Results

The fit of the model is now compared to Mexican and Canadian data for different sizes of the endogenous volatility elasticities η_p and η_z. The theoretical benchmark for our analysis is the model without time-varying volatility by Aguiar and Gopinath (2007) that uses larger trend than cycle shocks to generate countercyclical net exports and excess volatility in consumption.

We compare the fit of the different model specifications using the second moments. The simulated data of the model is HP-Filtered as in Hodrick and Prescott (1997) with a filter weight of 1600 for quarterly data. Each model is simulated for 288 periods with the baseline parameters in Table 2.3. We then drop the first 200 observations to get rid of initial conditions so that we are left with 88 observations, the same size as the empirical data. We replicate the model 1000 times with a different sequence of exogenous shocks and report the medians and the 5th to 95th percent confidence bands of the moments.

2.4.1 Simulated Method of Moments Estimates

We are now using a Simulated Method of Moments (SMM) approach to estimate the parameters for the elasticities of the debt premium and TFP volatility, η_p and η_z, by matching the model moments to the empirically observed moments.[15] In a first specification we estimate the elasticities of the debt premium and TFP volatility, η_p and η_z, to match the standard deviation of output and the relative standard deviation of consumption to output. In a second specification we additionally include the standard deviation of the exogenous TFP shock σ_{u_z} that gives us some flexibility to match the standard deviation of output in our model. We further include in this second specification the persistence parameter for the debt premium and TFP volatility process ρ_{σ_p} and ρ_{σ_z} and the capital

[15] We use SMM for two reasons. First, since we are using a third-order approximation of our model, theoretical moments are difficult to calculate. Only recently Andreasen et al. (2018) provide an approach using a pruned state space approximation. Second, we are comparing our model moments to empirically observed moments obtained from a finite sample period. We therefore aim to compare like with like and calculate the model moments from a simulation with the same sample length as its empirical counterpart.

adjustment cost parameter ϕ. In this specification we try to match all ten moments of the model. In both specifications we try to minimize the sum of squared percentage deviations of the model moments from the targeted empirical moments.[16]

Table 2.4 shows the estimated parameter values and the second moments when we use Mexico and Canada as the target countries together with the results of the model by Aguiar and Gopinath (2007) for comparison.[17] For data comparison we use the data moments calculated by Aguiar and Gopinath (2007) as the benchmark value. These are calculated using quarterly data from 1980Q1 to 2003Q1. As Fernandez and Gulan (2015) mention, they cannot find any change in EME moments when data for the last recession is included. We therefore conclude that the data sample calculated by Aguiar and Gopinath (2007) is still representative for many emerging markets. Most informative for our purpose are the relative standard deviations of consumption to output σ_C/σ_Y, the relative standard deviation of net exports to output σ_{NX}/σ_Y, as well as the correlation between output and net exports $\rho_{Y,NX}$ and the correlation between output and consumption $\rho_{Y,C}$. These moments are most informative in our context as especially the negative correlation of output and net exports as well as the excess volatility in consumption are defining features of emerging market business cycles. It is well known that standard business cycle models usually fail to produce countercyclical net exports and excess volatility in consumption. We therefore take these second moments as a natural benchmark to evaluate our model.

By matching the Mexican standard deviation of output and the relative standard deviation of consumption we obtain estimated elasticities of about 0.475 and 0.149 for η_p and η_z, respectively. Matching the Canadian standard deviation of output and the relative standard deviation of consumption to output on the other hand implies elasticities of 0.194 and 0.154 for η_p and η_z. The estimated parameter results when only η_p and η_z are estimated seem to indicate that Mexican business cycles are mainly driven by a higher degree of endogenous time-varying volatility in the debt premium rather than differences in the degree of time-varying TFP volatility.

When we match all moments using the six parameters described above we can substantially improve the fit of the model for Mexico. We obtain parameter estimates of 0.438 and 0.162 for η_p and η_z, respectively. Those estimates are of similar size as when we only target the standard deviation of output and the relative standard deviation of consumption to output. We also estimate the persistence of endogenous volatility as 0.966 and 0.935 for ρ_{σ_p} and ρ_{σ_z}. In addition we estimate the standard deviation of the TFP shock σ_{u_z} as 0.39 percent and the capital adjustment cost ϕ as 0.90. The lower standard deviation

[16] We minimize the sum of squared percentage deviations rather than the sum of squared residuals as our targeted moments are in different units and sizes. The main results stay the same when we minimize the sum of squared residuals.

[17] We show the results of Aguiar and Gopinath (2007) when they estimate the standard deviation of the cycle TFP shock and the standard deviation of the trend TFP shock as parameters as comparison for our first specification. For the second specification we compare our results with the results by Aguiar and Gopinath (2007) when they estimate the standard deviation of cycle and trend shocks, the persistence of cycle and trend shocks, and the growth rate of trend TFP as well as the capital adjustment cost.

of the TFP shocks hence makes up for the higher values of the persistence parameters ρ_{σ_p} and ρ_{σ_z} when compared to the values used in our baseline example of 0.90. Both the estimated standard deviation of TFP shocks as well as the capital adjustment cost are similar in size to what Aguiar and Gopinath (2007) find by matching all moments. Compared to Aguiar and Gopinath (2007) we only have one exogenous shock in the model. We nevertheless require a comparable size of the transitory TFP shock as the TFP shock in our model gets amplified by the presence of the endogenous time-varying volatility.[18] By comparing the sum of squared percentage deviations (SSPD) of the targeted empirical moments from the model moments it turns out that we can slightly increase the fit of the model for Mexico compared to the moments obtained by Aguiar and Gopinath (2007) when they match all moments with six parameters. Turning to the estimates for Canada it turns out that once we target all moments using six parameters we get similar estimates for the elasticities η_p and η_z compared to the values for Mexico. However, we get significantly different estimates for the persistence parameters ρ_{σ_p} and ρ_{σ_z}. The persistence parameters are much lower in the case of Canada than their Mexican counterparts. The estimated capital adjustment cost ϕ is higher than the corresponding value for Mexico as Canada shows much less mean volatility in investment.

[18]We shed more light on this fact in the next section.

TABLE 2.4
Simulated Method of Moments I

	Mexico				
		σ_Y, σ_C/σ_Y Used		All Moments Used	
	Data	AG2007	Model	AG2007	Model
η_p			0.475		0.438
η_z			0.149		0.162
σ_{u_z}					0.0039
ρ_{σ_p}					0.966
ρ_{σ_z}					0.935
ϕ					0.900
σ_Y	2.40	2.40	2.40	2.13	2.11
$\sigma_{\Delta Y}$	1.52	1.73	1.86	1.42	1.74
σ_C/σ_Y	1.26	1.26	1.26	1.10	1.35
σ_I/σ_Y	4.15	2.60	1.89	3.83	2.84
σ_{NX}/σ_Y	0.90	0.71	0.50	0.95	0.75
ρ_Y	0.83	0.78	0.72	0.82	0.75
$\rho_{\Delta Y}$	0.27	0.13	0.03	0.18	0.25
$\rho_{Y,NX}$	-0.75	-0.66	-0.64	-0.50	-0.64
$\rho_{Y,C}$	0.92	0.94	0.93	0.91	0.92
$\rho_{Y,I}$	0.91	0.92	0.97	0.80	0.78
SSPD		0.00	0.00	0.28	0.22

	Canada				
		σ_Y, σ_C/σ_Y Used		All Moments Used	
	Data	AG2007	Model	AG2007	Model
η_p			0.194		0.451
η_z			0.154		0.113
σ_{u_z}					0.0089
ρ_{σ_p}					0.810
ρ_{σ_z}					0.914
ϕ					3.130
σ_Y	1.55	1.55	1.55	1.24	0.95
$\sigma_{\Delta Y}$	0.80	1.14	1.24	0.82	0.73
σ_C/σ_Y	0.74	0.74	0.74	0.76	0.85
σ_I/σ_Y	2.67	1.99	1.96	3.14	2.28
σ_{NX}/σ_Y	0.57	0.41	0.20	0.65	0.42
ρ_Y	0.93	0.75	0.71	0.81	0.72
$\rho_{\Delta Y}$	0.55	0.04	-0.01	0.17	0.02
$\rho_{Y,NX}$	-0.12	0.18	0.02	-0.15	-0.13
$\rho_{Y,C}$	0.87	0.87	0.97	0.87	0.87
$\rho_{Y,I}$	0.74	0.94	0.98	0.82	0.96
SSPD		0.00	0.00	0.66	1.32

Note: Table 2.4 shows the model moments using estimated parameter values obtained from SMM for Mexico and Canada. σ denotes standard deviations of a variable and ρ denotes the correlation between two variables. Moments are the median of 1000 replications of the model. Data moments are as calculated in Aguiar and Gopinath (2007). If not estimated, parameters are as reported in Table 2.3. SSPD denotes the sum of squared percentage deviations of the targeted moments from the data moments.

Table 2.5 shows in addition the second moments when we match all ten moments of the model to the average moments of 13 emerging market economies and 13 developed economies, respectively.[19] It becomes clear that our model does a good job in replicating

[19]Averages are calculated by Aguiar and Gopinath (2007) and are unweighted averages of Argentina,

both EME and developed economies business cycle moments. Not only are we able to produce excess consumption volatility and countercyclical net exports in emerging market economies, we get in addition close to the empirically observed correlation of output and consumption $\rho_{Y,C}$ and output and investment $\rho_{Y,I}$. Our model also does a remarkably good job in creating the right level of persistence in output ρ_Y and output growth $\rho_{\Delta Y}$. However, in both specifications we slightly underestimate the mean volatility of net exports and investment. The SMM parameter estimates again indicate that differences in EME and developed economies are mainly driven by differences in the persistence of volatility rather than differences in the size of the elasticities once we estimate the full set of parameters.

TABLE 2.5
Simulated Method of Moments II

	Emerging Markets		Developed Economies	
	Data	Model	Data	Model
η_p		0.426		0.378
η_z		0.099		0.105
σ_{u_z}		0.0071		0.0116
ρ_{σ_p}		0.958		0.850
ρ_{σ_z}		0.938		0.897
ϕ		0.908		1.375
σ_Y	2.74	2.54	1.34	1.34
$\sigma_{\Delta Y}$	1.87	1.94	0.95	0.97
σ_C/σ_Y	1.45	1.74	0.94	0.95
σ_I/σ_Y	3.91	2.91	3.41	3.36
σ_{NX}/σ_Y	1.18	0.98	0.76	0.63
ρ_Y	0.76	0.78	0.75	0.76
$\rho_{\Delta Y}$	0.23	0.20	0.09	0.09
$\rho_{Y,NX}$	-0.51	-0.48	-0.17	-0.18
$\rho_{Y,C}$	0.72	0.81	0.66	0.74
$\rho_{Y,I}$	0.77	0.64	0.67	0.84
SSPD		0.21		0.11

Note: Table 2.5 shows the model moments using estimated parameter values obtained from SMM for the average of EME and developed economies. σ denotes standard deviations of a variable and ρ denotes the correlation between two variables. Moments are the median of 1000 replications of the model. Data moments are as calculated in Aguiar and Gopinath (2007). If not estimated, parameters are as reported in Table 2.3. SSPD denotes the sum of squared percentage deviations of the targeted moments from the data moments.

2.4.2 Baseline Model

We have shown that we can generate EME business cycle moments for a certain combination of values for the elasticities of the debt premium and TFP η_p and η_z, respectively. We now analyze the behavior of the model when only time-varying debt premium or TFP volatility are present. The benchmark model by Aguiar and Gopinath (2007) is again

Brazil, Ecuador, Israel, South Korea, Malaysia, Mexico, Peru, Philippines, Slovakia, South Africa, Thailand, and Turkey as emerging market economies and Australia, Austria, Belgium, Canada, Denmark, Finland, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, and Switzerland as developed economies.

compared to our model with time-varying volatility in the debt premium p_t and to the model with time-varying volatility in TFP z_t. We then continue and analyze the behavior of the model when time-varying volatility in TFP and the debt premium is jointly present. It turns out that only a model that features both, endogenous time-varying debt premium volatility and time-varying TFP volatility, is able to replicate EME business cycle moments.

The second column in Table 2.6 shows the results by Aguiar and Gopinath (2007) without time-varying volatility. In this model TFP cycle shocks are set to a standard deviation of 0.48 percent and trend shocks to TFP have a standard deviation of 2.81 percent. With such a specification net exports become countercyclical and consumption volatility is higher than output volatility.[20] Model (1) in Table 2.6 presents the results for the small open economy model with a single transitory TFP shock but without time-varying volatility. As mentioned before such a model fails to reproduce the main emerging market business cycle facts.

[20] Note that Aguiar and Gopinath (2007) match the standard deviation of output and the relative standard deviation of consumption to output by construction.

TABLE 2.6
Second Moments Baseline Model

	Data	AG2007	(1) No Volatility	(2) Premium Volatility	(3) TFP Volatility	(4) Premium and TFP Volatility
η_p			0.00	0.60	0.00	0.60
η_z			0.00	0.00	0.15	0.15
σ_Y	2.40	2.40	1.67	4.02	1.67	3.12
	(1.70;3.10)	(1.70;3.10)	(1.31;2.18)	(2.60;8.46)	(0.40;8.38)	(0.86;11.14)
$\sigma_{\Delta Y}$	1.52	1.73	1.31	3.15	1.28	2.45
	(1.02;2.02)	(1.21;2.25)	(1.14;1.49)	(2.15;6.68)	(0.32;6.20)	(0.70;8.68)
σ_C/σ_Y	1.26	1.26	0.49	0.44	0.54	1.49
	(1.10;1.42)	(1.10;1.42)	(0.41;0.56)	(0.18;0.76)	(0.46;0.62)	(1.00;2.70)
σ_I/σ_Y	4.15	2.60	1.88	1.97	1.90	1.85
	(3.57;4.73)	(2.40;2.80)	(1.78;1.99)	(1.71;2.23)	(1.76;2.02)	(1.44;2.27)
σ_{NX}/σ_Y	0.90	0.71	0.23	0.95	0.19	0.65
	(0.72;1.08)	(0.63;0.79)	(0.16;0.32)	(0.72;1.21)	(0.12;0.29)	(0.24;1.84)
ρ_Y	0.83	0.78	0.72	0.74	0.72	0.72
	(0.69;0.97)	(0.76;0.80)	(0.56;0.82)	(0.59;0.84)	(0.51;0.84)	(0.54;0.84)
$\rho_{\Delta Y}$	0.27	0.13	0.00	0.10	0.02	0.04
	(0.09;0.45)	(0.09;0.17)	(-0.18;0.19)	(-0.13;0.34)	(-0.21;0.28)	(-0.18;0.31)
$\rho_{Y,NX}$	-0.75	-0.66	0.91	0.98	0.84	-0.69
	(-0.91;-0.59)	(-0.86;-0.46)	(0.81;0.95)	(0.97;0.99)	(0.50;0.92)	(-0.90;0.05)
$\rho_{Y,C}$	0.92	0.94	1.00	-0.97	1.00	0.92
	(0.88;0.96)	(0.90;0.98)	(0.99;1.00)	(-0.99;-0.89)	(0.97;1.00)	(0.29;0.98)
$\rho_{Y,I}$	0.91	0.92	0.97	0.91	0.97	0.97
	(0.85;0.97)	(0.91;0.93)	(0.95;0.98)	(0.82;0.95)	(0.95;0.98)	(0.71;0.99)
σ_p	0.25		0.02	0.61	0.04	0.61
			(0.01;0.06)	(0.13;2.45)	(0.01;0.23)	(0.13;2.30)
σ_z	3.01		2.40	2.43	2.48	6.85
			(1.59;3.73)	(1.55;3.83)	(0.61;12.77)	(1.91;24.22)
$\rho_{Y,p}$	-0.40		0.05	-0.06	-0.02	0.04
			(-0.11;0.28)	(-0.30;0.12)	(-0.20;0.28)	(-0.21;0.33)
$\rho_{Y,z}$	0.53		0.69	0.70	0.67	0.62
			(0.45;0.89)	(0.42;0.89)	(0.30;0.90)	(0.26;0.87)

Note: Table 2.6 shows the second moments of the Mexican data and the different DSGE models. σ denotes standard deviations of a variable and ρ denotes the correlation between two variables. Moments are the median of 1000 replications of the model. The 5th to 95th percentile confidence bands are in parenthesis. Data moments are as calculated in Aguiar and Gopinath (2007).

Endogenous Debt Premium Volatility

Model (2) in Table 2.6 only contains time-varying debt premium volatility i.e. η_p is set to 0.60 and η_z is set to 0. The standard deviation of TFP shocks is set to 1 percent. A model that only features endogenous volatility to the debt premium is not able to generate countercyclical net exports or excess volatility in consumption. The correlation of net exports with output is clearly positive with 0.98 and the relative volatility of consumption to output is far below unity with 0.44. These observations are in line with the findings by Fernandez-Villaverde et al. (2011) who introduce exogenous stochastic volatility to the interest rate. Most striking in our findings is that endogenous debt premium volatility that is caused by the debt to output ratio induces a strongly countercyclical consumption behavior in the model with a correlation of output and consumption of -0.97. This is a result of the specification of the debt premium process which implies a non-monotonic behavior with respect to the debt to output ratio. After a positive TFP shock the debt to output ratio becomes negative and the debt premium hence turns negative. However, given the functional form, further decreases in the debt to output ratio lead to a convergence back to a zero debt premium. Given the parameter value for the debt premium elasticity ψ this results in a debt premium increase by up to 10 basis points in the long run and therefore negative consumption growth through the Euler equation. The result is then a strongly negative correlation between output and consumption. As a remedy we test an alternative functional form for the debt premium

$$p_t = \psi \left(e^{\sigma_{p_t}} e^{\widetilde{B}_t} - 1 \right). \tag{2.18}$$

Using this functional form the debt premium becomes strictly monotonic and the correlation between output and consumption remains close to unity.[21] Additionally, this functional form is able to generate countercyclical net exports when only endogenous time-varying debt premium volatility is present. However, it is not able to generate the desired excess consumption volatility. We can therefore rule out with certainty that a model with only time-variation in the debt premium volatility is suitable to describe business cycles in EME with both countercyclical net exports and excess consumption volatility.

Endogenous TFP Volatility

Model (3) in Table 2.6 contains only time-variation in the volatility of TFP i.e. η_p is set to 0 and η_z is set to 0.15. Time-varying volatility in TFP is again not able to generate countercyclical net exports and excess consumption volatility. Correlations of output and net exports are lower than in the previous case but still positive with a value of 0.84 and the relative volatility of consumption to output is again below unity with 0.54. However, compared to the model with only endogenous debt premium volatility, consumption is now highly correlated with output as the functional form of the debt premium remains strictly monotonic.

Endogenous Debt Premium and TFP Volatility

Finally, model (4) in Table 2.6 contains endogenous time-varying volatility in the debt premium and TFP simultaneously i.e. η_p is set to 0.60 and η_z is set to 0.15. Both parameter values are therefore set on the upper bound of the SMM estimates for illustrative purpose. Introducing time-varying volatility to the debt premium and TFP simultaneously allows for countercyclical net exports and excess volatility in consumption. Net exports become strongly countercyclical with a correlation of -0.69 and the ratio of consumption to output volatility is above unity with 1.49 which is even higher than the empirical value for Mexico with 1.26. The correlation of output and consumption matches now exactly the value for Mexico with 0.92. Similar to Aguiar and Gopinath (2007) our model underpredicts the autocorrelation of output growth and the relative volatility of investment

[21]See Figure 2.A1 in Appendix 2.A.2 for a graphical representation.

to output and net exports to output. However, in general our values closely match the empirical data and the moments obtained by Aguiar and Gopinath (2007). In addition this specification is able to generate a substantial mean volatility in the debt premium comparable to levels observed in emerging market economies. However the model falls short of delivering a strongly countercyclical debt premium as observed in the data.

Grid Space of Endogenous Debt Premium and TFP Volatility

Figure 2.2 then shows the most important moments for different combinations of η_p and η_z within the range of 0 to 0.80 for η_p and 0 to 0.20 for η_z that govern the elasticity of the endogenous time-varying volatility in the debt premium and TFP with respect to the deviation of the debt to output ratio from its steady state. We classify a model as being able to generate realistic EME business cycle moments when it satisfies certain assumption i.e. when the second moments are within a certain range. For this we calculate the 5th and 95th percentile of second moments reported by Aguiar and Gopinath (2007) for 13 emerging markets. The main features of these empirical bounds are, (1) NX are countercyclical i.e. correlations with output are between -0.82 and 0.10, (2) relative volatility of consumption to output is between 0.66 and 2.34, (3) relative volatility of investment to output is between 2.39 and 7.44, (4) relative volatility of net exports to output is between 0.39 and 3.29 and (5), the standard deviation of output is between 1.30 and 4.25. All conditions together with the value for Mexico are shown in Table 2.7. Moments that satisfy our restrictions are colored blue, whereas those that do not satisfy the restrictions are shown in red. Even after imposing these restrictions it becomes clear that there are combinations of η_p and η_z that can endogenously generate emerging market business cycle characteristics in the generated data and that satisfy all imposed restrictions except for the relative investment to output volatility which is generally too low in our model.

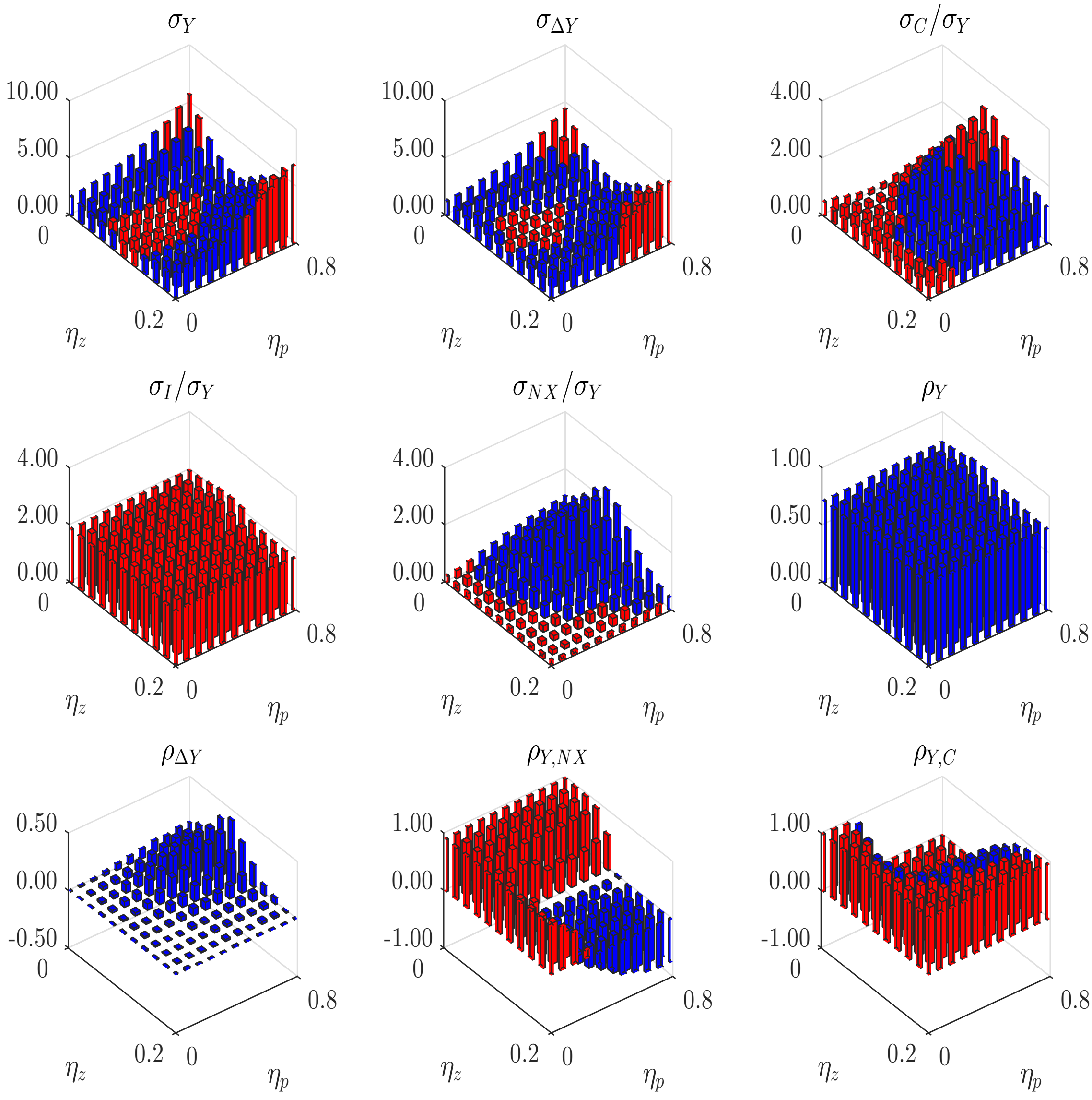
It is striking that neither time-varying volatility in the debt premium nor time-varying volatility in TFP can generate emerging market business cycles alone as shown in Table 2.6 and as visible in Figure 2.2. Only when time-varying volatility in the debt premium governed by η_p and in TFP as governed by η_z are present simultaneously, a negative correlation between output and net exports and excess volatility in consumption arises. The previously observed negative correlation between output and consumption after introducing debt premium volatility vanishes and turns strongly positive when TFP volatility is added to the model.

TABLE 2.7
Empirical Emerging Market Conditions

Moment	Description	Data	Min	Max
σ_Y	Standard Deviation of Output	2.40	1.30	4.25
$\sigma_{\Delta Y}$	Standard Deviation of Output Growth	1.52	0.88	2.96
σ_C/σ_Y	Relative Standard Deviation Consumption to Output	1.26	0.66	2.34
σ_I/σ_Y	Relative Standard Deviation Investment to Output	4.15	2.39	7.44
σ_{NX}/σ_Y	Relative Standard Deviation Net Exports to Output	0.90	0.39	3.29
ρ_Y	Autocorrelation of Output	0.83	0.52	0.89
$\rho_{\Delta Y}$	Autocorrelation of Output Growth	0.27	-0.26	0.60
$\rho_{Y,NX}$	Correlation Output and Net Exports	-0.75	-0.82	0.10
$\rho_{Y,C}$	Correlation Output and Consumption	0.92	0.41	0.92
ρ_{NX_1}	Autocorrelation of Net Exports Lag 1	0.82	0.23	0.88
ρ_{NX_2}	Autocorrelation of Net Exports Lag 2	0.55	-0.38	0.68
ρ_{NX_3}	Autocorrelation of Net Exports Lag 3	0.27	-0.27	0.44
ρ_{NX_4}	Autocorrelation of Net Exports Lag 4	0.04	-0.46	0.23

Note: Table 2.7 shows the conditions imposed on the second moments to classify as a DSGE model that is able to generate EME business cycles. Data moments shown are for Mexico. Min and Max represent the 5th and 95th percentile of the 13 EME moments reported by Aguiar and Gopinath (2007).

FIGURE 2.2
Second Moments Baseline Model



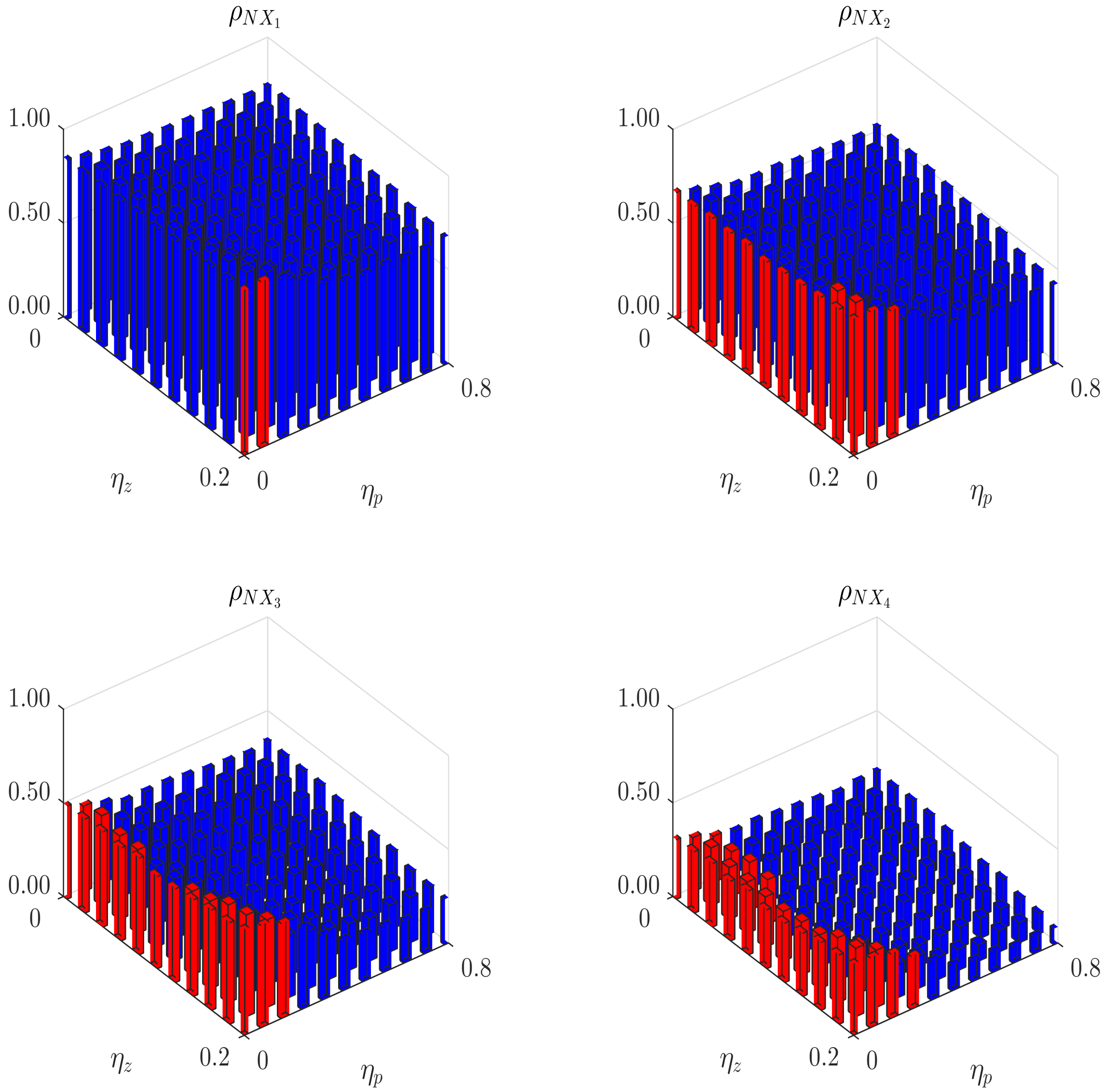
Note: Figure 2.2 shows the moments of the model for the baseline specification of parameters. Blue bars indicate parameter combinations of η_p and η_z that satisfy the characteristics of emerging market data, whereas red bars do not. Moments are the median of 500 replications of the model.

Autocorrelation of Net Exports

Garcia-Cicco et al. (2010) argue that standard real business cycle models produce an autocorrelation for net exports that is flat and close to unity so that net exports essentially follow a random walk. In the data however, autocorrelations are significantly less than unity. Figure 2.3 shows the autocorrelation of net exports for different lags. It becomes clear that our model does a good job in producing autocorrelations of net exports that are significantly below unity and most of the time within the 5th and 95th percentile of the EME data sample by Aguiar and Gopinath (2007) as indicated by the blue bars.[22] However, it becomes also clear that even in the absence of time-varying volatility in the debt premium and TFP the model generates autocorrelations in line with the empirical data. Nevertheless, time-variation in the volatility of the debt premium and TFP will however lower the autocorrelation even further as the actual debt premium increases as it is amplified by the time-varying volatility.

[22]See Table 2.7 for the value of the lower and upper bounds for the autocorrelation of net exports. We calculate these bounds using the original data by Aguiar and Gopinath (2007) for 13 EME countries. After calculating the autocorrelation for each country we construct the 5th and 95th percentile of the autocorrelations across countries.

FIGURE 2.3
Autocorrelation Net Exports Baseline Model



Note: Figure 2.3 shows the autocorrelation of net exports for different lags for the baseline specification of parameters. Blue bars indicate parameter combinations of η_p and η_z that satisfy the characteristics of emerging market data, whereas red bars do not. Moments are the median of 500 replications of the model.

No Persistence in Time-Varying Volatility

To rule out that our main results are driven by strong autoregressive forces in the AR(1) processes in Equation (2.9) and (2.17), i.e. in the persistence parameters ρ_{σ_p} and ρ_{σ_z}, we set these parameters equal to zero and estimate the model again. In this way only contemporaneous deviations of the debt to output ratio from its steady state $\widetilde{B}_t$ have an effect on volatility in the debt premium and TFP. Table 2.8 shows the results when we target the standard deviation output and the relative standard deviation of consumption and output using the two elasticities η_p and η_z. It turns out that we are still able to exactly match our two targeted moments. In addition, we get strongly countercyclical net exports even without targeting this moment explicitly. However, compared with our results when we allow for some persistence in the AR(1) process in Table 2.4 we can observe a slight deterioration in some of the non targeted moments.

TABLE 2.8
Second Moments Baseline Model - Robustness

	Data	AG2007	No Persistence
η_p			4.322
η_z			1.625
σ_Y	2.40	2.40	2.40
	(1.70;3.10)	(1.70;3.10)	(0.63;7.75)
$\sigma_{\Delta Y}$	1.52	1.73	1.91
	(1.02;2.02)	(1.21;2.25)	(0.51;5.79)
σ_C/σ_Y	1.26	1.26	1.26
	(1.10;1.42)	(1.10;1.42)	(0.83;2.13)
σ_I/σ_Y	4.15	2.60	2.01
	(3.57;4.73)	(2.40;2.80)	(1.54;2.48)
σ_{NX}/σ_Y	0.90	0.71	0.51
	(0.72;1.08)	(0.63;0.79)	(0.19;1.41)
ρ_Y	0.83	0.78	0.71
	(0.69;0.97)	(0.76;0.80)	(0.52;0.83)
$\rho_{\Delta Y}$	0.27	0.13	0.00
	(0.09;0.45)	(0.09;0.17)	(-0.22;0.25)
$\rho_{Y,NX}$	-0.75	-0.66	-0.72
	(-0.91;-0.59)	(-0.86;-0.46)	(-0.92;0.29)
$\rho_{Y,C}$	0.92	0.94	0.95
	(0.88;0.96)	(0.90;0.98)	(0.57;0.99)
$\rho_{Y,I}$	0.91	0.92	0.98
	(0.85;0.97)	(0.91;0.93)	(0.68;0.99)
σ_p	0.25		0.32
			(0.09;1.23)
σ_z	3.01		4.86
			(1.45;15.15)
$\rho_{Y,p}$	-0.40		0.06
			(-0.19;0.33)
$\rho_{Y,z}$	0.53		0.65
			(0.32;0.89)

Note: Table 2.8 shows the second moments of the Mexican data and the different DSGE models for robustness. σ denotes standard deviations of a variable and ρ denotes the correlation between two variables. Moments are the median of 1000 replications of the model. The 5th to 95th percentile confidence bands are in parenthesis. Data moments are as calculated in Aguiar and Gopinath (2007).

2.4.3 Monte Carlo Prior

We now want to test how robust our results are to certain variations in the model parameters. For this purpose we use a Monte Carlo prior for the main parameters of the DSGE model as shown in Table 2.3. The simulation of the model with different parameter combinations allows us to verify our findings and to test whether a subset of η_p and η_z exists for each of the draws of the DSGE parameters that can generate EME business cycle features.

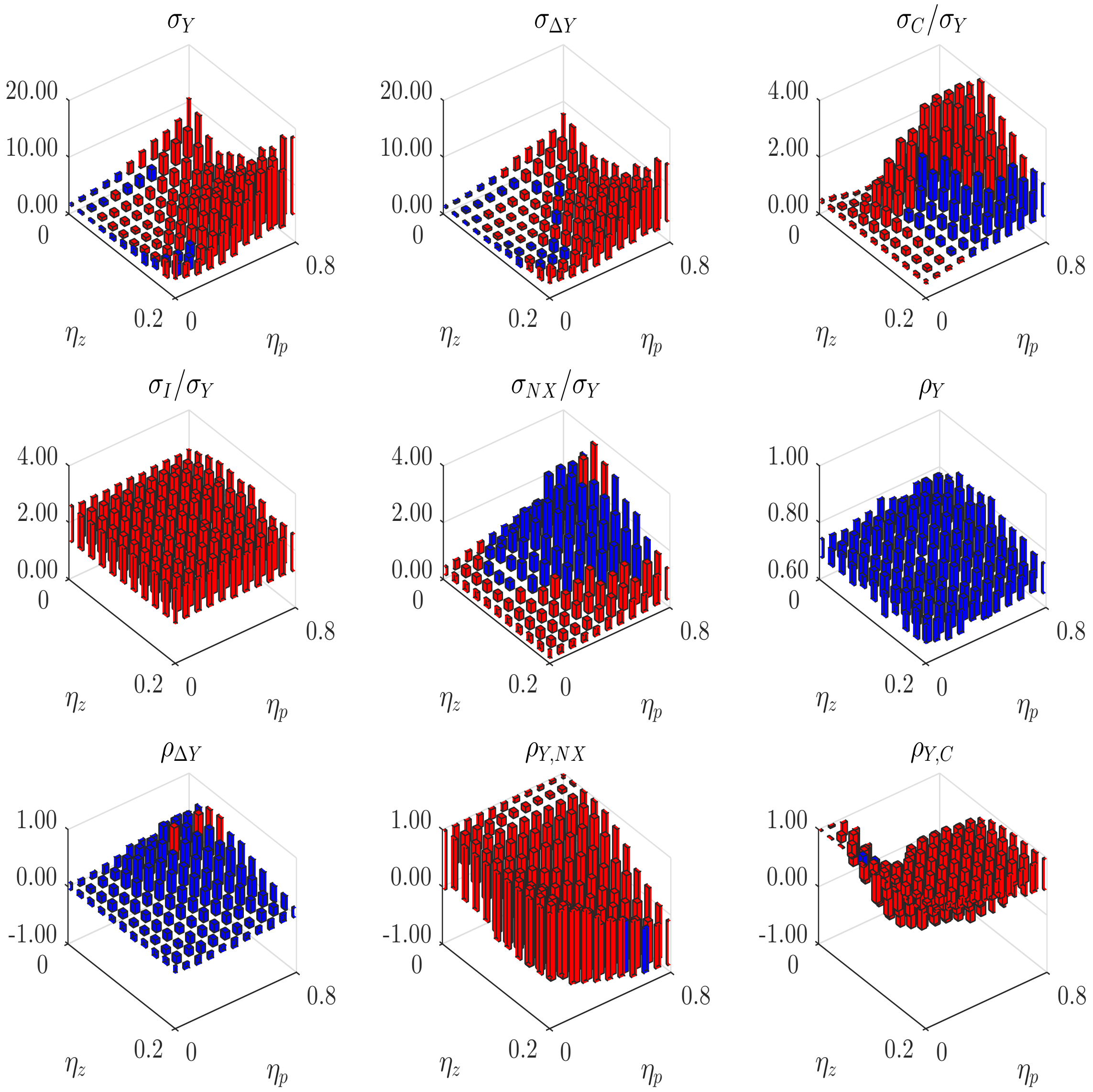
Figures 2.4 shows the moments of the model for the 5th and 95th percentile of the Monte Carlo draws.[23] Although we have chosen a fairly uninformative and wide prior for the model parameters we get relatively tight results i.e. a small distance between the 5th and 95th percentile of the Monte Carlo draws indicated by the height of the bars. Variations in the importance of endogenous time-varying volatility η_p and η_z seem to have a much larger impact in moving the moments of the model than the variation in the model parameters drawn from the uniform distribution. As previously, blue bars indicate parameter combinations for η_p and η_z that satisfy the EME conditions shown in Table 2.7 entirely between the 5th and 95th percentile of the Monte Carlo draws. Whereas red bars do not fully satisfy these restrictions. Especially for the relative standard deviation of consumption to output, the Monte Carlo draws are able to produce excess volatility between the 5th and 95th percentile for a large set of combinations of η_p and η_z. However, we find that for the correlation of output and net exports the set of possible combinations of η_p and η_z shrinks somewhat when we require the 5th to 95th percentile of the Monte Carlo draws to be negative. However, requiring only the 10th to 90th percentile to be negative creates many more parameter combinations that can generate countercyclical net exports. One can clearly observe that the confidence intervals for most moments increase when the time-varying volatility in the debt premium increases as this increases the overall variability in the model.

Figure 2.5 shows the Monte Carlo results for the autocorrelation of net exports. Especially for the first, second, and third lag of the autocorrelation of net exports the model is able to generate results where the 5th and 95th percentile of the Monte Carlo draws are mostly within the range of empirical results obtained from the original data by Aguiar and Gopinath (2007). Here again, similar to the baseline case, introducing time-varying volatility seems to slightly lower the autocorrelation of net exports. However, one has to say that our results are still on the upper range of the empirical observations.

Figure 2.6 finally shows the proportion of Monte Carlo draws that are able to generate EME business cycles for the combinations of η_p and η_z. The maximum frequency occurs for a value of 0.60 for the debt premium volatility parameter η_p and 0.15 for the TFP volatility parameter η_z. With this parameterization about 60 percent of the Monte Carlo draws generate EME business cycles under the set of restrictions in Table 2.7 once we exclude the relative standard deviation of investment to output.

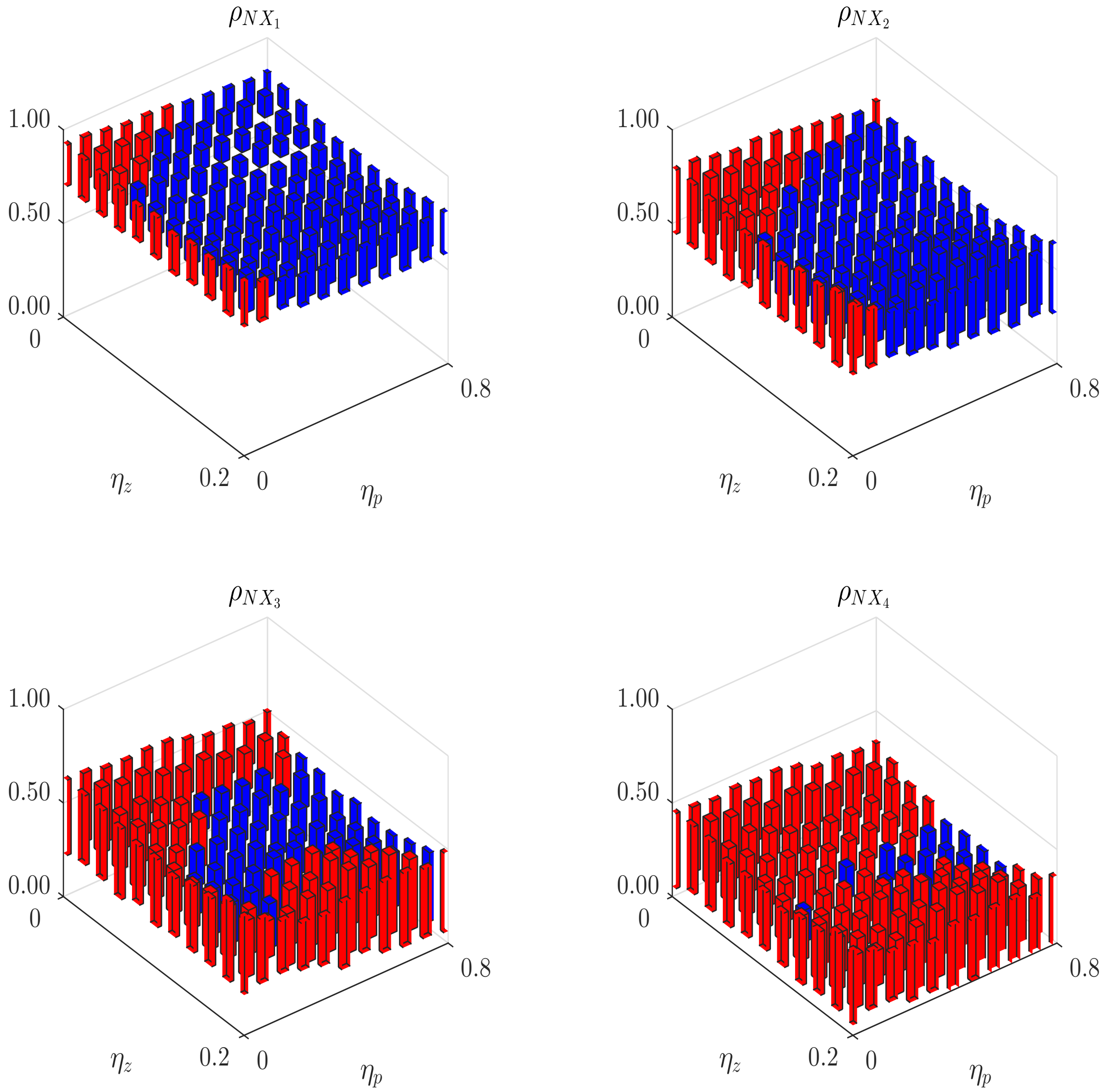
[23]We draw 50 independent parameter combinations for the DSGE parameters in Table 2.3 where each Monte Carlo draw is the median of 25 replications of the model with a different sequence of exogenous shocks.

FIGURE 2.4
Second Moments Monte Carlo Prior



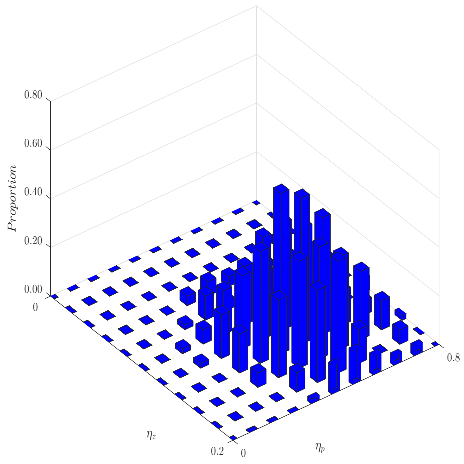
Note: Figure 2.4 shows the moments of the model for different combinations of η_p and η_z for the 5th and 95th percentile of the Monte Carlo draws. Blue bars indicate parameter combinations for η_p and η_z that satisfy EME conditions between the 5th and 95th percentile of the Monte Carlo draws. Whereas red bars do not fully satisfy these restrictions. Each Monte Carlo draw is the median of 25 replications of the model.

FIGURE 2.5
Autocorrelation Net Exports Monte Carlo Prior



Note: Figure 2.5 shows the autocorrelation of net exports for different combinations of η_p and η_z for the 5th and 95th percentile of the Monte Carlo draws. Blue bars indicate parameter combinations for η_p and η_z that satisfy EME conditions between the 5th and 95th percentile of the Monte Carlo draws. Whereas red bars do not fully satisfy these restrictions. Each Monte Carlo draw is the median of 25 replications of the model.

FIGURE 2.6
Relative Frequency with Monte Carlo Prior



Note: Figure 2.6 shows the proportion of Monte Carlo draws that satisfy all EME conditions for different values of the parameters η_p and η_z. Each Monte Carlo draw is the median of 25 replications of the model.

2.4.4 Exogenous Volatility Shocks

We use exogenous volatility processes to show the difference to a case when the volatility process for the debt premium and TFP are endogenous. We replace the endogenous driving process of volatility $\widetilde{B}_t$ for the debt premium and TFP volatility process in Equation (2.9) and (2.17) by exogenous shocks v_{p_t} and v_{z_t}, respectively.[24] The exogenous volatility shocks are drawn from a normally distributed random variable with mean zero and variance $\sigma^2_{v_p}$ and $\sigma^2_{v_z}$ of 1 percent

$$v_{p_t} \sim \mathcal{N}\left(0, \sigma^2_{v_p}\right) \tag{2.19}$$

$$v_{z_t} \sim \mathcal{N}\left(0, \sigma^2_{v_z}\right). \tag{2.20}$$

We control the actual size of the standard deviation of the stochastic volatility process by the parameters η_p and η_z that now control the size of the stochastic volatility in the debt premium and TFP, respectively. We choose different values for η_p and η_z in the range of 1 to 100 so that the actual size of stochastic volatility is comparable to Fernandez-Villaverde et al. (2011).[25] It turns out that once we use exogenous volatility shocks to the debt premium and TFP, the model is not able to generate the key EME business cycle

[24] Note that this setup is still different from Neumeyer and Perri (2005) who use exogenous level shocks to the debt premium.

[25] Fernandez-Villaverde et al. (2011) find values for η between 0.28 for Brazil and 0.46 for Argentina in the case of interest rate volatility when a unit variance is used. This is equivalent to values of η of 28 and 46 when a 1 percent standard deviation is used.

moments anymore. That is the correlation of net exports with output remains positive and the relative volatility of consumption to output stays below unity. If we increase the stochastic volatility beyond the given range the mean volatility of output, consumption, investment, and net exports increases beyond reasonable values without having a sizable effect on the correlation of net exports with output. It is hence crucial to endogenize the volatility processes for the debt premium and TFP in order to get EME business cycle characteristics. This observation is again caused by the fact our endogenous volatility is driven by a slowly moving process, namely the debt to output ratio. Only a slowly moving process can generate persistent increases and decreases in volatility that are then perceived as permanent by risk averse agents. Whenever volatility shocks become exogenous this persistence vanishes and net exports turn procyclical.

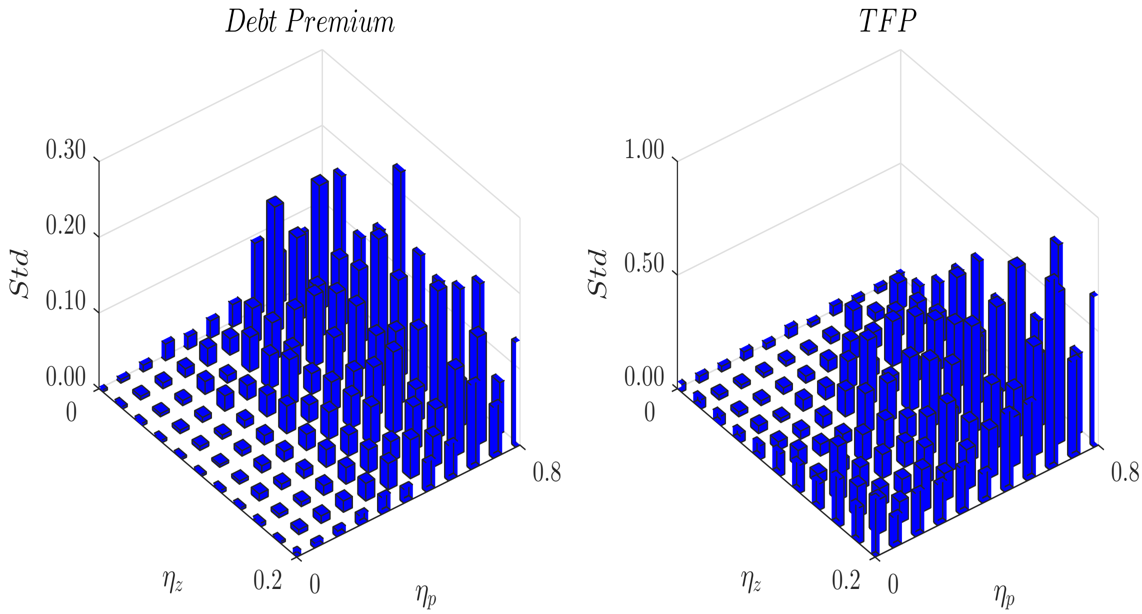
Time-Varying Volatility Increases Exogenous Shocks

Every non-zero value for the elasticity parameter η_z will increase the mean standard deviation of the corresponding TFP shocks u_{z_t} in a setup as in Fernandez-Villaverde et al. (2011) because of the non-linear nature of the exponential function.[26] In a setup with exogenous volatility shocks it is possible to control for these amplifying effects as the standard deviation of the innovations and hence the volatility term is known a priori. However, when volatility is endogenous, as in our work, the variance of the driving process is not known a priori and it is hence not possible to control for these amplifying effects.

Figure 2.7 shows the standard deviations of the debt premium and TFP process after amplification through the volatility process for different combinations of the elasticities η_p and η_z when volatility is endogenous. The standard deviation of the debt premium in the left panel increases strongly in the elasticity of premium volatility η_p and increases only slightly as TFP becomes more volatile when η_z increases. Higher volatility in TFP has only a minor effect on the size of debt premium as the transmission is only indirectly through higher volatility in the production function and hence output. The standard deviation of TFP in the right panel increases naturally in the degree of TFP volatility η_z but also strongly in the degree of debt premium volatility η_p through the endogenous deviation of the debt to output ratio from its steady state. This endogenous increase in the mean volatility of the debt premium and TFP is the main driver of the increasing mean volatility in output, consumption, and investment that can be observed in Figure 2.2. It is however important to stress that the pure increase in the standard deviation of TFP is not able to create countercyclical net exports or excess consumption volatility. The high correlation between standard deviations of the TFP process and the correlation of net exports with output is rather a side effect of the endogenous volatility process that drives both.

[26]For every normal random variable $Z \sim \mathcal{N}(0, \sigma^2)$ the mean of the exponential of Z is equivalent to $E[e^Z] = e^{\frac{\sigma^2}{2}}$.

FIGURE 2.7
Standard Deviation of the Debt Premium and TFP Baseline Model



Note: Figure 2.7 shows the standard deviations of the debt premium and TFP process after amplification through the volatility process for different combinations of η_p and η_z in the baseline model. Moments are the median of 500 replications of the model.

Shock Size Adjusted Exogenous Volatility Shocks

Once the volatility process is exogenous it becomes possible to resize the level shocks so that the level shocks get not augmented by the presence of time-varying volatility. This allows us to increase the stochastic volatility further without reaching too high values of mean volatility. The process for TFP then takes the form

$$z_t = \rho_z z_{t-1} + \Gamma e^{\sigma_{z_t}} u_{z_t} \tag{2.21}$$

where Γ is a time invariant scaling factor and u_{z_t} is a normally distributed random variable with mean zero and variance $\sigma^2_{u_z}$ and can be considered as a shock in levels to TFP

$$u_{z_t} \sim \mathcal{N}\left(0, \sigma^2_{u_z}\right). \tag{2.22}$$

As in the case of endogenous volatility, the variable σ_{z_t} is not assumed to be constant but instead follows an AR(1) process so that the volatility process then follows as

$$\sigma_{z_t} = \rho_{\sigma_z} \sigma_{z_{t-1}} + \eta_z w_{z_t} \tag{2.23}$$

where w_{z_t} is now a normally distributed random variable with mean zero and an a priori known variance $\sigma^2_{w_z}$ that can be regarded as a shock in TFP volatility

$$w_{z_t} \sim \mathcal{N}\left(0, \sigma^2_{w_z}\right). \tag{2.24}$$

By replacing the endogenous volatility process with an exogenous one we can now calculate the variance of the AR(1) process in Equation (2.23) as[27]

$$\Sigma^2_{\sigma_z} = \frac{\eta_z \sigma^2_{w_z}}{1 - \rho^2_{\sigma_z}}. \tag{2.25}$$

This allows us to resize the level shock u_{z_t} in Equation (2.21) accordingly by the factor

$$\Gamma = e^{-\frac{\eta_z \sigma^2_{w_z}}{2\left(1-\rho^2_{\sigma_z}\right)}}. \tag{2.26}$$

The process for the debt premium follows the same pattern. The result is that the mean standard deviation of the level shocks u_{z_t} is not augmented anymore by increases in the volatility parameters η_z and η_p. However, similarly to the case without the resizing of the level shocks, exogenous time-varying volatility is not able to generate EME business cycles. Resizing the level shocks helps to control the mean volatility of output, consumption, investment, and net exports. It does however little to create countercyclical net exports and excess consumption volatility as again the exogenous volatility shocks are not persistent enough.

2.4.5 Transmission Channels

We finally want to show how the endogenous time-varying volatility propagates through the model after an exogenous transitory shock in TFP and how it affects the correlation of output and net exports.

Figure 2.8 shows the transmission channels of endogenous time-varying volatility after a positive transitory shock in TFP and the effect on net exports. We identify four channels through which net exports, and therefore the correlation of net exports with output, might potentially change. One direct effect is shown as the solid line and an additional effect through the level of the debt premium is shown as the dotted line. These two effects are also present in standard models without volatility that feature a debt elastic interest rate as in Schmitt-Grohe and Uribe (2003). The other two channels are the effect through time-varying debt premium volatility shown as the dashed line and TFP volatility shown as the dashed-dotted line.

The direct effect shown as the solid line increases both output and consumption. However as agents perceive the shock as transitory they save in anticipation of lower

[27] We use $\Sigma^2_{\sigma_z}$ to denote the variance of the AR(1) process in Equation (2.23) and $\sigma^2_{w_z}$ to denote the variance of the exogenous error term of the AR(1) process.

future income. Hence the debt decreases and net exports turn positive resulting in procyclical net exports.

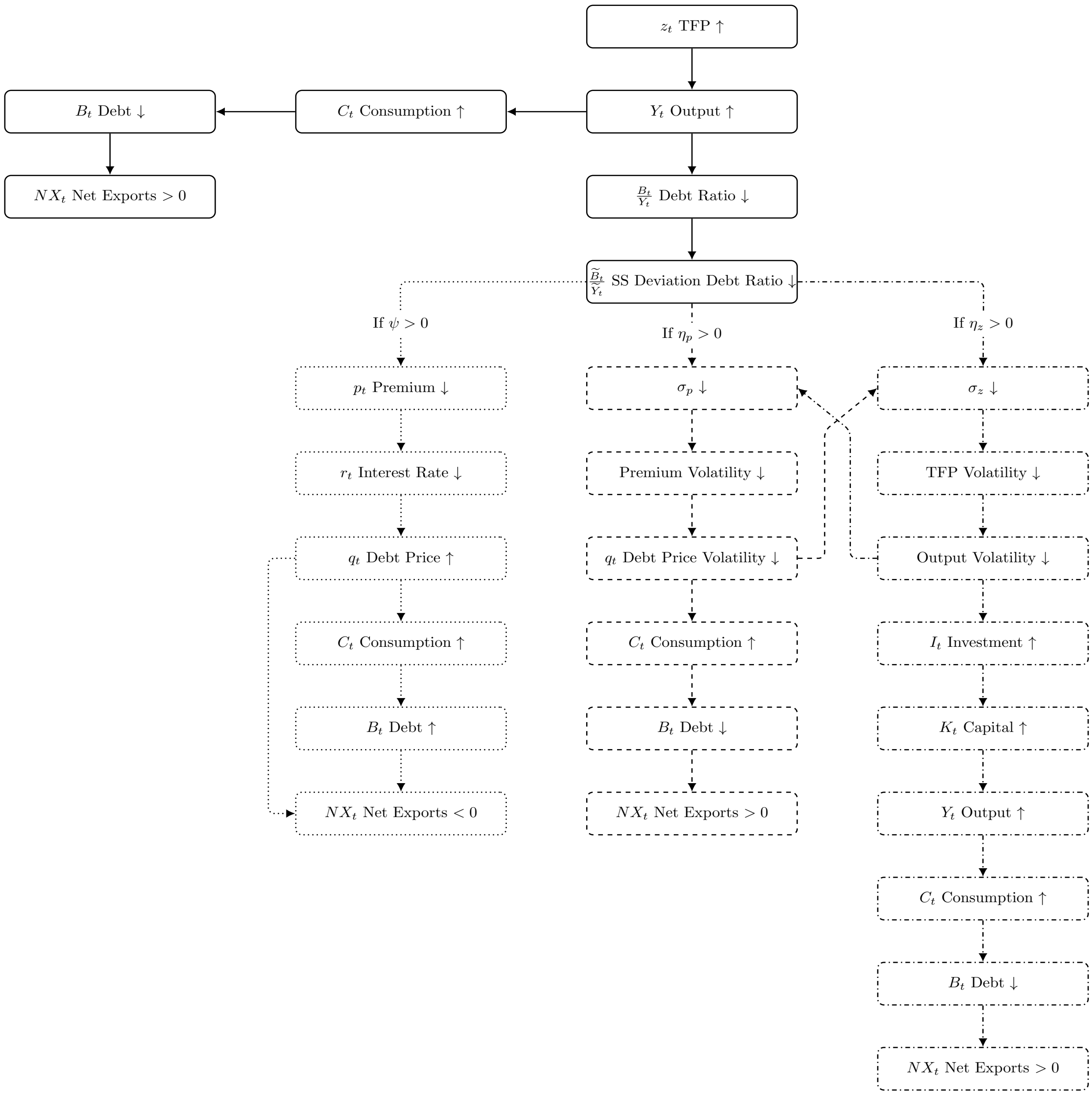
The effect through the level of the debt premium shown as the dotted line reduces the debt premium after a positive TFP shock leading to lower interest rates and higher debt prices. Higher debt prices and larger debt will lead to negative net exports and hence countercyclical net exports. For reasonable values of the debt premium elasticity ψ this channel has a rather limited effect on the overall behavior of net exports. So that standard small open economy models with an endogenous debt premium still have procyclical net exports.

In the debt premium volatility channel shown as the dashed line a positive TFP shock increases output and hence the volatility of the debt premium decreases as the debt to output ratio is below its steady state. Lower debt premium volatility lowers the debt price volatility and increases consumption and lowers the debt. As the change in volatility originates from a transitory shock in TFP, agents perceive the lower volatility to be of temporary nature, therefore leading to positive net exports and hence procyclical net exports.

Finally, in the TFP volatility channel shown as the dashed-dotted line a positive TFP shock reduces the volatility of TFP and hence the volatility of output. This in turn leads to higher investment and a higher capital stock which results in higher output and consumption and lower debt and positive net exports. Again, as the lower volatility originates from a transitory TFP shock, the effects on output are perceived as temporary and agents react by increased savings, or reduced debt, to prepare for future bad times resulting in procyclical net exports.

The debt premium and TFP volatility channels are connected to each other as lower debt price volatility will also reduce TFP volatility and lower output volatility will reduce the debt premium volatility. The following reduced output volatility will then have a dampening effect on the volatility of the deviation of the debt to output ratio from its steady state. This reduction in volatility then leads to a long lasting reduction in the premium volatility which in turn leads to an increase in consumption and previously positive net exports turn negative and thus become countercyclical. The connection between debt premium volatility and TFP volatility is hence leading to the observed effect that debt premium and TFP volatility need to be jointly present in order to generate countercyclical net exports.

FIGURE 2.8
Transmission Channels



Note: Figure 2.8 shows the different transmission channels when endogenous time-varying volatility is present in a model with a single exogenous TFP shock.

2.5 Conclusion

This paper introduced endogenous time-varying volatility into a standard small open economy model with transitory shocks to total factor productivity. A simplified version of the nowadays standard Aguiar and Gopinath (2007) model is augmented for additional time-varying volatility in the debt premium faced by a small open economy and by time-varying volatility in TFP. The time-varying volatility is driven by deviations of the debt to output ratio from its steady state. Introducing endogenous time-varying volatility into the debt premium and TFP can generate business cycle moments that are in line with emerging market economy business cycle data even when trend shocks in TFP are not more important than cycle shocks as required by Aguiar and Gopinath (2007) or in a case where they are not present at all. Our work is therefore in-line with the findings by Garcia-Cicco et al. (2010) who emphasize the importance of financial frictions to generate EME business cycles. However, our paper does not require any exogenous shocks on the debt premium.

By using a simulated method of moments approach we can find parameter values for the elasticities of the volatility process and its persistence that allow us to closely match business cycle moments for Mexico and Canada as well as the average of EME and developed economies. To verify our results we ran a Monte Carlo like prior for the main DSGE model parameters and find that the right combination of endogenous time-varying debt premium and TFP volatility can generate EME business cycle characteristics for most of the Monte Carlo draws. We find that to maximize the frequency of the Monte Carlo draws that satisfy the EME restrictions we impose, the debt premium volatility elasticity needs to be about four times larger than the TFP volatility elasticity.

We have deliberately chosen a rather simple reduced form process to generate endogenously time-variation in the volatility of our model, the deviation of the debt to output ratio from its steady state, which allowed us to focus more on the mechanics of the model. Future work might therefore consider a more sophisticated process to endogenize volatility.

2.A Appendix

2.A.1 Data Sources

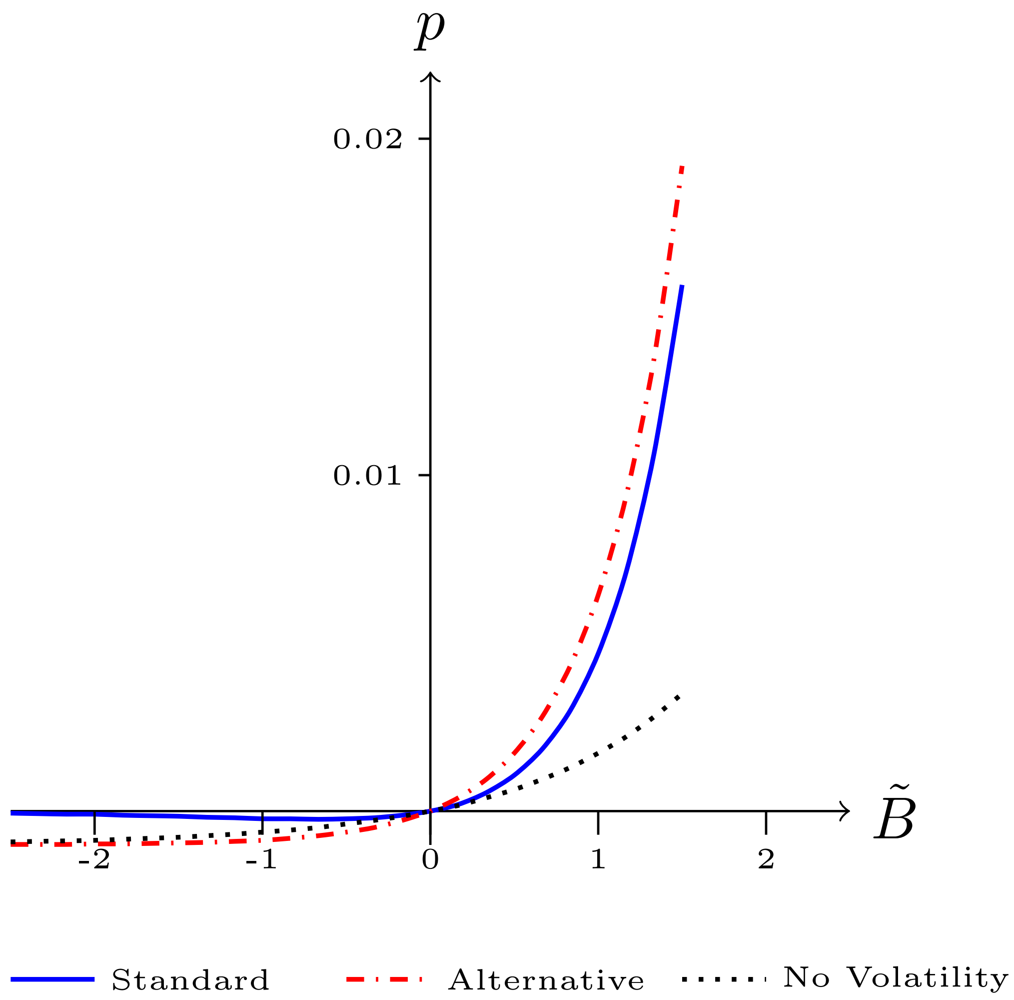
TABLE 2.A1
Data Sources

Variable	Series	FRED Identifier
Mexico GDP	GDP by Expenditure in Constant Prices: Total GDP for Mexico	NAEXKP01MXQ657S
Mexico Consumption	GDP by Expenditure in Constant Prices: Private Final Consumption	NAEXKP02MXQ657S
Mexico Exports	Exports of Goods and Services in Mexico	MEXEXPORTQDSMEI
Mexico Imports	Imports of Goods and Services in Mexico	MEXIMPORTQDSMEI
Mexico Capital	Capital Stock at Constant National Prices for Mexico	RKNANPMXA666NRUG
Mexico Labor	Number of Persons Engaged for Mexico	EMPENGMXA148NRUG
Mexico Hours	Monthly Hours Worked: Manufacturing for Mexico	HOHWMN03MXQ661N
Mexico Exchange Rate	Mexico / U.S. Foreign Exchange Rate	EXMXUS
Mexico Interest Rate 1	Interest Rates, Government Securities, Treasury Bills for Mexico	INTGSTMXM193N
Mexico Interest Rate 2	3-Month or 90-Day Rates and Yields: Treasury Securities for Mexico	IR3TTS01MXQ156N
Canada GDP	GDP by Expenditure in Constant Prices: Total GDP for Canada	NAEXKP01CAQ189S
Canada Consumption	GDP by Expenditure in Constant Prices: Private Final Consumption	NAEXKP02CAQ189S
Canada Exports	Exports of Goods and Services in Canada	CANEXPORTQDSMEI
Canada Imports	Imports of Goods and Services in Canada	CANIMPORTQDSMEI
Canada Capital	Capital Stock at Constant National Prices for Canada	RKNANPCAA666NRUG
Canada Labor	Number of Persons Engaged for Canada	EMPENGCAA148NRUG
Canada Hours	Weekly Hours Worked: Manufacturing for Canada	HOHWMN02CAQ065N
Canada Exchange Rate	Canada / U.S. Foreign Exchange Rate	EXCAUS
Canada Interest Rate 1	Interest Rates, Government Securities, Treasury Bills for Canada	INTGSTCAM193N
Canada Interest Rate 2	3-Month or 90-Day Rates and Yields: Interbank Rates for Canada	IR3TIB01CAQ156N
US Interest Rate 1	Interest Rates, Government Securities, Treasury Bills for United States	INTGSTUSM193N
US Interest Rate 2	3-Month or 90-Day Rates and Yields: Interbank Rates for United States	IR3TIB01USQ156N

Note: Table 2.A1 shows the used data series and their FRED database identifier.

2.A.2 Debt Premium

FIGURE 2.A1
Debt Premium



Note: Figure 2.A1 shows the debt premium for different values of the deviation of the debt to output ratio from its steady state when ρ_{σ_p} is set to zero.

2.A.3 Steady State

Since our model is a standard small open economy model we can provide analytical solutions for the steady state. Since we assumed that $\beta(1+r) = 1$ and $\frac{1}{q} = 1 + r$, as the debt premium is solely dependent on the deviation of the debt to output ratio from its steady state, we know that in the steady state

$$q = \beta \tag{2.A1}$$

and the output to capital ratio can be derived using the fact that the marginal product of capital equals $r + \delta$ as

$$\frac{Y}{K} = \frac{\left(\frac{1}{q} - (1 - \delta)\right)}{(1 - \alpha)}. \tag{2.A2}$$

The steady state consumption to output ratio follows from the budget constraint as

$$\frac{C}{Y} = 1 - \delta\frac{K}{Y} - (1 - q)\,\frac{B}{Y} \tag{2.A3}$$

where $\frac{B}{Y}$ is the exogenous debt to output ratio and $\frac{K}{Y}$ is the inverse of the output to capital ratio in Equation (2.A2). Steady state labor can be derived using the first-order condition for labor in Equation (2.A17) and the marginal utilities for consumption and labor in Equation (2.A12) and (2.A13) as

$$L = \frac{\alpha\gamma}{\frac{C}{Y} - \gamma\frac{C}{Y} + \alpha\gamma} \tag{2.A4}$$

where $\frac{C}{Y}$ is the consumption to output ratio from Equation (2.A3). Capital follows from the Cobb-Douglas production function as

$$K = \left[\frac{L^{\alpha}}{\frac{Y}{K}}\right]^{\frac{1}{\alpha}} \tag{2.A5}$$

where $\frac{Y}{K}$ is the output to capital ratio from Equation (2.A2). Steady state output follows then from the Cobb-Douglas production function as

$$Y = K^{1-\alpha}L^{\alpha}. \tag{2.A6}$$

Consumption follows from the consumption to output ratio in Equation (2.A3) and output in Equation (2.A6) as

$$C = \frac{C}{Y}Y. \tag{2.A7}$$

Steady state investment follows from the capital law of motion as

$$I = \delta K \tag{2.A8}$$

and net exports to output in steady state are defined as

$$NX = \frac{Y - C - I}{Y}. \tag{2.A9}$$

The debt level is derived from the exogenous debt to output ratio $\frac{B}{Y}$ and actual output in Equation (2.A6) as

$$B = \frac{B}{Y} Y. \tag{2.A10}$$

Finally, in steady state the debt premium p and TFP z as well as the debt premium and TFP volatility σ_p and σ_z are all zero. Table 2.A2 provides a numerical example for the steady state using the baseline parameters from Table 2.3.

TABLE 2.A2
Steady States Baseline Model

Variable	Description	Steady State
Y	Output	0.67
C	Consumption	0.52
I	Investment	0.15
K	Capital	3.07
L	Labor	0.33
B	Debt	0.07
NX	Net Exports	0.002
Y/K	Output to Capital Ratio	0.22
C/Y	Consumption to Output Ratio	0.77
I/Y	Investment to Output Ratio	0.227
q	Debt Price	0.98
p	Debt Premium	0.00
z	TFP	0.00
σ_p	Debt Premium Volatility	0.00
σ_z	TFP Volatility	0.00

Note: Table 2.A2 shows the steady states of the model using the baseline parameter values.

2.A.4 Equilibrium Conditions

We here give the full set of equilibrium conditions for the model. Utility is given by

$$U_t = \frac{\left[C_t^{\gamma} \left(1 - L_t\right)^{1-\gamma}\right]^{1-\sigma}}{1 - \sigma} \tag{2.A11}$$

The marginal utility for consumption U_{C_t} follows as

$$U_{C_t} = \frac{\gamma \left(1 - \sigma\right) U_t}{C_t} \tag{2.A12}$$

The marginal disutility for labor U_{L_t} follows as

$$U_{L_t} = -\frac{\left(1 - \gamma\right)\left(1 - \sigma\right) U_t}{\left(1 - L_t\right)} \tag{2.A13}$$

Budget constraint

$$C_t + K_{t+1} = Y_t + (1-\delta) K_t - \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t - B_t + q_t B_{t+1} \tag{2.A14}$$

Capital law of motion including capital adjustment costs

$$K_{t+1} = (1-\delta) K_t + I_t - \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t \tag{2.A15}$$

The first-order condition for capital follows as

$$\begin{aligned} &U_{C_t}\left[1 + \phi\left(\frac{K_{t+1}}{K_t} - 1\right)\right] = \\ &\beta U_{C_{t+1}}\left[(1-\delta) + (1-\alpha)\frac{Y_{t+1}}{K_{t+1}} - \frac{\phi}{2}\left(-2\left(\frac{K_{t+2}}{K_{t+1}} - 1\right)\frac{K_{t+2}}{K_{t+1}} + \left(\frac{K_{t+2}}{K_{t+1}} - 1\right)^2\right)\right] \end{aligned} \tag{2.A16}$$

The first-order condition for labor is

$$-U_{L_t} = U_{C_t}\alpha\frac{Y_t}{L_t} \tag{2.A17}$$

The Euler equation for the debt follows as

$$\frac{U_{C_{t+1}}}{U_{C_t}} = \frac{q_t}{\beta} \tag{2.A18}$$

Production is of standard Cobb-Douglas form

$$Y_t = e^{z_t} K_t^{1-\alpha} L_t^{\alpha} \tag{2.A19}$$

Price of debt

$$\frac{1}{q_t} = 1 + r^* + p_t \tag{2.A20}$$

Debt premium process and the endogenous volatility term are given by

$$p_t = \psi e^{\sigma_{p_t}}\left(e^{\widetilde{B}_t} - 1\right) \tag{2.A21}$$

$$\sigma_{p_t} = \rho_{\sigma_p}\sigma_{p_{t-1}} + \eta_p \widetilde{B}_t \tag{2.A22}$$

TFP process and the endogenous volatility term are given by

$$z_t = \rho_z z_{t-1} + e^{\sigma_{z_t}} u_{z_t} \tag{2.A23}$$

$$\sigma_{z_t} = \rho_{\sigma_z}\sigma_{z_{t-1}} + \eta_z \widetilde{B}_t \tag{2.A24}$$

Deviation of the debt to output ratio from steady state

$$\widetilde{B}_t = \frac{B_t}{Y_t} - \frac{B}{Y} \tag{2.A25}$$

Net exports

$$NX_t = \frac{B_t - q_t B_{t+1}}{Y_t} \tag{2.A26}$$

Output growth is defined as

$$\Delta Y_t = \log\left(Y_t\right) - \log\left(Y_{t-1}\right) \tag{2.A27}$$

Chapter 3

Countercyclical Risk Aversion and International Business Cycles

3.1 Introduction

There is increasing empirical evidence that agents' risk aversion is time-varying and countercyclical. In a recent contribution Cohn et al. (2015) show using a lab experiment with financial professionals that agents become less risk averse in a boom scenario and more risk averse in a bust scenario i.e. risk aversion appears to be countercyclical. In a closely related work Guiso et al. (2018) find countercyclical risk aversion in an experiment with Italian bank customers before and after the financial crises, therefore confirming the findings by Cohn et al. (2015).

At the same time international real business cycle models suffer from various problems or puzzles. Standard international business cycle models predict that (1) The real exchange rate is strongly positively correlated with relative consumption across countries. However, the data shows a correlation of close to zero or even a negative correlation. This anomaly is known in the literature as the Backus-Smith Puzzle.[1] (2) International real business cycle models predict a negative or close to zero correlation of investment and labor across countries. In the data however these correlations are significantly positive, a problem known as the International Comovement Puzzle. (3) Consumption shows a higher correlation across countries than output. In the data however, output is higher correlated than consumption. This problem is known as the Quantity Puzzle.[2] (4) In the model the real exchange rate has a much lower volatility than in the data, a problem widely known as the Price Puzzle.

In this paper we ask the question whether countercyclical risk aversion can help to solve some of these puzzles in international macroeconomics. We do so by introducing time-varying risk aversion into an otherwise standard international real business cycle (RBC) model with incomplete markets. In our reduced form model the risk aversion of agents decreases when output growth turns positive in reaction to a positive transitory total factor productivity (TFP) shock. The subsequent return back to steady state risk aversion increases consumption growth through an additional term in the Euler equation. Since we make both countries' risk aversion dependent on domestic output growth, countercyclical risk aversion alters the relative consumption behavior across countries after a transitory productivity shock. The introduction of a countercyclical risk aversion then leads to a break down of the correlation of the real exchange rate with relative consumption across countries, thus providing a plausible explanation for the Backus-Smith Puzzle. In addition the correlation across countries of investment and labor increases, thus making progress in the International Comovement Puzzle. A positive side effect of our model with countercyclical risk aversion is the additional increase in investment and labor volatility that now matches the empirically observed volatility more closely. The model with countercyclical risk aversion, however, does little to solve the Quantity and Price Puzzles i.e. consumption across countries is still higher correlated than out-

[1] See Backus and Smith (1993) and Kollmann (1996).

[2] See Backus et al. (1994).

put and the real exchange rate is still not volatile enough. We find that our results are robust across different functional forms for the utility function, namely Cobb-Douglas, Greenwood-Hercowitz-Huffman (GHH), and King-Plosser-Rebelo (KPR) preferences.

This papers combines two areas of economic research. The literature on behavioral economics featuring countercyclical risk aversion on the one hand and the literature on international real business cycle models on the other hand. On the side of the behavioral economics literature, this paper is based on the findings by Cohn et al. (2015) who use a lab experiment to show the existence of countercyclical risk aversion. Since the experiment is conducted in a lab setting it allows for a clear identification of the treatment effect and ensures that the results are not invalidated by measurement error or identification issues. Our paper is also based on the closely related findings by Guiso et al. (2018) who find countercyclical risk aversion in an experiment with Italian bank customers before and after the financial crises. They find that bank customers have a higher risk aversion after the financial crisis in 2009 than the same customers before the crisis in 2007. However, unlike Cohn et al. (2015) it is harder for them to disentangle the effect of a change in the risk aversion from other effects, like wealth effects, on the consumers actions.

There is also a growing literature that incorporates a time-varying risk aversion into macroeconomic and finance models. Among them Brandt and Wang (2003) who consider time-varying risk aversion caused by news about output growth and inflation in a consumption based asset pricing model. As Brandt and Wang (2003) point out, time-variation in the relative risk aversion is widespread in consumption based asset pricing models and can usually be found as habit formation to explain the equity premium puzzle as in Campbell and Cochrane (1999).[3] In a different setting Chu et al. (2014) introduce a state dependent risk aversion into a heterogeneous agent model where the risk aversion depends on the wealth of the household. They find that a model with a state dependent risk aversion predicts a larger wealth inequality than a model with a constant risk aversion. Finally, Benchimol (2014) introduces risk aversion shocks to a New Keynesian model and finds that risk aversion shocks increase inflation and decrease output.

From the perspective of international RBC models our paper is closely related to Mandelman et al. (2011) who introduce investment specific technology (IST) shocks to a two-country model and can break the positive correlation between the real exchange rate and relative consumption. However, after calibrating the model to the US and the rest of the world IST shocks turn out to be not strong enough. When it comes to explain the Backus-Smith Puzzle, our paper is also closely related to Stockman and Tesar (1995) and Heathcote and Perri (2013) who use taste shocks as an explanation. As an alternative explanation for the Backus-Smith Puzzle, Corsetti et al. (2008b) use non-traded goods that can break the correlation between the real exchange rate and relative consumption. By introducing non-traded goods they are also able to address the Price Puzzle of low real exchange rate volatility.

[3] With preferences of the form $U(C_t) = \frac{(C_t - \tau C_{t-1})^{1-\sigma}}{1-\sigma}$ the relative risk aversion becomes $RRA_t = \sigma \frac{C_t}{C_t - \tau C_{t-1}}$ so that the relative risk aversion depends on the habit stock.

Karabarbounis (2014) uses home production to solve for the Backus-Smith, International Comovement, and Quantity Puzzle. Bai and Rios-Rull (2015) introduce goods market frictions in a model with only demand shocks and are subsequently able to solve the Backus-Smith Puzzle. In addition they successfully address the Quantity Puzzle so that output is more correlated across countries than consumption. Dogan (2019) uses IST shocks originating in the US to explain Mexican business cycles. She finds that due to the high exchange rate volatility of Mexico, investment specific technology shocks in the US get transmitted to Mexico more strongly and are hence sufficient to explain the Backus-Smith and Quantity Puzzle.

Our paper is different from the previous international RBC literature in that we assume that the risk aversion is explicitly time-varying and countercyclical. Our way of introducing a time-varying risk aversion can therefore be seen as a reduced form to implement the empirical observations of countercyclical risk aversion into a standard macroeconomic RBC model. In this way we can introduce the behavioral finding of countercyclical risk aversion into a standard open economy model that does not require investment specific technology shocks like in Mandelman et al. (2011) or taste shocks like in Stockman and Tesar (1995) and Heathcote and Perri (2013) to match the empirically observed moments. We therefore overcome the potential problem of IST shocks not being strong enough or the shortcoming of taste shocks as being hard to empirically measure. Our paper is also different from papers like Benchimol (2014) in the sense that we assume an endogenous process for the risk aversion instead of an exogenous shock. In contrast to papers that look at the effect of a time-varying risk aversion in a consumption based asset pricing model like Brandt and Wang (2003) we look at the implications for international business cycles.

The rest of this paper is organized as follows. The second part will present an international real business cycle model with countercyclical risk aversion and only transitory TFP shocks. The third part will present the results and the fourth part will conclude and provide an outlook for possible future research.

3.2 Model

The model is a standard two-country open economy incomplete markets model similar to Heathcote and Perri (2002) or Mandelman et al. (2011). The standard international business cycle model is only augmented to include a time-varying countercyclical risk aversion process that depends on final output growth. For simplicity we will show the problem for the home economy, the foreign economy faces exactly the same problem. We follow the convention and indicate foreign variables by a *.

3.2.1 Households

Representative households maximize life-time utility

$$\max_{C_t, L_t, I_t, K_t, D_t} \qquad \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t, \sigma_t\right) \right] \tag{3.1}$$

where $\beta \in (0,1)$ is the discount factor of the household and $U\left(C_t, L_t, \sigma_t\right)$ is the period utility function, with C_t being consumption and L_t being labor at time t. Finally, σ_t is the time-varying risk aversion of the households that takes the following functional form

$$\sigma_t = \sigma + e^{\gamma_t} - 1 \tag{3.2}$$

where $\sigma \geq 2$ is the steady state risk aversion and γ_t follows a first-order autoregressive process of the form

$$\gamma_t = \rho_\sigma \gamma_{t-1} - \eta_\sigma \Delta Y_t \tag{3.3}$$

where ρ_σ is the persistence and η_σ is the elasticity of the risk aversion AR(1) process with respect to final output growth ΔY_t. This functional form of the time-varying risk aversion admits a steady state value for the risk aversion of σ and a lower bound of $\sigma - 1$. By only considering the case of $\sigma \geq 2$ we can avoid the case of logarithmic utility arising when $\sigma_t = 1$. This functional form using final output growth as the driving source of changes in risk aversion is motivated by the findings of Cohn et al. (2015) that risk aversion is time-varying and countercyclical. We explicitly allow for some persistence in the AR(1) process as it appears to be reasonable that agents, when forming their risk aversion, not only consider contemporaneous output growth but most likely also include past values in their decision making.

The capital law of motion for each country is

$$K_t = (1-\delta) K_{t-1} + I_t - \frac{\phi}{2} I_{t-1} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \tag{3.4}$$

where K_t denotes capital, I_t denotes investment, and $\delta \in (0,1)$ is the capital depreciation rate. $\phi > 0$ denotes further a capital adjustment cost to control the speed of investment. Households have the following budget constraint

$$P_t C_t + P_t I_t + P_{H,t} Q_t D_t \leq P_t W_t L_t + P_t R_t K_{t-1} + P_{H,t} D_{t-1} - P_{H,t} \frac{\zeta}{2} D_t^2 \tag{3.5}$$

where Q_t is the price of the bond D_t. W_t is the wage rate and R_t is the return on capital where both are defined in terms of the final good. Further P_t denotes the CES price aggregate of the home final good as in Equation (3.A19) and $P_{H,t}$ denotes the price of the home intermediate good, respectively. The last term induces a convex cost of holding bonds which is an increasing function in the bond level to ensure stationarity of the bond level D_t where $\zeta > 0$ is the elasticity of the cost of holding bonds.

3.2.2 Firms

Final output Y_t is a composite good consisting of the home country and the foreign country intermediate goods produced by the final good producer

$$Y_t = \left[\omega^{\frac{1}{\theta}} Y_{H,t}^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} Y_{F,t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{3.6}$$

where $Y_{H,t}$ is the home produced intermediate good, $Y_{F,t}$ is the foreign produced intermediate good, and $\omega \in (0,1)$ is the share of home intermediate goods in total output. Finally, $\theta > 0$ is the elasticity of substitution between home and foreign intermediate goods. The final goods producer in each country then solves the maximization problem

$$\max_{Y_t \geq 0, Y_{H,t} \geq 0, Y_{F,t} \geq 0} \qquad P_t Y_t - P_{H,t} Y_{H,t} - P_{F,t} Y_{F,t} \tag{3.7}$$

subject to the above production function in Equation (3.6). The intermediate goods producer maximizes profits

$$\max_{L_t \geq 0, K_{t-1} \geq 0} \qquad P_{H,t} Y_{H,t} + P_{H,t} Y_{H,t}^* - P_t W_t L_t - P_t R_t K_{t-1} \tag{3.8}$$

subject to the production function

$$Y_{H,t} + Y_{H,t}^* = e^{A_t} K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{3.9}$$

where A_t is the home country's productivity and $\alpha \in (0,1)$ is the elasticity of intermediate goods with respect to capital.

3.2.3 Productivity Shocks

The productivity processes in the two countries follow a VAR(1) process so that for the home country

$$A_t = \rho_A A_{t-1} + \rho_A^* A_{t-1}^* + u_t \tag{3.10}$$

and similarly in the foreign country

$$A_t^* = \rho_A A_{t-1}^* + \rho_A^* A_{t-1} + u_t^* \tag{3.11}$$

where ρ_A and ρ_A^* denote the persistence of the TFP process and u_t and u_t^* are the exogenous TFP shocks following

$$u_t \sim \mathcal{N}\left(0, \sigma_u^2\right) \tag{3.12}$$

$$u_t^* \sim \mathcal{N}\left(0, \sigma_{u^*}^2\right) \tag{3.13}$$

with mean zero and a known variance σ_u^2 and $\sigma_{u^*}^2$. In this way increases in productivity in one country can slowly spill over to the other country. We only consider transitory TFP shocks in our model as they are sufficient to generate the desired dynamics in the model. However, we verified the results of the model by including permanent shocks to TFP without a change in the general results of our model.

3.2.4 Market Clearing

Market clearing in each country implies that the final good Y_t in each country is either consumed or used for investment

$$C_t + I_t = Y_t \tag{3.14}$$

and since bonds are in zero net supply we require that

$$D_t + D_t^* = 0. \tag{3.15}$$

3.2.5 Preferences

We consider different utility functions for our two economies to analyze the effect of countercyclical risk aversion on business cycles. We start with a standard Cobb-Douglas form and then present Greenwood-Hercowitz-Huffman (GHH) and King-Plosser-Rebelo (KPR) preferences as two extreme cases of the Jaimovich and Rebelo preferences as in Jaimovich and Rebelo (2009). The form of the utility function and hence the marginal utility of consumption and labor affected by the time-varying risk aversion will enter the model in three ways. The first is via the labor supply decision as shown in Equation (3.A21). In this case the time-varying risk aversion parameter σ_t simply drops out and does not affect the labor supply directly. Nevertheless, in the case of Cobb-Douglas and KPR preferences where the labor supply depends on consumption, we can observe an increase in the labor volatility when countercyclical risk aversion is present. The second case is the effect of different marginal utilities of consumption in the consumption Euler equation as shown in Equation (3.A30). We will show later by log-linearizing the consumption Euler equations that, although all of our three preferences are of constant relative risk aversion (CRRA) form when σ_t becomes a constant, they clearly differ in the generated consumption growth when σ_t becomes time-variant.[4] That will be in addition to the already different expressions for consumption growth when the risk aversion is constant. The third effect is via the risk sharing equation as shown in Equation (3.A29).

[4]Note that the measure of relative risk aversion $RRA = -\frac{U''(C_t,L_t,\sigma_t)}{U'(C_t,L_t,\sigma_t)} F\left(C_t L_t\right)$, where $F\left(C_t, L_t\right)$ denotes a function that combines consumption and labor in the CRRA function case as $U\left(C_t, L_t, \sigma_t\right) = \frac{F(C_t,L_t)^{1-\sigma_t}}{1-\sigma_t}$, becomes in all our cases $RRA = \sigma_t$.

Cobb-Douglas

As our benchmark we consider Cobb-Douglas preferences of the form

$$U(C_t, L_t, \sigma_t) = \frac{\left[C_t^{\tau}(1-L_t)^{1-\tau}\right]^{1-\sigma_t}}{1-\sigma_t} \tag{3.16}$$

where C_t is consumption and L_t is labor. $\sigma_t > 1$ is the time-varying risk aversion from Equation (3.2) and $\tau \in (0,1)$ is the consumption share in the utility function. Cobb-Douglas preferences give the following marginal utilities for consumption

$$U_{t,C} = \frac{\tau}{C_t}\left[C_t^{\tau}(1-L_t)^{1-\tau}\right]^{1-\sigma_t} \tag{3.17}$$

and for labor

$$U_{t,L} = -\frac{(1-\tau)}{(1-L_t)}\left[C_t^{\tau}(1-L_t)^{1-\tau}\right]^{1-\sigma_t} \tag{3.18}$$

and hence labor supply becomes

$$\frac{(1-\tau)}{\tau}\frac{C_t}{(1-L_t)} = W_t. \tag{3.19}$$

Since the labor supply is in the Cobb-Douglas case dependent on consumption, changes in the consumption growth rate introduced by the time-varying risk aversion will directly affect labor supply.

GHH

We then consider GHH preferences as in Greenwood et al. (1988) that are known to have no wealth effect and labor supply only depends on wages

$$U(C_t, L_t, \sigma_t) = \frac{\left(C_t - \psi L_t^{\nu}\right)^{1-\sigma_t}}{1-\sigma_t} \tag{3.20}$$

where $\psi > 0$ and $\nu > 0$ with the following marginal utilities for consumption

$$U_{t,C} = \left(C_t - \psi L_t^{\nu}\right)^{-\sigma_t} \tag{3.21}$$

and for labor

$$U_{t,L} = -\left(C_t - \psi L_t^{\nu}\right)^{-\sigma_t}\left(\nu\psi L_t^{\nu-1}\right) \tag{3.22}$$

and hence labor supply becomes

$$\nu\psi L_t^{\nu-1} = W_t. \tag{3.23}$$

Since the labor supply is independent of both the time-varying risk aversion σ_t, and in the GHH case also of consumption, the behavior of labor will be the same for a constant risk aversion and a time-varying risk aversion.

KPR

We then analyze KPR preferences as in King et al. (1988) of the form

$$U(C_t, L_t, \sigma_t) = \frac{[C_t (1 - \psi L_t^\nu)]^{1-\sigma_t}}{1 - \sigma_t} \tag{3.24}$$

where $\psi > 0$ and $\nu > 0$ with the following marginal utilities for consumption

$$U_{t,C} = [C_t (1 - \psi L_t^\nu)]^{-\sigma_t} (1 - \psi L_t^\nu) \tag{3.25}$$

and for labor

$$U_{t,L} = -[C_t (1 - \psi L_t^\nu)]^{-\sigma_t} \left(\nu \psi C_t L_t^{\nu-1}\right) \tag{3.26}$$

and hence labor supply becomes

$$\frac{\nu \psi C_t L_t^{\nu-1}}{1 - \psi L_t^\nu} = W_t. \tag{3.27}$$

Similar to the Cobb-Douglas case in Equation (3.19), labor supply depends on consumption and hence faster consumption growth caused by a change in the risk aversion will be reflected in higher growth in the labor supply.

Preference Parameters

For all preferences we calibrate the labor elasticity of supply such that in steady state the labor supply equals 0.30 so that we can compare our results across preference specification. For the Cobb-Douglas preferences we get using the labor supply equation in Equation (3.19) and by solving for the Cobb-Douglas consumption share τ that[5]

$$\tau = \frac{L\frac{C}{Y}}{L\frac{C}{Y} + (1 - L)(1 - \alpha)} \tag{3.28}$$

where L is the exogenously fixed steady state labor supply. $\frac{C}{Y}$ denotes the implied steady state consumption to output ratio dependent on parameters α,β, and δ.[6] Given a standard parameterization this implies a value of about 0.33 for τ.[7] Following Raffo (2008) and

[5] See Appendix 3.A.4 for a derivation.

[6] Note that the consumption to output ratio can easily be expressed dependent on parameters as $\frac{C}{Y} = 1 - \frac{\alpha\beta\delta}{1-\beta-\beta\delta}$.

[7] See Table 3.1 for all parameter values of the model.

Mandelman et al. (2011) we derive the two parameters of the GHH and KPR preferences by imposing that the Frisch labor supply elasticities ϵ are equal across preferences such that

$$\epsilon_{CD} = \epsilon_{GHH} = \epsilon_{KPR}. \tag{3.29}$$

We continue by using that the Frisch labor supply elasticity in the Cobb-Douglas case can be defined as[8]

$$\epsilon_{CD} = \frac{(1-L)\,(1-\tau\,(1-\sigma))}{\sigma L} \tag{3.30}$$

where σ denotes the steady state risk aversion.[9] Similarly, in the GHH case one gets for the Frisch labor supply elasticity

$$\epsilon_{GHH} = \frac{1}{\nu_{GHH}-1} \tag{3.31}$$

so that we can calculate ν_{GHH} as

$$\nu_{GHH} = \frac{1+\epsilon_{GHH}}{\epsilon_{GHH}} \tag{3.32}$$

and using the labor supply equation in Equation (3.23) the parameter ψ_{GHH} follows as

$$\psi_{GHH} = \frac{WL^{1-\nu_{GHH}}}{\nu_{GHH}} \tag{3.33}$$

where W is the steady state wage. For the KPR utility we follow Holden et al. (2018) who derive the Frisch labor supply elasticity as

$$\epsilon_{KPR} = \left[\nu - 1 + \frac{\nu\psi L^{\nu}\,(2\sigma-1)}{\sigma\,(1-\psi L^{\nu})}\right]^{-1} \tag{3.34}$$

together with the labor supply equation in Equation (3.27)

$$\nu\psi C L^{\nu-1} = W\,(1-\psi L^{\nu}) \tag{3.35}$$

we can solve for the two unknown parameters ν_{KPR} and ψ_{KPR} in the KPR case. So that

$$\nu_{KPR} = \frac{1+\epsilon_{KPR}}{\epsilon_{KPR}} - \frac{WL\,(2\sigma-1)}{\sigma C} \tag{3.36}$$

[8]See e.g. Mandelman et al. (2011).

[9]In what follows, we denote steady state values of a variable by referring to that variable without a time subscript.

where C is steady state consumption. And finally

$$\psi_{KPR} = \frac{W}{\nu_{KPR} C L^{\nu_{KPR}-1} + W L^{\nu_{KPR}}}. \tag{3.37}$$

3.2.6 Equilibrium and Equilibrium Conditions

An equilibrium of this economy is characterized by a set of allocations for the consumer in each country consisting of consumption C_t, labor L_t, capital K_t, investment I_t, and bond D_t. The allocations for home and foreign intermediate goods producers $Y_{H,t}$, $Y^*_{H,t}$, $Y_{F,t}$, and $Y^*_{F,t}$, the allocations for both the home and foreign goods producers Y_t and Y^*_t and prices of the intermediate goods $P_{H,t}$, $P^*_{H,t}$, $P_{F,t}$, and $P^*_{F,t}$ as well as final prices P_t and P^*_t. Finally, we require the price of labor and capital W_t and R_t for each country and the price of the bond Q_t as well as the risk aversion for each country σ_t and σ^*_t such that (1) Households allocations solve the households' problem. (2) Intermediate goods producers' allocations solve the intermediate goods producers' problem. (3) Final good producers' allocation solve the final goods producers' problem. (4) All markets clear.

3.2.7 Solution Method, Steady State, and Equilibrium Conditions

To solve the model we use a first-order approximation around the steady state. We also approximate the model to second and third-order to verify our results. The obtained second moments of the model, however, show no noticeable difference between a first, second, or third-order approximation. We provide the steady states for all variables and a numerical example for the steady state in the Appendix 3.A.1. The full set of equilibrium conditions can be found in the Appendix 3.A.2.

3.2.8 Parameters

The model is parameterized as in Mandelman et al. (2011) to replicate the quarterly data for the US and the rest of the world. Both countries are assumed to be fully symmetric. All parameters of the model can be found in Table 3.1. The capital elasticity α is set to 0.36 and the discount factor β is set to 0.99 to match the quarterly empirical data moments. We set the depreciation rate δ to 0.025 and the steady state risk aversion σ to 2.00, respectively. The elasticity of substitution between the home and foreign intermediate goods θ is set to 0.62 which is consistent with the value used by Raffo (2010) but slightly higher than the value used by Corsetti et al. (2008a) and lower than the value used by Heathcote and Perri (2002).[10] The capital adjustment cost ϕ is set to 0.60. The share of home intermediate goods in the final good production function ω is set to 0.90 which matches the home bias found in the data. We assume further a low value of 0.001 for the cost of holding bonds ζ to ensure stationarity of the model. Following Mandelman et al. (2011) and their

[10] Since the elasticity of substitution between home and foreign intermediate goods is crucial for the cross-country dynamics of the model we will provide robustness checks with the higher value of 0.85 used by Heathcote and Perri (2002).

empirical findings for the US and the rest of the world we assume a correlation of the TFP innovations u_t and u_t^* of 0.29. The persistence of the home and foreign component of the TFP process ρ_A and ρ_A^* is set to 0.97 and 0.025. It thereby matches the empirically observed values by Mandelman et al. (2011) for the US and the rest of the world.[11]

When we fix the labor supply in the steady state L to 0.30 we obtain the following parameters in Table 3.1 for the three different utility functions. The Cobb-Douglas consumption share τ is calculated as 0.33 and the Frisch labor supply elasticity ϵ is 1.55 for all utility functions. In the case of GHH preferences we get that ν_{GHH} is 1.64 and ψ_{GHH} is 3.13. For the KPR preferences we obtain ν_{KPR} as 0.35 and ψ_{KPR} as 1.08.

Since it is difficult to come up with reasonable estimates of the parameters of the AR(1) process ρ_σ and η_σ for the risk aversion we estimate these two parameters using SMM to match the second moments of the model and the empirical data moments. We target the standard deviation of output to obtain reasonable values for the mean volatility of our model. Since we know that our model has some problems matching the relative standard deviation of consumption and output and the correlation between output and consumption we include these two moments in our SMM routine. We also include the correlation of the real exchange rate and relative consumption across countries and the correlation of output, consumption, investment, and labor across countries.[12] In this way we try to match eight moments with two parameters.

TABLE 3.1
Model Parameters

Parameter	Description	Value
α	Capital Elasticity	0.36
β	Discount Factor	0.99
δ	Capital Depreciation Rate	0.025
σ	Steady State Risk Aversion	2.00
θ	Elasticity of Substitutability between Intermediate Goods	0.62
ϕ	Capital Adjustment Cost	0.60
ω	Share of Home Intermediate Goods - Home Bias	0.90
ζ	Cost of Bond Holding	0.001
ρ_A	TFP Persistence Home Component	0.97
ρ_A^*	TFP Persistence Foreign Component	0.025
σ_u^2	TFP Shock Variance Home	0.01
$\sigma_{u^*}^2$	TFP Shock Variance Foreign	0.01
L	Steady State Labor Supply	0.30
τ	Cobb-Douglas Consumption Share	0.33
ϵ	Frisch Labor Supply Elasticity	1.55
ν_{GHH}	GHH Utility Parameter	1.64
ψ_{GHH}	GHH Utility Parameter	3.13
ν_{KPR}	KPR Utility Parameter	0.35
ψ_{KPR}	KPR Utility Parameter	1.08

Note: Table 3.1 shows the parameter values in the DSGE model.

[11] For robustness we will also try a correlation of the TFP shocks of zero and no spillovers of the TFP shocks across countries.

[12] Not including these cross-country moments in the SMM routine would lead to a case where the SMM predicts a constant risk aversion. However, we believe that a potential small loss in the standard deviation of consumption is worth accepting given the substantial gains in the cross-country moments we can achieve.

3.3 Results

We evaluate the performance of our model with countercyclical risk aversion using the second moments of the model. We linearize the model around the steady state and generate data series of 500 periods and subsequently discard the first 364 periods as a burn-in to get rid of initial conditions. This leaves us with 136 periods, the same number of periods as in the empirical data provided by Mandelman et al. (2011) which we use for comparison. We replicate the model 500 times using a different sequence of error terms and calculate the median moment and the 5th and 95th percent confidence bands of the moments. All data is in logarithms and HP-Filtered as in Hodrick and Prescott (1997) using a filter weight of 1600 to match the quarterly frequency of the empirical data. We use the empirical data presented by Mandelman et al. (2011) for comparison which ranges from 1973Q1 to 2006Q4 and represents the US and the rest of the world, where the rest of the world consists of the Euro area, the United Kingdom, Canada, Japan, and Australia.

3.3.1 Moments

Table 3.2 shows the second moments for different preference functions. The column "Constant RA" for each preference function shows the second moments when η_σ is set to zero and hence the risk aversion σ_t becomes a constant parameter σ with value 2.00. The column "Variable RA" shows the second moments when the two risk aversion parameters ρ_σ and η_σ are estimated using SMM and the eight moments described in the parameter section.[13]

Cobb-Douglas

When we are targeting the moments of the model described in the parameter section using our two risk aversion parameters ρ_σ and η_σ we obtain estimates for ρ_σ of about 0.80 and for η_σ of about 3.54 as can be seen in column (2) of Table 3.2. Once we add a countercyclical risk aversion to the model using a Cobb-Douglas utility function as in Equation (3.16) we observe that output, investment, and labor become more volatile. This is especially true for investment and labor where the relative standard deviation to output is now close to the empirical value and almost twice as large as in a model with a constant risk aversion. For the relative consumption to output volatility we observe, however, a decrease compared to a model with a constant risk aversion. The correlation of the real exchange rate and relative consumption across countries becomes virtually zero with a value of 0.11 and now matches the empirical value of -0.04 closely, thereby solving the Backus-Smith Puzzle. In addition, we can also observe that the comovement of investment and labor across countries increases and is now closer to the empirical data. We

[13] We minimize the sum of squared percentage deviations rather than the sum of squared residuals as our targeted moments are in different units and sizes. The main results stay the same when we minimize the sum of squared residuals.

are therefore able to make some progress in the International Comovement Puzzle. Using Cobb-Douglas preferences in combination with a countercyclical risk aversion leads however to a decline in the correlation of output and consumption due to a change in the reaction of consumption to a TFP shock. Interestingly, the introduction of a countercyclical risk aversion has no effect on the relative volatility of the real exchange rate and consumption, and on the autocorrelation of the real exchange rate. In both cases the real exchange rate does not show enough volatility and is not persistent enough. We thereby fail to make any grounds on the Price Puzzle. Similarly to a constant risk aversion model, our model fails to generate a higher correlation of output across countries than consumption correlation. We are therefore not able to solve the Quantity Puzzle.

GHH

Using a GHH preference specification as in Equation (3.20) again allows us to generate a correlation of the real exchange rate and relative consumption across countries which is close to zero with 0.10 as can be seen in column (4) of Table 3.2. The estimated risk aversion parameters η_σ is close to its counterpart in the Cobb-Douglas case with an estimated value of 4.40. However, the persistence parameter ρ_σ is now lower with a value of 0.55. Contrary to the Cobb-Douglas preferences the model is still able to produce a high correlation of output and consumption of 0.80 compared to an almost perfect correlation in the GHH model with a constant risk aversion and a value of 0.84 in the data. Similar to Cobb-Douglas preferences we see a significant increase in the relative volatility of investment and an, albeit, smaller decrease in the relative volatility of consumption to output. Since the labor supply in the GHH case is independent of both, the time-varying risk aversion and consumption, the relative standard deviation of labor to output is unchanged when we introduce time-variation in the risk aversion.

KPR

By using a KPR utility function as in Equation (3.24) gives a value of -0.05 for the correlation of the real exchange rate and relative consumption as can be seen in column (6) of Table 3.2. In addition, the cross-country correlations for investment and labor turn positive when the risk aversion becomes countercyclical with values of 0.18 and 0.26, respectively. Although these cross-country correlations are still below the empirically observed value, they mark a significant improvement over the model with a constant risk aversion which shows cross-country correlations for investment and labor of -0.15 and -0.26, respectively. Similar to Cobb-Douglas and GHH preferences, the introduction of a countercyclical risk aversion has only a small effect on the relative volatility of the real exchange rate and output which remains well below the empirically observed values or on the autocorrelation of the real exchange rate. However, contrary to Cobb-Douglas and GHH preferences the estimated risk aversion parameter ρ_σ and η_σ are significantly lower with values of 0.41 and 1.92, respectively.

TABLE 3.2
Second Moments

		Cobb-Douglas		GHH		KPR	
		(1)	(2)	(3)	(4)	(5)	(6)
	Data	Constant RA	Variable RA	Constant RA	Variable RA	Constant RA	Variable RA
ρ_σ		0.00	0.80	0.00	0.55	0.00	0.41
η_σ		0.00	3.54	0.00	4.00	0.00	1.92
$\sigma(Y)$	1.58	1.37	1.82	1.88	1.90	1.36	1.75
		(1.11;1.71)	(1.43;2.30)	(1.53;2.31)	(1.53;2.37)	(1.10;1.68)	(1.44;2.10)
$\sigma(C)/\sigma(Y)$	0.76	0.68	0.36	0.85	0.62	0.72	0.49
		(0.63;0.74)	(0.31;0.44)	(0.81;0.91)	(0.54;0.70)	(0.67;0.79)	(0.42;0.56)
$\sigma(I)/\sigma(Y)$	4.55	2.14	3.49	1.55	2.69	2.00	3.10
		(1.98;2.28)	(3.31;3.66)	(1.41;1.69)	(2.45;2.92)	(1.83;2.17)	(2.89;3.30)
$\sigma(L)/\sigma(Y)$	0.75	0.25	0.57	0.59	0.59	0.25	0.58
		(0.23;0.27)	(0.54;0.60)	(0.57;0.60)	(0.58;0.60)	(0.22;0.28)	(0.53;0.63)
$\sigma(RER)/\sigma(Y)$	3.06	1.38	1.39	1.29	1.52	1.57	1.77
		(1.09;1.72)	(1.10;1.77)	(1.02;1.60)	(1.19;1.87)	(1.25;1.96)	(1.46;2.14)
$\rho(Y,L)$	0.87	0.85	0.94	1.00	1.00	0.85	0.88
		(0.77;0.90)	(0.91;0.95)	(1.00;1.00)	(1.00;1.00)	(0.77;0.90)	(0.85;0.91)
$\rho(Y,C)$	0.84	0.94	0.51	0.98	0.80	0.95	0.69
		(0.92;0.96)	(0.37;0.62)	(0.97;0.99)	(0.73;0.86)	(0.93;0.97)	(0.59;0.77)
$\rho(Y,I)$	0.91	0.95	0.97	0.95	0.92	0.95	0.95
		(0.93;0.97)	(0.95;0.98)	(0.92;0.97)	(0.89;0.94)	(0.92;0.97)	(0.93;0.96)
$\rho(RER)$	0.82	0.61	0.61	0.54	0.49	0.59	0.47
		(0.48;0.71)	(0.48;0.70)	(0.40;0.66)	(0.36;0.61)	(0.46;0.70)	(0.35;0.57)
$\rho(RER,C/C^*)$	-0.04	0.99	0.11	0.98	0.10	1.00	-0.05
		(0.99;0.99)	(-0.08;0.30)	(0.97;0.98)	(-0.08;0.28)	(0.99;1.00)	(-0.26;0.15)
$\rho(Y,Y^*)$	0.44	0.38	0.40	0.55	0.61	0.36	0.42
		(0.08;0.61)	(0.09;0.63)	(0.31;0.72)	(0.37;0.77)	(0.06;0.60)	(0.16;0.61)
$\rho(C,C^*)$	0.36	0.68	0.63	0.81	0.81	0.75	0.69
		(0.54;0.79)	(0.38;0.79)	(0.69;0.88)	(0.61;0.91)	(0.61;0.84)	(0.45;0.84)
$\rho(I,I^*)$	0.28	0.03	0.19	0.00	0.35	-0.15	0.18
		(-0.32;0.36)	(-0.17;0.48)	(-0.31;0.31)	(0.08;0.55)	(-0.46;0.19)	(-0.06;0.38)
$\rho(L,L^*)$	0.40	0.09	0.28	0.67	0.71	-0.26	0.26
		(-0.28;0.43)	(-0.09;0.55)	(0.47;0.80)	(0.51;0.83)	(-0.55;0.09)	(0.03;0.43)

Note: Table 3.2 shows the moments of the model. σ denotes the standard deviation of a variable and ρ denotes the correlation between two variables. 5th and 95th percentile confidence bands in parenthesis. Moments are the median of 500 replications.

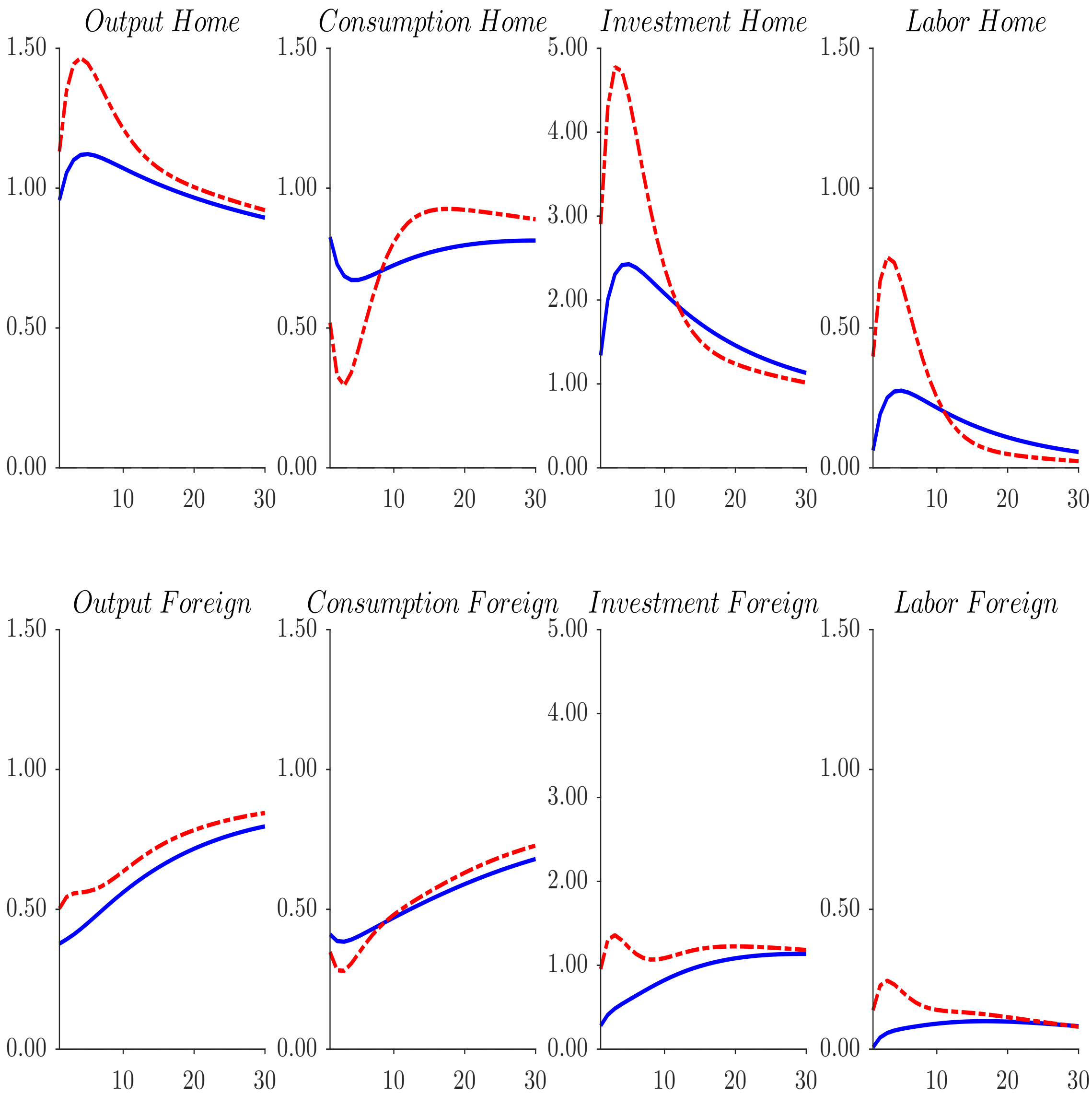
3.3.2 Impulse Responses

Figure 3.1 and Figure 3.2 show the impulse responses of a one percent transitory TFP shock in the home country for the case of Cobb-Douglas preferences.[14] There are two striking features of the impulse responses generated by the model with countercyclical risk aversion denoted by the dashed-dotted red line. First, the impulse response of all variables becomes more pronounced. And second, the impulse response for relative consumption shown in Figure 3.2 changes the shape. This change in the behavior of relative consumption is caused by the large increase in consumption in the home country in response to a technology shock, whereas the foreign country's consumption behavior changes only slightly. Examining the impulse response of consumption in the home country more closely reveals that consumption initially reacts less on impact but is subsequently subject to a higher growth rate. Together with the observed increase in output

[14] Although our TFP shock is only transitory, it is very persistent when applying the same parameters as Mandelman et al. (2011) and impulse responses consequently converge only slowly back to the steady state. Our main results stay the same once we use a less persistent process for TFP as we show in the robustness section.

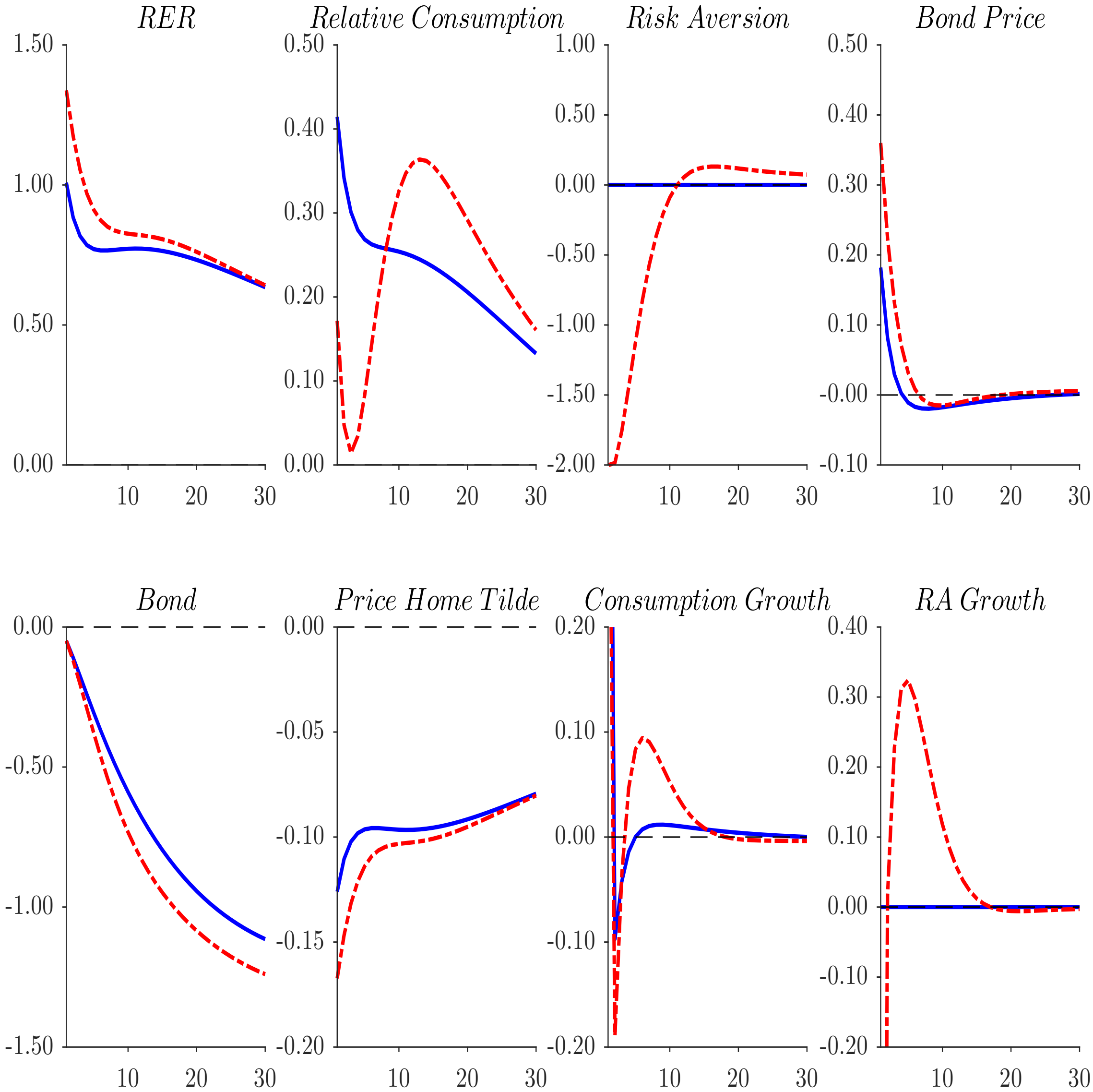
volatility this leads to a lower relative volatility of consumption to output as is evident in column (2) of Table 3.2. From Figure 3.2 it becomes also clear that the movements in the risk aversion σ_t induced by a home technology shock are rather small. A one percent TFP shock decreases risk aversion by about 2.0 percent from its steady state value of 2.00. A rather small change compared to the wide range of estimates for risk aversion parameters in macroeconomic models that usually include the case of log utility i.e. a risk aversion of 1 and substantially higher values in many asset pricing models. Figure 3.2 also shows clearly that the growth rate in the risk aversion and consumption growth in the home country share the same pattern after a couple of periods.

FIGURE 3.1
Impulse Responses I



Note: Figure 3.1 shows the impulse responses of home and foreign output, consumption, investment, and labor to a home country TFP shock using Cobb-Douglas preferences. Model with a constant risk aversion in blue and with time-varying risk aversion as the dashed-dotted red line. Scale is in percentage deviations from the steady state.

FIGURE 3.2
Impulse Responses II



Note: Figure 3.2 shows the impulse responses of additional variables to a TFP shock in the home country using Cobb-Douglas preferences. Model with a constant risk aversion in blue and with time-varying risk aversion as the dashed-dotted red line. Scale is in percentage deviations from the steady state.

3.3.3 Intuition

We now investigate why the consumption impulse response is more pronounced when the risk aversion becomes countercyclical and why the labor supply reacts more volatile to a TFP shock in that case when we use Cobb-Douglas or KPR preferences. In short, we find that the introduction of a time-varying and countercyclical risk aversion leads to an additional term in the log-linearized Euler equation that leads to higher consumption growth in response to a positive productivity shock. Since labor supply is directly dependent on consumption in the Cobb-Douglas and KPR case we can observe an amplification of the labor response after a productivity shock.

Log-Linear Euler Equation

The Euler equation of the economy is given by

$$\frac{Q_t + \zeta D_t}{\beta} = \mathbb{E}_t \left[\frac{U_{t+1,C}}{U_{t,C}} \frac{\tilde{P}_{H,t+1}}{\tilde{P}_{H,t}} \right] \tag{3.38}$$

where $\tilde{P}_{H,t}$ is the relative price of the home intermediate good to the final good and $U_{t,C}$ and $U_{t+1,C}$ denote marginal utility of consumption. Then for the case of Cobb-Douglas preferences we get

$$\frac{Q_t + \zeta D_t}{\beta} = \mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \frac{\left[C_{t+1}^{\tau} \left(1 - L_{t+1}\right)^{1-\tau} \right]^{1-\sigma_{t+1}}}{\left[C_t^{\tau} \left(1 - L_t\right)^{1-\tau} \right]^{1-\sigma_t}} \frac{\tilde{P}_{H,t+1}}{\tilde{P}_{H,t}} \right] \tag{3.39}$$

note that the risk aversion σ_t is now dependent on time t in the Euler equation. Taking logarithms of the Euler equation gives

$$\begin{aligned} \ln\left(Q_t + \zeta D_t\right) - \ln\left(\beta\right) + \ln\left(\tilde{P}_{H,t}\right) - \ln\left(\tilde{P}_{H,t+1}\right) &= \ln\left(C_t\right) - \ln\left(C_{t+1}\right) + \left(1 - \sigma_{t+1}\right) \tau \ln\left(C_{t+1}\right) \\ + \left(1 - \sigma_{t+1}\right)\left(1 - \tau\right) \ln\left(1 - L_{t+1}\right) &- \left(1 - \sigma_t\right) \tau \ln\left(C_t\right) - \left(1 - \sigma_t\right)\left(1 - \tau\right) \ln\left(1 - L_t\right) \end{aligned} \tag{3.40}$$

the log-linearization around the steady state follows then after canceling out redundant terms as

$$\begin{aligned} -\widetilde{r_t} + \zeta \widetilde{D_t} + \widetilde{P_{H,t}} - \widetilde{P_{H,t+1}} &= \widetilde{C_t}\left[1 - \tau + \sigma\tau\right] - \widetilde{C_{t+1}}\left[1 - \tau + \sigma\tau\right] - \widetilde{L_t} \frac{L}{1-L}\left[\left(\sigma - 1\right)\left(1 - \tau\right)\right] \\ + \widetilde{L_{t+1}} \frac{1}{1-L}\left[\left(\sigma - 1\right)\left(1 - \tau\right)\right] &- \tau \ln\left(C\right)\left(\sigma_{t+1} - \sigma_t\right) - \left(1 - \tau\right) \ln\left(1 - L\right)\left(\sigma_{t+1} - \sigma_t\right) \end{aligned} \tag{3.41}$$

where as usual $\widetilde{x_t} = \frac{x_t - x}{x}$ denotes percentage deviations from the steady state of a variable. Then after rearranging terms we get for consumption growth

$$\widetilde{C_{t+1}} - \widetilde{C_t} = \frac{\widetilde{r_t} - \zeta \widetilde{D_t} - \widetilde{\tilde{P}_{H,t+1}} + \widetilde{\tilde{P}_{H,t}}}{1 - \tau + \sigma\tau} + \Gamma\left(\widetilde{L_{t+1}} - \widetilde{L_t}\right) + \Lambda\left(\widetilde{\sigma_{t+1}} - \widetilde{\sigma_t}\right) \tag{3.42}$$

where

$$\Gamma = \frac{L}{1-L}\left[\frac{(\sigma - 1)(1 - \tau)}{1 - \tau + \sigma\tau}\right] \tag{3.43}$$

and Λ can be considered as the elasticity of consumption growth with respect to growth in the risk aversion and equals

$$\Lambda = -\left[\frac{\tau \ln(C) + (1-\tau)\ln(1-L)}{1 - \tau + \sigma\tau}\right]. \tag{3.44}$$

Compared to a model with a constant risk aversion, we have an additional term that affects consumption growth, namely the growth rate of the risk aversion. It is worth noting that time variation in the risk aversion also has general equilibrium effects on the behavior of the interest rate $\widetilde{r_t}$, the bond $\widetilde{D_t}$, and the price of home intermediate goods relative to final goods $\widetilde{\tilde{P}_{H,t}}$ in the Euler equation.[15] However, a look that the impulse responses suggests that these effects are rather small and possibly negligible. We therefore assume that changes in consumption growth are caused by the additional term in the log-linearized Euler equation in Equation (3.42).

After a positive TFP shock output growth increases and hence the risk aversion decreases suddenly as can be seen in Figure 3.2. After the initial decline in the risk aversion σ_t returns to its steady state value and shows a positive growth rate. Agents will react by increased consumption growth and decreasing labor supply while the risk aversion returns back to its steady state. Hence roughly between period 2 and 20 we observe positive consumption growth and a quick return of labor supply back to its steady state value. However, as the risk aversion increases and finally reaches its steady state, consumption growth will decline and return back to steady state.

As both countries' risk aversion is dependent on the growth rate of the final output of each country, the home country's consumption will show a more pronounced impulse response than the foreign country's consumption in response to a home productivity shock, which remains basically unchanged. This creates a relative consumption impulse response which is driven by the home country's impulse response pattern. As the impulse response of the real exchange rate remains almost unaffected by the introduction of a countercyclical risk aversion, the correlation of the real exchange rate and relative consumption breaks down.

In our model the initial drop in consumption further leads to an investment boom

[15]Note that $\tilde{P}_{H,t}$ refers to the relative price of the home intermediate good to the final good. Whereas $\widetilde{\tilde{P}_{H,t}}$ refers to the percentage deviation of that variable from the steady state.

which leads to higher capital and hence a higher marginal return of labor i.e. wages increase and through the labor supply equation labor increases more than in the case with a constant risk aversion. We provide the log-linearized Euler equation for the case of GHH and KPR preferences in the Appendix 3.A.3. The log-linearized Euler equation in these cases follows the same pattern and includes an additional term that accounts for the growth rate of the risk aversion.

Euler Equation Elasticities

Table 3.3 shows the Euler equation elasticities Γ and Λ for the parameters used in Table 3.1 and a steady state labor supply L of 0.30 for Cobb-Douglas, GHH, and KPR preferences. Setting these values into context of the impulse responses in Figure 3.2 suggests in the Cobb-Douglas case that at its peak 0.10 percent growth in the risk aversion directly causes consumption growth to be higher by 0.023 percent between two periods. The remaining portion of higher consumption growth in the presence of countercyclical risk aversion is explained by the change in the remaining terms in the Euler equation when countercyclical risk aversion is present.

To verify the sign of the elasticity of consumption growth with respect to growth in the risk aversion Λ we calculate the steady state values of consumption C and the Cobb-Douglas consumption share τ given a grid of values for steady state labor L.[16] We further test for robustness and draw 10000 values for α, β, and δ that affect steady state consumption C and the consumption share τ and therefore the size and sign of the elasticity Λ from a random uniform distribution.[17] Figure 3.3 shows the size of the elasticity of consumption growth with respect to growth in the risk aversion for different steady state values of labor for our three preference function specifications. We show the median, as well as the 5th and 95th percentile confidence bands of all calculated values for Λ. The simulations imply two outcomes. First, the elasticity of consumption growth with respect to growth in the risk aversion is positive for all three specifications for reasonable parameterizations of the model. Second, the elasticity of Cobb-Douglas preferences is the smallest, followed by GHH preferences and KPR preferences that imply the largest risk aversion elasticity. The relative size of the estimated elasticities are in line with the size of elasticities estimated for the risk aversion process. Specifically, the estimated parameter η_σ is lower in the KPR case than in the Cobb-Douglas or GHH case as the consumption Euler equation already features a higher multiplier Λ and therefore smaller changes in the risk aversion are required.

[16] Note that changes in steady state labor L imply different values in steady state consumption C and τ, respectively ψ and ν.

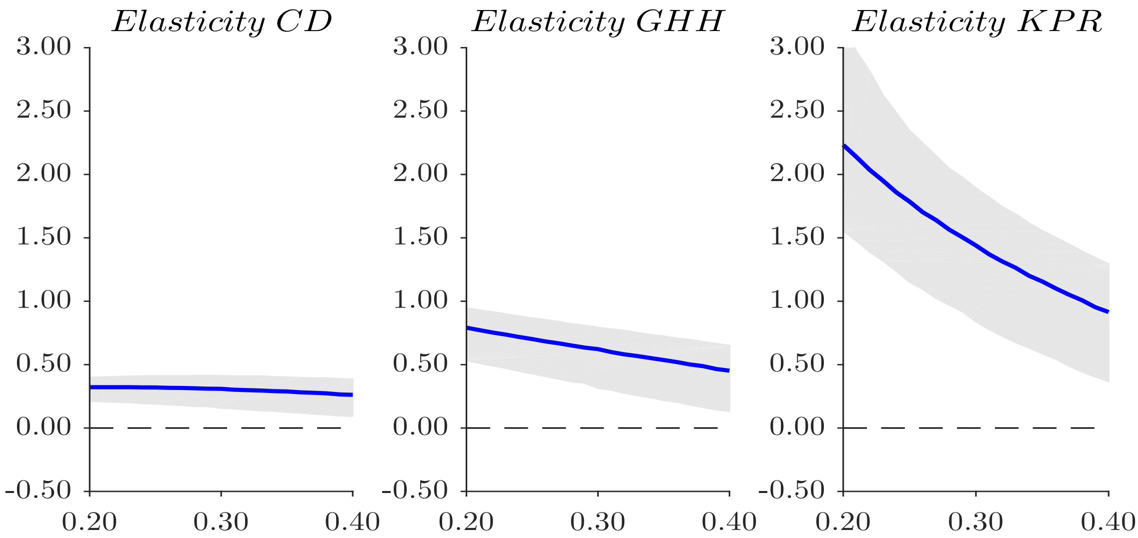
[17] We draw the capital elasticity α from between 0.30 and 0.42, the discount factor β from between 0.90 and 0.995, the depreciation rate δ from 0.01 to 0.05, and the steady state risk aversion σ from 1.50 to 2.50.

TABLE 3.3
Euler Equation Elasticities

Parameter	Description	Cobb-Douglas	GHH	KPR
Γ	Labor Growth Elasticity	0.21	0.86	0.43
Λ	Risk Aversion Growth Elasticity	0.23	0.44	1.43

Note: Table 3.3 shows the implied Euler equation elasticities using the DSGE parameters shown in Table 3.1.

FIGURE 3.3
Euler Equation Elasticities



Note: Figure 3.3 shows the size of the elasticity of consumption growth with respect to growth in the risk aversion Λ for different steady state values of labor. The blue line is the median and the 5th and 95th percentile confidence bands of 10000 draws for different values of α, β, δ, and σ drawn from a random uniform distribution are shown as the shaded area.

3.4 Robustness

We subject our model to various alternative specifications and to some variation in the key parameters of the model to verify the robustness of our results.

3.4.1 Different Model Specifications

Most importantly we test different forms of the risk aversion process in Equation (3.3). Our main results still hold when we replace the growth rate of final output ΔY_t i.e. of the CES aggregate of home and foreign produced intermediate goods in Equation (3.6) by the growth rate of the home intermediate good $\Delta Y_{H,t}$ or by the growth rate of GDP i.e. the sum of the home produced intermediate goods for domestic and foreign consumption $Y_{H,t} + Y^*_{H,t}$ in Equation (3.9). What is crucial in our model is that risk aversion is driven by a country specific process that induces a different degree of risk aversion in each country. Having a risk aversion process that depends on e.g. world output will have no impact on the relative consumption behavior across countries and therefore on the correlation of the real exchange rate and relative consumption.[18] However, since output growth is highly correlated across countries in the model, also the risk aversion shows a high degree of cross-country correlation. We also replaced the growth rate of the above mentioned variables by their deviations from the steady state. Although this introduces a slightly different dynamic into the risk aversion, our results still hold and we obtain a near zero correlation of the real exchange rate and relative consumption across countries.

3.4.2 Different Parameter Values

We now change some parameters of the model that can have a significant impact on cross-country business cycle moments to see whether our model with countercyclical risk aversion is still able to generate moments that are in line with the empirically observed data moments. We do so by changing one parameter from Table 3.1 at once leaving all other parameters unchanged and then re-estimate our two parameters ρ_σ and η_σ that govern the persistence and degree of countercyclical risk aversion in the model. We opt for showing the second moments for the case of GHH preferences as this kind of preferences are not suffering from the decrease in the correlation of output with consumption and therefore seem to be better suited for an application with time-varying risk aversion as in our model.

We start by using the elasticity of substitution between home and foreign intermediate goods θ of 0.85 as used by Heathcote and Perri (2002). Table 3.4 shows the second moments for the case of GHH utility when key parameters of the model are changed one by one. As can be seen in column (2), our model is still able to generate a close to zero correlation of the real exchange rate and relative consumption across countries of about 0.10. Compared to the GHH case with a low elasticity of substitution in column (4) of Table 3.2 our risk aversion parameters ρ_σ and η_σ are virtually unchanged.

In Equation (3.10) and (3.11) we allowed for a spillover effect of the TFP shocks to the other country. We now shut down this channel by assuming that ρ_ν^* is 0.00 hence making TFP only dependent on one country's TFP shocks. Column (4) in Table 3.4 indicates that TFP spillovers between the home and foreign country are not essential for our results. Estimating ρ_σ and η_σ to be 0.51 and 5.11 allows us again to generate a value of -0.02 for the correlation of the real exchange rate and relative consumption.

Finally, we again allow for a spillover in TFP shocks but assume this time that the TFP shocks ν_t and ν_t^* are completely uncorrelated instead of the previously assumed correla-

[18] Similarly, in a case where the final output good is used but the share of home intermediate goods ω is set to 0.50 our model would fail. However, note that since the law of one price holds for intermediate goods, that in such a case the real exchange rate would be one anyway as the price level is the same in both countries.

tion of 0.29. Clearly, column (6) shows that although the cross-correlation in consumption, investment, and labor decreases when we set the TFP shock correlation to zero, we are still able to account for the Backus-Smith Puzzle by generating a negative correlation between the real exchange rate and relative consumption across countries.

TABLE 3.4
Second Moments - Robustness

		High Elasticity of Substitution		No TFP Spillover		No TFP Correlation	
		(1)	(2)	(3)	(4)	(5)	(6)
	Data	Constant RA	Variable RA	Constant RA	Variable RA	Constant RA	Variable RA
ρ_σ		0.00	0.53	0.00	0.51	0.00	0.19
η_σ		0.00	4.05	0.00	5.11	0.00	7.52
$\sigma(Y)$	1.58	1.94	1.96	1.82	1.86	1.81	1.83
		(1.59;2.40)	(1.58;2.45)	(1.48;2.24)	(1.51;2.32)	(1.46;2.23)	(1.47;2.29)
$\sigma(C)/\sigma(Y)$	0.76	0.85	0.62	0.75	0.55	0.82	0.75
		(0.81;0.90)	(0.54;0.69)	(0.72;0.78)	(0.48;0.60)	(0.77;0.88)	(0.69;0.82)
$\sigma(I)/\sigma(Y)$	4.55	1.56	2.66	1.79	3.26	1.69	3.16
		(1.43;1.70)	(2.43;2.89)	(1.71;1.86)	(3.00;3.53)	(1.53;1.85)	(2.80;3.57)
$\sigma(L)/\sigma(Y)$	0.75	0.59	0.59	0.59	0.60	0.58	0.58
		(0.58;0.60)	(0.58;0.61)	(0.58;0.61)	(0.59;0.61)	(0.56;0.59)	(0.56;0.60)
$\sigma(RER)/\sigma(Y)$	3.06	0.83	1.07	1.60	1.82	1.59	2.02
		(0.66;1.02)	(0.86;1.31)	(1.26;2.00)	(1.43;2.29)	(1.28;1.94)	(1.63;2.45)
$\rho(Y,L)$	0.87	1.00	1.00	1.00	1.00	1.00	1.00
		(1.00;1.00)	(1.00;1.00)	(1.00;1.00)	(1.00;1.00)	(1.00;1.00)	(1.00;1.00)
$\rho(Y,C)$	0.84	0.98	0.81	0.99	0.57	0.97	0.59
		(0.97;0.99)	(0.74;0.86)	(0.98;0.99)	(0.44;0.68)	(0.95;0.98)	(0.42;0.71)
$\rho(Y,I)$	0.91	0.95	0.92	0.98	0.92	0.94	0.83
		(0.92;0.97)	(0.89;0.94)	(0.97;0.99)	(0.90;0.94)	(0.91;0.97)	(0.80;0.86)
$\rho(RER)$	0.82	0.47	0.43	0.60	0.55	0.54	0.44
		(0.33;0.59)	(0.30;0.54)	(0.47;0.71)	(0.42;0.66)	(0.39;0.65)	(0.31;0.56)
$\rho(RER,C/C^*)$	-0.04	0.95	0.10	0.99	-0.02	0.98	-0.28
		(0.94;0.95)	(-0.04;0.25)	(0.99;0.99)	(-0.22;0.18)	(0.97;0.98)	(-0.50;-0.05)
$\rho(Y,Y^*)$	0.44	0.45	0.52	0.67	0.72	0.32	0.39
		(0.19;0.65)	(0.25;0.71)	(0.47;0.80)	(0.53;0.84)	(0.02;0.56)	(0.08;0.62)
$\rho(C,C^*)$	0.36	0.72	0.74	0.78	0.75	0.68	0.55
		(0.56;0.82)	(0.49;0.88)	(0.65;0.87)	(0.50;0.88)	(0.49;0.79)	(0.23;0.73)
$\rho(I,I^*)$	0.28	-0.08	0.28	0.50	0.66	-0.29	0.20
		(-0.38;0.24)	(0.00;0.50)	(0.21;0.70)	(0.48;0.78)	(-0.56;0.03)	(-0.04;0.41)
$\rho(L,L^*)$	0.40	0.54	0.59	0.75	0.78	0.47	0.53
		(0.30;0.71)	(0.35;0.76)	(0.59;0.85)	(0.63;0.88)	(0.20;0.67)	(0.26;0.72)

Note: Table 3.2 shows the moments of the model for GHH preferences. σ denotes the standard deviation of a variable and ρ denotes the correlation between two variables. 5th and 95th percentile confidence bands in parenthesis. Moments are the median of 500 replications.

3.5 Conclusion

There is increasing empirical evidence that the risk aversion of agents is time-varying and countercyclical. In this paper we introduced countercyclical risk aversion into a standard international RBC model and analyzed its effects on international business cycle moments. Once the risk aversion becomes slightly countercyclical the correlation between the real exchange rate and relative consumption across countries vanishes and turns zero or slightly negative as observed in the data. The model is hence able to account for the Backus-Smith Puzzle. In addition our model induces a higher cross-country correlation of investment and labor and therefore makes some progress in the International-

Comovement Puzzle. Based on the behavioral findings by Cohn et al. (2015) and Guiso et al. (2018) we introduced countercyclical risk aversion in a reduced form process. We documented that our findings are robust to various functional forms of the preferences. Further research might want to be more explicit about how agents' risk aversion is affected by macroeconomic fundamentals e.g. by introducing a learning process for the risk aversion.

3.A Appendix

3.A.1 Steady State

We here derive the steady states for the home economy. Since both economies are fully symmetric the steady states for the home and the foreign economy are identical. We know that in the steady state

$$Q = \beta. \tag{3.A1}$$

The output to capital ratio can then be derived knowing that the marginal product of capital equals $\frac{1}{Q} - (1 - \delta)$

$$\frac{Y}{K} = \frac{\frac{1}{Q} - (1 - \delta)}{\alpha}. \tag{3.A2}$$

From the budget constraint we get by knowing that in steady state $D = 0$ and that the labor and wage share add up to one

$$\frac{C}{Y} = 1 - \delta \frac{K}{Y} \tag{3.A3}$$

where $\frac{K}{Y}$ is the inverse of the output to capital ratio in Equation (3.A2). Steady state labor is exogenously fixed at

$$L = 0.30. \tag{3.A4}$$

Capital follows from the Cobb-Douglas production function as

$$K = \left[\frac{L^{(1-\alpha)}}{\frac{Y}{K}} \right]^{\frac{1}{(1-\alpha)}} \tag{3.A5}$$

where $\frac{Y}{K}$ is the output to capital ratio from Equation (3.A2). Steady state output is produced using a Cobb-Douglas function and follows as

$$Y = K^{\alpha} L^{1-\alpha}. \tag{3.A6}$$

Consumption can be derived from the consumption to output ratio in Equation (3.A3) and output in Equation (3.A6)

$$C = \frac{C}{Y} Y. \tag{3.A7}$$

Investment in the steady state follows from the capital law of motion as

$$I = \delta K. \tag{3.A8}$$

The return on capital equals

$$R = \frac{1}{Q} - (1 - \delta) \,. \tag{3.A9}$$

Wages are the marginal product of labor

$$W = (1 - \alpha) K^{\alpha} L^{-\alpha} \tag{3.A10}$$

where K is steady state capital as in Equation (3.A5) and L is the exogenously fixed steady state labor supply from Equation (3.A4). The home intermediate good is

$$Y_H = \omega Y \tag{3.A11}$$

where ω is the share of home intermediate goods. The foreign intermediate good is then

$$Y_F = (1 - \omega) Y. \tag{3.A12}$$

For the foreign country we have that in steady state

$$Y_H^* = (1 - \omega) Y \tag{3.A13}$$

and

$$Y_F^* = \omega Y. \tag{3.A14}$$

Since in the steady state $\Delta Y = 0$ the AR(1) process for the risk aversion becomes

$$\gamma = 0 \tag{3.A15}$$

and the risk aversion in steady state becomes

$$\sigma = 2. \tag{3.A16}$$

We finally know that in steady state the price of the final good P, the price of the home intermediate good P_H, and the price of the foreign intermediate good P_F are 1 as well as the real exchange rate $RER = 1$. Table 3.A1 provides the steady states for the parameter values shown in Table 3.1. Since both countries are symmetric, the steady states are the same for both countries.

TABLE 3.A1
Steady States

Variable	Description	Steady State
Y	Output	1.11
Y_H	Home Intermediate Good	1.00
Y_F	Foreign Intermediate Good	0.11
C	Consumption	0.83
I	Investment	0.28
K	Capital	11.40
L	Labor	0.30
R	Return on Capital	0.035
W	Wage	2.37
D	Debt	0.00
Y/K	Output to Capital Ratio	0.10
C/Y	Consumption to Output Ratio	0.74
γ	Risk Aversion Process	0.00
σ	Risk Aversion	2.00
Q	Debt Price	0.98

Note: Table 3.A1 shows the steady states of the model using the parameters in Table 3.1.

3.A.2 Equilibrium Conditions

We give here the full set of equations to solve the model. Relative prices of home and foreign intermediate goods are defined as

$$\tilde{P}_{H,t} = \frac{P_{H,t}}{P_t} \tag{3.A17}$$

$$\tilde{P}^*_{F,t} = \frac{P_{F,t}}{P^*_t} \tag{3.A18}$$

Final good prices using that the law of one price holds so that $P_{H,t} = P^*_{H,t}$ and $P_{F,t} = P^*_{F,t}$

$$P_t = \left[\omega P_{H,t}^{1-\theta} + (1-\omega) P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{3.A19}$$

$$P^*_t = \left[\omega P_{F,t}^{1-\theta} + (1-\omega) P_{H,t}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{3.A20}$$

Labor supply

$$-\frac{U_{t,L}}{U_{t,C}} = W_t \tag{3.A21}$$

$$-\frac{U^*_{t,L}}{U^*_{t,C}} = W^*_t \tag{3.A22}$$

Marginal utilities for labor and consumption in the Cobb-Douglas, GHH, and KPR case are defined in the main text in Equation (3.17) and (3.18) for Cobb-Douglas preferences, in Equation (3.21) and (3.22) for GHH preferences, and in Equation (3.25) and (3.26) for KPR preferences. Euler equations for investment with λ_t and μ_t being the Lagrange multiplier of the budget constraint and the capital law of motion

$$\lambda_t = \mu_t \left[1 - \phi\left(\frac{I_t}{I_{t-1}} - 1\right)\right] + \beta\mathbb{E}_t\mu_{t+1}\left[\phi\left(\frac{I_{t+1}}{I_t} - 1\right)\frac{I_{t+1}}{I_t} - \frac{\phi}{2}\left(\frac{I_{t+1}}{I_t} - 1\right)^2\right] \tag{3.A23}$$

$$\lambda_t^* = \mu_t^* \left[1 - \phi\left(\frac{I_t^*}{I_{t-1}^*} - 1\right)\right] + \beta\mathbb{E}_t\mu_{t+1}^*\left[\phi\left(\frac{I_{t+1}^*}{I_t^*} - 1\right)\frac{I_{t+1}^*}{I_t^*} - \frac{\phi}{2}\left(\frac{I_{t+1}^*}{I_t^*} - 1\right)^2\right] \tag{3.A24}$$

Capital Euler

$$\mu_t = \beta\mathbb{E}_t\left[R_{t+1}\lambda_{t+1} + \mu_{t+1}(1-\delta)\right] \tag{3.A25}$$

$$\mu_t^* = \beta\mathbb{E}_t\left[R_{t+1}^*\lambda_{t+1}^* + \mu_{t+1}^*(1-\delta)\right] \tag{3.A26}$$

Capital law of motion

$$K_t = (1-\delta)K_{t-1} + I_t - \frac{\phi}{2}I_{t-1}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 \tag{3.A27}$$

$$K_t^* = (1-\delta)K_{t-1}^* + I_t^* - \frac{\phi}{2}I_{t-1}^*\left(\frac{I_t^*}{I_{t-1}^*} - 1\right)^2 \tag{3.A28}$$

Risk sharing where the real exchange rate $RER_t = \frac{P_t^*}{P_t}$

$$\mathbb{E}_t\left[\frac{U_{t+1,C}^*}{U_{t,C}^*}\frac{\tilde{P}_{H,t+1}}{\tilde{P}_{H,t}}\frac{RER_t}{RER_{t+1}}\right] = \mathbb{E}_t\left[\frac{U_{t+1,C}}{U_{t,C}}\frac{\tilde{P}_{H,t+1}}{\tilde{P}_{H,t}}\right] - \frac{\zeta}{\beta}D_t \tag{3.A29}$$

Consumption Euler

$$\frac{Q_t + \zeta D_t}{\beta} = \mathbb{E}_t\left[\frac{U_{t+1,C}}{U_{t,C}}\frac{\tilde{P}_{H,t+1}}{\tilde{P}_{H,t}}\right] \tag{3.A30}$$

Wages and capital returns derived from the intermediate goods producers optimization problem

$$R_t = \alpha e^{A_t}\tilde{P_{H,t}}K_{t-1}^{\alpha-1}L_t^{1-\alpha} \tag{3.A31}$$

$$R_t^* = \alpha e^{A_t^*}\tilde{P_{F,t}^*}\left(K_{t-1}^*\right)^{\alpha-1}\left(L_t^*\right)^{1-\alpha} \tag{3.A32}$$

$$W_t = (1-\alpha)e^{A_t}\tilde{P_{H,t}}K_{t-1}^{\alpha}L_t^{-\alpha} \tag{3.A33}$$

$$W_t^* = (1-\alpha)e^{A_t^*}\tilde{P_{F,t}^*}\left(K_{t-1}^*\right)^{\alpha}\left(L_t^*\right)^{-\alpha} \tag{3.A34}$$

Demand for intermediate goods derived from the final goods producers optimization problem

$$Y_{H,t} = \omega\tilde{P}_{H,t}^{-\theta}Y_t \tag{3.A35}$$

$$Y_{F,t} = (1-\omega)\left(\tilde{P}_{F,t}RER_t\right)^{-\theta}Y_t \tag{3.A36}$$

$$Y_{H,t}^{*} = (1-\omega)\left(\frac{\tilde{P}_{H,t}}{RER_t}\right)^{-\theta} Y_t^{*} \tag{3.A37}$$

$$Y_{F,t}^{*} = \omega \tilde{P}_{F,t}^{-\theta} Y_t^{*} \tag{3.A38}$$

Market clearing final good

$$Y_t = C_t + I_t \tag{3.A39}$$

$$Y_t^{*} = C_t^{*} + I_t^{*} \tag{3.A40}$$

Market clearing intermediate good

$$Y_{H,t} + Y_{H,t}^{*} = e^{A_t} K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{3.A41}$$

$$Y_{F,t} + Y_{F,t}^{*} = e^{A_t^{*}} \left(K_{t-1}^{*}\right)^{\alpha} \left(L_t^{*}\right)^{1-\alpha} \tag{3.A42}$$

Final good production

$$Y_t = \left[\omega^{\frac{1}{\theta}} Y_{H,t}^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} Y_{F,t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{3.A43}$$

$$Y_t^{*} = \left[\omega^{\frac{1}{\theta}} \left(Y_{F,t}^{*}\right)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} \left(Y_{H,t}^{*}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{3.A44}$$

Processes for TFP

$$A_t = \rho_A A_{t-1} + \rho_A^{*} A_{t-1}^{*} + u_t \tag{3.A45}$$

$$A_t^{*} = \rho_A A_{t-1}^{*} + \rho_A^{*} A_{t-1} + u_t^{*} \tag{3.A46}$$

Risk aversion

$$\sigma_t = \sigma + e^{\gamma_t} - 1 \tag{3.A47}$$

$$\sigma_t^{*} = \sigma + e^{\gamma_t^{*}} - 1 \tag{3.A48}$$

Risk aversion processes

$$\gamma_t = \rho_\sigma \gamma_{t-1} - \eta_\sigma \Delta Y_t \tag{3.A49}$$

$$\gamma_t^{*} = \rho_\sigma \gamma_{t-1}^{*} - \eta_\sigma \Delta Y_t^{*} \tag{3.A50}$$

Growth rate of final good

$$\Delta Y_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}} \tag{3.A51}$$

$$\Delta Y_t^{*} = \frac{Y_t^{*} - Y_{t-1}^{*}}{Y_{t-1}^{*}} \tag{3.A52}$$

Law of motion of the bond

$$P_{H,t}Q_tD_t = P_{H,t}Y^*_{H,t} - P_{F,t}Y_{F,t} + P_{H,t}D_{t-1} - P_{H,t}\frac{\zeta}{2}D_t^2 \tag{3.A53}$$

3.A.3 Log-Linear Euler Equations

GHH

The Euler equation with GHH preferences becomes

$$\frac{Q_t + \zeta D_t}{\beta} = \mathbb{E}_t\left[\frac{(C_t - \psi L_t^\nu)^{\sigma_t}}{\left(C_{t+1} - \psi L_{t+1}^\nu\right)^{\sigma_{t+1}}}\frac{\tilde{P}_{H,t+1}}{\tilde{P}_{H,t}}\right] \tag{3.A54}$$

or in logarithmic terms

$$\ln(Q_t + \zeta D_t) - \ln(\beta) + \ln\left(\tilde{P}_{H,t}\right) - \ln\left(\tilde{P}_{H,t+1}\right) = \sigma_t \ln(C_t - \psi L_t^\nu) - \sigma_{t+1}\ln(C_{t+1} - \psi L_{t+1}^\nu). \tag{3.A55}$$

Log-linearizing around the steady state gives

$$\begin{aligned} -\widetilde{r_t} + \zeta\widetilde{D_t} + \widetilde{\tilde{P}_{H,t}} - \widetilde{\tilde{P}_{H,t+1}} = {} & \frac{\sigma C}{C - \psi L^\nu}\widetilde{C_t} - \frac{\sigma C}{C - \psi L^\nu}\widetilde{C_{t+1}} - \frac{\sigma\psi\nu L^\nu}{C - \psi L^\nu}\widetilde{L_t} + \frac{\sigma\psi\nu L^\nu}{C - \psi L^\nu}\widetilde{L_{t+1}} \\ & + \ln(C - \psi L^\nu)\,\sigma\widetilde{\sigma_t} - \ln(C - \psi L^\nu)\,\sigma\widetilde{\sigma_{t+1}}. \end{aligned} \tag{3.A56}$$

Then after rearranging terms we get for consumption growth

$$\widetilde{C_{t+1}} - \widetilde{C_t} = \left(\widetilde{r_t} - \zeta\widetilde{D_t} - \widetilde{\tilde{P}_{H,t+1}} + \widetilde{\tilde{P}_{H,t}}\right)\frac{C - \psi L^\nu}{\sigma C} + \Gamma\left(\widetilde{L_{t+1}} - \widetilde{L_t}\right) + \Lambda\left(\widetilde{\sigma_{t+1}} - \widetilde{\sigma_t}\right) \tag{3.A57}$$

where

$$\Gamma = \frac{\psi\nu L^\nu}{C} \tag{3.A58}$$

and

$$\Lambda = -\ln(C - \psi L^\nu)\frac{C - \psi L^\nu}{C}. \tag{3.A59}$$

KPR

The Euler equation with KPR preferences becomes

$$\frac{Q_t + \zeta D_t}{\beta} = \mathbb{E}_t\left[\frac{\left[C_t(1 - \psi L_t^\nu)\right]^{\sigma_t}\left(1 - \psi L_{t+1}^\nu\right)}{\left[C_{t+1}\left(1 - \psi L_{t+1}^\nu\right)\right]^{\sigma_{t+1}}(1 - \psi L_t^\nu)}\frac{\tilde{P}_{H,t+1}}{\tilde{P}_{H,t}}\right] \tag{3.A60}$$

or in logarithmic terms

$$\ln\left(Q_t + \zeta D_t\right) - \ln\left(\beta\right) + \ln\left(\tilde{P}_{H,t}\right) - \ln\left(\tilde{P}_{H,t+1}\right) = \sigma_t \ln\left(C_t\right) + \sigma_t \ln\left(1 - \psi L_t^{\nu}\right)$$
$$+ \ln\left(1 - \psi L_{t+1}^{\nu}\right) - \sigma_{t+1} \ln\left(C_{t+1}\right) - \sigma_{t+1} \ln\left(1 - \psi L_{t+1}^{\nu}\right) - \ln\left(1 - \psi L_t^{\nu}\right). \quad (3.A61)$$

Log-linearizing around the steady state gives

$$-\widetilde{r_t} + \zeta\widetilde{D_t} + \widetilde{\tilde{P}_{H,t}} - \widetilde{\tilde{P}_{H,t+1}} = \sigma\widetilde{C_t} - \sigma\widetilde{C_{t+1}} + \widetilde{L_t}\left[\frac{\psi\nu L^{\nu} - \sigma\psi\nu L^{\nu}}{1 - \psi L^{\nu}}\right] - \widetilde{L_{t+1}}\left[\frac{\psi\nu L^{\nu} - \sigma\psi\nu L^{\nu}}{1 - \psi L^{\nu}}\right]$$
$$+ \sigma\widetilde{\sigma_t}\left[\ln\left(C\right) + \ln\left(1 - \psi L^{\nu}\right)\right] - \sigma\widetilde{\sigma_{t+1}}\left[\ln\left(C\right) + \ln\left(1 - \psi L^{\nu}\right)\right]. \quad (3.A62)$$

Then after rearranging terms we get for consumption growth

$$\widetilde{C_{t+1}} - \widetilde{C_t} = \frac{\widetilde{r_t} - \zeta\widetilde{D_t} - \widetilde{\tilde{P}_{H,t+1}} + \widetilde{\tilde{P}_{H,t}}}{\sigma} + \Gamma\left(\widetilde{L_{t+1}} - \widetilde{L_t}\right) + \Lambda\left(\widetilde{\sigma_{t+1}} - \widetilde{\sigma_t}\right) \quad (3.A63)$$

where

$$\Gamma = \frac{\sigma - 1}{\sigma}\left[\frac{\psi\nu L^{\nu}}{1 - \psi L^{\nu}}\right] \quad (3.A64)$$

and

$$\Lambda = -\left[\ln\left(C\right) + \ln\left(1 - \psi L^{\nu}\right)\right]. \quad (3.A65)$$

3.A.4 Derivation of the Cobb-Douglas Consumption Share

Combining the labor supply equation in Equation (3.A21) and the steady state expression for wages in Equation (3.A10) gives

$$\left(\frac{1-\tau}{1-L}\right)\frac{C}{\tau} = \left(1-\alpha\right)K^{\alpha}L^{-\alpha} \quad (3.A66)$$

dividing both sides by steady state production Y and substituting in

$$\left(\frac{1-\tau}{1-L}\right)\frac{C}{Y}\frac{1}{\tau} = \frac{\left(1-\alpha\right)K^{\alpha}L^{-\alpha}}{K^{\alpha}L^{1-\alpha}} \quad (3.A67)$$

so that canceling terms and multiplying both sides by $(1-L)$ gives

$$\left(\frac{1-\tau}{\tau}\right)\frac{C}{Y} = \frac{\left(1-L\right)\left(1-\alpha\right)}{L} \quad (3.A68)$$

so that when multiplying by Y, dividing by C, and subsequently adding one on both sides yields

$$\frac{1}{\tau} = \frac{CL}{CL} + \frac{Y\left(1-L\right)\left(1-\alpha\right)}{CL} \quad (3.A69)$$

and by taking the reciprocal one gets

$$\tau = \frac{CL}{CL + Y\left(1-L\right)\left(1-\alpha\right)} \tag{3.A70}$$

multiplying and dividing the right hand side by Y then gives

$$\tau = \frac{L\frac{C}{Y}}{L\frac{C}{Y} + \left(1-L\right)\left(1-\alpha\right)}. \tag{3.A71}$$

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