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Achievable Rates for Full-Duplex Massive MIMO Systems With Low-Resolution ADCs/DACs

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ABSTRACT This paper investigates the uplink and downlink achievable rates of full-duplex (FD) massive multi-input-multi-output (MIMO) systems in which low-resolution analog-to-digital converters/digital-to-analog converters (ADCs/DACs) are employed and maximum ratio combining/maximum ratio transmission processing are adopted. Then, employing an additive quantization noise model, we derive approximate expressions of the uplink and downlink achievable rates, in which the effect of the quantization error, the loop interference, and the inter-user interference is considered. The theoretical results show that using proper power scaling law and more antennas can eliminate the interference and the noise. Furthermore, under the fixed number of antennas, the uplink and downlink approximate achievable rates will become a constant, as the number of quantization bits tends to infinity. Increasing the resolution of ADCs/DACs will limitedly improve the system performance but cause excessive overhead and power consumption, so adopting low-resolution ADCs/DACs in FD massive MIMO systems is sensible.

INDEX TERMS Full-duplex, low-resolution ADCs/DACs, quantization error, achievable rate, additive quantization noise model.

I. INTRODUCTION

To further improve spectral efficiency (SE), massive MIMO system, in which the multi-user interference can be reduced to zero as the number of antennas at the base station (BS) is infinity, has been proposed [1], [2]. On the other hand, theoretically, full-duplex (FD) technology doubles spectral efficiency theoretically because it transmits and receives signals on the same frequency resources at the same time and it also enables two-way communication [3], [4]. Reference [4] analyzed the capacity of FD MIMO system and compared it with that of half-duplex MIMO system. However, one of the main problems is how to wipe out the loop interference (LI) caused by FD. To obtain the beneficial impacts brought by the two key techniques, one has conducted a study about the massive MIMO system with the FD functionality [5]–[7]. Wang *et al.* [6] studied the energy/spectral efficiency of multi-user FD massive MIMO systems over Rayleigh fading channels. Contrary to [6], [7] investigated the achievable rates of multi-user FD massive MIMO systems over Rician fading, which is more general and realistic than Rayleigh fading, with zero-forcing reception/zero-forcing transmission (ZFR/ZFT) and maximum ratio combining/maximum ratio transmission (MRC/MRT) processing.

However, there still exist some problems on further improvement in the practical implementation of massive MIMO. Too much hardware cost and radio frequency (RF) circuit power consumption will result from BS which has lots of antennas. Particularly, each receive antenna and transmit antenna needs one analog-digital converter (ADC) and digital-to-analog converter (DAC) unit, respectively.

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A typical flash ADC with b-bit and the sampling frequency f_s operates $f_s 2^b$ conversion steps per second. So, the use of high resolution ADCs can cause excessive overhead and power consumption, a significant bottleneck in applying large-scale MIMO systems. To solve the problem, adopting low-resolution ADCs is the most direct method [8]–[10]. At the same time, Zhang *et al.* [10] showed that increasing the number of receive antennas in massive MIMO systems can compensate the performance loss resulted from employing low-resolution ADCs. So employing low-resolution ADCs is a feasible solution for the power consumption problem in massive MIMO systems. Recently, the impact of low-resolution ADCs on large-scale MIMO systems has been studied. Also, some researchers have carried out studies about Massive MIMO systems with low-resolution ADCs and they found the uplink achievable rate under Rayleigh fading channel in [8]. Even they began to study the Rician fading channel from which the uplink SE was obtained in [10]. Jia *et al.* [11], Jiao *et al.* [12], Kong *et al.* [13], and Kong *et al.* [14] gave consideration to a more general FD massive MIMO relaying system in which the relay is fixed with low-resolution ADCs, and got the similar result of the sum rate. Reference [15] researched into the low-resolution DACs processing with multiple transmit antennas on the basis of minimum mean square error (MMSE).

The works [11], [12] assumed that the receive antennas of the FD relay are fixed with low-resolution ADCs and the works [13], [14] considered that low-resolution ADCs are applied at the relay as well as users. From the above literature review, we can find that FD massive MIMO relay with low-resolution ADCs could be another method to further increase the SE. But the literatures mentioned above did not show that the low-resolution DACs were simultaneously applied at the transmit antennas of the relay. In contrast to the above works, in this paper, we give consideration to a FD massive MIMO system in which low-resolution ADCs/DACs are used in both the receiver and transmitter so that the hardware cost and the power consumption will be reduced. Aided by additive quantization noise model (AQNM) which has been applied widely [10], the uplink and downlink approximate achievable rates are derived in this paper. Nevertheless, suppressing the impact of the LI and researching into the effect of the quantization error on the system performance are crucial if the receive and transmit antennas of the FD mode BS use the low-resolution ADCs/DACs at the same time. Therefore, the approximate expressions of the uplink and downlink achievable rates derived from the study indicate that it possible to eliminate the multi-user interference, LI, quantization noise and additive white Gaussian noise (AWGN) by using power scaling law appropriately and using more antennas. Moreover, we find that adopting low-resolution ADCs/DACs in FD massive MIMO systems is sensible. The main contributions of this paper are summarized as follows.

- We present the multi-user massive MIMO systems in which both the FD mode and low-resolution ADCs/DACs are simultaneously employed.
- We derive original closed-form approximate expressions for the uplink and downlink achievable rates of FD massive MIMO systems with low-resolution ADCs/DACs, respectively. The theoretical results display the effects of the number of quantization bits, the number of BS antennas and the transmit power of each user and BS on the achievable rate performance.
- We analyze the asymptotic performance, based on approximate achievable rates for uplink and downlink of FD massive MIMO systems with low-resolution ADCs/DACs, when the transmit power of BS, the number of quantization bits and the number of BS antennas tend to infinity, respectively. These results show the effects of LI, the inter-user interference (IUI) and the quantization error on the achievable rate performance.

The rest of this paper is organized as follows: system model is presented in Section II. The approximate achievable rates of uplink and downlink are analyzed in Section III. Numerical results are then provided in Section IV, and the paper is concluded in Section V.

In this paper, we use lower and upper case boldface to define vectors and matrices, respectively. Let $(\boldsymbol{A})^H$, $tr\{\boldsymbol{A}\}$, $\|(\boldsymbol{A})\|$ respectively stand for the matrix conjugate transpose, matrix trace, the Euclidean norm of matrix $\boldsymbol{A}$,; $\mathbb{E}\{\bullet\}$ represents the expectation; $[\boldsymbol{A}]_{nn}$ denotes the $n \times n$ diagonal entry of matrix $\boldsymbol{A}$; The $M \times M$ identity matrix reads as $\boldsymbol{I}_M$; $x_i \sim \mathcal{CN}\left(0, \sigma_i^2\right)$ whose mean and variance are 0 and σ_i^2,respectively, represents the ith entry of a circularly symmetric complex-Gaussian vector $\boldsymbol{x}$.

II. SYSTEM MODEL

Consider a multi-user FD massive MIMO system model shown in Fig. 1 which comprised of an FD radio BS and N users. It is assumed that the BS is equipped with M transmitter antennas and M receiver antennas, where each of the receive antennas is connected with low-resolution ADCs and each of transmit antennas is linked to low-resolution DACs, and each user has a transmit antenna and a receive antenna [17]. We define $\boldsymbol{G}_U = \left[\boldsymbol{g}_{U,1}, \cdots, \boldsymbol{g}_{U,N}\right] \in \mathbb{C}^{M\times N}$, $\boldsymbol{G}_D = \left[\boldsymbol{g}_{D,1}, \cdots, \boldsymbol{g}_{D,N}\right] \in \mathbb{C}^{M\times N}$ as the uplink and downlink channel matrix, respectively. To facilitate the next discussions in this paper, we decompose the uplink and downlink channel matrix as $\boldsymbol{G}_a = \boldsymbol{H}_a\boldsymbol{D}_a^{1/2}$ $(a \in \{U, D\})$, where $\boldsymbol{H}_a$ is the $M \times N$ small-scale fading channel matrix and has elements that are independent and identically distributed (i.i.d.) $\mathcal{CN}(0, 1)$, while $\boldsymbol{D}_a$ is the $N \times N$ large-scale channel fading diagonal matrix whose nth diagonal entry is $[\boldsymbol{D}_a]_{nn} = \beta_{a,n}$. $\boldsymbol{G}_{LI} \in \mathbb{C}^{M\times M}$ represents the LI channel matrix between transmit antennas and receive antennas of the BS, each of whose entries follows the distribution of $\mathcal{CN}\left(0, \mu_{LI}^2\right)$. Further, $\boldsymbol{G}_{IU} = \left[\boldsymbol{g}_{IU,1}, \cdots, \boldsymbol{g}_{IU,N}\right] \in \mathbb{C}^{N\times N}$ is used to denote the IUI channel with $[\boldsymbol{G}_{IU}]_{i,j}$ defined as the channel coefficient from the ith user to the jth user and $[\boldsymbol{G}_{IU}]_{nn} = g_{nn}$ denoted as the self-interference channel coefficient of the nth user. Each element in $\boldsymbol{G}_{IU}$ is presumed as the i.i.d. random variable with the distribution of $\mathcal{CN}\left(0, \delta_{ij}^2\right)$.

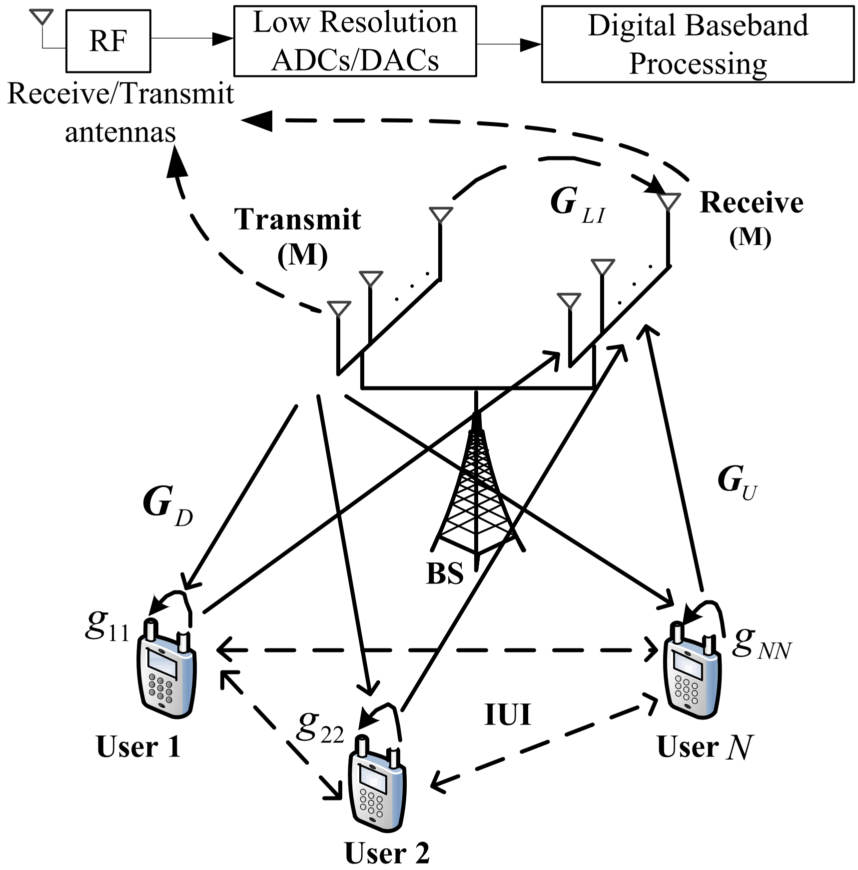

FIGURE 1. Illustration of a FD massive MIMO system with N users.

Before the ADCs for uplink data transmission, the received signal $\boldsymbol{y}_U$ at the BS is given as [4] by

$$\boldsymbol{y}_U = \sqrt{P_U}\boldsymbol{G}_U\boldsymbol{x}_U + \sqrt{\eta}\sqrt{P_D}\boldsymbol{G}_{LI}\tilde{\boldsymbol{y}}_D + \boldsymbol{n}_U \quad (1)$$

And the downlink un-quantized transmit signal $\boldsymbol{y}_D$ is given by

$$\boldsymbol{y}_D = \boldsymbol{W}\boldsymbol{x}_D \quad (2)$$

where the transmit signal vectors of the BS and users are represented by $\boldsymbol{x}_D = \left[x_{D,1}, \cdots, x_{D,N}\right]^T$ and $\boldsymbol{x}_U = \left[x_{U,1}, \cdots, x_{U,N}\right]^T$, satisfying $\mathbb{E}\left\{\boldsymbol{x}_a\boldsymbol{x}_a^H\right\} = \boldsymbol{I}_N$, $(a \in \{D, U\})$, P_D and P_U are denoted as the transmit power of BS and the users, respectively, $\tilde{\boldsymbol{y}}_D$ is the quantized transmit signal at the BS, $\boldsymbol{W} = [\boldsymbol{w}_1, \cdots, \boldsymbol{w}_N] \in \mathbb{C}^{M\times N}$ represents the downlink precoding matrix, η refers to the factor which is closely related to the LI, the term $\boldsymbol{n}_U \sim \left(0, \sigma_U^2\boldsymbol{I}_M\right)$ represents the additive white Gaussian noise (AWGN) vector at the BS.

As the quantization error can be well approximated as a linear gain with AQNM, the received signal $\boldsymbol{y}_U$ and the transmit signal $\boldsymbol{y}_D$ after the ADCs/DACs at the BS can be given, respectively, as follows

$$\tilde{\boldsymbol{y}}_U = \alpha_u\boldsymbol{y}_U + \boldsymbol{q}_U \quad (3)$$

$$\tilde{\boldsymbol{y}}_D = \alpha_d\boldsymbol{y}_D + \boldsymbol{q}_D \quad (4)$$

TABLE 1. $\rho_u(\rho_d)$ for different ADC quantization bits b ($b \leq 5$).

b	1	2	3	4	5
$\rho_u(\rho_d)$	0.3634	0.1175	0.03454	0.009497	0.002499

with $\alpha_u = 1 - \rho_u$, $\alpha_d = 1 - \rho_d$, where ρ_u and ρ_d denote the inverse of the signal-to quantization-noise ratios for the uplink and downlink, respectively, which are decided by the number of quantization bits. Table 1 gives the values of ρ_u and ρ_d for the number of quantization bits $b \leq 5$ for the non-uniform minimum mean-square-error quantizer. While it usually approximates ρ_u and ρ_d for $b > 5$ by computing $\rho_u(\rho_d) = \frac{\pi\sqrt{3}}{2}\cdot 2^{-2b}$ [15], [16]. $\boldsymbol{q}_U$ and $\boldsymbol{q}_D$ represent the additive Gaussian quantization noise for uplink and downlink signals, respectively. When channel matrices $\boldsymbol{G}_a$ $(a \in \{D, U\})$, $\boldsymbol{G}_{LI}$, $\boldsymbol{G}_{IU}$ and precoding matrix $\boldsymbol{W}$ are given, the covariance matrices of $\boldsymbol{q}_U$ and $\boldsymbol{q}_D$ can be written, respectively, as [8]

$$\begin{aligned}\boldsymbol{R}_{\boldsymbol{q}_U} &= \mathbb{E}\left\{\boldsymbol{q}_U\boldsymbol{q}_U^H\right\} \\ &= \alpha_u\left(1-\alpha_u\right) diag\left(P_U\boldsymbol{G}_U\boldsymbol{G}_U^H + \boldsymbol{T} + \sigma_U^2\boldsymbol{I}_M\right) \\ \boldsymbol{R}_{\boldsymbol{q}_D} &= \mathbb{E}\left\{\boldsymbol{q}_D\boldsymbol{q}_D^H\right\} \\ &= \alpha_d\left(1-\alpha_d\right) diag\left(\boldsymbol{W}\boldsymbol{W}^H\right)\end{aligned}$$

where $\boldsymbol{T} = \eta P_D\boldsymbol{G}_{LI}\left(\alpha_d^2\boldsymbol{W}\boldsymbol{W}^H + \boldsymbol{R}_{\boldsymbol{q}_D}\right)\boldsymbol{G}_{LI}^H$.

III. ANALYSIS OF UPLINK AND DOWNLINK ACHIEVABLE RATES WITH MRC/MRT PROCESSING

In this section, the tractable approximate achievable rates of uplink and downlink with perfect channel state information (CSI) are derived. Based on the derived approximate expressions, we try to find that employing proper power scaling law and using more BS antennas can eliminate the quantization error, LI and the IUI. Moreover, we find that when the number of quantization bit b tends to infinity under the fixed number of antennas M, the uplink and downlink approximate achievable rates will converge to a constant.

In this paper, without loss of generality, we will give consideration to the maximal-ratio combining / maximum ratio transmission (MRC/MRT), the beamforming and precoding matrix of which are given as [17]

$$\begin{cases} \boldsymbol{F} = \boldsymbol{G}_U \\ \boldsymbol{W} = \boldsymbol{G}_D \left(tr\left(\boldsymbol{G}_D^H \boldsymbol{G}_D \right) \right)^{-\frac{1}{2}} \end{cases} \tag{5}$$

where $\boldsymbol{F}$ stands for the uplink decoding matrix at the receiver side and $\boldsymbol{W}$ denotes the downlink precoding matrix at the transmit side.

First, we introduce a lemma in support of the following derivations. Reference [18, Lemma 3] can prove it easily when Rician K-factor is set as zero, and thus is omitted due to space limitation.

Lemma 1: The following results hold:

$$\begin{aligned} \mathbb{E}\left\{ \left\| \boldsymbol{g}_{a,n} \right\|^2 \right\} &= \beta_{a,n} M \\ \mathbb{E}\left\{ \left\| \boldsymbol{g}_{a,n} \right\|^4 \right\} &= \beta_{a,n}^2 \left(M + M^2 \right) \\ \mathbb{E}\left\{ \left| \boldsymbol{g}_{a,n}^H \boldsymbol{g}_{a,i} \right|^2 \right\} &= \beta_{a,n} \beta_{a,i} M \end{aligned} \tag{6}$$

A. ANALYSIS OF UPLINK ACHIEVABLE RATE

From (3) and (5), we can obtain the quantized signal vector after the MRC receiver as follows

$$\boldsymbol{r}_U = \boldsymbol{G}_U^H \tilde{\boldsymbol{y}}_U \tag{7}$$

Then substituting (1), (2), (3) and (4) into (7), the processed quantized signal vector $\boldsymbol{r}_U$ can be reshaped as follows

$$\begin{aligned} \boldsymbol{r}_U = & \alpha_u \sqrt{P_U} \boldsymbol{G}_U^H \boldsymbol{G}_U \boldsymbol{x}_U \\ & + \alpha_u \sqrt{\eta} \sqrt{P_D} \boldsymbol{G}_U^H \boldsymbol{G}_{LI} \left(\alpha_d \boldsymbol{W} \boldsymbol{x}_D + \boldsymbol{q}_D \right) \\ & + \alpha_u \boldsymbol{G}_U^H \boldsymbol{n}_U + \boldsymbol{G}_U^H \boldsymbol{q}_U \end{aligned} \tag{8}$$

From (8), the received signal of nth user can be expressed as follows

$$\begin{aligned} r_{U,n} = & \alpha_u \sqrt{P_U} \boldsymbol{g}_{U,n}^H \boldsymbol{g}_{U,n} x_{U,n} + \alpha_u \sqrt{P_U} \sum_{i=1, i \neq n}^{N} \boldsymbol{g}_{U,n}^H \boldsymbol{g}_{U,i} x_{U,i} \\ & + \alpha_u \alpha_d \sqrt{\eta} \sqrt{P_D} \sum_{i=1}^{N} \frac{\boldsymbol{g}_{U,n}^H \boldsymbol{G}_{LI}}{\sqrt{tr\left(\boldsymbol{G}_D^H \boldsymbol{G}_D \right)}} \boldsymbol{g}_{D,i} x_{D,i} \\ & + \alpha_u \sqrt{\eta} \sqrt{P_D} \boldsymbol{g}_{U,n}^H \boldsymbol{G}_{LI} \boldsymbol{q}_D + \alpha_u \boldsymbol{g}_{U,n}^H \boldsymbol{n}_U + \boldsymbol{g}_{U,n}^H \boldsymbol{q}_U \end{aligned} \tag{9}$$

where $\boldsymbol{g}_{U,n}$ stands for the nth column vector of $\boldsymbol{G}_U$.

Theorem 1: When FD mode with low-resolution ADCs/DACs and MRC/MRT processing with perfect CSI in massive MIMO systems are employed, the uplink achievable rate of the nth user can be approximated by

$$R_{U,n} \approx \log_2 \left(1 + \frac{\alpha_u P_U \beta_{U,n} (1 + M)}{\Upsilon_{U,n}} \right) \tag{10}$$

where

$$\begin{aligned} \Upsilon_{U,n} = & P_U \sum_{i=1, i \neq n}^{N} \beta_{U,i} + \alpha_d \eta P_D \mu_{LI} M + \sigma_U^2 \\ & + 2 (1 - \alpha_u) P_U \beta_{U,n} \end{aligned}$$

Proof: According to the preceding assumption in this paper, the ergodic achievable uplink rate of the nth user can be given by

$$R_{U,n} = \mathbb{E}\left\{ \log_2 \left(1 + \frac{\left| A_{U,n} \right|^2}{\Psi_{U,n}} \right) \right\} \tag{11}$$

where

$$A_{U,n} = \alpha_u \sqrt{P_U} \boldsymbol{g}_{U,n}^H \boldsymbol{g}_{U,n} x_{U,n} \tag{12}$$

$$\begin{aligned} \Psi_{U,n} = & \left| B_{U,n} \right|^2 + \left| C_{U,n} \right|^2 + \left| D_{U,n} \right|^2 \\ & + \left| E_{U,n} \right|^2 + \left| F_{U,n} \right|^2 \end{aligned} \tag{13}$$

with

$$\begin{aligned} B_{U,n} &= \alpha_u \sqrt{P_U} \sum_{i=1, i \neq n}^{N} \boldsymbol{g}_{U,n}^H \boldsymbol{g}_{U,i} x_{U,i} \\ C_{U,n} &= \alpha_u \alpha_d \sqrt{\eta} \sqrt{P_D} \sum_{i=1}^{N} \frac{\boldsymbol{g}_{U,n}^H \boldsymbol{G}_{LI}}{\sqrt{tr\left(\boldsymbol{G}_D^H \boldsymbol{G}_D \right)}} \boldsymbol{g}_{D,i} x_{D,i} \\ D_{U,n} &= \alpha_u \sqrt{\eta} \sqrt{P_D} \boldsymbol{g}_{U,n}^H \boldsymbol{G}_{LI} \boldsymbol{q}_D \\ E_{U,n} &= \alpha_u \boldsymbol{g}_{U,n}^H \boldsymbol{n}_U \\ F_{U,n} &= \boldsymbol{g}_{U,n}^H \boldsymbol{q}_U \end{aligned} \tag{14}$$

Applying [18, Lemma 1], (11) can be approximated as follows

$$R_{U,n} \approx \log_2 \left(1 + \frac{\mathbb{E}\left\{ \left| A_{U,n} \right|^2 \right\}}{\mathbb{E}\left\{ \Psi_{U,n} \right\}} \right) \tag{15}$$

where

$$\mathbb{E}\left\{ \left| A_{U,n} \right|^2 \right\} = \alpha_u^2 P_U \beta_{U,n}^2 \left(M + M^2 \right) \tag{16}$$

$$\begin{aligned} \mathbb{E}\left\{ \Psi_{U,n} \right\} = & \mathbb{E}\left\{ \left| B_{U,n} \right|^2 \right\} + \mathbb{E}\left\{ \left| C_{U,n} \right|^2 \right\} \\ & + \mathbb{E}\left\{ \left| D_{U,n} \right|^2 \right\} + \mathbb{E}\left\{ \left| E_{U,n} \right|^2 \right\} + \mathbb{E}\left\{ \left| F_{U,n} \right|^2 \right\} \end{aligned} \tag{17}$$

with

$$\mathbb{E}\left\{ \left| B_{U,n} \right|^2 \right\} = \alpha_u^2 P_U \sum_{i=1, i \neq n}^{N} \beta_{U,n} \beta_{U,i} M \tag{18}$$

$$\mathbb{E}\left\{\left|C_{U,n}\right|^2\right\} = \alpha_u^2\alpha_d^2\eta P_D\mu_{LI}\beta_{U,n}M^2 \tag{19}$$

$$\mathbb{E}\left\{\left|D_{U,n}\right|^2\right\} = \alpha_u^2\alpha_d\left(1-\alpha_d\right)\eta P_D\mu_{LI}\beta_{U,n}M^2 \tag{20}$$

$$\mathbb{E}\left\{\left|E_{U,n}\right|^2\right\} = \alpha_u^2\beta_{U,n}M\sigma_U^2 \tag{21}$$

$$\begin{aligned}\mathbb{E}\left\{\left|F_{U,n}\right|^2\right\} &= 2\alpha_u\left(1-\alpha_u\right)P_U\beta_{U,n}^2M \\ &\quad+\alpha_u\left(1-\alpha_u\right)\left(P_U\sum_{i=1,i\neq n}^{N}\beta_{U,n}\beta_{U,i}M\right) \\ &\quad+\alpha_u\left(1-\alpha_u\right)\Big(\alpha_d\eta P_D\mu_{LI}\beta_{U,n}M^2 \\ &\quad+\beta_{U,n}M\sigma_U^2\Big)\end{aligned} \tag{22}$$

The proof of Eq. (22) is given in Appendix A.

Based on Lemma 1, substituting (16), (17) to (15), (10) can be derived. □

From (10), we study the impact of three factors on the rate performance, which includes the number of quantization bits b, the number of antennas M and the transmit power of BS and each user. In contrast to [8], Theorem 1 involves the LI term caused by FD mode BS. We carry out an analysis of the following three asymptotic results respectively, so that we can have an intuitive understanding of Theorem 1.

Remark 1.1: As P_U, P_D and M are fixed and $b \to \infty$ ($\alpha_u = \alpha_d = 1$), $R_{U,n}$ can converge to the formula as follows

$$R_{U,n} \to \log_2\left(1+\frac{P_U\beta_{U,n}\left(1+M\right)}{P_U\sum\limits_{i=1,i\neq n}^{N}\beta_{U,i}+P_D\eta\mu_{LI}M+\sigma_U^2}\right) \tag{23}$$

From (23), we notice that the quantization error which is brought by ADCs/DACs can be eliminated once the number of quantization bits b is infinity. But FD leads to some effect of LI that cannot be wiped out. If a fixed M is given, the approximate achievable rate of uplink will become a constant. Thus, the system performance is limited although it can be improved by increasing ADCs/DACs resolution. Moreover, when the number of antennas is $M \to \infty$, we further obtain the result which is in agreement with [17, eq. (18)].

Remark 1.2: As b and M are fixed and $P_U = P_D \to \infty$, $R_{U,n}$ can converge to the formula as follows

$$R_{U,n} \to \log_2\left(1+\frac{\alpha_u\beta_{U,n}\left(1+M\right)}{\sum\limits_{i=1,i\neq n}^{N}\beta_{U,i}+\alpha_d\eta\mu_{LI}M+2\left(1-\alpha_u\right)\beta_{U,n}}\right) \tag{24}$$

(24) is in agreement with [8, eq. (25)] when the LI is wiped out thoroughly. From Remark 1.2, we find that the rate relies on the number of antennas M as well as the number of quantization bits b when the transmit power of BS and users tend to infinity. Furthermore, we also find that the uplink rate performance degradation cannot be compensated by increasing transmit power and we cannot ignore that FD really causes some effect of LI and low-resolution ADCs/DACs bring the quantization error.

Remark 1.3: If the transmit power of BS and each user are scaled with M, i.e., $P_U = E_U/M$ and $P_D = E_D/M$, where E_U, E_D is fixed, as $M \to \infty$, $R_{U,n}$ can converge to the formula as follows

$$R_{U,n} \to \log_2\left(1+\frac{\alpha_u E_U\beta_{U,n}}{\alpha_d\eta\mu_{LI}+\sigma_U^2}\right) \tag{25}$$

From (25), $R_{U,n} \to \log_2\left(1+\alpha_u E_U\beta_{U,n}\right)$ can be obtained when $\sigma_U^2 = 1$ which is consistent with [8, eq. (26)]. We found that using proper power scaling law and more antennas M can eliminate the quantization error and LI caused by FD. The number of quantization bits b determines the approximate uplink rate when M tends to infinity. If the number of quantization bits $b \to \infty$ and $\sigma_U^2=1$, we further get $R_{U,n} \to \log_2\left(1+E_U\beta_{U,n}\right)$ that is in agreement with [19, eq. (13)].

B. ANALYSIS OF DOWNLINK ACHIEVABLE RATE

From (2) and (4), the downlink received signal at the users can be expressed as follows

$$\begin{aligned}\mathbf{r}_D &= \sqrt{P_D}\mathbf{G}_D^H\tilde{\mathbf{y}}_D+\sqrt{P_U}\mathbf{G}_{IU,n}^H\mathbf{x}_U+\mathbf{n}_D \\ &= \alpha_d\sqrt{P_D}\mathbf{G}_D^H\mathbf{W}\mathbf{x}_D+\sqrt{P_D}\mathbf{G}_D^H\mathbf{q}_D+\sqrt{P_U}\mathbf{G}_{IU}^H\mathbf{x}_U+\mathbf{n}_D\end{aligned} \tag{26}$$

From (26), the received signal of nth user is expressed as follows

$$\begin{aligned}r_{D,n} &= \alpha_d\sqrt{P_D}\mathbf{g}_{D,n}^H\mathbf{w}_n x_{D,n}+\alpha_d\sqrt{P_D}\sum_{i=1,i\neq n}^{N}\mathbf{g}_{D,n}^H\mathbf{w}_i x_{D,i} \\ &\quad+\sqrt{P_D}\mathbf{g}_{D,n}^H\mathbf{q}_D+\sqrt{P_U}\mathbf{g}_{IU,n}^H\mathbf{x}_U+n_{D,n}\end{aligned} \tag{27}$$

Theorem 2: When FD mode with low-resolution ADCs/DACs and MRC/MRT processing with perfect CSI in massive MIMO systems are employed, the downlink achievable rate of the nth user can be approximated by

$$R_{D,n} \approx \log_2\left(1+\frac{\alpha_d^2P_D\beta_{D,n}^2(1+M)}{\alpha_d^2P_D\beta_{D,n}\sum\limits_{i=1,i\neq n}^{N}\beta_{D,i}+\Delta_1 tr\left(\mathbf{D}_D\right)+P_D\Delta_2}\right) \tag{28}$$

where $\Delta_1 = P_U\sum\limits_{i=1}^{N}\delta_{in}+\sigma_{D,n}^2$,

$$\Delta_2 = \alpha_d\left(1-\alpha_d\right)\beta_{D,n}\left(\beta_{D,n}+\sum_{i=1}^{N}\beta_{D,i}\right).$$

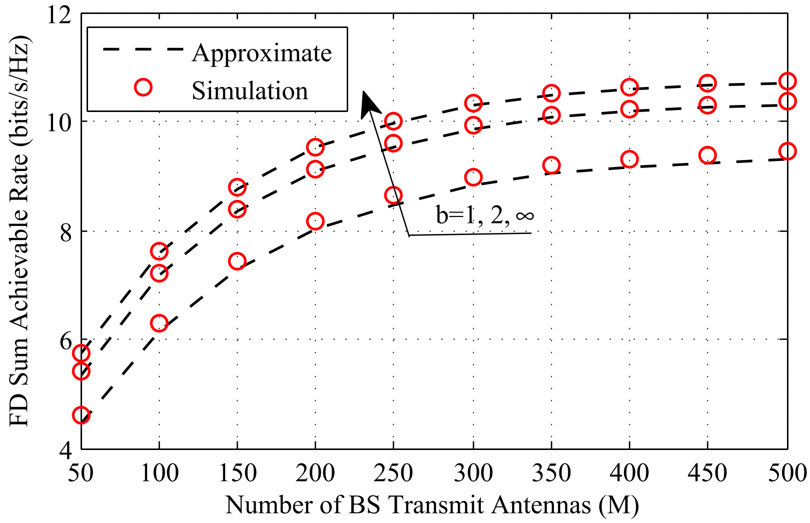

FIGURE 2. FD sum rate versus the number of antennas at the BS, where the transmit power of each user and the BS are scaled with M.

Proof: From (27), the downlink achievable rate of the nth user is expressed as follows

$$R_{D,n} = \mathbb{E}\left\{\log_2\left(1+\frac{\alpha_d^2 P_D \left|\mathbf{g}_{D,n}\mathbf{w}_n x_{D,n}\right|^2}{\Psi_{D,n}}\right)\right\} \tag{29}$$

where

$$\Psi_{D,n} = \alpha_d^2 P_D \sum_{i=1,i\neq n}^{N}\left|\mathbf{g}_{D,n}^H \mathbf{w}_i x_{D,i}\right|^2 + P_D \mathbf{g}_{D,n}^H \mathbf{R}_{\mathbf{q}_D}\mathbf{g}_{D,n} + P_U\left|\mathbf{g}_{IU,n}^H \mathbf{x}_U\right|^2 + \sigma_{D,n}^2$$

By substituting (5) into (29) and using [18, Lemma 1], (29) can be approximated as follows

$$R_{D,n} \approx \log_2\left(1+\frac{P_D\alpha_d^2\mathbb{E}\left\{\left\|\mathbf{g}_{D,n}\right\|^4\right\}\left(tr\left(\mathbf{G}_D^H\mathbf{G}_D\right)\right)^{-1}}{\mathbb{E}\{\Psi_{D,n}\}}\right) \tag{30}$$

Applying Lemma 1, we can obtain

$$\mathbb{E}\{\Psi_{D,n}\} = \alpha_d^2 P_D \sum_{i=1,i\neq n}^{N}\mathbb{E}\left\{\left|\mathbf{g}_{D,n}^H\mathbf{g}_{D,i}\right|^2\right\}\left(tr\left(\mathbf{G}_D^H\mathbf{G}_D\right)\right)^{-1} + P_U\sum_{i=1}^{N}\delta_{in} + \sigma_{D,n}^2 + \alpha_d(1-\alpha_d)P_D\, \mathbb{E}\left\{\mathbf{g}_{D,n}^H diag\left\{\mathbf{G}_D\mathbf{G}_D^H\right\}\mathbf{g}_{D,n}\right\}\left(tr\left(\mathbf{G}_D^H\mathbf{G}_D\right)\right)^{-1} \tag{31}$$

Then, we can derive the last term in (31) as follows

$$\mathbb{E}\left\{\mathbf{g}_{D,n}^H diag\left\{\mathbf{G}_D\mathbf{G}_D^H\right\}\mathbf{g}_{D,n}\right\}\left(tr\left(\mathbf{G}_D^H\mathbf{G}_D\right)\right)^{-1} = \sum_{m=1}^{M}\mathbb{E}\left\{\left|g_{D,mn}\right|^4\right\}\left(tr\left(\mathbf{D}_D\right)M\right)^{-1} + \sum_{m=1}^{M}\sum_{i=1,i\neq n}^{N}\mathbb{E}\left\{\left|g_{D,mn}\right|^2\right\}\mathbb{E}\left\{\left|g_{D,mi}\right|^2\right\}\left(tr\left(\mathbf{D}_D\right)M\right)^{-1} = \beta_{D,n}\left(\beta_{D,n}+\sum_{i=1}^{N}\beta_{D,i}\right)\left(tr\left(\mathbf{D}_D\right)\right)^{-1} \tag{32}$$

Substituting (31), (32) into (30), (28) can be derived. □

Next, we respectively study what the effect of three factors have on the downlink rate performance, which include the number of antennas M, the number of quantization bits b and the transmit power of BS and each user in the same way. Thus, we have the following remarks.

Remark 1.4: As P_U, P_D and M are fixed and $b \rightarrow \infty$ ($\alpha_d = 1$), $R_{D,n}$ can converge to the formula as follows

$$R_{D,n} \rightarrow \log_2\left(1+\frac{P_D\beta_{D,n}^2(1+M)}{P_D\beta_{D,n}\sum_{i=1,i\neq n}^{N}\beta_{D,i}+\Delta_1 tr\left(\mathbf{D}_D\right)}\right) \tag{33}$$

It is shown from (33) that the quantization error can be ignored as the number of quantization bits b tends to infinity. When M is fixed, the downlink approximate achievable rate will became a constant. Although the increasing DACs resolution can improve the system performance, it is still limited after b increases to a specific value. Moreover, we also obtain the result which agrees with [17, eq. (27)] when the number of antennas M is fixed.

Remark 1.5: As b and M are fixed and $P_U = P_D \rightarrow \infty$, $R_{D,n}$ can converge to the formula as follows

$$R_{D,n} \rightarrow \log_2\left(1+\frac{\alpha_d^2\beta_{D,n}^2(1+M)}{\alpha_d^2\beta_{D,n}\sum_{i=1,i\neq n}^{N}\beta_{D,i}+\sum_{i=1}^{N}\sigma_{in} tr\left(D_D\right)+\Delta_2}\right) \tag{34}$$

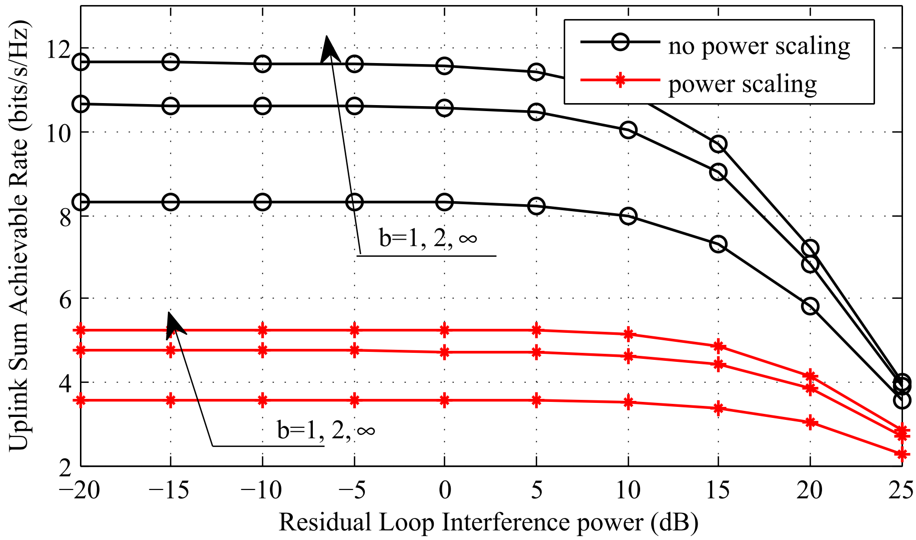

FIGURE 3. Uplink sum rate versus the number of quantization bits.

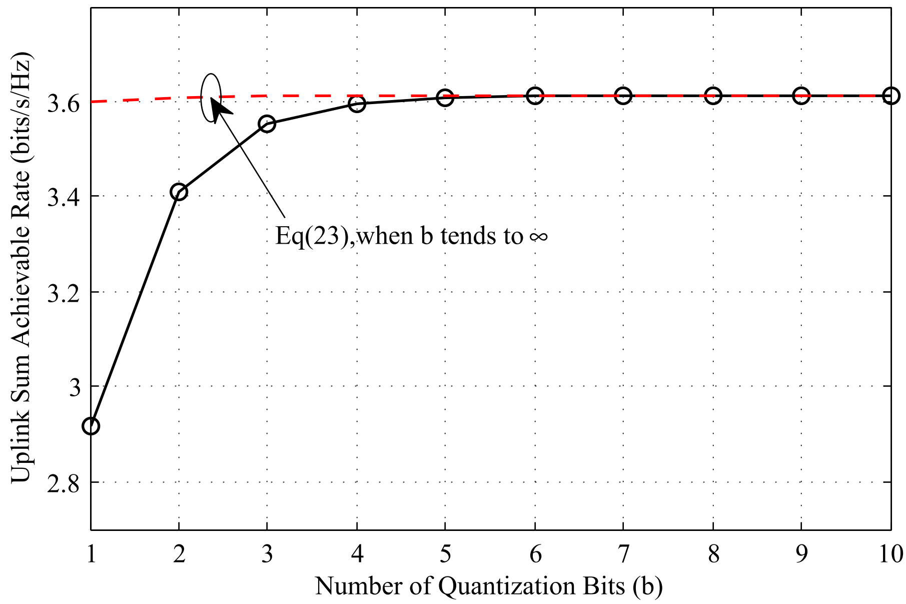

FIGURE 4. Uplink sum rate versus the number of quantization bits.

(34) indicates that the rate relies on the number of antennas M as well as the number of quantization bits b when the transmit power of BS and users tend to infinity. Furthermore, increasing transmit power makes it possible to ignore the quantization error caused by the ADCs and the effect of IUI caused by FD mode users.

Remark 1.6: If the transmit power of BS and each user are scaled with M, i.e., $P_U = E_U/M$ and $P_D = E_D/M$, where E_U, E_D is fixed, as $M \to \infty$, $R_{D,n}$ can converge to the formula as follows

$$R_{D,n} \to \log_2\left(1 + \frac{\alpha_d^2 E_D \beta_{D,n}^2}{\sigma_{D,n}^2 tr\left(D_D\right)}\right) \tag{35}$$

(35) implies that using proper power scaling law and more antennas M can eliminate the IUI caused by FD. The number of quantization bits b determines the approximate downlink rate when M tends to infinity.

IV. NUMERICAL RESULTS

We present the simulation results of sum achievable rates related to various system parameters that verify the derived results and show the effectiveness of FD massive MIMO system where low-resolution ADCs/DACs are employed. Assuming that the radius of a cell, in which N users are uniformly distributed, is $r = 1000m$. r_n is denoted as the distance from the nth user to the BS where the minimum

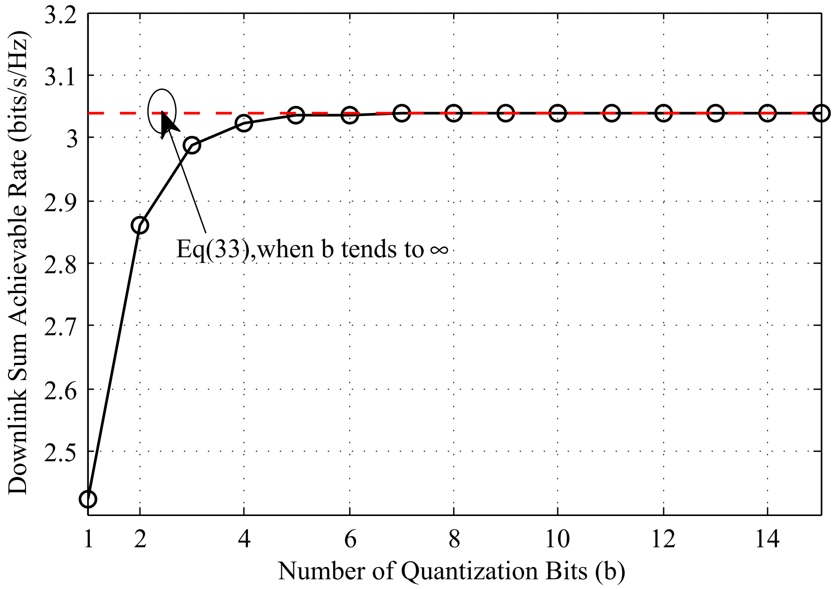

FIGURE 5. Downlink sum rate versus the number of quantization bits.

distance from the user to the BS is set as $r_{\min} = 100m$. a_n is defined as a log-normal random variable whose standard deviation $\sigma = 8dB$. $\upsilon = 3.8$ is specified as the path loss exponent [20], [21]. The large-scale fading can be modeled as $\beta_n = a_n (r_n/r_{\min})^{-\upsilon}$ [22]. Other simulation parameters are set as the variances of the noise are $\sigma_D = \sigma_U = 1$, $N = 10$, the residual LI power is $\rho{=}\beta_{LI} = 0dB$, $P_U = 10dB$ and $P_D = NP_U$.

When the transmit power of each user and the BS are scaled with M, Fig. 2 presents the FD sum achievable rate versus different numbers of antennas M at the BS. Simulation results are presented for three different numbers of quantization bits with 1, 2 and ∞. As shown in Fig. 2, the FD sum rates increase with the number of quantization bits b and the number of antennas M. For variable values of b, these corresponding curves eventually converge to fixed values with an increased M, which agree with (25).

Fig. 3 describes the uplink sum rates of FD massive MIMO systems with or without power scaling versus different residual LI power $\eta\mu_{LI}$ when $M = 200$ for three different numbers of quantization bits with 1, 2 and ∞. The results in Fig. 3 show that the uplink sum rate is a constant with small residual LI power $\eta\mu_{LI}$, and the uplink sum rate is on the decrease and the performance of the proposed system deteriorates as the residual LI power $\eta\mu_{LI}$ gradually increases.

Fig. 4 and Fig. 5 display the uplink and downlink sum achievable rate versus different b, respectively, assuming the number of antennas $M = 100$. As shown in Fig. 4 and Fig. 5, we see that the uplink and downlink rate increases with b and converges to a constant which can be obtained by (23) and (33) when $b \geq 5$. So that low-resolution ADCs/DACs can significantly decrease the system complexity. The figures demonstrate the rationality of adopting low-resolutions ADCs/DACs in FD massive MIMO systems.

V. CONCLUSION

In this paper, we have investigated a FD massive MIMO system with low-resolution ADCs/DACs in the receive antennas and transmit antennas at the FD mode BS. Using MRC/MRT processing and AQNM, the approximate achievable rates for uplink and downlink with perfect CSI have been obtained. And then we have discussed the asymptotic analyses under three cases. The results indicate the quantization error of the low-resolution ADCs/DACs and LI lead to the loss of system performance, however, the loss of system performance can be compensated by using proper power scaling law and more antennas, where this result shows the feasibility of adopting low-resolution ADCs/DACs in FD massive MIMO systems.

APPENDIX A
PROOF OF EQ.(22)

To calculate (11), we will singly compute $A_{U,n}$, $B_{U,n}$, $C_{U,n}$, $D_{U,n}$, $E_{U,n}$ and $F_{U,n}$ as follows, where $A_{U,n}$ denotes the desired signal, $B_{U,n}$ stands for the multi-user interference, $C_{U,n}$ represents the loop interference (LI), $D_{U,n}$ denotes the additive Gaussian quantization noise at the transmit antennas of the BS, $E_{U,n}$ represents the additive white Gaussian noise, $F_{U,n}$ stands for the additive Gaussian quantization noise at the receive antennas of the BS.

1) Compute $\mathbb{E}\left\{\left|A_{U,n}\right|^2\right\}$:

$$\begin{aligned}\mathbb{E}\left\{\left|A_{U,n}\right|^2\right\} &= \alpha_u^2 P_U \mathbb{E}\left\{\left|\mathbf{g}_{U,n}^H \mathbf{g}_{U,n} x_{U,n}\right|^2\right\} \\ &= \alpha_u^2 P_U \mathbb{E}\left\{\left\|\mathbf{g}_{U,n}\right\|^4\right\} \\ &= \alpha_u^2 P_U \beta_{U,n}^2 \left(M + M^2\right) \end{aligned} \tag{36}$$

2) Compute $\mathbb{E}\left\{\left|B_{U,n}\right|^2\right\}$:

$$\begin{aligned}\mathbb{E}\left\{\left|B_{U,n}\right|^2\right\} &= \alpha_u^2 P_U \sum_{i=1,i\neq n}^{N} \mathbb{E}\left\{\left|\mathbf{g}_{U,n}^H \mathbf{g}_{U,i}\right|^2\right\} \\ &= \alpha_u^2 P_U \sum_{i=1,i\neq n}^{N} \beta_{U,n}\beta_{U,i}M \end{aligned} \quad (37)$$

3) Compute $\mathbb{E}\left\{\left|C_{U,n}\right|^2\right\}$:

$$\begin{aligned}&\mathbb{E}\left\{\left|C_{U,n}\right|^2\right\} \\ &= \alpha_u^2\alpha_d^2\eta P_D \mathbb{E}\left\{\left|\sum_{i=1}^{N}\frac{\mathbf{g}_{U,n}^H\mathbf{G}_{LI}}{\sqrt{tr\left(\mathbf{G}_D^H\mathbf{G}_D\right)}}\mathbf{g}_{D,i}x_{D,i}\right|^2\right\} \\ &= \alpha_u^2\alpha_d^2\eta P_D \sum_{i=1}^{N}\frac{\mathbb{E}\left\{\left|\mathbf{g}_{U,n}^H\mathbf{G}_{LI}\mathbf{g}_{D,i}\right|^2\right\}}{tr\left(\mathbf{G}_D^H\mathbf{G}_D\right)} \\ &= \alpha_u^2\alpha_d^2\eta P_D \sum_{i=1}^{N}\frac{\mathbb{E}\left\{tr\left(\mathbf{g}_{U,n}^H\mathbf{G}_{LI}\mathbf{g}_{D,i}\mathbf{g}_{D,i}^H\mathbf{G}_{LI}^H\mathbf{g}_{U,n}\right)\right\}}{\sum_{j=1}^{N}\beta_{D,j}M} \\ &= \alpha_u^2\alpha_d^2\eta P_D \\ &\quad\times\frac{tr\left(\mathbb{E}\left\{\mathbf{g}_{U,n}^H g_{U,n}\right\}\mathbb{E}\left\{\mathbf{G}_{LI}\mathbf{G}_{LI}^H\right\}\sum_{i=1}^{N}\mathbb{E}\left\{\mathbf{g}_{D,i}\mathbf{g}_{D,i}^H\right\}\right)}{\sum_{j=1}^{N}\beta_{D,j}M} \\ &= \alpha_u^2\alpha_d^2\eta P_D\mu_{LI}\beta_{U,n}M^2 \end{aligned} \quad (38)$$

4) Compute $\mathbb{E}\left\{\left|D_{U,n}\right|^2\right\}$:

$$\begin{aligned}&tr\left(\mathbb{E}\left\{\mathbf{q}_D\mathbf{q}_D^H\right\}\right) \\ &= \alpha_d\left(1-\alpha_d\right)tr\left(diag\left(\mathbf{G}_D\mathbf{G}_D^H\right)\right)\left(tr\left(\mathbf{G}_D\mathbf{G}_D^H\right)\right)^{-1} \\ &= \alpha_d\left(1-\alpha_d\right) \end{aligned} \quad (39)$$

$$\begin{aligned}&\mathbb{E}\left\{\left|D_{U,n}\right|^2\right\} \\ &= \alpha_u^2\eta P_D\mathbb{E}\left\{\left|\mathbf{g}_{U,n}^H\mathbf{G}_{LI}\mathbf{q}_D\right|^2\right\} \\ &= \alpha_u^2\eta P_D\mathbb{E}\left\{tr\left(\mathbf{g}_{U,n}^H\mathbf{G}_{LI}\mathbf{q}_D\mathbf{q}_D^H\mathbf{G}_{LI}^H\mathbf{g}_{U,n}\right)\right\} \\ &= \alpha_u^2\eta P_D tr\left(\mathbb{E}\left\{\mathbf{g}_{U,n}^H\mathbf{g}_{U,n}\right\}\mathbb{E}\left\{\mathbf{G}_{LI}\mathbf{G}_{LI}^H\right\}\mathbb{E}\left\{\mathbf{q}_D\mathbf{q}_D^H\right\}\right) \\ &= \alpha_u^2\alpha_d\left(1-\alpha_d\right)\eta P_D\mu_{LI}\beta_{U,n}M^2 \end{aligned} \quad (40)$$

5) Compute $\mathbb{E}\left\{\left|E_{U,n}\right|^2\right\}$:

$$\begin{aligned}\mathbb{E}\left\{\left|E_{U,n}\right|^2\right\} &= \alpha_u^2\mathbb{E}\left\{\left|\mathbf{g}_{U,n}^H\mathbf{n}_U\right|^2\right\} \\ &= \alpha_u^2\beta_{U,n}M\sigma_U^2 \end{aligned} \quad (41)$$

6) Compute $\mathbb{E}\left\{\left|F_{U,n}\right|^2\right\}$:

$$\begin{aligned}\mathbb{E}\left\{\left|F_{U,n}\right|^2\right\} &= \mathbb{E}\left\{\left|\mathbf{g}_{U,n}^H\mathbf{q}_U\right|^2\right\} \\ &= \alpha_u\left(1-\alpha_u\right)\mathbb{E}\left\{tr\left(\mathbf{g}_{U,n}^H diag\left(P_U\mathbf{G}_U\mathbf{G}_U^H\right)\mathbf{g}_{U,n}\right)\right\} \\ &\quad+\alpha_u\left(1-\alpha_u\right) \\ &\quad\mathbb{E}\left\{tr\left(\mathbf{g}_{U,n}^H diag\left(\alpha_d^2\eta P_D\mathbf{G}_{LI}\mathbf{W}\mathbf{W}^H\mathbf{G}_{LI}^H\right)\mathbf{g}_{U,n}\right)\right\} \\ &\quad+\alpha_u\left(1-\alpha_u\right) \\ &\quad\mathbb{E}\left\{tr\left(\mathbf{g}_{U,n}^H diag\left(\eta P_D\mathbf{G}_{LI}\mathbf{q}_D\mathbf{q}_D^H\mathbf{G}_{LI}^H\right)\mathbf{g}_{U,n}\right)\right\} \\ &\quad+\alpha_u\left(1-\alpha_u\right)\mathbb{E}\left\{tr\left(\mathbf{g}_{U,n}^H diag\left(\mathbf{n}_U\mathbf{n}_U^H\right)\mathbf{g}_{U,n}\right)\right\} \end{aligned} \quad (42)$$

We can derive the first term in (42) as follows

$$\begin{aligned}&\mathbb{E}\left\{tr\left(\mathbf{g}_{U,n}^H diag\left(P_U\mathbf{G}_U\mathbf{G}_U^H\right)\mathbf{g}_{U,n}\right)\right\} \\ &= P_U\sum_{m=1}^{M}\mathbb{E}\left\{\left|g_{U,mn}\right|^4\right\} \\ &\quad+P_U\sum_{m=1}^{M}\sum_{i=1,i\neq n}^{N}\mathbb{E}\left\{\left|g_{U,mn}\right|^2\right\}\mathbb{E}\left\{\left|g_{U,mi}\right|^2\right\} \end{aligned} \quad (43)$$

Substituting $\mathbb{E}\left\{\left|g_{U,mn}\right|^2\right\} = \beta_{U,n}$ and $\mathbb{E}\left\{\left|g_{U,mn}\right|^4\right\} = 2\beta_{U,n}^2$ [8] to (43) yields

$$\begin{aligned}&\mathbb{E}\left\{tr\left(\mathbf{g}_{U,n}^H diag\left(P_U\mathbf{G}_U\mathbf{G}_U^H\right)\mathbf{g}_{U,n}\right)\right\} \\ &\qquad= 2P_U\beta_{U,n}^2M + P_U\sum_{i=1,i\neq n}^{N}\beta_{U,n}\beta_{U,i}M \end{aligned} \quad (44)$$

The second term in (42) can be derived as follows

$$\begin{aligned}&\mathbb{E}\left\{tr\left(\mathbf{g}_{U,n}^H diag\left(\alpha_d^2\eta P_D\mathbf{G}_{LI}\mathbf{W}\mathbf{W}^H\mathbf{G}_{LI}^H\right)\mathbf{g}_{U,n}\right)\right\} \\ &= \alpha_d^2\eta P_D tr \\ &\quad\times\left(\mathbb{E}\left\{\mathbf{g}_{U,n}^H\mathbf{g}_{U,n}\right\}\mathbb{E}\left\{diag\left(\mathbf{G}_{LI}\mathbf{G}_D\mathbf{G}_D^H\mathbf{G}_{LI}^H\right)\right\}\right) \\ &\quad\times\left(tr\left(\mathbf{D}_DM\right)\right)^{-1} \\ &= \alpha_d^2\eta P_D\beta_{U,n}M tr\left(\mathbb{E}\left\{diag\left(\mathbf{G}_{LI}\mathbf{G}_D\mathbf{G}_D^H\mathbf{G}_{LI}^H\right)\right\}\right) \\ &\quad\times\left(tr\left(\mathbf{D}_DM\right)\right)^{-1} \\ &= \alpha_d^2\eta P_D\beta_{U,n}M tr\left(\mathbf{D}_DM\mu_{LI}\right)\left(tr\left(\mathbf{D}_DM\right)\right)^{-1} \\ &= \alpha_d^2\eta P_D\mu_{LI}\beta_{U,n}M^2 \end{aligned} \quad (45)$$

Combine (40), (44) and (45), we have

$$\begin{aligned}\mathbb{E}\left\{\left|F_{U,n}\right|^2\right\} &= \alpha_u\left(1-\alpha_u\right) \\ &\quad\times\left(2P_U\beta_{U,n}^2M + P_U\sum_{i=1,i\neq n}^{N}\beta_{U,n}\beta_{U,i}M\right) \\ &\quad+\alpha_u\left(1-\alpha_u\right)\left(\alpha_d\eta P_D\mu_{LI}\beta_{U,n}M^2+\sigma_U^2\beta_{U,n}M\right) \end{aligned} \quad (46)$$

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