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Orbital similarity functions – application to asteroid pairs

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ABSTRACT

The paper expands the idea of Vokrouhlický and Nesvorný who used a modified Zappalà et al. metric with osculating elements in search for pairs of asteroids suspected of having a common origin. Using six different orbital similarity functions, we find that five of them display a similar excess of close pairs in the catalogue of osculating elements. The excess is even higher when mean orbital elements are used. Similarly, when the mean elements are applied, there is a better agreement between the closest pairs found in the same catalogue using different metrics. The common subset of 62 pairs from five lists of 100 closest pairs according to different distance functions is provided. Investigating an artificial sample of asteroid orbital pairs with a known initial orbital velocity difference we find that the Drummond metric best preserves orbital proximity over long time intervals.

Key words: methods: numerical – celestial mechanics – minor planets, asteroids: general.

1 INTRODUCTION

Southworth & Hawkins (1963) emphasized the requirement of a quantitative criterion for comparing meteor orbits. The criterion was supposed to determine stream membership. The orbits of meteors are treated as points in a five-dimensional osculating elements space, or a space created by any five independent functions of Keplerian elements: semimajor axis a, eccentricity e, inclination i, longitude of the ascending node Ω and argument of perihelion ω. The distance between the points serves to measure orbital similarity. Traditionally, a number of various orbital similarity functions, also termed D-criteria, were applied in two domains. In the identification of meteor streams and their parent bodies D-criteria are based upon five osculating elements (Southworth & Hawkins 1963; Drummond 1981; Jopek 1993; Jenniskens 2008; Jopek, Rudawska & Bartczak 2008).

For the identification of asteroid families only three proper elements, a, e and i, were used (Zappalà et al. 1990), until Nesvorný & Vokrouhlický (2006) and Nesvorný, Vokrouhlický & Bottke (2006) extended the D-criterion of Zappalà to be used with five osculating elements in the search of young asteroid clusters. The new criterion was used recently by Vokrouhlický & Nesvorný (2008). Their main goal was to find close pairs of possibly common origin, formed by collisional disruption of km-sized parent bodies, Yarkovsky–O'Keefe–Radzievski–Paddack (YORP)-induced rotational fission of fast rotating objects or splitting of unstable asteroid binaries. In their work, followed by Pravec & Vokrouhlický (2009), Vokrouhlický (2009); Vokrouhlický& Nesvorný (2009) and Pravec et al. (2010), they use orbital similarity as a selection rule providing candidates for more advanced dynamical studies.

The objective of the present work is to investigate how strongly the results of the close asteroid pairs searches depend on the adopted D-criterion. For this purpose we compare six orbital similarity functions presented in Section 2.

In Section 3 we follow Vokrouhlický & Nesvorný (2008), studying the existence of statistical excess of close asteroid pairs using various distance functions applied to osculating and mean orbital elements. We also check the agreement between the closest pairs found by various functions. Section 4 discusses the relation between breakup velocity and orbital distance. We study the range of velocity differences far beyond the close pair limit, so that the results can serve in other problems, like meteor stream identification.

2 REVIEW OF ORBITAL SIMILARITY FUNCTIONS

From the variety of D-criteria used in meteor stream searches we chose five functions that will be applied to asteroid orbital pairs search: $D_{\rm SH}$ of Southworth & Hawkins (1963), $D_{\rm D}$ of Drummond (1981), $D_{\rm H}$ of Jopek (1993), $D_{\rm V}$ of Jopek et al. (2008) and $D_{\rm B}$ of Jenniskens (2008). More details concerning their origins can be found in the cited papers; here we briefly recall the definitions.

Let A and B be indices that mark orbital elements of two bodies – meteors or asteroids. Southworth & Hawkins (1963) define the distance function $D_{\rm SH}$ using the formula for its square:

$$D^2_{\rm SH} = (e_B - e_A)^2 + (q_B - q_A)^2 + \left(2\sin\frac{I_{BA}}{2}\right)^2 + \left(\frac{e_B + e_A}{2}\right)^2 \left(2\sin\frac{\pi_{BA}}{2}\right)^2, \qquad (1)$$

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where I_{BA} is the angle between orbital planes of the bodies and π_{BA} is the difference of the longitudes of perihelia measured from the intersection of the orbits. We calculate I_{BA} from

$$\left(2\sin\frac{I_{BA}}{2}\right)^2 = \left(2\sin\frac{i_B - i_A}{2}\right)^2 + \sin i_A \sin i_B \left(2\sin\frac{\Omega_B - \Omega_A}{2}\right)^2, \tag{2}$$

and π_{BA} from

$$\pi_{BA} = \omega_B - \omega_A + 2\arcsin(S_{BA}), \tag{3}$$

$$S_{BA} = \cos\frac{i_B + i_A}{2}\sin\frac{\Omega_B - \Omega_A}{2}\sec\frac{I_{BA}}{2}. \tag{4}$$

In the original formulation of Southworth & Hawkins (1963), q stands for the perihelion distance; it is more correct, however, to interpret it as the perihelion distance divided by the unit length of 1 au, in order to maintain all terms in equation (1) dimensionless.

Drummond (1981) proposed a modification of the D_{SH} in the following form:

$$D_{\mathrm{D}}^2 = \left(\frac{e_B - e_A}{e_B + e_A}\right)^2 + \left(\frac{q_B - q_A}{q_B + q_A}\right)^2 + \left(\frac{I_{BA}}{180^\circ}\right)^2 + \left(\frac{e_B + e_A}{2}\right)^2\left(\frac{\theta_{BA}}{180^\circ}\right)^2. \tag{5}$$

The main difference between D_{D} and D_{SH} consists in introducing weights in the first two terms and using θ_{BA}, the angle between the lines of apsides of the two orbits. Instead of original formulae by Drummond (1981), we calculate I_{BA} as the angle between angular momentum vectors $\boldsymbol{G} = \boldsymbol{r} \times \boldsymbol{v}$ for orbits A and B:

$$I_{BA} = \arccos\left(\frac{\boldsymbol{G}_A \cdot \boldsymbol{G}_B}{G_A G_B}\right). \tag{6}$$

θ_{BA} is also derived from the scalar product of the two orbits' Laplace vectors $\boldsymbol{e}$:

$$\theta_{BA} = \arccos\left(\frac{\boldsymbol{e}_A \cdot \boldsymbol{e}_B}{e_A e_B}\right). \tag{7}$$

This way is computationally more efficient than the original formulation of Drummond (1981), who stacked spherical trigonometry identities.

In the work of Jopek (1993) we find a distance function which is a hybrid of the previous two. In the D_{H} function the second term of D_{SH} is replaced by a part of D_{D}, hence

$$D_{\mathrm{H}}^2 = (e_B - e_A)^2 + \left(\frac{q_B - q_A}{q_B + q_A}\right)^2 + \left(2\sin\frac{I_{BA}}{2}\right)^2 + \left(\frac{e_B + e_A}{2}\right)^2\left(2\sin\frac{\pi_{BA}}{2}\right)^2. \tag{8}$$

Jopek, Rudawska & Bartczak (2008) use vectorial elements for meteor stream identification. Their metric D_{V} substantially differs from the previous ones:

$$D_{\mathrm{V}}^2 = w_{h1}(h_{A1} - h_{B1})^2 + w_{h2}(h_{A2} - h_{B2})^2 + \frac{3}{2}w_{h3}(h_{A3} - h_{B3})^2 + w_{e1}(e_{A1} - e_{B1})^2 + w_{e2}(e_{A2} - e_{B2})^2 + w_{e3}(e_{A3} - e_{B3})^2 + 2w_E(E_A - E_B)^2, \tag{9}$$

Table 1. Weighting coefficients for equation (9) according to Jopek et al. (2008).

Coefficient	Value	Coefficient	Value
w_{e1}	3.19×10^4	w_{h1}	4.00×10^4
w_{e2}	3.19×10^4	w_{h2}	4.34×10^4
w_{e3}	6.25×10^4	w_{h3}	3.19×10^4
w_E	4.96×10^{11}		

involving the differences of the orbital energies $E = -\mu/(2a)$, and of Cartesian components of the Laplace vectors $\boldsymbol{e}$, and of dimensionless angular momenta $\boldsymbol{h} = \boldsymbol{G}/\sqrt{\mu a}$. The weights w are listed in Table 1.

The distance function introduced by Jenniskens (2008) is based upon three approximate dynamical invariants considered earlier by Babadzhanov (1989):

$$D_{\mathrm{B}}^2 = ((C_{1A} - C_{1B})/0.13)^2 + ((C_{2A} - C_{2B})/0.06)^2 + ((C_{3A} - C_{3B})/14.^\circ2)^2. \tag{10}$$

The first invariant, C_1, corresponds to the z-component of the orbital angular momentum:

$$C_1 = \left(1 - e^2\right)\cos^2 i. \tag{11}$$

The second comes form the secular model of Lidov,

$$C_2 = e^2\left(\frac{2}{5} - \sin^2 i \sin^2 \omega\right), \tag{12}$$

and the third is the longitude of perihelion,

$$C_3 = \omega + \Omega = \varpi. \tag{13}$$

Let us emphasize the absence of the semimajor axis in D_{B}, making it quite different from the four previous ones.

The five orbital similarity functions, originally designed for the study of meteoroids, will be confronted with the asteroidal metric of Zappalà et al. (1990) extended by Nesvorný & Vokrouhlický (2006) by adding the last two terms in

$$\left(\frac{D_{\mathrm{Z}}}{na}\right)^2 = k_a\left(\frac{a_B - a_A}{a}\right)^2 + k_e(e_B - e_A)^2 + k_i(\sin i_B - \sin i_A)^2 + k_\Omega(\Omega_B - \Omega_A)^2 + k_\varpi(\varpi_B - \varpi_A)^2. \tag{14}$$

Note that a was not explained by Nesvorný & Vokrouhlický (2006); we use the arithmetic mean $a = (a_A + a_B)/2$. The weights given by Pravec & Vokrouhlický (2009) are $k_a = 5/4, k_e = k_i = 2$ and $k_\varpi = k_\Omega = 10^{-4}$. Contrarily to the remaining metrics, D_{Z} is not dimensionless. Throughout the paper, the values of D_{Z} are always given in $\mathrm{m\,s^{-1}}$.

3 CLOSE ORBITAL PAIRS

3.1 Distribution of close pairs

We have studied a sample of 372 282 asteroids from the 2010 February release of the AstDys catalogue by Knežević, Lemaître & Milani (2002), including numbered and multi-opposition asteroids with semimajor axes from the range of 1.7–3.6 au. For the six functions presented in Section 2 we have computed mutual distances between the orbits using osculating Keplerian elements. The results served to plot the number $N(D)$ of pairs with orbital distances smaller than D, analogous to Vokrouhlický & Nesvorný (2008, fig. 1). Our plots are

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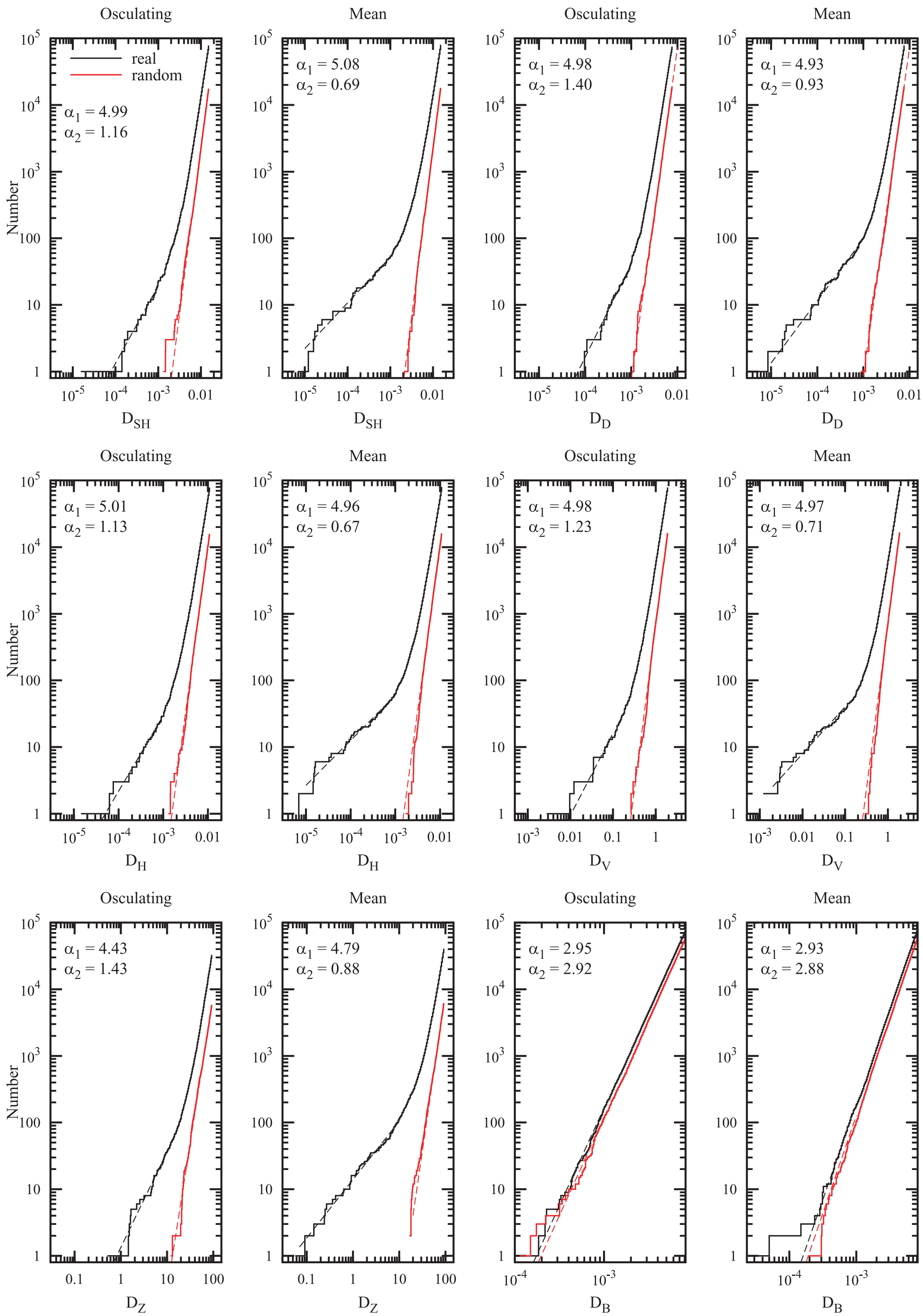

Figure 1. The number of orbital pairs found by each metric depending on the maximum value of D. A cumulative distribution of pairs found amongst the AstDys catalogue orbits (black) is compared with a random population (red). Red dashed line presents the fit of the power law D^{α_1} to the random distribution, black dashed line is the power law D^{α_2} fit to the excess of close pairs. Respective panels present distributions for six orbital similarity functions calculated for osculating and mean elements.

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presented in Fig. 1 (black solid line, 'osculating' panels). Similarly to Vokrouhlický & Nesvorný (2008) we have confronted $N(D)$ of real asteroids with the distribution of distances in a random population. The latter was generated considering each Keplerian element to be an independent variable with the probability density of the real sample. The resulting $N(D)$ is presented as a red line ('osculating' panels).

According to Vokrouhlický & Nesvorný (2008), a random distribution of points in a five-dimensional space should result in a power law of $N(D) \propto D^{\alpha_1}$, with $\alpha_1 = 5$. The red dashed line in Fig. 1 presents the power-law fit of orbital distance functions in the random population. Indeed, in most cases α_1 is close to 5 regardless of the metric. A notable exception is the distribution of D_B distance function with $\alpha_1 \approx 3$.

Using the distance function D_Z, Vokrouhlický & Nesvorný (2008) discovered the excess in the number of very close pairs with respect to the random model, and this observation is essential for the theories of asteroids evolution. Our results confirm that the excess is not related to a particular form of the orbital similarity function. As noted by Pravec & Vokrouhlický (2009), the distribution of close pairs in the excess domain should follow a power law $N(D) \propto D^{\alpha_2}$ with $\alpha_2 \approx 2$. The black dashed line in Fig. 1 corresponds to a power-law fit to the excess of close pairs. The values of α_2 vary between 2/3 for mean elements and 3/2 for osculating, so the value of α_2 is rather closer to 1. Once again, it does not apply to the D_B of Jenniskens (2008), for which random and real populations give the same $N(D) \propto D^3$ distribution.

Having confirmed and generalized the results concerning osculating elements of asteroids we decided to compare them with the statistics based upon the mean elements, liberated from the short periodic variations. The mean elements, calculated by the analytical theory of Knežević et al. (1988), are also available in the AstDys catalogue. The results are shown in Fig. 1 ('mean' panels). The sample of mean orbital elements consists of 371 385 orbits, i.e. about 99.8 per cent of the osculating elements set, but such a small difference should not influence the relative statistics.

Remarkably (with the usual exception of D_B) the excess of tight pairs is much higher in the mean elements space than in the osculating elements case. Moreover the minimum value obtained with each of the six distance functions is smaller. In other words very close pairs are even closer (up to factor of 0.1) when we study the mean elements distributions. A possible explanation of this property will be given in Section 3.3.

3.2 Closest pairs in different metrics

Although the statistics is similar for most of the distance functions, they may differ in the ranking of individual pairs. To shed more light on this issue, each orbital metric was applied to the real asteroids population in order to create a list of orbital pairs sorted according to their distance. Each of the six lists was truncated, retaining only 100 pairs with the smallest D. The number of common pairs in two lists measures a coherence between their associated metrics, although we skip the problem of different ordering of the pairs present in both lists. The results of such comparison are presented in Table 2 (osculating elements) and in Table 3 (mean elements). As expected only a few per cent of the pairs found by D_B were registered by different metrics. The best agreement can be observed between D_V, D_H and D_{SH}. This is understandable for the last two functions, since most of the terms in the hybrid distance function D_H were taken from D_{SH} (Jopek 1993). However, a particularly good agreement between D_V and D_H is quite surprising and its reasons are unclear. Comparing Tables 2 and 3, it is worth noting that the coherence of all distance functions is significantly better when the mean orbital elements are applied.

Table 2. Number of common pairs among first 100 pairs found amongst osculating orbits by each orbital similarity function.

D	D_{SH}	D_D	D_H	D_V	D_Z	D_B
D_{SH}	100	52	78	71	61	3
D_D	52	100	47	43	62	3
D_H	78	47	100	89	54	3
D_V	71	43	89	100	51	3
D_Z	61	62	54	51	100	3
D_B	3	3	3	3	3	100

Table 3. Number of common pairs among first 100 pairs found amongst mean orbits by each orbital similarity function.

D	D_{SH}	D_D	D_H	D_V	D_Z	D_B
D_{SH}	100	75	87	79	78	9
D_D	75	100	68	66	81	9
D_H	87	68	100	92	70	9
D_V	79	66	92	100	67	9
D_Z	78	81	70	67	100	10
D_B	9	9	9	9	10	100

Table 4 presents the subset of 62 pairs – the common part of five 'top 100' lists without D_B – based upon mean elements (a similar list for osculating elements involves only 30 pairs). The pairs are sorted according to the position on the D_Z list, but each value of the orbital distance is followed by the number in bracket that refers to the position of a given pair on the list associated with a given distance function. A comparison with table 7 of Pravec & Vokrouhlický (2009) reveals that the mean elements based values of D_Z are often smaller not only than the ones in osculating elements, but even in proper elements. Six pairs in Table 4 can be identified as belonging to young asteroid families listed in table 1 of Nesvorný & Vokrouhlický (2006) and could therefore be formed by different mechanisms than other asteroid pairs.

3.3 Evolution of a metric

In order to understand the superiority of mean orbital elements in the close pair search one should study the evolution of a distance function for a given, initially tight, pair of orbits. Fig. 2 presents time evolution of the orbital distance function D_Z for a synthetic orbital pair of two clones of 63440 2001 MD30 ($a \approx 1.94$ au, $e \approx 0.09$, $i \approx 20°$) with a small (4.82×10^{-7} au d$^{-1} \approx 0.84$ m s^{-1}) velocity difference.

As it was pointed out by Vokrouhlický & Nesvorný (2008, fig. 6) the major part of a distance function variations results from the detuning of short period perturbations. Asteroids forming a close pair are subject to similar short periodic perturbations, but the phases of the perturbations become different for both objects due to different mean anomaly values resulting from a small difference in the mean motions. In other words, a distance function computed from the osculating elements evolves periodically together with the difference of mean anomalies δM, as we see in Fig. 2 (red line), revealing the squared short periodic perturbations between subsequent minima. One may expect that this phenomenon will not be observed when using mean elements, because they do not posses short periodic perturbations by their definition. The black line in Fig. 2 clearly confirms

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Table 4. The common subset of pairs found by five distance functions in mean elements. Numbers in bracket refer to the position on the list for a specific *D*. The asteroids belonging to a young cluster are preceded by a letter D – Datura, Y – 1992 YC2 or L – Lucascavin.

	Pair members		D_{SH}		D_{D}		D_{H}		D_{V}		D_{Z}	
	2003YK39	21436	6.5e-06	(1)	8.5e-06	(2)	6.4e-06	(1)	1.2e-03	(1)	0.054	(1)
	2005UY97	229401	1.2e-05	(2)	5.1e-06	(1)	6.7e-06	(2)	1.2e-03	(2)	0.090	(2)
	2004TV14	63440	2.0e-05	(5)	2.1e-05	(5)	1.6e-05	(6)	3.2e-03	(6)	0.14	(3)
	180906	217266	1.7e-05	(4)	1.8e-05	(3)	1.4e-05	(3)	2.6e-03	(3)	0.23	(4)
	195479	2008WK70	2.5e-05	(6)	7.2e-05	(9)	1.6e-05	(5)	3.0e-03	(5)	0.25	(5)
	194083	92652	4.5e-05	(8)	3.0e-05	(6)	3.3e-05	(7)	5.9e-03	(7)	0.26	(6)
	54827	6070	1.6e-05	(3)	1.9e-05	(4)	1.5e-05	(4)	2.8e-03	(4)	0.35	(7)
	1999VA117	88259	4.4e-05	(7)	1.0e-04	(12)	3.6e-05	(8)	7.0e-03	(8)	0.41	(8)
	214954	26416	8.6e-05	(9)	2.0e-04	(24)	7.3e-05	(10)	1.4e-02	(10)	0.60	(9)
	2002UY20	57202	1.3e-04	(15)	7.4e-05	(10)	1.2e-04	(16)	2.1e-02	(16)	0.62	(10)
	1741	2002ER36	1.2e-04	(10)	1.0e-04	(11)	9.7e-05	(14)	1.9e-02	(15)	0.83	(11)
	184300	2001UU227	1.6e-04	(17)	7.2e-05	(8)	1.6e-04	(19)	2.9e-02	(19)	0.85	(12)
	165389	2001VN61	1.6e-04	(18)	1.4e-04	(19)	1.4e-04	(18)	2.6e-02	(18)	0.87	(13)
	2007RD148	95750	1.3e-04	(14)	7.1e-05	(7)	9.0e-05	(13)	1.7e-02	(13)	0.89	(14)
	2005GS180	2008FK107	1.3e-04	(13)	1.4e-04	(17)	1.0e-04	(15)	1.8e-02	(14)	0.89	(15)
	220143	54041	1.2e-04	(12)	1.2e-04	(15)	7.9e-05	(12)	1.4e-02	(12)	0.90	(16)
	2005QV114	2007OS5	2.9e-04	(23)	1.3e-04	(16)	2.9e-04	(25)	6.0e-02	(26)	1.1	(17)
	111962	2001UR224	1.4e-04	(16)	1.1e-04	(14)	6.7e-05	(9)	1.1e-02	(9)	1.1	(18)
	11842	228747	2.7e-04	(21)	1.1e-04	(13)	2.6e-04	(22)	5.5e-02	(24)	1.2	(19)
	2002TM148	67982	5.3e-04	(34)	1.7e-04	(22)	5.2e-04	(38)	1.1e-01	(43)	1.3	(20)
	2006KM53	99052	1.2e-04	(11)	2.9e-04	(31)	7.6e-05	(11)	1.4e-02	(11)	1.3	(21)
	129880	2008WL83	4.3e-04	(29)	1.4e-04	(18)	4.3e-04	(33)	8.0e-02	(34)	1.3	(22)
	106700	2007UV	8.2e-04	(47)	2.9e-04	(32)	8.2e-04	(54)	1.7e-01	(57)	1.4	(23)
	70287	8898	2.9e-04	(22)	1.5e-04	(20)	2.7e-04	(23)	4.9e-02	(23)	1.5	(24)
	142131	60677	2.2e-04	(19)	1.5e-04	(21)	1.7e-04	(20)	3.0e-02	(20)	1.7	(25)
	2006WG56	67620	5.5e-04	(36)	4.7e-04	(48)	5.4e-04	(39)	1.1e-01	(40)	1.8	(26)
	182259	2897	3.9e-04	(26)	2.7e-04	(26)	3.8e-04	(30)	6.9e-02	(29)	2.0	(27)
	125887	197706	3.9e-04	(25)	1.9e-04	(23)	3.4e-04	(27)	6.8e-02	(28)	2.0	(28)
	134789	2003RJ10	9.5e-04	(51)	3.4e-04	(37)	9.5e-04	(62)	1.8e-01	(67)	2.1	(29)
	2009EL11	2384	5.4e-04	(35)	2.9e-04	(30)	4.9e-04	(36)	9.1e-02	(35)	2.1	(30)
	157123	2002QM97	8.1e-04	(46)	2.8e-04	(28)	8.1e-04	(52)	1.5e-01	(53)	2.2	(31)
	10484	44645	3.9e-04	(27)	4.1e-04	(42)	3.5e-04	(28)	7.1e-02	(31)	2.2	(32)
	2001XO105	4765	4.4e-04	(31)	4.2e-04	(43)	3.9e-04	(32)	8.0e-02	(33)	2.3	(33)
	2005JY103	76111	4.1e-04	(28)	9.0e-04	(94)	3.0e-04	(26)	5.9e-02	(25)	2.4	(34)
	2008YV80	39991	4.4e-04	(30)	2.9e-04	(29)	3.7e-04	(29)	6.7e-02	(27)	2.4	(35)
D	1270	215619	9.1e-04	(50)	2.9e-04	(33)	9.1e-04	(60)	1.8e-01	(65)	2.8	(36)
D	203370	60151	3.5e-04	(24)	2.5e-04	(25)	2.7e-04	(24)	4.8e-02	(22)	3.1	(37)
Y	16598	218697	1.2e-03	(68)	4.4e-04	(47)	1.1e-03	(80)	2.0e-01	(75)	3.2	(38)
	13481	158395	6.0e-04	(37)	3.2e-04	(36)	4.9e-04	(35)	9.4e-02	(36)	3.5	(39)
Y	190603	218697	5.0e-04	(33)	3.2e-04	(35)	2.5e-04	(21)	4.2e-02	(21)	3.6	(40)
	2001RK103	2008RB40	7.3e-04	(41)	2.8e-04	(27)	6.9e-04	(47)	1.2e-01	(46)	3.6	(41)
	2009BR60	3749	1.1e-03	(59)	3.5e-04	(39)	1.1e-03	(76)	2.3e-01	(90)	3.8	(42)
	2003WZ36	53754	1.2e-03	(69)	5.8e-04	(54)	1.1e-03	(81)	2.2e-01	(85)	3.9	(43)
	2002SF64	2007AQ6	1.1e-03	(57)	3.5e-04	(41)	1.0e-03	(71)	2.1e-01	(81)	3.9	(44)
	2000SP31	2007TN127	1.2e-03	(67)	5.1e-04	(51)	1.1e-03	(83)	2.3e-01	(89)	3.9	(45)
	213471	80218	2.5e-04	(20)	7.8e-04	(81)	1.2e-04	(17)	2.1e-02	(17)	4.0	(46)
	2000SS4	84203	1.1e-03	(56)	4.3e-04	(45)	1.0e-03	(73)	2.1e-01	(83)	4.4	(47)
	17198	229056	4.7e-04	(32)	7.4e-04	(72)	3.8e-04	(31)	7.1e-02	(30)	4.4	(48)
	18777	57738	8.8e-04	(49)	3.5e-04	(40)	8.2e-04	(55)	1.7e-01	(59)	4.6	(49)
	2005WW113	5026	7.4e-04	(42)	3.1e-04	(34)	7.3e-04	(50)	1.3e-01	(48)	4.8	(50)
	226877	227658	8.0e-04	(45)	3.5e-04	(38)	7.1e-04	(48)	1.4e-01	(50)	4.9	(51)
L	209570	21509	6.1e-04	(38)	7.8e-04	(83)	5.1e-04	(37)	9.6e-02	(37)	5.0	(52)
	122173	2003UG220	1.0e-03	(55)	9.8e-04	(100)	1.0e-03	(72)	2.1e-01	(78)	5.4	(56)
	32957	38707	7.3e-04	(40)	8.6e-04	(90)	5.9e-04	(41)	1.1e-01	(42)	5.8	(57)
	140778	2008FL78	1.1e-03	(62)	5.9e-04	(55)	8.9e-04	(57)	1.8e-01	(69)	5.9	(58)
	2006BR54	39991	8.2e-04	(48)	7.3e-04	(71)	6.2e-04	(44)	1.1e-01	(41)	6.1	(59)
Y	16598	190603	1.3e-03	(73)	6.5e-04	(61)	1.0e-03	(65)	1.8e-01	(63)	6.5	(68)
	2005WH141	47866	7.9e-04	(44)	6.2e-04	(57)	5.5e-04	(40)	1.1e-01	(39)	6.5	(69)
	2006UT69	220015	1.1e-03	(63)	7.2e-04	(69)	9.7e-04	(63)	1.7e-01	(60)	6.9	(71)
	2007PD19	59184	1.4e-03	(80)	9.5e-04	(97)	1.3e-03	(100)	2.4e-01	(99)	7.9	(89)
	174725	2002QD132	1.3e-03	(72)	7.6e-04	(75)	1.1e-03	(74)	2.1e-01	(82)	8.0	(90)
	2002VY142	203069	9.8e-04	(53)	7.6e-04	(76)	4.6e-04	(34)	7.3e-02	(32)	8.2	(92)

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this conjecture. The mean orbital elements were obtained according to the theory of Knežević et al. (1988) with ORBIT9 software, a part of the ORBFIT9 package (http://hamilton.dm.unipi.it/astdys/). Extending the remark of Vokrouhlický & Nesvorný (2008) that

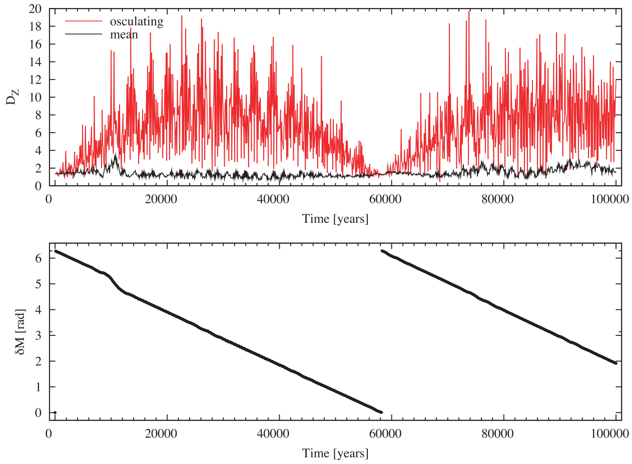

Figure 2. The time evolution of a test pair of clones of asteroid 63440 2001 MD30 generated with an orbital velocity difference of 0.84 m s^{-1}. The upper panel presents evolution of D_Z calculated with osculating (red line) and mean elements (black line). The lower panel shows how the difference in mean anomaly changes.

D attains the smallest value when $\delta M = 0$, we can add that this minimum value coincides with the mean elements based D.

Obviously, short periodic perturbations are not the only factor affecting the evolution of a distance function. Resonances, non-gravitational effects and long period perturbations will cause the growth of orbital distance even for the mean elements.

4 RELATION BETWEEN D AND δv

Southworth & Hawkins (1963) stated that an orbital similarity function of a pair should be the measure of the perturbation required to transform one orbit into another. The same idea is present behind the derivation of D_Z (Zappalà et al. 1990). In order to verify how well is this postulate fulfilled for various orbital similarity functions, we have performed simulations for the special case of a breakup of an object in an asteroidal orbit. The velocity of an object at a nominal orbit of the asteroid 63440 was perturbed at some random position on the orbit by adding a velocity increment $\delta \boldsymbol{v}$. The magnitude δv was uniformly distributed between 0 and 10^{-4} au $d^{-1} \approx 174$ m s^{-1}. The directions of $\delta \boldsymbol{v}$ had a uniform distribution on a sphere. For 500 pairs of clones the orbital distances were calculated at the epoch of the pair creation and after 100 kyr of orbital evolution, using both osculating and mean elements.

Points in Fig. 3 show values of all six distance functions at the epoch of pair creation (breakup) for given magnitudes of the breakup velocity δv. The lines mark the upper (solid) and the lower (dashed)

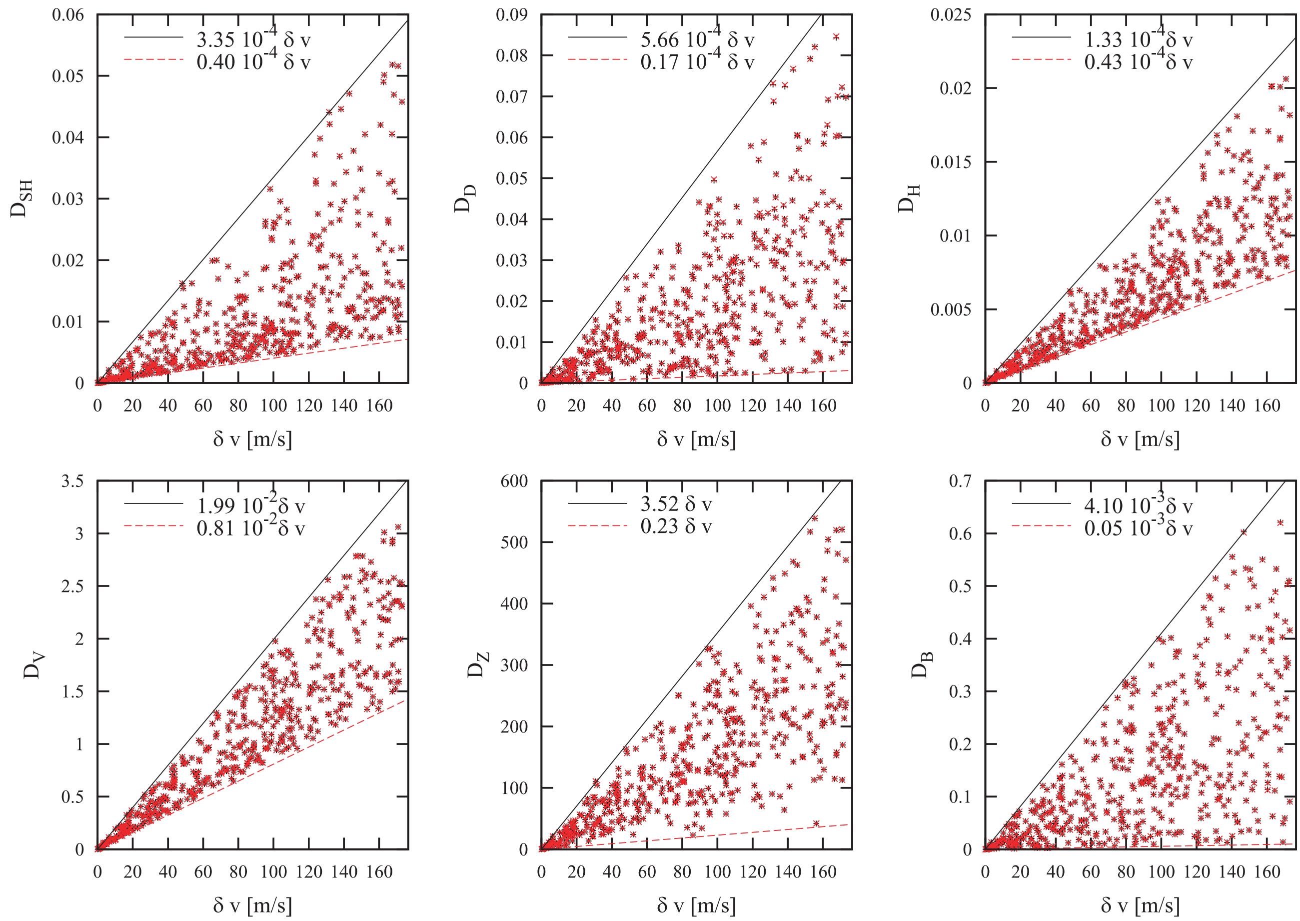

Figure 3. The relation between δv and D for each of the distance functions. The graphs are made for the epoch of pair creation. The red × and black + symbols show values of D-criteria calculated with mean and osculating elements, respectively. The dashed red line on each graph shows the estimate lower limit of the range of values taken by a given distance function, the solid black line is the upper limit, for the epoch of pair creation.

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envelope of the distance. Considering the expected correlation we examine values of δv allowed by a given small distance D. For the D_V and D_H we notice well-constrained ranges of δv. For the remaining orbital similarity functions, however, especially for D_D and D_B, only the lower limit of possible magnitude δv exists, except for unreasonably small values of D. From this point of view the hybrid metric D_H and the vectorial D_V may be considered superior compared to the remaining ones.

Fig. 4 shows the situation after 100 kyr, when we expect the dispersion of D with respect to the initial envelope. The latter is copied from Fig. 3 for the reference. The range of values of D widens for all the distance functions; more precisely the maximum value of D for a given δv increases. The lower envelope (dashed line) remains valid. The effect is relatively strongest for D_H and D_V (which offered the best confinement at the epoch of breakup) and relatively weakest for D_D. From the point of view of the range of δv allowed by a given value of D it means that the lower limit of δv is decreased while the upper one is not affected. If a small dispersion with respect to the initial envelope is taken for a criterion, D_D and D_Z will come out best in this respect.

One may notice the absence of a significant difference between the mean and osculating elements in Figs 3 and 4 apparently contradicting Fig. 2. It can be explained by the fact that most of the pairs considered in this section are not quite close in the context of Vokrouhlický & Nesvorný (2008) or Pravec & Vokrouhlický (2009); their 'close pairs' have D_Z not exceeding 36 m s^{-1}. The assumption about the crucial role of differential short periodic perturbations seems to hold true only for tight pairs, e.g. with $D_Z < 10$ m s^{-1} (this limit is also coherent with Fig. 1). In our simulated population, only a few pairs, concentrated in the lower left-hand corner of each panel, belong to this type.

5 CONCLUSIONS

We have verified that the excess of close asteroid pairs with respect to a random sample, considered to be the indication of some active pair formation mechanism, can be detected using five different distance functions. The excess is more pronounced in terms of mean orbital elements. The D_B metric of Jenniskens (2008) cannot be used for this kind of study; we suspect that the use of quasi-invariants brings it closer to the thee-dimensional proper elements space in spite of using osculating elements. On the other hand, we observed that the evolution of D_B is dominated by $C_3 = \varpi$.

The differences in mean orbital elements of a close pair change slower than respective differences in osculating elements, because they lack short periodic perturbations. The values of orbital distance functions, obtained with mean elements for very close pairs, are more stable than those obtained with osculating elements in short orbital evolution time (up to 100 kyr). Candidates for further dynamical examination, suspected to be young asteroid pairs, are more likely to be found when using mean orbital elements, regardless the distance function in use. Mean elements are probably the best compromise between osculating elements based pair selection

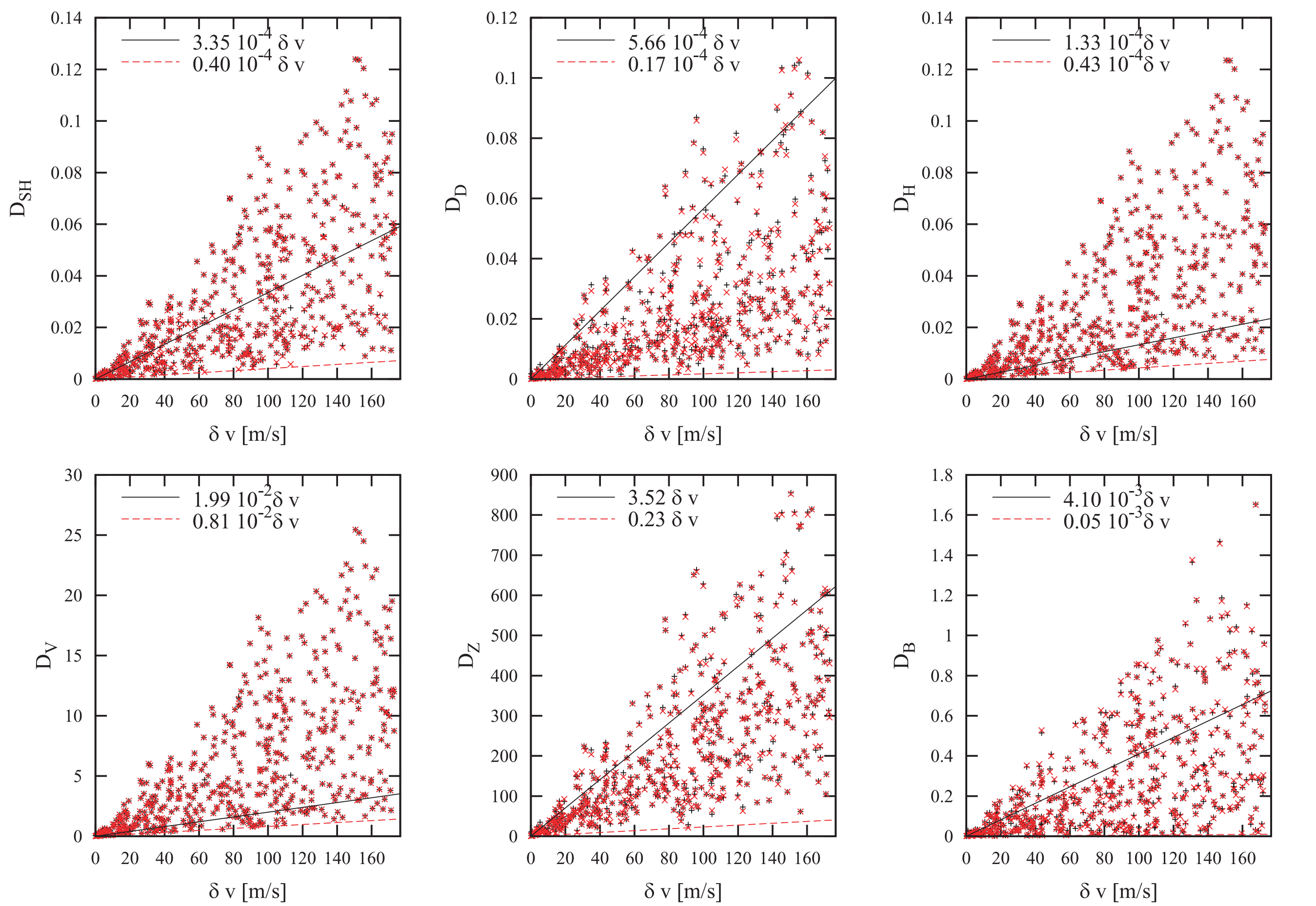

Figure 4. Same as Fig. 3 but after 100 kyr of orbital integration.

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algorithms and the close pairs search in the space of proper elements (preferred by Milani et al. 2010).

Although the distance functions, except for D_{B}, are statistically equivalent, there are differences between the results of close pair searches given by each orbital metric. Tight pairs according to one function could be more distant with respect to another. The coherence between the results of close pair search with different metrics is better when mean elements are considered. Repeating searches using few different functions would lower the risk of omitting a potentially interesting object.

For a close orbital pair, created with a small velocity difference, there is a range of values of D returned by each of the metrics. For recent breakups, the best constrains on δv that could create a close pair are given by D_{V} and D_{H}.

Time evolution of D helps to identify differences in sensitivity of various orbital similarity functions for changes in orbital elements. The values of functions strongly depending on changes of difference in Ω ($D_{\mathrm{SH}}, D_{\mathrm{H}}$ and D_{V}) grow faster than D_{D} or D_{Z}. Thus, D_{D}, dominated by e-related terms, shows the best time stability, at least in the main belt orbits case discussed in this paper. After 100 kyr of orbital evolution, D_{D} and D_{Z} could be recommended as far as the conservation of the lower bound of δv is concerned.

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