Second Comment on: 'The most energy efficient way to charge the capacitor in a RC circuit'

R Oven

School of Engineering and Digital Arts

The University of Kent

Canterbury

Kent, UK

CT2 7NT

R.Oven@kent.ac.uk

Abstract

In a recent reply to the paper (Comment on 'The most energy efficient way to charge the capacitor in a RC circuit') it was argued that the analysis in the above comment was flawed in that the circuit used for the current generator was incomplete. The reply also analyses a current generator circuit to support the arguments and reasserts that the energy loss involved in charging a RC circuit is dependent on the charging rate. Here, the counter arguments in the reply are shown to be in error. It is demonstrated that the conclusions reached in the above comment remain valid.

A comparison has been made between the efficiency in charging a capacitor in a series RC circuit using a constant current source and using a constant voltage source [1]. The paper concluded that it is more efficient to charge *C* to a given voltage using the constant current rather than the voltage source and that the inefficiency in using the voltage source was associated with the rapid changes in the current. These conclusions were argued to be incorrect when the internal construction of a current source was considered [2]. To counter the arguments presented in [2] it has been suggested that the model used for the current source circuit was incomplete in that it did not fully consider the control voltage used to control the current [3]. Further, a circuit that explicitly includes a control voltage source was presented and analysed [3], which purports to support of the conclusions made in paper [1]. Here these counter arguments are discussed and it is shown that the conclusions reached in [2] remain valid.

We first address the criticisms made in [3] of the model current generator circuit discussed in [2]. Figure 1 shows the model current generator circuit from [2] but with the control voltage explicitly added to the base of the transistor to answer the criticism. In fig. 1 the control voltage V_A is applied to the base of the bipolar transistor as outlined in [2]. Since the base current is small for a high gain transistor it is assumed, as stated in [2], that this does not supply significant power to the capacitor; only the source V_B supplies power. Voltage V_A is produced by other circuitry not shown in order to maintain the capacitor charging current at a predetermined level. The base-emitter voltage of the transistor is determined by the difference between V_B and V_A and it is this voltage that determines the collector current and hence charging current. It should be emphasised that devices other than a transistor may be used for the control element (for example a FET); the details are not important. Inspection of the current source circuits in [4] shows that they are all powered by a constant voltage source and have a series control element as outlined in [2]. Since the energy supplied by the constant voltage source V_B, ($U_B = V_B Q$) and the energy stored in the capacitor ($U_C = Q^2/2C$) are both a function of charge *Q* then the energy loss in the transistor and resistor combined $U_E = V_B Q - Q^2/2C$ is also a function of *Q* and independent of the charging rate *I*(*t*) provided of course that the integral of *I* is *Q*. Clearly U_E depends on the supply voltage V_B since the excess energy is dissipated in the transistor and *R*. The analysis of [2] is valid even if the control voltage V_A varies with time, which will vary the charging current *I*(*t*). However, the supply voltage V_B must be a constant as is the case in normal current source circuits [4]. In general the energy supplied by a voltage source is given by $U_B = \int_0^T V_B(t) I(t) dt$ which can only be written as $U_B = V_B Q$ if V_B is a constant.

We now address the circuit and analysis made in [3]. Fig. 2 reproduces the circuit to aid the discussion. We note that this circuit does not conform to the model current generator discussed in [2]. It can be seen that the control voltage V_A is now in series with the emitter rather than the base of the transistor and is thus performing two functions. Not only is V_A determining the current *I* by determining the base-emitter voltage of the transistor but it is also providing power to the circuit since the charging current flows through V_A. If V_A is a function of time as assumed in [3] then not only will the current be a function of time but so will the power supplied by the circuit. This is different to the situation described above where a constant voltage power source is used [2, 4]. The apparent dependence of the energy loss on the charging rate in [3] is thus due to the fact that the time dependent control voltage is also providing power to the circuit as well as controlling the transistor current.

A meaningful comparison can still be made with this circuit by considering the case where V_A is a constant. Since V_A is the base-emitter voltage of the transistor in fig. 2 then this implies that the collector current which is the charging current is a constant as required in [1]. We compare this circuit with the circuit shown in fig. 3 where *C* is charged via *R* with just a constant voltage source equal to V_A+V_B. It is well known that for this circuit *I*(*t*) decays exponentially with time. The energy supplied by V_A+V_B is $U_{TOTAL} = (V_A + V_B)Q$. Subtracting the energy stored in *C* gives the energy lost in *R*

$$U_E = (V_A + V_B)Q - \frac{Q^2}{2C} \tag{1}$$

Now consider the circuit in fig. 2 with constant V_A. The energy supplied by V_A and V_B is again $U_{TOTAL} = (V_A + V_B)Q$. This is valid since the current from V_A and V_B is the same to a very good approximation if the base current of the transistor is small. Subtracting the energy stored in *C* again gives eqn. (1) where U_E now represents the energy lost in the transistor and *R.* So the energy lost in charging *C* to the same *Q* for the circuits in fig. 3 and for fig. 2 with constant V_A is the same. For fig. 3 the current decays exponentially with time, for fig. 2 with constant V_A the current is a constant yet the energy loss is the same.

In summary, precisely because current source circuits are invariably powered by a constant voltage source [4], the energy supplied by the voltage source is a function of charge *Q*. Coupled with the fact that the energy stored in the capacitor is a function of *Q*, then the energy loss in the series control element and resistor combined is also a function of *Q* and independent of *I*(*t*) with the proviso of course that the integral of *I* is *Q*, as shown in [2].

References

[1] Wang. D. 2017 The most energy efficient way to charge the capacitor in a RC circuit Phys. Educ. **52** 065019

[2] Oven. R. 2018 Comment on ‘The most energy efficient way to charge the capacitor in a RC circuit’ Phys. Educ. **53** 046501

[3] Wang. D. (2018) Reply to Comment on ‘The most energy efficient way to charge the capacitor in a RC circuit’ Phys. Educ. **53** 046502

[4] The Art of Electronics. Horowitz and Hill (1980) Cambridge University Press. ISBN 0 521 29837 7 (Ch. 2 pp. 69-62, Ch. 5 pp. 216-218)

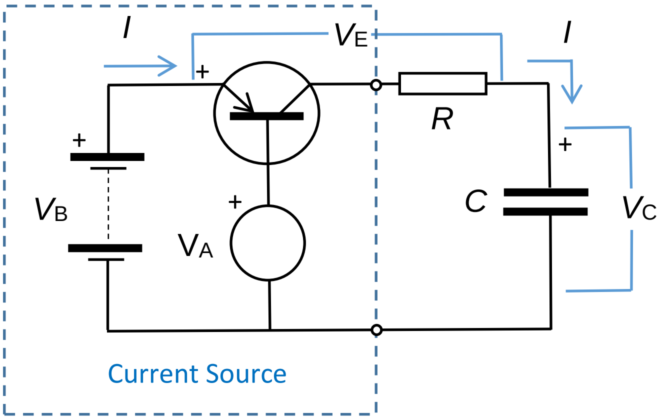

Figure 1. Showing the basic elements of a current source from [2] with V_A explicitly shown.

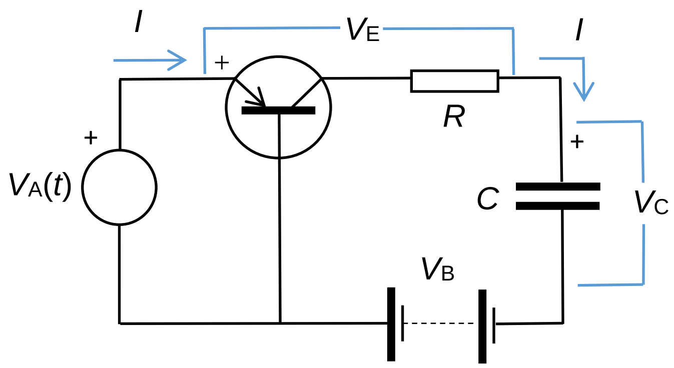

Figure 2. Showing the circuit from [3].

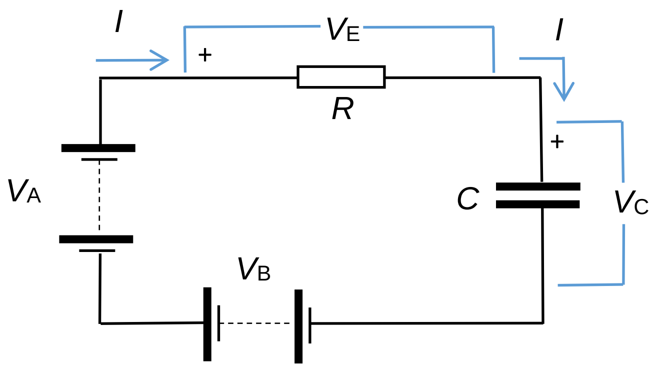

Figure 3. Circuit showing C charging from $V_A + V_B$ with R.