ESSAYS ON BOND PRICING AND GROWTH-INDEXED BONDS

WITHIN THE NEW KEYNESIAN FRAMEWORK

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Abstract:

The term structure of interest rate has long been a special topic of interest in both academia and the financial market. A plethora of models developed in both finance and macroeconomics literature are only partially useful for either macroeconomic analysis or bond pricing, but not for both at the same time. The second chapter of this thesis focuses on this issue from the macroeconomic viewpoints. Firstly, we survey the important papers in term structure of interest rates and asset pricing models and discuss their key features. We then examine the ability of standard DSGE models at replicating the stylized bond pricing facts especially focusing on the volatility of long-term bonds. Lastly, we survey various recent modifications made to the DSGE models and investigate whether and how each approach may (or may not) improve the ability of DSGE model in terms of replicating key bond pricing facts, either under the expectations hypothesis or with the help of term premium.

The third chapter focuses on nominal GDP growth-indexed bonds where their nominal payoffs are fully indexed to nominal GDP growth of the issuing country. The idea of indexing government debt to a country's growth rate goes back at least to the 1980s, and several papers have already illustrated the potential benefits of issuing such bonds. However, most of the analysis were conducted using partial equilibrium models. In addition, as there exists no actual market for such an asset, only few analyses exist for the price of growth-indexed bonds, and most of them are based on simple CAPM models. In this chapter, on the contrary, we try to calculate the theoretical price of nominal GDP growth-indexed bonds using a general equilibrium model. Based on a medium sized New Keynesian DSGE model estimated with the U.S. macroeconomic data, we show that the government may face lower borrowing cost when replacing conventional nominal bonds with nominal GDP growth-indexed

bonds, assuming the other premiums - such as novelty and liquidity premiums - are negligible. As a by-product of the analysis, we also show such a change may benefit the government by giving more room for countercyclical fiscal policy.

The fourth chapter examines the welfare effect of growth-indexed bonds within the framework of new Keynesian DSGE model. Even though there already exist papers that show issuing growth-indexed bond may help stabilize the debt process and give more room for countercyclical fiscal policy, the analysis on its welfare effect has not been actively conducted, especially within the framework of general equilibrium model. It was mostly because the standard DSGE models, where Ricardian equivalence holds, the choices of consumers are immune to the source of government finance. This chapter examines whether and how the use of growth-indexed bonds, instead of the conventional nominal bonds, affects the business cycle and welfare when Ricardian equivalence does not hold any more. More specifically, we augmented hand to mouth households, distortionary income taxes, and Epstein-Zin type recursive preference to the most widely used medium scale DSGE model of Smets and Wouters (2007). The results show that the growth-indexed bond can significantly increase the welfare of the hand-to-mouth households by stabilising their consumption and labour especially when the government cannot flexibly change its debt-to-GDP ratio.

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Contents

List of Tables

List of Figures

Chapter 1

Introduction

The idea of growth-indexed bonds goes back, at least, to the debt crises of emerging market countries in 1980s. After the crisis, several economists have argued that linking the payoffs of the government bonds to the economic performance of the issuing country would help prevent the surge of debt-to-GDP ratio in case of crisis (Bailey, 1983; Krugman, 1988). This idea has been regaining interest recently as many advanced countries are suffering from very high levels of government debt after the financial crisis of 2007.[1] Contrary to the discussions in the 1980s which focused mostly on emerging market countries, recent papers point out that the advanced countries may benefit from the GDP growth-indexed bonds as well. Furthermore, since the advanced countries are less likely to suffer from challenges in issuing GDP-indexed bonds such as data

[1]For example, over the last few years, both academics and central banks around worlds have actively published papers supporting the idea of GDP-linked bonds (see Barr et al., 2014; Blanchard et al., 2016; Bowman et al., 2016; Benford et al., 2016; Cabrillac et al., 2016; Bonfim and Pereira, 2018). Also, in the G20 Finance Ministers and Central Bank Governors Meeting of 2016 and 2017, G20 members called for further analysis for the state-contingent sovereign debt including GDP-linked bonds.

manipulation or adverse selection[2] problems, they may be able to issue them with lower novelty premium than the emerging market countries.

Previous works on the GDP growth-indexed bonds focused on two main benefits from the perspective of the government debt management. First of all, the use of GDP growth-indexed bonds may help stabilise debt-to-GDP dynamics, and thus reduce its tail-risk. When there exists upward pressure on debt-to-GDP ratio due to a slowdown in growth, a government should cope with the pressure by increasing its primary surplus. When the primary surplus cannot be increased sufficiently for various (usually political) reasons, the government is forced to face a sharp rise in its debt-to-GDP ratio. If the bond payoffs are linked to the GDP growth, however, the slower growth also leads to a smaller debt repayment, and thus the rise in debt-to-GDP ratio can be mitigated.

The second benefit is related with conducting fiscal policy. When a country's debt-to-GDP ratio reaches (or closely approaches to) its debt limit[3], the government faces pressures to conduct pro-cyclical fiscal policy. In other words, the government is forced to increase its primary surplus even when the economy is in a recession. In such cases, the use of growth-indexed bonds may play a role of mitigating the pressure of conducting pro-cyclical fiscal policy.

One thing the previous papers commonly pointed out is that these benefits may disappear if the premium on the GDP growth-indexed bonds (over the conventional government bonds) becomes excessively large. Therefore, pricing them accurately can also be an important topic. However, there are only a small

[2]When the market participants believe the government has private information that is not known to the public, they may see the issuance of GDP-indexed bonds as a signal of slowdown in growth and would demand more premium. The advanced countries in general are believed to be less subject to this problem.

[3]The debt limit can be defined in several ways. On one hand, Ostry et al. (2010) developed the concept of debt limit as follows. If a government cannot issue more debt when its debt-to-GDP ratio exceeds a certain level, mostly because its fiscal solvency is in doubt, that level of debt-to-GDP ratio is defined as the government's debt limit. On the other hand, the limits can be set politically such as the debt ceiling of the U.S. or the debt limit of 60% set by the Stability and Growth Pact among the EU countries.

number of academic studies conducted on this topic mainly because the GDP growth-indexed bonds have never been traded, and thus there is no historical data available for empirical analyses.

Most of the existing researches on the benefits and/or the price of GDP growth-indexed bonds depend on partial equilibrium models. More specifically, the benefits were analysed by simulating the well-known debt-to-GDP ratio identity using exogenously assumed joint processes of output growth, interest rates and primary balances. In the case of prices, the analyses mostly depend on CAPM model (Borensztein and Mauro, 2004; Kamstra and Shiller, 2009; Benford et al., 2016; Bowman et al., 2016), where the required expected return of the GDP growth-indexed bond is solely determined by its β with an arbitrarily chosen market portfolio. For this reason, the estimates from CAPM models vary widely across the selection of market portfolio.

The main focuses of the chapters in this thesis are pricing the nominal GDP-growth indexed bonds (NGDP-indexed bond) and investigating its benefits within the framework of New Keynesian DSGE model. Even though the bond prices and benefits from DSGE models also rely on the joint processes of the key variables, the joint processes are obtained by the optimal choices of rational agents, not by arbitrary assumptions. The fact that the DSGE models have become a dominant modelling framework both among academic researchers and practitioners in analysing the inextricably linked relationship among the key macro variables also justifies the use of DSGE models. Furthermore, from a more practical point of view, by using a DSGE model, we can find the answers to the questions particularly important to policy makers, such as on which deep parameters the growth risk premium depends, or under which conditions the government benefits more from the use of growth-indexed bonds.

In Chapter 2, before jumping into pricing GDP growth-indexed bonds, we surveyed various bond pricing and term structure models. First, we began looking at the models from finance literature that can replicate the bond pricing facts quite well, but lack of explanations for the fundamental macroeconomic forces, and we moved on to the macro-finance models which have been developed in efforts to overcome the shortcomings of the finance models, mainly by augmenting macroeconomic theories or variables to the finance models in several ways. Then we illustrated how poor the standard New Keynesian DSGE model is in terms of replicating bond prices, and explored various modifications suggested as an attempt to improve the DSGE model's ability of asset pricing.

In Chapter 3, we built a DSGE model estimated with the U.S. data to calculate the theoretical price of 10-year NGDP-indexed bonds and showed that the government may benefit from lower borrowing cost when replacing conventional nominal bonds with the NGDP-indexed bonds. As the investors purchasing NGDP-indexed bonds are immune to the inflation risk but are exposed to the risk in real GDP growth, the government should pay additional growth risk premium (GRP) but can save inflation risk premium (IRP) when issuing the NGDP-indexed bonds. We showed that both premiums are positive due to the fact that the business cycle in this model is driven mainly by supply shocks; and that the IRP is much larger than the GRP since the inflation is much more persistent than the output growth. As a by-product of the analysis, we also showed that the government may benefit from issuing NGDP-indexed bonds as it gives more room for counter-cyclical fiscal policy, particularly when the government cannot flexibly adjust its debt-to-GDP ratio.

In Chapter 4, we examined the welfare effect of NGDP-indexed bonds also within the New Keynesian framework. In Chapter 3, we briefly showed how the use of NGDP-indexed bonds affects the cyclicality of fiscal policy. However, as

Ricardian equivalence holds in the model, we were not able to see how it affects the business cycle or the welfare of the economy. By adding some households who cannot access financial and capital markets (hand-to-mouth households) to the standard DSGE model, we broke Ricardian equivalence, and showed that the government can use the NGDP-indexed bonds as an alternative fiscal policy tool to stabilise the business cycle and improve the welfare of the hand-to-mouth households.

There are many areas where we may extend our models in Chapter 3 and 4. One of them is from the fact that the models in this thesis ignore the possibility of default and the related costs. Our DSGE models are closed economy models, and they are estimated (or calibrated) with macroeconomic data from the U.S, which is taken to be a benchmark country with little likelihood of government default. Therefore, the conclusion from the model is valid only for the government with little possibility of default; for a small number of advanced economies. To make the conclusion more general, we may extend the model to a small open economy model. Once we explicitly incorporate foreign currency denominated debts and probability of default as Chamon and Mauro (2006) or Barr et al. (2014) did, then we will be able to discuss the cases of emerging market countries as well. If the use of GDP-indexed bonds lowers the overall borrowing cost of the government as Ostry et al. (2010) and Kim and Ostry (2018) argue, the model with such extensions may exhibit the benefits even more clearly.

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Chapter 2

An exposition of the forward rate volatility puzzle

2.1 Introduction

Understanding what moves the term structure of interest rates is important in many aspects. For example, long-term interest rates reflect how bond market participants forecast the future path of the economy. It is especially true under the expectations hypothesis under which the interest rate on a long-maturity bond is represented as an average of expected future short-term interest rates. This topic is also important in the area of finance. The term structure of interest rate is closely related to pricing interest rate derivatives as their theoretical prices are calculated using current and future yields for risk-free bonds (Campbell et al., 1997; Piazzesi, 2010). The relation between the short and long end of yield curves has long been a special topic of interest for both macroeconomists and monetary authorities. It is because, even though central banks, in general, are believed to be able to set the overnight interbank interest rate at whatever level they want (unless it is constrained by the zero lower bound), what central banks really want to affect is the long-term real interest rate to which aggregate demand responds (Blinder, 1997).

Despite of the importance of and the active research on this topic, there still exist several unsolved puzzles about the behaviour of term structure. The bond premium puzzle and the excess volatility puzzle are well known among them. The bond premium puzzle refers to the phenomenon where standard macroeconomic models cannot generate the size and volatility of risk premium on nominal bonds prices. Backus et al. (1989) is one of the early papers that drew attention to this puzzle using a consumption-based asset pricing model of an endowment economy. They showed that the artificial data from their theoretical model cannot reproduce the size and sign of the risk premium estimated from the US Treasury data under reasonable assumptions on consumption growth and risk aversion. They found that, to match the sign and size of term premium, a negative autocorrelation of consumption growth and a coefficient of relative risk aversion (CRRA) larger than 8 are needed, which are not supported by empirical data. Even more recent and sophisticated macroeconomic models have not been able to generate sizeable term premium without relying on implausible assumptions.

The excess volatility puzzle, which is the main interest of this chapter, refers to the excess volatility of the long-term interest rates under the expectations hypothesis. The standard macroeconomic models share several assumptions: In the long run, the level of certain variables such as inflation rate or real interest rate to be constant at their steady state level, the exogenous shocks are stationary and transitory, and all the agents are homogeneously forward-looking and fully informed. Under these assumptions, temporary shocks cannot be transmitted into the far-future expectations of the short-term interest rate, and that is why the long-term bond yield, which is represented as the (weighted) average of those expectations, should be very stable. Shiller (1979), however, originally showed that the long-term rates display a much larger volatility above the upper limit implied by the expectations hypothesis of a rational expectations model. In the paper, he calculated the theoretical upper limits of variances of the six different long-term interest rates covering from 1824 to 1977 under the expectations hypothesis. His results showed that the volatilities of the six long-term interest rates were around 1.2 to 4.4 time as large as their theoretical upper limits given by the expectations hypothesis[1].

This chapter examines the excess volatility puzzle specifically within the framework of New Keynesian DSGE model. Despite the success of models in finance literature in matching the stylized facts of bond prices, such models are not fully satisfactory from the viewpoint of macroeconomists. It is mainly because those models do not explain what the latent factors they rely on are, and what fundamental economic forces are behind those factors. Given the fact that many central banks around the world are actively using DSGE models to analyse the macroeconomic phenomena and to formulate monetary policies (Tovar, 2009; Dotsey et al., 2013), and the fact that the effectiveness of monetary policy relies highly on the relation between the short-term nominal policy rates and long-term real rates, there is a strong demand for a term structure model which is based on macroeconomic theory and, at the same time, able to match the bond pricing fact well.[2] However, as shown in Section 2.4, the medium scale DSGE model estimated with the recent data for macro variables and short-term interest rate does not generate volatile enough long-term interest

[1]More details about his methodology will be provided in Section 2.3.

[2]How central banks interpret the spread between short- and long-term interest rates is critical for their monetary policy responses, and the interpretation is closely related with the macroeconomic models they rely on. For example, under the expectations hypothesis, central banks may rely more on forward looking guidances to influence the expectations of market participants. On the contrary, if central banks see the spread reflects relative supply between short- and long-term bonds, they may rely more on asset purchase policies.

rate under the expectations hypothesis.[3] This may tell us that such DSGE models cannot transmit the effects of current shocks sufficiently into the future (if the expectations hypothesis holds), or we should modify the standard DSGE models such that they can generate sufficiently volatile term premiums (if the expectations hypothesis is abandoned).

This chapter also illustrates that, under the expectations hypothesis, incorporating time-varying inflation target to the DSGE model can be an effective way in generating volatile long-term interest rate. It is because that makes the response of short-term interest rate to current shocks significantly more persistent. This approach is also consistent with the empirical evidence provided by Gürkaynak et al. (2005) which shows that even far away forward rates respond to the unexpected shocks to various macroeconomic and monetary variables. However, under the expectations hypothesis, sensitivity analysis with other modifications, such as different composition of shocks or different value of key structural parameters, only slightly changed the relative volatility of long-term interest rates.

This chapter also examines the DSGE models which accommodate the existence of term premiums. Even though the standard DSGE models cannot generate sizeable term premium even when they are solved with second- or higher-order approximations, the models with Epstein and Zin (1989) type recursive preference successfully generated sizeable term premiums (Rudebusch and Swanson, 2012; Darracq Paries and Loublier, 2010). However, even this approach was not very effective in matching the relative volatility of long-term interest rates. As another approach, we also examined the DSGE models with asset market frictions so that the short- and long-term bonds are not perfect

[3] Rudebusch and Swanson (2008) also illustrated that the medium scale DSGE model with nominal rigidities, labour market frictions, and habit in consumption is poor at matching stylized facts on term structure of interest rates.

substitutes (Harrison, 2011, 2012; Carlstrom et al., 2017; Fuerst, 2015). If this is the case, the relative supply of these bonds affects the size and sign of term premiums. Among them, we examine the model of Harrison (2011, 2012) more in detail.

This chapter consists of 5 sections. Section 2.2 provides a detailed survey of literature on term structure and asset pricing models, and how they replicate key bond pricing facts such as upward sloping yield curves or volatile long-term interest rates. In Section 2.3, we reconfirm the existence of the excess volatility puzzle by conducting the same test as Shiller (1979) with newer data set. In section 2.4, the key stylized facts are provided and we examine how well the medium sized DSGE model can replicate those stylized facts. Also, various recent modifications to DSGE models are investigated both under the expectations hypothesis and when the hypothesis is abandoned. Section 2.5 concludes.

2.2 Literature Review

Early literature on term structure models were stemmed from the seminal papers of Vasicek (1977) and Cox et al. (1981, 1985). These models begin by specifying stochastic processes of the risk-free short-term rate, r_t, and the stochastic discount factor (SDF), m_{t+1} given by Equation (2.1) and (2.2)[4] respectively:

$$r_{t+1} = \varphi r_t + (1-\varphi)\,\theta + \Sigma\,(r_t, \sigma)\,\varepsilon_{t+1} \tag{2.1}$$

$$-\log m_{t+1} = \delta + r_t + \lambda \varepsilon_{t+1}, \tag{2.2}$$

[4]Most of the equations describing the (discrete time) affine term structure models in this chapter are borrowed from Backus et al. (1998) who translated the original continuous time version of models into the discrete time version.

where ε_{t+1} is distributed normally and independently with mean zero and variance one. The parameters $\varphi, \theta, \sigma^2$ are the autocorrelation coefficient, the mean, and the conditional variance of r_t, respectively. λ is the so-called market price of risk assumed to be constant in these models. The last term in Equation (2.1) is assumed as

$$\Sigma\left(r_t, \sigma\right) = \begin{cases} \sigma & \text{in Vasicek model} \\ \sigma\sqrt{r_t} & \text{in CIR model.} \end{cases}$$

They then derive the dynamics of the term structure of interest rates with an additional assumption of no arbitrage. Under the arbitrage-free assumption, there always exists m_{t+1} that satisfies the following general asset pricing condition,

$$p_t = E_t\left(m_{t+1} p_{t+1}\right), \tag{2.3}$$

where p_t denotes the price of any asset at time t. They then assume an affine functional form of the n-period zero coupon bond price, $p_t^{(n)}$ as

$$-\log p_t^{(n)} = A_n + B_n r_t. \tag{2.4}$$

Combining Equation (2.3) and (2.4), and applying the method of undetermined coefficients, the closed form expressions for A_{n+1} and B_{n+1} are explicitly given in terms of A_n and B_n. Since $A_0 = B_0 = 0$, and $A_1 = 0,\ B_1 = 1$, the price of any n-period zero coupon bond (and its yield to maturity) can be explicitly calculated given r_t. Since these models rely on only a single factor, r_t, these “single-factor” affine term structure models (ATSM) give closed form solutions and are easy for variety of applications.

However, as Backus et al. (1998) already showed, these models generate yield curves with much smaller curvature than is observed in the actual data[5]. Moreover, having a single factor also requires the correlations between bond prices for all the maturities close to one[6], which is not observed from the data either. Considering these shortcomings, models with multiple factors are more widely used in practice. Multi-factor ATSMs assume that long-term bond prices and SDFs are affine functions of several unobserved latent factors; and they require the dynamic evolution of yield curve consistent with cross sectional relation by adding no arbitrage condition. Two-factor CIR models (Chen and Scott, 1993; Longstaff and Schwartz, 1992) can be included in this category, and more general conditions for multi-factor ATSMs can be found from Duffie and Kan (1996) and Dai and Singleton (2000).[7]

Meanwhile, a demand for a more parsimonious model of term structure, especially from the financial market participants, leads to the development of so-called pure statistical models. Nelson and Siegel (1987) and Litterman and Scheinkman (1991) are the two most influential models of this group. These models are able to describe various shapes - monotonic, humped and, S-shaped - of yield curves with only a very small number of latent factors. Litterman and Scheinkman (1991) showed that only three orthogonal factors obtained from the principal component analysis are enough to explain most of the variabilities

[5]See Figure 2.1 which compares the average yield curve estimated from the US data (Gürkaynak et al., 2007) with those constructed using CIR model.

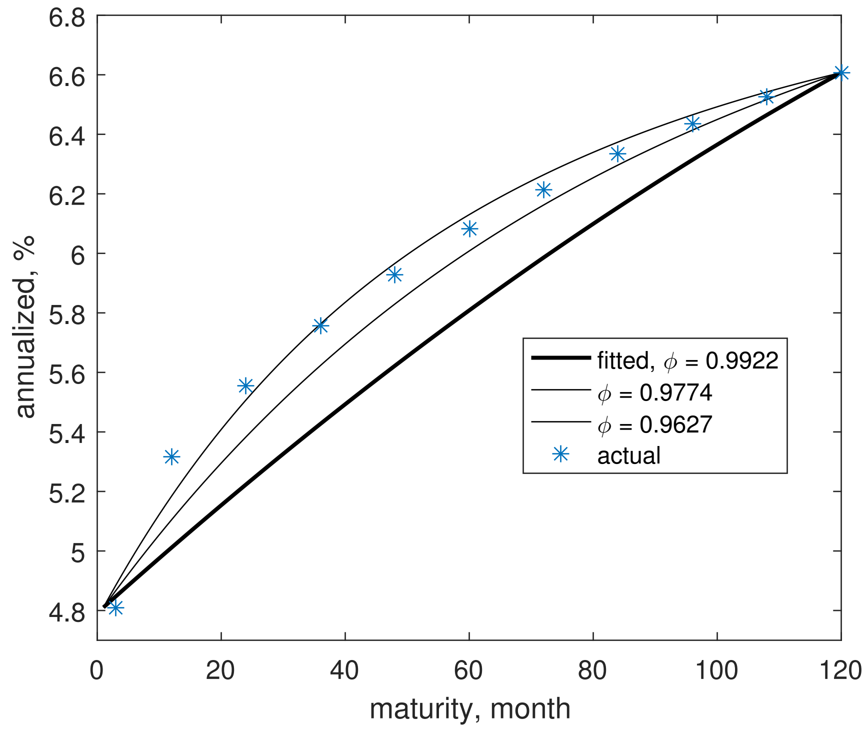
[6]For example, in a single factor Vasicek model, the n-period ahead forward rate is given as follows:

$$f_t^{(n)} = \left(1-\varphi^2\right)\theta + \frac{1}{2}\left(\lambda^2 - \left(\lambda + \frac{1-\varphi^n}{1-\varphi}\sigma\right)^2\right) + \varphi^n r_t.$$

Therefore, the correlations between r_t and $f_t^{(n)}$, or between forward rates of any horizons should all be equal to 1 (Backus et al., 1998). But as seen from Table 2.2 and 2.3, the correlations between the short-term rate and the forward rates from the historical data are smaller than 1.

[7]Duffie and Kan (1996) provided the necessary and sufficient conditions to represent the bond price as an affine function of state variables under a risk neutral measure, and Dai and Singleton (2000) extended these conditions under an actual physical measure.

Figure 2.1: **Average yield curve in the CIR model**



Note: CIR model implied yield curve (thick black line) used sample autocorrelation coefficient from the data ($\varphi = 0.9922$), and the other two curves used smaller φ. The asterisk shows the zero coupon yield curve estimated by Gürkaynak et al. (2007).

of the yield curve, and they named those three factors as "level," "slope," and "curvature" factors for they seem to affect the level, slope, and curvature of the yield curve, respectively. Nelson and Siegel (1987) developed a similar model where $m-$period ahead instantaneous forward rate, $f^{(m)}$, is given by

$$f^{(m)} = \beta_0 + \beta_1 \exp\left(-m/\tau\right) + \beta_2 \left[\left(m/\tau\right) \exp\left(-m/\tau\right)\right]. \tag{2.5}$$

In this model, β_0 determines the long-term components of the forward rate as $f^{(m)}$ approaches to β_0 when $m \to \infty$. Similarly, β_1 determines the short-end of the yield curve as it approaches to $\beta_0 + \beta_1$ when $m \to 0$. Lastly, the shape of "hump" - or medium term component - is determined by β_2.[8] The yield for the m-period zero coupon bond, $R^{(m)}$, is calculated by integrating $f^{(m)}$ from zero to m, and dividing it by m:

$$R^{(m)} = \beta_0 + (\beta_1 + \beta_2) \frac{[1 - \exp(-m/\tau)]}{(m/\tau)} - \beta_2 \exp(-m/\tau). \tag{2.6}$$

Svensson (1994) further improved the Nelson and Siegel (1987) model by augmenting one more term (with two more parameters, β_3 and τ_2):

$$f^{(m)} = \beta_0 + \beta_1 \exp(-m/\tau_1) + \beta_2 [(m/\tau_1) \exp(-m/\tau_1)] + \beta_3 [(m/\tau_2) \exp(-m/\tau_2)]. \tag{2.7}$$

This model is called Nelson-Siegel-Svensson (or NSS) model. By adding the fourth term, this model allows two humps in the yield curve. As the yield curves often show two humps, the new term significantly improved the fit of the model.[9]

The biggest advantage of these group of models are that they give better fit than the theory-based models without resorting to the complicated models with many variables. However, the fact that the earlier versions of these models give no information about the dynamics of yield curve was a drawback for macroeconomists. It is because that means such models cannot be used for forecasting purpose. This limitation motivated the development of the dynamic version of Nelson and Siegel (1987) by Diebold and Li (2006). They incorporated a vector autoregressive (VAR) process of the three latent factors to the model of Nelson

[8] Nelson and Siegel (1987) used 37 cross-sectional daily term structure samples, and each sample contains 30 or 31 pairs of yields and maturities for the date. For a given parameter τ, the best-fitting values of three factors are estimated using linear regression. By trying a grid of values for τ, they found the overall best-fitting values of the factors and τ.

[9] One of the widely used term structure data given by Gürkaynak et al. (2007) is also constructed using NSS model.

and Siegel (1987), and described the m-period interest rate at t as follows:

$$R_t^{(m)} = y(m)\,\theta_t + \varepsilon_t(m)\,, \tag{2.8}$$

where θ_t is a vector of latent factors at t, and $y(m)$ is a vector of factor loadings for a maturity m. In this sense, Equation (2.8) is just a vector representation of Equation (2.6). As the factor loadings, $y(m)$, can be simply calculated with given m and τ, they obtained the estimates of the factors, $\hat{\theta}_t$, for each time-t by conducting an ordinary linear regression of yield curve data, $R_t^{(m)}$, on factor loadings, $y(m)$. Once the factors at each point in time are obtained, they constructed the VAR model for the factors.

Lengwiler and Lenz (2010) improved this model one step further. They named their model as an intelligible factor model for it resolved one of the main drawbacks of the previous model: the lack of orthogonality among the innovations to the three factors. Since Diebold and Li (2006) model does not guarantee that the innovations in θ_t are mutually orthogonal, it is hard to conduct an impulse response analysis with the model. Lengwiler and Lenz (2010) transform the model such that

$$R_t^{(m)} = k(m)\,\phi_t + \varepsilon_t(m)\,, \tag{2.9}$$

where $\phi_t = B\theta_t$ and $k(m) = y(m)\,B^{-1}$, and impose restrictions on the matrix B in the way that guarantees the innovations to the factors are mutually orthogonal.

Despite the fact that these factor models from the finance literature are able to replicate the yield curve behaviour quite well (Dewachter and Iania, 2011), one of the weakest points of such models is that they cannot clearly explain how and which macroeconomic forces are behind the latent factors, and thus, the term structure behaviours. Moreover, for the same reason, the usage of these models for the purpose of policy analyses is very limited. For example, if a central bank tries to analyse the effect of a tighter monetary policy, it can be done by simply changing a relevant structural parameter in monetary policy equation if they use a structural macroeconomic model such as DSGE models. However, with the reduced-form models, one cannot simply change some coefficients since the reduced-form model coefficients are not structural, and the theoretical relation between the reduced-form model coefficients and the structural parameters are not clearly given by the model. Therefore, at least for the purpose of policy analysis, structural macroeconomic models are more widely used by the policy makers or central bankers.

To bridge the gap between the finance and the macroeconomics literatures, so-called macro-finance models have emerged. The first stage of the macro-finance models simply replaced the latent factors with macroeconomic variables (Bernanke et al., 2004), or added macroeconomic variables as additional factors for the Nelson-Siegel type dynamic models (Diebold et al., 2006) or for the multi-factor ATSMs (Ang and Piazzesi, 2003). For example, Diebold et al. (2006) extended Diebold and Li (2006) by simply adding a VAR representation of macroeconomic variables. Contrary to Nelson and Siegel (1987) type dynamic models on which arbitrage-free condition is not explicitly imposed, Ang and Piazzesi (2003) built a macro-finance model by adding macroeconomic variables into a multi-factor ATSM such that both the macro factors[10] and the latent factors simultaneously determine the short rate process, the SDF, and

[10] Ang and Piazzesi (2003) constructed the inflation factor by extracting the first principal component from three inflation measures (CPI, PPI and commodity price indices), and the real-activity factor from four variables capturing the real activity of the economy (help wanted advertising in newspapers, unemployment, employment growth, and the industrial production growth).

the market price of risk. Their model consists of the following four equations:

$$
\begin{aligned}
X_t &= \mu + \Phi X_{t-1} + \Sigma \varepsilon_t && (2.10) \\
r_t &= \delta_0 + \delta_1' X_t && (2.11) \\
-\log m_{t+1} &= \frac{1}{2}\lambda_t' \lambda_t + \delta_0 + \delta_1 X_t + \lambda_t' \varepsilon_{t+1} && (2.12) \\
\lambda_t &= \lambda_0 + \lambda_1 X_t. && (2.13)
\end{aligned}
$$

Equation (2.10) is a Gaussian VAR process of the vector of state factors, X_t, containing both macro and latent factors. Equation (2.11) shows the assumption that the short-term interest rate is an affine function of state factors. The assumption of no-arbitrage guarantees the existence of SDF, and its functional form is assumed to be Equation (2.12). The last equation assumes that the time-varying market price of risk, λ_t, is also an affine function of the state factors. As can be seen, it is almost same as the original multi-factor ATSMs except that its state vector contains the observable macro variables as well[11]. More recently, Dewachter and Iania (2011) introduced an extended version of Ang and Piazzesi (2003)'s model in that financial factors are augmented to Ang and Piazzesi (2003), and showed that adding the financial factors could improve the cross sectional fit of yield curves.

The macro-finance models introduced above have only one feedback channel, from macro variables to financial variables, but not the other way around. However, there also exist macro-finance models which are equipped with two-way feedback channel by incorporating macroeconomic structures into finance models. Hördahl et al. (2006) and Rudebusch and Wu (2007, 2008) are examples of this group of models. Here we look more closely at Rudebusch and Wu (2008). More specifically, Rudebusch and Wu (2008) used the linear-

[11] Dewachter and Lyrio (2006) also follows this approach.

ised equilibrium conditions from the standard New Keynesian macroeconomic model to specify the dynamics of the latent factors in their model (see Equation 2.18 and 2.19). Nonetheless, from the fact that the process of the price of risk is not derived from the optimal choices of consumers, this model is still just a partial equilibrium model. In this model, the functional form of a short-term interest rate, r_t, is assumed as an affine function of latent factors:

$$r_t = \delta_0 + L_t + S_t, \tag{2.14}$$

where L_t and S_t is the two latent factors corresponding to level and slope of an yield curve. The dynamics of these latent factors are specified such that they are affected by macroeconomic variables :

$$\begin{aligned} L_t &= \rho_L L_{t-1} + (1-\rho_L)\,\pi_t + \varepsilon_{L,t} && (2.15)\\ S_t &= \rho_S S_{t-1} + (1-\rho_S)\left[g_y y_t + g_\pi\left(\pi_t - L_t\right)\right] + u_{S,t} && (2.16)\\ u_{S,t} &= \rho_u u_{S,t-1} + \varepsilon_{S,t}, && (2.17) \end{aligned}$$

where π_t is inflation rate and y_t is output gap. Given the specification, L_t is interpreted as the time-varying inflation target of the central bank, and S_t is interpreted as the central bank's monetary policy stance. Most distinct feature of this model comes from Equation (2.18) and (2.19), which are similar to the dynamic IS equation and the Phillips curve, respectively:

$$\begin{aligned} y_t &= \mu_y E_t y_{t+1} + (1-\mu_y)\left(\beta_{y_1} y_{t-1} + \beta_{y_2} y_{t-2}\right) - \beta_r\left(r_{t-1} - L_{t-1}\right) + \varepsilon_{y,t} && (2.18)\\ \pi_t &= \mu_\pi L_t + (1-\mu_\pi)\left(\alpha_{\pi_1}\pi_{t-1} + \alpha_{\pi_2}\pi_{t-2}\right) + \alpha_y y_t + \varepsilon_{\pi,t}. && (2.19) \end{aligned}$$

These two equations mean that the macroeconomic variables y_t and π_t are also affected by the latent factors. Given the processes of the latent factors and the short-term interest rate assumed above, together with the assumption that the price of risk and the long-term bond prices are affine functions of the latent factors, one can derive the closed form relation between a short- and long-term bond prices, following the same way as the standard no-arbitrage ATSMs.

Another stream of macro-finance models derive SDF from the optimal choice of a rational consumer, while assuming exogenous processes of macro variables. To generate upward sloping yield curves and more volatile long-term interest rates, these models make modifications to the preference of consumers. Wachter (2006) proposed a consumption-based asset pricing model of an endowment economy augmented with an external habit formation. The idea of the external habit in asset pricing model is originally from Campbell and Cochrane (1999) who successfully matched various properties of equity prices. Intuitively, including an external habit, which is the weighted average of past consumptions, in a preference makes consumers prefer a smoother consumption stream, or dislike consumption volatility more. Therefore, consumers demand a higher risk premium when purchasing risky assets[12]. In Wachter (2006), consumers maximises the following life-time utility:

$$E\sum_{t=0}^{\infty}\delta^t\frac{(C_t-X_t)^{1-\gamma}-1}{1-\gamma}, \tag{2.20}$$

where the external habit, X_t, is indirectly defined through the surplus consumption defined as

$$S_t \equiv \frac{C_t-X_t}{C_t}.$$

From Equation (2.20), the real stochastic discount factor is derived as

$$M_{t+1}=\delta\left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma}. \tag{2.21}$$

[12] See De Paoli et al. (2010) for an example.

She then assumed the process of the log surplus consumption, s_t, as

$$s_{t+1} = (1-\phi)\,\bar{s} + \phi s_t + \lambda(s_t)\left(\Delta c_{t+1} - E(\Delta c_{t+1})\right), \tag{2.22}$$

where $\bar{s}$ is the long-run average of surplus consumption, and the sensitivity function, $\lambda(s_t)$, determines the volatility of surplus consumption. Once the processes of the consumption growth and inflation are added[13], the nominal stochastic discount factor can be calculated. Then, the price of nominal bonds can be given from the fundamental asset pricing relation.[14]

Instead of the external habit in the preference, Piazzesi and Schneider (2007) used Epstein and Zin (1989) type recursive preference, EZ preference hereafter, for their consumption-based asset pricing model of an endowment economy. In Epstein and Zin (1989), a consumer maximises the following recursively defined utility in each period :

$$V_t = \left[(1-\beta)\,C_t^{\left(\frac{1-\gamma}{\theta}\right)} + \beta\left(E_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}, \tag{2.23}$$

where $\theta \equiv (1-\gamma)\,/\,(1-1/\psi)$. $\gamma \geq 0$ is the parameter which controls the preference for consumption smoothing over the state of nature (or CRRA), and $\psi \geq 0$ is the parameter governing the preference for consumption smoothing over time (or IES). By assuming a unitary IES, Piazzesi and Schneider (2007) reduced Equation (2.23) to the following equation :

$$V_t = C_t^{1-\beta} CE_t\left(V_{t+1}\right)^{\beta} \tag{2.24}$$

[13] She simply assumed that the consumption growth follows a random walk with a drift, and the inflation follows an ARMA(1,1) process.

[14] Detailed solution method used in Wachter (2006) can be found from Wachter (2005).

where

$$CE_t\left(V_{t+1}\right) \equiv E_t\left[V_{t+1}^{1-\gamma}\right]^{1/(1-\gamma)}.$$

The stochastic discount factor is then derived as

$$M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-1}\left(\frac{V_{t+1}}{CE_t\left(V_{t+1}\right)}\right)^{1-\gamma}, \tag{2.25}$$

and once log-normality is assumed, the log of SDF can be derived as follows[15]:

$$\begin{aligned} m_{t+1} &= \log\beta - \Delta c_{t+1} + (1-\gamma)\left(\sum_{i=0}^{\infty}\beta^{i+1}\left(\Delta c_{t+i+1} - E_t\Delta c_{t+i+1}\right)\right) \\ &\quad -\frac{1}{2}(1-\gamma)^2\,\mathrm{var}_t\left(\sum_{i=0}^{\infty}\beta^{i+1}\Delta c_{t+i+1}\right). \end{aligned} \tag{2.26}$$

Equation (2.26) clearly shows the difference between the time-separable expected utility, EU preference hereafter, and EZ preference. The first two terms in Equation (2.26) commonly exist under both preferences, but the remaining two terms appear only when EZ preference is used. In general, an asset should pay a positive risk premium if it gives a smaller payoff when a bad event happens. With EU preference, a bad event means a lower consumption growth for one period, and the first two terms in Equation (2.26) imply this. On the contrary, a consumer with EZ preference considers the entire path of future consumption growth. That is, lower consumption growth of further future periods are also considered as bad events, which is captured in the last two terms of Equation (2.26). In Piazzesi and Schneider (2007), high inflation is regarded as a bad event as it is (exogenously) related with several periods of lower consumption growth in the future. Therefore, if an asset's payoff is negatively correlated with inflation, the asset should provide an additional

[15] This equation is a simplified version of Equation (6) of Piazzesi and Schneider (2007). We used a constant parameter β for the weight in Equation (2.24), while Piazzesi and Schneider (2007) assumed the weight is time-varying.

positive premium on top of the premium required when standard EU preference is assumed. This well explains the upward slope nominal yield curve because the payoffs of longer-term nominal bonds are more negatively correlated with inflation.

Moreover, by changing the value of γ, one can freely choose the size of the additional premium. Note that the EZ preference collapses to the standard EU preference when $\gamma = 1$. Here comes another advantage of EZ preference. EU preference assumes the two risk parameters, γ and ψ, are tightly related, $\gamma = 1/\psi$, with no clear evidence. Therefore, a very high level of CRRA means an unrealistically low level of IES in models with EU preference, and this ends up distorting the fit of macroeconomic variables. However, as the EZ preference breaks the linkage between the two parameters, one can choose any level of CRRA to match the term premium while holding the level of IES, or without comprising the fit of macroeconomic variables.

Even though all these macro-finance models (Rudebusch and Wu, 2008; Wachter, 2006; Piazzesi and Schneider, 2007) are quite successful in matching the bond pricing facts, there is still room for criticism in that they are all partial equilibrium models. More specifically, the process of price of risk in Rudebusch and Wu (2008) is not derived from the optimal choice of rational consumers, but arbitrarily assumed. Therefore, the price of risk does not reflect the consumer's expectations on consumption and inflation. On the contrary, Wachter (2006) and Piazzesi and Schneider (2007) derive the stochastic discount factor from consumer's utility maximisation, and thus the relations between SDF, consumption, and inflation are based on the macroeconomic theory. Nevertheless, they are imperfect since the processes for consumption and inflation are exogenous in these models. In other words, these models can generate sizeable and volatile term premium mostly because of their assumptions of negative correlation between consumption and inflation (both Wachter 2006; Piazzesi and Schneider 2007) or of inflation as the predictor for the future consumption growth path (Piazzesi and Schneider, 2007). The fact that the key ingredients of their successes are not based on the consumer's optimal choices, but just on the exogenous assumptions, can be an important drawback of these partial equilibrium models[16].

In order to overcome the drawbacks of these partial equilibrium macro-finance models, efforts to explain the anomalies of bond price behaviour within the framework of DSGE models have been being made. In contrast with Wachter (2006) who successfully generated sizeable term premium with the help of an external habit, the production economy model with habit formation was not very impressive in terms of replicating the bond pricing facts. The reason is already pointed out by many authors (Jermann, 1998; Boldrin et al., 2001; Rudebusch and Swanson, 2008; De Paoli et al., 2010). By incorporating habit formation, Wachter (2006) was able to amplify the curvature of the utility function and increase the consumer's distaste for volatile consumption. This consequently induced consumers to ask for a higher risk premium for a risky asset. However, contrary to the endowment economy model, consumers have an additional margin to smooth consumption in response to shocks by adjusting labour supply in production economy models. For this reason, the consumers in production economy models require less premium than in endowment models.[17] More recently, Rudebusch and Swanson (2012) showed that the DSGE model with EZ preference and very high level of CRRA can generate sizeable and

[16] Rudebusch and Wu (2008) also mentioned that their partial equilibrium model is "just an intermediate step between the purely empirical models and deep theoretical models".

[17] Despite Hördahl et al. (2008) argued that their DSGE model with habit formation successfully generated sizeable term premiums, their seemingly successful results are mainly from a very large and persistent technology shocks. In order to stabilise the volatile processes of macro variables resulted from the large and persistent technology shocks, they had to rely on extremely high level of monetary policy smoothing parameter and zero output gap coefficient.

volatile term premium without distorting the fit of macroeconomic variables. More details on this model will be discussed in Section 2.4.

2.3 Shiller test with recent data

Before jumping into the DSGE models, let us briefly sketch and apply Shiller (1979)'s method to document the existence of excess volatility. Under the expectations hypothesis, $n-$period interest rate, $R_t^{(n)}$, should be equal to the weighted average of the expected future short-term interest rates, $E_t(r_{t+K})$:

$$R_t^{(n)} = \frac{1-\gamma}{1-\gamma^n}\sum_{K=0}^{n-1}\gamma^K E_t\left(r_{t+K}\right), \tag{2.27}$$

where $\gamma \equiv 1/\left(1+\overline{R}\right)$, and $\overline{R}$ is a discount rate for the future. Also, 1-period holding period return of this bond is given by

$$H_t^{(n)} = \frac{P_{t+1}^{(n-1)} - P_t^{(n)} + C}{P_t^{(n)}}, \tag{2.28}$$

where $P_t^{(n)}$ denotes the price of $n-$period bond at time t, and C denotes the coupon. Substituting the price of the bond (assuming principal at maturity being 1) given below,

$$P_t^{(n)} = \frac{C}{R_t^{(n)}} + \frac{R_t^{(n)} - C}{R_t^{(n)}\left(1+R_t^{(n)}\right)^n}, \tag{2.29}$$

into Equation (2.28) gives:

$$H_t^{(n)} = \left\{C + \frac{C}{R_{t+1}^{(n-1)}} + \frac{R_{t+1}^{(n-1)} - C}{R_{t+1}^{(n-1)}\left(1+R_{t+1}^{(n-1)}\right)^{n-1}}\right\} \Big/ \left\{\frac{C}{R_t^{(n)}} + \frac{R_t^{(n)} - C}{R_t^{(n)}\left(1+R_t^{(n)}\right)^n}\right\} - 1. \tag{2.30}$$

The first order approximation of Equation (2.30) around $R_t^{(n)} = R_{t+1}^{(n-1)} = C = \overline{R}$ gives the following expression for linearised holding period return:

$$\tilde{H}_t^{(n)} = \left(R_t^{(n)} - \gamma_n R_{t+1}^{(n-1)} \right) / \left(1 - \gamma_n\right), \tag{2.31}$$

where $\gamma_n \equiv \left\{ 1 + \overline{R} \left[1 - 1/\left(1 + \overline{R}\right)^{n-1} \right]^{-1} \right\}^{-1} = \gamma \left(\frac{1-\gamma^{n-1}}{1-\gamma^n} \right)$. When the maturity becomes infinity, Equation (2.31) becomes

$$\tilde{H}_t = \left(R_t - \gamma R_{t+1}\right) / \left(1 - \gamma\right), \tag{2.32}$$

where superscript (∞) is abstracted. In a similar way, when the maturity becomes infinity, Equation (2.27) becomes

$$R_t = (1-\gamma) \sum_{K=0}^{\infty} \gamma^K E_t \left(r_{t+K} \right). \tag{2.33}$$

From Equation (2.33), Shiller defined "*ex post* rational long-term rate", R_t^*, as follows:

$$R_t^* \equiv (1-\gamma) \sum_{K=0}^{\infty} \gamma^K r_{t+K}. \tag{2.34}$$

Then from Equation (2.33) and (2.34), he defined forecast error for the *ex post* long-term rate, $R_t^* - E_t R_t^* = R_t^* - R_t$. As the forecast error is uncorrelated with time t information set, the following conditions hold:

$$\begin{aligned} E\left[\left(R_t^* - R_t\right) R_{t-\tau}\right] &= 0 \ \ \forall \tau \geq 0 \ \text{ and} \\ E\left[\left(R_t^* - R_t\right) r_{t-\tau}\right] &= 0 \ \ \forall \tau \geq 0. \end{aligned} \tag{2.35}$$

Now we are ready to derive the variance of (linearised) holding period return, $Var(\tilde{H}_t)$. Using Equation (2.31) and conditions (2.35), the variance can be expressed as[18]

$$Var(\tilde{H}_t) \quad = \quad \frac{1}{(1-\gamma)^2}\left[\left(-1+\gamma^2\right)Var(R_t)+2\left(1-\gamma\right)Cov\left(r_t,R_t\right)\right]. \quad (2.36)$$

By differentiating Equation (2.36) with respect to $Var(R_t)$, we are given the upper bound for $Var(\tilde{H})$ as follows:

$$V_{max}(\tilde{H}) = Var\left(r\right)\rho_{rR}^2\left(1-\gamma^2\right), \quad (2.37)$$

where ρ_{rR} is correlation coefficient between r_t and R_t; and from here, we get the condition, $\sigma(\tilde{H}) < \rho_{rR}\sigma\left(r\right)\sqrt{1-\gamma^2}$, or

$$\sigma(\tilde{H}) < a\sigma\left(\hat{r}\right), \quad (2.38)$$

where $a \equiv \sqrt{(1-\gamma^2)}$ and $\hat{r}$ is the fitted value of a regression of r_t on R_t.[19]

In this section, the same test was conducted with two recent long-term interest rate series from the US and the UK. For the US, 30-year zero coupon bond yields from 1986Q1 to 2016Q4 were used; and consol yields from 1960Q1 to 2016Q4 were used for the UK. For the short-term interest rates, 3 month T-bill rates were used for both countries. The results are summarised in Table 2.1. For the US, the standard deviation of the linearised holding period return, $\sigma(\tilde{H})$, is 1.8 times as large as the theoretical maximum level calculated under the expectations hypothesis. Similar results were obtained with the UK data where the volatility of long-term rates are 1.5 to 2.8 times as large as their theoretical upper limits depending on the time periods concerned. Such results are not quite different from the Shiller's original results where the volatility

[18]To derive this expression, first derive $R_t - R_t^* = \left(\frac{1-\gamma}{1-\gamma F}\right)\left(\tilde{H}_t - r_t\right)$ from Equation (2.33) and (2.34), then use the condition that $R_t - R_t^*$ is uncorrelated with $R_{t-\tau}$ for all $\tau \geq 0$. See footnote 13 from Shiller (1979) for more details.

[19]$\rho_{rR}^2 Var\left(r\right) = Var\left(\hat{r}\right)$ is used.

Table 2.1: **Shiller's test with recent data**

	UK				US
Periods	1960Q1 - 1979Q4	1980Q1 - 1999Q4	2000Q1 - 2016Q4	1960Q1 - 2016Q4	1986Q1 - 2016Q4
maturity(n)	∞	∞	∞	∞	120
γ_n	0.9776	0.9775	0.9894	0.9810	0.9826
a	4.75	4.74	6.88	5.16	5.38
$\sigma(\hat{r})$	2.28	2.54	1.24	2.28	2.08
$a\sigma(\hat{r})$	10.80	12.06	8.54	16.92	11.20
$\sigma(\tilde{H})$	27.50	21.19	23.57	25.51	20.32
$\sigma(\tilde{H})/a\sigma(\hat{r})$	2.55	1.75	2.76	1.51	1.81

Note: For the UK, quarterly averages of consol yields provided by the Bank of England are used. For the US, quarterly average of 30-year zero coupon yields constructed by Gürkaynak et al. (2007) are used. $\tilde{H}_t$ and γ_n are calculated from Equation (2.31) with n indicated. Sample average of long-term interest rates were used for $\overline{R}$.

of six different long-term rates were around 1.2 to 4.4 times as large as their theoretical upper limits.

2.4 Exposition of the Puzzle with DSGE model

In Section 2.3, we saw that there still exists the excess volatility puzzle using Shiller's methodology. In other words, the long-term interest rate were more volatile than their theoretical upper limit calculated under the expectations hypothesis. In this section, we examine whether the artificial data generated by a medium scale New Keynesian DSGE model gives the same results. We provide the stylized facts on bond prices in the US and the UK in subsection 2.4.1, and examine how well various New Keynesian DSGE models replicate the stylized facts in subsection 2.4.2, 2.4.3 and 2.4.4.

2.4.1 Stylized facts

Now let us look at the key stylized facts for the term structure of interest rates in the US and the UK. Table 2.2 and 2.3 provide the descriptive statistics for key indicators: per capita real GDP, Y_t, inflation rate, π_t, 3-month treasury bill rate, r_t, 2- and 10-year nominal zero coupon bond yields, $R_t^{(2)}$ and $R_t^{(10)}$, and 2- and 10-year ahead instantaneous forward rates, $f_t^{(2)}$ and $f_t^{(10)}$. The inflation denotes an annualized quarterly percentage change in personal consumption expenditure price index. The quarterly averages of daily (annualized) rates were used for the interest rates and the forward rates. The standard deviation of Y_t was calculated using Hodrick-Prescott filtered data, but all the other statistics in Table 2.2 and 2.3 were computed with raw data. The detailed sources of the data can be found in Table 2.4.

In Table 2.2 and 2.3, the full sample covers the period from 1971Q3 to 2016Q4 for the US, and from 1971Q2 to 2016Q4 for the UK. The full sample is divided into three sub-sample periods. The first subperiod, before 1986, can be characterized as the period of high inflation hand in hand with high interest rates in both countries. The second subperiod, from 1986 to 2007, is the period of so-called Great Moderation, where both economies experienced a long period of low volatility in business cycles and persistent growth in output, but more particularly in the US. The last subperiod, after 2008, covers the period of and after the Great recession when unconventional monetary policies have been actively conducted by the central banks of both countries[20]. The sub-periods

Table 2.2: **Stylized facts of the US term structure**

	1971Q3 - 1985:Q4			1986:Q1 - 2007:Q4		
	σ_x	$\frac{\sigma_x}{\sigma_r}$	ρ_{rx}	σ	$\frac{\sigma_x}{\sigma_r}$	ρ_{rx}
Y_t	2.17	0.74	0.13	1.05	0.56	0.50
π_t	2.45	0.84	0.32	0.90	0.48	0.34
r_t	2.92	1.00	1.00	1.87	1.00	1.00
$R_t^{(2)}$	2.62	0.90	0.95	1.87	1.00	0.96
$R_t^{(10)}$	2.39	0.82	0.84	1.52	0.82	0.75
$f_t^{(2)}$	2.46	0.84	0.85	1.74	0.93	0.87
$f_t^{(10)}$	2.38	0.82	0.79	1.39	0.74	0.51

	2008Q1 - 2016:Q4			Full sample		
	σ_x	$\frac{\sigma_x}{\sigma_r}$	ρ_{rx}	σ	$\frac{\sigma_x}{\sigma_r}$	ρ_{rx}
Y_t	1.06	2.29	0.42	1.50	0.44	0.17
π_t	0.82	1.76	0.26	2.44	0.71	0.67
r_t	0.47	1.00	1.00	3.44	1.00	1.00
$R_t^{(2)}$	0.53	1.13	0.89	3.47	1.01	0.98
$R_t^{(10)}$	0.83	1.79	0.46	2.85	0.83	0.92
$f_t^{(2)}$	0.63	1.35	0.65	3.31	0.96	0.95
$f_t^{(10)}$	1.16	2.50	0.31	2.46	0.72	0.85

Note: σ_x denotes the standard deviation of variable x, σ_x/σ_r is the standard deviation of variable x over that of short-term interest rate, and ρ_{rx} is the correlation between variable x and short-term interest rate. Y_t is the per capita GDP (HP-filtered), π_t is the annualized percentage change in personal consumption expenditure price index from the previous quarter, r_t is 3 month treasury bill rate, $R_t^{(2)}$ and $R_t^{(10)}$ are 2- and 10-year zero coupon yield, $f_t^{(2)}$ and $f_t^{(10)}$ are 2- and 10-year ahead instantaneous forward rate. All the interest rates and forward rates are quarterly averages of daily numbers.

Table 2.3: **Stylized facts of the UK term structure**

	1971Q2 - 1986Q4			1986:Q1 - 2007:Q4		
	σ	$\frac{\sigma_x}{\sigma_r}$	ρ_{rx}	σ	$\frac{\sigma_x}{\sigma_r}$	ρ_{rx}
Y_t	1.99	0.69	0.12	1.18	0.39	0.19
π_t	6.18	2.13	0.36	2.58	0.85	0.67
r_t	2.90	1.00	1.00	3.04	1.00	1.00
$R_t^{(2)}$	2.23	0.77	0.95	2.47	0.81	0.96
$R_t^{(10)}$	1.87	0.65	0.78	2.28	0.75	0.85
$f_t^{(2)}$	2.00	0.69	0.86	2.24	0.74	0.88
$f_t^{(10)}$	2.47	0.85	0.40	2.22	0.73	0.73

	2008Q1 - 2016:Q4			Full sample		
	σ	$\frac{\sigma_x}{\sigma_r}$	ρ_{rx}	σ	$\frac{\sigma_x}{\sigma_r}$	ρ_{rx}
Y_t	1.48	1.08	0.58	1.55	0.37	0.16
π_t	1.95	1.42	0.50	5.62	1.34	0.61
r_t	1.37	1.00	1.00	4.20	1.00	1.00
$R_t^{(2)}$	1.17	0.85	0.96	3.87	0.92	0.98
$R_t^{(10)}$	0.97	0.71	0.64	3.67	0.87	0.90
$f_t^{(2)}$	1.17	0.85	0.77	3.80	0.90	0.93
$f_t^{(10)}$	0.95	0.69	0.29	3.82	0.91	0.77

***Note**:* Refer to the note from Table 2.2.

selected in this chapter also roughly coincide with the personnel changes of the Fed[21].

The first columns in each panel in Table 2.2 and 2.3 give the standard deviations, σ_x, of the key variables for the full sample and the three sub-sample periods. Comparing the subperiods reveals a clear declining trend in volatility

[20] The first quantitative easing, QE hereafter, was initiated in November 2008 in the US, and March 2009 in the UK.

[21] The first subperiod can be named as the pre-Greenspan period, and the Fed was chaired by Alan Greenspan (from November 1987 to January 2006) in most of the second subperiod. During the last period, the Fed was under Ben Bernanke's presidency (from February 2006 to February 2014) for most of the time.

Table 2.4: **Data sources for stylized facts**

	Series	Source	Details
US	real GDP	BEA	NIPA table 1.1.6
	real consumption	BEA	NIPA table 2.2.3
	population (+16)	BLS	CNP16OV
	GDP deflator	BEA	NIPA table 1.1.9
	3 month T-bill rate	FRED	TB3MS
	zero coupon yields and forward rates	see note	-
UK	real GDP	ONS	ABMM
	real consumption	ONS	ABJR+HAYO
	population (+16)	ONS	MGSL
	GDP deflator	ONS	(ABJQ+HAYE)/(ABJR+HAYO)
	3 month T-bill rate	FRED	INTGSTGBM193N
	zero coupon yields and forward rates	BOE	BOE yield curve archive

Note: Gürkaynak et al. (2010b).

of the interest rates. The second column provides the relative volatilities, σ_x/σ_r, of the key variables defined as the standard deviation of each variable over that of the short-term interest rate. In general, the relative volatility decreased as the maturity increases, but even the 10-year ahead forward rates are around 80% as volatile as the short-term rates in the full sample in both countries. In contrast to the other two subperiods, the third subperiod shows clearly different pattern from the full sample. During the third subperiod, especially in the US, 10-year spot rate, $R_t^{(10)}$, and 10-year ahead forward rate, $f_t^{(10)}$, were much more volatile than the short-term interest rates, r_t. This seems mainly because the short-term rate was almost tied to zero since the beginning of QE, while the long-term rates were allowed to fluctuate[22]. The correlation

[22]It does not seem that the UK data shows the similar behaviour as the US data in this period. It may be attributable to the fact that the Bank of England began its QE several quarters later than the Fed. If we use the data from 2009Q1 to 2016Q4, the relative volatilities of $f^{(10)}$ become very large in both countries (1,156% in the US and 967% in the UK).

of each variable with the short-term interest rate, which is presented in the third column, shows the similar pattern as the relative volatility: The longer the maturity, the smaller the correlation with the short-term rates. Also note that the correlations in the third subsample periods were much lower than the other two subsample periods in both countries.

The two interesting findings from the stylized facts in Table 2.2 and 2.3 are the sizeable volatility in the long-term bond yields and their close positive correlation with the short-term interest rates. One of the possible explanation can be found from Gürkaynak et al. (2005) which shows that far-away (even up to 15 years ahead) forward rates respond to the unexpected shocks to various current macroeconomic variables such as consumer price index, GDP, unemployment, etc. Under the expectations hypothesis where no premium exists, forward rate should be equal to the expectation of short-term interest rate over the same horizon. If that is the case, the volatile long-term interest rates can only be explained by the expected future short-term rates being around 80% as volatile as current short-term rates. Put differently, in order to be able to generate volatile enough long-term interest rate under the expectations hypothesis, either the shocks in the model are very persistent (as in Hördahl et al., 2008) or the model is equipped with an appropriate mechanism which transfers the impacts on the short-term rate into its far-future expectations. These will be discussed in the following sections.

2.4.2 Baseline DSGE model

In this subsection, we use a slightly modified version of Smets and Wouters (2007) as our baseline model for the analysis under the expectations hypothesis. The baseline model of Smets and Wouters (2007) is most commonly used as the workhorse for the macroeconomic analysis. We also conduct sensitivity analyses using alternative parameter calibrations to see how they affect the relative volatility of long-term interest rates.

2.4.2.1 Model Summary

Our baseline model is mainly based on Smets and Wouters (2007), and assumes the similar frictions and shocks as they did. However, as we are going to use up to third-order approximation of the model in Section 2.4.3 to examine the role of time-varying term premium, we slightly simplify the model of Smets and Wouters (2007) such that we can explicitly and compactly write the non-linear equilibrium conditions recursively. As the two models are not very different, the description of the model will focus only on the differences between the two models.

(Households) The optimization problem of the representative household is same as Smets and Wouters (2007):

$$\max_{C_{t,j},h_{t,j}} E_t \sum_{k=0}^{\infty} \beta^k \varepsilon_t^b \left[\frac{1}{1-\sigma_c} \left(C_{t+k,j} - \lambda C_{t+k-1} \right)^{1-\sigma_c} \right] \exp \left[\frac{\sigma_c - 1}{1+\sigma_l} \left(h_{t,j} \right)^{1+\sigma_l} \right]. \quad (2.39)$$

Here we assume the preference shock, ε_t^b, following an AR(1) process, instead of the premium shock used in Smets and Wouters (2007). Each households faces the following budget constraint:

$$\begin{aligned} C_{t,j} + I_{t,j} + \frac{B_{t,j}}{R_t P_t} - T_{t,j} \quad = \quad & \frac{B_{t-1,j}}{P_t} + \frac{W_t^h h_{t,j}}{P_t} + \frac{R_t^k z_{t,j} K_{t-1,j}}{P_t} + \delta K_{t-1,j} \\ & - a\left(z_{t,j}\right) K_{t-1,j} + Div_{t,j}, \qquad (2.40) \end{aligned}$$

where all the households are given same wage, W_t^h, for they supply homogeneous labours to the union, and $Div_{t,j}$ denotes the profits from intermediate good producers and unions. The low of motion of capital is given as follows:

$$K_{t,j} = (1-\delta) K_{t-1,j} + \varepsilon_t^I \left[1 - S\left(I_{t,j}, I_{t-1,j}\right)\right] I_{t,j}. \tag{2.41}$$

In addition, we explicitly specified the functional form of the capital utilization cost:

$$a\left(z_{t,j}\right) = \delta_1 \left(z_{t,j} - 1\right) + \frac{\delta_2}{2} \left(z_{t,j} - 1\right)^2, \tag{2.42}$$

and the investment adjustment cost:

$$S\left(I_{t,j}, I_{t-1,j}\right) = \frac{\phi}{2} \left(\frac{I_{t,j}}{I_{t-1,j}} - \gamma\right)^2, \tag{2.43}$$

and the investment shock, ε_t^I, is assumed to follow an AR(1) process.

(Price and wage setting) The assumptions on labour and goods market structure are similar to Smets and Wouters (2003) with a minor modification on markup shocks. There is a competitive final good firm who aggregates intermediate goods produced by monotonically competitive intermediate good producers using a standard Dixit-Stiglitz aggregator following Smets and Wouters (2003):

$$Y_t = \left(\int_0^1 Y_{t,i}{}^{\frac{1}{1+\lambda_P}} di\right)^{1+\lambda_P}, \tag{2.44}$$

instead of using Kimball (1995) aggregator used in Smets and Wouters (2007). We also replaced the time-varying price markup, $\lambda_{p,t}$, in Smets and Wouters (2007) with constant price markup, λ_P, and added wedge type markup shock, ε_{t+s}^p, in the following price setting problem of intermediate good producers:

$$\max_{\hat{P}_{t,i}} E_t \sum_{s=0}^{\infty} \zeta_p^s M_{t,t+s} \left(\frac{1}{P_{t+s}}\right) \left[X_{t,s}^p \hat{P}_{t,i} - \varepsilon_{t+s}^p MC_{t+s}\right] Y_{t+s,i}, \tag{2.45}$$

where $\hat{P}_{t,i}$ denotes the optimal price of ith intermediate good and $X^p_{t,s}$ is the partial indexation factor given by

$$X^p_{t,s} \equiv \begin{cases} 1 & \text{if } s = 0 \\ \prod_{l=1}^{s} \left(\pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p} \right) & \text{if } s \geq 1. \end{cases}$$

The same changes are made in the wage setting procedure too. We assume that the competitive labour packer uses the standard Dixit-Stiglitz aggregator and the time-varying price markup, $\lambda_{w,t}$, is replaced with λ_w and ε^w_t. We also assume AR(1) processes for these two shocks in the same manner as the other shocks. These modifications are made just because it is tricky to write the non-linear price and wage setting equations recursively when there is a time-varying markup in the exponents. The productivity shock, ε^a_t, follows the same process as in Smets and Wouters (2007).

(Government and Monetary policy) The ratio of government spending to trend output, $\varepsilon^g_t = G_t/(y_* \gamma^t)$, is assumed to have the following process:

$$\log \left(\varepsilon^g_t / \varepsilon^g_* \right) = \rho_g \log \left(\varepsilon^g_{t-1} / \varepsilon^g_* \right) + \eta^g_t + \rho_{ga} \eta^a, \tag{2.46}$$

where G_t is real government spending, y_* is the steady state level of detrended real output, and ε^g_* is the steady state ratio of government spending over output. η^g_t and η^a_t are i.i.d. shocks on government spending and productivity, respectively. Monetary policy rule is borrowed from Smets and Wouters (2007) with some simplification:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*} \right)^{\rho} \left[\left(\frac{\pi_t}{\pi^*_t} \right)^{\psi_1} \left(\frac{Y_t}{Y^*_t} \right)^{\psi_2} \right] \left(\frac{Y_t / Y_{t-1}}{\gamma} \right)^{\psi_3} \varepsilon^r_t, \tag{2.47}$$

where ε_t^r follows AR(1) process. Here, we use the steady state level of real output Y_* instead of the flexible price/wage economy output level of Y_t^f used by Smets and Wouters (2007). More importantly, the inflation target is allowed to be time-varying. For this purpose, we added two more equations borrowed from Rudebusch and Swanson (2012) as below:

$$\pi_t^* = \rho_{\pi^*}\pi_{t-1}^* + \vartheta_{\pi^*}(\bar{\pi}_t - \pi_t^*), \tag{2.48}$$

and

$$\log \bar{\pi}_t = \theta_\pi \log \bar{\pi}_{t-1} + (1 - \theta_\pi) \log \pi_t. \tag{2.49}$$

Except for these differences, the model maintains most of the key features of Smets and Wouters (2007).

2.4.2.2 Baseline results

The baseline parameter values are reported in Table 2.5. Most of the structural parameters and size of shocks are equal to the estimates of Smets and Wouters (2007) except for (inverse of) elasticity of intertemporal substitution in consumption σ_c, steady state level of price and wage markup, λ_p and λ_w, and the standard deviation of price and wage markup shocks, σ_p and $\sigma_{\mathbf{w}}$. We set $\sigma_c = 2.0$ following Rudebusch and Swanson (2012) which is somewhat larger than the estimate of Smets and Wouters (2007), 1.39. This is just to avoid σ_{EZ} being to large when matching the term premium of 100 basis points in subsection 2.4.3. The steady state level of markups are set as $\lambda_p = 0.2$ and $\lambda_w = 0.5$ following Levin et al. (2006) and Smets and Wouters (2003), respectively. As we use a different specification for the markup shocks from those of Smets and Wouters (2007), we calibrated the size of these two shocks so that the model matches the volatility of output growth, consumption growth, inflation and short-term interest rates of the US data which are used to construct Table 2.2 (full sample, 1971Q3 - 2016Q4). As we use the first order approximation of the model and do not assume time-varying inflation target in this subsection, we let $\sigma_{EZ} = 0$, $\rho_{\pi^*} = 1$ and $\vartheta_{\pi^*} = 0$ for our baseline model.

Given that all the discussions in this subsection are made under the expectations hypothesis, we solved the first-order approximation of our baseline model, and then generated 10,000 periods of simulated data, including 39 forecasts for the future short-term rates in each period. Using the simulated data, we constructed the model implied $n-$period interest rates as follows:

$$R_t^{(n)} = \frac{1}{n}\sum_{k=0}^{n-1} E_t r_{t+k}, \tag{2.50}$$

where $R_t^{(n)}$ denotes $n-$period interest rate, and $E_t r_{t+k}$ denotes time t expectation of $k-$period ahead short-term interest rate from the model.

Table 2.6 provides the summary of the baseline results. The first two columns show the standard deviation and relative volatility of the key macro and bond price variables from the actual data, and the last two columns show those from the simulated data. Even though the baseline model well replicates the volatility of actual output growth, consumption growth, inflation and short-term interest rate, the model implied long-term interest rates (computed under the expectations hypothesis) seem to be too stable compared to the actual data. For example, the actual 10-year zero coupon bond yields were 83% as volatile as the short-term rates, but the standard deviation of the (baseline) model implied 10-year yield is only 21% of that of short-term rate. Similarly, the expectation for the 10-year ahead short-term rate given by the model ($E_t r_{t+39}$) is only 15% as volatile as the current short-term rates, but it was 72% in the actual data. In short, despite of the model's good fit to the macro variables, the expectations

Table 2.5: **Baseline calibration**

(Structural parameters)

Parameter		Description	Parameter		Description
σ_c	2.00	IES in consumption	ρ	0.81	policy rate smoothing
σ_l	1.90	labour supply elasticity	ψ_1	2.03	inflation gap coefficient
λ	0.71	degree of habit	ψ_2	0.08	output gap coefficient
ξ_p	0.65	price stickiness	ψ_3	0.22	growth rate coefficient
ξ_w	0.73	wage stickiness	ρ_{π^*}	0.99	AR(1) coefficient for π_t^*
ι_p	0.22	price indexation	ϑ_{π^*}	0.01	response of π^* to $\overline{\pi}_t$
ι_w	0.69	wage indexation	θ_π	0.7	controlling duration of $\overline{\pi}_t$
λ_p	0.20	steady state price markup	α	0.19	capital share in income
λ_w	0.50	steady state wage markup	$\overline{\pi}_*$	0.81	steady state inflation
Φ	1.61	fixed cost	$\overline{\beta}$	0.16	discount factor
ϕ	5.48	investment adj. cost	$\overline{\gamma}$	0.43	balanced growth rate
δ_1	0.035	first derivative of util. cost	ε_*^g	0.18	gov. spending over output
δ_2	0.019	second derivative of utili. cost	σ_{EZ}	-	EZ parameter

Note: $\overline{\beta} \equiv 100\left(\beta^{-1}-1\right)$ and $\overline{\gamma} \equiv 100 \times \log \gamma$. Note also that, for the baseline model, time-varying inflation target is not assumed, that is, $\rho_{\pi^*}=1$ and $\vartheta_{\pi^*}=0$. In subsection 2.4.2.2 and 2.4.2.3, $\sigma_{EZ}=0$ is used as we assume expected utility preference, and in subsection 2.4.2.4, $\sigma_{EZ}=-106$ is used to match the term premium of 100 basis points under EZ preference.

(Shocks)

Parameter		Parameter		Description
σ_a	0.45	ρ_a	0.95	productivity shock
σ_b	0.24	ρ_b	0.18	preference shock
σ_g	0.52	ρ_g	0.97	spending shock
σ_i	0.45	ρ_i	0.71	investment shock
σ_r	0.24	ρ_r	0.12	monetary shock
σ_p	0.40	ρ_p	0.90	price markup shock
σ_w	0.40	ρ_w	0.97	wage markup shock
		ρ_{ga}	0.52	correlation between a, g shocks

Table 2.6: **Too stable long-term rates from the baseline model**

	Actual (US)		Model implied	
	σ_x	σ_x/σ_r	σ_x	σ_x/σ_r
dy_t	3.19	0.93	3.28	0.93
dc_t	2.62	0.76	2.34	0.78
π_t	2.44	0.71	2.95	0.79
r_t	3.44	1.00	3.43	1.00
$R_t^{(2)}$	3.47	1.01	2.17	0.63
$R_t^{(10)}$	2.85	0.83	0.73	0.21
$f_t^{(2)}$ (or $E_t r_{t+7}$)	3.31	0.96	0.94	0.28
$f_t^{(10)}$(or $E_t r_{t+39}$)	2.46	0.72	0.51	0.15

Note *:* Moments for the actual data are calculated from the same data as in Table 2.2 for the period from 1971Q3 - 2016Q4. dy_t and dc_t denote quarterly output and consumption growth rates expressed in percent. The model moments are obtained by simulating the baseline model for 10,000 periods.

given by the model were too stable to replicate the behaviour of the actual long-term interest rates under the expectations hypothesis.

We also examined how well the current unexpected shocks are transmitted into the future using the baseline model simulated data. For this, we examined the relation between the response of current short-term rate to unexpected shocks and the responses of various (1 to 40 quarter ahead) expected short-term rates to the same unexpected shocks. This can be done by simply regressing the unexpected change in the expectation of k-period ahead short-term rate $(E_t r_{t+k} - E_{t-1} r_{t+k})$ on the unexpected change in the current short-term rate $(r_t - E_{t-1} r_t)$:

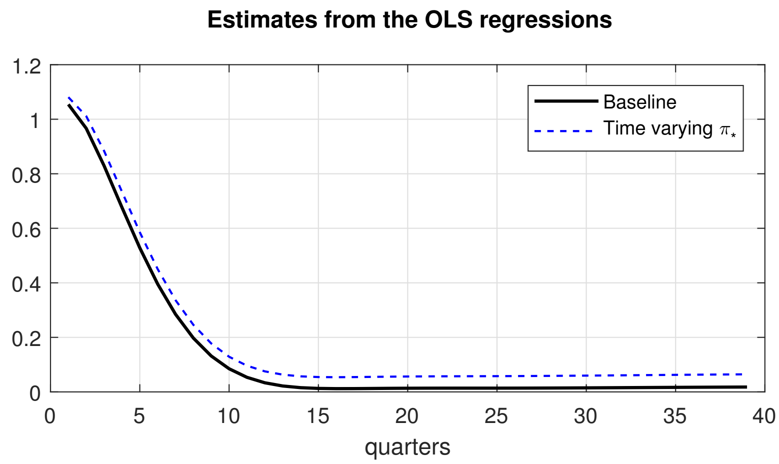
$$E_t r_{t+k} - E_{t-1} r_{t+k} = \alpha_k + \beta_k \left(r_t - E_{t-1} r_t\right) + \epsilon_t. \tag{2.51}$$

When there is no unexpected shock at time t, r_t and $E_t r_{t+k}$ should be equal to those are expected at time $t-1$; i.e., $r_t = E_{t-1} r_t$ and $E_t r_{t+k} = E_{t-1} r_{t+k}$. Therefore, $(r_t - E_{t-1} r_t)$ and $(E_t r_{t+k} - E_{t-1} r_{t+k})$ are the unexpected changes in current and expected short-term rates due to the unexpected shocks revealed at time t.

Figure 2.2 shows the β_k coefficients from all 40 regressions. The unexpected shocks at time-t that increases r_t by 1 percent point raises $E_t r_{t+1}$ by more than 100 basis points in the baseline model (solid black line), but β_k rapidly decreases as the expectation horizon extends, and it becomes less than 2 basis points when the expectation horizon is longer than 15 quarters. This result is quite different from the empirical evidence observed by other authors. As already mentioned, Gürkaynak et al. (2005) showed that even far-ahead forward rates strongly respond to current shocks on macro and monetary variables. They showed that, for example, 1 percent point surprise of monetary policy announcement changes 10 year ahead forward rates by 16 basis points. Nakamura and Steinsson (2013) also showed that a shock that increases 2-year nominal interest rate by around 1 percent point also raises the 5- and 10-year ahead real forward rates by 47 basis points and 12 basis points, respectively.[23] Gilchrist et al. (2014) and Hanson and Stein (2015) reported similar results. We show, however, that our baseline DSGE model cannot replicate such empirical findings. In order to match these empirical findings, a lot stronger responses of $E_t r_{t+39}$ are required to the current shocks. For comparison, we also presented the results when time-varying inflation target is incorporated to the baseline model (setting $\rho_{\pi^*} = 0.99$ and $\vartheta_{\pi^*} = 0.01$) in Figure 2.2, and the result suggests that allowing time varying inflation target may help increase the responses of far future expectations.

[23] Note that Gürkaynak et al. (2005) and Nakamura and Steinsson (2013) are different in interpreting their results. The former argue that the current shocks affect far-ahead nominal forward rates through inflation expectations. However, Nakamura and Steinsson (2013) argue that most of the responses of the far-ahead nominal forward rates are from the responses of the real forward rates, not the inflation expectations.

Figure 2.2: **Responses of expected short-term rates to unexpected shocks**



2.4.2.3 Sensitivity analysis with alternative parameters

In this subsection, we conducted a sensitive analysis using alternative sets of parameters to see how different parameters affect the relative volatility of long-term interest rates, i.e., whether they may be able to increase the volatility of far future expectations large enough to match the actual data. Specifically, we first examined how different composition of shocks affect the relative volatility of the long-term interest rates. We simulated the model for 10,000 periods with different set of shocks and compared the relative volatilities calculated in each case. We also tried different values of parameters controlling degree of monetary policy smoothing (ρ), intertemporal elasticity of substitution (σ_c), price and wage rigidities (ζ_p, ζ_w), degree of external habit (λ), and time-varying inflation target (ρ_{π^*}). Note again that all these analysis are made under the expectations hypothesis.

(Composition of shocks) Table 2.7 shows how different shocks affect the relative volatility of long-term interest rates. The panel (I) and (II) provide the statistics from the actual data and the baseline model. Panel (III) shows the

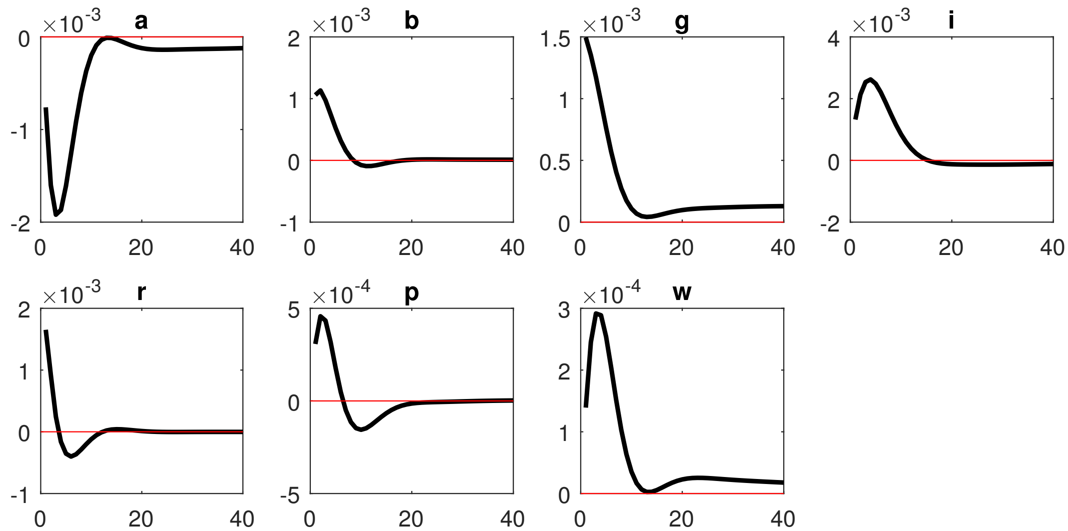
Table 2.7: **Sensitivity analysis with different shocks**

			dy	dc	π	r	$R^{(2)}$	$R^{(10)}$	$E_t r_{t+7}$	$E_t r_{t+39}$
(I)	**Actual**	σ_x	3.19	2.62	2.44	3.44	3.47	2.85	3.31	2.46
		σ_x/σ_r	0.93	0.76	0.71	1.00	1.01	0.83	0.96	0.72
(II)	**Baseline**	σ_x	3.28	2.34	2.95	3.43	2.17	0.73	0.95	0.51
		σ_x/σ_r	0.96	0.68	0.86	1.00	0.63	0.21	0.28	0.15
(III)	**Demand**	σ_x	2.97	2.13	1.93	3.00	1.91	0.56	0.82	0.40
		σ_x/σ_r	0.99	0.71	0.64	1.00	0.64	0.19	0.27	0.13
	Supply	σ_x	1.30	0.96	2.27	1.73	1.07	0.43	0.44	0.27
		σ_x/σ_r	0.75	0.56	1.31	1.00	0.62	0.25	0.25	0.16
(IV)	**shock** a	σ_x	1.23	0.89	2.16	1.65	1.03	0.40	0.38	0.25
		σ_x/σ_r	0.74	0.54	1.31	1.00	0.62	0.25	0.23	0.15
	shock b	σ_x	1.32	1.98	0.57	0.85	0.36	0.07	0.08	0.02
		σ_x/σ_r	1.55	2.34	0.67	1.00	0.42	0.08	0.10	0.03
	shock g	σ_x	2.14	0.33	0.52	1.20	0.69	0.44	0.44	0.38
		σ_x/σ_r	1.79	0.28	0.44	1.00	0.58	0.37	0.37	0.31
	shock i	σ_x	1.47	0.20	1.45	2.56	1.79	0.40	0.74	0.23
		σ_x/σ_r	0.57	0.08	0.57	1.00	0.70	0.16	0.29	0.09
	shock r	σ_x	0.68	0.72	1.06	0.83	0.19	0.03	0.10	0.00
		σ_x/σ_r	0.82	0.86	1.27	1.00	0.23	0.04	0.12	0.00
	shock p	σ_x	0.40	0.31	0.62	0.36	0.15	0.03	0.14	0.01
		σ_x/σ_r	1.13	0.88	1.75	1.00	0.43	0.09	0.38	0.03
	shock w	σ_x	0.20	0.24	0.34	0.26	0.16	0.06	0.06	0.03
		σ_x/σ_r	0.78	0.92	1.29	1.00	0.63	0.24	0.23	0.13

Note: Demand shocks include b, g, i, r shocks and supply shocks consists of a, p, w shocks. Model implied volatilities are calculated using 10,000 simulated data with different set of shocks.

simulated results when only demand shocks (preference, government spending, investment, and monetary policy shocks) or supply shocks (productivity, price markup, and wage markup shocks) are used. When there exist only demand shocks in the model, the volatilities of macro variables are not very different from the baseline model, and the relative volatility of $E_t r_{t+39}$ becomes a little bit smaller than the baseline result (15%→13%). On the contrary, when there are only supply shocks in the model, the relative volatility of $E_t r_{t+39}$ becomes

Figure 2.3: **Impulse response functions (Baseline model)**



a bit larger the baseline model (16%), but still far lower than the actual data (72%).

We also examined the seven cases where only one shock exists in the model (see panel IV). Among the seven cases, only government-shock-only case generates the relativity volatility of $E_t r_{t+39}$ larger than the baseline model. Even the results from the government-shock-only case is, however, still far lower than the actual level of 72%. Figure 2.3 which provides the impulse response of short-term interest rate to each of the seven shocks up to 40 periods tells the same story. The results in table 2.7 and Figure 2.3 suggest that some combinations of the shocks may increase the relative volatility of $E_t r_{t+39}$, but it seems hard to reach the level from the actual data.

(Different value of structural parameters) Table 2.8 shows how the changes in some parameter values affect the relative volatility of long-term interest rates. Same as the Table 2.7, the top two panels show the statistics from the actual data and the baseline model. The rest of the panels, from (III)

Table 2.8: **Sensitivity analysis with different parameters**

			dy	dc	π	r	$R^{(2)}$	$R^{(10)}$	$E_t r_{t+7}$	$E_t r_{t+39}$
(I)	**Actual**	σ_x	3.19	2.62	2.44	3.44	3.47	2.85	3.31	2.46
		σ_x/σ_r	0.93	0.76	0.71	1.00	1.01	0.83	0.96	0.72
(II)	**Baseline**	σ_x	3.28	2.34	2.95	3.43	2.17	0.73	0.95	0.51
		σ_x/σ_r	0.96	0.68	0.86	1.00	0.63	0.21	0.28	0.15
(III)	$\rho = 0.20$	σ_x	3.09	2.02	2.81	5.34	3.04	0.98	1.29	0.57
		σ_x/σ_r	0.58	0.38	0.53	1.00	0.57	0.18	0.24	0.11
	$\rho = 0.99$	σ_x	11.79	11.84	25.47	7.10	4.64	1.21	2.09	0.38
		σ_x/σ_r	1.66	1.67	3.59	1.00	0.65	0.17	0.29	0.05
(IV)	$\zeta_p = 0.01$	σ_x	3.38	2.28	4.57	3.82	2.11	0.72	0.85	0.51
		σ_x/σ_r	0.88	0.60	1.20	1.00	0.55	0.19	0.22	0.13
	$\zeta_p = 0.99$	σ_x	3.50	2.43	0.13	2.32	1.51	0.55	0.90	0.33
		σ_x/σ_r	1.51	1.05	0.05	1.00	0.65	0.24	0.39	0.14
(V)	$\zeta_w = 0.01$	σ_x	3.82	1.91	3.79	3.74	2.12	0.73	0.86	0.51
		σ_x/σ_r	1.02	0.51	1.01	1.00	0.57	0.19	0.23	0.14
	$\zeta_w = 0.99$	σ_x	3.45	2.77	1.78	2.51	1.57	0.50	0.82	0.32
		σ_x/σ_r	1.37	1.10	0.71	1.00	0.62	0.20	0.33	0.13
(VI)	$\sigma_c = 0.7$	σ_x	3.17	2.99	1.88	1.46	0.92	0.36	0.61	0.14
		σ_x/σ_r	2.18	2.05	1.29	1.00	0.63	0.25	0.42	0.10
	$\sigma_c = 5.0$	σ_x	3.51	3.09	4.45	5.66	3.71	1.77	2.02	1.53
		σ_x/σ_r	0.62	0.55	0.79	1.00	0.66	0.31	0.36	0.27
(VII)	$\lambda = 0.01$	σ_x	3.00	3.22	2.23	2.30	1.63	0.61	0.90	0.42
		σ_x/σ_r	1.31	1.40	0.97	1.00	0.71	0.26	0.39	0.18
	$\lambda = 0.98$	σ_x	3.26	2.40	4.61	5.39	3.51	1.77	1.88	1.47
		σ_x/σ_r	0.60	0.44	0.86	1.00	0.65	0.33	0.35	0.27
(VIII)	time-varying	σ_x	3.29	2.36	5.16	5.50	4.73	4.19	4.32	4.20
	inf. target	σ_x/σ_r	0.60	0.43	0.94	1.00	0.86	0.76	0.78	0.76

***Note**:* For the case of time-varying inflation target, we assumed that $\rho_{\pi^*} = 0.99$, $\vartheta_{\pi^*} = 0.01$ and $\theta_\pi = 0.7$ as in Table 2.5.

to (VII), display the relative volatilities for the cases where some structural parameters have extreme values while leaving other parameters unchanged. For example, from the panel (III), we see that the relative volatility of $E_t r_{t+39}$ does not increase significantly even when we use a very small ($\rho = 0.2$) or a very large ($\rho = 0.9$) values for the monetary policy smoothing parameter. The other parameters tell the similar stories. We also examined the two parameters which control the rigidities of price and wage, ζ_p and ζ_w. We chose to examine these two parameters because the rigidities in price and wage are directly related with the volatility in inflation, and thus in short-term interest rate as well. However, as the result shows (panel IV and V), they did not significantly affect the relative volatility of the long-term interest rate or far future expectations for the short-term interest rate.

The parameters related with the consumer's preference, σ_c and λ, also affect the volatility of current short-term interest rate. Higher level of σ_c means smaller intertemporal elasticity of substitution, and this implies a smaller change in consumption for a given change in real interest rate. In other words, the larger σ_c becomes, the more volatile the short-term interest rate becomes. When we increase σ_c from 2.0 (baseline value) to 5.0, the relative volatility also rises from 0.15 to 0.27 as seen from the panel (VI). However, it is still far lower than the actual level of 0.72. Higher degree of external habit, λ, gives the similar result. When λ increases, consumers respond to current shocks less sensitively, and thus other variables, including short-term interest rate, become more volatile. This change also increased the relative volatility, but not enough to match the actual data. As we can see from Figure 2.4, 2.5 and 2.6, the extreme values in these parameters affect mostly the instant (and near future) responses of short-term interest, but none of them seems to significantly change far-future expectations. In short, under the expectations hypothesis, it seems hard to replicate the relative volatility of actual data by simply changing the composition of shocks or with different set of parameter values.

(Allowing time-varying inflation target) The bottom panel (VIII) in Table 2.8 shows the results when we allowed time-varying inflation target using Equation (2.48) and (2.49), which were assumed to be constant in the previous cases. The idea of time-varying inflation target is first introduced by Kozicki and Tinsley (2001), and many authors accepted this idea for explaining their empirical findings (Gürkaynak et al., 2005), interpreting the latent factors in their macro-finance models (Dewachter and Lyrio, 2006; Rudebusch and Wu, 2008), or matching sizeable term premium in a DSGE based asset pricing models (Rudebusch and Swanson, 2012; Hördahl et al., 2008).[24] In our model, we set $\rho_{\pi^*} = 0.99$ and $\vartheta_{\pi^*} = 0.01$ following Rudebusch and Swanson (2012). In contrast to the previous cases where even extreme values can hardly increase the relative volatility of far-future expectations, the result in panel (VIII) shows that allowing time-varying inflation target increases the relative volatility of $E_t r_{t+39}$ to a significantly higher level.

The impulse response functions in Figure 2.7 help explain this result. Assuming time-varying inflation target hardly affects the instant or near future responses of short-term rates, but it significantly amplifies the responses of far-future expectations. The mechanism behind this is as follows. When a certain shock - e.g., a negative productivity shock - increases both current short-term interest rate and inflation at the same time; and it also increases the central bank's inflation target through Equation (2.48) and (2.49). Then,

[24]Gürkaynak et al. (2005) explained why current unexpected shocks affect far-ahead forward rates by assuming time-varying inflation target. In their model, the unexpected shocks that raise current short-term rate also increase long-run inflation target, which in turn affect the expectation for far-ahead nominal interest rate. In Dewachter and Lyrio (2006) and Rudebusch and Wu (2008), the level factors of their macro-finance models are interpreted as time-varying long-run inflation expectations. Hördahl et al. (2008) relied on the assumption of very persistent ($\rho_{\pi^*} = 0.999$) time-varying inflation target to generate sizeable term premium.

Figure 2.4: **Responses of short-term rate (alternative parameters)**

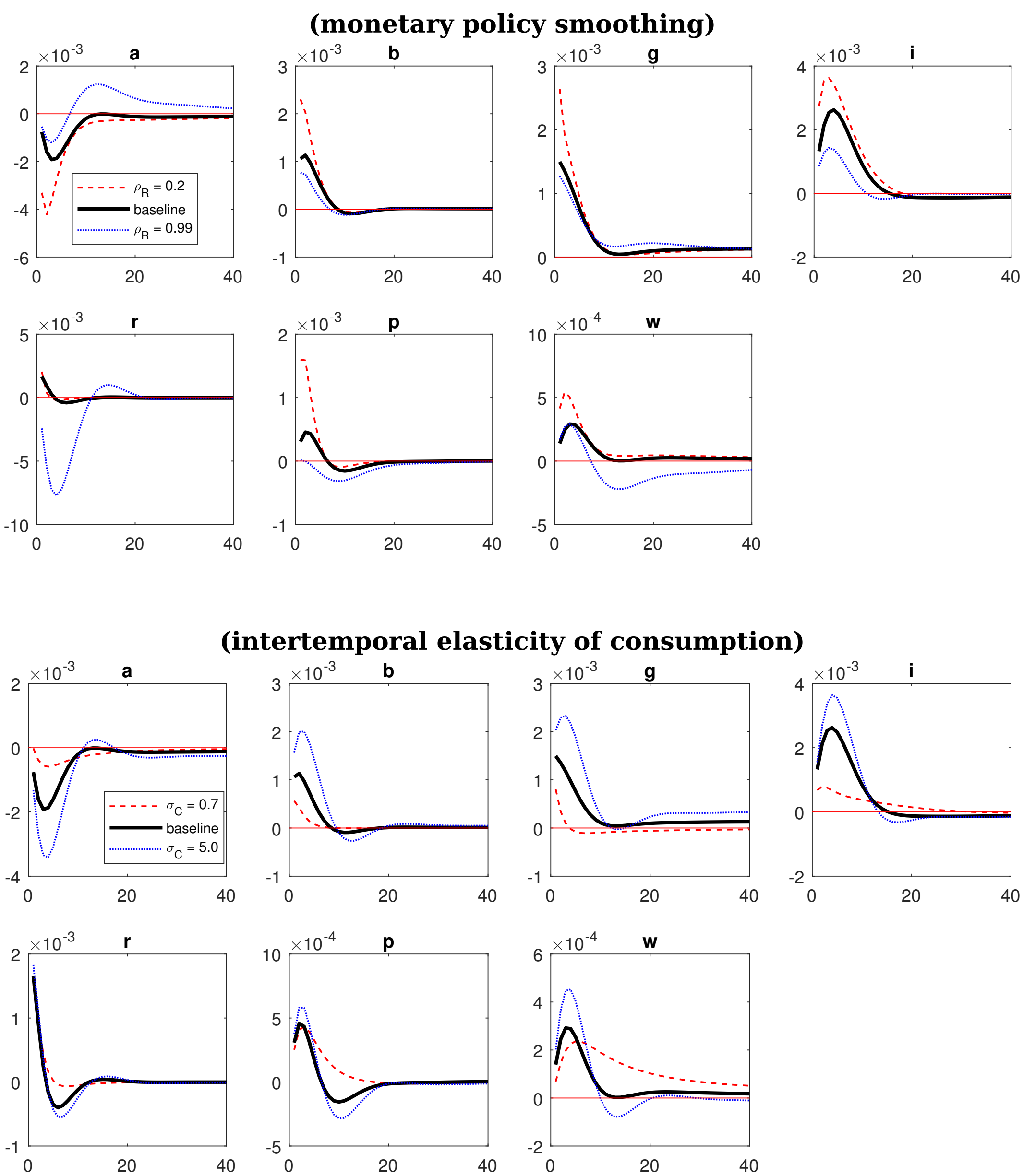
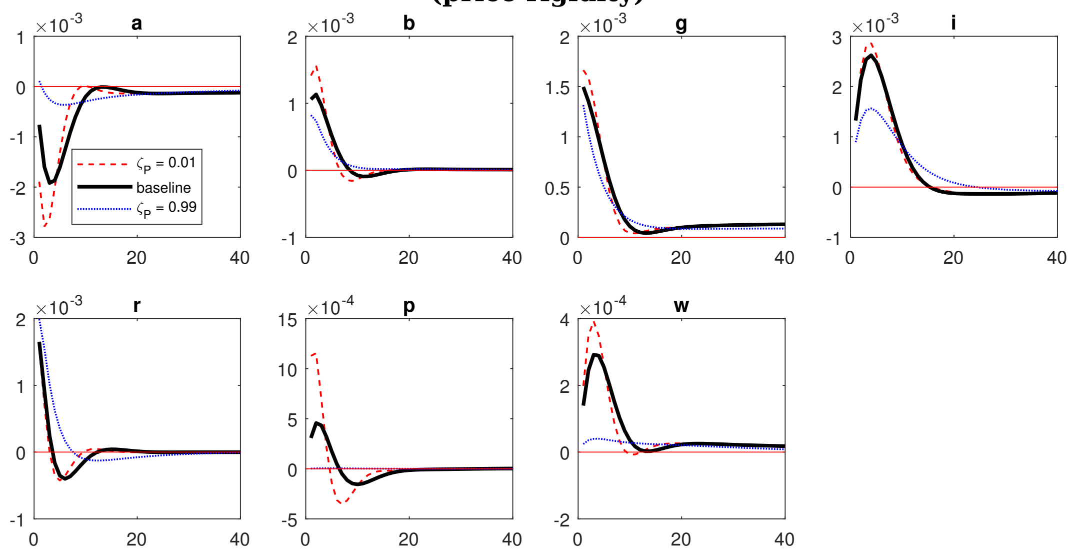


Figure 2.5: **Responses of short-term rate (alternative parameters)**

(price rigidity)



(wage rigidity)

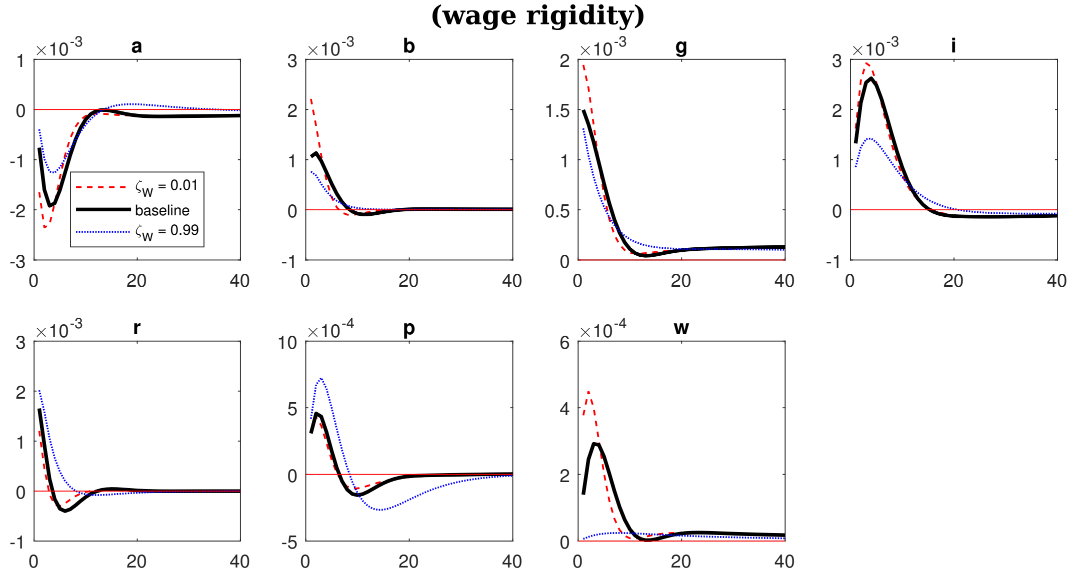
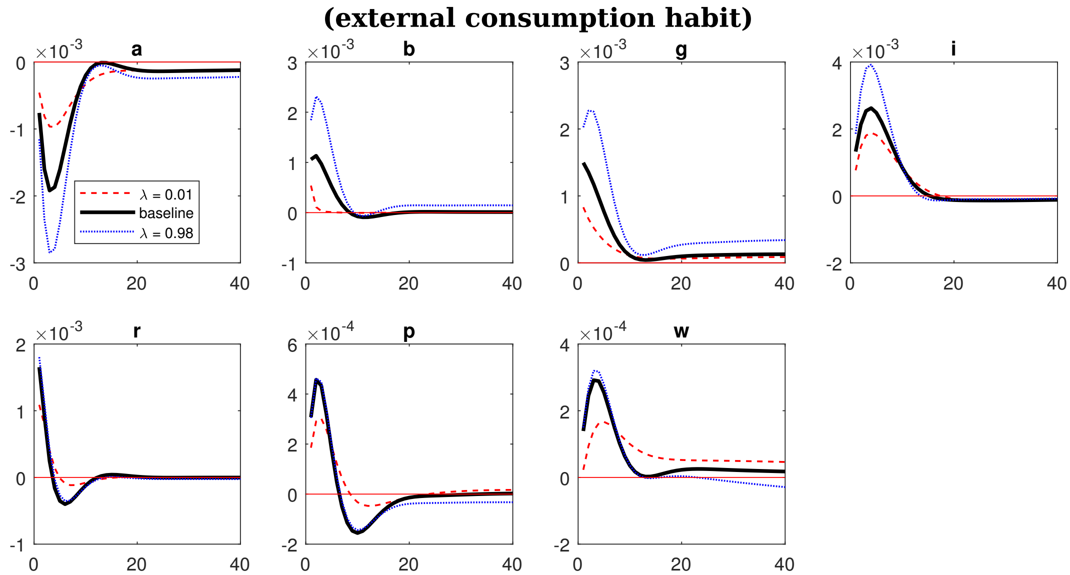


Figure 2.6: **Responses of short-term rate (alternative parameters)**



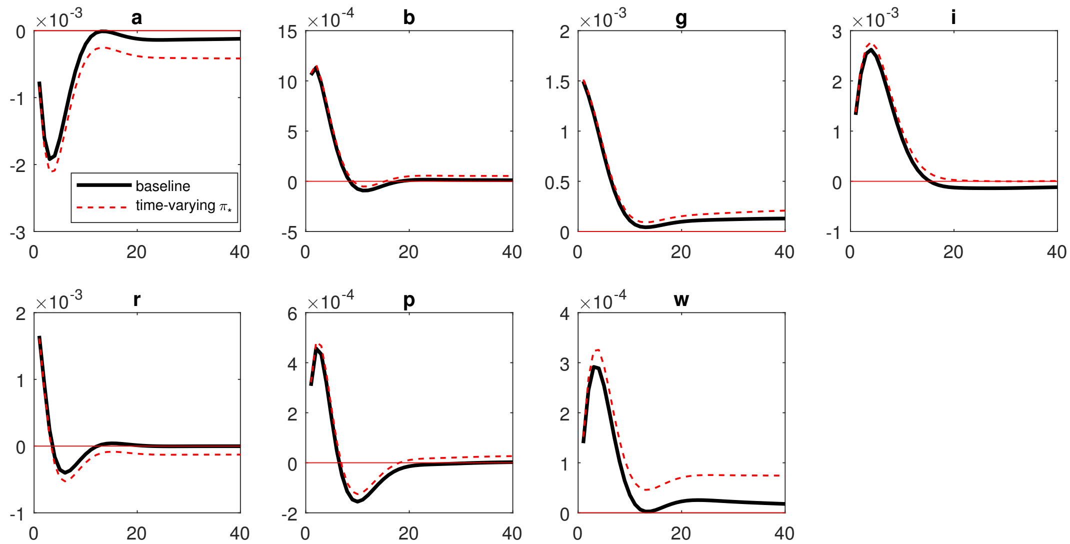
the increased inflation target raises the far-future expectations for nominal interest rates. This explains why incorporating time-varying inflation target enlarges the volatility of long-term interest rates disproportionately than that of the short-term interest rate.

Note, however, that there are criticisms on the assumption of time-varying inflation target. One of them is the lack of micro-foundation. In other words, even though our results rely heavily on Equation (2.48) and (2.49), there is no clear theory or empirical evidence that supports the assumption that central banks revise their inflation target following the two equations.

2.4.3 DSGE model with Recursive Preference

When the expectations hypothesis is assumed as in the previous sections, volatility of long-term interest rate is purely determined by the average expectations of future short-term rates. Therefore, the DSGE model implied relative volatility

Figure 2.7: **Responses of short-term rate (time-varying inflation target)**



of long-term interest rate is able to be enlarged only when the responses of short-term rates to current shocks became significantly more persistent, and this was well achieved when time-varying inflation target was incorporated to the model. When the expectations hypothesis does not hold, however, the volatility of long-term rate can be affected by term premiums as well.

To examine the role of term premium, we made two modifications to our baseline model in subsection 2.4.2. First, we assumed that the representative households optimise the welfare as follows:

$$\max V_t = u\left(c_t, h_t\right) + \beta \left(E_t V_{t+1}^{1-\sigma_{EZ}}\right)^{1/1-\sigma_{EZ}}, \tag{2.52}$$

where the period utility $u\left(c_t, h_t\right)$ and the budget constraint are same as those of the baseline model. This form of recursive preference is used by Rudebusch and Swanson (2012) who rewrote the recursive preference of Epstein and Zin (1989) for notational clarification. Under this assumption, the real stochastic discount factor (SDF) is derived as follows:

$$M_{t,t+1} \equiv \beta \left(\frac{V_{t+1}^{R}}{E_t \left[\left(V_{t+1}^{R} \right)^{1-\sigma_{EZ}} \right]^{\frac{1}{1-\sigma_{EZ}}}} \right)^{-\sigma_{EZ}} \left(\frac{\Xi_{t+1}}{\Xi_t} \right), \tag{2.53}$$

where Ξ_t is the marginal utility of consumption. When EZ preference is assumed, an additional term, $\left(V_{t+1}^{R} / E_t \left[\left(V_{t+1}^{R} \right)^{1-\sigma_{EZ}} \right]^{\frac{1}{1-\sigma_{EZ}}} \right)^{-\sigma_{EZ}}$, is added on top of the SDF used in the baseline model with EU preference; and this term determines how much consumers dislike consumption volatility across different states. In other words, the larger σ_{EZ} (in absolute terms) makes consumers become more reluctant to substitute consumption across different states (i.e., more risk-averse), and thus makes them ask more premium when investing in risky assets.

When the expectations hypothesis is abandoned, long-term interest rate cannot be represented as an average of future short-term rates. Instead, the price of n-period zero coupon bond, $P_t^{(n)}$, is calculated as the expected value of the nominal stochastic discount factor, $M_{t,t+n}^{\$}$:

$$P_t^{(n)} = E_t \left(M_{t,t+n}^{\$} \right) = E_t \left(M_{t,t+n} \Pi_{t,t+n}^{-1} \right), \tag{2.54}$$

where $\Pi_{t,t+n} \equiv \frac{P_{t+n}}{P_t}$ denotes cumulative gross inflation from t to $t+n$. Also, n-period zero coupon bond yield is given as

$$R_t^{(n)} = -\frac{1}{n} \log E_t \left(M_{t,t+n} \Pi_{t,t+n}^{-1} \right). \tag{2.55}$$

As there exist autocorrelation among real SDFs (real term premium) and covariance between SDF and inflation (inflation risk premium), $R_t^{(n)}$ is, in general, different from that is calculated under the expectations hypothesis. If the model is approximated higher than third order, these premiums (covariance terms) can be time-varying and become the source of volatility in long-term interest rate.

For the calculation of 10-year bond price and its term premium, we followed the method used by Rudebusch and Swanson (2012). In order to save calculation time[25], they used an infinitely lived consol-type bond where a unit of consol issued at time-t pays δ_c^k units of money as a coupon at $t+k$ for all $k \geq 0$. The price of the decaying consol, P_t^c, satisfies

$$P_t^c = 1 + \delta_c E_t M_{t+1}^{\$} P_{t+1}^c, \tag{2.56}$$

and the continuously compounded yield to maturity of the consol, R_t^c, is given by

$$R_t^c = \log\left(\frac{\delta_c P_t^c}{P_t^c - 1}\right). \tag{2.57}$$

By setting $\delta_c = 0.9848$, we set the duration of the decaying consol to 10 years following Rudebusch and Swanson (2012). One can also calculate risk neutral price (and yield to maturity) of the consol by replacing the stochastic discount factor with nominal short-term interest rate[26], and the difference between the two YTMs gives the term premium by definition.

We solved and simulated the third-order approximation of the model to generate time-varying term premium. The value of newly added parameter, $\sigma_{EZ} = -106$, is calibrated such that the average term premium of 10-year

[25]When we use the typical zero-coupon bonds, we need 40 variables to calculate 10-year bond price. However, when using the decaying consol following Rudebusch and Swanson (2012), one can calculate the 10-year bond price by just setting δ_c such that the Macaulay duration (or maturity) of the bond becomes 10-year, and thus one can save 39 variables.

[26]Under the expectations hypothesis, Equation (2.56) becomes $P_t^{c,rn} = 1 + \delta_c e^{-r_t} P_{t+1}^{c,rn}$ where the (nominal) stochastic discount factor is replaced with gross short-term rate, e^{-r_t}. Similarly, risk neutral yield to maturity is given by $R_t^{c,rm} = \log\left(\frac{\delta_c P_t^{c,rn}}{P_t^{c,rn}-1}\right)$. Then we can calculate the term premium as the difference between R_t^c and $R_t^{c,rm}$.

Table 2.9: **Sensitivity analysis with EZ preference**

			dy	dc	π	r	$R^{(10,EH)}$	$R^{(10,EZ)}$	TP
(I)	**Actual**	σ_x	3.19	2.62	2.44	3.44	2.85	n.a	n.a
		σ_x/σ_r	0.93	0.76	0.71	1.00	0.83	n.a	n.a
(II)	**Baseline**	σ_x	3.28	2.34	2.95	3.43	0.75	n.a	n.a
	(first order)	σ_x/σ_r	0.96	0.68	0.86	1.00	0.22	n.a	n.a
(III)	**EZ, third**	σ_x	3.32	2.33	2.98	3.42	0.55	0.64	0.13
	($\sigma_{EZ} = -106$)	σ_x/σ_r	0.97	0.68	0.87	1.00	0.16	0.19	
(IV)	**EZ, third**	σ_x	3.45	2.79	3.29	4.04	0.76	0.88	0.16
	($\sigma_{EZ} = -22.5$)	σ_x/σ_r	0.85	0.69	0.81	1.00	0.19	0.22	

Note: $R^{(10,EH)}$ denotes the risk-neutral long-term interest rate, and $R^{(10,EZ)}$ is the risky long-term interest rates with term premium. Both of them are calculated with decaying consol with $\delta_c = 0.9848$. TP denotes the standard deviation of term premium. In (IV), we set $\sigma_{EZ} = -22.5$ and $\sigma_c = 3$ to keep the mean of term premium at 100 basis points.

nominal bonds from the model matches 100 basis points leaving all the other parameters unchanged. Table 2.9 compares the results from actual data (I), the baseline results[27] with EU preference (II), and the EZ preference models (III, VI).

The panel (III) shows that the relative volatility of 10-year interest rate from the model with EZ preference, $R^{(10,EZ)}$, is only 0.19, which is far smaller than 0.83 from the actual data. Even though the model generated sizeable time-varying term premium with EZ preference and this made the volatility of long-term interest rate lager than that of risk-neutral rates, the standard deviation of term premium is only 13 basis points, which does not seem to make much difference. One more thing to note here is the trade-off between IES and CRRA. By setting lower IES (higher σ_c), it is possible to match the same size of term premium with lower CRRA (smaller σ_{EZ} in absolute term). As seen in the bottom panel of Table 2.9, when we increase σ_c to 3.0 and decrease the absolute value

[27]The standard deviation of risk neutral yield to maturity, $R^{(10,EH)}$, in Table 2.9 is slightly different from those in Table 2.7 and 2.8. It is just because the bond prices in this Table are calculated using Rudebusch and Swanson (2012)'s decaying consol, instead of the standard 10-year zero coupon bonds.

of σ_{EZ} to 22.5, the model generated more volatile long-term interest rates, keep matching the mean term premium of 100 basis points. This gives us a hint on how Rudebusch and Swanson (2012) successfully matched the volatility of long-term interest rate using EZ preference, which we failed to do here.[28]

In addition, we should also note that this approach is not immune to criticism as well. What is most frequently pointed out is the justification of the incredibly high level of risk aversion. Rudebusch and Swanson (2012) used around -150 for σ_{EZ}, and Darracq Paries and Loublier (2010) show that σ_{EZ} should be around -1,000 to generate term premium of 100 basis points if they use the exactly same model as Smets and Wouters (2007). Moreover, as Darracq Paries and Loublier (2010) also showed, the level of σ_{EZ} needed to match a certain level of term premium changes dramatically depending on the value of other parameters such as β, γ, and σ_c. Fuerst (2015) also pointed out that a simple change in period utility function significantly changes the size of term premium with given level of σ_{EZ}[29].

2.4.4 DSGE model with Bond Market Segmentation

In the DSGE models used in subsection 2.4.2 and 2.4.3, we assumed perfect substitutability among the assets. However, there also exist DSGE models where such assumption is abandoned. When imperfect substitutability among assets is assumed, the rate of returns of bonds with different maturities can vary even when investors are risk-neutral. Such models usually incorporate

[28] Note that Rudebusch and Swanson (2012) relied on very small IES ($\sigma_c \approx 9$) to match the volatility of long-term interest rates in their best-fit model. In their baseline model, where they set $\sigma_c = 2$, the standard deviation of 10-year interest rate was not much different from that of EU preference model. However, when they decreased the IES down to 0.11 in their best-fit model, the standard deviation became more than twice as large as that of the EU preference model and well matched the actual data.

[29] As an example, he showed that by simply adding a constant term in the utility function, a lot larger σ is required to obtain the same size of term premium.

imperfect substitutability by assuming that relative supply affects the sign and size of term premium[30]. Andrés et al. (2004), Harrison (2011, 2012), and Carlstrom et al. (2017) accepted this idea and modified the standard New Keynesian DSGE model. These papers assume in common that there exist some frictions in asset markets, and the frictions hinder the arbitrage ability of households (or financial intermediaries) between long and short-term bonds.[31]

Different assumptions about the existence of such frictions lead to different implications of the term premium. In section 2.4.3, we assumed frictionless bond market, and thus term premium does not have any effect on real activities. For example, we were able to choose any level of term premium by adjusting σ_{EZ} without distorting the fit of key variables. However, in the DSGE models with bond market segmentation, aggregate demand is affected not only by short-term interest rate, but also long-term interest rate. Therefore, the size of term premium can play an important role in business cycle. This can leads to the different stance of monetary policy channel as well. While monetary policy is conducted solely by changing short-term interest rates when no friction is assumed, the assumption on segmented asset market allows an additional monetary policy channel (Bernanke et al., 2002)[32]. For example, under the assump-

[30]This assumption reflects Tobin (1969)'s view that increasing the supply of one asset affects not only the return of the asset but also the spread between the asset and other assets.

[31]Harrison (2011, 2012) can be regarded one of the simplest among the models with such asset market frictions. Andrés et al. (2004), who introduced asset market segmentation in New Keynesian DSGE model ahead of Harrison, is similar in that the household utility function is augmented with an "arbitrary" portfolio adjustment cost. More micro-founded models for asset market segmentation include Carlstrom et al. (2017). This model uses a similar approach to Gertler and Karadi (2011) in order to limit the arbitrage ability of financial intermediary. More specifically, the size of deposit issuance by the financial intermediary is linked to its net worth, and it is costly for them to adjust the size of net worth.

[32]Keynes also made a similar argument in his book *Treatise on money*: "My remedy in the event of the obstinate persistence of the slump would consist, therefore, in the purchase of securities by the central bank until the long-term market rate of interest has been brought down to the limiting point, which we shall have to admit a few paragraphs further on. It should not be beyond the power of a central bank (international complications apart) to bring down the long-term market-rate of interest to any figure at which it is itself prepared to buy long-term securities" (Keynes, 1930)

tion of segmented bond market, central banks may justify their intervention in the long-term bond market (or quantitative easing) by arguing that a decrease in relative supply of long-term bonds can leads to a decline in term premium.

This subsection explains how the volatility of long-term interest rate is affected when the market segmentation assumption is incorporated into the standard New Keynesian DSGE model, using the models of Harrison (2011, 2012). Since the focus of this subsection is to examine the mechanism by which the term premium is determined in the DSGE model with the market segmentation hypothesis, the description of the model focuses only to the part that differs from the standard New Keynesian DSGE model by incorporating segmented bond markets.[33] The most distinctive difference of his models from the standard DSGE model is the existence of portfolio adjustment cost in the household utility function[34], which creates imperfect substitution between short- and long-term bonds. The households in Harrison (2012) solve the following optimization problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \phi_t \left[\frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \frac{n_t^{1+\psi}}{1+\psi} + \frac{\chi_m^{-1}}{1-\sigma_m^{-1}} \left(\frac{M_t}{P_t} \right)^{1-\sigma_m^{-1}} - \frac{\tilde{\nu}}{2} \left(\delta \frac{B_t}{B_{L,t}} - 1 \right)^2 \right]$$

subject to the budget constraint

$$B_{L,t} + B_t + M_t = R_{L,t} B_{L,t-1} + R_t B_{t-1} + M_{t-1} + W_t n_t + T_t + D_t - P_t c_t,$$

where $R_{L,t}$ denotes 1-period holding period return of the long-term bond (consol) from $t-1$ to t, and $B_{L,t}$ is its nominal amount. The household in this model

[33]Originally, Harrison (2011, 2012) developed his models to analyse the effect of asset purchase policy of a central bank. However, we are only interested in the effect of market segmentation on relative volatility of long-term interest rate, so we set the size of long-term bond purchase, $q_t = 0$, in our model.

[34]In Harrison (2011), which is a bit more complicated than Harrison (2012), the portfolio adjustment cost exists in financial intermediary's profit function. However, the two models give very similar results.

can save via both short- and long-term bonds[35]. The portfolio adjustment cost, however, implies that the households have tendency to keep their portfolio mix to a certain level, δ^{-1}. This forces the household to tolerate the difference in returns between the two assets. The parameter $\tilde{\nu}$ determines the size of friction (or degree of market segmendation). The household optimization problem gives the following (log-linearised) dynamic IS relation (Equation 2.58), term premium (Equation 2.59), and a money demand relation (Equation 2.60):

$$\hat{y}_t = E_t\hat{y}_{t+1} - \sigma\left[\frac{1}{1+\delta}\hat{R}_t + \frac{\delta}{1+\delta}E_t\hat{R}_{L,t+1} - E_t\hat{\pi}_{t+1} - r_t^*\right] \quad (2.58)$$

$$E_t\hat{R}_{L,t+1} - \hat{R}_t = -(1+\delta)\frac{\tilde{\nu}c_*^{1/\sigma}}{b_{L,*}}\left[\hat{b}_t - \hat{b}_{L,t}\right] \quad (2.59)$$

$$\hat{m}_t = \frac{\sigma_m}{\sigma}\hat{c}_t - \frac{\beta\sigma_m}{1-\beta}\hat{R}_t + \frac{\beta\sigma_m}{1-\beta}\frac{\tilde{\nu}c_*^{1/\sigma}}{b_{L,*}}\left[\hat{b}_t - \hat{b}_{L,t}\right], \quad (2.60)$$

where lower-case letters denote real variables, variables with an asterisk are steady states, and the hat is used to denote a log-deviation from their steady states[36]. r_t^* is the natural real interest rate which is assumed to follow the exogenous AR(1) process:

$$r_t^* = \rho_* r_{t-1}^* + \eta_t^r. \quad (2.61)$$

Equation (2.58) and (2.59) most distinctly show the difference between this model and the standard New Keynesian DSGE model. Because of the friction,

[35] In this model, the long-term bond denotes consol, and a unit of consol pays one unit of currency each period for the infinite future.

[36] The following three first order conditions and $\hat{y}_t = \hat{c}_t$ constitute Equation (2.58) and (2.59):

$$(\partial c_t) \quad \hat{c}_t = E_t\hat{c}_{t+1} - \sigma\left[\hat{R}_t - E_t\hat{\pi}_{t+1} + E_t\left(\phi_{t+1} - \phi_t\right)\right] + \frac{\tilde{\nu}\delta c_*^{1/\sigma}\sigma}{b_{L,*}}\left[\hat{b}_t - \hat{b}_{L,t}\right]$$

$$(\partial B_t) \quad \hat{\Lambda}_t = \frac{\tilde{\nu}c_*^{1/\sigma}}{b_{L,*}}\left[\hat{b}_t - \hat{b}_{L,t}\right] + E_t\left[\hat{\Lambda}_{t+1} - \hat{\pi}_{t+1} + \hat{R}_{L,t+1}\right]$$

$$(\partial B_{L,t}) \quad \hat{\Lambda}_t = -\frac{\tilde{\nu}c_*^{1/\sigma}}{b_{L,*}}\left[\hat{b}_t - \hat{b}_{L,t}\right] + E_t\left[\hat{\Lambda}_{t+1} - \hat{\pi}_{t+1} + \hat{R}_t\right]$$

where $\hat{\Lambda}$ is real Lagrange multiplier.

aggregate demand relies not only on the short-term interest rate, but on the weighted average of both interest rates. Note that Equation (2.58) collapses to the standard dynamic IS equation when there is no friction ($\tilde{\nu} = 0$). Equation (2.59) shows that the term premium - the difference between the expected 1-period holding period return of long-term bonds, $E_t \hat{R}_{L,t+1}$, and the short-term interest rate, $\hat{R}_t$ - depends on the relative suppy of the two bonds, $\hat{b}_t - \hat{b}_{L,t}$.

There exist monopolistically competitive intermediate good firms. They hire only labours to produce their intermediate goods and set their prices under Calvo (1983) pricing scheme where $\alpha = 0.75$ of them cannot change the price in each term. Wage rigidity is not included in this model. This gives the following Phillips curve:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t, \tag{2.62}$$

where $\kappa \equiv \frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\left(\psi - \sigma^{-1}\right)$. Finally the model is closed with two additional equations: monetary policy rule,

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left(\phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t\right) + \eta_t^R, \tag{2.63}$$

and government budget constraint[37],

$$\hat{b}_t + \frac{m_*}{b_*}\hat{m}_t = -\left[\frac{m_*}{b_*} + \frac{1+\delta}{\beta}\right]\hat{\pi}_t + \frac{m_*}{b_*}\hat{m}_{t-1} + \left(\frac{1}{\beta} - \theta\right)\hat{b}_{t-1}. \tag{2.64}$$

We generated 10,000 simulated data from this model using two i.i.d. exogenous shocks - real interest rate shock and monetary policy shock - whose

[37] To stabilise the total debt stock, the following transfer rule,

$$\frac{\tau}{b}\hat{\tau}_t = -\beta^{-1}\hat{R}_{t-1} - \theta\hat{b}_{t-1},$$

where τ_t denotes real government transfer to households, is incorporated into the government budget constraint.

Table 2.10: **Parameter values**

Parameter		Description	Parameter		Description
σ	6	IES	θ	0.025	feedback in tax rule
β	0.9925	discount factor	ρ_*	0.85	AR(1) of natural real rate
κ	0.024	slope of Phillips curve	σ_{r^*}	0.01	s.d. of natural real rate shock
σ_m	6	money demand elasticity	ϕ_π	1.5	policy response to inflation
m/b	0.001	steady state m/b	ϕ_y	0.5	policy response to output gap
δ	3	steady state b/b_L	ρ	0.85	policy rate inertia
π_*	0	steady state inflation	σ_R	0.01	s.d. of monetary policy shock

Note: $\kappa = 0.024$ is from $\alpha = 0.75$ and $\psi = 0.11$.

Table 2.11: **Effect of bond market frictions on relative volatility**

	$sd(r_t)$	$sd(R_t^{(10)})$	$sd(R_t^{(10)})/sd(r_t)$
$\tilde{\nu} = 0$	6.47	1.20	0.19
$\tilde{\nu} = 0.1$	7.87	1.06	0.13
$\tilde{\nu} = 0.2$	8.88	0.99	0.11

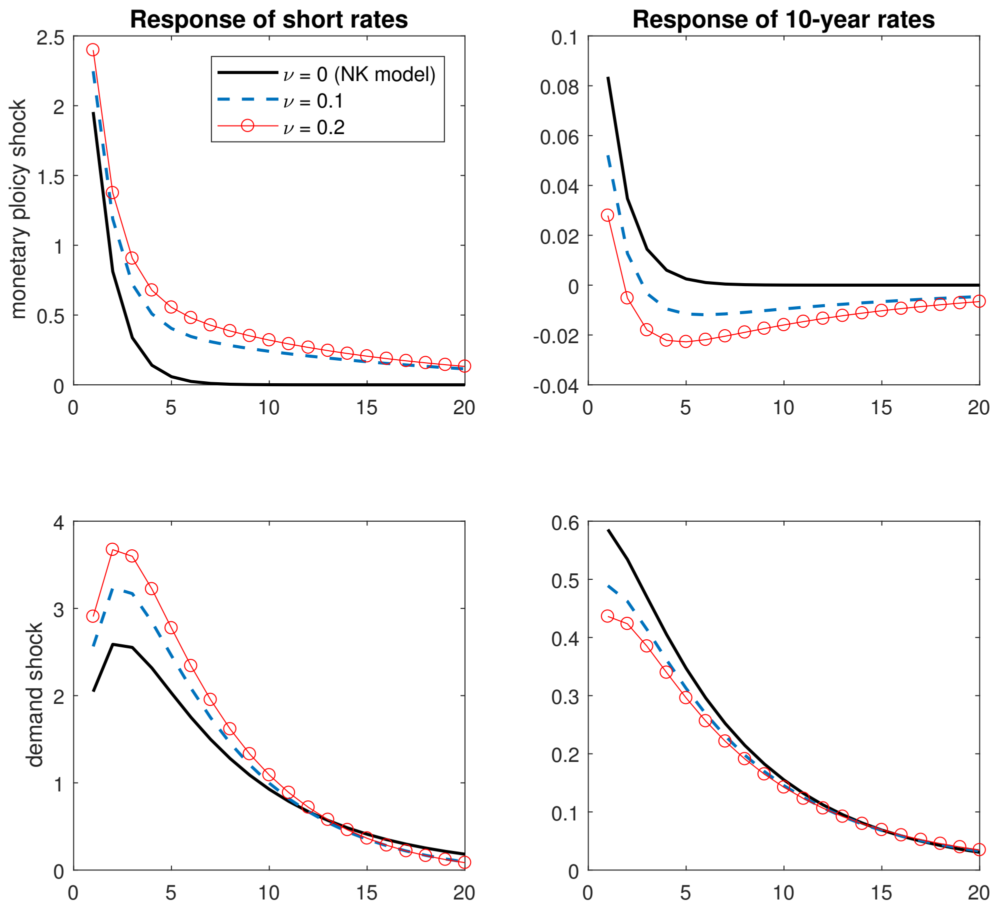
Note: $\tilde{\nu} = 0$ means New Keynesian model with no friction. The 40-period interest rate, $R_t^{(10)}$, is calculated using the average of expected 1-period holding period returns of the consol: $\frac{1}{40}\sum_{i=1}^{40} E_t\hat{R}_{L,t+i}$.

standard deviations are assumed to be 1%. The parameter values are provided in Table 2.10, and they are same as those in Harrison (2012)[38].

The results are summarized in Table 2.11. In the table, we compared the three cases where $\tilde{\nu} = 0$, 0.1, and 0.2. When $\tilde{\nu} = 0$, there is no friction in the economy and the two bonds are perfect substitutes, and thus the model becomes the standard New Keynesian DSGE model. The table shows that, as the bond market becoms more and more segmented, i.e., $\tilde{\nu}$ gets larger, the standard deviation of short-term interest rate becomes larger, but at the same time, that of long-term interest rate becomes smaller. This result suggests that the friction specified in this model actually lowers the relative volatility of long-term interest rate than that of the standard New Keynesian model.

[38]Harrison took most of the parameter values from Levin et al. (2010) except for the steady state ratio of long-term bonds to short-term bonds, δ, which is calibrated from the U.S. data.

Figure 2.8: **The IRFs from segmented market model**



Note: Expressed in annualized percent unit.

These results can be reconfirmed from the IRFs in Figure 2.8. Regardless of the type of shocks, the larger friction leads to the greater response of the short-term interest rate, and the smaller response of the long-term interest rate. This can be explained from the two key equations of the model; Equation (2.58) and (2.59). These equations tell that the aggregate demand depends solely on short-term interest rate when there exists no friction ($\tilde{\nu} = 0$), but it depends on the weighted average of the two interest rates when the friction exists ($\tilde{\nu} > 0$). For this reason, the short-term interest rates respond less and less sensitively to shocks as friction gets bigger.

Let us think more deeply about the results with the example of a positive monetary policy shock. In this model, a positive monetary policy shock means an unexpected increase in short-term bond supply, or an unexpected decrease in relative supply of long-term bonds[39]. Therefore, the shock has a negative effect on long-term interest rate when $\tilde{\nu} > 0$. This negative effect on long-term interest rate mitigates the negative effect of a positive monetary policy shock on aggregate demand, and such an effect becomes stronger as $\tilde{\nu}$ becomes larger. In general, to equate monetary policy rule (Equation 2.63) again after a positive monetary policy shock, we need a higher $\hat{R}_t$ and/or lower aggregate demand. That is to say, as the market segmentation gets stronger, a larger increase in $\hat{R}_t$ is required to equate Equation (2.63) again given the same size of shocks. A similar explanation can be made for the case of demand shock as well. In short, given the same size of shocks, the friction magnifies the required change in short-term interest rate, but it allows smaller change in long-term interest rate, and thus, increases (decreases) the volatility of short-term (long-term) interest rate.

In terms of the relatively volatility, this example illustrates that the volatility of long-term interest from the New Keynesian DSGE model with asset market segmentation can even be smaller than those from the standard models. Even though this conclusion may not be true to all the models with market segmentation, at least Harrison (2011, 2012) - the models where a relative supply affects the size of term premium - are not successful in generating more volatile long-term interest than the standard DSGE models.

[39]The real stock of consol is assumed to be held fixed in this model.

2.5 Conclusion and Summary

This chapter surveyed the key papers in the field of term structure of interest rate and asset pricing, and examined the characteristics of the models from a macroeconomic standpoint. Especially from the viewpoint of central bankers, a general equilibrium model which can explain the behaviour of both macroeconomic variables and bond pricing facts is necessary to conduct successful monetary policies. Nevertheless, even the most widely used models are only partially useful for either macroeconomic analysis or bond pricing.

This chapter re-confirmed that the most widely praised medium scale DSGE model of Smets and Wouters (2007) is also very poor at replicating volatile long-term interest rate observed from the data. From here, we surveyed various modifications made to the DSGE model and investigated whether and how each approach may or may not improve the ability of DSGE model in terms of replicating key bond pricing facts.

Under the expectations hypothesis, one of the effective way seems to be to incorporate the concept of time-varying inflation target. If a positive shock to current short-term rate also has a positive effect on inflation target, its effect on short-term rates becomes more persistent. This can increase the response of long-term interest rate even under the expectations hypothesis. However, consensus on such an assumption on central bank behaviour does not seem to exist yet.

Generating more volatile term premia may help when the expectations hypothesis is abandoned. We examined two ways of incorporating term premium into DSGE models. When perfect substitution between assets is assumed, one can rely on EZ preference and very high level of risk aversion to match the bond pricing facts while keeping the fit of macro variables. However, this modification was not very successful in matching the relative volatility of long-term interest rate from the actual data. Moreover, this approach is receiving criticism as there is not enough justification for the extreme level of risk aversion.

Alternatively, one may explain the existence of term premium with the DSGE models augmented with asset market frictions. The frictions in these models enable asset market segmentation by limiting investor's arbitrage ability between assets with different maturities. However, the results from Harrison (2012) show that models with such frictions may generate even more stable long-term interest rate than those from the standard New Keynesian DSGE model.

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Chapter 3

Nominal GDP growth-indexed bonds: prices and benefits within the framework of New Keynesian DSGE model

3.1 Introduction

The idea of issuing GDP-indexed government bonds goes back at least to the 1980s. After the debt crises of emerging market countries in 1980s, Bailey (1983) suggested to link a government's debt payments to the export of issuing country, and Krugman (1988) also suggested the idea of indexing government debt cash flows to some indices such as oil prices or world interest rates which is hard to be manipulated by the government. This idea was further developed by Shiller (1993, 2003) who proposed to issue consol type state-contingent debt instruments named as 'Trills'. The name came from the fact that one unit of Trill pays one trillionth of the US nominal GDP at each period perpetually. The idea has been gaining interest again recently as many advanced countries are suffering from high level of government debt (see Barr et al., 2014; Blanchard et al., 2016; Bowman et al., 2016; Benford et al., 2016; IMF, 2017).

The previous works on GDP-indexed government bonds focus mainly on its benefits in terms of government debt management. The two major benefits to the government from issuing growth-indexed bonds are common in the literature. First of all, growth-indexed bonds may stabilise the government debt dynamics particularly when the government cannot flexibly adjust its primary surplus in response to slower growth or increase in debt-to-GDP ratio. Second benefit is that it helps to reduce the pressure of conducting pro-cyclical fiscal policy.

For the first benefit, consider the following debt-to-GDP ratio identity which shows the evolution of a sovereign's debt stock:

$$d_{t+1} - d_t = \frac{(r_t - g_{t+1})}{1 + g_{t+1}} d_t - s_{t+1}, \tag{3.1}$$

where d_t is the debt-to-GDP ratio at time t, r_t is the real interest rate on debt at time t to be repaid at $t+1$, g_{t+1} is the real GDP growth rate from t to $t+1$, and s_{t+1} is the primary balance as a proportion of real GDP at period $t+1$. According to Equation (3.1), all else being equal, a slower GDP growth raises the debt-to-GDP ratio. If the government cannot increase its primary surplus enough to stabilise its debt-to-GDP for some reasons[1], that may eventually lead to a debt explosion. On the contrary, If the government debt repayments are fully or partially indexed to GDP growth, a slower GDP growth also lowers the repayment burden, and thus helps to stabilise the debt-to-GDP dynamics.

[1]As examples, we can think of emerging market countries which are usually forced to maintain a higher level of primary surplus in downturns. For the case of advanced countries, we can think of the EU member countries whose ability to increase their primary deficit was constrained by the Stability and Growth Pact.

Similarly, if a government is forced to keep its debt-to-GDP ratio lower than a certain level[2], a slower GDP growth forces the government to increase its primary surplus accordingly. That is to say, the government is forced to undertake a pro-cyclical fiscal policy. In this case, the use of GDP growth-indexed bonds can help mitigate the pressure on pro-cyclical fiscal policy. This is the second benefit.

Most of the previous papers illustrate these benefits by simulating Equation (3.1), and their conclusions are heavily dependent on the exogenous assumptions for the process of key variables, g_t, r_t, and d_t or s_t. For example, Borensztein and Mauro (2004) illustrated the benefits of indexation under restrictive assumptions on these variables. They assumed a hypothetical country where its interest rate and primary surplus over GDP are constant at 3.2% and 0.5% respectively, initial debt-to-GDP level is 30%, and the interest rate of indexed bond, r_t^G, is given by

$$r_t^G = r_t + \alpha \left(g_{t+1} - \overline{g} \right) + premium, \tag{3.2}$$

where the trend growth rate, $\overline{g}$, is assumed to be 3.0%, the degree of indexation ,α, is 0.7, and the premium required for GDP-indexed bond is 40 basis points. Under these assumptions, they simulated the path of debt-to-GDP ratio for twenty years with two different scenarios, where the GDP growth deviates its trend by ± 4 percent points, and in both scenarios with and without indexation. From this simple simulation they showed that the debt-to-GDP ratio can have much narrower distribution when the debts are financed with GDP-indexed bonds.

[2]For example, the Stability and Growth Pact requires EU member states to keep their debt-to-GDP lower than 60%. In the case of the U.S., there exist a legislative limit, so-called debt ceiling, on the amount of government debt issued by the Treasury. Recent empirical analysis by Ostry et al. (2010) also suggests that many advanced countries have already or almost reached their debt limits.

They also showed that the use of GDP-indexed bonds could have allowed many (including both advanced and emerging market) countries to conduct less pro-cyclical fiscal policy during 1992-2001. They produced the series for s_t by conducting counterfactual simulations with the assumption that all the government debts were financed through GDP-indexed bonds, leaving all other variables - d_t, g_t, r_t - intact. Their results showed that the primary surplus and GDP growth could have had much stronger positive correlation (implying less pro-cyclical fiscal policy) when GDP-indexed bonds were used, for both advanced and emerging countries. More recently, Bonfim and Pereira (2018) conducted a similar analysis using the data from France, Spain and Portugal from 2000 to 2015. They also showed that these countries could have experienced more counter-cyclical fiscal policy if they had used GDP-indexed bonds.

Blanchard et al. (2016) also focused on the case of advanced countries. They forecasted the distribution of debt-to-GDP ratios of the three European countries - Spain, Italy and Germany - from 2015 to 2035, and showed that the use of GDP-indexed bonds may significantly narrow the distribution of the debt-to-GDP ratio. Their simulations were also conducted using exogenously assumed paths of g_t, s_t and r_t (or r_t^G)[3]. To construct those series, they assumed that their expected paths are same as those forecasted by IMF's World Economic Outlook, and also assumed that there exist independently and identically distributed shocks for these three variables, whose covariance matrix is constructed with the historical data. Benford et al. (2016) extended the results of Blanchard et al. (2016) to both advanced and emerging market economies using similar approach. By conducting one million simulations of debt-to-GDP ratio for 20-year horizon, they showed that the 99% tail for the debt-to-GDP

[3]They specified r_t^G a little bit differently from Borensztein and Mauro (2004). In their model, the expected return on GDP-indexed bond is same as the expected growth rate (g_{t+1}) plus a constant, κ. Therefore, one may freely choose the size of premium required for GDP-indexed bonds over conventional bonds by adjusting κ.

ratio for both advanced and emerging countries can be significantly reduced by using GDP-indexed bonds.

While the papers mentioned above simply showed that the use of GDP-indexed bonds narrows the distribution of debt-to-GDP ratio as the benefit of the GDP-indexed bonds (e.g., Borensztein and Mauro, 2004; Blanchard et al., 2016; Benford et al., 2016), some papers more explicitly show that the use of GDP-indexed bonds can lower the probability of default (Chamon and Mauro, 2006; Ostry et al., 2010; Barr et al., 2014). For example, to explicitly incorporate the possibility of sovereign default, Barr et al. (2014) assumed a primary surplus rule with an upper limit[4] and that the government in the model declares default when its debt-to-GDP ratio reaches a so-called debt limit, $\overline{d}$, a concept developed by Ostry et al. (2010) and Ghosh et al. (2013). More specifically, when a positive shock to debt-to-GDP ratio occurs, the government responds by raising primary surplus. However, if the shock is too large, at some point, even the maximum level of primary surplus cannot stabilise its debt-to-GDP ratio. They define that level of debt-to-GDP ratio as the debt limit. Then, the probability of sovereign debt default in the next period is calculated as the probability of $d_{t+1} \geq \overline{d}$, and this default probability is explicitly reflected to the overall borrowing cost of the government (default risk premium). With this model, they showed that the use of GDP-indexed bonds lowers the probability of default since it reduces the tail risk of debt-to-GDP ratio and at the same time pushes up the debt limit. The reduced probability of default lowers the overall borrowing cost, and this further reduces the probability default.

So far, we summarised how the previous papers illustrated the potential benefits of GDP-indexed bonds. However, those aforementioned benefits may

[4] This is a simplification of the concept of 'fiscal fatigue' also developed by Ghosh et al. (2013), who defined it as a situation where the primary surplus responds more and more slowly as debt grows.

disappear if replacing conventional government bonds with GDP-indexed bonds excessively raises borrowing costs of the government. For example, Barr et al. (2014) and Blanchard et al. (2016) argued that, if the additional cost is larger than 150~300 basis points, the benefits to the government may disappear. Comparing with the conventional government bonds, there may exist three kinds of additional risk premiums associated with the GDP-indexed bonds. Investors usually ask for a higher expected return for a new, unfamiliar, and more complicated investment product (i.e., novelty risk premium). If it seems more costly or to take more time to convert the GDP-indexed bond into cash than for the case of the conventional bonds, such a risk should also be compensated (i.e., liquidity risk premium). Lastly, as GDP-indexed bond investors take the risk associated with uncertainty in GDP growth, there should be compensation for that risk as well (i.e., growth risk premium).

However, from the experience of similar investment products traded in the market, such as inflation-indexed government bonds[5], it seems that the first two among these three risks may disappear or become quite small in a short time. According to Chamon et al. (2008), the novelty premium on Argentina's GDP-linked warrants have fallen by half in less than two years[6]. Given the complexity of Argentina's GDP-linked warranty[7], the novelty premium on GDP-indexed bonds with much simpler structure may be much lower and become

[5]The inflation-indexed government bonds are actively being traded around the world. According to IMF (2017), in many advanced countries the share of inflation-indexed government bonds accounts for more than 10% of total government debts (the U.K., France, Italy, Sweden, etc.), and it even takes more than half in Chile.

[6]Chamon et al. (2008) show that the novelty premium was around 1,200 basis points when it was first issued, but it declined by more than 600 basis points in less than two years.

[7]The GDP warrants issued by Argentine government pays 5% of 'excess GDP', which is the difference between actual GDP and Base Case GDP, only if actual GDP growth and level of reference year (1 year before the payment occurs) satisfy the following three conditions: (1) actual GDP is greater than Base Case GDP, (2) actual GDP grows faster than Base Case GDP does, and (3) the cumulative payment on the GDP warrants should be smaller than 48% of notional amount. However, on the contrary, the GDP-indexed bonds considered in this papers lacks of such complexity and has much simpler and symmetric payoff structures.

Table 3.1: **Estimates of growth risk premiums from the literature**

Authors	**Method**	**Premium**	**Country/Region**
Borensztein and Mauro (2004)	CAPM	100	Argentina
Kamstra and Shiller (2009)	CAPM	150	The United States
Benford et al. (2016)	CAPM	140	Advanced countries
		80	Emerging countries
Barr et al. (2014)	consumption-based asset pricing model	35	Advanced countries
Bowman et al. (2016)	D-CAMP	-500 to 1500	19 countries

Note: premiums are expressed in basis points.

smaller more quickly. Additionally, the experiences of Treasury Inflation Protected Securities (TIPS) of the U.S. and the index-linked gilt of the U.K. show that liquidity premium may also disappear or become very small[8] as the size of the market grows. This implies that it is important to assess the likely size of the premium associated with the uncertainty on future GDP growth, or the growth risk premium. However, there are only few academic studies conducted on this topic mainly because the GDP-indexed bonds have not existed yet and thus there is no data available for an empirical analysis.

Most of the previous papers have estimated the growth risk premium using the Capital Asset Pricing Model, or CAPM. Table 3.1 summarises their estimates. Borensztein and Mauro (2004) calculated the risk premium of hypothetical GDP-indexed bond of Argentina using the historical data from 1970-2001. To take the international risk sharing into consideration, they used S&P500 index as the market portfolio. Since the β of the country's growth with respect to

[8]For example, the estimation of d'Amico et al. (2018) shows that 10-year TIPS liquidity premiums were around 150 basis points in 1999, but fell below 100 basis points in 2000, and then dropped steadily to close to zero around 2004. Gürkaynak et al. (2010b) also shows that the liquidity premium was around 50 basis points when it was first introduced but steadily decreased and almost disappeared around 2004. See also Chamon et al. (2008) and Campbell and Viceira (2009).

the return on S&P500 index was around 0.22 and the excess return of the market portfolio over the risk-free rate was around 5%, the required excess return on Argentina's GDP-indexed bond was calculated to be around 100 basis points over risk-free asset. Benford et al. (2016) updated the results of Borensztein and Mauro (2004) with more recent data, and showed that the premiums are expected to be on average 80 basis points for the emerging market countries, and 140 basis points for the advanced countries.[9] The estimate of Kamstra and Shiller (2009) for the Trills is 150 basis points, also based on CAPM, which is similar to the Benford et al. (2016)'s advanced country estimation.

Even though CAPM has been widely used to estimate the risk premium on GDP-indexed bonds, there also exist different approaches. For example, Chamon and Mauro (2006) suggested a simulation method for pricing growth-indexed bonds relying on the joint stochastic process of growth, real exchange rate, and primary balances constructed from the historical data[10]. Barr et al. (2014) calculated the growth risk premium using consumption-based asset pricing model calibrated with average of advanced country data. More specifically, assuming all the assets in their model are perfectly substitutable, the price of GDP-indexed bond is determined such that the expected utility of investing in GDP-indexed bond equals that of investing in risk-free assets (no arbitrage condition). Assuming the standard power utility function with CRRA of 4, the

[9]The larger average growth risk premium for the advanced countries reflects the fact that their GDP growths are more closely correlated with the market portfolio.

[10]They simulated the debt-to-GDP dynamics of 10-year horizon for 250,000 repetitions using the shocks extracted from the joint stochastic process. They assumed that a government defaults on its debts when the debt-to-GDP ratio exceeds a trigger level, and further assumed that the investors can recover only a certain proportion (recovery ratio) of the promised payments. In this setting, the simulation can provide the distribution of future payoffs of the 10-year government bond investment, and thus the price of the bond for a given trigger level. They also assumed that the trigger ratio is the debt-to-GDP ratio which makes the bond price equal to its face value. They showed that, given the same trigger ratio, both the probability of default and the borrowing cost of the government are lowered as the share of growth-indexed bond increases.

growth risk premium (for an average advanced country) was around 35 basis points.

One of the shortcomings of the previous analyses on both growth risk premium and the benefits of GDP-indexed bond is that they rely on partial equilibrium models. For example, the required expected return of a risky asset calculated with CAPM depends solely on the historical relationship between the return of the asset and the market portfolio, and the relationship is summarised in the asset's β. Therefore the choice of the market portfolio may significantly change the required return of GDP-indexed bonds (Bowman et al., 2016)[11]. However, there is no consensus on how to choose the market portfolio.

Similarly, the benefits of using GDP-indexed bonds rely heavily on the assumptions about the joint process of the key variables in debt-to-GDP dynamics identity, Equation (3.1). Even though the premiums and benefits calculated from DSGE models also depend on the relationship between the key variables, the relationships are obtained by the results of optimal choices of rational, forward-looking agents, not just by arbitrary assumptions. In particular, as the theoretical prices of NGDP-indexed bonds and growth risk premiums are determined by the relationship between stochastic discount factor (SDF) and the growth; and given the fact that the DSGE models have become a dominant modelling framework both in academia and central banks in analysing the

[11] Bowman et al. (2016) calculated the growth risk premiums of 19 selected countries with four different measures as the market portfolio: US equity index, US GDP growth, world equity index, and world GDP growth rate. Their estimates show that there exists considerable uncertainty surrounding the estimates of the growth risk premium; they vary widely not only across the countries, but also across the selection of market portfolio. Furthermore, the uncertainty gets even larger when they used a downside CAPM (see Table 3.1). Compared to the standard CAPM, D-CAPM focuses on the distribution of the below-average return. More specifically, beta of D-CAPM (β_i^D) for a risky asset i is defined as

$$\beta_i^D \equiv \frac{\text{cov}\left(r_i, r_m | r_m < \overline{r_m}; r_i < \overline{r_i}\right)}{\text{var}\left(r_m | \overline{r_m}\right)}.$$

where r_i, r_m are the rate of return on an asset i and market portfolio, respectively, and $\overline{r_m}$ and $\overline{r_i}$ are their sample means.

inextricably linked relationship between the growth and other variables, we may justify the use of DSGE models in pricing NGDP-indexed bonds and investigating its potential benefits. Furthermore, from a more practical point of view, DSGE models can give answers to the questions particularly important to policy makers, such as which deep parameters the growth risk premium depends on or under which conditions the government should use the NGDP-indexed bonds. Partial equilibrium models cannot answer such questions.

For these reasons, in this paper, we calculate the theoretical price of nominal GDP growth-indexed bonds (NGDP-indexed bonds) within the framework of a medium scale New Keynesian DSGE model based largely on Smets and Wouters (2007) and Rudebusch and Swanson (2012). We first examine whether issuing NGDP-indexed bonds may or may not lower the borrowing cost of the government using our baseline DSGE model estimated with the U.S. data. The model shows that, if the other two risk premiums are ignored, replacing the conventional government bonds with the NGDP-indexed bonds may reduce the borrowing costs of the government as the inflation risk premium is greater than the growth risk premium, at least, in this particular model. The model also allows us to explore how the structural changes in the economy affects the signs and sizes of the two premiums associated with uncertainty on growth and inflation. More specifically, we examine how different specifications for shock processes; parameters on consumer preference and nominal rigidity; and monetary policy rules may affect the two premiums. A similar analysis has also been conducted by De Paoli et al. (2010) who investigated how introducing real and nominal rigidity into a typical New Keynesian model may affect bond returns and term premium, but their analysis does not include growth-indexed bonds. Our analysis extends their results by using more sophisticated model with more frictions and shocks; and by adding analyses on NGDP-indexed bonds and growth risk premium. As we are interested in the size of risk premiums, we solve the DSGE model using second order perturbation method.

Additionally, we try to see whether there exist aforementioned benefits to the government using NGDP-indexed bonds. By simulating our baseline model augmented with a fiscal policy rule where the presence of positive outstanding debts are allowed, we show that the government in our baseline model benefits from issuing NGDP-indexed bonds as it helps mitigate the pressure of conducting pro-cyclical fiscal policy.

The remainder of this chapter is organised as follows. In section 3.2 and Section 3.3, we outline the model and its parametrisation. In section 3.4, we calculate the price of NGDP-indexed bonds, and show that it may lower the borrowing cost of the government. We also explore how different shocks and parameters affect the sign and size of risk premiums. In section 3.5, the benefits of NGDP-indexed bonds to the government is investigated, and section 3.6 concludes this chapter.

3.2 Summary of the DSGE model

The medium scale New Keynesian DSGE model used in this paper is based on Smets and Wouters (2007), which is one of the most frequently cited paper in the field of macroeconomics, augmented by Epstein-Zin preference following Rudebusch and Swanson (2012) which successfully matched term premium of the 10-year U.S. Treasury bond without compromising the fit of macroeconomic variables. The model presented here will contain similar ingredients as Smets and Wouters (2007) such as external habit formation, price and wage stickiness with partial indexation, capital utilization cost, investment adjustment cost, etc., with some minor modifications. These modifications will be explained in the following subsections.

3.2.1 Households

In this model, there is a continuum of households who has the following non-separable period utility function borrowed from Smets and Wouters (2007):

$$U_t(i) = \varepsilon_t^b \left[\frac{1}{1-\sigma_c} \left(C_t(i) - \lambda C_{t-1} \right)^{1-\sigma_c} \right] \exp\left[\frac{\sigma_c - 1}{1+\sigma_l} \left(h_t(i) \right)^{1+\sigma_l} \right], \tag{3.3}$$

where $U_t(i)$ is the period utility for household indexed by $i \in [0,1]$, $C_t(i)$ and C_{t-1} are individual and aggregate consumption of final goods in real terms, respectively, and $h_t(i)$ denotes the supply of the homogeneous labour. Following Smets and Wouters (2003), we assume the preference shock ε_t^b follows a simple AR(1) process. In each period, each household obtains utility from consumption of final goods relative to a proportion of aggregate consumption in the previous period (i.e., there exist external habit), and disutility from supply of homogeneous labour.

The most distinctive difference between this model and Smets and Wouters (2007) is the assumption of Epstein-Zin type recursive preference (or EZ preference), which is commonly used in the asset pricing models in the New Keynesian literature in these days as this helps to generate more realistic term premiums (Tallarini, 2000; Bansal and Yaron, 2004; Darracq Paries and Loublier, 2010; Rudebusch and Swanson, 2012). In our model, an individual household i maximises its welfare $V_t(i)$ recursively given as below,

$$V_t(i) = U_t(i) + \beta E_t \left[\left(V_{t+1}(i) \right)^{1-\sigma_{EZ}} \right]^{\frac{1}{1-\sigma_{EZ}}}, \tag{3.4}$$

where β is the discount factor, and σ_{EZ} controls the coefficient of relative risk aversion (CRRA). This functional form is borrowed from Rudebusch and Swanson (2012). As Rudebusch and Swanson (2012) has shown, by adjusting the size of σ_{EZ}, modellers can freely choose the size of term premium without distorting the fit of other variables[12]. Note that the larger the absolute size of σ_{EZ}, the more the household dislikes consumption volatility between states of $t+1$, and note also that Equation (3.4) collapses to the standard expected utility model when $\sigma_{EZ} = 0$.

Each household faces the following intertemporal budget constraint:

$$C_t(i) + I_t(i) + \frac{B_t(i)}{R_t P_t} + \frac{Q_{g,t} B_{g,t}(i)}{P_t} - T_t \tag{3.5}$$

$$= \frac{B_{t-1}(i)}{P_t} + \frac{B_{g,t-1}(i)}{P_t}\left(\frac{Y_t P_t}{Y_{t-1} P_{t-1}}\right) + \frac{\overline{W}_t h_t(i)}{P_t} + \frac{R_t^k}{P_t} z_t(i) K_{t-1}(i) - a(z_t(i)) K_{t-1}(i) + D_t,$$

where P_t is the price of a final good, $I_t(i)$ is real investment, $B_t(i)$ is units of nominal bonds purchased at t, T_t is lump-sum government transfer in real terms which is evenly distributed to all the households, $\overline{W}_t$ is the homogeneous nominal wage paid by labour unions, R_t^k is nominal rent of the capital, and $z_t(i)$ is the level of capital utilization. D_t denotes sum of the profits from the union and intermediate firms, which is also evenly distributed. As we assume the existence of nominal GDP growth-indexed bonds, one more term is added to both side of the budget constraint, $\frac{Q_{g,t} B_{g,t}(i)}{P_t}$ and $\frac{B_{g,t-1}(i)}{P_t}\left(\frac{Y_t P_t}{Y_{t-1} P_{t-1}}\right)$. $B_{g,t}(i)$ denotes units of NGDP-indexed bonds purchased at t, and $Q_{g,t}$ is its unit price.

[12]Expected utility preference (EU preference) implicitly and arbitrarily assumes a tight inverse relationship between risk aversion and intertemporal elasticity of substitution (IES). Therefore, under the assumption of EU preference, a very high level of risk aversion (i.e., very low level of IES) which is required to match the level of premium observed in the data, seriously distorts the fit of real variables such as consumption and output. The use of EZ preference solves this problem by breaking the link between risk aversion and IES. By adjusting σ_{EZ} under a given IES, any level of risk aversion can be chosen without compromising the fit of the macroeconomic variable.

In the right-hand-side, a unit of the NGDP-indexed bond purchased at $t-1$ pays $\left(\frac{Y_t P_t}{Y_{t-1} P_{t-1}}\right)$ units of money at t, which is same as the nominal output growth rate from $t-1$ to t.

There also exist capital adjustment costs and capital utilization costs following the standard specification from the literature. That is, the capital of individual household is accumulated as follows:

$$K_t\left(i\right) = \left(1-\delta\right) K_{t-1}\left(i\right) + \varepsilon_t^I \left[1 - \frac{\phi}{2}\left(\frac{I_t\left(i\right)}{I_{t-1}\left(i\right)} - \gamma\right)^2\right] I_t\left(i\right), \tag{3.6}$$

and there exists the following capital utilisation cost:

$$a(z_t\left(i\right)) = \delta_1\left(z_t\left(i\right) - 1\right) + \frac{\delta_2}{2}\left(z_t\left(i\right) - 1\right)^2. \tag{3.7}$$

Solving the household's problem gives the first order conditions which are almost same as those in Smets and Wouters (2007), except for the (real) stochastic discount factor, $M_{t,t+1}$, defined as

$$M_{t,t+1} \equiv \beta\left(\frac{V_{t+1}}{E_t\left[\left(V_{t+1}\right)^{1-\sigma_{EZ}}\right]^{\frac{1}{1-\sigma_{EZ}}}}\right)^{-\sigma_{EZ}}\left(\frac{\Xi_{t+1}}{\Xi_t}\right), \tag{3.8}$$

where Ξ_t denotes the marginal utility of consumption given by

$$\Xi_t \equiv \varepsilon_t^b\left(C_t - \lambda C_{t-1}\right)^{-\sigma_c}\exp\left[\frac{\sigma_c - 1}{1+\sigma_l}\left(h_t^H\right)^{1+\sigma_l}\right]. \tag{3.9}$$

Compared with the expected utility model, an additional term, $\left(\frac{V_{t+1}}{E_t\left[\left(V_{t+1}\right)^{1-\sigma_{EZ}}\right]^{\frac{1}{1-\sigma_{EZ}}}}\right)^{-\sigma_{EZ}}$, is added due to the EZ preference.

3.2.2 Labour market

There is a continuum of monopolistically competitive unions indexed over the same range as the households, $j \in [0,1]$, that purchase homogeneous labour from the household at the hourly wage of $\overline{W}_t$, differentiate them, and sell the differentiated labour, $l_t(j)$, to the labour packer at $W_t(j)$. The labour packer then aggregates them into an aggregate labour index, l_t, using the following standard Dixit-Stiglitz aggregator,

$$l_t = \left(\int_0^1 l_t(j)^{\frac{1}{1+\lambda_w}} dj \right)^{1+\lambda_w}, \tag{3.10}$$

and then sell it to the intermediate good firms at W_t through a perfectly competitive factor market. The union for type-j labour sets its optimal wage, $\hat{W}_t(j)$, in order to maximise the sum of stochastically discounted future profits. The profit of the union, the difference between differentiated wage $\hat{W}_t(j)$ and homogeneous wage $\overline{W}_t$, is fully transferred to the households and evenly distributed. Following Calvo (1983), only $1-\zeta_w$ of the unions are given a chance to re-optimise their wages in each period, and the rest of them partially index their wages using partial indexation factor given as below:

$$X_{t,s}^w \equiv \begin{cases} 1 & \text{if } s = 0 \\ \prod_{l=1}^s \left(\Pi_{t+l-1}^{\iota_p} \Pi_*^{1-\iota_p} \right) & \text{if } s \geq 1, \end{cases} \tag{3.11}$$

where Π_* denotes steady state level of gross inflation. Combining all these conditions, each union solves the following problem[13]:

$$\max_{\hat{W}_t(j)} E_t \sum_{s=0}^{\infty} \zeta_w^s M_{t,t+s} \left(\frac{1}{P_{t+s}} \right) \left[W_{t+s}(j) - \varepsilon_{t+s}^w \overline{W}_{t+s} \right] l_{t+s}(j),$$

[13] Note that the wage at $t+s$ set by the union j who optimised its wage at t is $W_{t+s}(j) = \hat{W}_t(j) \gamma^s X_{t,s}^w$.

where $\hat{W}_t(j)$ denotes the optimal wage when given the chance of re-optimising, subject to the demand schedule for type-j labour,

$$l_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} l_t,$$

given from the labour packer's optimisation problem. This problem is almost same as that from Smets and Wouters (2007) except that the stochastic discount factor which is derived under the assumption of EZ preference. One more thing to note is the existence of a wedge type wage markup shock, ε_t^w in the problem. Smets and Wouters (2003, 2007) used time-varying wage markup, i.e., $\lambda_{w,t}$ in Equation (3.10), in order to incorporate the wage markup shock into the model. On the contrary, we use a constant parameter, λ_w, and thus we need to specify a wage markup shock separately, using ε_t^w which follows an AR(1) process. The equilibrium equations about the wage setting in the appendix show how this shock works in more detail.

3.2.3 Goods market

In the production sector, a continuum of firms indexed by $z \in [0,1]$ produces intermediate goods in monopolistic competition using capital and the aggregate labour supplied from perfectly competitive factor markets. They then provide their intermediate goods to the final good firm which aggregates them into final goods with a standard Dixit-Stiglitz aggregator analogous to what the labour packer does in the labour market. The intermediate good producers also set their intermediate good prices following Calvo price setting rules in the same way as the unions do, and the discussion on wage markup shock applies to price markup shock, ε_t^p, as well. In short, an individual intermediate good producer solves the following problem,

$$\max_{\hat{P}_t(z)} E_t \sum_{s=0}^{\infty} \zeta_p^s M_{t,t+s} \left(\frac{1}{P_{t+s}} \right) \left[X_{t,s}^p \hat{P}_t(z) - \varepsilon_{t+s}^p MC_{t+s} \right] Y_{t+s}(z),$$

where $1 - \zeta_p$ is the probability of getting a chance of optimization, $\hat{P}_t(z)$ is an optimal price when given the chance, $X_{t,s}^p$ is the partial indexation factor for non-optimising firms, and MC_{t+s} is the nominal marginal cost of intermediate good production.

3.2.4 Monetary policy and government sector

The monetary policy rule also follows that in Smets and Wouters (2007) with some simplifications:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*} \right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi_*} \right)^{\psi_\pi} \left(\frac{Y_t}{Y_t^*} \right)^{\psi_y} \right] \left(\frac{Y_t / Y_{t-1}}{\gamma} \right)^{\psi_{\Delta y}} \varepsilon_t^r, \tag{3.12}$$

where R^* and Π_* are the steady state levels of nominal policy rate and inflation respectively, and $Y_t^* \equiv y_* \gamma^t$ is trend level of output, y_* is steady state level of de-trended aggregate real output, and γ is deterministic trend growth rate. Please note that we define the output gap in the monetary policy function in a different way than Smets and Wouters (2007) did. In this paper, the output gap is defined as a deviation of output from its deterministic trend as Taylor did in his seminal paper (Taylor, 1993) and Rudebusch and Swanson (2012) more recently did. In other words, the potential output increases with time with certainty. On the contrary, Smets and Wouters (2007) defined the output gap using so called an "efficient potential output" which is defined as the output level from a hypothetical economy where prices and wages are fully flexible. Therefore, the efficient potential output is affected by the shocks in the model unlike the potential output used in this paper. There exist some criticisms about using deterministic trend as a potential output as it may cause inflation become less stable than when the efficient potential output is used.[14] However, we choose to use this definition of potential output as it reduces the number of variables used in the model and thus helps relieve the computations burden of solving the model caused mostly by the fact that we calculate the long-term bond prices in a standard recursive method (see subsection 3.2.5). Lastly, ε_t^r is a monetary policy shock following an AR(1) process.

Finally, the government faces the following intertemporal budget constraint:

$$G_t + T_t + \frac{B_{t-1}}{P_t} + \left(\frac{Y_t P_t}{Y_{t-1} P_{t-1}}\right) \frac{B_{g,t-1}}{P_t} \quad = \quad \frac{B_t}{R_t P_t} + \frac{Q_{g,t} B_{g,t}}{P_t} \tag{3.13}$$

where government spending is defined as below

$$G_t \equiv \varepsilon_t^g y_* \gamma^t,$$

and government spending shock, ε_t^g, follows an exogenous AR(1) process. The full list of equilibrium conditions, and steady state equations can be found in Appendix 3.A and 3.B. Please note that the mix between government transfer and debt is indeterminate and irrelevant as Ricardian equivalence holds in

[14]As Woodford (2001) and Orphanides (2001) already pointed out, a monetary policy rule with deterministic output trend may not be optimal in terms of welfare. For example, suppose an economy where its business cycles is mainly driven by supply shocks. A positive productivity shock decreases output and increases price. The changes in price and output are larger when the prices are sticky than when they are fully flexible. Under this circumstance, an expansionary monetary policy is more desirable as it pushes the output closer to the efficient potential output and stabilises the price level as well. On the contrary, the monetary policy response becomes contractionary if the deterministic trend is used, and it may causes less stable inflation process.

this model, and therefore, these variables do not appear in the equilibrium conditions[15].

3.2.5 Bond prices and premiums

Theoretically, under no arbitrage assumption, the price of any asset should be equal to the expected value of stochastically discounted state contingent payoff of the asset. This gives the price of an n-period conventional zero-coupon bond and its real return as below:

$$Q_t^{(n)} = E_t\left[M_{t,t+n}^{\$}\right] = E_t\left[M_{t,t+n}\Pi_{t,t+n}^{-1}\right] \tag{3.14}$$

$$R_t^{(n)} = \left(\frac{\Pi_{t,t+n}^{-1}}{Q_t^{(n)}}\right)^{\frac{1}{n}}, \tag{3.15}$$

where $M_{t,t+n}$ denotes the real stochastic discount factor (SDF) from t to $t+n$, and $M_{t,t+n}^{\$}$ is the nominal SDF defined as

$$M_{t,t+n}^{\$} \equiv M_{t,t+n}\Pi_{t,t+n}^{-1}, \tag{3.16}$$

where $\Pi_{t,t+n} \equiv P_{t+n}/P_t$ denotes the cumulative inflation of final good price from t to $t+n$.

Similarly, the price of inflation-indexed bond and its real return are given by

$$Q_{i,t}^{(n)} = E_t\left[M_{t,t+n}\right] \tag{3.17}$$

$$R_{i,t}^{(n)} = \left(\frac{1}{Q_{i,t}^{(n)}}\right)^{\frac{1}{n}}, \tag{3.18}$$

[15] In section 3.4, we examine how the process of government transfer is affected by the use of NGDP-indexed bonds. To pin down the size of government transfer and two types of bonds, we add three more equations (Equation 3.25, 3.26, and 3.27) in Section 3.4.

and lastly, the price of NGDP-indexed bond and its real return are given as follows

$$Q_{g,t}^{(n)} = E_t\left[M_{t,t+n}^{\$}G_{t,t+n}\Pi_{t,t+n}\right] = E_t\left[M_{t,t+n}G_{t,t+n}\right] \tag{3.19}$$

$$R_{g,t}^{(n)} = \left(\frac{G_{t,t+n}}{Q_{g,t}^{(n)}}\right)^{\frac{1}{n}}, \tag{3.20}$$

where $G_{t,t+n}$ denotes cumulative growth in real output from t to $t+n$.[16]

Note that holders of the inflation indexed bond are exposed neither output growth risk nor inflation risk. On the other hand, holders of the conventional nominal bonds are exposed only to the inflation risk, and the NGDP-indexed bond holders are exposed only to the growth risk. That means, the expected return of the conventional bond contains inflation risk premium, and that of the NGDP-indexed bonds contains growth risk premium. Therefore, we can calculate the inflation risk premium and growth risk premium in terms of the expected returns given above. The Inflation risk premium (IRP) can be defined as the difference between the *ex-ante* expected real return of the conventional

[16]Unlike Rudebusch and Swanson (2012) which used so-called 'decaying consol' when calculating n-period bond prices, we calculated the price in a rather standard way. For example, in a standard way, the price of n-period inflation-indexed bond is calculated recursively as follows:

$$\begin{aligned}
Q_{i,t}^{(1)} &= E_t\left[M_{t,t+1}\right] \\
Q_{i,t}^{(2)} &= E_t\left[M_{t,t+2}\right] = E_t\left[M_{t,t+1}Q_{i,t+1}^{(1)}\right] \\
&\vdots \\
Q_{i.t}^{(n)} &= E_t\left[M_{t,t+n}\right] = E_t\left[M_{t,t+1}, Q_{i,t+1}^{(n-1)}\right].
\end{aligned}$$

Therefore, to get the price of 10-year bonds, $Q_{i,t}^{(40)}$, we had to calculated additional 39 more bond prices, $Q_{i,t}^{(1)}, ..., Q_{i,t}^{(39)}$ too. As Rudebusch and Swanson (2012) is interested in both size and volatility of term premium, they had to solve the third or higher order approximation of the model, and in that case, the standard way required too much calculation time. On the order hand, as the main focus of this paper is only the size of growth risk premium, the second-order approximation is enough, and thus we followed the standard way of recursive calculation.

government bond, $R_t^{(n)}$, and that of the inflation-indexed bond, $R_{i,t}^{(n)}$:

$$IRP_t^{(n)} \equiv E_t \left[\log R_t^{(n)} - \log R_{i,t}^{(n)} \right], \tag{3.21}$$

and similarly, the growth risk premium is defined as the difference between the *ex-ante* expected real return of the NGDP-indexed bond, $R_{g,t}^{(n)}$, and that of the inflation-indexed bond, $R_{i,t}^{(n)}$:

$$GRP_t^{(n)} = E_t \left[\log R_{g,t}^{(n)} - \log R_{i,t}^{(n)} \right]. \tag{3.22}$$

Furthermore, once log-normality is assumed and Jensen's inequality terms are ignored, the two premiums in Equation (3.21) and (3.22) can be expressed in terms of the covariance between logs of SDF and inflation, and negative of the covariance between logs of SDF and real output growth:

$$IRP_t^{(n)} \approx \frac{1}{n} \mathrm{cov}_t \left(m_{t,t+n}, \pi_{t,t+n} \right) \tag{3.23}$$

$$GRP_t^{(n)} \approx -\frac{1}{n} \mathrm{cov}_t \left(m_{t,t+n}, g_{t,t+n} \right). \tag{3.24}$$

where $m_{t,t+n} \equiv \log M_{t,t+n}$, $g_{t,t+n} \equiv \log G_{t,t+n}$, and $\pi_{t,t+n} \equiv \log \Pi_{t,t+n}$.

Let us briefly explain the implication of Equation (3.23) and (3.24). If an asset provides an insurance against a bad event, risk averse investors require a negative premium on the asset. If inflation and SDF are positively correlated, holders of the conventional bonds get a smaller real payoff when a bad event (i.e. higher SDF) happens, and thus a positive inflation risk premium should be given as in Equation (3.23). In an analogous manner, if output growth covaries positively with SDF, holders of NGDP-indexed bonds get a greater real payoff when a bad event occurs. That is to say, in this case, NGDP-indexed bonds provide insurance against bad events, and thus, the growth risk premium should be negative as in Equation (3.24).

An additional advantage of the expressions in Equation (3.23) and (3.24) is that the covariances can be easily decomposed into standard deviations and correlation coefficients as in Table 3.4, and that allows us to examine how much each factor contributed to the premiums when we conduct sensitivity analysis in the following sections. For example, if a certain change in the business cycle affects mostly the volatility of SDF, that change may affects the absolute size of both inflation and growth risk premiums to the same direction. On the contrary, if the change makes inflation more volatile leaving other things hardly changed, it would only increase the absolute size of inflation risk premium.

3.3 Parametrisation

Most of the structural parameters and the seven exogenous shock processes are estimated using the seven U.S. macroeconomic data[17] from the first quarter of 1971 to the fourth quarter of 2008[18], except for the three parameters in consumer preference, which are borrowed from the previous papers. These parameters are the curvature of period utility function with respect to relative consumption for constant labour σ_c, the elasticity of labour supply σ_l, and the Epstein-Zin parameter σ_{EZ} that controls CRRA. We set $\sigma_c = 2.0$ following Rudebusch and Swanson (2012) implying the elasticity of intertemporal substitution in consumption of 0.5. It is a bit larger than the estimate of Smets and Wouters (2007), 1.39, but still consistent with the estimates in micro and macro literature (e.g., Vissing-Jørgensen, 2002; Havránek, 2015). We also set

[17]The detailed source of data can be found in Appendix 3.C

[18]Because our model does not explicitly incorporate zero-lower-bound for the short-term interest rate, we excluded the data after 2008 for the estimation.

$\sigma_l = 1.9$, borrowing the estimate of Smets and Wouters (2007). The steady state ratio of government spending over output, ε_*^g, is set as 0.18 following Smets and Wouters (2007). This is also standard in the literature where it is usually in the range of 0.15 to 0.2. Finally, we calibrated $\sigma_{EZ} = -89$ such that the term premium of 10-year nominal government bonds would be 100 basis points[19].

Except for these four parameters, all the other structural parameters and exogenous shock processes are estimated using the same Bayesian methodology used by Smets and Wouters (2007). The estimates of the parameter values are reported in Table 3.2. On top of the parameters estimated in Smets and Wouters (2007), we had to estimate the steady state ratios of price and wage markups, λ_p and λ_w, because we used different assumptions on makrup shocks. The estimates for λ_p and λ_w are 0.37 and 0.30, respectively. They are a bit larger than those from many of the New Keynesian and fiscal policy literature, where both of the parameters are usually assumed to be around 0.1 or 0.2 (Levin et al., 2006; Christiano et al., 2005; etc.). As our model structure and data are only slightly different from Smets and Wouters (2007), and as we used the same estimation methodology as they have used, most of the parameters and shock processes are quite similar to those from Smets and Wouters (2007) except for the degree of external habit in consumption, $\lambda = 0.39$, which is somewhat smaller than the range of literature estimates of 0.5~0.7. The fact that we exogenously set σ_c higher than the estimate of Smets and Wouters (2007) played a role of making more stable consumption dynamics than when using their estimate of $\sigma_c = 1.39$, and thus it was possible to match the data with a lower degree of external habit than their estimate of $\lambda = 0.71$. Another difference is found in the standard deviations of the two markup shocks. They

[19] Note that when matching the term premium, we used the decaying consol used by Rudebusch and Swanson (2012). For more detailed explanation, refer to Rudebusch and Swanson (2008).

Table 3.2: **List of parameters**

(Structural parameters)

	Baseline	**SW2007**	**Description**
σ_c	2.00	1.39	IES in consumption
σ_l	1.90	1.90	elasticity of labour supply
σ_{EZ}	-89	n.a.	EZ parameter
λ	0.39	0.71	degree of consumption habit
λ_p	0.37	n.a	steady state price markup
λ_w	0.30	n.a	steady state wage markup
ξ_p	0.79	0.65	degree of price stickiness
ξ_w	0.69	0.73	degree of wage stickiness
ι_p	0.24	0.22	price indexation
ι_w	0.76	0.59	wage indexation
ϕ_p	1.49	1.61	fixed costs
φ	6.31	5.48	capital adjustment cost
ψ	0.64	0.54	elasticity of util. adj. cost ($\equiv \frac{\delta_2}{\delta_1}$)
ρ_R	0.80	0.81	degree of policy rate smoothing
ψ_π	1.94	2.03	policy response to inflation
ψ_y	0.06	0.08	policy response to output gap
$\psi_{\Delta y}$	0.15	0.22	policy response to current output growth
α	0.18	0.19	capital share in income
ε_*^g	0.18	0.18	steady state spending over output
π_*	0.60	0.81	steady state inflation
$100\left(\beta^{-1}-1\right)$	0.21	0.16	discount factor
$\overline{\gamma}$	0.40	0.43	trend growth rate

(Shock processes)

	Baseline	**SW2007**	**Description**
σ_a	0.43	0.45	Standard deviation of tech shock
σ_b	0.34	0.24	Standard deviation of preference shock
σ_g	0.51	0.52	Standard deviation of spending shock
σ_i	0.58	0.45	Standard deviation of investment shock
σ_r	0.23	0.24	Standard deviation of monetary policy shock
σ_p	2.18	0.14	Standard deviation of price markup shock
σ_w	4.10	0.24	Standard deviation of wage markup shock
ρ_a	0.98	0.95	AR(1) for tech shock
ρ_b	0.47	0.18	AR(1) for preference shock
ρ_g	0.97	0.97	AR(1) for spending shock
ρ_i	0.55	0.71	AR(1) for investment shock
ρ_r	0.26	0.12	AR(1) for monetary policy shock
ρ_p	0.80	0.90	AR(1) for price markup shock
ρ_w	0.89	0.97	AR(1) for wage markup shock
ρ_{ga}	0.52	0.52	Correlation between spending and tech shock

Note: σ_c, σ_l, σ_{EZ} and ε_*^g are not estimated. $\pi_* \equiv 100 \times \log \Pi_*$ and $\overline{\gamma} \equiv 100 \times \log \gamma$. Note also that the estimated standard deviations for preference, spending, and investment shocks do not correspond to the standard deviation in Section 3.2 as they are rescaled so that they can be easily compared with the estimates from Smets and Wouters (2003, 2007).

Table 3.3: **Fit of the model**

	actual	baseline		actual	baseline
σ_{g_y}	0.80	1.00	ρ_{g_y,g_c}	0.65	0.71
σ_{g_c}	0.65	0.89	$\rho_{g_y,\pi}$	-0.18	-0.36
σ_{π}	0.61	0.67	$\rho_{g_c,\pi}$	-0.24	-0.40
σ_{r}	0.86	0.61	AR1(g_y)	0.34	0.34
$\sigma_{g_c}/\sigma_{g_y}$	0.82	0.89	AR1(π)	0.65	0.83
$\sigma_{\pi}/\sigma_{g_y}$	0.76	0.67			

Note: σ_{g_y}, σ_{g_c} and σ_{π} denote standard deviations of output growth, consumption growth, and inflation in quarterly percentage term. $\rho_{x,y}$ denotes correlation coefficient between x and y.

look much larger than those from Smets and Wouters (2003, 2007). However, as already explained in Section 3.2, we specified the two markup shocks quite differently from them, and it does not seem appropriate to directly compare them.

In order to generate sizeable premiums, we solved the model numerically to a second-order approximation using Dynare[20]. Table 3.3 shows the fit of the baseline model. It provides the standard deviations and AR(1) coefficients of output growth, consumption growth, and inflation; and the correlations between them calculated using the simulated data for 10,000 periods. The standard deviations are slightly larger than actual data, but they well replicate the negative correlation between inflation and the other two variables, and the fact that inflation is more persistent than output growth.

[20]We provided the non-linear equilibrium conditions and deterministic steady states to Dynare, and let it solve the model using a second order approximation.

Table 3.4: **IRP and GRP, by different shocks**

	Baseline	**a**	**p**	**w**	**b**	**g**	**i**	**r**	**supply**	**demand**
$\frac{1}{\sqrt{40}}\sigma_m$	18.99	15.49	0.90	1.70	1.52	10.81	3.29	0.38	15.60	10.81
$\frac{1}{\sqrt{40}}\sigma_\pi$	1.90	1.39	0.39	0.91	0.14	0.35	0.38	0.59	1.72	0.79
$\frac{1}{\sqrt{40}}\sigma_{g_y}$	0.89	0.57	0.31	0.53	0.13	0.18	0.21	0.16	0.84	0.35
$\rho_{m,\pi}$	0.46	0.65	0.96	0.90	0.97	0.55	-0.83	-0.66	0.60	0.19
$\rho_{m,y}$	-0.44	-0.85	-0.56	-0.61	0.20	0.58	-0.34	-0.65	-0.65	0.24
IRP_{40}	16.40	14.25	0.28	0.98	0.20	2.12	-1.09	-0.33	15.51	0.90
GRP_{40}	7.03	7.24	0.14	0.54	-0.04	-1.13	0.26	0.03	7.91	-0.88

Note: Each column provides the decomposition of IRP and GRP under different composition of shocks. The standard deviation of each shock is equal to the estimates in Table 3.2. Supply shock denotes productivity (a), price and wage markup (p and w) shocks. Demand shock includes preference (b), government spending (g), investment (i), and monetary policy (r) shocks. All the numbers are quarterly average of 10-year variables (not annualized). Premiums are expressed in basis points.

3.4 Simulated premiums

3.4.1 Baseline model

Table 3.4 shows the inflation risk premium (IRP) and the growth risk premium (GRP) calculated from the baseline model simulation; and their decomposition into standard deviations of SDF, inflation, and output growth, and the correlation coefficients between them are also provided. To construct the table, we used 10,000 periods of simulated data. Note that we compared the premiums associated with the two kinds (conventional and NGDP-indexed) of government bonds with maturity of 40 periods. That is, the first group of three rows provide the standard deviations of 10-year average quarterly (not annualized) SDF (σ_m), inflation (σ_π), and output growth (σ_{g_y}); and the next group of rows show the correlation coefficients between them ($\rho_{m,\pi}$ and $\rho_{m,y}$). The final group of rows show the IRP for 10-year conventional bonds and GRP for 10-year NGDP-indexed bonds (IRP_{40} and GRP_{40}).

The baseline model results (first column) show that, for the maturity of 10 years, both premiums are positive, and the IRP is more than twice as large as the GRP. When a government finances all its debt with NGDP-indexed bonds, it needs to pay additional premium to compensate the growth risk, but at the same time, it can save the inflation risk premium required when issuing the conventional government bond. Therefore, theoretically, the additional borrowing cost for issuing NGDP-indexed bonds equals GRP minus IRP. This implies that the government in the baseline model may save around 38 basis points (annualised)[21] in borrowing costs through issuing NGDP-indexed bonds instead of the conventional bonds[22].

The second to tenth columns of Table 3.4 show how the two premiums are affected when a different composition of shocks is assumed. Each column shows the premiums when only one of the seven shocks exists, and the last two columns are the cases where there exist only demand shocks (preference, government spending, investment and monetary policy) or supply shocks (productivity, price markup, and wage markup). When the main driver of the business cycle changes, the joint process of consumption (thus SDF), output and inflation are also changed, and so do the premiums, consistently.

Among the seven shocks the productivity shock plays the most significant role; it accounts for most of the two premiums as seen in the table. This is consistent with the analysis of Smets and Wouters (2007) where supply shocks have been the dominant drivers for long-term business cycle of the U.S. economy. The positive IRP and GRP of the baseline model can be explained from this fact. When supply shocks are dominant, consumption growth covaries negatively (positively) with inflation (output growth) as in Figure 3.1. The

[21] $(16.40 - 7.03) \times 4$

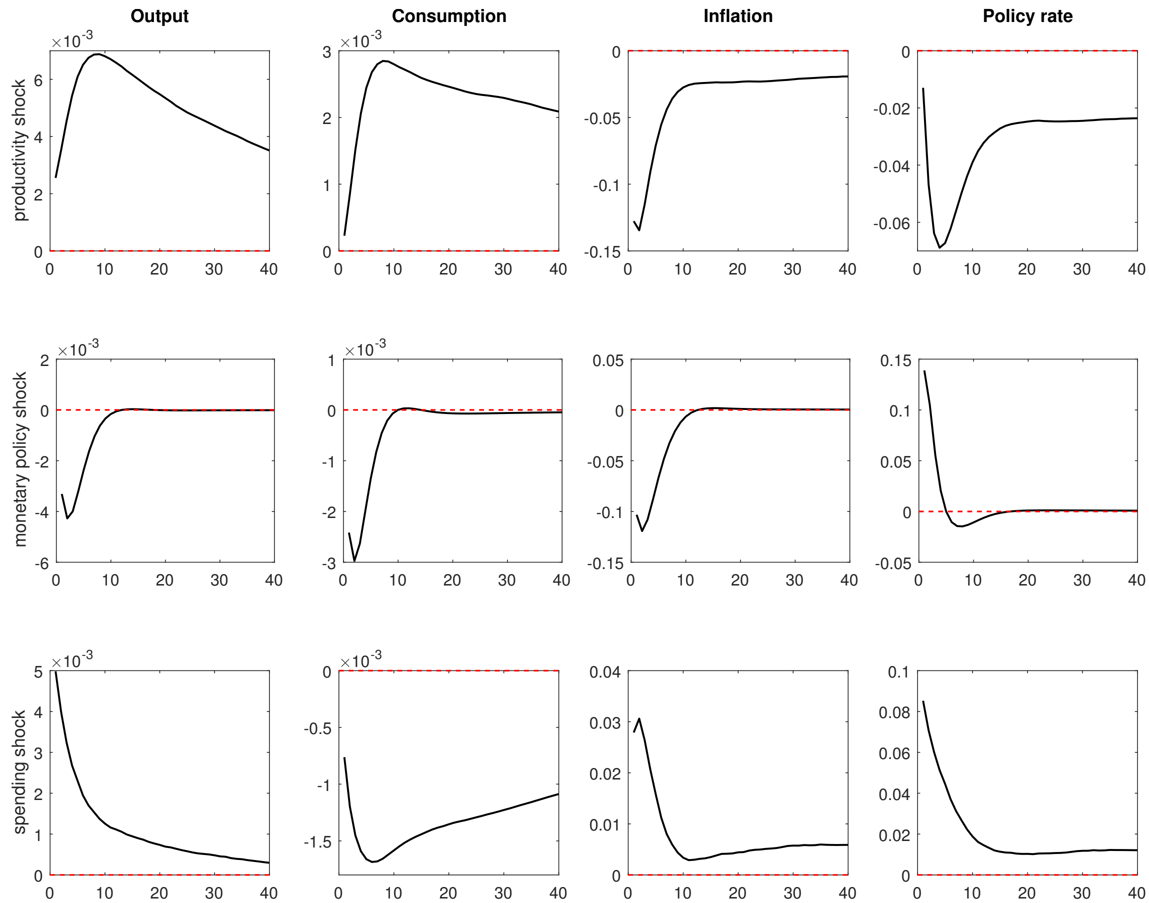
[22] Even when we consider "real" GDP growth-indexed bonds, whose holders are not immune to inflation uncertainty, the additional borrowing cost is less than 30 basis points in annualised terms (7.03×4), which is a lot smaller than the estimates from the literature in Table 3.1

positive covariance between inflation and SDF is consistent with positive IRP, and the negative covariance between output growth and SDF results in positive GRP as in Equation (3.23) and (3.24).

Figure 3.1 also explains why IRP is greater than GRP in the baseline model when the productivity shock dominates. As seen from the top panel of the figure, the response to the productivity shock is much more persistent in inflation than in output growth. In fact, Table 3.3 already showed that the AR(1) coefficient of inflation from the baseline model, 0.83, is a lot larger than that of output growth, 0.34. This directly explains why the standard deviation of the annualized 10-year average inflation is much larger than that of the output growth in the first rows of Table 3.4, and the difference in the absolute size of the two premiums too.

In general, the sign of both IRP and GRP are determined by how the output, consumption and inflation respond to the main driver(s) of the business cycle. The results from the baseline parametrisation are mainly due to the fact that the main driver of the business cycle is the productivity shock, or differently speaking, the characteristics of the business cycle data used for our estimation are consistent with the case where productivity shock dominates. That is to say, if the business cycle is driven by different shocks, the results can be different, and Table 3.4 tells us how they will be changed. For example, if the monetary policy shock dominates the long-term business cycle, as seen from the middle panels of Figure 3.1, all the responses of inflation, output and consumption to monetary policy shock have the same direction. This implies that both the covariance of SDF and inflation and the covariance of SDF and output growth are negative, and in such cases, as in the eighth column of Table 3.4, IRP becomes negative and GRP becomes positive. On the other hand, if the government spending shock is dominant the two premiums can have the

Figure 3.1: **Impulse responses function for key variables**



opposite signs (positive IRP and negative GRP). As an increase in government spending has a negative effect on consumption and a positive effect on inflation, both the covariance of SDF and inflation and the covariance of SDF and output growth become positive. That explains the positive IRP and negative GRP in the sixth column of Table 3.4.

3.4.2 Alternative parametrisation

In this chapter, we are also interested in how various assumptions used in New Keynesian models to better match the fit of business cycle affect the size and sign of the premiums. More specifically, we examined how the parameters

Table 3.5: **IRP and GRP, by different parameterisation (preference)**

	Baseline	$\sigma_{EZ} = -150$	$\sigma_c = 1.2$	$\sigma_c = 2.2$	$\lambda = 0.1$	$\lambda = 0.8$
$\frac{1}{\sqrt{40}}\sigma_m$	18.99	30.74	2.64	24.54	18.01	31.94
$\frac{1}{\sqrt{40}}\sigma_\pi$	1.90	1.91	1.77	1.94	1.94	2.18
$\frac{1}{\sqrt{40}}\sigma_{g_y}$	0.89	0.89	0.92	0.89	0.91	0.79
$\rho_{m,\pi}$	0.46	0.44	0.45	0.45	0.51	0.38
$\rho_{m,y}$	-0.44	-0.44	-0.52	-0.44	-0.51	-0.21
IRP_{40}	16.40	27.82	1.06	23.31	17.07	32.20
GRP_{40}	7.03	11.55	1.05	9.15	7.92	4.98

Note: Baseline parameters are $\sigma_{EZ} = -89$, $\sigma_c = 2.0$, $\lambda = 0.39$.

that affect the preference of consumers or nominal rigidity may affect the two premiums in our New Keynesian DSGE model. For example, the parameters in consumer preference such as σ_c, σ_{EZ} and λ affect how much consumers dislike consumption volatility between time or states, and thus we may expect that these parameters would affect the premiums mostly through the stochastic discount factor. The parameters that control the degree of nominal rigidity such as ζ_p and ζ_w would also affect the premiums, but mainly through the volatility of inflation and/or output growth. We also examined the role of monetary policy coefficients.

Table 3.5 shows how the changes in the parameters governing preference may affect the IRP and GRP given other parameters unchanged from the baseline parametrisation. As Rudebusch and Swanson (2012) have illustrated, an increase in the absolute size of σ_{EZ} linearly increases the risk aversion of consumers leaving other variables nearly unchanged. This is consistent with the second column of Table 3.5, where the increase in both premiums are mostly attributable to more volatile SDF when the absolute value of σ_{EZ} is increased from 89 to 150. The third and fourth columns show that the changes in σ_c positively affect both premiums, and similarly to the case of σ_{EZ}, most of the

Table 3.6: **IRP and GRP, by different parameterisation (price/wage stickiness, monetary policy)**

	Baseline	$\zeta_p = 0.0$	$\zeta_p = 0.9$	$\zeta_w = 0.0$	$\zeta_w = 0.9$	$\psi_y, \psi_{\Delta y} = -0.2$
$\frac{1}{\sqrt{40}}\sigma_m$	18.99	19.36	18.38	18.97	19.01	19.24
$\frac{1}{\sqrt{40}}\sigma_\pi$	1.90	2.46	1.59	1.97	1.82	4.14
$\frac{1}{\sqrt{40}}\sigma_{g_y}$	0.89	1.12	0.80	0.89	0.94	1.35
$\rho_{m,\pi}$	0.46	0.49	0.42	0.46	0.49	-0.39
$\rho_{m,y}$	-0.44	-0.42	-0.41	-0.44	-0.43	-0.40
IRP_{40}	16.40	21.50	13.09	16.54	16.93	-42.78
GRP_{40}	7.03	8.52	5.68	6.96	7.41	10.19

Note: Baseline parameters are $\zeta_p = 0.79$, $\zeta_w = 0.69$, $\psi_y = 0.06$ and $\psi_{\Delta y} = 0.15$.

changes are attributable to the changes in SDF leaving the volatility of inflation or output growth scarcely changed. Therefore, the increase in these two parameters, σ_{EZ} and σ_c, affect the absolute size of the two premiums, but not substantially their relative sizes or signs.

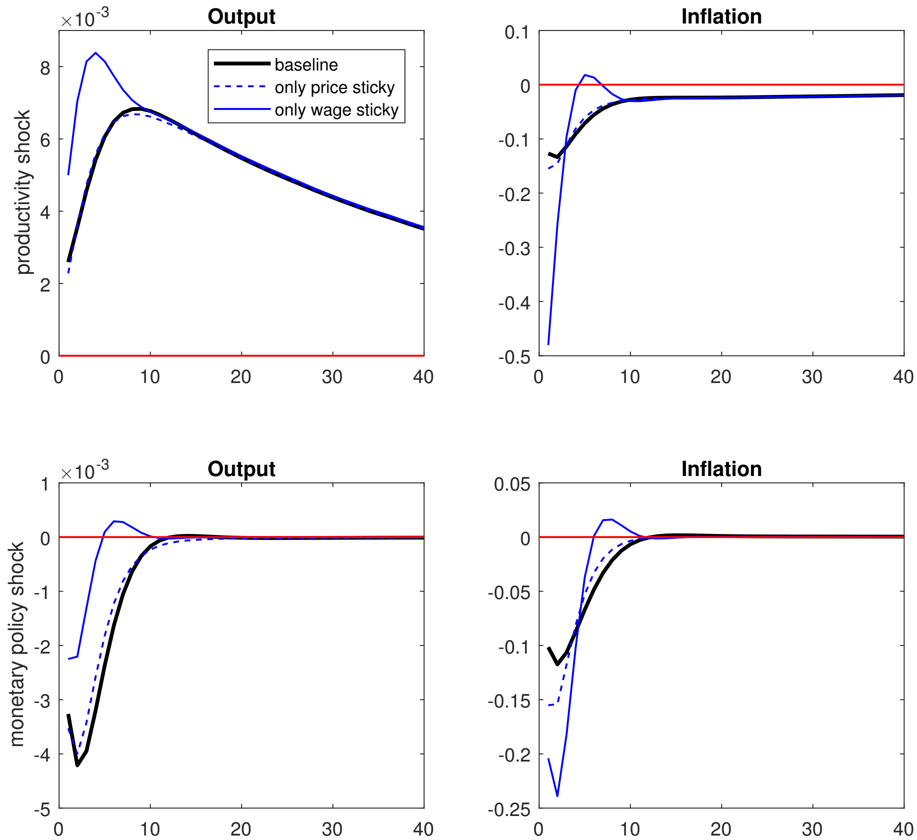
Meanwhile, column 5 and 6 show that an increase in the degree of external habit λ raises the IRP but decreases GRP in our baseline model. In general, macroeconomic models incorporate the habit in consumption in order to match the hump-shaped responses of variables to various shocks (Fuhrer, 2000). From the asset pricing point of view, on the other hand, an existence of the habit enables more volatile marginal utility of consumption leaving the volatility of consumption unchanged, which implies larger risk premiums in absolute terms. At the same time, higher λ lowers the correlation between output growth and consumption growth (and thus SDF), which has negative effects on the GRP. Overall, higher degree of habit increases the IRP, but its effect on the GRP is hard to predict.

Now let us move to Table 3.6. The second to fifth columns in the table show how price and wage stickiness affects the two premiums. We compared the two cases where there is no friction ($\zeta_p = 0$, $\zeta_w = 0$) and where only 10% of firms or unions are given chances to optimise their prices or wages ($\zeta_p = 0.9$, $\zeta_w = 0.9$). In our model, when other things are equal, the degree of price rigidity ζ_p has negative effects on both IRP and GRP (see column 2 and 3), and it can be explained from the fact that the productivity shock is the dominant driver of business cycle in our model[23]. A positive productivity shock in general increases output and decreases price regardless of the existence of nominal rigidity. However, when price becomes stickier, the responses of price and output become smaller and smaller (as the friction increase the price markup) than the flexible price case, and this makes the absolute size of both premiums smaller. The top two panels of Figure 3.2 support this explanation. They show the responses of output and inflation to a productivity shock with and without price/wage stickiness. Comparing the solid black line (baseline case) and the solid blue line (only-wage-sticky case) in the top panels shows that adding price rigidity (i.e. moving from the solid blue line to the solid black line) makes the responses of both output and inflation smaller.

However, the role of wage stickiness to the premiums are not clear in our model where productivity shock dominates. In general, wage rigidity amplifies the response of output to a productivity shock. It is because, as a fraction of households cannot increase their wages in response to the positive productivity shock, the increase in real wage in the economy becomes smaller than the flexible wage economy, and thus the output responses more strongly when wage is stickier. Nevertheless, the effect of wage rigidity seems negligible in this model as seen from the top panels in Figure 3.2. In the figure, there are only negligible differences between the dotted blue lines (only-price-sticky case) and the solid black lines (baseline case) for the responses of both output and inflation. The premiums in Table 3.6 are consistent with what the figure shows.

[23] This is also consistent with the analysis of De Paoli et al. (2010) about term premium and nominal rigidity.

Figure 3.2: **Impulse respones and role of nominal rigidity**



The differences in premiums between baseline case and flexible wage case (i.e., only-price-sticky case) seems negligible, and even when we increase the degree of wage rigidity further up to 0.9, we can only see very small differences in the standard deviations of output growth and inflation from the flexible wage economy (compare first and fifth columns of Table 3.6), and thus very little changes in both GRP and IRP.

It is also worth mentioning that the effects of these nominal rigidities on the premiums can be different when the main shock in the economy changes. As an example, the bottom panels in Figure 3.2 shows how the responses of inflation and output to a monetary policy shock are affected as the degrees of price and wage rigidity change. In the case of inflation, the figure shows that its response becomes smaller when the price becomes sticky (from wage-only-sticky case to baseline case), which is same as the effect of productivity shock. However, in the case of output, price stickiness makes its response to the monetary policy shock greater, which is opposite to the case of productivity shock in the top panel. If both price and wage are fully flexible, monetary policy shock has no effect on real variables because the changes in price and wage can fully absorb the shock and leave the real interest rate not being affected at all. However, if the price (or wage) becomes sticky, changes in price and wage cannot fully absorb the shock, and thus output and consumption begin to respond to the shock.

In the bottom panels of Figure 3.2, comparing the solid black lines (baseline case) and solid blue line (only-wage-sticky case), we see that, when the monetary policy prevails, price rigidity results in more volatile output and more stable inflation. As this makes consumption growth (and thus SDF) more volatile, the GRP is expected to increase when the price becomes stickier, but it is hard to predict how the IRP is affected by the price stickiness since the changes in volatilities of SDF and inflation are in opposite directions.

Now let us think about the role of the wage stickiness in the economy where monetary policy shock dominates. A tighter monetary policy shock gives downward pressure on both price and wage. If wage becomes stickier the average level of wage becomes higher compared to the case of flexible wage economy, and thus the negative effect of a monetary policy shock on output is amplified, but its negative effect on inflation is mitigated. This can be shown by comparing the dotted blue line (only-price-sticky case) and the solid black line (baseline case) in Figure 3.2. However, similar to the case of productivity shock, the effects of wage stickiness on output and inflation in this model are very small, implying its negligible impacts on the premiums.

The last column in Table 3.6 is about how an extreme monetary policy rules can change the premiums. We examined a particular example of a bad monetary policy that may significantly affect the premiums. In our example, the monetary policy responds negatively to both output gap and current output growth ($\psi_y = \psi_{\Delta y} = -0.2$), but its response to inflation gap is unchanged from the baseline model. In this case, the monetary policy plays a role of destabilizing the economy. Under this monetary policy rule, as seen from the table, both premiums become much larger in absolute terms mainly because the monetary policy rule substantially destabilises the business cycle. Furthermore, as the monetary policy shock becomes the main driver of business cycle, the IRP becomes negative as seen from the eighth column in Table 3.4.

3.5 Potential Benefits to the government

In section 3.4, we showed that the government in our baseline DSGE model estimated with the U.S. data may save the borrowing cost when it replaces the conventional government bonds with the nominal GDP growth-indexed bonds. We also showed that the result relies on the fact that our model economy is driven mainly by the productivity shock that causes a positive correlation between SDF and inflation and a negative correlation between SDF and output growth, and the fact that the inflation is much more persistent than the output growth.

In this section, we assess whether the benefits to the government aforementioned by the previous papers can be obtained from the baseline model of this chapter. Governments that cannot flexibly adjust its debt-to-GDP ratio may face the pressure of conducting more pro-cyclical fiscal policy when other things are equal, and the previous papers have illustrated that such governments can mitigate the pressure by issuing NGDP-indexed bonds.

Before examining the benefit, we admit that our analysis in this section is very limited. In our model, the government's structure of finance (conventional vs. NGDP-indexed bonds) only affects the mix between debt and primary surplus, but cannot affect the optimal choices of consumers. The agents in our model are rational and forward-looking; and there exists neither borrowing constraint nor distortionary tax in this model. In other words, the Ricardian equivalence holds in the model, and in such models, a particular time path of debt and/or primary surplus is irrelevant. Therefore, we limit our discuss to the pros and cons of issuing NGDP-indexed bonds from the perspective of the government debt management, but not its effect on household welfare.

As mentioned earlier, previous works suggest the benefits to the government from the use of NGDP-indexed bonds in two ways. On one hand, a government may reduce the likelihood of debt explosion (or default) for a given path of primary surplus. On the other hand, a government that is constrained to keep a certain level of debt-to-GDP ratio may get more room for counter-cyclical fiscal policy by issuing NGDP-indexed bonds. In this section, we examine the benefits of NGDP-indexed bonds from the latter point of view. That is, we focus on whether and under what conditions the use of NGDP-indexed bonds may mitigate the pressure of conducting pro-cyclical fiscal policy.

For the analysis on cyclicality of fiscal policy, we add one more assumption on government sector[24]. That is, we assume that our model government should

[24] As Ricardian equivalence holds in our model, the mix between debt and transfer is indeterminate, and thus b_t, $b_{g,t}$ and t_t disappear from the equilibrium conditions in the previous sections. In order to pin them down, we add three more equations (Equation 3.25, 3.26, and 3.27).

keep its debt-to-GDP ratio at some fixed level, $\overline{D}$, such that

$$
\begin{aligned}
b_t/R_t &= \left(1-\omega^G\right)\overline{D}y_t &\quad (3.25)\\
b_{g,t}Q_{g,t} &= \omega^G\overline{D}y_t, &\quad (3.26)
\end{aligned}
$$

where b_t (or $b_{g,t}$) denotes the unit of real, detrended conventional (or NGDP-indexed) government debt, and $\omega^G \in \{0,1\}$ is the fraction of NGDP-indexed bonds. In other words, we assume that total real amount of new debt issued at time t equals $\overline{D}y_t$ regardless of the choice of bond type. Under this assumption, the government faces the following budget constraint:

$$
\underbrace{\overline{D}\left[\left(1-\omega^G\right)\frac{y_{t-1}R_{t-1}}{\pi_t\gamma}+\omega^G\frac{y_t}{Q_{g,t-1}}\right]}_{\text{debt repayment}} = \underbrace{\overline{D}y_t}_{\text{new debt}} + \underbrace{\left(-t_t-\varepsilon_t^g y_*\right)}_{\text{primary surplus}}, \qquad (3.27)
$$

where t_t and $\varepsilon_t^g y_*$ are detrended real government transfer and government spending, respectively. Then we can rewrite the constraint as follows:

$$
t_t = \Delta D_t - \varepsilon_t^g y_*, \qquad (3.28)
$$

where ΔD_t denotes the net increase in the amount of debt (= new debt issuance - debt repayment) between $t-1$ and t. This implies that the amount of government transfer equals net increase in debt minus (exogenous) government spending. Under these assumptions, the only way the government can change the government transfer is by choosing the type of bond.

We simulated the baseline model augmented with the constant debt-to-GDP constraint for 10,000 periods. To compare the cyclicality of fiscal policy under the two different government financing structure (100% conventional bonds vs. 100% NGDP-indexed bonds), we calculated the sample correlation between

Table 3.7: **Correlation between output and government transfer**

	$\overline{D}$ =**0%**	**63%**	**100%**	**150%**	**200%**
Conventional ($\omega^G = 0$)	-0.38	-0.08	-0.00	0.05	0.08
NGDP-indexed ($\omega^G = 1$)	-0.38	-0.21	-0.14	-0.09	-0.05

Note: The numbers in this table show the correlation between output and government transfer.

output and lump-sum government transfer with the simulated data. Note that the smaller the correlation, the more counter-cyclical the fiscal policy. Table 3.7 summarises the results. The first row shows the correlation between output and government transfer with different level of $\overline{D}$ when the government finances all its debt with conventional government bonds, and the second row shows the correlations when it is financed with NGDP-indexed bonds.

The first column in Table 3.7 shows that, when there is no outstanding debt in the economy ($\overline{D} = 0$), fiscal policy is most counter-cyclical and type of bonds makes no difference. In this case, the government budget constraint, Equation (3.28), becomes $t_t = -\varepsilon_t^g y_*$. As the government spending, $\varepsilon_t^g y_*$, covaries positively with output, the lump-sum government transfer, t_t, should be negatively correlated with output. The table also shows that, when there exists positive outstanding debt, the government is forced to conduct less and less counter-cyclical (less negative correlation) fiscal policy as $\overline{D}$ becomes larger. Comparing the two rows in the table tells us that, when all the debts are financed by NGDP-indexed bonds, the correlation between output and government transfer increases more slowly than when the conventional bonds are used. In other words, indexation helps to relieve the burden of conducting less counter-cyclical fiscal policy.

Let us more closely see why and under what condition the use of NGDP-indexed bonds makes the fiscal policy more counter-cyclical. As $\varepsilon_t^g y_*$ and y_t is positively correlated, if we remove ΔD_t term from Equation (3.28), correlation between t_t and y_t becomes strongly negative. However, if ΔD_t is positively correlated with y_t, the existence of positive outstanding debt plays a role of making fiscal policy less counter-cyclical. Therefore, in order to clearly see the role of NGDP-indexed bonds, we have to examine how the correlation between ΔD_t and y_t varies depending on the type of bonds. The relation can be more clearly seen from the expression below:

$$\Delta D_t = \begin{cases} \overline{D}\left(y_t - \frac{y_{t-1}R_{t-1}}{\pi_t \gamma}\right) & \text{if } \omega^G = 0 \\ \overline{D}\left(y_t - \frac{y_t}{Q_{g,t-1}}\right) & \text{if } \omega^G = 1. \end{cases} \tag{3.29}$$

The expression above shows that, when conventional bonds are used ($\omega^G = 0$), an increase in y_t clearly increases the government's ability to issue new debt, $\overline{D}y_t$, but its effect on debt repayment, $\overline{D}y_{t-1}R_{t-1}/\pi_t\gamma$, is not clear. If the increase in y_t decreases the latter, or increases it but less than the new debt, ΔD_t covaries positively with y_t. If this is the case, the existence of positive debt ($\overline{D} > 0$) plays a role of making the correlation between y_t and t_t less negative (or less counter-cyclical), and the larger $\overline{D}$, the stronger this effect. The first row in Table 3.7 shows this.

When NGDP-indexed bonds are used ($\omega^G = 1$) instead, an increase in y_t increases both new debt issuance and debt repayment, thus its net effect on ΔD_t is hard to predict whether it would be positive or negative. Even if it is positive, however, it is likely to be smaller than the conventional bond case. In our particular model, Table 3.7 shows that the existence of positive debt in NGDP-indexed bonds cases also plays a role of making the fiscal policy less counter-cyclical, but as mentioned, at much smaller degree than conventional bond cases.

Table 3.8: **Correlation between output and government transfer**

	Baseline	**a**	**p**	**w**	**b**	**g**	**i**	**r**	**supply**	**demand**
$\omega^G = 0$	-0.08	-0.90	0.45	0.33	0.28	-0.64	-0.10	0.80	-0.39	-0.07
$\omega^G = 1$	-0.21	-0.89	0.48	0.34	-0.51	-0.91	-0.52	0.74	-0.39	-0.44

Note: $\overline{D} = 63\%$ is used for all the cases.

In summary, if current output and net debt changes (ΔD_t) covary positively, the existence of positive outstanding debt gives a pressure for less counter-cyclical fiscal policy at least for our baseline parametrisation. In other words, when growth slows, government's ability to issue additional debt (ΔD_t) also shrinks and the government should reduce its government transfer accordingly. In such economy, issuing NGDP-indexed bond weakens the positive relationship between y_t and ΔD_t, and thus helps mitigate the pressure of conducting less counter-cyclical fiscal policy (than conventional bond case).

Table 3.8 shows more general idea about the cyclicality of fiscal policy and the DSGE model; i.e., how different shocks change the correlation between output and government transfer. As already mentioned, given the assumption that the government spending is an exogenous process, the relation between output and government transfer is mainly affected by the relation between output, y_t, and net increase in debt, ΔD_t, and this relation makes the difference between the two bonds types. As the amount of new debt issuance is given by $\overline{D}y_t$, regardless of the type of the bond, the difference between the two type of bonds comes from how y_t and the burden of debt repayment ($\frac{y_{t-1}R_{t-1}}{\pi_t \gamma}$ or $\frac{y_t}{Q_{g,t-1}}$) is related. Equation (3.29) clearly shows that the relation between y_t and debt repayment depends on how y_t and π_t covary. In a nut shell, how the issuance of NGDP-indexed bond affects cyclicality of fiscal policy depends on the relation between y_t and π_t, and this relation fundamentally depends on the type of the main shock in the business cycle.

We simulated the model for 10,000 periods with baseline structural parameters, but with different set of shocks, and the sample correlation between y_t and t_t in each case is reported in Table 3.8. From the table, we can see that the government can significantly benefit from issuing NGDP-indexed bonds, in terms of counter-cyclical fiscal policy, when the economy is mainly driven by demand shocks (b, g, i, r). However, the benefit becomes negligible when the main driver of the business cycle is a supply shock (a, p, w).

Demand shocks move output and inflation to the same direction. When all the debts are financed by the conventional bonds, Equation (3.29) shows that an increase in y_t is associated with an increase in new debt issuance, and at the same time, with a decrease in debt repayment burden (as π_t increases as well), and thus results in an increase in ΔD_t. This means that the existence of a positive debt plays a role of making the correlation between y_t and t_t less negative (or more pro-cyclical). When we replace the conventional bonds with NGDP-indexed bonds, the effect from the positive correlation between y_t and π_t disappears, and the pressure of conducting pro-cyclical fiscal policy can be significantly reduced.

On the contrary, the responses of y_t and π_t to supply shocks are the opposite directions. In this case, even when all the debts are financed with conventional bonds, an increase in y_t is associated with an increase in both new debt issuance and debt repayment, and thus their impacts on ΔD_t cancel out each other. This explains why the government can expect much smaller benefit from issuing NGDP-indexed bonds when a supply shock is the main driver of the business cycle.

3.6 Conclusion and Summary

In this chapter, we tried to calculate the theoretical price of nominal GDP growth-indexed bonds within the framework of New Keynesian DSGE model largely based on Smets and Wouters (2007) augmented with EZ preference. We estimated the model with the U.S. macroeconomic data and matched the term premium for the 10-year U.S. Treasury bonds by calibrating EZ preference parameter accordingly. In other words, the main focus of this chapter is to examine whether an advanced country government like the U.S. may save the borrowing cost by issuing NGDP-indexed bonds instead of the conventional nominal bonds.

In our baseline model, the growth risk premium is around 30 basis points and the inflation risk premium is around 70 basis points for 10-year government bonds. The positive premiums are mainly due to the fact that the business cycle in our model is driven mostly by supply shocks in 10-year horizon, and the inflation risk premium is higher since the inflation is much more persistent than the growth in output. As the additional borrowing cost the government should bear by issuing nominal GDP growth-indexed bonds instead of conventional nominal bonds is same as the difference between growth risk premium and inflation risk premium (GRP - IRP), the additional cost is negative in our baseline model estimated with the U.S. data. This result may relieve the concern that excessive premiums may have to be paid when a government issues NGDP-indexed bonds.

Contrary to the previous papers mostly based on partial equilibrium models, we calculated the bond prices and premiums consistently with macroeconomic theories with the help of DSGE model. This also enabled us to analyse how different compositions of shocks and different parameters may affect the sign and size of the premiums. In DSGE models, the sign and size of both premiums are determined by the main driver of the business cycle, and our baseline results are mainly attributable to the fact that the productivity shock dominates. This also suggests that it may be possible that an economy where a different shock dominates can have different results.

We also have shown that the parameters related with consumer preference and nominal rigidity also play important roles in determining the sign and size of the premiums. The parameters on consumer preference determine how much consumers dislike volatility in consumption. In our model, the changes in these parameters affected the absolute size of premiums mostly via the stochastic discount factors, but their effects via the changes in volatility of output growth or inflation were not substantial. On the other hand, the effects of the two nominal rigidity parameters on the premiums were largely through the volatilities in inflation or output growth, and their signs varied depending on the main driver of the business cycle.

Lastly, we shortly examined the benefits of issuing NGDP-indexed bonds to the government, but only in terms of cyclicality of fiscal policy. In our model, the government with constant debt-to-GDP ratio benefits from issuing NGDP-indexed bonds as the use of NGDP-indexed bonds reduces the pressure of conducting pro-cyclical fiscal policy. However, as is the case of the premiums, the sensitivity analysis showed that the benefit also is strongly affected by the characteristics of the business cycle. That is to say, the government may benefit significantly from issuing NGDP-indexed bonds when the business cycle is driven mainly by demand shocks, but not much benefit is expected when supply shocks are dominant.

The reader should also keep in mind the limitations of our analysis, especially about the benefit. We showed that the government in our DSGE model can conduct more counter-cyclical fiscal policy when it uses NGDP-indexed bonds. However, as the cyclicality of fiscal policy cannot affect the level of consumption in our model (Ricardian equivalence), the model cannot explicitly show why more counter-cyclical fiscal policy is better from the household's perspective. In this paper, we made this implicit assumption. Explicit analysis on the benefit in terms of consumer welfare requires a model where Ricardian equivalence does not hold. One of the most widely supported assumption is the presence of hand-to-mouth households as in Galí et al. (2007) and/or distortionary taxes on wage and capital rental income. We are going to discuss this topic in the next chapter.

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Appendix

3.A List of equilibrium equations (detrended)

- Production sector[25]

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{w_t}{r_t^k}\right) = \left(\frac{z_t k_{t-1}/\gamma}{l_t}\right) \tag{3.30}$$

$$mc_t = \frac{1}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\varepsilon_t^a}\left[\left(w_t\right)^{1-\alpha}\left(r_t^k\right)^{\alpha}\right] \tag{3.31}$$

- Price setting[26]

$$g_t^1 = \hat{\Pi}_t^{-\frac{1}{\lambda_p}} y_t + \zeta_p E_t\left[M_{t,t+1}\gamma\left(\frac{\hat{\Pi}_t}{\hat{\Pi}_{t+1}}\right)^{-\frac{1}{\lambda_p}}\left(\frac{\Pi_t^{\iota_p}\Pi_*^{1-\iota_p}}{\Pi_{t+1}}\right)^{-\frac{1}{\lambda_p}} g_{t+1}^1\right] \tag{3.32}$$

$$g_t^2 = \varepsilon_t^p\cdot\hat{\Pi}_t^{-\frac{1+\lambda_p}{\lambda_p}} y_t\cdot mc_t + \zeta_p E_t\left[M_{t,t+1}\gamma\left(\frac{\hat{\Pi}_t}{\hat{\Pi}_{t+1}}\right)^{-\frac{1+\lambda_p}{\lambda_p}}\left(\frac{\Pi_t^{\iota_p}\Pi_*^{1-\iota_p}}{\Pi_{t+1}}\right)^{-\frac{1+\lambda_p}{\lambda_p}} g_{t+1}^2\right] \tag{3.33}$$

$$g_t^1 = \left(1+\lambda_p\right) g_t^2 \tag{3.34}$$

- Law of motion : price

$$1 = \left(1-\zeta_p\right)\hat{\Pi}_t^{-\frac{1}{\lambda_p}} + \zeta_p\left(\frac{\Pi_{t-1}^{\iota_p}\Pi_*^{1-\iota_p}}{\Pi_t}\right)^{-\frac{1}{\lambda_p}} \tag{3.35}$$

[25] $w_t \equiv \frac{W_t}{P_t\gamma^t}$, $r_t^k \equiv \frac{R_t^k}{P_t}$, $k_t \equiv \frac{K_t}{\gamma^t}$, $mc_t \equiv \frac{MC_t}{P_t}$

[26] $y_t \equiv \frac{Y_t}{\gamma^t}$, $\hat{\Pi}_t \equiv \hat{P}_t/P_t$

- Law of motion : capital[27]

$$k_t = \frac{(1-\delta)}{\gamma} k_{t-1} + \varepsilon_t^I \left[1 - \frac{\phi}{2} \left(\frac{i_t \gamma}{i_{t-1}} - \gamma \right)^2 \right] i_t \tag{3.36}$$

- Value function and period utility[28]

$$\begin{aligned} u_t &= \varepsilon_t^b \left[\frac{1}{1-\sigma_c} \left(c_t - \frac{\lambda}{\gamma} c_{t-1} \right)^{1-\sigma_c} \right] \exp \left[\frac{\sigma_c - 1}{1+\sigma_l} (h_t)^{1+\sigma_l} \right] && (3.37) \\ v_t &= u_t + \overline{\beta} \gamma E_t \left[(v_{t+1})^{1-\sigma_{EZ}} \right]^{\frac{1}{1-\sigma_{EZ}}} && (3.38) \end{aligned}$$

- First order conditions of households [29]

$$\begin{aligned} \lambda_t &= \left(c_t - \frac{\lambda}{\gamma} c_{t-1} \right)^{-\sigma_c} \exp \left[\frac{\sigma_c - 1}{1+\sigma_l} (h_t)^{1+\sigma_l} \right] && (3.39) \\ \frac{1}{R_t} &= E_t \left[M_{t,t+1} \Pi_{t+1}^{-1} \right] && (3.40) \\ Q_{g,t} &= E_t \left[M_{t,t+1} \frac{y_{t+1}\gamma}{y_t} \right] && (3.41) \\ \overline{w}_t &= \left(c_t - \frac{\lambda}{\gamma} c_{t-1} \right) (h_t)^{\sigma_l} && (3.42) \\ 1 &= q_t \varepsilon_t^I \left[1 - \frac{\phi}{2} \left(\frac{i_t\gamma}{i_{t-1}} - \gamma \right)^2 - \phi \left(\frac{i_t\gamma}{i_{t-1}} - \gamma \right) \frac{i_t\gamma}{i_{t-1}} \right] + && (3.43) \\ & \quad E_t \left[M_{t,t+1} q_{t+1} \varepsilon_{t+1}^I \phi \left(\frac{i_{t+1}\gamma}{i_t} - \gamma \right) \left(\frac{i_{t+1}\gamma}{i_t} \right)^2 \right] \\ q_t &= E_t \left[M_{t,t+1} \left\{ \begin{array}{c} r_t^k z_t - \delta_1 (z_{t+1} - 1) \\ -\frac{\delta_2}{2} (z_{t+1} - 1)^2 + q_{t+1} (1-\delta) \end{array} \right\} \right] && (3.44) \\ r_t^k &= \delta_1 + \delta_2 (z_t - 1) && (3.45) \end{aligned}$$

[27] $i_t \equiv \frac{I_t}{\gamma^t}$

[28] $u_t \equiv \frac{U_t}{\gamma^{t(1-\sigma_c)}}$, $v_t \equiv \frac{V_t}{\gamma^{t(1-\sigma_c)}}$, $c_t \equiv \frac{C_t}{\gamma^t}$, $\overline{\beta} \equiv \beta\gamma^{-\sigma_c}$

[29] $\overline{w}_t \equiv \frac{\overline{W}_t}{P_t\gamma^t}$, $\lambda_t \equiv \Xi_t \gamma^{\sigma_c t}$

- Wage setting[30]

$$f_t^1 = l_t \left(\frac{\hat{w}_t}{w_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} \hat{w}_t + \zeta_w E_t \left[M_{t,t+1}\gamma \left(\frac{\hat{w}_t}{\hat{w}_{t+1}}\right)^{-\frac{1}{\lambda_w}} \left(\frac{\Pi_t^{\iota_w}\Pi_*^{1-\iota_w}}{\Pi_{t+1}}\right)^{-\frac{1}{\lambda_w}} f_{t+1}^1\right] \quad (3.46)$$

$$f_t^2 = \varepsilon_t^w l_t \left(\frac{\hat{w}_t}{w_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} \overline{w}_t + \zeta_w E_t \left[M_{t,t+1}\gamma \left(\frac{\hat{w}_t}{\hat{w}_{t+1}}\right)^{-\frac{1+\lambda_w}{\lambda_w}} \left(\frac{\Pi_t^{\iota_w}\Pi_*^{1-\iota_w}}{\Pi_{t+1}}\right)^{-\frac{1+\lambda_w}{\lambda_w}} f_{t+1}^2\right] \quad (3.47)$$

$$f_t^1 = (1+\lambda_w) f_t^2 \quad (3.48)$$

- Law of motion : wage

$$w_t = \left[(1-\zeta_w)(\hat{w}_t)^{-\frac{1}{\lambda_w}} + \zeta_w \left(\frac{\Pi_{t-1}^{\iota_w}\Pi_*^{1-\iota_w}}{\Pi_t} w_{t-1}\right)^{-\frac{1}{\lambda_w}}\right]^{-\lambda_w} \quad (3.49)$$

- Monetary policy rule[31]

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi_*}\right)^{\psi_\pi} \left(\frac{y_t}{y_*}\right)^{\psi_y}\right]^{1-\rho_R} \left(\frac{y_t}{y_{t-1}}\right)^{\psi_{\Delta y}} \varepsilon_t^r \quad (3.50)$$

- Resource constraint

$$y_t = c_t + i_t + \varepsilon_t^g y_* + \left\{\delta_1 (z_t - 1) + \frac{\delta_2}{2}(z_t-1)^2\right\} \frac{k_{t-1}}{\gamma} \quad (3.51)$$

- Market clearing : final good market[32]

$$y_t = \frac{\varepsilon_t^a \left(\frac{z_t k_{t-1}}{\gamma}\right)^\alpha (l_t)^{1-\alpha} - y_*(\phi_p - 1)}{s_t^p} \quad (3.52)$$

[30] $\hat{w}_t \equiv \frac{\hat{W}_t}{P_t \gamma^t}$

[31] $y_* \equiv \frac{Y_t^*}{\gamma^t}$

[32] $\phi_p \equiv 1 + \frac{\Phi}{y_t}$, 1 plus share of fixed cost in the production.

- Law of motion : price dispersion

$$s_t^p = (1-\zeta_p)\left(\hat{\Pi}_t\right)^{-\frac{1+\lambda_p}{\lambda_p}} + \zeta_p \left(\frac{\Pi_{t-1}^{\iota_p}\Pi_*^{1-\iota_p}}{\Pi_t}\right)^{-\frac{1+\lambda_p}{\lambda_p}} s_{t-1}^p \tag{3.53}$$

- Market clearing : labour market

$$h_t = s_t^w l_t \tag{3.54}$$

- Law of motion : wage dispersions

$$s_t^w = (1-\zeta_w)\left(\frac{\hat{w}_t}{w_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} + \zeta_w \left(\frac{\Pi_{t-1}^{\iota_w}\Pi_*^{1-\iota_w}}{\Pi_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} \left(\frac{w_{t-1}}{w_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} s_{t-1}^w \tag{3.55}$$

- Shock processes

$$\log \varepsilon_t^a = \rho_a \log \varepsilon_t^a + \eta_t^a \tag{3.56}$$

$$\log \varepsilon_t^b = \rho_b \log \varepsilon_t^b + \eta_t^b \tag{3.57}$$

$$\log\left(\frac{\varepsilon_t^g}{\varepsilon_*^g}\right) = \rho_g \log\left(\frac{\varepsilon_{t-1}^g}{\varepsilon_*^g}\right) + \eta_t^g + \rho_{ga}\eta_t^a \tag{3.58}$$

$$\log \varepsilon_t^i = \rho_i \log \varepsilon_t^i + \eta_t^i \tag{3.59}$$

$$\log \varepsilon_t^r = \rho_r \log \varepsilon_t^r + \eta_t^r \tag{3.60}$$

$$\log \varepsilon_t^p = \rho_p \log \varepsilon_t^p + \eta_t^p \tag{3.61}$$

$$\log \varepsilon_t^w = \rho_w \log \varepsilon_t^w + \eta_t^w \tag{3.62}$$

3.B List of steady states

- $z_* = 1$ is assumed, Π_* is estimated from the data, and g_* is exogenously calibrated.

- Then, all the steady state conditions are analytically given:

$\hat{\Pi}_* = 1$, $q_* = 1$, $s_*^p = 1$, $s_*^w = 1$

$r_*^k = \frac{\overline{\beta}^{-1} - \delta\tau_r - (1-\delta)}{1-\tau_r}$

$mc_* = 1/\left(1+\lambda_p\right)$

$w_* = (1-\alpha)\left(mc_*\left(\frac{\alpha}{r_*^k}\right)^{\alpha}\right)^{\frac{1}{1-\alpha}}$

$\overline{w}_* = w_*/\left(1+\lambda_w\right)$

$\hat{w}_* = w_*$

$\frac{k_*}{l_*} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{w_*}{r_*^k}\right)\gamma$

$\frac{i_*}{k_*} = \frac{\gamma - 1 + \delta}{\gamma}$

$R_* = \Pi_*/\overline{\beta}$

$Q_{g,*} = \overline{\beta}\gamma$

$\frac{y_*}{k_*} = \left(\frac{k_*}{l_*}\right)^{\alpha-1}\gamma^{-\alpha}\phi_p^{-1}$

$\frac{c_*}{y_*} = 1 - \frac{i_*}{k_*}/\frac{y_*}{k_*} - \varepsilon_*^g$

$l_* = \left[\left(\frac{1}{1+\lambda_w}\right)\left(\frac{k_*/\gamma}{y_*}\right)r_*^k\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{c_*}{y_*}\right)\left(\frac{\gamma}{\gamma-\lambda}\right)\right]^{\frac{1}{1+\sigma_l}}$

$k_* = l_*\left(\frac{k_*}{l_*}\right)$

$i_* = k_*\left(\frac{i_*}{k_*}\right)$

$y_* = k_*\left(\frac{y_*}{k_*}\right)$

$c_* = y_*\left(\frac{c_*}{y_*}\right)$

$u_* = \left[\frac{1}{1-\sigma_c}\left(c_* - \frac{\lambda}{\gamma}c_*\right)^{1-\sigma_c}\right]\exp\left[\frac{\sigma_c - 1}{1+\sigma_l}h_*^{1+\sigma_l}\right]$

$$v_* = \frac{u_*}{1-\overline{\beta}\gamma}$$

$$\lambda_* = \left(c_* - \frac{\lambda}{\gamma}c_*\right)^{-\sigma_c} \exp\left[\frac{\sigma_c-1}{1+\sigma_l}h_*^{1+\sigma_l}\right]$$

$$f_*^1 = \frac{l_* w_*}{1-\zeta_w\overline{\beta}\gamma},\ f_*^2 = \frac{l_*\overline{w}_*}{1-\zeta_w\overline{\beta}\gamma}$$

$$g_*^1 = \frac{y_*}{1-\zeta_p\overline{\beta}\gamma},\ g_*^2 = \frac{y_* mc_*}{1-\zeta_p\overline{\beta}\gamma}$$

3.C Source of data for the estimation

For the estimation, we used the seven U.S. macroeconomic data following Smets and Wouters (2007); output growth, consumption growth, investment growth, inflation, wage growth, labour supply, and short-term interest rate. The output growth denotes quarterly growth in per capital real GDP. It is calculated by dividing the real GDP (NIPA table 1.1.6) by the civilian non-institutional population over 16 (BLS code: CNP16OV). Similarly, per capita consumption and the investment are also calculated by dividing the real personal consumption expenditure (NIPA table 2.2.3) and the real private fixed investment index (NIPA table 5.3.3) by the population. The inflation denotes quarterly changes in the GDP deflator (NIPA table 1.1.9). The real wage growth denotes quarterly changes in real hourly compensation, which is obtained by dividing the nominal hourly compensation (BLS code: PRS85006103) by the GDP deflator. The index of labour supply is obtained by dividing total hours by the population. The former denotes the average weekly hours worked (BLS code: PRS85006023) multiplied by the civilian employment (BLS code: CE16OV). The percentage deviations of this index from its sample mean is used for the index of labour supply. Lastly, the quarterly averages of the daily 3-month Treasury Bill rates are used as the short-term rates (FRED code: TB3MS).

Chapter 4

Welfare effects of nominal GDP growth-indexed bonds

4.1 Introduction

Since the financial crisis of 2007-2008, a number of countries, including both advanced and emerging, have been suffering from rapidly increasing government debt. As of the end of 2016, the government debt of the U.S. is more than 100% of its GDP, which is much higher than its post-war average of 63%, and that of the U.K has also rapidly increased and now approaching 90%. From this background, the interest in linking government debt cash flows to the growth rate of issuing country's GDP has been gradually growing both in academia and practitioners (see Barr et al. 2014; Bowman et al. 2016; Benford et al. 2016; Blanchard et al. 2016; Cabrillac et al. 2016; Kim and Ostry 2018).

The main advantage of issuing GDP-indexed bonds is that it helps reduce the upper tail risk of debt-to-GDP ratio by narrowing its distribution, and thus lowers the probability of sovereign default (Chamon and Mauro, 2006; Barr et al., 2014). For example, from the following debt-to-GDP dynamics,

$$d_{t+1} - d_t = \frac{(r_t - g_{t+1})}{1 + g_{t+1}} d_t - s_{t+1}, \tag{4.1}$$

we can see that a slow-down in growth, g_{t+1}, leads to a higher level of debt-to-GDP ratio, d_{t+1}, when the other variables - the interest rate, r_t, and primary surplus to GDP ratio, s_t - are unchanged. However, if the government finances its debt with GDP growth-indexed bonds, a slower growth also reduces the burden of interest payment, and thus mitigates the increase in debt-to-GDP ratio compared to the case where the conventional government debt is used.

Another advantage suggested by the literature is that the use of GDP growth-indexed bonds gives more room for conducting a counter-cyclical fiscal policy (Borensztein and Mauro 2004; Barr et al. 2014; Kim and Ostry 2018; Bonfim and Pereira 2018). If a government has very little or no fiscal space[1], the government has to increase the primary surplus to maintain debt-to-GDP ratio even when there is a negative shock on output (i.e., pro-cyclical fiscal policy). A government trapped in such a situation would get a larger room for conducting a counter-cyclical fiscal policy when its debts are fully or partially linked to the country's growth rate. Borensztein and Mauro (2004) showed this by conducting counterfactual simulations using the data from several advanced and emerging countries in 1990s[2], and Bonfim and Pereira (2018) also showed the similar results with recent data from France, Spain and Portugal.

The third chapter of this thesis showed that such an advantage can be found within the framework of a new Keynesian DSGE model as well. Unfortunately,

[1]Ostry et al. (2010) has developed a concept of 'debt limit' which means an upper bound on how high debt-to-GDP ratio of a country can increase before the default risk becomes too high. The fiscal space means the gap between current debt-to-GDP ratio and the debt limit.

[2]They showed that the correlation betwen GDP growth and primary surplus-to-GDP ratio could have been much higher if those countries had linked all their government debts to their GDP growth. Such results held for both advanced and emerging market countries.

however, the analysis of the third chapter was limited to the effect of issuing GDP growth-indexed bonds only in terms of the cyclicality of fiscal policy, but its impacts on welfare and business cycle were not able to be analysed. It is mainly because, in the third chapter, we assumed a rational, forward-looking representative household who is able to smooth consumption intertemporally by trading in both financial and capital markets. Under such assumptions, the consumption of the representative household is a function of permanent income rather than current disposable income, and thus the structure of government finance (the choice between the two bonds) only affects the mix between outstanding debt and fiscal balance, and a particular mix is irrelevant to the household's decision on consumption and the business cycle. Generally speaking, since Ricardian equivalence holds in the standard DSGE model used in the third chapter, the model was not suitable for analysing the effect of the counter-cyclicality of fiscal policy on business cycle.

However, there are plenty of empirical evidence which shows that consumption relies more strongly on current disposable income than the standard DSGE model suggests (Campbell and Mankiw, 1989; Mankiw, 2000). Based on such empirical evidence, Mankiw (2000) suggested a new model where some households follow the permanent income hypothesis and the rest of them are so-called rule-of-thumb households[3]. Galí et al. (2007) is the first paper that incorporated Mankiw's idea of rule-of-thumb households into the New Keynesian DSGE model with sticky-price in order to analyse the effect of government spending on consumption. Following the seminal paper of Galí et al. (2007), the idea of rule-of-thumb household has been widely used in the fiscal policy literature. Coenen and Straub (2004) extended one of the most famous medium

[3]His justification for the presence of such households can be either they are irrational, myopic, or have limited access to the financial or capital market. In this chapter, we assumed the presence of "hand-to-mouth households" who are rational, forward-looking, but has no access to those markets.

scale New Keynesian DSGE model of Smets and Wouters (2003) by incorporating rule-of-thumb households, various distortionary and lump-sum taxes, and a fiscal policy rule that stabilises debt-to-GDP process. They estimated the share of rule-of-thumb households in the Euro area with the Bayesian estimation methodology. Their estimates for the share of rule of thumb households range from 24% to 37% depending on their assumptions on the fiscal policy rule. Similarly, Cogan et al. (2010) used extended version of Smets and Wouters (2007) model augmented with rule-of-thumb households to analyse the role of fiscal policy with the zero-lower-bound in nominal interest rate. They also estimated the model with the Bayesian methodology with the U.S. data, and their estimate of rule-of-thumb household share was around 29%. More recently, Drautzburg and Uhlig (2015) also relied on similar model to examine how and whether the presence of zero-lower-bound affects the sign and size of government spending multiplier.

These models relied on the idea of rule-of-thumb consumers mostly in order to examine the role of government spending on consumption. In this chapter, we also adopted their assumption on the presence of rule-of-thumb households. However, the focus of this chapter is different from the previous papers in that we are intended to examine whether and how the type of government bonds (conventional nominal bonds and GDP growth-indexed bonds) affects the business cycle and the welfare of the economy. The model in this chapter is also based on Smets and Wouters (2003, 2007), augmented with hand-to-mouth households, lump-sum and distortionary taxes and Epstein and Zin type recursive preference.

We show that, under certain conditions, the use of GDP growth-indexed bond may help stabilise the business cycle and improve the welfare of hand-to-mouth households. In our model, the hand-to-mouth households are assumed to be rational and forward-looking, and they have desires for consumption smoothing, but they do not have access to either financial or capital market. That is to say, they cannot save, borrow, and invest in capital. As mentioned above, when there exist only Ricardian households, even if the government's choice on the type of bonds can affect the fiscal balance, it has no impact on business cycle and welfare. However, when there exist non-Ricardian or hand-to-mouth households, their consumption is directly affected by the changes in primary surplus.

Furthermore, in our model, the consumption and leisure choices of the two types of households are interconnected via labour market. More specifically, the increase in current disposable income of hand-to-mouth households leads to an increase in aggregate demand, and at the same time, to a decrease in labour supply from the hand-to-mouth households (i.e., intratemporal consumption smoothing). Therefore, the increased demand in aggregate labour should be met by an increase in labour supply from the Ricardian households. This is a more realistic assumption than the previous papers where the two types of households supply identical amount labour and the hand-to-mouth households can smooth their consumption neither intertemporally nor intratemporally. Through this channel, the changes in fiscal balance affects not only the consumption/leisure choices of the hand-to-mouth households, but also those of the Rational households.

The remainder of this chapter is organised as follows. Section 4.2 briefly outlines the model focusing on the differences from existing models, and the model parameters are discussed in section 4.3. The results from the baseline model, and the mechanism behind the results are in Section 4.4 with the sensitivity analysis with different key parameter values. Section 4.5 concludes.

4.2 The DSGE model

To analyse the effect of the use of GDP growth-indexed bonds on the business cycle and welfare, we built our DSGE model based on the medium scale New Keynesian DSGE model of Smets and Wouters (2003, 2007). We kept most of the key features of the Smets and Wouters (2003, 2007) models, which include two nominal frictions: sticky prices and wages; four real rigidities: external consumption habit, investment adjustment cost, variable capital utilisation, monopolistically competitive goods and labour markets; and seven exogenous shocks on productivity, preference, government spending, investment, monetary policy, price markup and wage markup shocks. On top of them, we further assumed that the households in our model have a recursive preference following Rudebusch and Swanson (2012) to better reflect the difference in the government's borrowing cost between the two type of bonds. We also assumed the presence of hand-to-mouth households following Galí et al. (2007) so that the Ricardian equivalence does not hold anymore. Lastly, we assumed that the government finances exogenous government spending and lump-sum government transfer via debts and distortionary taxes on labour and capital income.

4.2.1 Households

There exist a continuum of households with a unit mass indexed by $j \in [0, 1]$ grouped into two types - Ricardian and hand-to-mouth households - in this model economy. A fraction $1 - \omega$ of the households are Ricardian household who are rational, forward-looking and able to access to both financial and capital markets. The rest of the households, a fraction of ω, are hand-to-mouth households. They are also rational and forward-looking, but they have no vehicle to save or borrow as they cannot access those markets. That is to say, the hand-to-mouth households in this model have a desire to smooth their consumption, but their ability to do this is severely restricted as they cannot do it intertemporally. They can smooth their consumption only through changes in their labour supply. Therefore, in each period, they consume all their disposable income (= after-tax labour income plus government transfer). Such assumption is a bit different from the previous papers in the fiscal policy literature (Galí et al., 2007; Coenen and Straub, 2004; Drautzburg and Uhlig, 2015). In those papers, non-Ricardian households are assumed to simply take wages and working hours determined by the labour union, and they optimise neither intertemporally nor intratemporally. This also means that the labour supplies of the two household groups are identical at all times even though the consumption level of the two groups can be different.[4] On the contrary, in our model, we make a bit more realistic assumption. That is, the hand-to-mouth households optimise at least intratemporally, and thus the labour supplies of the two households need not be the same.

An individual household j in this model is assumed to have the following non-separable period utility function[5]:

$$U_{t,j}^{X} = \varepsilon_t^b \left[\frac{1}{1-\sigma_c} \left(C_{t,j}^{X} - \lambda C_{t-1}^{X} \right)^{1-\sigma_c} \right] \exp \left[\frac{\sigma_c - 1}{1+\sigma_l} \left(h_{t,j}^{X} \right)^{1+\sigma_l} \right] \tag{4.2}$$

where

$$X = \begin{cases} H & \text{if } j \in [0, \omega] \\ R & \text{if } j \in [\omega, 1] \, . \end{cases}$$

[4]In some papers, it is simply assumed with no justification (Coenen and Straub, 2004; Drautzburg and Uhlig, 2015), and in other papers, this is guaranteed by the assumption that the wage markup is large enough such that the wage is always higher than the MRS of both households (Galí et al., 2007).

[5]The functional form of the period utility is same as one in Smets and Wouters (2007).

The superscript H and R denote hand-to-mouth and Ricardian households, respectively. The household obtains utility from the difference between individual current consumption, $C^X_{t,j}$, and the group-wise aggregate consumption in the previous period, C^X_{t-1}. That is to say, there exists an external habit in consumption and each household in this model tries to keep up with the other households only in the same group. The household also obtains disutility from supplying homogeneous labour, $h^X_{t,j}$, to the union. As the two types of households are identical except for their ability to access financial and capital markets, the quality of their labours are homogeneous regardless of the household type, and thus same hourly wage rate, $\overline{W}_t$, are applied. Following Smets and Wouters (2003), we assume the preference shock ε^b_t that affects the intertemporal substitution of households follows a simple AR(1) process.

Following Rudebusch and Swanson (2012)[6], we assume that an individual household in this model maximises its welfare $V^X_{t,j}$ recursively given as below:

$$V^X_{t,j} = U^X_{t,j} + \beta^X E_t \left[\left(V^X_{t+1,j} \right)^{1-\sigma_{EZ}} \right]^{\frac{1}{1-\sigma_{EZ}}}, \tag{4.3}$$

where β^X is a discount factor for household type X, but we assume that the two household groups share the same discount factor, $\beta = \beta^R = \beta^H$. This assumption is also consistent with our key assumption that the two types of households are heterogeneous only in terms of their ability to save or borrow, not in terms of their preference.

[6]Rudebusch and Swanson (2012) rewrote the recursive preference suggested by Epstein and Zin (1991) as in Equation (4.3) for notational clarification.

(Ricardian households) A Ricardian household j faces the following intertemporal budget constraint:

$$\begin{aligned} & C^R_{t,j} + I_{t,j} + \frac{B_{t,j}}{R_t P_t} + \frac{Q^G_t B^G_{t,j}}{P_t} \\ \leq \quad & \frac{B_{t-1,j}}{P_t} + \frac{B^G_{t-1,j}}{P_t}\left(\frac{Y_t P_t}{Y_{t-1}P_{t-1}}\right) + (1-\tau_w)\frac{\overline{W}_t h^R_{t,j}}{P_t} + (1-\tau_r)\frac{R^k_t}{P_t} z_{t,j} K_{t-1,j} \\ & + \tau_r \delta K_{t-1,j} - a(z_{t,j}) K_{t-1,j} + D^f_{t,j} + (1-\tau_w) D^u_{t,j} + T_{t,j}. \end{aligned} \tag{4.4}$$

In the left-hand side, the household j consumes, invests, and saves by purchasing bonds. As only Ricardian households can invest or save, we abstract superscripts R from real investment $I_{t,j}$, capital $K_{t,j}$, the two bonds $B_{t,j}$ and $B^G_{t,j}$, and related variables such as their prices. $B_{t,j}$ is the units of 1-period nominal conventional government bond purchased at t at the unit price of $1/R_t$, and $B^G_{t,j}$ is the units of nominal GDP growth-indexed bonds (NGDP-indexed bond) purchased at the price of Q^G_t. On the right-hand side, the household finances its expenditure from the repayment of the bonds purchased from the previous period, after-tax labour and capital rental incomes, profits from intermediate firms and labour unions, and lump-sum transfer from the government. The bonds purchased in the previous period pays $\frac{B_{t-1,j}}{P_t}$ or $\frac{B^G_{t-1,j}}{P_t}\left(\frac{Y_t P_t}{Y_{t-1}P_{t-1}}\right)$ back at t in terms of the final goods[7]. The distortionary taxes are levied on labour and capital rental income, and the same constant tax rates, τ_w and τ_r, are applied to both household groups. $D^f_{t,j}$ is the real profit from the intermediate firms, and it is evenly distributed among the Ricardian households because we assume that the firms are owned by only the Ricardian households. $D^u_{t,j}$ denotes real profit from the unions, and it is evenly distributed to all the households regardless of the household type. Note that the profit from the labour union

[7] Note that a unit of NGDP-indexed bond purchased at $t-1$ pays $\left(\frac{Y_t P_t}{Y_{t-1}P_{t-1}}\right)$ units of money when it matures.

is also taxed at the rate of τ_w; and tax allowance is assumed to apply to costs due to depreciation of capital, $\tau_r \delta K_{t-1,j}$. T_t is lump-sum government transfer in terms of final goods, which is also evenly distributed to all the households regardless of the household types. $z_{t,j}$ is the level of capital utilisation, and $a(z_{t,j})$ is the quadratic cost of capital utilisation given as below:

$$a(z_t) = \delta_1 (z_t - 1) + \frac{\delta_2}{2} (z_t - 1)^2 . \tag{4.5}$$

Lastly, the Ricardian household accumulates its capital following the law of motion based on Christiano et al. (2005) as below :

$$K_{t,j} = (1 - \delta) K_{t-1,j} + \varepsilon_t^I \left[1 - \frac{\phi}{2} \left(\frac{I_{t,j}}{I_{t-1,j}} - \gamma \right)^2 \right] I_{t,j}, \tag{4.6}$$

where γ is trend productivity growth, and the investment shock, ε_t^I, follows a simple AR(1) process.

(Hand-to-mouth households) The hand-to-mouth households can neither trade bonds nor accumulate capital, and do not have the ownership of intermediate firms. Thus, the sources of their income are after-tax wages, profits from the unions, and the lump-sum transfer from the government only. This gives the following simple budget constraint of hand-to-mouth households:

$$C_{t,j}^H \leq (1 - \tau_w) \left(\frac{\overline{W}_t h_{t,j}^H}{P_t} + D_{t,j}^u \right) + T_{t,j}. \tag{4.7}$$

(First order conditions: Ricardian households) The Ricardian households maximise Equation (4.3) by choosing $C_{t,j}^R$, $B_{t,j}$, $B_{t,j}^G$, $h_{t,j}^R$, $I_{t,j}$, $K_{t,j}$ and $z_{t,j}$ subject to Equation (4.4) to (4.6). This gives the following seven first order conditions[8]:

$$\Xi_t^R = \varepsilon_t^b \left(C_t^R - \lambda C_{t-1}^R\right)^{-\sigma_c} \exp\left[\frac{\sigma_c - 1}{1+\sigma_l}\left(h_t^R\right)^{1+\sigma_l}\right] \tag{4.8}$$

$$\frac{1}{R_t} = E_t\left[M_{t,t+1}^R \Pi_{t+1}^{-1}\right] \tag{4.9}$$

$$Q_t^G = E_t\left[M_{t,t+1}^R\left(\frac{Y_{t+1}}{Y_t}\right)\right] \tag{4.10}$$

$$(1-\tau_w)\frac{\overline{W}_t}{P_t} = \left(C_t^R - \lambda C_{t-1}^R\right)\left(h_t^R\right)^{\sigma_l} \tag{4.11}$$

$$\begin{aligned} 1 = {} & q_t \varepsilon_t^I \left[1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - \gamma\right)^2 - \phi\left(\frac{I_t}{I_{t-1}} - \gamma\right)\frac{I_t}{I_{t-1}}\right] + \\ & E_t\left[M_{t,t+1}^R q_{t+1}\varepsilon_{t+1}^I \phi\left(\frac{I_{t+1}}{I_t} - \gamma\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \end{aligned} \tag{4.12}$$

$$q_t = E_t\left[M_{t,t+1}^R\left\{\begin{array}{c}(1-\tau_r)\frac{R_t^k}{P_t} z_t + \delta\tau_r \\ -a(z_{t+1}) + q_{t+1}(1-\delta)\end{array}\right\}\right] \tag{4.13}$$

$$(1-\tau_r)\frac{R_t^k}{P_t} = \delta_1 + \delta_2(z_t - 1) \tag{4.14}$$

where $\Pi_{t+1} \equiv P_{t+1}/P_t$, and $M_{t,t+1}^R$ is the real stochastic discount factor for the Ricardian households defined as

$$M_{t,t+1}^R \equiv \beta\left(\frac{V_{t+1}^R}{E_t\left[\left(V_{t+1}^R\right)^{1-\sigma_{EZ}}\right]^{\frac{1}{1-\sigma_{EZ}}}}\right)^{-\sigma_{EZ}} \frac{\Xi_{t+1}^R}{\Xi_t^R}, \tag{4.15}$$

$q_t \equiv \frac{\Xi_t^k}{\Xi_t^R}$ is so-called Tobin's q, and Ξ_t^k is the Lagrangian multiplier for the law of motion of capital.

(First order conditions: hand-to-mouth households) The hand-to-mouth households also maximises Equation (4.3), but by choosing only consumption $C_{t,j}^H$ and labour supply $h_{t,j}^H$ subject to Equation (4.7). This gives the two first

[8] The subscript index j is dropped as the household decisions within the group are symmetric.

order conditions:

$$\Xi_t^H = \varepsilon_t^b \left(C_t^H - \lambda C_{t-1}^H\right)^{-\sigma_c} \exp\left[\frac{\sigma_c - 1}{1+\sigma_l}\left(h_t^H\right)^{1+\sigma_l}\right] \tag{4.16}$$

$$(1-\tau_w)\frac{\overline{W}_t}{P_t} = \left(C_t^H - \lambda C_{t-1}^H\right)\left(h_t^H\right)^{\sigma_l} \tag{4.17}$$

4.2.2 Producers

The production sector in this model is very similar to that of Smets and Wouters (2003) except for some simplications. In order to recursively express the non-linear equilibrium conditions for the price setting (and wage setting as well), we made a modification on the price markup shock[9]. The perfectly competitive final good producer aggregates intermediate goods using the standard Dixit-Stiglitz aggregator:

$$Y_t = \left(\int_0^1 Y_{t,i}{}^{\frac{1}{1+\lambda_P}}\, di\right)^{1+\lambda_P}, \tag{4.18}$$

and the optimisation problem of the final good producer gives the following demand schedule for the ith intermediate good:

$$Y_{t,i} = \left(\frac{P_{t,i}}{P_t}\right)^{-\left(\frac{1+\lambda_p}{\lambda_p}\right)} Y_t, \tag{4.19}$$

where $P_{t,i}$ denotes the price of the ith intermediate good.

There exists a continuum of firms indexed by $i \in [0,1]$ operating under monopolistic competition, and an individual firm i produces its intermediate good using the technology below:

$$Y_{t,i} = \varepsilon_t^a \left(K_{t,i}^S\right)^{\alpha}\left(\gamma^t l_{t,i}\right)^{1-\alpha} - \gamma^t \Phi, \tag{4.20}$$

[9]Smets and Wouters (2003) assumed that the substitutability parameter, $\lambda_{p,t}$, is time-varying in order to incorporate price markup shocks into the model. Instead, we assumed the parameter to be constant, but added a wedge type markup shock, ε_t^p, to the price setting problem of intermediate good producers (see Equation 4.21). In both cases, the steady state level of price markup is given by λ_p.

where $K^S_{t,i} \equiv z_t K_{t-1,i}$ is the capital service rented from the Ricardian households, $l_{t,i}$ is the labour index supplied by the labour packer, and Φ is the fixed costs in production. The intermediate firm i maximises the sum of stochastically discounted future profits by choosing optimal price of ith good, $\hat{P}_{t,i}$. Following Calvo (1983) pricing scheme, we assume only $1 - \zeta_p$ of them are allowed to re-optimise their prices and the rest of the firms just partially index their prices by past inflation. That is to say, each individual intermediate good producer solves the following problem when given a chance of re-optimising:

$$\max_{\hat{P}_{t,i}} E_t \sum_{s=0}^{\infty} \zeta_p^s M^R_{t,t+s} \left(\frac{1}{P_{t+s}} \right) \left[X^p_{t,s} \hat{P}_{t,i} - \varepsilon^p_{t+s} MC_{t+s} \right] Y_{t+s,i} \tag{4.21}$$

subject to the demand schedule given in Equation (4.19). Note again that the intermediate good firms are owned only by the Ricardian households, thus all the profits are given only to them. Therefore, the future profits from the firms are discounted using the stochastic discount factor of the Ricardian households. MC_{t+s} denotes the nominal marginal cost for intermediate good production, and $X^p_{t,s}$ denotes the indexation factor defined as below:

$$X^p_{t,s} \equiv \begin{cases} 1 & \text{if } s = 0 \\ \prod_{l=1}^{s} \left(\Pi^{\iota_p}_{t+l-1} \Pi^{1-\iota_p}_{*} \right) & \text{if } s \geq 1, \end{cases}$$

where Π_* is the steady state level of gross inflation. Note also that there exists a price markup shock, ε^p_t, that follows an AR(1) process.

4.2.3 Labour market

The assumptions on the labour market structure are not much different from the standard New Keynesian DSGE model with sticky wages. We assumed that there exists a continuum of monopolistically competitive labour unions indexed by $z \in [0,1]$. Each union differentiates the homogeneous labours purchased from the households at the wage of $\overline{W}_t$, and provides the differentiated labour, $l_{t,z}$, to the labour packer at the wage of $W_{t,z}$. The household types and the labour types are independent each other[10], and the union cannot tell the household type. That is why all the households gets the same hourly wages and there is no superscript R or H on the differentiated labour. The labour packer aggregates the differentiated labour into the aggregate labour index, l_t, using the following Dixit-Stiglitz aggregator:

$$l_t = \left(\int_0^1 l_{t,z}^{\frac{1}{1+\lambda_w}} dz \right)^{1+\lambda_w}. \tag{4.22}$$

Analogous to the final good producer, the optimisation problem of labour packer gives the following demand schedule for z-type of labour:

$$l_{t,z} = \left(\frac{W_{t,z}}{W_t} \right)^{-\frac{1+\lambda_w}{\lambda_w}} l_t. \tag{4.23}$$

A union for type-z labour solves the following optimisation problem to maximise the stochastically discounted future profits by choosing optimal wage, $\hat{W}_{t,z}$:

$$\max_{\hat{W}_{t,z}} E_t \sum_{s=0}^{\infty} \zeta_w^s M_{t,t+s}^R \left(\frac{1}{P_{t+s}} \right) \left[\gamma^s X_{t,s}^w \hat{W}_{t,z} - \varepsilon_{t+s}^w \overline{W}_{t+s} \right] l_{t+s,z}, \tag{4.24}$$

subject to the demand schedule of Equation (4.23), where $X_{t,s}^w$ is the indexation factor defined as:

$$X_{t,s}^w \equiv \begin{cases} 1 & \text{if } s = 0 \\ \prod_{l=1}^{s} \left(\Pi_{t+l-1}^{\iota_w} \Pi_*^{1-\iota_w} \right) & \text{if } s \geq 1. \end{cases}$$

[10]In other words, the fraction of hand-to-mouth households and Ricardian households is uniformly distributed across unions.

As mentioned earlier, the union cannot tell from which group the individual household comes. However, the Ricardian households account for the majority of the population, we assume that the unions discount future profits using the stochastic discount factor of the Ricardian households.[11] There exists a wage markup shock, ε_t^w, that follows AR(1) process as well.

4.2.4 Monetary and fiscal policy

The central bank is assumed to set its policy rate following the monetary policy rule below:

$$\frac{R_t}{R_*} = \left(\frac{R_{t-1}}{R_*}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi_*}\right)^{\psi_1} \left(\frac{Y_t}{Y_t^*}\right)^{\psi_2}\right]^{1-\rho_R} \left(\frac{Y_t/Y_{t-1}}{\gamma}\right)^{\psi_3} \varepsilon_t^r, \tag{4.25}$$

where R_* and y_* are the steady state levels of nominal short-term interest rate and detrended output, respectively, and $Y_t^* \equiv y_* \gamma^t$ is trend level of output. This monetary policy rule is same as that in Smets and Wouters (2007) except that the output gap in this model is defined as the deviation from trend output rather than a deviation from the flexible-price-economy output. There exists the monetary policy shock, ε_t^r, that follows an AR(1) process.

The government faces the following intertemporal budget constraint:

$$G_t + T_t + \frac{B_{t-1}}{P_t} + \left(\frac{Y_t P_t}{Y_{t-1} P_{t-1}}\right) \frac{B_{t-1}^G}{P_t} = \frac{B_t}{R_t P_t} + \frac{Q_t^G B_t^G}{P_t} + \tau_w \frac{W_t l_t}{P_t} + \tau_r \frac{z_t K_{t-1} R_t^k}{P_t} - \tau_r \delta K_{t-1}, \tag{4.26}$$

where $G_t \equiv \varepsilon_t^g Y_t^*$ is the level of real government spending. In other words, the government consumes G_t units of final good in each period. Following Smets and Wouters (2007), we assume that the government spending is also affected

[11]This assumption is following Drautzburg and Uhlig (2015) who justified their assumption with a median-voter decision rule.

by the productivity shock as follows:

$$\log\left(\frac{\varepsilon_t^g}{\varepsilon_*^g}\right) = \rho_g \log\left(\frac{\varepsilon_{t-1}^g}{\varepsilon_*^g}\right) + \eta_t^g + \rho_{ga}\eta_t^a, \tag{4.27}$$

where ε_*^g denotes the steady state government spending over output ratio. Note also that $W_t l_t$ is the tax base for labour income, which equals the sum of the wages paid to the households, $\overline{W}_t\left(\omega h_t^H + (1-\omega)\, h_t^R\right)$, and the unions' nominal profits, $P_t D_t^u = W_t l_t - \overline{W}_t\left(\omega h_t^H + (1-\omega)\, h_t^R\right)$.[12]

We further assume that the fiscal authority is constrained to keep its debt-to-GDP ratio at a constant level, $\overline{D}$. Under this assumption, the model government has no autonomy in fiscal policy since the distortionary tax rates are constant, government spending is exogenous, the debt-to-GDP ratio is constant, and these three determine the size of lump-sum government transfer. This assumption seems a bit extreme. However, as the goal of this paper is to examine how and whether the government can rely on NGDP-indexed bonds as an alternative fiscal policy tool for stabilising business cycle (or improving welfare) when all the other fiscal policy tools are lost, such an extreme assumption can help us to see the effect more clearly. Furthermore, the experiences after the financial crisis of 2007 may support this assumption as well. After the crisis, the debt-to-GDP ratios in many advanced countries have approached closely to their debt limits[13] (Ostry et al., 2010), and this forced many countries to use austerity measures even when they were in recession. From the assumption of constant debt-to-GDP ratio, the value of newly issued debt at t should always be equal to

[12]The labour packer receives $W_t l_t$ from the intermediate good producers by supplying labour, and as the labour packer earns zero profit in the perfectly competitive labour market, the total revenue of the labour unions from the labour packer should be $W_t l_t$ as well, or $W_t l_t = \int_0^1 w_{t,z} l_{t,z} dz$.

[13]The empirical analysis by Ostry et al. (2010) shows that many advance countries have already or almost reached their debt limits, which are defined as the theoretical threshold level of debt-to-GDP where a government with debt-to-GDP ratio higher than this level is excluded from the bond market.

$\overline{D}Y_t$ such that

$$\frac{B_t}{R_t P_t} = \left(1-\omega^G\right)\overline{D}Y_t \tag{4.28}$$

$$\frac{Q_t^G B_t^G}{P_t} = \omega^G \overline{D}Y_t, \tag{4.29}$$

where $\omega^G = 0$ or 1 is the share of NGDP-indexed bonds.

4.2.5 Aggregation and Equilibrium

Aggregating the individual households' budget constraints, Equation (4.4) and (4.7), within each group gives the two group-wise aggregate budget constraints below:

$$\begin{aligned}(1-\omega)\, C_t^R + I_t + \frac{B_t}{R_t P_t} + \frac{Q_t^G B_t^G}{P_t} &= \frac{B_{t-1}}{P_t} + \frac{B_{t-1}^G}{P_t}\left(\frac{Y_t P_t}{Y_{t-1}P_{t-1}}\right) \\ &\quad + (1-\tau_r)\,\frac{R_t^k K_t^s}{P_t} + \tau_r \delta K_{t-1} + (1-\tau_w)\,\frac{\overline{W}_t\,(1-\omega)\, h_t^R}{P_t} \\ &\quad + D_t^f + (1-\tau_w)\,(1-\omega)\, D_t^u - a\,(z_t)\, K_{t-1} + (1-\omega)\, T_t\end{aligned} \tag{4.30}$$

$$\omega C_t^H = (1-\tau_w)\left\{\frac{\overline{W}_t \omega h_t^H}{P_t} + \omega D_t^u\right\} + \omega T_t, \tag{4.31}$$

where D_t^f and D_t^u are aggregate real profits from the intermediate firms and labour unions, respectively:

$$\begin{aligned}D_t^f &= \frac{1}{P_t}\left(P_t Y_t - W_t l_t - R_t^k K_t^s\right) \\ D_t^u &= \frac{W_t l_t}{P_t} - \frac{\overline{W}_t}{P_t}\left(\omega h_t^H + (1-\omega)\, h_t^R\right).\end{aligned}$$

Combining the two group-wise budget constraints, Equation (4.30) and (4.31), and the government's budget constraint, Equation (4.26), gives the following aggregate resource constraint:

$$Y_t = C_t + I_t + G_t + a\left(z_t\right) K_{t-1}. \tag{4.32}$$

Aggregating the demand schedules for the intermediate goods, Equation (4.19), gives the following goods market clearing condition:

$$Y_t = \frac{\varepsilon_t^a \left(z_t K_{t-1}\right)^{\alpha} \left(\gamma^t l_t\right)^{1-\alpha} - \gamma^t \Phi}{s_t^p}, \tag{4.33}$$

where s_t^p is the price dispersion with the following law of motion:

$$s_t^p = \left(1-\zeta_p\right)\left(\hat{\Pi}_t\right)^{-\frac{1+\lambda_p}{\lambda_p}} + \zeta_p \left(\frac{\Pi_{t-1}^{\iota_p} \Pi_*^{1-\iota_p}}{\Pi_t}\right)^{-\frac{1+\lambda_p}{\lambda_p}} s_{t-1}^p, \tag{4.34}$$

and similarly, the labour market clearing condition is given as follows:

$$l_t s_t^w = \omega h_t^H + \left(1-\omega\right) h_t^R, \tag{4.35}$$

where s_t^w is the wage dispersion with the following law of motion:

$$s_t^w \quad = \quad \left(1-\zeta_w\right)\left(\frac{\hat{W}_t}{W_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} + \zeta_w \left(\frac{\Pi_{t-1}^{\iota_w} \Pi_*^{1-\iota_w}}{\Pi_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} \left(\frac{W_{t-1}\gamma}{W_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} s_{t-1}^w. \tag{4.36}$$

The full list of equilibrium conditions are attached in the Appendix 4.A.

4.3 Parameters

To calibrate the parameters for our baseline model we assume that the government follows a flexible-debt-rule,

$$\log\left(\frac{t_t}{t_*}\right) = \alpha_1 \log\left(\frac{d_t/y_t}{\overline{D}}\right), \tag{4.37}$$

where t_t is government transfer and d_t denotes the real detrended value of new debt issuance at t:

$$d_t \equiv \left(1-\omega^G\right)\frac{b_t}{R_t} + \omega^G Q_t b_t^G,$$

instead of the fixed debt-to-GDP rule assumed in the previous section. This is simply because that the U.S. government had not been constrained by the fixed debt-to-GDP rule while the U.S. data we try to match was being produced. This flexible-debt-rule implies that the government tries to keep the debt-to-GDP ratio near its steady state, $\overline{D}$, by adjusting its government transfer. In Equation (4.37), $\alpha_1 < 0$ controls the volatility of debt-to-GDP ratio. The larger the absolute size of α_1, the more strongly the government tries to keep the debt-to-GDP ratio near its steady state. For example, when α_1 becomes an extremely large negative number, the model becomes similar to the baseline model with constant debt-to-GDP ratio. In this section, we set $\alpha_1 = -10$ in order to allow the debt-to-GDP ratio flexibly fluctuates[14].

Most of the parameters in our model are standard in the literature (see the list of parameters in Table 4.1 and 4.2). We set the curvature of period utility function with respect to relative consumption for constant labour, $\sigma_c = 2.0$. It is in the range of parameter values from most of the New Keynesian literature even though it is a bit larger than the estimates of Smets and Wouters

[14]This implies that 2% deviation of debt-to-GDP ratio leads to -20% deviation of government transfer. With $\alpha_1 = -10$, the highest level of simulated debt-to-GDP was around 30% higher than the steady state level in our simulation.

(2003, 2007) of around 1.4. The inverse of the elasticity of labour, $\sigma_l = 1.9$ is borrowed from Smets and Wouters (2007). We set the degree of external habit, λ, at 0.7. The fraction of firms and unions which are not given the chance of re-optimising, ζ_p and ζ_w, are set to be 0.78 and 0.75 respectively, which imply an average period of around four quarters between re-optimising, and the indexation parameter for the price and wage, ι_p and ι_w, are set to be 0.1 and 0.5, respectively. The steady state level of both price and wage markups are assumed to be 0.1. The monetary policy rule coefficients are also borrowed from Smets and Wouters (2007). $\alpha = 1/3$ implies a steady state share of labour income of 66%, and depreciation rate $\delta = 0.025$ means an annual depreciation of 10%. The discount factor, $\overline{\beta} \equiv \beta\gamma^{-\sigma_c} = 0.9905$ implies around 4% annual real interest rate in steady state. We assume the trend annual productivity growth rate slightly lower than 1%, $\gamma = 1.002$, and steady state gross inflation rate, $\pi_* = 1.008$, or around 3.2% annually. $\phi_p = 1.0$ implies that there is no fixed cost in the production of intermediate goods. The parameters for the investment adjustment cost ϕ and the elasticity of the capital utilisation cost ψ are assumed to be 5.5 and 0.5, respectively. All the parameters above are standard among New Keynesian literature (see Levin et al. 2006; Christiano et al. 2005; Smets and Wouters 2007).

The Epstein-Zin parameter σ_{EZ} is set to be -360 to match the term premium of 100 basis points on 10-year zero-coupon U.S government bonds. This is much larger (in absolute term) than the parameter value of -148 used in Rudebusch and Swanson (2012), but much smaller than that of Darracq Paries and Loublier (2010). There seems to be no consensus as to the size of this parameter. Darracq Paries and Loublier (2010) showed that the absolute size of Epstein-Zin parameter should be around 1,000 to generate term premium of 100 basis points if one uses the exactly same model as Smets and Wouters (2007). In

Table 4.1: **List of parameters**

Parameter	**Value**	**Description**
σ_c	2.0	IES in consumption
σ_l	1.9	labour supply elasticity
σ_{EZ}	-360	Epstein ann Zin parameter
λ	0.7	degree of consumption habit
ζ_p	0.78	price stickiness
ζ_w	0.75	wage stickiness
ι_p	0.1	price indexation
ι_w	0.5	wage indexation
λ_p	0.1	steady state price markup
λ_w	0.1	steady state wage markup
ρ_R	0.8	policy rate smoothing
ψ_1	2.0	inflation gap coefficient
ψ_2	0.1	output gap coefficient
ψ_3	0.2	output growth coefficient
α	1/3	share of capital
δ	0.025	depreciation
$\overline{\beta}$	0.9905	discount factor
γ	1.002	trend growth in productivity
π_*	1.008	steady state inflation
ϕ_p	1.0	parameter for fixed cost
ψ	0.5	utilisation adjustment cost
ϕ	5.5	investment adjustment cost
ε_*^g	0.17	share of government spending
$\overline{D}$	2.52	steady state debt to GDP ratio
ω	0.1	share of hand-to-mouth households
τ_r	0.36	labour income tax rate
τ_w	0.28	capital rental income tax rate

note: $\overline{\beta} \equiv \beta\gamma^{-\sigma_c}$, $\phi_p \equiv 1 + \Phi/y_t$ is 1 plus share of fixed cost in the production, and $\psi \equiv \delta_2/\delta_1$ is elasticity of the capital utilisation cost function.

Table 4.2: **List of shocks**

Parameter	**Value**	**Description**
ρ_a	0.95	AR(1) coefficient of productivity shock
ρ_b	0.20	AR(1) coefficient of preference shock
ρ_g	0.90	AR(1) coefficient of government spending shock
ρ_i	0.60	AR(1) coefficient of investment shock
ρ_r	0.20	AR(1) coefficient of monetary policy shock
ρ_p	0.80	AR(1) coefficient of price markup shock
ρ_w	0.89	AR(1) coefficient of wage markup shock
ρ_{ga}	0.52	correlation between a and g shocks
σ_a	0.45	standard deviation of productivity shock
σ_b	0.24	standard deviation of preference shock
σ_g	0.30	standard deviation of government spending shock
σ_i	0.45	standard deviation of investment shock
σ_r	0.24	standard deviation of monetary policy shock
σ_p	2.40	standard deviation of price markup shock
σ_w	2.40	standard deviation of wage markup shock

general, to match a given size of term premium, the smaller σ_c, the larger σ_{EZ} is required. Rudebusch and Swanson (2012) was able to match the term premium of 100 basis points with a relatively small σ_{EZ} with the help of a very large σ_c (≈ 9)[15]. On the contrary, σ_c in Smets and Wouters (2007) is estimated to be only 1.39, and this is why an extremely larger σ_{EZ} is required. In this paper, we set σ_c to be 2, which is larger than that of Smets and Wouters (2007) but still in the range of the models in the literature.

The steady state ratio of government spending over output, $\varepsilon_*^g = 0.17$, is from Trabandt and Uhlig (2011) who calibrated the value using the historical U.S. data. We set the baseline value for the fixed ratio of debt-to-GDP, $\overline{D} = 2.56$, or the level of debt being 63% of annual GDP, using the post-war average U.S. data.

[15] In fact, in their baseline model where $\sigma_c = 2$, the term premium is only a third of the best fit model where σ_c is around 9.

The constant labour and capital rental income tax rates, $\tau_r = 0.36$ and $\tau_w = 0.28$, are also from Trabandt and Uhlig (2011). Lastly, we set the fraction of the hand-to-mouth households, ω, to 10% of the population. This is somewhat smaller than the fraction of the rule-of-thumb households assumed (or estimated) in the literature. For example, Campbell and Mankiw (1989) estimated that the fraction is around 50% of the population, and Galí et al. (2007) also used the same ratio. Recent papers in fiscal policy literature estimated that the fraction falls between 20% to 33% (see Coenen et al. 2012; Erceg and Lindé 2014; Cogan et al. 2010; Drautzburg and Uhlig 2015). However, we chose to use a smaller fraction of 10%. Our definition of the hand-to-mouth households are those who are fully rational and forward-looking but does not have any tool for saving or investing. It is hard to believe such households take up more than 20% of the population. Our assumption of 10% is slightly above around 7~8% of the fraction of the U.S. population who do not have a bank account (FDIC, 2015). We will see how the results are affected by ω in subsection 4.4.3. The autocorrelation coefficients and standard deviations of the exogenous shock processes are provided in Table 4.2.

Table 4.3 presents the standard deviations, autocorrelations, and cross correlations for key macroeconomic variables using the simulated data (for 10,000 periods) from the baseline model with flexible debt rule and conventional bonds. Note that the simulated model is not different from standard New Keynesian DSGE models except that we assume that 10% of the households are non-Ricardian and there exist distortionary taxes. As the table shows, our model well replicates the actual data of the U.S. despite the assumption of hand-to-mouth households. It replicates the negative correlation between inflation and output growth, and the highly persistent inflation process of the actual data, even though the inflation is a little bit more persistent than the actual data.

Table 4.3: **Key moments of the benchmark model** $(\omega = 0)$

	$\sigma_{\Delta y}$	$\sigma_{\Delta c}$	σ_{π}	$\rho_{\Delta y,\pi}$	$\rho_{\Delta y,\Delta c}$	*AR1*(Δy)	*AR1*(π)
Model	0.79	0.64	0.64	-0.16	0.73	0.30	0.75
US data	0.80	0.65	0.61	-0.18	0.65	0.34	0.65

note: σ_x denotes the standard deviation of variable x, and Δx means the quarterly percentage growth of the variable. $\sigma_{x,y}$ denotes sample correlation coefficient between x and y. The U.S. actual data from 1971Q3 to 2016Q4 were used.

The impulse responses in Figure 4.1 also show the similar patterns given from the standard New Keynesian DSGE models[16].

4.4 Results

4.4.1 Baseline results

Table 4.4 compares the simulation results from the benchmark model where there exist only Ricardian households (first four columns) and the baseline model where 10% of the population are hand-to-mouth households. Note that the fixed debt-to-GDP ratio is assumed in this subsection. Let us first compare the benchmark and the baseline results for the case where only the conventional bonds are used (first and second columns vs. fifth and sixth columns). The presence of the hand-to-mouth households only slightly changes the mean of the key variables, but its effect on volatility is substantial even though the fraction of hand-to-mouth households is only 10%. Especially, the consumption and labour supply of hand-to-mouth households are much more volatile than those of the Ricardian households in the benchmark model. Even the Ricardian

[16] All the figures are in Appendix 4.C.

Table 4.4: **Baseline results**

	Benchmark $(\omega = 0)$				**Baseline** $(\omega = 0.1)$			
	Conv.		**NGDP**		**Conv.**		**NGDP**	
	Mean	**Std.**	**Mean**	**Std.**	**Mean**	**Std.**	**Mean**	**Std.**
output	3.54	2.87	3.54	2.87	3.53	3.12	3.54	2.98
consumption	2.21	2.42	2.21	2.42	2.20	2.85	2.21	2.68
(Ricardian)	2.21	2.42	2.21	2.42	2.25	2.59	2.25	2.48
(H2M)	n.a	n.a	n.a	n.a	1.76	7.54	1.77	6.30
inflation	0.68	0.63	0.68	0.63	0.70	0.64	0.71	0.63
labour supply	1.28	1.97	1.28	1.97	1.28	2.36	1.28	2.16
(Ricardian)	1.29	1.97	1.29	1.97	1.27	2.79	1.27	2.41
(H2m)	n.a	n.a	n.a	n.a	1.46	5.84	1.45	3.59
capital supply	27.08	3.76	27.08	3.76	27.12	4.01	27.15	3.91
wage	1.66	3.13	1.66	3.13	1.66	3.36	1.66	3.32
rent rate	0.04	1.91	0.04	1.91	0.04	2.08	0.04	2.01
interest rate	1.02	0.63	1.02	0.63	1.02	0.69	1.02	0.64
net transfer	-0.66	15.70	-0.66	12.36	-0.66	23.21	-0.66	14.78
corr(y**,**NT**)**	-0.00		-0.14		0.08		-0.10	
welfare cost, %								
(Ricardian)	2.75		2.75		2.92		2.89	
(H2m)	n.a		n.a		16.52		13.67	

Note: Conv. and NGDP denote the case where $\omega^G = 0$ and $\omega^G = 1$, respectively. The standard deviations are calculated using log of the variables except for the inflation, and they expressed in percent. The correlation coefficient between output and net transfer (= transfer minus distortionary taxes) is presented to show the cyclicality of fiscal policy. See Equation (4.40) for the definition of welfare cost of business cycle.

households also experience more volatile labour supply in the baseline model than in the benchmark model.

The mechanism behind the larger volatility when there exist hand-to-mouth households is explained as follows. Under the assumption of constants debt-to-GDP ratio in this model, the net government transfer, NT_t, which is defined as the lump-sum government transfer minus distortionary taxes, can be expressed as below:[17]

$$NT_t \equiv t_t - \tau_t = -\varepsilon_t^g y_* + \overline{D}\left[y_t - \frac{y_{t-1}R_{t-1}}{\Pi_t \gamma}\right]. \tag{4.38}$$

This equation is given by substituting Equation (4.28) into the government budget constraint (Equation 4.26)[18]. τ_t denotes sum of the tax revenues from both labour and capital rental incomes. Equation (4.38) shows that a decrease in current output reduces the government's capacity of issuing new debts as the size of new debts should be proportional to the current output. At the same time, as the decrease in current output lowers current inflation[19], the burden of debt repayment becomes larger. All in all, a negative shock on current output has a negative impact on NT_t. Up to this point, there is no difference between the benchmark and the baseline models. However, contrary to the benchmark model where changes in NT_t has no effect on the business cycle, it has substantial effects in the baseline model. As the hand-to-mouth households are lack of consumption smoothing tools, a substantial portion of the change in net transfer goes to their consumption via the change in their disposable income.

On top of the direct effect on the consumption of hand-to-mouth households, there also exist second round effects. Figure 4.3 summarises this. The decreased consumption in hand-to-mouth households means a reduced demand in aggregate output, and in turn reduced demands in labour and capital service as well. This further decreases the net transfer, and this cycle goes on and on. At

[17]This expression is for the case where only conventional bonds are used.

[18]Note that the lower case variables are detrended variables.

[19]In fact, this is not the case in the benchmark model. In the benchmark model with no hand-to-mouth households, supply shock dominates, and thus output and inflation covary negatively. In the baseline model, however, demand shocks play more important roles for the business cycle. This is also explained by Equation (4.38). The difference between the baseline and the benchmark model is whether the changes in net transfer affect the business cycle, and the terms that determine the response of net transfer in the baseline model are the terms in parentheses in Equation (4.38). As the demand shocks move output and inflation in the opposite direction, the terms in the parentheses react much more strongly to the demand shocks then the supply shocks.

the same time, the hand-to-mouth households try to smooth their consumption in response to the decrease in disposable income by supplying more labour. That means, the decreased demand in aggregate labour should be met by less labour supply from the Ricardian households. In equilibrium, the cycle ends up with lower C^H, higher h^H, and lower h^R. This is why the labour supply of not only the hand-to-mouth but also the Ricardian households become more volatile in the baseline model. Also, the opposite responses of the two labour supplies, h^H and h^R, explain why aggregate labour supply is less volatile than both of the group-wise labour supplies are. To sum up, when the government is forced to keep its debt-to-GDP ratio constant, a shock that changes output leads to a change in net transfer. If all the households are Ricardian, the business cycle is immune to this change, but when there exist hand-to-mouth households, the changes in net transfer can have significant impact on the business cycle through their consumption.

Let us then examine whether and how the use of NGDP-indexed bonds may stabilise the business cycle and improve the welfare of hand-to-mouth households in the baseline economy. The fifth to eighth columns of Table 4.4 contrast the simulation results from the baseline model under the two different financing structure: 100% conventional bonds vs. 100% NGDP-indexed bonds. The results show that using the NGDP-indexed bonds only slightly changes the mean of key variables, but it decreases their volatility significantly. When the government relies 100% on the NGDP-indexed bonds, the equation for the net transfer is given as follows:

$$NT_t \equiv t_t - \tau_t = -\varepsilon_t^g y_* + \overline{D}\left[y_t - \frac{y_t}{Q_{t-1}^G}\right]. \tag{4.39}$$

In this case, a decrease in y_t reduces not only the government's ability to issue new debts, but also the repayment burden of previously issued bonds. Therefore, given the same size of shock (e.g., a negative shock on output), the decrease in net transfer is smaller in size in the case of NGDP-indexed bonds than in the case of conventional bonds. This, in turn, reduces the decline in consumption of hand-to-mouth households, and thus, reduces the changes in output, labour and capital service as well following the cycle described in Figure 4.3. In short, NGDP-indexed bonds can be used as a kind of automatic stabiliser.

The impulse response functions in Figure 4.2 tell us more stories. Using NGDP-indexed bonds greatly reduces the responses to demand shocks (preference, government spending, investment and monetary policy shocks) of key variables, while the responses to supply shocks (productivity, price markup, and wage markup shocks) are not affected much. This can also be explained from Equation (4.38) and (4.39). When a demand shock comes, both output and inflation move to the same direction. This implies, from Equation (4.38) where only conventional government bonds are used, a positive demand shock increases the government's capacity of issuing new debts and reduces the burden of debt repayment in real terms. As a result, the shock significantly increases the lump-sum transfer, and through the increase in disposable income of hand-to-mouth households, destabilises the entire economy. When NGDP-indexed bonds are used instead, the positive demand shock increases both new and old debts as in Equation (4.39), and thus the response of the net transfer becomes a lot smaller, and so do the responses of the other variables. On the contrary, to a supply shock, output and inflation respond to the opposite directions. In case of the conventional bonds, a positive supply shock increases both new and old debts, and thus its impact on transfer cancel out each other.

For this reason, the business cycle stabilising effect of issuing NGDP-indexed bonds is also reduced.

Meanwhile, the last two rows in Table 4.4 show the welfare costs of business cycle under various assumptions. In order to measure the changes in welfare in terms of final good consumption, we defined the welfare cost of business cycle, WC, as follows:

$$v_{mean} = \frac{\left[\frac{1}{1-\sigma_c}\left(\left(1-\frac{\lambda}{\gamma}\right)c_*\left(1-WC\right)\right)^{1-\sigma_c}\right]\exp\left[\frac{\sigma_c-1}{1+\sigma_l}\left(h_*\right)^{1+\sigma_l}\right]}{1-\overline{\beta}\gamma}, \tag{4.40}$$

where v_{mean} is the simulated mean of detrended welfare level, v_t[20]; and c_* and h_* are deterministic steady state levels of detrended consumption and working hours, respectively. This definition implies that the welfare cost, WC, shows the welfare loss incurred from the existence of business cycle in terms of deterministic steady state consumption level. In other words, WC shows the amount of the steady state consumption loss needed to lower the deterministic steady state welfare level down to the average welfare level when there exist business cycles.

Because the presence of hand-to-mouth households significantly destabilises the business cycle, even the Ricardian households face higher welfare cost in the baseline model than in the benchmark model (2.75%→2.92%)[21]. When the NGDP-indexed bonds are used in the baseline model, while there is no notable change in the welfare cost of the Ricardian households, the hand-to-mouth households can benefit substantially in terms of welfare cost (16.52%→13.67%).

[20]Note that the welfare levels of both households are directly captured by the value functions, v_t^R and v_t^H.

[21]The welfare cost of around 3% in the benchmark model may seem a lot lager compared with the literature. For example, Lucas (1987) showed that the welfare loss from fluctuations in consumption is less than 0.01% under the assumption of logarithmic preference. However, it is known that the welfare cost of business cycle can be much larger for models with recursive preferences (see Dolmas, 1998; Tallarini, 2000; Barrillas et al., 2006).

Table 4.5: **Sensitive analysis (share of H2M households)**

	($\omega = 0.07$)				($\omega = 0.15$)			
	Conv.		**NGDP**		**Conv.**		**NGDP**	
	Mean	**Std.**	**Mean**	**Std.**	**Mean**	**Std.**	**Mean**	**Std.**
output	3.53	3.03	3.54	2.94	3.52	3.48	3.54	3.13
consumption	2.20	2.69	2.21	2.59	2.19	3.44	2.20	2.96
(Ricardian)	2.24	2.52	2.24	2.46	2.27	2.85	2.28	2.58
(H2M)	1.76	7.00	1.77	6.05	1.75	9.44	1.77	7.08
inflation	0.70	0.64	0.70	0.63	0.72	0.65	0.72	0.62
labour supply	1.28	2.22	1.28	2.09	1.28	2.88	1.28	2.37
(Ricardian)	1.28	2.47	1.28	2.26	1.26	4.21	1.26	2.96
(H2m)	1.45	5.10	1.45	3.31	1.47	9.13	1.45	4.85
capital supply	27.11	3.92	27.13	3.86	27.11	4.32	27.19	4.08
wage	1.66	3.29	1.66	3.26	1.66	3.54	1.67	3.47
rent rate	0.04	2.02	0.04	1.98	0.04	2.27	0.04	2.12
interest rate	1.02	0.67	1.02	0.64	1.02	0.76	1.02	0.66
net transfer	-0.66	20.20	-0.66	13.86	-0.66	42.67	-0.66	19.02
corr(y**,**NT**)**	0.05		-0.12		0.20		-0.02	
welfare cost, %								
(Ricardian)	2.85		2.84		3.07		3.06	
(H2m)	14.90		13.28		25.73		15.87	

note: See the note in Table 4.4

One thing to note is that the focus of this chapter is not calculating the exact magnitude of welfare gain by the use of NGDP-indexed bonds, especially because of the simplistic assumptions on the government sector. Instead, we focus more on the mechanism how the use of NGDP-indexed bonds can affect the business cycle and welfare, and the condition under which the government and the households can benefit. We will discuss this with the sensitivity analysis in the next subsection.

4.4.2 Sensitivity analysis

In the baseline model, we assumed that the fraction of hand-to-mouth households, ω, is only 10% of the population. As mentioned already in section 4.3, this is somewhat lower than the fractions used in the literature. Therefore, it would be worthwhile to examine how different fractions of hand-to-mouth households change the results. Table 4.5 presents the simulation results when the fraction is changed to 7% and to 15%, leaving all the other parameters unchanged from the baseline. Obviously, the results show that the business cycle becomes more volatile when the share of hand-to-mouth households grows. In this model, the key channel through which the changes in net transfer can affect the business cycle is the consumption of hand-to-mouth households. Therefore, given the same change in net transfer, it is natural the larger the fraction of hand-to-mouth households, the more volatile the economy becomes. This also explains why the welfare gain from the use of NGDP-indexed bond gets larger as ω grows.[22]

Another key assumption in our model is that the government should keep its debt-to-GDP ratio at a constant level, and we assumed that this ratio is 252% of quarterly GDP (or 63% of annual GDP) from the U.S. data. Table 4.6 shows how the baseline results are altered when we apply different debt-to-GDP ratios. In the first four columns, we assume that the government keeps no debt at all times ($\overline{D} = 0.0$), and the next four columns show the simulation results when the ratio is 90% of annual output ($\overline{D} = 3.6$).

When the government keeps no outstanding debt, most of the variables become more stable than the baseline model. This is because the existence

[22] Even though we did not mention in this paper, the assumption on how the government transfer is distributed between the two groups can also affect the results. Cogan et al. (2010) have showed that the government spending multiplier gets larger when the rule-of thumb households get more fraction of government transfer.

Table 4.6: **Sensitive analysis (debt-to-GDP ratio)**

	$(\overline{D}=0)$				$(\overline{D}=3.6)$			
	Conv.		**NGDP**		**Conv.**		**NGDP**	
	Mean	**Std.**	**Mean**	**Std.**	**Mean**	**Std.**	**Mean**	**Std.**
output	3.54	2.87	3.54	2.87	3.53	3.30	3.54	3.05
consumption	2.21	2.56	2.21	2.56	2.20	3.09	2.21	2.77
(rational)	2.25	2.41	2.25	2.41	2.25	2.72	2.26	2.53
(H2M)	1.81	5.13	1.81	5.13	1.75	9.83	1.75	7.38
inflation	0.66	0.64	0.66	0.64	0.72	0.64	0.72	0.62
labour supply	1.28	2.02	1.28	2.02	1.28	2.61	1.28	2.25
(rational)	1.27	2.21	1.27	2.21	1.27	3.40	1.27	2.59
(H2m)	1.43	1.55	1.43	1.55	1.48	10.37	1.46	5.55
capital supply	27.22	3.78	27.22	3.78	27.08	4.15	27.13	3.98
wage	1.66	3.25	1.66	3.25	1.66	3.41	1.66	3.35
rent rate	0.04	1.94	0.04	1.94	0.04	2.16	0.04	2.06
interest rate	1.02	0.67	1.02	0.67	1.02	0.70	1.02	0.63
net transfer	-0.60	4.88	-0.60	4.88	-0.69	53.49	-0.69	22.74
corr(y,NT)	-0.40		-0.40		0.17		-0.03	
welfare cost, %								
(rational)	2.76		2.76		2.94		2.93	
(H2m)	9.08		9.08		28.50		18.96	

note: See the note in Table 4.4

of positive debt plays a role in making fiscal policy more pro-cyclical as seen from Equation (4.38) and (4.39). When $\overline{D} = 0.0$, the two equations collapse into $NT_t = -\varepsilon_t^g y_*$, and net transfer becomes strongly negatively correlated with output. This allows the hand-to-mouth households to have more stable consumption and labour path. For the same reason, as $\overline{D}$ becomes higher, it puts more pressure of pro-cyclical fiscal policy, and thus makes the consumption of hand-to-mouth households more volatile. As the use of NGDP-indexed bonds mediates the pressure of conducting pro-cyclical fiscal policy, we may expect more welfare gain when $\overline{D}$ becomes higher.

Lastly, we examined how the baseline results may be affected if we relax the assumption of constant debt-to-GDP ratio. To see this, we replaced the constant

Table 4.7: **More flexible debt rule**

	Baseline				Flexible-debt-rule			
	Conv.		NGDP		Conv.		NGDP	
	Mean	**Std.**	**Mean**	**Std.**	**Mean**	**Std.**	**Mean**	**Std.**
output	3.53	3.12	3.54	2.98	3.54	2.88	3.54	2.87
consumption	2.20	2.85	2.21	2.68	2.21	2.52	2.21	2.51
(Ricardian)	2.25	2.59	2.25	2.48	2.26	2.40	2.26	2.39
(H2M)	1.76	7.54	1.77	6.30	1.76	4.73	1.77	4.62
inflation	0.70	0.64	0.71	0.63	0.69	0.64	0.70	0.64
labour supply	1.28	2.36	1.28	2.16	1.29	2.01	1.29	1.99
(Ricardian)	1.27	2.79	1.27	2.41	1.27	2.18	1.27	2.15
(H2m)	1.46	5.84	1.45	3.59	1.45	1.41	1.45	1.51
capital supply	27.12	4.01	27.15	3.91	27.19	3.77	27.18	3.76
wage	1.66	3.36	1.66	3.32	1.66	3.22	1.66	3.21
rent rate	0.04	2.08	0.04	2.01	0.04	1.93	0.04	1.92
interest rate	1.02	0.69	1.02	0.64	1.02	0.67	1.02	0.66
net transfer	-0.66	23.21	-0.66	14.78	-0.67	5.80	-0.67	6.96
corr(y,NT)	0.08		-0.10		-0.50		-0.50	
welfare cost, %								
(Ricardian)	2.92		2.89		2.77		2.80	
(H2m)	16.52		13.67		10.60		11.81	

debt-to-GDP rule in the baseline model with the flexible-debt-rule of Equation (4.37) in Section 4.3.

Table 4.7 compares the two cases: flexible-debt-rule model and the baseline model. We can see that the flexible-debt-rule significantly stabilises the consumption and labour of hand-to-mouth households even with conventional bonds. Under the flexible-debt-rule, the government still needs to adjust its transfer in response to the shocks that affect debt-to-GDP ratio, but as there is a leeway allowed in the debt-to-GDP ratio, the pressure of pro-cyclical fiscal policy can be much smaller than the constant debt-to-GDP case. This directly leads to more stable consumption path for hand-to-mouth households. In the meantime, as there is much smaller pressure of pro-cyclical fiscal policy under the flexible-debt-rule, the business cycle stabilising effects of NGDP-indexed bonds also becomes smaller (or disappear), and so does the welfare gain. This shows that our results rely highly on the assumption of the constant debt-to-GDP ratio.

4.5 Conclusion and Summary

In this chapter, we examined how a government can use NGDP-indexed bonds as an alternative fiscal policy tool when it is constrained to keep a constant debt-to-GDP within the new Keynesian framework. As Ricardian equivalence holds in the standard new Keynesian DSGE models, the assumption of constant debt-to-GDP is irrelevant to the business cycle in such models. However, when a fraction of the population is non-Ricardian, the constant debt-to-GDP assumption plays a role of making fiscal policy more pro-cyclical, and this makes the disposable income of non-Ricardian households very volatile. Since they are not able to smooth consumption intertemporally, their consumption becomes very volatile as well. Under this situation, NGDP-indexed bonds can play a role of an automatic stabiliser. That is to say, the use of NGDP-indexed bonds mitigates the pressure of pro-cyclical fiscal policy and helps stabilise the consumption of non-Ricardian households. This may increase the welfare of non-Ricardian households as well.

In addition, in contrast to the previous papers with the presence of non-Ricardian households, we assume that the hand-to-mouth households in our model have a desire for consumption smoothing and do it at least intratemporally. For this reason, the use of NGDP-indexed bonds stabilises not only the consumption of hand-to-mouth households, but also their supply of labour. Moreover, as the labour supply of the two group of households are closely interconnected through the labour market, the labour supply from the Ricardian households is stabilised as well. To sum up, the government with restricted fiscal policy tools can rely on NGDP-indexed bonds to stabilise business cycle and improve the welfare of at least a part of the households without damaging the others. We also showed that the larger benefits can be obtained in an economy with a larger share of hand-to-mouth households, a higher level of debt-to-GDP ratio, and when the business cycle is mainly driven by demand shocks.

One may point out several shortcomings of the analysis in this chapter. One of them is the fact that the conclusion of this chapter is strongly dependent on the assumption of constant debt-to-GDP ratio. In fact, we also showed that the benefits have disappeared in the model with more relaxed fiscal policy rule. Therefore, our results should not be interpreted that the government can benefit from the use of NGDP-indexed bonds unconditionally. Nevertheless, as many advanced countries are actually approaching their debt limits as Ostry et al. (2010) shows, it may be reasonable to consider NGDP-indexed bonds as part of their fiscal policy tools.

We want to close this chapter by discussing a few model extensions for the future. In this chapter, our model does not explicitly include the possibility of default. If there exists an endogenous mechanism through which a rise in debt-to-GDP ratio raises the probability of default and related risk premium, we can have a vicious cycle in which a positive shock to debt-to-GDP ratio raises the government's overall borrowing costs and further increases its debt-to-GDP ratio. When such a mechanism is included to the model, we may expect a lot larger benefits from the use of NGDP-indexed bonds as suggested by the previous papers (Chamon and Mauro, 2006; Ostry et al., 2010; Barr et al., 2014; Kim and Ostry, 2018).

Another shortcoming we acknowledge is that our model is a closed economy model and calibrated with the U.S. macroeconomic data which is believed to have little or no possibility of government default. Therefore, the analyses and results presented can be extended only to a set of advanced economies. By extending the model to a small open economy model and explicitly incorporating foreign currency denominated debts, we may be able to discuss the benefits of issuing NGDP-indexed bonds to the emerging market countries as well.

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Appendix

4.A List of detrended non-linear equilibrium conditions

- Production sector[23]

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{w_t}{r_t^k}\right)=\left(\frac{z_t k_{t-1}/\gamma}{l_t}\right) \tag{4.41}$$

$$mc_t=\frac{1}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\varepsilon_t^a}\left[\left(w_t\right)^{1-\alpha}\left(r_t^k\right)^{\alpha}\right] \tag{4.42}$$

- Price setting[24]

$$g_t^1 = \hat{\Pi}_t^{-\frac{1}{\lambda_p}} y_t+\zeta_p E_t\left[M_{t,t+1}^R\gamma\left(\frac{\hat{\Pi}_t}{\hat{\Pi}_{t+1}}\right)^{-\frac{1}{\lambda_p}}\left(\frac{\Pi_t^{\iota_p}\Pi_*^{1-\iota_p}}{\Pi_{t+1}}\right)^{-\frac{1}{\lambda_p}} g_{t+1}^1\right] \tag{4.43}$$

$$g_t^2 = \varepsilon_t^p\hat{\Pi}_t^{-\frac{1+\lambda_p}{\lambda_p}} y_t\cdot mc_t+\zeta_p E_t\left[M_{t,t+1}^R\gamma\left(\frac{\hat{\Pi}_t}{\hat{\Pi}_{t+1}}\right)^{-\frac{1+\lambda_p}{\lambda_p}}\left(\frac{\Pi_t^{\iota_p}\Pi_*^{1-\iota_p}}{\Pi_{t+1}}\right)^{-\frac{1+\lambda_p}{\lambda_p}} g_{t+1}^2\right] \tag{4.44}$$

$$g_t^1 = \left(1+\lambda_p\right)g_t^2 \tag{4.45}$$

[23] $w_t\equiv\frac{W_t}{P_t\gamma^t}$, $r_t^k\equiv\frac{R_t^k}{P_t}$, $k_t\equiv\frac{K_t}{\gamma^t}$, $mc_t\equiv\frac{MC_t}{P_t}$

[24] $y_t\equiv\frac{Y_t}{\gamma^t}$, $\hat{\Pi}_t\equiv\frac{\hat{P}_t}{P_t}$

- Law of motion: price

$$1 = (1-\zeta_p)\,\hat{\Pi}_t^{-\frac{1}{\lambda_p}} + \zeta_p \left(\frac{\Pi_{t-1}^{\iota_p} \Pi_*^{1-\iota_p}}{\Pi_t} \right)^{-\frac{1}{\lambda_p}} \tag{4.46}$$

- Law of motion: capital[25]

$$k_t = \frac{(1-\delta)}{\gamma} k_{t-1} + \varepsilon_t^I \left[1 - \frac{\phi}{2} \left(\frac{i_t \gamma}{i_{t-1}} - \gamma \right)^2 \right] i_t \tag{4.47}$$

- Value function and period utility[26]

$$u_t^R = \varepsilon_t^b \left[\frac{1}{1-\sigma_c} \left(c_t^R - \frac{\lambda}{\gamma} c_{t-1}^R \right)^{1-\sigma_c} \right] \exp \left[\frac{\sigma_c - 1}{1+\sigma_l} \left(h_t^R \right)^{1+\sigma_l} \right] \tag{4.48}$$

$$u_t^H = \varepsilon_t^b \left[\frac{1}{1-\sigma_c} \left(c_t^H - \frac{\lambda}{\gamma} c_{t-1}^H \right)^{1-\sigma_c} \right] \exp \left[\frac{\sigma_c - 1}{1+\sigma_l} \left(h_t^H \right)^{1+\sigma_l} \right] \tag{4.49}$$

$$v_t^R = u_t^R + \overline{\beta} \gamma E_t \left[\left(v_{t+1}^R \right)^{1-\sigma_{EZ}} \right]^{\frac{1}{1-\sigma_{EZ}}} \tag{4.50}$$

$$v_t^H = u_t^H + \overline{\beta} \gamma E_t \left[\left(v_{t+1}^H \right)^{1-\sigma_{EZ}} \right]^{\frac{1}{1-\sigma_{EZ}}} \tag{4.51}$$

- First order conditions: Ricardian households[27]

$$\lambda_t^R = \varepsilon_t^b \left(c_t^R - \frac{\lambda}{\gamma} c_{t-1}^R \right)^{-\sigma_c} \exp \left[\frac{\sigma_c - 1}{1+\sigma_l} \left(h_t^R \right)^{1+\sigma_l} \right] \tag{4.52}$$

$$\frac{1}{R_t} = E_t \left[M_{t,t+1}^R \Pi_{t+1}^{-1} \right] \tag{4.53}$$

[25] $i_t \equiv \frac{I_t}{\gamma^t}$

[26] $u_t \equiv \frac{U_t}{\gamma^{t(1-\sigma_c)}}$, $v_t \equiv \frac{V_t}{\gamma^{t(1-\sigma_c)}}$, $c_t \equiv \frac{C_t}{\gamma^t}$, $\overline{\beta} \equiv \beta \gamma^{-\sigma_c}$

[27] $\lambda_t^R \equiv \Xi_t^R \gamma^{\sigma_c t}$, $\overline{w}_t \equiv \frac{\overline{W}_t}{P_t \gamma^t}$

$$Q_t^G = E_t\left[M_{t,t+1}^R \frac{y_{t+1}\gamma}{y_t}\right] \tag{4.54}$$

$$(1-\tau_w)\,\overline{w}_t = \left(c_t^R - \frac{\lambda}{\gamma}c_{t-1}^R\right)\left(h_t^R\right)^{\sigma_l} \tag{4.55}$$

$$\begin{aligned} 1 &= q_t\varepsilon_t^I\left[1-\frac{\phi}{2}\left(\frac{i_t\gamma}{i_{t-1}}-\gamma\right)^2-\phi\left(\frac{i_t\gamma}{i_{t-1}}-\gamma\right)\frac{i_t\gamma}{i_{t-1}}\right]+ \qquad (4.56)\\ &\quad E_t\left[M_{t,t+1}^R q_{t+1}\varepsilon_{t+1}^I\phi\left(\frac{i_{t+1}\gamma}{i_t}-\gamma\right)\left(\frac{i_{t+1}\gamma}{i_t}\right)^2\right] \\ q_t &= E_t\left[M_{t,t+1}^R\left\{\begin{array}{c}(1-\tau_r)\,r_t^k z_t+\delta\tau_r-\delta_1\left(z_{t+1}-1\right)\\ -\frac{\delta_2}{2}\left(z_{t+1}-1\right)^2+q_{t+1}\left(1-\delta\right)\end{array}\right\}\right] \qquad (4.57)\end{aligned}$$

$$(1-\tau_r)\,r_t^k = \delta_1+\delta_2\left(z_t-1\right) \tag{4.58}$$

- First order conditions: hand-to-mouth households

$$\lambda_t^H = \varepsilon_t^b\left(c_t^H-\frac{\lambda}{\gamma}c_{t-1}^H\right)^{-\sigma_c}\exp\left[\frac{\sigma_c-1}{1+\sigma_l}\left(h_t^H\right)^{1+\sigma_l}\right] \tag{4.59}$$

$$(1-\tau_w)\,\overline{w}_t = \left(c_t^H-\frac{\lambda}{\gamma}c_{t-1}^H\right)\left(h_t^H\right)^{\sigma_l} \tag{4.60}$$

- Budget constraint of hand-to-mouth households[28]

$$\left(c_t^H - t_t\right) = (1-\tau_w)\left(\overline{w}_t h_t^H + \underbrace{w_t l_t - \omega\overline{w}_t h_t^H - (1-\omega)\,\overline{w}_t h_t^R}_{D_t^u:\text{ union profit}}\right) \tag{4.61}$$

[28] $t_t \equiv \frac{T_t}{\gamma^t}$

- Wage setting[29]

$$f_t^1 = l_t\left(\frac{\hat{w}_t}{w_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}}\hat{w}_t + \zeta_w E_t\left\{M_{t,t+1}^R\gamma\left(\frac{\hat{w}_t}{\hat{w}_{t+1}}\right)^{-\frac{1}{\lambda_w}}\left(\frac{\Pi_t^{\iota_w}\Pi_*^{1-\iota_w}}{\Pi_{t+1}}\right)^{-\frac{1}{\lambda_w}} f_{t+1}^1\right\} \quad (4.62)$$

$$f_t^2 = \varepsilon_t^w l_t\left(\frac{\hat{w}_t}{w_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}}\overline{w}_t + \zeta_w E_t\left\{M_{t,t+1}^R\gamma\left(\frac{\hat{w}_t}{\hat{w}_{t+1}}\right)^{-\frac{1+\lambda_w}{\lambda_w}}\left(\frac{\Pi_t^{\iota_w}\Pi_*^{1-\iota_w}}{\Pi_{t+1}}\right)^{-\frac{1+\lambda_w}{\lambda_w}} f_{t+1}^2\right\} \quad (4.63)$$

$$f_t^1 = (1+\lambda_w) f_t^2 \quad (4.64)$$

- Law of motion: wage

$$(w_t)^{-\frac{1}{\lambda_w}} = (1-\zeta_w)(\hat{w}_t)^{-\frac{1}{\lambda_w}} + \zeta_w\left(\frac{\Pi_{t-1}^{\iota_w}\Pi_*^{1-\iota_w}}{\Pi_t}w_{t-1}\right)^{-\frac{1}{\lambda_w}} \quad (4.65)$$

- Monetary policy rule

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*}\right)^{\rho_R}\left[\left(\frac{\Pi_t}{\Pi_*}\right)^{\psi_1}\left(\frac{y_t}{y_*}\right)^{\psi_2}\right]^{1-\rho_R}\left(\frac{y_t}{y_{t-1}}\right)^{\psi_3}\varepsilon_t^r \quad (4.66)$$

- Government budget constraint

$$\begin{aligned} & \varepsilon_t^g y_* + t_t + \frac{b_{t-1}}{\Pi_t\gamma} + \frac{b_{t-1}^G y_t}{y_{t-1}} \quad (4.67) \\ = \; & \frac{b_t}{R_t} + Q_t^G b_t^G + \tau_w w_t l_t + \tau_r z_t r_t^k\frac{k_{t-1}}{\gamma} - \tau_r\delta\frac{k_{t-1}}{\gamma} \end{aligned}$$

- Debt rules[30]

$$b_t/R_t = \left(1-\omega^G\right)\overline{D}y_t \quad (4.68)$$

$$Q_t^G b_t^G = \omega^G\overline{D}y_t \quad (4.69)$$

---

[29] $\hat{w}_t \equiv \frac{\hat{W}_t}{P_t\gamma^t}$

[30] $b_t \equiv \frac{B_t}{P_t\gamma^t}$, $b_t^G \equiv \frac{B_t^G}{P_t\gamma^t}$

- Aggregate consumption

$$c_t = (1-\omega)\, c_t^R + \omega c_t^H \tag{4.70}$$

- Aggregate resource constraint

$$y_t = c_t + i_t + \varepsilon_t^g y_* + \underbrace{\left\{\delta_1 (z_t - 1) + \frac{\delta_2}{2} (z_t - 1)^2\right\}}_{=a(z_t)} \frac{k_{t-1}}{\gamma} \tag{4.71}$$

- Market clearing condition: final goods

$$y_t = \frac{\varepsilon_t^a \left(\frac{z_t k_{t-1}}{\gamma}\right)^{\alpha} (l_t)^{1-\alpha} - y_* (\phi_p - 1)}{s_t^p} \tag{4.72}$$

- Law of motion: price dispersion

$$s_t^p = (1-\zeta_p) \left(\hat{\Pi}_t\right)^{-\frac{1+\lambda_p}{\lambda_p}} + \zeta_p \left(\frac{\Pi_{t-1}^{\iota_p} \Pi_*^{1-\iota_p}}{\Pi_t}\right)^{-\frac{1+\lambda_p}{\lambda_p}} s_{t-1}^p \tag{4.73}$$

- Market clearing condition: labour

$$\omega h_t^H + (1-\omega)\, h_t^R = s_t^w l_t \tag{4.74}$$

- Law of motion: wage dispersion

$$s_t^w = (1-\zeta_w) \left(\frac{\hat{w}_t}{w_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} + \zeta_w \left(\frac{\Pi_{t-1}^{\iota_w} \Pi_*^{1-\iota_w}}{\Pi_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} \left(\frac{w_{t-1}}{w_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} s_{t-1}^w \tag{4.75}$$

- Shock processes

$$\log \varepsilon_t^a = \rho_a \log \varepsilon_t^a + \eta_t^a \tag{4.76}$$

$$\log \varepsilon_t^b = \rho_b \log \varepsilon_t^b + \eta_t^b \tag{4.77}$$

$$\log \left(\frac{\varepsilon_t^g}{\varepsilon_*^g} \right) = \rho_g \log \left(\frac{\varepsilon_{t-1}^g}{\varepsilon_*^g} \right) + \eta_t^g + \rho_{ga} \eta_t^a \tag{4.78}$$

$$\log \varepsilon_t^i = \rho_i \log \varepsilon_t^i + \eta_t^i \tag{4.79}$$

$$\log \varepsilon_t^r = \rho_r \log \varepsilon_t^r + \eta_t^r \tag{4.80}$$

$$\log \varepsilon_t^p = \rho_p \log \varepsilon_t^p + \eta_t^p \tag{4.81}$$

$$\log \varepsilon_t^w = \rho_w \log \varepsilon_t^w + \eta_t^w \tag{4.82}$$

4.B Steady states

- $z_* = 1$ is assumed and Π_* is an exogenously given parameter.
- The following steady state conditions are analytically given with pencil and paper:

  $\hat{\Pi}_* = q_* = s_*^p = s_*^w = 1$

  $r_*^k = \frac{\left(\overline{\beta}\right)^{-1} - \delta\tau_r - (1-\delta)}{1-\tau_r}$

  $mc_* = 1/\left(1+\lambda_p\right)$

  $w_* = (1-\alpha)\left(mc_* \left(\frac{\alpha}{r_*^k}\right)^{\alpha}\right)^{\frac{1}{1-\alpha}}$

  $\overline{w}_* = w_*/\left(1+\lambda_w\right)$

  $\hat{w}_* = w_*$

  $\left(\frac{k_*}{l_*}\right) = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{w_*}{r_*^k}\right)\gamma$

  $\frac{i_*}{k_*} = \frac{\gamma - 1 + \delta}{\gamma}$

$R_* = \frac{\Pi_*}{\overline{\beta}}$

$Q^G_* = \overline{\beta}\gamma$

$\frac{y_*}{k_*} = \left(\frac{k_*}{l_*}\right)^{\alpha-1} \gamma^{-\alpha}\phi_p^{-1}$

$\frac{c_*}{y_*} = 1 - \frac{i_*}{k_*}\frac{k_*}{y_*} - \varepsilon^g_*$

$\frac{b_*}{y_*} = \left(1-\omega^G\right)\overline{D}R_*$

$\frac{b^G_*}{y_*} = \omega^G\overline{D}/Q^G_*$

$\frac{t_*}{y_*} = \frac{b_*}{y_*}\left(\frac{1}{R_*} - \frac{1}{\Pi_*\gamma}\right) + \frac{b^G_*}{y_*}\left(Q^G_* - 1\right) + \tau_w\left(\frac{k_* r^k_*}{y_*\gamma}\right)\left(\frac{1-\alpha}{\alpha}\right) + \tau_r\frac{\left(r^k_* - \delta\right)}{\gamma}\frac{k_*}{y_*} - \varepsilon^g_*$

- We get $\frac{c^H_*}{y_*}, \frac{c^R_*}{y_*}, h^H_*, h^R_*, l_*$ numerically from the following five equations:
  - From Equation (4.55) and (4.60):

$$\left(\frac{1-\tau_w}{1+\lambda_w}\right)\frac{w_* l_*}{y_*} = \left(1-\frac{\lambda}{\gamma}\right)\frac{c^R_*}{y_*}\left(h^R_*\right)^{\sigma_l} l_*$$

and

$$\left(\frac{1-\tau_w}{1+\lambda_w}\right)\frac{w_* l_*}{y_*} = \left(1-\frac{\lambda}{\gamma}\right)\frac{c^H_*}{y_*}\left(h^H_*\right)^{\sigma_l} l_*.$$

  - From Equation (4.70):

$$\frac{c_*}{y_*} = \omega\frac{c^H_*}{y_*} + (1-\omega)\frac{c^R_*}{y_*}.$$

  - From Equation (4.61):

$$\left(\frac{c^H_*}{y_*} - \frac{t_*}{y_*}\right) = (1-\tau_w)\left(\frac{w_* l_*}{y_*}\right)\left\{1 + \frac{(1-\omega)\left(h^H_* - h^R_*\right)}{(1+\lambda_w)\, l_*}\right\}.$$

  - From Equation (4.74):

$$l_* = \omega h^H_* + (1-\omega)\, h^R_*$$

- Then, we can find the rest of the steady state conditions analytically as well:

$$y_* = \frac{w_* l_*}{\left(\frac{w_* l_*}{y_*}\right)}$$

$$k_* = y_* \left(\frac{k_*}{y_*}\right)$$

$$i_* = k_* \left(\frac{i_*}{k_*}\right)$$

$$t_* = y_* \left(\frac{t_*}{y_*}\right)$$

$$b_* = y_* \left(\frac{b_*}{y_*}\right)$$

$$b_*^G = y_* \left(\frac{b_*^G}{y_*}\right)$$

$$c_* = y_* \left(\frac{c_*}{y_*}\right)$$

$$c_*^R = y_* \left(\frac{c_*^R}{y_*}\right)$$

$$c_*^H = y_* \left(\frac{c_*^H}{y_*}\right)$$

$$g_*^1 = \frac{y_*}{\left(1-\zeta_p \overline{\beta} \gamma\right)}$$

$$g_*^2 = \frac{y_* mc_*}{\left(1-\zeta_p \overline{\beta} \gamma\right)}$$

$$f_*^1 = \frac{l_* w_*}{\left(1-\zeta_w \overline{\beta} \gamma\right)}$$

$$f_*^2 = \frac{l_* \overline{w}_*}{\left(1-\zeta_w \overline{\beta} \gamma\right)}$$

$$u_*^R = \left[\frac{1}{1-\sigma_c} \left(c_*^R - \frac{\lambda}{\gamma} c_*^R\right)^{1-\sigma_c}\right] \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} \left(h_*^R\right)^{1+\sigma_l}\right)$$

$$u_*^H = \left[\frac{1}{1-\sigma_c} \left(c_*^H - \frac{\lambda}{\gamma} c_*^H\right)^{1-\sigma_c}\right] \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} \left(h_*^H\right)^{1+\sigma_l}\right)$$

$$v_*^R = \frac{u_*^R}{1-\overline{\beta}^R \gamma}$$

$$v_*^H = \frac{u_*^H}{1-\overline{\beta}^H \gamma}$$

$$\lambda_*^R = \left(c_*^R - \frac{\lambda}{\gamma} c_*^R\right)^{-\sigma_c} \exp\left[\frac{\sigma_c - 1}{1+\sigma_l} \left(h_*^R\right)^{1+\sigma_l}\right]$$

$$\lambda_*^H = \left(c_*^H - \frac{\lambda}{\gamma} c_*^H\right)^{-\sigma_c} \exp\left[\frac{\sigma_c - 1}{1+\sigma_l} \left(h_*^H\right)^{1+\sigma_l}\right]$$

4.C Figures

Figure 4.1: **IRFs for benchmark model (I)**

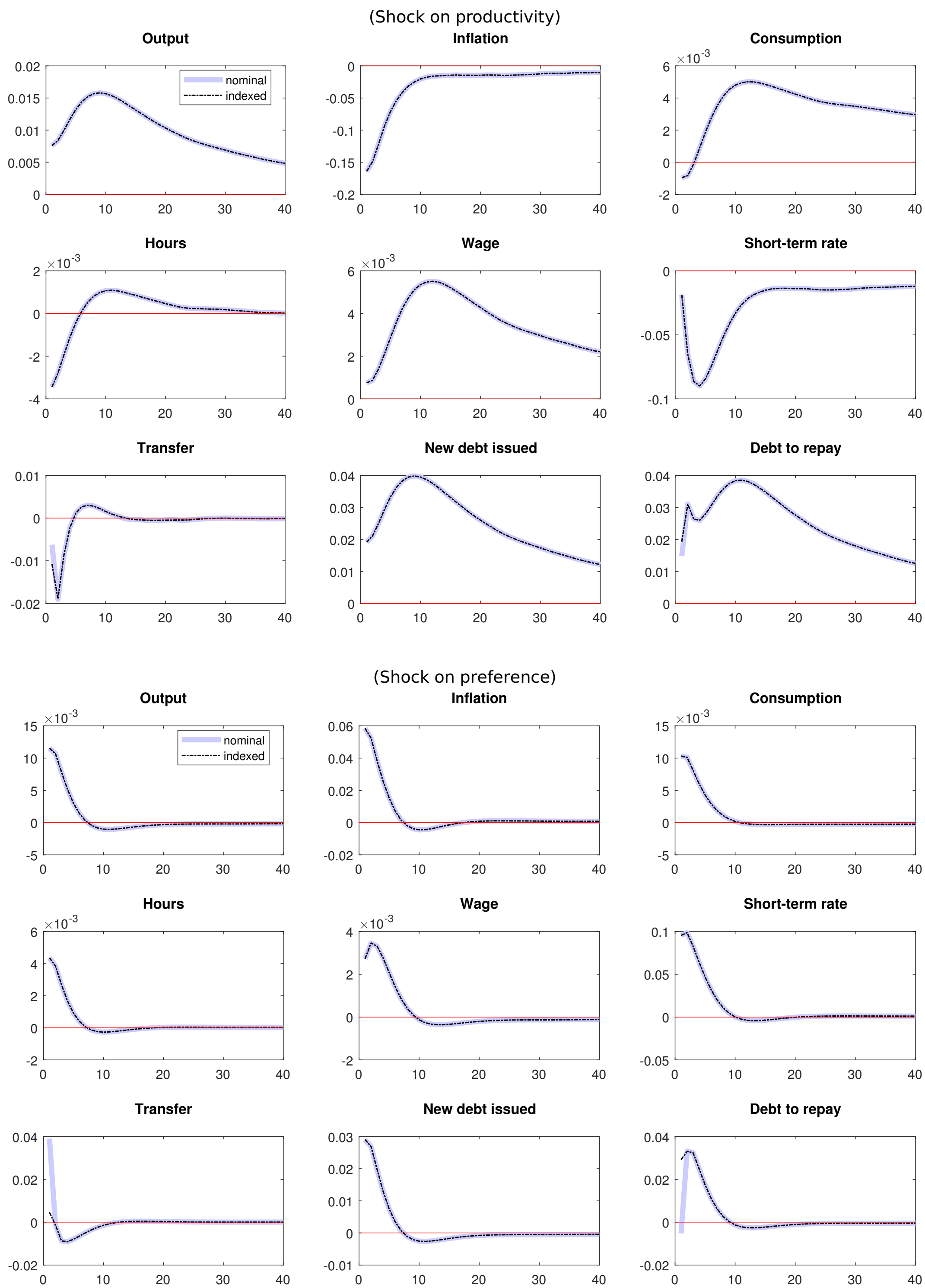

Figure 4.1: **IRFs for benchmark model (II)**

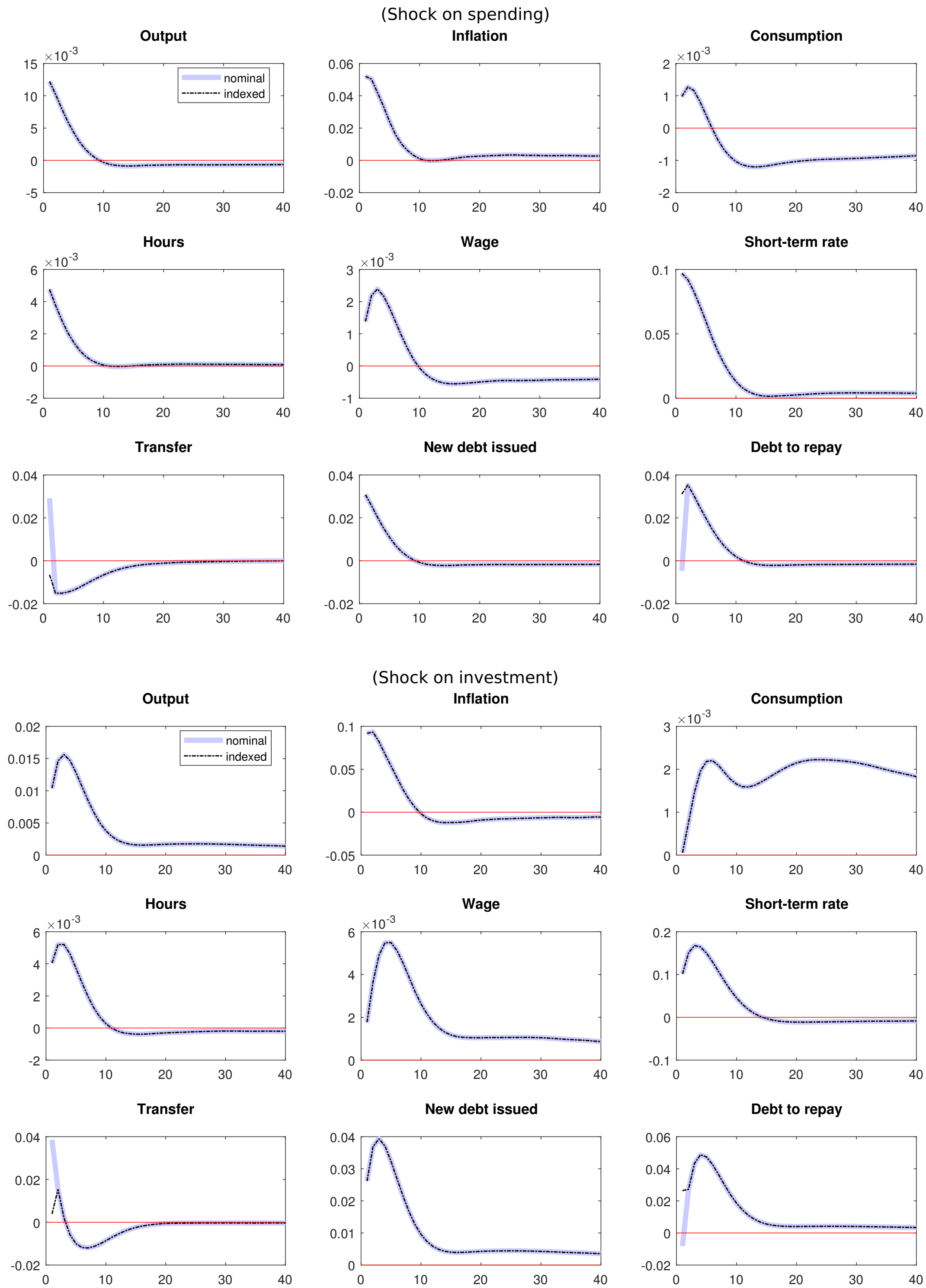

Figure 4.1: **IRFs for benchmark model (III)**

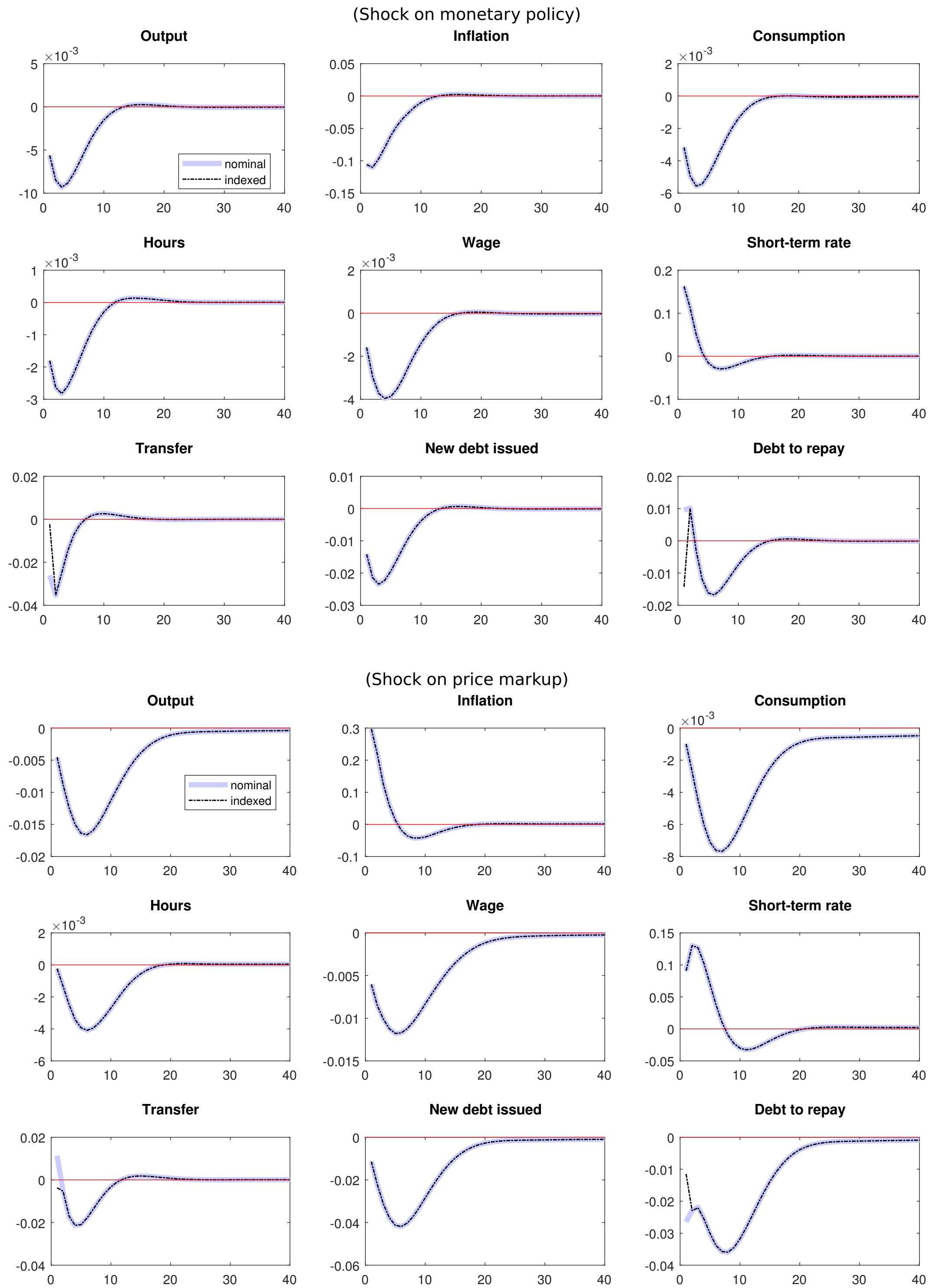

Figure 4.1: **IRFs for benchmark model (IV)**

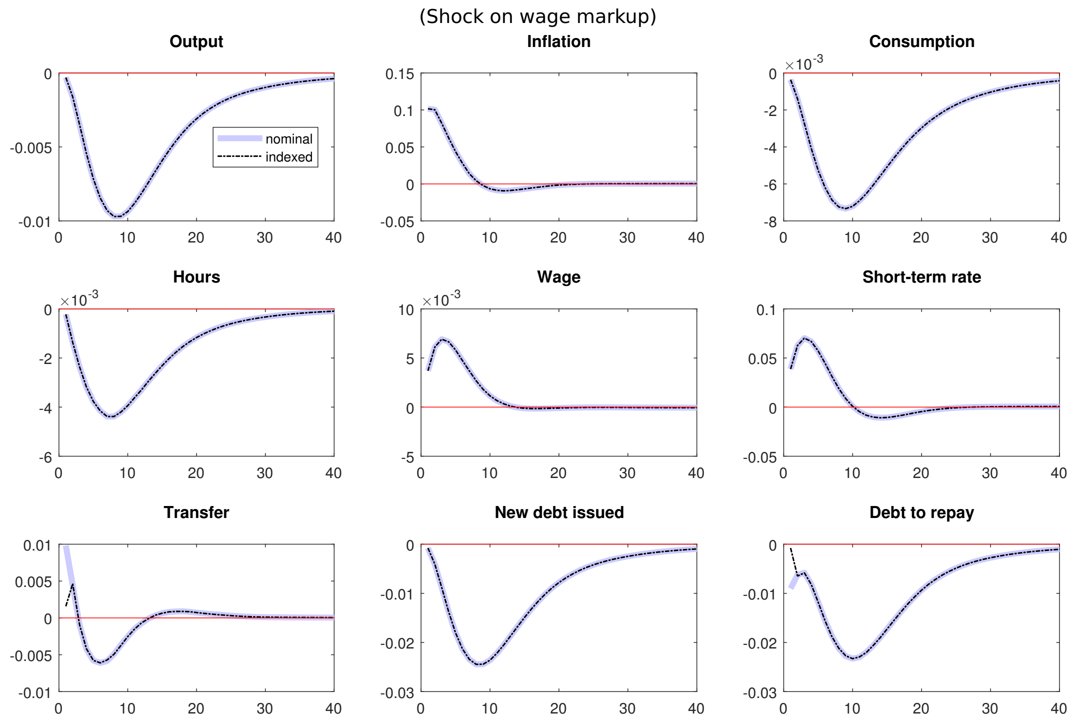

Figure 4.2: **IRFs for baseline model (I)**

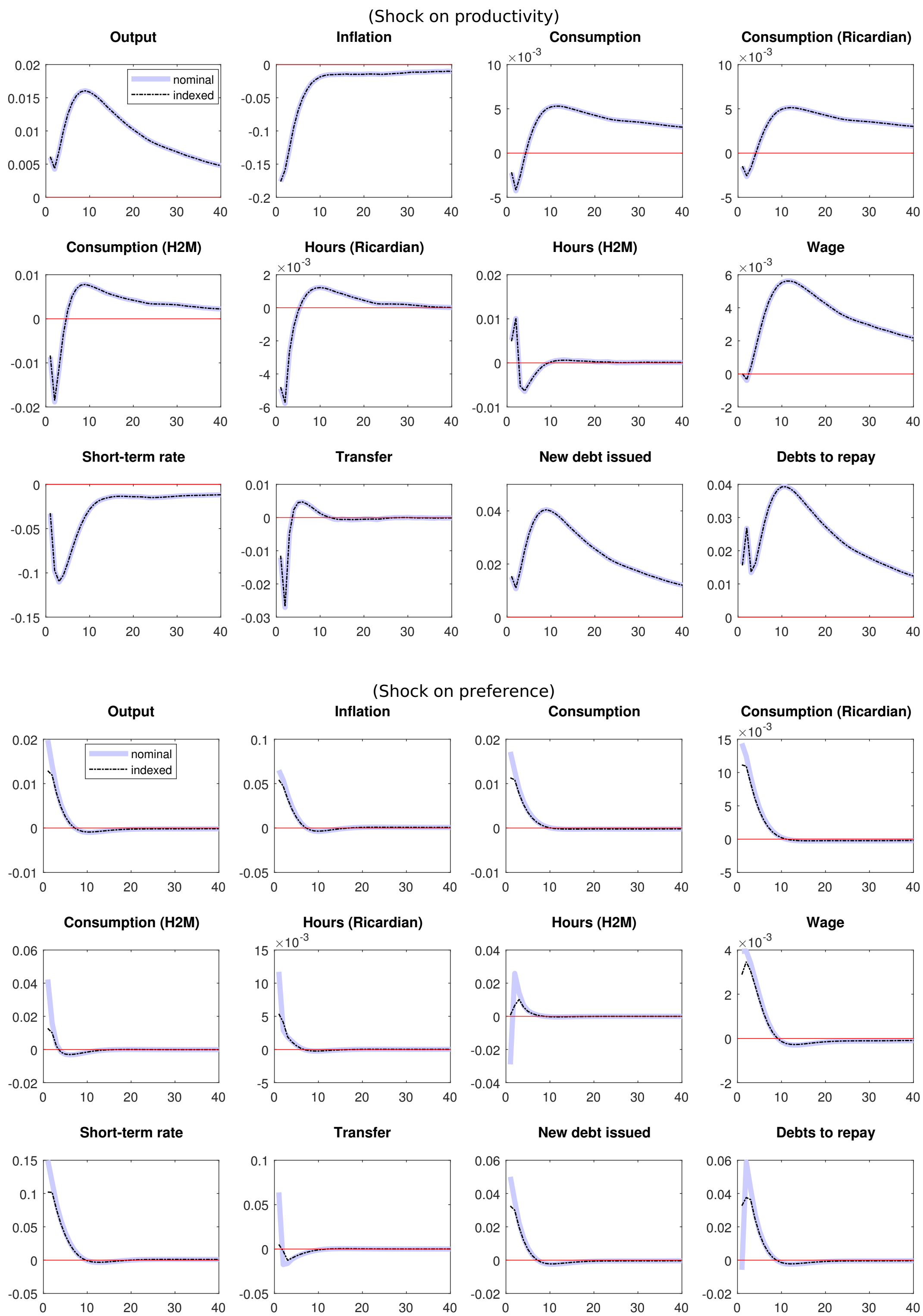

Figure 4.2: **IRFs for baseline model (II)**

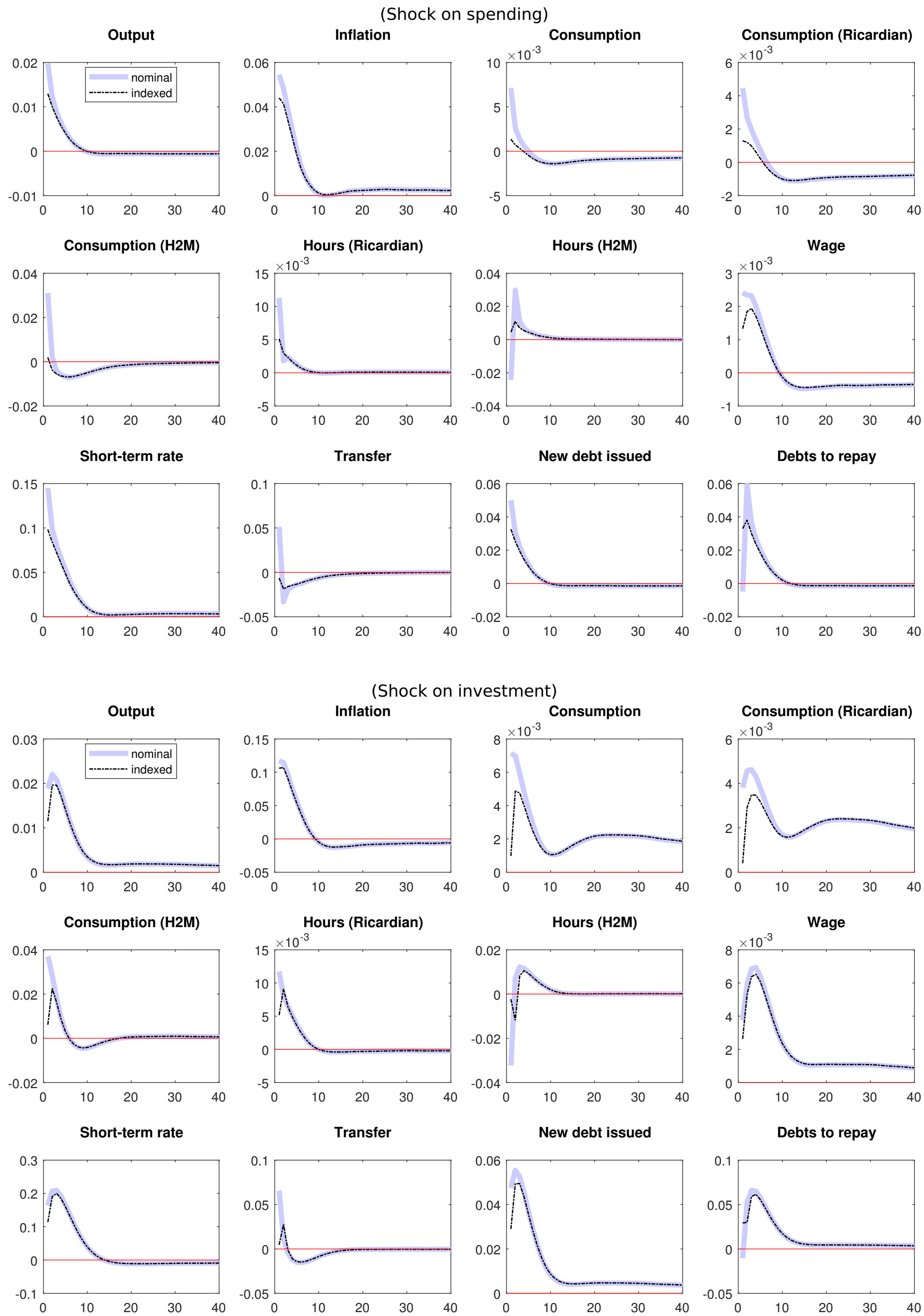

Figure 4.2: **IRFs for baseline model (III)**

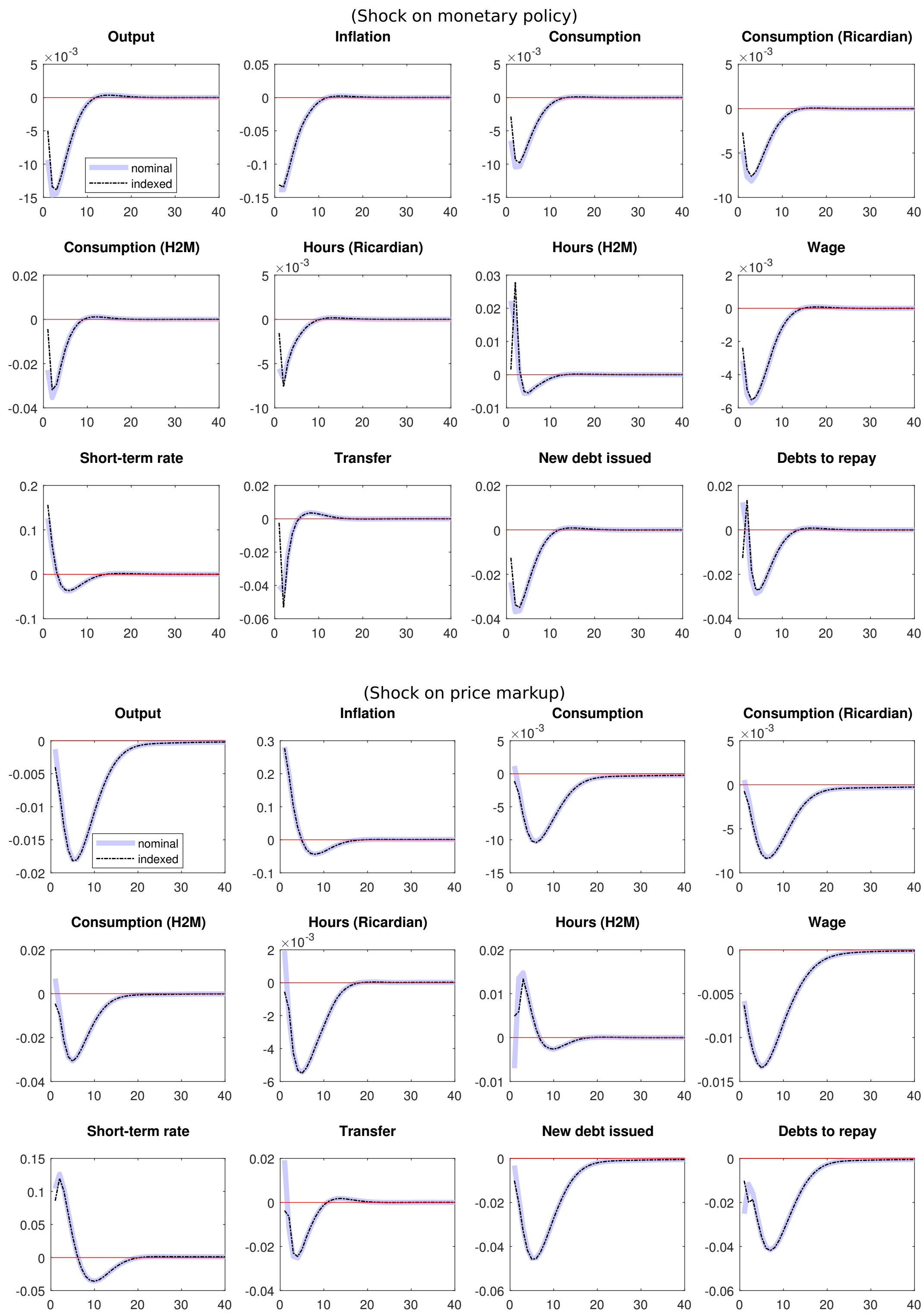

Figure 4.2: **IRFs for baseline model (IV)**

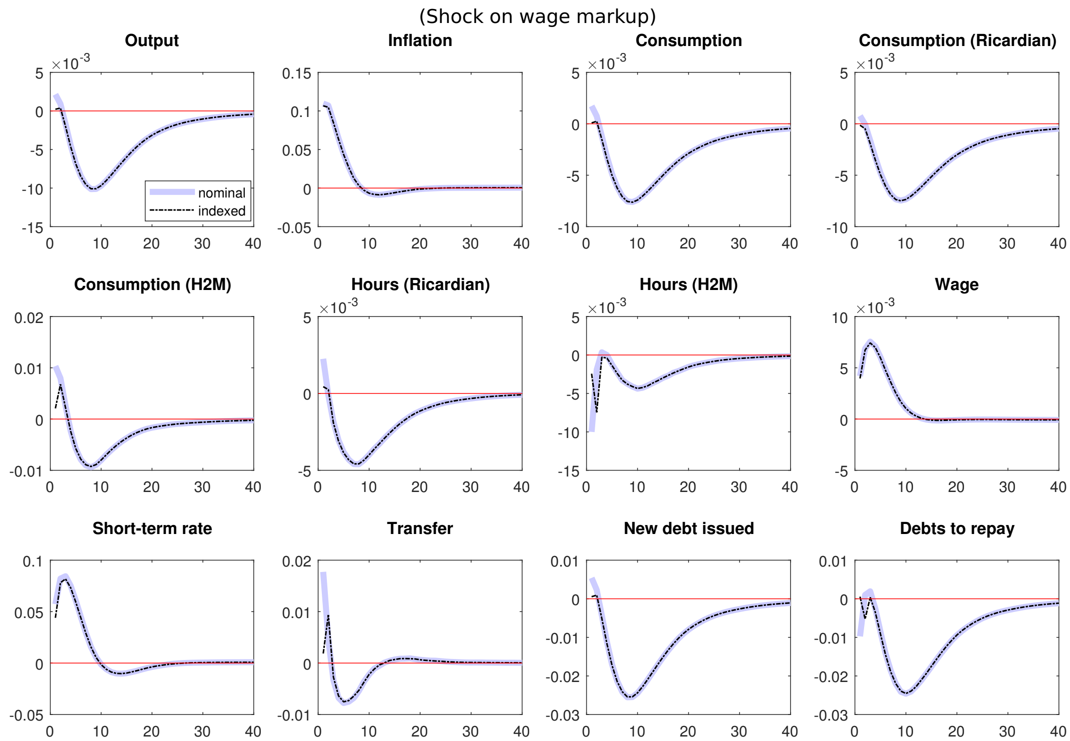

Figure 4.3: **Destabilising effect from the presence of hand-to-mouth households**