A unified solving approach for two and three dimensional coverage problems in sensor networks

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Abstract

The problem of designing a wired or a wireless *sensor network* to cover, monitor and/or control a region of interest has been widely treated in literature. This problem is referred to in literature as the *sensor placement problem* (*SPP*) and in the most general case it consists in determining the number and the location of one or more kind of sensors with the aim of covering all the region of interest or a significant part of it.

In this paper we propose a unified and stepwise solving approach for two and three dimensional coverage problems to be used in omni-directional and directional sensor networks.

The proposed approach is based on schematizing the region of interest and the sensor potential locations by a grid of points and representing the sensor coverage area by a circle or by a circular sector. On this basis, the *SPP* is reduced to an optimal coverage problem and can be formulated by integer linear programming (*ILP*) models. We will resume the main *ILP* models used in our approach, highlighting, for each of them, the specific target to be achieved and the design constraints taken into account.

The paper concludes with an application of the proposed approach to a real test case and a discussion of the obtained results.

1. Introduction

A *Sensor Network (SN)* can be defined as a group of spatially distributed sensors, linked by wire or by a wireless medium, performing sensing and acting tasks [1]. The theme of *SN* represents an important challenging topic for many research communities, due to the large number of application fields where they are employed (security and surveillance, industrial diagnostic, climate and environmental control and monitoring, design of communication systems, etc.). Moreover the number of its applications is still growing, given also the large number and variety of available sensors, the emergence of new and consolidated technologies and the great diffusion of low-cost and more effective sensors. Concerning the last point, with reference to surveillance applications, let us think for example to the wide diffusion, occurred in the last years, of the Complementary Metal-Oxide-Semiconductor cameras (CMOS) with respect to the more expensive Charge-Couple Device cameras (CCD).

Particular interest has been devoted to the design of a *SN*, which is the problem of defining the optimal number and the location of one or more sensors of different kinds with the aim of entirely/partially covering a region of interest or important parts of it.

The meaning of the term *coverage* depends on the specific kind of sensors under investigation. Indeed, in case of security and surveillance applications, it expresses the monitoring and control capability of a *SN*, whereas in case of communication systems, for instance, it could express the coverage capability of an antenna/repeater or similar devices.

The optimal coverage problem for *SN* is referred to in literature as the *sensor placement problem* (*SPP*). It is a complex problem where a very large set of sensor configurations and combinations (in case we use more than one type of sensors) have to be explored in order to determine the most efficient one, guaranteeing the highest coverage level and the economical sustainability. As discussed in [2], it is also important to highlight that *SPP* arises in and influences all the network definition stages, from design and deployment to the operational phase.

In order to have a good insight into the *SPP*, it is important to acquire some technical basic issues about the sensors. Generally speaking, a sensor is a device which responds to physical stimuli and converts them into recordable signals which are then digitalized to produce sensing data [2]. In [3] the main available sensors are surveyed and classified in function of their technological equipment (video, infrared and ultrasound sensors). However, regardless of the technological differences among the sensors, in the *SPP* literature the sensors are classified with respect to two main parameters: *coverage area* and *coverage function*. By *coverage area*, we mean the portion of the region of interest that can be covered by a sensor. By *coverage function* we mean a function expressing the geometric relation between a point to be covered and a sensor [2].

With reference to the first parameter, we can classify the sensors in two main groups: *omni-directional sensors* (*OS*) and *directional sensors* (*DS*). For the first group, in a two dimensional space, the *coverage area* can be schematized by a complete circle, for the second one, by a circular sector.

With reference to the second parameter, we can classify the sensors in two groups: *boolean coverage function sensors* and *general coverage function sensors*. The sensors of the first group are characterized by a coverage function assuming just values 0 and 1. The sensors of the second group are characterized by a coverage function assuming all the values in R_0^+. This function can express, for instance, the relation between the monitoring capability of a sensor (or its estimated coverage probability) and the distance between the sensor and the point to be covered. For a review of the main coverage functions, the interested reader is addressed to [2].

Given the previous sensor classification and from the literature review, we can affirm that a simple and straight categorization of the *SPP* can be done with respect to the following three main issues:

1. performance criteria adopted as objectives of the *SPP*;
2. dimensions of the region of interest, two-dimensional (*2D*) or three-dimensional (*3D*);
3. kinds of used sensors (*OS* and *DS*) and related coverage functions.

Concerning the first issue, Guvensan and Yavuz [3] surveyed the main contributions on the *SPP* considering four different objectives:

- targeted-based coverage solutions;
- region based coverage solutions;
- coverage solutions with guaranteed connectivity;
- network lifetime prolonging solutions.

In this work we will focus on the region based coverage solution. In literature, to the best of authors' knowledge, few contributions tackled the *3D* case or proposed general approaches to be used for both the *2D* and *3D* cases. In particular, most of the literature on the *SPP* is focused on the *2D region based coverage solutions* by both *OS* and *DS* with boolean coverage functions. The focus on the *2D* case, even if the sensors are generally deployed in three dimensional regions, can be explained by the higher complexity of the analysis and design problems imposed by the *3D* case. Moreover, the *2D* case well fits with the largely adopted idea of representing the region of interest by a grid of points, so reducing the *SPP* to a variant of the set covering problem (*SCP*). In the following we will refer to these contributions as *grid coverage based approaches*.

In this work we propose a simple unified grid coverage based approach to be used in the design of wired and wireless omni-directional (*OS)* and/or directional sensor *(DS)* networks for the coverage of *2D* or *3D* regions. Both the *OS* and *DS* will be characterized by boolean coverage functions. Our focus is on the coverage criterion, hence we do not consider the other design objectives arising for the *SPP*, related for example to the battery usage or coordination of the sensors, which could be relevant as well. Indeed within an integrated surveillance system, the complementary usage of ad-hoc activators for the deployed sensors is fundamental to design an energy efficient monitoring system. Let us consider, for example, the complementary usage of magnetic sensors deployed at the doors of an asset/room in order to activate a volumetric or a visual sensor.

This paper extends and integrates the approach presented in [4] and [5], where a *2D* grid coverage based approach is used for the design of a railway security baseline system, with *OS* and *DS*, in presence of occlusions. As it will be clearer in the following, the approach is based on a stepwise structure which allows to easily take into account additional features of the problem under investigation and can be integrated with more effective solution methods for the arising coverage problems. The approach has a modular structure which can be easily scaled in accordance with the size of the system under investigation.

Given that the present literature on the *SPP* is quite heterogeneous, this paper is also intended to provide a unifying framework which puts together *SPP* contributions coming from different research fields, aiming to be a platform for further developments and contributions.

The work is organized as follows. In *Section 2* we provide a short review of the main contributions present in literature for the *SPP* grid coverage based approaches. In *Section 3*, we present the proposed unified approach, providing a detailed description of the steps composing it and resuming the main *ILP* covering models to be used for the *SPP*. For each of them we highlight the specific target to be achieved and the design constraints taken into account, giving some hints to introduce additional specific features of the sensors. Finally, in *Section 4*, we present the results obtained by the proposed approach on a real test case related to one of the main station of a railway company operating in the western area of Naples district.

2. Literature review

At first the *SPP* has been considered as a variant of the art gallery problem (*AGP*), introduced in [6]. The *AGP* consists in opportunely distributing the minimum number of guards in an area such that all its points are observed. For a review of the main contributions on the *AGP* the interested reader is referred to [7] and [8]. However the basic assumptions of the *AGP* are unrealistic for both *OS* and *DS*. Indeed the guards have an unlimited omni-directional monitoring capacity. For this reason, after these first attempts, the *SPP* has been later treated by grid based approaches as a variant of the set covering problem (*SCP*), tackling it by optimal coverage integer linear programming (*ILP*) models and related exact and heuristic solution methods. Concerning the *SCP*, a review of the main contributions on covering problems can be found in [9] [10], [11], and [12]. In [10] the authors present several coverage heuristics and metaheuristics for path coverage problems, which could be easily adapted to the covering problems that we are going to present in this paper. For a complete review on the *SPP*, the interested reader is addressed to the survey works by Wang [2], Guvesan and Yavuz [3], and Mavrinac and Chen [13]. In the following, for the sake of the brevity, we will just summarize the main recent contributions on the grid based approach adopted in our work.

In [14] the authors tackle and solve by *ILP* models the optimal *OS* placement problem, taking into account sensors with different coverage functions and costs. Moreover, they also treat the problem of determining a sensor placement where each grid point is covered by a unique subset of sensors. In [15] the authors extend the work of [14], introducing the sensor detection probability. Then they solve the problem of locating the minimum number of sensors guaranteeing that every grid point is covered with a minimum confidence level. In [16] the authors further extend the idea proposed in [14] and tackle the *DS* location problem with the aim of coverage maximization with a certain resolution.

After these works, most of the literature focused on integrating additional features in the problem in order to take into account operational constraints of a sensor network. The work presented in [17] tackles the problem of locating *DS* in a region of interest characterized by the presence of occlusions and proposes an original *ILP* model where the points of the region are opportunely weighted in function of their importance. The orientation of *DS* within a *2D* plane is explicitly taken into account in [18]. The positioning error bounds are taken into account in [19]. The error bound concept and other operational constraints are then further investigated in [20].

Concerning the *3D* case, as discussed above, few works explicitly take it into account, and among them we just cite [21], where the problem is treated with reference to a *3D* region in an urban environment to be monitored by *OS*. The problem is solved by integer linear programming models reducing it to a two dimensional coverage problem.

3. A unified approach for the *SPP* in *2D* and *3D* regions

The main target of this paper is to provide a general framework in order to deal with *SPP* arising from different fields. The proposed approach adopts a modular structure which can be easily used for the *2D* and *3D* coverage, using also different kinds of sensors. Moreover it can be easily integrated, updated and modified. This makes the general framework of the methodology a valuable tool to be employed for designing a monitoring *SN* system. Moreover the focus is on the *3D* coverage case, since it has been scarcely treated in literature. Indeed, most of the papers present in literature, devoted to the *3D* coverage, reduce the problem to monitoring just one plane within a *3D* region of interest.

When tackling the *SPP* as a coverage optimization problem by a grid based approach, the following steps have to be performed (see also [4], [17], [21]):

1. Discretization of the region of interest.
2. Definition of the potential sensor locations.
3. Sensor coverage area schematization.
4. Coverage analysis.
5. Modeling of the coverage problem and choice of the solution approach.

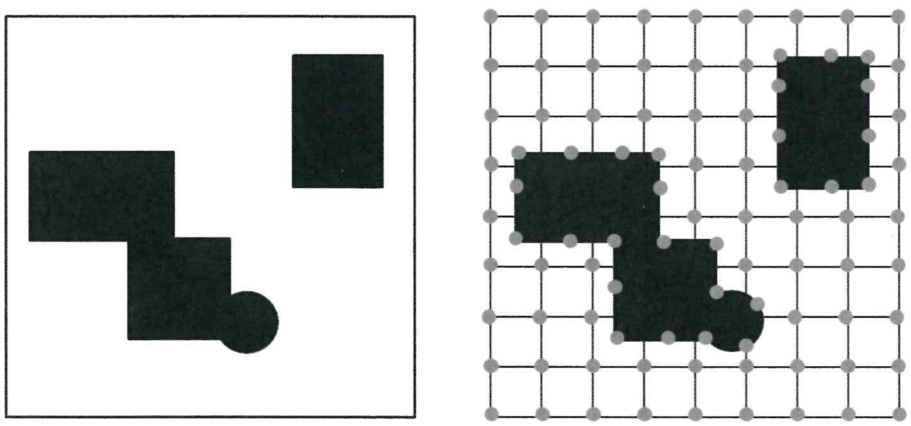
The coverage approaches proposed in literature for the *2D* and the *3D* cases differ in the way one or more of these steps are performed. In the following we provide a detailed description of each step.

3.1. Discretization of the region of interest

The discretization of the region of interest (*RI*) is based on a widely adopted procedure which overlaps a grid with step size *k* on the map of the asset under investigation. *Figure 1* reports an example of a two dimensional *RI* with obstacles (*Figure 1.a*) and of the related discretization (*Figure 1.b*). In the following the set of the grid points (in either the *2D* or *3D* case) will be referred to as *G*. It is important also to underline that, since the sizes of the *RI* and of the occlusions could not be a multiple integer of the step size *k*, then the set *G* is integrated with all the points of their boundaries (using the same step size *k*) and their corner points. Obviously, the smaller is the step size *k* the higher the quality of the grid representation and cardinality of *G*.

In order to have a complete representation of the *RI*, geometrical information about its shape, dimensions and presence of obstacles (in the following referred to as *occlusions*) are required. The only assumption about the asset and the occlusions is that their shapes can be reconstructed using elementary shapes (i.e. squares, rectangles, circles, etc.). Even if this assumption can be considered quite strong, it allows to simply obtain the discretization of unusual shapes, near to the ones that we can find in real applications. Moreover it is not context dependent, and can be easily performed by the usage of widely adopted tools, as discussed in [4, 5].

On this basis, a continuous three dimensional *RI* can be discretized just building a grid with a given step size *k* on several parallel planes having different heights with respect to the ground floor of the *RI*. The set of the grid points belonging to all the *2D* planes can be assumed as a *3D* grid. The choice of the plane height values and, consequently, of the number of parallel planes to be considered, has to be done in dependence of the specific region under investigation and of the monitoring tasks to be achieved. In a *3D* region, if the asset and/or the obstacles have particular shapes varying along the height, we just need to define the elementary shapes describing them at each height value.



(a) *(b)*

**Figure 1. a) Two-dimensional region of interest with occlusions.
b) Discretization of the region of interest and boundaries of the obstacles.**

It is important to highlight that the asset discretization is a fundamental and critical activity for this kind of approach, and could also require a significant effort. However, given the kind of the needed information (mainly geometrical), it can be easily replicable and could be performed using *ad hoc* graphical tools which take into account the specific geometries of the asset and occlusions (e.g. GIS for external spaces and CAD for internal ones).

3.2. Definition of the potential sensor locations

With the expression *sensor potential location* we refer to a specific *5-tuple* ($\boldsymbol{p}$, r, θ, α, β), representing a possible location of a sensor in the *RI,* where:

- $\boldsymbol{p}$, *sensor position*: vector expressing the coordinates $(\hat{x}_p, \hat{y}_p, \hat{z}_p)$ of the sensor, in a *3D* region of interest, with respect to a three dimensional reference coordinate system with its origin in one of the corners of the *RI.*
- r, *coverage ray*: maximum distance (expressed in meters) that a sensor can cover remaining still effective. For the video sensor it is the distance which guarantees to have sharp shapes.
- θ, *coverage angle*: width of the circle sector (expressed in degrees) that the sensor can cover remaining still effective.
- α, *orientation angle:* angle (expressed in degrees) between the horizontal axis and the sensor working direction in a three dimensional reference coordinate system with its origin in the position $\boldsymbol{p}$ of the sensor.
- β, *tilt angle*: angle (expressed in degrees) between the z axis and the sensor working direction with respect to a three dimensional reference coordinate system with its origin in the position $\boldsymbol{p}$ of the sensor.

The characterizing elements of the *5-tuple* can be integrated with additional parameters which, in some cases, have to be explicitly taken into account, since they significantly affect the size and the shape of the area covered by a sensor. Among them we cite here two additional and important features when dealing with video sensors:

- *Spatial Resolution (SP)*: ratio between the total number of pixels on its imaging element excited by the projection of a real object and the object size.
- *Hyperfocal distance* (*HD*): closest distance at which a lens can effectively work, producing sharp images. It changes the *CA* of a sensor in an annulus or in a sector of an annulus.

In the following sections we will provide more details about the way to take into account these two features in the proposed unified approach.

The *5-tuple* elements of a *DS* with and without *HD* effect and a representation of the area covered by the sensor are represented in *Figure 2* (sensor working direction is represented by a dotted line).

On this basis, the set of the sensor potential locations within a *RI* corresponds to all the possible *5-tuples* that we can obtain changing the values of their elements. This set will be referred to as *L*.

The values of r and θ are intrinsic technological characteristics of the sensor, hence their value just depend on its kind, quality and cost. The other parameters can instead vary and, in the most general case, they could assume all the values in the range where they are defined, but in the grid based approaches a discretization of these values is performed. In particular:

- For the coordinate vector $\boldsymbol{p}$, the set of potential positions of *DS* and *OS* is obtained building a grid of points in the *RI*. The grid can have the same or a different step size with respect to the one used for the *RI* discretization. This set is then integrated by adding also all the points placed along edges and corners of the occlusions. In some cases other specific positions should be added in order to satisfy specific monitoring tasks.
- For the orientation angle α and the tilt angle β, we will not consider all the possible values of these angles, but just the ones obtainable defining two step values, δ_α and δ_β respectively. Depending on these values, for each position *P* of each sensor, we will have different orientation and tilt angles, so generating overlapping and non-overlapping coverage areas. More precisely, with reference to δ_α, if $\delta_\alpha = \theta$, we generate just values of α which are integer multiple of θ and consequently the coverage areas are non-overlapping. On the contrary, if $\delta_\alpha < \theta$ (for simplicity a fraction of θ), overlapping coverage areas are generated. Regarding the step value δ_β, it can be chosen in the range [0° ÷ 360°], depending on the kind and on the specific targets of the *SN*.

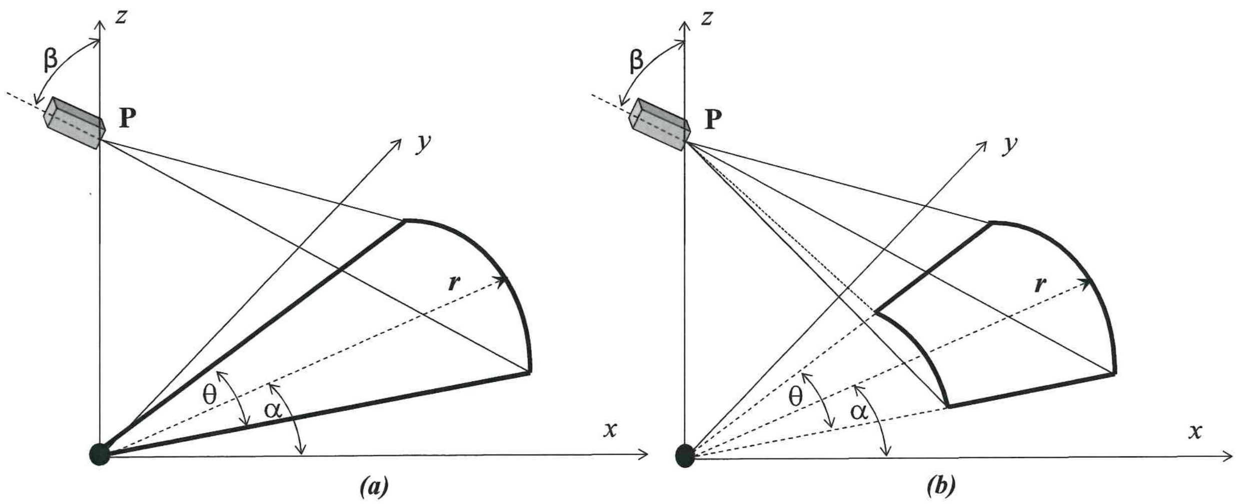


Figure 2. ***5- tuple*** **of a sensor: a) not affected by** ***HD*** **effect; b) affected by** ***HD*** **effect.**

As occurred in the discretization of the *RI*, it is easy to understand that the lowest the step size used for $\boldsymbol{p}$, α and β, the greatest the number of the solutions within the solution space. On the other side, higher values of the step size reduce the number of solutions to be explored, but also the quality of the *SN* system to be designed. Hence we have to find a good compromise taking into account the technological characteristics, the operational constraints and the dimensions of the *RI*.

3.3. Sensor coverage area schematization

As discussed above, the number and variety of *OS* and *DS* (differing for cost, equipment, performance, function, etc.) is significant, but one of the main features to take into account in *SPP* is the representation/schematization of the *coverage area* (*CA*) of a sensor. The *CA* can be defined as the portion of the *RI* that a sensor is able to cover. When tackling *2D* and *3D* coverage problems, the *CA* of both *OS* and *DS* can be obtained by the information contained in the *5-tuple* described in the previous section.

By the observation of *Figure 2*, it is possible to note that, in coverage problems, the width of the coverage area is determined by θ, but the specific portion of the *RI* that a sensor is able to cover strictly depends on the height, orientation and tilt angle values of the sensor. In most of the *2D* coverage problems treated in literature these aspects are missing since the height of the sensor is fixed and the tilt angle (β) is not taken into account. Indeed in most of the *SPP* contributions the *CA* of a sensor is schematized by a circle (for *OS*) or by a circular sector (for *DS*). These representations are generally obtained considering the sensors as placed on a plane and disregarding their real position in a *3D* space. In the following we will still use these schematizations of the *CA*, but we will also take into account the real position of the sensor in a *3D* space. It is easy to understand that *OS* in both *2D* and *3D* can be considered as a special case of the *DS* where the coverage angle θ is equal to 360° and both the orientation and tilt angle can be disregarded.

On this basis note that the *5-tuple* basically describes the sensor functioning mechanism, in both *2D* and *3D* cases, just using the position of the sensor and geometrical information which in general can be derived by the sensor technical sheet. Hence the construction of the coverage area is an easily replicable operation, regardless of the sensor technology.

3.4. Coverage Analysis

Coverage analysis consists in determining which are the points of the grid G that can be controlled by a sensor positioned at a given potential location (with a certain orientation and tilt angle). In the following this set of points will be referred to as S, $S \subseteq G$.

The coverage analysis has to be performed for each potential location of the set L and it can be done in two ways:

1. *Geometrical coverage*: the sub-set S for each potential location of a sensor is built without taking into account the presence of occlusions in the *RI*.
2. *Physical coverage*: the set S, determined by the geometrical coverage analysis, is filtered considering the presence of occlusions which can interdict the activity of the sensor. Hence a set $S' \subseteq S$ is generated, taking into account the coverage area of the sensor and the shapes and the sizes of the occlusions.

The concepts of geometrical and physical coverage apply to both the *2D* and *3D* cases. In the following we will describe the algorithmic procedure used for the *2D* coverage analysis and then we will give indications about the way it is modified for the *3D* case.

3.4.1. Coverage Analysis in a *2D* region of interest

In *Figure 3* we show a small example of discretization of a *2D* region of interest and the difference between the geometrical coverage and the physical coverage in case of presence of an occlusion. It is easy to observe that the red points cannot be covered because of the occlusion.

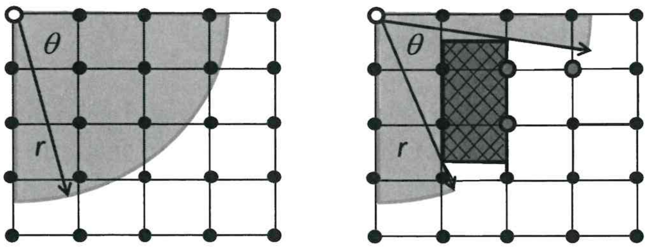


Figure 3. Geometrical and physical coverage of a sensor.

The geometrical coverage analysis, can be performed as follows: given a *5-tuple* characterizing a sensor, if a point falls within the coverage angle θ and the Euclidean distance between the sensor and the point to be covered is lower than or equal to the coverage ray r of the sensor, then the point is covered. In case, as for video sensors, the sensor coverage function is affected also by *hyperfocal distance* (*HD*) *effect* (as defined above), then it is also required that the Euclidean distance between the sensor and the point to be covered has to be equal to or higher than a pre-fixed value of HD. In this situation, it is important to note that the *CA* of an *OS* or a *DS* becomes an annulus or a sector of an annulus.

In case of occlusions which can interdict the normal functioning of the sensor, then it is necessary to perform the physical coverage as well. We recall that, in our approach an occlusion is generally schematized by a rectangle or by composing elementary shapes. To do this an algorithmic procedure has been developed. For the sake of the clarity, let us consider the example reported in *Figure 4*.

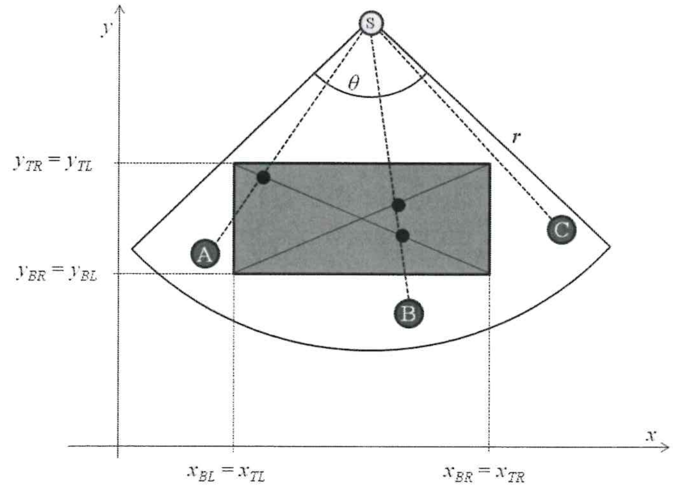


Figure 4. Physical coverage in a *2D* region of interest.

Let us define as a the straight line connecting the sensor location and the point to be covered and as d' and d'' the diagonals of the rectangle used to represent the occlusion. Finally, let us define $[x_{BL}, y_{BL}; x_{BR}, y_{BR}; x_{TL}, y_{TL}; x_{TR}, y_{TR}]$, respectively the coordinates of the bottom left and right corners and of the top left and top right corners of the occlusion. We have to check if the coordinate of the intersection points between a and d' and a and d'' are within the ranges defined by these four coordinates. If at least one intersection point is within this range (i.e. it falls within the rectangle representing the occlusion) then the point cannot be covered. If instead no intersection falls within the rectangle, then the point can be covered by the sensor. The different situations that can occur in case of presence of an occlusion are represented in *Figure 4*, where point A and B cannot be covered since at least one intersection point falls within the occlusion, whereas point C can be covered since no intersection falls within the rectangle. The proposed algorithm for the coverage analysis can be easily extended to other elementary and convex geometrical shapes.

3.4.2. Coverage analysis in a *3D* region of interest

It has been highlighted that the *RI* to be covered is really a *3D* space, but we restrict our analysis to a single plane in this *3D* space, hence performing a *2D* coverage analysis. Thus we can use the plane of the ground floor or another more significant plane within the *RI*. Moreover, as shown in *Figure 5*, generally the plane of the grid point set G and the plane of the grid point set L can be different. These statements are intuitive and can be easily confirmed by the sample case where a room has to be monitored by video sensors. Indeed, in this case it would be more effective to cover a plane which is orthogonal to the *vertical* axis in correspondence of a certain height (for example *1,70* meters) rather than at the ground floor, choosing a plane at a higher height to place the sensors. This assumption does not always hold in real applications and for all the sensors, but for specific covering targets it can be effective and significantly reduce the size of the problem under investigation (as explained in [20]).

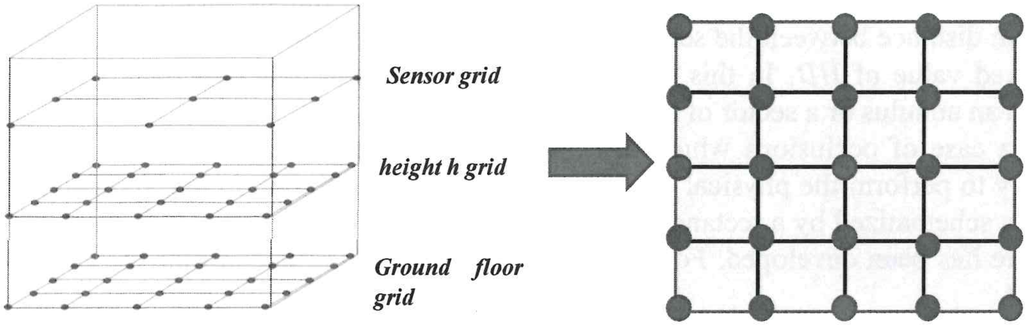
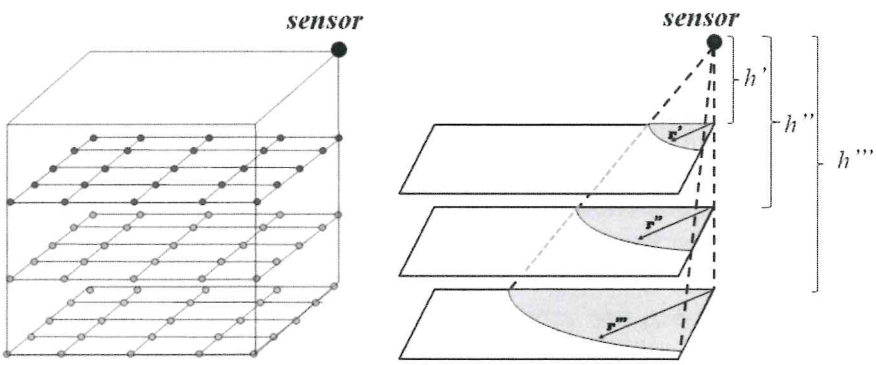


Figure 5. Grid point set *G* and grid point set *L* in a *3D* region and its projection on a *2D* plane.

Hence, in our unified approach the coverage analysis in a *3D* region of interest is performed by extending the algorithmic procedure presented in the previous section. More precisely, given a sensor identified by its *5-tuple*, we replicate the *2D* coverage analysis for each plane which is orthogonal to the z axis in the three dimensional reference coordinate system with its origin in a corner of the region of interest.

In order to perform this operation we use the following procedure. Let us consider a sensor characterized by a specific *5-tuple* $(\boldsymbol{p}, r, \theta, \alpha, \beta)$ represented by a black dot in *Figure 6.a*, where a *3D* region grid with three parallel planes to be covered is shown. We perform the *2D* coverage analysis on each plane orthogonal to the vertical axis, maintaining fixed all the values of the *5-tuple*, with the only exception of r, which decreases with the height of the plane. The value of r related to each plane can be easily computed just using the proportionality relations of a triangle as indicated in *Figure 6.b*.



(a) *(b)*

Figure 6. a) Discretization of a *3D* region grid using three parallel planes.
b) Multiple *2D* coverage analysis in a *3D* region.

3.4.3. Coverage matrix

The coverage analysis allows to generate the *Coverage Matrix* (*C*), i.e. a two dimensional matrix with L rows and G columns, which is the fundamental input for all the covering *ILP* models used in the proceeding of this work and related exact and heuristic solving methods. The value of the generic element c_{ij} of the C matrix is a measure of the sensor *coverage function*. Different coverage functions have been defined in [2], but as already discussed in *Section 1*, we just focus on the boolean coverage function. Using this function the generic element c_{ij} of C is equal to 1 if a sensor i, $i \in L$, characterized by its *5-tuple* can cover the point j, $j \in G$, it is equal to 0 otherwise.

When we define the coverage matrix, we can also consider in our problems the above introduced issue related to the spatial resolution of a video sensor. Indeed this can be easily done considering that, in order to have a c_{ij} value equal to 1, the sensor i, $i \in L$, has to cover ("geometrically" and "physically") the point j, $j \in G$, guaranteeing also a minimum pre-fixed value of the spatial resolution (where the *SP* is function of the distance between i and j and of the technological features of the sensor).

It is important to highlight here the differences, in terms of dimensions, between the coverage matrix obtained in the *2D* and *3D* cases. In both cases the number of the rows (L), i.e. the potential sensor locations, is determined by all the possible *5-tuples* that we can obtain changing the parameters with the pre-fixed step sizes. Concerning instead the number of the columns (G), i.e. the points to be covered, the cardinality of this set significantly increases in the switch from the *2D* to the *3D* case. Indeed, let G' be the set of points belonging to the grid of a single plane and N the number of planes orthogonal to the z axis. In the *2D* case, since we choose just one plane, $G = G'$ and hence C is an $L \times G$ matrix. Instead, in the *3D* case, since we have to take into account all the planes within the region of interest, then $G = G' \times N$ and C is an $L \times (G \times N)$ matrix, which can be effectively obtained juxtaposing the N matrices G' of the planes. In the following we will provide some considerations which can allow to reduce the size of the coverage matrix when tackling specific coverage problems in *3D* regions.

3.5. ILP coverage models

Covering problems used for the *SPP* can be classified in two main categories depending on the specific targets:

- Minimization of the number (or total cost) of the sensors to be installed with the aim of covering all the points of the *RI* under investigation or just important points of it;
- Maximization of the number of *RI* points covered by one (primary coverage) or more (multiple coverage) sensors with a constraint on the number of sensors to be installed.

Given this classification, four main covering problems can be individuated in literature, two for each category:

- ***Set Covering Problem* (*SCP*).** In this problem all the points of the *RI* have the same importance and have to be covered. Hence it consists in determining the optimal placement of the sensors which guarantees the coverage of the entire *RI*, minimizing the total installation cost [22].

- ***Weighted Demand Covering Problem* (*WDCP*).** In this problem we classify the points of the *RI* in two groups: important and non important points. All the important points have to be compulsorily covered. On the other side, the non important points are covered if they are within the coverage area of a sensor located to monitor at least one important point, or by a dedicated sensor if it is able to monitor a pre-defined minimum number of uncovered non important points. Hence it consists in determining the optimal placement of the sensors which guarantees a good trade-off between the coverage of all the important and non important points of the *RI*, minimizing the total installation cost [17].

- ***Maximal Covering Problem (MCP).*** In this problem all the points are characterized by a weight which expresses the importance of the point within the *RI*. Each point can be uncovered or covered by one or more sensors. Hence it consists in determining the optimal placement of a pre-fixed number of sensors, maximizing the weighted sum of the covered points [23], considering that multiple coverage of a point is allowed, but it is counted just once.

- ***Back-up Covering Problem (BCP).*** In this problem, as in the *MCP*, all the points are characterized by a weight, expressing their importance, and can be uncovered or covered by one or more sensors. Anyway, differently from the *MCP*, *multiple* (more precisely, *double*) counting of a covered point is allowed. In order to do this, we distinguish between the *primary coverage* and the *multiple/secondary coverage*, i.e. the coverage achieved when a point is monitored just once by a single sensor and the coverage achieved when the same point is monitored also by other sensors, respectively. Hence it consists in determining the optimal placement of a pre-fixed number of sensors maximizing the weighted sum of the primary and multiple covered weighted points [24].

It is important to underline that the concept of *important points* introduced for the *WDCP* is fundamental when dealing with security and safety problems, regardless of the specific target to be achieved in the *SN* system design. Indeed, some points as lifts, elevators, steps, etc., have to be compulsorily checked and, in many cases, they require the installation of dedicated sensors. Hence, in our approach, we guarantee the coverage of the important points independently of the covering problem to be tackled. In the following we will present just the *WDCP* and the *BCP* problems and related *ILP* models, since, as we will show, the *SCP* and *MCP* are special cases of the *WDCP* and *BCP* respectively.

3.5.1. Weighted Demand Covering Problem (*WDCP*)

In the formulation of the *WDCP*, the following sets and parameters will be adopted:

- $G = \{1,..,\|G\|\}$	set of points representing the *RI*;
- $L = \{1,.., \| L\|\}$	set of *5-tuples*;
- s_j	flag value defined for each j, $j \in G$. It is equal to *1* if the point j has to be compulsorily covered, 0 otherwise;
- h_i	installation cost of a sensor characterized by the *5-tuple* i, $i \in L$;
- γ	parameter between 0 and 1 regulating the placement of a new sensor in case a "significant" number of general points of the *RI* is controlled.

Moreover the following variables will be used:

- $y_i = \{0,1\}$:	binary variable associated to each *5-tuple* i, $i \in L$. It is equal to *1*, if a sensor i is installed, *0* otherwise.
- $x_j = \{0,1\}$:	binary variable associated to each point j to be covered, $j \in G$. It is equal to *1* if the point j is covered, *0* otherwise.

Given this problem setting and variables, the *WDCP* can be modeled as follows:

$$Min\ z = \sum_{i \in L} h_i y_i - \gamma \sum_{j \in G} (1 - s_j) x_j \qquad (1)$$

s.t.

$$\sum_{i \in L} c_{ij}\, y_i \geq 1 \qquad \forall j \in G \mid s_j = 1 \qquad (2)$$

$$\sum_{i \in L} c_{ij}\, y_i \geq x_j \qquad \forall j \in G \mid s_j = 0 \qquad (3)$$

$$y_i = \{0,1\} \qquad \forall i \in L \qquad (4.a)$$

$$x_j = \{0,1\} \qquad \forall j \in G \qquad (4.b)$$

The objective function (1) is composed by two terms. The first term minimizes the total installation cost of the sensors. The second term tries to locate additional sensors if their installation increase the number of controlled non important points of a minimum threshold value defined by the parameter γ. Constraints (2), impose that each important point j, $j \in G$, has to be covered at least by one sensor i, $i \in L$. Constraints (3) impose that a non important point is covered just in case a sensor able to control it has been installed. Constraints (4.a) and (4.b) are binary constraints for y_i, $i \in L$, and x_j, $j \in G$.

The *WDCP* reduces to the *SCP* if all the points of the *RI* are considered as important points. This model can be easily integrated with additional operational and design constraints taking into account:

- mutual distance between the sensor;
- orientation of the sensor with respect to the point to be covered (these constraints are required, for example, when video analysis algorithms have to be used). In order to impose such constraints we exploit the evaluation of the angle between the bisector of the coverage angle and the line connecting the point to be covered.
- multiple coverage of the *RI* points by sensors with different viewpoints/perspectives.

All these additional constraints can be straightforward formulated just introducing for both points of the region of interest and possible locations some conditions about the mutual distance and/or orientations.

3.5.2. Back-up covering problem (*BCP*)

In order to model the *BCP* we have to integrate the notations just introduced for the *WDCP* with the following parameters:

- d_j	weight associated to a point of the region j, $j \in G$;
- p	maximum number of sensors to be installed;
- ε	parameter between 0 and 1 weighting multiple coverage with respect to the primary coverage.

Moreover an additional binary variable is required:

- $u_j = \{0,1\}$	binary variable associated to each point j to be covered, $j \in G$. It is equal to 1 if the point j is covered by two or more sensors, 0 otherwise.

On this basis, the *BCP* can be modeled as follows:

$$Max\ z = (1-\varepsilon)\sum_{j \in G} d_j x_j + \varepsilon \sum_{j \in G} d_j u_j \qquad (5)$$

s.t.

$$\sum_{i \in L} c_{ij}\, y_i \geq x_j + u_j \qquad \forall j \in G \qquad (6)$$

$$\sum_{i \in I} y_i = p \qquad (7)$$

$$u_j \leq x_j \qquad \forall j \in G \qquad (8)$$

$$y_i = \{0,1\} \qquad \forall i \in L \qquad (9.a)$$

$$x_j = \{0,1\} \qquad \forall j \in G \qquad (9.b)$$

$$u_j = \{0,1\} \qquad \forall j \in G \qquad (9.c)$$

The objective function (5) maximizes the weighted sum of the primary and multiple coverage of the points of the *RI*. The relative weight of these two components is defined by the value of the parameter ε. Constraints (6) impose that a point j, $j \in G$, is controlled just in case at least one sensor i, $i \in L$, among the ones able to control it, is located. Constraint (7) imposes that the number of sensors to be located has to be exactly p. Constraints (8) impose that each point j, $j \in G$, is multiple covered just if it is also primary covered. In other words, constraints (6) and constraints (8) guarantee that if u_j is equal to 1, then the point j is covered by at least two sensors. Finally constraints (9.a), (9.b) and (9.c) are binary constraints for variables y_i, $i \in L$, x_j and u_j, $j \in G$.

This model can be easily generalized by replacing constraint (7) by the following budget constraint:

$$\sum_{i \in L'} h_i y_i \leq B \tag{10}$$

Moreover, the same integrations presented for the *WDCP* can be introduced in the *BCP*.

The *BCP* reduces to the *MCP* if the parameter ε is set to zero. Setting ε to zero means that, in the *MCP*, independently of the number of sensors covering a point of the region of interest, it is counted just once in the objective function. On the other side, in the *BCP*, each point covered by more than one sensor is counted twice, the first time by the primary coverage variable and the second time by the secondary coverage variable. If we define as many multiple coverage variables as the number of times we want to count the covered points, then the model can be easily extended to the multiple coverage case.

Moreover, we recall that, when tackling real safety and security problems, also for *MCP* and *BCP* it is important to take into account the constraints (2) of the *WDCP*, requiring that important points have to be compulsorily covered at least once. In this case, it is easy to understand, that the introduction of these constraints, if the number of available sensors is not sufficiently large, can make the problem unfeasible, as will be shown in the experimental result section.

3.5.3. ILP models and coverage matrix

Some considerations are needed to provide a deeper insight about the usage of the proposed models in solving the *SPP* and the elements to be taken into account, i.e. coverage matrix and discretization step.

As said above the *SPP* can be considered as an *SCP* (or a variant of it: *WDCP*, *MCP*, and *BCP*), which is *NP-hard* in the strong sense as discussed in Garey and Johnson [25]. The key issue of this hardness, which significantly affects the possibility of solving such complex problem and its variants, deals with the coverage matrix, and more precisely with its size. From this point of view we can distinguish two different situations.

If we know the coverage matrix, given or generated by us, then instances of significant sizes, with hundreds of rows and thousands of columns, can be effectively solved to optimality also using commercial optimization solvers, as discussed by Caprara et al. [9]. In this situation it is possible to exploit or develop algorithms for the reduction of the coverage matrix sizes, in order to get smaller instances and solve them more effectively. In this case, particular attention has to be given to the fact that these reduction algorithms could be time consuming, as highlighted in the same paper by Caprara et al. [9].

On the other side, if we do not know a-priori the coverage matrix and we are tackling very large covering problems for which it is very heavy to generate it, then the usage of column generation approach should be investigated.

The specific problem tackled in this work generally falls into or can be referred to the first case. Indeed, once defined the set of points to be covered and the set of the potential sensor locations, we can generate, by the described coverage analysis, the entire coverage matrix. Then, the possibility of optimally solving or effectively tackling the arising *SPP* by available optimization tools depends on the specific sizes of the problem under investigation. Indoor or small/medium size outdoor region of interest are generally described by a coverage matrix, whose sizes determine an *SPP* consistent with the current performances of the available optimization tools. Instead large outdoor spaces could be described by a coverage matrix with a huge number of rows and columns and, consequently, the development of ad-hoc heuristic approaches could be required to effectively solve the arising *SPP*.

In any case, as stated above, it is important to recall that the sizes of the coverage matrix depend on the discretization step *k*, which on one side can affect the *RI* schematization and consequently the quality of the obtained solution, on the other side it can affect the computational effort of the used approach. Indeed it is easy to understand that, using low values of the step size *k*, the size of *C* can significantly increase and consequently the difficulty in solving the *SPP* under investigation can arise. From this view point it is important to determine a value of the discretization step which could provide a good trade-off between these two opposite targets. Hence, to obtain a good solution quality with an effective steps size *k*, it would be important to reduce the size of the coverage matrix *C*, with particular reference to the *3D* case, where this size is equal to $|L|\times|G|$, $G| = |G'|\times|N|$ (as discussed in *Section 3.4.3*).

To this aim it is useful to introduce the geometrical properties represented in *Figure 7* (for sensors with or without *HD* effect). This figure represents the projection of the *CA* of an *OS* in a plane orthogonal to the *y* axis. We can note that, in case no *HD* effect is present (*Figure 7.a*), if a sensor is able to cover a point at a given height, for example the point A, then it is also able to cover all the points on its vertical with lower height values (A1, A2 and A3). The same occurs for the other points. Concerning instead the case with *HD* effect (*Figure 7.b*), we can repeat the same reasoning just done for the case with no *HD* effect, but referring just to the points that are in the *CA* band (strip).

On the basis of this discussion we can assert that in case we need to cover all the points of a *3D* region of interest adopting an *SCP* model by sensors (with no *HD* effect) located at a plane higher than all the ones to be covered, then the *3D* coverage problem reduces to a *2D* coverage problem where the only plane to be covered is the top plane of the *RI*.

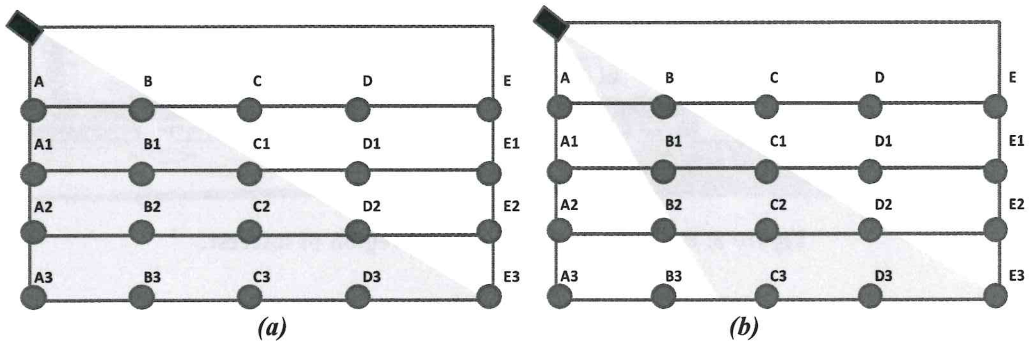


Figure 7. Representation of the *CA* of an *OS* in a *3D* region of interest in a plane parallel to the *z* axis: a) sensor with no *HD* effect; b) sensor with HD effect.

In the other cases, i.e. sensors with *HD* effect and/or with the other covering models, it is important to deploy effective algorithmic procedures which allow to reduce the size of the coverage matrix. These procedures, currently under investigation, are devoted to delete some columns of *C*.

4. Experimental results on a real test case

The presented unified approach for the *SPP* has been experienced on a real indoor *2D* and *3D* case related to a railway station located in the west area of Naples. In particular the aim was to design a video sensor networks to monitor the entire public area of the station (hence offices and private areas have not been taken into account). For the sake of the brevity, we just report here part of the obtained results in the *3D* case. It is important to highlight that this work proceeds what has been done by the authors during the METRIP project (*Methodological Tool for Railway Infrastructure Protection*) [4, 5, 26].

The region of interest has been schematized by a parallelepiped of dimensions *75 m*, *35 m* and *5 m*. A representation of the plant of the *RI* is reported in *Figure 8*, where the box highlights the public area under investigation.

In the construction of the *G* grid of the *RI* we used three different values of the step size along the length (*x* axis) and along the width (*y* axis): *2.5*, *5* and *10*. In this way we had the possibility of evaluating possible changes in the solution due to the quality of our discretization. Instead, concerning the height, we choose just three planes to be monitored: ground floor, *1 m* floor and *1,7 m* floor. We considered as important points all the ones in these planes corresponding to doors and windows, stairs, escalators and turnstiles. Moreover we considered as important all the points located along the yellow lines of the four tracks present in the station. All these points have to be always covered by at least one sensor.

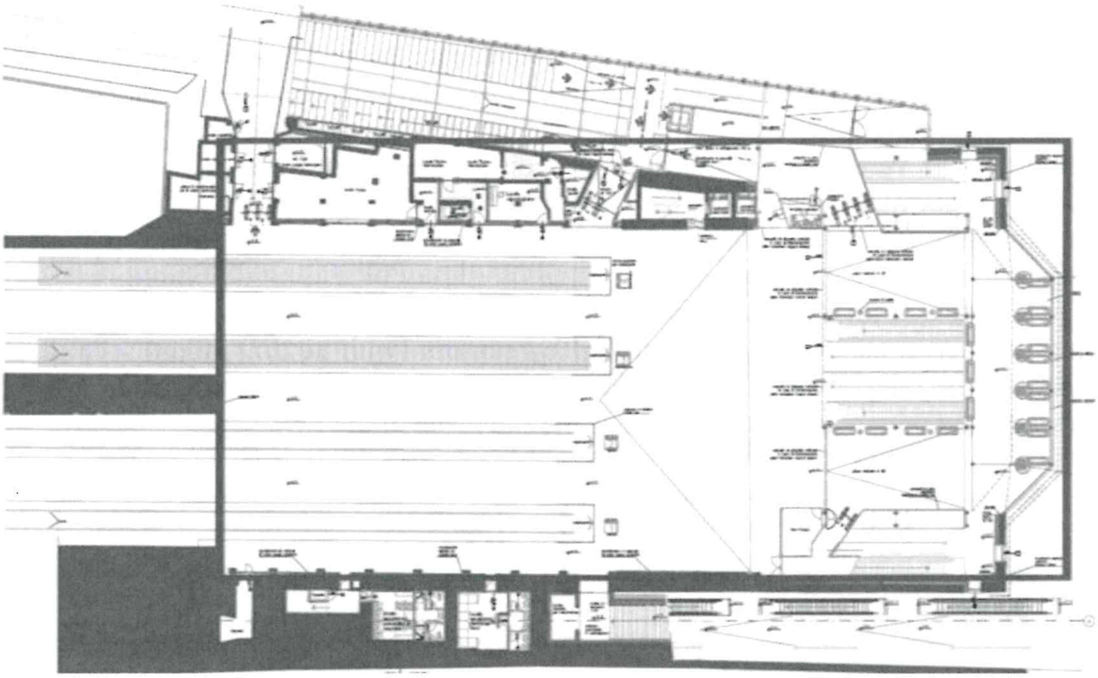


Figure 8. Railway station plant and region of interest.

In the construction of the *L* grid we used the same step size *k* adopted for the *RI* discretization in the definition of the $\hat{x}_p$, $\hat{y}_p$ coordinates. Concerning instead the $\hat{z}_p$ coordinate we used as possible height values just the ones imposed by the security legislation in dependence of the target to be covered. The sensors to be placed were characterized by the following parameters: $R = 25\ m$; $\theta = 90°$; $\alpha = 45°$ (hence we considered 8 orientations with overlapping coverage areas). The tilt angle β has been considered as fixed and equal to *45°* in all the *5-tuples*.

The procedure has been implemented in Java language and the coverage *ILP* models have been solved to optimality by the optimization software FICO™ Xpress-MP 7.6. The procedure has been run on an Intel® Core™ i7, 870, 2.93 GHz, 4GB RAM, Windows Vista™ 64 bit. We tested the procedure considering all the four previously introduced covering problems: *SCP*, *WDCP*, *MCP* and *BCP*.

In Table 1 we summarize the results obtained for the *SCP* and *WDCP* (with $\gamma = 0.5$), assuming the installation costs equal for all the possible locations. The table reports for each instance: the name of the instance (*Instance*); the discretization step of the grids (*k*); the value of *L*, *G'* and *N*; the solution obtained by the optimization software in terms of number of sensors to be located (column *OS*); the number of covered points , with no distinction between important and not important, (column *Covered Points*); the percentage of covered points (column *% Cov*); the computation time in seconds of the optimization software (*CPU time*).

It is easy to note that obviously the *SCP* provides solution covering all the points of the *RI* and independently from the value of the step size, it locates 7 sensors. Instead , the *WDCP* does not cover all the points of the *RI* and in order to cover just the important points locates always 6 sensors. It is also important to note, that, given the sizes of the arising covering problems (at most we have a coverage matrix with 3728 rows and 1650 columns) all the instances have been solved to optimality with very low computation time.

Instance	*K*	*L*	*G'*	*N*	*OS*	*Covered Points*	*% Cov*	*CPU time*
					SCP			
I1	2.5	3728	550	3	7	1650	100	20.7
I2	5	1336	188	3	7	564	100	6.6
I3	10	816	176	3	7	528	100	1.4
					WDCP ***(*** γ ***= 0.5)***			
I1	2.5	3728	550	3	6	1468	89.0	31.5
I2	5	1336	188	3	6	525	93.1	5.7
I3	10	816	176	3	6	476	90.3	3.5

Table 1. Results obtained for the SCP and WDCP

The fact that we use the same number of sensors independently of the discretization step, is mainly due to the specific features of the problem under investigation. Indeed, given the sizes of the *RI* under investigation and the *CA* of the used sensors, each camera is able to monitor a significant portion of the region of interest. For this, the definition of a larger number of potential locations does not provide any reduction in the number of installed sensors.

In *Figure 9* we provide, as an example, a two dimensional representation of the solution obtained by the optimization software Xpress-MP for the *SCP* with *k=2.5.*

In Table 2 we summarize the results obtained for the *MCP*, assuming a unitary weight for all the non important points. The table reports the same information reported in Table 1, with the only exception of the *OS* column which is replaced by the column *p*, indicating the number of sensors to be located. Finally in Table 3 we summarize the results obtained for the *BCP* (ε = 0.4), assuming a unitary weight for all the non important points. The table reports the same information of Table 3 with addition of two other columns: the column *(M-Covered points)* which reports the number of multiple covered points and the column *(% M-Cov)* which reports the percentage of M-covered points. For both problems (*MCP* and *BCP*), as occurred for the *SCP* and *WDCP*, we can easily note that all the instances have been solved to optimality with very low computation time, independently from the value of *p*. Moreover it is important to note that with *p = 5* and *k* values higher than or equal to *5*, we cannot determine a feasible solution. This is due to the fact that using all the positions of the grid, we are not able to find a solution which can cover at least all the important points of the *RI*. Hence, contrarily to what occurred for the *SCP* and *WDCP*, in this case the choice of a smaller and effective discretization step *k* significantly affects the solution of the *MCP* and *BCP*.

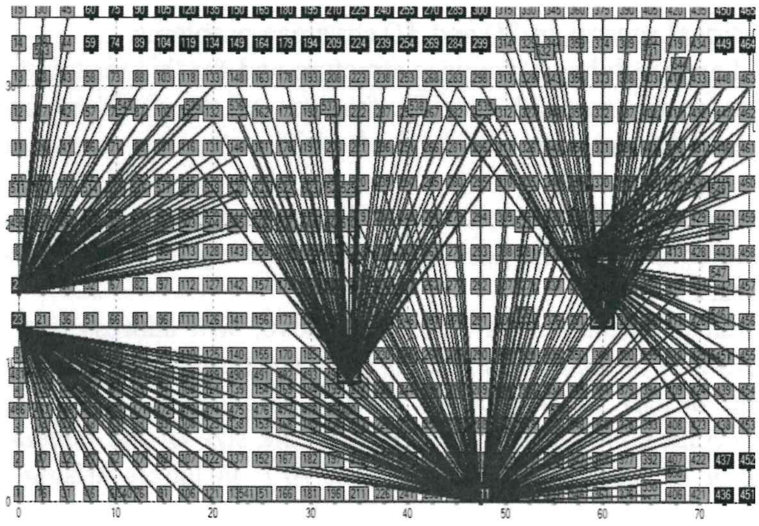


Figure 9. ***2D*** **representation of the** ***3D SCP*** **solution with** ***k = 2.5.***

Instance	*K*	*L*	*G'*	*N*	*P*	*Covered Points*	*% Cov*	*CPU time*
I1	2.5	3728	550	3	5	1387	84.1	15.6
					6	1470	89.1	18.7
					7	1650	100	27.6
I2	5	1336	188	3	5	No feasible solution	---	---
					6	527	93.6	5.9
					7	564	100	6.5
I3	10	816	176	3	5	No feasible solution	---	---
					6	488	92.6	1.7
					7	528	100	2.9

Table 2. Results obtained for the MCP

Instance	*k*	*L*	*G'*	*N*	*p*	*Covered Points*	*% Cov*	*M-Covered Points*	*% M-Cov*	*CPU time*
I1	2.5	3728	550	3	5	1356	82.2	168	10.2	17.9
					6	1386	84	590	35.8	21.7
					7	1478	89.6	737	44.7	33.8
I2	5	1336	188	3	5	No feasible integer solution	---	---	---	---
					6	507	89.9	179	31.9	9.6
					7	521	92.5	270	47.9	13.3
I3	10	816	176	3	5	No feasible integer solution	---	---	---	---
					6	488	92.6	144	27.3	5.1
					7	479	90.9	233	44.3	7.4

Table 3. Results obtained for the BCP

5. Conclusions

The *sensor placement problem (SPP)* has been largely studied given the great number of applications fields where it arises. Starting from the main contributions present in literature, our work has been devoted to propose a simple and unifying approach to be used for both omni-directional and directional sensors in *2D* and *3D* coverage problems.

The proposed approach has a stepwise structure and for this reason it can be easily modified to take into account additional and specific features of the problem under investigation and integrated with more effective solution methods with reference to the solution of the arising coverage problems.

The proposed approach has been successfully applied to a real medium size test case, related to an indoor space, so confirming the possibility of effectively tackling real world problems arising in railway infrastructure security.

Future research perspectives, as discussed above, will address two main issues. The first concerns the integration of the proposed models with additional features and constraints to be taken into account in the design of a security system, regardless of the specific context under investigation. The second concerns the reduction of the size of the coverage matrix, in order to optimally solve and/or effectively tackle large size *SPP* instances arising in the design of large outdoor spaces.

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