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# Adaptive Fault-Tolerant Formation Control for Quadrotors with Actuator Faults

Wanzhang Liu, Ke Zhang, Bin Jiang, Xinggang Yan

## ABSTRACT

In this paper, we investigate the fault-tolerant formation control of a group of quadrotor aircrafts with a leader. Continuous fault-tolerant formation control protocol is constructed by using adaptive updating mechanism and boundary layer theory to compensate actuator fault. Results show that the desired formation pattern and trajectory under actuator fault can be achieved using the proposed fault-tolerant formation control. A simulation is conducted to illustrate the effectiveness of the method..

***Key Words:*** Fault-tolerant control, Quadrotor, Adaptive control, Formation control

## I. INTRODUCTION

In recent decades, cooperative control of multi-agent systems, including numerous mobile robot systems [1], UAV [2, 3], and microsatellite attitude synchronization control [4, 5], has received considerable attention from the community given the potential practical application of these multi-agent systems. Formation control is an important branch of cooperative control, the goal is to form a certain predefined shape using a group of agents. For complex systems composed of numerous autonomous agents, the centralized control method is no longer applicable, and developing a distributed intelligent control strategy that is independent of global formation is crucial.

Many previous studies systematically investigated the problem of the cooperative control of multi-agent systems. Work [6] analyzed the coordinated control performance of the UAV formation system. In practical engineering, the control problem of a single quadrotor is challenging given its strong nonlinear coupling [7]. However, designing a controller for a quadrotor is difficult, the quadrotor is open-loop instable because of its rotary-wing and inherent nonlinearity, which requires a fast control response and a large operation range. The formation control problems of second-order multi-agent systems with a time-invariant topology were studied in [8, 9]. However, above literatures disregarded the cooperative control problems in the case of actuator faults, which are not negligible in formation control problems.

A FTC system can control the system with satisfactory performance even if one or several faults, or critically, one or several failures occur in this system. In general, FTC can be classified into two types: passive and active. An active FTC depends on the fault-diagnosis module that monitors the system health. [10] proposed a sliding mode observer(SMO) to recover the pitch rate information. A distributed fault estimation observer (DFEO) design for the multi-agent systems with switching topologies was studied in [11], and a active fault-tolerant scheme for a hypersonic gliding vehicle to conuteract actuator faults and model uncertainties was studied in [12]. In the present study, we focus on the passive FTC problem of a group of quadrotor aircrafts.

Wanzhang Liu, Ke Zhang and Bin Jiang(e-mail:kezhang@nuaa.edu.cn) are with the College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 211106, China.

Xinggang Yan is with School of Engineering and Digital Arts, University of Kent, Canterbury, Kent CT2 7NT, United Kindom.

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In practical cases, actuator faults affect the normal operation of each agent, thereby making the actual outputs of the actuator different from its control inputs. A distributed control strategy of a class of nonlinear second-order leader-following multi-agent systems against multiple actuator faults was considered in [13] by designing adaptive schemes, [14] develop an auxiliary system to ensure actuators behave within amplitude and rate limits under the influence of partial loss of control effectiveness. A fault tolerant control was studied in [15] for a two-dimensional airfoil with input saturation and actuator fault, and a robust nonlinear controller, which combines sliding-mode and backstepping control techniques, was considered in [16]. A novel fault tolerant control (FTC) scheme for hybrid systems modeled by hybrid Petri nets (HPNs) was proposed in [17]. A novel robust fault-tolerant controller for the attitude control problem of a quadrotor aircraft in the presence of actuator faults and wind gusts was developed in [18], and a fault-tolerant control of feedback linearizable systems with stuck actuators was studied in [19]. A fault-tolerant consensus control protocol for multi-agent system with actuator bias faults was developed in [20]; the consensus control problems under bias and loss of effectiveness faults for a group of second-order agents were considered in [21]. In the framework of a distributed cooperative control of a group of quadrotor aircrafts, the actuator faults of the inner-loop subsystem (attitude subsystem) in a single quadrotor can affect the normal operation of the outer-loop subsystem (position subsystem), which can spread to neighboring aircrafts via the interaction topology, thereby affecting the performance of the entire multi-agent system. Therefore, the FTC method for the multi-agent systems must be explored. The main contribution of this study is to solve the fault-tolerant formation problem for a group of quadrotor aircrafts, which can be simplified into two subproblems: (i) for position subsystem, a formation control strategy base on the global error is formulated so that the quadrotor can track the desired trajectory with a predefined shape. (ii) for actuator faults of attitude subsystem, a adaptive fault-tolerant controller is designed to ensure that three angles can track the desired value. To the best of our knowledge, the fault-tolerant formation control of a group of quadrotor aircafts with undirected topologies remains open.

Throughout this paper, $\|\cdot\|$ stands for the Euclidean norm of a vector, diag represents a diagonal matrix, $\mathrm{sgn}(\cdot)$ denote the sign function. A vector is considered positive if all its elements are greater than zero, $R^+$ represents a real number field.

## II. System Model and Problem Formulation

### 2.1. Graph Theory

This study mainly focus on the formation control of a group of quadrotor aircrafts with a leader-following architecture, which consists of a leader and $N$ followers. If the leader is a node labeled as 0, and each follower is also a node labeled as $1, 2, 3, \cdots, N.$, then the node indexes belong to a finite index set, . Thus, $i \in \Gamma = \{1, 2, \cdots, N\}$. Subsequently, an undirected graph $\mathcal{G} = \{V, E, A\}$ is denoted as the communication topology among $N$ quadrotors, where $V = \{v_i\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represent the set of nodes and the set of edges, respectively. $A \in R^{N\times N}$ is the weighted adjacency matrix of the graph $\mathcal{G}$. If an edge is observed between agent $i$ and $j$, that is, $(v_i, v_j) \in \mathcal{E}$, then $a_{ij} = a_{ji} > 0$; otherwise, $a_{ij} = a_{ji} = 0$. Moreover, we assume that $a_{ii} = 0$ for all $i \in \Gamma$. The set of neighbors of node $v_i$ is expressed by $\mathcal{N}_i = \{j : (v_i, v_j) \in \mathcal{E}\}$. The out-degree of node $v_i$ is defined as $\deg_{\text{out}}(v_i) = d_i = \sum_{j=1}^{N} a_{ij} = \sum_{j\in\mathcal{N}_i} a_{ij}$. The degree matrix of the unidirected graph $\mathcal{G}$ is $\mathcal{D} = \mathrm{diag}\{d_1, d_2, \cdots, d_N\}$, and the Laplacian matrix of the undirected graph $\mathcal{G}$ is $L = \mathcal{D} - A$. A path in the graph $\mathcal{G}$ from $v_i$ to $v_j$ is a sequence of distinct vertices that start with $v_i$ and end with $v_j$ for the consecutive vertices to be adjacent. The graph $\mathcal{G}$ is connected if a path exists between two vertices. The reference state is assumed to be a leader. The connection weight between the leader and the $N$ quadrotors can access the information of the leader, then $b_i > 0$, otherwise, $b_i = 0$. Let $B = \mathrm{diag}\{b_i, b_2 \cdots b_N\}$. Throughout the study, the network communication topology satisfies the following assumption.

**Assumption 1.** The communication topology for the $N$ followers is connected , and at least one path from the leader to the follower exists.

**Remark 1.** For undirected networks considered in this study, Assumption 1 can ensure that each node have paths from the leader node and thus can receive information from leader node. Then the formation control can be performed.

### 2.2. System Model and Problem Formulation

This study investigates the formation control of a group of $n$ quadrotor aircrafts. $\Gamma \in (1, 2, \cdots, n)$ is denoted. The quadrotor typically consists of four motor drive systems, which are fixed to a rigid cross structure. Degree-of-freedom variable, that is, position and attitude, is typically required to predict the motion of a three-dimensional quadrotor. Specifically, the coordinates of the quadrotor aircraft are expressed by:

$$(x_i, y_i, z_i, \phi_i, \theta_i, \psi_i)^T \in R^6, \quad i \in \Gamma, \tag{1}$$

where $\vartheta_i = (x_i, y_i, z_i)^T \in R^3$ represents the position of the aircraft mass center relative to the inertial coordinate system, $\Phi_i = (\phi_i, \theta_i, \psi_i)^T \in R^3$ represents the three Euler angles used to describe the posture of an aircraft relative to the inertial coordinate system. The rotational Euler angles around the x-, y-, and z-axis are represented by the roll angle $\phi$, pitch angle $\theta$, and yaw angle $\psi$.

The detailed description of the dynamic model of a quadrotor aircraft was introduced in [22]. In this study, to simplify the implementation of the control scheme, we apply the simplified model. Then, the dynamic model is expressed as:

$$\begin{cases} m_i\ddot{x}_i = -K_{i1}\dot{x}_i + T_i(\cos\phi_i \sin\theta_i \cos\psi_i + \sin\phi_i \sin\psi_i), \\ m_i\ddot{y}_i = -K_{i2}\dot{y}_i + T_i(\cos\phi_i \sin\theta_i \sin\psi_i - \sin\phi_i \cos\psi_i), \\ m_i\ddot{z}_i = -K_{i3}\dot{z}_i - mg + T_i(\cos\phi_i \cos\theta_i), \\ J_{i1}\ddot{\phi}_i = -K_{i4}l_i\dot{\phi}_i + l_i\tau_{i1}, \\ J_{i2}\ddot{\theta}_i = -K_{i5}l_i\dot{\theta}_i + l_i\tau_{i2}, \\ J_{i3}\ddot{\psi}_i = -K_{i5}l_i\dot{\psi}_i + l_i\tau_{i3}, \end{cases} \tag{2}$$

where $K_{ij} \in R^+$ for $j = 4, 5, 6$ denotes the aerodynamic damping coefficient. $J_{ij} \in R^+$ for $j = 1, 2, 3$ denotes the moment of inertia, $l_i \in R$ represents the distance between the center of the aircraft and the motor axis, $\tau_{ij}$ for $j = 1, 2, 3$ represents three rotational forces generated by the four rotors, $c_i \in R^+$ is a constant force-motor coefficient, $T_i \in R$ represents the total thrust generated by the four rotors.

**Remark 2.** Ideally, the dynamics model of the quadrotor includes a gyroscopic effect caused by the rotation of the space rigid body and the four propellers. These are often overlooked in previous works.

### 2.3. Control Objectives

The purpose of this study is to design a distributed control law for a set of quadrotor aircrafts (2) to achieve fault-tolerant formation control. Specifically, the fault-tolerant formation control requires that pattern and desired formation trajectory can be achieved.

The desired tracking trajectory $[x_d, y_d, z_d]^T$ is generated by:

$$\dot{x}_d = v_{xd},\ \dot{y}_d = v_{yd},\ \dot{z}_d = v_{zd}, \tag{3}$$

in addition, the desired geometric pattern in three-dimensional space is determined by vector

$$\Delta_{ij} = \Delta_i - \Delta_j = [\Delta_{ix}, \Delta_{iy}, \Delta_{iz}]^T - [\Delta_{jx}, \Delta_{jy}, \Delta_{jz}]^T, \tag{4}$$

which indicates the desired position deviation between quadrotor $i$ and $j$. We only consider the invariant formation mode to simplify the statement, that is, $\forall i \in \Gamma$ ,$\Delta_{ij}$ are constant vectors.

By using a mathematical statement, the control objective of this paper is to design a distributed control law in which, for any $i, j \in \Gamma$

$$\lim_{t\to\infty} \begin{bmatrix} x_i(t) - x_j(t) \\ y_i(t) - y_j(t) \\ z_i(t) - z_j(t) \end{bmatrix} = \Delta_{ij}, \tag{5}$$

and

$$\lim_{t\to\infty} \frac{1}{N} \sum_{i=1}^{N} [x_i(t), y_i(t), z_i(t)]^T = [x_d, y_d, z_d]^T. \tag{6}$$

**Remark 3.** The overall control objective is to design the control input signal for the quadrotor aircraft to track the time-varying desired trajectory $[x_d(t), y_d(t), z_d(t)]^T$ and maintain the given pattern.

## III. Formation Controller Design

In this section, the formation controller algorithm, which can be divided into two steps, is proposed. The control strategy for a group of quadrotor aircrafts is illustrated in Fig. 1.

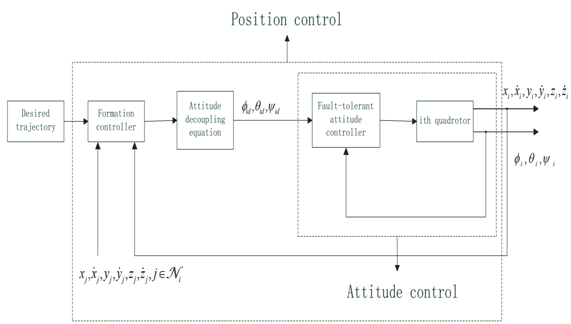


Fig. 1. The control block diagram for a group of quadrotor aircraft.

### 3.1. Controller Design of Position Subsystem

The position dynamics (2) is used to facilitate the design of the position controller:

$$\begin{cases} u_{ix} = \dfrac{T_i}{m_i}(\cos\phi_i \sin\theta_i \cos\psi_i + \sin\phi_i \sin\psi_i), \\ u_{iy} = \dfrac{T_i}{m_i}(\cos\phi_i \sin\theta_i \sin\psi_i - \sin\phi_i \cos\psi_i), \\ u_{iz} = \dfrac{T_i}{m_i}\cos\phi_i \cos\theta_i - g,\ i \in \Gamma, \end{cases}$$

Under these notations, the position dynamics equation for a quadrotor aircraft is rewritten as:

$$\begin{cases} \ddot{x}_i = -\dfrac{K_{i1}}{m_i}\dot{x}_i + u_{ix}, \\ \ddot{y}_i = -\dfrac{K_{i2}}{m_i}\dot{y}_i + u_{iy}, \\ \ddot{z}_i = -\dfrac{K_{i3}}{m_i}\dot{z}_i + u_{iz}. \end{cases} \tag{7}$$

**Theorem 1.** For position dynamic model (8), if the virtual control inputs $u_{iz}$, $u_{iy}$ and $u_{iz}$ have the same form, and $u_{ix}$ is designed as follows:

$$\begin{cases} u_{ix} = -\sum_{j\in\mathcal{N}_i} a_{ij}[k_1(x_i - x_j - \Delta_{ijx}) + k_2(v_{ix} - v_{jx})] \\ \quad - b_i[k_1(x_i - x_d - \Delta_{ix} + \frac{1}{N}\sum_{i=1}^{N}\Delta_{ix}) + k_2(v_{ix} \\ \quad - v_d)] + \frac{K_{i1}}{m_i}v_{xd} + \dot{v}_{xd}, \\ u_{iy} = -\sum_{j\in\mathcal{N}_i} a_{ij}[k_1(y_i - y_j - \Delta_{ijy}) + k_2(v_{iy} - v_{jy})] \\ \quad - b_i[k_1(y_i - y_d - \Delta_{iy} + \frac{1}{N}\sum_{i=1}^{N}\Delta_{iy}) + k_2(v_{iy} \\ \quad - v_d)] + \frac{K_{i2}}{m_i}v_{yd} + \dot{v}_{yd}, \\ u_{iz} = -\sum_{j\in\mathcal{N}_i} a_{ij}[k_1(z_i - z_j - \Delta_{ijz}) + k_2(v_{iz} - v_{jz})] \\ \quad - b_i[k_1(z_i - z_d - \Delta_{iz} + \frac{1}{N}\sum_{i=1}^{N}\Delta_{iz}) + k_2(v_{iz} \\ \quad - v_d)] + \frac{K_{i3}}{m_i}v_{zd} + \dot{v}_{zd}, \end{cases} \tag{8}$$

where $k_1 > 0, k_2 > 0$ are two positive constants, and $\dot{x}_i = v_{ix}, \dot{y}_i = v_{iy}, \dot{z}_i = v_{iz}$, then the task of the formation control can be achieved under the controller.

**Proof.** Without loss of generality, we provide proof only on the x-axis, $i$th quadrotor coordinate changes are as follows:

$$e_{ixp} = x_i - x_d - \Delta_{ix} + \frac{1}{N}\sum_{i=1}^{N}\Delta_{ix},$$

$$e_{ixv} = v_{ix} - v_{xd},\ i \in \Gamma.$$

Under these notations, it follows from (8) and (9) that the closed-loop system is:

$$\begin{aligned} \dot{e}_{ixp} &= e_{ixv}, \\ \dot{e}_{ixv} &= -\sum_{j\in\mathcal{N}_i} a_{ij}[k_1(x_{ixp} - x_{jxp}) + k_2(e_{ixv} - e_{jxv})] \\ &\quad - b_i[k_1 e_{ixp} + k_2 e_{ixv}] - \frac{K_{i1}e_{ixv}}{m_i}. \end{aligned} \tag{9}$$

We will show that the multi-agent system (9) is globally stable, and is verified as follows.

A candidate Lyapunov function for system (9) is constructed as:

$$V(t) = k_1\sum_{i=1}^{N}(\sum_{j=1}^{N}\frac{1}{2}a_{ij}(e_{ixp} - e_{jxp})^2 + b_i e_{ixp}^2) + \sum_{i=1}^{N} e_{ixv}^2, \tag{10}$$

where the derivative of $V$ along system (9) is:

$$\dot{V}(t) = k_1\sum_{i=1}^{N}(\sum_{j=1}^{N}(a_{ij}e_{ixp} - e_{jxp})(\dot{e}_{ixp} - \dot{e}_{jxp})) + 2b_i k_1 e_{ixp}e_{ixv} + 2\sum_{i=1}^{N} e_{ixv}\dot{e}_{ixv}, \tag{11}$$

by substituting (9) into (11), the second item to the right of the equal sign in (10) obtains the following expression:

$$\begin{aligned} \sum_{i=1}^{N} e_{ixv}\dot{e}_{ixv} &= \sum_{i=1}^{N} e_{ixv}(-\frac{K_{i1}}{m_i}e_{ixv} - \sum_{j\in\mathcal{N}_i} a_{ij}[k_1(e_{ixp} \\ &\quad - e_{jxp}) + k_2(e_{ixv} - e_{jxv})] - b_i[k_1(e_{ixp} \\ &\quad - e_{jxp}) + k_2(e_{ixv} - e_{jxv})]), \\ &= -\sum_{i=1}^{N}\frac{K_{i1}}{m_i}e_{ixv}^2 - \sum_{i=1}^{N} e_{ixv}\sum_{j\in\mathcal{N}_i} a_{ij}[k_1\times \\ &\quad (e_{ixp} - e_{jxp}) + k_2(e_{ixv} - e_{jxv})] \\ &\quad - \sum_{i=1}^{N} e_{ixv}b_i(k_1 e_{ixp} + k_2 e_{ixv}), \end{aligned} \tag{12}$$

owing to $A$ is a symmetric matrix, $a_{ij} = a_{ji}$, $i \in \Gamma$; thus, (12) can be rewritten as follows:

$$\sum_{i=1}^{N} e_{ixv}\sum_{j\in\mathcal{N}_i} a_{ij}[k_1(e_{ixp} - e_{jxp}) + k_2(e_{ixv} - e_{jxv})] = \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} a_{ij}[k_1(e_{ixv} - e_{jxv})(e_{ixp} - e_{jxp}) + k_2(e_{ixv} - e_{jxv})^2], \tag{13}$$

after taking the simplification of (13), $\sum_{i=1}^{N} e_{ixv}\dot{e}_{ixv}$ can be expressed as follows:

$$2\sum_{i=1}^{N} e_{ixv}\dot{e}_{ixv} = -2\sum_{i=1}^{N}\frac{K_{i1}}{m_i}e_{ixv}^2 - \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} a_{ij}[k_1\times (e_{ixv}-e_{jxv})(e_{ixp}-e_{jxp}) + k_2(e_{ixv} - e_{jxv})^2] - 2\sum_{i=1}^{N} b_i(k_1 e_{ixp}e_{exv} + k_2 e_{ixv}^2), \tag{14}$$

by using the time derivative of (10) and substituting (14) into (11), we obtain that:

$$\dot{V}(t) = -2\sum_{i=1}^{N}\frac{K_{i1}}{m_i}e_{ixv}^2 - \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} a_{ij}k_2(e_{ixv}-e_{jxv})^2 - 2\sum_{i=1}^{N} b_i k_2 e_{ixv}^2, \tag{15}$$

owing to $a_{ij}\geq 0$, $b_i \geq 0$, it can be concluded from (15) that

$$\dot{V}(t) \leq 0. \tag{16}$$

Define set $\Psi = \{(e_{ixp}, e_{ixv})|\dot{V}\equiv 0\}$. Owing to the connectivity of the communication topology graph, $\dot{V}\equiv 0$ implies that $e_{ixv}\equiv 0$ and then $\dot{e}_{ixv}\equiv 0$, $\forall i\in\Gamma$. Hence, based on LsSalle's invariance principle, it can be concluded that $(e_{ixp}(t), e_{ixv}(t))\to 0$ as $t\to\infty$ for any $i\in\Gamma$. In particular, the closed-loop system (11) is globally asymptotically stable. □

Simultaneously, $e_{iyp}, e_{iyv}, e_{izp}, e_{izv}$ can converge to 0 under the controllers $u_{iy}, u_{iz}$, thereby realizing the ideal formation control.

**Remark 4.** LaSalle invariance principle is an effective tool in studying nonlinear time-invariant systems, by using LaSalle invariance principle,the underlying mechanism that attains the algorithmic convergence is uncovered.

### 3.2. Attitude Controller Design

The desired attitude of the quadrotor aircraft can be generated by the three virtual control inputs $u_i = [u_{ix}, u_{iy}, u_{iz}]^T$. Specifically,

$$\begin{cases} T_{id} = m_i\sqrt{u_{ix}^2 + u_{iy}^2 + (u_{iz}+g)^2}, \\ \phi_{id} = \arcsin(\dfrac{m_i(u_{ix}\sin\psi_i^d - u_{iy}\cos\psi_i^d)}{T_i}), \\ \theta_{id} = \arctan(\dfrac{u_{ix}\cos\psi_i^d + u_{iy}\sin\psi_i^d}{u_{iz}+g}),\ i\in\Gamma. \end{cases} \tag{17}$$

The desired yaw angle can be set to $\psi_{id} = 0$ for easy analysis because the variable $\psi_{id}$ is a free variable, and the subscript $d$ means the desired value.

**Remark 5.** $u_{iz}+g$ should constantly be positive to avoid the discontinuity. It is feasible because proposed controller $u_{iz}$ is bounded by appropriately selecting the gains $k_1, k_2$ and the condition that $v_{zd}$, $\dot{v}_{zd}$ is in a certain range when we calculate $\theta_{id}$ from (17).

The dynamics of attitude subsystem can be expressed as a second-order system:

$$\begin{cases} \dot{\chi}(t) = \chi_v(t), \\ \dot{\chi_v}(t) = a_1\chi_v(t) + a_2\rho(t)u(t), \end{cases} \tag{18}$$

where $\chi(t), \chi_v(t)$, and $u(t)$ represent the position, velocity and control input vector, correspondingly. $\rho(t) = \text{diag}\{\rho_1(t), \rho_2(t), \ldots, \rho_m(t)\}$, where $0 < \rho_j(t) \leq 1$ denotes the unknown efficiency factor of actuator channel $j (j = 1, 2, \ldots, m)$.

**Assumption 2.** The unknown efficiency factor $\rho(t)$ is bounded, and an unknown positive constant $\underline{\rho}$ exists in which $0 < \underline{\rho} \leq \rho(t) \leq 1$.

**Remark 6.** Assumption 2 implies the following two cases: (i) $0 < \rho(t) < 1$ means that controller partially lose its effectiveness. (ii) $\rho(t) = 0$ indicate the outage of source, that is, input channel can no longer receive information from controller $u(t)$. (iii) $\rho(t) = 1$ correspond to the normal operation.

Owing to the fault-tolerant tracking control in the case of the actuator damage, the design controller $u(t)$ enables the system to implement the tracking control in case of actuator faults.

Let

$$\begin{cases} e_\chi(t) = \chi(t) - \chi_d(t), \\ e_{\chi_v}(t) = \chi_v(t) - \chi_{vd}(t), \\ \xi(t) = e_\chi(t) + e_{\chi_v}(t), \end{cases} \tag{19}$$

where $x_d, v_d$ is the desired position and velocity of the formation, respectively. For the second-order system, consider the following fault-tolerant time-varying control protocol:

$$u(t) = -\widehat{\alpha}(t)\xi(t) - f(\xi(t)), \tag{20}$$

where $\widehat{\alpha}(t)$ is an adaptive parameter updated by

$$\dot{\widehat{\alpha}}(t) = \gamma(-\eta\widehat{\alpha}(t) + a_2\|\xi(t)\|^2 + \|\xi(t)\|(\|\dot{\chi}_{vd}(t)\| + \|\chi_v(t)\| + \|e_{\chi_v}(t)\|)), \tag{21}$$

where $\widehat{\alpha}(t_0)\geq 0$, $\gamma$ is a positive constant, and $\eta$ is a small positive constant selected by the designer. The nonlinear function $f(\xi(t))$ is defined as

$$f(\xi(t)) = \begin{cases} \frac{\beta(t)\xi(t)}{\|\xi(t)\|}, & \beta(t)\xi(t) > k \\ \frac{\beta^2(t)\xi(t)}{k}, & \beta(t)\xi(t) \leq k \end{cases} \tag{22}$$

where $\beta(t) = \widehat{\alpha}(t)(\|\dot{\chi_{vd}}(t)\| + \|\chi_v(t)\| + \|e_{\chi_v}(t)\|)$, and $k$ is a positive constant. According to the boundary theory, the function $f(\xi(t))$ is the continuous approximation of the sign function, and $k$ represents the size of the boundary layers. If $k \to 0$, $f(\xi(t)) \to \beta(t)\text{sgn}(\xi(t))$.

In practical applications, the errors can hardly converge to zero exactly under the influences of the actuator failures. If the errors are uniformly ultimately bounded with sufficiently small bounds, system (18) is assumed to achieve the tracking control with a certain small error, which is acceptable in most practical circumstances.

**Theorem 2.** If Assumption 2 holds, then the errors $\xi(t)$ and the adaptive control gains $\widehat{\alpha}(t)$ are uniformly ultimately bounded under the FTC protocol (20) and converge exponentially to the bounded set:

$$D = \{\xi(t), \widehat{\alpha}(t) : V(t) < \frac{1}{2\delta}\underline{\rho}(ka_2 + \eta\alpha^2)\}, \quad (23)$$

where

$$V(t) = \frac{1}{2}\xi^T(t)\xi(t) + \frac{\widetilde{\alpha}^2(t)}{2\gamma}\underline{\rho}, \quad (24)$$

$\tilde{\alpha}(t) = \hat{\alpha}(t) - \alpha$, the positive constant $\delta$ satisfies $\delta < \eta\gamma$, and the positive constant $\alpha$ is sufficiently large; thus

$$\alpha > \max\{\frac{1}{\underline{\rho}}, \frac{a_1}{\underline{\rho}}, \frac{\delta}{2\underline{\rho}}\}. \quad (25)$$

**Proof.** On the basis of the definitions of $\xi(t)$, the second-order system can be rewritten in the following form:

$$\dot{\xi}(t) = e_{\chi_v}(t) + a_1\chi_v(t) + a_2\rho(t)u(t) - \dot{\chi}_{vd}(t), \quad (26)$$

The time derivative of $V(t)$ along the trajectory of (26) is obtained as

$$\begin{aligned}\dot{V}(t) = {} & \xi^T(t)(e_{\chi_v}(t) + a_1\chi_v(t) - a_2\rho(t)\hat{\alpha}(t)\xi(t) - a_2 \\ & \times \rho(t)f(\xi(t)) - \dot{\chi}_{vd}(t) + \frac{\dot{\tilde{\alpha}}(t)\widetilde{\alpha}(t)}{2\gamma}\underline{\rho} \le \xi^T(t)\times \\ & e_{\chi_v}(t) + a_1\xi^T(t)\chi_v(t) - a_2\underline{\rho}\hat{\alpha}(t)\|\xi(t)\|^2 - a_2\underline{\rho}\times \\ & \xi^T(t)f(\xi(t)) - \xi^T(t)\dot{\chi}_{vd}(t)\rho(t)f(\xi(t)) - \dot{\chi}_{vd}(t)) \\ & + \frac{\dot{\tilde{\alpha}}(t)\tilde{\alpha}(t)}{2\gamma}\underline{\rho} + \underline{\rho}\tilde{\alpha}(t)(-\eta\hat{\alpha}(t) + a_2\|\xi(t)\|^2 + \\ & \|\xi(t)\|(\|\dot{\chi}_{vd}(t)\| + \|\chi_v(t)\| + \|e_{\chi_v}(t)\|)).\end{aligned} \quad (27)$$

Two cases are considered in the following proof.

(i) If $\beta(t)\|\xi(t)\| > k$, then $\xi^T(t)f(\xi(t))$ can be expressed as

$$\xi^T(t)f(\xi(t)) = \beta(t)\|\xi(t)\|, \quad (28)$$

(ii) If $\beta(t)\|\xi(t)\| \le k$, then $\xi^T(t)f(\xi(t))$ can be expressed as

$$\xi^T(t)f(\xi(t)) = \frac{\hat{\alpha}^2(t)(\beta^2(t)\xi(t)}{k}, \quad (29)$$

furthermore, (28) and (29) can be obtained as

$$\underline{\rho}\beta(t)\|\xi(t)\| - a_2\underline{\rho}f(\xi(t))\xi(t) \le \frac{1}{4}\underline{\rho}ka_2, \quad (30)$$

where Young's inequality can be determined using the following form: $-\frac{a^2}{c} + a \le \frac{1}{4}c$ with $a \ge 0$ and $c > 0$ has been used.

Therefore, according to the above two cases (28) and (29), (27) can be denoted as

$$\begin{aligned}\dot{V}(t) \le & -\delta V(t) + \frac{1}{2}\delta\xi^T(t)\xi(t) + \frac{\underline{\rho}\delta\tilde{\alpha}^2(t)}{2\gamma} + \xi^T(t)\times \\ & e_{\chi_v}(t) + a_1\xi^T(t)\chi_v(t) - a_2\underline{\rho}\hat{\alpha}(t)\|\xi(t)\|^2 - \\ & a_2\underline{\rho}\xi^T(t)f(\xi(t)) - \xi^T(t)\dot{\chi}_{vd}(t) + \underline{\rho}\tilde{\alpha}(t)\times \\ & (-\eta\hat{\alpha}(t) + a_2\|\xi(t)\|^2 + \|\xi(t)\|(\|\dot{\chi}_{vd}(t)\| + \\ & \|\chi_v(t)\| + \|e_{\chi_v}(t)\|)),\end{aligned} \quad (31)$$

note that $-\tilde{\alpha}(t)(\tilde{\alpha}(t) + \alpha) \le -\frac{1}{2}\tilde{\alpha}^2(t) + \frac{1}{2}\alpha$. Then, (31) expresses

$$\begin{aligned}\dot{V}(t) \le & -\delta V(t) + \frac{1}{2}\delta\|\xi(t)\|^2 + \frac{\underline{\rho}\delta\tilde{\alpha}^2(t)}{2\gamma} + \|\xi(t)\|\times \\ & \|e_{\chi_v}(t)\| + a_1\|\xi(t)\|\|\chi_v(t)\| - a_2\underline{\rho}\tilde{\alpha}(t)\|\xi(t)\|^2 \\ & - a_2\underline{\rho}\xi^T(t)f(\xi(t)) + \|\xi^T(t)\|\|\dot{\chi}_{vd}(t)\| + \\ & \frac{1}{2}(\alpha^2 - \tilde{\alpha}^2)\underline{\rho}\eta + a_2 \times \underline{\rho}\tilde{\alpha}(t)\|\xi(t)\|^2 + \underline{\rho}\tilde{\alpha}(t)\times \\ & \|\xi(t)\|(\|\dot{\chi}_{vd}(t)\| + \|\chi_v(t)\| + \|e_{\chi_v}(t)\|),\end{aligned} \quad (32)$$

then

$$\begin{aligned}\dot{V}(t) \le & -\delta V(t) + (1 - \underline{\rho}\alpha)(\|\dot{\chi}_{vd}(t)\| + \|\chi_v(t)\|)\times \\ & \|\xi(t)\| + \tilde{\alpha}^2(t)(\frac{\underline{\rho}\delta}{2\gamma} - \underline{\rho}\eta) + (a_1 - \underline{\rho}\alpha)\|\chi_v(t)\| \\ & \times \|\xi(t)\| + \frac{1}{4}\underline{\rho}ka_2 + \frac{1}{2}\underline{\rho}\eta\alpha^2 + \|\xi(t)\|\times \\ & (\frac{1}{2}\delta - a_2\underline{\rho}\alpha),\end{aligned} \quad (33)$$

let the positive constant $\delta$ be sufficiently small; thus $\delta < \eta\gamma$, and design $\alpha$ is sufficiently large to satisfy $\alpha > \max\{\frac{1}{\underline{\rho}}, \frac{a_1}{\underline{\rho}}, \frac{\delta}{\underline{\rho}}\}$, then, (33) indicates that

$$\dot{V}(t) \le -\delta V(t) + \frac{1}{2}\underline{\rho}(ka_2 + \eta\alpha^2), \quad (34)$$

using the comparison lemma in [31], and let $\kappa = ka_2 + \eta\alpha^2$, the following equation can be obtained:

$$V(t) \leq (V(0) - \frac{1}{2\delta}\underline{\rho}\kappa)e^{-\delta t} + \frac{1}{2\delta}\underline{\rho}\kappa. \quad (35)$$

(35) expresses that $V(t)$ converges exponentially to the bounded set defined in (23) with a convergence rate that is faster than $e^{-\delta t}$, where the positive constant $\delta$ provides a lower bound of the convergence rate of $V(t)$. Therefore, $\xi(t)$ and $\hat{\alpha}(t)$ are uniformly ultimately bounded. These variables complete the proof. □

**Remark 7.** The formation errors are confirmed to uniformly converge to a bound defined in Theorem 2. From (23) and (25), the bounds of the actuator faults, the design parameters $k$ and $\eta$, and the parameters of the attitude subsystem can affect the bounded set $D$ in (23). We can design $k$ and $\eta$ to be relatively small to enable the formation errors to converge to a small neighborhood of zero, thereby satisfying the requirements of most practical applications. The nonlinear function $f(\xi(t))$ is a continuous approximation of discontinuous function $\beta(t)\text{sgn}(\xi(t))$ using boundary layer theory. A large chattering in the control input is prohibited in practice despite the convergence of the formation errors to zero by using the sign function. The continuous control protocol (20) can avoid the large chattering efficiently.

**Remark 8.** The system model mentioned in (21) is similar to the attitude subsystem in (2), that is, the proposed control scheme (23) can compensate for the actuator faults. Therefore, the three attitude angles in the inner-loop subsystem can still track the desired attitude angles generated by the three virtual control inputs of the outer-loop subsystem, and the formation control can still be accomplished when the actuator faults occur in the attitude subsystem.

## IV. Simulation Results

Three network systems that follow a quadrotor and a leader quadrotoris considered in this study. The exchange of information between quadrotor aircrafts is illustrated by the undirected topology depicted in Figure 3. The weights of the undirected edge are as follows: $a_{12} = a_{21} = 1, a_{23} = a_{32} = 1$, and $b_1 = 1$. The inertia matrices are selected as: $J_i = \text{diag}(0.0023, 0.0024, 0.0026)$, and the mass of the aircraft is $m_i = 0.468$, the distance is selected as $l_i = 0.3$, and the aerodynamic damping factor is selected as $K_{ij} = 0.01$, where $i = 1, 2, 3, j = 1, 2, 3, 4, 5, 6$.

In Figure 2, the regular triangle determined in the X-Z plane is selected as the desired formation pattern to

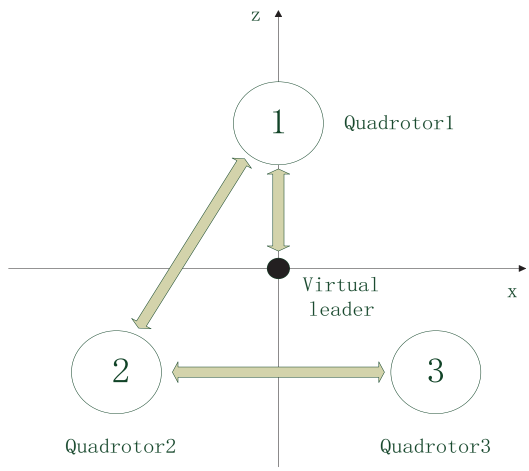


Fig. 2. The communication topology graph and the desired formation pattern.

facilitate analysis. The relative position deviations are presented as follows:

$$\Delta_1 = [0, 0, 1]^T,\ \Delta_2 = [\cos(\frac{5\pi}{6}), 0, \sin(\frac{-\pi}{6})]^T,$$
$$\Delta_3 = [\cos(\frac{-\pi}{6}), 0, \sin(\frac{-\pi}{6})]^T$$

The initial conditions for quadrotor1, quadrotor2 and quadrotor3 aircraft are $[0.46, 0, -3.6, 0, 0.46, 1.31]^T$, $[0, 0.76, 5, 0, 0.33, 0]^T$ and $[-0.54, 0.76, 0, 0, 0.51, 0]^T$ respectively, and the trajectory of the virtual leader is: $[x_d(t), y_d(t), z_d(t)]^T = [5\sin(0.2t), 5\cos(0.2t), 0.5t]^T$

Two cases will be considered.

**Case 1:** Absence of adaptive mechanism

By Theorem 1 and Theorem 2, the gains of distributed formation controller (9) and controller without adaptive mechanism are selected as follows:

$$\begin{cases} \tau_{i1} = -k_3\frac{J_{i1}}{l_i}(\phi_i - \phi_{id}) - k_4\frac{J_{i1}}{l_i}(\dot{\phi}_i - \dot{\phi}_{id}) + \frac{J_{i1}}{l_i}\ddot{\phi}_{id} \\ \quad + K_{i4}\dot{\phi}_{id}, \\ \tau_{i2} = -k_3\frac{J_{i2}}{l_i}(\theta_i - \theta_{id}) - k_4\frac{J_{i2}}{l_i}(\dot{\theta}_i - \dot{\theta}_{id}) + \frac{J_{i2}}{l_i}\ddot{\theta}_{id} \\ \quad + K_{i5}\dot{\theta}_{id}, \\ \tau_{i3} = -k_3\frac{J_{i3}}{l_i}(\psi_i - \psi_{id}) - k_4\frac{J_{i3}}{l_i}(\dot{\psi}_i - \dot{\psi}_{id}) + \frac{J_{i3}}{l_i}\ddot{\psi}_{id} \\ \quad + K_{i6}\dot{\psi}_{id}, \end{cases}$$

where

$$k_1 = 3.5, k_2 = 4.2, k_3 = 3.0, k_4 = 3.5.$$

In this case, under the proposed control algorithm, the response curves of errors between position and desired position for quadrotor1, quadrotor2 and quadrotor3 are respectively exhibited in Fig 3, Fig 4 and Fig5, because quadrotor2 and quadrotor3 are barely affected by the actuator fault, the response curves with adaptive mechanism for quadrotor2 and quadrotor3 can be omitted. The response curves for the attitude for quadrotor1 are displayed in Fig 6. Here, the following fault mode is considered:

The attitude system of quadrotor 1 before 37 s operates in the normal case. After 37s. The actuator of quadrotor 1 loses 50% of its effectiveness, whereas the other quadrotors are normal.

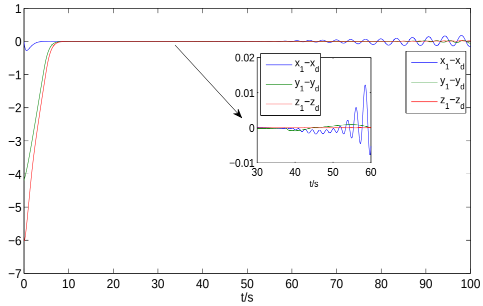


Fig. 3. The response curves of errors between position and desired position without adaptive mechanism.

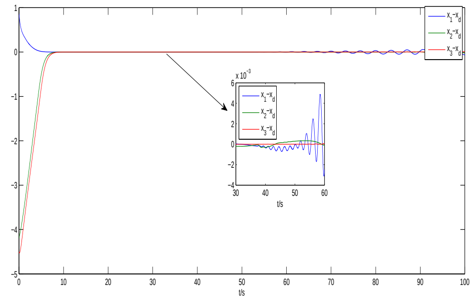


Fig. 4. The response curves of errors between position and desired position without adaptive mechanism.

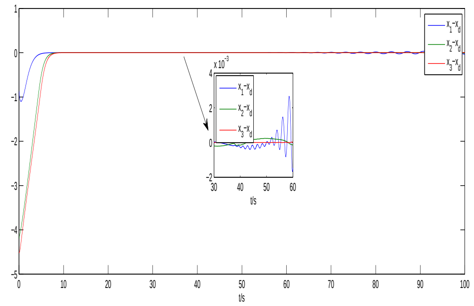


Fig. 5. The response curves of errors between position and desired position without adaptive mechanism.

**Case 2:** With adaptive mechanism

The fault mode is similar to Case 1. By Theorem 1 and Theorem 2, the position and the desired position for quadrotor 1 are presented in Figure 5. The response curves for the attitude and aircraft formation trajectory of quadrotor 1 are illustrated in Figures 6 and 7, correspondingly.

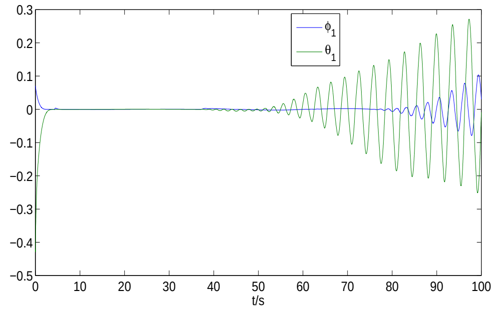


Fig. 6. The response curves of attitude for quadrotor1 without adaptive mechanism.

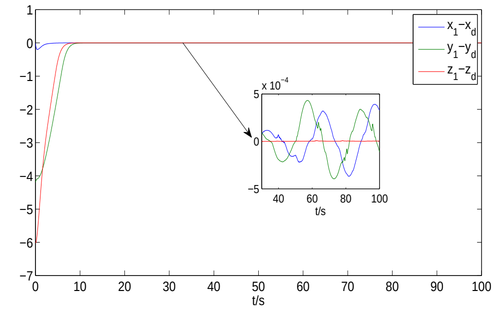


Fig. 7. The response curves of errors between position and desired position using proposed scheme.

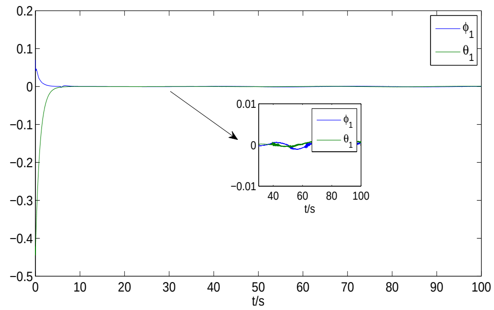


Fig. 8. The response curves of attitude for quadrotor1 using proposed scheme.

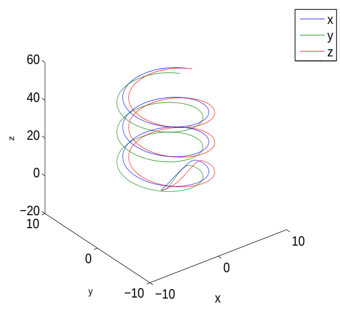


Fig. 9. The response curves of aircraft formation trajectory using proposed scheme.

The proposed adaptive controller can guarantee the tracking errors that converging to the bounded set in faulty cases.

## V. Conclusion

The main goal of this study is to solve the fault-tolerant formation control for a group of quadrotor aircrafts through the distributed control method. A distributed formation control algorithm that uses the global errors and a fault-tolerant control algorithm base on adaptive mechanism have been proposed. A precise theoretical analysis has indicated that the required formation pattern and the desired formation trajectory can be achieved through the proposed fault-tolerant formation control algorithm.

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**Wan-Zhang Liu** was born in Hebei province, China, in 1994. He was received B.Sc. degree in automation from Northwest Minzu University, Lanzhou, China, in 2016. He is currently working toward the M.Sc. degree at the College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, China.
His research interests cover fault-tolerant control and their applications in aircraft, and adaptive control.

**Ke Zhang** received the Ph.D. degree in control theory and engineering from Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 2012. He currently serves as Associate Editor for Nonlinear Analysis: Hybrid Systems. His research interests include fault diagnosis and fault tolerant control.

**Bin Jiang**(SM05) received the Ph. D. degree in automatic control from Northeastern University, Shenyang, China, in 1995.
He had been a Postdoctoral Fellow, a Research Fellow, and a Visiting Professor in Singapore, France, USA and Canada. He is currently a Chair Professor of the Cheung Kong Scholar Program, Ministry of Education, and the Dean of the College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, China. His research interests include fault diagnosis and fault-tolerant control and their applications in aircraft, satellite, and high-speed trains.
Prof. Jiang is currently an Associate Editor or an Editorial Board Member for a number of journals, such as the IEEE TRANSCATIONS ON CONTROL SYSTEMS TENOLOGY, Journal of Astronautics, Control and Decision, and Systems Engineering and Electronics Technologies.

**Xing-Gang Yan** received the B.Sc. degree from Shanxi Normal University, Xian, China, in 1985, the M.Sc. degree from Qufu Normal University, Jining, China, in 1991, and the Ph.D. degree in engineering from Northeastern University, Shenyang, China, in1997.
He was a lecturer with Qingdao University, China, from 1991 to 1994. He was a Research Fellow/Associate with the University of HongKong, HongKong, with Nanyang Technological University, Singapore, and with the University of Leicester, U.K. He is currently a Senior Lecturer with the University of Kent, Canterbury, U.K. His research interests include sliding mode control, decentralized control, fault detection and isolation, and control and observation of nonlinear systems and time-delay systems with applications.
Dr. Yan is the Editor-in-Chief of the International Journal of Engineering Research and Science and Technology.