Chapter 3
Stabilization

Abstract As one of the most important control problem, the stabilization problem is to design a controller such that the closed-loop system will be stable and has some desired performances. Due to singular Markovian jump systems containing singular derivative matrix and Markov property simultaneously, they usually complicate the synthesis, especially the underlying SMJSs have some general conditions. In this chapter, we will focus on the stabilization problem of SMJSs. Some kinds of controllers such that the closed-loop system is regular, stable and impulse-free are designed. A robust stabilizing controller guaranteeing the closed-loop systems robustly stochastically admissible is designed in the LMI setting. When an TRM can be designed, the stabilization for SMJSs is also discussed. The other kinds of controllers realized by noise control, proportional-derivative (PD) control and partially mode-dependent (PMD) control are put forward. Such stabilizing controller designs are formulated in terms of LMIs or LMIs with equation constraints, which can be solved easily.

3.1 Introduction

As one of the most important control problem, the stabilization problem is to design a controller such that the closed-loop system will be stable and has some desired performances. In this chapter, we will focus on the stabilization problem of SMJSs. Because singular derivative matrix and Markov property are included in SMJSs simultaneously, they usually make the synthesis for SMJSs with some general conditions complicated. The purpose is to design some kinds of controllers such that the closed-loop system is regular, stable and impulse-free. Based on the stability conditions proposed in Chapter 2, a robust stabilizing controller guaranteeing the closed-loop systems robustly stochastically admissible is designed in the LMI setting. When an TRM can be designed, the stabilization for SMJSs is also discussed. The other kinds of controllers realized by noise control, proportional-derivative (PD) control and partially mode-dependent (PMD) control are put forward. Such stabilizing controller designs are formulated in terms of LMIs or LMIs with equation constraints, which can be solved easily.

3.2 Robust Stabilization

Consider a class of SMJSs described as

$$E\dot{x}(t) = A(r_t)x(t) + B(r_t)u(t), \tag{3.1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input. Matrix $E \in \mathbb{R}^{n\times n}$ may be singular, which is assumed to be $\mathrm{rank}(E) = r \le n$. $A(r_t)$ and $B(r_t)$ are known matrices of compatible dimensions. Mode $\{r_t, t \ge 0\}$ is a continuous-time Markov process satisfying (2.2) and (2.3). In this section, TRM Π is obtained inexactly and described by Case 2.

Definition 3.1. Unforced SMJS in (3.1) is said to be robustly stochastically admissible, if there exists P_i, such that for all $i \in \mathbb{S}$

$$E_i^T P_i = P_i^T E \ge 0, \tag{3.2}$$

$$(A_i^T P_i)^\star + \sum_{j=1}^{N} \pi_{ij} E^T P_j < 0, \tag{3.3}$$

hold over admissible uncertainty (2.11).

Lemma 3.1. *[127] Let $\bar{P}_i \in \mathbb{R}^{n\times n}$ be symmetric such that $E_L^T \bar{P}_i E_L > 0$ and $\bar{Q}_i \in \mathbb{R}^{(n-r)\times(n-r)}$ is nonsingular for each $i \in \mathbb{S}$. Then, $\bar{P}_i E + U^T \bar{Q}_i V^T$ is nonsingular and its inverse is expressed as*

$$(\bar{P}_i E + U^T \bar{Q}_i V^T)^{-1} = \hat{P}_i E^T + V \hat{Q}_i U, \tag{3.4}$$

where $\hat{P}_i \in \mathbb{R}^{n\times n}$ is a symmetric matrix and $\hat{Q}_i \in \mathbb{R}^{(n-r)\times(n-r)}$ is a nonsingular matrix such that

$$E_R^T \hat{P}_i E_R = (E_L^T \bar{P}_i E_L)^{-1},\ \hat{Q}_i = (V^T V)^{-1} \bar{Q}_i^{-1} (U U^T)^{-1}, \tag{3.5}$$

where $U \in \mathbb{R}^{(n-r)\times n}$ is any matrix with full row rank and satisfies $UE = 0$; $V \in \mathbb{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $EV = 0$. Matrix E is decomposed as $E = E_L E_R^T$ with $E_L \in \mathbb{R}^{n\times r}$ and $E_R \in \mathbb{R}^{n\times r}$ are of full column rank.

In this section, a mode-dependent controller (MDC) is developed as follows:

$$u(t) = K(r_t)x(t), \tag{3.6}$$

where $K(r_t)$ is the designed control gain. When its operation mode is not available all time, a mode-independent controller (MIC) can be constructed as

$$u(t) = Kx(t), \tag{3.7}$$

where K is control gain to be determined.

Now, we will give an LMI condition for MDC (3.6).

Theorem 3.1. *Consider system (3.1), there exists an MDC (3.6) such that the closed-loop system is robustly stochastically admissible, if there exist $\hat{P}_i$, $\hat{Q}_i$, Y_i, $\bar{W}_i = \bar{W}_i^T$ and $\bar{T}_i > 0$, such that the following LMIs hold for all $i, j \in \mathbb{S}$, $j \neq i$,*

$$\begin{bmatrix} \bar{\Omega}_{i1} & \bar{W}_i & \bar{\Omega}_{i2} \\ * & -\bar{T}_i & 0 \\ * & * & \bar{\Omega}_{i3} \end{bmatrix} < 0, \tag{3.8}$$

$$\begin{bmatrix} -E^T \hat{P}_i E - \bar{W}_i & X_i^T E_R \\ * & -E_R^T \hat{P}_j E_R \end{bmatrix} \leq 0, \tag{3.9}$$

where

$$\bar{\Omega}_i = (A_i X_i + B_i Y_i)^\star + 0.25\varepsilon_{ii}^2 \bar{T}_i - \varepsilon_{ii}\bar{W}_i + \alpha_{ii} E \hat{P}_i E^T, X_i = \hat{P}_i E^T + V\hat{Q}_i U,$$
$$\bar{\Omega}_{i2} = \left[\sqrt{\alpha_{i1}} X_i^T E_R \cdots \sqrt{\alpha_{i(i-1)}} X_i^T E_R \ \sqrt{\alpha_{i(i+1)}} X_i^T E_R \cdots \sqrt{\alpha_{iN}} X_i^T E_R \right],$$
$$\bar{\Omega}_{i3} = -\mathrm{diag}\{E_R^T \hat{P}_i E_R, \ldots, E_R^T \hat{P}_{i-1} E_R, E_R^T \hat{P}_{i+1} E_R, \ldots, E_R^T \hat{P}_N E_R\}.$$

In this case, the gain of MDC (3.6) is given by

$$K_i = Y_i X_i^{-1}. \tag{3.10}$$

Proof. By Definition 3.1, system (3.1) is robustly stochastically admissible if (3.2) and (3.3) are satisfied. Especially, (3.3) is equivalent to

$$\begin{aligned} &(\bar{A}_i^T P_i)^\star + \sum_{j=1}^{N} \alpha_{ij}(E^T P_j - E^T P_i) - \Delta\tilde{\pi}_{ii} W_i - \varepsilon_{ii} W_i \\ &+ \sum_{j=1,\ j\neq i}^{N} (\Delta\tilde{\pi}_{ij} + \varepsilon_{ij})(E^T P_j - E^T P_i - W_i) < 0, \end{aligned} \tag{3.11}$$

where $\bar{A}_i = A_i + B_i K_i$ and $W_i = W_i^T$, which is guaranteed by

$$(\bar{A}_i^T P_i)^\star + \sum_{j=1}^{N} \alpha_{ij}(E^T P_j - E^T P_i) - \Delta\tilde{\pi}_{ii} W_i - \varepsilon_{ii} W_i < 0, \tag{3.12}$$

$$E^T P_j - E^T P_i - W_i < 0, j \neq i. \tag{3.13}$$

Moreover, for any $T_i > 0$, it is known that

$$\Delta\tilde{\pi}_{ii} W_i \leq 0.25(\Delta\tilde{\pi}_{ii})^2 T_i + W_i T_i^{-1} W_i \leq 0.25\varepsilon_{ii}^2 T_i + W_i T_i^{-1} W_i. \tag{3.14}$$

Taking into account (3.14), one has (3.12) got by

$$(\bar{A}_i^T P_i)^\star + \sum_{j=1}^{N} \alpha_{ij}(E^T P_j - E^T P_i) + 0.25\varepsilon_{ii}^2 T_i + W_i T_i^{-1} W_i - \varepsilon_{ii} W_i < 0, \tag{3.15}$$

Let

$$P_i \triangleq \bar{P}_i E + U^T \bar{Q}_i V^T, \tag{3.16}$$

where $\bar{P}_i > 0$ and $\bar{Q}_i$ is nonsingular. Then, one has

$$E^T P_i = P_i^T E = E^T \bar{P}_i E \geq 0, \tag{3.17}$$

always holds. Since $\bar{P}_i > 0$ and $\bar{Q}_i$ is nonsingular, we obtain $E_L^T \bar{P}_i E_L > 0$. Then via Lemma 3.1, we have

$$X_i \triangleq P_i^{-1} = \hat{P}_i E^T + V \hat{Q}_i U, \tag{3.18}$$

where $\hat{P}_i$ and $\hat{Q}_i$ are defined in Lemma 3.1. Denoting $\bar{W}_i = X_i^T W_i X_i$, pre- and post-multiplying (3.14) with X_i^T and X_i, one gets it is equivalent to (3.9). Let $\bar{T}_i = X_i^T T_i X_i$, pre- and post-multiplying (3.15) with $\mathrm{diag}\{X_i^T, X_i^T\}$ and its transpose, we obtain

$$\begin{bmatrix} \bar{\Omega}_i & \bar{W}_i \\ * & -\bar{T}_i \end{bmatrix} < 0, \tag{3.19}$$

where

$$\bar{\Omega}_i = (A_i X_i + B_i K_i X_i)^\star + 0.25\varepsilon_{ii}^2 \bar{T}_i - \varepsilon_{ii} \bar{W}_i + \sum_{j=1, j\neq i}^{N} \alpha_{ij} X_i^T E^T (P_j - P_i) X_i.$$

Taking into account (3.10) and (3.18), it is concluded that (3.8) implies (3.19). This completes the proof.

If TRM is got exactly, some sufficient conditions for MDC (3.6) were given.

Lemma 3.2. *[182] Consider system (3.1). There exists an MDC (3.6) such that the closed-loop system is stochastically admissible, if there exist $P_i > 0$, Y_i and scalar $\delta_i > 0$, such that the following LMIs hold for all $i \in \mathbb{S}$*

$$EP_i = P_i^T E^T \geq 0, \tag{3.20}$$

$$P_i^T E^T \leq \delta_i I, \tag{3.21}$$

$$\begin{bmatrix} (A_i P_i + B_i Y_i)^\star + \pi_{ii} P_i^T E^T & \hat{\Omega}_{i2} \\ * & \hat{\Omega}_{i3} \end{bmatrix} < 0, \tag{3.22}$$

where

$$\hat{\Omega}_{i2} = \left[\sqrt{\pi_{i1}} P_i^T \ \cdots \ \sqrt{\pi_{i(i-1)}} P_i^T \ \sqrt{\pi_{i(i+1)}} P_i^T \ \cdots \ \sqrt{\pi_{iN}} P_i^T \right],$$
$$\hat{\Omega}_{i3} = -\mathrm{diag}\{(P_1)^\star - \delta_1 I, \ldots, (P_{i-1})^\star - \delta_{i-1} I, (P_{i+1})^\star - \delta_{i+1} I, \ldots, (P_N)^\star - \delta_N I\}.$$

Then, the corresponding gain is given by

$$K_i = Y_i P_i^{-1}. \tag{3.23}$$

Lemma 3.3. *[11] Consider system (3.1). There exists an MDC (3.6) such that the closed-loop system is stochastically admissible, if there exist X_i, Y_i and $\delta_i > 0$, such that the following LMIs hold for all $i \in \mathbb{S}$*

$$EP_i = P_i^T E^T \geq 0, \tag{3.24}$$

$$\begin{bmatrix} (A_iX_i + B_iY_i)^\star + \pi_{ii}X_i^T E^T & \check{\Omega}_{i2} & \hat{\Omega}_{i3} \\ * & \Omega_{i4} & 0 \\ * & * & \Omega_{i5} \end{bmatrix} < 0, \tag{3.25}$$

where

$$\begin{aligned}
\check{\Omega}_{i2} &= \left[\sqrt{\pi_{i1}}X_i^T \ \cdots \ \sqrt{\pi_{i(i-1)}}X_i^T \ \sqrt{\pi_{i(i+1)}}X_i^T \ \cdots \ \sqrt{\pi_{iN}}X_i^T \right], \\
\hat{\Omega}_{i3} &= \left[\sqrt{\pi_{i1}}X_i^T E^T \ \cdots \ \sqrt{\pi_{i(i-1)}}X_i^T E^T \ \sqrt{\pi_{i(i+1)}}X_i^T E^T \ \cdots \ \sqrt{\pi_{iN}}X_i^T E^T \right], \\
\Omega_{i4} &= -4\mathrm{diag}\{\delta_1 I, \ldots, \delta_{i-1}I, \delta_{i+1}I, \ldots, \delta_N I\}, \\
\Omega_{i5} &= -\mathrm{diag}\{(X_1)^\star - \delta_1 I, \ldots, (X_{i-1})^\star - \delta_{i-1}I, (X_{i+1})^\star - \delta_{i+1}I, \ldots, (X_N)^\star - \delta_N I\}.
\end{aligned}$$

Then, the corresponding gain is given by (3.10).

Lemma 3.4. *[170] Let μ_i be given scalar. There exists an MDC (3.6) such that the closed-loop system is stochastically admissible, if there exist $\hat{P}_i > 0$, $\hat{Q}_i$, L_i and H_i, such that the following LMIs hold for all $i \in \mathbb{S}$*

$$\begin{bmatrix} \tilde{\Omega}_{i1} & \tilde{\Omega}_{i2} \\ * & \bar{\Omega}_{i3} \end{bmatrix} < 0, \tag{3.26}$$

where

$$\begin{aligned}
\tilde{\Omega}_{i1} &= [A_iX_i + B_i(L_iE^T + H_iV^T)]^\star + \pi_{ii}[\mu_i(EX_i)^\star - \mu_i^2 E\hat{P}_iE^T], X_i = \hat{P}_iE^T + V\hat{Q}_iU, \\
\tilde{\Omega}_{i2} &= \left[\sqrt{\pi_{i1}}X_i^T E_R \ \cdots \ \sqrt{\pi_{i(i-1)}}X_i^T E_R \ \sqrt{\pi_{i(i+1)}}X_i^T E_R \ \cdots \ \sqrt{\pi_{iN}}X_i^T E_R \right].
\end{aligned}$$

By investigating such results, it is seen that in order to stabilize an SMJS via MDC (3.6), some additional inequalities are introduced or some parameters are given beforehand. Based on Theorem 3.1, a corollary could be obtained directly, in which no more inequalities are used and no parameters are given in advance.

Corollary 3.1. *Consider system (3.1), there exists an MDC (3.6) such that the resulting closed-loop system is stochastically admissible, if there exist $\hat{P}_i$, $\hat{Q}_i$ and Y_i, such that the following LMIs hold for all $i \in \mathbb{S}$*

$$\begin{bmatrix} \Omega_{i1} & \Omega_{i2} \\ * & \bar{\Omega}_{i3} \end{bmatrix} < 0, \tag{3.27}$$

where

$$\begin{aligned}
\Omega_i &= (A_iX_i + B_iY_i)^\star + \tilde{\pi}_{ii}E\hat{P}_iE^T, \\
\Omega_{i2} &= \left[\sqrt{\pi_{i1}}X_i^T E_R \ \cdots \ \sqrt{\pi_{i(i-1)}}X_i^T E_R \ \sqrt{\pi_{i(i+1)}}X_i^T E_R \ \cdots \ \sqrt{\pi_{iN}}X_i^T E_R \right].
\end{aligned}$$

Then, the gain of MDC (3.6) is constructed by (3.10).

It is seen that the work of controller (3.6) requires its mode available online. However, in many practical applications, the data is usually transmitted through unreliable networks, which suffers packet dropout. As a result, controller (3.6) is too ideal. Instead, MIC (3.7) is usually constructed to deal with the above case. In order to obtain a common control gain K, the matrix related to K may be also a common matrix. That means the corresponding Lyapunov function is mode-independent, which is more conservative than mode-dependent ones. In the next, another sufficient condition is presented, which makes the requirements of mode-independent controller and mode-dependent Lyapunov function satisfy simultaneously.

Theorem 3.2. *Consider system (3.1), there exists an MDC (3.6) such that the closed-loop system is robustly stochastically admissible, if there exist $\hat{P}_i$, $\hat{Q}_i$, G_i, Y_i, $\bar{W}_i = \bar{W}_i^T$ and $\bar{T}_i > 0$, such that the following LMIs hold for all $i, j \in \mathbb{S}$, $j \neq i$,*

$$\begin{bmatrix} \Phi_{i1} & \Phi_{i2} & \bar{W}_i & \bar{\Omega}_{i2} \\ * & (-G_i)^\star & 0 & 0 \\ * & * & -\bar{T}_i & 0 \\ * & * & * & \bar{\Omega}_{i3} \end{bmatrix} < 0, \tag{3.28}$$

$$\begin{bmatrix} -E^T\hat{P}_iE - \bar{W}_i & X_i^T E_R \\ * & -E_R^T\hat{P}_jE_R \end{bmatrix} \leq 0, \tag{3.29}$$

where

$$\Phi_{i1} = (A_iG_i + B_iY_i)^\star + 0.25\varepsilon_{ii}^2\bar{T}_i - \varepsilon_{ii}\bar{W}_i + \alpha_{ii}E\hat{P}_iE^T,$$
$$\Phi_{i2} = (A_iG_i + B_iY_i)^\star + X_i^T - G_i^T.$$

In this case, the gain of MDC (3.6) is chosen as

$$K_i = Y_iG_i^{-1}. \tag{3.30}$$

Proof. Pre- and post-multiplying (3.28) with the following matrix

$$\begin{bmatrix} I & \bar{A}_i & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix},$$

and its transpose respectively, it is obvious that (3.28) implies (3.8). This completes the proof.

If the conditions in Theorem 3.2 with $G_i = G$ satisfy, a corollary is obtained directly.

Corollary 3.2. *Consider system (3.1), there exists an MIC (3.7) such that the closed-loop system is robustly stochastically admissible, if there exist $\hat{P}_i$, $\hat{Q}_i$, G, Y, $\bar{W}_i = \bar{W}_i^T$ and $\bar{T}_i > 0$, such that the following LMIs hold for all $i, j \in \mathbb{S}$, $j \neq i$,*

$$\begin{bmatrix} \bar{\Phi}_{i1} & \bar{\Phi}_{i2} & \bar{W}_i & \bar{\Omega}_{i2} \\ * & (-G)^\star & 0 & 0 \\ * & * & -\bar{T}_i & 0 \\ * & * & * & \bar{\Omega}_{i3} \end{bmatrix} < 0, \tag{3.31}$$

$$\begin{bmatrix} -E^T \hat{P}_i E - \bar{W}_i & X_i^T E_R \\ * & E_R^T \hat{P}_j E_R \end{bmatrix} \leq 0, \tag{3.32}$$

where

$$\bar{\Phi}_{i1} = (A_i G + B_i Y)^\star + 0.25\varepsilon_{ii}^2 \bar{T}_i - \varepsilon_{ii} \bar{W}_i + \alpha_{ii} E^T \hat{P}_i E,$$
$$\bar{\Phi}_{i2} = (A_i G + B_i Y)^\star + X_i^T - G^T.$$

Then, the gain of MIC (3.7) is computed by

$$K = YG^{-1}. \tag{3.33}$$

Example 3.1. Consider an SMJS of form (3.1) obtained by

$$A_1 = \begin{bmatrix} -0.2 & 1 & 0.3 \\ 2 & -1.2 & -6 \\ 2 & 1 & -1 \end{bmatrix}, \; B_1 = \begin{bmatrix} 1.5 \\ 0.4 \\ 1 \end{bmatrix}.$$

$$A_2 = \begin{bmatrix} 0.2 & 1.3 & -0.3 \\ 3 & -1.2 & -1 \\ 1 & 2 & 1 \end{bmatrix}, \; B_2 = \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}.$$

The singular matrix is given as

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The transition rates are given as $\tilde{\pi}_{11} = -5$ and $\tilde{\pi}_{22} = -7$, and its uncertainties are such that $|\Delta\tilde{\pi}_{12}| \leq \varepsilon_{12} \triangleq 0.5\tilde{\pi}_{12}$ and $|\Delta\tilde{\pi}_{21}| \leq \varepsilon_{21} \triangleq 0.5\tilde{\pi}_{21}$ respectively. Under initial condition $x_0 = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}^T$, the state of open-loop systems is illustrated in Fig. 3.1, which is not stable. When the system mode is always available to controller, by Theorem 3.2, an MDC can be computed as

$$K_1 = \begin{bmatrix} -0.3396 & 1.2769 & -1.1206 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -0.9982 & 0.6619 & 1.2338 \end{bmatrix}.$$

Applying the desired controller, the state response of the closed-loop system is shown in Fig. 3.2. It is stable over all the admissible uncertainties. If the system mode is not always available to controller, it means the controller mode accessibility is in terms of probability. For this example, the system mode received by controller is only about 30%. Fig. 3.3 gives the corresponding simulation, where $*$ denotes the current mode inaccessible. Via Corollary 3.2, an MIC can be designed as

$$K = \left[-0.3346 \ 0.4602 \ 0.7148 \right].$$

The response of the closed-loop system is given in Fig. 3.3, which shows the constructed controller can stabilize the system over all the admissible uncertainties.

3.3 Stabilization with TRM Design

Consider a class of linear SMJSs described as

$$E\dot{x} = A(r_t)x(t) + B(r_t)u(t), \tag{3.34}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input. Matrix $E \in \mathbb{R}^{n \times n}$ may be singular, which is assumed to be $\text{rank}(E) = r \leq n$. $A(r_t)$ and $B(r_t)$ are known matrices of compatible dimensions. The mode $\{r_t, t \geq 0\}$ is defined as (2.2) and (2.3).

In this section, the aim is to design a mode-dependent feedback controller

$$u(t) = K(r_t)x(t) \tag{3.35}$$

and an appropriate TRM Π such that the closed-loop system is stochastically admissible.

Theorem 3.3. *Consider system (3.34). There exist a controller (3.35) and an SPRM such that the closed-loop system (3.34) is stochastically admissible, if there exist*

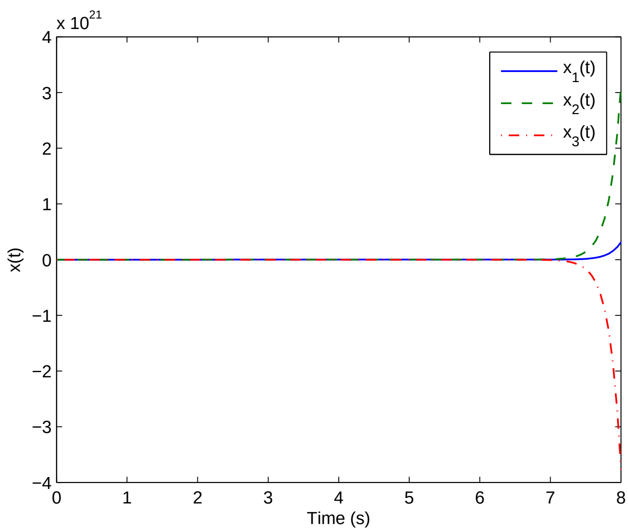

Fig. 3.1 The simulation of open-loop system

$\hat{P}_i > 0$, $W_i > 0$, $Z_i > 0$, $\hat{\pi}_{ij} \geq 0$, $i \neq j$, nonsingular matrix $\hat{Q}_i$ and matrix Y_i, such that the following LMIs hold for all $i, j \in \mathbb{S}$, $j \neq i$,

$$\begin{bmatrix} \Theta_{i1} & \Omega_{i2} \\ * & \Omega_{i3} \end{bmatrix} < 0, \tag{3.36}$$

$$\begin{bmatrix} \Phi_{i1} & \Phi_{i2} \\ * & \Phi_{j3} \end{bmatrix} < 0, \tag{3.37}$$

$$W_i Z_i = I, \tag{3.38}$$

where

$$\Theta_{i1} = (A_i \hat{P}_i E^T)^\star + (A_i V \hat{Q}_i U)^\star + (B_i Y_i)^\star$$
$$\Omega_{i2} = \left[\, \hat{\pi}_{i1} I \; \cdots \; \hat{\pi}_{i(i-1)} I \; \hat{\pi}_{i(i+1)} I \; \cdots \; \hat{\pi}_{iN} I \,\right],$$
$$\Omega_{i3} = -\mathrm{diag}\{Z_i, \cdots, Z_i\}, \Phi_{i1} = -E\hat{P}_i E^T - W_i,$$
$$\Phi_{i2} = E\hat{P}_i E_R + U^T \hat{Q}_i^T V^T E_R, \; \Phi_{j3} = -E_R^T \hat{P}_j E_R.$$

E is decomposed as $E = E_L E_R^T$ with $E_L \in \mathbb{R}^{n\times r}$ and $E_R \in \mathbb{R}^{n\times r}$ are of full column rank. Then the gain of controller (3.35) and the corresponding SPRM are obtained as

$$K_i = Y_i(\hat{P}_i E^T + V\hat{Q}_i U)^{-1}, \tag{3.39}$$

and

$$\pi_{ij} = \hat{\pi}_{ij}^2, \pi_{ii} = -\sum_{j\neq i} \pi_{ij}. \tag{3.40}$$

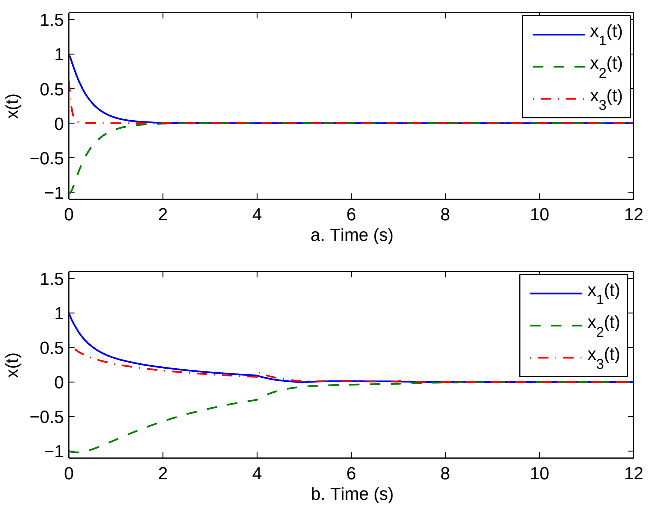

Fig. 3.2 The simulation of closed-loop system by MDC

Proof. Substituting A_i with $\bar{A}_i = A_i + B_i K_i$, we have the closed-loop system (3.34) is stochastically admissible if and only if (3.2) and (3.3) are satisfied which are equivalent to

$$X_i^T E_i^T = E X_i \geq 0, \tag{3.41}$$

$$(\bar{A}_i X_i)^{\star} + \sum_{j=1}^{N} \pi_{ij} X_i^T E^T P_j X_i < 0, \tag{3.42}$$

where $X_i = P_i^{-1}$. Based on Lemma 3.1, it is concluded that

$$X_i = (\bar{P}_i E + U^T \bar{Q}_i V^T)^{-1} = \hat{P}_i E^T + V \hat{Q}_i U, \tag{3.43}$$

$$E_R^T \hat{P}_i E_R = (E_L^T \bar{P}_i E_L)^{-1}, \tag{3.44}$$

with $\hat{P}_i > 0$ and $|\hat{Q}_i| \neq 0$. Then, (3.41) is obviously satisfied and (3.42) is transformed to

$$(\bar{A}_i X_i)^{\star} + \sum_{j=1,\ j\neq i}^{N} \pi_{ij} W_i + \sum_{j=1,\ j\neq i}^{N} \pi_{ij} (X_i^T E^T P_j X_i - X_i^T E^T - W_i) < 0. \tag{3.45}$$

Moreover, (3.45) is guaranteed by

$$(\bar{A}_i X_i)^{\star} + \sum_{j=1,\ j\neq i}^{N} \pi_{ij} W_i < 0, \tag{3.46}$$

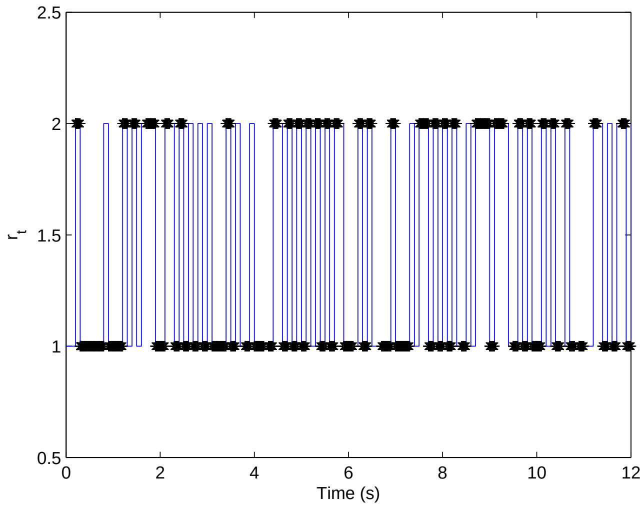

Fig. 3.3 The simulation of closed-loop system by MIC

$$X_i^T E^T P_j X_i - X_i^T E^T - W_i < 0. \tag{3.47}$$

Taking into account (3.43), (3.44) and by Schur complement, it is obtained that (3.36)-(3.38) with (3.39) and (3.40) imply (3.45). This completes the proof.

It is seen that Theorem 3.3 can also be extended to polytopic uncertainty case. Assume that system (3.34) has polytopic uncertainties, that is

$$\{A_i, B_i\} = \sum_{l=1}^{m} \alpha_l \{A_{il}, B_{il}\}, \ \sum_{l=1}^{m} \alpha_l = 1, \ \alpha_l \geq 0. \tag{3.48}$$

We will have the following corollary.

Corollary 3.3. *For system (3.34) with polytopic uncertainty, there exist a controller (3.35) and an SPRM such that the resulting system (3.34) is stochastically admissible, if there exist $\hat{P}_i > 0$, $W_i > 0$, $Z_i > 0$, $\hat{\pi}_{ij} \geq 0$, $i \neq j$, nonsingular matrix $\hat{Q}_i$ and matrix Y_i, such that the following LMIs hold for all $i, j \in \mathbb{S}$, $j \neq i$, and $l = 1, \cdots, m$*

$$\begin{bmatrix} \bar{\Theta}_{il} & \Omega_{i2} \\ * & \Omega_{i3} \end{bmatrix} < 0 \tag{3.49}$$

$$\begin{bmatrix} \Phi_{i1} & \Phi_{i2} \\ * & \Phi_{j3} \end{bmatrix} < 0, \tag{3.50}$$

$$W_i Z_i = I, \tag{3.51}$$

where

$$\bar{\Theta}_{il} = (A_{il} \hat{P}_i E^T)^\star + (A_{il} V \hat{Q}_i U)^\star + (B_{il} Y_i)^\star.$$

The other are given in Theorem 3.3. Then, the gain of controller (3.35) and TRM are computed by (3.39) and (3.40) respectively.

Example 3.2. Consider the stabilization problem via designing state feedback controller and TRM for system (3.34) with system parameters

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} 1 & 1 \\ 0 & -0.2 \end{bmatrix}, B_1 = \begin{bmatrix} 0.5 \\ -0.4 \end{bmatrix},$$

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -1 & 0.5 \\ 0.4 & 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1.2 \end{bmatrix}.$$

By Theorem 3.3 and CCL algorithm, we have the stabilizing controller

$$K_1 = \begin{bmatrix} 9.5251 & 3.7919 \end{bmatrix}, K_2 = \begin{bmatrix} -1.0979 & -1.3126 \end{bmatrix},$$

with the TRM

$$\Pi = \begin{bmatrix} -55.3788 & 55.3788 \\ 82.0548 & -82.0548 \end{bmatrix}.$$

Fig. 3.4 shows the state trajectories of the closed-loop system with initial value $x_0^T = \begin{bmatrix} 1 & -2 \end{bmatrix}^T$, where the simulation of the corresponding system mode is illustrated in Fig. 3.5.

3.4 Stabilization by Noise Control

Consider a class of Markovian jump singularly perturbed systems (MJSPSs) described as

$$E(\varepsilon)\mathrm{d}x(t) = A(r_t)x(t)\mathrm{d}t + [C(r_t)x(t) + D(r_t)u_\omega(t)]\mathrm{d}\omega(t), \tag{3.52}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u_\omega(t)$ is control input in the diffusion part, $\omega(t) \in \mathbb{R}^q$ is a q-dimensional Brownian motion or Wiener process. The underlying complete probability space is $(\Omega, \mathscr{F}, \mathscr{F}_t, \mathbb{P})$ with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. it is right continuous and $\mathscr{F}_0$ contains the $\mathbb{P}$-null sets). Matrices $A(r_t)$, $C(r_t)$ and $D(r_t)$ are known matrices of compatible dimensions. Without loss of generality, it is assumed that $E(\varepsilon) = \mathrm{diag}\{I_{n_1}, \varepsilon I_{n_2}\}$. Operation mode $\{(r_t), t \geq 0\}$ satisfying (2.2) and (2.3) is a stationary ergodic Markov process. For such Markov process, it is obtained that

$$\sum_{i=1}^{N} \pi_{\infty j} = 1, \ \pi_{\infty j} > 0, \tag{3.53}$$

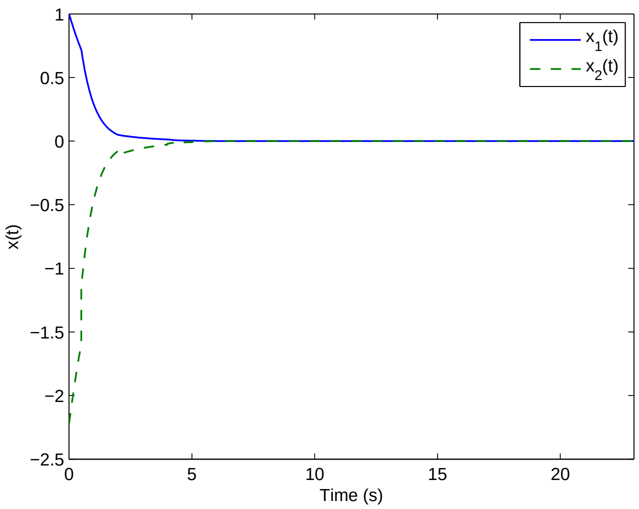

Fig. 3.4 The simulation of closed-loop system

where $\pi_{\infty\mu}$ is the μth element of vector $\pi_\infty = \mathscr{I}(\Pi + \mathbb{E})^{-1}$, and $\mathscr{I} = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}$, $\mathbb{E} = \begin{bmatrix} \mathscr{I}^T & \mathscr{I}^T & \cdots & \mathscr{I}^T \end{bmatrix}^T$.

In this section, the state feedback controller is restricted only in shift or diffusion part, which is ε-dependent and described as

$$u_\omega(t) = K(r_t, \varepsilon)x(t), \tag{3.54}$$

where control gain $K(r_t, \varepsilon)$ is to be designed. Then the considered problem is formulated as follows:

Proposition 3.1. *Given a stabilization bound $\bar{\varepsilon}$, determine a kind of stochastic controller (3.54) such that for any initial condition $x_0 \in \mathbb{R}^n$ and $r_0 \in \mathbb{S}$, the closed-loop system (3.52) is almost surely exponentially stable, whose solution is satisfied*

$$\lim_{t\to\infty} \sup \frac{1}{t} \log(|x(t, x_0)|) < 0 \quad \text{a.s.}$$

Theorem 3.4. *Given a scalar $\bar{\varepsilon} > 0$, the equilibrium of MJSPS (3.52) is almost surely exponentially stable with control gain $K_i(\varepsilon) = Y_i X^{-1}(\varepsilon)$ for any $\varepsilon \in (0, \bar{\varepsilon}]$, if there exists matrices $X_1 > 0$, $X_2 = X_2^T$, $X_3 = X_3^T$, $X_4 = X_4^T$, X_5, Y_{i1} and Y_{i2} such that the following LMIs hold for all $i \in \mathbb{S}$*

$$\Omega_1 \geq 0, \tag{3.55}$$

$$\Omega_1 + \bar{\varepsilon}\Omega_2 \geq 0, \tag{3.56}$$

$$\Omega_1 + \bar{\varepsilon}\Omega_2 + \bar{\varepsilon}^2\Omega_3 > 0, \tag{3.57}$$

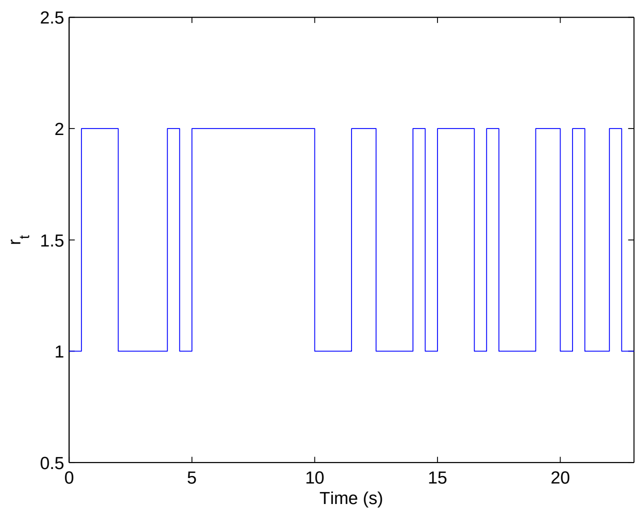

Fig. 3.5 The simulation of system mode

$$\begin{bmatrix} \Phi_{i1} & \Phi_{i2}\mathbb{I}^T \\ * & -X_1 \end{bmatrix} \leq 0, \tag{3.58}$$

$$\begin{bmatrix} \Phi_{i1} & \Phi_{i2} \\ * & -\Omega_1 \end{bmatrix} + \bar{\varepsilon} \begin{bmatrix} \Psi_{i1} & \Psi_{i2} \\ * & -\Omega_2 \end{bmatrix} \leq 0, \tag{3.59}$$

$$\begin{bmatrix} \Phi_{i1} & \Phi_{i2} \\ * & -\Omega_1 \end{bmatrix} + \bar{\varepsilon} \begin{bmatrix} \Psi_{i1} & \Psi_{i2} \\ * & -\Omega_2 \end{bmatrix} - \bar{\varepsilon}^2 \begin{bmatrix} \alpha_i \Omega_3 & 0 \\ * & \Omega_3 \end{bmatrix} < 0, \tag{3.60}$$

$$\sum_{j=1}^{N} \pi_{\infty j}(\alpha_j - 0.5\beta_j^2) < 0, \tag{3.61}$$

either

$$\Upsilon_{i1} \leq 0, \tag{3.62}$$

$$\Upsilon_{i1} + \bar{\varepsilon}\Upsilon_{i2} \leq 0, \tag{3.63}$$

$$\Upsilon_{i1} + \bar{\varepsilon}\Upsilon_{i2} + \bar{\varepsilon}^2 \beta_i \Omega_3 < 0, \tag{3.64}$$

or

$$\bar{\Upsilon}_{i1} \geq 0, \tag{3.65}$$

$$\bar{\Upsilon}_{i1} + \bar{\varepsilon}\bar{\Upsilon}_{i2} \geq 0, \tag{3.66}$$

$$\bar{\Upsilon}_{i1} + \bar{\varepsilon}\bar{\Upsilon}_{i2} - \bar{\varepsilon}^2 \beta_i \Omega_3 > 0, \tag{3.67}$$

where

$$\Omega_1 = \begin{bmatrix} X_1 & 0 \\ * & 0 \end{bmatrix},\ \Omega_2 = \begin{bmatrix} X_3 & X_5^T \\ * & X_2 \end{bmatrix},\ \Omega_3 = \begin{bmatrix} 0 & 0 \\ * & X_4 \end{bmatrix},$$
$$\Phi_{i1} = (A_i U)^\star - \alpha_i \Omega_1,\ \Phi_{i2} = U^T C_i^T + Y_{i1}^T D_i^T,$$
$$\Psi_{i1} = (A_i V)^\star - \alpha_i \Omega_2,\ \Psi_{i2} = V^T C_i^T + Y_{i2}^T D_i^T,$$
$$U = \begin{bmatrix} X_1 & 0 \\ X_5 & X_2 \end{bmatrix},\ V = \begin{bmatrix} X_3 & X_5^T \\ 0 & X_4 \end{bmatrix},$$
$$X_\varepsilon = U + \varepsilon V,\ Y_i = Y_{i1} + Y_{i2},\ \mathbb{I} = \begin{bmatrix} I & 0 \end{bmatrix},$$
$$\Upsilon_{i1} = (C_i U + D_i Y_{i1})^\star + \beta_i \Omega_1,\ \Upsilon_{i2} = (C_i V + D_i Y_{i2})^\star + \beta_i \Omega_2$$
$$\bar{\Upsilon}_{i1} = (C_i U + D_i Y_{i1})^\star - \beta_i \Omega_1,\ \bar{\Upsilon}_{i2} = (C_i V + D_i Y_{i2})^\star - \beta_i \Omega_2,$$

α_i and β_i are some nonnegative constants.

Proof. For any given initial condition $x_0 \neq 0$, it is known that $x(t) \triangleq x(t;x_0)$ will never reach zero with probability one. From the definition of $X(\varepsilon)$ and notation $E(\varepsilon)$, it is obtained that

$$E(\varepsilon)X(\varepsilon) = X^T(\varepsilon)E^T(\varepsilon) > 0,\ \forall \varepsilon \in (0, \bar{\varepsilon}], \tag{3.68}$$

which is guaranteed by Lemma 2.3 with conditions (3.55)-(3.57) and implies that $X(\varepsilon)$ is nonsingular $\forall \varepsilon \in (0, \bar{\varepsilon}]$. Let $P(\varepsilon) = X^{-1}(\varepsilon)$, we have that $E^T(\varepsilon)P(\varepsilon) = P^T(\varepsilon)E(\varepsilon) > 0$, and the corresponding Lyapunov function is defined as

$$V(x(t),t) = x^T(t)E^T(\varepsilon)P(\varepsilon)x(t), \tag{3.69}$$

Applying the Itô formula to $\log(V(x(t),t))$, one has that

$$\begin{aligned} \mathrm{d}[\log(V(x(t),t))] &= \frac{1}{V(x(t),t)}[\mathscr{L}V(x(t),t)\mathrm{d}t + \mathscr{H}V(x(t),t)\mathrm{d}\omega(t))] \\ &\quad - \frac{1}{2V^2(x(t),t)}|\mathscr{H}V(x(t),t)|^2\mathrm{d}t, \end{aligned} \tag{3.70}$$

where

$$\mathscr{L}V(x(t),t) = x^T(t)[(A_i^T P(\varepsilon))^\star + \bar{C}_i^T P(\varepsilon)E^{-1}(\varepsilon)\bar{C}_i]x(t),$$
$$\mathscr{H}V(x(t),t) = x^T(t)(\bar{C}_i^T P(\varepsilon))^\star x(t),$$
$$\bar{C}_i = C_i + D_i K_i(\varepsilon).$$

Taking into account (3.58)-(3.60), it is concluded that

$$\begin{bmatrix} (A_i X(\varepsilon))^\star - \alpha_i X^T(\varepsilon)E^T(\varepsilon) & X^T(\varepsilon)\bar{C}_i^T \\ * & -E(\varepsilon)X(\varepsilon) \end{bmatrix} < 0,\ \forall \varepsilon \in (0,\bar{\varepsilon}], \tag{3.71}$$

which implies

$$(A_i X(\varepsilon))^\star - \alpha_i X^T(\varepsilon)E^T(\varepsilon) + X^T(\varepsilon)\bar{C}_i^T(E(\varepsilon)X(\varepsilon))^{-1}\bar{C}_i X(\varepsilon) < 0, \tag{3.72}$$

Considering $P(\varepsilon) = X^{-1}(\varepsilon)$, we have that

$$(A_i^T P(\varepsilon))^\star + \bar{C}_i^T P(\varepsilon)E^{-1}(\varepsilon)\bar{C}_i < \alpha_i E^T(\varepsilon)P(\varepsilon),\ \forall \varepsilon \in (0,\bar{\varepsilon}]. \tag{3.73}$$

On the other hand, by (3.62)-(3.67), it is known that

$$(\bar{C}_i X(\varepsilon))^\star + \beta_i X^T(\varepsilon)E^T(\varepsilon) < 0,\ \forall \varepsilon \in (0,\bar{\varepsilon}], \tag{3.74}$$

or

$$(\bar{C}_i X(\varepsilon))^\star - \beta_i X^T(\varepsilon)E^T(\varepsilon) > 0,\ \forall \varepsilon \in (0,\bar{\varepsilon}], \tag{3.75}$$

which are equivalent to

$$(\bar{C}_i^T P(\varepsilon))^\star + \beta_i E^T(\varepsilon)P(\varepsilon) < 0, \tag{3.76}$$

or

$$(\bar{C}_i^T P(\varepsilon))^\star - \beta_i E^T(\varepsilon)P(\varepsilon) > 0. \tag{3.77}$$

Based on (3.72), (3.76) and (3.77), it is obtained from (3.70) that

$$\log(V(x(t),t)) \le \log(V(x_0,0)) + \int_0^t (\alpha(r(s)) - 0.5\beta^2(r(s)))\mathrm{d}s + M(t), \tag{3.78}$$

where $M(t) = \int_0^t \frac{\mathscr{H}V(x(s),s,r(s))}{V(x(s),s)}\mathrm{d}\omega(s)$ is a continuous martingale vanishing at $t = 0$. Let $\delta \in (0,1)$ arbitrarily, from the exponential martingale, it is seen that

$$\mathbb{P}\{\sup_{0\le t\le k}[M(t)-\frac{\delta}{2}\langle M(t),M(t)\rangle]>\frac{2}{\delta}\log k\}\le\frac{1}{k^2},\tag{3.79}$$

where $\langle M(t),M(t)\rangle=\int_0^t\frac{|\mathscr{H}V(x(s),s,r(s))|^2}{V^2(x(s),s)}\mathrm{d}s$ and $k=1,2,\cdots$. By using the Borel-Cantelli lemma, it is claimed that for almost all $\eta\in\Omega$, there always exists an integer $k_0=K_0(\eta)$ such that

$$M(t)\le\frac{2}{\delta}\log k+\frac{\delta}{2}\langle M(t),M(t)\rangle,\tag{3.80}$$

holds for $\forall t\in[0,k]$, if $k\ge k_0$. Based on this, it is concluded that

$$\log(V(x(t),t))\le\log(V(x_0,0))+\frac{2}{\delta}\log k+\int_0^t[\alpha(r(s))-0.5(1-\delta)\beta^2(r(s))]\mathrm{d}s\quad a.s.\tag{3.81}$$

holds for all $t\in[0,k]$, $k\ge k_0$. Then if $t\in[k-1,k]$ and $k\ge k_0$, one has

$$\begin{aligned}\frac{1}{t}\log(V(x(t),t))\le&\frac{1}{k-1}(\log(V(x_0,0))+\frac{2}{\delta}\log k)\\&+\frac{1}{t}\int_0^t[\alpha(r(s))-0.5(1-\delta)\beta^2(r(s))]\mathrm{d}s\quad a.s.\end{aligned}\tag{3.82}$$

which implies

$$\begin{aligned}\lim_{t\to\infty}\sup\frac{1}{t}\log(V(x(t),t))\le&\lim_{t\to\infty}\frac{1}{t}\int_0^t[\alpha(r(s))-0.5(1-\delta)\beta^2(r(s))]\mathrm{d}s\\=&\sum_{j=1}^N\pi_{\infty j}[\alpha_j-0.5(1-\delta)\beta_j^2]\quad\text{a.s.}\end{aligned}\tag{3.83}$$

Let $\delta\to0$, it is equivalent to (3.61), which implies $\lim_{t\to\infty}\sup\frac{1}{t}\log(V(x(t),t))<0$. This completes the proof.

Remark 3.1. It is worth mentioning that by using an ε-dependent Lyapunov function, an LMI condition for stochastic controller (3.54) is presented and is dependent of ε. It is seen that not only the almost surely exponential stability of the closed-loop system is guaranteed by noise control method, but also a stabilization bound $\bar{\varepsilon}$ is contained. Moreover, the proposed method can be extended to other problems such as mode-independent control problem.

From Theorem 3.4, it is seen that TPM π_∞ is assumed to be known exactly. The traditional results on stochastic stabilization or destabilization of stochastic Markovian jump systems [1, 30, 50, 91, 95] all require π_∞ accurately available. This ideal assumption will largely limit the scope of application. In the following, some general cases are considered, and some sufficient results are established. Firstly, TPM π_∞ is assumed to have admissible uncertainty, which is described as

$$\sum_{j=1}^{N}(\pi_{\infty j}+\Delta\pi_{\infty j})=1,\ \pi_{\infty j}\geq 0, \tag{3.84}$$

where $\pi_{\infty j}$ is the estimation and $\Delta\pi_{\infty j}\in[-\theta_j,\theta_j]$. Then, we have the following theorem.

Theorem 3.5. *Given a scalar $\bar{\varepsilon}>0$, the equilibrium of MJSPS (3.52) is almost surely exponentially stable with control gain $K_i(\varepsilon)=Y_iX^{-1}(\varepsilon)$ for any $\varepsilon\in(0,\bar{\varepsilon}]$, if there exists matrices $X_1>0$, $X_2=X_2^T$, $X_3=X_3^T$, $X_4=X_4^T$, X_5, Y_{i1}, Y_{i2}, $\delta_i>0$ and $\gamma_i>0$ such that LMIs (3.55)-(3.57), (3.58)-(3.60), (3.62)-(3.64) or (3.65)-(3.67) and the following LMIs hold*

$$\begin{bmatrix}\bar{\Omega} & \bar{\delta}\\ * & \bar{\gamma}\end{bmatrix}<0, \tag{3.85}$$

$$\alpha_j-0.5\beta_j^2-\delta_j\leq 0,\ j\in\mathbb{S}, \tag{3.86}$$

where

$$\bar{\Omega}=\sum_{j=1}^{N}[\pi_{\infty j}(\alpha_j-0.5\beta_j^2)+\theta_j\delta_j+\frac{1}{4}\theta_j^2\gamma_j],$$
$$\bar{\delta}=\begin{bmatrix}\delta_1 & \cdots & \delta_N\end{bmatrix},\bar{\gamma}=-\mathrm{diag}\{\gamma_1,\cdots,\gamma_N\}.$$

Proof. Based on the proof of Theorem 3.4, it is obtained that the changed condition only takes place in (3.61), that is

$$\sum_{j=1}^{N}(\pi_{\infty j}+\Delta\pi_{\infty j})(\alpha_j-0.5\beta_j^2)<0. \tag{3.87}$$

It is equivalent to

$$\sum_{j=1}^{N}[(\pi_{\infty j}-\theta_j)(\alpha_j-0.5\beta_j^2)+\theta_j\delta_j]+\sum_{j=1}^{N}\Delta\pi_{\infty j}\delta_j+\sum_{j=1}^{N}(\Delta\pi_{\infty j}+\theta_j)(\alpha_j-0.5\beta_j^2-\delta_j)<0, \tag{3.88}$$

which is guaranteed by

$$\sum_{j=1}^{N}[(\pi_{\infty j}-\theta_j)(\alpha_j-0.5\beta_j^2)+\theta_j\delta_j]+\sum_{j=1}^{N}\Delta\pi_{\infty j}\delta_j<0, \tag{3.89}$$

$$\sum_{j=1}^{N}(\Delta\pi_{\infty j}+\theta_j)(\alpha_j-0.5\beta_j^2-\delta_j)\leq 0. \tag{3.90}$$

For $\sum_{j=1}^{N}\Delta\pi_{\infty j}\delta_j$, it is obtained that

$$\sum_{j=1}^{N}\Delta\pi_{\infty j}\delta_j\leq\sum_{j=1}^{N}\frac{1}{4}\theta_j^2\gamma_j+\sum_{j=1}^{N}\delta_j^2\gamma_j^{-1}, \tag{3.91}$$

with $\gamma_j > 0$. Based on (3.89)-(3.91), we know (3.85) and (3.86) implies (3.87). The others are same to those in Theorem 3.4, which are omitted here. That completes the proof.

When π_∞ with property (3.53) is partially known or accessible, in which some elements are unknown. For example, a partly unknown π_∞ may be expressed as

$$\pi_\infty = \left[\, \pi_{\infty 1}\ ?\ \pi_{\infty 2}\ \pi_{\infty 3}\ ? \,\right],$$

where '?' represents the unknown elements. Based on this, for any $\mu \in \mathbb{S}$, define $\mathbb{S} = \mathbb{S}_k + \bar{\mathbb{S}}_k$ such that

$$\mathbb{S}_k = \{j : \pi_{\infty j} \text{ is known}\} \text{ and } \bar{\mathbb{S}}_k = \{j : \pi_{\infty j} \text{ is unknown}\}, \tag{3.92}$$

which are further described respectively as

$$\mathbb{S}_k = \{k_1, \cdots, k_m\} \text{ and } \bar{\mathbb{S}}_k = \{\bar{k}_1, \cdots, \bar{k}_{N-m}\} \tag{3.93}$$

where $k_i \in \mathbb{Z}^+$ is the index of the ith known element in π_∞, and $\bar{k}_i \in \mathbb{Z}^+$ is the index of the ith unknown element in π_∞. For this general case, we have the following result.

Theorem 3.6. *Given a scalar $\bar{\varepsilon} > 0$, the equilibrium of MJSPS (3.52) is almost surely exponentially stable with control gain $K_i(\varepsilon) = Y_i X^{-1}(\varepsilon)$ for any $\varepsilon \in (0, \bar{\varepsilon}]$, if there exists matrices $X_1 > 0$, $X_2 = X_2^T$, $X_3 = X_3^T$, $X_4 = X_4^T$, X_5, Y_{i1} and Y_{i2} such that LMIs (3.55)-(3.57), (3.58)-(3.60), (3.62)-(3.64) or (3.65)-(3.67) and the following LMIs hold for all $i \in \mathbb{S}$*

$$\sum_{i\in\mathbb{S}_k} \pi_{\infty i}(\alpha_i - 0.5\beta_i^2) + (1 - \sum_{i\in\mathbb{S}_k} \pi_{\infty i})(\alpha_j - 0.5\beta_j^2) < 0,\ j \in \bar{\mathbb{S}}_k. \tag{3.94}$$

Proof. Similar to the proof of Theorem 3.5, we only consider condition (3.61) under condition (3.92), which is equivalent to

$$\sum_{i\in\mathbb{S}_k} \pi_{\infty i}(\alpha_i - 0.5\beta_i^2) + (1 - \sum_{i\in\mathbb{S}_k} \pi_{\infty i}) \sum_{j\in\bar{\mathbb{S}}_k} \frac{\pi_{\infty j}}{1 - \sum_{i\in\mathbb{S}_k} \pi_{\infty i}} (\alpha_j - 0.5\beta_j^2) < 0 \tag{3.95}$$

It is also rewritten as

$$\sum_{j\in\bar{\mathbb{S}}_k} \frac{\pi_{\infty j}}{1 - \sum_{i\in\mathbb{S}_k} \pi_{\infty i}} [\sum_{i\in\mathbb{S}_k} \pi_{\infty i}(\alpha_i - 0.5\beta_i^2) + (1 - \sum_{i\in\mathbb{S}_k} \pi_{\infty i})(\alpha_j - 0.5\beta_j^2)] < 0, \tag{3.96}$$

which are guaranteed by (3.94). The next is same to the proof of Theorem 3.4, thus it is omitted here. This completes the proof.

In the following, we will discuss another general case that the underlying system is observable only in some system modes but not all. In this case, $\mathbb{S}$ is decomposed into two subsets $\mathbb{S}_1$ and $\mathbb{S}_2$ which satisfy $\mathbb{S} = \mathbb{S}_1 \cup \mathbb{S}_2$. For each $i \in \mathbb{S}_2$, the underlying system is not observable, which cannot be stabilized by state feedback control, while it can be stabilized for each $i \in \mathbb{S}_1$. Without loss of generality, we only consider the following SSPS which is with Markovian jump parameters and described as

$$E(\varepsilon)\mathrm{d}x(t) = A(r_t)x(t)\mathrm{d}t + D(r_t)u_\omega(t)\mathrm{d}\omega(t), \tag{3.97}$$

where the stochastic controller is satisfied

$$u_\omega(t) = \begin{cases} K_{r(t)}(\varepsilon)x(t), & \text{if } r_t \in \mathbb{S}_1, \\ 0, & \text{if } r_t \in \mathbb{S}_2. \end{cases} \tag{3.98}$$

Similar to Theorem 3.4, we have the following result.

Theorem 3.7. *Given a scalar $\bar{\varepsilon} > 0$, the equilibrium of MJSPS (3.97) is almost surely exponentially stable with control gain $K_i(\varepsilon) = Y_i X^{-1}(\varepsilon)$ satisfying (3.98) for any $\varepsilon \in (0, \bar{\varepsilon}]$, if there exists matrices $X_1 > 0$, $X_2 = X_2^T$, $X_3 = X_3^T$, $X_4 = X_4^T$, X_5, Y_{i1} and Y_{i2} such that LMIs (3.55)-(3.57) hold for all $i \in \mathbb{S}$ and the following LMIs hold*

$$\hat{\Phi}_{i1} \leq 0, \; i \in \mathbb{S}, \tag{3.99}$$

$$\hat{\Phi}_{i1} + \bar{\varepsilon}\hat{\Psi}_{i1} \leq 0, \; i \in \mathbb{S}, \tag{3.100}$$

$$\hat{\Phi}_{i1} + \bar{\varepsilon}\hat{\Psi}_{i1} - \bar{\varepsilon}^2 \alpha \Omega_3 < 0, \; i \in \mathbb{S}, \tag{3.101}$$

$$\begin{bmatrix} \hat{\Phi}_{i1} & Y_{i1}^T D_i^T \mathbb{I}^T \\ * & -X_1 \end{bmatrix} \leq 0, \; i \in \mathbb{S}_1, \tag{3.102}$$

$$\begin{bmatrix} \hat{\Phi}_{i1} & \hat{\Phi}_{i2} \\ * & -\Omega_1 \end{bmatrix} + \bar{\varepsilon} \begin{bmatrix} \hat{\Psi}_{i1} & \hat{\Psi}_{i2} \\ * & -\Omega_2 \end{bmatrix} \leq 0, \; i \in \mathbb{S}_1, \tag{3.103}$$

$$\begin{bmatrix} \hat{\Phi}_{i1} & \hat{\Phi}_{i2} \\ * & -\Omega_1 \end{bmatrix} + \bar{\varepsilon} \begin{bmatrix} \hat{\Psi}_{i1} & \hat{\Psi}_{i2} \\ * & -\Omega_2 \end{bmatrix} - \bar{\varepsilon}^2 \begin{bmatrix} \alpha\Omega_3 & 0 \\ * & \Omega_3 \end{bmatrix} < 0, \; i \in \mathbb{S}_1, \tag{3.104}$$

$$\alpha - 0.5 \sum_{j \in \mathbb{S}_1} \pi_{\infty j} \beta_j^2 < 0, \tag{3.105}$$

either

$$\hat{\Upsilon}_{i1} \leq 0, \; i \in \mathbb{S}_1, \tag{3.106}$$

$$\hat{\Upsilon}_{i1} + \bar{\varepsilon}\hat{\Upsilon}_{i2} \leq 0, \; i \in \mathbb{S}_1, \tag{3.107}$$

$$\hat{\Upsilon}_{i1} + \bar{\varepsilon}\hat{\Upsilon}_{i2} + \bar{\varepsilon}^2 \hat{\beta}_i \Omega_3 < 0, \; i \in \mathbb{S}_1, \tag{3.108}$$

or

$$\tilde{\Upsilon}_{i1} \geq 0, \; i \in \mathbb{S}_1, \tag{3.109}$$

$$\tilde{\Upsilon}_{i1} + \bar{\varepsilon}\tilde{\Upsilon}_{i2} \geq 0, \; i \in \mathbb{S}_1, \tag{3.110}$$

$$\tilde{\Upsilon}_{i1} + \bar{\varepsilon}\tilde{\Upsilon}_{i2} - \bar{\varepsilon}^2 \beta_i \Omega_3 > 0, \; i \in \mathbb{S}_1, \tag{3.111}$$

where

$$\hat{\Phi}_{i1} = (A_iU)^\star - \alpha\Omega_1,\ \hat{\Phi}_{i2} = Y_{i1}^T D_i^T,$$
$$\hat{\Psi}_{i1} = (A_iV)^\star - \alpha\Omega_2,\ \hat{\Psi}_{i2} = Y_{i2}^T D_i^T,$$
$$\hat{\Upsilon}_{i1} = (D_iY_{i1})^\star + \beta_i\Omega_1,\ \hat{\Upsilon}_{i2} = (D_iY_{i2})^\star + \beta_i\Omega_2,$$
$$\tilde{\Upsilon}_{i1} = (D_iY_{i1})^\star - \beta_i\Omega_1,\ \tilde{\Upsilon}_{i2} = (D_iY_{i2})^\star - \beta_i\Omega_2,$$

α and β_i are some nonnegative constants.

Proof. Taking into account (3.99)-(3.101) and by Lemma 2.3, it is seen

$$(A_iX(\varepsilon))^\star \le \alpha X^T(\varepsilon)E^T(\varepsilon),\ i \in \mathbb{S},\ \forall\varepsilon \in (0,\bar{\varepsilon}], \tag{3.112}$$

Similarly, by conditions (3.102)-(3.104), (3.106)-(3.108) or (3.109)-(3.111), one has

$$\begin{bmatrix} (A_iX(\varepsilon))^\star - \alpha X^T(\varepsilon)E^T(\varepsilon) & X^T(\varepsilon)\hat{C}_i^T \\ * & -E(\varepsilon)X(\varepsilon) \end{bmatrix} < 0,\ i \in \mathbb{S}_1,\ \forall\varepsilon \in (0,\bar{\varepsilon}], \tag{3.113}$$

$$(\hat{C}_iX(\varepsilon))^\star + \beta_i X^T(\varepsilon)E^T(\varepsilon) < 0,\ i \in \mathbb{S}_1,\ \forall\varepsilon \in (0,\bar{\varepsilon}], \tag{3.114}$$

or

$$(\hat{C}_iX(\varepsilon))^\star - \beta_i X^T(\varepsilon)E^T(\varepsilon) > 0,\ i \in \mathbb{S}_1,\ \forall\varepsilon \in (0,\bar{\varepsilon}], \tag{3.115}$$

where $\hat{C}_i = D_iK_i(\varepsilon)$. Based on (3.105) and by exploiting the similar process of Theorem 3.4, we can prove this theorem easily. This completes the proof.

Example 3.3. Consider a two-dimensional MJSPS of form (3.52) with $r(t) \in \mathbb{S} = \{1,2,3\}$, and its parameters are given by

$$A_1 = \begin{bmatrix} -0.1 & 0 \\ 0.1 & -0.7 \end{bmatrix},\ C_1 = \begin{bmatrix} 0.2 & 0.5 \\ 0 & -0.1 \end{bmatrix},\ D_1 = \begin{bmatrix} 0.2 & -0.5 \\ 1 & 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -1 & -0.8 \\ 0 & -0.1 \end{bmatrix},\ C_2 = \begin{bmatrix} 0.3 & 0.1 \\ 0.3 & 0.6 \end{bmatrix},\ D_2 = \begin{bmatrix} 0.3 & -1 \\ 0.7 & -0.5 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -0.2 & 0 \\ 1 & -0.7 \end{bmatrix},\ C_3 = \begin{bmatrix} 0.3 & 0 \\ 0.1 & 0.2 \end{bmatrix},\ D_3 = \begin{bmatrix} -1 & 0.6 \\ 0.2 & 0.7 \end{bmatrix},$$

with $E(\varepsilon)$ given by

$$E(\varepsilon) = \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix}.$$

First, the TRM is assumed to be given exactly, which is

$$\Pi = \begin{bmatrix} -1.5 & 0.6 & 0.9 \\ 0.7 & -1.2 & 0.5 \\ 1.5 & 1.4 & -2.9 \end{bmatrix},$$

where $\pi_\infty = \begin{bmatrix} 0.3883 & 0.4190 & 0.1927 \end{bmatrix}$. Under initial condition $x_0 = \begin{bmatrix} -1 & 1 \end{bmatrix}^T$ and $\varepsilon = 0.005 \in (0,\bar{\varepsilon}]$, the simulations with operation mode evolution are given in Fig. 3.6, Fig. 3.7 and Fig. 3.8 respectively. Based on the simulation, it is seen that this

MJSPS is not stable. For this case, by Theorem 3.4, we may design a controller of form (3.54) in the diffusion parts which makes the resulting closed-loop system almost surely exponentially stable. By Theorem 3.4, one has

$$U = \begin{bmatrix} 391.7339 & 0 \\ 224.4312 & 275.3567 \end{bmatrix}, \; V = 1.0e+003 * \begin{bmatrix} -1.1842 & 0.2244 \\ 0 & 0.0063 \end{bmatrix},$$

Then, the corresponding controllers are computed as

$$K_1(\varepsilon) = \begin{bmatrix} 0.1836 & -0.9740 \\ 0.9399 & -0.1246 \end{bmatrix},$$
$$K_2(\varepsilon) = \begin{bmatrix} 0.6039 & -0.2281 \\ -0.2229 & -0.3182 \end{bmatrix},$$
$$K_3(\varepsilon) = \begin{bmatrix} 0.6039 & -0.2281 \\ -0.2229 & -0.3182 \end{bmatrix},$$

where the stabilization bound is $\bar{\varepsilon} = 0.0109$. Applying such stochastic controllers to the above unstable MJSPS, the stabilization effect via noise controller (3.54) is presented in Fig. 3.9 and Fig. 3.10. From the simulations, it is said that presented stochastic controller can stabilize an unstable MJSPS in addition to check stabilization bound $\bar{\varepsilon}$. If π_∞ is with admissible uncertainty (3.84), where $\theta_j = 0.5\pi_{\infty j}$, $j = 1,2,3$, we have the corresponding controller

$$K_1(\varepsilon) = \begin{bmatrix} 0.3291 & -0.5630 \\ -0.3533 & 0.7339 \end{bmatrix},$$
$$K_2(\varepsilon) = \begin{bmatrix} 0.1841 & -0.9738 \\ 0.9411 & -0.1242 \end{bmatrix},$$
$$K_3(\varepsilon) = \begin{bmatrix} 0.6035 & -0.2277 \\ -0.2233 & -0.3178 \end{bmatrix}.$$

When π_∞ is partially unknown such as $\pi_\infty = \begin{bmatrix} ? & ? & 0.1927 \end{bmatrix}$, the desired controller gains only in the diffusion section can be obtained by Theorem 3.6 which are

$$K_1(\varepsilon) = \begin{bmatrix} 0.3293 & -0.5637 \\ -0.3534 & 0.7340 \end{bmatrix},$$
$$K_2(\varepsilon) = \begin{bmatrix} 0.1839 & -0.9739 \\ 0.9406 & -0.1243 \end{bmatrix},$$
$$K_3(\varepsilon) = \begin{bmatrix} 0.6045 & -0.2287 \\ -0.2224 & -0.3187 \end{bmatrix}.$$

Example 3.4. Consider an MJSPS of form (3.97) with $r(t) \in \mathbb{S} = \{1,2,3\}$, and its parameters are given by

$$A_1 = \begin{bmatrix} -0.6 & 0 \\ 0.1 & -0.1 \end{bmatrix},\ D_1 = \begin{bmatrix} 0.2 & -1 \\ -1 & 0.5 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.7 & -0.8 \\ 0 & -0.1 \end{bmatrix},\ D_2 = \begin{bmatrix} 1 & -1 \\ 0.7 & 0.5 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -0.2 & 0 \\ 1.2 & -0.7 \end{bmatrix},\ D_3 = \begin{bmatrix} 0.3 & 0.6 \\ 0.7 & -0.2 \end{bmatrix},$$

with $E(\varepsilon)$ given by

$$E(\varepsilon) = \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix}.$$

The transition rate matrix is assumed to be given exactly, which is

$$\Pi = \begin{bmatrix} -1.3 & 0.7 & 0.6 \\ 0.3 & -0.8 & 0.5 \\ 1.5 & 0.4 & -1.9 \end{bmatrix},$$

with $\pi_\infty = \begin{bmatrix} 0.3548 & 0.4220 & 0.2231 \end{bmatrix}$. For this example, it is firstly assumed that $\mathbb{S} = \mathbb{S}_1 \cup \mathbb{S}_2$ with $\mathbb{S}_1 = \{1\}$ and $\mathbb{S}_2 = \{2,3\}$. Let $\alpha = 0.1$, $\beta_1 = 0.8$, $\beta_2 = 0.4$, $\beta_3 = 0.6$ respectively, a kind of stochastic controller (3.98) is computed as

$$K_1(\varepsilon) = \begin{bmatrix} -0.2713 & -0.1313 \\ -0.6964 & -0.0250 \end{bmatrix},$$

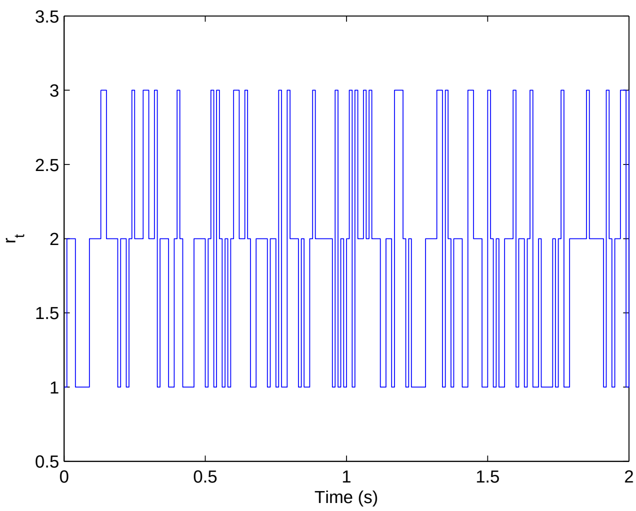

Fig. 3.6 Simulation of operation mode r_t

with stabilization bound $\bar{\varepsilon} = 0.105$. Under initial condition $x_0 = \begin{bmatrix} -1 & 1 \end{bmatrix}^T$ and applying the desired controller to the open-loop system, we have the state responses of the closed-loop system which are demonstrated in Fig. 3.11 and Fig. 3.12. From such simulations, it is claimed that though some subsystems of MJSPS are not observable, one can also design an effective stabilizing controller of form (3.98) by noise control. On the other hand, if $\mathbb{S}$ is decomposed into $\mathbb{S}_1 = \{3\}$ and $\mathbb{S}_2 = \{1,2\}$ respectively, under the same values of α and β_j with $j = 1,2,3$, we obtain that no matter what value $\bar{\varepsilon}$ choose, there is no solution to stochastic controller (3.98). From this fact, it is concluded that the partial observability of MJSPS (3.52) or (3.97) also plays an important role in its stabilization bound problem by noise control.

Example 3.5. Consider the following singularly perturbed system controlled by a DC motor, which is described by

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = \dfrac{g}{l}\sin x_1(t) + \dfrac{NK_m}{ml^2} z(t), \\ \dot{z}(t) = \dfrac{K_b N}{L_a} x_2(t) - \dfrac{R_{r(t)}}{L_a} z(t) + \dfrac{1}{L_a} u(t), \end{cases} \tag{3.116}$$

where $x_1(t) = \theta_p(t)$, $x_2(t) = \dot{\theta}_p(t)$, $z(t) = I_a(t)$, $u(t)$ is the control input, K_m is the motor torque constant, K_b is the back emf constant, N is the gear ratio, and $R(\eta_t)$ is defined by

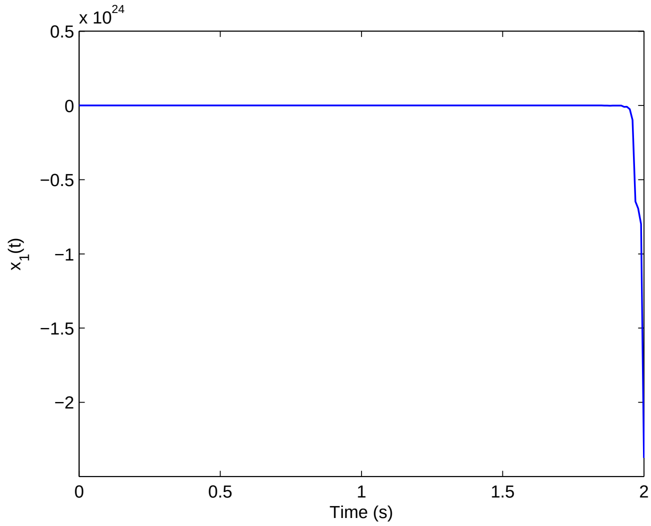

Fig. 3.7 Simulation of open-loop system state $x_1(t)$

$$R(r_t) = \begin{cases} R_a \; if \; r_t = 1, \\ R_b \; otherwise \; r_t = 2, \end{cases}$$

where $\{r(t), t \geq 0\}$ is a Markov process taking values in a finite set $\mathbb{S} = \{1, 2\}$. Let $L_a = \varepsilon H$, $g = 9.8m/s^2$, $l = 1m$, $m = 1kg$, $N = 10$, $l = 1m$, $K_m = 0.1Nm/A$, $K_b = 0.1Vs/rad$, $R_a = 1\Omega$, $R_b = 0.5\Omega$ and $u(t) = -20x_1 - 2x_2$, system (3.116) becomes

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = z(t) + 9.8\sin x_1(t), \\ \varepsilon\dot{z}(t) = -20x_1 - 3x_2 - R(r_t)z(t). \end{cases} \tag{3.117}$$

Its linearized model is

$$\dot{x}(t) = A(r_t)x(t), \tag{3.118}$$

where

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ z(t) \end{bmatrix}, E(\varepsilon) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \varepsilon \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 9.8 & 0 & 1 \\ -20 & -3 & -1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 9.8 & 0 & 1 \\ -20 & -3 & -0.5 \end{bmatrix}.$$

Firstly, TRM is assumed to be given exactly, which is

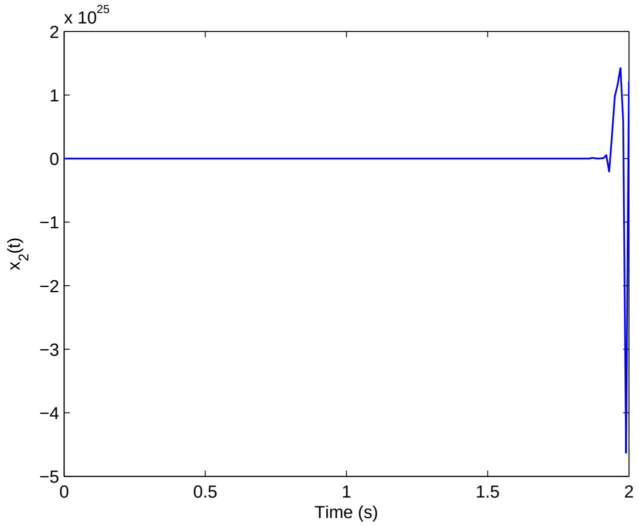

Fig. 3.8 Simulation of open-loop system state $x_2(t)$

$$\Pi = \begin{bmatrix} -1.5 & 1.5 \\ 0.7 & -0.7 \end{bmatrix}.$$

For this case, it is seen that the methods in [80, 76, 103] fail in giving an estimation of stability bound $\bar{\varepsilon}$. Based on the proposed criteria, it is shown that an SMJS can be stabilized by a stochastic controller (3.54). Without loss of generality, the corresponding system becomes

$$\mathrm{d}x(t) = A(r_t)x(t)\mathrm{d}t + D(r_t)u_\omega(t)\mathrm{d}\omega(t), \tag{3.119}$$

where

$$D_1 = \begin{bmatrix} 0.2 & -0.5 & -0.1 \\ 1 & 1 & 0 \\ -1 & 0.6 & 1 \end{bmatrix}, D_2 = \begin{bmatrix} 0.3 & -1 & 0.1 \\ 0.7 & -0.5 & 0.6 \\ 0.2 & 0.7 & -1 \end{bmatrix}.$$

More importantly, TPM is not necessary exactly which can be partially known. For this example, without loss of generality, Π is assumed to be totally unknown, whose elements are all unknown. By Theorem 3.6, one can design a stochastic controller which is computed as

$$K_1(\varepsilon) = \begin{bmatrix} -0.1495 & 0.3633 & -0.0298 \\ -1.4163 & 0.1504 & -0.0080 \\ 3.3560 & 0.4363 & 0.1173 \end{bmatrix}, K_2(\varepsilon) = \begin{bmatrix} -0.9978 & -0.7212 & -0.0285 \\ 1.5152 & -0.1707 & 0.0310 \\ 2.0664 & -0.2530 & 0.0724 \end{bmatrix},$$

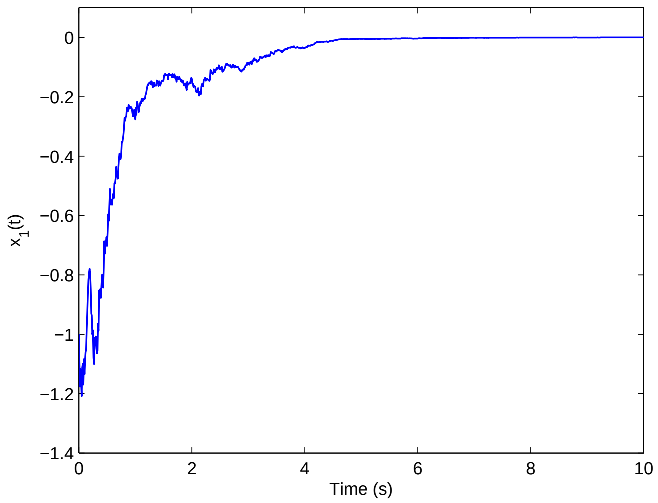

Fig. 3.9 Simulation of closed-loop system state $x_1(t)$

where an estimation bound of $\bar{\varepsilon}$ is obtained as $\bar{\varepsilon} = 0.045$. However, it is concluded that the methods in [95, 30, 92] cannot be applied to such stabilization problems. In this sense, it is said that our methods have larger application scope.

3.5 Stabilization by PD Control

Consider a class of uncertain stochastic singular Markovian jump systems (SSMJSs) described as

$$\begin{aligned}(E(r_t)+\Delta E(r_t))\mathrm{d}x = {} & [(A(r_t)+\Delta A(r_t))x(t)+(B(r_t)+\Delta B(r_t))u(t)]\mathrm{d}t \\ & +(H(r_t)+\Delta H(r_t))x(t)\mathrm{d}W(t),\end{aligned} \tag{3.120}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $W(t)$ is a q-dimension independent standard Wiener process. Matrix $E \in \mathbb{R}^{n\times n}$ may be singular, which is assumed to be $\mathrm{rank}(E) = r \leq n$. $A(r_t)$, $B(r_t)$ and $H(r_t)$ are known matrices of compatible dimensions. $\Delta E(r_t)$, $\Delta A(r_t)$, $\Delta B(r_t)$ and $\Delta H(r_t)$ are unknown matrices denoting the uncertainties of system. The mode $\{r_t, t \geq 0\}$ is a continuous-time Markov process given in (2.2) and (2.3).

In this section, without loss of generality, the above uncertainties are assumed as

$$[\Delta E(r_t), \Delta A(r_t), \Delta B(r_t), \Delta H(r_t)] = MF(t)[N_e(r_t), N_a(r_t), N_b(r_t), N_h(r_t)], \tag{3.121}$$

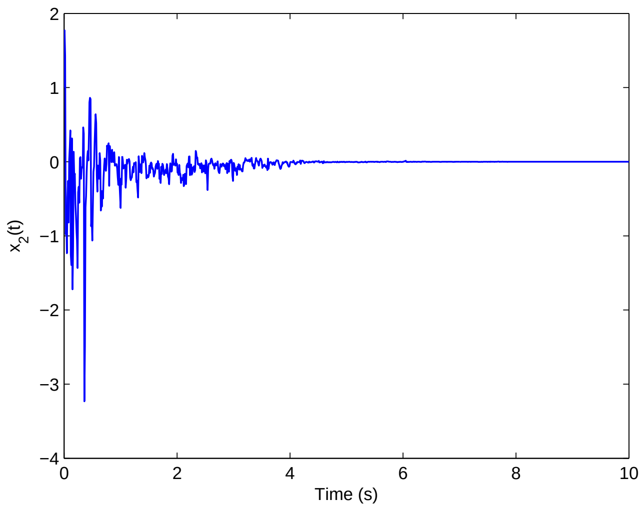

Fig. 3.10 Simulation of closed-loop system state $x_2(t)$

where $M, N_e(r_t), N_a(r_t), N_b(r_t)$ and $N_h(r_t)$ are known real constant matrices with appropriate dimensions. The uncertain matrix $F(t)$ satisfies $F^T(t)F(t) \leq I$. In addition, the TRM Π is with admissible uncertainty and described in Case 2.

Remark 3.2. It is seen that system (3.120) is very general, which covers many special systems studied very well. When there is no Wiener process and without jumping parameter, it is a singular system with uncertainties in system matrices [75, 124, 74, 109]. If there is no uncertainty and TRM is known exactly, it becomes a system in [53]. It also can be specialized into an MJS with or without uncertainties [178, 202, 150, 128], where the derivative matrix is nonsingular. In [53, 75, 124, 74, 109, 175], it has been shown that uncertainties in both derivative matrix and TRM and noise of a system play important effects, which make the system analysis and synthesis quite difficult. In one word, though system (3.120) is a general system combining the above mentioned systems, it can not be studied via combining the existing results directly and simply.

In this section, a proportional-derivative state feedback controller (PDSFC) depending on system mode is developed as follows:

$$u(t) = K_a(r_t)x(t) - K_e(r_t)\dot{x}(t), \tag{3.122}$$

where $K_a(r_t)$ and $K_e(r_t)$ are the designed control gains. Applying it to system (3.120) results the following closed-loop system, which is described by

$$E_c(r_t)\mathrm{d}x = A_c(r_t)x(t)\mathrm{d}t + \bar{H}(r_t)x(t)\mathrm{d}W(t), \tag{3.123}$$

where

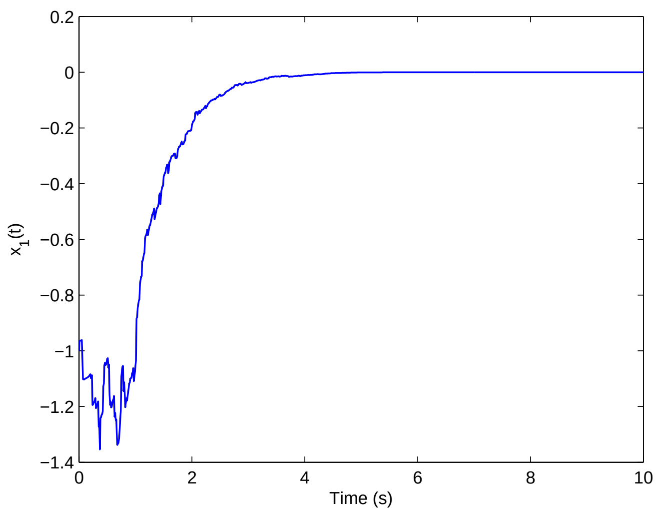

Fig. 3.11 Simulation of closed-loop system state $x_1(t)$

$$E_c(r_t) = \bar{E}(r_t) + \bar{B}(r_t)K_e(r_t), A_c(r_t) = \bar{A}(r_t) + \bar{B}(r_t)K_a(r_t),$$
$$\bar{E}(r_t) = E(r_t) + \Delta E(r_t), \bar{A}(r_t) = A(r_t) + \Delta A(r_t),$$
$$\bar{B}(r_t) = B(r_t) + \Delta B(r_t), \bar{H}(r_t) = H(r_t) + \Delta H(r_t).$$

Definition 3.2. Uncertain SSMJS

$$\bar{E}(r_t)dx = \bar{A}(r_t)x(t)\mathrm{d}t + \bar{H}(r_t)x(t)dW(t), \tag{3.124}$$

is said to be quadratically stochastically stable (QNQSS), if $\bar{E}_i, \forall\, i \in \mathbb{S}$ is nonsingular and there exists $P_i > 0$, such that for all $i \in \mathbb{S}$

$$(\bar{A}_i^T \bar{E}_i^{-T} P_i)^{\star} + \sum_{j=1}^{N} \pi_{ij} P_j + \bar{H}_i^T \bar{E}_i^{-T} P_i \bar{E}_i^{-1} \bar{H}_i < 0, \tag{3.125}$$

hold over admissible uncertainties (3.121) and (2.11).

Lemma 3.5. *[172] Given matrices H, U and V with appropriate dimensions and with $H = H^T$, then*

$$H + UF(t)V + (UF(t)V)^T < 0,$$

for all $F(t)$ satisfying $F^T(t)F(t) \le I$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$H + \varepsilon UU^T + \varepsilon^{-1} V^T V < 0.$$

Firstly, sufficient conditions of controller (3.122) are developed within LMI framework.

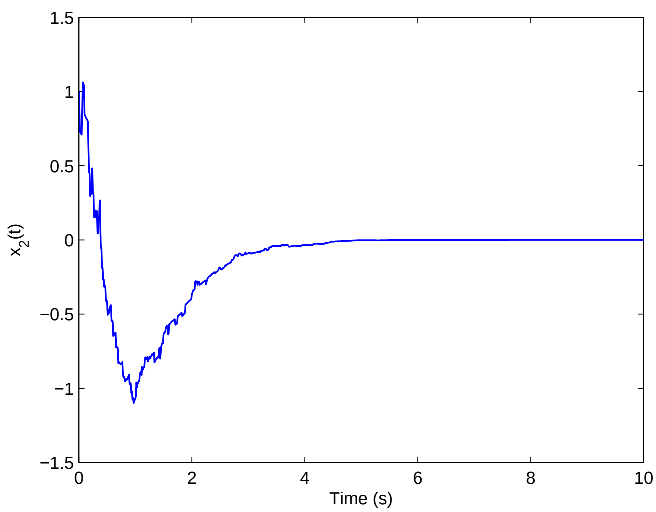

Fig. 3.12 Simulation of closed-loop system state $x_2(t)$

Theorem 3.8. *Consider uncertain SSMJS (3.120), there exists an PDSFC (3.122) such that the closed-loop system (3.123) is QNQSS, if there exist* $X_i > 0$, G_i, Y_i, Z_i, $\bar{W}_i = \bar{W}_i^T$, $V_i > 0$, $\bar{T}_i > 0$, $\varepsilon_1 > 0$ *and* $\varepsilon_2 > 0$, *such that the following LMIs hold for all* $i, j \in \mathbb{S}$, $j \neq i$,

$$\begin{bmatrix} \Omega_i + \varepsilon_1 \bar{M}\bar{M}^T & \bar{N}_i^T \\ * & -\varepsilon_1 I \end{bmatrix} < 0, \tag{3.126}$$

$$\begin{bmatrix} -X_i + \bar{W}_i & X_i \\ * & -X_j \end{bmatrix} \leq 0, \tag{3.127}$$

$$\begin{bmatrix} \Phi_i + \varepsilon_2 \tilde{M}\tilde{M}^T & \tilde{N}_i^T \\ * & -\varepsilon_2 I \end{bmatrix} < 0, \tag{3.128}$$

where

$$\Omega_i = \begin{bmatrix} \Omega_{i1} & \Omega_{i2} & \Omega_{i3} \\ * & (-G_i)^\star & 0 \\ * & * & \Omega_{i4} \end{bmatrix}, \bar{M} = \begin{bmatrix} M \\ 0 \\ 0 \end{bmatrix},$$

$$\begin{aligned}
&\bar{N}_i = \begin{bmatrix} N_{ai}G_i + N_{bi}Y_i & N_{ai}G_i + N_{bi}Y_i + N_{ei}X_i + N_{bi}Z_i & N_{ei}X_i + N_{bi}Z_i \end{bmatrix}, \\
&\Omega_{i1} = (A_iG_i + B_iY_i)^\star, \Omega_{i2} = A_iG_i + B_iY_i + E_iX_i + B_iZ_i - G_i^T, \\
&\Omega_{i3} = E_iX_i + B_iZ_i, \Omega_{i4} = (-X_i)^\star + V_i,
\end{aligned}$$

$$\Phi_i = \begin{bmatrix} \Phi_{i1} & \Phi_{i2} & \bar{W}_i & X_iH_i^T \\ * & \Phi_{i3} & 0 & 0 \\ * & * & -\bar{T}_i & 0 \\ * & * & * & \Phi_{i4} \end{bmatrix}, \tilde{M} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ M \end{bmatrix},$$

$$\begin{aligned}
&\tilde{N}_i = \begin{bmatrix} N_{hi}X_i & 0 & 0 & N_{ei}X_i + N_{bi}Z_i \end{bmatrix} \\
&\Phi_{i1} = -V_i + 0.25\varepsilon_{ii}^2 \bar{T}_i + \varepsilon_{ii}\bar{W}_i + \alpha_{ii}X_i, \\
&\Phi_{i2} = \begin{bmatrix} \sqrt{\alpha_{i1}}X_i & \cdots & \sqrt{\alpha_{i(i-1)}}X_i & \sqrt{\alpha_{i(i+1)}}X_i & \cdots & \sqrt{\alpha_{iN}}X_i \end{bmatrix}, \\
&\Phi_{i3} = -\mathrm{diag}\{X_1, \cdots, X_{i-1}, X_{i+1}, \cdots, X_N\}, \\
&\Phi_{i4} = (-E_iX_i - B_iZ_i)^\star + X_i.
\end{aligned}$$

In this case, the gains of controller (3.122) are given by

$$K_{ai} = Y_iG_i^{-1}, K_{ei} = Z_iX_i^{-1}. \tag{3.129}$$

Proof. From Definition 3.2, it is seen that system (3.123) is QNQSS if and only if there exists $P_i > 0$ such that

$$(A_{ci}^T E_{ci}^{-T} P_i)^\star + \sum_{j=1}^{N} \pi_{ij} P_j + \bar{H}_i^T E_{ci}^{-T} P_i E_{ci}^{-1} \bar{H}_i < 0. \tag{3.130}$$

Let $X_i = P_i^{-1}$, (3.130) is equivalent to

$$(A_{ci}X_iE_{ci}^T)^\star + E_{ci}X_i(X_iV_i^{-1}X_i)^{-1}X_iE_{ci}^T < 0, \tag{3.131}$$

$$X_i \sum_{j=1}^{N} \pi_{ij} P_j X_i^T + X_i \bar{H}_i^T E_{ci}^{-T} P_i E_{ci}^{-1} \bar{H}_i X_i - V_i \leq 0. \tag{3.132}$$

From (3.126), it is concluded that G_i is nonsingular. Then, the following condition implies (3.131), that is

$$\begin{bmatrix} (A_{ci}G_i)^{\star} & A_{ci}G_i + E_{ci}X_i - G_i^T & E_{ci}X_i \\ * & (-G_i)^{\star} & 0 \\ * & * & -X_i V_i^{-1} X_i \end{bmatrix} < 0, \tag{3.133}$$

by pre- and post-multiplying with

$$\begin{bmatrix} I & A_{ci} & 0 \\ 0 & 0 & I \end{bmatrix},$$

and its transpose respectively. It is seen that for any $R > 0$, one gets

$$-L^T R^{-1} L \leq (-L)^{\star} + R. \tag{3.134}$$

Taking into account (3.134) and substituting (3.121) into (3.133), via Lemma 3.5, we obtain that (3.126) with (3.129) implies (3.133). For any appropriate matrix $W_i = W_i^T$, it is obvious that

$$\sum_{j=1}^{N} (\Delta\tilde{\pi}_{ij} + \varepsilon_{ij}) W_i \equiv 0. \tag{3.135}$$

Then, (3.132) is equivalent to

$$\begin{aligned} &- V_i + \alpha_{ii} X_i + \varepsilon_{ii} X_i W_i X_i + \Delta\tilde{\pi}_{ii} X_i W_i X_i + X_i \sum_{j=1,\ j\neq i}^{N} \alpha_{ij} P_j X_i \\ &+ X_i \bar{H}_i^T (E_{ci} X_i E_{ci}^T)^{-1} \bar{H}_i X_i + \sum_{j=1,\ j\neq i}^{N} (\Delta\pi_{ij} + \varepsilon_{ij}) X_i (P_j - P_i + W_i) X_i < 0. \end{aligned} \tag{3.136}$$

Noting that for any $T_i > 0$, one gets

$$\Delta\tilde{\pi}_{ii} W_i \leq 0.25 (\Delta\tilde{\pi}_{ii})^2 T_i + W_i T_i^{-1} W_i \leq 0.25 \varepsilon_{ii}^2 T_i + W_i T_i^{-1} W_i. \tag{3.137}$$

Taking into account (3.137), and let $\bar{W}_i \triangleq X_i W_i X_i$ and $\bar{T}_i \triangleq X_i T_i X_i$, one has that conditions (3.127) and (3.128) with (3.129) imply (3.132) with substituting (3.121) into (3.136), since $\Delta\pi_{ij} + \varepsilon_{ij} \geq 0$ always holds, $\forall j \neq i \in \mathbb{S}$. This completes the proof.

Next, another condition on the existence of controller (3.120) for uncertain SS-MJS (3.120) is given.

Theorem 3.9. *Consider uncertain SSMJS (3.120), there exists an PDSFC (3.122) such that the closed-loop system (3.123) is QNQSS, if there exist $X_i > 0$, G_i, Q_i, Y_i, Z_i, $\bar{W}_i = \bar{W}_i^T$, $V_i > 0$, $\bar{T}_i > 0$, $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$, such that the following LMIs hold for all $i \in \mathbb{S}$*

$$\begin{bmatrix} \Theta_i + \varepsilon_1 \bar{M}\bar{M}^T & U_i^T \\ * & -\varepsilon_1 I \end{bmatrix} < 0, \tag{3.138}$$

$$\begin{bmatrix} -X_i + \bar{W}_i & X_i \\ * & -X_j \end{bmatrix} \leq 0, \tag{3.139}$$

$$\begin{bmatrix} \bar{\Phi}_i + \varepsilon_2 \hat{M}\hat{M}^T & S_i^T \\ * & -\varepsilon_2 I \end{bmatrix} < 0, \tag{3.140}$$

where

$$\Theta_i = \begin{bmatrix} \Theta_{i1} & \Theta_{i2} & \Theta_{i3} \\ * & (-Q_i)^\star & 0 \\ * & * & \Theta_{i4} \end{bmatrix},$$

$$U_i = \begin{bmatrix} N_{ei}G_i + N_{bi}Z_i & N_{ei}Q_i + N_{ai}X_i + N_{bi}Y_i & N_{ei}G_i + N_{bi}Z_i \end{bmatrix},$$
$$\Theta_{i1} = (E_iG_i + B_iZ_i)^\star, \Theta_{i2} = E_iQ_i + A_iX_i + B_iY_i - G_i^T,$$
$$\Theta_{i3} = E_iG_i + B_iZ_i, \Theta_{i4} = (-G_i)^\star + V_i,$$

$$\bar{\Phi}_i = \begin{bmatrix} \Phi_{i1} & \Phi_{i2} & \bar{W}_i & X_iH_i^T & G_i^T \\ * & \Phi_{i3} & 0 & 0 & 0 \\ * & * & -\bar{T}_i & 0 & 0 \\ * & * & * & \bar{\Phi}_{i4} & 0 \\ * & * & * & * & -X_i \end{bmatrix}, \hat{M} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ M \\ 0 \end{bmatrix},$$

$$S_i = \begin{bmatrix} N_{hi}X_i & 0 & 0 & N_{ei}G_i + N_{bi}Z_i & 0 \end{bmatrix}, \bar{\Phi}_{i4} = (E_iG_i + B_iZ_i)^\star,$$

In this case, the gains of controller (3.122) are given by

$$K_{ai} = Y_iX_i^{-1} - Z_iG_i^{-1}Q_iX_i^{-1}, K_{e_i} = Z_iG_i^{-1}. \tag{3.141}$$

Proof. Similar to the proof of Theorem 3.8, let $X_i = P_i^{-1}$, system (3.123) is QNQSS if and only if there exists $X_i > 0$ such that

$$(A_{ci}X_iE_{ci}^T)^\star + E_{ci}V_iE_{ci}^T < 0, \tag{3.142}$$

$$X_i \sum_{j=1}^{N} \pi_{ij}P_jX_i^T + X_i\bar{H}_i^T E_{ci}^{-T} P_i E_{ci}^{-1} \bar{H}_i X_i - V_i \leq 0, \tag{3.143}$$

From (3.138), it is seen that Q_i and G_i are all nonsingular. Then (3.142) could be obtained by

$$\begin{bmatrix} (E_{ci}G_i)^\star & E_{ci}Q_i + A_{ci}X_i - G_i^T & E_{ci}G_i \\ * & (-Q_i)^\star & 0 \\ * & * & -G_i^T V_i^{-1} G_i \end{bmatrix} < 0, \tag{3.144}$$

with pre- and post-multiplying with

$$\begin{bmatrix} I & E_{ci} & 0 \\ 0 & 0 & G_i^{-T} \end{bmatrix},$$

and its transpose respectively. Taking into account (3.134) and substituting the uncertainties into (3.144), we obtain that (3.138) with (3.141) implies (3.144). From (3.143), it is equivalent to

$$\begin{bmatrix} -V_i + X_i \sum_{j=1}^{N} \pi_{ij} P_j X_i^T & X_i \bar{H}_i^T \\ * & -E_{ci} X_i E_{ci}^T \end{bmatrix} < 0, \tag{3.145}$$

which could be guaranteed by

$$\begin{bmatrix} -V_i + X_i \sum_{j=1}^{N} \pi_{ij} P_j X_i^T & X_i \bar{H}_i^T \\ * & (E_{ci} G_i)^\star + G_i^T X_i^{-1} G_i \end{bmatrix} < 0. \tag{3.146}$$

The next is similar to the proof of (3.132). This completes the proof.

When matrix $\bar{E}(r_t)$ in (3.120) is mode-independent, that is

$$\begin{aligned} (E + \Delta E)\mathrm{d}x = {} & [(A(r_t) + \Delta A(r_t)x(t) + (B(r_t) + \Delta B(r_t))u(t)]\mathrm{d}t \\ & + (H(r_t) + \Delta H(r_t))x(t)\mathrm{d}W(t), \end{aligned} \tag{3.147}$$

where $\Delta E(t)$ is satisfied $\Delta E = MF(t)N_e$ and the other uncertainties are same to ones in (3.121). In this case, the corresponding controller becomes

$$u(t) = K_a(r_t)x(t) - K_e \dot{x}(t), \tag{3.148}$$

where $K_a(r_t)$ and K_e are the control gains to be determined. In this case, controller (3.148) is said to be partially mode-dependent, since both mode-dependent and mode-independent control gains are contained. The closed-loop system is

$$\tilde{E}_c(r_t)\mathrm{d}x = A_c(r_t)x(t)\mathrm{d}t + \bar{H}(r_t)x(t)\mathrm{d}W(t), \tag{3.149}$$

where

$$\tilde{E}_c(r_t) = \bar{E} + \bar{B}(r_t)K_e, \bar{E} = E + \Delta E.$$

The others are given in (3.123). By Theorem 3.9, a corollary can be obtained directly.

Corollary 3.4. *Consider uncertain SSMJS (3.120), there exists an PDSFC (3.148) such that the closed-loop system (3.149) is QNQSS, if there exist $X_i > 0$, G, Q_i, Y_i, Z, $\bar{W}_i = \bar{W}_i^T$, $V_i > 0$, $\bar{T}_i > 0$, $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$, such that the following LMIs hold for all $i \in \mathbb{S}$*

$$\begin{bmatrix} \bar{\Theta}_i + \varepsilon_1 \bar{M} \bar{M}^T & \bar{U}_i^T \\ * & -\varepsilon_1 I \end{bmatrix} < 0, \tag{3.150}$$

$$\begin{bmatrix} -X_i + \bar{W}_i & X_i \\ * & -X_j \end{bmatrix} \leq 0, \tag{3.151}$$

$$\begin{bmatrix} \check{\Phi}_i + \varepsilon_2 \hat{M} \hat{M}^T & \bar{S}_i^T \\ * & -\varepsilon_2 I \end{bmatrix} < 0, \tag{3.152}$$

where

$$\bar{\Theta}_i = \begin{bmatrix} \bar{\Theta}_{i1} & \bar{\Theta}_{i2} & \bar{\Theta}_{i3} \\ * & (-Q_i)^\star & 0 \\ * & * & \bar{\Theta}_{i4} \end{bmatrix},$$

$$\bar{U}_i = \begin{bmatrix} N_eG + N_{bi}Z & N_eQ_i + N_{ai}X_i + N_{bi}Y_i & N_eG + N_{bi}Z \end{bmatrix},$$
$$\bar{\Theta}_{i1} = (EG + B_iZ)^\star, \bar{\Theta}_{i2} = EQ_i + A_iX_i + B_iY_i - G^T,$$
$$\bar{\Theta}_{i3} = EG + B_iZ, \bar{\Theta}_{i4} = (-G)^\star + V_i,$$

$$\check{\Phi}_i = \begin{bmatrix} \Phi_{i1} & \Phi_{i2} & \bar{W}_i & X_iH_i^T & G^T \\ * & \Phi_{i3} & 0 & 0 & 0 \\ * & * & -\bar{T}_i & 0 & 0 \\ * & * & * & \check{\Phi}_{i4} & 0 \\ * & * & * & * & -X_i \end{bmatrix},$$

$$\bar{S}_i = \begin{bmatrix} N_{hi}X_i & 0 & 0 & N_eG + N_{bi}Z & 0 \end{bmatrix}, \check{\Phi}_{i4} = (EG + B_iZ)^\star.$$

In this case, the gains of controller (3.148) are given by

$$K_{ai} = Y_iX_i^{-1} - ZG^{-1}Q_iX_i^{-1}, K_e = ZG^{-1}. \tag{3.153}$$

Remark 3.3. It is seen that both two conditions on the existence of PDSFC (3.122) are obtained. In Theorem 3.8, matrix X_i coming from Lyapunov function is related to derivative matrix, while X_i is related to system matrix. Both of them can be seen as two independent kinds of methods for designing PDSFC (3.122). In some cases, Theorem 3.8 is less conservative than Theorem 3.9, which is illustrated via a numerical example. However, Theorem 3.9 can be used to deal with special case (3.147) by using an MD Lyapunov function due to X_i without correlation to MI derivative matrix. It is less conservative than Theorem 3.8 via taking a common X_i which comes from an MI Lyapunov function.

Example 3.6. Consider an SSMJS described in (3.120) with parameters as follows:

$$E_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, A_1 = \begin{bmatrix} -2 & 0.1 \\ 1 & -1 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0.4 \end{bmatrix}, H_1 = \begin{bmatrix} -0.5 & -1 \\ 0 & 0.7 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 2 & 1.3 \\ -0.3 & -2 \end{bmatrix}, B_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, H_2 = \begin{bmatrix} 0.9 & 0 \\ -1 & 0.2 \end{bmatrix}.$$

The norm-bounded uncertainties satisfying (3.121) are described as

$$M = \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix}, N_{e1} = \begin{bmatrix} 0.7 & 0.7 \end{bmatrix}, N_{a1} = \begin{bmatrix} 0.2 & 0.4 \end{bmatrix},$$

$$N_{b1} = 0.8, N_{h1} = \begin{bmatrix} 0.9 & 0.5 \end{bmatrix}, N_{e2} = \begin{bmatrix} 0.3 & 0.2 \end{bmatrix},$$
$$N_{a2} = \begin{bmatrix} 0.3 & 0.5 \end{bmatrix}, N_{b2} = 0.3, N_{h2} = \begin{bmatrix} 0.6 & 0.3 \end{bmatrix}.$$

The transition rates of $\tilde{\Pi}$ are given as $\tilde{\pi}_{11} = -5$ and $\tilde{\pi}_{22} = -7$, whose uncertainties satisfy $|\Delta\tilde{\pi}_{12}| \leq \varepsilon_{12} \triangleq 0.5\tilde{\pi}_{12}$ and $|\Delta\tilde{\pi}_{21}| \leq \varepsilon_{21} \triangleq 0.5\tilde{\pi}_{21}$ respectively. It is seen that there is no solution to an PDSFC via Theorem 3.9. However, by Theorem 3.8, an PDSFC can be computed as

$$K_{a1} = \begin{bmatrix} -0.2330 & -0.9867 \end{bmatrix}, \ K_{e1} = \begin{bmatrix} 1.5996 & 0.0788 \end{bmatrix},$$

$$K_{a2} = \begin{bmatrix} 4.3893 & 3.0188 \end{bmatrix}, \ K_{e2} = \begin{bmatrix} -2.1171 & 0.4571 \end{bmatrix}.$$

Example 3.7. Consider an SSMJS of (3.138) with uncertainties described as

Mode 1

$$E_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \ A_1 = \begin{bmatrix} -2 & 0.1 \\ 1 & -1 \end{bmatrix}, \ B_1 = \begin{bmatrix} 1 & 1 \\ 0.3 & -1 \end{bmatrix}, \ H_1 = \begin{bmatrix} -0.5 & 0 \\ -0.1 & 0.3 \end{bmatrix},$$

$$N_{e1} = \begin{bmatrix} 0.3 & 0.3 \end{bmatrix}, \ N_{a1} = \begin{bmatrix} 0.2 & 0.4 \end{bmatrix}, \ N_{b1} = \begin{bmatrix} 0.2 & 0.4 \end{bmatrix}, \ N_{h1} = \begin{bmatrix} 0.9 & 0.5 \end{bmatrix}.$$

Mode 2

$$E_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ A_2 = \begin{bmatrix} 2 & 1.3 \\ -0.3 & -2 \end{bmatrix}, \ B_2 = \begin{bmatrix} 0.2 & 1 \\ 1 & -1 \end{bmatrix}, \ H_2 = \begin{bmatrix} 0.3 & 0.1 \\ 0 & 0.2 \end{bmatrix},$$

$$N_{e2} = \begin{bmatrix} 0.3 & 0.3 \end{bmatrix}, \ N_{a2} = \begin{bmatrix} 0.3 & 0.5 \end{bmatrix}, \ N_{b2} = \begin{bmatrix} 0.3 & 0.4 \end{bmatrix}, \ N_{h2} = \begin{bmatrix} 0.6 & 0.3 \end{bmatrix}.$$

with $M = \begin{bmatrix} 0.3 & 0.1 \end{bmatrix}^T$. The estimated transition rate ares given as $\tilde{\pi}_{11} = -6$ and $\tilde{\pi}_{22} = -3$, whose uncertainties are such that $|\Delta\tilde{\pi}_{12}| \le \varepsilon_{12} \triangleq 0.5\tilde{\pi}_{12}$ and $|\Delta\tilde{\pi}_{21}| \le \varepsilon_{21} \triangleq 0.5\tilde{\pi}_{21}$ respectively. Via Theorem 3.9, we have

$$K_{a1} = \begin{bmatrix} 1.2171 & 2.5734 \\ 2.1874 & -1.4784 \end{bmatrix}, \ K_{e1} = \begin{bmatrix} -0.5036 & -1.3060 \\ -0.1201 & 1.1840 \end{bmatrix},$$
$$K_{a2} = \begin{bmatrix} -0.1753 & 1.5265 \\ -0.6353 & -1.5984 \end{bmatrix}, \ K_{e2} = \begin{bmatrix} -0.5183 & -1.1798 \\ -0.4975 & -0.7444 \end{bmatrix}.$$

When there is no jumping parameter in the derivative matrix, the singular matrix and its uncertainty are described as

$$E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ M = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, \ N_e = \begin{bmatrix} 0.3 & 0.3 \end{bmatrix}.$$

By Corollary 3.4, a partially mode-dependent PDSFC can be computed as

$$K_{a1} = \begin{bmatrix} 2.1051 & 2.3721 \\ 1.9304 & -1.1491 \end{bmatrix},$$
$$K_{a2} = \begin{bmatrix} -0.1077 & 1.5805 \\ -0.9033 & -2.0858 \end{bmatrix},$$
$$K_e = \begin{bmatrix} -0.6033 & -1.2701 \\ -0.2246 & -0.0340 \end{bmatrix}.$$

Example 3.8. Consider a special kind of SSMJS (3.120) without uncertainties, whose parameters are described as follows:

Mode 1

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ A_1 = \begin{bmatrix} 0.2 & -0.3 & 1 \\ 0.7 & -1 & -0.5 \\ 0.1 & 0 & 0.4 \end{bmatrix}, \ B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{bmatrix},$$

Mode 2

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ A_2 = \begin{bmatrix} 0.1 & -1 & 0 \\ -0.2 & -1 & 0.4 \\ 0 & 0.3 & 0.1 \end{bmatrix}, \ B_2 = \begin{bmatrix} 0 & -0.3 \\ -1 & 0 \\ 1 & 1 \end{bmatrix},$$

Mode 3

$$E_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ A_3 = \begin{bmatrix} 0.6+\rho & 0 & 0.4 \\ -0.5 & 0 & 0.7 \\ -0.2 & 0.1 & -0.3 \end{bmatrix}, \ B_3 = \begin{bmatrix} -1 & -1 \\ 0 & 0.6 \\ 1 & 0 \end{bmatrix},$$

where ρ is a positive parameter in matrix A_3. In order to do a comparison, the TRM Π is assumed to be obtained exactly, which is given as

$$\Pi = \begin{bmatrix} -2.9 & 1.5 & 1.4 \\ 1.0 & -2.2 & 1.2 \\ 1.0 & 0.9 & -1.9 \end{bmatrix}.$$

The aim is to design a state feedback controller such that the resulting closed-loop system is stochastically stable. From the method in reference [11], it is concluded that there is no solution to a mode-dependent controller if $\rho \geq 4.09$, where the designed controller is proportional. From Theorem 3.8, it is obtained that one can get an PDSFC of form (3.122), where ρ can suffer a large value. When $\rho = 10$ and by computation, the gains of controller (3.122) are given as

$$K_{a1} = \begin{bmatrix} -0.7686 & 0.0843 & 1.0850 \\ -1.2524 & 0.2762 & -2.1698 \end{bmatrix}, \ K_{e1} = \begin{bmatrix} -0.3117 & -0.1189 & -0.2195 \\ -0.1348 & -0.3076 & 0.2823 \end{bmatrix},$$
$$K_{a2} = \begin{bmatrix} -1.6348 & 1.2554 & -0.2209 \\ 1.3399 & -1.2974 & -1.0068 \end{bmatrix}, \ K_{e2} = \begin{bmatrix} -0.0220 & 0.5067 & 0.1025 \\ 0.1310 & -0.4656 & 0.4397 \end{bmatrix},$$
$$K_{a3} = \begin{bmatrix} 6.6885 & 0.9848 & 0.6920 \\ 24.5096 & 2.2914 & 3.9127 \end{bmatrix}, \ K_{e3} = \begin{bmatrix} -0.1006 & 0.1517 & -0.3836 \\ -0.3537 & -0.3491 & 0.2660 \end{bmatrix}.$$

With the same value of ρ and by Theorem 3.9, another group gains of controller (3.122) are got as

$$K_{a1}=\begin{bmatrix} 38.8072 & 139.2688 & -437.5656 \\ -183.7134 & -65.2039 & -456.2965 \end{bmatrix},\ K_{e1}=\begin{bmatrix} -1.2810 & 0.2790 & 0.0008 \\ 0.2785 & -1.2765 & 0.0009 \end{bmatrix},$$

$$K_{a2}=\begin{bmatrix} 68.2507 & -32.4988 & 355.5190 \\ -17.1552 & -138.8974 & -343.0098 \end{bmatrix},\ K_{e2}=\begin{bmatrix} 0.1773 & 1.0536 & -0.0018 \\ 5.1883 & 0.5605 & 0.0018 \end{bmatrix},$$

$$K_{a3}=\begin{bmatrix} -167.3929 & -39.3024 & -4.3013 \\ 178.2350 & -18.5276 & 119.5219 \end{bmatrix},\ K_{e3}=\begin{bmatrix} 0.0025 & -0.2528 & -1.1544 \\ -0.0025 & -2.3479 & -0.4160 \end{bmatrix}.$$

3.6 Stabilization by PMD Control

Consider a class of singular Markovian jump systems with time delay described as

$$\begin{cases} E\dot{x}(t)=A(r_t)x(t)+A_d(r_t)x(t-\tau)+F(r_t)u(t)+B(r_t)\omega(t), \\ y(t)=C(r_t)x(t)+D(r_t)\omega(t), \\ x(t)=\phi(t),\ \forall t\in[-\tau,0], \end{cases} \tag{3.154}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $u(t)\in\mathbb{R}^m$ is the control input, $\omega(t)\in\mathbb{R}^p$ is the disturbance input which belongs to $\mathscr{L}_2[0,\infty)$ and $y(t)\in\mathbb{R}^q$ is the measurement. Matrix $E\in\mathbb{R}^{n\times n}$ may be singular, which is assumed to be $\text{rank}(E)=r\le n$. $A(r_t), A_d(r_t), F(r_t), B(r_t), C(r_t)$ and $D(r_t)$ are known matrices of compatible dimensions. τ is an unknown constant delay and satisfies $0\le\tau\le\bar{\tau}$. The parameter r_t is a continuous-time Markov process with right continuous trajectory taking values in a finite set $\mathbb{S}$ with transition probabilities

$$\Pr\{r_{t+h}=j|r_t=i\}=\begin{cases} \pi_{ij}h+o(h) & i\ne j, \\ 1+\pi_{ii}h+o(h) & i=j, \end{cases} \tag{3.155}$$

where $h>0$, $\lim_{h\to 0^+}(o(h)/h)=0$ and the transition probability rate satisfies $\pi_{ij}\ge 0$, for $i,j\in\mathbb{S}, i\ne j$ and

$$\pi_{ii}=-\sum_{j=1,j\ne i}^{N}\pi_{ij}. \tag{3.156}$$

The traditional controller design methods for MJSs are generally classified into two categories: that are MDCs and MICs, which are $u(t)=K(r_t)x(t)$ and $u(t)=Kx(t)$ respectively. In this section, a kind of controller called as PMD controller is developed as follows:

$$u(t)=(\alpha(t)K(r_t)+K)x(t), \tag{3.157}$$

where $K(r_t)$ and K are control gains to be determined, and $\alpha(t)$ is an indicator function satisfying Bernoulli process and described as

$$\alpha(t)=\begin{cases} 1 & \text{if } r_t \text{ is transmitted successfully}, \\ 0 & \text{otherwise}. \end{cases} \tag{3.158}$$

Then, we have

$$\Pr\{\alpha(t)=1\}=\mathscr{E}(\alpha(t))=\alpha,\ \Pr\{\alpha(t)=0\}=1-\alpha. \tag{3.159}$$

Moreover, it can be readily verified that

$$\mathscr{E}(\alpha(t)-\alpha)=0,\ \beta^2\triangleq\Pr\{(\alpha(t)-\alpha)^2\}=\alpha(1-\alpha). \tag{3.160}$$

Remark 3.4. The introduction of stochastic variable $\alpha(t)$ could reflect the jam degree of network in which r_t is transmitted. That is the larger value of α means that the higher probability of mode signal transmitted successfully. Compared with traditional controller design methods, controller (3.157) has some advantageous: 1) Different from MDC needing its OM online, controller (3.157) can bear the mode lost with some probability. We may measure or drop the mode signal with some probability. In this sense, it could reduce the burden of data transmission; 2) In contrast to MIC totally ignoring OM, the probability of mode accessible to controller is considered. Due to method for MIC is to find a common controller for all modes, the solvable solution set is smaller than one generated by (3.157). When there is no solution to an MIC, we may still get an effective controller of form (3.157). In this sense, it is said that the method for MIC is overdesign and more conservative.

Applying controller (3.157) to system (3.154) results in the following continuous-time closed-loop system:

$$\begin{cases} E\dot{x}(t)=\bar{A}(r_t)x(t)+A_d(r_t)x(t-\tau)+(\alpha(t)-\alpha)\check{A}(r_t)x(t)+B(r_t)\omega(t),\\ y(t)=C(r_t)x(t)+D(r_t)\omega(t),\\ x(t)=\phi(t),\ \forall t\in[-\tau,0], \end{cases} \tag{3.161}$$

where

$$\bar{A}(r_t)=A(r_t)+F(r_t)(\alpha K(r_t)+K),\ \check{A}(r_t)=F(r_t)K(r_t).$$

For closed-loop system (3.161), some definitions are needed.

Definition 3.3. Singular Markovian jump system (3.161) with $\omega(t)\equiv 0$ is said to be:

1) regular and impulse free for any constant time delay τ satisfying $0\le\tau\le\bar{\tau}$, if pairs $(E,\bar{A}(r_t))$ and $(E,\bar{A}(r_t)+A_d(r_t))$ are regular and impulse free for every $r_t\in\mathbb{S}$;
2) stochastically stable, if there exists a constant $M(\phi(t),r_0)$ such that

$$\mathscr{E}\{\int_0^\infty x^T(t)x(t)dt|\phi(t),r_0\}\le M(\phi(t),r_0), \tag{3.162}$$

for any initial conditions $\phi(t)\in\mathbb{R}^n$ and $r_0\in\mathbb{S}$;
3) stochastically admissible, if it is regular, impulse free and stochastically stable.

Before giving the concept of dissipativity, an energy supply function related to system (3.154) is defined by

$$\Psi(\omega,y,\hat{T}) \triangleq \mathscr{E}\{\langle y,R(r_t)y\rangle_{\hat{T}}\}+2\mathscr{E}\{\langle y,S(r_t)\omega\rangle_{\hat{T}}\}+\mathscr{E}\{\langle \omega,T(r_t)\omega\rangle_{\hat{T}}\}, \quad (3.163)$$

where $R(r_t)$, $S(r_t)$ and $T(r_t)$ are real matrices of appropriate dimensions with $R(r_t)$ and $T(r_t)$ symmetric, $\hat{T} \geq 0$ is an integer, and $\langle u,v\rangle_{\hat{T}} = \int_0^{\hat{T}} u^T v dt$. Now, we will give the following definition.

Definition 3.4. System (3.161) with zero initial state x_0 is said to be strictly (R_i,S_i,T_i)-dissipative for $i \in \mathbb{S}$, if for any $\hat{T} \geq 0$ and some scalar $\mu > 0$, the following condition holds

$$\Psi(\omega,y,\hat{T}) \geq \mu\mathscr{E}\{\langle \omega,\omega\rangle_{\hat{T}}\}, \quad (3.164)$$

for any initial conditions $\phi(t) \in \mathbb{R}^n$ and $r_0 \in \mathbb{S}$.

From Definition 3.4, it is seen that strict (R_i,S_i,T_i)-dissipativity includes H_∞ performance and passivity as special cases. That is

1) When $R(r_t) = -I$, $S(r_t) = 0$ and $T(r_t) = \gamma^2 I$, for any $r_t = i \in \mathbb{S}$, (3.164) will be simplified to be an H_∞ performance constraint;
2) When $R(r_t) = 0$, $S(r_t) = I$ and $T(r_t) = 0$, for any $r_t = i \in \mathbb{S}$, (3.164) will be reduced to be a strict positive realness.

Before presenting the main results and without loss of generality, an assumption is given as follow:

Assumption 3.1. It is assumed that

1) $T(r_t)+(S(r_t)^T D(r_t))^\star + D(r_t)^T R(r_t) D(r_t) > 0$;
2) $\tilde{R}(r_t) = \tilde{R}^{\frac{1}{2}}(r_t)\tilde{R}^{\frac{1}{2}}(r_t) \triangleq -R(r_t) \geq 0$.

First of all, the strict dissipativity of system (3.161) is considered.

Theorem 3.10. *Let matrices R_i, S_i and T_i be given with R_i and T_i symmetric and Assumption 3.1 holds. Then system (3.161) via given controller (3.157) is stochastically admissible and strictly (R_i,S_i,T_i)-dissipative, if there exist matrices P_i, $Q > 0$ and $Z > 0$, such that the following coupled LMIs hold for all $i \in \mathbb{S}$*

$$E^T P_i = P_i^T E \geq 0, \quad (3.165)$$

$$\begin{bmatrix} \Omega_{i1} & \Omega_{i2} & \Omega_{i3} & \bar{\tau}\bar{A}_i^T & \hat{\tau}\check{A}_i^T \\ * & \Omega_{i4} & 0 & \bar{\tau}A_{di}^T & 0 \\ * & * & \Omega_{i5} & \bar{\tau}B_i^T & 0 \\ * & * & * & -Z^{-1} & 0 \\ * & * & * & * & -Z^{-1} \end{bmatrix} < 0, \quad (3.166)$$

where

$$\Omega_{i1} = (\bar{A}_i^T P_i)^\star + \sum_{j=1}^{N} \pi_{ij} E^T P_j + Q - E^T Z E - C_i^T R_i C_i,$$

$$\Omega_{i2} = P_i^T A_{di} + E^T Z E,\ \Omega_{i3} = P_i^T B_i - C_i^T R_i D_i - C_i^T S_i,$$

$$\Omega_{i4} = -Q - E^T Z E,\ \Omega_{i5} = -T_i - (S_i^T D_i)^\star - D_i^T R_i D_i,\ \hat{\tau} = \beta\bar{\tau}.$$

Proof. Firstly, we show that system (3.161) is regular and impulse free. From (3.166), we have

$$(\bar{A}_i^T P_i)^\star + \sum_{j=1}^{N} \pi_{ij} E^T P_j + Q - E^T Z E - C_i^T R_i C_i < 0. \tag{3.167}$$

Similar to [182], there always exists two nonsingular matrices M and N such that

$$MEN = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \; M\bar{A}_i N =: \begin{bmatrix} \hat{A}_{i1} & \hat{A}_{i2} \\ \hat{A}_{i3} & \hat{A}_{i4} \end{bmatrix}, \; M^{-T} P_i N =: \begin{bmatrix} \hat{P}_{i1} & \hat{P}_{i2} \\ \hat{P}_{i3} & \hat{P}_{i4} \end{bmatrix}. \tag{3.168}$$

Pre- and post-multiplying (3.165) by N^T and its transpose respectively, it is concluded that $N^T E^T M^T M^{-T} P_i N = N^T P_i^T M^{-1} MEN$, which implies $\hat{P}_{i2} = 0$. Similarly, pre- and post-multiplying (3.167) by N^T and N, we have

$$\begin{bmatrix} \divideontimes & \divideontimes \\ \divideontimes & (\hat{A}_{i4}^T \hat{P}_{i4})^\star + \hat{Q}_3 + \hat{R}_{i3} \end{bmatrix} < 0, \tag{3.169}$$

where $\divideontimes$ denotes the terms are not used in (3.169), and $\hat{Q}_3$ and $\hat{R}_{3i}$ come from the following:

$$N^T Q N =: \begin{bmatrix} \hat{Q}_1 & \hat{Q}_2 \\ \hat{Q}_2^T & \hat{Q}_3 \end{bmatrix} > 0, \; -N^T C_i^T R_i C_i N =: \begin{bmatrix} -\hat{R}_{i1} & -\hat{R}_{i2} \\ -\hat{R}_{i2}^T & -\hat{R}_{i3} \end{bmatrix} \leq 0, \tag{3.170}$$

which imply $\hat{Q}_3 > 0$ and $\hat{R}_{i3} \geq 0$. Taking into account (3.169) and (3.170), we obtain

$$\hat{A}_{i4}^T \hat{P}_{i4} + \hat{P}_{i4}^T \hat{A}_{i4} < 0, \tag{3.171}$$

which implies that $\hat{A}_{i4}$ is nonsingular. Then, for each $i \in \mathbb{S}$, pair $(E, \bar{A}_i)$ is regular and impulse free. Since LMI (3.166) holds, it is known that

$$\begin{bmatrix} I \\ I \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \begin{bmatrix} \Omega_{i1} & \Omega_{i2} & \Omega_{i3} & \bar{\tau}\bar{A}_i^T & \hat{\tau}\check{A}_i^T \\ * & \Omega_{i4} & 0 & \bar{\tau}A_{di}^T & 0 \\ * & * & \Omega_{i5} & \bar{\tau}B_i^T & 0 \\ * & * & * & -Z^{-1} & 0 \\ * & * & * & * & -Z^{-1} \end{bmatrix} \begin{bmatrix} I \\ I \\ 0 \\ 0 \\ 0 \end{bmatrix} < 0, \tag{3.172}$$

which implies

$$((\bar{A}_i + A_{di})^T P_i)^\star + \sum_{j=1}^{N} \pi_{ij} E^T P_j - C_i^T R_i C_i < 0. \tag{3.173}$$

Similar to that in (3.167), it follows from (3.173) that pair $(E, \bar{A}(r_t) + A_d(r_t))$ is regular and impulse free for every $i \in \mathbb{S}$. Then, from Definition 3.3, we have system (3.161) is regular and impulse free, for any time delay τ satisfying $0 \leq \tau \leq \bar{\tau}$.

Next, we show system (3.161) is stochastically stable. Let $x_t(s) = x(t+s), -2\tau \leq s \leq 0$, similar to [27], we know that $\{(x_t, r_t), t \geq \tau\}$ is a Markov process. Now, for $t \geq \tau$, choose a stochastic Lyapunov function as

$$V(x_t, r_t) = V_1(x_t, r_t) + V_2(x_t, r_t) + V_3(x_t, r_t), \tag{3.174}$$

where

$$V_1(x_t, r_t) = x^T(t)E^T P(r_t)x(t),$$
$$V_2(x_t, r_t) = \int_{t-\tau}^{t} x^T(s)Qx(s)ds,$$
$$V_3(x_t, r_t) = \tau \int_{-\tau}^{0} \int_{t+\theta}^{t} \dot{x}^T(s)E^T ZE\dot{x}(s)dsd\theta.$$

For each $r_t = i \in \mathbb{S}$, it is defined as

$$\mathscr{L}[V(x_t, r_t = i)] = \lim_{h \to 0^+} \frac{1}{h}[\mathscr{E}(V(x_{t+h}, r_{t+h})|x_t, r_t = i) - V(x_t, i)]. \tag{3.175}$$

Then under $\omega(t) \equiv 0$, we have

$$\begin{aligned}
\mathscr{L}[V(x_t, r_t)] \leq\ & 2x^T(t)P_i^T[\bar{A}_i x(t) + A_{di}x(t-\tau)] + x^T(t)\sum_{j=1}^{N} \pi_{ij}E^T P_j x(t) \\
& + x^T(t)Qx(t) - x^T(t-\tau)Qx(t-\tau) \\
& + \bar{\tau}^2[\bar{A}_i x(t) + A_{di}x(t-\tau)]^T Z[\bar{A}_i x(t) + A_{di}x(t-\tau)] \\
& + \hat{\tau}^2(\check{A}_i x(t))^T Z\check{A}_i x(t) - \tau \int_{t-\tau}^{t} \dot{x}^T(s)E^T ZE\dot{x}(s)ds.
\end{aligned} \tag{3.176}$$

For $-\tau \int_{t-\tau}^{t} \dot{x}^T(s)E^T ZE\dot{x}(s)ds$ and via Jensen inequality, we have

$$-\tau \int_{t-\tau}^{t} \dot{x}^T(s)E^T ZE\dot{x}(s)ds \leq \zeta^T(t) \begin{bmatrix} -E^T ZE & E^T ZE \\ * & -E^T ZE \end{bmatrix} \zeta(t), \tag{3.177}$$

where $\zeta^T(t) = \begin{bmatrix} x^T(t) & x^T(t-\tau) \end{bmatrix}$. Then from (3.176) and (3.177), we obtain

$$\mathscr{L}[V(x_t, r_t)] \leq \zeta^T(t)\Gamma(r_t)\zeta(t) < 0, \tag{3.178}$$

where

$$\Gamma_i = \begin{bmatrix} \Omega_{i1} + C_i^T R_i C_i & \Omega_{i2} \\ * & \Omega_{i4} \end{bmatrix} + \begin{bmatrix} \bar{\tau}\bar{A}_i^T \\ \bar{\tau}A_{di}^T \end{bmatrix} Z \begin{bmatrix} \bar{\tau}\bar{A}_i^T \\ \bar{\tau}A_{di}^T \end{bmatrix}^T + \begin{bmatrix} \hat{\tau}\check{A}_i^T \\ 0 \end{bmatrix} Z \begin{bmatrix} \hat{\tau}\check{A}_i^T \\ 0 \end{bmatrix}^T.$$

From (3.166), we conclude $\Gamma_i < 0$, which implies (3.178) holds. Since $\mathscr{L}[V(x_t, i, t)] < 0$, there always exists a sufficient small scalar $\varepsilon > 0$ for each $i \in \mathbb{S}$

$$\mathscr{L}[V(x_t, r_t, t)] \leq -\varepsilon x^T(t)x(t). \tag{3.179}$$

By using Dynkin's formula, we obtain that for all $t \geq \tau$

$$\mathscr{E}\{V(x_t, r_t, t)\} - \mathscr{E}\{V(x_\tau, r_\tau, \tau)\} \leq -\varepsilon \mathscr{E}\{\int_\tau^t x^T(s)x(s)ds\}. \tag{3.180}$$

Thus

$$\mathscr{E}\{\int_\tau^t x^T(s)x(s)ds\} \leq \varepsilon^{-1}\mathscr{E}\{V(x_\tau, r_\tau, \tau)\}. \tag{3.181}$$

Since $\hat{A}_{i4}$ is nonsingular for each $i \in \mathbb{S}$ and set

$$\tilde{M}_i \triangleq \begin{bmatrix} I & -\hat{A}_{i2}\hat{A}_{i4}^{-1} \\ 0 & I \end{bmatrix} M,\ \tilde{E}_i \triangleq \tilde{M}_i E N = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \tag{3.182}$$

$$\tilde{A}_i \triangleq \tilde{M}_i \bar{A}_i N = \begin{bmatrix} \tilde{A}_{i1} & 0 \\ \hat{A}_{i3} & \hat{A}_{i4} \end{bmatrix},\ \hat{A}_{di} \triangleq \tilde{M}_i A_{di} N = \begin{bmatrix} \hat{A}_{di1} & \hat{A}_{di2} \\ \hat{A}_{di3} & \hat{A}_{di4} \end{bmatrix}, \tag{3.183}$$

where $\tilde{A}_{i1} \triangleq \hat{A}_{i1} - \hat{A}_{i2}\hat{A}_{i4}^{-1}\hat{A}_{i3}$, and let $\xi^T(t) = \begin{bmatrix} \xi_1^T(t) & \xi_2^T(t) \end{bmatrix} = N^{-1}x(t)$, system (3.161) with $\omega(t) \equiv 0$ is equivalent to

$$\begin{cases} \dot{\xi}_1(t) = \tilde{A}_{i1}\xi_1(t) + \hat{A}_{di1}\xi_1(t-\tau) + \hat{A}_{di2}\xi_2(t-\tau), \\ \quad 0 = \hat{A}_{i3}\xi_1(t) + \hat{A}_{i4}\xi_2(t) + \hat{A}_{di3}\xi_1(t-\tau) + \hat{A}_{di4}\xi_2(t-\tau), \\ \xi(t) = N^{-1}\phi(t), \quad t \in [-\tau, 0]. \end{cases} \tag{3.184}$$

For any $0 \leq t \leq \tau$ and from (3.184), it is easy to see that there exists a scalar $k_1 > 0$ such that

$$\begin{aligned} \|\xi_1(t)\| &\leq \|\xi_1(0)\| + 2k_1 \int_0^t \sup_{s-\tau \leq \theta \leq s} \|\xi_1(\theta)\| ds + k_1 \tau \sup_{-\tau \leq s \leq 0} \|\xi_2(s)\| \\ &\leq (1 + k_1\tau)\|N^{-1}\phi(0)\| + 2k_1 \int_0^t \sup_{s-\tau \leq \theta \leq s} \|\xi_1(\theta)\| ds, \end{aligned} \tag{3.185}$$

where $k_1 = \max_{i \in \mathbb{S}}\{\|\hat{A}_{i1}\|, \|\hat{A}_{di1}\|, \|\hat{A}_{di2}\|\}$. Moreover, for any $0 \leq t \leq \tau$, by using Gronwall-Bellman lemma, we conclude

$$\begin{aligned} \sup_{0 \leq t \leq \tau} \|\xi_1(t)\| &\leq (1 + k_1\tau)\|N^{-1}\phi(0)\| + 2k_1 \int_0^t \sup_{s-\tau \leq \theta \leq s} \|\xi_1(\theta)\| ds \\ &\leq (1 + k_1\tau)\|N^{-1}\phi(0)\| e^{2k_1\tau}. \end{aligned} \tag{3.186}$$

For any $t \in [0, \tau]$ and from (3.184), we have

$$\sup_{0 \leq t \leq \tau} \|\xi_2(t)\| \leq k_2(\|\xi_1(t)\| + 2\|N^{-1}\phi(0)\|) \leq k_2[(1 + k_1\tau)e^{2k_1\tau} + 2]\|N^{-1}\phi(0)\|, \tag{3.187}$$

where $k_2 = \max_{i \in \mathbb{S}}\{\|\hat{A}_{i4}^{-1}\hat{A}_{i3}\|, \|\hat{A}_{i4}^{-1}\hat{A}_{di3}\|, \|\hat{A}_{i4}^{-1}\hat{A}_{di4}\|\}$. Moreover, it is concluded that

$$\sup_{0\le t\le\tau}\|\xi(t)\|^2 \le \sup_{0\le t\le\tau}\|\xi_1(t)\|^2 + \sup_{0\le t\le\tau}\|\xi_2(t)\|^2 \le k_3\|N^{-1}\phi(0)\|^2, \tag{3.188}$$

where $k_3 = (1+k_1\tau)^2 e^{4k_1\tau} + [k_2(1+k_1\tau)e^{2k_1\tau} + 2k_2]^2$. Since N is nonsingular and by (3.188), we have

$$\sup_{0\le t\le\tau}\|x(t)\|^2 \le k_3\frac{\sigma^2_{max}(N^{-1})}{\sigma^2_{min}(N)}\|\phi(0)\|^2. \tag{3.189}$$

Hence, we get there exists a scalar $k_4 > 0$ such that $V(x_\tau, r_\tau, \tau) \le k_4\|\phi(0)\|^2$. Thus, we obtain that

$$\mathscr{E}\{\int_0^T \hat{x}^T(t)\hat{x}(t)dt|\phi(0), r_0\} \le \rho\|\phi(0)\|^2. \tag{3.190}$$

Now, we will show the dissipativity property of system (3.161) for each $r_t = i \in \mathbb{S}$, that is

$$\begin{aligned}
&\mathscr{L}[(V(x_t,r_t,t))] - y^T(t)R(r_t)y(t) - 2y(t)S(r_t)\omega(t) - \omega^T(t)T(r_t)\omega(t)\\
&= \mathscr{L}[(V(x_t,r_t,t))] - (C(r_t)x(t)+D(r_t)\omega(t))^T R(r_t)(C(r_t)x(t)+D(r_t)\omega(t))\\
&\quad - 2(C(r_t)x(t)+D(r_t)\omega(t))^T S(r_t)\omega(t) - \omega^T(t)T(r_t)\omega(t)\\
&= \hat{\zeta}^T(t)\hat{\Gamma}(r_t)\hat{\zeta}(t) < 0,
\end{aligned} \tag{3.191}$$

where

$$\hat{\zeta}^T(t) = \left[\, x^T(t)\ x^T(t-\tau)\ \omega^T(t)\,\right],$$

$$\hat{\Gamma}_i = \begin{bmatrix} \Omega_{i1} & \Omega_{i2} & \Omega_{i3}\\ * & \Omega_{i4} & 0\\ * & * & \Omega_{i5}\end{bmatrix} + \begin{bmatrix}\bar{\tau}\bar{A}_i^T\\ \bar{\tau}A_{di}^T\\ 0\end{bmatrix} Z \begin{bmatrix}\bar{\tau}\bar{A}_i^T\\ \bar{\tau}A_{di}^T\\ 0\end{bmatrix}^T + \begin{bmatrix}\hat{\tau}\check{A}_i^T\\ 0\\ 0\end{bmatrix} Z \begin{bmatrix}\hat{\tau}\check{A}_i^T\\ 0\\ 0\end{bmatrix}^T.$$

Since (3.166) is equivalent to $\hat{\Gamma}_i < 0$, it implies (3.191) holds. Moreover, there always exists a sufficient small scalar $\mu > 0$ such that $\Omega_{i5} + \mu I < 0$. As a result, we get

$$\mu\omega^T(t)\omega(t) < -\mathscr{L}[(V(x_t,i,t))] + y^T(t)R_iy(t) + 2y(t)S_i\omega(t) + \omega^T(t)T_i\omega(t), \tag{3.192}$$

which is further deduced as

$$\begin{aligned}
\mu\mathscr{E}\{\int_0^T \omega^T(t)\omega(t)dt\} \le \mathscr{E}(V(x_0,r_0,0)) - \mathscr{E}(V(x_T,r_T,T)) + \mathscr{E}\{\int_0^T [y^T(t)R(r_t)y(t)\\
+ 2y^T(t)S(r_t)\omega(t) + \omega^T(t)T(r_t)\omega(t)]dt\}.
\end{aligned} \tag{3.193}$$

Under zero initial condition, we have (3.164) holds. This completes the proof.

Remark 3.5. Via giving controller (3.157) beforehand, Theorem 3.10 gives a sufficient condition for dissipativity of continuous-time SMJS (3.161) with time delay. However, it cannot be used to test the dissipativity directly due to the couplings among variables. More importantly, an PMD controller cannot be solved directly via pre- and post-multiplying P_i, where there is a distinct contradiction between the solution to PMD controller and the requirement of an MD Lyapunov function.

In the following, a condition to separate P_i from $\bar{A}_i$ is proposed, where the requirements of PMD controller and MD Lyapunov function are likely to be satisfied simultaneously.

Theorem 3.11. *Let matrices R_i, S_i and T_i be given with R_i and T_i symmetric and Assumption 3.1 holds. Then system (3.161) via given controller (3.157) is stochastically admissible and strictly (R_i, S_i, T_i)-dissipative, if there exist matrices X_i, G_i, Z_i, $Q > 0$ and $Z > 0$, such that the following coupled LMIs hold for all $i \in \mathbb{S}$*

$$X_i^T E^T = E X_i \geq 0, \tag{3.194}$$

$$\begin{bmatrix} \bar{\Omega}_{i1} & \bar{\Omega}_{i2} & \bar{\Omega}_{i3} & \bar{\Omega}_{i4} & \bar{\tau} G_i^T \bar{A}_i^T & \hat{\tau} G_i^T \check{A}_i^T \\ * & \bar{\Omega}_{i5} & 0 & 0 & \bar{\tau} Z_i^T \bar{A}_i^T & \hat{\tau} Z_i^T \check{A}_i^T \\ * & * & \bar{\Omega}_{i6} & 0 & \bar{\tau} X_i^T A_{di}^T & 0 \\ * & * & * & \Omega_{i5} & \bar{\tau} B_i^T & 0 \\ * & * & * & * & -Z^{-1} & 0 \\ * & * & * & * & * & -Z^{-1} \end{bmatrix} < 0, \tag{3.195}$$

where

$$\bar{\Omega}_{i1} = (G_i^T \bar{A}_i^T)^\star + X_i^T (\sum_{j=1}^{N} \pi_{ij} E^T X_j^{-1} + Q - E^T Z E - C_i^T R_i C_i) X_i,$$

$$\bar{\Omega}_{i2} = \bar{A}_i Z_i + X_i^T - G_i^T, \ \bar{\Omega}_{i3} = A_{di} X_i + X_i^T E^T Z E X_i,$$

$$\bar{\Omega}_{i4} = B_i - X_i^T C_i^T R_i D_i - X_i^T C_i^T S_i, \ \bar{\Omega}_{i5} = -(Z_i)^\star, \ \bar{\Omega}_{i6} = -X_i^T (Q + E^T Z E) X_i,$$

which are equivalent to ones in Theorem 3.10.

Proof. Let $X_i = P_i^{-1}$, pre- and post-multiplying (3.166) with $diag\{X_i^T, X_i^T, I, I, I\}$ and (3.165) with X_i^T and their transposes respectively, we have

$$X_i^T E^T = E X_i \geq 0, \tag{3.196}$$

$$\begin{bmatrix} \hat{\Omega}_{i1} & \bar{\Omega}_{i3} & \bar{\Omega}_{i4} & \bar{\tau} X_i^T \bar{A}_i^T & \hat{\tau} X_i^T \check{A}_i^T \\ * & \bar{\Omega}_{i6} & 0 & \bar{\tau} X_i^T A_{di}^T & 0 \\ * & * & \Omega_{i5} & \bar{\tau} B_i^T & 0 \\ * & * & * & -Z^{-1} & 0 \\ * & * & * & * & -Z^{-1} \end{bmatrix} < 0, \tag{3.197}$$

where

$$\hat{\Omega}_{i1} = (X_i^T \bar{A}_i^T)^\star + X_i^T (\sum_{j=1}^{N} \pi_{ij} E^T X_j^{-1} + Q - E^T Z E - C_i^T R_i C_i) X_i.$$

Sufficiency: Pre- and post-multiplying (3.195) with the following matrix

$$\begin{bmatrix} I & \bar{A}_i & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & \bar{\tau}\bar{A}_i & 0 & 0 & I & 0 \\ 0 & \hat{\tau}\check{A}_i & 0 & 0 & 0 & I \end{bmatrix}, \tag{3.198}$$

and its transpose, it is directly obtained that (3.195) implies (3.197).
Necessity: Since (3.197) holds, there always exists a sufficient small scalar $\varepsilon_i > 0$ such that

$$\begin{bmatrix} \hat{\Omega}_{i1} & \bar{\Omega}_{i3} & \bar{\Omega}_{i4} & \bar{\tau}X_i^T\bar{A}_i^T & \hat{\tau}X_i^T\check{A}_i^T \\ * & \bar{\Omega}_{i6} & 0 & \bar{\tau}X_i^TA_{di}^T & 0 \\ * & * & \Omega_{i5} & \bar{\tau}B_i^T & 0 \\ * & * & * & -Z^{-1} & 0 \\ * & * & * & * & -Z^{-1} \end{bmatrix} + \begin{bmatrix} \bar{A}_i \\ 0 \\ 0 \\ \bar{\tau}\bar{A}_i \\ \hat{\tau}\check{A}_i \end{bmatrix} \frac{\varepsilon_i}{2} \begin{bmatrix} \bar{A}_i \\ 0 \\ 0 \\ \bar{\tau}\bar{A}_i \\ \hat{\tau}\check{A}_i \end{bmatrix}^T < 0. \tag{3.199}$$

Let $\varepsilon_i I = Z_i$ and $X_i = G_i$ and via using congruent transformation, we have that (3.199) implies (3.195). This completes the proof.

Remark 3.6. It is seen that Theorem 3.11 is equivalent to Theorem 3.10. However, system matrix $\bar{A}_i$ is decoupled from Lyapunov function matrix P_i, where both of them could be solved separately. Unfortunately, there are still some solution problems, such as equation constraint (3.194), nonlinear terms, e.g., $X_i^TQX_i$ and $X_i^TE^TZEX_i$. Especially, there is also a nonlinear term $X_i^T\sum_{j=1,j\neq i}^{N}\pi_{ij}E^TX_j^{-1}X_i$, which results from the inherent characteristics of continuous-time SMJSs. Since, there are singular matrix E and no symmetric positive definite matrix X_j^{-1} in such terms, it cannot be dealt with by using Schur complement directly. As a result, in order to establish LMI conditions for controller (3.157), such conditions of Theorem 3.11 should be further handled.

Finally, we will give strict LMI conditions for the design of PMD controller of form (3.157).

Theorem 3.12. *Let matrices R_i, S_i and T_i be given with R_i and T_i symmetric and Assumption 3.1 holds. Then system (3.161) via controller (3.157) is stochastically admissible and strictly (R_i, S_i, T_i)-dissipative, if there exist matrices G, $\hat{P}_i$, $\hat{Q}_i$, $\hat{Q} > 0$, $\hat{Z} > 0$, Y_i and Y, such that the following LMIs hold for all $i \in \mathbb{S}$*

$$\begin{bmatrix} \hat{\Omega}_{i1} & \hat{\Omega}_{i2} & \hat{\Omega}_{i3} & \hat{\Omega}_{i4} & \hat{\Omega}_{i5} & \hat{\Omega}_{i6} & X_i^T & X_i^TC_i^T\tilde{R}_i^{\frac{1}{2}} & \hat{\Omega}_{i7} \\ * & -(G)^\star & 0 & 0 & \hat{\Omega}_{i5} & \hat{\Omega}_{i6} & 0 & 0 & 0 \\ * & * & \hat{\Omega}_{i8} & 0 & \bar{\tau}X_i^TA_{di}^T & 0 & 0 & 0 & 0 \\ * & * & * & -T_i-(S_i^TD_i)^\star & \bar{\tau}B_i^T & 0 & 0 & D_i^T\tilde{R}_i^{\frac{1}{2}} & 0 \\ * & * & * & * & -\hat{Z} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\hat{Z} & 0 & 0 & 0 \\ * & * & * & * & * & * & -\hat{Q} & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & \hat{\Omega}_{i9} \end{bmatrix} < 0, \tag{3.200}$$

where

$$\hat{\Omega}_{i1} = (A_iG + F_i(\alpha Y_i + Y) - E\hat{P}_iE^T)^{\star} + \lambda_{ii}E\hat{P}_iE^T + \hat{Z},$$
$$\hat{\Omega}_{i2} = A_iG + F_i(\alpha Y_i + Y) + E\hat{P}_i^T + U^T\hat{Q}_iV^T - G^T,$$
$$\hat{\Omega}_{i3} = A_{di}\hat{P}_iE^T + A_{di}V\hat{Q}_iU + (E\hat{P}_iE^T)^{\star} - \hat{Z},\ \hat{\Omega}_{i4} = B_i - X_i^TC_i^TS_i,$$
$$\hat{\Omega}_{i5} = \bar{\tau}G^TA_i^T + \bar{\tau}(\alpha Y_i^T + Y^T)F_i^T,\ \hat{\Omega}_{i6} = \hat{\tau}Y_i^TF_i^T,$$
$$\hat{\Omega}_{i7} = \left[\sqrt{\pi_{i1}}X_i^TE_R \cdots \sqrt{\pi_{i(i-1)}}X_i^TE_R\ \sqrt{\pi_{i(i+1)}}X_i^TE_R \cdots \sqrt{\pi_{iN}}X_i^TE_R\right],$$
$$X_i = \hat{P}_iE^T + V\hat{Q}_iU,\ \hat{\Omega}_{i8} = -(\hat{P}_iE^T + V\hat{Q}_iU + E\hat{P}_iE^T)^{\star} + \hat{Q} + \hat{Z},$$
$$\hat{\Omega}_{i9} = -diag\{E_R^T\hat{P}_iE_R, \ldots, E_R^T\hat{P}_{i-1}E_R, E_R^T\hat{P}_{i+1}E_R, \ldots, E_R^T\hat{P}_NE_R\}.$$

Then, the desired control gains of form (3.157) are given as

$$K_i = Y_iG^{-1},\ K = YG^{-1}. \tag{3.201}$$

Proof. Let

$$P_i \triangleq \bar{P}_iE + U^T\bar{Q}_iV^T, \tag{3.202}$$

where $\bar{P}_i > 0$ and $\bar{Q}_i$ is nonsingular. Moreover, we have

$$E^TP_i = P_i^TE = E^T\bar{P}_iE \geq 0, \tag{3.203}$$

always holds, which could be omitted. Then, there is no constraint (3.165) in Theorem 3.10. If such conditions of Theorem 3.10 hold, the closed-loop system (3.161) will be stochastically admissible and strictly (R_i, S_i, T_i)-dissipative. Since $\bar{P}_i > 0$ and $\bar{Q}_i$ is nonsingular, we obtain $E_L^T\bar{P}_iE_L > 0$. Then we get

$$X_i \triangleq (\bar{P}_iE + U^T\bar{Q}_iV^T)^{-1} = \hat{P}_iE^T + V\hat{Q}_iU, \tag{3.204}$$

where $\hat{P}_i$ and $\hat{Q}_i$ are defined in Lemma 3.1. If the conditions in Theorem 3.11 are satisfied with $G_i = G$ and $Z_i = G$, where matrices P_i and X_i are replaced by (3.202) and (3.204) respectively, we have Theorem 3.10 holds. It means that closed-loop systems (3.161) via controller (3.157) is stochastically admissible and strictly (R_i, S_i, T_i)-dissipative. That is

$$\begin{bmatrix} \tilde{\Omega}_{i1} & \tilde{\Omega}_{i2} & \bar{\Omega}_{i3} & \bar{\Omega}_{i4} & \bar{\tau}G^T\bar{A}_i^T & \hat{\tau}G^T\check{A}_i^T \\ * & \tilde{\Omega}_{i5} & 0 & 0 & \bar{\tau}G^T\bar{A}_i^T & \hat{\tau}G^T\check{A}_i^T \\ * & * & \bar{\Omega}_{i6} & 0 & \bar{\tau}X_i^TA_{di}^T & 0 \\ * & * & * & \Omega_{i5} & \bar{\tau}B_i^T & 0 \\ * & * & * & * & -Z^{-1} & 0 \\ * & * & * & * & * & -Z^{-1} \end{bmatrix} < 0, \tag{3.205}$$

where

$$\tilde{\Omega}_{i1} = (G^T\bar{A}_i^T)^{\star} + \pi_{ii}X_i^TE^T + X_i^T(\sum_{j=1,j\neq i}^{N}\pi_{ij}E^TP_j + Q - E^TZE - C_i^TR_iC_i)X_i,$$

$$\tilde{\Omega}_{i2} = \bar{A}_i G + X_i^T - G^T, \tilde{\Omega}_{i5} = -(G)^{\star}.$$

The others are given in Theorem 3.11. However, there are sill nonlinear terms in (3.205), such as $X_i^T \sum_{j=1,j\neq i}^{N} \pi_{ij} E^T P_j X_i$, $X_i^T E^T Z E X_i$ and $X_i^T Q X_i$, which cannot be dealt with directly. For nonlinear term $X_i^T \sum_{j=1,j\neq i}^{N} \pi_{ij} E^T P_j X_i$, it cannot be handled directly because of singular matrix E and no symmetric positive-definite matrix P_j. Based on (3.202), it is concluded that

$$\begin{aligned} X_i^T \sum_{j=1,j\neq i}^{N} \pi_{ij} E^T P_j X_i &= X_i^T \sum_{j=1,j\neq i}^{N} \pi_{ij} E^T (\bar{P}_i E + U^T \bar{Q}_i V^T) X_i \\ &= X_i^T \sum_{j=1,j\neq i}^{N} \pi_{ij} E_R E_L^T \bar{P}_i E_L E_R^T X_i \\ &= X_i^T \sum_{j=1,j\neq i}^{N} \pi_{ij} E_R (E_R^T \hat{P}_i E_R)^{-1} E_R^T X_i. \end{aligned} \tag{3.206}$$

For nonlinear terms $X_i^T Q X_i$ and $X_i^T E^T Z E X_i$, by letting $\hat{Q} = Q^{-1}$ and $\hat{Z} = Z^{-1}$, we get

$$-X_i Q X_i \leq -(X_i)^{\star} + \hat{Q}, \tag{3.207}$$

$$-X_i^T E^T Z E X_i \leq -(E X_i)^{\star} + \hat{Z}, \tag{3.208}$$

which is further used to deduce

$$\begin{aligned} \begin{bmatrix} -X_i^T E^T Z E X_i & X_i^T E^T Z E X_i \\ X_i^T E^T Z E X_i & -X_i^T E^T Z E X_i \end{bmatrix} &= \begin{bmatrix} I \\ -I \end{bmatrix} (-X_i^T E^T Z E X_i) \begin{bmatrix} I & -I \end{bmatrix} \\ &\leq \begin{bmatrix} I \\ -I \end{bmatrix} (-(E X_i)^{\star} + \hat{Z}) \begin{bmatrix} I & -I \end{bmatrix}. \end{aligned} \tag{3.209}$$

Via using Schur complement and taking into account (3.206)-(3.209), it is deduced that (3.200) with (3.201) implies (3.205) with (3.202). This completes the proof.

Remark 3.7. It is remarked that Theorem 3.12 presents a sufficient strict LMI condition for designing an PMD controller such that the resulting system with delay is stochastically admissible and strictly (R_i, S_i, T_i)-dissipative. Moreover, from Theorem 3.12, it can be seen that both time delay bound $\bar{\tau}$ and mode observation probability α are involved, which play important roles in PMD controller design.

When system mode is always unavailable to state feedback controller, an MIC describe by (3.7) is obtained by (3.157) with $\alpha = 0$. Then, we have the following corollary.

Corollary 3.5. *Let matrices R_i, S_i and T_i be given with R_i and T_i symmetric and Assumption 3.1 holds. Then system (3.161) via controller (3.7) is stochastically admissible and strictly (R_i, S_i, T_i)-dissipative, if there exist matrices G, $\hat{P}_i$, $\hat{Q}_i$, $\hat{Q} > 0$, $\hat{Z} > 0$, Y_i and Y, such that the following LMIs hold for all $i \in \mathbb{S}$*

$$\begin{bmatrix} \Xi_{i1} & \Xi_{i2} & \hat{\Omega}_{i3} & \hat{\Omega}_{i4} & \Xi_{i5} & X_i^T & X_i^T C_i^T \tilde{R}_i^{\frac{1}{2}} & \hat{\Omega}_{i7} \\ * & -(G)^{\star} & 0 & 0 & \Xi_{i5} & 0 & 0 & 0 \\ * & * & \hat{\Omega}_{i8} & 0 & \bar{\tau} X_i^T A_{di}^T & 0 & 0 & 0 \\ * & * & * & -T_i-(S_i^T D_i)^{\star} & \bar{\tau} B_i^T & 0 & D_i^T \tilde{R}_i^{\frac{1}{2}} & 0 \\ * & * & * & * & -\hat{Z} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\hat{Q} & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & \hat{\Omega}_{i9} \end{bmatrix} < 0, \tag{3.210}$$

where

$$\Xi_{i1} = (A_i G + F_i Y - E\hat{P}_i E^T)^{\star} + \lambda_{ii} E\hat{P}_i E^T + \hat{Z},$$

$$\Xi_{i2} = A_i G + F_i Y + E\hat{P}_i^T + U^T \hat{Q}_i V^T - G^T,\ \Xi_{i5} = \bar{\tau} G^T A_i^T + \bar{\tau} Y^T F_i^T,$$

$\hat{\Omega}_{i3}$, $\hat{\Omega}_{i4}$, X_i, $\hat{\Omega}_{i7}$, $\hat{\Omega}_{i8}$ and $\hat{\Omega}_{i9}$ are defined in Theorem 3.12. Then a desired control gain of form (3.7) is given by

$$K = YG^{-1}. \tag{3.211}$$

When there is no time delay in system (3.154), system (3.161) reduced to

$$\begin{cases} E\dot{x}(t) = \bar{A}(r_t)x(t) + (\alpha(t)-\alpha)\check{A}(r_t)x(t) + B(r_t)\omega(t), \\ y(t) = C(r_t)x(t) + D(r_t)\omega(t), \\ x(0) = \varphi(0), \end{cases} \tag{3.212}$$

where $\bar{A}(r_t)$ and $\check{A}(r_t)$ are defined in (3.161). By the similar methods, we easily have the following corollary.

Corollary 3.6. *Let matrices R_i, S_i and T_i be given with R_i and T_i symmetric and Assumption 3.1 holds. Then system (3.212) via controller (3.157) is stochastically admissible and strictly (R_i, S_i, T_i)-dissipative, if there exist matrices G, $\hat{P}_i$, $\hat{Q}_i$, Y_i and Y, such that the following LMIs hold for all $i \in \mathbb{S}$*

$$\begin{bmatrix} \bar{\Xi}_{i1} & \hat{\Omega}_{i2} & \hat{\Omega}_{i4} & X_i^T C_i^T \tilde{R}_i^{\frac{1}{2}} & \hat{\Omega}_{i7} \\ * & -(G)^{\star} & 0 & 0 & 0 \\ * & * & -T_i-(S_i^T D_i)^{\star} & D_i^T \tilde{R}_i^{\frac{1}{2}} & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & \hat{\Omega}_{i9} \end{bmatrix} < 0, \tag{3.213}$$

where

$$\bar{\Xi}_{i1} = (A_i G + F_i(\alpha Y_i + Y) - E\hat{P}_i E^T)^{\star} + \pi_{ii} E\hat{P}_i E^T,$$

X_i, $\hat{\Omega}_{i2}$, $\hat{\Omega}_{i4}$, $\hat{\Omega}_{i7}$ and $\hat{\Omega}_{i9}$ are given in Theorem 3.12. Then a desired controller gain of form (3.157) is given via (3.201).

Remark 3.8. It is noticed that the criteria obtained in this section are related to be SMJSs. However, since system matrix E satisfies $\text{rank}(E) = r \leq n$, the results of normal MJSs can be obtained easily via the similar methods.

In this section, two examples are used to demonstrate the applicability of the proposed approach.

Example 3.9. Consider an SMJS of form (3.154) with

$$A_1 = \begin{bmatrix} 1.2 & 1 \\ -2.5 & -0.6 \end{bmatrix}, A_{d1} = \begin{bmatrix} 0 & 1.1 \\ 0 & 0 \end{bmatrix}, F_1 = \begin{bmatrix} -0.5 \\ 0.4 \end{bmatrix}, B_1 = \begin{bmatrix} 0.2 \\ 1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -0.3 & 0 \end{bmatrix}, D_1 = -0.5, R_1 = -0.4, S_1 = -1, T_1 = 1.3,$$

$$A_2 = \begin{bmatrix} 0.5 & 1 \\ 0 & -0.7 \end{bmatrix}, A_{d2} = \begin{bmatrix} -0.8 & 2 \\ 0 & 0.4 \end{bmatrix}, F_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0 & 0.2 \end{bmatrix}, D_2 = -0.2, R_2 = -0.8, S_2 = 0.5, T_2 = 1.5.$$

The TRM is given as

$$\Pi = \begin{bmatrix} -0.5 & 0.5 \\ 1.5 & -1.5 \end{bmatrix},$$

and singular matrix

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$

The constant time delay satisfies $\tau \in [0, 0.25]$. By Corollary 3.5, there is no solution to a totally MIC. However, by Theorem 3.12, we see that there is no solution when $0 \leq \alpha < 0.827$. Under $\alpha = 0.83$ and via Theorem 3.12, we have the PMD control gains of form (3.157) as follows:

$$\begin{aligned} K_1 &= \begin{bmatrix} 6.4087 & -0.4905 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} -0.6307 & 1.2948 \end{bmatrix}, \\ K &= \begin{bmatrix} -0.5228 & -1.8551 \end{bmatrix}. \end{aligned}$$

The corresponding control gains of an MDC are constructed as

$$\begin{aligned} K_1 &= \begin{bmatrix} -136.7725 & -26.9473 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} -142.7656 & -25.7526 \end{bmatrix}, \\ K &= \begin{bmatrix} 141.9878 & 24.8340 \end{bmatrix}, \end{aligned}$$

which are equivalent to

$$\tilde{K}_1 = \begin{bmatrix} 5.2153 & -2.1133 \end{bmatrix}, \tilde{K}_2 = \begin{bmatrix} -0.7778 & -0.9186 \end{bmatrix}.$$

Illustrated by the proposed results, it is known that the system mode is not necessary to an PMD controller. For this example with given system matrices and τ, we see that the obtained PMD control gain of form (3.157) could discard system mode signal with 17%. In this sense, it could reduce the burden of data transmission and has more scope of application. Moreover, from the above explanation, we know that the larger α corresponding to the high probability of mode accessible, the less conservativeness of the obtained results in terms of the larger $\bar{\tau}$, which is also illustrated in Table 3.1.

Table 3.1 Allowable upper bounds of $\bar{\tau}$ with given α

α	0.1	0.2	0.3	0.4	0.5
$\bar{\tau}$	0.053	0.079	0.101	0.123	0.146
α	0.6	0.7	0.8	0.9	1
$\bar{\tau}$	0.171	0.20	0.238	0.288	0.352

Example 3.10. Consider an SMJS described by (3.212) with

$$A_1 = \begin{bmatrix} -0.2 & 1 & 0.3 \\ 2 & -1.2 & -3 \\ 2 & 1 & 1 \end{bmatrix}, F_1 = \begin{bmatrix} -0.5 \\ 0.4 \\ 1 \end{bmatrix}, B_1 = \begin{bmatrix} 1.2 \\ -1 \\ 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -0.2 & 0 & 0 \end{bmatrix}, D_1 = -0.5, R_1 = -0.4, S_1 = 0.7, T_1 = 2.2,$$

$$A_2 = \begin{bmatrix} 0.2 & 1.3 & -0.3 \\ 3 & -1.2 & -1 \\ 1 & 2 & 1 \end{bmatrix}, F_2 = \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0.4 \\ 0.6 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.3 & 0 & 0.2 \end{bmatrix}, D_2 = 0.7, R_2 = -0.5, S_1 = -0.7, T_2 = 2.3.$$

Its TRM is given as

$$\Pi = \begin{bmatrix} -5 & 5 \\ 7 & -7 \end{bmatrix},$$

and singular matrix is

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Let initial condition $\varphi(0) = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}^T$, and the state of the open-loop system is illustrated in Fig. 3.13, which is not stable. By Corollary 3.5, it is known that for this example, there is no solution to an MIC. With the same parameters, we will design a dissipative controller of form (3.157), which is partially mode-dependent. If the probability of mode available to controller is $\alpha = 0.6$, then by Corollary 3.6, we have the gains of PMD controller as follows

$$K_1 = \begin{bmatrix} 702.1877 & 284.9663 & -201.3750 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 662.3898 & 269.6700 & -186.8841 \end{bmatrix},$$
$$K = \begin{bmatrix} -398.8988 & -161.6548 & 113.6045 \end{bmatrix},$$

where the closed-loop system is not only stochastically admissible but also satisfies condition (3.164). Moreover, from Fig. 3.14, it is seen that the corresponding closed-loop system is stable. Though we may also design an MDC, it needs system mode obtained exactly online. Compared with MDC, the PMD controller of this example only needs system mode accessible with distribution probability $\alpha = 0.6$, where 40% of mode signal can be dropped out. The effect of the desired PMD controller is also demonstrated in Fig. 3.15, where $*$ denotes the corresponding controller mode unavailable.

3.7 Conclusion

This chapter has investigated the stabilization problem for SMJSs. An LMI approach has been developed to design robust stabilizing state feedback controller such that the closed-loop system is robustly stochastically admissible over uncertainties of TRM. Stabilizing conditions for SMJSs via designing TRM and state feedback controller are presented in terms of LMIs with some equation constraints. Moreover, the other stabilization cases such as noise control, PD control and PMD control have been solved, which are formulated as LMIs or LMIs with some equations. Part contents of this chapter is based on the work of the author [130, 133, 143, 152].

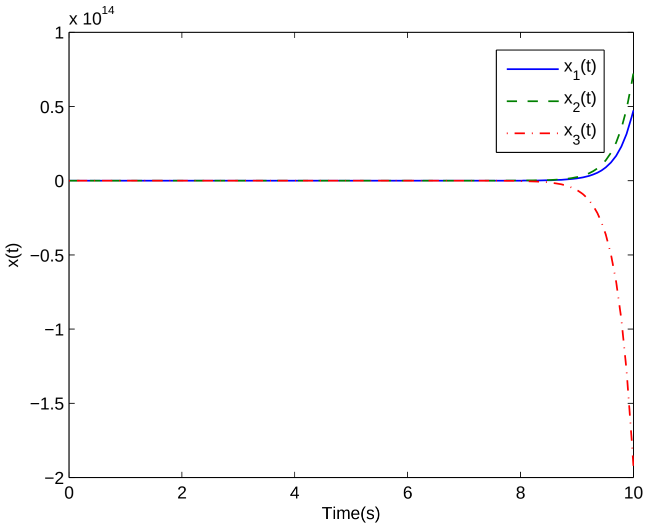

Fig. 3.13 Response of open-loop system

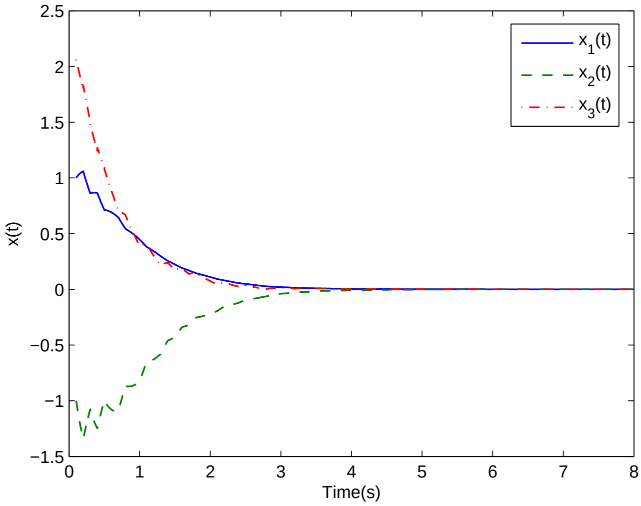

Fig. 3.14 Response of closed-loop system

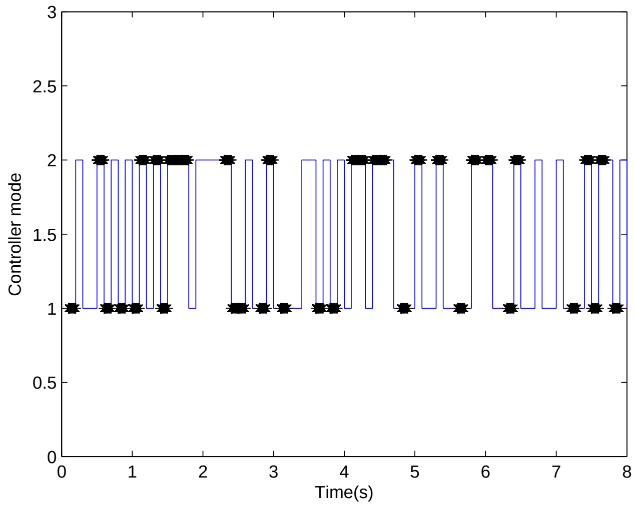

Fig. 3.15 The mode of controller with $\alpha = 0.6$