Chapter 10
Concluding Remarks

Study on complex control systems has received much attention in order to satisfy the increasing requirement for system performance in the modern world. This book has presented some of the recent research work of the authors along with associated fundamental knowledge in the area of variable structure control. There is great interests in the area of variable structure control, as high robustness are pursued by engineers working in a wide variety of application areas. The book has included various feedback framework including static output feedback control design, dynamical output feedback control design and reduced-order compensator based feedback control for complex systems. Both time delay dependent and independent control schemes have been presented for complex systems in the presence of time delays. Centralised control for nonlinear system and decentralised control for interconnected systems have been considered. Sliding mode observer based fault detection and isolation strategies have been discussed as well. Many examples and case studies with simulations have been provided to demonstrate the theoretical results, which also help readers to understand the theoretical results provided in this book.

This book focused on enhancing the robustness against uncertainties and reducing conservatism of the obtained theoretical results. All uncertainties considered in this book are nonlinear and bounded by nonlinear functions of system states and/or delayed states, or outputs and/or delayed outputs. This is in comparison with the other relevant work in which it is required that bounds on uncertainties satisfy linear growth condition [80, 89, 118, 139, 196, 210]. Both static and dynamic output feedback controllers are designed to stabilse complex control systems: the former is convenient for practical design but the developed results are usually conservative; the latter usually results in low conservatism but it require more resources in real implementation. All time delays involved in this book are time-varying, and Razumikhin Lyapunov approach is employed to deal with time delay. There is no limitation to the change rate of the time delay. The reconstruction/estimation for both system fault and sensor fault is considered using sliding mode observers. The results presented in this book are based on rigorous underpinning theory, but with wide practical applications.

It should be pointed out that nearly all of the designed controllers in this book are variable structure which usually results in discontinuous systems and thus chattering may occur. Chattering is usually harmful because it leads to low control accuracy and high wear of moving mechanical parts although lots of chattering is tolerable for many systems in reality. In order to overcome/attenuate chattering, the boundary layer approach was proposed in [14], which is at the cost of control accuracy. The other choice is to apply higher order sliding mode techniques which achieves finite time convergency and yields continuous closed-loop systems [5, 105, 10, 162]. This problem was not considered in this book.

Since systems considered in this book are complex and all developed results are mathematically rigorous, the proposed control schemes and fault detection and isolation strategies are complex and thus may be hard to implement in real systems like many other theoretical results. How to implement the various theoretical control schemes presented in this book in practical systems will be full of challenges for researchers and control engineers. Even from theoretical point of view, study on complex systems are far away from mature. All of the existing results are for a limited class of complex nonlinear systems and nearly all of obtained conditions are sufficient. Thus it is always related to the conservative levels: how large of the class of systems and how conservative the conditions. It should be noted that for many known nonlinear unforced systems, it is very difficult or even impossible to know whether the nonlinear system is stable or not, without saying the stabilisation problem of nonlinear control systems with uncertainties, delay and/or coupling. It is worth noting that this book, like most of existing efforts on complex systems, focused on reduction of conservatism or enhancement of robustness provided that the nominal systems have desired performance or assume that the controllers/observers have been well designed for nominal systems.

In the real world, there are myriad phenomena which need to discover and explore. Thus complex models are required to describe various phenomena, which will increase the complexity of research. It is impossible to find a systematic way to study all of complex systems like linear systems. Recall, at the beginning of the book, we mentioned, from general point of view, that nonlinearity, uncertainty/modeling error, time delay and interconnection are sources of complexity. Now, we conclude the book by providing a few specific examples and remarks to help readers to further understand complexity caused by these sources, and offer suggestions for possible future work.

Nonlinearity is one of the main characteristics of complex systems, which has been involved throughout the book. Systems studied in this book are either nonlinear or have nonlinear uncertainty (bounded by nonlinear functions). The behavior of a nonlinear system is usually very hard to predict or control even for a specific nonlinear system. Instead of study nonlinear system itself, the book has focused on developing less conservative results to tolerater/reject the effects of uncertainties by using possible available information of the uncertainties such that the systems considered have desired performance even in the presence of uncertainties provided that the nominal systems have the desired performance. Although study on linear systems has been very mature, many ideas/results for linear systems cannot be extended to nonlinear cases. In connection with this, the following simple example is provided.

Example 10.1. It is well known that a simple linear system

$$\dot{x} = Ax \tag{10.1}$$

where $x \in \mathbb{R}^n$ is system state and $A \in \mathbb{R}^{n\times n}$ is a constant matrix, is asymptotically stable if all the eigenvalues of the matrix A lie in the left half plan. However, this is not true for nonlinear systems. Consider the following 2nd-order nonlinear system

$$\dot{x}(t) = \underbrace{\begin{bmatrix} -1 & \frac{1}{x_2^2(t)} \\ 0 & -1 \end{bmatrix}}_{A(x)} x(t) \tag{10.2}$$

where $x = \mathrm{col}(x_1, x_2) \in \mathbb{R}^2$ is system state and the initial condition x_0 is given by

$$x_0 := \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix} \tag{10.3}$$

It is clear to see that both the eigenvalues of the matrix $A(x)$ in (10.2) are negative for any $x \in \mathbb{R}^2 \setminus \{0\}$. However, the solution to the equation (10.1) with respect to the initial condition x_0 is not stable.

Actually, it is easy to find that $x_2 = \frac{1}{2}e^{-t}$. Then using the integrating factor approach, it follows that $x_1 = e^t$. Therefore, the solution to system (10.1) with initial condition $x_0 = \mathrm{col}(1, \frac{1}{2})$ is

$$\begin{aligned} x_1(t) &= e^t \\ x_2(t) &= \frac{1}{2}e^{-t} \end{aligned}$$

which is not stable.

Remark 10.1. The example 10.1 shows that a nonlinear system

$$\dot{x} = A(x)x \tag{10.4}$$

where $x \in \mathbb{R}^n$ and $A(x) \in \mathbb{R}^{n\times n}$, may not be stable even if all the eigenvalues of the matrix $A(x)$ are negative in the considered domain. In order to guarantee the stability of nonlinear system (10.4), extra conditions are required. Detailed discussion is available in [3]. This is true for linear time varying systems as well, that is, a time varying system $\dot{x}(t) = A(t)x(t)$ may not be asymptotically stable even if for any $t \in \mathbb{R}$, all the real parts of the eigenvalues of matrix $A(t)$ lie on the open left half plane. Like the well known modern differential geometric approach for nonlinear systems proposed by Isidori a few decades ago [84], to explore new tools to study nonlinear control systems would be interesting and challenging.

An interconnected system can be considered as a system composed of many lower order subsystems interacted with each other, for which decentralised strategies are preferred. It is well known that even if all the isolated subsystems are stable/controllable/observable, the whole interconnected systems may not be stable/controllable/observable, which implies that interconnections affect performance of the whole interconnected systems. This book has shown that if the interconnections or the bounds on uncertain interconnections have 'superposition' property, their effects can be reduced/cancelled by design a proper sliding mode controller even if only decentralised scheme is employed. To deal with interconnections between the isolated subsystems is one of the main tasks for an interconnected system specifically when decentralised strategies are are considered. The following example shows how much interconnection terms affects the whole system performance.

Example 10.2. Consider the following special nonlinear interconnected systems

$$\dot{x}_1 = f(x_1) + \psi(x_1, x_2) \tag{10.5}$$

$$\dot{x}_2 = Ax_2 + Bu \tag{10.6}$$

where $x = \text{col}(x_1, x_2)$ with $x_1 \in \mathbb{R}^{n_1}$ and $x_2 \in \mathbb{R}^{n_2}$, and $u \in \mathbb{R}^m$ are system states and control respectively, the matrices A and B are constant with appropriate dimensions, and the term $\psi(\cdot)$ satisfies $\psi(x_1, 0) = 0$.

System (10.5)–(10.6) given in Example 10.2 can be considered as an interconnected system consisting of two subsystems where the interconnection exists in only the 1-st subsystem which is the term $\psi(x_1, x_2)$ in (10.5). The study in [155] disclosed that even if the subsystem

$$\dot{x}_1 = f(x_1)$$

is globally asymptotically stable and the matrix pair (A, B) is stabilisable, it is the interconnection term $\psi(x_1, x_2)$ determines whether the whole system (10.5)–(10.6) is stabilisable or not.

Remark 10.2. The Example 10.2 shows that interconnections not only affect the whole system performance but sometimes they may dominate whole interconnected system performance. This clearly demonstrates that the interconnections between subsystems greatly increase the complexity of the research. How to employ the structure and the possible known information of the interconnection terms to design decentralised controllers to reduce/reject effects of interconnections on the whole system is always significant for complex interconnected systems.

Uncertainties in a control system may destroy the system performance completely. A stable controlled system may become unstable if there is an uncertainty added to the system. To enhance performance of a control system, it is necessary to consider uncertainties experienced by the system when controllers are designed. This book has considered various uncertainties and controllers have been designed to reduce/reject the effects of the uncertainties when their bounds are known. The following example shows that an asymptotically stable controlled system will become unstable even if a "small" uncertainty is added to the controlled system.

Example 10.3. Consider the following simple control system

$$\dot{x} = f(x) + u \tag{10.7}$$

where $x \in \mathbb{R}$ and $u \in \mathbb{R}$ are system state and control respectively, and $f(x)$ is continuous in $\mathbb{R}$. Assume $x_0 = x(0)$ represents the initial condition.

The scalar system (10.7) can have any desired performance by designing an appropriate controller. It is straight forward to see that system (10.7) is globally stabilised by the controller, for example,

$$u = -x - f(x) \tag{10.8}$$

Now, consider the system

$$\dot{x} = f(x) + u + 2x^2 e^{-t}, \qquad x_0 = 2 \tag{10.9}$$

where $x_0 = 2$ is the initial condition. System (10.9) can be considered as a system by adding a nonlinear term $2x^2e^{-t}$ to the system (10.7) which can be considered as a disturbance on system (10.7). The term $2x^2e^{-t}$ has the following properties:

- It is vanished at the origin $x = 0$;
- It includes an exponentially damping factor e^{-t}.

However, system (10.9) cannot be stabilised by the controller (10.8) any more. Actually the corresponding closed-loop system by applying controller (10.8) to system (10.9) is

$$\dot{x} = -x + 2x^2 e^{-t}, \qquad x(0) = 2 \tag{10.10}$$

Letting $z = 1/x$, the system (10.10) can be expressed as a standard 1-st order linear differential equation. Then, using the integrating factor method, the solution of system (10.10) is given by

$$x = \frac{2}{-e^t + 2e^{-t}} \tag{10.11}$$

It is clear to see that $x(t) \to \infty$ when $t \to \frac{1}{2}\ln 2$ and thus it is not stable.

Remark 10.3. The example above shows that a "small" uncertainty may destroy system performance greatly. The corresponding relevant examples are available in [96]. This book has provided many results to deal with various uncertainties using bounds on uncertainties to enhance robustness. If bounds on the uncertainties are not available, some other approaches may be required to identify/estimate the bounds on uncertainties [58, 141].

Time delay widely exists in reality. It should be noted that sometimes even a small delay may greatly affect the performance of a system; a stable system may become unstable, or chaotic behaviour may appear due to delay in the system [137]. Time delay usually results in unpredictable results and thus increases the complexity of the research. This book has involved both delay dependent and delay independent control design. Delay dependent control in this book needs time delay is known which can be used in the design and thus the obtained results are usually less conservative when compared with delay independent control. However, delay independent control can be applied to the case when delay is unknown. The following example shows that a stable controlled system will become unstable if there is a delay in the input channel.

Example 10.4. Consider the 2nd order nonlinear control system

$$\dot{x}_1 = x_1 + x_1^4 x_2 \tag{10.12}$$

$$\dot{x}_2 = u(t) \tag{10.13}$$

where $\mathrm{col}(x_1, x_2) \in \mathbb{R}^2$ is state and $u \in \mathbb{R}$ is input. It is easy to check, using Lyapunov function $V = x_1^2 + x_2^2$, that the system (10.12)–(10.13) is stabilisable by feedback

$$u = -x_2 - x_1^5 \tag{10.14}$$

However, if the input has a constant delay $\tau > 0$, then system (10.12)–(10.13) is changed to the following time delay systems

$$\dot{x}_1 = x_1 + x_1^4 x_2 \tag{10.15}$$

$$\dot{x}_2 = u(t - \tau) \tag{10.16}$$

It is shown in [132] that the closed-loop system formed by applying control (10.14) to system (10.15)–(10.16) is not globally asymptotically stable.

Remark 10.4. In this book, only state delay is considered, and both delay dependent and delay independent results have been provided. However, input delay and output delay were not considered in this book. The example 10.4 shows that a delay in the input channel may destroy a controlled system performance. It will be interesting to study complex systems in the presence of input delay and/or output delay based on the skills and knowledge provided in this book in the future.

It should be noted that there are many sources which may result in complexity in control systems and only a few of them have been considered in this book. The examples and remarks have shown that nonlinearities, uncertainties/disturbances, time delay and interconnections, indeed, make behavior of systems very hard to predict and increase the complexity of the research greatly. However, in order to describe various phenomena existing in the real world, and also satisfy the increasing requirement for system performance, it is necessary to consider complex systems from both theoretical research and practical application points of view. It is helpful and feasible to build a research framework for a class of complex systems. The study on complex systems will be a long term task for control researchers and engineers.