Chapter 9
Applications of Decentralised Sliding Mode Control to Multimachine Power Systems

In this chapter, a robust stabilization problem for multimachine power systems is considered using only output information. The power system is formed from an interconnected set of lower order systems through a network transmission which is nonlinear and has an associated nonlinear bound. Under some mild conditions, a decentralized sliding mode control scheme is developed. Simulation results for a 3-machine power system are presented to show the effectiveness of the proposed method.

9.1 Introduction

With the development of scientific technology, the demand for electrical energy has increased greatly. Various complex power systems have been built to satisfy this demand. These systems are often modelled as dynamic equations composed of the interconnection of a set of lower-dimensional subsystems through a network transmission.

The complexity of the multimachine power system comes from its high dimensionality (if there are more generators), strong nonlinearity (each motor behaves nonlinearly) and strong interconnection between the subsystems (all the generators usually interact with each other), which makes traditional linear centralized control schemes difficult and sometimes impossible to implement. In fact, multimachine power systems are often widely distributed in space, and thus the information transfer among subsystems may be very difficult due to high cost, or even impossible due to practical limitations. These factors motivate the development of decentralized control which can avoid such shortcomings.

Power systems have played an important role in the practical world and many stabilizing control schemes have been proposed for such systems. In [115], using modern geometric methods, Lu and Sun proposed a nonlinear control scheme for a multimachine power system. However, the approach is based on a mathematical model with fixed structure and without uncertainty. Wang *et al* [193] studied a class of single machine systems, which was later extended to multimachine power systems in [192].

Decentralized control is an effective approach for the control of large-scale interconnected systems (see, for example [221, 213]), and many authors have successfully applied these techniques to multimachine power systems. Based on estimated states, a decentralized control strategy is presented for multimachine power systems in [20]. Recently, robust decentralized controllers have been designed for multimachine systems in [71] exploiting the systems lower-triangular structure. In [71], however, parametric uncertainty is not considered and only matched interconnections are dealt with. Xie *et al* [202] have developed a control scheme to deal with parametric uncertainty using LMIs. However, it should be pointed out that in all these results it is required that the interconnections are bounded by linear functions of the norm of the system state. Furthermore the uncertainty structure is not used in the control design, which may result in unnecessary conservatism. All the results mentioned above [20, 71, 202] are state variable based.

However, usually, all the system state variables are not fully available. Sometimes it may be possible to use an observer to estimate unknown states, but unfortunately, this approach not only requires more hardware resources, but also makes the dimension of the corresponding closed-loop system increase greatly. This may cause further difficulties, especially for large-scale power systems and thus it should be avoided if possible. Therefore, it is pertinent to study decentralized control for multimachine power systems using static output feedback.

In this chapter, as in previous work [202, 114], only the excitation control problem for multimachine power systems is considered. Not only are nonlinear interconnections considered, parametric disturbances are dealt with as well. Furthermore, the interconnections are allowed to be nonlinear and have nonlinear bounds. Mismatched uncertain interconnections are dealt with and parametric uncertainties in the direct axis transient short circuit time constants, which affect the subsystem input distribution matrix, are considered. By using the approach outlined in Section 2.5, an output sliding surface is synthesized which has stable sliding dynamics when the system is restricted to the surface. The approach used in this chapter is practical when compared with previous theoretical output feedback sliding mode control strategies which require some strong geometric conditions on the nominal subsystems. A robust decentralized sliding mode controller is proposed, using only system output information, such that the system can reach the sliding surface in a finite time. Robustness is enhanced by using the sliding mode technique and conservatism is reduced by fully using system output information and the available structure of the uncertainties.

The proposed approach can deal with interconnection terms and parametric disturbances with large magnitude. It also allows significant nonlinearity to be present in the interconnection terms. Furthermore, the obtained results hold in a large region of the origin if the control gain is high enough. This allows the operating point of the multimachine power system to vary to satisfy different load demands. Finally, simulation results of a three-machine power system are presented to illustrate the control scheme.

9.2 Dynamical Model for Multimachine Power Systems

The exciter is one of the main control systems which directly affect the performance of multimachine power systems. It can be approximately depicted by Figure 9.1.

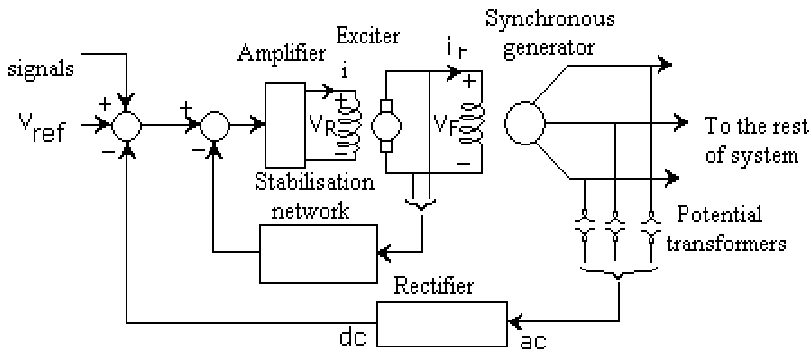

Fig. 9.1 Excitation System

The classical model of power systems was given by Bergen [9] (see e.g. Section 1.5.5). Based on the model in Bergen [9], multimachine power systems consisting of N synchronous generators interconnected through a transmission network can be modelled, as in [192, 71, 202, 114], by:

$$\dot{x}_i = (A_i + \Delta A_i)x_i + (B_i + \Delta B_i)v_{fi} + M_i(x) + \Delta M_i(x) \tag{9.1}$$

$$y_i = C_i x_i, \qquad i = 1, 2, \ldots, N \tag{9.2}$$

where $x = \mathrm{col}(x_1, x_2, \cdots, x_N)$ with

$$x_i = \mathrm{col}(x_{i1}, x_{i2}, x_{i3}) := \mathrm{col}\big(\delta_i - \delta_{i0}, \omega_i, P_{ei} - P_{mi0}\big)$$

for $i = 1, 2, \ldots, N$; $v_{fi} \in \mathbb{R}$ and $y_i \in \mathbb{R}^{p_i}$ are the input and the output of the i-th subsystems respectively; $C_i \in \mathbb{R}^{p_i \times 3}$ with $p_i \leq 3$ is the system output matrix ; $M_i(x)$ is the interconnection term; and $\Delta M_i(x)$ includes the network transmission disturbance, the torque disturbance acting on the rotating shaft, the electromagnetic disturbances entering the excitation winding and other unstructural uncertainties.

The nominal system and input distribution matrices are

$$A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{D_i}{2H_i} & -\frac{\omega_0}{2H_i} \\ 0 & 0 & -\frac{1}{T'_{doi}} \end{bmatrix}, \qquad B_i = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T'_{doi}} \end{bmatrix} \tag{9.3}$$

The uncertainty is described by

$$\Delta A_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \theta_i \end{bmatrix}, \qquad \Delta B_i = \begin{bmatrix} 0 \\ 0 \\ -\theta_i \end{bmatrix} \tag{9.4}$$

where

$$\theta_i = \frac{1}{T'_{doi}} - \frac{1}{T'_{doi} + \Delta T'_{doi}} \tag{9.5}$$

The interconnection term is given as

$$M_i(x) = \begin{bmatrix} 0 \\ 0 \\ \Phi_i(x) \end{bmatrix} \tag{9.6}$$

where

$$\Phi_i(x) = E'_{qi} \sum_{j=1}^{N} \dot{E}'_{qj} B_{ij} \sin(\delta_i - \delta_j) - E'_{qi} \sum_{j=1}^{N} E'_{qj} B_{ij} \cos(\delta_i - \delta_j)\omega_j, \tag{9.7}$$

The input control variables are

$$v_{fi} = I_{qi} K_{ci} u_{fi} - (x_{di} - x'_{di}) I_{qi} I_{di} - P_{mi0} - T'_{doi} Q_{ei} \omega_i \tag{9.8}$$

where u_{fi} is the actual input of the amplifier of the i-th generator for $i = 1, 2, \ldots, N$. The physical meanings of all the symbols used above are shown in Appendix E.1.

In this work, $P_{mi} = P_{mi0} = constant$ since only excitation control is considered. It should be noted that direct feedback linearisation compensation for the power system representation has been used to obtain the system model (9.1)–(9.2) as described in [192]. The feedback transformation (9.8) is nonsingular since $I_{qi} K_{ci} \neq 0$ for a generator working in the normal region.

From the work in [71]:

$$|\Phi_i(x)| \leq \sum_{j=1}^{N} \left(\gamma^{I}_{ij} |\sin\delta_j| + \gamma^{II}_{ij} |\omega_j| \right), \tag{9.9}$$

where the constants γ^{I}_{ij} and γ^{II}_{ij} are defined by

$$\gamma^{I}_{ij} = \frac{4}{|T'_{doj}|_{min}} |P_{ei}|_{max} \tag{9.10}$$

$$\gamma^{II}_{ij} = |Q_{ei}|_{max} \tag{9.11}$$

Therefore, for $i = 1, 2, \cdots, N$

$$\|M_i(x)\| = |\Phi_i(x)| \leq \sum_{j=1}^{N} \left(\gamma^{I}_{ij} |\sin x_{j1}| + \gamma^{II}_{ij} |x_{j2}| \right). \tag{9.12}$$

Remark 9.1. From (9.7) and (9.12) it is observed that the interconnections $M_i(x)$ are nonlinear and their bounds also take nonlinear forms instead of constants as in the work described in [71]. By using the nonlinear bounds, a control scheme with less conservatism will be presented.

9.3 Sliding Motion Analysis and Control Design

In this section, a sliding surface will be synthesized using the approach proposed by Edwards and Spurgeon [39, 40]. Then, under some mild conditions, the stability of the sliding mode dynamics is analysed and a decentralized output feedback sliding mode control strategy is proposed to guarantee that the system (9.1)–(9.2) can reach the sliding surface in finite time and remain on it thereafter.

9.3.1 Basic assumptions

In this section, some basic assumptions are imposed on the system (9.1)–(9.2).

Assumption 9.1. The matrices C_i and B_i satisfy $C_iB_i \neq 0$ for $i = 1,2,\ldots,N$.

From Section 2.6, it follows that Assumption 9.1 implies that there exists a nonsingular linear coordinate transformation such that the triple (A_i, B_i, C_i) with respect to the new coordinates has the structure

$$\widetilde{A}_i = \begin{bmatrix} \widetilde{A}_{i1} & \widetilde{A}_{i2} \\ \widetilde{A}_{i3} & \widetilde{A}_{i4} \end{bmatrix}, \qquad \widetilde{B}_i = \begin{bmatrix} 0 \\ \widetilde{b}_i \end{bmatrix}, \qquad \widetilde{C}_i = \begin{bmatrix} 0 & \widetilde{C}_{i2} \end{bmatrix} \tag{9.13}$$

where $\widetilde{A}_{i1} \in \mathbb{R}^{2\times 2}$, $\widetilde{b}_i \in \mathbb{R}$ and $\widetilde{C}_{i2} \in \mathbb{R}^{p_i\times p_i}$ for $i = 1,2,\ldots,N$. Furthermore $\widetilde{b}_i \neq 0$ and $\det(\widetilde{C}_{i2}) \neq 0$.

Assumption 9.2. The triple $(\widetilde{A}_{i1}, \widetilde{A}_{i2}, \Xi_i)$ is output feedback stabilisable where the matrix pair $(\widetilde{A}_{i1}, \widetilde{A}_{i2})$ is given by (9.13) and the matrix $\Xi_i =: \begin{bmatrix} 0_{(p_i-1)\times(n_i-p_i)} & I_{p_i-1} \end{bmatrix}$ for $i = 1,2,\ldots,N$.

Under Assumptions 9.1 and 9.2, Edwards and Spurgeon [39, 40] show that there exists a coordinate transformation $x_i \mapsto z_i = T_ix_i$ where

$$T_i = \begin{bmatrix} I & 0 \\ -K_i\Xi_i & I \end{bmatrix}$$

such that in the new coordinates system (A_i, B_i, C_i) has the following structure

$$\begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix}, \quad \begin{bmatrix} 0 \\ b_i \end{bmatrix}, \quad \begin{bmatrix} 0 & C_{i2} \end{bmatrix} \tag{9.14}$$

where $A_{i1} = \widetilde{A}_{i1} - \widetilde{A}_{i2} K_i \Xi_i$ is stable, $b_i \neq 0$ and $C_{i2} \in \mathbb{R}^{p_i \times p_i}$ is nonsingular.

Remark 9.2. Assumptions 9.1 and 9.2 are limitations on the isolated nominal subsystems. They ensure the existence of the output sliding surface. Notably, Assumption 9.2 requires $(\widetilde{A}_{i1}, \widetilde{A}_{i2}, \Xi_i)$ instead of (A_i, B_i, C_i) to be output feedback stabilisable. This is in contrast with other output feedback control results for interconnected systems (see, for example [205, 221]). It should be emphasized that all the matrices in (9.13) and (9.14) can be obtained directly from (A_i, B_i, C_i) using the algorithm given in [39, 40].

Assumption 9.3. There exist positive constants $\alpha_i < 1$ and known continuous functions $\beta_{ij}(x_j)$ such that

$$|T'_{doi}\theta_i| \leq \alpha_i \tag{9.15}$$

$$\|\Delta M_i(x)\| \leq \sum_{j=1}^{N} \beta_{ij}(x_j)\|x_j\|. \tag{9.16}$$

for $i,\, j = 1, 2, \ldots, N$.

Remark 9.3. Assumption 9.3 is a limitation on the uncertainties that can be tolerated by the system. From the work in [202, 114], these assumptions are fundamental and reasonable. The structural requirement on the interconnection bounds in (9.16) is not essential because it can be easily extended to a more general case (see for example [222]).

9.3.2 Stability of sliding motion

Based on the assumptions above, the stability of the sliding mode is analysed in this section. Suppose Assumptions 9.1 and 9.2 are satisfied. From Section 2.6, there exist matrices

$$F_i = \begin{bmatrix} K_1 & 1 \end{bmatrix} \widetilde{C}_{i2}^{-1} \tag{9.17}$$

such that for $i = 1, 2, \ldots, N$ the system

$$\dot{x}_i = A_i x_i + B_i v_{fi}$$

when restricted to

$$F_i C_i x_i = 0$$

is stable, where $F_i C_i x_i = 0$ is called the *switching surface*. Consider the composite sliding surface for the interconnected system (9.1)–(9.2) as

$$S(x) = 0 \tag{9.18}$$

with $S(x) =: \mathrm{col}(S_1(x_1), S_2(x_2), \ldots S_N(x_N))$ and

$$S_i(x_i) = F_i C_i x_i = F_i y_i \tag{9.19}$$

where the F_i can be obtained from the algorithm given in [39, 40]. Next the stability of the system (9.1)–(9.2) when restricted to the sliding surface (9.18) will be considered.

From the structure of ΔA_i in (9.4), it follows that

$$T_i \Delta A_i T_i^{-1} z_i = T_i \begin{bmatrix} 0 \\ 0 \\ \theta_i x_{i3} \end{bmatrix} = (T_i B_i) T'_{doi} \theta_i (P_{ei} - P_{mi0}). \tag{9.20}$$

In the new coordinates $z = \mathrm{col}(z_1, z_2, \ldots, z_N)$, system (9.1)–(9.2) has the following form

$$\begin{aligned} \dot{z}_i = & \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix} z_i + \begin{bmatrix} 0 \\ b_i \end{bmatrix} \Big((1 - T'_{doi}\theta_i) v_{fi} + \theta_i T'_{doi} (P_{ei} - P_{mi0}) \\ & + T'_{doi} \Phi_i(x) \Big) + T_i \Delta M_i(x) \end{aligned} \tag{9.21}$$

$$y_i = \begin{bmatrix} 0 & C_{i2} \end{bmatrix} z_i, \qquad i = 1, 2, \ldots, N, \tag{9.22}$$

where A_{i1} is stable, $b_i \neq 0$ and $C_{i2} \in \mathbb{R}^{p_i \times p_i}$ is nonsingular with

$$F_i \begin{bmatrix} 0_{p_i \times (n_i - p_i)} & C_{i2} \end{bmatrix} = \begin{bmatrix} 0_{1\times 2} & f_i \end{bmatrix} \tag{9.23}$$

where $f_i \neq 0$ is a real constant.

Since A_{i1} is stable for $i = 1, \ldots N$, for any $\Lambda_i > 0$, the following Lyapunov equation has a unique solution $\Pi_i > 0$ such that

$$A_{i1}^{\tau} \Pi_i + \Pi_i A_{i1} = -\Lambda_i, \qquad i = 1, 2, \ldots, N. \tag{9.24}$$

For convenience, partition

$$T_i =: \begin{bmatrix} T_{i1} \\ T_{i2} \end{bmatrix}, \qquad T_i^{-1} =: \begin{bmatrix} W_{i1} & W_{i2} \end{bmatrix} \tag{9.25}$$

where $T_{i1} \in \mathbb{R}^{2\times 3}$ and $W_{i1} \in \mathbb{R}^{3\times 2}$. Then, system (9.21)–(9.22) can be rewritten as

$$\dot{z}_{i1} = A_{i1} z_{i1} + A_{i2} z_{i2} + T_{i1} \Delta M_i(T^{-1} z) \tag{9.26}$$

$$\begin{aligned} \dot{z}_{i2} = & A_{i3} z_{i1} + A_{i4} z_{i2} + (1 - T'_{doi}\theta_i) v_{fi} + \theta_i T'_{doi} \Delta P_{ei} \\ & + T'_{doi} \Phi_i(x) + T_{i2} \Delta M_i(T^{-1} z) \end{aligned} \tag{9.27}$$

$$y_i = \begin{bmatrix} 0 & C_{i2} \end{bmatrix} z_i, \qquad i = 1, 2, \ldots, N \tag{9.28}$$

where $T^{-1} =: \mathrm{diag}\{T_1^{-1}, T_2^{-1}, \ldots, T_N^{-1}\}$, $z_{i1} \in \mathbb{R}^2$ and $z_{i2} \in \mathbb{R}$. Now, consider the sliding surface (9.19) in the new coordinate system. From (9.23),

$$F_i \begin{bmatrix} 0 & C_{i2} \end{bmatrix} z_i = f_i z_{i2}$$

and since $f_i \neq 0$ it follows that the sliding surface (9.18) becomes

$$z_{i2} = 0, \qquad i = 1, 2, \ldots, N. \tag{9.29}$$

When system (9.26)-(9.28) is restricted to the sliding surface (9.29), it has the following form

$$\dot{z}_{i1} = A_{i1} z_{i1} + T_{i1} \Delta M_i (W z_1), \qquad i = 1, 2, \ldots, N \tag{9.30}$$

where $z_1 =: \text{col}(z_{11}, 0, z_{21}, 0, \ldots, z_{N1}, 0)$, and $W =: \text{diag}\{W_{11}, 0, W_{21}, 0, \ldots, W_{N1}, 0\}$.

Theorem 9.1. *For system (9.1)–(9.2), suppose Assumptions 9.1-9.3 are satisfied. Then, the sliding mode is asymptotically stable if there exists a domain* $\Omega \subset \mathbb{R}^{N \times (n-m)}$ *including the origin, such that*

$$L^\tau + L > 0$$

in $\Omega \setminus \{0\}$ *where* $L \in \mathbb{R}^{N \times N}$ *is given element-wise by*

$$L_{ij} = \begin{cases} \lambda_{min}(\Lambda_i) - 2\|\Pi_i T_{i1}\| \, \|W_{i1}\| \beta_{ii}(W_{i1} z_{i1}, 0), & i = j \\ -2\|\Pi_i T_{i1}\| \, \|W_{j1}\| \beta_{ij}(W_{j1} z_{j1}, 0), & i \neq j \end{cases}$$

where Π_i *and* Λ_i *are defined in (9.24), and* $\lambda_{min}(\cdot)$ *denotes the minimum eigenvalue of the matrix for* $i, j = 1, 2 \ldots, N$.

Proof: From the analysis above, all that needs to be proved is that system (9.30) is asymptotically stable. For system (9.30), consider the Lyapunov function candidate

$$V = \sum_{i=1}^{N} (z_{i1})^\tau \Pi_i z_{i1}$$

The time derivative of V along the trajectories of system (9.30) is given by

$$\dot{V}\,|_{(9.30)} = \sum_{i=1}^{N} \Big\{ -(z_{i1})^\tau \Lambda_i z_{i1} + 2 (z_{i1})^\tau \Pi_i T_{i1} \Delta M_i (W z_1) \Big\} \tag{9.31}$$

where (9.24) is used to obtain the first term in the bracket. From Assumption 9.3

$$\begin{aligned} \dot{V} &\leq \sum_{i=1}^{N} \Big\{ -\lambda_{min}(\Lambda_i) \|z_{i1}\|^2 + 2\|z_{i1}\| \, \|\Pi_i T_{i1}\| \sum_{j=1}^{N} \|\Delta M_i (W z_1)\| \Big\} \\ &\leq -\sum_{i=1}^{N} \lambda_{min}(\Lambda_i) \|z_{i1}\|^2 + 2\sum_{i=1}^{N} \Big\{ \|z_{i1}\| \, \|\Pi_i T_{i1}\| \sum_{j=1}^{N} \beta_{ij}(W_{j1} z_{j1}, 0) \|W_{j1}\| \, \|z_{j1}\| \Big\} \end{aligned}$$

$$= -\sum_{i=1}^{N}\Big\{\lambda_{min}(\Lambda_i) - 2\beta_{ii}(W_{i1}z_{i1},0)\|\Pi_i T_{i1}\|\,\|W_{i1}\|\Big\}\|z_{i1}\|^2$$
$$+2\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\beta_{ij}(W_{j1}z_{j1},0)\|\Pi_i T_{i1}\|\,\|W_{j1}\|\,\|z_{i1}\|\,\|z_{j1}\|$$
$$= -\frac{1}{2}\left[\|z_{11}\|\;\|z_{21}\|\;\cdots\;\|z_{N1}\|\right](L^\tau + L)\begin{bmatrix}\|z_{11}\|\\ \|z_{21}\|\\ \vdots\\ \|z_{N1}\|\end{bmatrix} \tag{9.32}$$

Then, the conclusion follows since $L^\tau + L > 0$ for $\mathrm{col}(z_{11}, z_{21}, \ldots, z_{N1}) \in \Omega \setminus \{0\}$. #

It should be emphasised that in Theorem 9.1, $L^\tau + L > 0$ only depends on the partial state variables z_{i1} instead of the entire state variables z_i (actually x_i). This is in contrast with the work [221, 205, 222]. As such, this result is less conservative.

Theorem 9.1 presents a condition under which the sliding mode dynamics is asymptotically stable. The next objective is to design a decentralized output feedback sliding mode control law such that the system state is driven to and maintained on the sliding surface.

9.3.3 Sliding mode control synthesis

Traditionally, the reachability condition (see for example [40, 182]) is described by

$$S^\tau(t)\dot{S}(t) < 0$$

for small scale systems with switching function $S(t)$. However, for the interconnected system (9.1)–(9.2), the corresponding condition is described by

$$\sum_{i=1}^{N}\frac{S_i^\tau(x_i)\dot{S}_i(x_i)}{\|S_i(x_i)\|} < 0 \tag{9.33}$$

where $S_i(x_i)$ is defined by (9.19). For details see [74]. This condition is called composite reachability condition for interconnected systems.

From (9.12) and (9.16), for $i = 1, 2, \ldots, N$

$$\|M_i(x) + \Delta M_i(x)\| \;\le\; \sum_{j=1}^{N}\left(\gamma_{ij}^{I}|\sin x_{j1}| + \gamma_{ij}^{II}|x_{j2}|\right) + \sum_{j=1}^{N}\beta_{ij}(x_j)\|x_j\|$$
$$=: \sum_{j=1}^{N}\eta_{ij}(x_j) \tag{9.34}$$

In order to fully use system output information, consider the output matrix C_i. Comparing system (9.1)–(9.2) with (9.21)–(9.22), it follows that

$$C_i = \begin{bmatrix} 0 & C_{i2} \end{bmatrix} T_i = C_{i2} \begin{bmatrix} 0 & I_{p_i} \end{bmatrix} T_i, \qquad i = 1,2,\ldots,N. \tag{9.35}$$

where C_{i2} is nonsingular and satisfies (9.23). Splitting $T_i x_i$ into two components $(T_i x_i)_1 \in \mathbb{R}^{(3-p_i)}$ and $(T_i x_i)_2$, it follows that

$$x_i = T_i^{-1} T_i x_i = T_i^{-1} \begin{bmatrix} (T_i x_i)_1 \\ (T_i x_i)_2 \end{bmatrix} = T_i^{-1} \begin{bmatrix} (T_i x_i)_1 \\ C_{i2}^{-1} y_i \end{bmatrix} \tag{9.36}$$

Further, let

$$F_i C_i A_i T_i^{-1} =: \begin{bmatrix} \Upsilon_{i1} & \Upsilon_{i2} \end{bmatrix} \tag{9.37}$$

$$F_i C_i \begin{bmatrix} 0 \\ T_{i,3}^{-1} \end{bmatrix} =: \begin{bmatrix} \Gamma_{i1} & \Gamma_{i2} \end{bmatrix} \tag{9.38}$$

where $T_{i,3}^{-1}$ denotes the 3rd row of the matrix T_i^{-1}, $\Upsilon_{i1} \in \mathbb{R}^{1\times(3-p_i)}$ and $\Gamma_{i1} \in \mathbb{R}^{1\times(3-p_i)}$ for $i = 1,2,\ldots,N$. Since

$$F_i C_i B_i = F_i C_i T_i^{-1} T_i B_i = F_i \begin{bmatrix} 0 \ C_{i2} \end{bmatrix} \begin{bmatrix} 0 \\ b_i \end{bmatrix} = \begin{bmatrix} 0 \ f_i \end{bmatrix} \begin{bmatrix} 0 \\ b_i \end{bmatrix} = f_i b_i$$

it follows that $F_i C_i B_i$ is nonsingular due to $f_i \neq 0$ and $b_i \neq 0$ for $i = 1,2,\ldots,N$.

The objective is to satisfy the composite reachability condition (9.33). Consider system (9.1)–(9.2) in the domain $\mathscr{D} =: \mathscr{D}_1 \times \mathscr{D}_2 \times \cdots \times \mathscr{D}_N$ where $\mathscr{D}_i \in \mathbb{R}^3$ and explicitly

$$\mathscr{D}_i =: \left\{ x_i \mid x_i \in \mathbb{R}^3, \quad \|(T_i x_i)_1\| \le \mu_i \right\}, \qquad i = 1,2,\ldots,N \tag{9.39}$$

for some positive constant μ_i.

Then, the following control law is proposed for $i = 1,2,\cdots,N$

$$v_{fi} = -\frac{1}{1-\alpha_i} (F_i C_i B_i)^{-1} \mathrm{sign}(F_i y_i) \left[\|\Upsilon_{i2} C_{i2}^{-1} y_i\| + \frac{\alpha_i}{T'_{doi}} \|\Gamma_{i2} C_{i2}^{-1} y_i\| + k_i(y_i) \right] \tag{9.40}$$

where sign$(\cdot)$ represents the signum function, F_i is defined by (9.19) and can be designed by the approach in [39, 40], α_i is determined by Assumption 9.3, and $k_i(y_i) \ge 0$ is a control gain to be designed later. Obviously, the control law (9.40) depends only on system outputs and is decentralized.

Theorem 9.2. *Consider the nonlinear interconnected system (9.1)–(9.2). Under Assumptions 9.1-9.3, the decentralized sliding mode control (9.40) drives the system (9.1)–(9.2) to the composite sliding surface (9.18) and maintains a sliding motion in the domain $\mathscr{D}$ if the control gain function $k_i(y_i)$ satisfies*

$$k_i(y_i) > \left(\|\Upsilon_{i1}\| + \frac{\alpha_i}{T'_{doi}} \|\Gamma_{i1}\| \right) \mu_i + \sum_{j=1}^{N} \|F_j C_j\| \eta_{ji}(x_i) \tag{9.41}$$

where F_i and η_{ji} are determined by (9.19) and (9.34) respectively for $i, j = 1, 2, \ldots, N$ and $\mathscr{D}$ is defined by (9.39).

Proof: It is necessary to prove that the composite reachability condition (9.33) is satisfied.

From (9.19), (9.37), (9.38) and the structures of B_i and ΔB_i, the sliding mode dynamics of the system (9.1)–(9.2) can be described by

$$\begin{aligned} \dot{S}_i(x_i) &= F_i C_i (A_i + \Delta A_i) x_i + F_i C_i (B_i + \Delta B_i) v_{fi} + F_i C_i \left[M_i(x) + \Delta M_i(x) \right] \\ &= (\Upsilon_{i1} + \theta_i \Gamma_{i1}) (T_i x_i)_1 + (\Upsilon_{i2} + \theta_i \Gamma_{i2}) C_{i2}^{-1} y_i + F_i C_i B_i \left(1 - \theta_i T'_{doi}\right) v_{fi} \\ &\quad + F_i C_i \left[M_i(x) + \Delta M_i(x) \right] \end{aligned} \tag{9.42}$$

for $i = 1, 2 \ldots, N$. Substituting (9.40) into (9.42), it follows that

$$\begin{aligned} &\sum_{i=1}^{N} \frac{S_i^\tau(x_i) \dot{S}_i(x_i)}{\|S_i(x_i)\|} \\ &= \sum_{i=1}^{N} \frac{(F_i y_i)^\tau}{\|F_i y_i\|} \Bigg\{ (\Upsilon_{i2} + \theta_i \Gamma_{i2}) C_{i2}^{-1} y_i - \frac{1 - \theta_i T'_{doi}}{1 - \alpha_i} \operatorname{sign}(F_i y_i) \Big(\|\Upsilon_{i2} C_{i2}^{-1} y_i\| \\ &\quad + \frac{\alpha_i}{T'_{doi}} \|\Gamma_{i2} C_{i2}^{-1} y_i\| \Big) + (\Upsilon_{i1} + \theta_i \Gamma_{i1}) (T_i x_i)_1 + F_i C_i \left[M_i(x) + \Delta M_i(x) \right] \\ &\quad - \frac{1 - \theta_i T'_{doi}}{1 - \alpha_i} \operatorname{sign}(F_i y_i) k_i(y_i) \Bigg\} \end{aligned} \tag{9.43}$$

From Assumption 9.3,

$$1 - \theta_i T'_{doi} \geq 1 - \left| \theta_i T'_{doi} \right| \geq 1 - \alpha_i > 0. \tag{9.44}$$

Then, for $i = 1, 2, \ldots, N$

$$
\begin{aligned}
&\frac{(F_i y_i)^\tau}{\|F_i y_i\|}\left\{(\Upsilon_{i2}+\theta_i\Gamma_{i2})\,C_{i2}^{-1}y_i-\frac{1-\theta_i T'_{doi}}{1-\alpha_i}\operatorname{sign}(F_i y_i)\Big(\|\Upsilon_{i2}C_{i2}^{-1}y_i\|\right.\\
&\left.+\frac{\alpha_i}{T'_{doi}}\|\Gamma_{i2}C_{i2}^{-1}y_i\|\Big)\right\}\\
&=\frac{(F_i y_i)^\tau}{\|F_i y_i\|}\left(\Upsilon_{i2}C_{i2}^{-1}y_i+\theta_i\Gamma_{i2}C_{i2}^{-1}y_i\right)-\frac{1-\theta_i T'_{doi}}{1-\alpha_i}\Big(\|\Upsilon_{i2}C_{i2}^{-1}y_i\|\\
&+\frac{\alpha_i}{T'_{doi}}\|\Gamma_{i2}C_{i2}^{-1}y_i\|\Big)\\
&\le\|\Upsilon_{i2}C_{i2}^{-1}y_i\|+|\theta_i|\,\|\Gamma_{i2}C_{i2}^{-1}y_i\|-\|\Upsilon_{i2}C_{i2}^{-1}y_i\|-\frac{\alpha_i}{T'_{doi}}\|\Gamma_{i2}C_{i2}^{-1}y_i\|\\
&=\left(|\theta_i|-\frac{\alpha_i}{T'_{doi}}\right)\|\Gamma_{i2}C_{i2}^{-1}y_i\|\\
&\le 0, \qquad (9.45)
\end{aligned}
$$

and from (9.34)

$$
\begin{aligned}
&\frac{(F_i y_i)^\tau}{\|F_i y_i\|}\left\{(\Upsilon_{i1}+\theta_i\Gamma_{i1})\,(T_i x_i)_1+F_iC_i\,[M_i(x)+\Delta M_i(x)]\right.\\
&\left.-\frac{1-\theta_i T'_{doi}}{1-\alpha_i}\operatorname{sign}(F_i y_i)k_i(y_i)\right\}\\
&=\frac{(F_i y_i)^\tau}{\|F_i y_i\|}\Big[(\Upsilon_{i1}+\theta_i\Gamma_{i1})\,(T_i x_i)_1+F_iC_i\,[M_i(x)+\Delta M_i(x)]\Big]-\frac{1-\theta_i T'_{doi}}{1-\alpha_i}k_i(y_i)\\
&\le\left(\|\Upsilon_{i1}\|+\frac{\alpha_i}{T'_{doi}}\|\Gamma_{i1}\|\right)\|(T_i x_i)_1\|+\|F_iC_i\|\,\|M_i(x)+\Delta M_i(x)\|-k_i(y_i)\\
&\le\left(\|\Upsilon_{i1}\|+\frac{\alpha_i}{T'_{doi}}\|\Gamma_{i1}\|\right)\|(T_i x_i)_1\|+\|F_iC_i\|\sum_{j=1}^{N}\eta_{ij}(x_j)-k_i(y_i) \qquad (9.46)
\end{aligned}
$$

where (9.44) is used to establish the first inequality.

Now, substituting (9.45) and (9.46) into (9.43), in the domain $\mathscr{D}$

$$
\begin{aligned}
&\sum_{i=1}^{N}\frac{S_i^\tau(x_i)\dot S_i(x_i)}{|S_i(x_i)|}\\
&\le\sum_{i=1}^{N}\left\{\Big(\|\Upsilon_{i1}\|+\frac{\alpha_i}{T'_{doi}}\|\Gamma_{i1}\|\Big)\mu_i+\|F_iC_i\|\sum_{j=1}^{N}\eta_{ij}(x_j)-k_i(y_i)\right\}\\
&=\sum_{i=1}^{N}\left\{\Big[\Big(\|\Upsilon_{i1}\|+\frac{\alpha_i}{T'_{doi}}\|\Gamma_{i1}\|\Big)\mu_i+\sum_{j=1}^{N}\|F_jC_j\|\eta_{ij}(x_i)\Big]-k_i(y_i)\right\}. \qquad (9.47)
\end{aligned}
$$

Then, if $k_i(y_i)$ is chosen to satisfy (9.41), it follows that in the domain $\mathscr{D}$

$$\sum_{i=1}^{N}\frac{S_i^\tau(x_i)\dot S_i(x_i)}{|S_i(x_i)|}<0.$$

Hence, the result follows. #

Remark 9.4. It should be noted that inequality (9.41) can be satisfied globally only in some specific cases. However, it can always be satisfied in the arbitrarily large domain $\mathscr{D}$ with $\mu_i < \infty$ for $i = 1,2,\dots,N$ if the control gain $k_i(y_i)$ is sufficiently high. In fact, one conservative choice of $k_i(y_i)$ is

$$k_i(y_i) > \left(\|\Upsilon_{i1}\| + \frac{\alpha_i}{T'_{doi}}\|\Gamma_{i1}\|\right)\mu_i + \sum_{j=1}^{N}\|F_jC_j\| \max_{x_i\in\mathscr{D}_i}\left\{\eta_{ji}\left((T_ix_i)_1, C_{i2}^{-1}y_i\right)\right\}$$

for $i = 1,2,\dots,N$.

Remark 9.5. From the analysis above, it is observed that there is no special requirement on the interconnections $M_i(x_i)$ for $i = 1,2,\dots,N$. Only their bounds are assumed to be known. This shows that the approach is applicable to the multimachine power system which has high nonlinearity and coupling.

Remark 9.6. From (9.8) and (9.40), the designed excitation control for the original multimachine power system is as follows

$$\begin{aligned} u_{fi} = &-\frac{1}{I_{qi}K_{ci}}\left[\frac{1}{1-\alpha_i}(F_iC_iB_i)^{-1}\text{sign}(F_iy_i)\Big(\|\Upsilon_{i2}C_{i2}^{-1}y_i\| \right.\\ &+\frac{\alpha_i}{T'_{doi}}\|\Gamma_{i2}C_{i2}^{-1}y_i\| + k_i(y_i)\Big) + (x_{di}-x'_{di})I_{qi}I_{di} + P_{mi0} \\ &\left. +T'_{doi}Q_{ei}\omega_i\right], \quad i = 1,2,\dots,N. \end{aligned} \tag{9.48}$$

Remark 9.7. According to sliding mode control theory, Theorems 9.1 and 9.2 show that the closed loop system resulting from the designed control law (9.48) and system (9.1)–(9.2) is asymptotically stable. Moreover, under Assumptions 9.1-9.3, the multimachine power system is globally stabilized by (9.48) if for $i, j = 1,2,\dots,N$,

i) $L^T + L > 0$ is satisfied globally;
ii) $\Upsilon_{i1} = 0$ and $\Gamma_{i1} = 0$;
iii) $\eta_{ij}(x_i)$ is bounded by a function of y_i.

9.4 Simulation on Three Machine Power Systems

Consider the three-machine power system shown in Figure 9.2 where the generator 3 is an infinite busbar being used as a reference.

This system is also called the two-machine infinite bus power system (see [71]). The simulation parameters listed in Appendix E.2 are chosen as in [71, 202]. Then it follows that

$$|P_{e1}|_{\max} = |Q_{e1}|_{\max} = 1.4, \qquad |P_{e2}|_{\max} = |Q_{e2}|_{\max} = 1.5$$
$$|T'_{do1}|_{\min} = 6.21\text{s}, \qquad |T'_{do2}|_{\min} = 7.614\text{s}$$

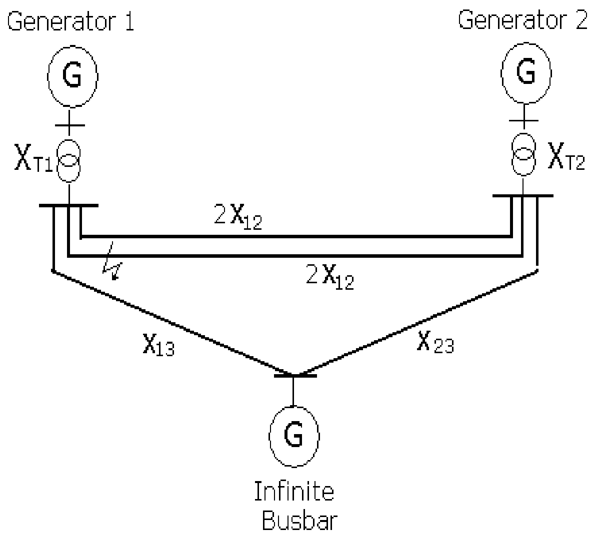

Fig. 9.2 A three-machine power system

As in [202], take

$$\Delta T'_{doi} = 0.1T'_{doi}$$

for $i = 1,2$. With the chosen value of $\Delta T'_{doi}$, it follows that equation (9.15) is satisfied for

$$\alpha_1 = \alpha_2 = 0.1$$

In addition, assume

$$\|\Delta M_1\| = \|\Delta M_2\| \leq (x_{13} - 0.0025x_{11})^2\|x_1\|^2 + 0.006\|x_2\|$$

Then, from (9.1)

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.625 & -39.27 \\ 0 & 0 & -0.1449 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 0.1449 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.2941 & -30.8 \\ 0 & 0 & -0.1256 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0.1256 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

where C_1 and C_2 are assumed to be the system output matrices.

Obviously, Assumption 9.1 is satisfied. Let

$$K_1 = -0.0025, \qquad K_2 = -0.0008$$

Then according to the algorithm given by Edwards and Spurgeon [39, 40], it can be verified that Assumption 9.2 is satisfied, and the appropriate transformation matrices (9.25) are given by

$$T_1 = \begin{bmatrix} T_{11} \\ \hline T_{12} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ \hline -0.0025 & 0 & 1 \end{bmatrix}, \qquad T_2 = \begin{bmatrix} T_{21} \\ \hline T_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ \hline -0.0008 & 0 & 1 \end{bmatrix}$$

and consequently

$$W_{11} = \begin{bmatrix} 0 & 1.0000 \\ 1.0000 & 0 \\ 0 & 0.0025 \end{bmatrix}, \qquad W_{21} = \begin{bmatrix} 0 & 1.0000 \\ 1.0000 & 0 \\ 0 & 0.0008 \end{bmatrix}$$

In the new z_i coordinate system the special representation of the triple in (9.14) takes the form

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{13} & A_{14} \end{bmatrix} = \left[\begin{array}{cc|c} -0.6250 & -0.0982 & -39.27 \\ 1.0000 & 0 & 0 \\ \hline -0.0025 & -0.0004 & -0.1449 \end{array}\right]$$

$$\begin{bmatrix} A_{21} & A_{22} \\ A_{23} & A_{24} \end{bmatrix} = \left[\begin{array}{cc|c} -0.2941 & -0.0246 & -30.8000 \\ 1.0000 & 0 & 0 \\ \hline -0.0008 & -0.0001 & -0.1256 \end{array}\right]$$

and

$$C_{12} = \begin{bmatrix} 0.0025 & 1 \\ 1 & 0 \end{bmatrix} \qquad C_{22} = \begin{bmatrix} 0.0008 & 1 \\ 1 & 0 \end{bmatrix}$$

The associated switching functions matrices from (9.17) are

$$F_1 = \begin{bmatrix} 1 & -0.0025 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} 1 & -0.0008 \end{bmatrix}$$

Choosing $\Lambda_1 = I_2$, $\Lambda_2 = 0.1 I_2$ and solving the Lyapunov equations (9.24) yields

$$\Pi_1 = \begin{bmatrix} 8.9466 & 5.0916 \\ 5.0916 & 4.0608 \end{bmatrix}$$

and

$$\Pi_2 = \begin{bmatrix} 7.0810 & 2.0325 \\ 2.0325 & 0.7720 \end{bmatrix}$$

Since in the sliding surface

$$x_{13} - 0.0025x_{11} = 0$$

it is easy to observe that

$$\beta_{11}(W_{11}z_{11}, 0) = 0, \quad \text{and} \quad \beta_{21}(W_{11}z_{11}, 0) = 0$$

and further

$$\beta_{12}(W_{21}z_{21}, 0) = 0.006, \quad \text{and} \quad \beta_{22}(W_{21}z_{21}, 0) = 0.006$$

By direct computation,

$$L + L^\tau = \begin{bmatrix} 2.0000 & -0.1458 \\ -0.1458 & 0.0157 \end{bmatrix} > 0$$

Then, from Theorem 9.1 the designed sliding mode is globally asymptotic stable. By Theorem 9.2, the 3-machine power system is stabilized by the control law

$$\begin{aligned} v_{f1}(y_1) = & -\frac{1}{0.9 \times 0.1449} \text{sign}(y_{11} - 0.0025y_{12}) \Big(0.1449|y_{11}| \\ & + \frac{1}{69} |y_{11} - 0.0025y_{12}| + k_1(y_1) \Big) \end{aligned} \tag{9.49}$$

$$\begin{aligned} v_{f2}(y_2) = & -\frac{1}{0.9 \times 0.1256} \text{sign}(y_{21} - 0.0008y_{22}) \Big(0.1256|y_{21}| \\ & + \frac{10}{796} |y_{21} - 0.0008y_{22}| + k_2(y_2) \Big) \end{aligned} \tag{9.50}$$

where

$$\begin{aligned} k_1(y_1) &= 2.9025\mu_1 + 1.8036|\sin y_{12}| + (y_{11} - 0.0025y_{12})(y_{11}^2 + y_{12}^2 + \mu_1^2) + 0.5, \\ k_2(y_2) &= 2.9008\mu_2 + 1.471|\sin y_{22}| + 0.012\sqrt{y_{21}^2 + y_{22}^2 + \mu_2^2} + 0.5. \end{aligned}$$

The original control signals u_{f1} and u_{f2} can be obtained from (9.48).

For simulation purposes, let

$$\mu_1 = \mu_2 = 5$$

The operating point is chosen as

$$\begin{aligned} &\delta_{10} = 60.98^o, \quad &&\delta_{20} = 58.62^o \\ &\omega_{10} = \omega_{20} = 0 \text{ r/s}, \quad &&P_{m10} = 1.1 \text{ p.u.}, \quad P_{m20} = 1.0 \text{ p.u.} \end{aligned}$$

Simulation results with initial conditions $x_0 = (0.05, -0.5, 0.3, 0.1, 2, 0.4)$ are presented in Figure 9.3 to verify that the results are effective as it is expected.

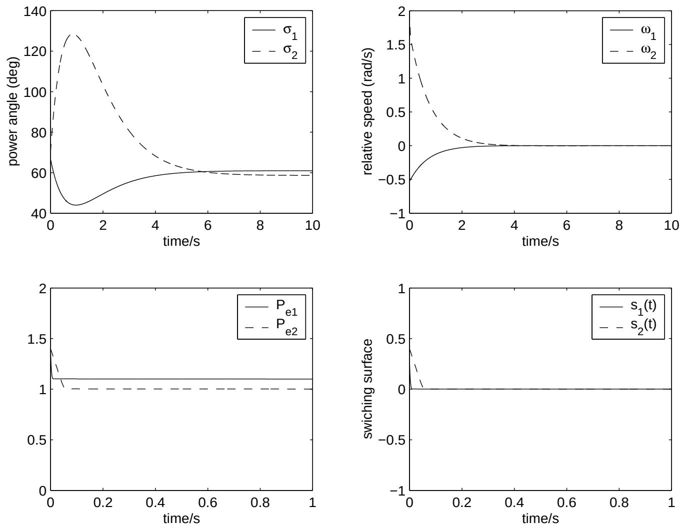

Fig. 9.3 The time responses of the three-machine power system under control (9.49)–(9.50)

9.5 Summary

This chapter has presented a sliding mode control strategy to stabilize multimachine power systems using only static output feedback. A composite sliding surface is formed at first and then, a decentralized control scheme is synthesized which guarantees the reachability condition for the whole interconnected system. The developed results are convenient for practical design due to their static output feedback nature. It allows large matched uncertainty and nonlinearities in the interconnection terms. Simulation shows that the results are effective and valuable.