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Adaptive Sliding Mode Observer for Nonlinear Interconnected Systems with Time Varying Parameters*

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ABSTRACT

In this paper, a class of nonlinear interconnected systems with uncertain time varying parameters (TVPs) is considered. Both the interconnections and the isolated subsystems are nonlinear. Sliding mode control method and adaptive techniques are employed together to design an observer to estimate the state variables of the systems in presence of unknown TVPs. The Lyapunov direct method is used to analysis the stability of the sliding motion and it is not required to solve the so-called constrained Lyapunov problem (CLP). A set of conditions is developed under which the augmented systems formed by the error dynamical systems and the designed adaptive laws, are globally uniformly ultimately bounded. A simulation example is presented and the results show that the method proposed in this paper is effective.

Key Words: Sliding Mode Observer, Nonlinear Interconnected Systems, Adaptive Techniques, Time Varying Parameters

I. INTRODUCTION

In the modern world, it is required to deal with advanced systems using advanced technologies, which has resulted in many large-scale complex systems. With the increasing requirement for the system performance, it needs to develop novel techniques to achieve novel design to satisfy the requirements. It should be noted that control design for large-scale interconnected systems has obtained great achievement [1, 2]. However, lots of results are based on the fact that all system state variables are available for design, which does not always hold and actually only partial state variables are usually available in reality [1]. Therefore, observer design is one of the main topics to estimate system states using the available system inputs and outputs in control engineering.

The concept of the observer was first introduced by Luenberger in 1964 where the observation error between the output of the actual plant and the output of the observer converges to zero when time goes to infinity. Subsequently, many approaches have been developed to design observers for different systems to estimate system states (see e.g. [3, 4, 5, 6]). However, the problem becomes more challenging when some parameters in the model of the system are unknown, particularly when these parameters are time varying [7]. Over the last few decades, much literature has been devoted to the design of adaptive observers for linear and nonlinear systems. The early results are mainly for linear systems (see e.g. [8] and references therein). Later, many authors have focused on the development of adaptive observer design for nonlinear systems (see e.g. [9, 10]). In these results, the designed adaptive observers are able to maintain bounded parameters estimation error under the persistence of excitation condition and it is required that the unknown parameters are bounded with some extra constraints imposed on the system.

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More recently, adaptive observers using different techniques have been proposed in (see e.g. [11, 12]) where the unknown parameters are limited to be constant. Compared with much existing work in adaptive observer design with unknown constant parameters, the corresponding observation results for unknown time varying parameters (TVPs) are very limited. The approach for nonlinear time varying systems proposed in [13] is based on the fact that the nonlinear systems can be transformed to a particular observable canonical form, and the unknown parameters are bounded. The authors in [14] proposed a sampled output high gain observer for a class of uniformly observable nonlinear systems where the unknown parameters are bounded. An adaptive estimator is proposed in [15] to estimate TVPs for nonlinear systems. However, all the system states are assumed available. In [16] an adaptive observer for a class of nonlinear interconnected systems with uncertain TVPs has been developed. It is required to solve the well known constrained Lyapunov problem (CLP) (see e.g. [17, 18]). The authors in [19] used an adaptive unscented Kalman filter approach to estimate the time varying parameters and system states of a class of nonlinear high-speed objects. This technique requires the assumption that the additive noise vectors are Gaussian uncorrelated white noises. If the assumption is not satisfied, the estimation process accuracy will be significantly affected [20].

Sliding mode techniques have been successfully used in control design and state estimation due to its attractive features such as high robustness to uncertainties in input channel and to parameters variations (see e.g. [21, 22]). An adaptive observer applying sliding mode techniques have been developed in [23] to enhance the performance of the adaptive observer proposed by [24]. Adaptive sliding mode observer based fault reconstruction for nonlinear systems with parametric uncertainties is considered in [25]. However, the unknown parameters considered in these papers are constant. Many adaptive observers have been developed using sliding mode techniques for particular applications and for particular purposes (see e.g. [26, 27]) and thus corresponding specific conditions need to be imposed on the systems considered. Sliding mode techniques with super twisting algorithm are used in [28] to design adaptive observers for nonlinear systems where the unknown parameter vector is assumed to be constant. Sliding mode synchronization method is combined with adaptive techniques in [29] to estimate the unknown parameters for multiple chaotic systems where the system states are assumed to be known and the unknown parameters are constant. To the best of authors' knowledge, this is the first contribution where sliding mode techniques are applied to design an adaptive observer for nonlinear interconnected systems with unknown TVPs.

In this paper, an adaptive sliding mode observer is established for a class of nonlinear interconnected systems with unknown TVPs, in which both the isolated subsystems and interconnections are nonlinear. It is not required that the bounds on the TVPs are known, but the rate of change of these unknown parameters needs to be bounded. Sliding mode techniques and adaptive techniques are employed together to estimate system states with unknown TVPs. In addition, it is not required to solve the CLP. Sufficient conditions are developed such that the augmented systems formed by the error dynamical system and the designed adaptive laws, are globally uniformly ultimately bounded. Simulation results for a numerical nonlinear interconnected system are presented to demonstrate the effectiveness of the developed results.

II. System Description and Preliminaries

Consider a nonlinear interconnected system composed of N subsystems as follows

$$\begin{aligned} \dot{x}_i &= A_i x_i + g_i(x_i, u_i) + \phi_i(y_i, u_i)\Theta_i(t) \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N} H_{ij}(x_j) \qquad (1) \\ y_i &= C_i x_i \qquad (2) \end{aligned}$$

where $x_i \in \Omega_i \subset R^{n_i}$ (Ω_i are neighborhoods of the origin), $u_i \in U_i \in R^{m_i}$ (U_i are the admissible control sets) and $y_i \in R^{p_i}$ with $m_i \le p_i \le n_i$ are the state variables, inputs and outputs of the i-th subsystem respectively, $g_i(x_i, u_i) \in R^{n_i}$ are nonlinear known functions, $\phi_i(y_i, u_i) \in R^{n_i}$ are known functions and $\Theta_i(t) \in R$ are unknown TVPs. The matrix triples (A_i, C_i) are constant with appropriate dimensions and C_i are full row rank. The terms $\sum_{\substack{j=1 \\ j\neq i}}^{N} H_{ij}(x_j)$ are the known interconnections for $i = 1, \cdots, N$.

Since the C_i are full row rank, there exist nonsingular matrices T_{c_i} such that

$$\bar{A}_i = \begin{bmatrix} \bar{A}_{i1} & \bar{A}_{i2} \\ \bar{A}_{i3} & \bar{A}_{i4} \end{bmatrix} := T_{c_i} A_i T_{c_i}^{-1}, \qquad (3)$$

$$\bar{C}_i = \begin{bmatrix} 0 & I_{p_i} \end{bmatrix} := C_i T_{c_i}^{-1} \qquad (4)$$

where $\bar{A}_{i1} \in R^{(n_i - p_i)\times(n_i - p_i)}$ for $i = 1, \cdots, N$. Then in the new coordinates $\bar{x}_i$ defined by

$$\bar{x}_i = T_{c_i} x_i \tag{5}$$

system (1)-(2) can be rewritten as

$$\begin{aligned}
\dot{\bar{x}}_{i1} &= \bar{A}_{i1}\bar{x}_{i1} + \bar{A}_{i2}\bar{x}_{i2} + \bar{g}_{i1}(\bar{x}_i, u_i) \\
&\quad + \bar{\phi}_{i1}(y_i, u_i)\Theta_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij}^a(\bar{x}_j) \qquad (6)\\
\dot{\bar{x}}_{i2} &= \bar{A}_{i3}\bar{x}_{i1} + \bar{A}_{i4}\bar{x}_{i2} + \bar{g}_{i2}(\bar{x}_i, u_i) \\
&\quad + \bar{\phi}_{i2}(y_i, u_i)\Theta_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij}^b(\bar{x}_j) \qquad (7)\\
y_i &= \bar{x}_{i2} \qquad (8)
\end{aligned}$$

where $\bar{x} = \mathrm{col}(\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_N)$, $\bar{x}_i = \mathrm{col}(\bar{x}_{i1}, \bar{x}_{i2})$, $\bar{x}_{i1} \in R^{n_i - p_i}$, $\bar{x}_{i2} \in R^{p_i}$, and

$$\begin{bmatrix} \bar{g}_{i1}(\bar{x}_i, u_i) \\ \bar{g}_{i2}(\bar{x}_i, u_i) \end{bmatrix} := \bar{g}_i(\bar{x}_i, u_i) = T_{c_i}[g_i(x_i, u_i)]_{x_i = T_{c_i}^{-1}\bar{x}_i} \tag{9}$$

$$\begin{bmatrix} \bar{\phi}_{i1}(y_i, u_i) \\ \bar{\phi}_{i2}(y_i, u_i) \end{bmatrix} := T_{c_i}\phi_i(y_i, u_i), \tag{10}$$

$$\begin{bmatrix} H_{ij}^a(x_j) \\ H_{ij}^b(x_j) \end{bmatrix} := T_{c_i}[H_{ij}(x_j)]_{x_j = T_{c_j}^{-1}\bar{x}_j} \tag{11}$$

Assumption 1. The uncertain TVPs $\Theta_i(t)$ satisfy

$$|\dot{\Theta}_i(t)| \leq \mu_i \tag{12}$$

where μ_i are known constants and $\mu_i > 0$.

Assumption 1 means the bounds on the unknown TVPs are not required, but the rate of changes of these parameters are required to be bounded.

Assumption 2. The matrix pairs $(\bar{A}_i, \bar{C}_i)$ in (3)-(4) are observable for $i = 1, 2, \cdots, N$.

Under Assumption 2, there exist matrices L_i such that $\bar{A}_i - L_i\bar{C}_i$ are stable, and thus for any $Q_i > 0$ the Lyapunov equations

$$(\bar{A}_i - L_i\bar{C}_i)^T P_i + P_i(\bar{A}_i - L_i\bar{C}_i) = -Q_i \tag{13}$$

have unique solutions $P_i > 0$ for $i = 1, 2, \cdots, N$.

For further analysis, introduce partitions of P_i and Q_i which are conformable with the decomposition in (6)-(8) as follows

$$P_i = \begin{bmatrix} P_{i1} & P_{i2} \\ P_{i2}^T & P_{i3} \end{bmatrix}, \qquad Q_i = \begin{bmatrix} Q_{i1} & Q_{i2} \\ Q_{i2}^T & Q_{i3} \end{bmatrix} \tag{14}$$

where $P_{i1} \in R^{(n_i - p_i)\times(n_i - p_i)}$, $Q_{i1} \in R^{(n_i - p_i)\times(n_i - p_i)}$. Then, from $P_i > 0$ and $Q_i > 0$, it follows that $P_{i1} > 0$, $P_{i3} > 0$, $Q_{i1} > 0$ and $Q_{i3} > 0$. The following result is required for further analysis.

Lemma 1. The matrices $\bar{A}_{i1} + P_{i1}^{-1}P_{i2}\bar{A}_{i3}$ are Hurwitz stable, where P_{i1} and P_{i2} are defined in (14) and $\bar{A}_{i1}$ and $\bar{A}_{i3}$ are defined in (3), if the Lyapunov equations (13) are satisfied.

Proof. See Lemma 2.1 in [30].

Assumption 3. The functions $\bar{g}_i(\bar{x}_i, u_i)$ defined in (9) satisfy the Lipschitz condition with respect to $\bar{x}_i \in R^{n_i}$ and uniformly for $u_i \in U_i \in R^{m_i}$ for $i = 1, 2, \cdots, N$.

Assumption 3 implies that there exist nonnegative functions $\ell_{\bar{g}_{i1}}$ and $\ell_{\bar{g}_{i2}}$ such that

$$\begin{aligned}
\|\bar{g}_{i1}(\bar{x}_i, u_i) - \bar{g}_{i1}(\hat{\bar{x}}_i, u_i)\| &\leq \ell_{\bar{g}_{i1}}(u_i)\|\bar{x}_i - \hat{\bar{x}}_i\| \qquad (15)\\
\|\bar{g}_{i2}(\bar{x}_i, u_i) - \bar{g}_{i2}(\hat{\bar{x}}_i, u_i)\| &\leq \ell_{\bar{g}_{i2}}(u_i)\|\bar{x}_i - \hat{\bar{x}}_i\| \qquad (16)
\end{aligned}$$

for $i = 1, 2, \cdots, N$.

Remark 1. Assumption 3 shows that the functions $\bar{g}_i(\bar{x}_i, u_i)$ defined in (9) satisfy the Lipschitz condition with respect to only $\bar{x}_i$ instead of $(\bar{x}_i, u_i)$. Such an Assumption is reasonable because control inputs u_i are usually known in observer design, and may relax the limitation to the functions $\bar{g}_i(\bar{x}_i, u_i)$.

III. Adaptive Sliding Mode Observer Design

Consider the system in (6)-(8). Introduce a linear coordinate transformation

$$z_i = \underbrace{\begin{bmatrix} I_{n_i - p_i} & K_i \\ 0 & I_{p_i} \end{bmatrix}}_{T_i} \bar{x}_i \tag{17}$$

where $K_i = P_{i1}^{-1}P_{i2}$. In the new coordinate system z_i, system (6)-(8) has the following form

$$\begin{aligned}
\dot{z}_{i1} &= (\bar{A}_{i1} + K_i\bar{A}_{i3})z_{i1} + (\bar{A}_{i2} - \bar{A}_{i1}K_i + K_i(\bar{A}_{i4} \\
&\quad - \bar{A}_{i3}K_i)z_{i2} + \bar{g}_{i1}(T_i^{-1}z_i, u_i) + \bar{\phi}_{i1}(\cdot)\Theta_i(t) \\
&\quad + K_i\bar{g}_{i2}(T_i^{-1}z_i, u_i) + \sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij}^a(T^{-1}z) \\
&\quad + K_i\bar{\phi}_{i2}(y_i, u_i)\Theta_i(t) + K_i\sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij}^b(T^{-1}z) \qquad (18)\\
\dot{z}_{i2} &= \bar{A}_{i3}z_{i1} + (\bar{A}_{i4} - \bar{A}_{i3}K_i)z_{i2} + \bar{g}_{i2}(T_i^{-1}z_i, u_i) \\
&\quad + \bar{\phi}_{i2}(\cdot)\Theta_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij}^b(T^{-1}z) \qquad (19)\\
y_i &= z_{i2} \qquad (20)
\end{aligned}$$

where $z_i = \mathrm{col}(z_{i1}, z_{i2})$ with $z_{i1} \in R^{n_i - p_i}$. For system (18)-(20), consider a dynamical system

$$\begin{aligned}\dot{\hat{z}}_{i1} &= (\bar{A}_{i1}+K_i\bar{A}_{i3})\hat{z}_{i1}+(\bar{A}_{i2}-\bar{A}_{i1}K_i+K_i(\bar{A}_{i4}\\ &\quad -\bar{A}_{i3}K_i))y_i+\bar{g}_{i1}(T_i^{-1}\hat{z}_i,u_i)+\bar{\phi}_{i1}(\cdot)\hat{\Theta}_i(t)\\ &\quad +\sum_{\substack{j=1\\ j\neq i}}^{N}H_{ij}^a(T^{-1}\hat{z})+K_i\bar{g}_{i2}(T_i^{-1}\hat{z}_i,u_i)\\ &\quad +K_i\bar{\phi}_{i2}(\cdot)\hat{\Theta}_i(t)+K_i\sum_{\substack{j=1\\ j\neq i}}^{N}H_{ij}^b(T^{-1}\hat{z}) \qquad (21)\\ \dot{\hat{z}}_{i2} &= \bar{A}_{i3}\hat{z}_{i1}+(\bar{A}_{i4}-\bar{A}_{i3}K_i)y_i+\bar{g}_{i2}(T_i^{-1}\hat{z}_i,u_i)\\ &\quad +\bar{\phi}_{i2}(\cdot)\hat{\Theta}_i(t)+\sum_{\substack{j=1\\ j\neq i}}^{N}H_{ij}^b(T^{-1}\hat{z})+d_i(\cdot) \qquad (22)\\ \hat{y}_i &= \hat{z}_{i2} \qquad (23)\end{aligned}$$

where $\hat{z}=\text{col}(\hat{z}_1,y)$, and the injection term $d_i(\cdot)$ is defined by

$$d_i(\cdot) = \rho_i \text{sgn}(y_i-\hat{y}_i) \qquad (24)$$

where ρ_i are positive constants for $i=1,2,\cdots,N$, with adaptive laws

$$\begin{aligned}\dot{\Gamma}_i &= -\sigma_i[\dot{\hat{y}}_i-d_i(\cdot)] \qquad (25)\\ \hat{\Theta}_i(t) &= \Gamma_i+\sigma_i y_i \qquad (26)\end{aligned}$$

where $d_i(\cdot)$ is given in (24), and σ_i are positive constants. Let $e_{i1}=z_{i1}-\hat{z}_{i1}$, $e_{y_i}=y_i-\hat{y}_i$ and $e_{\Theta_i}=\Theta_i(t)-\hat{\Theta}_i(t)$. Then from (18)-(20) and (21)-(23), the error dynamics are described by

$$\begin{aligned}\dot{e}_{i1} &= (\bar{A}_{i1}+K_i\bar{A}_{i3})e_{i1}+[\bar{g}_{i1}(\cdot)-\bar{g}_{i1}(\hat{\cdot})]+\bar{\phi}_{i1}(\cdot)[\Theta_i(t)\\ &\quad -\hat{\Theta}_i(t)]+\sum_{\substack{j=1\\ j\neq i}}^{N}[H_{ij}^a(\cdot)-H_{ij}^a(\hat{\cdot})]+K_i[\bar{g}_{i2}(\cdot)-\bar{g}_{i2}(\hat{\cdot})]\\ &\quad +K_i\bar{\phi}_{i2}(\cdot)[\Theta_i(t)-\hat{\Theta}_i(t)]+K_i\sum_{\substack{j=1\\ j\neq i}}^{N}[H_{ij}^b(\cdot)-H_{ij}^b(\hat{\cdot})] \qquad (27)\\ \dot{e}_{y_i} &= \bar{A}_{i3}e_{i1}+[\bar{g}_{i2}(\cdot)-\bar{g}_{i2}(\hat{\cdot})]+\bar{\phi}_{i2}(\cdot)[\Theta_i(t)-\hat{\Theta}_i(t)]\\ &\quad +\sum_{\substack{j=1\\ j\neq i}}^{N}[H_{ij}^b(\cdot)-H_{ij}^b(\hat{\cdot})]-d_i(\cdot) \qquad (28)\end{aligned}$$

where $d_i(\cdot)$ is given in (24) for $i=1,2,\cdots,N$, and

$$\begin{aligned}\bar{g}_{i1}(T_i^{-1}z_i,u_i) &= \bar{g}_{i1}(\cdot),\ \ \bar{g}_{i1}(T_i^{-1}\hat{z}_i,u_i)=\bar{g}_{i1}(\hat{\cdot})\\ \bar{g}_{i2}(T_i^{-1}z_i,u_i) &= \bar{g}_{i2}(\cdot),\ \ \bar{g}_{i2}(T_i^{-1}\hat{z}_i,u_i)=\bar{g}_{i2}(\hat{\cdot})\\ H_{ij}^a(T^{-1}z) &= H_{ij}^a(\cdot),\ \ H_{ij}^a(T^{-1}\hat{z})=H_{ij}^a(\hat{\cdot})\\ H_{ij}^b(T^{-1}z) &= H_{ij}^b(\cdot),\ \ H_{ij}^b(T^{-1}\hat{z})=H_{ij}^b(\hat{\cdot})\end{aligned}$$

From (25) and (26)

$$\begin{aligned}\dot{e}_{\Theta_i} &= \dot{\Theta}_i(t)-\dot{\hat{\Theta}}_i(t)=\dot{\Theta}_i(t)-\{\dot{\Gamma}_i+\sigma_i\dot{y}_i\}\\ &= \dot{\Theta}_i(t)-\{\{-\sigma_i[\bar{A}_{i3}\hat{z}_{i1}+(\bar{A}_{i4}-\bar{A}_{i3}K_i)y_i\\ &\quad +\bar{g}_{i2}(\hat{\cdot})+\bar{\phi}_{i2}(\cdot)\hat{\Theta}_i(t)+\sum_{\substack{j=1\\ j\neq i}}^{N}H_{ij}^b(\hat{\cdot})+d_i(\cdot)\\ &\quad -d_i(\cdot)]\}+\{\sigma_i[\bar{A}_{i3}z_{i1}+(\bar{A}_{i4}-\bar{A}_{i3}K_i)z_{i2}\\ &\quad +\bar{g}_{i2}(\cdot)+\bar{\phi}_{i2}(\cdot)\Theta_i(t)+\sum_{\substack{j=1\\ j\neq i}}^{N}H_{ij}^b(\cdot)]\}\}\\ &= -\sigma_i\bar{A}_{i3}e_{i1}-\sigma_i[\bar{g}_{i2}(\cdot)-\bar{g}_{i2}(\hat{\cdot})]-\sigma_i\bar{\phi}_{i2}(\cdot)e_{\Theta_i}\\ &\quad -\sum_{\substack{j=1\\ j\neq i}}^{N}\sigma_i[H_{ij}^b(\cdot)-H_{ij}^b(\hat{\cdot})]+\dot{\Theta}_i(t) \qquad (29)\end{aligned}$$

From the structure of the transformation matrix T_i in (17) and the fact that $\hat{z}_i=\text{col}(\hat{z}_{i1},y_i)$, it follows that

$$\begin{aligned}\|T_i^{-1}z_i-T_i^{-1}\hat{z}_i\| &= \|T_i^{-1}(z_i-\hat{z}_i)\|\\ &= \left\|T_i^{-1}\begin{bmatrix}e_{i1}\\ 0\end{bmatrix}\right\|=\|e_{i1}\| \qquad (30)\end{aligned}$$

From the analysis above, it is straightforward to see

$$\|T^{-1}z-T^{-1}\hat{z}\|=\|e_1\| \qquad (31)$$

where

$$e_1 := \text{col}(e_{11},e_{21},\cdots,e_{N1}) \qquad (32)$$

Therefore, from (15), (16), (30) and (31)

$$\begin{aligned}\|\bar{g}_{i1}(T_i^{-1}z_i,u_i)-\bar{g}_{i1}(T_i^{-1}\hat{z}_i,u_i)\| &\le \ell_{\bar{g}_{i1}}(u_i)\|e_{i1}\| \qquad (33)\\ \|\bar{g}_{i2}(T_i^{-1}z_i,u_i)-\bar{g}_{i2}(T_i^{-1}\hat{z}_i,u_i)\| &\le \ell_{\bar{g}_{i2}}(u_i)\|e_{i1}\| \qquad (34)\\ \|H_{ij}^a(T^{-1}z)-H_{ij}^a(T^{-1}\hat{z})\| &\le \ell_{H^a}\|e_1\| \qquad (35)\\ \|H_{ij}^b(T^{-1}z)-H_{ij}^b(T^{-1}\hat{z})\| &\le \ell_{H^b}\|e_1\| \qquad (36)\end{aligned}$$

where $\ell_{\bar{g}_{i1}}(u_i)$ and $\ell_{\bar{g}_{i2}}(u_i)$ are nonnegative functions, and ℓ_{H^a} and ℓ_{H^b} are constants.

Remark 2. It is well known that sliding mode is a reduced order system. In this paper, the sliding motion governs by the error dynamical systems (27) with adaptive laws (25) - (26) while the error dynamical systems (28) does not affect the sliding motion, which makes the obtained results less conservative.

IV. Stability of the Error Dynamical Systems

Theorem 1. Under Assumptions $1-3$, the error dynamical systems (27) with adaptive laws (25) - (26)

are globally uniformly ultimately bounded if the matrix $W^T + W$ is positive definite, where

$$W = \begin{bmatrix} w^a & w^b \\ w^c & w^d \end{bmatrix}_{2N\times 2N} \tag{37}$$

where $w^a = (w^a_{ij})_{N\times N}$, $w^b = (w^b_{ij})_{N\times N}$, $w^c = (w^c_{ij})_{N\times N}$, $w^d = (w^d_{ij})_{N\times N}$, and

$$w^a_{ij} = \begin{cases} \{\lambda_{min}(Q_{i1}) - 2\|P_{i1}\|[\ell_{g_{i1}} + \|K_i\|\ell_{g_{i2}}] \\ -2\|P_{i1}\|[\ell_{H^a} + \|K_i\|\ell_{H^b}]\}, & i = j \\ -\|P_{i1}\|[\ell_{H^a} + \|K_i\|\ell_{H^b}], & i \neq j \end{cases}$$

$$w^b_{ij} = w^c_{ij} = \begin{cases} -\{\|P_{i1}\|\alpha_{i1} + \sigma_i\|\bar{A}_{i3}\| \\ +\sigma_i\ell_{g_{i2}} + \sigma_i\ell_{H^b}\}, & i = j \\ \sigma_i\ell_{H^b}, & i \neq j \end{cases}$$

$$w^d_{ij} = \begin{cases} 2\sigma_i\alpha_{i2}, & i = j \\ 0, & i \neq j \end{cases} \tag{38}$$

where P_{i1} and Q_{i1} are given in (14), and

$$\|\bar{\phi}_{i1}(\cdot) + K_i\bar{\phi}_{i2}(\cdot)\| \leq \alpha_{i1} \tag{39}$$

$$\|\bar{\phi}_{i2}(\cdot)\| \leq \alpha_{i2} \tag{40}$$

for $i, j = 1, 2, \cdots, N$.

Proof. For systems (27) and (29), consider the candidate Lyapunov function

$$V = \sum_{i=1}^{N} e_{i1}^T P_{i1} e_{i1} + \sum_{i=1}^{N} e_{\Theta_i}^T e_{\Theta_i} \tag{41}$$

The time derivative of $V(\cdot)$ along the trajectories of system (27) and (29) is given by

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{N}[\dot{e}_{i1}^T P_{i1} e_{i1} + e_{i1}^T P_{i1}\dot{e}_{i1} + \dot{e}_{\Theta_i}^T e_{\Theta_i} + e_{\Theta_i}^T \dot{e}_{\Theta_i}] \\ &= \sum_{i=1}^{N}\Big\{e_{i1}^T[(\bar{A}_{i1}+K_i\bar{A}_{i3})^T P_{i1}+P_{i1}(\bar{A}_{i1}+K_i\bar{A}_{i3})]e_{i1} \\ &+2e_{i1}^T P_{i1}[\bar{g}_{i1}(\cdot)-\bar{g}_{i1}(\hat{\cdot})]+2e_{i1}^T P_{i1}\bar{\phi}_{i1}(\cdot)e_{\Theta_i}+2e_{i1}^T P_{i1} \\ &\times\sum_{\substack{j=1\\j\neq i}}^{N}[H^a_{ij}(\cdot)-H^a_{ij}(\hat{\cdot})] + 2e_{i1}^T P_{i1}K_i[\bar{g}_{i2}(\cdot) - \bar{g}_{i2}(\hat{\cdot})] \\ &+2e_{i1}^T P_{i1}K_i\bar{\phi}_{i2}(\cdot)e_{\Theta_i}+2e_{i1}^T P_{i1}K_i\sum_{\substack{j=1\\j\neq i}}^{N}[H^b_{ij}(\cdot)-H^b_{ij}(\hat{\cdot})] \\ &-2e_{\Theta_i}^T\sigma_i\bar{A}_{i3}e_{i1}-2e_{\Theta_i}^T\sigma_i[\bar{g}_{i2}(\cdot)-\bar{g}_{i2}(\hat{\cdot})] - 2e_{\Theta_i}^T\sigma_i \\ &\times\sum_{\substack{j=1\\j\neq i}}^{N}[H^b_{ij}(\cdot)-H^b_{ij}(\hat{\cdot})]-2e_{\Theta_i}^T\sigma_i\bar{\phi}_{i2}(\cdot)e_{\Theta_i}+2e_{\Theta_i}^T\dot{\Theta}_i(t)\Big\}\end{aligned}$$

From (33)-(36),

$$\begin{aligned}\dot{V} \leq &\sum_{i=1}^{N}\Big\{-e_{i1}^T Q_{i1}e_{i1}+2\|P_{i1}\|[\ell_{g_{i1}}+\|K_i\|\ell_{g_{i2}}]\|e_{i1}\|^2 \\ &+2\|P_{i1}\|\|[\bar{\phi}_{i1}(\cdot)+K_i\bar{\phi}_{i2}(\cdot)]\|\|e_{i1}\|\|e_{\Theta_i}\|+2\|e_{i1}\| \\ &\times\|P_{i1}\|[\ell_{H^a}+\|K_i\|\ell_{H^b}]\|e_1\|-2\|e_{\Theta_i}\|\sigma_i\|\bar{A}_{i3}\|\|e_{i1}\| \\ &-2\|e_{\Theta_i}\|\sigma_i\ell_{g_{i2}}\|e_{i1}\|-2\|e_{\Theta_i}\|\sigma_i\ell_{H^b}\|e_1\|-2\|e_{\Theta_i}\| \\ &\times\sigma_i\|\bar{\phi}_{i2}(\cdot)\|\|e_{\Theta_i}\| + 2\|e_{\Theta_i}\|\|\dot{\Theta}_i(t)\|\Big\}\end{aligned} \tag{42}$$

From the definition of e_1 in (32)

$$\|e_1\| \leq \sum_{j=1}^{N}\|e_{j1}\| = \|e_{i1}\| + \sum_{\substack{j=1\\j\neq i}}^{N}\|e_{j1}\| \tag{43}$$

Then, from (42) and (43)

$$\begin{aligned}\dot{V} \leq &\sum_{i=1}^{N}\Big\{-e_{i1}^T Q_{i1}e_{i1}+2\|P_{i1}\|[\ell_{g_{i1}}+\|K_i\|\ell_{g_{i2}}]\|e_{i1}\|^2 \\ &+2\|P_{i1}\|\|[\bar{\phi}_{i1}(\cdot)+K_i\bar{\phi}_{i2}(\cdot)]\|\|e_{i1}\|\|e_{\Theta_i}\|+2\|P_{i1}\| \\ &\times[\ell_{H^a}+\|K_i\|\ell_{H^b}]\|e_{i1}\|^2+\sum_{\substack{j=1\\j\neq i}}^{N}[2\|P_{i1}\|[\ell_{H^a} + \|K_i\| \\ &\times\ell_{H^b}]\|e_{i1}\|\|e_{j1}\|]-2\sigma_i\|\bar{A}_{i3}\|\|e_{i1}\|\|e_{\Theta_i}\| \\ &-2\sigma_i\ell_{g_{i2}}\|e_{i1}\|\|e_{\Theta_i}\|-2\sigma_i\ell_{H^b}\|e_{i1}\|\|e_{\Theta_i}\| \\ &-\sum_{\substack{j=1\\j\neq i}}^{N}2\sigma_i\ell_{H^b}\|e_{\Theta_i}\|\|e_{j1}\| - 2\sigma_i\|\bar{\phi}_{i2}(\cdot)\| \\ &\times\|e_{\Theta_i}\|^2 + 2\|e_{\Theta_i}\|\|\dot{\Theta}_i(t)\|\Big\}\end{aligned} \tag{44}$$

From Assumption 1, (39) and (40)

$$\begin{aligned}\dot{V} \leq -&\sum_{i=1}^{N}\Big\{\{\lambda_{min}(Q_{i1}) - 2\|P_{i1}\|[\ell_{g_{i1}} + \|K_i\|\ell_{g_{i2}}] \\ &-2\|P_{i1}\|[\ell_{H^a} + \|K_i\|\ell_{H^b}]\}\|e_{i1}\|^2 - \{2\|P_{i1}\|\alpha_{i1} \\ &+2\sigma_i\|\bar{A}_{i3}\| + 2\sigma_i\ell_{g_{i2}} + 2\sigma_i\ell_{H^b}\}\|e_{i1}\|\|e_{\Theta_i}\| \\ &-\sum_{\substack{j=1\\j\neq i}}^{N}[2\|P_{i1}\|[\ell_{H^a} + \|K_i\|\ell_{H^b}]\|e_{i1}\|\|e_{j1}\|] \\ &+\sum_{\substack{j=1\\j\neq i}}^{N}2\sigma_i\ell_{H^b}\|e_{\Theta_i}\|\|e_{j1}\| + 2\sigma_i\alpha_{i2}\|e_{\Theta_i}\|^2\Big\}\end{aligned}$$

$$+\sum_{i=1}^{N} 2\mu_i \|e_{\Theta_i}\| \tag{45}$$

Then, from the definition of the matrix W in Theorem 1 and the inequality above, it follows that

$$\begin{aligned}\dot{V} &\leq -\frac{1}{2}X^T[W^T+W]X+\gamma\|X\| \\ &\leq -\left(\frac{1}{2}\lambda_{min}(W^T+W)\|X\|-\gamma\right)\|X\| \end{aligned}\tag{46}$$

where $\gamma = 2\mu_i$ and $X = [\|e_{11}\|, \|e_{21}\|, \cdots, \|e_{N1}\|, \|e_{\Theta_1}\|, \|e_{\Theta_2}\|, \cdots, \|e_{\Theta_N}\|]^T$.

From the definition of Lyapunov function in (41), it is straightforward to see that

$$\lambda_{min}(P_{i1})\|X\|^2 \leq \quad V \leq \lambda_{max}(P_{i1})\|X\|^2$$

where $X = [\|e_{11}\|, \|e_{21}\|, \cdots\cdots, \|e_{N1}\|, \|e_{\Theta_1}\|, \|e_{\Theta_2}\|, \cdots, \|e_{\Theta_N}\|]^T$, for all $X \in R^n$. It can been seen clearly that $\lambda_{min}(P_{i1})\|X\|^2$ belongs to class K_∞.

Therefore, from the condition that W^T+W is positive definite, system (27) is globally uniformly ultimately bounded. Hence the result follows. △

Remark 3. From Theorem 1, it follows that e_1 and e_{Θ_i} are bounded and thus there exist constants $\beta_1 > 0$ and $\beta_2 > 0$ such that

$$\|e_1\| \leq \beta_1, \qquad \|e_{\Theta_i}\| \leq \beta_2 \tag{47}$$

where β_1 can be estimated using the approach given in [30] by slightly modification.

For system (27)-(28), consider a sliding surface

$$S_i \quad = \quad \{(e_{i1}, e_{y_i}, e_{\Theta_i}) \big| e_{y_i} = 0\} \tag{48}$$

From the structure of the error dynamical system (27)-(28), it follows that the sliding mode of the error system (27)-(28) with respect to the sliding surface (48) is the system (27) when limited to the sliding surface (48). All that remains is to determine the gains ρ_i in (24) such that the system (27)-(28) can be driven to the sliding surface S_i in finite time and a sliding motion maintained thereafter.

Theorem 2. Under Assumptions 1-3 and the inequality (40), system (27)-(28) is driven to the sliding surface (48) in finite time and remains on it thereafter if

$$\rho_i \geq (\|\bar{A}_{i3}\| + \ell_{\bar{g}_{i2}} + \ell_{H^b} + \alpha_{i2}\beta_2)\beta_1 + \eta \tag{49}$$

where $\eta > 0$ is constant, β_1 and β_2 satisfy (47).

Proof. From (28)

$$\begin{aligned}\sum_{i=1}^{N} e_{y_i}^T \dot{e}_{y_i} &= \sum_{i=1}^{N} e_{y_i}^T \Big\{ \bar{A}_{i3}e_{i1} + [\bar{g}_{i2}(\cdot) - \bar{g}_{i2}(\hat{\cdot})] + \bar{\phi}_{i2}(\cdot) \\ &\times[\Theta_i(t) - \hat{\Theta}_i(t)] + \sum_{\substack{j=1\\ j\neq i}}^{N} [H_{ij}^b(\cdot) - H_{ij}^b(\cdot)] - d_i(\cdot) \Big\} \\ &\leq \sum_{i=1}^{N} \Big\{ \|\bar{A}_{i3}\|\|e_{i1}\|\|e_{y_i}\| + \ell_{g_{i2}}\|e_{i1}\|\|e_{y_i}\| + \ell_{H^b} \\ &\times\|e_1\|\|e_{y_i}\| + \|\bar{\phi}_{i2}(\cdot)\|\|e_{\Theta_i}\|\|e_{y_i}\| - \rho_i \mathrm{sgn}(e_{y_i}) \Big\} \\ &\leq \sum_{i=1}^{N} \Big\{ \{(\|\bar{A}_{i3}\| + \ell_{\bar{g}_{i2}} + \ell_{H^b} + \alpha_{i2}\beta_2)\beta_1 \\ &\quad -\rho_i\}\|e_{y_i}\| \Big\} \end{aligned}\tag{50}$$

Applying (49) into (50)

$$\sum_{i=1}^{N} e_{y_i}^T \dot{e}_{y_i} \quad = \quad -\eta \sum_{i=1}^{N} \|e_{y_i}\| \tag{51}$$

which implies that $e_y^T \dot{e}_y \leq -\eta\|e_y\|$. where $e_y = \mathrm{col}(e_{y_1}, \ e_{y_2}, \cdots, e_{y_N})$ and the inequality $\|e_y\| \leq \sum_{i=1}^{N} \|e_{y_i}\|$ is applied to obtain the inequality above. This shows that the reachability condition is satisfied. Hence the conclusion follows. △

Remark 4. From sliding mode theory, Theorems 1 and 2 show that system (21)-(23) is an approximate observer for the system (18)-(20) and the estimation error enters a bounded domain in finite time.

V. Simulation Example

Consider a nonlinear interconnected system as follows:

$$\begin{aligned}\dot{x}_1 &= \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \begin{bmatrix} u_1 \\ \sin x_{12} \end{bmatrix} \\ &+ \begin{bmatrix} x_{11} \\ 0 \end{bmatrix} \theta_1(t) + \begin{bmatrix} 0 \\ 0.1x_{21}^2 \end{bmatrix} \end{aligned}\tag{52}$$

$$y_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} \tag{53}$$

$$\begin{aligned}\dot{x}_2 &= \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} + \begin{bmatrix} u_2 \\ 0.7\cos x_{22} \end{bmatrix} \\ &+ \begin{bmatrix} x_{21} \\ 0 \end{bmatrix} \theta_2(t) + \begin{bmatrix} 0 \\ 0.7\sin x_{11} \end{bmatrix} \end{aligned}\tag{54}$$

$$y_2 = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} \tag{55}$$

where $\mathrm{col}(x_1, x_2)$ are the system states, y_1 and y_2 are the system outputs. Let

$$T_{c_i} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad i = 1, 2 \tag{56}$$

The system (52)-(55) can be transformed to

$$\dot{\bar{x}}_1 = \underbrace{\begin{bmatrix} -3 & 2 \\ 1 & 0 \end{bmatrix}}_{\bar{A}_1} \begin{bmatrix} \bar{x}_{11} \\ \bar{x}_{12} \end{bmatrix} + \underbrace{\begin{bmatrix} \sin \bar{x}_{12} \\ u_1 \end{bmatrix}}_{\bar{g}_1(\cdot)} + \underbrace{\begin{bmatrix} 0 \\ \bar{x}_{11} \end{bmatrix}}_{\bar{\phi}_1(\cdot)} \theta_1(t) + \underbrace{\begin{bmatrix} 0.1\bar{x}_{21}^2 \\ 0 \end{bmatrix}}_{H_{12}(\bar{x}_2)} \quad (57)$$

$$y_1 = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{\bar{C}_1} \begin{bmatrix} \bar{x}_{11} \\ \bar{x}_{12} \end{bmatrix} \quad (58)$$

$$\dot{\bar{x}}_2 = \underbrace{\begin{bmatrix} -3 & 2 \\ 1 & 0 \end{bmatrix}}_{\bar{A}_2} \begin{bmatrix} \bar{x}_{21} \\ \bar{x}_{22} \end{bmatrix} + \underbrace{\begin{bmatrix} 0.7\cos \bar{x}_{22} \\ u_2 \end{bmatrix}}_{\bar{g}_2(\cdot)} + \underbrace{\begin{bmatrix} 0 \\ \bar{x}_{21} \end{bmatrix}}_{\bar{\phi}_2(\cdot)} \theta_2(t) + \underbrace{\begin{bmatrix} 0.7\sin \bar{x}_{11} \\ 0 \end{bmatrix}}_{H_{21}(\bar{x}_1)} \quad (59)$$

$$y_2 = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{\bar{C}_2} \begin{bmatrix} \bar{x}_{21} \\ \bar{x}_{22} \end{bmatrix} \quad (60)$$

Choose $L_i = [1 \ \ 1]$ and $Q_i = 8I$ for $i = 1, 2$. Then, the Lyapunov equation (13) has unique solution:

$$P_i = \begin{bmatrix} 1 & 0.2 \\ 0.2 & 4.2 \end{bmatrix}, \qquad i = 1, 2 \quad (61)$$

Therefore, under the transformation $x_i = (T_i T_{c_i})^{-1} z_i$ with T_{c_i} defined in (56) and T_i given by

$$T_i = \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}, \qquad i = 1, 2 \quad (62)$$

The system can be described in z coordinates as follows

$$\begin{aligned} \dot{z}_{11} &= -2.8z_{11} + 2.2z_{12} + \sin z_{12} \\ &\quad +0.1(z_{21} - 0.2z_22)^2 & (63) \\ \dot{z}_{12} &= z_{11} - 0.2z_{12} + (z_{11} - 0.2z_{12})\theta_1(t) + u_1 & (64) \\ y_1 &= z_{12} & (65) \\ \dot{z}_{21} &= -2.8z_{21} + 2.2z_{22} + 0.7\cos z_{22} \\ &\quad +0.7\sin(z_{11} - 0.2z_{12}) & (66) \\ \dot{z}_{22} &= z_{21} - 0.2z_{22} + (z_{21} - 0.2z_{22})\theta_2(t) + u_2 & (67) \\ y_2 &= z_{22} & (68) \end{aligned}$$

For simulation purposes, the controllers are chosen as $u_i = -k_i x_i$ and $k_i = [8 \ \ 2]$ for $i = 1, 2$.

By direct computation, it follows that the matrix $W^T + W$ is positive definite. Thus, all the conditions of Theorem 1 are satisfied. Therefore the following dynamical system is an asymptotic observer of the system (63) − (68)

$$\begin{aligned} \dot{\hat{z}}_{11} &= -2.8\hat{z}_{11} + 2.2y_1 + \sin \hat{z}_{12} \\ &\quad +0.1(\hat{z}_{21} - 0.2\hat{z}_{22})^2 & (69) \\ \dot{\hat{z}}_{12} &= \hat{z}_{11} - 0.2\hat{z}_{12} + (\hat{z}_{11} - 0.2\hat{z}_{12})\hat{\theta}_1(t) \\ &\quad +u_1 + d_1(\cdot) & (70) \\ \hat{y}_1 &= \hat{z}_{12} & (71) \\ \dot{\hat{z}}_{21} &= -2.8\hat{z}_{21} + 2.2y_2 + 0.7\cos \hat{z}_{22} \\ &\quad +0.7\sin(\hat{z}_{11} - 0.2\hat{z}_{12}) & (72) \\ \dot{\hat{z}}_{22} &= \hat{z}_{21} - 0.2\hat{z}_{22} + (\hat{z}_{21} - 0.2\hat{z}_{22})\hat{\theta}_2(t) \\ &\quad +u_2 + d_2(\cdot) & (73) \\ \hat{y}_2 &= \hat{z}_{22} & (74) \end{aligned}$$

where $d_1(\cdot) = 9 \ \mathrm{sgn}(y_1 - \hat{y}_1), d_2(\cdot) = 9 \ \mathrm{sgn}(y_2 - \hat{y}_2)$. The parameters are chosen as $\beta_1 = 6.5, \eta = 2.5$ and $\sigma_1 = \sigma_2 = 1$. Then, from (25) and (26), the designed adaptive laws are given by

$$\begin{aligned} \dot{\Gamma}_1 &= -[\dot{\hat{y}}_1 - d_1(\cdot)] & (75) \\ \hat{\Theta}_1(t) &= \Gamma_1 + y_1 & (76) \\ \dot{\Gamma}_2 &= -[\dot{\hat{y}}_2 - d_2(\cdot)] & (77) \\ \hat{\Theta}_2(t) &= \Gamma_2 + y_2 & (78) \end{aligned}$$

Simulation in Figures 1-2 shows the system state variables and their estimations in presence of unknown time varying parameters $\Theta_1(t) = \Theta_2(t) = 0.3t$, and simulation in Figures 3-4 shows that the system state variables and their estimations in presence of unknown time varying parameters $\Theta_1(t) = \Theta_2(t) = 0.6t$. The estimation error between the states of the system (63)-(68) and the states of the observer (69)-(74) converges to zero globally ultimately bounded. Therefore, $\hat{z}_i = \mathrm{col}(\hat{z}_{i1}, \hat{z}_{i2})$ in (69)-(74) is an asymptotic estimation of $z_i = \mathrm{col}(z_{i1}, z_{i2})$ in (63)-(68).

Remark 5. It should be noted that the states $\hat{z}_i = \mathrm{col}(\hat{z}_{i1}, \hat{z}_{i2})$ in (69)-(74) give estimations of the variable $z_i = \mathrm{col}(z_{i1}, z_{i2})$ in (63)-(68) for $i = 1, 2$. From the analysis in Sections II and III, it is straightforward to see that $\hat{x}_i = (T_i T_{c_i})^{-1} \hat{z}_i$ are estimations of the states $x_i = [x_{i1} \ \ x_{i2}]^T$ of the system (52)-(55) where T_{c_i} and T_i are defined in (56) and (62) respectively for $i = 1, 2$.

VI. Conclusion

An adaptive sliding mode observer for a class of nonlinear large scale interconnected systems with unknown TVPs has been proposed based on Lyapunov

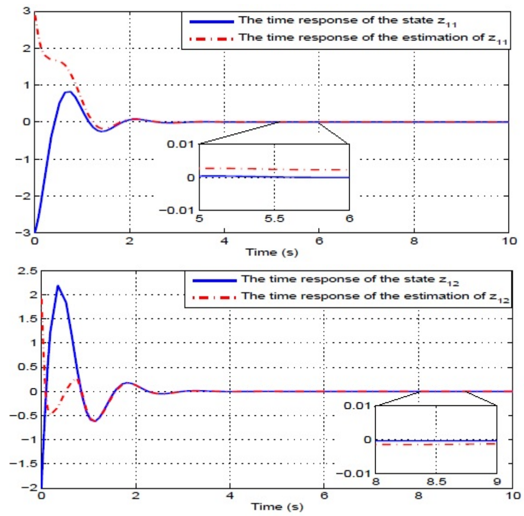

Fig. 1. The time response of the 1st subsystem states and their estimates with $\Theta_1(t) = \Theta_2(t) = 0.3t$

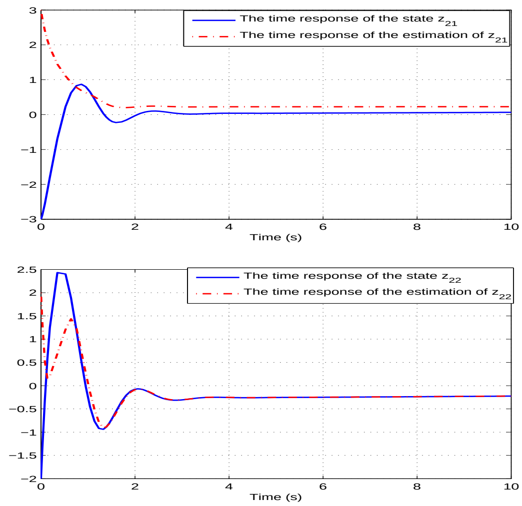

Fig. 2. The time response of the 2nd subsystem states and their estimates with $\Theta_1(t) = \Theta_2(t) = 0.3t$

direct method. Although bounds on the unknown TVPs are not required, the rate of changes of these parameters are bounded. The technique that used in this paper is combined of sliding mode techniques and adaptive techniques to guarantee the ultimate boundedness of the estimation error of the designed observer. Simulation example has shown that the method is effective.

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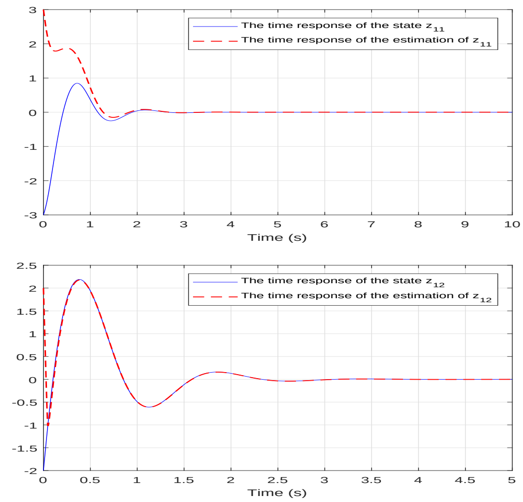

Fig. 3. The time response of the 1st subsystem states and their estimates with $\Theta_1(t) = \Theta_2(t) = 0.6t$

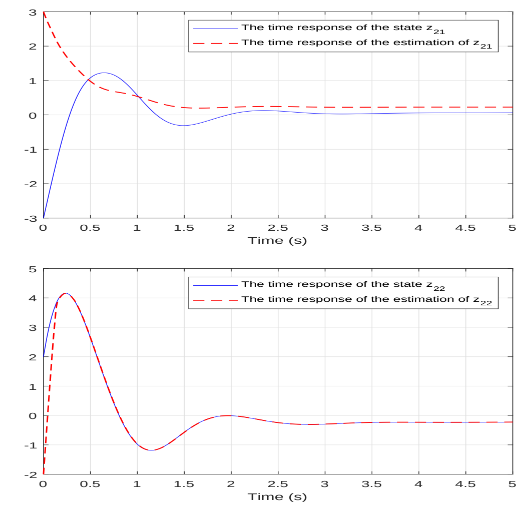

Fig. 4. The time response of the 2nd subsystem states and their estimates with $\Theta_1(t) = \Theta_2(t) = 0.6t$

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