Kent Academic Repository

Zhang, Kangkang, Jiang, Bin, Yan, Xinggang, Mao, Zehui and Shen, Jun (2018) *State and Parameter Estimation for a Class of Nonlinearly Parameterized Systems Using Sliding Mode Techniques.* In: 2018 Annual American Control Conference (ACC). IEEE, pp. 2378-5861. ISBN 978-1-5386-5427-9.

Downloaded from

https://kar.kent.ac.uk/69220/ The University of Kent's Academic Repository KAR

The version of record is available from

https://doi.org/10.23919/ACC.2018.8430753

This document version

Author's Accepted Manuscript

DOI for this version

Licence for this version

UNSPECIFIED

Additional information

Versions of research works

Versions of Record

If this version is the version of record, it is the same as the published version available on the publisher's web site. Cite as the published version.

Author Accepted Manuscripts

If this document is identified as the Author Accepted Manuscript it is the version after peer review but before type setting, copy editing or publisher branding. Cite as Surname, Initial. (Year) 'Title of article'. To be published in *Title of Journal* , Volume and issue numbers [peer-reviewed accepted version]. Available at: DOI or URL (Accessed: date).

Enquiries

If you have questions about this document contact ResearchSupport@kent.ac.uk. Please include the URL of the record in KAR. If you believe that your, or a third party's rights have been compromised through this document please see our Take Down policy (available from https://www.kent.ac.uk/guides/kar-the-kent-academic-repository#policies).

State and Parameter Estimation for a Class of Nonlinearly Parameterized Systems Using Sliding Mode Techniques

Kangkang Zhang[1] Bin Jiang[1], Xinggang Yan[2], Zehui Mao[1] and Jun Shen[1]

***Abstract*— In this study, a class of nonlinear parameterized systems is considered where the unknown parameters are parameterized nonlinearly. A stability criteria for time-varying systems is developed based on Perron-Frobenius theorem, and used for designing observer gains. A particular sliding mode observer with an update law, which can ensure that the sliding motion is driven to origin in finite time, is designed to estimate states and unknown parameters without estimation errors. The developed result is applied to a three-phase inverter system used by China high-speed trains to verify the effectiveness.**

I. INTRODUCTION

State and parameter estimation for a dynamical systems has been a central theme not only in observer issues but also in control issues. For example, state and fault parameter estimation is the most important component for fault diagnoses and fault-tolerant controls, which provides the essential information for making fault detection and isolation decision and also provides basic signals for fault compensation and tolerance. The usage of state and parameter estimation in fault-diagnose and fault-tolerant control areas refers [1], [2], [3], [4] and [5]. In addition, state and parameter estimation is also used for many practical systems, such as high-speed railways [6] and aircrafts [7].

State and parameter estimation for the class of nonlinearly parameterized systems remains a relative unexplored field. Most of existing design techniques are restricted to systems that are linear in the unknown parameters. Through work on nonlinearly parameterized systems remains rare, Annaswamy and co-workers have done excellent work such as [8], [9] and [10] on state and parameter estimation for nonlinearly parameterized systems using adaptive techniques. Their developed estimation techniques use Min-Max approach to yield the global minimum value of the objective quadratic function. In [11], a simple and modular design is introduced to construct an update law which asymptotically inverts a nonlinear equation. An uncertainty set-update approach is proposed for adaptive estimation of unknown parameters in [12].

For decades, sliding mode approaches have been applied for state and parameter estimation widely [13], [14] and [15]. The paper [13] presents a state and unknown fault parameter estimation method using equivalent output injection signals in sliding mode observers. In [14], the unknown inputs, which is a class of unknown parameters, is estimated using high order sliding mode observers. It should be pointed out that the unknown parameters appear in the system linearly in these two papers, and nonlinearly parameterized systems are not considered, which is also the current state of most papers that using sliding mode techniques to estimate states and unknown parameters. While, [15] develops a sliding mode observer with an adaptive law to estimate states and unknown parameters for a class of nonlinearly parameterized systems. In the considered nonlinear terms of [15], only measurable signals such as system outputs and inputs are included, however, system states are not covered, which restricts its applicable. On the other hand, [15] does not provide the method to design the observer gain that ensures the stability of the sliding motion. The main difficulty to design this gain is that the sliding motion is a time-varying system. In this study, the authors are motivated by [15] and would develop more general results about state and parameter estimation for a class of nonlinearly parameterized systems using sliding mode techniques.

In this study, a stability criteria for time-varying systems is developed based on Perron-Frobenius theorem, which is then used for observer gain design. A particular structure of sliding mode observer with an update law is designed. Specifically, the sliding motion is added a discontinuous function associated with system outputs, which can ensure that the sliding motion is driven to origin in finite time, and states and unknown parameters are estimated without estimation errors.

Notion: For a square matrix A, $\lambda(A)$ represents the one eigenvalue of A and $\rho(A)$ represents the spectral radius of A. For any matrix $A \in R^{n\times n}$, $A \gg 0$ ($A \ll 0$) means that A is a strictly positive (negative) matrix, and $A > 0$ ($A < 0$, $A \geq 0$, $A \leq 0$) means that all the elements of A are positive (negative, nonnegative, nonpositive). In addition, $\mathcal{E}_n \in R^{n\times n}$ and $\mathcal{I}_n \in R^n$ represent a n-dimension square matrix and n-dimension vector with all elements being one.

II. PRELIMINARIES AND ASSUMPTIONS

A. Preliminaries

A matrix $M \in R^{n\times n}$ is called a Metzler matrix if all off-diagonal elements of M are nonnegative. The following lemmas are presented to summarized some properties of Metzler matrices.

Lemma 1: [16] Suppose that $M \in R^{n\times n}$ is a Metzler matrix. Then

*This work was not supported by any organization

[1] College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing210016, China KangZhang359@163.com; binjiang@nuaa.edu.cn; zehuimao@nuaa.edu.cn; junshen2009@gmail.com.

[2]School of Engineering and Digital Arts, University of Kent, Canterbury, Kent CT2 7NT, United Kingdom x.yan@kent.ac.uk.

- (Perron-Frobenius) The spectral radius $\rho(M)$ is an eigenvalue of M, and there exists a nonnegative eigenvector $x \neq 0$ such that $Mx = \rho(M)x$.
- Given $\alpha \in R$, there exists a nonzero vector $x \geq 0$ such that $Mx \geq \alpha x$ if and only if $\rho(M) \geq \alpha$.

Lemma 2: [17] Suppose that $M \in R^{n\times n}$ is a Metzler matrix. Then the following statements are equivalent

- M is a Hurwitz matrix;
- $\rho(M) < 0$;
- $Mx \ll 0$ for some $x \in R^n$, $x \gg 0$.

Lemma 3: Suppose that $M \in R^{n\times n}$ is a Metzler matrix and $M = Q - \beta I$ for some positive constant β and some nonnegative matrix Q. Then the following statements are equivalent

- M is a Hurwitz matrix;
- $\beta > \rho(Q)$.

Proof: The representation $M = Q - \beta I$ implies that every eigenvalue of M is of the form $\lambda(Q) - \beta$ where $\lambda(Q)$ is one eigenvalue of Q. Based on properties of nonnegative matrices, $\rho(Q)$ is an eigenvalue of Q (see Theorem 8.3.1 in [18]). Every eigenvalue of M lies in the disc $\{z \in \mathbb{C} : |z + \beta| \leq \rho(Q)\}$. Therefore, M is a Hurwitz matrix means that the real part of every eigenvalue of M is negative which implies that $\beta > \rho(Q)$. Also, $\beta > \rho(Q)$ can ensure that the real part of every eigenvalue of M is negative which means that M is a Hurwitz matrix. Hence, the result follows. ■

The the following lemma will be derived from Lemma 3.

Lemma 4: Let $Z \in \mathcal{Z} \subset R^{n\times n}$ be a matrix variable where $\mathcal{Z} = \{Z \in R^{n\times n} | Z - Z_0| \leq \Delta\}$ with $Z_0 = Z_0^\top \in R^{n\times n}$ being a Hurwitz matrix and $\Delta \in R_+^{n\times n}$. Construct a matrix M which has same eigenvalues with Z_0 and is represented by $M = Q - \beta I$ where $\beta = \rho(Z_0)$ and $Q^\top = Q > 0$. If

- $Q_{ij} \geq n\|\Delta\|_\infty$ for all $i > j$,
- $\beta > \rho(Q) + n^2\|\Delta\|_\infty$,

then

- the matrix M is Metzler and Hurwitz,
- there exists a Metzler and Hurwitz matrix $\bar{M}$ and an orthonormal matrix T such that the matrices $T^\top ZT \leq \bar{M}$.

Proof: Since the matrices Z_0 and Q are symmetric by their definition, there exist two orthonormal matrices $T_z \in R^{n\times n}$ and $T_M \in R^{n\times n}$ such that $T_z^\top Z_0 T_z = T_M^\top M T_M$. Then there exists an orthonormal matrix $T = T_z T_M^\top$ such that $\|S\|_2 = 1$ and $M = T^\top Z_a T$. For any $Z \in \mathcal{Z}$, $T^\top ZT = T^\top(Z_a + \Pi)T$ where $\Pi \in R^{n\times n}$ and $-\Delta \leq \Pi \leq \Delta$. Then

$$T^\top ZT = M + T^\top \Pi T = Q + T^\top \Pi T - \beta I.$$

In light of $\|T^\top \Pi T\|_\infty \leq n\|\Delta\|_\infty$,

$$T^\top \Pi T \leq n\|\Delta\|_\infty E_n.$$

Then,

$$T^\top ZT \leq \bar{M}$$

where $\bar{M} = Q - \beta I + n\|\Delta\|_\infty E_n$. Thus, the first condition that $Q_{ij} \geq n\|\Delta\|_\infty$ for all $i > j$ can guarantee that $Q + T^\top \Pi T \geq 0$ which means that M and $\bar{M}$ are Metzler matrices.

From Theorem 8.1.2 in [18],

$$\min_{1\leq i\leq n} \sum_{j=1}^{n} Q_{ij} \leq \rho(Q) \leq \max_{1\leq i\leq n} \sum_{j=1}^{n} Q_{ij}.$$

Then

$$\begin{aligned} \rho(Q + T^\top \Pi T) &\leq \max_{1\leq i\leq n} \sum_{j=1}^{n} Q_{ij} + n^2\|\Delta\|_\infty \\ &= \rho(Q) + n^2\|\Delta\|_\infty. \end{aligned}$$

Therefore, based on Lemma 2, the second condition that $\beta > \rho(Q) + n^2\|\Delta\|_\infty$ ensures that M and $\bar{M}$ are Hurwitz matrices. ■

Remark 1: The existence of Metzler and Hurwitz matrix $\bar{M}$ implies the stability of the systems $\dot{x} = T^\top Z(t)Tx$ where $Z(t)$ is pantagrated by the interval $\mathcal{Z} = \{Z(t) \in R^{n\times n} | Z(t) - Z_0| \leq \Delta\}$ with $Z_0 = Z_0^\top \in R^{n\times n}$ being a Hurwitz matrix and $\Delta \in R_+^{n\times n}$ (See corollary 2.4 in [17]). Therefore, Lemma 4 provides a stability criteria for time-varying systems $\dot{z} = Z(t)z$.

B. Assumptions

Consider a nonlinear system with nonlinearly parameterized nonlinear terms described by

$$\dot{x} = Ax + f(x, u, \theta) + Dd, \tag{1}$$

$$y = Cx \tag{2}$$

where $x \in \mathcal{X} \subset R^n$($\mathcal{X}$ is the state space), $u \in \mathcal{U} \subset R^m$ ($\mathcal{U}$ is an admissible control) and $y \in \mathcal{Y} \subset R^p$ ($\mathcal{Y}$ is the output space) are state variable, inputs and outputs, respectively, $d \in R^q$ represents the lumped disturbances and uncertainties which is norm bounded by $\bar{d}$, i.e. $\|d\| \leq \bar{d}$; $A \in R^{n\times n}$, $C \in R^{p\times n}$ and $D \in R^{q\times n}$ are constant matrices with C being full row rank and D being full column rank, and $\theta \in \Theta \subset R^r$ are unknown parameters.

The objective of this study is to design a dynamical system and an update law such that the corresponding error dynamical systems converge to origin in finite time by using sliding mode techniques. In addition, the local case will be addressed in this study, however, the developed results can be extended to global case directly.

Assumption 1: The pair (A, C) is observable.

From Lemma 1 in [15], system (1)-(2) satisfies Assumption 1 and output matrix C is full row rank implies that there exists a nonsingular matrix T_1 such that in the new coordinates $z = T_1 x$, system (1)-(2) can be described by

$$\begin{aligned} \dot{z}_1 &= (A_1 - LA_3)\, z_1 \\ &\quad + (A_2 - LA_4 + (A_1 - LA_3)\, L)\, z_2 \\ &\quad + f_1(z_1, y, u, \theta) + D_1 d_1, \end{aligned} \tag{3}$$

$$\begin{aligned} \dot{z}_2 &= A_3 z_1 + (A_4 + A_3 L)\, z_2 \\ &\quad + f_2(z_1, y, u, \theta) + D_2 d_2, \end{aligned} \tag{4}$$

$$y = C_2 z_2 \tag{5}$$

where $z_1 \in R^{n-p}$ and $z_2 \in R^p$, the square matrix C_2 is nonsingular, the pair (A_1, A_3) is observable, the gain matrix $L \in R^{(n-p)\times p}$ is chosen such that $A_1 - LA_3$ is Hurwitz, and

$$\begin{bmatrix} f_1(\cdot) \\ f_2(\cdot) \end{bmatrix} := T_1 f(\cdot)|_{x=T_1^{-1}z},$$
$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} := T_1 D,$$

where $f_1 \in R^{n-p}$ and $D_1 \in R^{(n-p)\times q}$.

Suppose that $z_1 \in \mathcal{Z}_1$ in new coordinate. Two assumptions on $f_1(\cdot)$ and $f_2(\cdot)$ are given in the following.

Assumption 2: The nonlinear function vectors $f_1(\cdot)$ and $f_2(\cdot)$ are differentiable Lipschitz continuous, and have bounded Jacobian matrices with respect to θ for $\theta \in \Theta$, $z_1 \in \mathcal{Z}_1$, $y \in \mathcal{Y}$ and $u \in \mathcal{U}$, i.e.,

$$\underline{F}_1 \leq \frac{\partial f_1}{\partial \theta} \leq \bar{F}_1, \ \underline{F}_2 \leq \frac{\partial f_2}{\partial \theta} \leq \bar{F}_2. \tag{6}$$

Also, there exists a continuous function $\Xi(\cdot) \in R^{r\times p}$ such that for any $(z_1, y, u, \theta) \in \mathcal{Z}_1 \times \mathcal{Y} \times \mathcal{U} \times \Theta$

$$\Xi(\cdot)\frac{\partial f_2}{\partial \theta} < 0. \tag{7}$$

Using the differential Mean Value Theorem (DMVT), it follows from (6) in Assumption 2 that the differential function $f(z_1, y, u, \theta) - f(z_1, y, u, \hat{\theta})$ ($\hat{\theta} \in \Theta$) is proportional to $\theta - \hat{\theta}$ with bounded proportion matrix, i.e.,

$$\begin{aligned} &f_1(z_1, y, u, \theta) - f_1(z_1, y, u, \hat{\theta}) = F_1(t)(\theta - \hat{\theta}), \\ &\underline{F}_1 \leq F_1(t) \leq \bar{F}_1. \end{aligned} \tag{8}$$

Analogously,

$$\begin{aligned} &f_2(z_1, y, u, \theta) - f_2(z_1, y, u, \hat{\theta}) = F_2(t)(\theta - \hat{\theta}), \\ &\underline{F}_2 \leq F_2(t) \leq \bar{F}_2. \end{aligned} \tag{9}$$

Assumption 3: The nonlinear function vector $f_1(\cdot)$ is differentiable Lipschitz continuous, and have bounded Jacobian matrices with respect to z_1 for $z_1 \in \mathcal{Z}_1$, $\theta \in \Theta$, $y \in \mathcal{Y}$ and $u \in \mathcal{U}$, i.e.,

$$\underline{G}_1 \leq \frac{\partial f_1}{\partial z_1} \leq \bar{G}_1. \tag{10}$$

It follows from Assumption 3 that

$$\begin{aligned} &f_1(z_1, y, u, \hat{\theta}) - f_1(\hat{z}_1, y, u, \hat{\theta}) = G_1(t)(z_1 - \hat{z}_1), \\ &\underline{G}_1 \leq G_1(t) \leq \bar{G}_1. \end{aligned} \tag{11}$$

III. MAIN RESULTS

A. Error Dynamical System Formulation

For system (3)-(5), construct the following dynamical system

$$\begin{aligned} \dot{\hat{z}}_1 &= (A_1 - LA_3)\hat{z}_1 \\ &\quad + (A_2 - LA_4 + (A_1 - LA_3)L)C_2^{-1}y \\ &\quad + f_1\left(\hat{z}_1, y, u, \hat{\theta}\right) + K_1\nu, \end{aligned} \tag{12}$$

$$\begin{aligned} \dot{\hat{z}}_2 &= A_3\hat{z}_1 + (A_4 + A_3L)\hat{z}_2 - K(y - \hat{y}) \\ &\quad + f_2\left(\hat{z}_1, y, u, \hat{\theta}\right) + k_2\nu, \end{aligned} \tag{13}$$

$$\hat{y} = C_2\hat{z}_2 \tag{14}$$

where the gain matrix K is chosen such that $A_4 + A_3L + KC_2$ is symmetric negative definite, which is always exist because C_2 is nonsingular. The function ν is defined by

$$\nu = C_2^{-1}\text{sign}(y - \hat{y}), \tag{15}$$

and the gains k_1 is a positive scalar and $K_2 \in R^{(n-p)\times p}$ to be determined later. The vector $\hat{\theta}$ develops along the following update law:

$$\begin{aligned} \dot{\hat{\theta}} &= -\Xi(\cdot)\left(A_3\hat{z}_1 + (A_4 + A_3L)C_2^{-1}y \right. \\ &\quad \left. + f_2\left(\hat{z}_1, y, u, \hat{\theta}\right) - C_2^{-1}\dot{y}\right) + K_\theta\nu \end{aligned} \tag{16}$$

where $\Xi \in R^{r\times p}$ is a designed parameter which satisfies Assumption 2 and $K_{1\theta} \in R^{r\times p}$ is to be determined later.

Let $e_1 = z_1 - \hat{z}_1$, $e_\theta = \theta - \hat{\theta}$ and $e_2 = z_2 - \hat{z}_2$. Since θ is a constant vector, $\dot{e}_\theta = -\dot{\hat{\theta}}$. From (16) and by comparing system (3)-(4) with dynamical system (12)-(13), the error dynamical system is obtained by

$$\begin{aligned} \dot{e}_1 &= (A_1 - LA_3)e_1 + F_1(t)e_\theta \\ &\quad + G_1(t)e_1 + D_1d - K_1\nu, \end{aligned} \tag{17}$$

$$\dot{e}_\theta = \Xi A_3e_1 + \Xi(F_2(t)e_\theta + \delta_2) - K_\theta\nu, \tag{18}$$

$$\begin{aligned} \dot{e}_2 &= A_3e_1 + (A_4 + A_3L + K)e_2 \\ &\quad + F_2(t)e_\theta + \delta_2 + D_2d - k_2\nu. \end{aligned} \tag{19}$$

where F_1 and F_2 are given in (8) and (9) respectively,

$$\delta_2 = f_2\left(\hat{z}_1, y, u, \hat{\theta}\right) - f_2\left(z_1, y, u, \hat{\theta}\right). \tag{20}$$

The continuity of $f_2(\cdot)$ for $\hat{z}_1 \in \mathcal{Z}_1$ provided in Assumption 3 can ensure that δ_2 are norm bounded, that is there exists a constant δ_2 such that

$$\|\delta_2\| \leq \bar{\delta}_2. \tag{21}$$

For error dynamical system (17)-(19), consider a sliding surface

$$\mathcal{S} = \{(e_1, e_\theta, e_2)|e_2 = 0\}. \tag{22}$$

From the structure of system (17)-(19) and the definition of the sliding surface (22), it follows that the sliding motion associated with the sliding surface (22) is governed by subsystem (17)-(18).

B. Stability Analysis of Sliding Motion

Firstly, introduce a new orthogonal coordinate-transformation matrix $T_2^\top$ such that $e_m = T_2^\top e_{1\theta}$ where $e_{1\theta} = \text{col}(e_1, e_\theta)$. The sliding motion (17)-(18) in new coordinates can be written in a compact form as

$$\dot{e}_m = A_me_m + \eta_m + \delta_m - K_m\nu \tag{23}$$

where $e_m \in R^{n-p+r}$, $A_m = T_2^\top A_{1\theta}T_2$, $\eta_m = T_2^\top D_{1\theta}d$, $\delta_m = T_2^T\delta_{1\theta}$ and $K_m = T_2^\top K_{1\theta}$ with

$$A_{1\theta} = \begin{bmatrix} A_1 - LA_3 + G_1(t) & F_1(t) \\ \Xi(\cdot)A_3 & \Xi(\cdot)F_2(t) \end{bmatrix},$$
$$K_{1\theta} = \begin{bmatrix} K_1 \\ K_\theta \end{bmatrix}, \ D_{1\theta} = \begin{bmatrix} D_1 \\ 0 \end{bmatrix}, \ \delta_{1\theta} = \begin{bmatrix} 0 \\ \Xi(\cdot)\delta_2 \end{bmatrix}.$$

From $\|d\| \leq \bar{d}$, there exists a $\bar{\eta}_m$ such that $\|\eta_m\| \leq \bar{\eta}_m$. Also, from (21), there exists a $\bar{\delta}_m$ such that $\|\delta_m\| \leq \bar{\delta}_m$. From (6) and (7) in Assumption 2, it follows that there exists an interval constraints $\mathcal{A}_{1\theta}$ such that $A_{1\theta} \in \mathcal{A}_{1\theta}$ where

$$\mathcal{A}_{1\theta} = \{A_{1\theta} \in R^{n\times n} | A_{1\theta} - A_0| \leq \Delta_{1\theta}\} \qquad (24)$$

with $A_0 = A_0^\top \in R^{n\times n}$ being a Hurwitz matrix and $\Delta \in R_+^{n\times n}$. Now the matrix M which has the same eigenvalues with A_0 is constructed by $M = Q - \beta I$ where $\beta = \rho(A_0)$ and $Q = Q^\top \geq 0$. If $\beta > \rho(Q) + (n-p+r)^2\|\Delta_{1\theta}\|_\infty$ for any Q with $Q_{ij} \geq (n-p+r)\|\Delta_{1\theta}\|_\infty$ for all $i > j$, then based on Lemma 4, M is a Metzler and Hurwitz matrix, and there exists a Metzler and Hurwitz matrix $\bar{M}$ satisfying

$$A_m \leq \bar{M} \qquad (25)$$

where $\bar{M} = Q - \beta I + (n-p+r)\|\Delta\|_\infty \mathcal{E}_n$ and $\bar{M}$ is a Metzler and Hurwitz matrix.

Then, the following theorem is ready to be presented.

Theorem 1: Suppose that Assumptions 1-3 hold and $\sup\{\|\Xi(\cdot)F_2(t)\|\} < \infty$, if there exists a gain L such that $\beta > \rho(Q) + n^2\|A_0\|_\infty$ for any Q with $Q_{ij} \geq n\|A_0\|_\infty$ for all $i > j$, then subsystem (17)-(18) is ultimately bounded, and $e_{1\theta}$ ultimately converges to the sphere $\mathcal{B}$ which is given later.

Proof: Choose a candidate Lyapunov function $V = e_m^\top P_m e_m$ for (25) where $P_m = P_m^\top \geq 0$. Then

$$\begin{aligned} \dot{V}_1 &= e_m^\top \left(A_m^\top P_m + P_m A_m\right) S e_m + 2e_m^\top P_m \eta_m \\ &\quad +2e_m^\top P_m \delta_m - 2e_m^\top P_m K_m \nu \\ &\leq e_m^\top \left(\bar{M}^\top P_m + P_m \bar{M}\right) e_m + 2e_m^\top P_m \eta_m \\ &\quad +2e_m^\top P_m \delta_m - 2e_m^\top P_m K_m \nu. \end{aligned} \qquad (26)$$

Let $Q_m = -(\bar{M}^T P_m + P_m \bar{M})$ and α is a small scalar satisfying $\lambda_{\min}(Q_M) - \alpha\|P_m\| > 0$, which always exists for Hurwitz matrix $\bar{M}$. Then it follows from (26) that

$$\begin{aligned} \dot{V}_1 &\leq -\lambda_{\min}(Q_m)\|\bar{e}_m\|^2 \\ &\quad +2\|e_m\|\|P_m\|(\bar{\eta}_m + \bar{\delta}_m + \sqrt{p}\|K_m\|) \\ &\leq -(\lambda_{\min}(Q_m) - \alpha\|P_m\|)\|e_m\|^2 \\ &\quad +\alpha^{-1}\|P_m\|^{-1}(\bar{\eta}_m + \bar{\delta}_m + \sqrt{p}\|K_m\|)^2 \end{aligned}$$

Thus, the vector e_m will ultimately converge to the sphere $\mathcal{B}$ represented by

$$\mathcal{B} = \left\{ \|e_m\|^2 \leq \frac{\alpha^{-1}\|P_m\|^{-1}(\bar{\eta}_m + \bar{\delta}_m + \sqrt{p}\|K_m\|)^2}{\lambda_{\min}(Q_m) - \alpha\|P_m\|} \right\}. \qquad (27)$$

From Lemma 4, T_2 is an orthonormal matrix, $\|T_2\| = 1$ and $\|e_{1\theta}\| \leq \|e_m\|$. Hence, $e_{1\theta}$ will also ultimately converge to the sphere $\mathcal{B}$. ■

C. Reachability Analysis

Consider a system given by

$$\dot{\bar{e}}_m = \bar{M}\bar{e}_m + (\bar{\eta}_m + \bar{\delta}_m + \|K_m\|\sqrt{p})\mathcal{I}_n, \ \bar{e}_m(0) \geq |e_m(0)|. \qquad (28)$$

Then for e_{mi}, $i = 1, \cdots, n-p+r$, which is the ith element of e_m,

$$\begin{aligned} \frac{d\,|e_{mi}|}{dt} &= \operatorname{sign}(e_{mi})\,\dot{e}_{mi} \\ &\leq (A_m)_{ii}|e_{mi}| + \sum_{j=1,j\neq i}^{n-p+r}(A_m)_{ij}|e_{mj}| \\ &\quad \bar{\eta}_m + \bar{\delta}_m + \|K_m\|\sqrt{p} \\ &\leq \bar{M}_{ii}\,|e_{mi}| + \sum_{j=1,j\neq i}^{n-p+r}\bar{M}_{ij}\,|e_{mj}| \\ &\quad \bar{\eta}_m + \bar{\delta}_m + \|K_m\|\sqrt{p} \end{aligned} \qquad (29)$$

Due to $D^+\,|e_{mi}(t)| := \limsup\limits_{h\to 0^+} \frac{|e_{mi}(t+h)| - |e_{mi}(t)|}{h} = \limsup\limits_{h\to 0^+} \frac{1}{h}\int_t^{t+h} \frac{d}{ds}\,|e_{mi}(s)|\,ds$ where D^+ denotes the Dini upper-right derivative, by using integral mean value theorem, it follows from (29) that

$$\begin{aligned} D^+\,|e_{mi}| &\leq \bar{M}_{ii}\,|e_{mi}| + \sum_{j=1,j\neq i}^{n-p+r}\bar{M}_{ij}\,|e_{mj}| \\ &\quad +\bar{\eta}_m + \bar{\delta}_m + \|K_m\|\sqrt{p}. \end{aligned} \qquad (30)$$

Then, based on Comparison Principle in [19],

$$|e_{mi}(t)| \leq \bar{e}_{mi}(t), \ \|e_m(t)\| \leq \|\bar{e}_m(t)\|. \qquad (31)$$

From Assumption 2, there exists a positive constant $\mathcal{F}_2$ such that $\|F_2(t)\| \leq \mathcal{F}_2$. Then it follows from (31) that

$$\|[A_3, F_2(t)]e_{1\theta}\| \leq (\|A_3\| + \mathcal{F}_2)\|\bar{e}_m\|. \qquad (32)$$

Note that $\|D_2 d\| \leq \|D_2\|\|d\| \leq \|D_2\|\bar{d}$. Choose a candidate Lyapunov function $V_2 = e_2^\top e_2$. Then the time derivative along subsystem (19) is presented as follows

$$\begin{aligned} \dot{V}_2 &= e_2^\top(A_4 + A_3 L + KC_2)e_2 \\ &\quad +e_2^\top(A_3 e_1 + F_2(t)e_\theta + \delta_2 + D_2 d) - k_2 e_2^\top \nu. \end{aligned}$$

If the gain k_2 in (14) is chosen to satisfy $k_2 \geq \underline{k}_{21}$ where

$$\underline{k}_{21} = (\|A_3\| + \mathcal{F}_2)\|\bar{e}_m\| + \bar{\delta}_2 + \|D_2\|\bar{d} + \varpi \qquad (33)$$

with ϖ being a positive constant, then $\dot{V}_2 \leq -\varpi V_2^{\frac{1}{2}}$ which shows that the reachability condition holds and thus the error system (17)-(19) is driven to sliding surface (22) in finite time and maintains on it thereafter.

After sliding motion occurs, $\dot{e}_2 = e_2 = 0$. It follows from (19) that

$$[A_3, F_2(t)]e_{1\theta} = k_2\nu - \delta_2 - D_2 d. \qquad (34)$$

Choose the gain matrix K_m as

$$K_m = P_m^{-1}[A_3, \mathcal{F}_2]^\top k_{1\theta} \qquad (35)$$

where $k_{1\theta}$ is a positive scalar. Substituting (34) and (35) to (26), it follows that

$$\begin{aligned}\dot{V}_1 &\leq e_m^\top\left(\bar{M}^\top P_m + P_m\bar{M}\right)e_m + 2e_m^\top P_m(\bar{\eta}_m + \bar{\delta}_m)\\ &\quad -2e_m^\top[A_3, F_2(t) + (\mathcal{F}_2 - F_2(t))]^\top k_{1\theta}\nu\\ &\leq -e_m^\top Q_m e_m + 2{e_m}^\top P_m(\bar{\eta}_m + \bar{\delta}_m)\\ &\quad -2(k_2\nu - \delta_2 - D_2 d)^\top k_{1\theta}\nu\\ &\quad -2e_m^\top[A_3, \mathcal{F}_2 - F_2(t)]^\top k_{1\theta}\nu\\ &\leq -\lambda_{\min}(Q_m)\|e_m\|^2 - 2pk_2k_{1\theta}\\ &\quad +2(\bar{\delta}_2 + \|D_2\|\bar{d} + \|e_m\|\|\mathcal{A}_3\|)\sqrt{p}k_{1\theta}\\ &\quad +2\|e_m\|\|P_m\|(\bar{\eta}_m + \bar{\delta}_m)\end{aligned}$$

where $\mathcal{A}_3 = [A_3, \mathcal{F}_2 - F_2(t)]$. Since $\|e_m\| \leq \|\bar{e}_m\|$, it follows that

$$\begin{aligned}\dot{V}_1 &\leq -\lambda_{\min}(Q_m)\|e_m\|^2 - 2pk_2k_{1\theta}\\ &\quad +2\|\bar{e}_m\|\|P_m\|(\bar{\eta}_m + \bar{\delta}_m)\\ &\quad +2(\bar{\delta}_2 + \|D_2\|\bar{d} + \|\bar{e}_m\|\|\mathcal{A}_3\|)\sqrt{p}k_{1\theta}.\end{aligned}$$

Denote ζ_m as a scalar which satisfies that $\zeta_m = (\bar{\delta}_2 + \|D_2\|\bar{d} + \|\bar{e}_m\|\|\mathcal{A}_3\|)$. Then if k_2 and $k_{1\theta}$ are chosen to satisfy that $k_2 > \underline{k}_{22}$ and $k_{1\theta} > \underline{k}_{1\theta}$ respectively where

$$\underline{k}_{22} = \frac{\zeta_m}{\sqrt{p}},\ \underline{k}_{1\theta} = \frac{2\|\bar{e}_m\|\|P_m\|(\bar{\eta}_m + \bar{\delta}_m)}{\sqrt{p}(k_2 - \delta_m)}, \tag{36}$$

then there exists a small positive constant ϵ such that $\dot{V}_1 \leq -\frac{\lambda_{\min}(Q_m)}{\lambda_{\min}(P_m)}V_1 - \epsilon \leq -2\sqrt{\frac{\lambda_{\min}(Q_m)\epsilon}{\lambda_{\min}(P_m)}}V_1^{\frac{1}{2}}$. Therefore, $e_{1\theta}$ is driven to $e_{1\theta} = 0$ in finite time and maintains zero thereafter.

Then, the following theorem is ready to be presented.

Theorem 2: Supposed that Assumptions 1-3 hold, error system (17)-(19) is driven to the sliding surface (22) in finite time and maintains on it thereafter, furthermore, after this sliding motion occurs, error subsystem (17)-(18) is also driven to $\mathrm{col}(e_1, e_\theta) = 0$ in finite time and maintains zero thereafter, if $k_2 > \max(\underline{k}_{21}, \underline{k}_{22})$ and $k_{1\theta} > \underline{k}_{1\theta}$ where $\underline{k}_{21}$, $\underline{k}_{22}$ and $\underline{k}_{1\theta}$ are given in (33) and (36) respectively.

Proof: The result is straightforward obtained from above analysis and omitted here. ■

IV. SIMULATION

Consider the state-space model of the three-phase inverters used in China high-speed railways described in [20]. Let $x_1 = v_{od}$, $x_2 = v_{oq}$, $x_3 = i_{Ld}$, $x_4 = i_{Lq}$ where v_{od}, v_{oq}, i_{Ld}, i_{Lq} represent positive-sequence and negative-sequence voltages and currents, respectively. The state-space model is given as follows:

$$\begin{aligned}\dot{x} &= Ax + f(x,\omega) + Bu + Dd,\\ y &= Cx\end{aligned}$$

where $x = \mathrm{col}(x_1, x_2, x_3, x_4)$. The nonlinear term $g(x,\omega)$ is given by

$$f(x,\omega) = [\omega x_2, -\omega x_1, \omega x_4, -\omega x_3]^\top$$

where ω represents the operating frequency of the inverter, and also is the parameter to be estimated in this simulation. The system matrices are presented as follows

$$A = \begin{bmatrix} 0 & 0 & \frac{1}{C_f} & 0\\ 0 & 0 & 0 & \frac{1}{C_f}\\ -\frac{1}{L_f} & 0 & -\frac{r}{L_f} & 0\\ 0 & -\frac{1}{L_f} & 0 & -\frac{r}{L_f}\end{bmatrix},\ D = \mathcal{I}_4,$$

$$B = \begin{bmatrix} 0 & 0\\ 0 & 0\\ \frac{V_{dc}}{L_f} & 0\\ 0 & \frac{V_{dc}}{L_f}\end{bmatrix},\ C = \begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

where $r = 0.144\Omega$, $L_f = 1.417 \times 10^{-3}H$ and $C_f = 6.0 \times 10^{-3}F$ and $V_{dc} = 3600v$. The disturbance d is set by $d = 20\sin(500t)$. Based on [15], the coordinate transformation matrix $T_1 = I_4$. Then $z_1 = \mathrm{col}(x_1, x_2)$, $z_2 = \mathrm{col}(x_3, x_4)$ and $y = z_2$. The nonlinear terms $f_1(x,\omega) = [\omega x_2, -\omega x_1]^\top$ and $f_2(x,\omega) = [,\omega x_4, -\omega x_3]^\top$. Thus,

$$\begin{aligned}\frac{\partial f_1}{\partial \omega} &= \begin{bmatrix} x_2\\ -x_1\end{bmatrix},\ \frac{\partial f_2}{\partial \omega} = \begin{bmatrix} x_4\\ -x_3\end{bmatrix},\\ \frac{\partial f_1}{\partial z_1} &= \begin{bmatrix} 0 & \omega\\ -\omega & 0\end{bmatrix}.\end{aligned}$$

The nonlinear function $\Xi(\cdot) = [-y_2, y_1]$. Based on the practical experience, $\omega \in [0, 1000]Hz$, $x_1, x_2 \in [-1000, 1000]v$ and $x_3, x_4 \in [-200, 200]A$. Then, $\bar{F}_1$, $\underline{F}_1$, $\bar{F}_2$ and $\underline{F}_2$ in Assumption 2, and $\bar{G}_1$, $\underline{G}_1$ in Assumption 3 can be obtained based on [21]. From Theorem 1, it can be calculated that

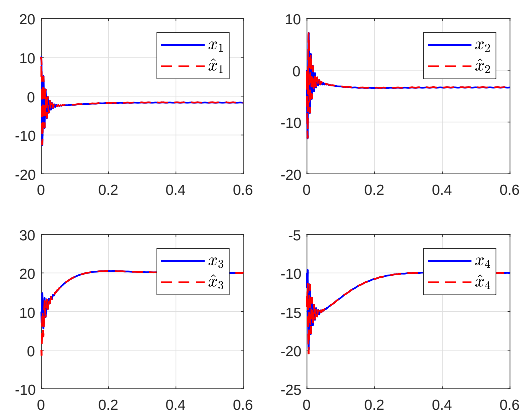

Fig. 1. State estimates

$$\begin{aligned}L &= \begin{bmatrix} 5017 & 0\\ 0 & 5017\end{bmatrix},\\ T_2 &= \begin{bmatrix} -0.8165 & 0.4082 & 0.4082\\ 0 & -0.7071 & 0.7071\\ -0.5774 & -0.5774 & -0.5774\end{bmatrix},\\ \bar{M} &= \begin{bmatrix} -3.1171 & 0.4234 & 0.4234\\ 0.4234 & -3.1171 & 0.4234\\ 0.4234 & 0.4234 & -3.1171\end{bmatrix} \times 10^6\end{aligned}$$

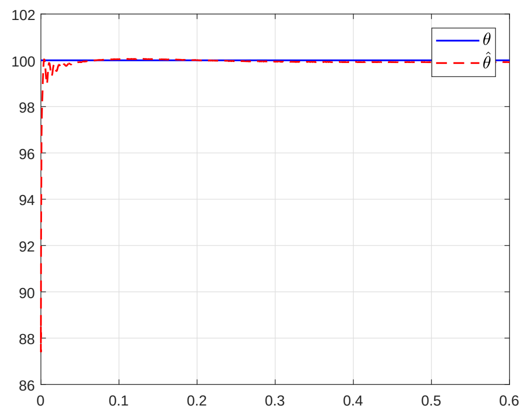

Fig. 2. Parameter estimates

Then, K_1, K_θ and k_2 are chosen based on Theorem 2. Figs. 1 and 2 illustrate the estimates for system states and parameters. This simulation shows the effectiveness of the proposed approach.

V. CONCLUSIONS

This paper has developed a state and unknown parameter estimation technique for a class of nonlinear systems with nonlinear parameterization. A new stability criteria for time-varying systems has been proposed and used for designing observer gains. A particular sliding mode observer with an update law, which can ensure that the sliding motion is driven to origin in finite time, has been designed to estimate states and unknown parameters without estimation errors. Furthermore, the unknown parameters and system states exist in the nonlinear terms simultaneously. The developed result has been applied to a inverter used by China high-speed railways.

REFERENCES

[1] Bin J., Staroswiecki M, Cocquempot V. Fault accommodation for nonlinear dynamic systems. IEEE Transactions on Automatic Control, 2006, 51(9): 1578-1583.

[2] Xinggang, Y., Edwards C. Nonlinear robust fault reconstruction and estimation using a sliding mode observer. Automatica, 2007; 43(9): 1605-1614.

[3] Keliris C, Polycarpou M M, Parisini T. A robust nonlinear observer-based approach for distributed fault detection of inputoutput interconnected systems. Automatica, 2015, 53: 408-415.

[4] Dan Y, Yang G H. Adaptive fault-tolerant tracking control against actuator faults with application to flight control. IEEE Transactions on control systems technology, 2006, 14(6): 1088-1096.

[5] Xiaodong Z, Polycarpou M M, Parisini T. Fault diagnosis of a class of nonlinear uncertain systems with Lipschitz nonlinearities using adaptive estimation. Automatica, 2010, 46(2): 290-299.

[6] Kangkang Zhang, Jiang B, Yan X G, et al. Incipient Voltage Sensor Fault Isolation for Rectifier in Railway Electrical Traction Systems. IEEE Transactions on Industrial Electronics, 2017, 64(8):6763-6774.

[7] Yu X, Liu Z, Zhang Y. Fault-tolerant flight control design with finite-time adaptation under actuator stuck failures. IEEE Transactions on Control Systems Technology, 2017, 25(4): 1431-1440.

[8] Annaswamy A M, Skantze F P, Loh A P. Adaptive control of continuous time systems with convex/concave parametrization. Automatica, 1998, 34(1): 33-49.

[9] Loh A P, Annaswamy A M, Skantze F P. Adaptation in the presence of a general nonlinear parameterization: An error model approach. IEEE Transactions on Automatic Control, 1999, 44(9): 1634-1652.

[10] Cao C, Annaswamy A M, Kojic A. Parameter convergence in nonlinearly parameterized systems. IEEE Transactions on automatic control, 2003, 48(3): 397-412.

[11] Grip H F, Johansen T A, Imsland L, et al. Parameter estimation and compensation in systems with nonlinearly parameterized perturbations. Automatica, 2010, 46(1): 19-28.

[12] Adetola V, Guay M, Lehrer D. Adaptive estimation for a class of nonlinearly parameterized dynamical systems. IEEE Transactions on Automatic Control, 2014, 59(10): 2818-2824.

[13] Edwards C, Spurgeon S K, Patton R J. Sliding mode observers for fault detection and isolation. Automatica, 2000, 36(4): 541-553.

[14] Shtessel Y B, Baev S, Edwards C, et al. HOSM observer for a class of non-minimum phase causal nonlinear MIMO systems. IEEE Transactions on Automatic Control, 2010, 55(2): 543-548.

[15] Xinggang Yan, Spurgeon S K, Edwards C. State and parameter estimation for nonlinear delay systems using sliding mode techniques. IEEE Transactions on Automatic Control, 2013, 58(4): 1023-1029.

[16] Son N K, Hinrichsen D. Robust stability of positive continuous time systems. Numerical functional analysis and optimization, 1996, 17(5-6): 649-659.

[17] Ngoc P H A. New criteria for exponential stability of nonlinear time-varying differential systems. International Journal of Robust and Nonlinear Control, 2014, 24(2): 264-275.

[18] Horn R G, Johnson C R. Matrix Analysis. 1985.

[19] Khalil H K. Noninear Systems. Prentice-Hall, New Jersey, 1996.

[20] Kangkang Zhang, Jiang B, Yan X G, et al. Incipient sensor fault estimation and accommodation for inverter devices in electric railway traction systems. International Journal of Adaptive Control and Signal Processing, 2017, 31(5): 785-804.

[21] Phanomchoeng G, Rajamani R, Piyabongkarn D. Nonlinear observer for bounded Jacobian systems, with applications to automotive slip angle estimation. IEEE Transactions on Automatic Control, 2011, 56(5): 1163-1170.