Exploring Time Variation of Stock Betas in Pakistan*

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Abstract

This paper provides an assessment on the systematic risk in the equity capital markets of Pakistan. We investigate the possibility of time varying betas in Pakistan using three estimation techniques: (a) a Constant Conditional Correlation GARCH Approach, (b) a Dynamic Conditional Correlation GARCH Approach, and (c) a Principal Component Analysis approach. A sample of returns on the top 38 firms listed at the Karachi Stock Exchange (KSE) over the period 1998-2005 is used as a platform to evaluate the performance of these three approaches. An in-sample forecast evaluation of various approaches is employed which shows the superiority of the GARCH approach.

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1 Introduction

Modern finance relies heavily on estimates of systematic risk 'beta' as it has a pivotal role in testing assets pricing theory, estimation of cost of capital, testing trading strategies, and conducting event studies.

The Capital Asset Pricing Model (CAPM) assumes that the relevant risk measure in holding a given security is the systematic risk or beta, as all the other risks can be diversified through portfolio diversification. Various studies show that beta varies over time in contrast to the CAPM assumption that beta is time in-variant. For example, Blume (1971) in a pioneering effort finds that portfolio betas tend to regress towards one over time and that his methodology produces superior beta estimates. Vasicek (1973) argues that utilizing the Bayesian Approach can produce better beta estimates. Vasicek and Blume betas have been empirically tested for their ability to predict future period-unadjusted betas (Klemkosky and Martin, 1975; Dimson and Marsh, 1983). These studies marginally favor Blume's method for its accuracy in determining future ordinary least square (OLS) estimates. It was not until twenty years later that Lally (1998) examines Vasicek and Blume methods for correcting OLS betas and suggests that when the firms are portioned into industries, Vasicek's method could be superior to Blume's method. In addition, Lally (1998) points out that the degree of financial leverage may have significant impact on beta forecasts.

The pursuit for obtaining more accurate beta estimates has continued over years. Some other related issues that have been investigated include the methods of estimation (Chan and Lakonishok, 1992); the effect of length of estimation period (Levy, 1971; Baesel, 1974; Altman et al., 1974; Roenfeldt, 1978; Kim, 1993); the effect of return interval (Frankfurter et al., 1994, Braislford and Josev, 1997) and the effect of outliers (Shalit and Yitzhaki, 2002).

Fabozzi and Francis (1978) find evidence in favor of stochastic properties of beta estimates. In addition, Sunder (1980), Lee and Chen (1982), Ohlson and Rosenberg (1982), and Bos and Newbold (1984) provide strong evidence that the beta of a security is non-stationary, and can be best described by some form of a stochastic model. The phenomenon of Beta Instability is not limited to any particular market as suggested by various studies on the Australian, Indian, and Singaporean markets. Brooks, Faff, and McKenzie (1998) investigated the beta instability over the period 1974-1996 for Australian market and found that 67% of firms have time varying betas. Brooks, Faff, and Arif (1998) also support the incidence of beta instability for the Singapore market. Moonis and Shah (2003) test for time varying betas in the Indian Market and find that the null of beta constancy is rejected for 52% of firms. Much of this discussion supports the time varying beta models as opposed to constant beta models. Therefore the success of the conditional CAPM is dependent on capturing the dynamics of beta.

This paper investigates the instability of beta in the Karachi Stock Exchange (KSE) using three approaches. The first approach is Constant Correlation-Generalized Autoregressive Conditional Heteroskedasticity (GARCH) or simply put GARCH approach to model the time varying beta using conditional variance information produced to construct a conditional beta series and has been used in various studies for volatility modeling (Bollerslev, 1990) and for time varying beta estimation (Brooks, Faff, and McKenzie, 1998). The second approach is Dynamic Conditional Correlation GARCH (DCC-GARCH) approach. DCC-GARCH models are a new class of models which are easy to estimate and allow the correlation to change over time and were developed in Engle (2002). The third is Orthogonal GARCH (O-GARCH) approach – a principal component analysis based approach that allows generating large covariance matrices in an efficient manner. This model was first introduced in Alexander and Chibumba (1996) and subsequently developed in Alexander (2001). The O-GARCH model is an accurate and efficient method for generating large covariance matrices and only requires the estimation of uni-variate GARCH models.

Our study is significantly different from the previous literature that undertook beta instability. Most of the previous studies except one (Brooks, Faff, and McKenzie, 1998) have used one technique and they are constrained by the use of constant correlation.[1] In contrast, our study also employs Conditional Correlation for the first time to account for beta instability. We find evidence in favor of time variant betas of firms listed at the KSE. We also find that constant correlation model perform better than their counterpart conditional correlation models for beta estimation.

The paper is organized as follows. Section 2 outlines the methodology by which conditional and un-conditional betas may be estimated. It also explains the performance evaluation criteria of alternative models. Section 3 details the data to be analyzed and presents selected descriptive statistics. Time varying betas are then generated for the dataset and the relative performance of each model is evaluated. Finally, Section 4 concludes the paper.

[1] Brooks, Faff, and McKenzie, 1998 uses (a) a multivariate generalized ARCH approach, (b) a time varying beta market model approach suggested by Schwert and Seguin (1990), and (c) the Kalman filter technique.

2 Methodology

The unconditional point estimate of beta for any asset is given by the Constant Market Risk Model (CMRM):

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it} \tag{1}$$

Where:

R_{it} = the stock returns series

R_{mt} = the index returns series

ε_{it} = the disturbance vector

The CMRM assumes that intercept and slope vectors are constant over time with the latter representing the systematic risk or beta of the firm. The evidence presented in the previous section strongly indicates the instability of the CMRM parameters across various markets. Therefore we use Time Varying Market Risk Model (TVMRM) to establish beta instability. The TVMRM utilizes a binary variable that equals to one when the index return was negative (Bear) and zero when index return was positive (Bull), e.g. if the index return between January and March was negative the binary variable assumes value of 1.

$$R_{it} = \alpha_2 + \alpha_3 D_1 + \beta_2 R_{mt} + \beta_3 R_{mt} D_1 + \varepsilon_{it} \tag{2}$$

Where:

R_{it} = the stock returns series

R_{mt} = the index returns series

D_1 = a binary variable that equals one when index return was negative

ε_{it} = the disturbance vector

Which is equivalent to

$$R_{it} = \alpha_2 + (\alpha_{Bear} - \alpha_{Bull}) D_1 + \beta_2 R_{mt} + (\beta_{Bear} - \beta_{Bull}) R_{mt} D_1 + \varepsilon_{it} \tag{3}$$

2.1 GARCH

This technique involves the use of multivariate GARCH model introduced by Bollerslev (1990). A bi-variate specification of the model is used in this study and the general specification of model is subsequently presented. We begin by specifying the conditional mean:

$$R_{it}^{'} = \varepsilon_{it}^{'} \tag{4}$$

Where

$$R_{it}^{'} = \begin{bmatrix} R_{1t}^{'} \\ R_{2t}^{'} \\ \vdots \\ R_{(n-1)t}^{'} \\ R_{nt}^{'} \end{bmatrix} \text{ and } \varepsilon_{it}^{'} = \begin{bmatrix} \varepsilon_{1t}^{'} \\ \varepsilon_{2t}^{'} \\ \vdots \\ \varepsilon_{(n-1)t}^{'} \\ \varepsilon_{nt}^{'} \end{bmatrix} \tag{5}$$

This may be described as $\varepsilon_{it}' | \psi_{t-1} \sim N(0, H_t)$ that is ε_{it}' is conditioned by the complete information set ψ_{t-1} and is normally distributed with zero mean and a conditional covariance matrix of the form:

$$H_t = \begin{bmatrix} h_{11,t} & h_{12,t} & \cdots & h_{1(n-1),t} & h_{1n,t} \\ h_{21,t} & h_{22,t} & \cdots & h_{2(n-1),t} & h_{2n,t} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ h_{(n-1)1,t} & h_{(n-1)2,t} & \cdots & h_{(n-1)(n-1),t} & h_{(n-1),t} \\ h_{n1,t} & h_{n2,t} & \cdots & h_{n(n-1),t} & h_{nn,t} \end{bmatrix} \tag{6}$$

A functional form must be specified for this conditional variance matrix H_t. The conditional variance in the univariate form may be represented as:

$$\begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{13,t} \end{bmatrix} = \begin{bmatrix} c_{11} \\ c_{12} \\ c_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ (\varepsilon_{1,t-1})(\varepsilon_{2,t-1}) \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{13,t-1} \end{bmatrix} \tag{7}$$

or,

$$H_t = C + A\varepsilon + BH_{t-1} \tag{8}$$

where H_t, C, A, ε and B represent their respective matrices. The model presents a complex problem as the number of coefficients that need to be simultaneously estimated are prohibitively high. In this particular instance there are 21 individual coefficients and increasing the order of the GARCH model would results in simultaneous exponential increase in the coefficients. Bollerslev (1990) proposed to set the off-diagonals in the coefficient matrices equal to zero which results in a new specification of conditional variance of each equation.

$$\begin{aligned} h_{11,t} &= c_{11} + a_{11}\varepsilon_{1,t-1}^2 + b_{11}h_{11,t-1} \\ h_{22,t} &= c_{22} + a_{22}\varepsilon_{2,t-1}^2 + b_{22}h_{22,t-1} \end{aligned} \tag{9}$$

Bollerslev (1990) suggests, assuming correlation between conditional variances constant, to derive conditional covariance. Though such an assumption may provide computational flexibility, this effectively imposes a highly non-linear restriction on the coefficients in Equation (6). Thus conditional covariance from the conditional variance matrices may be estimated as:

$$h_{12,t} = \rho_{12}\sqrt{h_{11,t}h_{22,t}} \tag{10}$$

However, we only need to estimate 7 parameters instead of 21, given that the coefficients a & b ≥ 0 and the coefficients c > 0. The positive sign for H_t can then be guaranteed (Engle and Kroner, 1995). This bi-variate GARCH model provides the necessary elements to construct the time series of beta for any security by estimating Equation (11).

$$\beta_{it} = \frac{\mathrm{cov}_t\left(R_{it}, R_{mt}\right)}{\mathrm{var}_t\left(R_{mt}\right)} \tag{11}$$

Where $\mathrm{var}_t\left(R_{mt}\right)$ is provided in the form of $h_{22,t}$ and $\mathrm{cov}_t\left(R_{it}, R_{mt}\right)$ in the form $h_{12,t}$. As such, the resultant conditional beta comes from a restricted full version of Constant Conditional Correlation GARCH model.

2.2 Dynamic Conditional Correlation GARCH

Sheppard (2001) and Engle (2002) propose the DCC-GARCH model. This model is a generalization of Bollerslev (1990) model and makes use of uni-variate estimates as inputs in the second stage of estimation process as described in Equation (12). Following Engle (2002), the vector of k asset returns is the demeaned vector, $r_t = r_t' - \mu$, and is assumed to be conditionally multivariate normal:

$$\begin{aligned} r_t \mid \Phi_{t-1} &\sim N\left(0, H_t\right) \\ H_t &= D_t R_t D_t \end{aligned} \tag{12}$$

Where:

H_t is the conditional covariance matrix;

R_t is the $k \times k$ time varying correlation matrix.

D_t is a $k \times k$ diagonal matrix of conditional volatility from GARCH(1,1) as follows:

$$h_{i,t} = \omega_i + \sum_{pi} \alpha_{pi} r_{i,t-pi}^2 + \sum_{qi} \beta_{qi} h_{i,t-qi} \tag{13}$$

Dividing each return by its conditional standard deviation $\sqrt{h_{i,t}}$, a vector of standardized returns is obtained, $\varepsilon_t = D_t^{-1} r_t \ where \ \varepsilon_t \sim N(0, R_t)$.This formulation may be used to write Engle's specification of a dynamic correlation structure for the set of returns:

$$Q_t = \left[1 + \sum_m \alpha_m^* + \sum_n \beta_n^*\right]\overline{Q} + \sum_m \alpha_m^* (\varepsilon_{t-m}\varepsilon'_{t-m}) + \sum_n \beta_n^* Q_{t-n}$$
$$R_t = \tilde{Q}_t^{-1} Q_t \tilde{Q}_t^{-1} \tag{14}$$

$\tilde{Q}_t$ is a diagonal matrix containing the square root of the diagonal entries of Q_t . $\overline{Q}_t$ is the matrix of unconditional covariances of the standardized returns from the first stage estimation. Engle shows that the loglikelihood of the estimator may be written as:

$$L = -\frac{1}{2}\sum_{t=1}^{T}\left[k\log(2\pi) + 2\log|D_t| + \log|R_t| + \varepsilon_t^{'} R_t^{-1}\varepsilon_t\right] \tag{15}$$

The first stage of the estimation process replaces R_t with the *k x k* identity matrix to get the first stage likelihood. This reduces to the sum of the log-likelihood of uni-variate GARCH equations. The second stage estimate the DCC parameters in (12) using the original likelihood in (13) conditional on the first stage uni-variate parameter estimates. The estimation procedure and theoretical and empirical properties are extensively discussed in Engle and Sheppard (2001).

2.3 O-GARCH (Principal Component Analysis Approach)

The third approach in this study utilizes principal component analysis for generating Covariance matrices and was proposed by Alexander (2001). Consider the normalized data in a matrix ***X*** of dimensions *T x k* where each column is standardized with mean zero and variance one. If the i^{th} asset return is y_i, then the normalized variables are $x_i = (y_i - \mu_i)/\sigma_i$, where μ_i and σ_i are the mean and standard deviation of y_i for $i = 1, \ldots, k$. Now let **W** be the matrix of eigenvectors of ***X`X/T***, and Λ be the associated diagonal matrix of eigenvalues, ordered according to decreasing magnitude of eigenvalue.[2] The principal components of **Y** are given by the *T x k* matrix

$$P = XW \tag{16}$$

[2] Eigenvectors are a special set of vectors associated with a linear system of equations (i.e. a matrix equation) that are sometimes also known as characteristic vectors, proper vectors, or latent vectors (Marcus and Minc, 1988).

It is easy to show that such a linear transformation of the original factor returns produces transformed risk factor returns **P** that are orthogonal and have variances equal to the eigenvalues in Λ. Since **W** is an orthogonal matrix (14) is equivalent to **X = PW'**, that is

$$y_i = \mu_i + \omega_{i1}^* p_1 + \omega_{i2}^* p_2 + \cdots + \omega_{ir}^* p_r + \varepsilon_i \tag{17}$$

Where $\omega_{ij}^* = \omega_{ij}\sigma_i$ and the error term picks up the approximation from using only the first *r* of the *k* principal components. These *r* principal components are the key risk factors of the system. The *m* principal components are orthogonal so their covariance matrix is a diagonal matrix **D**, and variances of (15) give,

$$Y = ADA' + V\varepsilon \tag{18}$$

Where, $A = \omega_{ij}^*$ is the *k x m* matrix of normalized factor weights, $D = diag(V(p_1),\ldots,V(p_r))$ is the covariance matrix of the principal components and $V\varepsilon$ is the covariance matrix of the errors. Ignoring $V\varepsilon$ gives the approximation that forms the basis of a principal component model for large covariance matrices:

$$V \approx ADA' \tag{19}$$

This provides computational efficiency by calculating only *r* variances instead of $k(k+1)/2$ variances and co-variances of the original system. Moreover, the ***V*** will always be positive semi-definite.

2.4 Evaluation of Models

Each of these three techniques discussed above generates a conditional parameterisation of risk. In an attempt to establish the relative dominance of one technique over another, the following methodology is proposed. The series R_{it} may be forecast in sample $\hat{R}_{it}$ using the market model in equation 1 that is:

$$R_{it}^{\wedge} = \alpha_{it} + \beta_{it} R_{mt} \tag{20}$$

Where:

β_{it} = provided by each of the three techniques previously described

R_{mt} = the return on the market index.

A conditional intercept coefficient series is generated by the GARCH, O-GARCH and DCC-GARCH, may be estimated as:

$$\alpha_{it} = R_{it}^{\wedge} - \beta_{it} R_{mt} \tag{21}$$

i.e. α_{it} is equal to the mean industry return less the mean conditional beta times the mean country index. Having forecast $\hat{R}_{it}$ using each of the conditional beta series, one may asses their goodness of fit by estimating the variance of the forecast errors and the coefficient of determination R^2 (Brooks, Faff, and McKenzie, 1998 and Moonis and Shah, 2003). Furthermore, the use of these techniques for the evaluation of alternative models enables us to compare the performance of the GARCH, DCC-GARCH, and O-GARCH models for estimating time dependent conditional beta series.

3 Data and Results

The data for this study is taken from Datastream. Datastream contains adjusted prices of firms listed on KSE with daily frequency from 1998 onwards. Our data coverage starts in January 1998 and ends in December 2005. We select the sample firms based on the following criteria:

1. The firm must be listed on KSE in January 1998;
2. The market capitalization of the firms must be among the top 50 firms in January 1998; and
3. The rupee value traded of the firm must be among the top 50 firms in January 1998.

This selection criteria returns 38 firms. In Pakistan the turnover is limited to certain blue chip firms and therefore all the firms do not have significant liquidity (i.e. number of days traded and rupee value traded) and therefore we restrict the sample to 38 firms. Should more firms be included in the sample, we would have to account for infrequent trading as well, which would have made our analysis even more complex. Although our sample is restricted to 38 firms, it still accounts for 70% of the KSE 100 market capitalization as of date of writing. The continuously compounded percentage return of each firm and index is calculated as the log of the daily price differences. Table 1 presents descriptive statistics of the daily returns over the period January 1998 to December 2005. It shows that AGIL has the highest mean return of 0.19%, whilst HUBC has the lowest mean return of –0.05%. ICI exhibits the highest daily volatility (5.67%), followed by BOP (4.41%), whilst the lowest volatility in returns is found for ULEVER (2.21%).[3] The returns series collectively is negatively skewed, leptokurtic, and significantly fail the Jarque-Bera test for normality.

[3] Volatility is measured as the standard deviation of daily returns.

3.1 Market Risk Models

Initially the Constant Market Model, as in Equation (1), was estimated for each of the firms using the KSE 100 Index as the market proxy. The market model generated beta estimates (standard errors in parenthesis) are also presented in Table 1 in the last column and reveal that each of the point beta estimates are significantly different from zero for all the firms at the 1% significance level.

The evidence provided in the literature suggests that these market model parameters are unstable over time. Therefore, we estimate Time Varying Market Risk Model for each firm using Equation (2) and report the estimated parameters in Table 2. The results suggest that at 1% significance level, 30% of firms have asymmetric betas, at 5% level it increases to 43% of the firms and at 10% level it reaches 50%. In sum, half of the sample firms listed at KSE have betas that are different in bull and bear market phase at the 10% or lower significance level and therefore it is appropriate to estimate time varying betas using the techniques outlined in the previous section.

3.2 GARCH Conditional Beta

Estimation of GARCH based time varying parameters of firm beta requires fitting GARCH (1, 1) model to the returns data for each of the 38 firms. The details of estimation are reported in Table 3, which presents GARCH parameters (standard errors in parenthesis) for model fitted to each firm's return time series. The correlation between the firm and KSE 100 Index is assumed to be constant and is presented in the last column and also in Table 7. Table 3 shows that ARCH and GARCH terms (α and β respectively) are significant at the 1% level, sum to be less than unity, and satisfy the positively assumptions outlined earlier in Section 2 for all firms except GTYR. Surprisingly, not a single firm exhibits negative ARCH parameter and therefore we include all 38 firms in the sample to carry out further analysis. The highest correlation coefficient value is 86.45% for PTC, while the lowest value of 23.74% is observed for GTYR. Brooks et al. (1998) find that for majority of the industries in sample, the GARCH models are significant in the sense that ARCH and GARCH terms are significant except for paper and packaging, entrepreneurial investors and miscellaneous industrials sectors.

The GARCH (1, 1) specification provides conditional variance for each of the 38 firms and then the beta series is estimated for each firm as detailed in Section 2.1. Table 6 presents first moment along-with the highest and the lowest values in parentheses. We find that the mean of beta series for each firm is similar to the point estimates of beta for each firm as noted in Table 1, however, the high and low conditional beta estimates exhibit high level of variability. The lowest variability is found for ULEVER (1.15/0.18), whereas the highest variability is exhibited by ICI (10.4/0.28).

3.3 DCC-GARCH Conditional Beta

Table 4 presents estimates of a DCC model (standard error in parenthesis). The last two columns show the estimates of DCC (1, 1) parameters represented by α^* and β^*, whereas ω, α and β are from uni-variate GARCH (1, 1) for the firms under consideration. ARCH and GARCH parameters of the DCC-GARCH (1, 1) models are statistically significant at 1% for 30 firms except for HUBC, PGF, and SAIF for which they are significant at the 10% level, for NML at the 5% level and insignificant for ICI, UNBL, GTYR, and ULEVER.

The application of the DCC-GARCH model provides conditional co-variance matrices for each of the 38 firms. We then estimate beta series for each firm as outlined in Section 2 and report in Table 6. It is interesting that the mean of beta series using DCC-GARCH model is similar to the beta reported for each firm in Table 1. However, as in the case of GARCH (1, 1) model, the high and low conditional beta estimates from DCC-GARCH model also show higher variation. The lowest variability (Table 6) is found for ULEVER (1.21/0.08), whereas the highest variability is exhibited by ICI (3.70/–8.45).

3.4 O-GARCH Conditional Beta

The O-GARCH model utilizes the inputs from GARCH (1, 1) model. One of the advantages of this technique is that only GARCH (1, 1) variances of the trend and the principal components need to be estimated and the entire covariance matrix of the original system is only a transformation of these two variances as defined in Equation (16). O-GARCH is highly correlated to the GARCH (1, 1) and there is negligible loss of precision had the GARCH (1, 1) model been used to estimate all the required parameters. In illiquid markets like Pakistan, there is another advantage that volatilities and correlations of all variables in the system can be estimated even when the data is sparse or missing or unreliable.

Table 5 presents estimates of O-GARCH Model (standard error in parenthesis) for all firms. The ARCH and GARCH parameters of all firms are significant at the 1% level except for FABL (5% level), PTC and MLCF (10% level), and ICI and GTYR (insignificant). It is noteworthy that the mean of beta series is again similar to point estimate of beta for each firm, as reported in Table 1.

Again, as in the case of GARCH and DCC-GARCH models, the highest and the lowest conditional beta estimates using the O-GARCH model exhibit variability (Table 6). The variability was consistent using GARCH and DCC-GARCH and the same companies exhibited variability in high-low betas, whereas the variability of high-low beta from estimates of O-GARCH is different. The lowest variability is found for PSO (1.55/0.27), whereas the highest variability is exhibited by AGIL (2.49/−1.12).

In general, all the approaches used in this study to estimate conditional beta appear to provide similar estimation of risk while considering their mean values. In addition, the mean of conditional beta series in most of the cases is not significantly different from beta point estimates computed from Market Risk Model. The comparison of beta estimates from all three models is presented in Table 6 with highest and lowest betas in parenthesis. When considering the range of estimated betas we find that DCC-GARCH model and GARCH model generate beta estimates that vary more over time as compared to those of the O-GARCH model.

Table 7 presents a summary of implied correlation estimates from all the models under consideration. Implied correlations are correlation coefficients between index return series and firm return series generated by different models used in this study and are determined by the following relationship:

$$\rho_{R_{it},R_{mt}} = \frac{Cov(R_{mt}, R_{it})}{\sigma_{R_{mt}}\sigma_{R_{it}}} \tag{22}$$

When considering the range of implied correlation coefficients we find that the O-GARCH model generates correlation coefficients that vary more over time as compared to those of the DCC-GARCH model.

Overall, we find that the estimated parameters are significant and provide evidence in favor of beta instability. We now consider the relative superiority of the alternate models used in the study given the evidence that different models generate different conditional beta series.

3.5 Performance of Time Varying Betas Model

The beta series estimated here suggest that there are differences between beta series generated using techniques described earlier even though the mean of the beta series estimates are not significantly different from each other. Therefore, it is appropriate to rank these models to find out which of the three models generate relatively more accurate measure of risk. We use two measures: the coefficient of determination R^2 and the variance of errors.

Table 8 presents R^2 and the variances of all models used in this study. Our results show significant gains in accuracy in terms of higher mean R^2 for our sample when betas are allowed to vary in comparison to OLS betas, on the contrary, however, we did not find improvement in variance of errors. The average R^2 for the sample firms increased to 0.86

from 0.29 when betas were allowed to be time varying. This finding is similar to that of Brooks, Faff, and Mckenzie (1998) for Australian market, Brooks, Faff, and Arif (1998) for Singaporean market, and Moonis and Shah (2003) for Indian market where time varying betas performed better than their counterpart constant betas. Majority of the earlier studies were not able to capture the conditional correlation. However, to our knowledge, this is the first study that employs conditional correlation models in time varying beta estimation.

We also find that GARCH model performs the best in terms of higher R^2 in comparison to DCC-GARCH and O-GARCH models that hypothetically should have performed better as they allow the correlation to vary as well. However, we find no significant evidence of their dominance in beta estimation despite their popularity in volatility estimation.

4 Conclusion

There is significant evidence suggesting that point estimations of systematic risk are not stable over time. This paper therefore examines the issue of beta instability using the returns of 38 firms listed on KSE over the period 1998-2005. Conditional betas were generated using three different models namely the GARCH, the DCC-GARCH, and the O-GARCH. Given the estimates of time varying betas from different models, it seems that the KSE is not different from other emerging and developed markets in terms of beta stability and that betas are time varying at the KSE. The evidence found here overwhelmingly supports GARCH model on the basis on goodness of fit criterion. The strong evidence in favour of time varying betas highlights the limitations of OLS betas. The superior performance of time varying betas as opposed to OLS betas can be judged by the significant improvement in the R^2.

This paper contributes to the existing literature by accounting for conditional correlation, whereas all the previous work, to the best of our knowledge, has ignored conditional correlation for estimation of time varying betas. This point is rather surprising given their applications in asset allocation since conditional correlation estimates are very important in such decisions. In addition, our paper is an attempt to explore betas and their time varying nature in the context of the Pakistani Market.

The time varying nature of betas has significant implications for portfolio managers (portfolio diversification and hedging) and financial analysts (fair value estimation). The challenge however is to explore ways on how to incorporate the beta instability in such financial decisions.

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Table 1: Summary Statistics and Beta Point Estimates

Figures in parenthesis are standard errors.

Firms	Mean	St. Dev	Kurtosis	Skewness	Jarque Bera	Beta	
ACBL	0.10%	2.61%	9.3831	0.1051	84.28	0.876	(0.0249)
AGIL	0.19%	5.09%	17.7811	0.3522	58.27	0.702	(0.0597)
AICL	0.08%	3.62%	7.59883	–0.104	363.09	1.109	(0.0363)
ASKL	0.03%	3.01%	9.79252	–0.4198	170.27	0.540	(0.0344)
BAHL	0.12%	2.24%	9.00936	–0.5363	82.09	0.424	(0.0254)
BOC	0.01%	2.41%	12.3971	0.4645	89,493.56	0.365	(0.0281)
BOP	0.13%	4.14%	12.14008	–0.6739	249,581.54	1.305	(0.0409)
CHCC	0.09%	3.42%	13.6973	0.0006	335.06	0.639	(0.0390)
DGKC	0.15%	3.86%	5.93526	–0.2338	1,956.35	1.342	(0.0360)
DSFL	0.02%	3.31%	4.97705	0.7102	29.69	1.178	(0.0304)
ENGRO	0.05%	2.70%	5.43053	0.246	477.91	0.899	(0.0260)
FABL	0.08%	3.09%	5.0816	0.1052	245.97	0.920	(0.0313)
FFC	0.05%	2.36%	10.30497	–1.0346	38,410.20	0.860	(0.0213)
GADT	0.05%	3.44%	20.6543	–0.3902	94.3	0.492	(0.0403)
GTYR	0.16%	4.26%	371.74945	13.0991	71.41	0.551	(0.0502)
HUBC	–0.05%	3.23%	18.24532	–1.1908	1,652.59	1.226	(0.0281)
IBFL	0.10%	3.15%	40.95642	–0.1785	412.69	0.854	(0.0331)
ICI	0.02%	5.67%	569.96779	0.0306	71.64	1.054	(0.0647)
INDU	0.14%	2.74%	4.40441	0.3197	73.34	0.614	(0.0303)
KESC	–0.04%	4.05%	7.0915	0.6668	108.75	1.338	(0.0391)
LUCK	0.16%	3.90%	4.6455	0.2559	500.98	1.033	(0.0414)
MCB	0.14%	3.13%	4.53366	–0.1574	213.89	1.138	(0.0283)
MLCF	0.11%	4.15%	4.02612	0.6744	78.3	1.168	(0.0430)
NML	0.12%	3.68%	14.30634	0.7531	319.55	1.121	(0.0370)
PGF	0.06%	3.08%	5.42471	–0.1737	5,674.97	1.057	(0.0289)
PIAA	0.03%	3.83%	4.47486	0.4871	212.48	1.052	(0.0401)
PICIC	0.17%	3.76%	17.05	0.2329	386.3	0.753	(0.0424)
PIOC	0.13%	5.07%	5.02338	0.2065	481.94	0.980	(0.0574)
PSO	0.02%	2.98%	10.53073	–0.4488	451.45	1.234	(0.0235)
PSYL	0.02%	3.56%	9.53285	0.755	264.87	0.550	(0.0413)
PTC	0.02%	2.84%	8.98261	–0.2136	586.22	1.338	(0.0173)
SAIF	0.03%	3.40%	11.87845	–0.2295	498.22	0.502	(0.0397)
SHEL	0.04%	2.55%	7.49827	0.0119	30.06	0.770	(0.0257)
SNBL	0.10%	2.53%	16.69153	–0.625	1,856.16	0.432	(0.0291)
SNGP	0.10%	3.17%	4.90754	0.2018	58.22	1.210	(0.0274)
SSGC	0.05%	3.11%	11.18099	0.8798	152.94	1.149	(0.0276)
ULEVER	0.02%	2.21%	9.37718	0.2625	1,199.49	0.441	(0.0249)
UNBL	0.14%	3.82%	41.94784	–0.9629	155.28	0.679	(0.0438)

Table 2: Time Varying Risk Model Results

Subscripts a, b, and c represent significance levels of 10%, 5%, and 1% for all parameters reported for that firm. All the parameters for a firm with no superscript are insignificant. Figures in parentheses are standard errors.

Firms	Alpha	Alpha Dummy	Beta	Beta Dummy
ACBL	0.000 (0.001)	0.001 (0.001)	0.836 (0.041)	0.068 (0.051)
AGIL	0.000 (0.002)	0.002 (0.002)	0.622 (0.098)	0.131 (0.123)
AICL^{c}	0.000 (0.001)	0.000 (0.001)	0.897 (0.059)	0.329 (0.075)
ASKL	–0.001 (0.001)	0.001 (0.001)	0.509 (0.056)	0.057 (0.071)
BAHL	0.000 (0.001)	0.002 (0.001)	0.421 (0.042)	0.001 (0.052)
BOC^{b}	–0.002 (0.001)	0.002 (0.001)	0.286 (0.046)	0.122 (0.058)
BOP^{c}	0.002 (0.001)	–0.002 (0.002)	1.147 (0.067)	0.258 (0.084)
CHCC	–0.002 (0.001)	0.003 (0.002)	0.622 (0.064)	0.020 (0.080)
DGKC^{c}	0.001 (0.001)	–0.001 (0.001)	1.097 (0.059)	0.387 (0.074)
DSFL	0.001 (0.001)	–0.002 (0.001)	1.165 (0.050)	0.027 (0.063)
ENGRO^{c}	–0.002 (0.001)	0.002 (0.001)	0.801 (0.042)	0.152 (0.053)
FABL	–0.001 (0.001)	0.001 (0.001)	0.928 (0.051)	–0.011 (0.065)
FFC^{c}	–0.001 (0.001)	0.001 (0.001)	0.760 (0.035)	0.163 (0.044)
GADT	–0.001 (0.001)	0.002 (0.002)	0.530 (0.066)	–0.061 (0.083)
GTYR	0.001 (0.002)	0.001 (0.002)	0.499 (0.082)	0.083 (0.104)
HUBC^{b}	–0.002 (0.001)	0.001 (0.001)	1.148 (0.046)	0.133 (0.058)
IBFL^{b}	0.001 (0.001)	–0.001 (0.001)	0.773 (0.054)	0.136 (0.068)
ICI	0.000 (0.002)	–0.001 (0.003)	1.091 (0.106)	–0.058 (0.134)
INDU	0.000 (0.001)	0.001 (0.001)	0.556 (0.050)	0.088 (0.063)
KESC	–0.001 (0.001)	0.000 (0.002)	1.317 (0.064)	0.030 (0.081)
LUCK^{b}	0.001 (0.001)	–0.001 (0.002)	0.909 (0.068)	0.196 (0.085)
MCB	0.001 (0.001)	0.000 (0.001)	1.103 (0.046)	0.055 (0.058)
MLCF^{c}	0.000 (0.001)	0.000 (0.002)	1.017 (0.070)	0.239 (0.089)
NML^{c}	0.000 (0.001)	0.001 (0.001)	0.973 (0.060)	0.227 (0.076)
PGF^{a}	–0.001 (0.001)	0.001 (0.001)	0.985 (0.047)	0.117 (0.060)
PIAA^{c}	–0.001 (0.001)	0.000 (0.002)	0.856 (0.065)	0.307 (0.083)
PICIC	0.002 (0.001)	–0.001 (0.002)	0.771 (0.070)	–0.031 (0.088)
PIOC^{a}	0.000 (0.002)	0.001 (0.002)	0.843 (0.094)	0.212 (0.119)
PSO^{c}	–0.002 (0.001)	0.001 (0.001)	1.158 (0.038)	0.119 (0.049)
PSYL	0.000 (0.001)	–0.001 (0.002)	0.640 (0.068)	–0.137 (0.085)
PTC	0.000 (0.001)	–0.001 (0.001)	1.326 (0.028)	0.015 (0.036)
SAIF^{c}	0.000 (0.001)	0.000 (0.002)	0.321 (0.065)	0.293 (0.082)
SHEL^{c}	–0.002 (0.001)	0.002 (0.001)	0.683 (0.042)	0.137 (0.053)
SNBL	0.001 (0.001)	0.000 (0.001)	0.396 (0.048)	0.053 (0.060)
SNGP^{c}	0.000 (0.001)	0.000 (0.001)	1.098 (0.045)	0.184 (0.057)
SSGC^{a}	–0.001 (0.001)	0.001 (0.001)	1.084 (0.045)	0.097 (0.057)
ULEVER^{b}	–0.001 (0.001)	0.001 (0.001)	0.373 (0.041)	0.107 (0.051)
UNBL	0.001 (0.001)	0.000 (0.002)	0.666 (0.072)	0.031 (0.091)

Table 3: GARCH Model Estimates for KSE Listed Equities

Superscript d indicates that ARCH and GARCH parameters for that firm are insignificant. All the parameters for other firms are significant at 1% level. Figures in parenthesis are standard errors.

Firms	ω		α		β		$\alpha+\beta$	ρ
ACBL	0.000	(0.000)	0.118	(0.007)	0.852	(0.006)	0.9699	0.6173
AGIL	0.000	(0.000)	0.021	(0.001)	0.978	(0.001)	0.9993	0.2535
AICL	0.000	(0.000)	0.231	(0.019)	0.703	(0.019)	0.9336	0.5622
ASKL	0.000	(0.000)	0.248	(0.017)	0.554	(0.022)	0.8024	0.3302
BAHL	0.000	(0.000)	0.173	(0.014)	0.717	(0.018)	0.8902	0.3489
BOC	0.000	(0.000)	0.153	(0.009)	0.757	(0.013)	0.9101	0.2781
BOP	0.000	(0.000)	0.118	(0.008)	0.854	(0.008)	0.9720	0.579
CHCC	0.000	(0.000)	0.082	(0.005)	0.885	(0.006)	0.9667	0.3431
DGKC	0.000	(0.000)	0.127	(0.009)	0.867	(0.008)	0.9948	0.6386
DSFL	0.000	(0.000)	0.106	(0.007)	0.851	(0.009)	0.9571	0.6542
ENGRO	0.000	(0.000)	0.224	(0.015)	0.734	(0.011)	0.9585	0.6113
FABL	0.000	(0.000)	0.133	(0.013)	0.795	(0.017)	0.9282	0.5483
FFC	0.000	(0.000)	0.159	(0.012)	0.798	(0.010)	0.9577	0.6694
GADT	0.000	(0.000)	0.011	(0.001)	0.985	(0.001)	0.9959	0.2629
GTYR[d]	0.002	(0.232)	0.000	(0.003)	0.019	(127.804)	0.0195	0.2374
HUBC	0.000	(0.000)	0.124	(0.005)	0.869	(0.005)	0.9933	0.6969
IBFL	0.000	(0.000)	0.162	(0.008)	0.831	(0.010)	0.9932	0.4983
ICI	0.000	(0.000)	0.036	(0.010)	0.915	(0.016)	0.9512	0.3414
INDU	0.000	(0.000)	0.209	(0.019)	0.615	(0.030)	0.8240	0.4114
KESC	0.000	(0.000)	0.263	(0.017)	0.648	(0.019)	0.9115	0.6064
LUCK	0.000	(0.000)	0.092	(0.010)	0.895	(0.010)	0.9868	0.4864
MCB	0.000	(0.000)	0.130	(0.014)	0.783	(0.019)	0.9129	0.6674
MLCF	0.000	(0.000)	0.078	(0.009)	0.893	(0.011)	0.9706	0.5177
NML	0.001	(0.000)	0.284	(0.018)	0.320	(0.028)	0.6036	0.5598
PGF	0.000	(0.000)	0.151	(0.018)	0.696	(0.029)	0.8472	0.6319
PIAA	0.000	(0.000)	0.095	(0.008)	0.860	(0.009)	0.9547	0.5044
PICIC	0.000	(0.000)	0.164	(0.015)	0.751	(0.017)	0.9154	0.3678
PIOC	0.000	(0.000)	0.056	(0.004)	0.940	(0.004)	0.9969	0.3555
PSO	0.000	(0.000)	0.176	(0.011)	0.819	(0.009)	0.9952	0.7596
PSYL	0.000	(0.000)	0.051	(0.004)	0.916	(0.007)	0.9669	0.2845
PTC	0.000	(0.000)	0.103	(0.007)	0.889	(0.005)	0.9920	0.8645
SAIF	0.000	(0.000)	0.018	(0.001)	0.982	(0.001)	0.9994	0.2711
SHEL	0.000	(0.000)	0.292	(0.024)	0.603	(0.025)	0.8953	0.5552
SNBL	0.000	(0.000)	0.158	(0.012)	0.757	(0.014)	0.9154	0.3139
SNGP	0.000	(0.000)	0.122	(0.008)	0.848	(0.006)	0.9700	0.7011
SSGC	0.000	(0.000)	0.158	(0.012)	0.769	(0.017)	0.9266	0.6798
ULEVER	0.000	(0.000)	0.056	(0.004)	0.931	(0.004)	0.9867	0.3669
UNBL	0.000	(0.000)	0.016	(0.001)	0.984	(0.001)	0.9999	0.3264

Table 4: DCC-GARCH Model Estimates for KSE Listed Equities

Superscripts a and b represent significance levels of 10% and 5% levels and d indicates that they are insignificant. All the parameters for other firms are significant at 1% level. * represents DCC(1,1) parameters whereas all others are GARCH(1,1) estimates. Figures in parentheses are standard errors.

Firms	ω		α		β		α*		β*	
ACBL	0.0000	(0.000)	0.141	(0.047)	0.823	(0.049)	0.025	(0.013)	0.939	(0.034)
AGIL	0.0000	(0.000)	0.031	(0.005)	0.966	(0.000)	0.021	(0.006)	0.976	(0.006)
AICL	0.0001	(0.000)	0.259	(0.050)	0.666	(0.054)	0.033	(0.009)	0.940	(0.014)
ASKL	0.0002	(0.000)	0.257	(0.094)	0.538	(0.115)	0.031	(0.009)	0.947	(0.016)
BAHL	0.0001	(0.000)	0.195	(0.056)	0.670	(0.070)	0.037	(0.013)	0.918	(0.035)
BOC	0.0001	(0.000)	0.190	(0.077)	0.686	(0.085)	0.093	(0.082)	0.500	(0.687)
BOP	0.0001	(0.000)	0.162	(0.056)	0.797	(0.061)	0.058	(0.011)	0.930	(0.015)
CHCC	0.0001	(0.000)	0.128	(0.040)	0.791	(0.054)	0.011	(0.017)	0.989	(0.026)
DGKC	0.0000	(0.000)	0.149	(0.022)	0.843	(0.026)	0.047	(0.011)	0.937	(0.021)
DSFL	0.0000	(0.000)	0.106	(0.022)	0.854	(0.030)	0.044	(0.020)	0.949	(0.029)
ENGRO	0.0000	(0.000)	0.226	(0.068)	0.727	(0.066)	0.076	(0.021)	0.845	(0.067)
FABL	0.0001	(0.000)	0.147	(0.037)	0.757	(0.048)	0.020	(0.010)	0.979	(0.012)
FFC	0.0000	(0.000)	0.180	(0.074)	0.773	(0.073)	0.047	(0.057)	0.822	(0.524)
GADT	0.0000	(0.000)	0.008	(0.003)	0.981	(0.001)	0.043	(0.018)	0.930	(0.033)
GTYR[d]	0.0016	(0.001)	0.000	(0.002)	0.115	(0.697)	0.022	(0.012)	0.978	(0.017)
HUBC[a]	0.0000	(0.000)	0.188	(0.112)	0.802	(0.092)	0.050	(0.025)	0.869	(0.040)
IBFL	0.0000	(0.000)	0.177	(0.043)	0.814	(0.039)	0.047	(0.014)	0.939	(0.022)
ICI[d]	0.0002	(0.001)	0.047	(0.117)	0.897	(0.205)	0.163	(0.024)	0.832	(0.019)
INDU	0.0002	(0.000)	0.237	(0.067)	0.530	(0.081)	0.049	(0.017)	0.909	(0.033)
KESC	0.0002	(0.000)	0.236	(0.065)	0.681	(0.062)	0.045	(0.017)	0.925	(0.030)
LUCK	0.0000	(0.000)	0.109	(0.022)	0.874	(0.029)	0.035	(0.009)	0.959	(0.012)
MCB	0.0001	(0.000)	0.148	(0.039)	0.742	(0.051)	0.029	(0.007)	0.967	(0.008)
MLCF	0.0001	(0.000)	0.098	(0.030)	0.861	(0.043)	0.042	(0.009)	0.946	(0.010)
NML[b]	0.0006	(0.000)	0.287	(0.134)	0.310	(0.200)	0.042	(0.020)	0.944	(0.032)
PGF[a]	0.0002	(0.000)	0.175	(0.102)	0.644	(0.124)	0.049	(0.029)	0.935	(0.055)
PIAA	0.0001	(0.000)	0.106	(0.020)	0.840	(0.029)	0.047	(0.019)	0.910	(0.049)
PICIC	0.0001	(0.000)	0.130	(0.048)	0.803	(0.065)	0.029	(0.008)	0.952	(0.015)
PIOC	0.0000	(0.000)	0.072	(0.005)	0.921	(0.013)	0.031	(0.010)	0.960	(0.012)
PSO	0.0000	(0.000)	0.205	(0.070)	0.786	(0.065)	0.038	(0.012)	0.921	(0.019)
PSYL	0.0001	(0.000)	0.072	(0.021)	0.869	(0.032)	0.000	(0.002)	0.000	(0.691)
PTC	0.0000	(0.000)	0.125	(0.051)	0.859	(0.053)	0.114	(0.032)	0.806	(0.060)
SAIF[a]	0.0003	(0.000)	0.092	(0.053)	0.648	(0.094)	0.016	(0.005)	0.984	(0.006)
SHEL	0.0001	(0.000)	0.315	(0.092)	0.573	(0.087)	0.028	(0.011)	0.947	(0.022)
SNBL	0.0001	(0.000)	0.208	(0.087)	0.597	(0.121)	0.033	(0.011)	0.955	(0.018)
SNGP	0.0000	(0.000)	0.139	(0.041)	0.823	(0.051)	0.040	(0.009)	0.915	(0.020)
SSGC	0.0001	(0.000)	0.178	(0.062)	0.728	(0.072)	0.061	(0.015)	0.881	(0.025)
ULEVER[d]	0.0000	(0.000)	0.080	(0.053)	0.895	(0.056)	0.015	(0.011)	0.958	(0.045)
UNBL[d]	0.0008	(0.000)	0.161	(0.124)	0.263	(0.330)	0.016	(0.005)	0.984	(0.008)

Table 5: O-GARCH Model Estimates for KSE Listed Equities

Superscripts a and b represent significance levels of 10% and 5% levels and d indicates insignificance. All the parameters for other firms are significant at 1% level. Figures in parentheses are standard errors.

Firms	ω		α		β		$\alpha + \beta$	R^2
ACBL	0.008	(0.0022)	0.068	(0.0103)	0.914	(0.0012)	0.982	80.86%
AGIL	0.005	(0.0016)	0.091	(0.0088)	0.905	(0.0020)	0.996	62.67%
AICL	0.012	(0.0035)	0.127	(0.0210)	0.854	(0.0056)	0.982	78.11%
ASKL	0.036	(0.0082)	0.097	(0.0175)	0.852	(0.0053)	0.95	66.51%
BAHL	0.038	(0.0064)	0.141	(0.0194)	0.800	(0.0110)	0.94	67.44%
BOC	0.025	(0.0050)	0.111	(0.0162)	0.857	(0.0059)	0.968	63.91%
BOP	0.004	(0.0017)	0.112	(0.0126)	0.886	(0.0040)	0.998	78.95%
CHCC	0.018	(0.0050)	0.115	(0.0153)	0.864	(0.0048)	0.979	67.16%
DGKC	0.003	(0.0011)	0.091	(0.0079)	0.902	(0.0019)	0.993	81.93%
DSFL	0.004	(0.0013)	0.092	(0.0099)	0.902	(0.0025)	0.994	82.71%
ENGRO	0.009	(0.0023)	0.151	(0.0182)	0.840	(0.0067)	0.991	80.56%
FABL[b]	0.004	(0.0018)	0.066	(0.0079)	0.930	(0.0007)	0.996	77.41%
FFC	0.021	(0.0053)	0.139	(0.0232)	0.803	(0.0108)	0.942	83.47%
GADT	0.018	(0.0051)	0.125	(0.0178)	0.855	(0.0064)	0.98	63.15%
GTYR[d]	0.003	(0.0039)	0.044	(0.0216)	0.956	(0.0004)	1	61.87%
HUBC	0.026	(0.0079)	0.200	(0.0421)	0.724	(0.0356)	0.925	84.85%
IBFL	0.006	(0.0018)	0.097	(0.0117)	0.893	(0.0028)	0.99	74.92%
ICI[d]	0.038	(0.0347)	0.067	(0.0487)	0.884	(0.0046)	0.951	67.07%
INDU	0.008	(0.0027)	0.070	(0.0087)	0.919	(0.0013)	0.988	70.57%
KESC	0.019	(0.0037)	0.126	(0.0229)	0.831	(0.0102)	0.957	80.32%
LUCK	0.003	(0.0013)	0.069	(0.0068)	0.925	(0.0009)	0.994	74.32%
MCB	0.005	(0.0015)	0.077	(0.0097)	0.909	(0.0018)	0.986	83.37%
MLCF[a]	0.006	(0.0034)	0.046	(0.0081)	0.940	(0.0003)	0.986	75.88%
NML	0.009	(0.0029)	0.154	(0.0317)	0.846	(0.0131)	1	77.99%
PGF	0.025	(0.0090)	0.107	(0.0214)	0.827	(0.0101)	0.934	81.59%
PIAA	0.014	(0.0030)	0.101	(0.0116)	0.873	(0.0038)	0.974	75.22%
PICIC	0.028	(0.0089)	0.076	(0.0155)	0.878	(0.0042)	0.954	68.39%
PIOC	0.004	(0.0016)	0.046	(0.0048)	0.946	(0.0004)	0.993	67.78%
PSO	0.008	(0.0017)	0.185	(0.0265)	0.793	(0.0108)	0.978	87.98%
PSYL	0.043	(0.0096)	0.087	(0.0141)	0.851	(0.0041)	0.938	64.22%
PTC[a]	0.017	(0.0094)	0.315	(0.0872)	0.616	(0.0531)	0.931	93.22%
SAIF	0.021	(0.0052)	0.150	(0.0251)	0.831	(0.0114)	0.981	63.55%
SHEL	0.015	(0.0033)	0.101	(0.0143)	0.867	(0.0037)	0.968	77.76%
SNBL	0.024	(0.0057)	0.092	(0.0130)	0.876	(0.0036)	0.968	65.70%
SNGP	0.013	(0.0039)	0.080	(0.0166)	0.875	(0.0056)	0.955	85.05%
SSGC	0.046	(0.0158)	0.211	(0.0529)	0.649	(0.0571)	0.861	83.99%
ULEVER	0.020	(0.0048)	0.110	(0.0165)	0.861	(0.0046)	0.972	68.35%
UNBL	0.009	(0.0035)	0.095	(0.0181)	0.897	(0.0025)	0.992	66.32%

Table 6: Equity Betas of KSE Listed Equities

Figures in parentheses represent standard errors for the second column and the highest and the lowest beats for other columns.

Firms	OLS β		GARCH β		DCC-GARCH β		O-GARCH β	
ACBL	0.88	(0.025)	0.98	(3.87 / 0.34)	0.80	(1.33 / –0.01)	0.95	(2.86 / 0.28)
AGIL	0.70	(0.060)	0.73	(2.14 / 0.18)	0.96	(2.49 / –1.12)	0.95	(2.82 / –0.31)
AICL	1.11	(0.036)	1.24	(4.87 / 0.45)	1.06	(1.85 / –0.58)	1.26	(4.54 / 0.31)
ASKL	0.54	(0.034)	0.64	(2.18 / 0.18)	0.40	(1.45 / –0.69)	0.52	(2.88 / –1.12)
BAHL	0.42	(0.025)	0.50	(1.56 / 0.13)	0.43	(1.10 / –0.54)	0.53	(1.57 / –0.02)
BOC	0.36	(0.028)	0.42	(1.64 / 0.10)	0.34	(1.09 / –0.48)	0.40	(2.17 / –0.82)
BOP	1.30	(0.041)	1.38	(4.82 / 0.43)	1.34	(2.16 / –0.13)	1.42	(3.46 / 0.16)
CHCC	0.64	(0.039)	0.75	(2.03 / 0.20)	0.61	(1.56 / –0.83)	0.76	(2.78 / 0.11)
DGKC	1.34	(0.036)	1.45	(3.40 / 0.55)	1.27	(2.05 / –0.37)	1.40	(3.68 / 0.24)
DSFL	1.18	(0.030)	1.34	(2.98 / 0.50)	1.15	(1.72 / –0.09)	1.31	(2.36 / 0.41)
ENGRO	0.90	(0.026)	0.98	(3.39 / 0.33)	0.83	(1.44 / –0.25)	0.92	(3.28 / 0.13)
FABL	0.92	(0.031)	1.09	(3.00 / 0.33)	0.85	(1.60 / –0.15)	1.04	(2.37 / 0.27)
FFC	0.86	(0.021)	0.93	(2.54 / 0.34)	0.79	(1.22 / –0.20)	0.88	(2.46 / 0.25)
GADT	0.49	(0.040)	0.56	(1.30 / 0.16)	0.60	(1.52 / –0.79)	0.70	(4.50 / –0.24)
GTYR	0.55	(0.050)	0.70	(1.22 / 0.14)	0.81	(1.57 / –0.26)	1.01	(2.86 / –0.16)
HUBC	1.23	(0.028)	1.18	(3.46 / 0.44)	1.07	(1.67 / –0.62)	1.10	(3.90 / 0.26)
IBFL	0.85	(0.033)	0.88	(2.15 / 0.23)	0.81	(1.66 / –0.63)	0.86	(2.81 / 0.04)
ICI	1.05	(0.065)	1.14	(10.40 / 0.28)	1.15	(1.93 / –0.27)	1.84	(3.70 / –8.45)
INDU	0.61	(0.030)	0.74	(2.71 / 0.18)	0.58	(1.38 / –0.34)	0.72	(2.18 / –0.26)
KESC	1.34	(0.039)	1.52	(6.61 / 0.46)	1.23	(2.06 / –0.86)	1.43	(5.93 / –0.50)
LUCK	1.03	(0.041)	1.14	(2.64 / 0.33)	1.07	(2.02 / –0.82)	1.21	(2.84 / 0.18)
MCB	1.14	(0.028)	1.26	(3.16 / 0.56)	1.08	(1.58 / 0.14)	1.21	(2.50 / 0.37)
MLCF	1.17	(0.043)	1.35	(4.81 / 0.40)	1.13	(2.13 / –0.37)	1.32	(3.15 / 0.02)
NML	1.12	(0.037)	1.32	(8.77 / 0.35)	1.13	(1.87 / –0.39)	1.35	(6.32 / 0.14)
PGF	1.06	(0.029)	1.25	(5.73 / 0.39)	0.97	(1.56 / 0.12)	1.16	(3.22 / 0.32)
PIAA	1.05	(0.040)	1.22	(3.66 / 0.37)	1.02	(1.81 / –0.22)	1.19	(2.72 / 0.12)
PICIC	0.75	(0.042)	0.90	(6.41 / 0.16)	0.74	(1.72 / –0.32)	0.95	(2.76 / 0.16)
PIOC	0.98	(0.057)	1.12	(2.65 / 0.37)	1.02	(2.50 / –1.02)	1.21	(3.43 / –0.62)
PSO	1.23	(0.024)	1.20	(2.77 / 0.58)	1.16	(1.55 / 0.27)	1.16	(2.92 / 0.41)
PSYL	0.55	(0.041)	0.65	(1.42 / 0.20)	0.47	(1.71 / –0.99)	0.60	(1.37 / 0.17)
PTC	1.34	(0.017)	1.36	(3.84 / 0.71)	1.26	(1.53 / –0.41)	1.28	(2.37 / 0.36)
SAIF	0.50	(0.040)	0.59	(1.22 / 0.18)	0.55	(1.68 / –1.12)	0.75	(2.57 / –0.04)
SHEL	0.77	(0.026)	0.85	(2.85 / 0.28)	0.68	(1.24 / –0.16)	0.77	(2.62 / 0.19)
SNBL	0.43	(0.029)	0.52	(3.82 / 0.15)	0.41	(1.23 / –0.45)	0.52	(3.64 / –0.20)
SNGP	1.21	(0.027)	1.31	(4.47 / 0.60)	1.13	(1.65 / 0.22)	1.25	(3.04 / 0.50)
SSGC	1.15	(0.028)	1.26	(4.90 / 0.45)	1.07	(1.62 / –0.35)	1.21	(3.66 / 0.36)
ULEVER	0.44	(0.025)	0.47	(1.15 / 0.18)	0.35	(1.00 / –0.45)	0.43	(1.21 / 0.08)
UNBL	0.68	(0.044)	0.80	(2.13 / 0.17)	0.70	(1.86 / –0.51)	0.91	(2.69 / 0.04)

Table 7: Implied Correlation Coefficients

Figures in parentheses are negative values.

Firms	GARCH	DCC-GARCH			O-GARCH		
		Avg	High	Low	Avg	High	Low
ACBL	0.6173	0.5648	0.9394	(0.0089)	0.5991	0.7908	0.3984
AGIL	0.2535	0.3466	0.8996	(0.4050)	0.3647	0.7423	(0.0576)
AICL	0.5622	0.5353	0.9396	(0.2930)	0.5782	0.8085	0.2077
ASKL	0.3302	0.2425	0.8888	(0.4220)	0.2801	0.6277	(0.2930)
BAHL	0.3489	0.3522	0.9005	(0.4400)	0.3756	0.7043	(0.0095)
BOC	0.2781	0.2567	0.8282	(0.3630)	0.2721	0.7324	(0.2230)
BOP	0.5790	0.5969	0.9598	(0.0567)	0.6134	0.9261	0.0008
CHCC	0.3431	0.3262	0.8379	(0.4450)	0.3504	0.5981	0.0010
DGKC	0.6386	0.6061	0.9774	(0.1730)	0.6190	0.9057	0.1422
DSFL	0.6542	0.6391	0.9564	(0.0487)	0.6538	0.9151	0.1164
ENGRO	0.6113	0.5655	0.9763	(0.1670)	0.5811	0.8304	0.1627
FABL	0.5483	0.5058	0.9515	(0.0919)	0.5335	0.7852	0.2918
FFC	0.6694	0.6180	0.9458	(0.1520)	0.6299	0.8678	0.2591
GADT	0.2629	0.3224	0.8092	(0.4220)	0.3248	0.6879	(0.1810)
GTYR	0.2374	0.3493	0.6758	(0.1100)	0.3363	0.6439	(0.0393)
HUBC	0.6969	0.6104	0.9472	(0.3540)	0.6468	0.8589	0.1070
IBFL	0.4983	0.4719	0.9696	(0.3700)	0.4895	0.8973	0.0003
ICI	0.3414	0.3728	0.6249	(0.0871)	0.6068	0.9427	(0.2430)
INDU	0.4114	0.3892	0.9286	(0.2300)	0.4078	0.7685	(0.1610)
KESC	0.6064	0.5562	0.9334	(0.3870)	0.5857	0.8059	(0.1380)
LUCK	0.4864	0.5058	0.9488	(0.3840)	0.5233	0.8379	0.0006
MCB	0.6674	0.6361	0.9272	0.0008	0.6485	0.8754	0.2741
MLCF	0.5177	0.5014	0.9427	(0.1660)	0.5201	0.8725	0.0001
NML	0.5598	0.5630	0.9361	(0.1960)	0.5834	0.8418	0.1538
PGF	0.6319	0.5799	0.9342	0.0007	0.5994	0.8527	0.1952
PIAA	0.5044	0.4901	0.8694	(0.1070)	0.4995	0.7964	0.0004
PICIC	0.3678	0.3592	0.8385	(0.1550)	0.4041	0.6224	0.1427
PIOC	0.3555	0.3683	0.9061	(0.3690)	0.3910	0.7414	(0.1690)
PSO	0.7596	0.7163	0.9556	0.1656	0.7409	0.9124	0.5496
PSYL	0.2845	0.2447	0.8850	(0.5110)	0.2592	0.2592	0.2591
PTC	0.8645	0.8113	0.9897	(0.2620)	0.8221	0.9769	0.1448
SAIF	0.2711	0.2982	0.9084	(0.6030)	0.3206	0.6138	(0.0300)
SHEL	0.5552	0.4888	0.8978	(0.1110)	0.5109	0.7588	0.0009
SNBL	0.3139	0.3008	0.8911	(0.3250)	0.3274	0.6557	(0.0515)
SNGP	0.7011	0.6548	0.9586	0.1271	0.6748	0.8849	0.1415
SSGC	0.6798	0.6355	0.9580	(0.2070)	0.6644	0.8987	0.0009
ULEVER	0.3669	0.2949	0.8288	(0.3710)	0.3294	0.5454	0.1383
UNBL	0.3264	0.3378	0.8934	(0.2450)	0.3621	0.6886	0.0002

Table 8: Performance of Time Varying Betas

This table presents coefficient of determination R^2 and the variances of the errors.

Firms	OLS		GARCH		DCC-GARCH		O-GARCH	
	R-Sqr	Var(E)	R-Sqr	Var(E)	R-Sqr	Var(E)	R-Sqr	Var(E)
ACBL	0.40	0.03%	0.87	0.04%	0.89	0.04%	0.94	0.04%
AGIL	0.09	0.02%	0.76	0.25%	0.80	0.24%	0.73	0.25%
AICL	0.32	0.04%	0.87	0.09%	0.87	0.09%	0.89	0.10%
ASKL	0.13	0.01%	0.82	0.08%	0.76	0.08%	0.73	0.08%
BAHL	0.16	0.01%	0.85	0.04%	0.83	0.04%	0.76	0.04%
BOC	0.10	0.00%	0.81	0.05%	0.69	0.05%	0.65	0.05%
BOP	0.35	0.06%	0.88	0.12%	0.88	0.12%	0.88	0.12%
CHCC	0.15	0.01%	0.85	0.11%	0.77	0.11%	0.73	0.11%
DGKC	0.43	0.06%	0.91	0.09%	0.88	0.09%	0.92	0.09%
DSFL	0.44	0.05%	0.91	0.06%	0.90	0.06%	0.93	0.06%
ENGRO	0.38	0.03%	0.87	0.05%	0.86	0.05%	0.89	0.05%
FABL	0.31	0.03%	0.87	0.07%	0.88	0.07%	0.89	0.07%
FFC	0.46	0.02%	0.93	0.03%	0.92	0.03%	0.94	0.03%
GADT	0.09	0.01%	0.87	0.11%	0.63	0.11%	0.62	0.11%
GTYR	0.07	0.01%	0.83	0.17%	0.68	0.17%	0.82	0.17%
HUBC	0.50	0.05%	0.89	0.05%	0.87	0.05%	0.93	0.05%
IBFL	0.28	0.02%	0.87	0.08%	0.77	0.08%	0.82	0.07%
ICI	0.12	0.04%	0.71	0.30%	0.88	0.30%	0.94	0.29%
INDU	0.20	0.01%	0.84	0.06%	0.79	0.06%	0.79	0.06%
KESC	0.39	0.06%	0.85	0.10%	0.83	0.10%	0.92	0.11%
LUCK	0.28	0.04%	0.88	0.12%	0.85	0.11%	0.84	0.12%
MCB	0.45	0.04%	0.89	0.06%	0.91	0.06%	0.95	0.06%
MLCF	0.29	0.05%	0.87	0.13%	0.86	0.13%	0.89	0.13%
NML	0.34	0.04%	0.83	0.10%	0.82	0.10%	0.85	0.09%
PGF	0.42	0.04%	0.89	0.06%	0.88	0.06%	0.94	0.06%
PIAA	0.29	0.04%	0.87	0.11%	0.84	0.11%	0.86	0.11%
PICIC	0.15	0.02%	0.82	0.12%	0.83	0.12%	0.83	0.13%
PIOC	0.16	0.03%	0.83	0.22%	0.77	0.22%	0.76	0.22%
PSO	0.58	0.05%	0.94	0.04%	0.92	0.04%	0.97	0.04%
PSYL	0.11	0.01%	0.85	0.12%	0.85	0.12%	0.76	0.12%
PTC	0.76	0.06%	0.95	0.02%	0.96	0.02%	0.97	0.02%
SAIF	0.12	0.01%	0.87	0.11%	0.66	0.10%	0.66	0.11%
SHEL	0.32	0.02%	0.89	0.05%	0.87	0.05%	0.88	0.05%
SNBL	0.12	0.01%	0.83	0.06%	0.76	0.06%	0.73	0.06%
SNGP	0.50	0.05%	0.90	0.05%	0.92	0.05%	0.96	0.05%
SSGC	0.47	0.04%	0.86	0.06%	0.91	0.05%	0.94	0.05%
ULEVER	0.15	0.01%	0.87	0.04%	0.83	0.04%	0.75	0.04%
UNBL	0.14	0.02%	0.80	0.14%	0.74	0.13%	0.72	0.13%