Essays on Banking and Finance

Jungu Yang

A thesis presented for the degree of
Doctor of Philosophy

Department of Economics
University of Kent
United Kingdom
Nov. 2017

Essays on Banking and Finance

Jungu Yang

Abstract

The first chapter studies how two different financial institutions– asset markets and financial intermediaries– allow consumers to allocate their resources over a period and provide liquidity to them when facing liquidity preference shocks and random returns on illiquid assets. For that purpose, an overlapping generations model in which agents are subject to random relocation shocks is formed. Fiat money is introduced explicitly as a tool for consumers to address limited communication and random relocation problems. This article also analyses the mechanism of asset price determination when the economy is hit by several stochastic shocks by drawing together the results of the scattered studies into a comprehensive view. The framework and outcomes of this chapter will also be used as the basis for chapters 2 and 3.

In Chapter 2 it is shown that under asymmetric information and limited participation in financial markets, bank runs (or bankruptcy) may help uninformed agents to achieve an efficient allocation since bank runs can reveal hidden information. The production of information is made efficiently without cost, at which point there is a distinction between this paper and most other related studies. The efficient bank run provides a new ground for the coexistence of banks and financial markets. Even when all the agents deposit their whole endowment goods with the bank, financial intermediaries and markets coexist once bank runs happen. Allowing a run implies that investment in liquidity can be minimised. This effect is strengthened when agents have limited access to a market.

In Chapter 3 a general equilibrium model is made by applying the *Lucas island model* and the *random-relocation model* and the following results are derived: (1) The amount of liquid assets held by the whole economy, as well as the regions where securitization is present, decreases significantly with securitization. As the economy can invest its resources more efficiently, both consumption and welfare increase. The impact of securitization on banks' liquidity, consumption, and welfare, however, depend on the population structure of each region. This result may have important policy implications with regards to the effect of global asset shortages on bubbles and global imbalances. (2) Expansionary monetary policy can affect the bank's portfolio composition, but this outcome depends on the magnitude of the elasticity of substitution coefficients. (3) A randomised monetary policy combined with other real shocks cause banks to suffer from a signal extraction problem.

Declaration

I certify that the thesis I have presented for examination for the PhD degree of the University of Kent is solely my own work.

Acknowledgements

Firstly, I would like to express my sincere gratitude to my advisor Prof. Jagjit S. Chadha for the continuous support of my PhD study and related research, for his patience, motivation, and immense knowledge.

Besides my advisor, I would like to thank the rest of my thesis committee: Prof. Miguel León-Ledesma and Prof. Herakles Polemarchakis for their insightful comments and encouragement, but also for the hard question which incented me to widen my research from various perspectives. Prof. Miguel León-Ledesma was willing to discuss me whenever I was in trouble and encouraged me warmly.

A special thanks to my family. Words cannot express how grateful I am to my mother and father. Without them, this dissertation could not have begun. I would also like to thank my mother-in-law, father-in-law who supported and encouraged me to strive towards my goal. I would like express appreciation to my beloved wife Chae In who spent sleepless nights with and was always my support in the moments when there was no one to answer my queries. Without her I could not have finished this journey. Last but not the least, I dedicate this PhD thesis to my lovely son Seonou who was born when I started to write my thesis and grows more brightly and lovingly than anyone else.

Contents

List of Figures

List of Tables

Thesis Introduction

One of the biggest lessons we learned during the 2008 financial crisis is that we need to better understand the relationship between liquidity, the financial intermediaries, and financial markets. The financial crises that occurred at different times are not so different from each other in their causes and process (Reinhart and Rogoff, 2009). Often the crises are accompanied by a bubble in asset prices and their collapses, and associated with the banking crisis. Also, many instances of crises have been preceded by financial innovations which accelerate asset price bubbles. The collapse of the bubble and the resulting sharp fall in asset prices, on the one hand, increase the demand for liquidity of financial institutions and on the other, they lead to a decline in liquidity supply in the financial market. This liquidity shortage may force financial institutions to sell their assets at fire-sale prices, which in turn accelerates the price collapse.

This thesis studies the relationship between financial intermediaries[1] and markets. Liquidity and asset prices play a vital role in establishing relationships between two different financial systems. In some cases, markets and banks will be analysed separately, and in some cases, we will see how equilibria are achieved when two institutions coexist. Consumers face liquidity preference shocks in the middle of their period, and thus they need to hold liquid assets in order to cope with immediate consumption even though liquid assets earn low returns. Both market and banks function to provide liquidity to them. Their role, however, has a different impact on consumers' consumption and well-being, depending on various circumstances.

[1]Financial intermediaries are defined as the voluntary coalition by agents in this chapter, and intermediaries, coalitions are used interchangeably with 'banks'.

Let's take a closer look at the relationship between financial intermediaries and markets before entering each chapter in earnest. There have been long debates on the role of financial intermediaries and markets in providing liquidity to consumers since the studies of Bryant (1980) and Diamond and Dybvig (1983). However, the argument for the role and functions of financial intermediaries and markets, and the relationship between the two have yet to be clearly resolved. This is partly because, as von Thadden (1999) pointed out, banking operation is varied and financial intermediaries provide several different functions at the same time.[2] Therefore, questions and debates about the role of banks and markets often appear to be contradictory, and "can only be productive if the context is spelled out clearly."(von Thadden, 1999, p.992)

On the one hand, the mere fact that financial intermediaries exist may imply that direct trade in a market between lenders and borrowers is dominated (Gorton and Winton, 2003). This is because those institutions provide their customers with some special services which cannot be easily replicated in capital markets. The financial intermediaries, for example, dominate the market where securities, issued by borrowers, are traded (Diamond, 1984). Another advantage of allocating resources through banks is that banks are able to provide more liquidity[3] than markets if the degree of relative risk aversion, σ,[4] is greater than 1. According to Diamond and Dybvig (1983), banks make this possible through cross-subsidization between agents facing liquidity shocks. Therefore, the liquidity provision by banks allows consumers to achieve higher utility level than the one which can be attained when they directly invest in the production technology or use financial markets only. This discussion implies that financial intermediaries dominate financial markets, and there is no rational for the existence of financial markets.

[2]Gorton and Winton (2003) classifies the role of banks as delegated monitors, information producers, consumption smoothers, and liquidity provision.

[3]While liquidity usually means marketability, the meaning of 'liquid asset' in Diamond and Dybvig (1983) refers to its allowing for intertemporal consumption by economic agents when they wish to. When I mean by 'liquid asset' from now on indicates the same meaning as in Diamond and Dybvig (1983).

[4]σ is defined as $\sigma = -\frac{u''(c)c}{u'(c)}$.

The role of banks in Diamond and Dybvig (1983), however, may exist only "in an environment in which people are isolated from each other" (Wallace, 1988, p.3). If there exist full-participation markets, no cross-subsidization among early and late consumers is possible, and no beneficial role of banks exists. According to Jacklin (1987),[5] banks are able to enhance liquidity through demand deposits only when the direct holding of assets is restricted. If markets exist where equity shares are traded, then this trading mechanism will weakly dominate intermediation.

Banks, on the other hand, would have no role in an economy where perfect financial markets exist. For example, if agents have access to complete market, then "in a competitive equilibrium, the size and composition of banks' balance sheet have no effect on other economic agents." (Freixas and Rochet, 2008, p.10). Therefore, introducing both financial intermediaries and markets into an economy is assuming either explicitly or implicitly the imperfect financial market. Informational asymmetries[6] and limited participation[7] in a market are used often in the literature to model incomplete markets structure. For example, Diamond (1997), faced with the criticism described above, presents a model of limited participation where there is still a scope for banks. With limited participation, banks create more liquidity than financial markets and make the market more liquid. Financial intermediaries and markets coexist, and the amount of cross-subsidization is reduced as more agents participate in the market, but as long as there is a limited-participation market, it is not eliminated. This discussion will be dealt with in more detail in chapter 2.

Chapter 1 analyses how markets and banks allocate resources over a period and if each institution may provide liquidity efficiently to consumers who face uncertainties. Consumers have preferences for liquidity because they are subject to relocations shocks in the middle of their lives and thus uncertain about the place of their consumption when an investment decision has to be made. The model used throughout the thesis contrasts with the standard Diamond and Dybvig (1983)'s

[5]See Haubrich and King (1990) and Hellwig (1994) in a similar vein

[6]The related papers are, for example, Townsend (1979), Diamond (1984), Williamson (1986), and Gorton and Pennacchi (1990)

[7]See Allen and Gale (1994), Diamond (1997), and Allen and Gale (2009).

model in that liquidity shocks are represented by the uncertainty about the place of consumption rather than the timing of it. There is a maturity structure between liquid and illiquid assets; more liquid assets yield a lower rate of return than that of less liquid assets. Investors are willing to hold liquid assets despite the maturity structure because of liquidity preference. The amount of liquidity provided to agents differs depending on the availability of different institutional frameworks (for example, whether they have access to markets and financial intermediaries or not, and how the two institutions interplay each other), their preferences for liquidity (attitude toward risk), and the information structure.

Also, this chapter provides the mechanism of asset prices determination, and how asset prices affect the market equilibrium and the agent's welfare. Many researchers have studied these important problems, but their studies tend to be scattered around the author's interests. This chapter is meaningful in that it draws together the results of the scattered studies into a comprehensive view. The framework and outcomes of this chapter will also be used as the basis for chapters 2 and 3.

The purpose of the second chapter is to examine whether uninformed consumers who face uncertainty can prevent losses arising from asymmetric information by constructing a bank and operating it in a particular way. For that, this chapter presents a theoretical model showing that financial intermediaries can play a special role, that of revealing information without investing resources in order to identify information when financial markets are imperfect. While this article shares some similarities with standard models in that banks manage the problems resulting from asymmetric information, the mechanism of processing or revealing information is quite different from them.

The biggest difference between the previous studies and mine is that the financial coalition by agents in this model does not produce information deliberately by investing some of its resources. The production of information is done efficiently without cost throughout bank runs under certain conditions. In the process of motivating the rationale for the existence of financial intermediation, information-asymmetric problems of the uninformed agents are solved in relation to informed agents. I argue that under asymmetric information and limited participation in financial markets, bank runs (or bankruptcy) may help agents to achieve an efficient allocation since bank runs can reveal hidden information, to which only a fraction of agents in the economy are assumed to have access.

Therefore, the financial intermediaries in this model not only serve the standard role of liquidity provision but also have an alternative justification based on information processing when the economy faces uncertainty. This chapter also provides a new ground for the coexistence of banks and financial markets. Even when all the agents deposit their whole endowment goods with the bank, financial intermediaries and markets coexist once (efficient) bank runs happen. The scale of banks under asymmetric information and limited participation in relation to financial markets are also considered.

The third chapter aims to analyse how monetary policy can affect the composition of banks' portfolios and whether monetary policy can change the composition of assets in the direction desired by the monetary authorities. The second part of this chapter aims to examine how securitization affects the need for liquid assets of individual banks, and the economy as a whole. To address the problem, I used a simple overlapping generations model with fiat money, following the Lucas' island model and its simplified version, Wallace (1980). However, this chapter deals with the general equilibrium problem, unlike Lucas (1972) and Wallace (1980), all of which construct partial equilibrium models. The general equilibrium problem arises because of the framework whereby the two islands are linked to each other. Some of the members of each island move to different islands at some point after random relocation shocks, which, in combination with other nominal and real uncertainties, produces interesting results. Later in this chapter, we set up a model that explains the relationship between securitization and the bank's liquid asset holdings, as well as the process by which banks in two different regions trade securities. This framework provides a detailed description of how the changes in demand and supply structure of securities affect various key economic variables.

The first major finding of the third chapter is that expansionary monetary policy can affect the bank's portfolio composition and thus its depositors' consumption and welfare, but the outcome depends on the magnitude of the elasticity of substitution coefficients. When the consumer is less (resp. more) risk averse, the substitution effect dominates (resp. is dominated by) the income effect and banks are willing to hold more (resp. less) liquid assets in response to higher price levels than would otherwise be the case. However, if a randomised monetary policy combines with other real shocks, the bank suffers from a signal extraction problem. Since banks cannot differentiate between the two elements of real and nominal factors that determine the current price level, they react to changes in the nominal supply of money, not real ones. This results in banks holding too many, and in some cases too few, liquid assets when compared to under full information. Another interesting question is whether monetary policy can change the composition of assets in the direction desired by the monetary authorities. It is shown that the benefits from inflation surprises disappear when economic agents understand the monetary authorities' intention and act accordingly. This leads only to inflation bias as Kydland and Prescott (1977), Barro and Gordon (1983a,b)'s research shows.[8]

The second part of this chapter aims to examine how securitization affects the need for liquid assets of individual banks, and the economy as a whole. First of all, securitization not only allows banks to transform illiquid assets into liquid assets but it also provides an alternative source for investment in liquid assets. In addition to the liquid asset holdings in the region where securitization is possible, the liquid asset reserves of the whole region combined are significantly reduced compared with before securitization. This implies the economy uses its resources in a more productive way to increase the total output across the two islands. One of the main conclusions is

[8]Kydland and Prescott (1977), Barro and Gordon (1983a), and Barro and Gordon (1983b) showed the positive correlation between output and inflation cannot arise systematically when agents have rational expectations. The benefits from inflation surprises disappear when economic agents understand the monetary authorities' intention and act accordingly and lead only to inflation bias.

that the impact of securitization on banks' liquidity, consumption, and welfare is affected by the population structure of each island. If we assume two regions with different demographics, the demographic structure affects not only the supply of securities in an area where securitization is possible but also the size of demand in the other region, which may lead to a different equilibrium through adjustment of securities prices.

These results have important policy implications, especially when looking at the work of Caballero (2006), Caballero et al. (2008), and Caballero and Krishnamurthy (2009), which analyse the effect of global asset shortages on bubbles and global imbalances. Also, these results have great similarities with the study of Bencivenga and Smith (1991). Based on the random-relocation economy, Bencivenga and Smith (1991) showed that the introduction of financial intermediaries might affect the composition of savings toward more productive assets, capital, and thus intermediaries enhance growth by reducing socially unnecessary capital liquidation. Securitization introduced in this chapter shows that the need for liquid assets, required for movers facing liquidity shocks but provide low rates of return, can be reduced significantly in a general equilibrium setting.

Chapter 1

Liquidity, Asset Prices and Welfare

1.1 Introduction

One of the biggest lessons we learned during the 2008 financial crisis is that we need to better understand the relationship between liquidity, the financial intermediaries, and financial markets. Liquidity is one of the investor's main concerns because they may face liquidity preference shocks; thus they need to hold liquid assets in order to cope with immediate consumption, even though liquid assets earn low returns. Depositing with a bank has been regarded as a solution to this maturity structure problem since the seminal studies of Bryant (1980) and Diamond and Dybvig (1983) on banking. However, this maturity structure may cause financial intermediaries to be vulnerable to consumers' sudden demands for liquidity because of the maturity mismatch between the bank's liquid liabilities and illiquid assets. Liquidity also affects financial markets via changes in asset prices because liquidity affects the demand for, and supply of, the assets directly. Changing asset prices influence the expected returns on the illiquid assets; thus change consumers' portfolio decisions. Investors' changing preferences toward more liquid assets (for example, from commercial papers or ABS toward U.S. government securities) would affect consumption, borrowing firms' financing needs, and growth. There have been long debates on the role of financial intermediaries and markets in providing liquidity to consumers. However, the argument for the role and functions of financial intermediaries and markets, and the relationship between the two have yet to be clearly resolved.

This chapter aims to analyse the role of markets and banks in allocating resources over the period, and in providing liquidity to consumers who face uncertainties. Consumers have a preference for liquidity because they are subject to relocation shocks in the middle of their lives, and are thus uncertain about the place of their consumption when an investment decision has to be made. There is a maturity structure between liquid and illiquid assets; more liquid assets yield a lower rate of return than less liquid assets. Investors are willing to hold liquid assets despite the maturity structure because of liquidity preference. The amount of liquidity provided to agents differs depending on the availability of different institutional frameworks (for example, whether they have access to markets and financial intermediaries or not, and how the two institutions interplay each other), and their preference (attitude toward risk).

In addition, this paper provides the mechanism of asset prices determination, and how asset prices affect the market equilibrium and the agent's welfare. These significant problems have been studied by many researchers, but their studies tend to be scattered around the author's interests. This paper is meaningful in that it draws together the results of the scattered studies to a comprehensive view. The framework and outcomes of this chapter will also be used as the basis for chapters 2 and 3.

The first issue to be addressed in this chapter is to look at the properties of asset prices. In this discussion, we examine what role liquidity plays in determining asset prices and how various stochastic shocks affect them. The definition of an asset's market liquidity is used with different meaning according to the context. Generally, an asset's liquidity means the asset's ability to be bought and sold quickly without having to reduce its price at low transaction costs (Keynes, 1930). However, in the widely-used, standard banking model of Diamond and Dybvig (1983), the asset's liquidity is related to the intertemporal consumption allocation. Thus, if a specific asset is called a 'liquid' asset, it implies that it allows consumers to consume over intertemporally when they wish to. A 'liquid asset' in this chapter is used in the same sense to refer to its allowing for intertemporal consumption by economic agents when they want to.

An asset price is determined by the demand and supply of the asset within the return of the asset. According to Allen and Gale (1994, 2009), the price of an asset is simply equal to the discounted value of future dividends (a fundamental value) when the demand for assets is large enough. However, if the available liquidity (the demand for the assets) is insufficient, then assets are traded at prices lower than the fundamental value, and the market undergoes 'cash-in-the-market pricing'. The first part of this chapter summarizes the findings of Allen and Gale (1994, 1997, 2009), Allen et al. (2009), Gorton and Pennacchi (1990), and Diamond (1997) and applies them to an overlapping generations (OLGs) model with fiat money.[1] The economy faces two different shocks: random relocation shocks and shocks on the return of illiquid assets. It shows that the demand and supply of assets are determined by market liquidity. While the amount of liquidity is determined in the process of an individual agent's optimising behaviour, it is closely related to how stochastic shocks are realised.[2]

The second part of this chapter discusses how financial markets and banks address the agent's intertemporal optimisation problem. When a new generation appears in the economy at date t, each member of it constructs her portfolio consisting of liquid and illiquid assets to optimise her consumption allocation at the beginning

[1]All these models deal with Walrasian equilibrium to determine asset prices. There exists, however, a new stream of study on asset-pricing. For example, Duffie et al. (2005) use search and bargaining model to study how asset prices are determined.

[2]Asset prices are also affected by the number of market participants and information structure. This chapter deals only with the effect of liquidity preferences and uncertainty on asset returns on the asset price. The effect of limited participation and the role of asymmetric information in the determination of asset prices are discussed in detail in Chapter 2. Chapter 3 discusses the effect of the population structure on the asset price.

of period t. The new-born agents construct their portfolios by trading their endowment maximisations with fiat money or investing in illiquid assets. However, the economy has no complete markets where Arrow securities are traded. Therefore, consumers cannot trade contingent commodities corresponding to states of nature.[3] The financial market is held after relocation shocks are revealed, and before agents are relocated. Trades occur between agents who belong to the same generation but have different preferences for liquidity, and the portfolio is rebalanced to fit their interests accordingly. Markets can achieve the efficient allocation if the degree of relative risk aversion, σ,[4] is 1 (the log case). However, if $\sigma > 1$ (resp. $\sigma < 1$), then the market provides insufficient (resp. more) liquidity than efficient one. Financial intermediaries may address this problem efficiently and thus provide consumers with efficient risk-sharing if it were not for bank runs.[5] In particular, if the return on assets is uncertain, banks can provide more efficient risk sharing than financial markets by allowing bank runs. This result is obtained by applying Allen and Gale (1998) to the stochastic relocation model.[6]

The main configuration of the model used in this chapter is a modified form of the Champ et al. (1996) model. Champ et al. (1996) constructs an OLGs model where agents are born on one of two islands. The model considers the role of currency and they show monetary factors play a specific role in banking panics. Except for assumptions in the basic set-up, however, this paper is not directly related to Champ et al. (1996) in its content. Based on the random-relocation economy, Bencivenga and Smith (1991) showed that the introduction of financial intermediaries might affect the composition of savings toward more productive assets (illiquid assets),

[3] Except for this, the market is assumed to be complete. Therefore, markets are perfectly competitive, there are neither externalities nor asymmetric information, and no transactions cost is present.

[4] The degree of relative risk aversion is defined as $\sigma = -\frac{u''(c)c}{u'(c)}$.

[5] An in-depth discussion of the relationship between financial intermediaries and financial markets is addressed in Chapter 2. It is discussed that the amount of liquidity provided to agents is varied when both institutions coexist facing more than one exogenous shocks and asymmetrical information.

[6] Allen and Gale (1998) is representative of several papers dealing with efficient bank runs. Related papers are Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988), and Diamond and Rajan (2001), just to name a few. Chapter 2 discusses in detail how banks provide effective risk sharing through these bank runs under asymmetric information.

and thus intermediaries enhance growth by reducing socially unnecessary capital liquidation.[7] Bhattacharya and Singh (2008) formed OLGs model, in which limited communication and stochastic relocation exist, to study the optimal choice of monetary policy instruments.

There are many papers which introduce Diamond and Dybvig (1983)'s model into an OLGs model. The OLGs model makes the studies possible on dynamic effects of shocks and monetary policy on the equilibrium and welfare belonging to different generations. For example, while Diamond and Dybvig (1983) explain that a run on the bank is due to excessive withdrawals from the late consumers, who belong to the same generation as early consumers, Qi (1994) shows that it could be the result of the lack of new deposits from posterior generations. Allen and Gale (1997) introduces banks which conduct a role for smoothing intertemporal and intergenerational risks when risky assets, which are assumed to last forever, pays a random payment in each period.[8]

Another feature of this chapter is the explicit introduction of fiat money in the framework of OLGs model. In an OLGs model, the young do not have any incentive to trade with the old because the old cannot provide anything valuable to the young. Money is used for means that make intergenerational transactions possible by removing the friction which exists between generations. According to Kocherlakota (1998), decentralised solution in which inter-generational transfers occur may achieve an efficient allocation without using money, if perfect social record keeping ("memory") is possible and the economy can enforce sufficient punishment for transgressions. However, if these requirements are not met, then money plays a pivotal role to support the intergenerational trade. In other words, "money is memory."

We assume in this chapter that the claims on capital are useless due to limited communication between islands. With this limited communication assumption, along with stochastic relocation shocks, money is valued and essential for old-age

[7] In Chapter 3 of this doctoral thesis I have shown that the introduction of securitization can bring about similar results to Bencivenga and Smith (1991)'s conclusions.

[8] Freeman (1988) presents an OLGs model as well to introduce banks as an optimal financial coalition.

consumption. Fiat money is not only used as a medium of exchange and a store of value, but it is assumed to be accepted in any place and time. Capital is assumed to be illiquid because it is hindered from moving between the two different locations, and communication between places is limited. Suppose a person carries a paper claim saying, for example, that she has a certain amount of capital in the place she just left, but communication between two spatially separated places is limited. Limited communication implies that verifying that the claims she holds are accurate is so costly to the potential buyers that there is no way to trade private claims against the capital a person holds.

The role of fiat money as a store of value and medium of exchange using OLGs model was first discussed in Samuelson (1958). Lucas (1972) introduced fiat money as well to show how a positive short-term correlation existed between the rate of change in money expansion and the level of GDP, and argued that the relationship may disappear when the government tries to exploit it.[9] Wallace (1980) presents an OLGs model in which many important monetary issues are dealt with. Especially, Wallace (1980) formed micro-founded model of money applying Samuelson (1958)'s model, in which money has a role of facilitating transactions across time and generations.

The relatively new field of studying, which is known by the name New Monetarist economics, uses search theory to study the micro-foundations of money. Some of its main questions are: "What is liquidity?", "Is money essential in achieving desirable outcomes?", "When can asset prices differ from fundamentals?" and so forth (Lagos et al., 2015, Abstract).[10] However, this chapter concentrates on the standard Walrasian competitive equilibrium. If the results of Chapters 2 and 3 and the findings of New Monetarist literature are synthesised, then we may have more interesting results, but it is postponed to a later study.

[9] In Chapter 3, I study how monetary policy can affect the composition of banks' portfolios and whether monetary policy can change the composition of assets to the direction desired by the monetary authorities following Lucas (1972) and Wallace (1980).

[10] Also see Waller (2015) in which he illustrates five main results which distinguish the New Monetary economics model from the standard cash-in-advance model.

The remainder of this chapter is organised as follows. Section 1.2 describes the structure of the physical set-up, information, and money and financial market equilibrium conditions. In section 1.3 the process of asset price determination is shown in detail. Section 1.4 discusses how market equilibrium is achieved in the face of various shocks. Section 1.5 deals with how banks can promote a consumer's welfare through bank runs when the economy is hit with shocks on the asset return. Section 1.6 illustrates numerical examples discussed in Section 1.4 and 1.5. Concluding remarks are contained in Section 1.7.

1.2 The Structure of the Economy

1.2.1 Random Relocation Model

The economy consists of two distinct, spatially separated locations (islands). Each island is populated by two age cohorts: the young and the old. A young person, the size of which on each island is normalised to one, is born on one of the two islands. The population of the initial old is also normalised to unit mass. Each young agent, *ex-ante* identical, is born with a unit of endowment goods at date t and has nothing at a subsequent date. The goods can be used for consumption or investment, but are restricted from moving across the two islands. Each young person faces privately observable, stochastic relocation shocks in the middle of date t.

If a young agent, who is born on the home island, is subject to relocation shocks, she is called a mover and is relocated to the foreign island to spend her old age there. Let λ be the probability that a young person is subject to relocation shocks. This assumption plays the same role as the preference shocks which divide agents into 'early' and 'late' consumers in standard Diamond and Dybvig (1983) type model. We assume that the probability of being relocated is the same on both islands. If there were no aggregate relocation shocks, there were large numbers of young agents, and their relocation shocks are assumed to be independent, then the fraction of movers are equal to the probability of being a mover according to the Law of Large Numbers.

All agents are assumed to consume only in their old age and thus are uncertain about the precise place where to consume. The agent's problem born at date t will be that of maximizing the following von Neumann-Morgenstern utility function

$$E_t[U(c_{m,t+1}, c_{n,t+1}; \lambda)] = \lambda u(c_{m,t+1}) + (1-\lambda)u(c_{n,t+1}),$$

where $c_{m,t+1}$ (resp. $c_{n,t+1}$) denotes the consumption of a mover (resp. a nonmover) who is born at date t and consume at $t+1$. The utility function satisfies the usual neoclassical properties (increasing, strictly concave, and twice continuously differentiable).

1.2.2 Portfolio Decision

Each person needs to save for the old-age consumption. There are two different assets that she is able to access: a liquid asset (fiat money) and an illiquid asset (capital). Let p_t be the nominal price of one unit of consumption goods at date t. Then one unit of the goods which is traded for fiat money at date t yields p_t/p_{t+1} units of consumption goods. p_t/p_{t+1} is assumed to be less than or equal to 1 because p_{t+1} is expected to be greater than or equal to p_t (total money supply is expected to grow or remain unchanged.). The illiquid asset (capital) takes two periods to mature[11] and allows one unit of the goods invested in the current period to be converted into $R > 1$ units of the goods at date $t+1$.[12] There exists a trade-off between an asset's liquidity and its return. The illiquid asset is immobile and takes two periods to mature, but pays a higher return. The liquid asset is mobile, universally accepted, and takes no time to mature, but yields a lower return.

[11] Therefore, the illiquidity of capital is captured in two dimensions: the mismatch between asset maturity and liquidity preferences, and limited communication due to its immobility.

[12] The linear capital production technology (constant returns-to-scale technology) was introduced into the overlapping generations model by Cass and Yaari (1966) and the assumption is widely used in the literature.

Portfolio Allocation with Financial Markets and Banks

The portfolio decision depends on the agent's (prior) expectations about risk and the attitude toward risk. This decision is made irrevocably before the preference shocks are revealed. The consumer spends the fraction q for fiat money among her endowment and acquires $p_t q$ units of fiat money from the trade because the nominal price of one unit of the goods at date t is p_t. The remaining fraction k of her endowment is invested in the illiquid asset (capital). Then the agent's budget constraint is given as

$$q + k = 1 \tag{1.1}$$

The consumption of the agent depends on her choice on q and how uncertainty is realised. However, the economy has no complete markets where Arrow securities are traded. Therefore, consumers cannot trade contingent commodities corresponding to states of nature when they construct their portfolio. The financial market is held after relocation shocks are revealed, and before agents are relocated. The market is complete because now all uncertainties are resolved. Trades occur between agents who belong to the same generation but have different preferences for liquidity, and the portfolio is rebalanced to fit their interests accordingly.

The welfare of consumers is enhanced when they have access to financial markets than they are in autarky. This is because the market expands the feasible set of the individual agent by allowing a consumer to trade her private claims with other cohorts.[13] Banks can increase consumers' welfare as well. The young people deposit their endowment goods with a competitive bank which uses the proceeds to acquire assets on behalf of them. Banks provide its depositors insurance against relocation shocks because it can choose an efficient portfolio by using the law of large numbers.

[13] Without financial markets, the capital investment is useless to movers because of the assumption of the limited communication.

1.2.3 Structure of Uncertainty, Information

There are two main views to explain the causes of bank runs or banking crises. The first perspective is that a bank run is caused by panics. The most influential study taking this perspective is Diamond and Dybvig (1983) in which they developed a model of bank runs as self-fulfilling prophecies. The other view sees bank runs as a natural result of the business cycle. Depositors who expect the economic fundamentals to deteriorate in the near future withdraw their deposits in anticipation of the insolvency of the bank. Gorton (1988) raised the view that the financial crisis is the result of the business cycle from the empirical study on the National Banking Era. Allen and Gale (1998) formed a theoretical model to discuss how the business cycle causes banking distress.

Therefore, we can roughly divide uncertainty into two broad categories, depending on the reasons for the bank run: intrinsic or extrinsic uncertainty. According to Allen and Gale (2004), intrinsic uncertainty indicates that the fundamentals of the economy fluctuate stochastically, such as a preference for liquidity or a change in the return of assets. Extrinsic uncertainty, on the other hand, implies that endogenous variables are influenced by extraneous variables (sunspots) that do not directly affect fundamentals.

In this chapter, we ignore the possibility of a bank run equilibrium due to sunspots and focus only on the intrinsic uncertainty that causes a bank run. In other words, this article deals only with the case where agents ex-ante expect the possibility of runs because of the uncertainty associated with fundamentals.[14] Banks construct their portfolio weighing the costs of avoiding defaults (by holding sufficient liquidity) and the benefits allowing it to happen. Banks find the optimal deposit contract to maximise the expected utility of depositors considering this cost and benefit.[15]

[14]The bankruptcy caused by the panic as in Diamond and Dybvig (1983) is an entirely unexpected event at the time when a bank constructs its portfolio. If the bank run had been anticipated in advance, the bank would have built a different portfolio to reflect this fact. See Allen and Gale (2009, Ch.3) for more details.

[15]Because of free entry into the banking industry and competition in it, banks will offer demand deposit contracts so as to maximise the agent's expected utility.

We deal with three different sources of intrinsic uncertainty following Allen and Gale (1998, 2004) and Allen et al. (2009). First, individual consumers face idiosyncratic relocation shocks. Second, the economy as a whole faces the aggregate uncertainty about the number of movers. Finally, the rate of return on the capital is uncertain. Uncertainty about the return on capital is introduced to reflect the impact of the business cycle on the asset value of the bank.

All uncertainties are resolved in the middle of each period. Each consumer learns whether she is a mover or a nonmover, which is private information and thus not observed publicly. Except for that information, the actual values of each random variable are publicly observed, i.e., other information about fundamentals is observed with accuracy.

In the asymmetrical information situation, only some consumers (known as informed agents) are assumed to have access to the information on the rate of returns on assets and the total fraction of movers. This case is discussed in detail in Chapter 2. The following subsections analyse each uncertainty as isolated, taking other information is given. More complicated cases combining two different uncertainties are dealt in Chapter 2 and Chapter 3.

Preference Uncertainty

Inhabitants of both islands are *ex-ante* identical, but each island faces aggregate relocation shocks in each period. The number of movers, λ_θ, is a stochastic variable which is assumed to take the following form[16]

$$\lambda_\theta = \alpha + \varepsilon\theta$$

α is a probability of the young being a mover without aggregate shocks and is

[16] If individual bank faces idiosyncratic liquidity shocks on the fraction of movers, which is uncorrelated with other island's uncertainty, then λ_θ can be represented as

$$\lambda_{i\theta} = \alpha_i + \varepsilon\theta$$

following Allen and Gale (2004) and Allen et al. (2009). In this case α_i is a random variable and can take different supports according to the formation of a model.

equal to the fraction of movers following the law of large numbers. θ is a random variable which denotes the aggregate uncertainty about the fraction of the mover of the economy and ε is a positive constant. Let the random variable θ take the following values:

$$\theta = \begin{cases} 0, & \text{with probability } \omega_l \\ 1, & \text{with probability } \omega_h \end{cases}$$

Note that the population of two different islands remains the same when they suffer aggregate relocations shocks because both islands experience the same shocks at the same time.

Uncertainty on Asset Returns

The capital invested at date t using a unit of endowment goods transforms it into R_H units of consumption goods with probability ρ_H or R_L with probability ρ_H, where $R_H > R_L > 0$.

$$R_j = \begin{cases} R_H, & \text{with probability } \rho_H \\ R_L, & \text{with probability } \rho_L \end{cases}$$

The relations of the rate of returns between different assets are assumed to be following:

Assumption 1 *Money Supply is constant over periods.*

Assumption 2

$$E[R_j] = \rho_H R_H + \rho_L R_L > E_t[p_t/p_{t+1}] \tag{1.2}$$

The above assumption tells us that the expected rate of return on an illiquid asset is greater than that of fiat money, which is needed for the agent to hold capital. She would hold the risky asset if and only if the expected rate of return on capital is greater than the rate of return on currency. Even though p_t and p_{t+1} is determined endogenously within the model, the rate of return on fiat money is just equal to 1 because the money supply is assumed to remain constant in this chapter.[17] Therefore Assumption 1.2 is valid if $E[R_j]$ is assumed to be greater than 1.

Assumption 3

$$u'(0)\frac{p_t}{p_{t+1}} > u'(R_j)R_j \tag{1.3}$$

This assumption is needed for ensuring $q > 0$, which is used in Allen and Gale (1998). If $(q, k) = (0, 1)$, then movers will consume nothing and nonmovers consume R_j. The preceding assumption tells us that the consumption of a mover will be increased more than the reduction on the consumption of a nonmover if an agent increases in investment in q by reducing the equal amount in k. (1.2) and (1.3) are a necessary and sufficient condition for interior solution (i.e., for having both $q > 0$ and $k > 0$).

Assumption 4

$$0 < R_L < p_t/p_{t+1} < R_H \tag{1.4}$$

The preceding equation assumes specific relations between rates of returns of different assets. Even though the average returns from the illiquid asset is greater than liquid asset, there is a chance of capital return being lower than that of fiat money.

1.2.4 Equilibrium

In this section, we will look at the equilibrium conditions of markets.

Money Market Equilibrium

No one in the future generations is born with fiat money. For young agents to acquire fiat money, they must trade with the initial old who live on the same island as them and are endowed in total with M units of fiat money at time t. Let the real demand for fiat money (the number of goods each young agent chooses to sell his fraction of endowment for fiat money) be q. The price level at date t, p_t, is determined at the beginning of period t by the coincidence of the demand for fiat

[17] See Money Market Equilibrium part.

money by the young and the supply of it by the initial old.

$$p_t = \frac{M}{q} \tag{1.5}$$

In this chapter, we assume the constant money supply, so $M_{t+s} = M, \forall s \geq 0$. Since agents are ex-ante identical and the population of each island is assumed to be constant over periods,[18] all generations face the same decision problem. Therefore, in a stationary equilibrium where the money supply is constant, q and thus p_t have constant values, i.e., $p_{t+s} = p$ for all $s \geq 0$.[19]

Asset Market Equilibrium

After uncertainties are revealed in the middle of period t, an asset market is held, and movers and nonmovers trade with each other before movers are relocated. The prevailing (nominal) price of the illiquid asset, B_t, clears the asset market at date t. In each period t, the total amount of capital supplied by movers (the amount of fiat money demanded) is $S_t^k (= D_t^q) = \lambda_\theta k$, and the total amount of fiat money supplied by the nonmovers (the demand for capital) is $S_t^q (= D_t^k) = (1 - \lambda_\theta) p_t q$. Therefore for the asset market to be cleared it is required that

$$D_t^q B_t \leq S_t^q$$

Goods Market Equilibrium

The goods market clearing condition requires in each period t that

$$\lambda_\theta c_{m,t} + (1 - \lambda_\theta) c_{n,t} \leq \frac{p_t q}{p_{t+1}} + kR$$

The left hand side of the equation denotes the total consumption of movers and nonmovers combined, and the right hand side represents the sum of available con-

[18] This assumption cannot be sustained in situations where each island faces different population structure, or where monetary authorities play an active role to affect the price level via expansionary or contractionary monetary policy. This issue is discussed in Chapter 3.

[19] However, subscripts denoting time will sometimes be used for analysis convenience.

sumption goods.

1.3 Properties of Asset Price

This section contains a detailed exposition on how the price of capital, an illiquid and risky asset, is determined when the economy faces uncertainty on the fundamentals. This discussion is based on the work of Gorton and Pennacchi (1990), Allen and Gale (1994), and Diamond (1997), which discuss the process of asset price determination in different situations.

1.3.1 Fundamental Asset Price

Assume that an asset market is held after the relocation shocks are revealed and before movers are relocated. Let $B_t^{\theta j}$ be the (nominal) price of one unit of capital traded in the asset market in period t facing uncertainties. θ and j denote the state of nature where $\theta = \{0, 1\}$, and $j = \{H, L\}$ respectively. We have the following proposition:

Proposition 1.3.1

$$B_t^{\theta j} = \min\left\{p_{t+1}R_j, \frac{\gamma(1-\lambda_\theta)p_t q}{\lambda_\theta k}\right\} \tag{1.6}$$

Proof The supply of capital comes from movers. They inelastically supply their holdings of illiquid assets, whatever the price is. So there is a vertical supply curve, and the quantity supplied is:

$$S^K (= D^M) = \lambda_\theta k$$

The demand for illiquid assets comes from nonmovers. Nonmovers need to decide whether to hold fiat money and pass it over to the next period. For the decision, they compare the expected rate of return on fiat money with that of capital. Let γ be the ratio of money exchanged with capital. Then, $1-\gamma$ fraction among their money holdings ($p_t q$) is passed over to the date $t+1$. γ takes a value between 0 and 1, and $\gamma = 1$ implies that nonmovers inelastically supply their whole fiat money to the market. However, $\gamma = 0$ (all storage of fiat money) will not occur in an equilibrium since as $\gamma \to 0$, $B_t^{\theta j} \to 0$. So, as long as $R_j > 0$ there exists fiat money provider to trade for illiquid assets.

If $R_j < \frac{B_t^{\theta j}}{p_{t+1}}$, then no nonmovers would hold capital from date t to $t+1$, which implies $\gamma = 0$. If $R_j = \frac{B_t^{\theta j}}{p_{t+1}}$, then the two assets are perfect substitutes to nonmovers. Thus, any γ between $0 \leq \gamma \leq 1$ is possible. If $R_j > \frac{B_t^{\theta j}}{p_{t+1}}$, no nonmovers would hold fiat money from date t to $t+1$, which implies $\gamma = 1$. Then, for the nonmovers to trade fiat money for capital, it is required that $B_t^{\theta j} \leq p_{t+1} R_j$. The maximum price of the the asset is $B_t^{\theta j} = p_{t+1} R_j$.

Therefore, the aggregate demand for capital of nonmovers is

$$D^K (= S^M) = (1 - \lambda_\theta) \gamma p_t q$$

The asset market clearing condition requires that

$$\begin{aligned} \lambda_\theta k B_t^{\theta j} &\leq \gamma (1 - \lambda_\theta) p_t q \\ \Leftrightarrow \quad B_t^{\theta j} &\leq \frac{\gamma (1 - \lambda_\theta) p_t q}{\lambda_\theta k} \end{aligned} \tag{1.7}$$

If the market has enough liquidity (i.e.,$S^M > D^M$), then the asset price would reach its highest possible point, $p_{t+1} R_j$. However, if the market suffers liquidity shortage for any reason, the price will be determined by the amount of cash supplied. This is known as 'cash-in-the-market pricing' in which the asset price is determined as the ratio of total available 'cash' to the amount of asset provided (Allen and Gale, 1994, 2009). ■

1.3.2 With No Uncertainty

Proposition 1.3.2 *Suppose an economy which faces no uncertainty. The price of an illiquid asset, B_t, is determined uniquely such that $B_t = p_t$.*

Proof Proposition 1.3.2 implies that the price of capital sold at date t is equal to the price of a unit of consumption goods. To prove that, let us consider the consumer's decision at date t. First, suppose that $B_t > p_t$. Then capital would dominate the liquid asset investment and thus $k = 1$ ($q = 0$). If $k = 1$, a mover consumes $\frac{kB_t}{p_{t+1}} = \frac{B_t}{p_{t+1}} > \frac{p_t}{p_{t+1}}$, and a nonmover consumes R. However, since all agents hold only capital in the first place and have no fiat money, the demand for capital is zero. The price of capital will drop to zero as well, which is a contradiction.

Now suppose $B_t < p_t$, then the liquid asset dominates the illiquid asset and no one will hold capital at the beginning, which implies $q = 1$ ($k = 0$). A mover consumes $\frac{qp_t}{p_{t+1}} = \frac{p_t}{p_{t+1}}$, and a nonmover consumes $\frac{qp_t}{B_t}R = \frac{p_t}{B_t}R > R$. However, when nonmovers try to trade fiat money for capital, there will be no supply of capital, and thus the price will increase to its fundamental value, $p_{t+1}R$. This leads to $R = \frac{B_t}{p_{t+1}} < \frac{p_t}{p_{t+1}}$, another contradiction. ■

1.3.3 Uncertainty on Asset Returns

Proposition 1.3.3 *Suppose the following*

1. *No uncertainty on liquidity shocks exis, i.e.,* $\lambda_\theta = \alpha$.

2. *The random variable* R_j *takes on the value of* R_H *with probability of* ρ_H *and* R_L *with probability of* ρ_L*,*

3. *All assets are held by consumers*

Then, given α q, k, p_t*, and* p_{t+1}*, the following holds*

1. *The date* t *price of one unit of capital,* B_t^j*, is less than or equal to* $p_{t+1}R_j$*. When* $R_j = R_H$ *the price of capital is determined such that* $B_t^H = \min\left\{p_{t+1}R_H, \frac{\gamma^H(1-\alpha)p_tq}{\alpha k}\right\}$*, and when* $R_j = R_L$*,* $B_t^L = p_{t+1}R_L$*.*[20]

[20] Strictly speaking, the notation of B_t^j should be $B_t^{\theta j}$. However, for simplicity of notation, superscripts are used for denoting stochastic variables that are currently in question only. The superscripts which are not shown in $B_t^{\theta j}$ imply that the random variable concerning the uncertainty is not stochastic.

2. *γ^H is equal to 1 as long as $p_{t+1}R_H > \frac{\gamma^H(1-\alpha)p_t q}{\lambda_\theta k}$, and γ^L is determined such that $p_{t+1}R_L = \frac{\gamma^L(1-\alpha)p_t q}{\alpha k}$, which implies $\gamma^L = \frac{\lambda_\theta k R_L}{(1-\alpha)q}\frac{p_{t+1}}{p_t}$.*

3. *Combining the above two results, we have the following relationship.*

$$R_H \geq \frac{B_t^H}{p_{t+1}} > \frac{B_t^L}{p_{t+1}} = \frac{p_t}{p_{t+1}}\frac{\gamma^L(1-\alpha)q}{\alpha k} = R_L \tag{1.8}$$

Proof One unit of consumption goods is exchanged for p_t units of currency, and p_t units of fiat money purchases p_t/B_t^j units of capital. p_t/B_t^j units of capital produce $\frac{p_t R_j}{B_t^j}$ units of consumption goods at date $t+1$. If a consumer acquires p_t units of fiat money at date t and holds until the date $t+1$, she is able to purchase p_t/p_{t+1} units of consumption goods at date $t+1$.

All fiat money is inelastically supplied to the market for buying capital and cash is dominated between the period t and $t+1$, and thus γ becomes 1, if and only if the rate of return on capital is greater than or equal to that of fiat money, i.e.,

$$\frac{p_t}{B_t^j}R_j \geq \frac{p_t}{p_{t+1}}$$

Rewriting the preceding equation, we have:

$$p_{t+1}R_j \geq B_t^j$$

The preceding equation implies that agents will want to hold only capital from period t to period $t+1$ when the rate of return on capital is high. Since the rate of return on capital takes two different values, it is assumed that the preceding relation holds only when $R_j = R_H$ without loss of generality. The value of γ with R_H, γ^H, becomes 1. Utilizing (2.5), we may express B_t^H in the form

$$B_t^H = \min\left\{p_{t+1}R_H, \frac{(1-\alpha)p_t q}{\alpha k}\right\} \tag{1.9}$$

A similar discussion can be made when the random variable R_j takes the value R_L.

Some money reserves will be handed over to the next period as long as the following relation holds,

$$\frac{p_t}{B_t^L}R_L = \frac{p_t}{p_{t+1}}$$

Rearranging the preceding equation we have

$$p_{t+1}R_L = B_t^L = \min\left\{p_{t+1}R_L, \frac{\gamma^L(1-\alpha)p_t q}{\alpha k}\right\}$$

γ^L is adjusted such that $p_{t+1}R_L = \frac{\gamma^L(1-\alpha)p_t q}{\alpha k}$. Therefore, the price of capital in the state of L, B_t^L, is

$$B_t^L = p_{t+1}R_L = \frac{\gamma^L(1-\alpha)p_t q}{\alpha k} \tag{1.10}$$

Note that γ^L takes on any value between $0 < \gamma^L < 1$ depending on parameter values.

■

Note that in any case of either $R_j = \frac{B_t^j}{p_{t+1}}$ or $R_j = \frac{p_t}{p_{t+1}}\frac{\gamma^j(1-\alpha)q}{\alpha k}$, the asset price in this section cannot be higher than the fundamental value, $p_{t+1}R_j$. The market always undergoes "underpricing" (Allen and Gale, 1994).[21]

1.3.4 Liquidity Uncertainty

Proposition 1.3.4 *Suppose that the economy suffers aggregate uncertainty on the fraction of movers, then the fraction of movers at date t is given as*

$$\lambda_\theta = \alpha + \varepsilon\theta$$

The random variable θ takes on the value of 0 with probability ω_l and 1 with probability ω_h. Also assume that capital returns have a fixed value at $R = R_H$, then the capital prices at date t facing aggregate liquidity shocks, B_t^θ, have the following

[21] Asking if the asset price always takes the value less than the fundamental value is questioning whether 'bubbles' may exist or not. This is an interesting question but is not dealt with in this article. See Prize (2013) for an introductory description of whether financial assets reflect fundamental values or bubbles exist.

relationship:

$$\frac{B_t^1}{p_{t+1}} < \frac{p_t}{p_{t+1}} < \frac{B_t^0}{p_{t+1}} \leq R, \tag{1.11}$$

Proof From Proposition 1.3.1 we have

$$\begin{aligned} B_t^0 &= \min\left\{p_{t+1}R, \frac{\gamma^j(1-\lambda_0)p_t q}{\lambda_0 k}\right\} \\ B_t^1 &= \min\left\{p_{t+1}R, \frac{\gamma^j(1-\lambda_1)p_t q}{\lambda_1 k}\right\}, \end{aligned} \tag{1.12}$$

Note that $\gamma^j = 1$ irrespective of the state j because the asset return is given as a definite value $R = R_H \geq \frac{B_t^\theta}{p_{t+1}}$. and thus no fiat money is stored (see Proposition 1.3.3). By construction, we have the following

$$\lambda_0 = \alpha < \lambda_1 = \alpha + \varepsilon$$

because ε is a positive constant. Since $\frac{(1-\lambda_0)p_t q}{\lambda_0 k} > \frac{(1-\lambda_1)p_t q}{\lambda_1 k}$,[22] the price of capital in the state 0, B_t^0, is higher than or equal to that in the state of h, B_t^h. Therefore, we have

$$p_{t+1}R \geq B_t^0 \geq B_t^1, \quad (=, \text{ if } B_t^0 = B_t^1 = p_{t+1}R), \tag{1.13}$$

Note that, however, B_t^0 cannot be equal to B_t^1. If $B_t^0 = B_t^1$, then we have $B_t^0 = B_t^1 = p_{t+1}R$ [23], which implies that capital dominates fiat money. If agents hold only capital, i.e., $k = 1$, then movers sell capital at a price of B_t^θ and consume $\frac{B_t^\theta k}{p_{t+1}} = \frac{B_t^\theta}{p_{t+1}} = R > \frac{p_t}{p_{t+1}}$. No one would hold fiat money in the first place, which cannot be an equilibrium. Therefore we should have the relationship: $p_{t+1}R \geq B_t^0 > B_t^1$.

Additional condition, $B_t^1 < p_t < B_t^0$, is required because otherwise fiat money is (weakly) dominated by capital as well. If $B_t^0 > B_t^1 \geq p_t$, then a consumer holding only capital sells it at a price B_t^θ and consume $\frac{B_t^0}{p_{t+1}} > \frac{B_t^1}{p_{t+1}} \geq \frac{p_t}{p_{t+1}}$ (see

[22] $\left[\frac{(1-\lambda_i)q}{\lambda_i k}\right]$ is decreases in λ_i, i.e., $\frac{\partial}{\partial \lambda_i}\left[\frac{(1-\lambda_i)q}{\lambda_i k}\right] < 0$

[23] Because $\frac{(1-\lambda_0)p_t q}{\lambda_0 k} \neq \frac{(1-\lambda_1)p_t q}{\lambda_1 k}$, the only way $B_t^0 = B_t^1$ is both B_t^0 and B_t^1 having the fundamental value.

Proposition 1.3.2). Also, if $B_t^1 < B_t^0 \leq p_t$, capital is weakly dominated by fiat money since $\frac{B_t^1}{p_{t+1}} < \frac{B_t^0}{p_{t+1}} \leq \frac{p_t}{p_{t+1}}$. Therefore we have $\frac{B_t^1}{p_{t+1}} < \frac{p_t}{p_{t+1}} < \frac{B_t^0}{p_{t+1}}$. All these explanations yield the conclusion that $\frac{B_t^1}{p_{t+1}} < \frac{p_t}{p_{t+1}} < \frac{B_t^0}{p_{t+1}} \leq R$.

■

1.4 Equilibrium with Markets

Consider an economy that does not have financial intermediaries and has only asset markets, which are incomplete because of lacking Arrow securities. This section deals with the relationship between asset prices and market liquidity and shows how individual agents optimize their utility using markets.

1.4.1 Consumption

A consumer, who turns out to be a mover, sells her entire holdings of capital at a nominal price $B_t^{\theta j}$ before she is relocated. She inelastically supplies her capital holdings, whatever the price is. The total unit of currency the mover accumulates after the trade is:

$$p_t q + B_t^{\theta j} k$$

The second term denotes the amount of fiat money the mover obtains from selling her illiquid assets to nonmovers who belongs to the same generation as her. Then the old-age consumption of a mover at date $t+1$ is given as:

$$c_{m,t+1} = \frac{p_t q + B_t^{\theta j} k}{p_{t+1}} \tag{1.14}$$

The consumption depends on: the price of an illiquid asset at date t, $B_t^{\theta j}$, and the gross real rate of return on fiat money over one-period, p_t/p_{t+1}.

The consumption of a nonmover is

$$c_{n,t+1} = \left\{ \frac{\gamma^j p_t q}{B_t^{\theta j}} + k \right\} R_j + \frac{(1-\gamma^j) p_t q}{p_{t+1}} \tag{1.15}$$

Nonmovers need to decide how much capital to buy from movers considering the expected rate of returns of the two different assets, unlike movers facing a trivial decision problem. If they use γ^j ratio of its cash holdings to purchase illiquid assets, then they could buy $\frac{\gamma^j p_t q}{B_t^L}$ units of capital. According to Proposition 1.3.3, $\gamma^j = 1$ when $R_j = R_H$ and $0 < \gamma^j < 1$ when $R_j = R_L$.

1.4.2 Efficient Risk Sharing

Let's take a look at the case of the efficient allocation achieved by the social planner as a baseline model. All variables are assumed to be deterministic and uncertainty is assumed to be absent. The number of movers on each island is assumed to be $\lambda_\theta = \alpha$ and the illiquid asset return is given as $R_j = R$.

Suppose that the social planner maximises the welfare of the young agents born at date t, then the problem will be

$$\max_{c_{m,t+1}, c_{n,t+1}} E_t[U(c_{m,t+1}, c_{n,t+1})] = \alpha u(c_{m,t+1}) + (1-\alpha) u(c_{n,t+1}) \tag{1.16}$$

subject to the budget constraint (1.1), the feasibility conditions

$$\alpha c_{m,t+1} \leq \frac{p_t q}{p_{t+1}} \tag{1.17}$$

$$\alpha c_{m,t+1} + (1-\alpha) c_{n,t+1} = \frac{p_t q}{p_{t+1}} + kR, \tag{1.18}$$

and the incentive constraint

$$c_{m,t+1} \leq c_{n,t+1} \tag{1.19}$$

The incentive constraint is needed if the individual type is a private information. Each individual will reveal her true type if and only if (1.19) is satisfied. By substituting the budget constraint into objective function, the Lagrangian function of this optimization model is given as

$$\begin{aligned}\mathcal{L} =& \alpha u\left(c_{m,t+1}\right)+(1-\alpha)u\left(c_{n,t+1}\right)\\&+\mu_1\left[\frac{p_t q_t}{p_{t+1}}-\alpha c_{m,t+1}\right]\\&+\mu_2\left[\frac{p_t q}{p_{t+1}}+(1-q)R-\alpha c_{m,t+1}-(1-\alpha)c_{n,t+1}\right]\end{aligned}$$

where μ_1 and μ_2 are Lagrange multipliers which both have non-negative values. Note that the incentive constraint (1.19) are not included in the preceding equation. As shown below, this is because the incentive constraint is always satisfied automatically when we optimize this problem only with the first three constraints.[24]

We have the following set of simultaneous equations of the first-order conditions with respect to consumption:

$$\begin{aligned}(c_{m,t+1}) \quad & u'\left(c_{m,t+1}\right)=\mu_1+\mu_2\\(c_{n,t+1}) \quad & u'\left(c_{n,t+1}\right)=\mu_2\end{aligned}$$

Note that α terms disappear, which shows the reason why the social planner achieves the efficient allocation; the social planner is able to use the law of large numbers to choose the efficient portfolio irrespective of the individual agent' preference shocks. Non-binding feasibility condition (1.17) implies that $c_{m,t+1}=c_{n,t+1}$ because $\mu_1=0$. When (1.17) is binding, it implies $c_{m,t+1}<c_{n,t+1}$ because $\mu_1>0$. Therefore, the incentive constraint is always satisfied in any case. The first order condition with respect to q yields the following:

$$(\mu_1+\mu_2)\frac{p_t}{p_{t+1}}=\mu_2 R$$

Combining all these first order conditions we have the following efficient risk-sharing

[24] See the Appendix of Allen and Gale (1998) where they showed this result.

condition.

$$u'(c_{m,t+1})\frac{p_t}{p_{t+1}} = u'(c_{n,t+1})R \tag{1.20}$$

1.4.3 Equilibrium with No Uncertainty

Following Proposition 1.3.2, the capital is traded at the price of $B_t = p_t$. The young born at date t maximises their expected utility given $\lambda_\theta = \alpha$ and $R_j = R$

$$\max_{c_m, c_n} E_t[U(c_{m,t+1}, c_{n,t+1})] = \alpha u(c_{m,t+1}) + (1-\alpha)u(c_{n,t+1}) \tag{1.21}$$

subject to (1.1), (1.14), and (1.15). γ^j is 1 from Proposition 1.3.3. (1.14) and (1.15) is simplified as following using the fact $B_t = p_t$, and assuming constant money supply and stationary and symmetric allocation

$$\begin{aligned} c_{m,t+1} &= \frac{p_t}{p_{t+1}} = 1 \\ c_{n,t+1} &= R \end{aligned} \tag{1.22}$$

Efficient solution requires satisfying (1.20). However, the market allocation (1.22) is efficient only when the degree of the relative risk aversion (RRA) is equal to 1. Substituting (1.22) into efficient solution (1.20), we have

$$u'\left(\frac{p_t}{p_{t+1}}\right)\frac{p_t}{p_{t+1}} = Ru'(R)$$

With log utility the preceding equation holds, which implies the market allocation is efficient.

Suppose that agents have CRRA utility function

$$u(c) = \frac{c^{1-\sigma}-1}{1-\sigma},$$

and assume $\sigma > 1$. Then we have

$$u'\left(\frac{p_t}{p_{t+1}}\right)\frac{p_t}{p_{t+1}} = \left(\frac{p_t}{p_{t+1}}\right)^{1-\sigma} < R^{1-\sigma} = Ru'(R)$$

The previous equation implies that the consumption of a mover should be increased to hold efficiency condition, which is unattainable in a market. This result suggests that the markets provide consumers with insufficient liquidity than the efficient allocation.

Table 1.1 shows the numerical examples with three different values of $\sigma = 0.5, 1, 1.5$ to examine the effect of different σ on equilibria. The number of movers is given as $\lambda_\theta = \alpha = 0.5$, the rate of return on capital, R, is 2, and the money supply at date t is normalized to 1.

Table 1.1: Equilibria for Variables

σ		Market	Efficient
0.5	q	0.5000	0.4000
	p_t	2.0000	2.5000
	B_t	2.0000	—
	$c_{m,t+1}$	1.0000	0.8000
	$c_{n,t+1}$	1.5000	1.8000
	EU	0.2247	0.2361
1	q	0.5000	0.5000
	p_t	2.0000	2.0000
	B_t	2.0000	—
	$c_{m,t+1}$	1.0000	1.0000
	$c_{n,t+1}$	1.5000	1.5000
	EU	0.2027	0.2027
1.5	q	0.5000	0.5337
	p_t	2.0000	1.8736
	B_t	2.0000	—
	$c_{m,t+1}$	1.0000	1.0675
	$c_{n,t+1}$	1.5000	1.3988
	EU	0.1835	0.1866

1.4.4 Equilibrium with Random Asset Returns

Consumers try to maximise their expected utility by choosing the optimal portfolio (q, k), taking the price function B_t^j and liquidity shock λ_θ as given. Assume that no other uncertainties exist except for R_j. Assume $\lambda_\theta = \alpha$ and let $c^j_{m,t+1}$, $c^j_{n,t+1}$, and B^j_t be the consumption of movers, nonmovers, and the price of capital facing random capital returns R_j respectively. The problem posed will be that of maximizing the following expected utility function

$$\begin{aligned}\max_{c_{m,t+1},c_{n,t+1}} \; & E_t[U(c_{m,t+1},c_{n,t+1})] \\ & = \sum_{j=H,L} \rho_j \left[\alpha u(c^j_{m,t+1}) + (1-\alpha)u(c^j_{n,t+1})\right]\end{aligned} \tag{1.23}$$

$$\begin{aligned} s.t. \quad (i) \quad & q + k = 1, \\ (ii) \quad & c^j_{m,t+1} = \frac{p_t q + B^j_t k}{p_{t+1}}, \text{ for } j = H, L \\ (iii) \quad & c^H_{n,t+1} = \left\{\frac{p_t q}{B^H_t} + k\right\} R_H \\ & c^L_{n,t+1} = \left\{\frac{\gamma^L p_t q}{B^L_t} + k\right\} R_L + \frac{(1-\gamma^L)q}{\Pi_{t,t+1}} - c^L_{n,t+1} \end{aligned}$$

In a state of R_H, nonmovers hold only capital from t to $t+1$ by trading all their fiat money for capital since $R_H \geq \frac{B^H_t}{p_{t+1}}$. On the other hand, when $R_j = R_L$ they may keep some currency until the date $t+1$.

Substituting $B^L_t = p_{t+1}R_L$ (see Proposition 1.3.3) into the constraint (iii), we have

$$\begin{aligned} c^L_{m,t+1} &= \frac{p_t q + B^L_t k}{p_{t+1}} = \frac{q p_t}{p_{t+1}} + R_L k \\ c^L_{n,t+1} &= \left\{\frac{\gamma^L p_t q}{B^L_t} + k\right\} R_L + \frac{(1-\gamma^L)p_t q}{p_{t+1}} = \frac{q p_t}{p_{t+1}} + R_L k, \end{aligned} \tag{1.24}$$

where

$$\begin{aligned} B^H_t &= \min\left\{p_{t+1}R_H, \frac{(1-\alpha)p_t q}{\alpha k}\right\} \\ B^L_t &= p_{t+1}R_L = \frac{\gamma^L(1-\alpha)p_t q}{\alpha k} \end{aligned}$$

Note that the consumption of both movers and nonmovers in the state of R_L is equal to each other. Substituting (1.24) and (1.1) into the objective function, (1.23), and differentiating it with respect to q, we get

$$\rho_H\left[\alpha u'(c^H_{m,t+1})\left(\frac{p_t - B^H_t}{p_{t+1}}\right) + (1-\alpha)u'(c^H_{n,t+1})\left(\frac{p_t}{B^H_t} - 1\right)R_H\right],$$
$$+\,\rho_L\left[\alpha u'(c^L_{m,t+1})\left(\frac{p_t - B^L_t}{p_{t+1}}\right) + (1-\alpha)u'(c^H_{n,t+1})\left\{\left(\frac{\gamma^L p_t}{B^L_t} - 1\right)R_L - \frac{\gamma^L p_t}{p_{t+1}}\right\}\right] = 0$$

Solving the preceding equation gives the optimal value of q. The numerical example for this problem is illustrated in Section 1.6.

1.4.5 Equilibrium with Aggregate Relocation Shocks

This part discusses the market equilibrium with aggregate relocation shocks. The fraction of movers is given as $\lambda_\theta = \alpha + \varepsilon\theta$, and θ takes the value of 0 (resp. 1) with the probability of ω_l (resp. ω_h). The individual agent's maximisation problem is

$$\begin{aligned}\max_{c_{m,t+1},c_{n,t+1}} \; & E_t[U(c_{m,t+1}, c_{n,t+1}); \lambda_\theta] \\ &= \sum_{\theta=0,1}\omega_\theta\left[\lambda_\theta u(c^\theta_{m,t+1}) + (1-\lambda_\theta)u(c^\theta_{n,t+1})\right]\end{aligned} \tag{1.25}$$

$$\begin{aligned} s.t. \quad & (i) \quad q + k = 1, \\ & (ii) \quad \sum_{\theta=0,1}\omega_\theta\left(c^\theta_{m,t+1} - \frac{p_t q + B^\theta_t k}{p_{t+1}}\right) = 0 \\ & (iii) \quad \sum_{\theta=0,1}\omega_\theta\left(c^\theta_{n,t+1} - \left\{\frac{p_t q}{B^\theta_t} + k\right\}R\right) = 0,\end{aligned} \tag{1.26}$$

where B^θ_t is

$$B^\theta_t = \min\left\{p_{t+1}R, \frac{(1-\lambda_\theta)p_t q}{\lambda_\theta k}\right\}$$

The solution of this maximisation is given as

$$\sum_{\theta=0,1}\omega_\theta\left[\lambda_\theta u'(c^\theta_{m,t+1})\left(\frac{p_t - B^\theta_t}{p_{t+1}}\right) + (1-\lambda_\theta)u'(c^\theta_{n,t+1})\left(\frac{p_t}{B^\theta_t} - 1\right)R\right] = 0$$

which gives us the optimal value of q.

1.5 Bank Equilibrium

Now suppose that young agents born at date t can organise an institution called a bank. They deposit all their endowment goods with a bank, and it uses the proceeds to acquire assets in favour of its members.[25] Banks function as liquidity providers. Free entry into the banking industry forces banks to compete by offering deposit contracts that maximise the expected utility of the consumers.

At the beginning of each period, financial intermediaries take deposits from typical individuals and provide a standard deposit contract, promising consumers a fixed amount of fiat money if they withdraw at date t, and consumption goods, $c_{n,t+1}$, if they withdraw at date $t+1$. More specifically, the arrangement is comprised of giving d units of fiat money if she withdraws at the end of period t irrespective of the occurrence of the state of nature if it has enough liquid assets, or $c_{n,t+1}$ units of the consumption goods if she withdraws at date $t+1$. d is the face value of the deposit at t, thus movers will consume $c_{m,t+1} = d/p_{t+1}$ in period $t+1$.

The standard deposit contract, however, cannot be made contingent on the asset returns or the liquidity shocks because the contract is made before the uncertainties are revealed. Therefore, if the bank does not have enough liquid assets to make the promised payment, it pays out all available liquid assets and capital, divided equally among those withdrawing. In dealing with situations of bank runs, I follow the assumption in Allen and Gale (1998) rather than the 'sequential service' assumption in Diamond and Dybvig (1983).

The consumption of a nonmover depends on the amount of d promised to movers, and the fraction of nonmovers who join the withdrawal at date t along with movers. Since the information on the ex-post agent's type is assumed to be private, and thus the bank cannot observe it publicly, the bank cannot tell if the withdrawer is a mover or not. Therefore, a nonmover has an incentive to pretend to be a mover unless the consumption of a nonmover is at least as much as that of a mover. We require the following constraint

[25] See Chapter 2 where agents may hold capital directly instead of depositing in a bank.

$$c^{\theta j}_{n,t+1} \geq c^{\theta j}_{m,t+1} \tag{1.27}$$

The preceding constraint, which is known as incentive compatibility constraint, tells us that a nonmover does not have any incentive to pretend to be a mover as long as she would get a higher consumption at their old-age. However, the incentive constraints may not be satisfied, depending on how uncertainty is realised, and this makes banks face defaults.

Suppose the following consumption function, which takes different values conditional on whether the equation (1.27) is met or not.

$$c^{\theta j}_{m,t+1} = \begin{cases} \frac{p_t}{p_{t+1}} \frac{q}{\lambda_\theta} = \frac{d}{p_{t+1}}, \text{ if (1.27) is satisfied} \\ \frac{q p_t}{p_{t+1}} + kR_L, \text{ otherwise} \end{cases} \tag{1.28}$$

$$c^{\theta j}_{n,t+i+1} = \begin{cases} \frac{1}{1-\lambda_\theta} \left(kR_j + \frac{p_t q - \lambda_\theta d}{p_{t+1}} \right), \text{ if (1.27) is satisfied} \\ \frac{q p_t}{p_{t+1}} + kR_L, \text{ otherwise} \end{cases} \tag{1.29}$$

The bank maximises its depositors' expected utility

$$\max_{c_{m,t+1}, c_{n,t+1}} E_t[U(c^{\theta j}_{m,t+1}, c_{n,t+1}); \lambda_\theta, R_j] = \lambda_\theta u(c^{\theta j}_{m,t+1}) + (1-\lambda_\theta) u(c_{n,t+1}) \tag{1.30}$$

subject to (1.27), (1.28), (1.29), and a budget constraint,

$$q + k = 1 \tag{1.31}$$

1.5.1 No Uncertainty

First, let us consider the case where the optimizing bank faces no uncertainty. With no uncertainty, the bank can achieve the first best allocation as in Section 1.4.2. A bank signs a deposit contract with its depositors to pay a fixed nominal amount d to anyone withdrawing at date t. The bank's maximisation problem is expressed in the form

$$\max_{c_{m,t+1},c_{n,t+1}} E_t[U(c_{m,t+1},c_{n,t+1})] = \alpha u(c_{m,t+1}) + (1-\alpha)u(c_{n,t+1}) \tag{1.32}$$

$$\begin{aligned} s.t. \quad & (i) \quad q + k = 1, \\ & (ii) \quad c_{m,t+1} \leq \frac{d}{p_{t+1}} \\ & (iii) \quad \alpha c_{m,t+1} + (1-\alpha)c_{n,t+1} = \frac{d}{p_{t+1}} + kR \\ & (iv) \quad c_{m,t+1} \leq c_{n,t+1} \end{aligned} \tag{1.33}$$

This maximisation problem is exactly the same as that in Section 1.4.2 if the bank chooses d such that $\alpha d = p_t q$. Therefore, the bank choosing $d = \frac{p_t q}{\alpha}$ may achieve the efficient allocation as shown in Section 1.4.2.

1.5.2 Equilibrium with Random Asset Returns

Assume that no other uncertainties exist except for R_j. All other variables are assumed to be deterministic.

Contingent Deposit Contract

Suppose the case in which the deposit contract is possible to be conditional on the return to the risky asset. More specifically, assume the contingent deposit contract to give movers and nonmovers $\frac{d}{p_{t+1}}$ and kR_H respectively in the state of $R_j = R_H$, and $\frac{qp_t}{p_{t+1}} + kR_L$ equally to movers and nonmovers when $R_j = R_L$. A bank maximises its depositors ex-ante expected utility

$$\begin{aligned} \max_{c^j_m, c^j_n} \quad & E_t[U(c^j_{m,t+1}, c^j_{n,t+1}; R_j)] \\ = & \sum_{j=H,L} \rho_j \left\{ \alpha u\left(c^j_{m,t+1}\right) + (1-\alpha) u\left(c^j_{n,t+1}\right) \right\} \end{aligned} \tag{1.34}$$

subject to the budget constraint (1.31), the feasibility conditions

$$\begin{aligned} c^j_{m,t+1} &\leq \frac{d}{p_{t+1}} = \frac{p_t}{p_{t+1}} \frac{q}{\alpha} \\ \alpha c^j_{m,t+1} + (1-\alpha)c^j_{n,t+1} &= \frac{qp_t}{p_{t+1}} + (1-q)R_j, \end{aligned} \tag{1.35}$$

and the incentive constraint (1.27). The Lagrangian function of this optimization is

$$\begin{aligned} \mathcal{L} =& \alpha u(c^j_{m,t+1}) + (1-\alpha)u(c^j_{n,t+1}) + \mu_1 \left(\frac{qp_t}{p_{t+1}} - \alpha c^j_{m,t+1} \right) \\ &+ \mu_2 \left(\frac{qp_t}{p_{t+1}} + (1-q)R_j - \alpha c^j_{m,t+1} - (1-\alpha)c^j_{n,t+1} \right) \end{aligned}$$

where μ_1 and μ_2 are Lagrange multipliers both of which have non-negative values. As the first-order conditions, we have the following set of simultaneous equations:

$$u'(c^j_{m,t+1}) = \mu_1 + \mu_2$$

$$u'(c^j_{n,t+1}) = \mu_2$$

$$\mu_1 \frac{p_t}{p_{t+1}} = \mu_2 R_j$$

From the first two first order conditions, we have

$$u'(c^j_{m,t+1}) \geq u'(c^j_{n,t+1}) \Leftrightarrow c^j_{m,t+1} \leq c^j_{n,t+1}$$

Thus incentive constraint is satisfied for all $j = \{H, L\}$. When the second constraint is binding (i.e., $\mu_1 > 0$),

$$u'(c^j_{m,t+1}) > u'(c^j_{n,t+1}) \Leftrightarrow c^j_{m,t+1} < c^j_{n,t+1}$$

Without loss of generality, let us call this state of R_j as R_H. Then it follows that

$$c^H_{m,t+1} = \frac{p_t}{p_{t+1}}\frac{q}{\alpha} < c^H_{n,t+1} = kR_H \tag{1.36}$$

When the second constraint is non-binding (i.e., $\mu_1 = 0$), we have

$$u'(c^j_{m,t+1}) = u'(c^j_{n,t+1}) \Leftrightarrow c^j_{m,t+1} = c^j_{n,t+1}$$

Call this state of of R_j as R_L. From the second feasibility condition in (1.35), and using the fact $c^L_{m,t+1} = c^L_{n,t+1}$, we have the following relation

$$c^L_{m,t+1} = c^L_{n,t+1} = \frac{qp_t}{p_{t+1}} + kR_L \tag{1.37}$$

In this way, contingent deposit contract makes it possible to give movers and non-movers $\frac{d}{p_{t+1}}$ and kR_H respectively in the state of $R_j = R_H$, and $\frac{qp_t}{p_{t+1}} + kR_L$ equally to movers and nonmovers when $R_j = R_L$. Substituting (1.36) and (1.37) into the objective function, then we have

$$\begin{aligned}\max_q \quad & \rho_H \left\{\alpha u\left(\frac{p_t}{p_{t+1}}\frac{q}{\alpha}\right) + (1-\alpha)u\left(\frac{(1-q)R_H}{1-\alpha}\right)\right\} \\ & + \rho_L \left\{\alpha u\left(\frac{qp_t}{p_{t+1}} + (1-q)R_L\right) + (1-\alpha)u\left(\frac{qp_t}{p_{t+1}} + (1-q)R_L\right)\right\}\end{aligned} \tag{1.38}$$

We get the optimal portfolio by differentiating (1.38) with respect to q, which yields the following

$$\sum_{j=H,L} \rho_j u'(c^j_{m,t+1})\frac{p_t}{p_{t+1}} = \sum_{j=H,L} \rho_j R_j u'(c^j_{n,t+1})$$

1.5.3 Equilibrium with Bank Runs

Deposit contract cannot be made state contingent, however, because the contract is made before the uncertainty on the illiquid asset return is revealed. This section shows whether banks can achieve the same allocation as contingent deposit contracts via (partial) bank runs in a specific state of nature when facing random asset returns.[26]

The bank's problem is the same as in (1.34) except that the amount of fiat money promised to give to the person who withdraws at date t is not contingent on the illiquid asset returns. Low capital return R_L may imply the incentive constraint (1.27) is not satisfied, in which case some nonmovers may wish to withdraw early at date t along with movers. From (1.28) and (1.29), we have

$$c_{m,t+1}^{j} \leq \frac{p_t}{p_{t+1}} \frac{q}{\alpha} = \frac{d}{p_{t+1}} \text{ and } c_{m,t+1}^{j} = c_{n,t+1}^{j} \text{ if } c_{m,t+1}^{j} < \frac{d}{p_{t+1}}$$

Note that the consumption goods always remain at date $t+1$ amounting $R_L k$ because of the assumption that the capital cannot be liquidated early and no financial markets available exist. The nonmovers who do not join a run withdraw their deposits at date $t+1$. Let δ denote the proportion of nonmovers who join to withdraw their deposits early, then we have the followings

$$\begin{aligned}
&\alpha c_{m,t+1}^{L} + \delta(1-\alpha) c_{n,t+1}^{L1} = \frac{p_t q}{p_{t+1}} = \frac{\alpha d}{p_{t+1}}, \\
&(1-\delta)(1-\alpha) c_{n,t+1}^{L2} = (1-q)\, R_L = \left(1 - \frac{\alpha d}{p_t}\right) R_L, \\
&c_{m,t+1}^{L} = c_{n,t+1}^{L1} = c_{n,t+1}^{L2}
\end{aligned}$$

$c_{n,t+1}^{L1}$ denotes the consumption of early withdrawing nonmovers and $c_{n,t+1}^{L2}$ is the consumption of nonmovers not withdrawing early. The first equation tells us that in a situation of low return of capital, the consumption of movers and some fraction of nonmovers combined is equal to the amount of liquid asset. The nonmovers who do not join the early withdrawal will consume from the returns on capital investment. In the end, three different types of consumers will end up consuming the same amount of goods.

[26] This section is based on Allen and Gale (1997).

Combining these relations leads to

$$c^L_{m,t+1} = c^L_{n,t+1} = \frac{qp_t}{p_{t+1}} + (1-q)R_L = \frac{\alpha d}{p_{t+1}} + \left(1 - \frac{\alpha d}{p_t}\right) R_L \tag{1.39}$$

On the other hand, when R_H happens (which implies the incentive constraint is met with strict inequality), agents consume the amount promised by the standard deposit contract as following:

$$\begin{aligned} c^H_{m,t+1} &= \frac{d}{p_{t+1}} = \frac{p_t}{p_{t+1}}\frac{q}{\alpha} \\ &< c^H_{n,t+1} = \frac{1}{1-\alpha}\left[\left(1 - \frac{\alpha d}{p_t}\right) R_H\right] = \frac{(1-q)\,R_H}{1-\alpha} \end{aligned} \tag{1.40}$$

Substituting (1.39), (1.40), and the budget constraint (1.31) into the objective function (1.34), we have

$$\begin{aligned} \max_q \quad & \rho_H \left\{\alpha u\left(\frac{p_t}{p_{t+1}}\frac{q}{\alpha}\right) + (1-\alpha)u\left(\frac{(1-q)R_H}{1-\alpha}\right)\right\} \\ & + \rho_L \left\{\alpha u\left(\frac{p_t}{p_{t+1}}\frac{q}{\alpha} + (1-q)R_L\right) + (1-\alpha)u\left(\frac{qp_t}{p_{t+1}} + (1-q)R_L\right)\right\} \end{aligned} \tag{1.41}$$

We get the optimal level of q by differentiating (1.41) with respect to q, which yields the following.

$$\begin{aligned} & \rho_H \left\{u'\left(c^H_{m,t+1}\right)\frac{p_t}{p_{t+1}} - u'\left(c^H_{n,t+1}\right) R_H\right\} \\ & \quad + \rho_L \left\{\alpha u'\left(c^L_{m,t+1}\right)\left(\frac{p_t}{p_{t+1}} - R_L\right) + (1-\alpha)u'\left(c^L_{n,t+1}\right)\left(\frac{p_t}{p_{t+1}} - R_L\right)\right\} = 0 \\ \Leftrightarrow & \rho_H \left\{u'\left(c^H_{m,t+1}\right)\frac{p_t}{p_{t+1}} - u'\left(c^H_{n,t+1}\right) R_H\right\} + \rho_L\left\{u'\left(c^L_{t+1}\right)\left(\frac{p_t}{p_{t+1}} - R_L\right)\right\} = 0 \\ \Leftrightarrow & \sum_{j=H,L} \rho_j u'(c^j_{m,t+1})\frac{p_t}{p_{t+1}} = \sum_{j=H,L} \rho_j R_j u'(c^j_{n,t+1}) \end{aligned}$$

This implies that the solution to this problem using the standard deposit contract is the same as in contingent deposit contract and thus efficient[27] when the bank choose $q^* = \frac{\alpha d^*}{p_t}$.

[27] Since ρ_L is not zero and $c^L_{m,t+1} = c^L_{m,t+1} = c^L_{t+1}$, the second term in the last equation can be

1.5.4 Equilibrium without Runs

Now suppose that a bank chooses its portfolio (q, k) and thus d such that runs never happen, which implies that the bank chooses either a small d and/or a high q to prevent default in equilibrium.[28] In an equilibrium without default, the bank promises the fixed nominal payment d to movers irrespective of the states $j = (H, L)$ at date t and nonmovers will receive the residue of the bank's assets at date $t + 1$, the amount of which should satisfy the incentive constraint, $c^j_{n,t+1} \geq c^j_{m,t+1}$ for any j. More specifically, the incentive constraint requires the following for all j:

$$c^j_{m,t+1} = \frac{d}{p_{t+1}} \leq c^j_{n,t+1} = \frac{1}{1-\alpha}\left[\frac{p_t q - \alpha d}{p_{t+1}} + kR_j\right]$$

Without loss of generality, we assume that $c^j_{m,t+1} = c^j_{n,t+1}$ if $j = L$.

From the preceding equation we obtain the following:

$$\begin{aligned} &\frac{(1-\alpha)d}{p_{t+1}} \leq \frac{p_t q}{p_{t+1}} - \frac{\alpha d}{p_{t+1}} + kR_j \\ \Leftrightarrow &\frac{d}{p_{t+1}} \leq \frac{p_t q}{p_{t+1}} + kR_j \quad (=,\ \text{if } j = L) \end{aligned}$$

Therefore, the consumption of movers is

$$c^H_{m,t+1} = c^L_{m,t+1} = \frac{d}{p_{t+1}} = \frac{p_t q}{p_{t+1}} + kR_L \tag{1.42}$$

The consumption of nonmovers, however, depends on the state R_j. When the capital returns are low, the consumption of a nonmover is the same as that of a mover. The amount of consumption is illustrated in (1.42), but the actual consumption of movers is not like that because movers can consume only with their fiat money.

rewritten as

$$\begin{aligned} &u'\left(c^L_{t+1}\right)\frac{p_t}{p_{t+1}} = R_L u'\left(c^L_{t+1}\right) \\ \Leftrightarrow\ &u'\left(c^L_{m,t+1}\right)\frac{p_t}{p_{t+1}} = R_L u'\left(c^L_{n,t+1}\right) \end{aligned}$$

[28] Allen and Gale (2004) call these types of banks *safe banks*.

Let M_{c_m} denote the aggregate money held by movers. Among total money holdings, $M = p_t q = 1$,[29] M_{c_m} is given to movers. d is determined such that $d = \frac{M_{c_m}}{\alpha}$, and each mover consumes d/p_{t+1}. The remaining fiat money, $M - M_{c_m}$ is passed over next period for the consumption of nonmovers. Therefore, we have

$$c^L_{n,t+1} = \left(\frac{M - M_{c_m}}{p_{t+1}} + kR_L\right)/(1-\alpha)$$

Since $c^L_{n,t+1}$ is equal to $c^L_{m,t+1}$, we have

$$c^j_{m,t+1} = \frac{d}{p_{t+1}} = \left(\frac{M - M_{c_m}}{p_{t+1}} + kR_L\right)/(1-\alpha) = \frac{p_t q}{p_{t+1}} + kR_L$$

Therefore, the consumption of a nonmover can be written alternatively as

$$c^L_{n,t+1} = \frac{p_t q}{p_{t+1}} + kR_L \tag{1.43}$$

The total amount of resources available at date $t+1$ when capital returns is high is

$$\frac{p_t q}{p_{t+1}} + kR_H$$

The share of consumption of movers among the total available resources is

$$\alpha c^H_{m,t+1} = \alpha \left[\frac{p_t q}{p_{t+1}} + kR_L\right]$$

Therefore, consumption of a nonmover is

$$\begin{aligned} c^H_{n,t+1} = & \frac{\frac{p_t q}{p_{t+1}} + kR_H - \alpha\left[\frac{p_t q}{p_{t+1}} + kR_L\right]}{1-\alpha} \\ = & \frac{p_t q}{p_{t+1}} + \frac{k(R_H - \alpha R_L)}{1-\alpha} \end{aligned} \tag{1.44}$$

Substituting the bank's budget constraint (1.31), and the consumption functions (1.42), (1.43), and (1.44) into the objective function, we have the following La-

[29]The total money supply of an island is normalized to 1.

grangian function

$$\mathcal{L} = \rho_H \left\{ \alpha u \left(\frac{p_t q}{p_{t+1}} + (1-q) R_L \right) + (1-\alpha) u \left(\frac{p_t q}{p_{t+1}} + \frac{(1-q)(R_H - \alpha R_L)}{1-\alpha} \right) \right\}$$
$$+ \rho_L \left\{ \alpha u \left(\frac{p_t q}{p_{t+1}} + (1-q) R_L \right) + (1-\alpha) u \left(\frac{p_t q}{p_{t+1}} + (1-q) R_L \right) \right\}$$

The first order condition with respect to q yields:

$$\rho_H \left\{ \alpha u' \left(c^H_{m,t+1} \right) \left(\frac{p_t}{p_{t+1}} - R_L \right) + (1-\alpha) u' \left(c^H_{n,t+1} \right) \left(\frac{p_t}{p_{t+1}} - \frac{R_H - \alpha R_L}{1-\alpha} \right) \right\}$$
$$+ \rho_L \left\{ \alpha u' \left(c^L_{m,t+1} \right) \left(\frac{p_t}{p_{t+1}} - R_L \right) + (1-\alpha) u' \left(c^L_{n,t+1} \right) \left(\frac{p_t}{p_{t+1}} - R_L \right) \right\} = 0$$

Solving the preceding equation gives the optimal q.

The consumption of a mover is always constant regardless of the state of nature, but the consumption of a nonmover, on the other hand, depends on the value of the random variable, R_j. Therefore, the solution to this problem that the bank's using a safe strategy and not allowing a run is not always efficient.

1.5.5 Equilibrium with Liquidity Shocks

The fraction of movers is a stochastic variable depending on the realisation of the state θ. If interbank deposit contracts are available across different islands and the shocks are correlated, then contingent contracts may be available as Bhattacharya and Gale (1985) and Allen and Gale (2000) show.[30] This chapter assumes that there are no exchanges between banks located on different islands, so I will not discuss how banks should deal with these issues further. The effect of liquidity shocks on the risk-sharing when an economy has both markets and banks is discussed in detail in Chapter 2. In Chapter 3 it is shown that how the two islands can be interconnected when one of them starts securitization.

[30]However, allowing interbank deposit markets may also cause a systemic crisis as studied in their papers.

1.6 Numerical Example

Let's look at numerical examples analysed in Section 1.4 and 1.5 using specific parameter values. Suppose that consumers have log utility and the economy faces random asset return.

The initial money supply M_t is normalized to 1. Since no monetary expansion or shrinkage are assumed, the current price level p_t is equal to the next price level, p_{t+1}. Let the fraction of movers at date t be $\lambda_\theta = \alpha = 0.5$, and assume the random variable R_j has the following two-point supports.

$$R_j = \begin{cases} R_H = 2.5, & \text{with probability } \rho_H = 1/2 \\ R_L = 0.5, & \text{with probability } \rho_L = 1/2 \end{cases}$$

The results with these parameter values are illustrated in Table 1.2. The row (I) shows the equilibrium values with markets. When the capital return is low, the price of capital is equal to the rate of return of the capital times future price level, i.e, $B_t^L = p_{t+1}R_L = 2\times 0.5 = 1$. Nonmovers exchange 50% of its cash holdings for capital and keep the remainder for the next period consumption ($\gamma^L = 0.5$). When the capital return is high, the capital is traded at a price $B_t^H = \min\left\{p_{t+1}R_H, \frac{(1-\alpha)p_t q}{\alpha k}\right\}$. Because of the insufficient liquidity, the price is determined following 'cash-in-the-market' price $\frac{(1-\alpha)p_t q}{\alpha k} = 2.0$ rather than maximum possible price, $p_{t+1}R_H = 3.5$. Since $\frac{p_t q}{B_t^H}R_H = \frac{2\times 0.5}{2}\times 1.5 = 0.75 > \frac{p_t q}{p_{t+1}} = \frac{2\times 0.5}{2} = 0.5$, nonmovers exchange all their fiat money for capital, and thus $\gamma^H = 1$.

The row (II) illustrates the equilibrium values with contingent deposit contract. When $R_j = R_H$, the bank promises to give movers $d = 2$ and they consume $\frac{d}{p_{t+1}} = \frac{2}{1.5616} = 1.2808$. Nonmovers consume $\frac{kR_H}{1-\alpha} = \frac{0.3596}{0.5} = 1.7981$ at date $t+1$. In the state of $R_j = R_L$, the bank gives $\frac{p_t q}{p_{t+1}} + kR_L = \frac{2\times 0.6404}{2} + 0.3596\times 0.5 = 0.8202$ both to movers and nonmovers equally.

The row (III) shows that the bank can achieve the same results as contingent deposit contract if it allows a run. When $R_j = R_L$, 56.16% of nonmovers withdraw at date t along with movers. The aggregate fiat money held by the bank is $p_t q = 1.5616 \times 0.6404 = 1$, and 56.16% of nonmovers withdraw early. Since the aggregate fraction of the young withdrawing at date t is $\alpha+\delta(1-\alpha) = 0.7808$, the consumption per person is $\frac{p_t q}{p_{t+1}(\alpha+\delta(1-\alpha))} = \frac{1}{1.5616\times 0.7808} = 0.8202$. Nonmovers who do not join the run consume $\frac{kR_L}{(1-\delta)(1-\alpha)} = 0.8202$ as well. The market provides insufficient liquidity than bank allocation when the economy faces random asset return. The bank holds more liquid asset $q = 0.6404$ than the market $q = 0.5$ when the rate of return on capital is stochastic.

Row (IV) shows the result of equilibrium values without a run. The safe bank holds much higher liquid asset ($q = 0.7143$), but lower $d = 1.2$[31] than the case allowing a run (III). Therefore, irrespective of the state of R_j, it can provide movers with $\frac{d}{p_{t+1}} = \frac{1.2}{1.4} = 0.8571$. This is the sum of $\frac{p_t q}{p_{t+1}} = 0.7143$ and $kR_L = 0.2857 \times 0.5 = 0.1429$. The remaining cash balances after movers' withdrawing is $p_t q - M_{c_m} = 1 - 0.6 = 0.4$. Therefore, a nonmover's consumption is $\left(\frac{p_t q - M_{c_m}}{p_{t+1}} + kR_L\right)/(1-\alpha) = \frac{0.4}{1.4} + 0.2857 \times 0.5 = 0.8571$ as well.

Table 1.2: Equilibrium Values with Random Asset Returns

	q	d	$\delta, \begin{bmatrix} \gamma^H \\ \gamma^L \end{bmatrix}$	$\begin{bmatrix} B_t^H \\ B_t^L \end{bmatrix}$	p_t	$\begin{bmatrix} c_{m,t+1}^H \\ c_{m,t+1}^L \end{bmatrix}$	$\begin{bmatrix} c_{n,t+1}^H \\ c_{n,t+1}^L \end{bmatrix}$	EU
(I)	0.5000	—	$\begin{bmatrix} 1.0000 \\ 0.5000 \end{bmatrix}$	$\begin{bmatrix} 2.0000 \\ 1.0000 \end{bmatrix}$	2.0000	$\begin{bmatrix} 1.0000 \\ 0.7500 \end{bmatrix}$	$\begin{bmatrix} 2.5000 \\ 0.7500 \end{bmatrix}$	0.0852
(II)	0.6404	2.0000	—	$\begin{bmatrix} - \\ - \end{bmatrix}$	1.5616	$\begin{bmatrix} 1.2808 \\ 0.8202 \end{bmatrix}$	$\begin{bmatrix} 1.7981 \\ 0.8202 \end{bmatrix}$	0.1094
(III)	0.6404	2.0000	0.5616	$\begin{bmatrix} - \\ - \end{bmatrix}$	1.5616	$\begin{bmatrix} 1.2808 \\ 0.8202 \end{bmatrix}$	$\begin{bmatrix} 1.7981 \\ 0.8202 \end{bmatrix}$	0.1094
(IV)	0.7143	1.2000	0.0000	$\begin{bmatrix} - \\ - \end{bmatrix}$	1.4000	$\begin{bmatrix} 0.8571 \\ 0.8571 \end{bmatrix}$	$\begin{bmatrix} 2.0000 \\ 0.8571 \end{bmatrix}$	0.0577

Notes. (I) Market, (II) Contingent Contract, (III) With Run, (IV) No Run

[31] $d = \frac{p_t q}{\alpha} = \frac{1.4\times 0.7143}{0.5} = 1.2$. Let denote M_{c_m} be the aggregate money held by movers. Then $\frac{M_{c_m}}{p_{t+1}\alpha} = c_{m,t+1}^L = 0.8571$, which yields $M_{c_m} = 0.6$. I.e., among total money holding $M = p_t q = 1$, 60% is given to movers.

1.7 Conclusion

This chapter studied the processes of how asset prices are determined when the economic agents face various stochastic shocks. Based on that, this article analysed how financial markets and intermediaries work to address the consumer's problem: allocating resources efficiently over a period of her life. Due to the maturity structure and maturity mismatch between liquid and illiquid assets, the portfolio decision by a consumer who are facing liquidity shocks must be made with caution.

The mechanism of asset prices determination is described in detail by drawing together the results of the scattered studies to a comprehensive view. Facing random returns of an illiquid asset and liquidity preference shocks, consumers trade their assets at a price realized in the asset market according to the shocks. This article also showed that forming a bank sometimes allows consumers to achieve higher utility than using financial markets. Especially, when the agents are subject to random returns on illiquid assets and bank runs are allowed to occur in a particular state of nature, banks are shown to achieve efficient allocation as if contingent deposit contract were made.

This chapter assumes complete markets, other than the fact that the Arrow securities are not traded. Also, it was assumed that the market and the bank do not coexist together, and thus allocation and welfare were analysed separately. If incomplete markets were assumed, and both financial intermediaries and markets coexist, then asset price and the amount of liquidity to be provided with consumers would take a different form from the results analysed in this chapter. More in-depth studies into the relationship between financial intermediaries and financial markets are discussed in Chapter 2, where both institutions coexist, and the economy faces more than one exogenous shock at the same time and asymmetrical information situations.

Another weakness of this chapter is that, by assuming a constant money supply, it does not take into account how the monetary policy will affect portfolio decisions. Chapter 3 discusses whether the expansionary monetary policy can affect the bank's portfolio composition, and how the amount of liquidity held by the whole economy is influenced when securitization is introduced.

Chapter 2

A Swansong by Banks: Banking and Financial Markets under Asymmetric Information and Limited Participation

2.1 Introduction

Consumers facing uncertainties participate in asset markets to allocate their resources over periods. If true asset prices are fully revealed in the market, they can obtain efficient consumption allocation. However, if some of the consumers (as known as uninformed consumers) facing uncertainty have access to only limited information, then informed consumers might exploit the uninformed and gain profits by utilising their knowledge and influencing the prices systematically.

The purpose of this chapter is to examine whether uninformed consumers who face uncertainty can prevent losses arising from asymmetric information by constructing a bank and operating it in a particular way. More specifically, I show that bank runs may serve to communicate information across agents and, thus, enhance rather than thwart the efficiency of the allocations. Bank runs can be efficient because they reveal new information to uninformed investors and thus prevent informed agents from affecting asset prices. This eventually makes asset prices be determined by fundamentals.

For that, this chapter presents a theoretical model showing that financial intermediaries[1] can play a special role, that of revealing information even without investing resources in order to identify information when financial markets are imperfect. The role of banks as information producers under information asymmetries has been discussed widely in the literature. What I mean by 'information processing' in this chapter, however, is different from the concept the most related studies are adopting. While this article shares some common features with standard models in that banks manage the problems resulting from asymmetric information,[2] the mechanism of processing or revealing information is quite different from them.

It is generally assumed that, for example, the production of information in the market "will not be done efficiently or at least cost (Campbel and Kracaw, 1980, p.881)." Thus, the studies dealing with banks facing information problem focus on the need for those institutions to invest their resources in order to produce valuable information. The seminal paper on this issue by Brealey et al. (1977), and Boyd and Prescott (1986), for example, show that financial intermediaries can produce reliable information which is only known to a borrower ex-ante and thus results in an adverse selection problem.

The striking difference between these studies and mine is that the financial coalition by agents in this model does not produce information deliberately by investing some of its resources. The production of information is done efficiently without cost throughout bank runs under certain conditions. Let me explain this result in an intuitive way. Suppose there exist two kinds of uncertainties in the world: liquidity preference shocks and shocks on illiquid assets' return, both of which are assumed

[1] Financial intermediaries are defined as the voluntary coalition by agents in this chapter, and intermediaries, coalitions are used interchangeably with 'banks'.

[2] For example, financial intermediaries can provide efficient lending mechanisms when there exist asymmetric information between lenders and borrowers. See Diamond (1984).

to be known only to informed agents.[3] Suppose also the informed agents can manipulate illiquid asset prices, which are determined endogenously and affected by random shocks, only in a certain state of nature by adjusting the supply of and demand for them. As a result, Uninformed consumers who participate in the market suffer losses due to these distorted prices. To prevent these losses, the uninformed form a bank, which makes a specific type of deposit contract with its members and let it go bankrupt if a particular state of nature occurs. The existence of bank runs itself reveals that a distinct state of nature happens (e.g., the fraction of uninformed agents who are subject to preference shocks are revealed through bank runs), which again makes them conjecture still unknown information (e.g., the rate of return on illiquid assets) with accuracy. Now that the asymmetric information problem has been resolved, they will be able to trade in the asset market without loss.

More specifically, uninformed agents form intermediaries, and each bank makes a standard deposit contract promising a fixed amount of fiat money if she turns out to be a mover at date $t+1$. The bank, which maximises the ex-ante expected utility of the typical uninformed agent, will hold only minimal fiat money hoping the number of movers to be low. Then it goes bankrupt when the fraction of movers is revealed higher than expected. Even though agents' relocation shocks are private information, the happening of bank runs itself will disclose the information that the fraction of movers is high. Once the uninformed agents acquire this information, they know that the true prices of illiquid assets will be revealed. Thus, they can exchange their assets with others without suffering the loss caused by asymmetric information, in accordance with their true liquidity preferences. This process shows one of the functions banking serves as information processor by allowing for bank runs.

In the process of motivating the rationale for the existence of financial intermediation, information-asymmetric problems of the uninformed agents are solved in relation to informed agents. I argue that under asymmetric information and lim-

[3]Therefore, there are four possible states of nature, and hence four different forms of asset prices.

ited participation in financial markets, bank runs (or bankruptcy) may help agents to achieve an efficient allocation since bank runs can make hidden information revealed, to which only a fraction of agents in the economy are assumed to have access. Therefore, the financial intermediaries in this model not only serve the standard role of liquidity provision, which has been widely studied since the 1980s pioneered by Bryant (1980) and Diamond and Dybvig (1983), but also have an alternative justification based on information processing when the economy faces uncertainty.

This chapter also provides a new ground for the coexistence of banks and financial markets. Even when all the agents deposit their whole endowment goods with the bank, financial intermediaries and markets coexist once (efficient) bank runs happen. The scale of banks under asymmetric information and limited participation in relation to financial markets are also considered. Allowing a run implies that the scale of the banking sector is minimized to the lower fraction of consumers who face liquidity shocks, and thus potentially increase welfare.

The model in this chapter assumes that the financial market is imperfect due to informational asymmetries and limited participation. I follow assumptions used in Gorton and Pennacchi (1990) to model information asymmetry and Diamond (1997) for introducing the effect of limited participation in financial markets and intermediation. This chapter, however, provides a different rationale for the coexistence of markets and banks because of the possibility of a run. Moreover, Diamond (1997) assumes that all assets pay a rate of return that is known with complete certainty and, thus, has not considered the effects of uncertainty on the equilibrium and the possibility of bank runs, which are dealt with importance in this chapter.

Gorton and Pennacchi (1990) present a model explaining how financial intermediaries arise endogenously and how security contracts are made when uninformed agents, facing asymmetric information, need to transact. Informed agents can exploit the uninformed by their superiority in information[4]. Facing this problem,

[4]They used the standard three-period model as in Diamond and Dybvig (1983). Uninformed agents are endowed with 1 unit of capital good at their initial period and consumption good at their middle period. Agents who are subject to liquidity shocks become early consumers. The trade between early and later consumers is made in imperfect financial markets where early consumers

uninformed agents form a coalition to protect themselves from losses and let it issue debt and equity simultaneously. Throughout this arrangement, financial intermediaries 'can attract informed agents to hold equity and uninformed agents to hold debt which they then use for trading purposes (Gorton and Pennacchi, 1990, p.50)' if the number of informed agents is sufficiently larger than that of uninformed agents. In this way, 'the existence of our intermediary does not rely on providing risk-sharing or resolving inefficient interruption of production. Our notion of liquidity as providing protection from insiders is fundamentally different (Gorton and Pennacchi, 1990, p.51).' It seems that Gorton and Pennacchi (1990) *implicitly* concludes that financial intermediaries dominate markets if certain conditions are met.[5]

For some similarities between Gorton and Pennacchi (1990) and mine, there are some significant differences. In Gorton and Pennacchi (1990), the institution resolves the problem on asymmetric information by providing different bespoke assets with uninformed and informed agents respectively, irrespective of the information structure. In this chapter, however, the bank just plays the standard role of providing risk sharing with consumers. The mechanism through which the intermediary reveal the information asymmetries to the uninformed, unlike Gorton and Pennacchi (1990), is a simple standard deposit contract allowing bank runs to occur under certain circumstances. Moreover, both institutions coexist unlike Gorton and Pennacchi (1990).

Unlike the traditional costly bank runs,[6] one of the conclusions in this article is that a bank run is an efficient phenomenon in certain circumstances. The literature

sell their capital good in return for consumption goods from late consumers. Their results are valid in this chapter in that the informed agents exploit the uninformed agents systematically when markets have information asymmetries. The difference in environment between Gorton and Pennacchi (1990) and this chapter is that this article takes (i) the OLGs model having two separate islands, (ii) agents are subject to relocation shocks, (iii) consumers use fiat money as liquid assets the rate of return of which is not fixed but determined in the interaction between the old and the young, and (iv) endowment goods are only available at agents' initial period. These are explained in detail in Sec 2.4.2.

[5]For details on the relationship between markets and banks, see Thesis Introduction and Chapter 1.

[6]For the recent studies on costly bank runs, see Martin et al. (2014), Angeloni and Faia (2013), Gertler and Kiyotaki (2015), and Gertler et al. (2016).

dealing with efficient bank runs deals with information based runs.[7] They focus on the fact that bank runs are the reflection of the business cycle, and they come from agents' optimising behaviour expecting the poor performance of the banks rather than from the result of sunspots.

The related literature with this chapter is Allen and Gale (1998). According to Allen and Gale (1998), bank runs could be efficient in the case where a standard deposit contract cannot be made contingent on the 'leading economic indicator', such as the return on illiquid assets, which yields a random return. A bank run makes consumption contingent on the state of nature. The indicator can be observed with accuracy at the middle period and thus, 'the possibility of equilibrium bank runs allows banks to hold the first-best portfolio and produces just the right contingencies to provide first-best risk sharing (p.1250).' No information problem arises at least at a time when bank runs actually happen in Allen and Gale (1998) since each agent can observe the indicator with precision by assumption.[8] Bank runs happen, in this chapter, however, irrespective of whether agents have access to the information about the rate of return on risky asset. In other words, banks runs may happen even when the higher rate of return on illiquid assets is expected. And the run is efficient not because it makes contingent consumption possible but because it reveals the unknown information to which uninformed agents do not have access. The studies I reviewed until now share one common feature; those papers all deal with the economy focused on real factors in which a role for currency is ignored. Champ et al. (1996), on the other hand, constructs a monetary model in which a role for currency is considered and monetary factors play a specific role in banking panics. In Champ et al. (1996), the bank provides insurance with agents who have random needs for liquidity because of the possibility of relocation. The 'relocation shock' takes the place of 'preference shock' in Diamond and Dybvig (1983). One

[7] The related papers are Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988), Allen and Gale (1998), and Diamond and Rajan (2001), just to name a few.

[8] In a similar vein, Gorton (1985) provides a model where agents obtain, at the middle period, some information regarding the rate of return on risky asset of which expected value is realized at the final period.

of the reasons why they consider it seriously is because the defining fractures of banking distress involve the currency in a central way. Except assumptions used in the basic set-up of the model (three-period OLGs model with relocation shock where fiat money is used as liquid assets), this chapter, however, is not directly related to the Champ et al. (1996).

The rest of the chapter is organized as follows. The environment is described in Section 2.2. In Section 2.3 the process of asset price determination is shown in detail. In Section 2.4 the asset market equilibrium with and without asymmetric information is described. Then, in Section 2.5 the financial intermediary is introduced and considers how the intermediary deals with the problems regarding the asymmetric information. In Section 2.6, I show that financial intermediaries are still needed when we introduce another friction into financial markets, limited participation, even though each agent has access to the perfect information. And then, I consider an economy, having both asymmetric information and the limited participation, and study the role of banks. Concluding remarks are contained in Section 2.7.

2.2 Environment and Behaviour of Agents

There[9] are infinitely many overlapping generations of agents who live for three periods (call them young-middle-old respectively). The population in this economy is equally divided by two distinct locations– two different islands. At the middle of each period stochastic relocation shocks occur and a fraction of movers among uninformed agents, λ^i, is relocated to the different island at the end of the same period and will spend her final period on the island. λ^i is assumed to take two different values: λ^h, with probability μ_h, or λ^l, with probability μ_l, where $0 < \lambda^l < \lambda^h < 1$. The probability of relocation is assumed to be the same across both islands so that the population in each island remains constant.[10] There are two different types of

[9]The model in this section where only financial markets exist is based on models by Champ et al. (1996), Gorton and Pennacchi (1990) and Diamond (1997). Champ et al. (1996) built an OLGs model with random relocation where agents live for two periods.

[10]If there were no aggregate relocation shocks, there were large numbers of uninformed agents, and their relocation shocks are assumed to be independent, the fraction of movers is equal to the

assets for savings: capital and fiat money. The capital is an illiquid asset for two reasons. Firstly, the capital invested in production needs two periods of time before it transformed into consumption goods. Secondly, it is assumed not to be transported across islands, and moreover, due to the 'limited communication', claims against capital is assumed to be useless. For movers to consume after being relocated, they need fiat money which is identical in the two islands and universally accepted as a means of exchange.

2.2.1 Agents

There are four types of agents as of date t (see Figure 2.1).

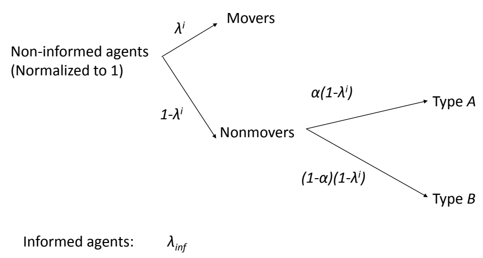

Figure 2.1: Types of Agents

Agents are divided broadly into two groups: the informed and the uninformed. The measure of the entire set of uninformed agents in each island is normalised to the unit interval $[0, 1]$, and the fraction of informed agents in terms of the scale of the uninformed is denoted as λ_{inf}. Thus, in each island, $1 + \lambda_{inf}$ number of agents live.

Each uninformed agents are subject to random relocation shocks when they are middle age. After learning they should move, movers are relocated to the other island at the beginning of date $t + 2$ (at the beginning of their old). Before they are relocated, they trade their holdings of capital for fiat money in a financial market. They exchange this accumulated money with consumption goods from the new-born young (or the coalition of the young if any) at date $t + 2$. The informed are assumed

probability of being a mover according to the Law of Large Numbers.

not to be subject to the relocation shocks and thus stay on the same island. Another difference between the informed and the uninformed is that informed agents have access to the information whether the rate of return on capital would be high or low, and the fraction of movers, who belong to the same generation as them, is large or small when they are in the middle age (at date 1).

Agents, who are turned out to stay on the home islands, exchange some or all of their holdings of money with capital the movers hold. They consume the return from any capital investment made when young, and the capital they bought from movers when middle, plus consumption goods, for which they trade any fiat money holdings (if any left) with the newly born young at $t+2$. The timing of events is illustrated in Figure 2.

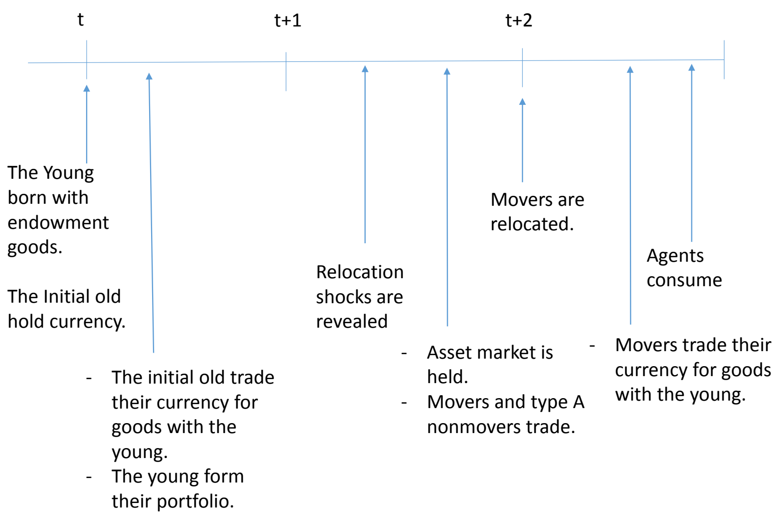

Figure 2.2: Timing of Events

The nonmovers are classified again into two subgroups. The fraction among nonmovers, $\alpha(1-\lambda^i)$, born at date t and called type A, have access to a financial market at date $t+1$. On the other hand, the fraction of $(1-\alpha)(1-\lambda^i)$ is assumed not to join the market. Thus, α can be used as an index to show financial market development as in Diamond (1997). Except for the possibility of participating in the market, there are no essential differences between types A and B. The fraction of type B, $(1-\alpha)(1-\lambda^i)$, is assumed to be zero in this section, Section 2.4 and

2.5. The effect of the existence of type B on the role and scale of banking will be discussed in detail in Section 2.6.

2.2.2 Preferences

All agents are risk-neutral and consume at their final period. The reason why we taking linear utility function is for calculating a closed-form solution of the optimizing agent. Uninformed agents, who are born at time t, are ex-ante identical and individually observe relocation shocks at the middle of date $t+1$. This information is privately held and not observable by other agents.[11] Preferences of the generation born at time t are represented by:

$$U(c_{\tau,t+2}) = \begin{cases} c_{m,t+2}, & \text{if movers} \\ c_{n,t+2}, & \text{if nonmovers} \\ c_{inf,t+2}, & \text{if the informed} \end{cases}$$

where $c_{\tau,t+2}$ denotes the amount of good that is consumed in the final period of life by a type of τ born in period t.

2.2.3 Endowments and Technology

Each agent is born with one unit of endowment goods, which can be used both for consumption and investment and cannot be stored from one period to the next. Endowments are used to form a portfolio consisting of two types of assets: fiat money, M and capital, k. Each agent born at date t chooses to allocate her endowment into q_t and k_t to maximise her expected utility.[12] Let v_t be the value of 1 unit of

[11] If this information can be observed by anyone living on the island, the information problem is removed. Movers deal only with nonmovers and nonmovers only with movers. Also, with 'open communication', where the claims against the capital can be verified costless and can be traded without cost, all agents will only hold capital since its rate of return is expected to be greater than that of fiat money. The next footnote explains that such an equilibrium is not possible.

[12] If agents are risk-averse, they will both acquire fiat money and capital even though the rate of return on capital is expected to be higher than that of currency. If agents are assumed to be risk-neutral, all might want to hold only capital. If financial markets take place in the middle period of their life, however, at least some should keep some fiat money for trades to occur. Some fraction of agents who are turned out to be a mover will try to sell their holdings of capital. However, since no one holds money, the asset price will go to zero. Thus, there will be at least some agents

fiat money in terms of goods at time t. Then $q_t(\equiv v_t M_t)$ denotes the real demand for fiat money at date t. Uninformed agents will save some of their endowment in the form of currency despite its lower rate of return than its alternative (capital) because some of them are subject to relocation shocks. Thus, two different forms of savings–fiat money and capital–are not regarded as perfect substitutes to each other.

All uninformed agents are homogeneous ex-ante and decide how to divide their savings between currency and capital with their one unit of endowment goods at the beginning of their first period (at date t). Their budget constraint is given as

$$q_t + k_t = 1$$

k_t units of investment at date t yields $R_j k_t$[13] units of output at date $t+2$, and R_j is a random variable taking two different values: R_H, with probability ρ_H,[14] or R_L, with probability ρ_L where $R_H > R_L > 0$. Even though v_t and v_{t+1} is determined endogenously within the model, the rate of return on fiat money is just equal to 1 if money supply is assumed to remain constant and two different islands are symmetric.[15] $E[R_j]$ is assumed to be greater than two periods rate of return on fiat money,[16] v_2/v_0, i.e., it is assumed that $E[R_j] > v_2/v_0$. It is also assumed that $R_H > v_2/v_0 > R_L$, i.e., the rate of return on capital could be lower than that of fiat money when the random return is low. The assumption on technology is

who might want to hold currency to exploit this arbitrage opportunity. In this section, agents are assumed to hold both fiat money and currency for this reason, i.e., in equilibrium both $q_t > 0$ and $k_t > 0$.

[13]The linear capital production technology was introduced into the overlapping generations model by Cass and Yaari (1966) and the assumption is widely used in the literature.

[14]For simplicity I assume that the probability of each possible rate of return on capital is equal as $\rho_L = \rho_H = 1/2$.

[15]In this chapter, I consider stationary and symmetric allocation only, so I drop the time scripts t from now on. The consumption of agents born at date t is written as $c_{\tau,2}$, and the value of fiat money at date $t+s$ is denoted as v_s for all $s \geq 0$. The notations without time subscript denote the amount determined at date $t = 0$.

[16]Thus, the fiat money is used for both medium of exchange and store of values in this model.

summarized as follows:

$$E[R_j] = \frac{1}{2}(R_H + R_L) > \frac{v_2}{v_0}, \quad R_H > \frac{v_2}{v_0} > R_L > 0$$

2.2.4 Money Market Equilibrium

No one in the future generations is born with fiat money. For young agents to acquire fiat money per each island, they must trade with the initial old living on the same island with them who are endowed in total with M units of fiat money at date t. The initial value of fiat money, v_0, is determined at the beginning of period t by the coincidence of the demand for fiat money by the young and the supply of it by the initial old.[17] The (real) demand for fiat money (the number of goods each young agent chooses to sell for fiat money) is given as:

$$D^M = (\lambda^i + (1 - \lambda^i))q = q$$

The supply of fiat money measured in goods comes from the initial old.

$$S^M = v_0 M$$

Therefore, the value of one unit of fiat money at date t is determined such that

$$v_0 = \frac{q}{M} \tag{2.1}$$

In a stationary equilibrium where the money supply is constant,[18] all generations face the same decision problem. Therefore, q and thus v have constant values, i.e.,

[17] The initial middle-aged agents, holding both currency and capital at date t, are assumed not to participate in this market since the market takes place before the relocation shocks are revealed. Under this assumption, the initial middle have no incentive to change their portfolio.

[18] This assumption cannot be sustained in situations where each island faces different idiosyncratic relocation shocks, or where monetary authorities play an active role to affect the economy by adjusting the money supply. If the two separate islands had a different fraction of movers, then the population of each island would be distinct from each other and thus, will have dynamic effects on consumption and the value of money after their relocation. The effect of these incidents on equilibria will be discussed in the further study.

$v_{t+s} = v$ for all $s \geq 0$.

2.2.5 Movers

A consumer, who is turned out to be a mover at her middle age (at date 1), sells her entire holdings of capital at a price B_1 before she moves. She inelastically supplies her capital holdings, whatever the price is. The total unit of currency the mover accumulates after the trade at date 1 is represented as:

$$M + \frac{B_1 k}{v_1} \tag{2.2}$$

The second term of the preceding equation denotes the amount of fiat money the mover acquires by selling her capital to nonmovers who belongs to the same generation with her. Then the old-age consumption of a mover at date $t+2$ is given as:

$$c_{m,2} = \left(M + \frac{B_1 k}{v_1}\right) v_2 = \frac{v_2}{v_0} q + \frac{v_2}{v_1} B_1 k = q + B_1 k \tag{2.3}$$

The consumption depends on agents' portfolio decision and the price of an illiquid asset at date 1, B_1.

2.2.6 Nonmovers

Non-movers must decide to trade either some or all of their holdings of fiat money for illiquid assets (capital) at date $t+1$ weighing the difference between the expected returns of the two assets, unlike movers facing a trivial decision problem whether to sell their capital or not. So, the total supply of fiat money (S^M) by nonmovers, which equals to the demand for capital, D^K, takes the following form.

$$S^M = D^K = (1 - \lambda^i)\gamma M$$

where γ ($0 \leq \gamma \leq 1$) is the ratio of cash reserves exchanged with capital. $\gamma = 1$ implies that nonmovers trade their whole currency for capital at date $t+1$ and do not pass it over to the next period. Then consumption of a nonmover at date $t+2$ is represented as:

$$\begin{aligned} c_{n,2} &= \left(\frac{\gamma M v_1}{B_1} + k\right) R_j + (1-\gamma) M v_2 \\ &= \left(\frac{\gamma v_1}{v_0}\frac{q}{B_1} + k\right) R_j + (1-\gamma)\frac{v_2}{v_0} q \\ &= \left(\frac{\gamma q}{B_1} + k\right) R_j + (1-\gamma) q \end{aligned} \tag{2.4}$$

2.3 Determination of Asset Prices and Equilibrium

This section contains a detailed exposition on how the price of capital, an illiquid and risky asset, is determined when the economy faces uncertainty on the fundamentals. This discussion is based on the work of Gorton and Pennacchi (1990), Allen and Gale (1994), and Diamond (1997), which discuss the process of asset price determination in different situations.

2.3.1 Fundamental Asset Price

Assume that an asset market at date 1 is held after the relocation shocks are revealed and before movers are relocated. Let $B_1^{i,j}$ be the (nominal) price of one unit of capital traded in the asset market at date 1 facing uncertainties. i and j denote the state of nature where $i = \{h, l\}$, and $j = \{H, L\}$ respectively. We have the following proposition:

Proposition 2.3.1

$$B_t^{i,j} = \min\left\{\frac{R_j}{v_2/v_1}, \frac{v_1}{v_0}\frac{\gamma(1-\lambda^i)q}{\lambda^i k}\right\} \tag{2.5}$$

Proof See Appendix 2-1.

The proposition tells us that if the market has enough liquidity (i.e.,$S^M > D^M$), then the asset price would reach its highest possible point, $\frac{R_j}{v_2/v_1}$. However, if the market suffers liquidity shortage for any reason, the price will be determined by the amount of cash supplied. This is known as 'cash-in-the-market pricing' in which the asset price is determined as the ratio of total available 'cash' to the amount of asset provided (Allen and Gale, 1994, 2009).

2.3.2 Asset Prices with Uncertainties

Proposition 2.3.2 *Suppose that the random variable R_j takes on the value of R_H with probability of ρ_H and R_L with probability of ρ_L, where $R_H > R_L > 0$, and all assets are held by consumers. Also assume the random variable λ^i has the following two-point supports.*

$$\lambda^i = \begin{cases} \lambda^h, & \text{with probability } \mu_h \\ \lambda^l, & \text{with probability } \mu_l, \end{cases}$$

where $0 < \lambda^l < \lambda^h < 1$.

Then the capital prices at date 1 facing uncertainties, $B_t^{i,j}$, have the following relationship:

1. *When $R_j = R_H$ the price of capital is determined such that $B_t^{i,H} = \min\left\{\frac{R_H}{v_2/v_1}, \frac{v_1}{v_0}\frac{\gamma^H(1-\lambda^i)q}{\lambda^i k}\right\}$, which is strictly greater than $\frac{v_1}{v_0}$, and when $R_j = R_L$, $B_t^{i,L} = \frac{R_L}{v_2/v_1}$, which is strictly less than $\frac{v_1}{v_0}$. ($i = \{h, l\}$).*

2. *γ^H is equal to 1 as long as $R_H > \frac{v_2}{v_0}\frac{\gamma^H(1-\lambda^i)q}{\lambda^i k}$, and γ^L is determined such that $R_L > \frac{v_2}{v_0}\frac{\gamma^L(1-\lambda^i)q}{\lambda^i k}$, which implies $\gamma^L = \frac{v_2}{v_0}\frac{\lambda^i k R_L}{(1-\lambda^i)q}$.*

3. *In the equilibrium we have*

$$B_1^{l,H} > B_1^{h,H} > v_1/v_0 > B_1^{i,L} \tag{2.6}$$

4. *Combining the above results, we have the following relationship.*

$$R_H \geq \frac{v_2}{v_1}B_1^{l,H} > \frac{v_2}{v_1}B_1^{h,H} > \frac{v_2}{v_0} > \frac{v_2}{v_1}B_1^{h,L} = \frac{v_2}{v_1}B_1^{l,L} = R_L \tag{2.7}$$

where $B_t^{i,H} = \min\left\{\frac{R_H}{v_2/v_1}, \frac{v_1}{v_0}\frac{(1-\lambda^i)q}{\lambda^i k}\right\}$.

Proof See Appendix 2-2.

Note that in any case of either $R_j = \frac{B_t^j}{p_{t+1}}$ or $R_j = \frac{p_t}{p_{t+1}}\frac{\gamma^j(1-\alpha)q}{\alpha k}$, the asset price in this section cannot be higher than the fundamental value, $p_{t+1}R_j$. The market always undergoes "underpricing" (Allen and Gale, 1994).[19]

Assumption 5 *When the economy has large fraction of movers, i.e., when* $\lambda^i = \lambda^h$*, then the asset price is determined by the amount of liquidity in the market.*

When the economy suffers high liquidity needs which implies less provision of liquidity by relatively lower numbers of nonmovers, the asset price is assumed to determined by the 'cash-in-the-market'. This assumption is needed to calculate the exact amount of capital provided to the market by informed agents (See Proposition 2.4.1).

Corollary 2.3.3 *Applying the previous assumption, we have the following relationship.* $B_t^{l,H} = \min\left\{\frac{R_H}{v_2/v_1}, \frac{v_1}{v_0}\frac{(1-\lambda^l)q}{\lambda^l k}\right\}$ *and* $B_t^{h,H} = \frac{v_1}{v_0}\frac{(1-\lambda^h)q}{\lambda^h k}$.

2.3.3 Equilibrium

Formally, a imperfectly competitive rational expectations equilibrium for agents born at date $t = 0$ consists of

1. The price system (a price vector): $\{B_1^{i,j}\} = (B_1^{h,H}, B_1^{h,L}, B_1^{l,H}, B_1^{l,L})$.

2. The vectors of the value of fiat money: $v_0 = v_1 = v_2 = v$

3. The expected consumption of agents:

$$E^A[C_{\tau,2}] = \begin{cases} E[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] \\ E[c_{inf,2}] \end{cases}$$

[19] Asking if the asset price always takes the value less than the fundamental value is questioning whether 'bubbles' may exist or not. This is an interesting question but is not dealt with in this article. See Prize (2013) for an introductory description of whether financial assets reflect fundamental values or bubbles exist.

where $E[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}]$ denotes the expected utility of the uninformed agent.

4. A specification of storage strategies for (market participating) nonmovers: γ^L, where $0 < \gamma^L \leq 1$.

5. A specification of insider coalition strategies : $\lambda^{i,j}_{inf}$ (discussed in detail in Section 2.4.2).

such that,

1. Agents' respective utilities are maximised ($E[c_{\tau,2}]$ maximises agent τ's expected utility)

2. $\{B_1^{i,j}\}$ and γ clear the asset market in all states of $\{i,j\}$, where $i = \{l,h\}$ and $j = \{L,H\}$.

3. $\lambda^{i,j}_{inf}$ is self-enforcing.

2.4 Market Equilibrium

2.4.1 Full Participation & Complete Information

Let $\widehat{B}_1^{i,j}$ be the full-information prices of an illiquid asset for states $\{i,j\}$ which is determined from Proposition 2.3.2. And let $E^F[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}]$ be expected utility of uninformed agents with full information and full participation, then $E^F[\cdot]$

is given as:[20]

$$
\begin{aligned}
&E^F[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] \\
&\quad = \frac{\mu_h}{2}\left[\lambda^h\left(\frac{v_2}{v_0}q + \frac{v_2}{v_1}\widehat{B}_1^{h,H}k\right) + (1-\lambda^h)\left(\frac{v_1}{v_0}\frac{q}{\widehat{B}_1^{h,H}}R_H + R_H k\right)\right] \\
&\quad + \frac{\mu_l}{2}\left[\lambda^l\left(\frac{v_2}{v_0}q + \frac{v_2}{v_1}\widehat{B}_1^{l,H}k\right) + (1-\lambda^l)\left(\frac{v_1}{v_0}\frac{q}{\widehat{B}_1^{l,H}}R_H + R_H k\right)\right] \\
&\quad + \frac{\mu_h}{2}\left[\lambda^h\left(\frac{v_2}{v_0}q + \frac{v_2}{v_1}\widehat{B}_1^{h,L}k\right) + (1-\lambda^h)\left(\frac{v_1}{v_0}\frac{\gamma^h q}{\widehat{B}_1^{h,L}}R_L + R_L k + (1-\gamma^h)\frac{v_2}{v_0}q\right)\right] \\
&\quad + \frac{\mu_l}{2}\left[\lambda^l\left(\frac{v_2}{v_0}q + \frac{v_2}{v_1}\widehat{B}_1^{l,L}k\right) + (1-\lambda^l)\left(\frac{v_1}{v_0}\frac{\gamma^l q}{\widehat{B}_1^{l,L}}R_L + R_L k + (1-\gamma^l)\frac{v_2}{v_0}q\right)\right] \\
&\quad = q + E[R_j]k
\end{aligned}
\tag{2.8}
$$

Lines between the second and fifth in (2.8) represents the consumption level when each state $\{i, j\}$ happens. In what follows, I study how the expected welfare of each type is affected when consumers have asymmetric information.

2.4.2 Markets with Asymmetric Information

As before,[21] movers sell their holdings of capital, k, regardless of the realized states of nature, while nonmovers need to decide whether to store fiat money or to trade all or some units of currency for capital. Only informed agents have access to the information at date $t+1$ on the precise returns on capital, R_j, and the proportion of movers, λ^i, which will be known to the uninformed at date $t+2$.

Informed agents hold only capital at date 0 since they know that they are not relocated. Then the total amount of capital informed agents' holding at date t is simply λ_{inf}, where λ_{inf} is the relative proportion of informed agents with respect to the uninformed. The informed form a coalition at date 1 after the uncertainty is revealed. The coalition collectively decides whether to provide capital of its members

[20] See the appendix 2-3 for computational details.

[21] This section is based on Gorton and Pennacchi (1990) and closely follows their assumptions and shows similar results. As they point out, the informed may exploit uninformed agents and systematically benefit at the expense of them when prices are not fully revealing.

holding in a state $\{i,j\}$, where $i=\{h,l\}$ and $j=\{H,L\}$ to the market. Let $\lambda_{inf}^{i,j}$ be the amount (proportion) of capital the coalition decides to provide with the financial market at date $t+1$, where $0 \leq \lambda_{inf}^{i,j} \leq \lambda_{inf}$. The question posed in this situation is: will $\lambda_{inf}^{i,j}$ be positive in some information sets $\{i,j\}$ at date $t+1$? The coalition will trade capital for fiat money only when they get additional gains from the trade by manipulating the asset price by providing its holdings of capital. The following proposition[22] shows when that arbitrage opportunity is possible.

Proposition 2.4.1

- *Let $\bar{R}=\mu_h' R_H+\mu_l' R_L$ denote the informed nonmover's posterior expectation on the rate of return on capital when the state $\{l,L\}$ happens, and μ_h' and μ_l' are their posterior probabilities on the fraction of movers.*

- *Suppose the total money supply remains constant. If (i) $\widehat{B}_1^{h,H}=\frac{v_1}{v_0}\frac{(1-\lambda^h)q}{\lambda^h k} \leq \frac{\bar{R}}{v_2/v_1}$, and (ii) the number of informed agents compared to that of the uninformed is equal or greater than $\lambda_{inf} \geq \overline{\lambda}_{inf}=\frac{k[\lambda^h-\lambda^l]}{1-\lambda^h}$, then the coalition of informed agents can mimic the price $\widehat{B}_1^{h,H}$ at date $t+1$ when the state $\{l,L\}$ happens. Let $\widetilde{B}_1^{l,L}$ be the price which is manipulated by the informed agents when the state $\{l,L\}$ occurs. Then we have the following:*

$$\widetilde{B}_1^{l,L}=\widehat{B}_1^{h,H}>\widehat{B}_1^{l,L}$$

  $\overline{\lambda}_{inf}$ is the minimal size of the informed to influence the asset price. Uninformed nonmovers trade their whole fiat money for capital because they misunderstand that the state $\{h,H\}$ has occurred when they observe $\widetilde{B}_1^{l,L}$, and thus $\gamma^l=1$.

- *Except for the state of $\{l,L\}$, the true prices are revealed as in Proposition 2.3.2 and $\lambda_{inf}^{i,j}=0$, where $\{i,j\} \neq \{l,L\}$.*

[22] This proposition is a variant of Proposition 1 of Gorton and Pennacchi (1990). We modified it for the OLGs model with random relocation and fiat money.

Proof See Gorton and Pennacchi (1990) to see how the informed agents manipulate the asset price when a state $\{l, L\}$ occurs to make it looks like $\widehat{B}_1^{h,H}$.

Let $\widetilde{B}_1^{i,j}$ be the asymmetric-information price for states $\{i, j\}$ when $\lambda_{inf}^{i,j} > 0$. Now, let us look at how much capital is provided by the coalition of the informed. Let $\lambda_{inf}^{l,L}$ be the amount of capital supplied to the market by the coalition in the state of $\{l, L\}$. The specific $\lambda_{inf}^{l,L}$ value can be obtained from the market equilibrium condition, using $\widetilde{B}_1^{l,L} = \widehat{B}_1^{h,H}$.

$$\lambda^l k \widetilde{B}_1^{l,L} + \lambda_{inf}^{l,L} \widetilde{B}_1^{l,L} = (1 - \lambda^l) \frac{v_1}{v_0} q$$

The left-hand side of the preceding equation denotes the total supply of capital in the state of $\{l, L\}$, and the right hand side of it is the demand for capital. Substituting $\frac{(1-\lambda^h)q}{\lambda^h k}$ for $k\widetilde{B}_1^{l,L} (= k\widehat{B}_1^{h,H})$ using Assumption 5, and in the stationary equilibrium $v_{t+s} = v, \forall s \geq 0$, we have

$$\begin{aligned}
&\lambda_{inf}^{l,L} \frac{(1-\lambda^h)q}{\lambda^h k} + \lambda^l \frac{(1-\lambda^h)q}{\lambda^h} = (1-\lambda^l)q \\
\Leftrightarrow &\lambda_{inf}^{l,L}(1-\lambda^h) = (1-\lambda^l)\lambda^h k - \lambda^l(1-\lambda^h)k \\
\Leftrightarrow &\lambda_{inf}^{l,L}(1-\lambda^h) = k[\lambda^h - \lambda^l] \\
\Leftrightarrow &\lambda_{inf}^{l,L} = \frac{k[\lambda^h - \lambda^l]}{1-\lambda^h}
\end{aligned} \tag{2.9}$$

As long as $\lambda_{inf} \geq \overline{\lambda}_{inf} = \frac{k[\lambda^h - \lambda^l]}{1-\lambda^h}$, the coalition of the informed is able to it can mimic the price $\widehat{B}_1^{h,H}$ when a state $\{l, L\}$ actually occurs. ■

The preceding proposition implies that when the state $\{l, L\}$ actually happens, the consumption of the uninformed agents will be

$$\lambda^l \left(\frac{v_2}{v_0} q + \frac{v_2}{v_1} \widetilde{B}_1^{l,L} k \right) + (1-\lambda^l) \left(\frac{v_1}{v_0} \frac{q}{\widetilde{B}_1^{l,L}} R_L + R_L k \right)$$

Now let $E^A[\lambda^i c_{m,2} + (1-\lambda^i) c_{n,2}]$ denote the expected utility of the uninformed agents with asymmetric information, then $E^A[\cdot]$ is given as

$$
\begin{aligned}
&E^A[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] \\
&\quad = \frac{\mu_h}{2}\left[\lambda^h\left(\frac{v_2}{v_0}q + \frac{v_2}{v_1}\widehat{B}_1^{h,H}k\right) + (1-\lambda^h)\left(\frac{v_1}{v_0}\frac{q}{\widehat{B}_1^{h,H}}R_H + R_H k\right)\right] \\
&\quad + \frac{\mu_l}{2}\left[\lambda^l\left(\frac{v_2}{v_0}q + \frac{v_2}{v_1}\widehat{B}_1^{l,H}k\right) + (1-\lambda^l)\left(\frac{v_1}{v_0}\frac{q}{\widehat{B}_1^{l,H}}R_H + R_H k\right)\right] \\
&\quad + \frac{\mu_h}{2}\left[\lambda^h\left(\frac{v_2}{v_0}q + \frac{v_2}{v_1}\widehat{B}_1^{h,L}k\right) + (1-\lambda^h)\left(\frac{v_1}{v_0}\frac{\gamma^h q}{\widehat{B}_1^{h,L}}R_L + R_L k + (1-\gamma^h)\frac{v_2}{v_0}q\right)\right] \\
&\quad + \frac{\mu_l}{2}\left[\lambda^l\left(\frac{v_2}{v_0}q + \frac{v_2}{v_1}\widetilde{B}_1^{l,L}k\right) + (1-\lambda^l)\left(\frac{v_1}{v_0}\frac{q}{\widetilde{B}_1^{l,L}}R_L + R_L k\right)\right]
\end{aligned} \tag{2.10}
$$

Since the difference between eqs (2.8) and (2.10) exists only when the state $\{l, L\}$ happens, we have the following:

$$
\begin{aligned}
E^F[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] - E^A[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] \\
= \frac{\mu_l}{2}\left[\lambda^l k(\widehat{B}_1^{l,L} - \widetilde{B}_1^{l,L}) + (1-\lambda^l)q\left(1 - \frac{R_L}{\widetilde{B}_1^{l,L}}\right)\right] \\
= \frac{\mu_l}{2}\frac{k(\lambda^h - \lambda^l)}{1-\lambda^h}(\widehat{B}_1^{h,H} - R_L) > 0
\end{aligned} \tag{2.11}
$$

The preceding equation tells us that $E^A[\cdot]$ is always lower than $E^F[\cdot]$ when asymmetric information exists (See Appendix 2-4 for the detailed computation).

Finally, look at the expected utility of the informed agents. Under full information where trades only occurs at true prices, the expected utility of the informed is just

$$
E^F[c_{inf,2}] = E[R_j]
$$

On the other hand, under asymmetric information,

$$
\begin{aligned}
E^A[c_{inf,2}] &= \frac{1}{2}R_H + \frac{\mu_h}{2}R_L + \frac{\mu_l}{2}\left(\left[1 - \frac{\lambda_{inf}^{l,L}}{\lambda_{inf}}\right]R_L + \frac{\lambda_{inf}^{l,L}}{\lambda_{inf}}\frac{\widetilde{B}_1^{l,L}}{v_1}v_2\right) \\
&= E[R_j] + \frac{\mu_l}{2}\frac{\lambda_{inf}^{l,L}}{\lambda_{inf}}\frac{(\widehat{B}_1^{h,H} - R_L)}{v_1}v_2 > E[R_j]
\end{aligned} \tag{2.12}
$$

The coalition now can exchange for currency $\frac{\lambda_{inf}^{l,L}}{\lambda_{inf}}$ per unit of the capital good at date $t+1$ at a higher price, $\widetilde{B}_1^{l,L} = \widehat{B}_1^{h,H}$, when a state $\{l, L\}$ happens. This gain comes from at the expense of consumption by nonmovers. Trading losses by uninformed agents associated with information asymmetry is equal to the trading gains by informed agents.[23] The results are summarized in the following proposition.

Proposition 2.4.2 *The imperfectly competitive rational expectations (temporary) equilibrium with asymmetric information and without a coalition of the uninformed young is given as*

1. *The Price system:* $\{B_1^{ij}\} = (\widehat{B}_1^{h,H}, \widehat{B}_1^{l,H}, \widehat{B}_1^{h,L}, \widetilde{B}_1^{l,L} = \widehat{B}_1^{h,H})$

2. *The expected consumption of agents:*

$$E^A[C_{\tau,2}] = \begin{cases} E^A[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] < E^F[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] \\ E^A[c_{inf,2}] = E[R_j] + \frac{1}{4}\frac{\lambda_{inf}^{l,L}}{\lambda_{inf}}\frac{(\widehat{B}_1^{h,H} - R_L)}{v_1}v_2 > E^F[c_{inf,2}] \end{cases}$$

3. *The specification of storage strategies for (market participating) nonmovers:*

$$\begin{cases} 0 < \gamma < 1, \ \textit{when a state } \{h, L\} \textit{ happens.} \\ \gamma = 1, \ \textit{otherwise.} \end{cases}$$

4. *The specification of insider coalition strategies:*

$$\lambda_{inf}^{i,j} = \begin{cases} \frac{k[\lambda^h - \lambda^l]}{1-\lambda^h}, \ \textit{if } \{i, j\} = \{l, L\} \\ 0, \ \textit{otherwise.} \end{cases}$$

2.5 Forming a Bank

Now suppose that young, uninformed agents who are born at date t can organize an institution, which is called a bank, and see if the institution can protect them-

[23] Since there are $\lambda_{inf}^{l,L}$ number of informed agents with respect to that of the uninformed agents (which is normalized to 1), total gains from trade of informed agents is exactly equal to the total loss by uninformed agents.

selves from the loss induced by asymmetric information. They deposit some or all of their endowment goods with a bank, and it uses the proceeds to acquire assets in favour of its members. Free entry into the banking industry forces banks to compete by offering deposit contracts that maximise the expected utility of consumers. In dealing with bank runs situation, I follow the assumptions in Allen and Gale (1998) rather than 'sequential service' assumption in Diamond and Dybvig (1983). The bank makes a standard deposit contract with its members. The contract promises a nominal amount of fiat money, d, at date 1 if it has enough liquid assets. If the bank does not have enough liquid assets to make the promised amount, however, it pays out all available liquid assets and capital, divided equally among those withdrawing.

2.5.1 The Model

At the beginning of time 0, the uninformed young deposit their endowment goods to a bank in exchange for a promised amount of fiat money at date 1 if they turn out to be movers. If they are nonmovers they become residual claimants and share whatever left over equally with other nonmovers. Then, the bank's problem is to choose how much capital and fiat money to acquire to maximise the ex-ante expected utility of the typical uninformed agent, and the standard deposit contract is made in the process.[24]

$$\max_{c_{m,2},c_{n,2}} \Psi = E[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] \tag{2.13}$$

$$\begin{aligned} s.t. \quad (i) \quad & q^i + k^i \leq 1, \\ (ii) \quad & \lambda^i d \leq \frac{q^i}{v_0}, \\ (iii) \quad & (1-\lambda^i)c_{n,2} \leq v_2\left(q^i - \lambda^i v_0 d\right) + R_j k^i, \\ (iv) \quad & c_{n,2} \geq c_{m,2} = v_2 d \end{aligned} \tag{2.14}$$

[24] To make the transaction sequence as simple as possible I assume the initial old at date 0 and old movers after date 1 consume after the coalition accepts deposits from the young uninformed.

The first constraint is the budget constraint facing a bank.[25] The bank determines how to divide its member's endowment goods between liquid and illiquid assets.

The second decision made by the bank is about how much it should pay to movers who must withdraw at date 1. The bank makes a standard deposit contract with its members. The contract promises to give a fixed amount of fiat money, d, at date 1[26] by paying out all available liquid assets, divided equally among those withdrawing. Movers withdrawing d at date 1 consume $v_2 d$ at date 2. If the bank does not have enough liquid assets to make the promised amount to movers at date 1, the bank will go bankrupt. The constraint (ii) says that the bank's holding of fiat money must be sufficient to provide movers with d according to deposit contracts. Non-movers are paid whatever is available at their third period which is shown in constraint (iii). The constraint (iii) says that the consumption of nonmovers are limited by the total value of the risky asset, capital, plus the amount of fiat money left over after the movers are paid off, if any.

The constraint (iv), which denotes incentive compatibility constraint, says that consumption by nonmovers must be at least as much as that of movers. Movers will withdraw their deposits in forms of fiat money from the bank at their middle age before they are relocated and then take the money to the bank in the foreign island and trade them for consumption goods. Each mover will consume $v_2 d = \frac{v_2}{v_0}\frac{q^i}{\lambda^i}$. Non-movers will get whatever left over in a bank when they are old. Since whether a certain agent is relocated or not is private information, and thus, the bank cannot identify who is a mover and who is a nonmover, nonmovers have an incentive to pretend to be a mover unless this constraint holds. The incentive constraint tells us that nonmovers do not have any incentive to pretend to be a mover as long as they would get a higher level of the consumption good at their final period.

[25] In this section I suppose each uninformed young agent deposits her whole endowment goods when she is young. However, we do not necessarily need to assume this way. For example, agents may decide to hold capital directly and deposit only part of the endowment goods in the bank. Diamond (1997) explains this type of deposit agreement. I will consider how the lower bound of banking is changed under different assumptions applying the study of Diamond (1997).

[26] q^i/v_0 denotes the amount of fiat money the bank holds at date $t+1$.

In the optimum, $\lambda^i d$ must be equal to q^i/v_0, otherwise (i.e., $\lambda^i d < q^i/v_0$) the bank could increase expected utility by reducing q because the rate of return on capital is expected to be higher than that of fiat money. Note that a contingent deposit contract is not possible to make because the portfolio decision is made before relocation shocks are revealed. Since there are two possible states about the fraction of movers, either of the following relations holds depending on how much a bank chooses q:

$$\lambda^h d = \frac{q^h}{v_0} > \lambda^l d \tag{2.15}$$

or,

$$\lambda^l d = \frac{q^l}{v_0} < \lambda^h d \tag{2.16}$$

If the bank chooses to spend q^h instead of q^l, and if a state l actually happens, it will have excess liquidity which is passed over to the next period. On the other hand, if the real money demand is equal to q^l rather than q^h and the state h happens, then it will experience the shortage of liquidity and suffers bankruptcy.

2.5.2 Equilibrium

As noted in the preceding subsection, the optimal amount of q is not determined as a fixed amount because the optimal q does not take a single value and depends on how many agents are subject to the relocation shocks. Therefore, regarding the bank's choice of how much to pay to movers, there exist two possible choices between q^l and q^h.[27] Whether q^i is the optimal amount of liquidity banks hold depends on the realization of λ^i as shown in (2.15) and (2.16). Now let us consider the effect of the bank's two different choices on its members' utility. The analysis shows that the uninformed young will enjoy higher consumption level when the bank spends a minimum on purchasing liquid assets at date t. In other words, the bank always chooses q^l instead of q^h. Note that, however, in any case, the welfare of the

[27] In a situation where bank runs are costly, in that physical liquidation incurs cost (Allen and Gale, 2000), or individuals do not have access to financial markets. In the latter case, not allowing bank runs by holding enough liquidity can be optimal. Allen et al. (2009) takes this position.

uninformed is increased when they form a bank.

Equilibrium with Bankruptcy

Suppose that the bank makes a contract with its depositors expecting the state l would happen and holds liquid assets such that $q^l = \lambda^l$, i.e., the bank determines to sell minimal goods for fiat money. There is a good reason for choosing $q^l = \lambda^l$. In fact, with linear utility function, the expected utility of the uninformed agents will be greater the less q is. However, if $q^l < \lambda^l$ the bank always suffers bankruptcy, and so the bankruptcy's unique role of revealing unknown states of nature is no longer being performed.[28]

The bank's choice of how much to spend to retain liquid assets has two consequences, depending on how the actual state of nature i is realized. If the state l happens at date $t+1$, then the bank will hold sufficient amount of fiat money to distribute it to movers. The expected consumption level by each agent is given as

$$\text{movers} \quad c_{m,2} \leq v_2 d = \frac{v_2}{v_0}\frac{q^l}{\lambda^l}$$

$$\text{nonmovers} \quad c_{n,2} \leq \frac{1}{1-\lambda^l}\left(v_2\left(q^l - \lambda^l v_0 d\right) + k^l R_j\right)$$

Carrying over fiat money to date 2, i.e., $c_{m,2} < v_2 d$, is not optimal because the bank can increase the consumption of nonmovers by reducing q without affecting the consumption of movers. Therefore, in the optimal plan, the bank always choose q and thus d such that

$$c_{m,2} = v_2 d = \frac{v_2}{v_0}\frac{q^l}{\lambda^l} \tag{2.17}$$

[28] In the standard Diamond and Dybvig type model with log utility, the optimal amount of q is just equal to λ. See Chapter 1. Another reason why banks must have at least λ^l in liquidity is that there might be liquidity regulation by central banks. However, the issue of regulation of the central bank on liquidity or capital is not the subject of this paper, so we will not discuss it any further.

which determines the consumption of nonmovers as

$$c_{n,2} = \frac{k^l R_j}{1 - \lambda^l} \tag{2.18}$$

If the state h happens at date 1, on the other hand, then the bank will not hold enough liquidity to be provided with movers since $\lambda^h d > \frac{q^l}{v_0} = \lambda^l d$. The bank goes bankrupt (bank runs happens) inevitably. If bank runs happen, then movers and nonmovers equally divide the fiat money and capital the bank hold, $q^l + k^l$, and movers and nonmovers trade these assets in financial markets with one another.

From the fact that bank runs happen, the agent now can realize that the state h occurs instead of the state l. From this newly acquired knowledge, they can confine the remaining possible asset prices into two distinct cases. In other words, they know the only viable remaining states of nature will be either $\{h, H\}$ or $\{h, L\}$, both of which cases the true values of asset prices are expected to be revealed (see Proposition 2.4.1). So, nonmovers can trade their holdings of fiat money for capital without risk incurred by asymmetric information.

$\widehat{B}_1^{h,L}$ is always equal to R_L. Note that, however, the asset price in the state of $\{h, H\}$ takes a different form from that in (**??**). The bank has chosen q^l but since a state h happens, the actual asset price in the state of $\{h, H\}$ is expressed as

$$\bar{B}_1^{h,H} = \frac{v_1}{v_0} \frac{(1 - \lambda^h) q^l}{\lambda^h k^l} \tag{2.19}$$

The different asset price in the state $\{h, H\}$ does not affect the expected utility of the uninformed agent since the agents are assumed to be risk-neutral.

Then, the consumption of a mover in each state can be represented as:

$$\begin{aligned} \{h, H\} \quad c_{m,2} &= \frac{v_2}{v_0} q^l + \frac{v_2}{v_1} \bar{B}_1^{h,H} k_0^l \\ \{h, L\} \quad c_{m,2} &= \frac{v_2}{v_0} q^l + \frac{v_2}{v_1} \widehat{B}_1^{h,L} k_0^l \end{aligned} \tag{2.20}$$

And the consumption of a nonmover:

$$\begin{aligned} \{h, H\} \quad c_{n,2} &= \left(\frac{v_1}{v_0} \frac{q^l}{\bar{B}_1^{h,H}} + k_0^l \right) R_H \\ \{h, L\} \quad c_{n,2} &= \left(\frac{v_1}{v_0} \frac{\gamma q^l}{\widehat{B}_1^{h,L}} + k_0^l \right) R_L + (1-\gamma) \frac{v_2}{v_0} q^l \end{aligned} \tag{2.21}$$

Substituting (2.17), (2.18), (2.20), and (2.21) into the objective function (2.13), and assuming constant money supply, we can calculate expected utility of the uninformed agents.

$$\begin{aligned} & E^l[\lambda^i c_{m,2} + (1-\lambda^i) c_{n,2}] \\ &\quad = \mu^l \left[q^l + k^l E[R_j] \right] \\ &\quad + \frac{\mu_h}{2} \left[\lambda^h \left(q^l + \bar{B}_1^{h,H} k^l \right) + (1-\lambda^h) \left(\frac{q^l}{\bar{B}_1^{h,H}} + k^l \right) R_H \right] \\ &\quad + \frac{\mu_h}{2} \left[\lambda^h \left(q^l + \widehat{B}_1^{h,L} k^l \right) + (1-\lambda^h) \left\{ \left(\frac{\gamma q^l}{\widehat{B}_1^{h,L}} + k^l \right) R_L + (1-\gamma) q^l \right\} \right] \\ &\quad = q^l + E[R_j] k^l \end{aligned} \tag{2.22}$$

where $E^l[\lambda^i c_{m,2} + (1-\lambda^i) c_{n,2}]$ is expected utility of uninformed agents when agents organize a bank and the bank chooses q^l. Losses that occur under asymmetric information do not occur in this situation,[29] and thus, the uninformed agents have higher expected utility than under asymmetric information.[30]

Equilibrium without Bankruptcy

Now suppose that the bank makes an alternative choice expecting the state h would happen and hold $q^h = \lambda^h$. In other words, the bank chooses q expecting the number of movers will be higher. There exist two possible scenarios according to the realization of state i as in the previous subsection.

If the state h happens at date $t+1$, then the bank will hold sufficient amount of

[29] See Appendix 2-5 for the computational details.

[30] As long as q^l is less than or equal to $E[R_j]k^l$, the incentive constraint is met.

fiat money to distribute to movers. Unlike the previous section, bank runs do not occur even when the state h occurs. The expected consumption level by each agent is given as

$$\begin{aligned} \text{movers} \quad c_{m,2} &= v_2 d = \frac{v_2}{v_0}\frac{q^h}{\lambda^h} \\ \text{nonmovers} \quad c_{n,2} &= \frac{k^h R_j}{1-\lambda^h} \end{aligned} \tag{2.23}$$

If the state l happens at date 1, then the bank holds excessive liquidity, which is unintended when the bank first chooses its portfolio at date 0 amounting $(q^h - \lambda^l v_0 d)$.[31]

The expected consumption level of agents when state l happens is given as

$$\begin{aligned} \text{movers} \quad c_{m,2} &= \frac{v_2}{v_0}\frac{q^h}{\lambda^h} = 1 \\ \text{nonmovers} \quad c_{n,2} &= \frac{v_2\big[d(\lambda^h - \lambda^l)\big] + R_j k^h}{1-\lambda^l} \\ &= \frac{v_2}{v_0}\frac{\big(\lambda^h - \lambda^l\big) + R_j k^h}{1-\lambda^l} \\ &= \frac{\big(\lambda^h - \lambda^l\big) + R_j k^h}{1-\lambda^l} < \frac{R_j k^l}{1-\lambda^l} \end{aligned} \tag{2.24}$$

The first term on the right hand side of $c_{n,2}$, $v_2\big[d(\lambda^h - \lambda^l)\big]$, represents the increased amount of consumption due to fiat money carried over from date 1 to date 2. The second term, $R_j k^h$, is the consumption from illiquid assets. Note that the expected consumption of nonmovers is lower than that in (2.18) because the bank holds excessive fiat money than necessary and less capital (note that $k^h = 1 - \lambda^h < 1 - \lambda^l = k^l$). This implies that inefficiency is caused by the bank's holding excessive liquidity at date 0. This inefficiency arises because the decision by a bank is made at date 0, which expects the specific state i will happen, but a different state actually happens at date 1.

Therefore, when the state l happens, the bank holding excessive liquidity at date 1 may wish to trade this excessive liquidity for capital with the informed if $\frac{E[R_j]}{B_1^{h,j}} > \frac{v_2}{v_1}$

[31] This is because $\lambda^h d = \frac{q^h}{v_0} > \lambda^l d$.

is expected. Does it buy additional capital from the informed at date 1? It turns out that it will never end up trading with informed because of the following reason.

By the fact that it holds excessive liquidity after movers' withdrawing, the bank now gets a new information that the state l happened instead of the state h. It still does not know, nevertheless, the exact return on capital which will be realized at date 2, which is known only to the informed. Now that the coalition acquires the information about the total fraction of movers, it knows that the possible asset prices will be confined only to two distinct cases. i.e., it knows only remaining possible states of nature are $\{i, j\} = \{l, H\}$ or $\{l, L\}$. It also knows that informed agents will try to sell their capital only when the returns on capital is low, i.e., only in the state of $\{l, L\}$ (see Proposition 2.4.1). The financial market now can be seen as "the market for lemons" (Akerlof, 1970). The bank chooses to keep the currency to the next period, and thus, trades in the financial market are not made at date $t + 1$. The uninformed agents will consume according to (2.23) and (2.24).

Substituting (2.23) and (2.24) into the objective function (2.13), and letting $E^h[\lambda^i c_{m,2} + (1 - \lambda^i)c_{n,2}]$ denote the expected utility of the uninformed agents when the bank chosses q^h, we have[32]

$$\begin{aligned}
&E^h[\lambda^i c_{m,2} + (1 - \lambda^i)c_{n,2}] \\
=&\mu_h \left[q^h + k^h E[R_j]\right] + \mu_l \left[\frac{\lambda^l q^h}{\lambda^h} + (\lambda^h - \lambda^l) + k^h E[R_j]\right] \\
=&\mu_h \left[q^h + k^h E[R_j]\right] + \mu_l \left[q^h + k^h E[R_j]\right] \\
=&q^h + k^h E[R_j]
\end{aligned} \tag{2.25}$$

The second term in the third line of the preceding equation is calculated using $\lambda^l = q^l$ and $\lambda^h = q^h$. Losses that occur under asymmetric information do not occur when the bank chooses q^h as well. However, the expected utility with q^l is greater than

[32]The constant money supply is assumed as well.

that with q^h because from (2.22) and (2.25)

$$\begin{aligned} &E^l[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] - E^h[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] \\ &= (q^h - q^l)(E[R_j] - 1) > 0 \end{aligned} \tag{2.26}$$

Therefore, the coalition will always choose $q^* = \lambda^l$ rather than $q^* = \lambda^h$.

I showed that the coalition of uninformed agents can effectively eliminate the loss due to asymmetric information by allowing a run and thus revealing the true prices of capital. Also note that financial markets and banks coexist after a bank run occurs.

I assumed until now that all uninformed young agents deposit their whole endowment with a bank at date t. However, the assumption is not necessarily needed. Uninformed agents can deposit only parts of their endowment with a bank and hold capital directly. Let the scale of banking be β, the fraction of endowment goods deposited with a bank, then the lower bound on the date t scale of banking under asymmetric information, β^{AI}, is the amount of goods needed to buy fiat money $\beta \geq \beta^{AI} = \lambda^l$. When $\beta = \lambda^l$, each agent directly hold the capital amounting to $1-\beta$. Summing up all the findings until now can be summarized as the following proposition.

Proposition 2.5.1 *The imperfectly competitive rational expectations equilibrium when the uninformed young form a bank is given as*

1. *The Price system:* $\{B_1^{ij}\} = (\bar{B}_1^{h,H}, \widehat{B}_1^{l,H}, \widehat{B}_1^{h,L}, \widehat{B}_1^{l,L})$, *where* $\bar{B}_1^{h,H} = \frac{v_1}{v_0}\frac{(1-\lambda^h)q^l}{\lambda^h k^l}$.

2. *The expected consumption of agents:*

$$E[C_{\tau,2}] = \begin{cases} E^l[\lambda^i c_{m,2} + (1-\lambda^i)c_{n,2}] = q^l + k^l E[R_j] \\ E^l[c_{inf}] = E[R_j], \end{cases}$$

where $q^l = \lambda^l$ *and* $k^l = 1 - \lambda^l$.

3. *The specification of storage strategies for (market participating) nonmovers:*

$$\begin{cases} \gamma < 1, \ \textit{in states of } \{i, L\} \\ \gamma = 1, \ \textit{in states of } \{i, H\}, \textit{ where } i = \{h, l\}. \end{cases}$$

4. *The specification of insider coalition strategies:* $\lambda_{inf}^{i,j} = 0$

5. *The lower bound on the date* 0 *scale of banks:* $\beta \geq \beta^{AI} = q^l = \lambda^l$

6. *Bank runs are inevitable and efficient when state* $\{h, j\}$ *happens, where* $j = \{H, L\}$. *Financial markets and banks coexist when the states* $\{h, j\}$ *occurs even when* $\beta = 1$ *(when uninformed agents deposit their whole endowments with the bank).*

2.6 Limited Participation in Markets & Imperfect Information

Until this point, the analysis has assumed that uninformed agents have access to the market without limitation. This section analyses an incomplete asset market caused by asymmetric information and limited participation at the same time. Suppose that only α fraction among nonmovers have access to financial markets. Then, the total fraction of nonmovers participating in markets will be $\alpha(1 - \lambda^i)$ and that of nonmovers who do not have access to a market will be $(1 - \alpha)(1 - \lambda^i)$.

Let $\breve{B}_1^{i,j}$ be the price of an illiquid asset which would prevail with limited participation and complete information. Then, asset prices take the following relations.

$$\begin{aligned} R_H > &\widehat{B}_1^{l,H} > \breve{B}_1^{l,H} = \alpha \widehat{B}_1^{l,H}, \\ &\widehat{B}_1^{h,H} > \breve{B}_1^{h,H} = \alpha \widehat{B}_1^{h,H} > 1 > R_L = \breve{B}^{l,L} = \breve{B}^{h,L} \end{aligned} \tag{2.27}$$

The price in the state of $\{h, H\}$, under full information, $\widehat{B}_1^{h,H}$, can be higher than that in the state of $\{l, H\}$ under imperfect information, $\breve{B}_1^{l,H}$, if the fraction of nonmovers who have access to the market is lower than some point, i.e.,

$$\breve{B}_1^{l,H} < \widehat{B}_1^{h,H}, \text{ if } \alpha < \frac{\lambda^l}{\lambda^h}\frac{1-\lambda^h}{1-\lambda^l}$$

As before, $\breve{B}_1^{l,H}$ and $\breve{B}_1^{h,H}$ are equated through the adjustment of γ^i, such that,[33]

$$\gamma^i = \frac{R_L \lambda^i k}{\alpha(1-\lambda^i)q}$$

Now look at the consumption of a mover at date $t+2$:

$$c_{m,2} = \left(M + \frac{\breve{B}_1 k}{v_1}\right) v_2 = \frac{v_2}{v_0} q + \frac{v_2}{v_1}\breve{B}_1 k = q + \breve{B}_1 k$$

The consumption of type A nonmovers at date $t+2$ is given as:

$$\begin{aligned} c_{nA,2} &= \left(\frac{\gamma^i M v_1}{\breve{B}_1} + k\right) R_j + (1-\gamma^i) M v_2 \\ &= \left(\frac{\gamma^i v_1}{v_0}\frac{q}{\breve{B}_1} + k\right) R_j + (1-\gamma^i)\frac{v_2}{v_0} q \\ &= \left(\frac{\gamma^i q}{\breve{B}_1} + k\right) R_j + (1-\gamma^i) q \end{aligned}$$

The consumption of type B nonmovers at date $t+2$ is

$$c_{nB,2} = v_2 M + kR_j = \frac{v_2}{v_0} q + kR_j = q + kR_j$$

It would be interesting to ask whether the uninformed young still would like to form a coalition at date 0 with full-information but some of its members are isolated from the market. The answer to the question is that the uninformed young will set up a bank to minimise losses that could arise from some of them being excluded from the market.

[33] As long as $\alpha < 1$ and other things being equal, the money holding ratio γ tends to be bigger than under full participation case. Due to the reduced demand for capital (because of the limited participation), the asset prices in the state $\{i, L\}$ could be lower than R_L. The lower asset prices will induce type B nonmovers, who used to hold some currency if all agents have access to markets, want to trade more of their currency holdings for capital.

Direct calculation shows the following holds (see the Appendix 2-6 for the detailed computation.).

$$E^{F'}[c_{m,2} + c_{A,2} + c_{nB,2}] < q + E[R_j]k = E^F[c_{m,2} + c_{n,2}], \tag{2.28}$$

where $E^{F'}[\cdot]$ denotes expected utility under full information and limited participation. Even though the calculation process is messy, the economic intuition behind this result is straightforward. Limited participation in markets implies the type B nonmovers end up holding inefficient liquidity assets, which is passed over from period $t+1$ to period $t+2$ for consumption. Thus, the economy would keep excessive liquidity than necessary eventually.

Total losses from this limited participation constraint are always greater than 0 as long as $\alpha < 1$. This loss is a decreasing function of α and the maximum loss incurred from limited participation happens when $\alpha = 0$ (see Appendix 2-6 for computational details.).

To address this problem, the young uninformed agents organize a bank at date 0. The role of a bank which maximises the expected utility of the uninformed young is gathering the need for holding fiat money together and avoiding the inefficient holdings of fiat money by type B nonmovers at date 1. The bank will minimize its of fiat money holding as a fraction of movers λ^i.[34]

In the following, I will study the effects of characteristics of asymmetric information and limited participation on equilibria dividing it into two parts. The first deals with the resource allocation problem when there is only the asset market, and the second analyses the case where the asset market and the bank are together.

Note that the results below will be quite different depending on whether α is known to the uninformed agents or not. Moreover, the asset prices, which the informed agents can mimic when the low rate of return on capital is expected, depend on the magnitude of α. When α is small, the coalition of the informed can manipulate any prices as it wishes. Therefore, the optimal level of q is not be given

[34] Since complete information is assumed here λ^i is not a random variable.

as a single value if there exists limitation on market participation. Rather it will depend on the magnitude of nonmovers who have access to the market, α. The results will be also different from that in Section 2.5, where the optimal deposit contract are made expecting the state l happens and choosing to give movers the amount of fiat money, d, by holding a minimal fiat money. From now on, however, I assume that the fraction of α is known to all agents and α is not low enough for the coalition to mimic any price, and the price manipulation is possible only in the state of $\{h, H\}$ as in Section 2.5.

2.6.1 Market Equilibrium

As discussed in Section 2.4.2, type A nonmovers will not keep fiat money for the consumption at date 2 when $j = H$ (irrespective of the state of i), and some storage happens when $j = L$, i.e., some storages will occur only in the states $\{h, L\}$ or $\{l, L\}$.

Let $0 \leq \breve{\lambda}_{inf}^{i,j} \leq \lambda_{inf}$ be the amount (proportion) of capital the coalition decides to provide with the financial market at date $t+1$ when markets are imperfect. In a state $\{l, L\}$, the coalition of the informed can mimic state $\{h, H\}$ by selling their capital holdings and thus increasing the total supply of illiquid assets to equate the prices such that,

$$\breve{B}_1^{h,H} = \frac{v_1}{v_0}\frac{\alpha(1-\lambda^h)q}{\lambda^h k} = \widetilde{B}^{l,L} = \frac{v_1}{v_0}\frac{\alpha(1-\lambda^l)q}{\lambda^l k + \breve{\lambda}_{inf}^{l,L}} > \breve{B}_1^{l,L} \tag{2.29}$$

This happens when all the agents including uninformed agents know the value of α the fraction of whom have access to the markets.[35]

We can show that the coalition by informed agents can mimic the price $\breve{B}_1^{h,H}$ ($> \breve{B}_1^{l,L}$) at date 1 when the state $\{l, L\}$ happens as we did in Proposition 2.4.1.

[35] If α is unknown to the uninformed, then the informed can mimic even $\widehat{B}_1^{h,H}$, which is greater than $\breve{B}_1^{h,H}$ if their capital holding is big enough.

$\breve{\lambda}^{l,L}_{inf}$ can be obtained from the market equilibrium condition, using $\widetilde{B}^{l,L}_1 = \breve{B}^{h,H}_1$

$$\lambda^l k \widetilde{B}^{l,L}_1 + \breve{\lambda}^{l,L}_{inf} \widetilde{B}^{l,L}_1 = \alpha(1-\lambda^l)\frac{v_1}{v_0} q$$

The least fraction of informed agents compared to the uninformed, which is needed to affect the price is given as by substituting $\frac{v_1}{v_0}\frac{(1-\lambda^h)q}{\lambda^h k}$ for $\widetilde{B}^{l,L}_1$ using (2.27):

$$\begin{aligned} &\breve{\lambda}^{l,L}_{inf}\frac{v_1}{v_0}\frac{\alpha(1-\lambda^h)q}{\lambda^h k} + \lambda^l \frac{v_1}{v_0}\frac{\alpha(1-\lambda^h)q}{\lambda^h} = \alpha(1-\lambda^l)\frac{v_1}{v_0}q \\ \Leftrightarrow &\breve{\lambda}^{l,L}_{inf} = \frac{k[\lambda^h - \lambda^l]}{1-\lambda^h} = \widetilde{\lambda}^{l,L}_{inf} \end{aligned} \tag{2.30}$$

Note that $\breve{\lambda}^{l,L}_{inf}$ is the same with $\lambda^{l,L}_{inf}$ in (2.9). The minimal number of informed agents needed to manipulate the price is the same as in the full participation case.[36] As discussed in Section 2.4.2, the welfare of the informed agents become higher at the expense of uninformed agents' welfare.

2.6.2 Bank Equilibrium

The bank takes the similar step in optimising its depositors' expected utility as in Section 2.5. As analysed in Section 2.5, the bank faces two possible choices about how much to invest for real money balances, q, at date 1. Following the same approach as one used in Section 2.5, let us consider the effect of the bank's two different choices about q on its members' utility. The analysis is summarized as the following proposition.

Proposition 2.6.1 *The Proposition 2.5.1 holds with limited participation when agents have linear utility function.*

Proof To prove the preceding proposition, I analyse two different possible cases of a bank and see which behaviour will give higher consumption profile of uninformed

[36] As I mentioned before, this fraction can be changed a lot depending on the assumption regarding α, and under certain situations, the existence of the informed agents might result in Pareto improvement. However, these issues are not discussed in this chapter.

as in Section 2.5.

Equilibrium with Bankruptcy

The bank makes a deposit contract expecting the state l to happen and chooses to invest $q^l = \lambda^l$ for liquid assets and promises to give $d = \frac{q^l}{v_0 \lambda^l}$ to any person who withdraw at date 1. This implies that the bank determines to sell the minimal amount of goods for fiat money and hold as much as $k^l = 1 - \lambda^l$.

If the state l happens at date $t+1$, then the expected consumption level by each agent is given as

$$\begin{aligned} \text{movers} \quad c_{m,2} &= v_2 d = \frac{v_2}{v_0} \frac{q^l}{\lambda^l} \\ \text{nonmovers} \quad c_{nA,2} = c_{nB,2} &= \frac{k^l R_j}{1-\lambda^l} \end{aligned} \tag{2.31}$$

If the state h happens at date $t+1$, however, the bank goes bankrupt (bank runs happen) because $\lambda^h d > \frac{q^l}{v_0} = \lambda^l d$. If runs occur, then movers and nonmovers equally divide the fiat money and capital the bank holding, $q^l + k^l$. Movers and type A nonmovers will trade these assets in a financial market with one another. The bank run itself instructs the uninformed agents that total fraction of movers is λ^h. This information will confine the possible remaining states of nature to two distinct cases: either $\{i, j\} = \{h, H\}$ or $\{h, L\}$. We already show that the true asset prices are expected to be revealed in both cases.

Let $\bar{B}_1^{h,H}$ be the asset price in state $\{h, H\}$ when the bank chooses q^l. Then $\bar{B}_1^{h,H}$ is expressed as

$$\bar{B}_1^{h,H} = \frac{v_1}{v_0} \frac{\alpha(1-\lambda^h) q^l}{\lambda^h k^l} \tag{2.32}$$

The Consumption of a mover, then, can be represented depending on the realized state of nature as:

$$\begin{aligned} \{h, H\} \quad c_{m,2} &= \frac{v_2}{v_0} q^l + \frac{v_2}{v_1} \bar{B}_1^{h,H} k^l \\ \{h, L\} \quad c_{m,2} &= \frac{v_2}{v_0} q^l + \frac{v_2}{v_1} \breve{B}_1^{h,L} k^l \end{aligned} \tag{2.33}$$

The consumption of a type A nonmover is:

$$\begin{aligned} \{h,H\} \quad c_{nB,2} &= \left(\frac{v_1}{v_0}\frac{q^l}{\bar{B}_1^{h,H}} + k_0^l\right) R_H \\ \{h,L\} \quad c_{nB,2} &= \left(\frac{v_1}{v_0}\frac{\gamma q^l}{\breve{B}_1^{h,L}} + k_0^l\right) R_L + (1-\gamma)\frac{v_2}{v_0} q^l \end{aligned} \tag{2.34}$$

And the consumption of a type B nonmover:

$$\{h,j\} \quad c_{nB,2} = \frac{v_2}{v_0} q^l + k^l E[R_j] \tag{2.35}$$

Type B nonmovers are not affected by the state of j since they do not have access to the financial market.

Substituting (2.31), (2.33), (2.34), and (2.35) into the objective function (2.13), and assuming constant money supply, we can calculate expected utility of the uninformed agents as follows.[37]

$$\begin{aligned} & E^{l'}[\lambda^i c_{m,2} + (1-\lambda^i) c_{n,2}] \\ &= \mu^l \left[q^l + k^l E[R_j]\right] + \frac{\mu_h}{2}\Bigg[\lambda^h \left(q^l + \bar{B}_1^{h,H} k^l\right) \\ &\quad + (1-\lambda^h)\left\{\alpha\left(\frac{q^l}{\bar{B}_1^{h,H}} + k^l\right) R_H + (1-\alpha)\left(q^l + k^l R_H\right)\right\}\Bigg] \\ &\quad + \frac{\mu_h}{2}\Bigg[\lambda^h \left(q^l + \breve{B}_1^{h,L} k^l\right) \\ &\qquad + (1-\lambda^h)\left(\alpha\left\{\left(\frac{\gamma q^l}{\breve{B}_1^{h,L}} + k^l\right) R_L + (1-\gamma) q^l\right\} + (1-\alpha)\left\{q^l + k^l R_L\right\}\right)\Bigg] \\ &= q^l + E[R_j] k^l = E^l[c_{m,2} + c_{n,2}] \end{aligned} \tag{2.36}$$

where $E^{l'}[c_{m,2} + c_{n,2}]$ denotes the uninformed agent's expected utility with limited market participation and asymmetric information when they formed a bank. The preceding equation shows that the restriction on market participation does not affect the uninformed agents' welfare without reference to how big the parameter α is if

[37] See the Appendix 2-7 for computational details.

they could form a bank. This rather weird result comes because of the assumption that agents are all risk-neutral. Since the portfolio decision by the bank is made at date t, their expected utility is the same as long as no loss occurs in trade in the financial markets. Once we assume a standard, risk-averse preference, however, the result might be different.

There will be no gap in consumption between type A and B nonmovers when the state $\{h, L\}$ happens. They all consume $q^l + k^l R_L$. However, when the state $\{h, H\}$ occurs, the difference between the consumption between type A and B is:

$$\left(\frac{q^l}{\bar{B}_1^{h,H}} + k^l\right) R_H - \left(q^l + k^l R_H\right) = \left(\frac{R_H}{\bar{B}_1^{h,H}} - 1\right) q^l > 0$$

The first term on the left-hand side is the consumption of a type A nonmover and the second term is a type B's consumption in the state of $\{h, H\}$. Since $R_H > \bar{B}_1^{h,H}$, the difference is always positive. This implies that the expected utility of nonmovers will be lower if only limited fraction of nonmovers has access to markets. The loss from limited participation is a decreasing function of α. If α were very low, then allowing bank runs may not be efficient. Then, investing in liquid assets more in order to avoid bank runs might enhance the uninformed agents' welfare, which I do not discuss further.

Equilibrium without Bankruptcy

The results are the same as in Sec 2.5.2. Thus,

$$E^{h'}[\lambda^i c_{m,2} + (1 - \lambda^i) c_{n,2}] < E^{l'}[\lambda^i c_{m,2} + (1 - \lambda^i) c_{n,2}]$$

Therefore, as long as the agents are regarded as risk-neutral, the limitation on market participation does not affect the optimal behaviour of banks.

■

2.7 Conclusions

With asymmetric information and restriction in market participation, financial intermediaries can make the unknown information revealed without needing to invest its resources in order to identify it by just allowing bank runs to occur. Then, uninformed agents make transactions each other knowing that the true asset prices are available. In this way, financial intermediaries and markets coexist after banks go bankruptcy. This mechanism of 'a bank run first' and 'transactions in the market second', thus, enhance the expected utility of the uninformed, otherwise facing lower expected consumption generated by information asymmetries. The arrangement of bank's achieving this goal is made through standard deposit contract. Without restriction in market participation, i.e., when all uninformed agents have access to financial markets, the intermediary would hold the minimal amount of goods for fiat money expecting the lower fraction of movers to be realized. Thus, the scale of the banking sector is minimized to the lower fraction of movers, λ^l. Even though there exists restriction in accessing to the market among the uninformed, the lower bound of banking scale remain the same as long as agents have risk-neutral preferences and the parameter indicating the financial market development, α, is a common knowledge.

This conclusion, however, might be limited when α is unknown to uninformed agents or when α has very low value. Also, bank runs might be costly if we assume the standard, strictly concave utility function because the type B nonmovers' loss due to the limited participation can be critical. Then, the optimal spending on liquid assets would be increased to the level where bank runs does not happen, and the spending will be the function of α. If the value is lower than the critical point, then the intermediary will not allow bank runs to occur by holding more liquid assets amounting to λ^h, even though it incurs some cost due to a lower rate of return on fiat money. The lower bound of banking scale is higher than under full participation case.

The conclusion might be also limited when we allow the rate of return on the risky asset to be a random variable having general probability density function, $f(R)$. Then the asset prices also take continuous values rather than discrete.

Moreover, this chapter is only concerned with a stationary allocation and a constant money supply, and thus assumes that the rate of return on fiat money is constant each period. If a central bank having authority to print money exists, the rate of return on fiat money will be affected by the monetary policy, and this potentially will change the bank's portfolio decision and thus the behaviour of agents and the intermediary.[38] Another interesting question would be to ask that the central bank is able to remove the inefficiency induced by an asymmetric information.

Also, I considered the symmetric allocation between islands. Each island faces the same stochastic shocks, so the population of each island remains constant. What would happen if the relocation shock is idiosyncratic across each island, and so the population of each region changes as each region sends the small number of movers and take large number of movers? Would there be contagion or systemic risk in interbank markets? It is also important to know what will happen to the mutual relations between the two regions, which have different financial market structures or financial market developments, and whether financial exchange between these two regions can bring systematic risks or contagion. These questions ask us to consider the interbank markets between islands and to solve a general equilibrium problem. Some of these questions is dealt with in the following chapter.

[38] I discuss this issues in the following Chapter 3.

Appendices

2-1 Proof of Proposition 2.3.1

Proof The supply of capital comes from movers. They inelastically supply their holdings of illiquid assets, whatever the price is. So there is a vertical supply curve, and the quantity supplied (the demand for money) is:

$$S^K(= D^M) = \lambda^i k$$

The demand for illiquid assets comes from nonmovers. Nonmovers need to decide whether to hold fiat money and pass it over to the next period. For the decision, they compare the expected rate of return on fiat money with that of capital. Let γ be the ratio of money exchanged with capital at date 1. Then, $1 - \gamma$ fraction among their money holdings (q/v_0) is passed over to the date 1. γ takes a value between 0 and 1, and $\gamma = 1$ implies that nonmovers inelastically supply their whole fiat money to the market. However, $\gamma = 0$ (all storage of fiat money) will not occur in an equilibrium since as $\gamma \to 0$, $B_t^{i,j} \to 0$. So, as long as $R_j > 0$ there exists fiat money provider to trade for illiquid assets.

One unit of fiat money will purchase $1/B_1^{i,j}$ units of claim on consumption goods at date 1, and this will produce $R_j/B_1^{i,j}$ units of consumption goods at date 2. One unit of the good is traded for $1/v_1$ units of fiat money at date 1 and this will purchase v_2/v_1 units of consumption good at date 2.

If $R_j < \frac{v2}{v1} B_1^{i,j}$, then no nonmovers would hold capital from date 1 to 2, which implies $\gamma = 0$. If $R_j = \frac{v2}{v1} B_1^{i,j}$, then the two assets are perfect substitutes to nonmovers. Thus, any γ between $0 < \gamma \leq 1$ is possible. If $R_j > \frac{v2}{v1} B_1^{i,j}$, no nonmovers would hold fiat money from date 1 to 2, which implies $\gamma = 1$. Then, for the nonmovers to trade fiat money for capital, it is required that $B_1^{i,j} \leq \frac{R_j}{v_2/v_1}$. The maximum price of the the asset is $B_1^{i,j} = \frac{v_2}{v_1} R_j$.

Therefore, the aggregate demand for capital (supply of money) of nonmovers is

$$D^K(= S^M) = (1 - \lambda^i)\gamma M v_1 = \frac{v_1}{v_0}(1 - \lambda^i)\gamma q$$

The asset market clearing condition requires that

$$\begin{aligned} \lambda^i k B_1^{i,j} &\leq \frac{v_1}{v_0}\gamma(1-\lambda^i)q \\ \Leftrightarrow \quad B_1^{i,j} &\leq \frac{v_1}{v_0}\frac{\gamma(1-\lambda^i)q}{\lambda^i k} \end{aligned} \tag{37}$$

If the market has enough liquidity (i.e.,$S^M > D^M$), then the asset price would reach its highest possible point, $\frac{R_j}{v_2/v_1}$. However, if the market suffers liquidity shortage for any reason, the price will be determined by the amount of cash supplied. This is known as 'cash-in-the-market pricing' in which the asset price is determined as the ratio of total available 'cash' to the amount of asset provided (Allen and Gale, 1994, 2009). ■

2-2 Proof of Proposition 2.3.2

Proof No fiat money is passed over to the date 2, i.e., $\gamma = 1$, if and only if the rate of return on capital is greater than or equal to that of fiat money, i.e.,

$$\frac{R_j}{B_1^{i,j}} \geq \frac{v_2}{v_1}$$

Rewriting the preceding equation, we have:

$$R_j \geq \frac{v_2}{v_1}B_1^{i,j} = \frac{v_2}{v_1}\frac{(1-\lambda^i)Mv_1}{\lambda^i k} = \frac{v_2}{v_0}\frac{(1-\lambda^i)q}{\lambda^i k}$$

This implies when the rate of return on capital is high, agents will want to hold only capital from period 1 to period 2. Since the rate of return on capital takes two values, it is assumed that the preceding relation holds only when $R_j = R_H$ without loss of generality. The value of γ with R_H, γ^H, becomes 1.

On the other hand, some fiat money is stored for the consumption of date 2, i.e.,

$0 < \gamma < 1$, only when

$$\frac{R_j}{B_1^{i,j}} = \frac{v_2}{v_1}$$

In other words, agents will hold some currency and pass it over to the next period only when the rate of return on capital is low. So I assume that $0 < \gamma^L < 1$ when $R_j = R_L$.

Since $\frac{(1-\lambda^l)q}{\lambda^l k} > \frac{(1-\lambda^h)q}{\lambda^h k}$,[39] the price of capital in the state l, $B_t^{l,j}$, is higher than or equal to that in the state of h, $B_t^{h,j}$. Therefore, we have

$$\frac{R_j}{v_2/v_1} \geq B_1^{l,j} \geq B_t^{h,j}, \quad \left(=, \text{ if } B_t^{l,j} = B_t^{h,j} = \frac{R_j}{v_2/v_1}\right), \tag{38}$$

Note that, however, B_1^l cannot be equal to B_1^h. If $B_1^l = B_1^h$, then we have $B_1^l = B_1^h = \frac{R_j}{v_2/v_1}$ [40], which implies that capital dominates fiat money. If agents hold only capital, i.e., $k = 1$, then movers sell capital at a price of $B_1^{i,j}$ and consume $\frac{v_2}{v_1}B_1^{i,j}k = \frac{v_2}{v_1}B_1^{i,j} = R > \frac{v_2}{v_0}$. No one would hold fiat money in the first place, which cannot be an equilibrium. Therefore we should have the relationship: $\frac{R_j}{v_2/v_1} \geq B_t^l > B_t^h$.

Additional condition, $B_1^{h,j} < v_1/v_0 < B_1^{l,j}$, is required because otherwise fiat money is (weakly) dominated by capital as well. If $B_1^{l,j} > B_1^{h,j} \geq v_1/v_0$, then a consumer holding only capital sells it at a price $B_1^{i,j}$ and consume $\frac{v_2}{v_1}B_1^{l,j} > \frac{v_2}{v_1}B_1^{h,j} \geq \frac{v_2}{v_0}$. Also, if $B_1^{h,j} < B_1^{l,j} \leq v_1/v_0$, capital is weakly dominated by fiat money since $\frac{v_2}{v_1}B_1^{h,j} < \frac{v_2}{v_1}B_1^{l,j} \leq \frac{v_2}{v_0}$. Therefore we have $B_1^{h,j} < v_1/v_0 < B_1^{l,j}$. All these explanations yield the conclusion that $\frac{B_t^1}{p_{t+1}} < \frac{p_t}{p_{t+1}} < \frac{B_t^0}{p_{t+1}} \leq R$.

$$R_L = \frac{v_2}{v_1}B_1^{i,L} < \frac{v_2}{v_0} < \frac{v_2}{v_1}B_1^{i,H} \leq R_H$$

[39] $\left[\frac{(1-\lambda^i)q}{\lambda^i k}\right]$ is decreases in λ^i, i.e., $\frac{\partial}{\partial \lambda^i}\left[\frac{(1-\lambda^i)q}{\lambda^i k}\right] < 0$

[40] Because $\frac{(1-\lambda^l)q}{\lambda^l k} \neq \frac{(1-\lambda^h)q}{\lambda^h k}$, the only way $B_1^l = B_1^h$ is both B_1^l and B_1^h having the fundamental value.

where $i = \{l, h\}$. γ^L takes on any value between $0 < \gamma^L < 1$ depending on parameter values. ■

2-3 Expected utility of uninformed agents

$$
\begin{aligned}
&E^F[c_{m,2} + c_{n,2}] \\
&\quad = \frac{\mu_h}{2}\left[\lambda^h\left(\frac{v_2}{v_0}q + \frac{v_2}{v_1}\widehat{B}_1^{h,H}k\right) + (1-\lambda^h)\left(\frac{v_1}{v_0}\frac{q}{\widehat{B}_1^{h,H}}R_H + R_H k\right)\right] \\
&\quad + \frac{\mu_l}{2}\left[\lambda^l\left(\frac{v_2}{v_0}q + \frac{v_2}{v_1}\widehat{B}_1^{l,H}k\right) + (1-\lambda^l)\left(\frac{v_1}{v_0}\frac{q}{\widehat{B}_1^{l,H}}R_H + R_H k\right)\right] \\
&\quad + \frac{\mu_h}{2}\left[\lambda^h\left(\frac{v_2}{v_0}q + \frac{v_2}{v_1}\widehat{B}_1^{h,L}k\right) + (1-\lambda^h)\left(\frac{v_1}{v_0}\frac{\gamma^h q}{\widehat{B}_1^{h,L}}R_L + R_L k + (1-\gamma^h)\frac{v_2}{v_0}q\right)\right] \\
&\quad + \frac{\mu_l}{2}\left[\lambda^l\left(\frac{v_2}{v_0}q + \frac{v_2}{v_1}\widehat{B}_1^{l,L}k\right) + (1-\lambda^l)\left(\frac{v_1}{v_0}\frac{\gamma^l q}{\widehat{B}_1^{l,L}}R_L + R_L k + (1-\gamma^l)\frac{v_2}{v_0}q\right)\right]
\end{aligned} \tag{39}
$$

Let the sum of the third and fourth line on the right-hand side be A. Using the fact $\widehat{B}_1^{i,L} = R_L$ and by the assumption that $v_{t+s} = v, \forall s \geq 0$, $\widehat{B}_1^{i,L}$ is equal to R_L and then A becomes

$$
\begin{aligned}
A &= \frac{\mu_h}{2}\left[\lambda^h(q + R_L k) + (1-\lambda^h)(q + R_L k)\right] \\
&\quad + \frac{\mu_l}{2}\left[\lambda^l(q + R_L k) + (1-\lambda^l)(q + R_L k)\right] \\
&= \frac{\mu_h}{2}(q + R_L k) + \frac{\mu_l}{2}(q + R_L k) \\
&= \frac{q}{2} + \frac{1}{2}R_L k
\end{aligned}
$$

Let the sum of the first and second line on the right-hand side be B, then B becomes by utilizing the fact $\widehat{B}_1^{i,H} = \frac{v_1}{v_0}\frac{(1-\lambda^i)q}{\lambda^i k}$,

$$\begin{aligned}
B =& \frac{\mu_h}{2}\left[\lambda^h\left(q+\frac{(1-\lambda^h)q}{\lambda^h}\right)+(1-\lambda^h)\left(\frac{\lambda^h}{(1-\lambda^h)}R_H k + R_H k\right)\right] \\
&+\frac{\mu_l}{2}\left[\lambda^l\left(q+\frac{(1-\lambda^l)q}{\lambda^l}\right)+(1-\lambda^l)\left(\frac{\lambda^l}{(1-\lambda^l)}R_H k + R_H k\right)\right] \\
=& \frac{\mu_h}{2}\left[\lambda^h q + (1-\lambda^h)q + \lambda^h R_H k + (1-\lambda^h)R_H k\right] \\
&+\frac{\mu_l}{2}\left[\lambda^l q + (1-\lambda^l)q + \lambda^l R_H k + (1-\lambda^l)R_H k\right] \\
=& \frac{\mu_h}{2}\left[q+R_H k\right] + \frac{\mu_l}{2}\left[q+R_H k\right] \\
=& \frac{q}{2}+\frac{1}{2}R_H k
\end{aligned}$$

Therefore,

$$A + B = q + E[R_j]k$$

2-4 Uninformed agents' loss due to information asymmetries

$$\begin{aligned}
&E^F[c_{m,2}+c_{n,2}] - E^A[c_{m,2}+c_{n,2}] \\
=&\frac{\mu_l}{2}\left[\lambda^l\left(\frac{v_2}{v_0}q+\frac{v_2}{v_1}\widehat{B}_1^{l,L}k\right)+(1-\lambda^l)\left(\frac{v_1}{v_0}\frac{\gamma^l q}{\widehat{B}_1^{l,L}}R_L + R_L k + (1-\gamma^l)\frac{v_2}{v_0}q\right)\right] \\
&-\frac{\mu_l}{2}\left[\lambda^l\left(\frac{v_2}{v_0}q+\frac{v_2}{v_1}\widetilde{B}_1^{l,L}k\right)+(1-\lambda^l)\left(\frac{v_1}{v_0}\frac{q}{\widetilde{B}_1^{l,L}}R_L + R_L k\right)\right] \\
=&\frac{\mu_l}{2}\left[\lambda^l k(\widehat{B}_1^{l,L}-\widetilde{B}_1^{l,L}) + (1-\lambda^l)q\left(1-\frac{R_L}{\widetilde{B}_1^{l,L}}\right)\right]
\end{aligned}$$

Using the fact $\widetilde{B}_1^{l,L} = \widehat{B}_1^{h,H} = \frac{(1-\lambda^h)q}{\lambda^h k}$ and $\widehat{B}_1^{l,L} = R_L$, the terms in the square bracket becomes:

$$\begin{aligned}
\lambda^l k R_L - &\frac{\lambda^l(1-\lambda^h)q}{\lambda^h} + (1-\lambda^l)q - \frac{(1-\lambda^l)\lambda^h k R_L}{(1-\lambda^h)} \\
&= \frac{q(\lambda^h-\lambda^l)}{\lambda^h} - \frac{kR_L(\lambda^h-\lambda^l)}{1-\lambda^h} \\
&= (\lambda^h-\lambda^l)\left[\frac{(1-\lambda^h)q}{\lambda^h(1-\lambda^h)} - \frac{kR_L}{1-\lambda^h}\right] \\
&= (\lambda^h-\lambda^l)\left[\frac{k\widehat{B}_1^{h,H}}{(1-\lambda^h)} - \frac{kR_L}{1-\lambda^h}\right] \\
&= \frac{k(\lambda^h-\lambda^l)}{1-\lambda^h}(\widehat{B}_1^{h,H} - R_L) < 0
\end{aligned}$$

The first term in the second line is the sum of the second and third terms in the first line, and the second term is the sum of the first and fourth terms.

2-5 Expected utility with banks

Expected utility with banks having q^l is given as

$$\begin{aligned}
&E^l[c_{m,2}+c_{n,2}] \\
&\quad =\mu^l\left[q^l + E[R_j]\,k^l\right] \\
&\quad\quad + \frac{\mu_h}{2}\left[\lambda^h\left(q^l + \bar{B}_1^{h,H}k^l\right) + (1-\lambda^h)\left(\frac{q^l}{\bar{B}_1^{h,H}} + k^l\right)R_H\right] \\
&\quad\quad + \frac{\mu_h}{2}\left[\lambda^h\left(q^l + \widehat{B}_1^{h,L}k^l\right) + (1-\lambda^h)\left\{\left(\frac{\gamma q^l}{\widehat{B}_1^{h,L}} + k^l\right)R_L + (1-\gamma)q^l\right\}\right]
\end{aligned}$$

The third line of the preceding equation are simplified by using $\bar{B}_1^{h,H} = \frac{(1-\lambda^h)q^l}{\lambda^h k^l}$ and $\widehat{B}_1^{h,L} = R_L$,

$$\begin{aligned}
&\frac{\mu_h}{2}\left[\lambda^h q^l + (1-\lambda^h)q^l + \lambda^h k^l R_H + (1-\lambda^h)k^l R_H\right] \\
&\quad = \frac{\mu_h}{2}\left[q^l + k^l R_H\right]
\end{aligned}$$

The fourth line is simplified as:

$$\frac{\mu_h}{2}\left[q^l + k^l R_L\right]$$

Thus,

$$\begin{aligned}
&E^l[c_{m,2} + c_{n,2}] \\
&\quad = \mu^l\left[q^l + E[R_j]\,k^l\right] + \mu^h\left[q^l + E[R_j]k^l\right] \\
&\quad = q^l + E[R_j]k^l
\end{aligned}$$

2-6 Expected uility with limited participation

$$\begin{aligned}
&E^{F'}[c_{m,2} + c_{nA,2} + c_{nB,2}] \\
&= \frac{\mu_h}{2}\Bigg[\lambda^h\left(\frac{v_2}{v_0}q + \frac{v_2}{v_1}\breve{B}_1^{h,H}k\right) \\
&\quad + \alpha(1-\lambda^h)\left(\frac{v_1}{v_0}\frac{q}{\breve{B}_1^{h,H}}R_H + R_H k\right) + (1-\alpha)(1-\lambda^h)\left(\frac{v_2}{v_0}q + R_H k\right)\Bigg] \\
&+ \frac{\mu_l}{2}\Bigg[\lambda^l\left(\frac{v_2}{v_0}q + \frac{v_2}{v_1}\breve{B}_1^{l,H}k\right) \\
&\quad + \alpha(1-\lambda^l)\left(\frac{v_1}{v_0}\frac{q}{\breve{B}_1^{l,H}}R_H + R_H k\right) + (1-\alpha)(1-\lambda^h)\left(\frac{v_2}{v_0}q + R_H k\right)\Bigg] \\
&+ \frac{\mu_h}{2}\Bigg[\lambda^h\left(\frac{v_2}{v_0}q + \frac{v_2}{v_1}\breve{B}_1^{h,L}k\right) \\
&\quad + \alpha(1-\lambda^h)\left(\frac{v_1}{v_0}\frac{\gamma^h q}{\breve{B}_1^{h,L}}R_L + R_L k + (1-\gamma^h)\frac{v_2}{v_0}q\right) + (1-\alpha)(1-\lambda^h)\left(\frac{v_2}{v_0}q + R_L k\right)\Bigg] \\
&+ \frac{\mu_l}{2}\Bigg[\lambda^l\left(\frac{v_2}{v_0}q + \frac{v_2}{v_1}\breve{B}_1^{l,L}k\right) \\
&\quad + \alpha(1-\lambda^l)\left(\frac{v_1}{v_0}\frac{\gamma^l q}{\breve{B}_1^{l,L}}R_L + R_L k + (1-\gamma^l)\frac{v_2}{v_0}q\right) + (1-\alpha)(1-\lambda^h)\left(\frac{v_2}{v_0}q + R_L k\right)\Bigg] \\
&\quad = q + \alpha E[R_j]k + (1-\alpha)(q + kE[R_j])
\end{aligned}$$

Let the sum of the third and fourth line be A, and using the fact $\breve{B}_1^{i,L} = R_L$ and the assumption that $v_t = v$, then A becomes

$$
\begin{aligned}
A =& \frac{\mu_h}{2}\left[\lambda^h\left(q+R_Lk\right)+\alpha(1-\lambda^h)\left(q+R_Lk\right)+(1-\alpha)(1-\lambda^h)\left(q+R_Lk\right)\right]\\
&+\frac{\mu_l}{2}\left[\lambda^l\left(q+R_Lk\right)+\alpha(1-\lambda^l)\left(q+R_Lk\right)+(1-\alpha)(1-\lambda^l)\left(q+R_Lk\right)\right]\\
=& \frac{\mu_h}{2}(q+R_Lk)+\frac{\mu_l}{2}(q+R_Lk)\\
=& \frac{q}{2}+\frac{1}{2}R_Lk
\end{aligned}
$$

Let the sum of the first and second line be B, and using the fact $\breve{B}_1^{i,H} = \frac{v_1}{v_0}\frac{\alpha(1-\lambda^i)q}{\lambda^i k}$, then B becomes:

$$
\begin{aligned}
B =& \frac{\mu_h}{2}\Bigg[\lambda^h\left(q+\frac{\alpha(1-\lambda^h)q}{\lambda^h}\right)\\
&+\alpha(1-\lambda^h)\left(\frac{\lambda^h}{\alpha(1-\lambda^h)}R_Hk+R_Hk\right)+(1-\alpha)(1-\lambda^h)\left(q+R_Hk\right)\Bigg]\\
+&\frac{\mu_l}{2}\Bigg[\lambda^l\left(q+\frac{\alpha(1-\lambda^l)q}{\lambda^l}\right)\\
&+\alpha(1-\lambda^l)\left(\frac{\lambda^l}{\alpha(1-\lambda^l)}R_Hk+R_Hk\right)+(1-\alpha)(1-\lambda^l)\left(q+R_Hk\right)\Bigg]\\
=& \frac{\mu_h}{2}\left[\lambda^hq+\alpha(1-\lambda^h)q+\lambda^hR_Hk+\alpha(1-\lambda^h)R_Hk+(1-\alpha)(1-\lambda^h)\left(q+R_Hk\right)\right]\\
&+\frac{\mu_l}{2}\left[\lambda^lq+\alpha(1-\lambda^l)q+\lambda^lR_Hk+\alpha(1-\lambda^l)R_Hk+(1-\alpha)(1-\lambda^l)\left(q+R_Hk\right)\right]\\
=& \frac{\mu_h}{2}\left[\alpha(q+R_Hk)+(1-\alpha)(1-\lambda^h)\left(q+R_Hk\right)\right]\\
&+\frac{\mu_l}{2}\left[\alpha(q+R_Hk)+(1-\alpha)(1-\lambda^l)\left(q+R_Hk\right)\right]\\
=& \frac{\mu_h}{2}(q+R_Hk)[\alpha+(1-\alpha)(1-\lambda^h)]\\
&+\frac{\mu_l}{2}(q+R_Hk)[\alpha+(1-\alpha)(1-\lambda^l)]\\
=& \left(\frac{q}{2}+\frac{1}{2}R_Hk\right)\underbrace{[\alpha+(1-\alpha)(1-\lambda^h)]}_{<1}\\
<& \frac{q}{2}+\frac{1}{2}R_Hk
\end{aligned}
$$

Therefore,

$$A + B < q + E[R_j]k$$

and B is a increasing function of α.

2-7 Non-movers' Expected Utility under Limited Participation & Imperfect Information

$$\begin{aligned}
&E^{l'}[c_{m,2} + c_{n,2}] \\
&=\mu^l \left[\lambda^l + (1-\lambda^l)E[R_j]\right] \\
&\quad + \frac{\mu_h}{2}\left[\lambda^h\left(q^l + \bar{B}_1^{h,H}k^l\right) + (1-\lambda^h)\left\{\alpha\left(\frac{q^l}{\bar{B}_1^{h,H}} + k^l\right)R_H + (1-\alpha)\left(\frac{v_2}{v_0}q^l + k^l R_H\right)\right\}\right] \\
&\quad + \frac{\mu_h}{2}\left[\lambda^h\left(q^l + \breve{B}_1^{h,L}k^l\right)\right. \\
&\qquad \left. + (1-\lambda^h)\left(\alpha\left\{\left(\frac{\gamma q^l}{\breve{B}_1^{h,L}} + k^l\right)R_L + (1-\gamma)q^l\right\} + (1-\alpha)\left\{q^l + k^l R_L\right\}\right)\right]
\end{aligned}$$

The bank chooses q^l and since state h happens, the asset price in state $\{h, H\}$ is expressed as $\bar{B}_1^{h,H}$

$$\bar{B}_1^{h,H} = \frac{\alpha(1-\lambda^h)q^l}{\lambda^h k^l}$$

and $\breve{B}_1^{h,L} = R_L$.

The third line is simplified as:

$$\begin{aligned}
&\frac{\mu_h}{2}\Bigg[(\lambda^h q^l + \alpha(1-\lambda^h)q^l) + (\lambda^h k^l R_H + \alpha(1-\lambda^h)k^l R_H) \\
&\qquad + \left\{\underbrace{(1-\lambda^h)(q^l + k^l R_H)}_{=q^l + k^l R_H - \lambda^h q^l - \lambda^h k^l R_H} - \underbrace{\alpha(1-\lambda^h)(q^l + k^l R_H)}_{=\alpha(1-\lambda^h)q^l + \alpha(1-\lambda^h)k^l R_H}\right\}\Bigg] \\
&=\frac{\mu_h}{2}\left[q^l + k^l R_H\right]
\end{aligned}$$

The fourth and fifth line is simplified as:

$$\begin{aligned}
&\frac{\mu_h}{2}\Big[\lambda^h(q^l + R_L k^l) + (1-\lambda^h)\alpha(q^l + k^l R_L) + (1-\lambda^h)(1-\alpha)(q^l + k^l R_L)\Big] \\
=&\frac{\mu_h}{2}\left[\lambda^h(q^l + R_L k^l) + (1-\lambda^h)(q^l + k^l R_L)\right] \\
=&\frac{\mu_h}{2}\left[q^l + R_L k^l\right]
\end{aligned}$$

Therefore,

$$E^{l'}[c_{m,2} + c_{n,2}] = q^l + E[R_j]k^l = E^l[c_{m,2} + c_{n,2}]$$

Chapter 3

Bank Liquidity, Securitization, and the Efficacy of Monetary Policy

3.1 Introduction

Since the seminal studies of Bryant (1980) and Diamond and Dybvig (1983), the maturity structure trade-off between liquid and illiquid assets of a bank, and the liquidity preference resulting from uncertainty about the timing of consumption have provoked long debates on how banks should hold their assets. Also, the tendency of excessive securitization, which is considered to be one of the principal causes of the 2008 financial crisis, raises interest in the relationship between the degree of securitization and the liquid asset holdings of banks. All these issues have important implications for the effective implementation of monetary policy, given that monetary policy is implemented through the banking system.

This chapter aims to analyse how the need for liquid assets of individual banks, and the economy as a whole is affected by securitization. Also, it examines how monetary policy can affect the composition of banks' portfolio decision and whether monetary policy can change the composition of assets in the direction desired by it. Many empirical papers show the effect of securitization on bank liquidity and the relationship between the efficacy of monetary policy and securitization. Loutskina

(2011) shows the empirical evidence that bank's ability to securitize loans reduces its liquid asset holding. Maddaloni and Peydró (2011) illustrates the relationships between the short-term interest rate, securitization, and banks' asset choices using Euro area and U.S data. Estrella et al. (2002) and Loutskina (2011) analyse the effect of securitization on the efficacy of monetary policy using empirical macroeconomic models. **?** argues that securitization alters the effectiveness of the bank lending channel and increases the capacity to supply new loans. This chapter contributes to the strand of research by providing theoretical grounds of the effect of securitization on the economy's need for liquid assets, which has not been discussed much in the literature.

In the first part of this paper, I present a model which explain the relationship between securitization and the bank's liquid asset holdings, as well as the process by which banks in two different regions trade securities. This framework provides a detailed description of how the changes in demand and supply structure of securities from different islands affect various key economic variables. Securitization not only allows banks to transform illiquid assets into liquid assets but it also provides an alternative source of investment in liquid assets. In addition to the liquid asset holdings in the region where securitization is possible, the liquid asset reserves of the whole region are significantly reduced compared with before securitization. This implies the economy uses its resources in a more productive way to increase the total output across the two islands. One of the main conclusions is that the impact of securitization on banks' liquidity, consumption, and welfare is affected by the population structure of each island. The demographic structure between two regions affects not only the supply of securities in an area where securitization is present but also the size of demand in the other region, which may lead to a different equilibrium through adjustment of securities prices.

These results may have important policy implications, especially when looking at the work of Caballero (2006), Caballero et al. (2008), and Caballero and Krishnamurthy (2009), which analyse the effect of global asset shortages on bubbles and global imbalances. Caballero (2006) analyse the effect of global asset shortages on bubbles and global imbalances. Since only some regions of the world are capable of supplying good quality financial assets, the global shortage of assets may lead to capital movement from emerging markets to advanced economy, and potentially cause bubbles. This chapter assumes only one of two islands produce safe securities and shows how the different population structure affects risk sharing and the welfare of consumers. The possibility of producing more output through capital inflows from the island which does not issue securities, however, is limited due to the assumption that movement of goods between islands is not possible.[1]

Also, these results have significant similarities with the study of Bencivenga and Smith (1991). Based on the random relocation economy, Bencivenga and Smith (1991) showed that the introduction of financial intermediaries might affect the composition of savings toward more productive assets (capital) and thus intermediaries enhance growth by reducing socially unnecessary capital liquidation. Securitization introduced in this chapter shows that the need for liquid assets, required for movers facing liquidity shocks but provide low rates of return, can be reduced significantly in a general equilibrium setting.

In the second part of this chapter, I examine how monetary policy can affect the composition of banks' portfolios and whether monetary policy can change the composition of assets in the direction desired by the monetary authorities. Expansionary monetary policy can affect the bank's portfolio composition and thus its depositors' consumption and welfare, but the outcome depends on the magnitude of the elasticity of substitution coefficients. When the consumer is less (resp. more) risk-averse, the substitution effect dominates (resp. is dominated by) the income effect and banks are willing to hold more (resp. less) liquid assets in response to higher price levels than would otherwise be the case. However, if a randomised monetary policy combines with other real shocks, the bank suffers from a signal ex-

[1] If the assumption is relaxed and capital accumulation is possible, then capital flows toward the island A, the asset producing region, can result in so-called *global imbalances* (See Caballero (2006), Caballero et al. (2008), Caballero and Krishnamurthy (2009)).

traction problem. Since banks cannot differentiate between the two elements of real and nominal factors that determine the current price level, they react to changes in the nominal supply of money, not real ones. This results in banks holding too many, and in some cases too few, liquid assets when compared to under full information.

Another interesting question is whether monetary policy can change the composition of assets in the direction desired by the monetary authorities. More specifically, let us consider the question of whether monetary authorities will be able to increase capital investment via expansionary monetary policy if consumers are highly risk-averse. This chapter shows that the benefits from inflation surprises disappear when economic agents understand the monetary authorities' intention and act accordingly. This leads only to inflation bias as Kydland and Prescott (1977), Barro and Gordon (1983a,b)'s research shows.[2]

To address these issues, I used a simple overlapping generations model with fiat money, following the Lucas' island model and its simplified version Wallace (1980). Lucas (1972) showed that a positive short-term correlation between the rate of change in nominal prices and the level of real output exists and argues that the relationship may disappear when the government tries to exploit it. In Lucas (1972), consumers live in two spatially separated places and are subject to two different shocks. The first shock is the stochastic changes in the quantity of money which may adjust the nominal price levels. The second source of disturbance is an island-specific population shock. The population of the young in each period is random; thus it may cause a difference in relative prices between the two places. Information on these real and monetary shocks is conveyed by the local price, but in the special framework, prices provide only imperfect information with people. Those consumers who wish to respond only to real disturbances fail to distinguish between them; thus nominal monetary shocks have an effect on output.[3]

[2] Kydland and Prescott (1977), Barro and Gordon (1983a,b) showed the positive correlation between output and inflation could not arise systematically when agents have rational expectations. The benefits from inflation surprises disappear when economic agents understand the monetary authorities' intention and act accordingly and lead only to inflation bias.

[3] Wallace (1980) formed a simple model to stress the effect of Lucas (1972)'s incomplete information on time series correlation between output and inflation by way of a simple example.

This chapter and Lucas (1972) have two significant differences. First, this article examines the linkages between monetary policy and bank holding portfolio composition. Second, I deal with spillovers between islands, unlike Lucas (1972)' and Wallace (1980)'s analysis where no linkages between islands exist.

The effect of spillovers arises because of the framework whereby the two islands are linked to each other. Some of the members (movers) of each island move to different islands at some point after random relocation shocks, which, in combination with other nominal and real uncertainties, produces interesting results. If two separate regions are interconnected, then the outcome of the decision made in each region is related to the choice of another region, and therefore the issue of general equilibrium is essential. It is of particular importance to look at the interrelationships between the two regions, especially if one region can trade securities with other parts through the securitization of assets.

For movers to consume after being relocated, they need fiat money, the supply of which is identical in the two islands and universally accepted as a means of exchange.[4]. In order to reflect the preference for liquidity due to the uncertainty of the consumption place, I followed the setting of Champ et al. (1996).[5] In Champ et al. (1996), the bank provides insurance to agents who have random needs for liquidity because of the possibility of relocation. The 'relocation shock' takes the place of the 'preference shock' in Diamond and Dybvig (1983).

The remainder of this chapter is organised as follows. Section 3.2 describes the structure of the physical set-up, information, and money market equilibrium. Section **??** introduces securitization and analyses the effect of it on banks' and the overall needs for liquid assets of the whole economy. Section 3.4 studies the effect of monetary policy on the bank's portfolio decision and how a signal extraction problem arises during the process. Section 3.5 analyses whether the monetary authorities can

[4]The role of fiat money as a store of value and medium of exchange using OLGs model is discussed in Samuelson (1958)

[5]Champ et al. (1996) constructs a monetary model in which a role for currency is considered, and monetary factors play a particular role in banking panics. However, except for the basic assumptions and settings, this chapter is not directly related to Champ et al. (1996).

influence the bank's portfolio decision when the authorities have a specific goal to achieve. Concluding remarks are contained in Section 3.6.

3.2 The Structure of the Economy

3.2.1 Environment

There are infinitely many overlapping generations of agents who live for two periods and thus the economy is populated by two age cohorts: the young and the old. People live on two spatially separated islands, where communication across the two islands is limited. The population of the young and the old respectively are normalised to unity. Therefore, the total population of the economy are added up to 2. The number of the old in this economy is equally divided by two distinct locations. The young, however, are distributed unequally across the islands. Let the population of the young in each island at date t be N_t^i where $i = \{H, L\}$. Let N_t^H denote the island having large population and N_t^L being an island with small number of the young. Since the population of the young are normalized to 1, $N_t^H + N_t^L = 1$. The probability of the island i has N_t^H (resp. N_t^L) is ω_H (resp. ω_L). According to Lucas (1972) notation, let $N_t^H = \theta/2$ and $N_t^L = 1 - \theta/2$, where $1 < \theta < 2$ is a constant. So we have the following distribution.

$$N_t^i = \begin{cases} N_t^L, & \text{with probability } \omega_L \\ N_t^H, & \text{with probability } \omega_H, \end{cases}$$

The information about N_t^i is unknown to agents.

At the end of each period the stochastic relocation shocks occur and a fraction of movers among the young agents, λ, are relocated to the other island at the end of the same period and will spend their final time on the island. For N_t^i and N_{t+1}^j are assumed to be serially uncorrelated, the young who are born at date t in an island i, and subject to being relocated to the island j, will face two different population distribution of the young at date $t + 1$. The assumption that the old population of the two islands are equally distributed implies that the parameter λ is $1/2$.[6]

There are two separate types of assets for savings: capital and fiat money. The capital is regarded as an illiquid asset for two reasons. Firstly, the capital investment needs two periods of time before it transformed into consumption goods. Secondly, capital is assumed not to be transported across islands, and moreover, due to the 'limited communication', claims against capital is assumed to be useless. For movers to consume after being relocated, they need fiat money which is identical in the two islands and universally accepted as a means of exchange.

Total money supply is expected to grow or remain constant. The nominal price of one unit of the goods at date t is p_t and thus one unit of the good invested in fiat money at date t yields $p_t/p_{t+1} \leq 1$ units of the consumption good because p_{t+1} is expected to be greater than p_t. The illiquid asset represented by capital is a constant-returns-to-scale technology that takes two periods to mature and allows one unit of the good invested in the current period to be converted into $R > 1$ units of the good at date $t+1$. R is considered to have a definite value, in order to focus more on the uncertainty of the rate of return on money and the effect of monetary policy on the agent's portfolio decision.

3.2.2 Preferences

Agents are risk-averse and consume only at their final period. Agents, who are born at time t, are ex-ante identical and individually observe relocation shocks in the middle of date t after making a savings decision. This information is privately held and not observable by other agents. Expected utility of the generation born at time

[6]Unless $\lambda = 1/2$, the population of the two different islands is not equal to each other. The population of the old on the island H after the transfer of movers is $N^H - \lambda(N^H - N^L)$ and that of the island L is $N^L - \lambda(N^L - N^H)$. Since the elderly population of the two islands should be the same, the following holds true.

$$\begin{aligned} & N^H - \lambda(N^H - N^L) = N^L + \lambda(N^H - N^L) \\ \Leftrightarrow & N^H - N^L = 2\lambda(N^H - N^L) \\ \Leftrightarrow & \lambda = 1/2 \end{aligned}$$

t are represented by:

$$E_t[U(c_{m,t+1}, c_{n,t+1})] = \lambda u(c_{m,t+1}) + (1-\lambda)u(c_{n,t+1}),$$

where $c_{m,t+1}$ and $c_{n,t+1}$ denote consumption of a mover and a nonmover respectively, and λ is the fraction of movers among the young. A period utility function is characterized by CRRA.

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \tag{3.1}$$

where $\sigma = -\frac{cu''(c)}{u'(c)}$ denotes the degree of relative risk aversion. The utility function satisfies the usual neoclassical properties.

3.2.3 Supply of Fiat Money

The central bank controls a monetary instrument, the aggregate money supply, which enables it to choose the current price level, p_t. Let M_t be the total money supply of the economy at date t. The stock of fiat money increases according to the rule

$$M_t = z_t^k M_{t-1}$$

where M_{t-1} is a predetermined variable and z_t^k denotes the gross rate of money supply expansion which is greater than or equal to 1. z_t^k is a random variable with

$$z_t^k = \begin{cases} z_t^L, & \text{with probability } \varepsilon_L \\ z_t^H, & \text{with probability } \varepsilon_H, \end{cases}$$

where $1 \leq z_L < z_H$. Then new printed units of fiat money at date t is

$$M_t - M_{t-1} = \left(z_t^k - 1\right) M_{t-1}$$

Increases in the fiat money at date t is assumed to be distributed to the old proportional to an individual's balances of fiat money as in Lucas (1972). Wallace (1980) assumes differently that the increases in the stock of fiat money are subsidies which are allocated through lump-sum subsidies to each old person in each period t. In this model the subsidies are distributed to the old because only the movers among old people hold fiat money unlike Lucas (1972) and Wallace (1980).

Assumption 6

$$\frac{N_t^H}{N_t^L} = \frac{z_t^H}{z_t^L} > 1$$

This special assumption is used because the model uses a discrete sample space. See Wallace (1980) for a detailed explanation.

3.2.4 Money Market Equilibrium

No one in the future generations is born with fiat money. For young agents born at date t to acquire fiat money, they must trade with the initial old who are endowed in total with M_t units of fiat money at date t. The supply of fiat money in each island in period t is $M_t/2$ because the population of the old is distributed equally on each island. The demand for the fiat money comes from the young. Let q_t^i be the real demand for money from the individual young agent born on an island i at time t. In other words, q_t^i denotes the number of goods each young agent chooses to sell her fraction of endowment for fiat money. Then the total real demand for fiat money in the island i is $N_t^i q_t^i$.

The local price of consumption goods of the island i at date t, p_t^i, is determined at the beginning of period t by the coincidence of the demand for fiat money by the young and the supply of it by the old. Since uncertainties in the determination of current price level are the population of the young on the island i, and the money supply growth rate z_t^k, the current price level of island i is a function of N_t^i and z_t^k. Therefore, from now on denote the price level of the island i with the gross rate of money supply expansion, z_t^k, as $p_t^{ik} = p(N_t^i, z_t^k)$. The money market equilibrium condition implies the following

$$\begin{aligned} \frac{z_t^k M_{t-1}}{2p_t^i} &= N_t^i q_t^{ik} \\ \Leftrightarrow \quad p_t^{ik} &= \frac{0.5 z_t^k M_{t-1}}{N_t^i q_t^{ik}} \end{aligned} \tag{3.2}$$

Similarly, the expected future price level at date $t+1$ is expressed as

$$p_{t+1}^{jl} = \frac{0.5 z_{t+1}^l M_t}{N_{t+1}^j q_{t+1}^j}, \tag{3.3}$$

where $j, l = \{L, H\}$.

Let q_t^{ik} be the real money demand of agents living on the island i and facing money supply shock, z_t^k at date t. Then the rate of return from holding q_t^{ik} is $\frac{p_t^{ik}}{p_{t+1}^{jl}} q_t^i$. From (3.2) and (3.3) we have the rate of return on fiat money as following

$$\frac{p_t^{ik}}{p_{t+1}^{jl}} = \frac{N_{t+1}^j}{N_t^i} \frac{q_{t+1}^{jl}}{q_t^{ik}} \frac{1}{z_{t+1}^l} \tag{3.4}$$

The distribution of the rate of return on fiat money will depend on the realizations of random variables: N_t^i, N_{t+1}^j, z_t^k, and z_{t+1}^l. The price level at date $t+1$ is assumed to be serially uncorrelated with the current period t, and thus the price level at date $t+1$ is independent of that at date t. Therefore, the rate of return on fiat money depends only on the current price level. Therefore, q_t^{ik} is expressed as a function of p_t^{ik}.

3.2.5 Complete Information

I will divide the information structure that banks face into two different parts following Wallace (1980). In this part I deal with complete information case where the individual bank, h, on the island i optimises its depositors' expected utility knowing the value of z_t^k. The knowledge of z_t^k and the observed price p_t^{ik} gives the precise information on N_t^i from (3.2). For the future variables such as N_{t+1}^j and z_{t+1}^l, they have access to the distributions of them. The second case deals with the incomplete information set where banks do not observe z_t^k, and only have access to the distributions of z_t^k and N_t^i in Section 3.4.

Suppose that the young agents deposit all of their endowment goods with a bank, and the bank uses the proceeds to acquire assets in favour of its depositors. Banks compete for offering deposit contract to maximise the expected utility of their depositors due to free entry into the banking industry. Therefore, the bank's problem is to choose how much capital and fiat money to acquire in order to maximise the ex-ante expected utility of the young.

The financial intermediary h on the island i chooses its asset holdings $(q_t^{ik,h}, k_t^{ik,h})$[7] to maximise its depositor' expected utility, knowing the value of z_t^k. Then the rate of return on fiat money, (3.4), may take four different values because only N_{t+1}^j are p_{t+1}^{jl} are random variables. The probability of each event occurring is $\omega^j \varepsilon^l$.

Consumption

A mover consumes with her posttransfer fiat money balances. Consumption of a mover can be expressed as the distribution of the rate of return on fiat money.

$$c_{m,t+1}^{ik,jl} = \frac{q_t^{ik,h}}{\lambda} \frac{p_t^{ik}}{p_{t+1}^{jl}} + (z_{t+1}^l - 1) \frac{q_t^{ik,h}}{\lambda} \frac{p_t^{ik}}{p_{t+1}^{jl}}$$

$c_{m,t+1}^{ik,jl}$ denotes the consumption of a mover who is born on the island i and moves to the island j at date $t+1$ with the gross rate of money supply expansion z_t^k at date t and z_{t+1}^l during period $t+1$. The first term on the right-hand side denotes pretransfer money balances and the second term is subsidies transferred from the central bank proportional to their pretransfer balances. The preceding equation is simplified as

[7] k in the superscript denotes the states of the gross rate of money supply expansion.

$$c_{m,t+1}^{ik,jl} = z_{t+1}^{l} \frac{q_t^{ik,h}}{\lambda} \frac{p_t^{ik}}{p_{t+1}^{jl}} \tag{3.5}$$

The expected consumption of movers, then, will be

$$E[c_{m,t+1}^{ik,jl}] = \sum_j \sum_l \omega_j \varepsilon_l z_{t+1}^{l} \frac{q_t^{ik,h}}{\lambda} \frac{p_t^{ik}}{p_{t+1}^{jl}}$$

On the other hand, the consumption of nonmovers is not affected by the expected rate of return on money since they hold only real assets. Therefore, a nonmover's consumption is

$$c_{n,t+1}^{ik} = \frac{(1 - q_t^{ik,h})R}{1 - \lambda} \tag{3.6}$$

The Bank's Problem

The individual bank h on the island i maximises its depositor's expected utility

$$\max_{c_{m,t+1}, c_{n,t+1}} E_t[U(c_{m,t+1}, c_{n,t+1})] = \sum_j \sum_k \omega_j \varepsilon_k \left[\lambda u(c_{m,t+1}^{ik,jl}) + (1-\lambda) u(c_{n,t+1}^{ik}) \right] \tag{3.7}$$

subject to a budget constraint,

$$q_t^{ik,h} + k_t^{ik,h} = 1 \tag{3.8}$$

the feasibility conditions,

$$c_{m,t+1}^{ik,jl} - (z_{t+1}^{l} - 1) \frac{q_t^{ik,h}}{\lambda} \frac{p_t^{ik}}{p_{t+1}^{jl}} \leq \frac{q_t^{ik,h}}{\lambda} \frac{p_t^{ik}}{p_{t+1}^{jl}} \tag{3.9}$$

$$c_{m,t+1}^{ik,jl} + c_{n,t+1}^{ik} - (z_{t+1}^{l} - 1) \frac{q_t^{ik,h}}{\lambda} \frac{p_t^{ik}}{p_{t+1}^{jl}} = \frac{q_t^{ik,h}}{\lambda} \frac{p_t^{ik}}{p_{t+1}^{jl}} + \frac{k_t^{ik} R}{1-\lambda} \tag{3.10}$$

and the incentive constraint

$$c_{n,t+1}^{ik} \geq c_{m,t+1}^{ik,jl}, \tag{3.11}$$

and (3.5), (3.6).

First, the bank determines how to divide its member's endowment goods between real money demand and capital at date t. The second decision made by the bank is about how much it should pay to movers who must withdraw at the end of date t. The constraint (3.9) shows how this decision is made. The bank makes a standard deposit contract with its members. The contract promises to give a fixed amount of fiat money, $\frac{p_t^{ik} q_t^{ik,h}}{\lambda}$, at date t by paying out all available liquid assets, divided equally among those withdrawing.[8] Nonmovers are paid whatever is available at their final period which is shown in constraint (3.10). The constraint (3.10) says that the consumption by nonmovers are limited by the total value of the risky asset, capital, plus the amount of fiat money left over after the movers are paid off if any.

The constraint (3.11), which denotes incentive compatibility constraint, says that consumption by nonmovers must be at least as much as the promised real money to movers. Since whether a certain agent is relocated or not is private information, and thus, the bank cannot identify who is a mover and who is a nonmover, nonmovers have an incentive to pretend to be a mover unless this constraint holds. The incentive constraint tells us that a nonmover does not have any incentive to pretend to be a mover as long as she would get a higher level of the consumption good at their final period.

Since all members of the young on the island i are identical, and so are the behaviour of banks located on the island i. Therefore, the bank h's decision on $q_t^{ik,h}$ is identical to per capita real money holdings living on the island i, $\overline{q}_t^{ik}$. By substituting the budget constraint into objective function and letting $q_t^{ik,h} = \overline{q}_t^{ik}$, the

[8] If the bank does not have enough liquid assets to make the promised amount to movers at date t, the bank will go bankruptcy. However, bank runs are not considered in this chapter, and we assume banks do not suffer runs.

Lagrangian function of this optimization model is

$$\begin{aligned}\mathcal{L} =& \lambda u\left(c_{m,t+1}^{ik,jl}\right) + (1-\lambda)u\left(c_{n,t+1}^{ik}\right) \\ &+ \mu_1\left[z_{t+1}^{l}\frac{\overline{q}_t^{ik}}{\lambda}\frac{p_t^{ik}}{p_{t+1}^{jl}} - c_{m,t+1}^{ik,jl}\right] \\ &+ \mu_2\left[z_{t+1}^{l}\frac{\overline{q}_t^{ik}}{\lambda}\frac{p_t^{ik}}{p_{t+1}^{jl}} + \frac{(1-\overline{q}_t^{ik})R}{1-\lambda} - c_{m,t+1}^{ik,jl} - c_{n,t+1}^{ik}\right]\end{aligned}$$

where μ_1 and μ_2 are Lagrange multipliers which both have non-negative values. Note that the incentive constraint (3.11) are not included in the preceding equation. As shown below, this is because the incentive constraint is always satisfied automatically when we optimize this problem only with the first three constraints.[9]

As the first-order conditions with respect to consumption, we have the following set of simultaneous equations:

$$\begin{aligned}\left(c_{m,t+1}^{ik,jl}\right) \quad & \lambda u'\left(c_{m,t+1}^{ik,jl}\right) = \mu_1 + \mu_2 \\ \left(c_{n,t+1}^{ik}\right) \quad & (1-\lambda)u'\left(c_{n,t+1}^{i,jl}\right) = \mu_2\end{aligned}$$

Non-binding feasibility condition (3.9) implies that $c_{m,t+1}^{ik,jl} = c_{n,t+1}^{ik}$ because $\mu_1 = 0$ and $\lambda = 1/2$. When (3.9) is binding, it implies $c_{m,t+1}^{ik,jl} < c_{n,t+1}^{ik}$ because $\mu_1 > 0$. Therefore, the incentive constraint is always satisfied in any case.

First order condition with respect to $\overline{q}_t^{ik}$ yields the following:

$$(\mu_1 + \mu_2)\frac{z_{t+1}^{l}}{\lambda}\frac{p_t^{ik}}{p_{t+1}^{jl}} = \frac{\mu_2 R}{1-\lambda}$$

Combining all these first order conditions we have

$$\sum_j \sum_l \omega_j \varepsilon_l u'\left(c_{m,t+1}^{ik,jl}\right)\frac{z_{t+1}^{l} p_t^{ik}}{p_{t+1}^{jl}} = u'\left(c_{n,t+1}^{ik}\right) R, \tag{3.12}$$

Solving the preceding equation gives us the optimal value of $\overline{q}_t^{ik}$ where $i = \{L, H\}$.

[9]See Allen and Gale (1998).

3.3 Financial Innovation: securitization

securitization[10] is the process of taking illiquid assets and transforming them into liquid assets, and thus it allows banks to liquidate illiquid assets to finance their liquidity need. This section studies how financial development, represented by securitization, changes the financial intermediary's portfolio decision and how it alters the aggregate needs for liquid assets used for the consumption of agents facing liquidity shocks.[11] Also, I show that securitization not only allows banks to transform illiquid assets into liquid but also provides an alternative source for the investment in liquid assets.

Now let us call the two islands as the island A and island B.[12] Suppose that only the A island has an ability to generate (produce) liquid financial assets[13] appealing to (demanded by) foreign consumers who face liquidity shocks. The reason for making this assumption is to reflect the reality that not all countries in the world can supply financial assets. The population structure of the islands is important because it changes the relative size of demand for and supply of securities, and thus alters the bank's portfolio decision and the relevant equilibria values. A small population of the young on the island A means that an insufficient supply of quality financial assets increases the security price and potentially leads to bubbles and crisis, the possibility of which is not discussed in this chapter.

This section deals with the global equilibrium, in which the demand for and supply of ABS is equated in the securities market. The equilibrium of each island is affected by the decision made by the other island. I assume that the total money

[10] Financial innovation is a term widely used in relation to the act of creating new financial institutions, markets, instruments as well as inventing a new process for distributing financial services. Among the many contents related to financial innovation, this section explores securitization in particular and investigates how securitization affects the liquidity needs of banks.

[11] There are many potential benefits both to the issuer and investors when illiquid assets are securitized. For example, the issuer may manage interest rate risk better, improve her liquidity level, and diversify funding sources, and so forth. I do not develop the detailed process and various benefits of securitization, nor cover the important features regarding theoretical issues of security design happening in the asymmetric information situation. For these matters see Gorton and Metrick (2012).

[12] Therefore, i and j are expressed as $i, j = \{A, B\}$ rather than $i, j = \{L, H\}$

[13] Future returns that capital generates are used as the collateral for these securities. From now on, let us call these securities ABS.

supply does not change in this section. The reason for taking this assumption is to concentrate on the analysis on how securitization affects the amount of liquid assets held by the banks, by assuming that the amount of liquid assets required by the banks fluctuates only by real factors.[14] More specifically, I assume that $z_{t+s} = 1, \forall s \geq 0$. Then the price level the bank on the island i at date t and island j at $t+1$ are given as

$$p_t^i = \frac{0.5 z_t M_{t-1}}{N_t^i q_t^i(p_t^i)} = \frac{0.5 M_{t-1}}{N_t^i q_t^i(p_t^i)}$$
$$p_{t+1}^j = \frac{0.5 z_{t+1} M_t}{N_{t+1}^j q_{t+1}^j(p_{t+1}^j)} = \frac{0.5 M_{t-1}}{N_{t+1}^j q_{t+1}^j(p_{t+1}^j)}$$

which implies the expected rate of return on fiat money

$$\frac{p_t^i}{p_{t+1}^j} = \frac{N_{t+1}^j}{N_t^i} \frac{\overline{q}_{t+1}^j}{\overline{q}_t^i} \tag{3.13}$$

The rate of return on fiat money is only determined by real factors, N and q.

3.3.1 The securitization Market

Banks located on the island A (originators) begin the securitization process by gathering a series of illiquid assets (capital) and issue securities, which are backed by the assets (capital) they hold. They then sell the securities (ABS) to the banks located on the island B. The originator will have received proceeds from the securitization, and the proceeds (fiat money) are used for the consumption of movers. The returns generated by the underlying assets (capital on the island A) are then transferred to the investors who purchase the asset-backed securities, the bank B. More specifically, the returns are consumed by movers from the island B. During the process, the A bank is able to increase overall liquidity by securitizing illiquid assets into liquid assets and to generate immediate proceeds from their assets. These proceeds

[14] If there is no uncertainty in the economy and the population of both islands does not change, then the island A, which can securitize illiquid assets, can enjoy perfect risk-sharing. See the Appendix 3-1.

give the bank the potential to reduce the amount invested in liquid assets.[15]

Let k_t^A be the investment in capital which the bank on the island A chooses, and assume that the A bank securitizes its capital at a rate of α among its holdings of capital k_t^A. And let b_t be the (nominal) price of ABS which promises to provide R units of consumption goods per unit of securities at date $t+1$. Then, the total supply of ABS are $N_t^A \alpha k_t^A R b_t$.

Banks on the B island invest in securities with fiat money they hold. Suppose that the B bank uses β percentage of its total currency among its holdings of fiat money $M_t/2$ to purchase securities. Then, the supply of fiat money for buying the securities are $\frac{\beta M_t}{2} = \beta N_t^B q_t^B p_t^B$.

Therefore, the price of ABS, b_t, is determined so that the demand for and supply of securities match.[16]

$$\begin{aligned} N_t^A \alpha k_t^A R b_t &= \frac{\beta M_t}{2} = \beta N_t^B q_t^B p_t^B \\ \Rightarrow b_t &= \frac{\beta N_t^B q_t^B p_t^B}{N_t^A \alpha k_t^A R} \end{aligned} \tag{3.14}$$

The price of the securities depends on the bank A's decision on k_t^A and α, and the bank B's portfolio decision about q_t^B and β. Moreover, it relies on the parameter values of N_t^A and N_t^B. Since N_t^i is a random variable so b_t is.[17] When the population of the young on the island A is large, i.e., $N_t^A = N_t^H$, then the supply of securities will be bigger than the case of $N_t^A = N_t^L$, which leads to lower securities price. The value of N_t^i also affects the size of α and β, as illustrated in the example described below.

[15] Typically, there exist several players in the securitization process, such as originators, special purpose vehicles, credit rating agencies, and investors, and so forth. I simplified the process by assuming the securities issued by originators (banks on the island A) are sold directly to investors (the banks on the island B) and no uncertainties arise during the process since the rate of return on the securities are certain. This simplification is intended to reveal more clearly the relationship between securitization and bank's liquidity need, without considering irrelevant details. See Gorton and Metrick (2012) for a rigorous securitization process and related issues.

[16] It is assumed that neither short selling nor interbank lending are allowed, which implies $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$

[17] The island A can have the population of the young either N_t^H or N_t^L.

3.3.2 The Supply Side of securitization: Island A

The bank on the island A determines how to construct a portfolio (q_t^A, k_t^A) and how much capital to securitize in order to maximise its depositors' expected utility. Since banks on the island A securitize their capital investment at the rate of α, the total amount of capital to be securitized is αk_t^A. Then the consumption of a nonmover is

$$c_{n,t+1}^A = \frac{(1-\alpha)k_t^A R}{1-\lambda} \tag{3.15}$$

Total investment in liquid asset in the island A is

$$N_t^A q_t^A = \frac{M_t}{2p_t^A}$$

The proceeds the A bank will have received from the securitization (fiat money) are

$$\frac{\beta M_t}{2} = \beta N_t^B p_t^B q_t^B$$

because the B bank uses β percentage of its fiat money to purchase securities. Then, the aggregate consumption of movers who are born in the island A and are relocated to the island B is

$$\begin{aligned}
N_t^A \lambda c_{m,t+1}^{A,j} &= \left(\frac{M_t}{2} + \frac{\beta M_t}{2}\right)\frac{1}{p_{t+1}^j} \\
&= \frac{N_t^A q_t^A p_t^A + \beta N_t^B q_t^B p_t^B}{p_{t+1}^j} \\
&= \frac{N_t^A p_t^A q_t^A + N_t^A \alpha k_t^A b_t R}{p_{t+1}^j},
\end{aligned}$$

where $A, j = \{L, H\}$. The last line of the above equation is derived from the ABS market equilibrium condition (3.14).

The per capita consumption of movers is

$$c_{m,t+1}^{A,j} = \frac{q_t^A p_t^A + \beta(N_t^B/N_t^A)q_t^B p_t^B}{\lambda p_{t+1}^j} = \frac{p_t^A q_t^A + \alpha k_t^A b_t R}{\lambda p_{t+1}^j} \tag{3.16}$$

The preceding analysis shows that the securitization can reduce the total liquid asset holdings of the A bank in its portfolio by $\alpha k_t^A b_t R$ than before capital is securitized. Thus, the bank on the A island can invest its resources in a more efficient place, i.e., in capital, and thus produce more output in total than before securitization. If the young population of the A island can be used as a substitute for the A bank's ability to securitize illiquid assets, the larger the population of the island A (N_t^A), the smaller the amount of liquid assets (q_t^A) that must be retained.

Using the above facts, we can construct the maximization problem as follows. The A bank maximizes its depositors' expected utility

$$\max_{c_{m,t+1}, c_{n,t+1}} E_t[U(c_{m,t+1}, c_{n,t+1})] = \sum_{j=H,L} \omega_j \left[\lambda u(c_{m,t+1}^{A,j}) + (1-\lambda) u(c_{n,t+1}^{A}) \right] \tag{3.17}$$

subject to (3.15), (3.16), the budget constraint,

$$q_t^A + k_t^A = 1, \tag{3.18}$$

the feasibility conditions,[18]

$$c_{m,t+1}^{A,j} \leq \frac{p_t^A q_t^A + \alpha k_t^A b_t R}{\lambda p_{t+1}^j} \tag{3.19}$$

$$c_{m,t+1}^{A,j} + c_{n,t+1}^{A} \leq \frac{p_t^A q_t^A + \alpha k_t^A b_t R}{\lambda p_{t+1}^j} + \frac{(1-\alpha) k_t^A R}{1-\lambda}, \tag{3.20}$$

and the incentive constraint

$$c_{n,t+1}^{A} \geq c_{m,t+1}^{A,j} \tag{3.21}$$

[18]No subsidies are given to movers since there is no money expansion.

The Lagrangian function of this optimization model is

$$\begin{aligned}
\mathcal{L} =& \lambda u\left(c_{m,t+1}^{A,j}\right) + (1-\lambda)u\left(c_{n,t+1}^{A}\right) \\
&+ \mu_1\left[\frac{p_t^A q_t^A + \alpha k_t^A b_t R}{p_{t+1}^j} - \lambda c_{m,t+1}^{A,j}\right] \\
&+ \mu_2\left[\frac{p_t^A q_t^A + \alpha k_t^A b_t R}{p_{t+1}^j} + (1-\alpha)k_t^A R - \lambda c_{m,t+1}^{A,j} - (1-\lambda)c_{n,t+1}^{A}\right]
\end{aligned}$$

First order conditions with respect to consumptions are given

$$\begin{aligned}
\left(c_{m,t+1}^{A,j}\right) \quad & u'\left(c_{m,t+1}^{A,j}\right) = \mu_1 + \mu_2 \\
\left(c_{n,t+1}^{A}\right) \quad & u'\left(c_{n,t+1}^{A}\right) = \mu_2
\end{aligned}$$

Non-binding feasibility condition (3.9) implies that $c_{m,t+1}^{A,j} = c_{n,t+1}^{A}$ because $\mu_1 = 0$. When (3.20) is binding, the consumption becomes $c_{m,t+1}^{A,j} = c_{n,t+1}^{A}$ because $\mu_1 > 0$. Therefore, the incentive constraint is always satisfied.

First order conditions with respect to q_t^A and α yield the following:

$$(\mu_1 + \mu_2)\left(\frac{p_t^A - \alpha b_t R}{p_{t+1}^j}\right) = \mu_2(1-\alpha)R$$

$$(\mu_1 + \mu_2)\frac{(1-q_t^A)b_t R}{p_{t+1}^j} = \mu_2(1-q_t^A)R$$

Combining all these first order conditions we have

$$\sum_{j=H,L} \omega_j u'\left(c_{m,t+1}^{A,j}\right)\left(\frac{p_t^A - \alpha b_t R}{p_{t+1}^j}\right) = (1-\alpha)u'\left(c_{n,t+1}^{A}\right)R, \tag{3.22}$$

$$\sum_{j=H,L} \omega_j u'\left(c_{m,t+1}^{A,j}\right)\frac{b_t}{p_{t+1}^j} = u'\left(c_{n,t+1}^{A}\right), \tag{3.23}$$

Solving the equations (3.22) and (3.23) gives us the optimal value of q_t^A and α, given the price of ABS, b_t.

3.3.3 The Demand Side of securitization: Island B

Total investment in liquid assets of the island B is

$$N_t^B q_t^B = \frac{M_t}{2p_t^B}$$

Among fiat money obtained, the island B trades β fraction of the money for ABS issued in the A island. The amount of money transferred to the island A is

$$\frac{\beta M_t}{2} = \beta N_t^B p_t^B q_t^B \tag{3.24}$$

Then, aggregate consumption of moves born in the island B is

$$\begin{aligned} N_t^B \lambda c_{m,t+1}^{B,j} &= \frac{(1-\beta)M_t}{2p_{t+1}^j} + N_t^A \alpha k_t^A R \\ &= \frac{p_t^B(1-\beta)N_t^B q_t^B}{p_{t+1}^j} + \frac{\beta N_t^B q_t^B p_t^B}{b_t}, \end{aligned}$$

where $B, j = \{L, H\}$. The first term on the right hand side of the first line denotes the consumption due to real money holdings, excluding β rate used to purchase securities. The second term is returns generated by the underlying assets (capital in the island A). The second line is obtained using (3.24) and (3.14). The aggregate consumption of nonmovers is

$$N_t^B(1-\lambda)c_{n,t+1}^B = N_t^B k_t^B R$$

Therefore, the consumption per capita by type are given as

$$c_{m,t+1}^{B,j} = \frac{1}{\lambda}\left[\frac{(1-\beta)q_t^B p_t^B}{p_{t+1}^j} + \frac{\beta q_t^B p_t^B}{b_t}\right] \tag{3.25}$$

$$c_{n,t+1}^B = \frac{k_t^B R}{1-\lambda} \tag{3.26}$$

The bank's problem is to choose q_t^B and β to maximize its depositors' expected utility

$$\max_{c_{m,t+1},c_{n,t+1}} E_t[U(c_{m,t+1},c_{n,t+1})] = \sum_{j=H,L} \omega_j \left[\lambda u(c^{B,j}_{m,t+1}) + (1-\lambda)u(c^{B}_{n,t+1})\right] \tag{3.27}$$

subject to subject to (3.25), (3.26), the budget constraint,

$$q_t^B + k_t^B = 1,$$

the feasibility conditions,

$$c^{B,j}_{m,t+1} \leq \frac{1}{\lambda}\left[\frac{(1-\beta)q_t^B p_t^B}{p^j_{t+1}} + \frac{\beta q_t^B p_t^B}{b_t}\right]$$

$$c^{B,j}_{m,t+1} + c^{B}_{n,t+1} \leq \frac{1}{\lambda}\left[\frac{(1-\beta)q_t^B p_t^B}{p^j_{t+1}} + \frac{\beta q_t^B p_t^B}{b_t}\right] + \frac{k_t^B R}{1-\lambda},$$

and the incentive constraint

$$c^{B}_{n,t+1} \geq c^{B,j}_{m,t+1}$$

The Lagrangian function of this optimization model is

$$\begin{aligned}
\mathcal{L} =& \lambda u\left(c^B_{m,t+1}\right) + (1-\lambda)u\left(c^B_{n,t+1}\right) \\
&+ \mu_1\left[\frac{p_t^B(1-\beta)q_t^B}{p^A_{t+1}} + \frac{\beta q_t^B p_t^B}{b_t} - \lambda c^B_{m,t+1}\right] \\
&+ \mu_2\left[\frac{p_t^B(1-\beta)q_t^B}{p^A_{t+1}} + \frac{\beta q_t^B p_t^B}{b_t} + k_t^B R - \lambda c^B_{m,t+1} - (1-\lambda)c^B_{n,t+1}\right]
\end{aligned}$$

First order conditions with respect to consumption are given as

$$\begin{aligned}
\left(c^{B,j}_{m,t+1}\right) \quad & u'\left(c^B_{m,t+1}\right) = \mu_1 + \mu_2 \\
\left(c^{B}_{n,t+1}\right) \quad & u'\left(c^B_{n,t+1}\right) = \mu_2
\end{aligned}$$

First order conditions with respect to q_t^B and β yields the following:

$$(\mu_1 + \mu_2)\left(\frac{(1-\beta)p_t^B}{p_{t+1}^A} + \frac{\beta p_t^B}{b_t}\right) = \mu_2 R$$
$$(\mu_1 + \mu_2)\frac{p_t^B q_t^B}{p_{t+1}^j} = (\mu_1 + \mu_2)\frac{p_t^B q_t^B}{b_t}$$

Combining all these first order conditions, we obtain

$$\sum_{j=H,L} \omega_j u'\left(c_{m,t+1}^{B,j}\right)\left(\frac{(1-\beta)p_t^B}{p_{t+1}^j} + \frac{\beta p_t^B}{b_t}\right) = u'\left(c_{n,t+1}^B\right) R \tag{3.28}$$

$$\sum_{j=H,L} \omega_j \left[u'\left(c_{m,t+1}^{B,j}\right)\frac{1}{p_{t+1}^j} - u'\left(c_{m,t+1}^{B,j}\right)\frac{1}{b_t}\right] = 0 \tag{3.29}$$

Solving the equations (3.28) and (3.29) gives us the optimal value of q_t^B and β given the price of ABS, b_t.

3.3.4 Numerical Example

In this example, we use the same parameter values as in Section 3.4.3 to see how securitization affects various equilibrium values. The results are illustrated in Table 3.1 and 3.2. The column I deals with the case of relatively large supply of securities ($N_t^A = N_t^H$), and the column II shows the equilibrium values when the supply is small, and demand is high ($N_t^A = N_t^L$). As in the numerical examples in Section 3.4.3, the liquid asset holdings is increased as agents tend to be more risk-averse.

When $N_t^A = N_t^H$, the supply of ABS is large (and the demand for it is small) enough so that the price of the securities is lower than when $N_t^A = N_t^L$ for any given σ. The willingness to narrow the gap in consumption between movers and nonmovers is increased as σ is bigger, and thus the banks on the island A issues more ABS to finance the mover's consumption. This is reflected in the increase in α as σ increases in column I. Sufficient supply of securities, together with relatively lower price, the banks on the island B trade all their fiat money for ABS. This results in $\beta = 1$ in the column I. When $\beta = 1$, movers on the island B do not hold money but hold only securities. Therefore, they are not affected by the price level of the island j at date $t+1$ and thus face no uncertainty on their consumption. This result is illustrated as $c^{B,H}_{m,t+1} = c^{B,L}_{m,t+1}$ when $\beta = 1$.

Table 3.1: Equilibria for Variables

		I		II	
σ		$A = H$	$B = L$	$A = L$	$B = H$
0.5	q^i_t	0.1793	0.4458	0.2274	0.2851
	p^i_t	4.1827	3.3647	6.5959	2.6306
	b_t	2.0914		3.2979	
	α	0.2185		0.2944	
	β	1.0000		1.0000	
1	q^i_t	0.2500	0.5000	0.3333	0.5000
	p^i_t	3.0000	3.0000	4.5000	1.5000
	b_t	1.5000		2.2500	
	α	0.3333		0.2500	
	β	1.0000		0.5000	
1.5	q^i_t	0.2818	0.4880	0.3990	0.5985
	p^i_t	2.6618	3.0737	3.7595	1.2532
	b_t	1.3309		1.8797	
	α	0.3923		0.1983	
	β	1.0000		0.2987	

Let us compare the Table 3.1 with the column I in Table 3.4 and Table 3.6b. The A island's real demand for fiat money is decreased substantially with any σ and irrespective of whether $A = H$ or $A = L$. The aggregate need for the real money demand of the economy, $q^A_t + q^B_t$, is reduced significantly as well, which implies the economy uses its resources more productive way to increase the total output across the two islands. Table 3.1 also shows that, given σ value, the demand for real money, q^A_t, is lower when N^A_t is large than $N^A_t = N^L_t$, which implies that the greater the scope of securitization of the bank, the smaller the amount of liquid assets it requires.

Table 3.2 illustrates the consumption of each type of agents and expected utility. The overall welfare of all islands increases significantly due to more capital holdings. However, note that the impact of securitization on consumption and welfare is also affected by the population structure of each island. The price of securities, b_t, is higher when the population of the island issuing securities is small, i.e., when the

Table 3.2: Consumption and Utility with securitization

		σ	(I) $A = H, B = L$	(II) $A = L, B = H$
$c^{i,j}_{m,t+1} =$	$\begin{bmatrix} c^{A,H}_{m,t+1} & c^{B,H}_{m,t+1} \\ c^{A,L}_{m,t+1} & c^{B,L}_{m,t+1} \end{bmatrix}$		$\begin{bmatrix} 0.7172 & 1.4345 \\ 0.8916 & 1.4345 \end{bmatrix}$	$\begin{bmatrix} 2.2809 & 0.4548 \\ 0.9097 & 0.4548 \end{bmatrix}$
$c^{i}_{n,t+1} =$	$\begin{bmatrix} c^{A}_{n,t+1} & c^{B}_{n,t+1} \end{bmatrix}$	0.5	$\begin{bmatrix} 2.0693 & 2.4653 \end{bmatrix}$	$\begin{bmatrix} 2.1807 & 2.8596 \end{bmatrix}$
$EU =$	$\begin{bmatrix} C^{A}_{t+1} & C^{B}_{t+1} \\ sum & \end{bmatrix}$		$\begin{bmatrix} 0.4973 & 0.6866 \\ 0.5604 & \end{bmatrix}$	$\begin{bmatrix} 0.7087 & 0.3654 \\ 0.4799 & \end{bmatrix}$
$c^{i,j}_{m,t+1} =$	$\begin{bmatrix} c^{A,H}_{m,t+1} & c^{B,H}_{m,t+1} \\ c^{A,L}_{m,t+1} & c^{B,L}_{m,t+1} \end{bmatrix}$		$\begin{bmatrix} 1.0000 & 2.0000 \\ 1.0000 & 2.0000 \end{bmatrix}$	$\begin{bmatrix} 3.0000 & 0.8333 \\ 1.0000 & 0.5000 \end{bmatrix}$
$c^{i}_{n,t+1} =$	$\begin{bmatrix} c^{A}_{n,t+1} & c^{B}_{n,t+1} \end{bmatrix}$	1.0	$\begin{bmatrix} 2.0000 & 2.0000 \end{bmatrix}$	$\begin{bmatrix} 2.0000 & 2.0000 \end{bmatrix}$
$EU =$	$\begin{bmatrix} C^{A}_{t+1} & C^{B}_{t+1} \\ sum & \end{bmatrix}$		$\begin{bmatrix} 0.3466 & 0.6931 \\ 0.4621 & \end{bmatrix}$	$\begin{bmatrix} 0.6212 & 0.1277 \\ 0.2922 & \end{bmatrix}$
$c^{i,j}_{m,t+1} =$	$\begin{bmatrix} c^{A,H}_{m,t+1} & c^{B,H}_{m,t+1} \\ c^{A,L}_{m,t+1} & c^{B,L}_{m,t+1} \end{bmatrix}$		$\begin{bmatrix} 1.1270 & 2.2541 \\ 0.9760 & 2.2541 \end{bmatrix}$	$\begin{bmatrix} 3.1091 & 1.0778 \\ 1.0364 & 0.5182 \end{bmatrix}$
$c^{i}_{n,t+1} =$	$\begin{bmatrix} c^{A}_{n,t+1} & c^{B}_{n,t+1} \end{bmatrix}$	1.5	$\begin{bmatrix} 1.7459 & 2.0479 \end{bmatrix}$	$\begin{bmatrix} 1.9273 & 1.6060 \end{bmatrix}$
$EU =$	$\begin{bmatrix} C^{A}_{t+1} & C^{B}_{t+1} \\ sum & \end{bmatrix}$		$\begin{bmatrix} 0.2661 & 0.6352 \\ 0.3891 & \end{bmatrix}$	$\begin{bmatrix} 0.5050 & 0.0347 \\ 0.1915 & \end{bmatrix}$

supply of securities is relatively small compared to when it is large. It is natural that the value of relatively few existing assets should increases. Due to the high-security prices, both the consumption and welfare of the movers born on the island B fall, compared to when the securities prices are low (column II).

3.4 Effect of Monetary Policy with Incomplete Information

The financial intermediary now chooses q^i_t not knowing z^k_t. The realized value of z^k_t is kept unknown from the young until the date t ends, but the bank knows the distributions of z^k_t, z^l_{t+1} and N^j_{t+1}. Prices are the only thing the bank can directly observe.

The bank needs to infer the population of the young who are born on the island i by observing the current price level, p^{ik}_t. Rewriting the equation.(3.2), using the fact $q^{ik,h}_t = \overline{q}^{ik}_t$, then the price level of the island i and the rate of return on fiat money are given as

$$p_t^{ik} = \frac{0.5 z_t^k M_{t-1}}{N_t^i \overline{q}_t^{ik}(p_t^{ik})} \tag{3.30}$$

$$\frac{p_t^{ik}}{p_{t+1}^{jl}} = \frac{N_{t+1}^j}{N_t^i} \frac{\overline{q}_{t+1}^{jl}}{\overline{q}_t^{ik}} \frac{1}{z_{t+1}^{jl}} \tag{3.31}$$

Since z_t^k and N_t^i take two different values respectively, p_t^{ik} may have four different forms. By Assumption 6, however, p_t^{ik} can take only three different values depending on the distributions of z_t^k and N_t^i. Table 3.3 shows the distribution of the price levels which depend on the realizations of z_t^k and N_{t+1}^j.

Table 3.3: Distribution of Price

	Population of the Young	
Money Growth Rate	N_t^L	N_t^H
z_t^L	$p_t^{LL} = \frac{0.5 z_t^L M_{t-1}}{N_t^L \overline{q}_t^{LL}(p_t^{LL})}$	$p_t^{HL} = \frac{0.5 z_t^L M_{t-1}}{N_t^H \overline{q}_t^{HL}(p_t^{HL})}$
z_t^H	$p_t^{LH} = \frac{0.5 z_t^H M_{t-1}}{N_t^L \overline{q}_t^{LH}(p_t^{LH})}$	$p_t^{HH} = \frac{0.5 z_t^H M_{t-1}}{N_t^H \overline{q}_t^{HH}(p_t^{HH})}$

As shown in (3.30), the knowledge of the price level of the island i, p_t^{ik}, implies knowledge of z_t^k/N_t^i. When the bank observes the price p_t^{HL} or p_t^{LH}, it is able to infer the correct states of nature it faces. In other words, when the bank observes the price p_t^{ik} when $i \neq k$, the observed p_t^{ik} provides the precise values of z_t^k and N_t^i. More specifically, the observation of p_t^{HL} allows the bank to understand the states of nature as $N_t^i = N_t^H$ and $z_k = z_L$. Likewise, if the price level p_t^{LH} is observed, the bank has access to the exact information that $N_t^i = N_t^L$ and $z_k = z_H$.

Note that, however, p_t^{HH} is equal to p_t^{LL}, which implies that the observed price p_t^{HH} is indistinguishable from p_t^{LL}. This result is due to Assumption 6 stating that $\frac{N^H}{N^L} = \frac{z_H}{z_L}$. This analysis shows that banks face a signal extraction problem. The bank cannot elicit the correct signal of the random variable by observing the price because there are two unknown variables (N_t^i and z_k) while the bank has access to only one information, p_t^{ik}. The revealed price p_t^{HH} means that the demand for money is high due to large young population, but the supply of it is also high because of high z_t^k. And when the island watches the price p_t^{LL}, both the demand for and supply of fiat money are low. If the revealed price level is $p_t^{LL} = p_t^{HH} \equiv \widehat{p}_t$, then the true state of nature is either $N_t^i = N_t^L$ and $z_k = z_L$, or $N_t^i = N_t^H$ and $z_k = z_H$ with probability $\frac{\omega_L \varepsilon_L}{\omega_L \varepsilon_L + \omega_H \varepsilon_H}$ and $\frac{\omega_H \varepsilon_H}{\omega_L \varepsilon_L + \omega_H \varepsilon_H}$ respectively. In other words,

$$\Pr[p_t^{LL}|\widehat{p}_t] = \frac{\omega_L \varepsilon_L}{\omega_L \varepsilon_L + \omega_H \varepsilon_H}$$
$$\Pr[p_t^{HH}|\widehat{p}_t] = \frac{\omega_H \varepsilon_H}{\omega_L \varepsilon_L + \omega_H \varepsilon_H}$$

Unlike Section 3.2.5, the price level does not reveal the true state of nature if $i \neq k$. Now divide the consumption conditional on the distribution of inflation into two different cases following Wallace (1980): (1) the actual state of nature revealed ($i \neq k$), and (2) the true states of nature are not revealed ($i = k$).

The Case of $i \neq k$

Let the consumption of a mover (a nonmover) be $c_{m,t+1}^{ik,jl}$ ($c_{n,t+1}^{ik}$), who is born on the island i, deposits her endowment with a bank h, and face the money supply shock z_t^k. $c_{m,t+1}^{ik,jl}$ and $c_{n,t+1}^{ik}$ take the following form after substituting $\overline{q}_t^{ik}$ for $q_t^{ik,h}$.

$$c_{m,t+1}^{ik,jl} = z_{t+1}^l \frac{\overline{q}_t^{ik}}{\lambda} \frac{p_t^{ik}}{p_{t+1}^{jl}} \tag{3.32}$$

$$c_{n,t+1}^{ik} = \frac{(1-\overline{q}_t^{ik})R}{1-\lambda} \tag{3.33}$$

Since the true values of N_t^i and z_t^k are revealed from observing the price p_t^{ik} when $i \neq k$, the consumption of movers has four different distributions for each i depending on the state j and l. The consumption of movers born in H island (resp. L island) is expressed as $c_{m,t+1}^{HL,jl}$ (resp. $c_{m,t+1}^{LH,jl}$). When a bank's objective is to maximize its depositors' expected utility, the problem posed will be that of maximizing the following von Neumann-Morgenstern utility function

$$\max_{c_{m,t+1}, c_{n,t+1}} E_t[U(c_{m,t+1}, c_{n,t+1})] = \sum_j \sum_l \omega_j \varepsilon_l \left[\lambda u(c_{m,t+1}^{ik,jl}) + (1-\lambda) u(c_{n,t+1}^{ik})\right], \tag{3.34}$$

subject to (3.8), (3.9), (3.10),[19] (3.11), (3.32), and (3.33).

The Lagrangian function of this optimization model is

$$\begin{aligned}\mathcal{L} =&\lambda u\left(c_{m,t+1}^{ik,jl}\right) + (1-\lambda)u\left(c_{n,t+1}^{ik}\right)\\ &+\mu_1\left[z_{t+1}^{l}\frac{\overline{q}_t^{ik}}{\lambda}\frac{p_t^{ik}}{p_{t+1}^{jl}} - c_{m,t+1}^{ik,jl}\right]\\ &+\mu_2\left[z_{t+1}^{l}\frac{\overline{q}_t^{ik}}{\lambda}\frac{p_t^{ik}}{p_{t+1}^{jl}} + \frac{(1-\overline{q}_t^{ik})R}{1-\lambda} - c_{m,t+1}^{ik,jl} - c_{n,t+1}^{ik}\right]\end{aligned}$$

First order conditions yields the following.[20]

$$\sum_j\sum_l \omega_j\varepsilon_l u'\left(c_{m,t+1}^{ik,jl}\right)\frac{z_{t+1}^{l}p_t^{ik}}{p_{t+1}^{jl}} = u'\left(c_{n,t+1}^{i}\right)R, \tag{3.35}$$

Note that the preceding equation has the same form as (3.12) in complete information case. Solving the preceding equation gives us the optimal values of $\overline{q}_t^{HL}$ and $\overline{q}_t^{LH}$.

The Case of $i = k$

When $i = k$, we know that the information on p_t^{ik} does not reveal the precise values of N_t^i and z_t^k because $p_t^{ik} = \widehat{p}_t$ may imply either $(i,k) = (L,L)$ or $(i,k) = (H,H)$. The consumption of a mover, when $i = k$, is given as

$$c_{m,t+1}^{ii,jl} = z_{t+1}^{l}\frac{\overline{q}_t^{ii}}{\lambda}\frac{p_t^{ii}}{p_{t+1}^{jl}},\ \text{where } i = \{L,H\} \tag{3.36}$$

The preceding equation has 4 different values depending on the realizations of j and l for each $i = \{L,H\}$. The state of i can be either H or L and the bank cannot distinguish them since the true state of i is not revealed. Therefore, the mover's consumption can take eight different values depending on the states of i, j and l, and the probability of each event happening is $\omega_i\omega_j\varepsilon_k$, where $i, j, k = \{L,H\}$. We

[19] The feasibility conditions are also rewritten using $q_t^{ik,h} = \overline{q}_t^{ik}$.
[20] Incentive constraint is satisfied as well.

may express the expected consumption of movers in the form

$$E[c^{ii,jl}_{m,t+1}] = \sum_i \sum_j \sum_l \omega_i \omega_j \varepsilon_l z^l_{t+1} \frac{\overline{q}^{ii}_t}{\lambda} \frac{p^{ii}_t}{p^{jl}_{t+1}} \tag{3.37}$$

And the nonmover's consumption as

$$c^{ii}_{n,t+1} = \frac{(1-\overline{q}^{ii}_t)R}{1-\lambda} \tag{3.38}$$

The h bank maximizes the following objective function

$$\max_{c_{m,t+1},c_{n,t+1}} E_t[U(c_{m,t+1}, c_{n,t+1})] = \sum_i \sum_j \sum_l \omega_i \omega_j \varepsilon_l \left[\lambda u(c^{ii,jl}_{m,t+1}) + (1-\lambda)u(c^{ii}_{n,t+1})\right] \tag{3.39}$$

subject to (3.8), (3.9), (3.10), (3.11), (3.36), and (3.38). The Lagrangian function of this optimization model is

$$\begin{aligned}
\mathcal{L} =& \lambda u\left(c^{ii,jl}_{m,t+1}\right) + (1-\lambda)u\left(c^{ii}_{n,t+1}\right) \\
&+ \mu_1 \left[z^l_{t+1} \frac{\overline{q}^{ii}_t}{\lambda} \frac{p^{ii}_t}{p^{jl}_{t+1}} - c^{ii,jl}_{m,t+1}\right] \\
&+ \mu_2 \left[z^l_{t+1} \frac{\overline{q}^{ii}_t}{\lambda} \frac{p^{ii}_t}{p^{jl}_{t+1}} + \frac{(1-\overline{q}^{ii}_t)R}{1-\lambda} - c^{ii,jl}_{m,t+1} - c^{i}_{n,t+1}\right]
\end{aligned}$$

First order conditions yields the following.[21]

$$\sum_i \sum_j \sum_l \omega_i \omega_j \varepsilon_l u'\left(c^{ii,jl}_{m,t+1}\right) \frac{z^l_{t+1} p^{ii}_t}{p^{jl}_{t+1}} = \sum_i \omega_i u'\left(c^{ii}_{n,t+1}\right) R \tag{3.40}$$

Solving the preceding equation gives us the optimal level of investment in fiat money $\overline{q}^{ii}_t = \overline{q}^{HH}_t = \overline{q}^{LL}_t$.

[21] Incentive constraint is satisfied as well.

3.4.1 The Effect of Monetary Policy on Portfolio Decision

The main question in this section is to study how the bank's portfolio decision is affected by the current price level. Let G define an implicit function

$$G(p_t^{ik}, \overline{q}_t^{ik}) = u'\left(c_{m,t+1}^{ik,jl}\right)\frac{z_{t+1}^{l}p_t^{ik}}{p_{t+1}^{jl}} - u'\left(c_{n,t+1}^{ik,jl}\right)R = 0, \tag{3.41}$$

The partial differentiation of G with respect to p_t^{ik} yields

$$\begin{aligned}\frac{\partial G}{\partial p_t^{ik}} =& u''(c_{m,t+1}^{ik,jl})\frac{c_{m,t+1}^{ik,jl}z_{t+1}^{l}}{p_{t+1}^{jl}} + u'\left(c_{m,t+1}^{ik,jl}\right)\frac{z_{t+1}^{l}}{p_{t+1}^{jl}}\\ =& u'\left(c_{m,t+1}^{ik,jl}\right)(1-\sigma)\frac{z_{t+1}^{l}}{p_{t+1}^{jl}}\end{aligned}$$

The second line of the preceding equation is obtained using $\sigma = -\frac{cu''(c)}{u'(c)}$. And the partial differentiation of G with respect to $\overline{q}_t^{ik}$

$$\frac{\partial G}{\partial \overline{q}_t^{ik}} = u''(c_{m,t+1}^{ik,jl})\frac{\left(z_{t+1}^{l}\right)^2}{\lambda}\left(\frac{p_t^{ik}}{p_{t+1}^{j}}\right)^2 + \frac{u''(c_{n,t+1}^{ik})R}{1-\lambda}$$

The sign of the preceding equation is negative because $u''(\cdot) < 0$. Using the implicit function theorem, we get the following

$$\frac{\partial \overline{q}_t^{ik}}{\partial p_t^{ik}} = -\frac{\partial G/\partial p_t^{ik}}{\partial G/\partial \overline{q}_t^{ik}} = \frac{u'\left(c_{m,t+1}^{ik,jl}\right)(\sigma-1)}{\partial G/\partial \overline{q}_t^{ik}}\frac{z_{t+1}^{l}}{p_{t+1}^{jl}}\begin{cases} > 0, & \text{when } \sigma < 1\\ = 0, & \text{when } \sigma = 1\\ < 0, & \text{when } \sigma > 1\end{cases} \tag{3.42}$$

The current price level, given the future price of goods, p_{t+1}^{jl}, and the rate of return on capital, R, has a different effect on the demand for fiat money, which is conditional on the degree of relative risk aversion.[22] When agents are less risk averse ($\sigma < 1$), a higher local price, p_t^{ik}, leads to an increase in investment in fiat money. The substitution effect dominates income effect because agents are willing to substitute more fiat money for capital. Therefore, the amount of $\overline{q}_t^{ik}$ rises as p_t^{ik}

[22] Diamond (1965) used the similar approach like this to analyse the effect of changes in real wage on the interest rate.

increases. On the other hand, if agents are more risk averse ($\sigma > 1$), the income effect dominates the substitution effect; a higher price level will reduce the willingness to hold currency. $\overline{q}_t^{ik}$ is reduced as p_t^{ik} decreases. When $\sigma = 1$ (the log case), the real money demand does not depend on the current price level.

This result has an important implication for the central bank who may wish to affect the bank's portfolio decision by changing the current price level. Suppose that a central bank has a capital investment target that is higher than the natural level of capital investment at which the economy would return to normally. Then, (3.42) tells us that the monetary authorities' effort to enhance capital investment is effective only when consumers have higher relative risk aversion parameter, that is, only when $\sigma > 1$.[23]

3.4.2 The Effect of Inflation on Consumption

Applying the period utility function, (3.1), to the implicit function, (3.41), we have the following

$$\left(c_{m,t+1}^{ik,jl}\right)^{-\sigma} \frac{z_{t+1}^{l} p_t^{ik}}{p_{t+1}^{jl}} = \left(c_{n,t+1}^{ik}\right)^{-\sigma} R$$

Raising both sides to the power $-1/\sigma$ and rearranging terms yield

$$\begin{aligned} c_{m,t+1}^{ik,jl} \left(z_{t+1}^{l}\right)^{-1/\sigma} \left(\frac{p_t^{ik}}{p_{t+1}^{jl}}\right)^{-1/\sigma} &= c_{n,t+1}^{ik} R^{-1/\sigma} \\ \Leftrightarrow c_{m,t+1}^{ik,jl} \left(z_{t+1}^{l}\right)^{-1/\sigma} \left(\frac{R}{p_t^{ik}/p_{t+1}^{jl}}\right)^{1/\sigma} &= c_{n,t+1}^{ik} \end{aligned} \tag{3.43}$$

Also, we may express the resource constraint the bank facing, (3.8), by taking relevant discount factor in the form

$$\frac{\lambda c_{m,t+1}^{ik,jl}}{p_t^{ik}/p_{t+1}^{jl}} + \frac{(1-\lambda) c_{n,t+1}^{ik}}{R} = 1 + (z_{t+1}^{l} - 1)\overline{q}_t^{ik}$$

[23] In Section 3.5 I show that even in the case of $\sigma > 1$, however, the central bank's attempt is not effective when agents have rational expectations.

The second term on the right-hand side of the preceding equation denotes the present value of subsidies given to the old which is not related to the bank's budget constraint, but is necessary to calculate the consumption of a mover.[24] Therefore, the right hand side represents the present value of consumption of a mover and nonmover combined. Rearranging the preceding equation and using the fact $\lambda = 1/2$, we have

$$c_{m,t+1}^{ik,jl} = -c_{n,t+1}^{ik}\left(\frac{R}{p_t^{ik}/p_{t+1}^{jl}}\right)^{-1} + S \tag{3.44}$$

where $S = \left(1 + (z_{t+1}^{l} - 1)\overline{q}_t^{ik}\right)\frac{2p_t^{ik}}{p_{t+1}^{jl}}$ denotes the future value of the endowment goods along with the government subsidies. Substituting (3.43) into the modified resource constraint, (3.44), we obtain the consumption function of movers at date $t+1$

$$\begin{aligned}
&c_{m,t+1}^{ik,jl} = -c_{n,t+1}^{ik}\left(\frac{R}{p_t^{ik}/p_{t+1}^{jl}}\right)^{-1} + S \\
\Leftrightarrow &c_{m,t+1}^{ik,jl} = -c_{m,t+1}^{ik,jl}\left(z_{t+1}^{l}\right)^{-1/\sigma}\left(\frac{R}{p_t^{ik}/p_{t+1}^{jl}}\right)^{1/\sigma}\left(\frac{R}{p_t^{ik}/p_{t+1}^{jl}}\right)^{-1} + S \\
\Leftrightarrow &c_{m,t+1}^{ik,jl}\left[1 + \left(z_{t+1}^{l}\right)^{-1/\sigma}\left(\frac{R}{p_t^{ik}/p_{t+1}^{jl}}\right)^{1/\sigma-1}\right] = S \\
\Leftrightarrow &c_{m,t+1}^{ik,jl} = \frac{S}{1 + \left(z_{t+1}^{l}\right)^{-1/\sigma}\left(\frac{R}{p_t^{ik}/p_{t+1}^{jl}}\right)^{1/\sigma-1}}
\end{aligned} \tag{3.45}$$

$\frac{R}{p_t^{ik}/p_{t+1}^{jl}}$ denotes the relative ratio of rate of returns between illiquid and liquid assets. By construction, $\frac{R}{p_t^{ik}/p_{t+1}^{jl}}$ is always greater than 1. The preceding consumption function shows how a change in the current price level affects the consumption of movers.

When $\sigma > 1$, the income effect from the increase in p_t^{ik} dominates the substitution effect.[25] The bank is willing to decrease the investment in liquid assets and thus

[24]The future value of subsidies is $(z_{t+1}^{l} - 1)\overline{q}_t^{ik}p_t^{ik}/p_{t+1}^{jl}$. For a discount factor we use the rate of return on fiat money, p_t^{ik}/p_{t+1}^{jl}.

[25] $\left(\frac{R}{p_t^{ik}/p_{t+1}^{jl}}\right)^{1/\sigma-1}$ is increasing in p_t^{ik}.

the consumption of movers declines. When $\sigma = 1$, the bank's decision on the portfolio choice does not depend on the current price level. If the agent's coefficient of relative risk aversion is less than 1, the substitution effect dominates the income effect[26]: the bank wishes to hold more liquid assets, fiat money, to substitute mover's consumption for nonmover's consumption.

The effect of changes in the current price level on nonmover's consumption can be simply obtained by differentiating nonmover's consumption with respect to p_t^{ik}, which yields the following

$$
\begin{aligned}
\frac{\partial c_{n,t+1}^{ik}}{\partial p_t^{ik}} &= -\frac{R}{1-\lambda}\frac{\partial \overline{q}_t^{ik}}{\partial p_t^{ik}} \\
&= -\frac{R}{1-\lambda}\frac{u'\left(c_{m,t+1}^{ik,jl}\right)(\sigma-1)}{\partial G/\partial \overline{q}_t^{ik}}\frac{z_{t+1}^{l}}{p_{t+1}^{jl}}
\begin{cases}
< 0, & \text{when } \sigma < 1 \\
= 0, & \text{when } \sigma = 1 \\
> 0, & \text{when } \sigma > 1
\end{cases}
\end{aligned}
$$

The second line of the preceding equation is obtained from (3.42). When $\sigma < 1$, the rise in p_t^{ik} increases the rate of return on money, thus increases $\overline{q}_t^{ik}$ and decreases k_t^{ik} by the same amount. On the other hand, if $\sigma > 1$, then the higher current price level reduces the liquid asset holding and increases the capital investment. This increases the consumption of nonmovers.

3.4.3 Numerical Example

Let's look at numerical examples analysed in this section using specific parameter values. Let $\theta = 4/3$, then the population of the young takes the following values.

$$
N_t^i = \begin{cases}
N_t^L = 1-\theta/2 = 1/3, & \text{with probability } \omega_L = 1/2 \\
N_t^H = \theta/2 = 2/3, & \text{with probability } \omega_H = 1/2,
\end{cases}
$$

[26] Now, $\left(\frac{R}{p_t^{ik}/p_{t+1}^{jl}}\right)^{1/\sigma-1}$ is an decreasing function of p_t^{ik}.

The gross rate of money supply expansion is

$$z_t^k = \begin{cases} z_t^L = 1, & \text{with probability } \varepsilon_L = 1/2 \\ z_t^H = 2, & \text{with probability } \varepsilon_H = 1/2, \end{cases}$$

which satisfies Assumption 6. The rate of return on capital, R, is assumed to be 2. The predetermined money supply M_{t-1} is normalized to 1. I analyse three different values of $\sigma = 0.5, 1, 1.5$ respectively to examine the effect of different σ on equilibria.

Complete Information & No Uncertainty on Monetary Shocks

First of all, let us look at the case where the money supply does not change at all and remains constant, which implies $z_t = z_{t+1} = 1$. Secondly, suppose the case of a once-and-for-all higher level of money supply causing the high current price level, i.e., $z_t = 2$ and $z_{t+1} = 1$. Those results are shown in column I and II of the Table 3.4 and 3.5 with three different values of σ. The only uncertainty consumers face is the population of the young at date $t+1$, N_{t+1}^j.

The price level of the island with a large young population is lower than that of the island with a small population, the result of which is natural from (3.2). The high demand for money causes a higher price level than otherwise. Given the population of the young, the money demand is greater as the agents become more risk-averse. When agents are less risk averse ($\sigma < 1$), the higher local price, p_t^{ik}, leads to more investment in fiat money. In the column I of Table 3.4, the local price of the island L, $p_t^L = 3.9096$, is higher than $p_t^H = 2.5775$. This leads to higher real demand for money in island L, $q_t^L = 0.3837$, than $q_t^H = 0.2910$. This is because, as explained in Section 3.4.1, the substitution effect dominates income effect, and thus agents are willing to substitute more fiat money for capital. Therefore, the amount of $\overline{q}_t^i$ rises as p_t^{ik} increases.

On the other hand, if agents are more risk averse ($\sigma > 1$), the higher price level will reduce the willingness to hold currency. This is illustrated as a lower local price $p_t^{H1} = 1.2699 < p_t^{L1} = 2.8446$ and leads to a higher $q_t^H = 0.5939 > q_t^L = 0.5273$.

Income effect dominates the substitution effect and thus q_t^i is reduced as p_t^i decreases. When $\sigma = 1$ (the log case),[27] the real money demand doesn't depend on the current price level. Therefore, even though $p_t^H = 1.5 < p_t^L = 3.0$, the real demand for money on both islands are the same at $q_t^H = q_t^L = 0.5$. These results are consistent with the conclusions analysed in Section 3.4.1, and (3.42).

Table 3.5 illustrates the consumption of each type of agent and expected utility. When $\sigma > 1$, i.e., $cu'(c)$ is decreasing in c, the optimal deposit contract gives movers more consumption at date $t+1$ than the market solution. The bank invests more in the liquid asset at the expense of capital investment (i.e., q_t^i increases and k_t^i decreases), and therefore movers enjoy higher consumption than in the cases of $\sigma = 1$ or $\sigma < 1$. Cross-subsidy between movers and nonmovers occur.

Note that the values in the column II in Table 3.4 and 3.5 are exactly the same as the column I. This is because the once-and-for-all increase in money stocks does not affect the rate of return on fiat money[28] and thus returns on illiquid assets. Therefore, money is neutral in this specific example.

Table 3.4: Equilibria for Numerical Example

σ		I $z_t = z_{t+1} = 1$ H	I $z_t = z_{t+1} = 1$ L	II $z_t = 2, z_{t+1} = 1$ H	II $z_t = 2, z_{t+1} = 1$ L
0.5	q_t^i	0.2910	0.3837	0.2910	0.3837
	p_t^i	2.5775	3.9096	5.1550	7.8193
	p_{t+1}^i	2.5775	3.9096	5.1550	7.8193
1	q_t^i	0.5000	0.5000	0.5000	0.5000
	p_t^i	1.5000	3.0000	3.0000	6.0000
	p_{t+1}^i	1.5000	3.0000	3.0000	6.0000
1.5	q_t^i	0.5939	0.5273	0.5939	0.5273
	p_t^i	1.2629	2.8446	2.5258	5.6892
	p_{t+1}^i	1.2629	2.8446	2.5258	5.6892

[27] This result is equal to the market solution as studied in detail in Chapter 1.
[28] The total money supply at date t, M_t is equal to M_{t+1}.

Table 3.5: Consumption and Utility under Complete Information

		σ	I $z_t = z_{t+1} = 1$	II $z_t = 2, z_{t+1} = 1$
$c^{ij}_{m,t+1} =$	$\begin{bmatrix} c^{HH}_{m,t+1} & c^{HL}_{m,t+1} \\ c^{LL}_{m,t+1} & c^{LH}_{m,t+1} \end{bmatrix}$		$\begin{bmatrix} 0.5820 & 0.3837 \\ 0.7673 & 1.1639 \end{bmatrix}$	$\begin{bmatrix} 0.5820 & 0.3837 \\ 0.7673 & 1.1639 \end{bmatrix}$
$c^{i}_{n,t+1} =$	$\begin{bmatrix} c^{H}_{n,t+1} & c^{L}_{n,t+1} \end{bmatrix}$	0.5	$\begin{bmatrix} 2.8361 & 2.4653 \end{bmatrix}$	$\begin{bmatrix} 2.8361 & 2.4653 \end{bmatrix}$
$EU =$	$\begin{bmatrix} EU(c^H) & EU(c^L) \\ sum & \end{bmatrix}$		$\begin{bmatrix} 0.3752 & 0.6615 \\ 0.4706 & \end{bmatrix}$	$\begin{bmatrix} 0.3752 & 0.6615 \\ 0.4706 & \end{bmatrix}$
$c^{ij}_{m,t+1} =$	$\begin{bmatrix} c^{HH}_{m,t+1} & c^{LL}_{m,t+1} \\ c^{HL}_{m,t+1} & c^{LH}_{m,t+1} \end{bmatrix}$		$\begin{bmatrix} 1.0000 & 0.5000 \\ 1.0000 & 2.0000 \end{bmatrix}$	$\begin{bmatrix} 1.0000 & 0.5000 \\ 1.0000 & 2.0000 \end{bmatrix}$
$c^{i}_{n,t+1} =$	$\begin{bmatrix} c^{H}_{n,t+1} & c^{L}_{n,t+1} \end{bmatrix}$	1.0	$\begin{bmatrix} 2.0000 & 2.0000 \end{bmatrix}$	$\begin{bmatrix} 2.0000 & 2.0000 \end{bmatrix}$
$EU =$	$\begin{bmatrix} EU(c^H) & EU(c^L) \\ sum & \end{bmatrix}$		$\begin{bmatrix} 0.1733 & 0.5199 \\ 0.2888 & \end{bmatrix}$	$\begin{bmatrix} 0.1733 & 0.5199 \\ 0.2888 & \end{bmatrix}$
$c^{ij}_{m,t+1} =$	$\begin{bmatrix} c^{HH}_{m,t+1} & c^{HL}_{m,t+1} \\ c^{LL}_{m,t+1} & c^{LH}_{m,t+1} \end{bmatrix}$		$\begin{bmatrix} 1.1878 & 0.5273 \\ 1.0546 & 2.3755 \end{bmatrix}$	$\begin{bmatrix} 1.1878 & 0.5273 \\ 1.0546 & 2.3755 \end{bmatrix}$
$c^{i}_{n,t+1} =$	$\begin{bmatrix} c^{H}_{n,t+1} & c^{L}_{n,t+1} \end{bmatrix}$	1.5	$\begin{bmatrix} 1.6245 & 1.8908 \end{bmatrix}$	$\begin{bmatrix} 1.6245 & 1.8908 \end{bmatrix}$
$EU =$	$\begin{bmatrix} EU(c^H) & EU(c^L) \\ sum & \end{bmatrix}$		$\begin{bmatrix} 0.0681 & 0.4041 \\ 0.1801 & \end{bmatrix}$	$\begin{bmatrix} 0.0681 & 0.4041 \\ 0.1801 & \end{bmatrix}$

3.4.4 Incomplete Information

The results of the numerical example under incomplete information are described in Tables 3.6 and 3.7. The results that the local prices depend on the demographic structure and the changes in the demand for money conditional on the different values of σ is the same as that under complete information. The main difference between complete and incomplete information is that under incomplete information banks face three different price levels depending on how uncertainties are realised.

First of all, as can be seen in Table 3.6a, the three different prices have the following relationship:

$$p_t^{HL} < p_t^{HH} = p_t^{LL} < p_t^{LH}$$

Since $p_t^{LL} = p_t^{HH}$, the real demand for money in $(i, k) = (L, L)$ is equal to the real demand under $(i, k) = (H, H)$. Thus, when $i = k$, the real demand for money $\overline{q}_t^{HH}$ is identical with $\overline{q}_t^{LL}$, regardless of the value of σ.

On the other hand, when $i \neq k$, the real demand for money takes different values as σ fluctuates. More specifically, $\overline{q}_t^{ik}$ has the following relationship according to the

σ		N^H	N^L
0.5	z_L	2.3912	3.9088
	z_H	3.9088	7.3006
1	z_L	1.5000	3.0000
	z_H	3.0000	6.0000
1.5	z_L	1.2779	2.6470
	z_H	2.6470	5.7700

(a) price, p_t^{ik}

σ		N^H	N^L
0.5	z^L	0.3136	0.3837
	z^H	0.3837	0.4109
1	z^L	0.5000	0.5000
	z^H	0.5000	0.5000
1.5	z^L	0.5869	0.5667
	z^H	0.5667	0.5199

(b) Real demand for money, q_t^{ik}

Table 3.6: Price and real money demand

different σ values (see Table 3.6b).

$$\overline{q}_t^{HL} < \overline{q}_t^{HH} = \overline{q}_t^{LL} < \overline{q}_t^{LH}, \text{ if } \sigma < 1$$

$$\overline{q}_t^{HL} = \overline{q}_t^{HH} = \overline{q}_t^{LL} = \overline{q}_t^{LH}, \text{ if } \sigma = 1$$

$$\overline{q}_t^{HL} > \overline{q}_t^{HH} = \overline{q}_t^{LL} > \overline{q}_t^{LH}, \text{ if } \sigma > 1$$

These results are in accordance with (3.42). When $i = k$ and $\sigma = 0.5$, banks on both islands equally holds as much money as $\overline{q}_t^{HH} = \overline{q}_t^{LL} = \widehat{q}_t = 0.3837$. This amount of $\widehat{q}_t$ is less than what a bank would have if it knew it had a smaller population and less if it had known it had a larger population. Similarly, when $i = k$ and $\sigma = 1.5$, the amount of money held by each bank is equal to $\widehat{q}_t$, which is more than the amount that the bank would have had if it had known it had a larger population, and less than the amount that would have been retained if the bank had known it had a smaller population. Since banks cannot differentiate between the two elements of real and nominal factors that determine the current price level, they react to changes in nominal supply of money, not real ones.[29]

[29] These results are similar to those of Lucas (1972) and Wallace (1980). They show that randomised monetary policy does not always increase output because consumers are confused by "money illusion".

Table 3.7: Consumption and EU for Incomplete Information

		σ	$z^L = 1, z^H = 2$
$c^{ik,jl}_{m,t+1} =$	$\begin{bmatrix} c^{HH,HH}_{m,t+1} & c^{LL,HH}_{m,t+1} \\ c^{HH,HL}_{m,t+1} & c^{LL,HL}_{m,t+1} \\ c^{HH,LH}_{m,t+1} & c^{LL,LH}_{m,t+1} \\ c^{HH,LL}_{m,t+1} & c^{LL,LL}_{m,t+1} \\ c^{HL,HH}_{m,t+1} & c^{LH,HH}_{m,t+1} \\ c^{HL,HL}_{m,t+1} & c^{LH,HL}_{m,t+1} \\ c^{HL,LH}_{m,t+1} & c^{LH,LH}_{m,t+1} \\ c^{HL,LL}_{m,t+1} & c^{LH,LL}_{m,t+1} \end{bmatrix}$		$\begin{bmatrix} 0.5132 & 0.5132 \\ 0.5250 & 0.5250 \\ 0.9894 & 0.9894 \\ 0.2566 & 0.2566 \\ 0.5016 & 1.0032 \\ 0.4475 & 0.8951 \\ 0.2815 & 0.5629 \\ 0.2507 & 0.5014 \end{bmatrix}$
$c^{ik}_{n,t+1} =$	$\begin{bmatrix} c^{HH}_{n,t+1} & c^{LL}_{n,t+1} \\ c^{HL}_{n,t+1} & c^{LH}_{n,t+1} \end{bmatrix}$	0.5	$\begin{bmatrix} 1.9472 & 1.9472 \\ 2.2098 & 1.7482 \end{bmatrix}$
EU	$\begin{bmatrix} EU(c^H) & EU(c^L) \\ sum & \end{bmatrix}$		$\begin{bmatrix} 0.0086 & 0.0512 \\ 0.0228 & \end{bmatrix}$
$c^{ik,jl}_{m,t+1} =$	$\begin{bmatrix} c^{HH,HH}_{m,t+1} & c^{LL,HH}_{m,t+1} \\ c^{HH,HL}_{m,t+1} & c^{LL,HL}_{m,t+1} \\ c^{HH,LH}_{m,t+1} & c^{LL,LH}_{m,t+1} \\ c^{HH,LL}_{m,t+1} & c^{LL,LL}_{m,t+1} \\ c^{HL,HH}_{m,t+1} & c^{LH,HH}_{m,t+1} \\ c^{HL,HL}_{m,t+1} & c^{LH,HL}_{m,t+1} \\ c^{HL,LH}_{m,t+1} & c^{LH,LH}_{m,t+1} \\ c^{HL,LL}_{m,t+1} & c^{LH,LL}_{m,t+1} \end{bmatrix}$		$\begin{bmatrix} 0.6667 & 0.6667 \\ 0.4630 & 0.4630 \\ 1.0430 & 1.0430 \\ 0.3333 & 0.3333 \\ 0.5132 & 1.0263 \\ 0.6667 & 1.3333 \\ 0.3333 & 0.6667 \\ 0.2568 & 0.5136 \end{bmatrix}$
$c^{ik}_{n,t+1} =$	$\begin{bmatrix} c^{HH}_{n,t+1} & c^{LL}_{n,t+1} \\ c^{HL}_{n,t+1} & c^{LH0}_{n,t+1} \end{bmatrix}$	1.0	$\begin{bmatrix} 1.3333 & 1.3333 \\ 1.3333 & 1.3333 \end{bmatrix}$
$EU =$	$\begin{bmatrix} EU(c^H) & EU(c^L) \\ sum & \end{bmatrix}$		$\begin{bmatrix} -0.2163 & -0.0430 \\ -0.1586 & \end{bmatrix}$
$c^{ik,jl}_{m,t+1} =$	$\begin{bmatrix} c^{HH,HH}_{m,t+1} & c^{LL,HH}_{m,t+1} \\ c^{HH,HL}_{m,t+1} & c^{LL,HL}_{m,t+1} \\ c^{HH,LH}_{m,t+1} & c^{LL,LH}_{m,t+1} \\ c^{HH,LL}_{m,t+1} & c^{LL,LL}_{m,t+1} \\ c^{HL,HH}_{m,t+1} & c^{LH,HH}_{m,t+1} \\ c^{HL,HL}_{m,t+1} & c^{LH,HL}_{m,t+1} \\ c^{HL,LH}_{m,t+1} & c^{LH,LH}_{m,t+1} \\ c^{HL,LL}_{m,t+1} & c^{LH,LL}_{m,t+1} \end{bmatrix}$		$\begin{bmatrix} 0.7307 & 0.7307 \\ 0.5058 & 0.5058 \\ 1.0050 & 1.0050 \\ 0.3653 & 0.3653 \\ 0.5123 & 1.0246 \\ 0.7546 & 1.5092 \\ 0.3523 & 0.7046 \\ 0.2564 & 0.5128 \end{bmatrix}$
$c^{ik}_{n,t+1} =$	$\begin{bmatrix} c^{HH}_{n,t+1} & c^{LL}_{n,t+1} \\ c^{HL}_{n,t+1} & c^{LH}_{n,t+1} \end{bmatrix}$	1.5	$\begin{bmatrix} 1.0773 & 1.0773 \\ 0.9816 & 1.1815 \end{bmatrix}$
EU	$\begin{bmatrix} EU(c^H) & EU(c^L) \\ sum & \end{bmatrix}$		$\begin{bmatrix} -0.3894 & -0.1174 \\ -0.2987 & \end{bmatrix}$

3.5 Monetary Policy

An interesting question is whether the central bank (CB) is able to affect the bank's portfolio decision by adjusting the price level through money supply expansion. Suppose that the capital investment at date t is determined according to the following equation, which resembles the standard Phillips curve. Moreover, assume that $\sigma > 1$, which implies a higher current price level will increase the investment in capital by reducing liquid asset investment (see (3.42)).

$$k_t = \bar{k} + \alpha(p_t - p_t^e) \tag{3.46}$$

where $\alpha(> 0)$ is a parameter showing how much capital investment responds to unexpected changes in the price level, p_t^e is private sector expected price level formed at the beginning of period t, and $\bar{k}$ denotes the natural level of capital investment. If $\sigma > 1$, then a higher price level may reduce the willingness to hold currency and thus surprised increase in the price level may bring higher capital investment than natural level (see (3.43)).

he CB wishes to minimise the sum of the differences between the price level and its target, and between capital investment and its target by choosing the current price level of date t, p_t. The loss function takes the following form following a standard quadratic loss function a la Barro and Gordon (1983).

$$L_t = (k_t - \phi\bar{k})^2 + \beta(p_t - \bar{p})^2, \quad \beta > 0, \phi > 1 \tag{3.47}$$

where β is a parameter capturing the weight of the inflation objective relative to capital investment, and $\phi > 1$ means that the CB has a capital investment target that is higher than the level of capital investment at which the economy would return normally.

In the following, I study the effect of monetary policy on the bank's portfolio choice under the two different regimes: policy under discretion and rule.

3.5.1 Policy under Discretion

Let us suppose the CB acts in a discretionary way and minimizes its loss function in period t taking p_t^e as given. Using (3.46) to substitute out k_t in (3.47), and differentiating the loss function with respect to the price, we have

$$\begin{aligned} \frac{\partial L_t}{\partial p_t} &= 2\alpha\left(\bar{k} + \alpha(p_t - p_t^e) - \phi\bar{k}\right) + 2\beta(p_t - \bar{p}) = 0 \\ \Leftrightarrow p_t &= \frac{1}{\alpha^2 + \beta}\left(\beta\bar{p} + \alpha^2 p_t^e + \alpha(\phi - 1)\bar{k}\right) \end{aligned} \tag{3.48}$$

Let p_t^d denote the optimal level of price in period t set by the policymaker. If the private sector expectations about the price level is equal to the price level target, i.e., $p_t^e = \bar{p}$, then we obtain

$$p_t^d = \bar{p} + \frac{\alpha(\phi - 1)\bar{k}}{\alpha^2 + \beta}, \tag{3.49}$$

since the second term on the right hand side of the previous equation, $\frac{\alpha(\phi-1)\bar{k}}{\alpha^2+\beta}$, which is called a *price bias*, is positive, p_t^d is greater than $\bar{p}$. The level of price set by the policymaker is higher than the one expected by the private sector.

Let k_t^d be the equilibrium capital investment at date t when the CB enforces the discretionary monetary policy, then k_t^d is obtained by substituting p_t^d in (3.49) for p_t in (3.46) and by putting $\bar{p} = p_t^e$,

$$\begin{aligned} k_t^d &= \bar{k} + \alpha(p_t - p_t^e) = \bar{k} + \alpha\left(\bar{p} + \frac{\alpha(\phi - 1)\bar{k}}{\alpha^2 + \beta} - \bar{p}\right) \\ &= \bar{k} + \frac{\alpha^2(\phi - 1)\bar{k}}{\alpha^2 + \beta} > \bar{k} \end{aligned} \tag{3.50}$$

Let L_t^d be the social welfare loss when $k_t = k_t^d$ and $p_t = p_t^d$. Inserting these equilibria values into the loss function, (3.47), we obtain the level of social welfare loss that this policy can achieve,

$$\begin{aligned} L_t^d &= (k_t - \phi\bar{k})^2 + \beta(p_t - \bar{p})^2 \\ &= \left(\bar{k} + \frac{\alpha^2(\phi-1)\bar{k}}{\alpha^2+\beta} - \phi\bar{k}\right)^2 + \beta\left(\bar{p} + \frac{\alpha(\phi-1)\bar{k}}{\alpha^2+\beta} - \bar{p}\right)^2 \\ &= \left(\frac{\beta(\phi-1)\bar{k}}{\alpha^2+\beta}\right)^2 + \beta\left(\frac{\alpha(\phi-1)\bar{k}}{\alpha^2+\beta}\right)^2 \\ &= \frac{\beta}{\alpha^2+\beta}((\phi-1)\bar{k})^2 \end{aligned} \tag{3.51}$$

3.5.2 Policy under a Rule

Let L_2^r denote the social welfare loss when the CB follows the rule maintaining $p_t = \bar{p}$. Then the private sector expectations on the price level, p_t^e, is equal to $\bar{p}$. The welfare loss from the policy under a rule is given as

$$L_t^r = \left((\phi-1)\bar{k}\right)^2, \tag{3.52}$$

because $p_t^e = \bar{p}$ and thus $k_t = \bar{k} + \alpha(p_t - p_t^e) = \bar{k}$. Comparing (3.51) with (3.52), it is clear that $L_t^r > L_t^d$ and a policy commitment to target price level, $\bar{p}$, have caused higher social losses.

This analysis may justify the use of discretionary monetary policy rather than following policy rules. It is wrong, however, to conclude that because private sector expectations are actually wrong. In fact, the public can anticipate the policymaker's incentive, and they can adjust their expectations so that $p_t^e = p_t^d$.

Then, under rational expectations, the agent's expectation on the price level is given as

$$p_t^e = p_t^d = \bar{p} + \frac{\alpha(\phi-1)\bar{k}}{\alpha^2+\beta},$$

and thus the optimal price level in period t is obtained by substituting the preceding equation into (3.48), which yields

$$p_t^d = \frac{1}{\alpha^2 + \beta} \left(\beta \bar{p} + \alpha^2 p_t^d + \alpha(\phi - 1)\bar{k} \right)$$

Let p_t^{d*} be the price level when the previous equation is solved for p_t^d. Then we have

$$p_t^{d*} = \bar{p} + \frac{\alpha(\phi - 1)\bar{k}}{\beta} \tag{3.53}$$

The equilibrium output k_t^{d*} with p_t^{d*} is obtained by substituting p_t^{d*} in (3.53) for p_t in equation (3.46) and put $p_t^e = p_t^{d*}$, which yields the following

$$k_t^{d*} = \bar{k} \tag{3.54}$$

Inserting these equilibria values into the loss function, (3.47), we obtain the level of social welfare that this policy can achieve, L_t^{d*},

$$\begin{aligned} L_t^{d*} &= (k^{d*} - \phi\bar{k})^2 + \beta(p_t^{d*} - \bar{p})^2 \\ &= \left((\phi - 1)\bar{k}\right)^2 + \left(\frac{\alpha^2}{\beta}\right) \left((\phi - 1)\bar{k}\right)^2 \\ &= \left(\frac{\alpha^2 + \beta}{\beta}\right) \left((\phi - 1)\bar{k}\right)^2 > \left((\phi - 1)\bar{k}\right)^2 = L_t^r \end{aligned} \tag{3.55}$$

Now, the social welfare loss under discretion, (3.55) is bigger than that of rule, (3.52). The analysis here shows that the positive correlation between the current price level and the capital investment cannot arise systematically when agents have rational expectations when $\sigma > 1$.

3.6 Conclusion

In this chapter I formed a general equilibrium model using the *Lucas island model* and the *random-relocation model.* In the first part of this chapter, I dealt with the effect of securitization on banks liquidity needs. I showed that the amount of liquid assets held by the overall economy as well as the regions where securitisation begins, decrease significantly with securitisation than before securitization. As the economy can invest its resources in a more efficient place, both consumption and welfare increase. However, the impact of securitization on banks' liquidity, consumption, and welfare is affected by the population structure of each region. This result may have important policy implications on the effect of global asset shortages on bubbles and global imbalances. The main results of the second part of this chapter are that the expansionary monetary policy can affect the bank's portfolio composition, but the outcome depends on the magnitude of the elasticity of substitution coefficients. A randomised monetary policy combined with other real shocks cause banks to suffer a signal extraction problem as explained in Lucas (1972).

This chapter answers some interesting questions such as how banks should hold their assets when facing the maturity structure trade-off and its depositor's liquidity preference shocks, and the effect of securitization on a bank's portfolio choice. Nevertheless, this article has the following limitations. First of all, in this chapter, I assumed the rate of return on illiquid assets is deterministic. While that assumption makes the exposition as simple as possible on the one hand, interesting questions such as optimal security design resulting from asymmetric information on the asset values, and the effect of those uncertainties on banks runs, systemic risk and stability in financial networks could be answered with the relaxation of this assumption. Also, the issues regarding banking distress that banks may be more susceptible to liquidity crisis with securitisation market distress are not dealt with in this chapter. Moreover, since capital is assumed to depreciate completely, the dynamic effect of capital accumulation is ignored. These issues will be dealt with in further research.

Appendices

3-1 Complete Information & No Uncertainty

In this appendix it is shown that the agents who are born on the island A are able to obtain perfect risk sharing under certain assumptions.

Suppose that the money market is held in the beginning of each period where the initial old and the young gather in the specific place to trade fiat money for goods. This allows the price structure to be simpler because now we have a single price that is common to both islands. Also, assume that money supply is constant for all periods. Then the price level the bank on the island i faces at date t is given as

$$p_t = \frac{z_t M_{t-1}}{N_t^A q_t^A(p_t^k) + N_t^B q_t^B(p_t^k)}$$

Each island $i = (A, B)$ holds $N_t^i q_t^i$ units of money in total respectively. The population structure of each island at date t is uncertain, which means the population of the island A can be large or small with equal probability. However, suppose that the population at date $t+1$ is the same as that at date t, i.e., assume that $N_t^A = N_{t+1}^A$ and $N_t^B = N_{t+1}^B$. The population of the young of the island i is assumed to be constant, which means $N_t^A = N_{t+1}^A$ and $N_t^B = N_{t+1}^B$. Then, the the rate of return on fiat money is

$$\frac{p_t}{p_{t+1}} = \frac{N_{t+1}^A q_{t+1}^A + N_{t+1}^B q_{t+1}^B}{N_t^A q_t^A + N_t^B q_t^B} \frac{1}{z_{t+1}^l} = 1$$

because $N_{t+1}^i q_{t+1}^i = N_t^i q_t^i$ and $z_{t+1}^l = 1$. Except for the structure of the rate of return on money, all other premises are the same as before. From ((3.29)), the securities price is given as the price level at date $t+1$, which equal to p_t, is given ash .

$$b_t = p_{t+1} \tag{56}$$

So, the first order condition, (3.28), is simplified as

$$u'\left(c_{m,t+1}^{B,A}\right) = u'\left(c_{n,t+1}^{B}\right) R$$

β becomes 1. Then from (3.25), the consumption of the nonmover of the island B is given as

$$c_{m,t+1}^{B,A} = \frac{\overline{q}_t^B}{\lambda}$$

Also, from (3.23), we have

$$u'\left(c_{m,t+1}^{A,B}\right) = u'\left(c_{n,t+1}^{A}\right)$$

Perfect risk sharing is attained for people born on the island A. From the feasibility condition and using the fact $c_{m,t+1}^{A,B} = c_{n,t+1}^{A}$, we have

$$c_{m,t+1}^{A,B} = c_{n,t+1}^{A} = q_t^A + k_t^A R$$

implying the optimal value of $q_t^{A*} = 0$ and $c_{m,t+1}^{A,B} = c_{n,t+1}^{A} = R$ by letting $\alpha = 0.5$. Whether the island A has large or small population does not affect the equilibrium values and only changes the price level.

Bibliography

Akerlof, G. A. (1970). The market for "lemons": Quality uncertainty and the market mechanism, *The Quarterly Journal of Economics* **84**(3): 488–500.
URL: *http://www.jstor.org/stable/1879431*

Allen, F., Carletti, E. and Gale, D. (2009). Interbank market liquidity and central bank intervention, *Journal of Monetary Economics* **56**(5): 639–652.

Allen, F. and Gale, D. (1994). Limited market participation and volatility of asset prices, *The American Economic Review* pp. 933–955.

Allen, F. and Gale, D. (1997). Financial markets, intermediaries, and intertemporal smoothing, *Journal of political Economy* **105**(3): 523–546.

Allen, F. and Gale, D. (1998). Optimal financial crises, *The Journal of Finance* **53**(4): 1245–1284.

Allen, F. and Gale, D. (2000). Financial contagion, *Journal of political economy* **108**(1): 1–33.

Allen, F. and Gale, D. (2004). Financial fragility, liquidity, and asset prices, *Journal of the European Economic Association* **2**(6): 1015–1048.

Allen, F. and Gale, D. (2009). *Understanding financial crises*, OUP Oxford.

Angeloni, I. and Faia, E. (2013). Capital regulation and monetary policy with fragile banks, *Journal of Monetary Economics* **60**(3): 311–324.

Barro, R. J. and Gordon, D. B. (1983a). A positive theory of monetary policy in a natural rate model, *Journal of political Economy* **91**(4): 589–610.

Barro, R. J. and Gordon, D. B. (1983b). Rules, discretion and reputation in a model of monetary policy, *Journal of monetary economics* **12**(1): 101–121.

Bencivenga, V. R. and Smith, B. D. (1991). Financial intermediation and endogenous growth, *The Review of Economic Studies* **58**(2): 195–209.

Bhattacharya, J. and Singh, R. (2008). Optimal choice of monetary policy instruments in an economy with real and liquidity shocks, *Journal of Economic Dynamics and Control* **32**(4): 1273–1311.

Bhattacharya, S. and Gale, D. (1985). Preference shocks, liquidity, and central bank policy, *Liquidity and crises* **35**.

Boyd, J. H. and Prescott, E. C. (1986). Financial intermediary-coalitions, *Journal of Economic Theory* **38**(2): 211–232.

Brealey, R., Leland, H. E. and Pyle, D. H. (1977). Informational asymmetries, financial structure, and financial intermediation, *The journal of Finance* **32**(2): 371–387.

Bryant, J. (1980). A model of reserves, bank runs, and deposit insurance, *Journal of banking & finance* **4**(4): 335–344.

Caballero, R. J. (2006). On the macroeconomics of asset shortages, *Technical report*, National Bureau of Economic Research.

Caballero, R. J., Farhi, E. and Gourinchas, P.-O. (2008). Financial crash, commodity prices and global imbalances, *Technical report*, National Bureau of Economic Research.

Caballero, R. J. and Krishnamurthy, A. (2009). Global imbalances and financial fragility, *American Economic Review* **99**(2): 584–88.

Campbel, T. S. and Kracaw, W. A. (1980). Information production, market signalling, and the theory of financial intermediation, *The Journal of Finance* **35**(4): 863–882.

Cass, D. and Yaari, M. E. (1966). A re-examination of the pure consumption loans model, *The Journal of Political Economy* pp. 353–367.

Champ, B., Smith, B. D. and Williamson, S. D. (1996). Currency elasticity and banking panics: Theory and evidence, *Canadian Journal of Economics* pp. 828–864.

Chari, V. V. and Jagannathan, R. (1988). Banking panics, information, and rational expectations equilibrium, *The Journal of Finance* **43**(3): 749–761.

Diamond, D. W. (1984). Financial intermediation and delegated monitoring, *The Review of Economic Studies* **51**(3): 393–414.

Diamond, D. W. (1997). Liquidity, banks, and markets, *Journal of Political Economy* **105**(5): 928–956.

Diamond, D. W. and Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity, *The journal of political economy* pp. 401–419.

Diamond, D. W. and Rajan, R. G. (2001). Liquidity risk, liquidity creation, and financial fragility: A theory of banking, *Journal of Political Economy* **109**(2).

Diamond, P. A. (1965). National debt in a neoclassical growth model, *The American Economic Review* **55**(5): 1126–1150.

Duffie, D., Gârleanu, N. and Pedersen, L. H. (2005). Over-the-counter markets, *Econometrica* **73**(6): 1815–1847.

Estrella, A. et al. (2002). Securitization and the efficacy of monetary policy, *Economic Policy Review* **8**(1): 243–255.

Freeman, S. (1988). Banking as the provision of liquidity, *Journal of Business* pp. 45–64.

Freixas, X. and Rochet, J. (2008). Microeconomics of banking.

Gertler, M. and Kiyotaki, N. (2015). Banking, liquidity, and bank runs in an infinite horizon economy, *American Economic Review* **105**(7): 2011–43.

Gertler, M., Kiyotaki, N. and Prestipino, A. (2016). Wholesale banking and bank runs in macroeconomic modelling of financial crises, *Technical report*, National Bureau of Economic Research.

Gorton, G. (1985). Bank suspension of convertibility, *Journal of monetary Economics* **15**(2): 177–193.

Gorton, G. (1988). Banking panics and business cycles, *Oxford economic papers* **40**(4): 751–781.

Gorton, G. and Metrick, A. (2012). Securitization, *Technical report*, National Bureau of Economic Research.

Gorton, G. and Pennacchi, G. (1990). Financial intermediaries and liquidity creation, *The Journal of Finance* **45**(1): 49–71.

Gorton, G. and Winton, A. (2003). Financial intermediation, *Handbook of the Economics of Finance* **1**: 431–552.

Haubrich, J. G. and King, R. G. (1990). Banking and insurance, *Journal of Monetary Economics* **26**(3): 361–386.

Hellwig, M. (1994). Liquidity provision, banking, and the allocation of interest rate risk, *European Economic Review* **38**(7): 1363–1389.

Jacklin, C. J. (1987). Demand deposits, trading restrictions, and risk sharing, *Contractual arrangements for intertemporal trade* **1**.

Jacklin, C. J. and Bhattacharya, S. (1988). Distinguishing panics and information-based bank runs: Welfare and policy implications, *The Journal of Political Economy* pp. 568–592.

Keynes, J. M. (1930). A treatise on money.

Kocherlakota, N. R. (1998). Money is memory, *journal of economic theory* **81**(2): 232–251.

Kydland, F. E. and Prescott, E. C. (1977). Rules rather than discretion: The inconsistency of optimal plans, *Journal of political economy* **85**(3): 473–491.

Lagos, R., Rocheteau, G. and Wright, R. (2015). Liquidity: A new monetarist perspective, *Journal of Economic Literature, forthcoming* pp. 1–108.

Loutskina, E. (2011). The role of securitization in bank liquidity and funding management, *Journal of Financial Economics* **100**(3): 663–684.

Lucas, R. E. (1972). Expectations and the neutrality of money, *Journal of economic theory* **4**(2): 103–124.

Maddaloni, A. and Peydró, J.-L. (2011). Bank risk-taking, securitization, supervision, and low interest rates: Evidence from the euro-area and the us lending standards, *Review of Financial Studies* **24**(6): 2121–2165.

Martin, A., Skeie, D. and Von Thadden, E.-L. (2014). Repo runs, *Review of Financial Studies* **27**(4): 957–989.

Prize, N. (2013). Understanding asset prices, *the Sveriges Riksbank Prize in Economic Sciences* .

Qi, J. (1994). Bank liquidity and stability in an overlapping generations model, *Review of Financial Studies* **7**(2): 389–417.

Reinhart, C. M. and Rogoff, K. S. (2009). *This time is different: Eight centuries of financial folly*, princeton university press.

Samuelson, P. (1958). An exact consumption-loan model of interest with or without the social contrivance of money, *Journal of Political Economy* **66**.
URL: *http://EconPapers.repec.org/RePEc:ucp:jpolec:v:66:y:1958:p:467*

Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification, *Journal of Economic theory* **21**(2): 265–293.

von Thadden, E.-L. (1999). Liquidity creation through banks and markets: Multiple insurance and limited market access, *European Economic Review* **43**(4): 991–1006.

Wallace, N. (1980). The overlapping generations model of fiat money, in models of monetary economies (j. kareken and n. wallace, eds.), 49, *Minneapolis: Federal Reserve Bank of Minneapolis* .

Wallace, N. (1988). Another attempt to explain an llliquid banking system: With sequential service taken seriously, *Quarterly Review* .

Waller, C. J. (2015). Microfoundations of money: why they matter, *Browser Download This Paper* .

Williamson, S. D. (1986). Costly monitoring, financial intermediation, and equilibrium credit rationing, *Journal of Monetary Economics* **18**(2): 159–179.