Dynamic Time Warping as a Similarity Measure: Applications in Finance

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Abstract
This paper presents the basic DTW-algorithm and the manner it can be used as a similarity measure for two different series that might differ in length. Through a simulation process it is being showed the relation of DTW-based similarity measure, dubbed ρ_{DTW}, with two other celebrated measures, that of the Pearson's and Spearman's correlation coefficients. In particular, it is shown that ρ_{DTW} takes lower (greater) values when other two measures are great (low) in absolute terms. In addition a dataset composed by 8 financial indices was used, and two applications of the aforementioned measure are presented. First, through a rolling basis, the evolution of ρ_{DTW} has been examined along with the Pearson's correlation and the volatility. Results showed that in periods of high (low) volatility similarities within the examined series increase (decrease). Second, a comparison of the mean similarities across different classes of months is being carried. Results vary, however a statistical significant greater similarity within Aprils is being reported compared to other months, especially for the CAC 40, IBEX 35 and FTSE MIB indices.

1. Introduction

Various measures can be used in order to measure the similarity between two series of observations, like the Pearson's r, the Spearman's ρ, Kendall's τ and Kruskal's Γ. However, in view of finance applications, it might be required to measure the similarity between two series that differ in length (e.g. measuring the similarity between two different months). One solution to this problem might be found in the context of data mining by using the Dynamic Time Warping (DTW).

DTW is an algorithmic technique mainly used to find an optimal alignment between two given (time-dependent) sequences under certain restrictions (Muller 2007). First introduced in 1960s, DTW initially became popular in the context of speech recognition (Sakoe and Chiba 1978), and then in time series data mining, in particular in pattern recognition and similarity measurement (Berndt and Clifford 1994). Indicatively, we refer to two of the few academic papers that implement DTW in finance applications. First, in (Wang et al. 2012), DTW was used to study the topology of similarity networks among 35 currencies in international FX markets, by using the minimal spanning tree approach. Second, Tsinaslanidis and Kugiumtzis (2014) used perceptually important points (Chung et al. 2001; Fu et al. 2007) to dynamically segment price series into subsequences and DTW to find similar historical subsequences. Subsequently predictions were made from the mappings of the most similar subsequences.

This paper highlights the manner that DTW can be used as a similarity measure, while presenting simulated and empirical applications as cases. In particular Section 2 presents the DTW algorithm with a simplified example. Section 3 presents the DTW as a similarity measure and its relation with the Spearman's and Pearson's correlation coefficient. Section 4 presents an application whereby DTW is used to measure and compare the similarity across daily returns of different classes of months. Finally Section 5 makes a conclusion.

2. The Dynamic Time Warping Algorithm

Dynamic Time Warping (DTW) is an efficient scheme giving the distance (or similarity) of two sequences $Q \equiv \{q_1, q_2, \dots, q_n, \dots, q_N\}$ and $Y \equiv \{y_1, y_2, \dots, y_m, \dots, y_M\}$, where their lengths N and M may not be equal. An example of two sequences Q and Y is given by (1) and (2):

$$q_n = \sin(x_n) + 0.2\varepsilon_n,\ \ \varepsilon_n \sim IID(0,1), x_n \epsilon [0,2\pi]\ and\ N = 35 \tag{1}$$

$$y_m = \sin(x_m) + 0.2\varepsilon_m,\ \ \varepsilon_m \sim IID(0,1), x_m \epsilon [0,2\pi]\ and\ M = 50 \tag{2}$$

Clearly, both (1) and (2) represent a sine with Gaussian white noise in the closed interval $[0,2\pi]$ but with different lengths. First, a distance between any two components q_n and y_m of Q and Y is defined, forming the distance (or cost) matrix $\boldsymbol{D} \in \mathbb{R}^{N\times M}$ (Fig. 1b). Various distance measures can be used for this purpose, however for this simplified illustration we use the absolute value of the difference, i.e. $d(q_n, y_m) = |q_n - y_m|$.

The goal is to find the optimal alignment path between Q and Y of minimum overall cost (cumulative distance). A valid path is a sequence of elements $Z \equiv \{z_1, z_2, \dots, z_k, \dots, z_K\}$ with $z_k = (n_k, m_k)$, $k = 1, \dots, K$, denoting the positions in the distance matrix $\boldsymbol{D}$ that satisfy the boundary, monotonicity and step size conditions. The boundary condition ensures that the first and the last element of Z are $z_1 = (1,1)$ and $z_K = (N, M)$, respectively (i.e. the bottom left and the top right corner of $\boldsymbol{D}$, see Fig. 1b). The other two conditions ensure that the path always moves up, right or up and right of the current position in $\boldsymbol{D}$, i.e. $z_{k+1} - z_k \in \{(1,0),(0,1),(1,1)\}$.

To compute the total distance of each valid path, first the cost matrix of accumulated distances $\widetilde{\boldsymbol{D}} \in \mathbb{R}^{N\times M}$ is constructed with initial condition $\tilde{d}(1,1) = d(1,1)$, and accumulated distance for every other element of $\widetilde{\boldsymbol{D}}$ defined as

$$\tilde{d}(n,m) = d(n,m) + \min\{\tilde{d}(n-1,m), \tilde{d}(n,m-1), \tilde{d}(n-1,m-1)\}, \tag{3}$$

where $\tilde{d}(0,m) = \tilde{d}(n,0) = +\infty$ in order to define the accumulated distances for all elements of $\widetilde{\boldsymbol{D}}$ (see Fig. 1c). At this stage we keep the indexation regarding the adjacent cell with the minimum distance, and then starting from $\tilde{d}(N,M)$ we identify backwards the optimal path. In particular, if the optimal warping path is a sequence of elements $Z^* \equiv \{z_1^*, z_2^*, \dots, z_k^*, \dots, z_K^*\}$ with $z_K^* = (N, M)$, then conditioning on $z_k^* = (n, m)$, we choose z_{k-1}^* as

$$z_{k-1}^* = \begin{cases} (1, m-1), \ if \ n = 1 \\ (n-1, 1), \ if \ m = 1 \\ \operatorname{argmin}\{\tilde{d}(n-1, m-1), \tilde{d}(n-1, m), \tilde{d}(n, m-1)\}, \ otherwise. \end{cases} \quad (4)$$

The process terminates when $n = m = 1$ and $z_k^* = (1,1)$ (Muller 2007). The optimal path for our example is illustrated in Fig. 1b,c with the white solid line. Having identified the optimal path the initial sequences Q and Y are aligned by warping their time axis (Fig. 1d).

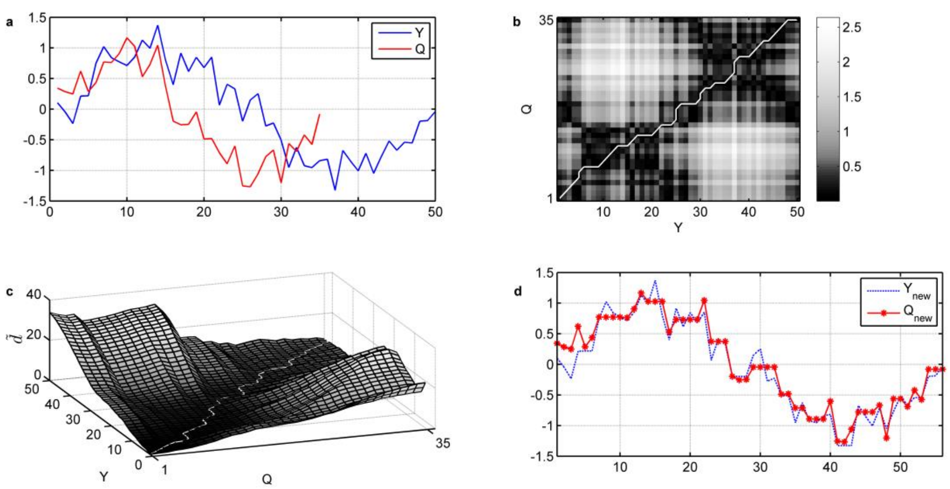

Fig. 1 **a** Q and Y price series of unequal length, **b** Colormap of the distance (cost) matrix, **c** 3D illustration of the accumulated distance (cost) matrix, **d** Sequences Q and Y aligned with DTW. In **b** and **c** the white solid line is the optimal warping path.

3. DTW as a similarity measure

In this section we use the DTW algorithm to measure diachronically the similarity evolution across 9 different major financial indexes Table 1. Daily values for the trading years 2005 until 2012 were downloaded from the Bloomberg data base. In this experiment we used daily logarithmic returns, defined as,

$$r_{i,t} = \ln P_{i,t} - \ln P_{i,t-1} = \ln(P_{i,t}/P_{i,t-1}). \quad (5)$$

Here, $r_{i,t}$ $(P_{i,t})$ is the logarithmic return (price) of the i^{th} index at time t.

Regarding the cleaning data process we followed, missing values where filled with linear interpolation, whereas outliers where winsorized by adopting a compressing algorithm, pulling them towards the mean and replacing them with a value at a precpesified limit of three standard deviations. This process was implemented on a rolling basis, with a window length of 250 trading days and a one-day step, in order to consider time-varying volatility exhibited in the examined dataset (Kumiega and Van Vliet 2008).

Table 1. Major EMEA indexes

idx_i	Bloomberg Ticker	Index
idx_1	SX5E Index	EURO Stoxx
idx_2	UKX Index	FTSE 100
idx_3	CAC Index	CAC 40
idx_4	DAX Index	DAX
idx_5	IBEX Index	IBEX 35
idx_6	FTSEMIB Index	FTSE MIB
idx_7	AEX Index	AEX
idx_8	OMX Index	OMX STKH30
idx_9	SMI Index	SWISS MKT

Empirical evidence suggests a link between correlation and volatility of financial assets' returns. In particular, correlations between returns on financial assets tend to be greater in highly volatile periods, compared with those observed in less volatile periods (Loretan and English 2000). This change in correlation may be attributed to structural breaks in the underlying return generating mechanisms, like contagion effects between markets. However, Boyer et al (1999) proved that when random variables evolve with more volatility, their sampling correlations should also increase even if the underlying generating mechanism remains unchanged. This implies that there is a "natural" relation between correlation and volatility, and thus correlation patterns can be predicted, by simply modelling volatility. Implications of this relation are significant, especially for finance practitioners dealing with the portfolio construction, and risk management.

For the indexes presented in Table 1, three different measures were computed on a rolling basis with a window of 21 days and a step of one day. First, an equally weighted theoretical portfolio consisting of the $\lambda = 9$ examined indexes was constructed and 21-day variance σ^2 was estimated as,

$$\sigma^2 = \boldsymbol{W}\boldsymbol{\Sigma}\boldsymbol{W}^T \tag{6}$$

In (6) $\boldsymbol{W}$ is a $(1 \times \lambda)$ row vector containing the weights attributed to each index, $\boldsymbol{\Sigma}$ is the $(\lambda \times \lambda)$ covariance matrix and $\boldsymbol{W}^T$ is the transpose of $\boldsymbol{W}$, with a size of $(\lambda \times 1)$. Second for each subperiod we calculated the $(\lambda \times \lambda)$ correlation matrix $|\boldsymbol{\rho}|$ where each component $|\rho_{i,j}|$ is the absolute value of the correlation coefficient between indexes i and j. The mean similarity measure we define equals with,

$$\overline{|\rho|} = \frac{2}{\lambda(\lambda - 1)} \sum_{i=1}^{\lambda-1} \sum_{j=i+1}^{\lambda} |\rho_{i,j}|. \tag{7}$$

The correlation coefficient measures the relation between the returns of two financial assets in a linear manner. Averaging values with different signs would result in meaningless measures. For example assume that $\rho_{1,2} = 1, \rho_{1,3} = -1$ *and* $\rho_{2,3} = -1$. Taking the averages would result in a value of -0.33 whereas we are interesting in a measure that tells as whether there are linear relations between the examined series, which in this hypothetical example there are ($\overline{|\rho|} = 1$). Finally, the DTW algorithm measures the similarity in the examined series in a nonlinear manner. At each subperiod an $(\lambda \times \lambda)$ DTW-based similarity matrix $\boldsymbol{c}$ is constructed, where each components $c_{i,j}$ is the total average similarity cost, $c_{i,j} = \tilde{d}(N, M)/K$, between indexes i and j, $\tilde{d}(N, M)$ is the total cost of the optimal warping path identified by the accumulated cost matrix and K is the length of the optimal warping path Z^*. The greater the similarity between two subsequences the lower the $c_{i,j}$ and apparently, $c_{i,j} = 0$ when $i = j$. In a similar manner with (7) the mean DTW-similarity measure for our examined dataset equals with,

$$\bar{c} = \frac{2}{\lambda(\lambda - 1)} \sum_{i=1}^{\lambda-1} \sum_{j=i+1}^{\lambda} c_{i,j} \,. \tag{8}$$

Fig. 2 illustrates the evolution of logarithmic returns, σ^2 (6), $\overline{|\rho|}$ (7) and $\bar{c}$ (8) of the examined indexes. Obviously, periods of high volatility, are characterized by high $\overline{|\rho|}$ values and low $\bar{c}$.

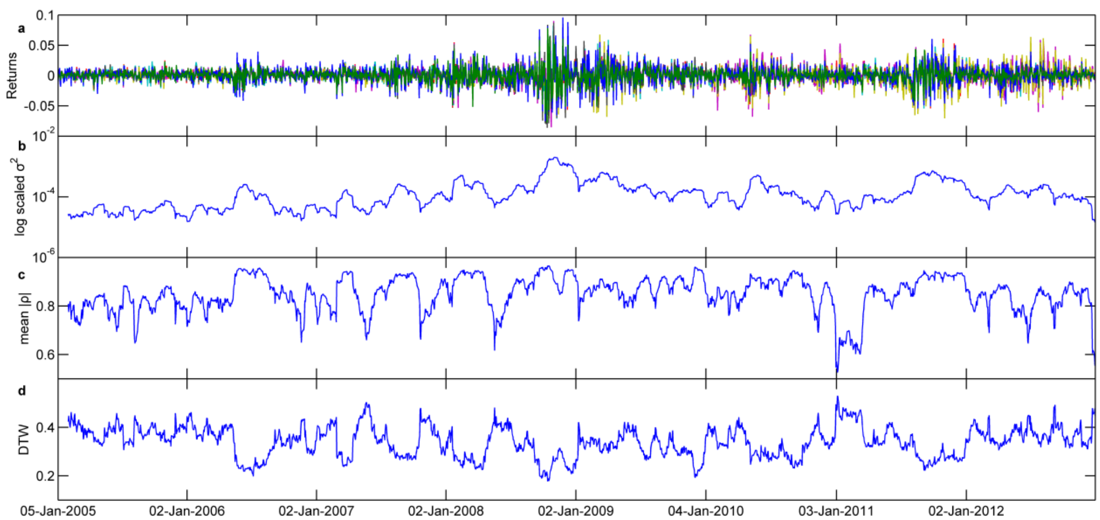

Fig. 2 a logarithmic returns, **b** logarithmic scaled variance, **c** mean Pearson similarity measure, **d** mean DTW-similarity measure. For **a, b** and **c** a 21-days rolling window was adopted with a rolling step of one day.

Fig. 3 shows the relation between the $\overline{|\rho|}$ and $\bar{c}$ for the examined dataset. As expected there is a negative curve relation between these two measures. This implies that when great in values linear correlations occur within a number of series their nonlinear similarity as measured by the DTW increases (recall that the lower the $\bar{c}$ the greater the similarity).

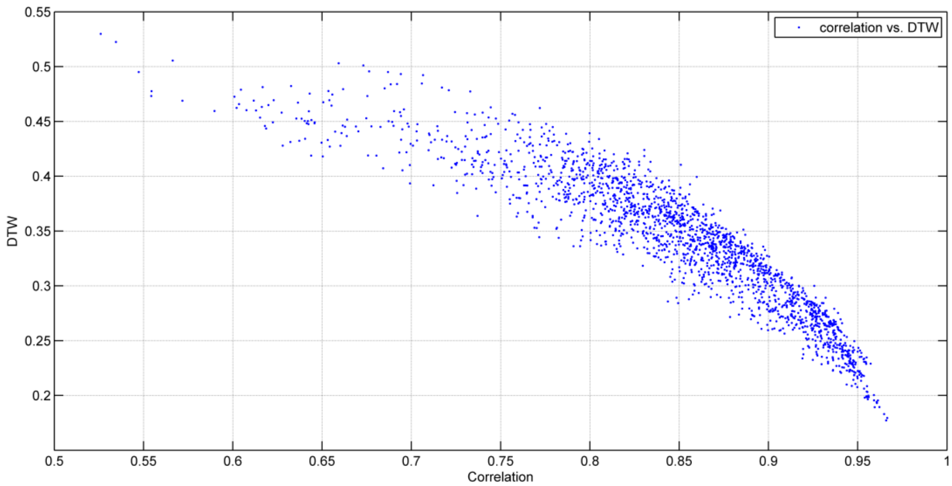

Fig. 3 Scatter plot of $\overline{|\rho|}$ against $\bar{c}$.

Except from the empirical results presented above, it was also examined the relation between the similarity measure derived by the DTW and the Pearson correlation coefficient, ρ_p, as well as the Spreaman's rho coefficient, ρ_s, through a simulation experiment. While ρ_p measures the linear relation between two variables, ρ_s is a nonparametric measure and assesses whether the relation between two variables can be described using a monotonic function (Best and Roberts 1975; Hollander and Wolfe 1973). For this simulation experiment, 2,000 pairs of randomly generated series with 21 observations each, were generated in a manner that they exhibit various ρ_p within the closed interval $[-1,1]$. For each pair the ρ_p, ρ_s and the DTW-based similarity measure, dubbed ρ_{DTW} were calculated. Let Q and Y be the randomly created series, with a predefined ρ_p. The ρ_{DTW} was defined as the minimum total average cost of the two optimal warping paths obtained by comparing series Q with Y and Q with $-Y$. Formally this is:

$$\rho_{DTW} = \min(DTW(Q,Y), DTW(Q,-Y)) \quad (9)$$

The reason for considering (9) is that series that exhibit perfect negative (or generally negative and great in absolute values) correlation, either with ρ_p or with ρ_s are classified as dissimilar when compared with the DTW algorithm. Our aim is that DTW should be able to identify similar series, where similarity is defined by great in absolute values ρ_p and ρ_s regardless their sign. The relation between the three similarities measures is presented in Fig. 4, where DTW approaches zero (takes maximum values), i.e. indicating great (low) similarity, when ρ_p and ρ_s take great (low) absolute values.

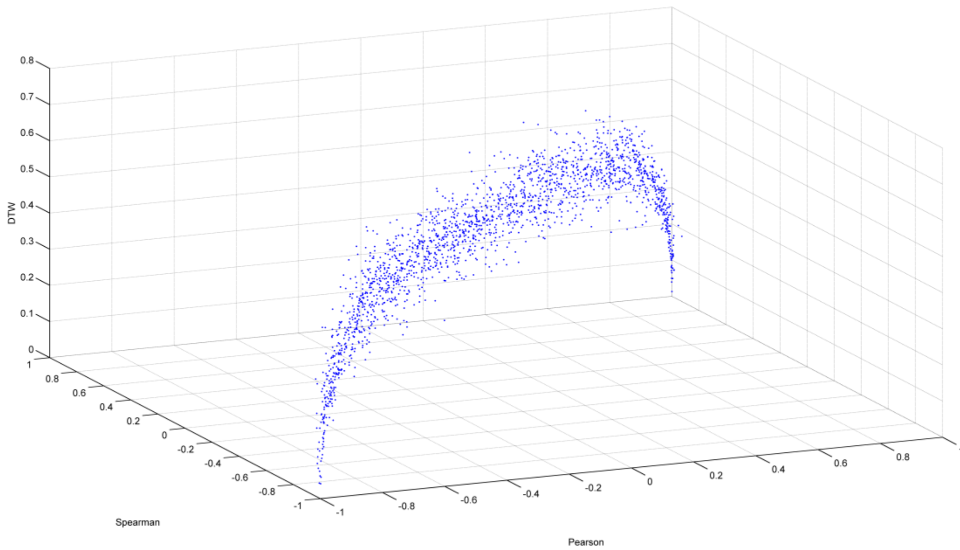

Fig. 4. 3D scatter plot illustrating the relation between Pearson's correlation coefficient, Spearman's correlation coefficient and DTW-based similarity measure.

4. Measuring similarities in months

The benefit of using DTW to measure similarities between two time series is mainly apparent when the series considered differ in length. For example, adopting DTW methodology allows a similarity comparison between months, where trading days observed from month to month may differ. Implications of this allowance are significant, especially for applications where we need to assess the existence of calendar effects. In this section an additional experiment on the same dataset is carried where similarities across different months are compared.

To be more specific, for each idx_i we compared with (9) all series of daily returns that correspond to the same month but different year (i.e. we measured the similarity between all Januaries pairs, all Februaries pairs and so on for each index). Since the examined years were 8 (2005-2012) for each index and for each month we performed $8 \times (8-1)/2 = 28$ comparisons. This means that for each index 12 distributions (one for each month) of 28 observed similarities measures where obtained. Subsequently we multi-compared pairwise[1] these 12 distributions by a two sample, one-tailed, unequal variance student's t-test and the resulted *p-values* are presented in Table 2. *P-values* in bold highlight significant cases at the 95% significance level where the mean of ρ_{DTW} obtained from one class of months m_{Low}, is lower than the mean of ρ_{DTW} obtained from another, different class of months m_{High}. Comparisons between different classes of months that did not reject the null hypothesis in any index were omitted for brevity reasons.

[1] For each index $12 \times 11 = 132$ comparisons were implemented between different classes of months.

Table 2. P-values from two-sample, one-tailed, unequal variance (heteroscedastic) Student's t-test. The null hypothesis states that there is no difference in ρ_{DTW} means of classes m_{Low} and m_{High}, whilst the alternative hypothesis states that $\bar{\rho}_{DTW}$ of m_{Low} is lower than that of m_{High}.

m_{Low}	m_{High}	idx_1	idx_2	idx_3	idx_4	idx_5	idx_6	idx_7	idx_8	idx_9
Jan	May	0,949	0,113	0,772	0,996	0,828	0,963	0,760	0,991	**0,012**
	Dec	0,107	0,337	0,059	0,637	0,381	0,478	0,291	0,365	**0,017**
Feb	Jan	0,748	**0,028**	0,290	0,758	0,706	0,285	0,489	0,470	0,936
	May	0,997	**0,004**	0,594	1,000	0,937	0,880	0,762	0,978	0,309
	Dec	0,233	**0,028**	**0,013**	0,833	0,575	0,281	0,276	0,349	0,278
Mar	Jan	0,571	0,266	0,314	**0,011**	0,765	0,453	0,406	0,372	0,137
	Feb	0,340	0,856	0,476	**0,002**	0,589	0,673	0,413	0,417	**0,009**
	May	0,952	0,063	0,547	0,620	0,948	0,951	0,678	0,992	**0,001**
	Aug	0,613	0,499	0,476	0,638	0,928	0,547	0,704	0,766	**0,013**
	Sept	0,893	0,970	0,778	0,799	0,998	0,972	0,735	0,990	**0,024**
	Dec	0,163	0,193	**0,035**	**0,044**	0,649	0,434	0,212	0,209	**0,002**
Apr	Jan	**0,032**	**0,023**	**0,000**	**0,000**	**0,000**	**0,004**	0,311	**0,042**	0,557
	Feb	**0,002**	0,564	**0,001**	**0,000**	**0,000**	**0,015**	0,313	0,069	0,082
	Mar	**0,031**	0,157	**0,011**	0,120	**0,000**	**0,005**	0,396	0,051	0,888
	May	0,297	**0,003**	**0,002**	0,179	**0,000**	0,095	0,566	0,710	**0,017**
	Jun	0,051	0,227	**0,005**	**0,005**	**0,000**	**0,001**	0,358	0,294	0,342
	Jul	0,276	0,278	**0,008**	0,127	**0,000**	0,299	0,242	0,438	0,732
	Aug	**0,033**	0,123	**0,002**	0,247	**0,000**	**0,005**	0,594	0,201	0,149
	Sept	0,212	0,849	**0,035**	0,374	**0,034**	0,234	0,641	0,755	0,181
	Oct	0,073	0,276	**0,000**	0,054	**0,000**	**0,000**	0,634	0,395	0,297
	Nov	0,142	0,082	**0,008**	**0,006**	**0,001**	0,137	0,704	**0,035**	0,606
	Dec	**0,001**	**0,027**	**0,000**	**0,003**	**0,000**	**0,006**	0,147	**0,008**	**0,023**
May	Jan	0,051	0,887	0,228	**0,004**	0,172	**0,037**	0,240	**0,009**	0,988
	Feb	**0,003**	0,996	0,406	**0,000**	0,063	0,120	0,238	**0,022**	0,691
	Mar	**0,048**	0,937	0,453	0,380	0,052	**0,049**	0,322	**0,008**	0,999
	Jun	0,084	0,970	0,564	**0,034**	0,136	**0,016**	0,287	0,132	0,967
	Oct	0,113	0,994	0,098	0,229	0,212	**0,005**	0,589	0,200	0,890
	Nov	0,224	0,929	0,493	**0,045**	0,614	0,524	0,673	**0,006**	0,986
	Dec	**0,002**	0,723	**0,009**	**0,021**	0,121	**0,047**	0,097	**0,001**	0,437
Jun	May	0,916	**0,030**	0,436	0,966	0,864	0,984	0,713	0,868	**0,033**
	Dec	**0,047**	0,122	**0,008**	0,386	0,457	0,607	0,248	**0,043**	**0,041**

Continued…

Table 2. (continue)

Jul	Jan	0,096	0,079	0,166	**0,002**	0,185	**0,012**	0,560	0,054	0,319
	Feb	**0,014**	0,753	0,304	**0,000**	0,068	**0,043**	0,575	0,085	**0,023**
	Mar	0,084	0,302	0,369	0,399	0,056	**0,016**	0,662	0,067	0,775
	May	0,576	**0,011**	0,389	0,538	0,533	0,226	0,835	0,768	**0,002**
	Jun	0,155	0,409	0,454	**0,027**	0,146	**0,005**	0,616	0,345	0,135
	Aug	0,105	0,273	0,314	0,571	0,435	**0,018**	0,855	0,243	**0,029**
	Oct	0,169	0,517	0,070	0,228	0,227	**0,001**	0,853	0,455	0,144
	Nov	0,307	0,198	0,399	**0,035**	0,649	0,275	0,883	**0,046**	0,391
	Dec	**0,006**	0,070	**0,006**	**0,017**	0,130	**0,016**	0,328	**0,012**	**0,005**
Aug	Jan	0,456	0,232	0,303	**0,008**	0,227	0,405	0,218	0,174	0,887
	Feb	0,204	0,884	0,501	**0,001**	0,089	0,635	0,215	0,220	0,251
	May	0,948	**0,042**	0,589	0,462	0,595	0,945	0,467	0,927	0,075
	Jun	0,608	0,622	0,644	**0,045**	0,179	0,274	0,263	0,607	0,739
	Dec	0,081	0,169	**0,017**	**0,030**	0,160	0,390	0,085	0,071	0,085
Sept	Jan	0,123	**0,001**	0,074	**0,001**	**0,010**	**0,021**	0,198	**0,010**	0,854
	Feb	**0,020**	0,228	0,144	**0,000**	**0,002**	0,069	0,195	**0,020**	0,297
	Mar	0,107	**0,030**	0,222	0,201	**0,002**	**0,028**	0,265	**0,010**	0,976
	May	0,663	**0,000**	0,206	0,284	0,058	0,308	0,405	0,417	0,128
	Jun	0,197	**0,047**	0,261	**0,012**	**0,008**	**0,009**	0,236	0,115	0,716
	Jul	0,574	**0,050**	0,299	0,231	**0,046**	0,584	0,136	0,197	0,941
	Aug	0,136	**0,014**	0,158	0,346	**0,034**	**0,031**	0,431	0,066	0,524
	Oct	0,207	**0,040**	**0,028**	0,103	**0,018**	**0,003**	0,499	0,171	0,598
	Nov	0,367	**0,008**	0,234	**0,015**	0,091	0,350	0,589	**0,007**	0,865
	Dec	**0,008**	**0,003**	**0,002**	**0,007**	**0,008**	**0,027**	0,081	**0,001**	0,126
Oct	Jan	0,406	0,055	0,720	**0,027**	0,478	0,731	0,207	0,069	0,743
	Feb	0,190	0,755	0,871	**0,005**	0,290	0,889	0,205	0,103	0,240
	May	0,887	**0,006**	0,902	0,771	0,788	0,995	0,411	0,800	0,110
	Dec	0,081	0,056	0,189	0,086	0,370	0,694	0,088	**0,018**	0,106
Nov	Feb	0,059	0,919	0,428	**0,050**	**0,035**	0,138	0,164	0,521	0,062
	Mar	0,183	0,589	0,463	0,910	**0,030**	0,064	0,220	0,625	0,815
	May	0,776	0,071	0,507	0,955	0,386	0,476	0,327	0,994	**0,014**
	Jun	0,324	0,705	0,561	0,412	0,086	**0,025**	0,197	0,888	0,257
	Oct	0,313	0,830	0,130	0,830	0,147	**0,010**	0,413	0,939	0,232
	Dec	**0,022**	0,233	**0,020**	0,304	0,078	0,059	0,071	0,343	**0,018**

Results vary across different indices, but we can spot some consistent cases. For example, similarity observed within Aprils is statistically significant greater than similarities observed within Decembers in 8 out of 9 indices. The second most consistent difference in similarities is observed when Aprils and Januaries are compared (7 out of 9 cases). For ease of observation, and in order to get an aggregate picture, we counted the number of significant cases reported in Table 2 by rows and we present the corresponding counts in Fig. 5.

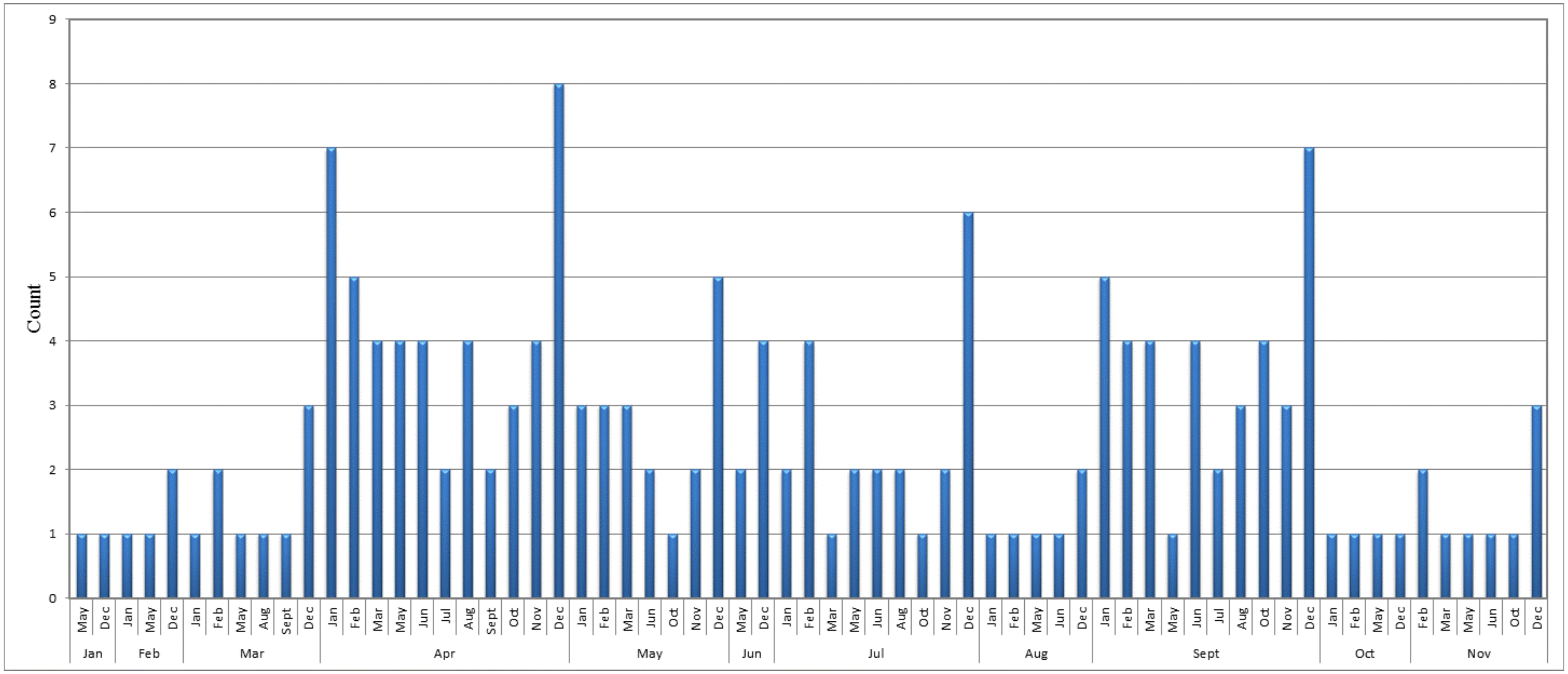

Fig. 5. Aggregate significant differences in $\bar{\rho}_{DTW}$ across different months for all indices examined.

Results indicate that similarities within Aprils are statistically significant greater than those observed within other classes of months and more than any other comparison (47 significant cases). This implies that generally, predictability within daily returns for an April based on historical returns of an earlier April can be superior to predictability for another month. This implication is more apparent for indices idx_3, idx_5 and idx_6 where Aprils' $\bar{\rho}_{DTW}$ is significantly lower than the corresponding mean similarity measures obtained from most other months.

5. Conclusions

In this paper we briefly presented the DTW algorithm and described the manner it can be used as a similarity measure between two series of observations. Initially we presented diachronically, on a rolling basis, the evolution of the DTW-based similarity measure, dubbed ρ_{DTW}, along with the volatility and the Pearson's correlation coefficient, ρ_P , for 6 financial market indices. Our results corroborate previous empirical findings, and show that in periods of higher volatility financial indices present greater similarity, both in terms of linear relation as expressed with the ρ_P but also in terms of nonlinear relation as described by the ρ_{DTW}. Subsequently, the relation of ρ_{DTW} with two celebrated similarity measures, ρ_P and Spearman's ρ_S has been examined through a simulation, and we showed that ρ_{DTW} approaches zero when ρ_P and ρ_S take greater absolute values whilst ρ_{DTW} takes its maximum values when correlation approaches zero.

The benefit, of using DTW as a similarity measure can be traced in cases where the candidate series differ in length whereby the implementation of traditional correlation measures in not possible. Implications of this characteristic in finance applications are significant, since DTW can be used to study market seasonalities by comparing the dynamics of returns series evolutions across different months which might differ in length. Subsequently, it might be possible to develop prediction algorithms based on this notion. But these are left for future investigation. Finally we presented an empirical assessment, by measuring pair-wisely similarities within same months of different years. Our results, showed that similarities within Aprils are greater compared with other months especially for CAC 40, IBEX 35 and FTSE MIB indices.

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