Non-Homothetic Growth Models for the Environmental Kuznets Curve*

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Abstract

We show the role of the elasticity of substitution between general consumption and the environment in environmental degradation. Specifically, our numerical experiments demonstrate, for a wide range of models, exponential utility generates the environmental Kuznets curve without adding any special assumptions. With exponential utility, the elasticity of substitution and hence the substitution effect between consumption and the environment are both decreasing in income. Hence, when income is low, society (the government) readily gives up environmental quality in return for more consumption, but it does not want to substitute consumption for the environment anymore, once it becomes wealthy enough.

RUNNING HEAD: EKC and Elasticity of Substitution

KEYWORDS: Environmental Kuznets Curve, Economic Growth, Non-Homothetic Preferences, Generalized Isoelastic Preferences.

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1 Introduction

In this paper, we employ a diverse set of general equilibrium growth models to study the environmental Kuznets curve (EKC). The EKC is the empirical observation that environmental degradation increases when income levels are low, but, after passing a certain level, it decreases as income increases (e.g., Dinda, 2004, Stern, 2004, Brock and Taylor, 2005, and Carson, 2010). The EKC is important because, if it holds in reality, economic growth is consistent with, rather than contradictory to, environmental sustainability (Stern, 2004).

In this paper, we numerically demonstrate the importance of the elasticity of substitution between general consumption goods and the environment quality. To date the importance of the elasticity of substitution has been somewhat overlooked within the EKC literature which is surprising given its central role within the literature on sustainability. By focussing on the elasticity of substitution, our analysis provides an interesting link to other sustainability related areas of research such as the literature on climate-economy modelling. For example, Sterner and Persson (2008) show that the elasticity of substitution is as important as the discount rate in their climate-economy models. Specifically, if the elasticity of substitution is less than one, then this can and does influence model results by at least as much as, if not more than, a very low discount rate. The recognition of the importance of the elasticity of substitution is also prominently highlighted in work by Neumayer (1999), Traeger (2011) and Guéant et al. (2012) in relation to economic sustainability.

Within the context of the EKC, by reframing the relationship between consumption and the environment in terms of the elasticity of substitution, we generate two important observations. *First, at each income level, if the elasticity of substitution is sufficiently high, the quality of the environment deteriorates as income increases*, and vice versa. To understand why, note that the elasticity of substitution shows how the demand for the environment is sensitive to changes in the relative price of the environment for general consumption. Thus, if it is elastic, people are willing to give up a large amount of the environment to enjoy an additional unit of consumption as compensation, when the price of the environment increases. Suppose hypothetically that the relative price is unchanged; in this case, as income increases, people demand more of the environment due to the standard income effect. However, as productivity improves, general consumption can be produced at a lower cost, meaning that the relative price of the environment increases. Since the environment becomes more expensive relative to general consumption, this change in the relative price reduces the demand for a higher quality environment due to the substitution effect. The relative strength of these two effects hinges on the elasticity of substitution. If the elasticity of substitution is low enough, the substitution effect is weak and is dominated, and vice versa.

Second, and as a natural consequence of the first finding, *if the elasticity of substitution decreases fast enough as income increases, it generates the EKC.* That is, when income is low, the elasticity of substitution should be high so that the environment is deteriorating, but, after income passes a certain level, the elasticity should be low so that the environment improves. We show that, with exponential (DES) utility,[2] the elasticity of substitution is decreasing in income, generating the EKC in a wide class of model settings. In contrast, with power (CES) utility,[3] the elasticity of substitution is constant, and as a result the environment is either monotonically increasing or decreasing in income depending on the elasticity. While CES utility is quite popular because of its analytical tractability, DES utility also has its own appeal (see Section 4.3). Thus, the only requirement to generate the EKC in our framework is a preference structure with decreasing elasticity of substitution, and, with DES utility, the EKC naturally arises under a wide class of model specifications without adding any other assumptions. Hence, the most important empirical implication of this paper is whether the elasticity of substitution is constant or decreasing.

Our main observations are driven by the existence of non-homotheticity in our growth model specifications of utility. The importance of non-homotheticity in generating the EKC is already discussed and explained in detail by López (1994) and Plassmann and Khanna (2006). Specifically, López (1994, section 1.3) discusses the role of non-homothetic preference like us, whereas Plassmann and Khanna (2006, lemma 1) show how non-homothetic preferences or a non-homothetic pollution function are a necessary condition for the existence the EKC. Plassmann and Khanna (2006) generate their results within a very general framework and as such their results do not depend on specific model assumptions or choice of functional form, like López (1994). This means that López (1994) and Plassmann and Khanna (2006) provide an overarching framework with which we can classify the

[2]Exponential utility is also known as constant absolute risk aversion (CARA) utility. In this paper, we refer to it as "decreasing elasticity of substitution (DES) utility" to emphasize the fact that the elasticity of substitution is decreasing under this utility. We also note that exponential utility is not the only form of utility that exhibits a decreasing elasticity of susbstitution.

[3]Power utility is also known as constant relative risk aversion (CRRA) utility and constant elasticity of substitution (CES) utility.

existing EKC models in terms of how non-homotheticity is introduced. The first group of models, such as Stokey (1998) and Hartman and Kwon (2005), rely on a constraint which is binding either before or after the peak of environmental degradation to generate non-homotheticity. In these constraint driven models, the environmental degradation takes inverted V-shape path, rather than an inverted U-shape. The second type of models, as with the model presented in this paper, employ no constraint like the first group; instead non-homotheticity emerges from gradual changes in the curvature of the production or utility function. This group also includes Andreoni and Levinson (2001), where the abatement technology exhibits increasing returns to scale, which is able to generate an inverted U-shape EKC.

To motivate our numerical experiments, Section 2 in this paper contains some theoretical results, which are equivalent to those of Plassmann and Khanna (2006), and as such adds little *qualitatively* to the literature. Instead, we study the role of the elasticity of substitution in understanding the EKC by undertaking numerical analyses. Methodologically, therefore, our approach is totally different from López (1994) and Plassmann and Khanna (2006). Our main results are presented in Section 3, where we demonstrate that the EKC can emerge with *quantitatively* plausible economic parameters under a wide range of model specifications. This means that we generate results that can be used to draw practical as well as empirical implications about the conditions, under which we may observe the EKC. To this end, our paper yields two key practical implications.

First, although the inverted U-shape of the EKC is important from a theoretical perspective, policy makers and the public are essentially more interested in the fate of the environment in the long-run. So as already noted, unless there are exogenous binding constraints, the elasticity of substitution must be low enough (i.e., lower than a threshold) to yield improving environmental quality when income is high enough. This is indeed a key observation found in the sustainability literature, as noted above.

Second, there are many types of pollutants and many dimensions of environmental quality, meaning that the elasticity of substitution and its threshold will take different values for different types of pollutants. In this regard, our numerical experiments suggest that pollutants such as CO_2 are likely to keep increasing in the future, because the depreciation rate of the stock of CO_2 is very low (Stern, 2007). In our models, not surprisingly, pollutants with a low stock depreciation rate (i.e., the speed at which the environment assimilates the pollution) have a low (tight) threshold with respect to the elasticity of substitution, below which the environment improves as income increases. This implication is supported by the observations reported in the literature (e.g., Lieb, 2004 and Brock and Taylor, 2005); the EKC is observed for flow pollutants (because the environment can deal with this type of pollution quickly) rather than slowly depreciating stock pollutants.

The plan of this paper is as follows. Section 2 motivates the importance of the elasticity of substitution by using a small model. Section 3 first shows the Slutsky decomposition for simple models, and then numerically demonstrates that DES utility generates the EKC in a wide range of models. Section 4 discusses some additional key issues and the last section concludes.

2 A Model With An Analytical Solution

In this section, we introduce and examine a simple general equilibrium model, where the only source of growth is exogenous productivity change W. The main purpose of Section 2 is to motivate the growth models we employ in the subsequent numerical exercises. Although the model in this section differs from Plassmann and Khanna (2006) in that we consider an economy expressed in terms of consumption and environmental quality, as opposed to consumption and abatement in their paper, the results are essentially equivalent to theirs.

Throughout this paper, following the convention in the literature, we assume there is no pollution emission externality. This means that emissions of pollution are priced correctly, as we implicitly assume that the government successfully employs a policy mechanism to internalise any such externalities. In contrast, if the costs of pollution damage are totally external and pollution is treated as an input to production, there is no incentive for firms to cut their emissions and, hence, we cannot observe the EKC. Also, by assuming that pollution externalities are internalised, we are able to solve our models as a social planner's problem, and we do so in Section 3. In this section, however, to isolate the effects of preferences from those of production, we solve the household's and firm's optimization problems separately.

In addition, this paper assumes that pollution emissions work as if they are a production factor. Although modeling pollution as a by-product is intuitively more appealing, as Stokey (1998) shows, under reasonable regularity assumptions, modelling it as a by-product or as a production factor does not make any difference. Finally, we measure the income level by either the level of exogenous technology or accumulated capital stock, depending on the context.

2.1 The Household

We start with a representative household. Its utility U is increasing in both general consumption C and the environment R. Here, R shows the quality of the environment, and its service flow is higher for higher R. The household takes the price of the environment P_R as given, and the price of C is the numéraire. Household income is the compensation for the pollution emission $P_R X$, which is used to purchase C. The environmental resource constraint implies that higher emissions lead to lower R, and it also implies that the entire endowment is effectively owned by the household.

$$\max \quad U(C, R) \tag{1a}$$

$$\text{s.t.} \quad C = P_R X \qquad \text{(budget constraint)} \tag{1b}$$

$$X = 1 - R \qquad \text{(environmental resource constraint)} \tag{1c}$$

The environmental resource constraint (1c) implies that (i) the upper and lower limits of R are 1 and 0, respectively; (ii) the price charged for the pollution emission is equal to the price of the environment i.e., $P_X = P_R$; and (iii) $\frac{\mathrm{d}R}{\mathrm{d}W}\frac{W}{R} = -\frac{X}{R}\frac{\mathrm{d}X}{\mathrm{d}W}\frac{W}{X}$. Solving $\max U(P_R X, 1 - X)$, we obtain one first order condition (FOC) for X, and C and R are determined by the above two constraints. This means that we can write R, X and C as functions of P_R (as well as utility parameters). Recognizing that $\mathrm{d}(R/C)/\mathrm{d}P_R$ is the derivative of the ratio R/C, the elasticity of substitution η between C and R can be decomposed into the price elasticities of R and C.

$$\eta = -\frac{\mathrm{d}(R/C)}{\mathrm{d}P_R}\frac{P_R}{R/C} = \frac{\mathrm{d}C}{\mathrm{d}P_R}\frac{P_R}{C} - \frac{\mathrm{d}R}{\mathrm{d}P_R}\frac{P_R}{R} \tag{2}$$

The key trick here is that we do not solve the utility maximization explicitly. Instead, we summarize the household's optimal behavior by η.[4] Note that η is not necessarily a constant unless the household has CES utility. Also, our definition of η is only based on the observed changes in quantities and their

[4] Note that P_R is the only signal that the household receives exogenously. Hence, we take total derivatives with respect to it. This also implies that technology W can only affect the household's behavior through P_R. Since there is no other channel, through which W affects the household's decision, the sign of $\frac{\mathrm{d}X}{\mathrm{d}W}\frac{W}{X}$ in (5) is solely determined by the curvature (η) of utility at the optimum.

relative price; see Section 2.3 for more details.

By applying the chain rule,[5] we now decompose the elasticity of R with respect to technology W as follows.

$$\frac{\mathrm{d}R}{\mathrm{d}W}\frac{W}{R} = \frac{\mathrm{d}C}{\mathrm{d}W}\frac{W}{C} - \eta\frac{\mathrm{d}P_R}{\mathrm{d}W}\frac{W}{P_R} \tag{3}$$

It is worth noting that equation (3) corresponds to equations (7) and (7e) in Plassmann and Khanna (2006). To see this, our (3) can be rewritten as $\frac{\mathrm{d}R}{\mathrm{d}W}\frac{W}{R} > 0 \Leftrightarrow \frac{\mathrm{d}C}{\mathrm{d}W}\frac{W}{C} > \eta\frac{\mathrm{d}P_R}{\mathrm{d}W}\frac{W}{P_R}$. Both Plassmann and Khanna (2006) and our (3) do not assume a specific utility function U, and both have a linear resource constraint. They have an anonymous pollution function, while we have an anonymous production function. Also, their (7) and (7e) are derived solely from the pollution function and the resource constraint, whereas our (3) is derived solely from the definition of the elasticity of substitution. Because of this, our derivation parallels that in Plassmann and Khanna (2006).

2.2 Production

Next, we turn to the production side of the economy. To produce C, the representative firm must emit pollution X with Hicks neutral productivity W.[6] As noted above, we treat X as a factor of production.

$$\begin{aligned} C &= Y && \text{(market clearing)} \\ Y &= Wf(X) && \text{(production)} \end{aligned}$$

At the firm's optimum, $P_R = \partial C/\partial X = Wf_X(X)$. Hence, it is straightforward to show that

$$\frac{\mathrm{d}C}{\mathrm{d}W}\frac{W}{C} = \bar{\eta}\frac{\mathrm{d}Y}{\mathrm{d}W}\frac{W}{Y} \quad \text{where } \bar{\eta} = 1 \text{ in this case} \tag{4a}$$

$$\frac{\mathrm{d}Y}{\mathrm{d}W}\frac{W}{Y} = 1 + \frac{P_R X}{Y}\frac{\mathrm{d}X}{\mathrm{d}W}\frac{W}{X} \tag{4b}$$

$$\frac{\mathrm{d}P_R}{\mathrm{d}W}\frac{W}{P_R} = 1 + \varepsilon_{XX}\frac{\mathrm{d}X}{\mathrm{d}W}\frac{W}{X} \tag{4c}$$

[5] That is, $\frac{\mathrm{d}C}{\mathrm{d}P_R}\frac{P_R}{C} = \frac{\mathrm{d}C}{\mathrm{d}W}\frac{W}{C}\frac{\mathrm{d}W}{\mathrm{d}P_R}\frac{P_R}{W}$ and $\frac{\mathrm{d}R}{\mathrm{d}P_R}\frac{P_R}{R} = \frac{\mathrm{d}R}{\mathrm{d}W}\frac{W}{R}\frac{\mathrm{d}W}{\mathrm{d}P_R}\frac{P_R}{W}$.

[6] For a non-Hicksian technology, we can develop a similar analysis, although the results become less sharp. For example, if technology is emission-augmenting, it has a stronger emission saving effect (to produce a certain amount, a better technology requires less emission), but it has a more direct impact on the marginal product of emissions. In this case, since both offset each other, although η still plays a key role, the net effect is uncertain.

Note that the second order optimality condition implies that $\varepsilon_{XX} = \frac{f_{XX}X}{f_X} \leq 0$. For example, $\varepsilon_{XX} = -\alpha$ under a Cobb-Douglas production function, $Y = Wf(X) = W\bar{K}^{\alpha}X^{1-\alpha}$ where $\bar{K}$ is a fixed production factor. Although $\bar{\eta}$ is always one in this section regardless of the exact functional forms of U and f, $\bar{\eta}$ can take different values depending on the exact model specification, as will be evident in Section 3.

2.3 The Equilibrium

Gathering both household and firm optimality conditions together, in equilibrium,

$$\left(\frac{X}{R} + \bar{\eta} - \eta\varepsilon_{XX}\right)\frac{\mathrm{d}X}{\mathrm{d}W}\frac{W}{X} = \eta - \bar{\eta} \tag{5}$$

Because the inside of the bracket on the left hand side is positive,[7] if $\eta > \bar{\eta}$, environmental quality deteriorates as W increases (i.e., $\frac{\mathrm{d}X}{\mathrm{d}W}\frac{W}{X} > 0$), and vice versa.

There are several remarks we can make about this model.

First, as discussed in the Introduction, if η decreases as income grows and passes through $\bar{\eta}$ from above, we observe the EKC, which we investigate in the next section.

Second, the definition of η here is only based on the observed quantity and price changes. However, the change in P_R also implies a change in wealth because, while the environmental endowment is fixed at one, which is the only wealth in this model, its price P_R changes. However, although our η mixes up both income and substitution effects in general, η exactly corresponds to the Hicks substitution effect for CES utility, because the income effects on R and C are exactly the same and they offset each other in ratio R/C. For our DES utility, in Section 3 we show that the change in η is mostly driven by the Hicks substitution effects under reasonable parameter assumptions. Because of this, η can be negative without violating any second order optimality conditions, especially if preferences are non-homothetic.

Third, to provide some intuition, consider (3). To avoid any ambiguity due to the gap between

[7] This is obvious if $\eta > 0$. For $\eta < 0$, assuming C is a normal good, $\eta = \left(\frac{\mathrm{d}C}{\mathrm{d}W}\frac{W}{C} - \frac{\mathrm{d}R}{\mathrm{d}W}\frac{W}{R}\right)\frac{\mathrm{d}W}{\mathrm{d}P_R}\frac{P_R}{W}$ takes its minimum value, when the preference is such that $\frac{\mathrm{d}C}{\mathrm{d}W}\frac{W}{C} = 0$; i.e., households choose to use all additional wealth to buy back R. Since $C = Y = P_R X$, from (4b) and (4c), we find $-1 = \frac{\mathrm{d}X}{\mathrm{d}W}\frac{W}{X} = -\frac{R}{X}\frac{\mathrm{d}R}{\mathrm{d}W}\frac{W}{R}$ and $\frac{\mathrm{d}P_R}{\mathrm{d}W}\frac{W}{P_R} = 1 - \varepsilon_{XX}$, implying $\eta = -\frac{X}{R}\left(\frac{1}{1-\varepsilon_{XX}}\right)$. Hence, $\frac{X}{R} - \eta\varepsilon_{XX} = -\eta > 0$.

η and Hicks substitution effect, assume CES preferences. Suppose temporarily that counterfactually $\frac{\mathrm{d}P_R}{\mathrm{d}W}\frac{W}{P_R} = 0$ (no change in relative price) so that there is no substitution effect operating. In this case, (3) implies that $\frac{\mathrm{d}R}{\mathrm{d}W}\frac{W}{R} = \frac{\mathrm{d}C}{\mathrm{d}W}\frac{W}{C} = 1$; i.e., both R and C increase at the same rate as W increases. This is the direct income effect (the first term in (3)). However, as W increases, P_R also increases in general. Since the production cost of C decreases as the production technology improves, it is natural to think that P_R is increasing in W.[8] Unless $\eta = 0$ (Leontief utility, extremely inelastic), an increase in P_R induces the household to shift its demand mix from R to C. This substitution effect is stronger when η is larger (the second term in (3)).[9] All in all, as W increases, the income effect leads to an improvement in R, whereas the opposite is true for the substitution effect; and, when η is large enough, the substitution effect is dominating, and vice versa. Intuitively, if C and R are close substitutes (η large enough), as the production cost of C decreases (i.e., as W increases), people want to exploit it by tilting their demand mix toward C simply because C is now cheaper; that is, they give up the quality of R to produce more C.

Fourth, non-homotheticity on the production side can also generate the EKC. To see this, assume that $0 < \eta < \bar{\eta}$ is a constant so that the right hand side of (5) is negative. Assume also that ε_{XX} is positive and variable. While $\varepsilon_{XX} > 0$ implies the violation of the second order optimality condition for individual firms, it can be justified in aggregate if, say, there is a production externality. In this case, if $\varepsilon_{XX} > \frac{1}{\eta}\left(\frac{X}{R} + \bar{\eta}\right)$, the environment deteriorates as W increases (i.e., $\frac{\mathrm{d}X}{\mathrm{d}W}\frac{W}{X} > 0$), and vice versa;[10] obviously, the EKC requires ε_{XX} to be decreasing in income. This is quite intuitive because $-1/\varepsilon_{XX}$ is the price elasticity of X, but with $\varepsilon_{XX} > 0$, an increase in P_R does not discourage firms to cut the emission. In this way, non-homotheticity in any part of the economy can be the source of the EKC at least potentially. See Andreoni and Levinson (2001) for an increasing returns to scale in abatement technology (see also Appendix A.1 of this paper to see how non-homotheticity works in their model).

[8] It can be shown that $\frac{\mathrm{d}P_R}{\mathrm{d}W}\frac{W}{P_R} \geq 0$ for a general utility function, because from (4c) and (5) we have $\frac{\mathrm{d}P_R}{\mathrm{d}W}\frac{W}{P_R} = 1 - \frac{\eta - 1}{\eta - (X/R + P_R X/C)/\varepsilon_{XX}} \geq 0$, which is zero only when $\eta = \infty$ and is strictly positive otherwise.

[9] The substitution effect here should be understood as the sum of Hicks income and Hicks substitution effects. But, these statements hold even if the substitution effect is defined as the Hicks substitution effect only. This is again because the Hicks income effects on C and R exactly offset each other in η for CES utility.

[10] Note that the production function per se is still homothetic (since there is only one production factor). But, the Lagrangian of the profit maximization $\mathcal{L}(C, X; \lambda) = C - P_R X + \lambda (W f(X) - C)$ is non-homothetic (in C and X). Note that non-homotheticity involves a subtlety, in the sense that a set of homothetic equations can be reduced to be a non-homothetic function. For example, two functions $y = x + z$ and $x = w^2$ are both homothetic. But, substituting out x, they reduce to one non-homothetic function: $y = w^2 + z$.

2.4 Summary

The main message of this section is that, at each income level, R deteriorates as income increases if η is low enough. Rephrasing this, for each W, there exists a threshold $\bar{\eta}$ such that $\mathrm{d}R/\mathrm{d}W \gtrless 0$ if $\eta \lessgtr \bar{\eta}$. Note that this section focuses on the *local* behavior of R, meaning that it describes the direction of R (i.e., R deteriorates or improves) *at each income level.* Although the EKC is a *global* phenomenon *over a range of income*, naturally we conjecture that if η is decreasing in income it shows the EKC, which we confirm in the next section.

3 Models with Numerical Solution

This section provides further results with some numerical examples. First, we study two models (I and II), which are special cases of the model developed in Section 2, by applying Slutsky decomposition. Model I (DES utility) demonstrates the importance of decreasing η in generating the EKC, whereas Model II (CES utility) works as a good benchmark because, for CES utility, our η and Hicksian effects are totally consistent. We then develop a third specification (Model III) to demonstrate that our main findings hold even with capital accumulation. The final model we introduce (Model IV) employs generalised isoelastic (GIE) preferences to show that it is the substitutability between C and R, rather than other curvature parameters, such as the coefficient of relative risk aversion, that determines the fate of R.

3.1 Model I: Exponential (DES) Utility

Model I studies the property of exponential (DES) utility in a static formulation (6a). In this model, output Y is produced by a linear production function with pollution emission X as a production factor, while productivity W increases exogenously (7a). All output is consumed as C (7c). The environmental endowment is normalized to be one, and the quality of the environment R is one minus X (7b). This simple model can be fully analytically solved and hence offers detailed analyses such as Slutsky decomposition of demand changes. In the next subsection, we apply the same analyses to

(6b).

$$\text{DES utility} \quad : \quad U(C,R) = -e^{-\alpha_C C} - \phi e^{-\alpha_R R} \tag{6a}$$

$$\text{CES utility} \quad : \quad U(C,R) = \frac{C^{1-1/\eta}}{1-1/\eta} + \phi \frac{R^{1-1/\eta}}{1-1/\eta} \tag{6b}$$

subject to

$$\text{production} \quad : \quad Y = WX \tag{7a}$$

$$\text{environmental endowment} \quad : \quad 1 = X + R \tag{7b}$$

$$\text{resource constraint} \quad : \quad Y = C \tag{7c}$$

For (6a) and (7), the Slutsky decompositions of R and C are:[11]

$$\frac{\mathrm{d}R}{\mathrm{d}W} = \frac{-1/W}{\alpha_R + \alpha_C W} - \frac{\alpha_C R}{\alpha_R + \alpha_C W} + \frac{\alpha_C}{\alpha_R + \alpha_C W}$$

$$\frac{\mathrm{d}C}{\mathrm{d}W} = \frac{1}{\alpha_R + \alpha_C W} - \frac{\alpha_R R}{\alpha_R + \alpha_C W} + \frac{\alpha_R}{\alpha_R + \alpha_C W}$$

For both of these decompositions, the first, second and third terms show Hicks substitution, Hicks income and the direct income effects, respectively (see Table 1). The Slutsky decomposition for R shows the positive income effect and the two negative Hicks effects, and these effects all shrink in absolute terms as W increases.

It can be shown that (a) the price of the environment is equal to technology level, $P_R = W$, and (b) the value of total wealth is also W. These two observations are true in Model II as well. The former is because of our simple production function and the latter is because the only wealth in this economy is the environmental endowment, which is normalized to be one and its price is $P_R = W$. The direct income effect is the effect of a change in W as the price of wealth. This is positive because, as wealth increases, demand for R increases, because R is a normal good. However, P_R increases as the marginal product of pollution increases. Hence, due to negative Hicks effects, the demand for R is

[11] The derivation is straightforward but tedious. The technical appendix is available from the authors upon request. Hicks income effect captures the effect of the change in real income because of the change in a general price level due to a change in a price. Hicks substitution effect is the effect of the change in a relative price after adjusting for the Hicks income effect. The direct income effect simply means the effect of the change in wealth, keeping relative prices unchanged.

suppressed because it is now more expensive. Note that this substitution effect (the sum of two Hicks effects) is due to a change in W as a (shadow) price. The direct income effect and two Hicks effects offset each other, which itself is true even for CES utility (see Table 2 below). Hence, whether $\mathrm{d}X/\mathrm{d}W$ ($= -\mathrm{d}R/\mathrm{d}W$) changes its sign from positive to negative or not depends on the relative strength of these two effects.

In this respect, Figure 1 shows model behavior with DES utility, where we set $\alpha_C = 1.0$, $\alpha_R = 1.0$ and $\phi = 0.5$. The upper left panel plots X, which we regard as environmental degradation, for each technology level W. As the EKC hypothesis postulates, when income is low (which is represented by low W), the economy accepts lower environment quality. However, once W exceeds a certain level, the economy starts cutting X. The upper right panel shows that such a turning point coincides with η being 1, at which $\mathrm{d}X/\mathrm{d}W$ changes its sign. The lower right panel shows the Slutsky decomposition of $\mathrm{d}R/\mathrm{d}W$. It is now obvious that the decrease in the two Hicks substitution effects in absolute term is fast enough relative to that of the direct income effect. That is, as discussed in Section 2, the substitution effect is first stronger but later weaker. The lower left panel shows the locus of the equilibrium. An increase in W (both as the shadow price and as wealth) is represented by the clockwise rotation of the budget constraint (straight lines). The optimum points are the tangency points between the indifference curves and the budget constraints for different W. As is visually clear, non-homotheticity of the preference is the key to generating the inverted U-curve.

Finally, asymptotically (i.e., for very large W), R approaches its upper limit 1 in this model. In a sense, the production of C is squeezed by the conservation of R. That is, C is increasing without limit but increasingly slowly.[12] Also, technically, when W is too small, the lower bound of R is binding; $R = 0$. Figure 1 plots the results only for W large enough for which the model has an interior solution.

[Figure 1: For Exponential (DES) Utility around here]

[Table 1: For Exponential (DES) Utility around here]

[12] Note that, $\mathrm{d}C/\mathrm{d}W \to 0$ does not necessarily imply that there is a saturation point of C. The situation is somewhat like a logarithmic function; for $y = \ln x$, $\mathrm{d}y/\mathrm{d}x \to 0$ as $x \to \infty$, but y is unbounded.

3.2 Model II: Power (CES) Utility

To enable a comparison with Section 3.1, Figures 2 and 3 show the results of the essentially same exercises as in Section 3.1 except we now employ power (CES) utility with $\eta = 3.0$ and 0.7, respectively; see (6b) and (7). Here, the two right panels show the elasticities, rather than the derivatives.[13] Unlike DES utility, they do not generate an inverted U-curve. Rather, they generate a monotone improvement or deterioration of R. The Slutsky decompositions are:

$$\frac{\mathrm{d}R}{\mathrm{d}W}\frac{W}{R} = -\eta\,(1-R) - R + 1 \tag{8a}$$

$$\frac{\mathrm{d}C}{\mathrm{d}W}\frac{W}{C} = \eta R - R + 1 \tag{8b}$$

Again, the first, second and third terms show Hicks substitution, Hicks income and the direct income effect, respectively (see Table 2). The difference between these two expressions shows that, with CES utility, η captures Hicks substitution effect only (substitute these into (2)). With CES utility the income effect is 1 for both C and R, meaning that C and R increase at the same rate as W if there is no price change. For $\eta > 1$, the sum of the two Hick effects is always dominating the direct income effect. Contrarily, for $\eta < 1$, it is always dominated by the direct income effect. In this way, $\frac{\mathrm{d}R}{\mathrm{d}W}\frac{W}{R}$ is always negative (positive) for $\eta > 1$ ($\eta < 1$). From Figures 2 and 3, it is now clear that the change in substitutability is not enough to overturn the direction of R. This is in sharp contrast with DES utility.

Setting aside the EKC, Model II once again confirms that the key parameter is η in determining the sign of $\frac{\mathrm{d}R}{\mathrm{d}W}\frac{W}{R}$ at each level of income. If utility is flexible (high η), R is a close substitute to C. In this case, the substitution effect is strong (i.e., a small change in the relative price induces a large shift in R/C) and hence it is dominating, which discourages the demand for R as the production cost of C decreases (i.e., as W increases). If preferences are inflexible (low η), people do not want to switch from R to C very much, even though R becomes more expensive (relative to C). In this case, the income effect is dominating.

[13] Because power utility functions exhibit quasi-multiplicative separability, it is more natural to show the elasticities. On the contrary, since exponential utility functions exhibit quasi-additive separability, it is more straightforward to show derivatives. We discuss this further in Section 4.3. The key reference for this is Behrens and Murata (2007); see also Barde (2010) for further discussions.

[Figure 2: Power (CES) Utility with $\eta = 3.0$ around here]

[Figure 3: Power (CES) Utility with $\eta = 0.7$ around here]

[Table 2: For Power (CES) Utility around here]

3.3 Model III: Exponential Utility with Capital and Pollution Stocks

We now consider a dynamic model with capital and pollution stocks, i.e., with two endogenous state variables $\{K_{t-1}, X_{t-1}\}$. Since we discretize the model to aid computation,[14] we formulate the model in discrete time from the beginning. Here, we measure the income level by accumulated capital stock K_{t-1}.

$$
\begin{aligned}
V(K_{t-1}, X_{t-1}) &= \max \sum_{t=0}^{\infty} \beta^t \left\{ -e^{-\alpha_C C_t} - \phi e^{-\alpha_R R_t} \right\} && \text{(9a)} \\
\text{s.t.} \quad Y_t &= A K_t^{\alpha} Z_t^{1-\alpha} && \text{(9b)} \\
1 &= X_t + R_t && \text{(9c)} \\
K_t - K_{t-1} &= Y_t - C_t - \delta_K K_{t-1} && \text{(9d)} \\
X_t - X_{t-1} &= Z_t - \delta_X X_{t-1} && \text{(9e)}
\end{aligned}
$$

In this model, the production of output Y_t takes flow pollutant Z_t and capital K_t as inputs (9b). Pollution stock X_t is the accumulation of Z_t, where, if there is no Z_t, X_t decreases at rate δ_X, because of the assimilative capacity of the environment (9e). Pollution stock X_t reduces the environmental quality R_t (9c). The parameter values are; $\delta_K = 0.1$ (10% annual capital depreciation rate), $\alpha = 0.5$ (capital share in production is one half), $\beta = 1/(1 + 0.06)$ (6% annual risk-free rate), $\alpha_C = \alpha_R = 1$ and $\phi = 1$. We experiment with several values for δ_X.

Figure 4 shows results for $\delta_X = 1$, where X_{t-1} is not a state variable anymore and $X_t = Z_t$ (flow pollutant). The upper two panels show that the shape of the EKC is similar to Model I. Since the peak of X_t appears to the left of the steady state, starting from a low level of capital and output, we observe the inverted U-shape of X_t. Also, as predicted, η is decreasing and the shadow price of the

[14] We implement the standard Euler equation iteration (see Appendix for details) for Model III. Also all dynamic models (Models III and IV) have a steady state at which economic growth stops. One may be tempted to seek a long-run balanced growth path instead of a short-run dynamics around the steady state, but given the non-monotonic nature of the EKC it is hard to construct a model with a balanced growth path.

environment P_R is increasing in K_t (lower left panel). One important difference is that the threshold value of η is now around 2. We discuss the significance of this value further in Section 4.1. Under this parameter assumption, starting with $K_0 = 0.49$, it takes 10 to 15 years for Y_t (as it does for K_t) to arrive at the steady state, while the peak of environmental degradation X_t is reached in the third year (lower right panel). Although arriving at the peak in three years may sound a bit quick, it is quite easy to delay the peak of X_t, for example, by setting ϕ higher than 1. Note that we do not target any special peak year in this paper, because the exact shape of the EKC is different for different pollutants as we mentioned in the Introduction. All in all, the qualitative implication is the same as that of Model I, as η decreases the speed of environmental deterioration decreases and at a certain point it becomes negative.

[Figure 4: With Capital Stock around here]

Figure 5 shows the equilibrium pollution emission for $\delta_X < 1$ (stock pollutant).[15] There are several observations worth mentioning. First, not surprisingly, when δ_X is close to 1, model behavior resembles that of the flow pollutant case (Figure 4). For $\delta_X = 0.9$, pollution emission Z_t is almost unaffected by pollution stock X_{t-1} (the surface is almost flat along X-axis in the upper left panel). Second, as δ_X decreases, the inverted U-shape becomes weaker, and for δ_X low enough it disappears for a reasonable range of W_t. This is again not surprising, because, at the limit $\delta_X \to 0$, the behavior of the pollution stock is one sided;[16] i.e., the possibility of the inverted U-shape is physically eliminated. Third, output is low when X_{t-1} is high, because there is little leeway to emit additional Z_t. In a sense, having high stock pollution is similar to having a high debt level. Fourth, capital and output in the steady state are strongly affected by δ_X. Intuitively, in the steady state, the economy can only emit pollution at an amount the environment can assimilate; $Z = \delta_X X$. However, the optimal size of X is limited by preferences. Hence, as δ_X decreases towards zero, allowable pollution emission Z for production decreases. In most parameter ranges, this level effect of the depreciation rate is quite strong. Under our parameter assumption, moving from $\delta_X = 0.9$ to $\delta_X = 0.3$, the steady state output becomes one third.

[Figure 5: With Pollution Stock around here]

[15] The vertical lines in the three panels show the steady state, and the lines on the x, y-plane show the contour sets.

[16] This is an irreversibility case. For $\delta_X < 1$, if in addition there is uncertainty in the model, the real option kicks in.

3.4 Model IV: GIE Preference

To complement the models above, we now examine generalized isoelastic (GIE) preferences, pioneered by Epstein and Zin (1989, 1991), Svensson (1989) and Weil (1990), which have previously been applied to environmental issues by Smith and Son (2005). With GIE preferences we can separate the following three economic concepts; (i) η is the elasticity of substitution between R and C, (ii) θ is the elasticity of intertemporal substitution and (iii) γ is the coefficient of relative risk aversion.[17] In this version of the model, since period utility is homothetic as in Model II, we do not observe the EKC. Rather, our intention is to demonstrate that it is η, but not θ or γ, that determines the fate of R. To make γ and θ meaningful, the model is dynamic and stochastic.

$$V_t\left(W_t, K_{t-1}\right) = \max\left\{U\left(C_t, R_t\right)^{1-1/\theta} + \beta E_t\left[V_{t+1}\left(W_{t+1}, K_t\right)^{1-\gamma}\right]^{\frac{1-1/\theta}{1-\gamma}}\right\}^{\frac{1}{1-1/\theta}} \tag{10a}$$

$$\text{where} \quad U\left(C_t, R_t\right) = \left(C_t^{1-1/\eta} + \phi R_t^{1-1/\eta}\right)^{\frac{1}{1-1/\eta}} \tag{10b}$$

subject to

$$Y_t = W_t K_{t-1}^{\alpha} Z_t^{1-\alpha} \tag{11a}$$

$$1 = X_t + R_t \tag{11b}$$

$$K_t - K_{t-1} = Y_t - C_t - \delta_K K_{t-1} \tag{11c}$$

$$X_t - X_{t-1} = Z_t - \delta_X X_{t-1} \tag{11d}$$

$$\ln W_t = \left(1-\rho_W\right)\ln A + \rho_W \ln W_{t-1} + \xi_t \text{ where } \xi_t \sim N\left(0, \sigma_\xi\right) \tag{11e}$$

Most of the parameter values are the same as before; $A = 1$ (steady state technology level), $\beta = 1/\left(1+0.06\right)$, $\alpha = 0.5$, $\delta_K = 0.1$, and $\phi = 1$. For simplicity, we assume the flow pollutant; $\delta_X = 1$ (hence, X_{t-1} is not a state variable). Technology shock is fairly persistent ($\rho_W = 0.6$) but very volatile ($\sigma_\xi = 0.2$).[18] To demonstrate the importance of η we use two values for it; $\eta = 3.0$ and 0.5.

[17] Note that, if $\theta = 1/\gamma$, GIE reduces to von Neumann-Morgenstern's expected utility (vNM). Thus, even under vNM preferences, η can be set independently from $\theta = 1/\gamma$.

[18] This is very large compared to the convention in business cycle literature, in which $\sigma_\xi = 0.01$ or lower (see Cooley and Prescott, 1995, for example). Also, $\gamma = 24$ for our sensitivity analysis is extremely large. For example, Mehra and Prescott (1985) suggest that a reasonable value for γ is 10 or less. These extreme choices are because otherwise it is hard to see the effect of changing γ visually, since the effect of uncertainty is very small.

We set $\theta = 2.0$ and $\gamma = 4.0$ (see Barro, 2009) as the baseline case, but we also check the sensitivity of the results to these parameters.

The bold lines in Figure 6 show that, measuring the income level by capital accumulation K_{t-1}, X_t is decreasing for η low enough, and vice versa. Figure 6 also shows the results of different values of θ and γ, and it demonstrates that they have no qualitative effects. In this formulation, their effects are small even quantitatively,[19] because of the assumption $\delta_X = 1$. If $\delta_X < 1$, today's choice of Z_t depends not only on today but also future utility through the accumulation of the pollution stock X_t, and the optimal choice of Z_t is strongly affected by uncertainty in the future; in this case, the effects of γ should be larger. Similarly, we know from standard saving theory that, with lower θ, people strongly prefer a smooth consumption path. If $\delta_X < 1$, the household can use X_t as a (dis)saving tool like K_t, but such an effect is absent for flow pollutants. In summary, although θ and γ can have much stronger quantitative effects depending on the value of δ_X, only η changes model behavior qualitatively.

[Figure 6: With GIE preference around here]

This result is, as noted in the Introduction, different from López (1994) in that he discusses the role played by the coefficient of relative risk aversion. In many cases, at least in constructing theoretical models, this difference is rather trivial; it is well known when employing CES utility, the elasticity of substitution among goods, the elasticity of intertemporal substitution and the coefficient of relative risk aversion are often all governed by the same single parameter[20]. But, the property that a single parameter governs different economic concepts has long been criticised by many empirical studies, because often the data suggests that, while people must be very risk averse, their intertemporal elasticity is fairly high; see Barro (2009) for example. This observation led to the generalization

[19]Note that the lines should not (and actually do not) pass through the *non-stochastic* steady state. However, since the effect of uncertainty is very small in this formulation, it is almost visually impossible to see that they do not pass through it.

[20]Even within the vNM framework, it is possible to disentangle the elasticity of substitution among goods from the other two, but not possible to separate the intertemporal elasticity of substitution and the relative risk aversion coefficient. For example, $U[C, R] = \left(C^{1-1/\eta} + \phi R^{1-1/\eta}\right)^{\frac{1-1/\theta}{1-1/\eta}}$, where the additive separability between C and R is lost. In the notation of López (1994) this means $\mu_{21} \neq 0$. Note algebraically that, different from our GIE specification (10), there are two separability assumptions in his indirect welfare function (equation (4) in López, 1994); (a) the separability assumption ($\mu_{21} = 0$) between revenue and emissions, which is imposed to facilitate the interpretation of the results (see footnote 10 on p.170); and (b) time separability, which is implicit in his static model setup. In our model II, for example, these two separability conditions are satisfied, and indeed η can also be interpreted as the risk aversion coefficient.

of the von-Neumann Morgenstern by Epstein and Zin (1989, 1991) and Weil (1990). This in turn has significant empirical implications for model simulation. For example, López (1994, p.172) states, interpreting η as the coefficient of relative risk aversion, which *"typically ranges between one and two"*, pollution would not increase if the elasticity of substitution on the production side is greater than 0.5. However, using the estimated value of the coefficient of relative risk aversion to evaluate model behavior can generate inappropriate quantitative results. Instead, we suggest to use the estimated value of the elasticity of substitution between C and R in utility. Also, a hidden implication in López (1994) is that the preference parameters of each pollutant are irrelevant, which is different from our view (see Section 4.1).

4 Discussions

4.1 Types of Pollutants and the EKC

As Carson (2010) and many other empirical studies report, the EKC is not a common observation for all types of pollutants (e.g., CO_2) or environmental goods and services (e.g., biodiversity). It is also the case that the exact shape and turning point of the EKC differs among pollutants as well as geographical and administrative locations. This section discusses how our results apply to various types of pollutant and the resulting optimal policy response of the government.

First of all, as already shown in Figure 5, threshold $\bar{\eta}$ is mostly affected by the depreciation rate of pollution δ_X, and its effects seem to dominate those of the other parameters. Although $\bar{\eta}$ does not capture the entire effect of a parameter change, it is important because the environment degrades if η is above $\bar{\eta}$ and vice versa. Hence, the most important empirical implication is, not surprisingly, that we are less likely to observe the EKC for a pollutant with low δ_X.

For the other parameters, Figure 7 shows the effect of changing parameters in model III with a flow pollutant. Next to δ_X, the depreciation rate of capital δ_K affects $\bar{\eta}$ most; indeed, as shown in Figure 7, near the benchmark parameter assumption, as δ_K increases from 0.1 to 0.4, $\bar{\eta}$ decreases from near 2.0 to around 1.2. It seems that δ_K affects $\bar{\eta}$ mainly via the level of capital K. As δ_K increases, the optimal K decreases, which in turn reduces the marginal product of emission X.

For preference parameters α_R, α_C and ϕ, they mainly affect the level of emission, while they affect

$\bar{\eta}$ only a little. If society puts more weight on the environment (higher α_R, lower α_C and/or higher ϕ), the level of X tends to be higher for given capital accumulation. This is quite intuitive and not surprising. For lower ρ, the government (or society) chooses higher X. This has a similar intuition to δ_K; $\bar{\eta}$ is higher for lower ρ, because K is higher. However, this result can be overturned for a stock pollutant. If δ_X is low enough, the quality of the environment becomes an asset, and hence the present value of R $(= 1-X)$ is decreasing in discount rate ρ. Finally, the effect of capital share α in production is complicated near the benchmark parameter set. As α increases, the line of X rotates anti-clockwise. However, for α large enough (say, larger than 0.65), as α increases, the level of X decreases for the whole range of K. This is again intuitive; $1 - \alpha$ is the share of a pollutant and, if it is smaller, the firm needs to emit less X.[21]

[Figure 7: Sensitivity Analysis around here]

In sum, preference parameters and capital share in production mainly affect the level of emission but not the threshold $\bar{\eta}$. The threshold becomes lower (tighter), if the depreciation of capital increases and/or if that of pollutant decreases. Of these two, the latter seems to produce the stronger effect. Indeed, even under DES utility, if the depreciation of a pollutant is low enough, it seems that the EKC disappears for a reasonable range of W, as shown in Figure 5 above.

In terms of the characteristics of each pollutant,[22] setting aside the level of emission, the direction of the environmental degradation almost entirely depends on the depreciation rate of a pollutant. More specifically, a pollutant with lower δ_X is less likely to decrease in the future. For example, we conjecture that CO_2 emission is going to keep increasing as discussed in the Introduction, because, as documented in Stern (2007), its depreciation rate δ_X is quite low (i.e., the nature cannot reduce it quickly). Certainly, since we do not explicitly model any catastrophic disasters that could happen for extremely high CO_2 levels, and international coordination failure (we assume that the price of the emission is efficiently imposed on firms), this prediction might be premature. However, understanding these limitations, our model still suggests that it is likely that the government *optimally* allows CO_2

[21] On the production side, though we do not experiment numerically, if the elasticity of substitution ε_{KZ} between capital and emission is smaller, the threshold value $\bar{\eta}$ becomes smaller (the condition to reduce the environmental degradation becomes tighter). This means that, given the preference being DES, if a pollutant has small ε_{KZ}, it is less likely to exhibit the EKC. In our Cobb-Douglas production, $\varepsilon_{KZ} = 1$.

[22] Note that the depreciation of capital δ_K is irrelevant to the characteristics of each pollutant. Also, ρ and α_C are also independent from the types of pollutants. However, α, α_R and ϕ as well as δ_X should be different for different pollutants.

to increase in the future, given its low depreciation rate. The choice of adverb *optimally* may sound wrong, but *optimally* here can be understood that there is an incentive in the economy to allow CO_2 to keep increasing, or it is too painful for industries to cut CO_2, as implied by the lower-right panel of Figure 5.

4.2 Antecedent Literature

Having derived our various model results, we now place these findings within the context of the existing theoretical literature on the EKC. We find that it is useful to classify the literature into four groups; three of them are classes of models and the remaining group does not rely on any specific models or functional forms.

We start with López (1994) and Plassmann and Khanna (2006), because we can use the findings of these two papers to frame our discussion of the other groups. Unlike the other three groups, they do not have models, in the sense that they do not explicitly analyse specific economic structures with specific functional forms. Without explicit modelling, it is hard if not impossible to draw quantitative empirical implications. Also, a model that satisfies their conditions are not necessarily economically plausible. Nonetheless, their general conditions to generate the EKC are useful to understand the literature. And, among others, the most important condition that they identify is non-homotheticity, as already discussed. Hence, in the following, we review the existing models of EKC in light of non-homotheticity.

The first group of models is initiated by Stokey (1998), which we call "constraint driven models". This group includes papers such as Chimeli and Braden (2005), Lieb (2004), Hartman and Kwon (2005), Smulders (2006) and Smulders et al. (2011), to name a few. The most important feature of the constraint driven models is that an exogenous constraint is binding only either before or after the peak of the environmental degradation; i.e., the equilibrium is a corner solution only before or after the peak. Perhaps, Smulders (2006) makes this point most clearly, and we basically follow his explanation here. Suppose that an economic agent has CES utility with $\eta < 1$, which implies that, as demonstrated in Model II, people demand more R as they become richer. If there is no exogenously given constraint, R monotonically improves as the technology improves, which captures the decreasing right tail of the EKC. However, he adds a technical constraint, which states that, when the technology level is too low, the economy does not have enough ability to exploit environmental quality fully. Hence, until a certain technology level is attained, the maximum possible level of destroying R is limited as an increasing function of the technology level. This binding technological constraint forces R to follow a gradual increasing path before the peak of the EKC, which generates the increasing left tail of the EKC. Alternatively, it can be assumed that $\eta > 1$ (R deteriorates freely until W arrives at a certain point), and the regulator imposes some emission regulation once the economy reaches a certain level of technology (so that R improves). For the constraint driven approach, we do not need to assume non-homothetic preferences or production functions per se. Instead, non-homotheticity arises from the change in the mode of operation of the economy. Hence, this class of models show the inverted V-shape, rather than the inverted U-shape, at the point where a constraint becomes binding; it exhibits a sharp peak in the environmental degradation. This type of models are also powerful candidates to explain the EKC. First, though the empirical data shows an inverted U-shape, in our opinion, given the noisy nature of the data used in many empirical studies, the theoretically predicted sharp pointed inverted V-shape is not a caveat. Second, the assumption imposed in these models is often quite convincing. Smulders (2006, p.12), for example, argues that *"Loggers in a poor village simply lack the technical means to cut all trees in the rain forest surrounding the village. Prehistoric man could hunt many deer, but lacked the capacity to destroy the ozone layer."*

In the second group, which we refer to as interior solution models, there are two models that explicitly focus on economic primitives; one is Andreoni and Levinson (2001) (and Egli and Steger, 2007 as its dynamic extension), and the other is ours.[23] In this class of models, the source of the non-homotheticity is a (gradual) change in the curvature of a function. In the case of Andreoni and Levinson (2001), the issue reduces to whether the abatement technology exhibits increasing returns to scale (IRS) or not, while, in our case, it reduces to whether η is decreasing fast enough or not.

As the third group, Brock and Taylor (2005, 2010) extract the essence of the EKC by considering somewhat ad hoc models to match stylized facts. For example, the Brock and Taylor (2010) model is not an optimisation model, but an extension of the Solow growth model, such that; (i) emissions are proportionally increasing in output, (ii) the resource allocated to abatement is a constant share of output, and (iii) the speed of exogenous technological growth of abatement is sufficiently fast to ensure

[23] See Appendix A.1 for how non-homotheticity works in Andreoni and Levinson (2001)

that environmental quality is improving asymptotically (i.e., in balanced growth or in the long-run). Point (i) ensures that environmental degradation is fast when capital accumulation is low (because the output growth rate is higher in earlier periods as in the standard Solow model). This means that at some point in time the effects generated by (i) and (iii) are equated; i.e., environmental degradation must reach a peak. In the current context, however, point (ii) is the most important because the constant saving rate implies that, regardless of the amount of existing capital and environmental quality, society will allocate a constant share of resources to abatement. Although the model presented by Brock and Taylor (2010) does not explicitly include a utility function, it is possible to conjecture that, if there were preferences that exhibit constant resource allocation to abatement regardless of the changes in the amount of capital and environmental quality, they would be non-homothetic, because, given changes in quantities, presumably, the relative price of the environment should change, but people still keep paying a constant share of output to improve the environment. In this sense, their model has an affinity with the non-homothetic models discussed.

In summary, we can classify the existing EKC models based on how they generate non-homotheticity. In this relation, we have a couple of remarks. First, each class of models has its own merits (and demerits). If optimization is not of interest and constant saving rate is an acceptable assumption, the simplicity of Brock and Taylor (2010), for example, may be appealing. Between constraint driven and interior solution models, perhaps the choice is an empirical matter and some pollutants are suitable for constraint driven models but others are for interior solution models. Indeed, one important implication of Section 4.1 is that the characteristics of each pollutant are important. Second, whatever the source of non-homotheticity is, it must be economically meaningful. This coincides with the generality of López (1994) and Plassmann and Khanna (2006). For example, a model that assumes a sudden shift of the economic structure without any justification can generate the EKC. Such a model can satisfy the conditions identified by them, which however itself does not guarantee any empirical relevance.

Finally, setting aside the classification of the papers, Brock and Taylor (2010) also marks an important development in the EKC literature in that it is attempting to adopt a research strategy that has come to dominate macro economics over the last twenty years (Smith, 2012). Specifically, Brock and Taylor (2010) require exogenous technological progress in abatement to drive the emergence of the EKC. They support this argument by employing data on carbon emission rates. However, as Smith (2012) observes, their method is not equivalent to the macro calibration approach. What is highly relevant as a result of Brock and Taylor (2010) and the comment by Smith (2012) is the need to consider societal preferences for the environment and how they shape pollution control efforts given observed levels of income. In our view, by focusing on the elasticity of substitution, we have provided a basis for examining specific forms of preferences and what this means for the environment.

4.3 Implication from Competitive Limit

This paper is partly motivated by the literature of competitive limit. The idea of it is that, under exponential (DES) utility family, as the number of varieties available to consumers increases, monopolistic competition approaches perfect competition (see Behrens and Murata, 2007). That is, as the number of varieties increases the demand elasticity increases (to positive infinity at the limit). This is perhaps intuitively convincing; for example, the firm that produces blue widgets has relatively strong monopolistic power if there is only one competitor, the red-widget producer, but if, say, a purple-widget producer enters into the market, the blue firm's monopolistic power may be undermined. However, under power (CES) utility, this decrease in monopolistic power does not take place; the monopolistic power is constant regardless of the number of competitors. One property of the exponential utility is that, as the level of consumption of each good decreases, its elasticity of substitution increases. At the competitive limit, since, without income growth, as the number of varieties increases, money spent for each good decreases, and hence the demand for each type of goods becomes more elastic. In our case, as W increases, money spent for C and R both increases, and hence the demand for each of C and R becomes less elastic.

In the growth literature, researchers almost always assume power utility, perhaps because of its tractability. However, we would like to emphasize that power utility is not necessarily empirically more plausible than exponential utility. Setting aside the analytical tractability, the choice between them is totally an empirical issue. If exponential utility is plausible to a certain degree, then the EKC is equally plausible because the only requirement for it to exist is decreasing η in income. As demonstrated above, the models do not require any other specific assumptions other than that.

5 Conclusion

This paper demonstrates that, at each income level, environmental degradation is decreasing as people become richer if η is small enough, and vice versa. Here, the key parameter η is the elasticity of substitution between consumption and the service flow from the environment. Intuitively, η shows how easily the quality of the environment can be substituted by consumption in the household's preferences. As shown above, as people become richer, there are two main effects; (i) given a price of the environment, people demand more environment (income effect); and (ii) the environment becomes more expensive, which induces people to accept a lower quality of the environment (substitution effect). The latter is stronger when η is higher (i.e., societal preferences are more flexible). In this regard, we can understand the EKC such that the substitution effect is dominating when the economy is poor, but the income effect overwhelms the substitution effect when societies are richer. Indeed, this paper demonstrates that, exponential (DES) utility, with which η is decreasing in income, generates the EKC in a wide class of model formulations. Thus, the importance of this paper lies in the fact that the only requirement to have the EKC is that preferences are such that η is decreasing in income.

There are, however, other economic models that generate the EKC. We find that such models can be classified into four groups, of which two explicitly provides optimisation models for the EKC. One group employs an exogenous constraint which is binding only before or after the peak of environmental degradation. The other group includes non-homotheticity in a part of the economy as that it changes the response of the economy as income increases. Our model belongs to the latter group. Specifically, our non-homothetic preference (exponential utility) specification exhibits decreasing η, which is the main driver of the emergence of the EKC. Although this paper does not provide detailed analyses, non-homotheticity in any other part of the economy can, at least potentially, generate the EKC as well (see Andreoni and Levinson, 2001). As we have explained, this point has previously been identified by López (1994) and Plassmann and Khanna (2006).

Apart from the EKC, in terms of policy implications, to predict the long-run fate of the environment, we need to know whether η is low enough or not relative to its threshold value for a high income level. The exact values of η and its threshold depend on economy wide parameters such as the discount rate as well as the characteristics of each pollutant. Not surprisingly, our study suggests that a pollutant with a low depreciation rate, such as CO_2, tends to have a low (tight) threshold. This implies that it is likely that society (or the government) will allow such a pollutant to keep increasing in the long-run (or till a catastrophic phenomenon takes place).

In this relation, the most important empirical implication of this paper is that, for each pollutant, whether η is decreasing fast enough or not is the key to determining if its emission is going to increase or not. We argue that the non-monotonic behaviour of EKC is theoretically interesting but practically it is much more important to know whether environmental quality is improving or deteriorating as the economy grows. It is interesting to test whether η is decreasing fast or not, because, if so, economic success is consistent with environmental conservation.

A Appendix

A.1 Non-Homotheticity in Andreoni and Levinson (2001) and N- & M-Shaped EKCs

This section reviews Andreoni and Levinson's (2001) model *as a model building procedure.* Their model looks as follows.

$$\begin{aligned} \max U &= C - X \\ \text{s.t. } X &= C - A \\ A &= C^{\alpha} E^{\beta} \\ M &= C + E \end{aligned}$$

where utility U is increasing in consumption C and decreasing in pollution X and their elasticity of substitution is infinite (linear utility). Pollution X is proportionally increasing in C but can be reduced by employing abatement technology A, which is an increasing function of C and abatement effort E. The total resource M available, which is increasing at an exogenous rate, can be used either for consumption or abatement effort. In this model, if the abatement technology exhibits increasing returns to scale (i.e., $\alpha + \beta > 1$), the optimal X shows an inverted U-shape.

After substituting out some variables, it is easy to reformulate the model without X

$$\begin{aligned} \max U &= C^{\alpha} E^{\beta} \\ \text{s.t. } C &= Y = M - E \end{aligned} \tag{12}$$

where we can reinterpret E as environmental quality and $-E$ as pollution emission which contributes to the production of output Y. Here, obviously Cobb-Douglas utility and linear production functions both show homotheticity. One of the good points of this model is, since it has homothetic functions only, it is easy to generate the balanced growth path. Indeed, the solution to this optimization problem shows C/M and E/M are constants.

$$C = \frac{\alpha}{\alpha+\beta} M \text{ and } E = \frac{\beta}{\alpha+\beta} M$$

Non-homotheticity in this model does not enter into the core part of the model (12). Instead, it appears in X, which can be obtained after solving the core part of the model.

$$X = C - C^{\alpha} E^{\beta} = \alpha \frac{M}{\alpha+\beta} - \alpha^{\alpha} \beta^{\beta} \left(\frac{M}{\alpha+\beta} \right)^{\alpha+\beta} \tag{13}$$

For $\alpha + \beta > 1$ and $\beta < 1$, X shows an inverted U-curve in M. Up to here, we reviewed Andreoni and Levinson (2001).

Next, motivated by the fact that Andreoni and Levinson (2001) can be written as a balanced growth model, we re-define (13). Any re-definition is fine as long as it exhibits an inverted U-shape. A straightforward example is a quadratic function.

$$X = M_{+} - \left(E - \frac{\beta}{\alpha+\beta} M_{*} \right)^2 = M_{+} - \left(\frac{\beta}{\alpha+\beta} \right)^2 (M - M_{*})^2 \tag{14}$$

where M_{+} is a large positive number and M_{*} is the threshold income level; X is increasing in M for $M < M_{*}$, but X is decreasing in M for $M > M_{*}$. If we substitute $E = (M_{+} - X)^{1/2} + \frac{\beta}{\alpha+\beta} M_{*}$ back into the original formulation, we obtain

$$\begin{aligned} \max U &= C^{\alpha} \left((M_{+} - X)^{1/2} + \frac{\beta}{\alpha+\beta} M_{*} \right)^{\beta} \\ \text{s.t. } C &= Y = M - \left((M_{+} - X)^{1/2} + \frac{\beta}{\alpha+\beta} M_{*} \right) \end{aligned}$$

This model shows an inverted U-curve by construction, and can be solved first for intermediate variable E and then solve for X as a function of E. If we want to have N-shaped environmental degradation, we should have a proper cubic polynomial instead of (14), and, by having X as a fourth order polynomial of E, we can even construct even M-shaped curve.

In sum, this way of constructing a model can be summarized as follows. First, write a model with homothetic preference and production; they are homothetic in "transformed" environmental quality

E. Without non-homotheticity, it is relatively easy to find the balanced growth path. Second, define the "true" environmental degradation X as a function of E so that X shows an inverted U-shape. Finally, substitute E back into the original model, which shows non-homotheticity in X (but it shows homotheticity in E). Unlike Andreoni and Levinson (2001), the economic intuition of the models constructed in this way may be vague in general. However, if the main interest is not revealing the mechanism that generates the EKC but investigating the consequence of the EKC, this way of model building can be a good device, because it can generate EKC and the balanced growth path in an easy way (indeed, an analytical solution is frequently available).

A.2 Computational Details for Model III

◊ Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} -e^{-\alpha_C C_t} - \phi e^{-\alpha_R (1-X_t)} \\ +\lambda_t \left\{ A K_{t-1}^{\alpha} Z_t^{1-\alpha} - C_t + (1-\delta_K) K_{t-1} - K_t \right\} \\ +\mu_t \left\{ X_t - Z_t - (1-\delta_X) X_{t-1} \right\} \end{array} \right\}$$

◊ Equilibrium Equations (FOCs and constraints):

$$\begin{aligned}
\partial C &: \lambda_t = \alpha_C e^{-\alpha_C C_t} \\
\partial K_t &: \lambda_t = \beta \lambda_{t+1} \left\{ \alpha A \left(Z_{t+1}/K_t \right)^{1-\alpha} + (1-\delta_K) \right\} \\
\partial Z_t &: \mu_t = \lambda_t (1-\alpha) A \left(Z_t / K_{t-1} \right)^{-\alpha} \\
\partial X_t &: \mu_t = \mu_{t+1} \beta (1-\delta_X) + \alpha_R \phi e^{-\alpha_R (1-X_t)} \\
lomK &: K_t = A K_{t-1}^{\alpha} Z_t^{1-\alpha} - C_t + (1-\delta_K) K_{t-1} \\
lomX &: X_t = Z_t + (1-\delta_X) X_{t-1}
\end{aligned}$$

where lom stands for the law of motion.

◊ Non-Stochastic Steady State:

Defining $\rho = \frac{1-\beta}{\beta}$,

$$\begin{aligned}
\rho + \delta_K &= \alpha A \left(Z/K\right)^{1-\alpha} \\
\mu\beta\left(\rho + \delta_X\right) &= \alpha_R \phi e^{-\alpha_R(1-X)} \\
\frac{\mu}{\lambda} &= \left(1-\alpha\right) A \left(Z/K\right)^{-\alpha} \\
AK^{\alpha}Z^{1-\alpha} - C &= \delta_K K \\
Z &= \delta_X X
\end{aligned}$$

Hence, eliminating Lagrange multipliers, $\left(1-\alpha\right) A \left(Z/K\right)^{-\alpha} \left(\rho + \delta_X\right) = \frac{\alpha_R \phi e^{-\alpha_R(1-X)}}{\alpha_C e^{-\alpha_C C}}$. Taking the log of this,

$$\begin{aligned}
K &= \frac{\ln\left(\beta\frac{\mu}{\lambda}\frac{\alpha_C}{\alpha_R\phi}\left(\rho+\delta_X\right)\right) + \alpha_R}{\alpha_C\frac{C}{K} + \alpha_R\frac{X}{K}} \\
&\quad \text{where } \frac{Z}{K} = \left(\frac{\rho+\delta_K}{\alpha A}\right)^{\frac{1}{1-\alpha}},\ \frac{C}{K} = A\left(\frac{Z}{K}\right)^{1-\alpha} - \delta_K,\ \frac{X}{K} = \frac{1}{\delta_X}\frac{Z}{K}
\end{aligned}$$

◊ Euler Equation Iteration:

Define node points on the state space (K_{t-1}, X_{t-1}). Suppose that we preliminarily know optimal C_t and Z_t as functions of states K_{t-1} and X_{t-1} from the previous iteration step; that is, we have

$C_t = C[K_{t-1}, X_{t-1}]$ and $Z_t = Z[K_{t-1}, X_{t-1}]$. Then, we sequentially obtain

$$\begin{aligned}
K_t &= AK_{t-1}^{\alpha} Z_t^{1-\alpha} - C[K_{t-1}, X_{t-1}] + (1-\delta_K) K_{t-1} \\
X_t &= Z[K_{t-1}, X_{t-1}] + (1-\delta_X) X_{t-1} \\
C_{t+1} &= C[K_t, X_t] \qquad \text{(interpolate to adjust node points)} \\
Z_{t+1} &= Z[K_t, X_t] \qquad \text{(interpolate to adjust node points)} \\
\lambda_{t+1} &= \alpha_C e^{-\alpha_C C_{t+1}} \\
\mu_{t+1} &= \lambda_{t+1} (1-\alpha) A (Z_{t+1}/K_t)^{-\alpha} \\
\lambda_t^{new} &= \beta \lambda_{t+1} \left\{ \alpha A (Z_{t+1}/K_t)^{1-\alpha} + (1-\delta_K) \right\} \\
\mu_t^{new} &= \beta \mu_{t+1} (1-\delta_X) + \alpha_R \phi e^{-\alpha_R (1-X_t)} \\
C_t^{new} &= \frac{\ln(\lambda_t^{new}/\alpha_C)}{-\alpha_C} \\
Z_t^{new} &= \left(\frac{\mu_t^{new}}{\lambda_t (1-\alpha) A} \right)^{\frac{-1}{\alpha}} K_{t-1}
\end{aligned}$$

Iterate this until $C_t^{new} = C_t$ and $Z_t^{new} = Z_t$ at each node.

A.3 Computational Details for Model IV

◊ Non-Stochastic Steady State:

Since without any stochasticity, GIE preference reduces to vNM preference, it is straightforward, though tedious, to find the non-stochastic steady state. For the value function iteration, the steady state values are not necessary but often useful in, say, determining node points. Defining $\rho = \frac{1-\beta}{\beta}$, we find the ratios.

$$\begin{aligned}
\frac{Y}{K} &= \frac{\rho + \delta_K}{\alpha} \quad , \quad \frac{C}{K} = \frac{Y}{K} - \delta_K \quad , \quad \frac{Z}{K} = \left(\frac{\rho + \delta_K}{\alpha A} \right)^{\frac{1}{1-\alpha}} \quad , \quad \frac{X}{K} = \frac{1}{\delta_X} \frac{Z}{K} \\
\frac{\mu}{\lambda} &= (1-\alpha) \frac{Y/K}{Z/K} = (1-\alpha) A^{\frac{1}{1-\alpha}} \left(\frac{\rho + \delta_K}{\alpha} \right)^{\frac{-\alpha}{1-\alpha}}
\end{aligned}$$

With these ratios on hand, we can find the steady state value of K.

$$\frac{1}{K} = \left(\frac{\mu}{\lambda} \frac{\rho + \delta_X}{\phi/\beta} \right)^{-\eta} \frac{C}{K} + \frac{X}{K}$$

◇ Value Function Iteration:

Substitute out some variables to obtain

$$
\begin{aligned}
F_t\left(W_t, K_{t-1}, X_{t-1}\right) &= \max_{K_t, X_t}\left\{U_t\left(W_t, K_{t-1}, X_{t-1}; K_t, X_t\right) + \beta E_t\left[F_{t+1}\left(W_{t+1}, K_t, X_t\right)^{1-\gamma}\right]^{\frac{1-1/\theta}{1-\gamma}}\right\}^{\frac{1}{1-1/\theta}} \\
U_t\left(W_t, K_{t-1}, X_{t-1}; K_t, X_t\right) &= \left(\begin{array}{c}\left[W_t K_{t-1}^{\alpha}\left(\left(1-\delta_X\right)X_{t-1}-X_t\right)^{1-\alpha}+\left(1-\delta_K\right)K_{t-1}-K_t\right]^{1-1/\eta} \\ +\phi\left(1-X_t\right)^{1-1/\eta}\end{array}\right)^{\frac{1-1/\theta}{1-1/\eta}}
\end{aligned}
$$

Suppose that we have the functional form of F_{t+1} from the previous Iteration step; then we can maximize the RHS with respect to K_t and X_t to obtain the functional form of F_t. Replacing F_{t+1} with F_t, and repeat this until F_t converges.

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B Tables and Figures

Table 1: **Summary Table for Exponential (DES) Utility**

	analytical expression	for low W	for high W	$W \to \infty$
elas of subs η	$1 - \frac{\alpha_C C - 1}{W(\alpha_R + \alpha_C W)}\left(\frac{W}{R} + \frac{W}{1-R}\right)$	>1.0	<1.0	0.0
$\mathrm{d}R/\mathrm{d}W$	$\frac{\alpha_C C - 1}{R(\alpha_R + \alpha_C W)}$	-ve	+ve	0.0
Hicks Sub	$\frac{-1/W}{\alpha_R + \alpha_C W} < 0$	-ve		0.0
Hicks Income	$-\frac{\alpha_C \alpha_R}{\alpha_R + \alpha_C W}\frac{R}{\alpha_R} < 0$	-ve		0.0
Direct Income	$\frac{\alpha_C}{\alpha_R + \alpha_C W} > 0$	+ve		0.0
R	$\frac{\ln \phi \alpha_R / W \alpha_C + \alpha_C W}{\alpha_R + \alpha_C W}$	decreasing	increasing	1.0

Note: Because $\frac{W}{R} + \frac{W}{1-R} > 0$, the above analytical expressions show $\eta > 1 \Leftrightarrow \mathrm{d}R/\mathrm{d}W < 0$.

Table 2: **Summary Table for Power (CES) Utility**

	$\boldsymbol{\eta > 1}$			$\boldsymbol{\eta < 1}$		
	$W \to 0$	middle W	$W \to \infty$	$W \to 0$	middle W	$W \to \infty$
elas of subs η		constant at η			constant at η	
$\frac{\mathrm{d}R/R}{\mathrm{d}W/W}$	0.0	-ve	$1-\eta < 0$	$1-\eta > 0$	+ve	0.0
Hicks Sub	0.0	-ve	$-\eta < 0$	$-\eta < 0$	-ve	0.0
Hicks Income	-1.0	$-R < 0$	0.0	0.0	$-R < 0$	-1.0
Direct Income		constant at 1.0			constant at 1.0	
R	1.0	decreasing	0.0	0.0	increasing	1.0

Note: See equations (8) for the algebraic expression of the decomposition.

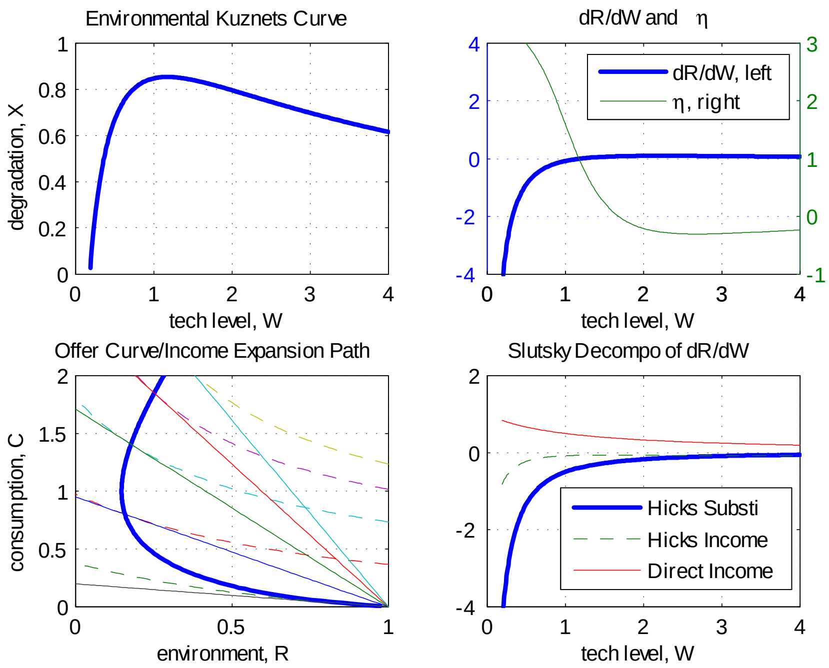

Figure 1: Model I Exponential (DES) Utility ($\alpha_R = \alpha_C = 1.0$ and $\phi = 0.5$).

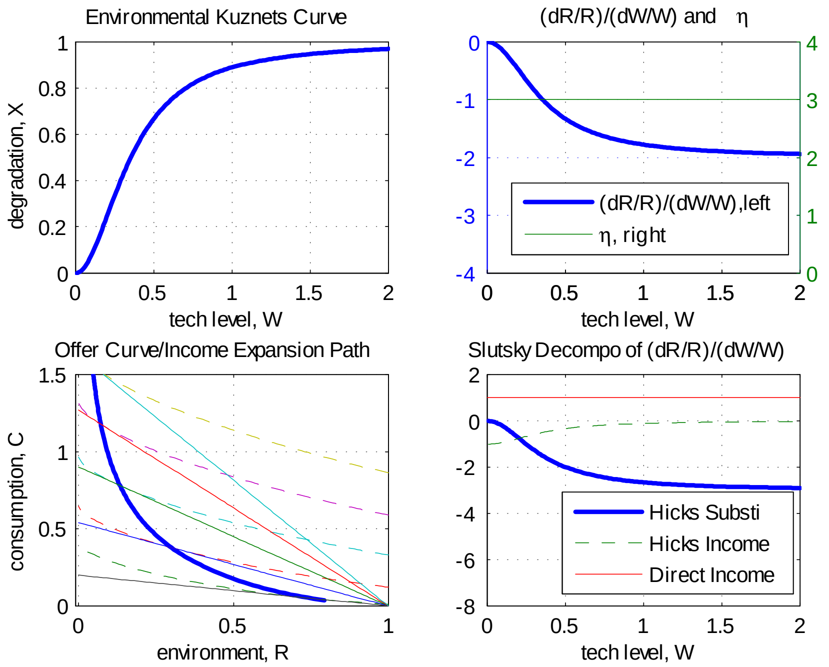

Figure 2: Model II Power (CES) Utility ($\eta = 3$ and $\phi = 0.5$).

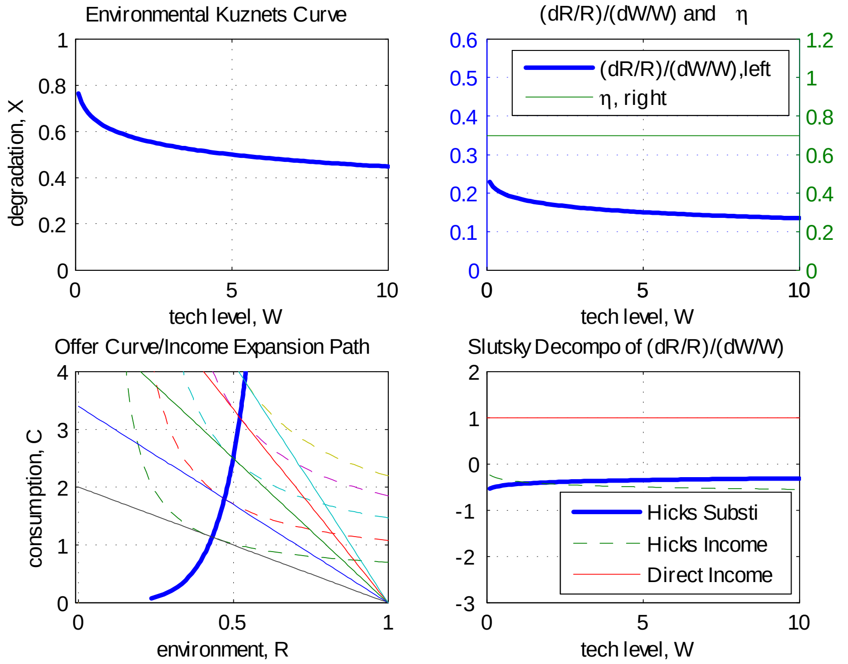

Figure 3: Model II Power (CES) Utility ($\eta = 0.7$ and $\phi = 0.5$).

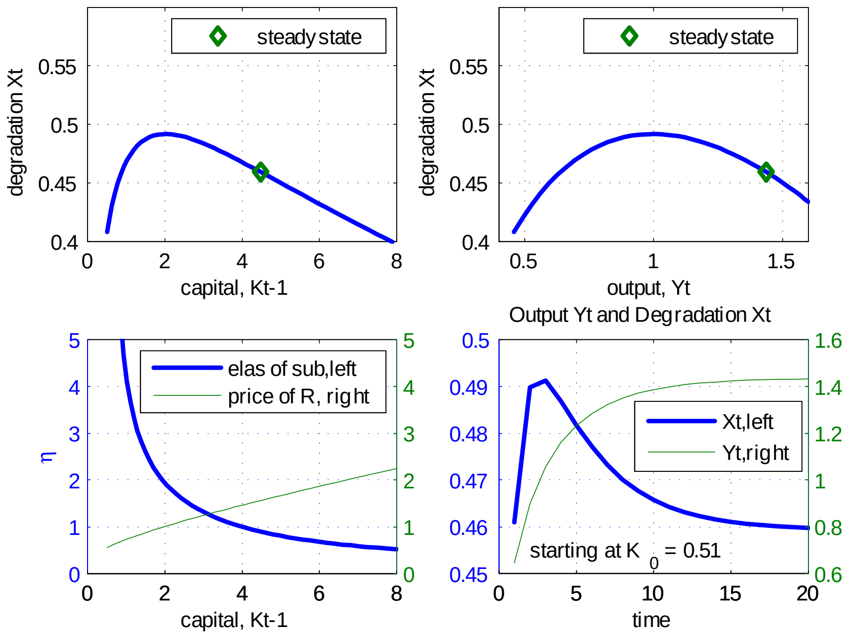

Figure 4: Model III Capital Accumulation and Flow Pollutant.

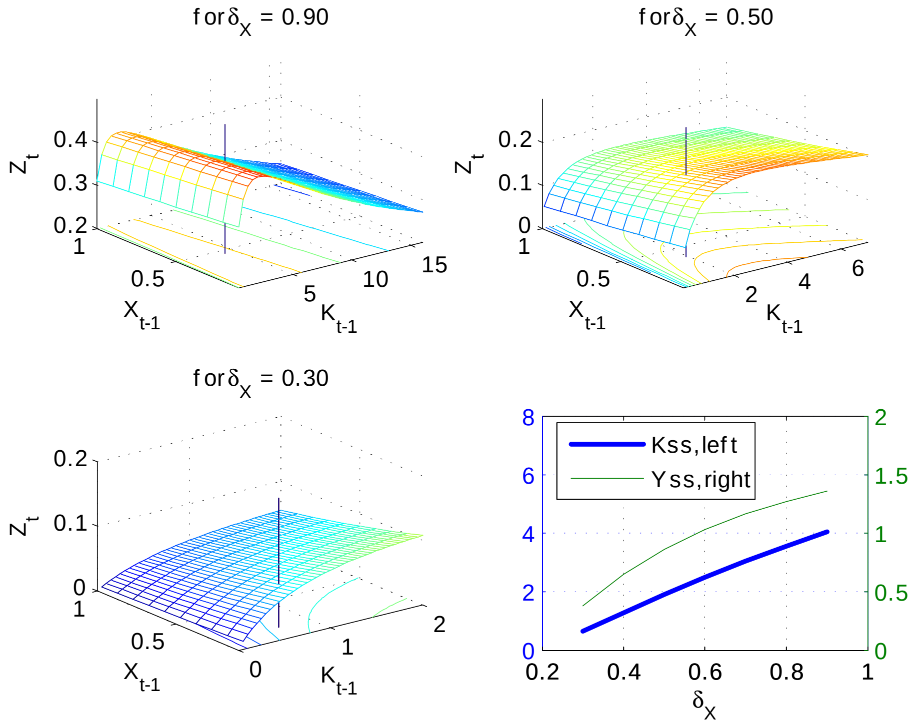

Figure 5: Model III Capital and Pollution Stocks for Selected Values of δ_X.

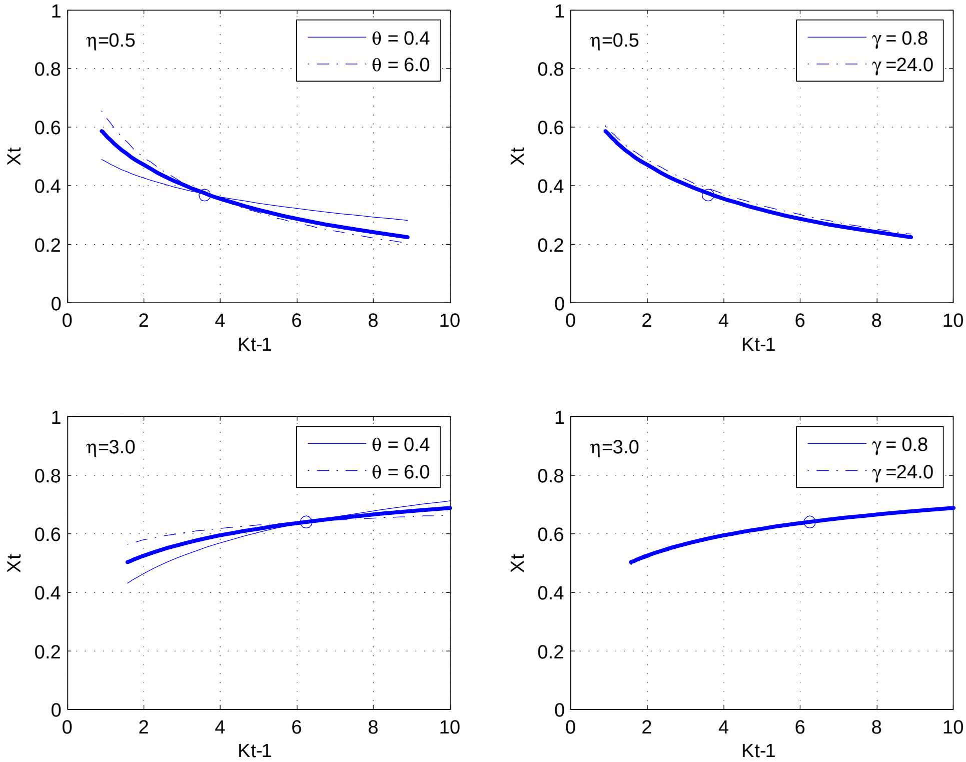

Figure 6: Model IV GIE Preference. The optimal level of emission X_t is shown as a function of capital K_{t-1} with $W_t = A$ for selected values of θ and γ. Note that the bold line and the circle in each panel show the baseline case and its steady state, respectively.

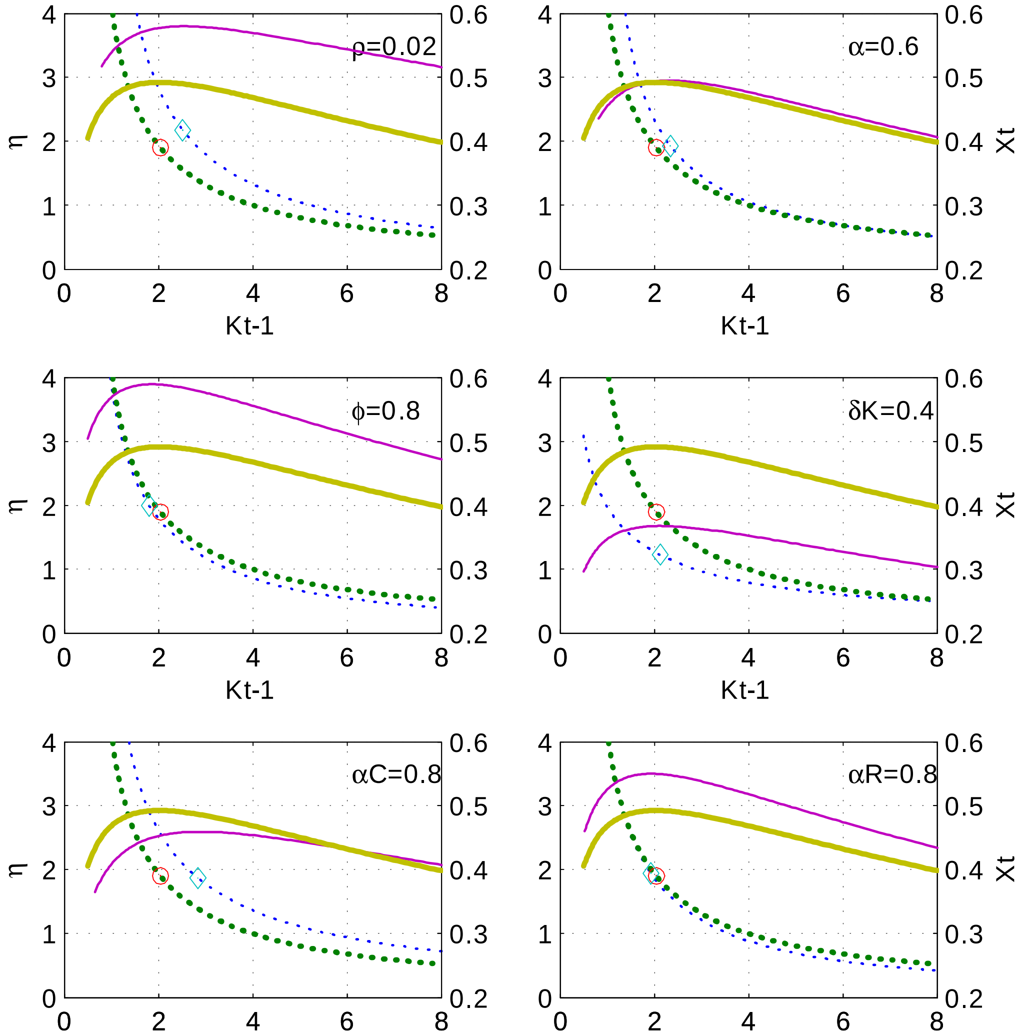

Figure 7: Sensitivity Analysis for Model III (flow pollutants). The dotted and solid lines show η_t (left axis) and X_t (right axis) as functions of K_{t-1}, respectively. The bold lines are the baseline case and thin lines show the effect of changing parameters. Circles and diamonds show threshold η.