Decentralised Delay-Dependent Static Output Feedback Variable Structure Control[1]

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Abstract

In this paper, an output feedback stabilisation problem is considered for a class of large scale interconnected time delay systems with uncertainties. The uncertainties appear in both isolated subsystems and interconnections. The bounds on the uncertainties are nonlinear and time delayed. It is not required that either the known interconnections or the uncertain interconnections are matched. Under the assumption that the time delay is known, a decentralised static output feedback variable structure control is synthesised to stabilise the system globally uniformly asymptotically using Lyapunov Razumikhin approach. A case study relating to a river pollution control problem is presented to illustrate the proposed approach.

Keywords: Decentralised control, interconnected systems, static output feedback, time delay, variable structure system.

1. Introduction

Interconnected systems exist widely in the real world. Examples include power networks, cellular systems, ecological systems and river pollution systems. Such systems are often widely distributed in space. A fundamental

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characteristic of interconnected systems, which holds for both natural and engineered systems, is that they tend to operate in a decentralised manner. For interconnected systems, the presupposition of centrality generally fails to hold due to the lack of centralised information or the lack of a centralised decision making focus. Issues such as the economic cost and reliability of communication links, particularly when systems are characterised by geographical separation, limit the appetite for centralised control. This has motivated the development of a wide literature in the area of decentralised control, see, for example, [1, 14, 24].

Interconnected systems are often modelled as dynamical equations composed of interconnections between a collection of lower-dimensional subsystems. A fundamental property of any interconnected system is that a perturbation of one subsystem can affect the other subsystems as well as the overall performance of the network. The purpose of control paradigms from the domain of engineering within an interconnected systems architecture is thus to minimise the effect of any uncertainty on the overall system behaviour. Moreover, interconnections between two or more subsystems in a network are often accompanied by phenomena such as material transfer, energy transfer and information transfer, which from a mathematical point of view, can be represented by delay elements [16]. However, for such a time delay system, the presence of even a small delay may greatly affect the performance of the system; a stable system may become unstable, or chaotic behaviour may result [16]. Therefore, the study of large scale interconnected systems in the presence of time delay is very important.

It should be noted that large scale interconnected systems with time delay is full of challenge especially when decentralised strategy is considered [17]. A class of time delay interconnected systems is considered by Mahmoud and Bingulac in [15] where time delay is not involved in the interconnections. Although many results have been achieved for time delay interconnected systems, most of them assume that the system states are available [1, 6]. The associated results based on decentralised output feedback control for time-delayed interconnected systems are few. An output feedback decentralised control scheme is given in [13] where discrete interconnected systems are considered. A class of nonlinear interconnected systems with triangular structure is considered in [8], and an interconnected system composed of a set of single input single output subsystems with dead zone input is considered in [25]. In both [8] and [25], dynamical output feedback control is employed which increases the computation greatly due to the associated closed-loop system possessing possibly double the order of the actual plant. A decentralised model reference adaptive control scheme is proposed in [11] where the considered interconnections are linear and matched. In all of the existing output feedback control strategies for large-scale interconnected time delay systems, it is required that the bounds on the uncertainties are functions of the outputs and/or largely linear [7, 13, 25]. Building on the work in the area of control of delay systems [21] and interconnected systems [23], a global decentralised output feedback sliding mode control scheme for time delay interconnected systems has been proposed in [22]. However, it is required that the known interconnections is linear to delayed outputs and the uncertain interconnections bounded by a functions of outputs and delayed outputs in [22].

In this paper, a class of time delay interconnected systems with nonlinear uncertainties is considered. The bounds on the uncertainties are nonlinear and time delayed. The same as [22] where sliding mode techniques are employed, it is assumed that the time delay is known. Unlike the work in [22], both the known interconnections and the bounds on the uncertain interconnections are allowed to involve delayed states which including the interconnections considered in [22] as a special case. Both the isolated subsystems and the interconnections involve multiple time varying delays. A static decentralised variable structure control strategy is proposed using only output information, and sufficient conditions are derived such that the corresponding closed-loop variable structure systems formed by the control and the considered interconnected systems are globally uniformly asymptotically stable based on the well-known Lyapunov Razumikhin approach. The study shows that the effects of the uncertainties can be largely rejected if the uncertainties bounded by functions of system outputs and delayed outputs. The limitation that the rate of change of the time delay is relaxed when compared with the ones when Lyapunov Krasovski approach is employed ([4, 12]). A compensator, which increases the required computation levels for large-scale interconnected systems as in [8, 23, 25], is not required in this paper. Case study of a river pollution control problem is given to demonstrate the work. Simulations show the effectiveness of the obtained results.

2. Preliminaries

Notation: In this paper, $\mathcal{R}^+$ denotes the nonnegative set of real numbers $\{t \mid t \geq 0\}$. The symbol $\mathcal{C}_{[a,b]}$ represents the set of $\mathcal{R}^n$-valued continuous function on $[a, b]$ and I_n denotes the unit matrix with dimension n. For a matrix A, the expression $A > 0$ ($A < 0$) means that A is symmetric positive (negative) definite and $\lambda_{\max}(A)$ ($\lambda_{\min}(A)$) represents its maximum (minimum) eigenvalue. The symbol $\mathrm{diag}\{A_1, A_2, \cdots, A_n\}$ represents diagonal/block-diagonal matrix with diagonal entries $A_1, A_2, \cdots, A_n$. For vectors $x = (x_1, x_2, \ldots, x_{n_1})^T \in \mathcal{R}^{n_1}$ and $y = (y_1,\ y_2, \ldots, y_{n_2})^T \in \mathcal{R}^{n_2}$, the expression $f(x, y)$ denotes a function $f(x_1, x_2, \ldots, x_{n_1}, y_1, y_2, \ldots, y_{n_2})$ defined on $\mathcal{R}^{n_1+n_2}$. Finally, $\|\cdot\|$ denotes the Euclidean norm or its induced norm.

Consider a time-delay system

$$\dot{x}(t) = f(t, x(t - d(t)) \tag{1}$$

with initial condition

$$x(t) = \phi(t), \qquad t \in [-\overline{d}, 0]$$

where $f : \mathcal{R}^+ \times \mathcal{C}_{[-\overline{d},0]} \mapsto \mathcal{R}^n$ takes $\mathcal{R}\times$ (bounded sets of $\mathcal{C}_{[-\overline{d},0]}$) into bounded sets in $\mathcal{R}^n$; $d(t)$ is the time-varying delay and $\overline{d} := \sup_{t\in\mathcal{R}^+}\{d(t)\} < \infty$.

Lemma 1. Consider system (1). If there exists a quadratic function $V_0(x) = x^T P x$ with $P > 0$ such that for $d \in [-\overline{d}, 0]$, the time derivative of V_0 along the solution of system (1) satisfies

$$\dot{V}_0(t, x) \leq -q_1\|x\|^2 \quad \text{if} \quad V_0(x(t + d)) \leq q_2 V_0(x(t)) \tag{2}$$

for some $q_1 > 0$ and $q_2 > 1$, then system (1) is globally uniformly asymptotically stable.

Proof: See Lemma 2 in Appendix 1 in [22]. $\triangle$

Lemma 2. Assume that matrix $B \in \mathcal{R}^{n\times m}$ is of full column rank and $C \in \mathcal{R}^{m\times n}$ is full of row rank. If there exists a matrix $F \in \mathcal{R}^{m\times m}$ such that $PB = C^T F$ where $P \in \mathcal{R}^{n\times n}$ is nonsingular, then F is nonsingular.

Proof: From the condition that $B \in \mathcal{R}^{n\times m}$ is of full column rank and $C \in \mathcal{R}^{m\times n}$ is full of row rank,

$$\text{rank}(B) = m, \qquad\qquad \text{rank}(C) = m$$

Since P is nonsingular, it is clear that $\text{rank}(PB) = \text{rank}(B) = m$. Then from $PB = C^T F$ where $F \in \mathcal{R}^{m\times m}$, it follows that

$$\text{rank}(F) \geq \text{rank}(C^T F) = \text{rank}(PB) = m$$

which implies that $\text{rank}(F) = m$ and thus F is nonsingular. Hence the conclusion follows. $\triangle$

Lemma 3. Assume the matrix/vector functions $H_{ij}(t, x_j) \in \mathcal{R}^{n_i\times m_j}$ with n_i and m_j positive integral numbers, and $x = \text{col}(x_1, x_2, \cdots, x_n)$ where $x_i \in \mathcal{R}^{n_i}$ for $i, j = 1, 2, \ldots, n$. Then

$$\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} H_{ij}(t, x_j) = \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} H_{ji}(t, x_i)$$

Proof: From the fact that

$$\sum_{i=1}^{n}\sum_{j=1}^{n} H_{ij}(t, x_j) = \sum_{j=1}^{n}\sum_{i=1}^{n} H_{ij}(t, x_j)$$

The conclusion follows by the similar proof as in the result 2 of Lemma 4 in Appendix 2 in [22]. $\triangle$

The results presented in this section will be used in the later analysis.

3. System Description and Problem Formulation

Consider a class of interconnected systems with time-varying delays composed of n n_i-th order subsystems described by

$$\begin{aligned}
\dot{x}_i &= A_i x_i + B_i\left(u_i + g_i(t, x_i, x_{id_i})\right) + \sum_{\substack{j=1\\ j\neq i}}^{n}\Big(D_{ij}x_{jd_j} + E_{ij}x_j \\
&\quad + \phi_{ij}(t, x_j, x_{jd_j})\Big) && (3)\\
y_i &= C_i x_i, \qquad\qquad i = 1, 2, \ldots, n, && (4)
\end{aligned}$$

where $x := \mathrm{col}(x_1, \ldots, x_n)$, $x_i \in \mathcal{R}^{n_i}$ and $u_i, y_i \in \mathcal{R}^{m_i}$ are the states, inputs and outputs of the i-th subsystem respectively, and A_i, B_i, C_i, D_{ij} and E_{ij} $(i \neq j)$ represent constant matrices of appropriate dimensions with B_i of full column rank and C_i of full row rank. The functions $g_i(\cdot)$ are matched nonlinear uncertainties in the i-th subsystem. The terms

$$\sum_{\substack{j=1\\j\neq i}}^{n}(D_{ij}x_{jd_j} + E_{ij}x_j) \qquad \text{and} \qquad \sum_{\substack{j=1\\j\neq i}}^{n}\phi_{ij}(t, x_j, x_{jd_j})$$

are, respectively, the known and uncertain interconnections of the i-th subsystem; $x_{id_i} := x_i(t - d_i)$ are the delayed states, and the symbols $d_i := d_i(t)$ denote the time-varying delays which are assumed to be known, nonnegative and bounded in $\mathcal{R}^+$, that is

$$\overline{d}_i := \sup_{t\in\mathcal{R}^+}\{d_i(t)\} < \infty, \qquad i = 1, 2, \ldots, n$$

The initial conditions associated with the time delays are given by

$$x_i(t) = \psi_i(t), \qquad t \in [-\overline{d}_i, 0]$$

where $\psi_i(\cdot)$ are continuous in $[-\overline{d}_i, 0]$ for $i = 1, 2, \ldots, n$. All the nonlinear functions are assumed to be smooth enough such that the unforced interconnected system has a unique continuous solution.

Definition 1. Consider system (3)–(4). The systems

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i(u_i + g_i(t, x_i, x_{id_i})) \\ y_i &= C_i x_i, \qquad i = 1, 2, \ldots, n, \end{aligned}$$

are called the i-th isolated subsystems of the system (3)–(4), and the systems

$$\dot{x}_i = A_i x_i + B_i u_i \tag{5}$$

$$y_i = C_i x_i, \qquad i = 1, 2, \ldots, n, \tag{6}$$

are called the i-th nominal isolated subsystems of the system (3)–(4).

For the interconnected system (3)–(4), the following basic conditions are imposed on the system firstly.

Assumption 1. The triple (A_i, B_i, C_i) are output feedback stabilisable.

Assumption 1 is a basic requirement for the triple (A_i, B_i, C_i). Under Assumption 1, there exist matrices $K_i \in \mathcal{R}^{m_i \times m_i}$ such that for any $Q_i > 0$, the Lyapunov equations

$$(A_i - B_i K_i C_i)^T P_i + P_i(A_i - B_i K_i C_i) = -Q_i < 0 \tag{7}$$

have unique solutions $P_i > 0$ for $i = 1, 2, \cdots, n$.

Assumption 2. For the input distribution matrices B_i and the output distribution matrices C_i, the matrix equations

$$P_i B_i = C_i^T F_i \tag{8}$$

are solvable for F_i, where P_i satisfy (7) for $i = 1, 2, \cdots, n$.

Remark 1. Assumption 2 is a limitation on the solution P_i of the Lyapunov equation (7). Assumptions 1 and 2 together forms the standard Constrained Lyapunov Problem (CLP) [5]. A similar condition has been imposed by many authors (see e.g, [5, 20]). A discussion for solving the CLP is available in [5, 3]. Since B_i are full column rank, C_i are full row rank and $P_i > 0$, it follows from Lemma 2 that the solutions F_i to equations (8) are nonsingular.

Assumption 3. There exist known continuous functions $\xi_i(\cdot)$, $\eta_i(\cdot)$, $\alpha_{ij}(\cdot)$ and $\beta_{ij}(\cdot)$ such that for $i, j = 1, 2, \ldots, n$

$$\begin{aligned} \|g_i(t, x, x_{id_i})\| &\leq \xi_i(t, y_i, y_{id_i}) + \eta_i(t, y_i, y_{id_i})\|x_{id_i}\| && (9) \\ \|\phi_{ij}(t, x_j, x_{jd_j})\| &\leq \alpha_{ij}(t, y_j, y_{jd_j})\|y_j\| + \beta_{ij}(t, x_j)\|x_{jd_j}\|, \quad (i \neq j) && (10) \end{aligned}$$

Remark 2. Assumption 3 is a limitation on the uncertainties that can be tolerated by the system. It is not required that the interconnections are described or bounded by functions of the system outputs, which is in comparison with the work in [22, 18, 25]. Moreover, both the known interconnections and the uncertain interconnections involve time delayed states in this paper, which makes the work applicable to a wide class of large scale interconnected systems.

The objective of this paper is, under the assumption that all the nominal isolated subsystems are output feedback stabilisible, to design a control law in form of

$$u_i = u_i(t, y_i, y_{id_i}), \qquad i = 1, 2, \ldots, n \tag{11}$$

and to develop a set of conditions under which the resultant closed-loop system formed by applying the control law in (11) to the large scale interconnected system (3)–(4), is globally uniformly asymptotically stable even in the presence of the uncertainties and time delays. It is straight forward to see that the control u_i in (11) are only dependent on time t, time delay d_i and local outputs y_i. Such class of controllers is called decentralised static output feedback control. In order to largely cancel the effects from the uncertainties, a variable structure control will be proposed subsequently and the bounds on the uncertainties will be fully employed in control design to reduce the conservatism.

Remark 3. Clearly the controller (11) requires that the time delays are known as in much of the existing work [17, 9, 22]. This may limit the application of the work. However, in some industrial systems such as flow through pipes and web forming processes, the delay existing in the process is known, and can thus be employed in the control design and/or the observer design [19]. Moreover, if the time delay is unavailable, the approach proposed in [2] can be employed to identify the time delay.

4. Main Results

In this section, a decentralised output feedback variable structure controller will be synthsised for the interconnected systems (3)–(4).

Under Assumption 2, it follows form Lemma 2 that the matrix F_i satisfying equation (8) is nonsingular. Then, consider the control law

$$u_i = -K_i y_i - \frac{1}{2\varepsilon_i} F_i^T y_i \eta_i^2(t, y_i, y_{id_i}) + u_i^a(\cdot) + u_i^b(\cdot) \tag{12}$$

where $K_i \in \mathcal{R}^{m_i \times m_i}$ satisfy Assumption 2, $\varepsilon_i > 0$ are constant, and $u_i^a(\cdot)$ and $u_i^a(\cdot)$ are defined by

$$u_i^a(\cdot) \quad := \quad \begin{cases} -\xi_i(t, y_i, y_{id_i}) \|F_i\| F_i^{-1} \frac{y_i}{\|y_i\|}, & y_i \neq 0 \\ 0, & y_i = 0 \end{cases} \tag{13}$$

$$u_i^b(\cdot) \quad := \quad -F_i^{-1} y_i \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{1}{\varepsilon_{ji}} \|P_j\|^2 \alpha_{ji}^2(t, y_i, y_{id_i}) \tag{14}$$

where F_i satisfy (8) and $\varepsilon_{ji} > 0$ ($j \neq i$) are constants for $i, j = 1, 2, \ldots, n$.

Note the structure of the control u_i in (12) are variable due to the term $u_i^a(\cdot)$ defined in (13). It is clear that u_i in (12) are decentralised because u_i are only dependent on time t, time delay d_i and local output information y_i. Thus the u_i in (12) are called time dependent decentralised output feedback variable structure controllers.

The following result is now ready to be presented.

Theorem 1. Under Assumptions 1-3, the closed-loop system formed by applying control (12) into system (3)–(4) is globally uniformly asymptotically stable if $\gamma := \inf_{x_i \in \mathcal{R}^{n_i}} \{\lambda_{\min}\left(\Gamma^T(\cdot) + \Gamma(\cdot)\right)\} > 0$ where the matrix

$$\Gamma := \begin{bmatrix} \Gamma_1 & \Gamma_2 \\ \Gamma_3 & \Gamma_4 \end{bmatrix}$$

is defined by

$$\Gamma_1 := \begin{bmatrix} \Theta_1^a & -2P_1D_{12} & \cdots & -2P_1D_{1n} \\ -2P_2D_{21} & \Theta_2^a & \ddots & \vdots \\ \vdots & \ddots & \ddots & -2P_{n-1}D_{(n-1)n} \\ -2P_nD_{n1} & \cdots & -2P_nD_{n(n-1)} & \Theta_n^a \end{bmatrix}$$

$$\Gamma_2 := \begin{bmatrix} 0 & -2P_1E_{12} & \cdots & -2P_1E_{1n} \\ -2P_2E_{21} & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & -2P_{n-1}E_{(n-1)n} \\ -2P_nE_{n1} & \cdots & -2P_nE_{n(n-1)} & 0 \end{bmatrix}$$

$$\Gamma_3 := \begin{bmatrix} 0 & -2P_2E_{21} & \cdots & -2P_nE_{n1} \\ -2P_1E_{12} & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & -2P_{n-1}E_{(n-1)n} \\ -2P_nE_{n1} & \cdots & -2P_nE_{n(n-1)} & 0 \end{bmatrix}$$

and

$$\Gamma_4 := \mathrm{diag}\left\{\Theta_1^b, \Theta_2^b, \cdots, \Theta_n^b\right\}$$

where

$$\Theta_i^a := Q_i - qP_i - 2\sum_{\substack{j=1 \\ j\neq i}}^{n} \varepsilon_{ij} I_{n_i}, \quad \Theta_i^b := P_i - \Big(\varepsilon_i + \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{1}{\varepsilon_{ji}}\beta_{ji}^2(\cdot)||P_j\|^2\Big) I_{n_i}$$

for $i = 1, \ldots, n$ and $q > 1$.

Proof: Applying the control (12) into system (3)–(4), the closed-loop system is described by

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i\Big(- K_i C_i x_i - \frac{1}{2\varepsilon_i} F_i^T y_i \eta_i^2(t, y_i, y_{id_i}) + u_i^a(t, y_i, y_{id_i}) + u_i^b(\cdot) \\ &\quad + g_i(t, x_i, x_{id_i})\Big) + \sum_{\substack{j=1 \\ j\neq i}}^{n} \Big(D_{ij} x_{jd_j} + E_{ij} x_j + \phi_{ij}(t, x_j, x_{jd_j})\Big) \end{aligned} \tag{15}$$

where $u_i^a(\cdot)$ and $u_i^b(\cdot)$ are defined by (13) and (14) respectively for $i = 1, 2, \ldots, n$. For the closed-loop system (15), consider the Lyapunov function candidate

$$V(x(t)) = \sum_{i=1}^{n} x_i^T(t) P_i x_i(t) \tag{16}$$

where $P_i > 0$ satisfy (7) for $i = 1, 2, \ldots, n$. Then, the time derivative of $V(\cdot)$ along the trajectories of system (15) is given by

$$\begin{aligned} \dot{V} &= -\sum_{i=1}^{n} x_i^T Q_i x_i + 2\sum_{i=1}^{n} x_i^T P_i B_i\Big(- \frac{1}{2\varepsilon_i} F_i^T y_i \eta_i^2(t, y_i, y_{id_i}) + u_i^a(t, y_i, y_{id_i})\Big) \\ &\quad + 2\sum_{i=1}^{n} x_i^T P_i B_i u_i^b(t, y_i, y_{id_i}) + 2\sum_{i=1}^{n} x_i^T P_i B_i g_i(\cdot) + 2\sum_{i=1}^{n}\sum_{\substack{j=1 \\ j\neq i}}^{n} x_i^T P_i D_{ij} x_{jd_j} \\ &\quad + 2\sum_{i=1}^{n}\sum_{\substack{j=1 \\ j\neq i}}^{n} x_i^T P_i E_{ij} x_j + 2\sum_{i=1}^{n}\sum_{\substack{j=1 \\ j\neq i}}^{n} x_i^T P_i \phi_{ij}(t, x_j, x_{jd_j}) \end{aligned} \tag{17}$$

From (9), (8) and Young's inequality ($ab \leq \frac{1}{2\varepsilon}a^2 + \frac{\varepsilon}{2}b^2$ for $\varepsilon > 0$), it follows that for any $\varepsilon_i > 0$

$$\begin{aligned} & x_i^T P_i B_i g_i(t, x_i, x_{id_i}) = (F_i^T y_i)^T g_i(t, x_i, x_{id_i}) \\ \leq\ & \|F_i\|\, \|y_i\| \xi_i(t, y_i, y_{id_i}) + \|F_i^T y_i\| \eta_i(t, y_i, y_{id_i}) \|x_{id_i}\| \\ \leq\ & \xi_i(t, y_i, y_{id_i}) \|F_i\|\, \|y_i\| + \frac{1}{2\varepsilon_i} \|F_i^T y_i\|^2 \eta_i^2(t, y_i, y_{id_i}) + \frac{\varepsilon_i}{2} \|x_{id_i}\|^2 \end{aligned} \tag{18}$$

From (8) and the definition of $u_i^a(\cdot)$ in (13), it follows that

i) if $y_i = 0$, then $u_i^a(\cdot) = 0$ and thus

$$x_i^T P_i B_i u_i^a(t, y_i, y_{id_i}) + \|F_i\| \, \|y_i\| \xi_i(t, y_i, y_{id_i}) = 0$$

ii) if $y_i \neq 0$, from the definition of $u_i^a(\cdot)$ in (13),

$$\begin{aligned}
& x_i^T P_i B_i u_i^a(t, y_i, y_{id_i}) + \|F_i\| \, \|y_i\| \xi_i(t, y_i, y_{id_i}) \\
\leq & -y_i^T F_i \xi_i(t, y_i, y_{id_i}) \|F_i\| F_i^{-1} \frac{y_i}{\|y_i\|} + \|F_i\| \, \|y_i\| \xi_i(t, y_i, y_{id_i}) \\
= & -\xi_i(t, y_i, y_{id_i}) \|F_i\| \frac{y_i^T y_i}{\|y_i\|} + \|F_i\| \, \|y_i\| \xi_i(t, y_i, y_{id_i}) \\
= & \ 0
\end{aligned}$$

Thus, from i) and ii) above,

$$x_i^T P_i B_i u_i^a(t, y_i, y_{id_i}) + \|F_i\| \, \|y_i\| \xi_i(t, y_i, y_{id_i}) \leq 0, \qquad i = 1, 2, \cdots, n \quad (19)$$

Further, from (8),

$$\begin{aligned}
& -\frac{1}{2\varepsilon_i} x_i^T P_i B_i F_i^T y_i \eta_i^2(t, y_i, y_{id_i}) + \frac{1}{2\varepsilon_i} \|F_i^T y_i\|^2 \eta_i^2(t, y_i, y_{id_i}) \\
= & -\frac{1}{2\varepsilon_i} x_i^T C_i^T F_i F_i^T y_i \eta_i^2(t, y_i, y_{id_i}) + \frac{1}{2\varepsilon_i} \|F_i^T y_i\|^2 \eta_i^2(t, y_i, y_{id_i}) \\
= & -\frac{1}{2\varepsilon_i} (F_i^T y_i)^T F_i^T y_i \eta_i^2(t, y_i, y_{id_i}) + \frac{1}{2\varepsilon_i} \|F_i^T y_i\|^2 \eta_i^2(t, y_i, y_{id_i}) = 0 \quad (20)
\end{aligned}$$

Therefore, from (18), (19) and (20)

$$\begin{aligned}
2 \sum_{i=1}^n x_i^T P_i B_i \Big(-\frac{1}{2\varepsilon_i} F_i^T y_i \eta_i^2(t, y_i, y_{id_i}) + u_i^a(\cdot) \Big) + 2 \sum_{i=1}^n x_i^T P_i B_i g_i(t, x_i, x_{id_i}) & \\
\leq \sum_{i=1}^n \varepsilon_i \|x_{id_i}\|^2 & \quad (21)
\end{aligned}$$

From (10) and Young's inequality,

$$\begin{aligned}
& x_i^T P_i \phi_{ij}(t, x_j, x_{jd_j}) \leq \|x_i\| \, \|P_i\| (\alpha_{ij}(t, y_j, y_{jd_j}) \|y_j\| + \beta_{ij}(t, x_j) \|x_{jd_j}\|) \\
= & \ \alpha_{ij}(t, y_j, y_{jd_j}) \|P_i\| \, \|y_j\| \, \|x_i\| + \beta_{ij}(t, x_j) \|P_i\| \, \|x_i\| \|x_{jd_j}\| \\
\leq & \ \frac{1}{2\varepsilon_{ij}} \alpha_{ij}^2(t, y_j, y_{jd_j}) \|P_i\|^2 \, \|y_j\|^2 + \frac{\varepsilon_{ij}}{2} \|x_i\|^2 + \frac{1}{2\varepsilon_{ij}} \beta_{ij}^2(t, x_j) \|P_i\|^2 \, \|x_{jd_j}\|^2
\end{aligned}$$

$$
\begin{aligned}
&+\frac{\varepsilon_{ij}}{2}\|x_i\|^2\\
=\;& \frac{1}{2\varepsilon_{ij}}\alpha_{ij}^2(t,y_j,y_{jd_j})\|P_i\|^2\,\|y_j\|^2+\varepsilon_{ij}\|x_i\|^2\\
&+\frac{1}{2\varepsilon_{ij}}\beta_{ij}^2(t,x_j)\|P_i\|^2\,\|x_{jd_j}\|^2 \qquad (22)
\end{aligned}
$$

Consider the uncertain interconnection $\sum_{\substack{j=1\\ j\neq i}}^{n}\phi_{ij}(t,x_j,x_{jd_j})$. From (22)

$$
\begin{aligned}
& 2\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} x_i^TP_i\phi_{ij}(t,x_j,x_{jd_j})\\
\leq\;& 2\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} \varepsilon_{ij}\|x_i\|^2+\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{1}{\varepsilon_{ij}}\alpha_{ij}^2(t,y_j,y_{jd_j})\|P_i\|^2\,\|y_j\|^2\\
&+\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{1}{\varepsilon_{ij}}\beta_{ij}^2(t,x_j)\|P_i\|^2\,\|x_{jd_j}\|^2\\
=\;& 2\sum_{i=1}^{n}\Big(\sum_{\substack{j=1\\ j\neq i}}^{n} \varepsilon_{ij}\Big)\|x_i\|^2+\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{1}{\varepsilon_{ji}}\alpha_{ji}^2(t,y_i,y_{id_i})\|P_j\|^2\|y_i\|^2\\
&+\sum_{i=1}^{n}\Big(\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{1}{\varepsilon_{ji}}\beta_{ji}^2(t,x_i)\|P_j\|^2\Big)\|x_{id_i}\|^2 \qquad (23)
\end{aligned}
$$

where Lemma 3 is used to obtain the last equality. From the definition of $u_i^b(\cdot)$ in (14),

$$
\begin{aligned}
& x_i^TP_iB_iu_i^b(\cdot)+\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{1}{\varepsilon_{ji}}\alpha_{ji}^2(t,y_i,y_{id_i})\|P_j\|^2\|y_i\|^2\\
\leq & -x_i^TC_i^TF_iF_i^{-1}y_i\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{1}{\varepsilon_{ji}}\alpha_{ji}^2(t,y_i,y_{id_i})\|P_j\|^2+\|y_i\|^2\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{1}{\varepsilon_{ji}}\alpha_{ji}^2(\cdot)\|P_j\|^2\\
=\;& 0 \qquad (24)
\end{aligned}
$$

Therefore, from (23) and (24)

$$
\begin{aligned}
& 2\sum_{i=1}^{n}x_i^TP_iB_iu_i^b(t,y_i,y_{id_i})+2\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} x_i^TP_i\phi_{ij}(t,x_j,x_{jd_j})\\
&\leq\; 2\sum_{i=1}^{n}\Big(\sum_{\substack{j=1\\ j\neq i}}^{n} \varepsilon_{ij}\Big)\|x_i\|^2+\sum_{i=1}^{n}\Big(\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{1}{\varepsilon_{ji}}\beta_{ji}^2(t,x_i)\|P_j\|^2\Big)\|x_{id_i}\|^2 \quad (25)
\end{aligned}
$$

Applying (21) and (25) to equation (17) yields

$$\begin{aligned}\dot{V} \leq & -\sum_{i=1}^{n} x_i^T Q_i x_i + \sum_{i=1}^{n} \varepsilon_i \|x_{id_i}\|^2 + 2\sum_{i=1}^{n} \sum_{\substack{j=1\\ j\neq i}}^{n} x_i^T P_i D_{ij} x_{jd_j} \\ & +2\sum_{i=1}^{n} \sum_{\substack{j=1\\ j\neq i}}^{n} x_i^T P_i E_{ij} x_j + 2\sum_{i=1}^{n} \Big(\sum_{\substack{j=1\\ j\neq i}}^{n} \varepsilon_{ij} \Big) \|x_i\|^2 \\ & +\sum_{i=1}^{n} \Big(\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{1}{\varepsilon_{ji}} \beta_{ji}^2(t, x_i) \|P_j\|^2 \Big) \|x_{id_i}\|^2 \end{aligned} \tag{26}$$

From the definition of $V(\cdot)$ in (16), it is clear that

$$V(x_{1d_1}, x_{2d_2}, \ldots, x_{nd_n}) \leq qV(x_1, x_2, \ldots, x_n), \qquad (q > 1)$$

implies that

$$q\sum_{i=1}^{n} x_i^T P_i x_i - \sum_{i=1}^{n} x_{id_i}^T P_i x_{id_i} \geq 0 \tag{27}$$

Therefore, from (27) and (26), it follows that when $V(x_{1d_1}, \ldots,\ x_{nd_n}) \leq qV(x_1, \ldots, x_n)$,

$$\begin{aligned}\dot{V} \ \leq \ & -\sum_{i=1}^{n} x_i^T Q_i x_i + \sum_{i=1}^{n} \varepsilon_i \|x_{id_i}\|^2 + 2\sum_{i=1}^{n} \sum_{\substack{j=1\\ j\neq i}}^{n} x_i^T P_i D_{ij} x_{jd_j} \\ & +2\sum_{i=1}^{n} \sum_{\substack{j=1\\ j\neq i}}^{n} x_i^T P_i E_{ij} x_j + 2\sum_{i=1}^{n} \Big(\sum_{\substack{j=1\\ j\neq i}}^{n} \varepsilon_{ij} \Big) \|x_i\|^2 \\ & +\sum_{i=1}^{n} \Big(\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{1}{\varepsilon_{ji}} \beta_{ji}^2(t, x_i) \|P_j\|^2 \Big) \|x_{id_i}\|^2 \\ & +q\sum_{i=1}^{n} x_i^T P_i x_i - \sum_{i=1}^{n} x_{id_i}^T P_i x_{id_i} \\ \leq \ & -\sum_{i=1}^{n} x_i^T \Big(Q_i - qP_i - 2(\sum_{\substack{j=1\\ j\neq i}}^{n} \varepsilon_{ij}) I_{n_i} \Big) x_i \\ & -\sum_{i=1}^{n} x_{id_i}^T \Big\{ P_i - \Big(\varepsilon_i + \sum_{\substack{j=1\\ j\neq i}}^{n} \frac{1}{\varepsilon_{ji}} \beta_{ji}^2(t, x_i) \|P_j\|^2 \Big) I_{n_i} \Big\} x_{id_i} \\ & +2\sum_{i=1}^{n} \sum_{\substack{j=1\\ j\neq i}}^{n} x_i^T P_i D_{ij} x_{jd_j} + 2\sum_{i=1}^{n} \sum_{\substack{j=1\\ j\neq i}}^{n} x_i^T P_i E_{ij} x_j \end{aligned}$$

$$
\begin{aligned}
&= -\frac{1}{2} Z^T (\Gamma^T(\cdot) + \Gamma(\cdot)) Z \\
&\le -\frac{1}{2} \lambda_{\min}(\Gamma^T(\cdot) + \Gamma(\cdot)) \|Z\|^2 \\
&\le -\frac{1}{2} \gamma \|x\|^2
\end{aligned}
$$

where $Z := \mathrm{col}(x_1, \cdots, x_n, x_{1d_1}, \ldots, x_{nd_n})$ and $x = \mathrm{col}(x_1, \cdots, x_n)$.

Hence, by applying Lemma 1, the conclusion follows from $\gamma > 0$. ∇

Remark 4. Consider (10) in Assumption 3. The bounds on the uncertain interconnections in system (3) are dependent on the systems states, and thus they cannot be employed in the control design since static output feedback is used in this paper. The effects of such interconnections have been reflected through $\beta_{ij}(\cdot)$ in the matrix Γ.

Next, consider that the uncertain interconnections $\phi_{ij}(\cdot)$ satisfy

$$
\|\phi_{ij}(t, x_j, x_{jd_j})\| \le \alpha_{ij}(t, y_j, y_{jd_j}) \|y_j\| \tag{28}
$$

which is a special case of (10) when $\beta_{ij}(\cdot) = 0$. The inequality of (28) implies that the bounds on the uncertain interconnections can be expressed in functions of outputs and delayed outputs. In this case, the effects of the uncertain interconnection $\phi_{ij}(\cdot)$ can be largely rejected by the control law designed in (12). This is shown by the following result.

Corollary 1. For system (3)–(4), it is assumed that $g_i(\cdot)$ satisfy (9) and ϕ_{ij} satisfy (28). Then under Assumptions 1-2, the closed-loop system formed by applying the control (12) to the system (3)–(4) is globally uniformly asymptotically stable if $W^T + W > 0$ where the matrix

$$
W := \begin{bmatrix} W_1 & W_2 \\ W_3 & W_4 \end{bmatrix}
$$

is defined by

$$
W_1 := \begin{bmatrix}
\Pi_1^a & -2P_1 D_{12} & \cdots & -2P_1 D_{1n} \\
-2P_2 D_{21} & \Pi_2^a & \ddots & \vdots \\
\vdots & \ddots & \ddots & -2P_{n-1} D_{(n-1)n} \\
-2P_n D_{n1} & \cdots & -2P_n D_{n(n-1)} & \Pi_n^a
\end{bmatrix}
$$

$$W_2 := \Gamma_2, \qquad\qquad W_3 := \Gamma_3$$

and

$$W_4 := \mathrm{diag}\left\{\Pi_1^b, \Pi_2^b, \cdots, \Pi_n^b\right\}$$

where Γ_2 and Γ_3 are defined in Theorem 1, $\Pi_i^a := Q_i - qP_i - (\sum_{\substack{j=1\\ j\neq i}}^n \varepsilon_{ij})I_{n_i}$ and $\Pi_i^b := P_i - \varepsilon_i I_{n_i}$ for $i = 1, \ldots, n$ and $q > 1$.

Proof: Consider the uncertain interconnection terms $\sum_{\substack{j=1\\ j\neq i}}^n \phi_{ij}(t, x_j, x_{jd_j})$. From the condition (28) and Young's inequality,

$$\begin{aligned}
& 2x_i^T P_i \phi_{ij}(t, x_j, x_{jd_j}) \\
\leq\ & 2\|P_i\|\,\|x_i\|\alpha_{ij}(t, y_j, y_{jd_j})\|y_j\| \\
\leq\ & \varepsilon_{ij}\|x_i\|^2 + \frac{1}{\varepsilon_{ij}}\|P_i\|^2\,\alpha_{ij}^2(t, y_j, y_{jd_j})\|y_j\|^2
\end{aligned} \tag{29}$$

for constant scalars $\varepsilon_{ij} > 0$. By comparing the difference between (22) and (29), the conclusion is obtained directly from the proof of Theorem 1. ∇

Remark 5. Corollary 1 shows that if the uncertain interconnections are bounded by functions of system outputs and delayed outputs, then their effects can be largely rejected by the control (12). Actually, from (29), it is straightforward to see that the effects of the uncertain interconnections $\phi_{ij}(\cdot)$ are reflected through ε_{ij} and $\alpha_{ij}(\cdot)$. As the second term in (29) has been completely canceled by the designed control (12) (actually the term $u_i^b(\cdot)$ in (14)). Thus ε_{ij} will be the only term resulted from the uncertain interconnections which appear in the matrix W_1 through Π_i^a. However, the terms $\sum_{\substack{j=1\\ j\neq i}}^n \varepsilon_{ij}$ will be very small if the parameters ε_{ij} is chosen to be small enough although small ε_{ij} usually result in high gain control.

Remark 6. Comparing the matrix Γ in Theorem 1 and the matrix W in Corollary 1, it is clear to see that Γ is a function matrix while W is a constant matrix. The difference between Γ and W lies in the diagonal entries. From the structure of the diagonal entries Γ_1, Γ_4, W_1 and W_4, it is straightforward to see that $\Gamma^T + \Gamma > 0$ implies that $W^T + W > 0$. Therefore, the results in Corollary 1 is less conservative than the result in Theorem 1 but Theorem 1 is applicable to a class of large-scale interconnected systems with a wider class of uncertain interconnections.

5. Case Study — River Pollution Control Problem

Consider a two-reach model of a river pollution control problem [10]. It is assumed that the concentration of biochemical oxygen demand (BOD) for the first subsystem is perturbed by a time delay. Then, the system can be described by (See, [22])

$$\dot{x}_1 = \begin{bmatrix} -1.32\delta & 0 \\ -0.32 & -1.2 \end{bmatrix} x_1 + \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} \Big(u_1 - 13.2(1-\delta)y_{1d_1}\Big) + \phi_{12}(\cdot) \tag{30}$$

$$\begin{aligned} \dot{x}_2 = & \begin{bmatrix} -1.32 & 0 \\ -0.32 & -1.2 \end{bmatrix} x_2 + \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} (u_2 + g_2(\cdot)) + \begin{bmatrix} 0.9\delta & 0 \\ 0 & 0 \end{bmatrix} x_{1d_1} \\ & + \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix} x_1 + \begin{bmatrix} -0.9\delta y_1 \\ 0 \end{bmatrix} \end{aligned} \tag{31}$$

$$y_1 = [1 \quad 0]\, x_1, \qquad y_2 = [1 \quad 0]\, x_2 \tag{32}$$

where $x_1 := \mathrm{col}(x_{11}, x_{12})$ and $x_2 := \mathrm{col}(x_{21}, x_{22})$. The variables x_{i1} and x_{i2} represent the concentration of the BOD and the concentration of dissolved oxygen respectively, and the control u_i are the BOD of the effluent discharge into the river for $i = 1, 2$. The constant $\delta \in [0, 1]$ is the retarded coefficient. The uncertainties $g_2(\cdot)$ and $\phi_{12}(\cdot)$ are added to illustrate the obtained results.

In order to fully use system output information, it is necessary to rewrite the system (30)–(32) in the following form

$$\begin{aligned} \dot{x}_1 = & \underbrace{\begin{bmatrix} -1.32\delta & 0 \\ -0.32 & -1.2 \end{bmatrix}}_{A_1} x_1 + \underbrace{\begin{bmatrix} 0.1 \\ 0 \end{bmatrix}}_{B_1} \Big(u_1 + \underbrace{(-13.2(1-\delta))y_{1d_1}}_{g_1(\cdot)}\Big) \\ & + \phi_{12}(\cdot) \end{aligned} \tag{33}$$

$$\dot{x}_2 = \underbrace{\begin{bmatrix} -1.32 & 0 \\ -0.32 & -1.2 \end{bmatrix}}_{A_2} x_2 + \underbrace{\begin{bmatrix} 0.1 \\ 0 \end{bmatrix}}_{B_2} (u_2 + g_2(\cdot)) + \underbrace{\begin{bmatrix} 0.9\delta & 0 \\ 0 & 0 \end{bmatrix}}_{D_{21}} x_{1d_1}$$

$$+\underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0.9 \end{bmatrix}}_{E_{21}} x_1 + \underbrace{\begin{bmatrix} (1-\delta)0.9y_1 \\ 0 \end{bmatrix}}_{\phi_{21}(\cdot)} \tag{34}$$

$$y_1 = \underbrace{[1 \quad 0]}_{C_1} x_1, \qquad y_2 = \underbrace{[1 \quad 0]}_{C_2} x_2 \tag{35}$$

It is assumed that

$$|g_2(\cdot)| \leq \underbrace{1+\sin y_2}_{\xi_2} + \underbrace{|y_2|}_{\eta_2} \|x_{2d_2}\|, \qquad \|\phi_{12}\| \leq \underbrace{|y_2 y_{2d_2}| \sin^2 t}_{\alpha_{12}} |y_2| \tag{36}$$

Let $\xi_1 = 0$, $\eta_1(\cdot) = 13.2(1-\delta)$ and $\alpha_{21} = 0.9(1-\delta)$. It is clear to see that Assumption 3 holds. Then choose $\sigma = 0.20$, $K_1 = 20$, $K_2 = 30$ and

$$Q_1 = \begin{bmatrix} 4.5280 & 0.3200 \\ 0.3200 & 2.4000 \end{bmatrix}, \qquad Q_2 = \begin{bmatrix} 8.6400 & 0.3200 \\ 0.3200 & 2.4000 \end{bmatrix}$$

The solutions to the Lyapunov equations in (7) are $P_1 = P_2 = I_2$ and the equations (8) are satisfied with $D_1 = D_2 = 0.1$. Comparing system (33)–(34) with the system (3)–(4), it is straightforward to see that $D_{12} = E_{12} = 0$. Let $\varepsilon_1 = \varepsilon_2 = 0.5$ and $\varepsilon_{12} = \varepsilon_{21} = 0.1$. Clearly both the unknown interconnections $\phi_{12}(\cdot)$ and $\phi_{21}(\cdot)$ are bounded by functions of the local outputs and time delay. Consider the Corollary 1. By direct computation,

$$W_1 = \begin{bmatrix} 3.418 & 0.32 & 0 & 0 \\ 0.320 & 1.29 & 0 & 0 \\ 0 & 0 & 7.53 & 0.32 \\ 0 & -1.80 & 0.32 & 1.29 \end{bmatrix}, \qquad W_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.36 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$W_3 = \begin{bmatrix} 0 & 0 & -0.36 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad W_4 = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}$$

and the matrix $W^T + W$ where W is defined in Corollary 1, is positive definite. Then, from Corollary 1, the controllers (12)–(14) which are well defined, stabilise the system (30)-(32) globally uniformly asymptotically.

For simulation purposes, the delays are chosen as

$$d_1(t) = 3 - 2\sin t \qquad \text{and} \qquad d_2(t) = 2 - \cos t$$

and the delay related initial conditions are chosen as

$$\psi_1(t) = \text{col}(2\cos t, 1) \qquad \text{and} \qquad \psi_2(t) = \text{col}(0, 1 - \sin t)$$

The simulation results shown in Figure 1 are as expected.

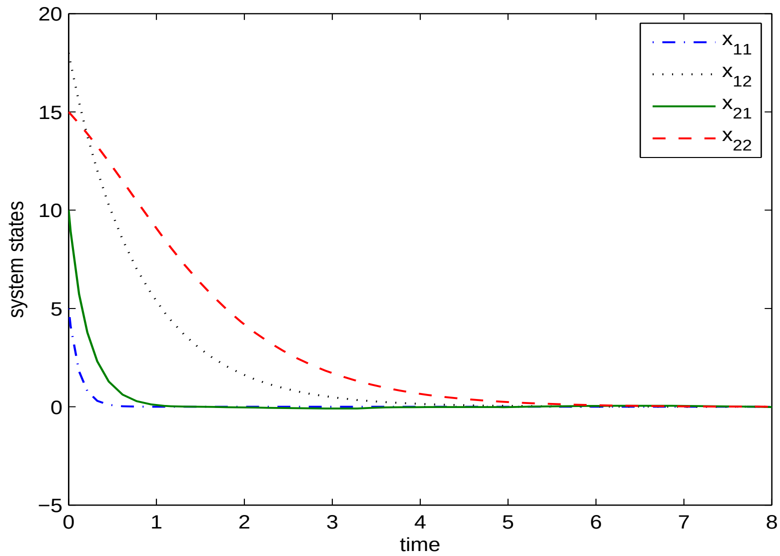

Figure 1: The time responses of the state variables of system (30)–(31)

6. Conclusions

This paper has presented delay dependent control strategies for a class of uncertain interconnected systems with time-varying delays. The proposed controllers are decentralised and based on only output information, which is convenient for real implementation. A set of sufficient conditions has been developed to guarantee that the resultant closed-loop system is globally uniformly asymptotically stable. The study also shows that if the uncertain interconnections are bounded by the functions of outputs and delayed outputs, then their effects can be largely rejected by designing appropriate controllers. The proposed approach can be used to accommodate mismatched uncertain interconnections.

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