Variable markups and capital-labor substitution[*]

Wei Jiang   Miguel León-Ledesma

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Abstract

We provide new estimates of the elasticity of capital-labor substitution (σ) and the bias in technical change allowing price markups to change over time as shown in De Loecker and Eeckhout (2017). Our estimate of σ is in the region of 0.8 and technical change is net capital-augmenting.

JEL Classification: E23, E25, O47.

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1 Introduction

The elasticity of capital-labor substitution (σ) and the bias in technical change are two of the most important objects in growth theory. Recent debates around the decline in the labor share, for instance, have focused on the role of technical change coupled with the substitutability between capital and labor.[1] Some have attributed the decline in the labor share not to technological forces, but to the structure of goods markets. Specifically, Barkai (2017) and De Loecker and Eeckhout (2017) have shown that the main driving force behind the labor (and capital) share decline is an increase in the price markup in the US economy which they relate to decreased competition.[2]

We estimate a normalised supply side system for the US economy between 1950-2014 following León-Ledesma et al. (2010) but introducing a variable price markup. This system is able to recover estimates of σ and average labor- and capital-augmenting technical progress parameters. However, when capital and labor shares are driven by factors unrelated to the production side of the economy such as markups, estimation of these parameters will be contaminated. Increasing markups, for instance, may be captured in technology trends leading to biased estimates of σ. To this end, we take as given the estimates of De Loecker and Eeckhout (2017) from firm level data as an approximation for the evolution of aggregate markups. We correct labor and capital shares using these aggregate markups and estimate a normalised supply side system. We then compare our estimates with those in which a constant markup is assumed as in Klump et al. (2007), León-Ledesma et al. (2015), and Mućk (2017).

Our key results are the following. First, estimates of σ are higher than when assuming a constant markup: our estimates range between 0.82 and 0.86 as opposed to 0.34-0.53 with a constant markup. Secondly, technical change appears to be net capital-augmenting. This resolves the counter-intuitive result in previous studies that found capital-augmenting efficiency to have regressed in the US since the post-war period.

The rest of the paper is organised as follows: Section 2 specifies the firm problem and estimation system; Section 3 explains the data and gives our estimation results; Section 4 concludes.

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[1]See Piketty (2013), Karabarbounis and Neiman (2013), Elsby et al. (2013) amongst many others.

[2]See also Autor et al. (2017) who attribute the fall in the labor share to increasing concentration driven by superstar firms.

2 Firm problem and estimation system

Consider the Lagrangian associated with the cost minimisation problem of a firm within competitive factor markets:

$$\mathcal{L}(H_t, K_t, \lambda_t) = W_t H_t + R_t^K K_t - \lambda_t(F(A_t H_t, B_t K_t) - Y_t), \qquad (1)$$

where output Y_t is given by a constant returns to scale time-invariant production function $F(A_t H_t, B_t K_t)$ on labor H_t and capital K_t, where A_t and B_t are the levels of labor-augmenting and capital-augmenting efficiencies. W_t are nominal wages, R_t^K is the user cost of capital, and λ_t is the Lagrange multiplier, which is the marginal cost of production (the value of the cost function as we relax the production constraint). Standard first order conditions (FOCs) yield:

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\begin{aligned}
\frac{\partial \mathcal{L}}{\partial H_t} &= W_t - \lambda_t F_{H,t} = 0, \\
\frac{\partial \mathcal{L}}{\partial K_t} &= R_t^K - \lambda_t F_{K,t} = 0, \\
\frac{\partial \mathcal{L}}{\partial \lambda_t} &= F(A_t H_t, B_t K_t) - Y_t = 0.
\end{aligned}
$$

Multiplying both sides of the FOCs for labor and capital times $\frac{H_t}{P_t Y_t}$ and $\frac{K_t}{P_t Y_t}$ respectively, where P_t is the price of output, we obtain the first order conditions in terms of cost shares:

$$S_{H,t} = \frac{W_t H_t}{P_t Y_t} = \frac{\lambda_t}{P_t} \frac{F_{H,t} H_t}{Y_t} = \frac{1}{\mu_t} \frac{F_{H,t} H_t}{Y_t}, \qquad (2)$$

$$S_{K,t} = \frac{R_t^K K_t}{P_t Y_t} = \frac{\lambda_t}{P_t} \frac{F_{K,t} K_t}{Y_t} = \frac{1}{\mu_t} \frac{F_{K,t} K_t}{Y_t}, \qquad (3)$$

where we have defined the markup as $\mu_t = \frac{P_t}{\lambda_t}$, the inverse of the real marginal cost. We assume the production function takes the form of a Constant Elasticity of Substitution (CES) production function:

$$F(A_t H_t, B_t K_t) = [(1-\alpha)(A_t H_t)^\rho + \alpha(B_t K_t)^\rho]^{\frac{1}{\rho}}, \qquad (4)$$

where $-\infty < \rho < 1$ is the substitution factor that relates to the elasticity of substitution as $\sigma = \frac{1}{1-\rho}$ with $0 < \sigma < \infty$. The loadings of capital and labor are determined by parameter $0 < \alpha < 1$. We assume, as is common in the literature, that technologies follow a linear trend in logs: $A_t = A_0 e^{\gamma_H t}$ and $B_t = B_0 e^{\gamma_K t}$.

Given these functional forms, the supply side system to estimate can be written as:

$$\ln Y_t = \frac{1}{\rho} \ln \left[(1-\alpha)(A_0 e^{\gamma_H t} H_t)^\rho + \alpha(B_0 e^{\gamma_H t} K_t)^\rho \right],$$

$$\ln(S_{H,t}\mu_t) = \ln(1-\alpha) + \rho \ln A_0 + \rho\gamma_H t - \rho \ln \frac{Y_t}{H_t},$$

$$\ln(S_{K,t}\mu_t) = \ln(\alpha) + \rho \ln B_0 + \rho\gamma_K t - \rho \ln \frac{Y_t}{K_t}.$$

Following the literature on "normalization"[3] we represent the system in index form, i.e. relative to a point of normalization. Variables at the point of normalization are expressed with an over-bar ($\bar{X}$). With the normalized system, parameter α at the normalization point ($\bar{\alpha}$) has a direct interpretation as the capital share in value added at the point of normalization.

$$\ln \frac{Y_t}{\bar{Y}} = \ln \xi + \frac{1}{\rho} \ln \left[(1-\bar{\alpha})(e^{\gamma_H(t-\bar{t})} \frac{H_t}{\bar{H}})^\rho + \bar{\alpha}(e^{\gamma_H(t-\bar{t})} \frac{K_t}{\bar{K}})^\rho \right], \quad (5)$$

$$\ln(S_{H,t}\mu_t) = \ln(1-\bar{\alpha}) + \rho\gamma_H(t-\bar{t}) - \rho \ln \frac{Y_t/\bar{Y}}{H_t/\bar{H}}, \quad (6)$$

$$\ln(S_{K,t}\mu_t) = \ln \bar{\alpha} + \rho\gamma_K(t-\bar{t}) - \rho \ln \frac{Y_t/\bar{Y}}{K_t/\bar{K}}, \quad (7)$$

where ξ is a normalization constant whose value should be close to unity.[4] The system (5)-(7) allows us to estimate the parameters of interest ρ, γ_H, and γ_K.

3 Data and estimation results

We obtain annual data for the US private sector from the Fernald (2014) database where we retrieve hours worked, capital services (adjusted for utilization), and output. The markup is directly taken from De Loecker and Eeckhout (2017)and is the weighted arithmetic mean of firms' markups.[5] Because their estimates range from 1950 to 2014, this determines our sample period. Although this is one possible estimate of the markup, our aim here is to understand the biases when we observe

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[3]See La Grandville (1989), Klump et al. (2007), and León-Ledesma et al. (2010).

[4]For details, see Klump et al. (2007).

[5]When preferences are CES, the aggregate markup should ideally be calculated using the harmonic mean of firms' markups. De Loecker and Eeckhout (2017) make available only the arithmetic mean. However, given the joint distribution of markups and firm sizes, the difference between both means is unlikely to be large and to have changed differently between the beginning and end of the sample. A simulation exercise, available on request, confirms this point. We thank an anonymous referee for pointing this out.

an increasing markup over time, for which this measure suffices.[6] We assume that the markup is related to market structure and competition and is independent from technology and factor inputs. Finally, the labor share for the private sector is measured following Gomme and Rupert (2004). It measures the labor share for the private business sector attributing ambiguous income components (proprietors' income and indirect net taxes) to capital and labor income in the same proportion as unambiguous components.[7]

Because of the constant returns to scale (CRS) assumption, we can write output as $P_t Y_t = P_t F_{H,t} H_t + P_t F_{K,t} K_t$. Given the FOC's above, this can be re-written as $1 = \frac{W_t H_t}{P_t Y_t} \mu_t + \frac{R_t^K K_t}{P_t Y_t} \mu_t$. Since we can observe $\frac{W_t H_t}{P_t Y_t}$ and μ_t, we can then derive $\frac{R_t^K K_t}{P_t Y_t}$ as a residual.[8] Figure 1 plots $1/\mu_t$, $S_{K,t}$, and $S_{H,t}$. The markup experienced a rapid increase during the sample period, especially since 1980. The labor share, as reported in many previous studies, fell since the mid-1990s, but it did so less than the increase in the markup. This led to a fall in the capital share as reported by Barkai (2017).[9]

We estimate first a system where we assume a constant markup which is fixed at the average value of the estimates of De Loecker and Eeckhout (2017). Then we estimate it allowing for time varying markups. The system is estimated using a non-linear SUR (NLSUR) method and (over-identified) three-stage non-linear least squares (3SNLLS) where we used lagged first differences of the variables as instruments. Table 1 provides the results for the constant markup and variable markup cases for both estimation methods. The table presents parameter estimates and standard errors as well as a test for the null of a unitary σ, the Log-determinant of the system, and the J-test for over-identifying restrictions in the case of the 3SNLLS estimates.

The results for constant markups in Table 1 are standard in the literature. The elasticity of substitution is estimated to be below unity, labor augmenting technical

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[6]A limitation of using this measure is that De Loecker and Eeckhout (2017) use firm level data for listed companies from Compustat. This clearly does not correspond to the whole US private sector. However, as already mentioned, there is widespread evidence of increasing markups in the US economy and we use this measure as a proxy to understand the consequences of markup trends on estimates of key production function parameters.

[7]We used two alternative labor share measures and the results did not change significantly.

[8]Given the markups estimated by De Loecker and Eeckhout (2017), this residual method would lead to negative values for $\frac{R_t^K K_t}{P_t Y_t} \mu_t$ in several periods. To correct for this, we scaled their measure by 0.85 simply to ensure that the capital share is always positive. This scaling could be interpreted as the difference in markups between the aggregate business sector and listed companies. Note that this constant is irrelevant for the estimates of σ and the technical progress coefficients as it will be absorbed in the constant. Changing this scaling factor to a different value did not change our results.

[9]Note that, had we assumed a constant markup, then the capital share calculated as a residual would appear as increasing.
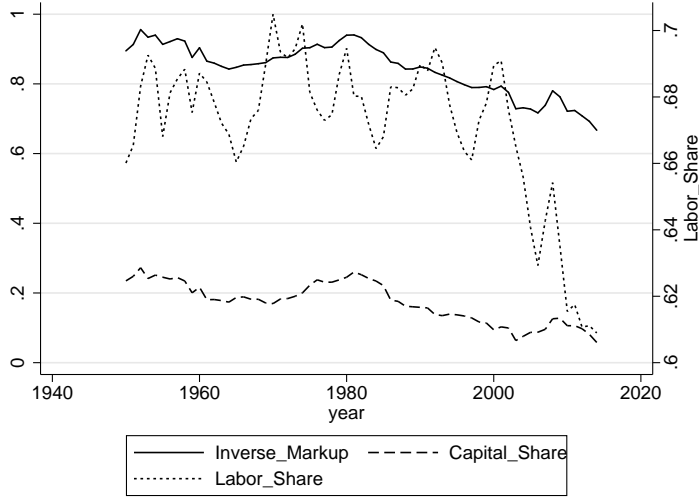
Figure 1: Inverse markups $1/\mu_t$, capital share ($S_{K,t}$), and labor share ($S_{H,t}$) (right scale) , U.S. 1950-2014.

progress is in the region of 2% per year, and capital augmenting technical progress is negative, although only marginally significant. Recently, Mućk (2017) uses a normalized supply side system to estimate σ and the bias in technical change for 12 advanced economies and finds that a negative γ_K is a robust finding. This result, however, is difficult to justify theoretically, as a regress in the level of efficiency of capital is unlikely to have taken place during this sample period.

For the time-varying markup estimates, however, the results show a very different picture. Whereas estimates of σ are still significantly below one, its estimated value is higher and in the region of 0.8. This is a higher value than that found in the previous literature using the normalised system approach. Perhaps more interesting, technical progress is now net capital augmenting. The reason behind this result is as follows. When assuming constant markups, $S_{H,t}\mu_t$ decreases over time (especially since the mid-1990s) and, as a consequence, $S_{K,t}\mu_t$ increases. If σ is below unity, for a given capital-labor ratio, γ_K is forced to be negative to fit the increasing capital share. The increase in the markup, however, leads to the opposite result. As the markup increase is stronger than the fall in $S_{H,t}$, $S_{H,t}\mu_t$ increases. Hence $S_{K,t}\mu_t$ increases leading to a positive estimate of γ_K.[10]

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[10]It is worth noting that the system with time-varying markups achieves a better fit for output, reducing the residual sum of squares by almost 22% in the case of NLSUR and 75% for the NL3SLS case. The fit for the capital and labor shares also improve substantially.

Table 1: Estimation results. U.S. 1950-2014

	Constant markup		Varying markup	
	NLSUR	NL3SLS	NLSUR	NL3SLS
ξ	0.987	1.005	1.013	1.006
	(0.011)	(0.005)	(0.013)	(0.011)
σ	0.532	0.336	0.864	0.816
	(0.005)	(0.001)	(0.023)	(0.065)
γ_H	0.023	0.018	0.005	0.01
	(0.003)	(0.000)	(0.003)	(0.005)
γ_K	-0.021	-0.007	0.059	0.040
	(0.001)	(0.000)	(0.014)	(0.018)
	Tests [p-values]			
$\sigma = 1$	[0.000]	[0.000]	[0.000]	[0.005]
J-test	–	[0.000]	–	[0.008]
Log-determinant	-20.02	-18.16	-18.99	-18.94

Robust standard errors in parentheses. P-values of tests in brackets for J-test of over-identifying restrictions and test for $\sigma = 1$.

4   Conclusions

We revisit the estimation of the aggregate elasticity of substitution (σ) and the bias in technical change for the US economy for the 1950-2015 period. Given the large increase in price markups reported in recent literature, assuming a constant markup, as previous studies have done, would bias the estimates of these deep parameters. When we correct for a time-varying markup, we find that σ is in the region of 0.8 and, contrary to previous literature, capital-augmenting technical progress is positive and larger than labor-augmenting technical progress.

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