Preventive maintenance models with random maintenance quality

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Abstract

In real-world environments it is usually difficult to specify the quality of a preventive maintenance (PM) action precisely. This uncertainty makes it problematic to optimise maintenance policy. This problem is tackled in this paper by assuming that the quality of a PM action is a random variable following a probability distribution. Two frequently studied models are, a failure rate PM model and an age reduction PM model, and then will be investigated. The optimal PM policies are presented and optimised. Numerical examples are also given.

Key words: Preventive maintenance; long-run average cost; failure rate; maintenance quality

1 Introduction

Maintenance actions can generally be divided into two types: corrective maintenance (CM) and preventive maintenance (PM). The quality of maintenance

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actions in both CM and PM is an interesting research topic in the reliability literature, and is also vitally important when maintenance policies are being developed in practice.

The state of a piece of equipment after a maintenance action is performed is assumed to be one of the three situations: *perfect*, *imperfect*, and *minimal*. A *perfect* maintenance action is assumed to restore the equipment to be as good as new; an *imperfect* maintenance action may bring the equipment to any condition between as good as new and as bad as previously, and a *minimal* maintenance action is assumed to restore the equipment to a state the same as before the action. Examples of models for perfect, imperfect and minimal maintenance actions are *Renewal Processes*, *Generalized Renewal Processes* and *Non-Homogeneous Poisson Processes*, respectively. More comprehensive discussion in maintenance from both theoretical and application points of view can be found in Pham and Wang[1]; Dekker[2] and Scarf[3].

The assumption that the equipment can be restored imperfectly, or imperfect maintenance, is closer to many practical scenarios than the other two assumptions. For modelling the quality of a PM action, two approaches have often been studied: a failure rate PM model by Lie and Chun [4] and Nakagawa [5], and an age reduction PM model by Canfield [6] and Malik [7]. Based on these two models, Lin et. al. [13,14] introduced a hybrid PM model that combines the failure rate PM model and the age reduction PM model.

Assume that PM actions on the equipment are carried out at every time interval T independent of the failure history of the equipment, and CM actions are conducted upon failures. The failure rate PM model, the age reduction PM model and the hybrid PM model are defined as follows.

- *Failure Rate PM Model* [4,5]. The failure rate after the kth PM becomes $h_k(t) = \theta h_{k-1}(t)$ for $t \in (0, T)$, where $\theta(> 1)$ is the adjustment factor, $h_k(t)$ $(t \in (0, T))$ is the failure rate after the kth PM, and T is the time interval between two adjacent PM actions. Each PM resets the failure rate to zero and the rate of increase of the failure rate gets higher after each additional PM. This model considers the change of the slope of the failure rate function. In this model, the adjustment factor θ is an index for measuring the quality of PM.
- *Age Reduction PM Model* [6,7]. Canfield[6] and Malik[7] introduced age reduction models. In the age reduction model introduced by Canfield[6], the effective age after the kth PM reduces to $t_k - \eta$ if the equipment's effective age was t_k just prior to this PM, where $\eta(< t_k)$ is the restoration interval in the effective age of the equipment due to the kth PM. The restoration interval η in this model is an index for measuring the quality of PM. This model has been widely used to optimize cost for maintenance policies under various assumptions[8,9]. In the age reduction model introduced by Malik[7], the effective age after the kth PM reduces to bt_k if the equipment's effective age was t_k just prior to this PM, where $b < 1$. Applications of the Malik model on the maintenance effect and optimizing maintenance policy can be found in [11,12].
- *Hybrid PM Model* [13]. The failure rate after the kth PM becomes $a_k h(bt_k + x)$, where t_k is the time when the kth PM is conducted, $1 = a_0 \leq a_1 \leq a_2 \leq ... \leq a_{N-1}$, $0 = b_0 \geq b_1 \geq b_2 \geq ... \geq b_{N-1} < 1$, $x > 0$, and $h(t)$ is the failure rate of the equipment when there is no CM or PM. Here, parameter a_k plays the same role as the parameter θ in the failure rate PM model, and parameter b_k functions similarly as the parameter b in the Malik's age reduction PM model.

All of the above three models assume that the failure rate of the equipment is increasing with time when no PM is conducted. This paper only studies the failure rate PM model and the Canfield age reduction PM model.

The parameters that determine the PM quality are the adjustment factor θ in the failure rate PM model and the restoration interval η in the Canfield age reduction PM model. They are important because they impact on the frequency of PM's and therefore the long-run average cost. These parameters can be estimated based on a domain expert's suggestion [7] or real data [6]. It is assumed by prior research on the above two models that the parameters θ and η are fixed constant. This assumption may be violated in many scenarios, especially in the case when the parameters are estimated by domain experts. It can be more practical to assume that these two parameters are random variables following certain probability distributions. Most maintenance engineers in building service systems, for example, usually do not indicate that the restoration interval of a PM is 2 years, they tend to estimate the restoration interval falls within an interval (1, 3) years instead. In this case, it can assume that the restoration interval is a random variable with a uniform probability distribution.

This paper considers the scenarios when the maintenance quality is a random variable. It assumes that both the adjustment factor θ and the restoration interval η are random variables with certain probability distributions. Optimal PM policies for these two models are then obtained.

The paper is organized as follows. Section 2 introduces two novel PM models that consider failure rate PM models and age reduction PM models whose parameters for measuring the maintenance quality are random variables, and

provides with algorithms for optimising PM policies. Further discussions on the quality of PM's are made in Section 3. Section 4 investigates two cases where the quality of PM's are assumed to be uniformly distributed and the failure time to be Weibull distributions. Finally, in the last section, concluding remarks are given.

2 MODEL FORMULATIONS

In this section, we investigate the failure rate PM model and the Canfield age reduction PM model, when the adjustment factor and the restoration interval are random variables. Model A assumes that the failure rate of a piece of equipment after a PM action become a product of a maintenance quality and the failure rate before the PM action. Model B assumes that the equipment's age become younger than before a PM action.

2.1 Model A

Assumptions

A. PM actions are performed at time kT for $k = 1, ..., N$. The equipment is replaced at time NT, where T is the time interval between two adjacent PM actions and N is the number of PM actions before a replacement.

B. When there is no PM or CM, the failure rate of the equipment, denoted by $h(t)$, is strictly increasing.

C. The equipment has the failure rate $h_k(t) = \theta^{k-1}h(t)$ after the kth PM, where $t \in (0, T)$, and θ is a random variable with a cumulative distribution function, denoted by $F(\theta)$ with $\theta \geq 1$. Assume that the Nth moment about

the random variable θ exists.

D. A replacement can restore the equipment to be as good as new. The equipment undergoes minimal repair upon failures between two adjacent PM's. The failure rate remains unchanged by minimal repair.

E. The times for PM, minimal repair and replacement are negligible.

F. The costs on a minimal repair, a PM and a replacement are c_m, c_p and c_r, respectively.

The long-run average cost rate is

$$C_A(T,N) = \frac{1}{NT}\left\{c_m \sum_{k=1}^{N} \int_0^T \left(\int_1^\infty \theta dF(\theta)\right)^{k-1} h(t)dt + (N-1)c_p + c_r\right\} \tag{1}$$

Denote $\gamma_k = \left(\int_1^\infty \theta dF(\theta)\right)^{k-1}$ and $r_k(t) = \gamma_k h(t)$. Eq. (1) can be re-written as

$$C_A(T,N) = \frac{1}{NT}\left\{\left(c_m \sum_{k=1}^{N} \int_0^T r_k(t)dt\right) + (N-1)c_p + c_r\right\} \tag{2}$$

To determine the optimal values of N and T that minimize $C_A(T,N)$ in Eq. (2), one can solve the following optimisation problem.

$$C_A(T_0,N_0) = \min_{T,N} C_A(T,N) \tag{3}$$

According to Nakagawa [5], there exists optimal T_0 and N_0 when Eq. (4) and (5) are satisfied.

$$\int_0^T \left\{(N-1)r_N(t) - \sum_{k=1}^{N-1} r_k(t)\right\} dt < \frac{c_r - c_p}{c_m}$$
$$\leq \int_0^T \left\{N r_{N+1}(t) - \sum_{k=1}^{N} r_k(t)\right\} dt \tag{4}$$

and

$$\sum_{k=1}^{N}\left\{Tr_k(T)-\int_0^T r_k(t)dt\right\}=\frac{(N-1)c_p+c_r}{c_m} \tag{5}$$

Table 1 shows an algorithm for searching the optimal T_0 and N_0 of Model A.

When the parameter θ is assumed to be a constant

2.2 Model B

Assume that Canfield's [6] age reduction PM model is utilized to maintain the equipment. Under Canfield's model, each PM reduces operational stress to that existing time units previous to the PM intervention, where the restoration interval is less than or equal to the PM intervention interval. Thus, each PM restores the failure rate of the equipment at time t to the one at $t-\eta$, where $\eta \leq T$, while its shape remains unchanged. That is, the failure rate keeps monotonically increasing, although the rate of degradation is reduced after each PM. Canfield [6] derived the following failure rate:

$$h_k(t)=\begin{cases} h(t) & \text{for } 0\leq t\leq T \\ w_k(T,\eta)+h(t-k\eta) & \text{for } kT\leq t\leq (k+1)T \end{cases} \tag{6}$$

where $0<\eta\leq T$, and $w_k(T,\eta)=\sum_{i=1}^{k}\{h(iT-(i-1)\eta)-h(i(T-\eta))\}$.

Assumptions

A. Assumptions A, B, D, E and F are the same as for Model A.

B. The equipment has the failure rate shown as Eq. (6), where η is a random

variable with cumulative distribution function $G(\eta)$ and $0 < \eta \leq T$.

Based on the above assumption, we can obtain the following long-run average rate

$$C_B(T,N) = \frac{1}{NT}\{c_m \int_0^T \left(\sum_{k=1}^{N-1} w_k(T,\eta)T + \sum_{k=0}^{N-1} \int_{kT}^{(k+1)T} h(t-k\eta)dt \right) dG(\eta) + (N-1)c_p + c_r\} \tag{7}$$

For a fixed N, by solving the following equation, the time period T_0 that achieves the minimal long-run average cost rate can be obtained.

$$C_B(T_0,N) = \min_T C_B(T,N) \tag{8}$$

To determine the values of N and T that minimize $C_B(T,N)$ in Eq. (8), denote

$$L(T,N) = N \int_0^T \left\{ w_N(T,\eta)T + \int_0^T h(t+NT-N\eta)dt \right\} dG(\eta) - \int_0^T \left\{ \sum_{k=1}^{N-1} w_k(T,\eta)T + \sum_{k=0}^{N-1} \int_0^T h(t+kT-k\eta)dt \right\} dG(\eta)$$

Let $C_B(T,N+1) \geq C_B(T,N)$ and $C_B(T,N-1) > C_B(T,N)$, which implies

$$L(T,N) \geq \frac{c_r - c_p}{c_m} \text{ and } L(T,N-1) < \frac{c_r - c_p}{c_m} \tag{9}$$

Let $\frac{\partial C_B(T,N)}{\partial T} = 0$, then

$$\begin{aligned} &\frac{N(N-1)}{2}T^2 h(T)g(T) + Tg(0)\int_0^{NT} h(t)dt + NTg(T)\int_0^T h(t)dt \\ &- \int_0^T \left(\sum_{k=1}^{N-1} w_k(T,\eta)T + \sum_{k=0}^{N-1} \int_0^T h(t+kT-k\eta)dt \right) dG(\eta) \\ &= \frac{(N-1)c_p + c_r}{c_m} \end{aligned} \tag{10}$$

where $g(T) = \frac{dG(t)}{dt}|_{t=T}$, and $g(0) = \frac{dG(t)}{dt}|_{t=0}$. Therefore, there exists optimal T_0 and N_0 when Eq. (9) and (10) are satisfied.

Table 2 shows an algorithm for searching the optimal T_0 and N_0 of Model B.

3 DISCUSSION

In the failure rate PM model and the age reduction PM model, both the adjustment factor θ and the restoration interval η are assumed to be constant. The above assumption that parameters θ and η are random variables provides an alternative approach to investigating the two parameters. The restoration interval η in the age reduction model can be estimated by either domain experts [7] or collected data [6]. As domain experts tend to offer a rough estimate of parameters, the above assumption may be more suitable for the situation when the domain experts are asked to estimate the parameters.

For the failure rate PM model, this paper only considers Canfield's age reduction model. Similarly, the parameter b in the Malik's age reduction model can be regarded as a random variable and analysed. As above mentioned, the hybrid PM model [13,14] is derived on the basis of the failure rate PM model and the age reduction PM model. The PM policies based on the hybrid PM model can be optimised if parameters a_k and b_k are random variables.

Further research can be considered in the case when the equipment's age may impact the adjustment factor or the restoration interval. Chan and Shaw[15], and Gu[16] considered the scenarios where the adjustment factor varies with the equipment's age; sequential PM modelling (see [13,14,5]) is a solution for such scenarios. However, each PM action in sequential PM modelling has a

different quality, which requires a large number of data to estimate it. In practical use, it may be very hard to provide system designers or maintenance engineers with a huge number of data.

4 NUMERICAL EXAMPLES

Suppose that the failure time of a piece of equipment has a Weibull distribution, i.e. $h(t) = \beta t^{\beta-1}$, where $\beta = 1.6$.

4.1 Model A

Let the adjustment factor θ in Model A has a uniform distribution as follows:

$$F(\theta) = \begin{cases} \frac{\theta-1}{u-1} \text{ for } 1 \le \theta \le u \\ \\ 0 \quad \text{otherwise} \end{cases} \tag{11}$$

Then for $\gamma_k = (\frac{1+u}{2})^{k-1}$ and $r_k(t) = \beta\gamma_k t^{\beta-1}$. The optimal solution N_0 and T_0 should satisfy the following inequality and equation:

$$(N_0-1){T_0}^{\beta}\gamma_{N_0} - {T_0}^{\beta}\sum_{k=1}^{N_0-1}\gamma_k < \frac{c_r - c_p}{c_m} \le N_0{T_0}^{\beta}\gamma_{N_0+1} - {T_0}^{\beta}\sum_{k=1}^{N_0}\gamma_k \tag{12}$$

and

$$T_0 = \left\{ \frac{(N_0-1)c_p + c_r}{c_m(\beta-1)\sum_{k=1}^{N_0}\gamma_k} \right\}^{\frac{1}{\beta}} \tag{13}$$

Then the minimal long-run average cost is

$$C_A(T_0, N_0) = \frac{1}{N_0 T_0}\left\{ c_m {T_0}^{\beta}\sum_{k=1}^{N_0}\gamma_k + (N_0-1)c_p + c_r \right\} \tag{14}$$

Based on research in the office building services systems, Evans et al(see [17]) have identified the relationships among three costs incurred within the systems' whole lifecycle span: initial capital costs, maintenance and building operating costs, and business operating costs. They found that the operation and maintenance of the building services systems costs can be five times the capital costs; while the business operating costs will be two hundred times the capital costs over the life of the building. In other words, business losses are forty times as large as maintenance costs, namely, corrective maintenance costs (that includes business losses) is forty times as large as preventive maintenance costs.

We therefore assume that $c_p = 1$, $c_m = 40$ and $c_r = 1000$. When u in the cumulative distribution function Eq. (11) changes from 1.1 to 2.0, one can obtain the following results shown in Table 3 and Figure 1 based on the above discussion. In order to show the data in a figure, the long-run average cost is divided by 10 in Figure 1.

From Figure 1, if u increases, the replacement times N_0 decreases, and both the time intervals T_0 and $C_A(T_0, N_0)$ increase.

4.2 Model B

Let the restoration interval η in Model B has a uniform distribution as follows:

$$G(\eta) = \begin{cases} \eta/T \text{ for } 0 \le \eta \le T \\ 0 \quad \text{otherwise} \end{cases} \tag{15}$$

The distribution indicates that a PM action can bring the equipment to the condition between before this PM and after the last PM. Denote

$$D_{N_0} = \sum_{k=2}^{N_0-1} \sum_{i=2}^{k} \frac{i^\beta - 1}{i-1} - \sum_{k=1}^{N_0-1} \left(\sum_{i=1}^{k} i^{\beta-1} - \frac{(k+1)^{\beta+1} - k^{\beta+1} - 1}{k(\beta+1)} \right) + (N_0 - 1)\beta + 1 \tag{16}$$

Then, the optimal solution N_0 should satisfy the following inequality:

$$(N_0 - 1)D_{N_0} - N_0 D_{N_0-1} \leq \frac{c_r - c_p}{c_m T_0^\beta} < N_0 D_{N_0+1} - (N_0 + 1)D_{N_0} \tag{17}$$

And T_0 can be obtained from the following equation:

$$T_0 = \left\{ \frac{(N_0 - 1)c_p + c_r}{c_m(\beta - 1)D_{N_0}} \right\}^{1/\beta} \tag{18}$$

The long-run average cost is

$$C_B(T_0, N_0) = \frac{c_m T_0^\beta D_{N_0} + (N_0 - 1)c_p + c_r}{N_0 T_0} \tag{19}$$

We assume that $c_p = 1$ and $c_m = 40$. When c_r changes from 500 to 5000, we have the following results shown in Table 4 and Figure 2 based on the above discussion. In order to show the data in a figure, the long-run average cost is divided by 10 in Figure 2.

Model A and model B are developed based on the assumption that the parameters θ and η are random variable. If the parameters are set to the expected values of the random variables, the number of the PM actions before a replacement and the time interval between two adjacent PM actions can be optimised. For example, set a fixed parameter $\eta = \frac{T}{2}$ for model B, then the optimal values of N_0 and T_0 are shown in Table 5. Compare Table 4 and Table 5, one can find that the cost in Table 4 is lower than in Table 5, which

indicates the effect on the results of choosing the uniform distributions for the parameter η of model B is better than that of choosing a fixed value. Figure 3 shows the change of the long-run average cost divided by 10 when $\eta = \frac{T}{2}$.

5 CONCLUSIONS

In both the reliability literature and practical use, the quality of preventive maintenance (PM) is an interesting topic because it is vitally important during optimizing maintenance policies and lifecycle costing.

Prior research assumes that the quality of PM is a fixed constant, which is usually not true in many scenarios. This paper investigated the optimization problem of PM policies for the situations where the quality of PM is a random variable with a certain probability distribution, which would be more practical than the situation when the quality of maintenance is assumed to be a fixed constant. The optimal maintenance policies for failure rate PM models and age reduction PM models were obtained in the paper.

When the life distribution of a piece of equipment is a Weibull distribution and the quality of PM distributes uniformly, explicit expressions of the optimal time interval of PM can be obtained. The numerical example shows how the long-run average cost changes with the quality of PM; it also shows that the effect on the long-run average cost of choosing the uniform distributions for the restoration interval of the age reduction model is lower than that of choosing a fixed value.

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Fig. 1. Changes of the optimal solution $\boldsymbol{N}_0$, T_0 and $C_A(T_0, N_0)$ with the change of u

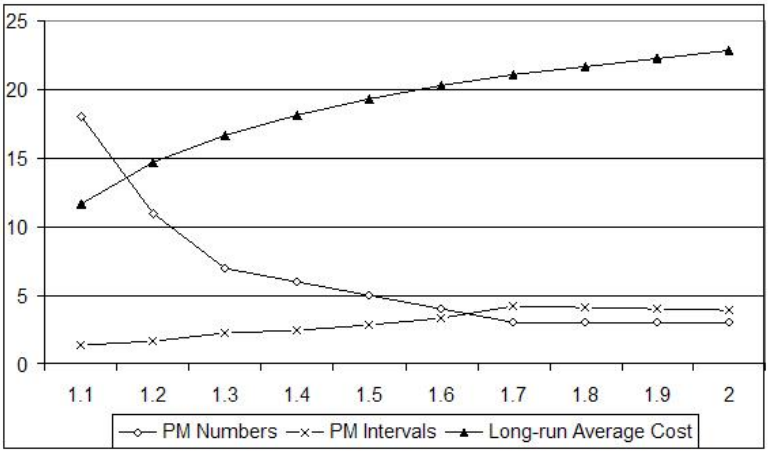


Fig. 2. Changes of the optimal solution $\boldsymbol{N}_0$, T_0 and $C_A(T_0, N_0)$ with the change of c_r when eta is a random variable with a uniform distribtuion as Eq. (15)

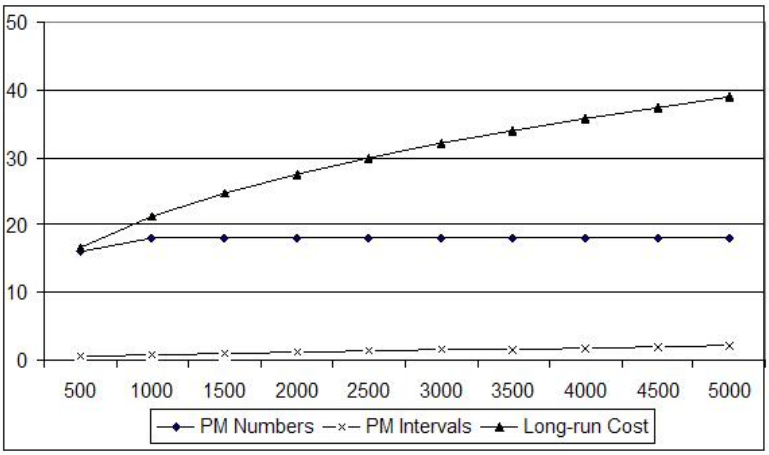


Fig. 3. Changes of the optimal solution $\boldsymbol{N}_0$, T_0 and $C_A(T_0, N_0)$ the change of c_r when $\eta = \frac{T}{2}$

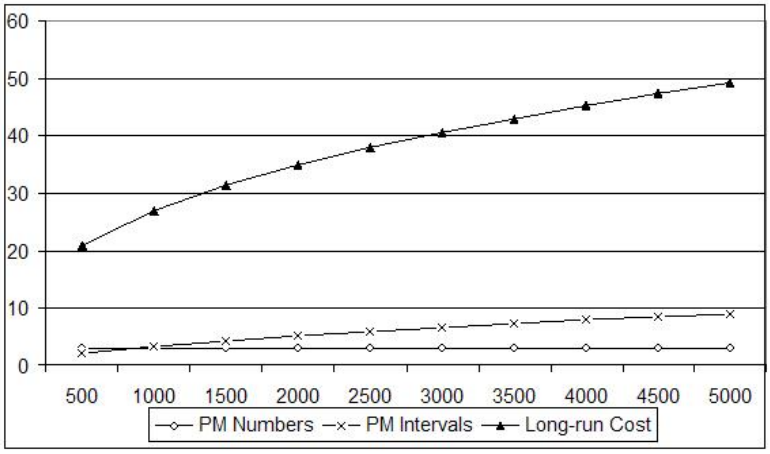


Table 1

Searching the optimal T_0 and N_0 for Model A

Inputs

$h(t)$: the failure rate c_m: the cost of minimal repair;

c_p: the cost of PM; c_r: the cost of replacement;

$F(\theta)$: the cumulative distribution function of θ

Outputs

N_0: the cumulative distribution function of θ

T_0: the optimal time interval for periodic PM;

1: **for** $N = \{1, 2, ..., \infty\}$ **do**

2: obtain T by solving Eq. (5);

3: **if** inequality (4) is satisfied, **then**

4: calculate $C_A(T, N)$ from Eq. (2);

5: $T_0 \Leftarrow T$; $N_0 \Leftarrow N$;

6: break;

7: **end**

8: **end**

Table 2

Searching the optimal T_0 and N_0 for Model B

For Inputs, replace $F(\theta)$ with $G(\eta)$, and θ with η. Outputs are the same as Algorithm 1. The calculating steps are the same as Algorithm 1 after the follow interchanges have been made: Eq. (4) with Eq. (9); Eq. (5) with Eq. (10); Eq. (2) with Eq. (7), respectively.

Table 3

Changes of the optimal solution $\boldsymbol{N}_0$, T_0 and $C_A(T_0, N_0)$ with the change of u

u	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
N_0	18	11	7	6	5	4	3	3	3	3
T_0	1.4	1.7	2.3	2.5	2.8	3.3	4.2	4.1	4	3.9
$C_A(T_0, N_0)$	116.8	146.6	166.7	181.7	193.5	202.6	211.1	217.1	223.0	228.9

Table 4

Changes of the optimal solution $\boldsymbol{N}_0$, T_0 and $C_B(T_0, N_0)$ with the change of c_r when *eta* is a random variable with a uniform distribtuion as Eq. (15)

c_r	500	1000	1500	2000	2500	3000	3500	4000	4500	5000
N_0	3	3	3	3	3	3	3	3	3	3
T_0	2.2	3.3	4.2	5.1	5.9	6.6	7.2	7.9	8.5	9.0
$C_B(T_0, N_0)$	207.6	268.9	313.0	348.6	378.9	405.8	429.9	451.9	472.4	491.4

Table 5

Changes of the optimal solution $\boldsymbol{N}_0$, T_0 and $C_B(T_0, N_0)$ with the change of c_r when $\eta = \frac{T}{2}$

c_r	500	1000	1500	2000	2500	3000	3500	4000	4500	5000
N_0	16	18	18	18	18	18	18	18	18	18
T_0	0.6	0.8	1.0	1.2	1.4	1.6	1.7	1.9	2.0	2.2
$C_B(T_0, N_0)$	167.3	213.6	247.8	275.8	299.9	321.6	339.7	357.8	373.3	389.1