A replacement policy for a repairable system with its repairman having multiple vacations

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Abstract

This paper considers a replacement policy for a repairable system with a repairman, who can have multiple vacations. If the system fails and the repairman is on vacation, it will wait for repair until the repairman is available. Assuming that the system can not be repaired "as good as new" and a repair upon failure can be performed immediately with a probability of *p*, we optimise replacement policy using geometric processes. The explicit expression of the expected cost rate is derived, and the corresponding optimal policy can be determined analytically or numerically. Finally, a numerical example is given to illustrate the theoretical results of the model.

Keywords: Geometric process; Multiple vacation; Replacement policy; Maintenance policy

1. Introduction

A repairable system is a system which, after failing to perform one or more of its functions satisfactorily, can be restored to fully satisfactory performance by any method, rather than the replacement of the entire system (Ascher and Feingold, 1984). Repair models developed upon successive inter-failure times have been employed in many applications such as the optimisation of

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maintenance policies, decision making and whole life cycle cost analysis. With different repair levels, repair can be broken down into three categories (Yamez et al, 2002): perfect repair, normal repair and minimal repair. A perfect repair can restore a system to an “as good as new” state, a normal repair is assumed to bring the system to any condition, and a minimal repair, or imperfect repair, can restore the system to the exact state it was before failure. Example models for perfect, normal, and minimal repair are renewal process (RP) models or homogeneous Poisson process (HPP) models, generalised renewal processes, and non-homogeneous Poisson process (NHPP) models, respectively. On the basis of the relationship between failure intensities and time, repair models fall into three categories: models with a constant failure intensity (e.g. HPP models), models with an operating-time dependent failure intensity (e.g. NHPP models) and models with a repair-time dependent failure intensity (e.g. geometric processes (GP) models (Lam, 1988)).

In reality, the survival times of a system after each repair can become shorter and shorter due to various reasons such as ageing and deterioration. The working times and repair times can be modeled by geometric processes as many authors have studied (Lam, 1988; Wu and Clements-Croome, 2005; Zhang and Wang, 2007). The geometric process introduced by Lam (1988) defines an alternative to the non-homogeneous Poisson process: a sequence of random variables $\{X_k, k=1,2,...\}$ is a geometric process if the distribution function of X_k is given by $F(a^{k-1}t)$ for k=1,2,... and a is a positive constant. Wang and Pham (1996) later refer the geometric process as a quasi-renewal process. Finkelstein (1993) develops a model: he defines a general deteriorating renewal process such that $F_{k+1}(t) \leq F_k(t)$. Wu and Clements-Croome (2006) extend the geometric process by replacing its parameter a^{k-1} with $a_1a^{k-1}+ b_1b^{k-1}$, where a>1 and 0<b<1. The geometric process has been applied to reliability analysis and maintenance policy optimisation for various systems by authors; for example, Wu and Clements-Croome (2005), Castro and Pérez-Ocón (2006), Zhang and Wang (2007), and Braun et al (2008).

The existing research mainly concentrates on the reliability analysis or maintenance optimisation with a consideration of the behaviours of repairable systems themselves. Little work has been conducted to consider reliability analysis for a system where the repairman might take a sequence of vacations of random durations and a repair on a failure is a normal repair. Here we emphasize that the durations of vacations can be different. Such a vacation policy is called a multiple vacation policy, which has attracted attention in queuing theory (for example, Lee, 1988; Krishna et al, 1998; Chang and Choi, 2005).

The applications of such situations where a repairman can take multiple vocations can be found in practice. In some situations, a repairman can have two roles: one for caring a system and one for other duties, which can happen in a small/median firm that wants to use the repairman effectively. If the repairman is assigned to look after only one system, he might have plenty of idle time. In this paper, *vocation* can mean period when the repairman is on other duties. The repairman can periodically check the status of the system: if the system fails, he repairs it; if the system is working, he goes back to the other duties. Allocating the manpower of the repairman in such a way is more realistic and more profitable than simply assigning him a single role of being a repairman.

This paper presents the formulations of the expected long-run profit per unit time for a repairable system with a repairman. We assume that the repairman takes multiple vacations. When the system fails, the repairman will be called in to bring the system back to a certain state. The time to repair is composed of two different periods: waiting and real repair periods. The waiting time starts from the component's failure to the start to repair, and the real repair time is the time between the start to repair and the completion of the repair. Both the working and real repair times are assumed to be a type of stochastic processes: *geometric processes*, and the waiting times are subject to a renewal process. The probability that a failed system can be immediately repaired is assumed to be p. The expected long-run profit per unit time is derived and a numerical example is given to illustrate the theoretical results of the model.

The paper is structured as follows. The coming section introduces geometric processes defined by Lam (1988), and assumptions. Sections 3 and 4 derives the expected long-run profit per unit time, and discusses special cases, respectively. Section 5 offers numerical examples. Concluding remarks are offered in the last section.

2. Definitions and Model Assumptions

This section first borrows the definition of geometric processes from Lam (1988), and then makes assumptions for model development.

2.1 Definition

Definition 1 Assume ξ, η are two random variables. For arbitrary real number α, there is

$$P(\xi \geq \alpha) > P(\eta \geq \alpha)$$

then ξ is called stochastically bigger than η. Similarly, if

$$P(\xi \geq \alpha) < P(\eta \geq \alpha)$$

then ξ is called stochastically smaller than η.

Definition 2 (Lam, 1988) Assume that $\{X_n, n=1,2,\ldots\}$ is a sequence of independent non-negative random variables. If the distribution function of X_n is $F(a^{n-1}t)$, for some a>0 and all, n=1,2, …, then $\{X_n, n=1,2,\ldots\}$ is called a geometric process.

Obviously,

if $a>1$, then $\{X_n, n=1,2,\ldots\}$ is stochastically decreasing,

if $a<1$, then $\{X_n, n=1,2,\ldots\}$ is stochastically increasing, and

if $a=1$, $\{X_n, n=1,2,\ldots\}$ is a renewal process.

2.2 Assumptions

The following assumptions are assumed to hold in what follows.

A. At time t=0, the system is new.

B. The system starts to work at time t=0, and it is maintained by a repairman. The repairman takes his first vacation after the system has started. After his vacation ends, there will be two situations.

(a) If the system has failed and is waiting for repair, the repairman will repair it. He will then take his second vacation after the repair is completed.

(b) If the system is still working, the repairman will take his second vacation. This operating policy continues until a replacement takes place.

C. After the repairman finishes his vacation, the probability that he can immediately repair the failed system is p. Denote V_n as the waiting time after the nth failure occurs, where $\{V_n, n=1,2,\ldots\}$ are independently and identically distributed with distribution $S(t)$ $(t \geq 0)$ and $\tau = EV_n < +\infty$.

D. The time interval from the completion of the (n-1)th repair to that of the nth repair of the system is called the nth cycle of the system, where $n = 1,2,\ldots$. Denote the working time and the repair time of the system in the nth cycle ($n = 1,2,\ldots$) as X_n and Y_n, respectively. Denote the length of the ith vacation during the nth cycle as $\{Z_n^i, n=1,2,\cdots\}$. Denote the cumulative distribution functions of X_n, Y_n, Z_n^i and $F_n(x)$ as $G_n(y)$, and $H_n(z)$, respectively, where $F_n(x) = F(a^{n-1}x)$, $G_n(y) = G(b^{n-1}y)$, and $H_n(z) = H(d^{n-1}z)$. Denote $E(X_1) = \lambda$, $E(Y_1) = \mu$, and $E(Z_1^1) = \gamma$.

E. X_n, Y_n, Z_n^i, and V_n (i=1,2,... and $n = 1,2,\ldots$) are statistically independent.

F. When a replacement is required, a brand new but identical component will be used, and the length of a replacement time is negligible.

G. The following costs are considered:

- C_1: repair cost per unit time;
- C_2: reward per unit time when the system is working;
- C_3: cost incurred for a replacement;

- C_4: reward per unit of the repairman when he is taking vacation or other duties, which can produce profits for the firm;
- C_5: cost per unit time when the system is waiting for repair; and
- C_6: cost per unit time incurred in the waiting time after the system has failed.

3. Expected profit under replacement policy *N*

Denote η_n the times of vacations of the repairman during the nth cycle of the system. A typical progress is given in Figure 1.

Figure 1 here

Figure 1. A typical progress of the system

Let T_1 be the time before the first replacement, T_n be the time between the (*n*-1)th and *n*th replacement with *n*=2,3,…. The process $\{T_n, n=1, 2,\ldots\}$ forms a renewal process. Denote $P(N)$ as the expected long-run profit per unit time under replacement policy N, then we have

$$P(N)=\lim_{t\to\infty}\frac{\text{Expected profit within }[0,\text{t}]}{\text{t}}$$

Since $\{T_n, n=1,2,\cdots\}$ is a renewal process, the time between two adjacent replacements is the length for a replacement. Hence

$$P(N)=\frac{\text{expected profit within a replacement cycle}}{\text{expected length of a cycle}}=\frac{ER}{EW} \tag{1}$$

Lemma 1. The probability of η_n is given by

$$P(\eta_n=m)=\int_0^{+\infty}[S_{m-1}(t)-S_m(t)]dF(a^{n-1}t),\quad m=1,2,\cdots,\ n=1,2,\cdots,\ N$$

and

$$E\eta_n=\int_0^{+\infty}[\sum_{m=1}^{\infty}S_m(t)]\,dF(a^{n-1}t).$$

where $S_m(t)$ is the cumulative distribution function of $\sum_{i=1}^{m}Z_n^i$.

Proof According to the law of total probability, we have

$$P(\eta_n=m)=P[\sum_{i=1}^{m-1}Z_n^i<X_n<\sum_{i=1}^{m}Z_n^i]=\int_0^{+\infty}P\,[\sum_{i=1}^{m-1}Z_n^i<t<\sum_{i=1}^{m}Z_n^i,\ X_n\le t]dF(a^{n-1}t)$$

$$=\int_0^{+\infty}[S_{m-1}(t)-S_m(t)]dF(a^{n-1}t),$$

and

$$E\eta_n=\sum_{m=1}^{\infty}mP(\eta_n=m)=\sum_{m=1}^{\infty}m\int_0^{+\infty}[S_{m-1}(t)-S_m(t)]dF(a^{n-1}t)$$

$$=\int_0^{+\infty}\sum_{m=1}^{\infty}m\,[S_{m-1}(t)-S_m(t)]dF(a^{n-1}t)=\int_0^{+\infty}[\sum_{m=1}^{\infty}S_m(t)]\,dF(a^{n-1}t)\,.$$

From the assumptions, the length of a replacement cycle is given by

$$W=\sum_{n=1}^{N}\sum_{k=1}^{\eta_n}Z_n^k+\sum_{i=1}^{N-1}[Y_iI\{A_i\}+(Y_i+V_i)I\{B_i\}]$$

$$=\sum_{n=1}^{N-1}Y_n+\sum_{n=1}^{N}\sum_{k=1}^{\eta_n}Z_n^k+\sum_{i=1}^{N-1}V_iI\{B_i\}\,.$$

where $I\{A\}=1$ if event A occurs, otherwise 0. Denote $A_i=\{$the system can be repaired immediately after the *i*th failure$\}$, and $B_i=\{$the system can not be repaired immediately after the *i*th failure$\}$.

Hence,

$$E[\sum_{k=1}^{\eta_n}Z_n^k]=E[E(\sum_{k=1}^{\eta_n}Z_n^k|\eta_n]=\sum_{m=1}^{\infty}[\sum_{k=1}^{m}E(Z_n^k)]P(\eta_n=m)$$

$$=\sum_{m=1}^{\infty}\sum_{k=1}^{m}\frac{\gamma}{d^{n-1}}P(\eta_n=m)=\frac{\gamma}{d^{n-1}}\sum_{m=1}^{\infty}mP(\eta_n=m)=\frac{\gamma}{d^{n-1}}E(\eta_n)$$

$$=\frac{\gamma}{d^{n-1}}\int_0^{+\infty}[\sum_{m=1}^{\infty}S_m(t)]\,dF(a^{n-1}t),\text{ and}$$

$$E[\sum_{i=1}^{N-1}V_iI\{B_i\}]=\sum_{i=1}^{N-1}E[V_iI\{B_i\}]=(N-1)(1-p)\tau\,.$$

The expected time for a replacement is

$$EW=\sum_{n=1}^{N-1}EY_n+\sum_{n=1}^{N}E\,[\sum_{k=1}^{\eta_n}Z_n^k]+\sum_{i=1}^{N-1}E[V_iI\{B_i\}]$$

$$=\sum_{n=1}^{N-1}\frac{\mu}{b^{n-1}}+\sum_{n=1}^{N}\frac{\gamma}{d^{n-1}}\int_0^{+\infty}[\sum_{m=1}^{\infty}S_m(t)]\,dF(a^{n-1}t)+(N-1)(1-p)\tau \qquad (2)$$

and the profit within a cycle is

$$R=C_2\sum_{n=1}^{N}X_n+C_4\sum_{n=1}^{N}\sum_{k=1}^{\eta_n}Z_n^k-C_1\sum_{n=1}^{N-1}Y_n-C_5\sum_{n=1}^{N}(\sum_{k=1}^{\eta_n}Z_n^k-X_n)-C_6E[\sum_{i=1}^{N-1}V_iI\{B_i\}]-C_3$$

$$=(C_2+C_5)\sum_{n=1}^{N}X_n+(C_4-C_5)\sum_{n=1}^{N}\sum_{k=1}^{\eta_n}Z_n^k-C_1\sum_{n=1}^{N-1}Y_n-C_6E[\sum_{i=1}^{N-1}V_iI\{B_i\}]-C_3$$

The expected profit within a cycle is given by

$$ER=(C_2+C_5)\sum_{n=1}^{N}\frac{\lambda}{a^{n-1}}+(C_4-C_5)\sum_{n=1}^{N}\frac{\gamma}{d^{n-1}}\int_0^{+\infty}[\sum_{m=1}^{\infty}S_m(t)]\,dF(a^{n-1}t)-C_1\sum_{n=1}^{N-1}\frac{\mu}{b^{n-1}}$$
$$-C_6(N-1)(1-p)\tau-C_3 \qquad (3)$$

If we consider equations (1), (2) and (3), we obtain the expected long-run profit per unit time as

$$P(N)=\frac{(C_2+C_5)\sum_{n=1}^{N}\frac{\lambda}{a^{n-1}}+(C_4-C_5)\sum_{n=1}^{N}\frac{\gamma}{d^{n-1}}\int_0^{\infty}[\sum_{m=1}^{\infty}S_m(t)]dF(a^{n-1}t)-C_1\sum_{n=1}^{N-1}\frac{\mu}{b^{n-1}}-C_6(N-1)(1-p)\tau-C_3}{\sum_{n=1}^{N-1}\frac{\mu}{b^{n-1}}+\sum_{n=1}^{N}\frac{\gamma}{d^{n-1}}\int_0^{\infty}[\sum_{m=1}^{\infty}S_m(t)]dF(a^{n-1}t)+(N-1)(1-p)\tau} \quad (4)$$

4. Special cases

We assume that the cumulative distribution functions of X_n, Y_n, Z_n^i, and V_n are

$$F_n(t)=F(a^{n-1}t)=1-\exp(-\frac{a^{n-1}}{\lambda}t)$$

$$G_n(t)=G(b^{n-1}t)=1-\exp(-\frac{b^{n-1}}{\mu}t),$$

$$H_n(t)=H(d^{n-1}t)=1-\exp(-\frac{d^{n-1}}{\gamma}t)$$

and

$$S(t)=1-\exp(-\frac{t}{\tau})$$

respectively, where $t\geq 0$.

As we assume that Z_n^i ($i=1,2,\cdots,m$) are mutually independent, the probability density function of $\sum_{i=1}^{m}Z_n^i$ is a hypo-exponential distribution (Ross, 1997).

Lemma 2. Assume that random variables V_1, V_2, $\cdots$, V_n are independently and identically distributed with an exponential distribution of parameter λ_0, then the probability density function of $\sum_{i=1}^{n}V_i$ is

$$\phi_n(t)=\frac{\lambda_0(\lambda_0 t)^{n-1}}{(n-1)!}e^{-\lambda_0 t} \quad (4)$$

Denote the cumulative distribution function of $\sum_{i=1}^{n}V_i$ as $\Phi_n(t)$**,** then

$$\sum_{n=1}^{\infty}\Phi_n(t)=\lambda_0 t \quad (5)$$

Proof. From Ross (1997), we have $\phi_n(t)=\frac{\lambda_0(\lambda_0 t)^{n-1}}{(n-1)!}e^{-\lambda_0 t}$**.** Then

$$\sum_{n=1}^{\infty}\Phi_n(t)=\sum_{n=1}^{\infty}\int_0^t\phi_n(t)\,dt=\int_0^t\lambda_0(\sum_{n=1}^{\infty}\frac{(\lambda_0 t)^{n-1}}{(n-1)!})e^{-\lambda_0 t}dt=\int_0^t\lambda_0\, e^{\lambda_0 t}\; e^{-\lambda_0 t}dt=\lambda_0 t\,.$$ †

Theorem 1. The expected long-run profit per unit time is given by

$$P(N)=\frac{(C_2+C_4)\sum_{n=1}^{N}\frac{\lambda}{a^{n-1}}-C_1\sum_{n=1}^{N-1}\frac{\mu}{b^{n-1}}-C_6(N-1)(1-p)\tau-C_3}{\sum_{n=1}^{N-1}\frac{\mu}{b^{n-1}}+\sum_{n=1}^{N}\frac{\lambda}{a^{n-1}}+(N-1)(1-p)\tau}$$

There exists an optimal N^* that maximizes the value $P(N)$.

Proof. Since Z_n^i ($i=1,2,\cdots,m$) are independently and identically distributed with an exponential distribution of parameter $\frac{d^{n-1}}{\gamma}$, then the probability distribution of $\sum_{i=1}^{m} Z_n^i$ is a gamma distribution with scale parameter $\frac{\gamma}{d^{n-1}}$ and shape parameter m, the probability density function of $\sum_{i=1}^{m} Z_n^i$ is given by

$$f_m(t)=\frac{1}{(m-1)!}\left(\frac{d^{n-1}}{\gamma}\right)^m t^{m-1} e^{-\frac{d^{n-1}}{\gamma}t} \quad (t\geq 0)$$

Hence, the cumulative distribution function of $\sum_{i=1}^{m} Z_n^i$ is given by

$$S_m(t)=\int_0^t f_m(u)\,du$$

Hence

$$\sum_{m=1}^{\infty} S_m(t)=\int_0^t \sum_{m=1}^{\infty}\left[\frac{1}{(m-1)!}\left(\frac{d^{n-1}}{\gamma}\right)^m u^{m-1} e^{-\frac{d^{n-1}}{\gamma}u}\right]du$$

$$=\int_0^t \left[\sum_{m=1}^{\infty}\frac{\left(\frac{d^{n-1}u}{\gamma}\right)^{m-1}}{(m-1)!}\right]\left(\frac{d^{n-1}}{\gamma}\right)e^{-\frac{d^{n-1}}{\gamma}u}\,du$$

$$=\int_0^t \left[\frac{d^{n-1}}{\gamma}\right]du=\frac{d^{n-1}}{\gamma}t$$

Then,

$$\sum_{n=1}^{N}\frac{\gamma}{d^{n-1}}\int_0^{+\infty}\left[\sum_{m=1}^{\infty}S_m(t)\right]dF(a^{n-1}t)=\sum_{n=1}^{N}\int_0^{\infty} t\,dF(a^{n-1}t)=\sum_{n=1}^{N}EX_n=\sum_{n=1}^{N}\frac{\lambda}{a^{n-1}}$$

Hence, the expected long-run profit per unit time is given by

$$P(N)=\frac{(C_2+C_4)\sum_{n=1}^{N}\frac{\lambda}{a^{n-1}}-C_1\sum_{n=1}^{N-1}\frac{\mu}{b^{n-1}}-C_6(N-1)(1-p)\tau-C_3}{\sum_{n=1}^{N-1}\frac{\mu}{b^{n-1}}+\sum_{n=1}^{N}\frac{\lambda}{a^{n-1}}+(N-1)(1-p)\tau} \qquad (6)$$

Since $a>1$, $0<b<1$, the expected long-run profit per unit time is monotonously increasing when the number N is small, and the expected long-run profit per unit time is monotonously decreasing when the number N is large. $\lim_{N\to\infty} P(N) = -C_1$. Therefore, there exists a maximum value in $P(N)$, or we can find the optimum replacement policy N^*, which maximizes the value of $P(N^*)$.

This proves the theorem.

5. Numerical examples

In this section, we will give examples to demonstrate the theoretical results of our model.

5.1 Sensitivity analysis for the repair times influencing the profit

If we set $a = 1.1, b = 0.98, \lambda = 100, \mu = 1, C_1 = 20, C_2 = 500, C_3 = 5000, C_4 = 200, C_6 = 100, \tau = 0.2$, and $p = 0.8$, then the optimum number for a replacement will be N=8, and the corresponding expected long-run profit per unit time is 535.09. The change of value $P(N)$ with repair times N is shown in Figure 2. The value $P(N)$ increases rapidly when repair times changes from 1 to 8, and then decreases slowly when repair times increases. This indicates that the expected long-run profit per unit time is more sensitive to big values of N^*. In case it is not possible to undertake a replacement when repair times reaches N^*(=8), we can replace the system after more repairs have been conducted, rather than less. This is because larger N^* ($3 < N^* < 13$, say) tends to have greater profit, whereas smaller N^* might not have good profits ($N^*<4$).

Figure 2 here

Figure 2 The expected long-run profit per unit time $P(N)$ against repair times N.

a	*N**	*P(N*)*	*a*	*N**	*P(N*)*	*a*	*N**	*P(N*)*
1.01	17	665.03	1.18	6	459.75	1.35	5	367.5
1.02	15	650.42	1.19	6	452.3	1.36	5	363.7
1.03	13	634.58	1.2	6	445.19	1.37	5	360.03
1.04	12	618.46	1.21	6	438.4	1.38	4	356.47
1.05	11	602.68	1.22	6	431.9	1.39	4	353.03
1.06	10	587.58	1.23	6	425.68	1.4	4	349.69
1.07	10	573.27	1.24	6	419.72	1.41	4	346.46
1.08	9	559.78	1.25	5	414.01	1.42	4	343.33

1.09	9	547.07	1.26	5	408.52	1.43	4	340.29
1.1	8	535.09	1.27	5	403.24	1.44	4	337.34
1.11	8	523.79	1.28	5	398.17	1.45	4	334.47
1.12	8	513.12	1.29	5	393.3	1.46	4	331.69
1.13	7	503.02	1.3	5	388.6	1.47	4	328.98
1.14	7	493.46	1.31	5	384.07	1.48	4	326.35
1.15	7	484.38	1.32	5	379.7	1.49	4	323.8
1.16	7	475.76	1.33	5	375.49	1.5	4	321.31
1.17	7	467.56	1.34	5	371.43			

Table 1: The expected long-run profit per unit time against the values of *a* and *N**.

5.2 Sensitivity analysis for parameters a and N

If we keep the values of parameters in Section 5.1, apart from the parameter *a*, we obtain results shown in Table 1. Table 1 shows how the optimum repair times *N** and the expected long-run profit per unit time change when parameter *a* changes from 1.01 to 1.5. From Table 1, we have the following results.

- We can see that the optimum *N** is sensitive to a small change of parameter *a* when *a* is smaller than 1.1: the optimum *N** change from 17 to 9. The optimum *N** becomes stable when *a* is larger than 1.1: it changes from 8 to 7 when *a* changes from 1.11 to 1.21. The *N** remains even more stable when *a* is larger than 1.21.
- The expected long-run profit per unit time for smaller *a*, for example, changing from 1.01 to 1.05, changes faster than that for larger *a*. As we can image, smaller *a*’s are more profitable than larger *a*’s. This is because they require fewer replacements and earn greater profit.

Figure 3 shows all of the changes over parameter *a* and repair times *N*, which gives a visual description on the changes of the expected long-run profits, parameter *a* and failure times *N*.

Figure 3 here

Figure 3 The expected long-run profit per unit time *P*(*N*) against repair times *N* and parameter *a*.

6. Conclusions

Searching an optimal replacement point for a system maintained by a repairman with multiple vocations is of interest and importance. This paper derived the expected long-run profit per unit time for such a system. We also considered a special scenario where the working times, real repair times, and vacation times are geometric processes. A numerical example is given to illustrate the theoretical results of the model.

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References

H. Ascher and H. Feingold, Repairable system modelling, inference, misconceptions and their causes. New York: Marcel Dekker (1984).

W. J. Braun, W. Li and Y.Q. Zhao, Some theoretical properties of the geometric and α-series processes, Communications in Statistics - Theory and Methods **37** (9) (2008), pp. 1483-1496.

I. T. Castro and R. Pérez-Ocón, Reward optimization of a repairable system, Reliability Engineering and System Safety **91**(2006), pp, 311-319.

S. H. Chang and D. W. Choi, Performance analysis of a finite-buffer discrete-time queue with bulk arrival, bulk service and vacations, Computers and Operations Research **32**(2005), pp, 2213-2234.

M. S. Finkelstein, A scale model of general repair, Microelectronics Reliability **33**(1993), pp, 41-44.

C. V. Krishna, R. Nadarajan and R. Arumuganathan, Analysis of a bulk queue with N-policy multiple vacations and setup times, Computers and Operations Research **25**(1998), pp, 957-967.

Y. Lam, A note on the optimal replacement problem, Advances in Applied Probability 20(1988), pp, 479-482.

H. W. Lee, M/G/1 Queue with Exceptional First Vacation, Computers and Operations Research 15(1988), pp, 441-445.

S. M. Ross, Introduction to Probability Models. Sixth edition. Academic Press, San Diego, CA (1997).

H. Wang and H. Pham, A quasi renewal process and its applications in imperfect maintenance, International Journal of Systems Science 27(1996), pp, 1055-1062.

S. Wu and D. Clements-Croome, A novel repair model for imperfect maintenance, IMA Journal Management Mathematics 17(2006), pp, 235-243.

S. Wu and D. Clements-Croome, Optimal maintenance policies under different operational schedules, IEEE Transactions on Reliability 54(2005), pp, 338-346.

M. Yanez, F. Joglar and M. Modarres, Generalized renewal process for analysis of repairable systems with limited failure experience, Reliability Engineering and System Safety 77(2002), pp, 167–180.

Y. L. Zhang and G. J. Wang, A deteriorating cold standby repairable system with priority in use, European Journal of Operational Research 183(2007), pp, 278-295.

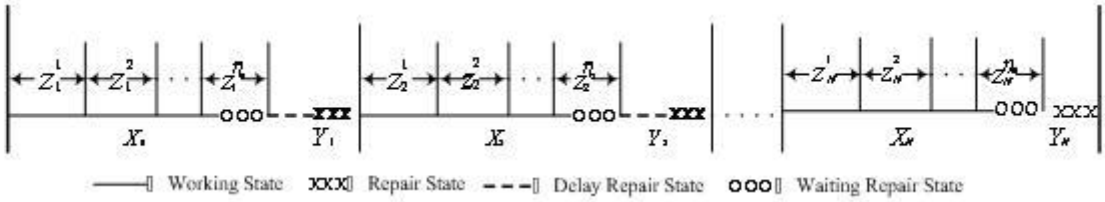

Figure 1

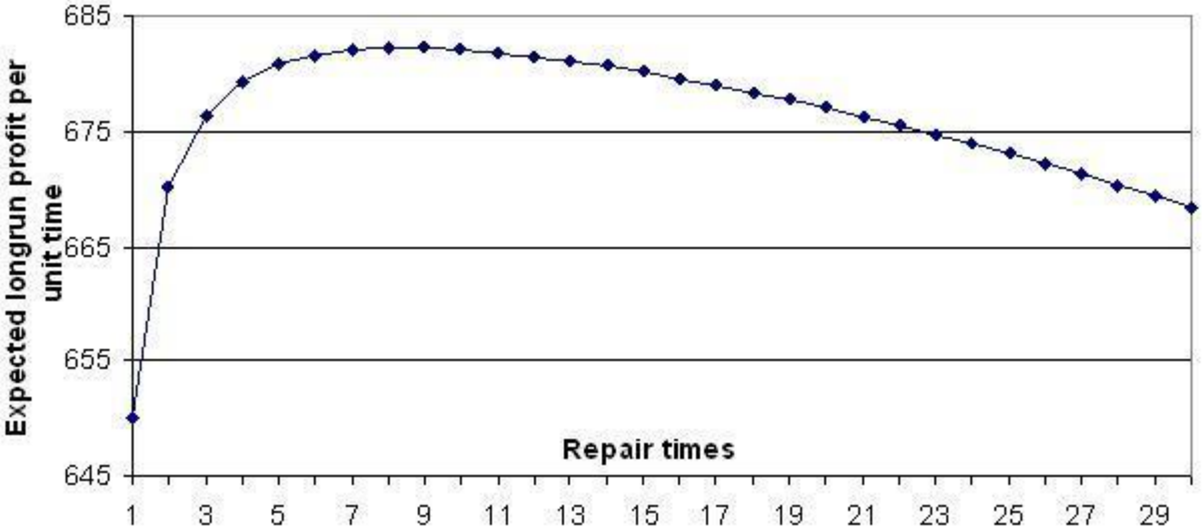

Figure 2

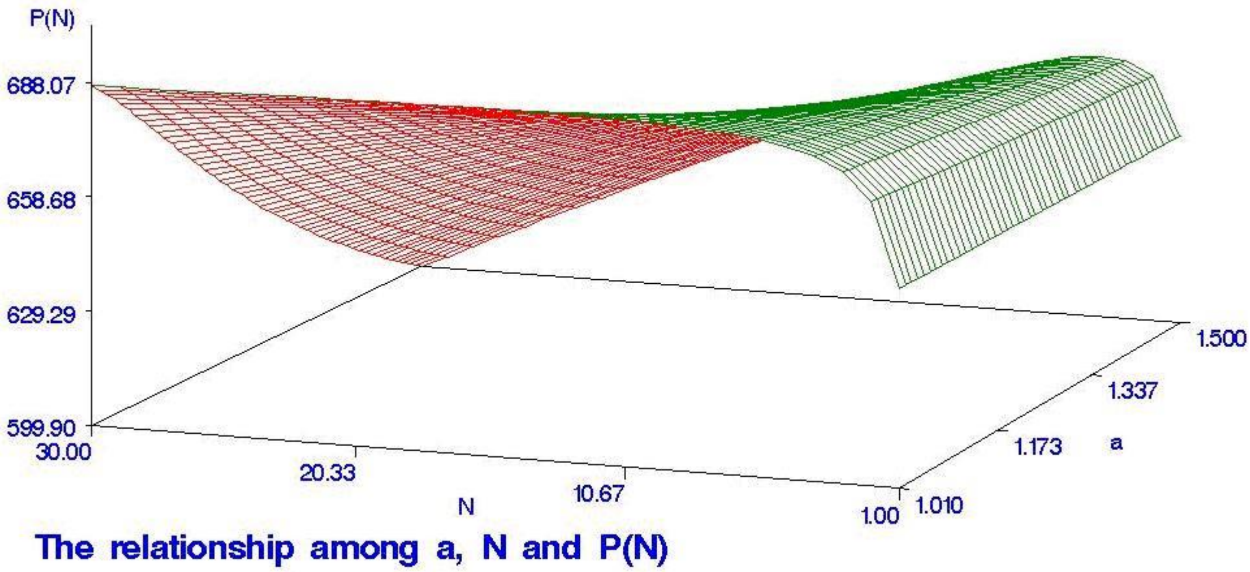

Figure 3