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Warranty claim analysis considering human factors

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Abstract

Warranty claims are not always due to product failures. They can also be caused by two types of human factors. On the one hand, consumers might claim warranty due to misuse and/or failures caused by other human factors. Such claims might account for more than 10% of all reported claims; on the other hand, consumers might not be bothered to claim warranty for failed items that are still under warranty, or claim warranty after they have experienced several intermittent failures. These two types of human factors can affect warranty claim costs. However, research in this area has received rather little attention.

In this paper, we propose three models to estimate the expected warranty cost when the two types of human factors are included. We consider two types of failures, intermittent and fatal failures, which might result in different claim patterns. Consumers might report claims after a fatal failure has occurred, and upon intermittent failures they might report claims after a number of failures have occurred. Numerical examples are given to validate the results derived.

Keywords: warranty claim, non-failed but reported (NFBR), failed but not reported (FBNR), human factor, intermittent failure, fatal failure.

Nomenclature

$\lambda_1(t)$	Intensity function of a non-homogeneous Poisson process of fatal failures
$\lambda_2(t)$	Intensity function of a non-homogeneous Poisson process of intermittent failures
$F_2(t)$	Cumulative distribution function for lifetime t due to intermittent failures
p_{2k}	Probability that the cause of an intermittent failure at the k th warranty claim is not successfully detected
$S_2(t)$	Probability of successfully identifying and then repairing the cause of intermittent failures at time t
$H_3(t)$	Cumulative distribution function due to a NFBR claim
$q_1(t)$	Probability of a claim being made at time t , given that a fatal failure has occurred
q_2	Probability that an intermittent failure results in a warranty claim
c_1	Cost on a claim due to a fatal failure
c_2	Cost on detecting the cause of intermittent failures
\tilde{c}_2	Cost on fixing the cause of an intermittent failure
c_{20}	Cost on detecting and fixing the cause of intermittent failures per unit time
c_{31}	Administration cost on per NFBR claim
c_{32}	Expected cost on fixing the cause of an NFBR claim
w	Length of warranty periods

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22 **1. Introduction**

23 A warranty is a contractual obligation incurred by a manufacturer (vendor or seller) in connection with the
24 sale of a product. In broad terms, the purpose of warranty is to establish liability in the event of a premature
25 failure of an item or the inability of the item to perform its intended function [1]. Product warranty has become
26 increasingly more important in consumer and commercial transactions, and is widely used to serve many
27 different purposes [2]. The US Congress has enacted several acts (UCC, Magnusson Moss Act, Tread Act, etc.)
28 over the last 100 years. The European Union (EU) passed legislation requiring a two-year warranty for all
29 products sold in Europe [3].

30 Analysing warranty claims can provide manufacturers with useful information on their products, as
31 warranty claim data are collected from the field that reflects the real operating conditions and usage intensity.
32 Research on analysing warranty claims data has mainly been concentrated on dealing with incomplete warranty
33 claims data (see [4-6], for example) and developing improved techniques to model warranty claims data (see
34 [7-10], for example). After the field reliability of products has been estimated, warranty servicing cost analysis
35 becomes another topic that needs to be focused. In this area, optimising warranty policies under different cost
36 settings (see [11-13], for example), and selecting maintenance policies for given warranty policies are two main
37 research focuses (see [14,15], for example). For more detailed information on warranty claims data analysis and
38 warranty servicing cost analysis, the reader is referred to the review papers [3,16-20] and the three books
39 [19,21,22].

40 However, our literature review shows that most of the existing research makes the following two
41 assumptions:

- 42 (i) failed products will be reported warranty, and
- 43 (ii) claims reported are due to product failures.

44 The above two assumptions do not necessarily hold and are associated with consumers' behaviours
45 towards warranty claims.

46 On Assumption (i), most of the research assumes that an absence of warranty claim is a 'no failure'
47 situation [2,5,23-26]. Only two publications, from the same authors, consider the cases where all of reported
48 products are not failed (see [27,28], for example). Patankar and Mitra [27,28] consider consumer behaviour in
49 exercising warranties and describe it with warranty execution functions. They assume that all consumers may
50 not exercise the warranty even if the product fails during the warranty period, or FBNR (failed be not reported).

51 For Assumption (ii), most authors assume that reported claims are due to product failures. Little research
52 considers situations where reported products might be due to misuse, other human factors, or even non-failed
53 products. In this paper, all of such claims are called non-failed but reported (NFBR) claims.

54 Consumers might also execute warranty claims after they have experienced a number of intermittent
55 failures. An item might stop working due to a fatal failure or an intermittent failure. A product with a fatal
56 failure will stop working until it is repaired. In practice, not all of failures are fatal; some are intermittent. An
57 intermittent failure is the loss of some functions or performance characteristics of a product for a limited period
58 of time until subsequent recovery of the function. In the case of intermittent failures, consumers may experience

59 a failure and restart the product (for example, computers) and it runs OK. When the product is taken to a service
60 agent, the repairman might not experience this failure when the item is being detected. The claims due to
61 intermittent failures can constitute a quite large proportion of the entire claim population, as the percentage of
62 the “no-fault found” (NFF) event can be as high as 50% of all failures in electronic products while intermittent
63 failure is one of the main causes of NFF [29]. An intermittent failure example can be as follows.

64 A global variable in an electronic product is read and rewritten over another global variable; a
65 miscalculation can then arise and lead to product failure. The users can therefore mistakenly believe that the
66 product has failed and claim warranty, the product is then sent to a service agent. However, when the global
67 variables are reset, perhaps upon rebooting the computer, the product can return to normal function.

68 Although intermittent failures are a main cause of failures, little research has been found to model their
69 warranty costs.

70 In this paper, we derive warranty costs for three situations: NFBR claims, FBNR phenomenon, and their
71 combination, and we assume two types of failures: fatal and intermittent failures.

72 The novelty of this work lies in:

- 73 • to our knowledge, it is the first paper modelling warranty claim cost due to NFBR claims;
- 74 • it considers manufacturer’s ability to rectify intermittent failures and assume such ability is increasing
75 over time; and
- 76 • warranty claims are usually considered for individual products. This paper assumes that a
77 manufacturer’s ability to fix the cause of intermittent failures develops over a batch of products.

78 The paper is structured as follows. Section 2 discusses human factors in warranty claims. Section 3
79 develops three models considering both physical reliability and human factors for repairable products. Section 4
80 offers numerical examples to validate the models developed above. Section 5 concludes the findings.

81 **2. Human factors in warranty claims**

82 In this section, we discuss three factors that might cause warranty claims: non-failed but reported (NFBR),
83 failed but not reported (FBNR), and claims arising from intermittent failures.

84 **2.1 Non-failed but reported (NFBR) claims**

85 There are situations when consumers might claim warranty, although failures are not due to product
86 reliability but due to human factors. Some examples are as follows.

- 87 • Claims for failures due to misuse, damage, accident, neglect, or lack of care. For example, a consumer
88 by accident poured water into a laptop computer, which makes the computer failed. The consumer then
89 claims warranty. If a product is damaged by human factors, consumer can be responsible for part of
90 fees, for example, returning and shipping fee.
- 91 • Fraudulent claims. Consumer might be driven by warranty or insurance claims, and replace multiple
92 items to repair one fault or resort to fraudulent reporting of a problem which never occurred. For
93 example, the AAA insurance stated “At least 10 percent of all reported claims are fraudulent in some
94 way, according to industry reports” [29].

- Some other forms of such claims can arise due to various reasons such as complexity of products, lack of sufficient training on product usage or faulty operational manual, product users might not be able to operate the products correctly, all of which can cause the products unable to work properly. The users can therefore mistakenly believe that the products have failed, and then claim warranty.

Here, we refer non-failed but reported claims (NFBR) to those claims that are purely due to human factors. It differs from claims upon intermittent failures that can occur but cannot be verified, replicated at will, or attributed to a specific failure site, mode, and mechanism. It also differs from claims arising from fatal failures that are due to products themselves but not human factors.

Responses of the manufacturers to NFBR claims can be different: (1) some manufacturers might even cease the warranty contract with consumers with NFBR claims, and (2) some manufacturers might not cease the warranty contract, as it is not easy to judge if a NFBF claim is intentionally or unintentionally committed.

However, a common feature is that both can incur costs to the manufacturers, and therefore should be considered in estimating warranty claim cost.

2.2 Failed but not reported (FBNR) events

Failed items might not be reported warranty. This can happen due to various reasons, for example, technological advances cause some products (especially electronic products) to be updated frequently or become obsolete quickly. Although such items are protected by warranty contracts, their users might not bother to claim warranty for failed items, especially when failed items have been served for a quite long time and/or they are not expensive. We call such a phenomenon as Failed But Not Reported (FBNR). Patankar and Mitra [27,28] investigate consumer behaviour when they do not execute the full execution of warranty. They listed a number of influencing factors such as costs of executing the warranty, the type of rebate plan, etc.

There is a trend towards long-term warranties [20]. Rapid technological advances in many industries, especially the electronics manufacturing industry, make products obsolete quicker than before, which requires product manufacturers to provide long-term warranties to protect consumers' profits. Long term warranty and complex products will make FBNR events occur more often than short term warranty. This presents an incentive to estimate warranty claims for the FBNR claims.

2.3 Warranty claims arising from intermittent failures

As intermittent failures might involve more testing to find the causes of the failures, costs on intermittent failures are different from those on fatal failures. Consumers might also claim warranty after intermittent failures have occurred for several of times. The claim patterns upon intermittent failures are therefore different from those claims arising from fatal failures. Hence, it is vitally important to consider consumer behaviour in analysing warranty claim data. For more discussion on intermittent failures or NFF, the reader is referred to [30,31], in which the authors discuss the concept, causes and impact of the "trouble not identified" phenomenon in the electronics industry, but they did not develop mathematical models to estimate warranty claims cost.

From a manufacturer's perspective, the ability to identify troubles arising from intermittent failures develops when more and more intermittent failures are investigated. It is reasonable to assume that the

131 probability of successfully detecting the causes of intermittent failures is increasing over time or over the
132 number of claims. Such improvement might not affect the number of warranty claims as products have already
133 been sold to consumers. The cost on repairing the intermittent failures, however, can decrease.

134 **2.4 Warranty costs due to human factors**

135 Normally, warranty cost incurs due to

- 136 • labour on diagnosing and repairing the failure;
- 137 • parts/materials used to repair the failed product;
- 138 • shipping including shipping new parts for replacement and/or failed products for repair.

139 Costs incurred by the above three types, claims upon NFBR claims, FBNR events, and intermittent failures
140 differ from those claim costs due to fatal failures.

141 From a manufacturer's perspective, costs due to NFBR might only include costs on reporting (for example,
142 delivering the non-failed products to their manufacturers), and cost due to FBNR might be zero as no report is
143 conducted for a failed product.

144 In developing warranty claims models considering human factors such as NFBR and FBNR, an important
145 requirement is that we should be able to differentiate claims due to NFBR from those due to FBNR in the
146 models. This is because we need to estimate the cost on claims due to NFBR and FBNR.

147 The impact of intermittent failures can be profound. Due to their characteristics, manufacturers may
148 assume a cause(s) rather than spend the time and cost to determine a root-cause. This can result in increased
149 maintenance costs, decreased equipment availability, increased consumer inconvenience, reduced consumer
150 confidence, damaged company reputation, and in some cases potential safety hazards [29].

151 The probability distributions of NFBR claims and FBNR events also have their own characteristic features.
152 The proportion of NFBR claims in the whole product population might decrease over time since the products
153 have entered service, whereas the proportion of FBNR events might increase over time. The NFBR for
154 repairable products might seldom occur after the products have failed and repaired once, because the users can
155 be assumed to have learnt how to operate the products from this failure and shall not make more mistakes of
156 reporting non-failed products. It is therefore reasonable to assume that a consumer makes at most one NFBR
157 claim.

158 **3. Model development**

159 Suppose that the following general assumptions hold.

- 160 (1) Two types of failures are considered: fatal and intermittent failures. Fatal failures require rectification to
161 restore the products to operational state, and intermittent failures do not require rectification action to
162 make it operational but need action to rectify the cause of such failures. The occurrences of fatal and
163 intermittent failures are assumed to be statistically independent.
- 164 (2) Three types of claims are considered: claims upon fatal failures, claims upon intermittent failures, and
165 claims arising from NFBR events.

- 166 (3) Time on repair is negligible. Repairs on fatal failures are minimal, that is, a product with a fatal failure
 167 is restored to the state where it was exactly before it failed. If an intermittent failure of a product cannot
 168 be verified, an identical product with the same age as the failed one will be used to replace it.
- 169 (4) An individual consumer makes at most one NFBR claim. Upon NFBR claims, only administration cost
 170 is incurred to the manufacturer.
- 171 (5) Only non-renewing warranty policy is considered.

172 In the rest of this section, we consider warranty costs for three situations: NFBR claims, FBNR
 173 phenomenon, and their combination. In all of the three situations, we assume two types of failures: fatal and
 174 intermittent failures.

175 **3.1 Expected cost with fatal failure, intermittent failure, and NFBR**

176 This section derives the expected warranty claim costs.

177 **3.1.1 Expected cost on fatal failures**

178 Based on Assumption (3) above, time on repair is negligible and repairs on fatal failures are minimal. Then
 179 the expected warranty cost with only fatal failures is given by

$$WC_1(w) = c_1 \int_0^w \lambda_1(t) dt \quad (1)$$

180 **3.1.2 Expected cost on intermittent failures**

181 We assume that

- 182 • every intermittent failure results in a warranty claim;
- 183 • the service agent can either detect the cause or not; and
- 184 • once the cause of intermittent failures has been detected and fixed, failures due to this cause will not
 185 occur again.

186 It should be noted that a manufacturer might receive warranty claims due to intermittent failures reported by
 187 different consumers, and then it tries to detect and fix the cause based on all of the claims. Hence, the
 188 manufacturer's ability to detect and further fix the cause develops over their experience learnt from treating all
 189 of claims. For this reason, it might not be correct to assume that the ability to detect and fix the cause of
 190 intermittent failures simply depends on claims from a single product/consumer. One should consider claims
 191 from all of the products sold. An alternative approach might be to assume that for an individual product,
 192 manufacturer's ability to detect and further fix the cause develops over time.

193 As such, we can consider the following two cases: the probability of successfully identifying and then
 194 repairing intermittent failures depends on (1) the number of claims due to intermittent failures; and (2) time.

195 In case that the probability of successfully identifying and then repairing intermittent failures is dependent
 196 on the number of claims, we can estimate the expected claim cost as follows.

197 Assume that n products are sold at the same date, and claims upon intermittent failures from all of the n
 198 products are reported according to a NHPP (nonhomogeneous Poisson process) or HPP (homogeneous Poisson
 199 process) with intensity function $\lambda_n(t)$. For example, if intermittent failures of an individual product occur

200 according to a HPP with intensity function is λ_0 , then $\lambda_n(t)=n\lambda$. Denote p_{2j} (where $p_{2j} > 0$ and $j \geq 1$) as the
 201 probability that the cause at the j th warranty claim is not detected. Then the probability of the first success in
 202 detecting the cause of the intermittent failures at the k th claim is $P_{2k} = (1 - p_{2k}) \prod_{j=1}^{k-1} p_{2j}$ with $k > 1$ (where
 203 $P_{21} = 1$ and $\sum_{k=1}^{\infty} P_{2k} = 1$). Note that k failures to occur in $[0, t)$ is given by the probability $[\Lambda(t)]^k/k!$, where
 204 $\Lambda_n(t) = \int_0^t \lambda_n(u)du$. Then the total expected warranty cost due to intermittent failures for an individual product
 205 is given by

$$\begin{aligned}
 WC_2(w) = & \frac{1}{n} \sum_{k=1}^{\infty} \left([(k-1)c_2 + \tilde{c}_2] P_{2k} \frac{e^{-\Lambda_n(w)} [\Lambda_n(w)]^k}{k!} \right) \\
 & + \frac{1}{n} \sum_{k=1}^{\infty} \sum_{m=k+1}^{\infty} \left([(m-1)c_2 + \tilde{c}_2] (1 - p_{2m}) \frac{e^{-\Lambda_n(w)} [\Lambda_n(w)]^k}{k!} \prod_{j=1}^{m-1} p_{2j} \right) \quad (2)
 \end{aligned}$$

206 The first term in Eq. (2) is the expected warranty claim cost if the cause of intermittent failures is detected
 207 and fixed within warranty period; it implies that the manufacturer fails to detect the cause of the intermittent
 208 failures arising from the first $k-1$ claims but it is successful at the k th claim. The second term in Eq. (2) is the
 209 expected warranty claim cost if detecting the cause of intermittent failures has not been successful during
 210 warranty and it continues after warranty expires.

211 **Remarks.** In some cases, detecting the cause of intermittent failures might start from the first claim and
 212 from then such effort might continue until the cause is eventually detected and fixed or a new model of products
 213 is launched to replace the old ones. In this case, the probability of successfully detecting and then fixing the
 214 cause depends on time, instead of the number of intermittent failures. If we can set the time when the n products
 215 were sold to be 0, then the cumulative distribution function of time to the first failure (and then claim) is
 216 $F_2^{(n)}(t) = 1 - (1 - F_2(t))^n$. The probability that an intermittent failure occurs during the warranty period is
 217 given by $\int_0^w dF_2^{(n)}(t)$. If it occurs, the expected time length to fix or to a new generation is $\int_t^{T_n} u dS_2(u)$. Then
 218 the expected cost on warranty claims is given by

$$WC_{21}(w) = \frac{c_{20}}{n} \int_0^w \int_t^{T_n} u dS_2(u) dF_2^{(n)}(t) \quad (3)$$

219 where $S_2(t)$ is the cumulative distribution function of time to detect and remove the cause of intermittent
 220 failures from all of claims of n products, T_n is an estimated time when the manufacturer might give up trying to
 221 detect the cause (or the time when a new model of products is launched), and c_{20} is the cost on detecting and
 222 fixing the cause per unit time.

223 In what follows, we shall concentrate on $WC_2(w)$ in Eq. (2), $WC_{21}(w)$ in Eq. (3) will be analysed in our
 224 future work.

225 3.1.3 Expected cost on NFBR claims

226 We assume that time to a NFBR claim is a random variable Z with distribution function $H_3(t)$. We consider the
 227 following two scenarios.

228 **Scenario 1**-- A NFBR claim will not cause warranty to be ceased. Then the expected warranty cost is
 229 given by

$$WC_{31}(w) = c_{31}H_3(w) \quad (4)$$

230 where c_{31} is the administration cost per NFBR claim.

231 **Scenario 2**-- A NFBR claim will cause warranty to be ceased. Once the warranty ceases, there are no
 232 further costs to the manufacturer. Then the expected warranty cost given by

$$WC_{32}(w) = c_{32}H_3(w) \quad (5)$$

233 where c_{32} is the expected cost on fixing the cause of an NFBR claim

234 **3.2 Model I --- combined effects from fatal, intermittent failures and NFBR** 235 **claims**

236 In this section, we assume that all failures are reported over the entire warranty period, and examine the
 237 combined effects from fatal failures, intermittent failures and NFBR, considering the two scenarios discussed in
 238 Section 3.1.3.

239 **Scenario 1**-- A NFBR claim will not cause warranty to be ceased. Then the expected warranty cost is
 240 given by

$$EC_{11}(w) = WC_1(w) + WC_2(w) + WC_{31}(w) \quad (6)$$

241 **Scenario 2**-- A NFBR claim will cause warranty to be ceased. If a NFBR claim occurs within warranty
 242 period, then the warranty ceases and no more claims on fatal failures or intermittent failures occur. The
 243 probability that this will occur is $H_3(z)$, where $z < w$, and the expected warranty claim cost is $WC_1(z) + WC_2(z)$
 244 with $z < w$. If a NFBR claim occurs after warranty expires, then the expected warranty claim cost is $WC_1(w) +$
 245 $WC_2(w)$ with $z \geq w$.

246 On removing the conditioning we have the expected warranty cost

$$EC_{12}(w) = \int_0^w [WC_1(z) + WC_2(z) + C_{32}]dH_3(z) + [WC_1(w) + WC_2(w)](1 - H_3(w)) \quad (7)$$

247 **3.3 Model II --- FBNR claims**

248 Due to reasons such as technological advances, some products (especially electronic products) can become
 249 obsolete quickly. Although such products might sometimes be protected by a long-term warranty contract, their
 250 consumers might not claim warranty for failed products, especially when failed products have served for a quite
 251 long time and/or they are not expensive. In this section, we consider the situation when warranty claims are
 252 partially executed.

253 Upon fatal failures, the willingness of consumers to claim warranty might diminish with time. The
 254 probability of consumers being inclined to claim warranty for products due to a fatal failure is assumed to be
 255 $q_1(t)$, which is a decreasing function in time t .

256 Upon intermittent failures, the following three human factors need consideration.

- 257 • Consumers' willingness to claim warranty might diminish with time;

258 • Manufacturer's capability to identify the causes of intermittent failures is improving with time. Hence,
 259 cost on dealing with such claims can decrease.

260 Let q_2 denote the probability that an intermittent failure results in a warranty claim. Then we have
 261 a thinning process with intensity function (for warranty claims) given by $q_2\lambda_n(t)$. Hence, similar to the
 262 derivation of Eq. (2), we have the expected warranty cost given by

$$\begin{aligned}
 EC_2(w) = & c_1 \int_0^w q_1(t)\lambda_1(t)dt + \frac{1}{n} \sum_{k=1}^{\infty} \left([(k-1)c_2 + \tilde{c}_2] P_{2k} \frac{e^{-q_2\Lambda_n(w)} [q_2\Lambda_n(w)]^k}{k!} \right) \\
 & + \frac{1}{n} \sum_{k=1}^{\infty} \sum_{m=k+1}^{\infty} \left([(m-1)c_2 + \tilde{c}_2] (1-p_{2m}) \frac{e^{-q_2\Lambda(w)} [q_2\Lambda(w)]^k}{k!} \prod_{j=1}^{m-1} p_{2j} \right) \quad (8)
 \end{aligned}$$

263 The first term in Eq (8) is the expected claim cost due to fatal failures. It considers both the probability of
 264 fatal failures and the probability of consumers being inclined to claim warranty for products due to these fatal
 265 failures. The meaning of the second and the third term are similar to those given for Eq. (2).

266 Patankar and Mitra [28] consider consumer behaviour in warranty execution and develop four warranty
 267 execution functions (WEFs), or called FBNR rates in this paper. Mathematically, these four WEFs can be
 268 categorised into the following two classes.

$$q_{11}(t, w_1, w, \varphi_1) = \begin{cases} 1 & 0 \leq t \leq w_1 \\ \frac{w - \varphi_1 w_1}{w - w_1} - \frac{(1 - \varphi_1)t}{w - w_1} & w_1 \leq t \leq w, \\ 0 & t > w \end{cases} \quad (9)$$

270 where $0 \leq \varphi_1 \leq 1$ and $0 \leq w_1 < w$, and

$$q_{12}(t, w_1, w, \varphi_2) = \begin{cases} 1 & 0 \leq t \leq w_1 \\ e^{-(t-w)/\varphi_2} & w_1 \leq t \leq w, \\ 0 & t > w \end{cases} \quad (10)$$

272 where $0 \leq w_1 < w$.

273 For fatal failures, the number of warranty claims, or WEF, is assumed to be

$$q_1(t) = e^{-\gamma_1 - \gamma_2 t}, \quad (11)$$

275 where $\gamma_1, \gamma_2 \geq 0$.

276 3.4 A hybrid model --- integrating both NFBR and FBNR cases

277 One can also combine both situations of NFBR claims and FBNR phenomenon and derive the expected
 278 cost as follows.

279 **Scenario 1**-- A NFBR claim will not cause warranty to be ceased. Then the expected warranty cost is
 280 given by

$$EC_{31} = EC_2(w) + WC_{31}(w) \quad (12)$$

281 **Scenario 2**-- A NFBR claim will cause warranty to be ceased. Then we have the following expected
 282 warranty cost.

$$EC_{32}(w) = \int_0^w [EC_2(z) + c_{32}]dH_3(z) + EC_2(w)(1 - H_3(w)) \quad (13)$$

283 3.5 Discussion

284 For the expected costs, we have the following special cases.

285 (1) If $p_{2j} = 1$, $q_1(t) = 1$, and $q_2 = 1$, where $j=1,2,\dots$, then the above expected costs can be obtained for
 286 the following situation, where

- 287 • all of failed products (including fatal and intermittent failures) are reported claims; and
- 288 • all of intermittent failures can be identified from the first instance.

289 (2) If $p_{2j} = 0$, $q_1(t) = 1$, and $q_2 = 0$, where $j=1,2,\dots$, then the above expected costs can be obtained for
 290 the following situations, where

- 291 • only fatal failure is considered; and
- 292 • all of failed products are reported claims.

293 (3) If $c_1, c_2+\tilde{c}_2, c_{20}, c_{31}$, and c_{32} are set to 1, then the expected costs in Eqs. (1) -- (8) become the
 294 expected numbers of warranty claims for corresponding scenarios, respectively.

295 Apart from the human factors considered above, Rai and Singh [6,32] consider the fact that consumers
 296 experiencing non-critical failures might delay reporting of warranty claims till the coverage is about to expire,
 297 which can introduce a bias into the dataset.

298 4. Numerical data analysis

299 Assume that

$$300 \lambda_1(t) = \frac{\xi_2}{\xi_1} \left(\frac{t}{\xi_1}\right)^{\xi_2-1}, \quad (14)$$

$$301 \lambda_2(t) = \frac{t}{\nu_1}, \quad (15)$$

$$302 H_3(t) = 1 - e^{-(t/\alpha_1)^{\alpha_2}}, \quad (16)$$

$$p_{2j} = \eta^j \quad (17)$$

303 and set the values of the parameters in the above equations as in Table 1.

304 **Table 1.** A list of parameters

c_1	c_2	\tilde{c}_2	c_{31}	c_{32}	ξ_1	ξ_2	γ_1	γ_2	η	ν_1	α_1	α_2	n	q_2	w
100	200	11	5	2	65	0.8	0.1	0.5	0.4	250	1000	1	50	0.8	36

305
 306 If we change one of the parameters, we can investigate the relationship between the parameter and its
 307 impact on the expected cost. For simplicity, we investigate the following three situations:

- 308 • Situation 1 – Change parameter η in $p_{2j}(= \eta^j)$ in Eqs. (6)—(7), (12), and (13).
- 309 • Situation 2 – Change parameter q_2 in Eqs. (10)—(13), and
- 310 • Situation 3 – change parameters η_1 and η_2 in Eq. (11).

311 4.1 Expected costs against parameters η and n

312 If we change η from 0.1 to 1 with a step 0.1, and keep the other parameters fixed as shown in Table 1, the
 313 expected costs are shown in Table 2.

314 **Table 2.** The expected costs $EC_{11}(w)$, $EC_{12}(w)$, $EC_2(w)$, $EC_{31}(w)$, and $EC_{32}(w)$ against η and n .

η	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$EC_{11}(w)$ ($n=5$)	64.20	66.59	70.44	76.67	86.92	104.36	135.68	193.79	275.01	62.51
$EC_{12}(w)$ ($n=5$)	63.16	65.56	69.42	75.65	85.86	103.19	134.25	191.84	272.28	61.43
$EC_2(w)$ ($n=5$)	8.20	11.25	15.86	22.89	33.69	50.13	73.37	99.05	102.91	5.83
$EC_{31}(w)$ ($n=5$)	8.38	11.43	16.04	23.07	33.87	50.30	73.54	99.23	103.08	6.00
$EC_{32}(w)$ ($n=5$)	8.27	11.32	15.93	22.96	33.76	50.20	73.44	99.12	102.98	5.90
$EC_{11}(w)$ ($n=25$)	62.51	62.51	62.51	62.51	62.53	62.62	63.13	65.51	72.42	62.51
$EC_{12}(w)$ ($n=25$)	61.44	61.44	61.45	61.47	61.51	61.66	62.27	64.84	72.00	61.43
$EC_2(w)$ ($n=25$)	5.83	5.83	5.83	5.84	5.89	6.02	6.31	6.81	7.17	5.83
$EC_{31}(w)$ ($n=25$)	6.00	6.00	6.01	6.02	6.07	6.20	6.49	6.98	7.34	6.00
$EC_{32}(w)$ ($n=25$)	5.90	5.90	5.90	5.91	5.96	6.09	6.39	6.88	7.24	5.90

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 316 From Table 2, when the number n of products sold is smaller, the values of $EC_{11}(w)$, $EC_{12}(w)$, $EC_2(w)$,
 317 $EC_{31}(w)$, and $EC_{32}(w)$ change quickly over η . However, when $n > 30$, the values of $EC_{11}(w)$, $EC_{12}(w)$, $EC_2(w)$,
 318 $EC_{31}(w)$, and $EC_{32}(w)$ change very slowly. It also shows that the expected warranty claim costs $EC_{11}(w)$ and
 319 $EC_{12}(w)$ are much larger than $EC_2(w)$, $EC_{31}(w)$, and $EC_{32}(w)$, which implies that the FBNR phenomenon takes
 320 effects.

321 Figure 1 shows the values of $EC_{32}(w)$ against η , for the cases when $n=5$ and $n=25$. It can be seen that $EC_{32}(w)$
 322 increases much faster for the case $n=5$ than that for the case $n=25$. It can also be seen that $EC_{32}(w)$ reaches the
 323 smallest value when $\eta=1$. $\eta=1$ implies that the cause of intermittent failures can be detected immediately.

324
 325 **Figure 1:** $EC_{32}(w)$ against η and n

326 4.2 Expected costs against parameter q_2 and n

327 If we change q_2 from 0.1 to 1 with a step 0.1, respectively, and keep the rest of the parameters unchanged
 328 as shown in Table 1, then we obtained the expected costs as shown in Table 3.

329 Figure 2 shows that the values of $EC_{32}(w)$ increase when q_2 changes from 0.1 to 0.6, then they decrease
 330 when q_2 becomes larger, say, when q_2 changes from 0.6 to 1. It can also be seen that the values of $EC_2(w)$ are
 331 much larger in the case of $n=5$ than those where the case of n becomes larger.

332 The findings about the relationship between parameter q_2 and $EC_2(w)$ is interesting. As one might expect,
 333 the larger values of q_2 imply more reports arising from intermittent failures and therefore can incur larger cost

334 to a manufacturer. From both Table 3 and Figure 2, we find that this is not always the case, due to the
 335 nonlinearity nature of the component $e^{-q_2 \Lambda_n(w)} [q_2 \Lambda_n(w)]^k$ in Eq. (8).

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Table 3. The expected costs $EC_2(w)$, $EC_{31}(w)$, and $EC_{32}(w)$ against q_2 and n .

q_2	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
$EC_2(n=5)$	14.42	19.99	23.30	24.95	25.41	25.04	24.13	22.89	21.46	19.96
$EC_{31}(n=5)$	14.59	20.17	23.48	25.13	25.59	25.22	24.31	23.07	21.64	20.14
$EC_{32}(n=5)$	14.49	20.06	23.37	25.02	25.48	25.11	24.21	22.96	21.53	20.04
$EC_2(n=85)$	6.15	5.84	5.83	5.83	5.83	5.83	5.83	5.83	5.83	5.83
$EC_{31}(n=85)$	6.33	6.02	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
$EC_{32}(n=85)$	6.22	5.91	5.90	5.90	5.90	5.90	5.90	5.90	5.90	5.90
$EC_2(n=185)$	5.83	5.83	5.83	5.83	5.83	5.83	5.83	5.83	5.83	5.83
$EC_{31}(n=185)$	6.01	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
$EC_{32}(n=185)$	5.90	5.90	5.90	5.90	5.90	5.90	5.90	5.90	5.90	5.90

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Figure 2: $EC_2(w)$, $EC_{31}(w)$ and $EC_{32}(w)$ against q_2 for $n=5$, $n=85$ and $n=185$, respectively.

341 4.3 Expected costs against parameters γ_1 , γ_2 and n

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If we change γ_1 and γ_2 from 0.1 to 1.9 with a step 0.2, respectively, and keep the rest of the parameters fixed as shown in Table 1, all of the expected costs of $EC_{11}(w)$, $EC_{12}(w)$, $EC_2(w)$, $EC_{31}(w)$, and $EC_{32}(w)$ decrease, as shown in Table 4 and Table 5. We also notice that the gradient of the changes in $EC_{11}(w)$, $EC_{12}(w)$, $EC_2(w)$, $EC_{31}(w)$, and $EC_{32}(w)$ become very similar when n is larger. For example, in Figure 3 and Figure 4, the values are very close for the cases when $n=85$ and $n=185$.

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Figure 5 shows how the expected cost $EC_{32}(w)$ changes over γ_1 and γ_2 . It shows that $EC_{32}(w)$ reaches the smallest value when both γ_1 and γ_2 are the smallest.

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Table 4. The expected costs $EC_{11}(w)$, $EC_{12}(w)$, $EC_2(w)$, $EC_{31}(w)$, and $EC_{32}(w)$ against γ_1 (when $\gamma_2=0.5$)

γ_2	0.1	0.3	0.5	0.7	0.9	1.1	1.3	1.5	1.7	1.9
$EC_2(n=5)$	22.89	21.83	20.97	20.26	19.68	19.21	18.82	18.50	18.24	18.03
$EC_{31}(n=5)$	23.07	22.01	21.15	20.44	19.86	19.38	19.00	18.68	18.42	18.20
$EC_{32}(n=5)$	22.96	21.90	21.04	20.33	19.75	19.28	18.89	18.57	18.31	18.10
$EC_2(n=85)$	5.83	4.77	3.91	3.20	2.62	2.14	1.75	1.44	1.18	0.96
$EC_{31}(n=85)$	6.00	4.95	4.08	3.37	2.79	2.32	1.93	1.61	1.35	1.14
$EC_{32}(n=85)$	5.90	4.84	3.98	3.27	2.69	2.21	1.83	1.51	1.25	1.03
$EC_2(n=185)$	5.83	4.77	3.91	3.20	2.62	2.14	1.75	1.44	1.18	0.96
$EC_{31}(n=185)$	6.00	4.95	4.08	3.37	2.79	2.32	1.93	1.61	1.35	1.14
$EC_{32}(n=185)$	5.90	4.84	3.98	3.27	2.69	2.21	1.83	1.51	1.25	1.03

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Table 5. The expected costs $EC_{11}(w)$, $EC_{12}(w)$, $EC_2(w)$, $EC_{31}(w)$, and $EC_{32}(w)$ against γ_2 (when $\gamma_1=0.1$)

γ_2	0.1	0.3	0.5	0.7	0.9	1.1	1.3	1.5	1.7	1.9
$EC_2(n=5)$	36.21	25.52	22.89	21.66	20.93	20.44	20.09	19.83	19.62	19.44
$EC_{31}(n=5)$	36.39	25.70	23.07	21.84	21.11	20.62	20.27	20.00	19.79	19.62
$EC_{32}(n=5)$	36.28	25.59	22.96	21.73	21.00	20.51	20.16	19.90	19.69	19.51
$EC_2(n=85)$	19.15	8.46	5.83	4.59	3.87	3.38	3.03	2.76	2.55	2.38
$EC_{31}(n=85)$	19.33	8.63	6.00	4.77	4.04	3.56	3.21	2.94	2.73	2.56
$EC_{32}(n=85)$	19.22	8.53	5.90	4.67	3.94	3.45	3.10	2.83	2.62	2.45
$EC_2(n=185)$	19.15	8.46	5.83	4.59	3.87	3.38	3.03	2.76	2.55	2.38
$EC_{31}(n=185)$	19.33	8.63	6.00	4.77	4.04	3.56	3.21	2.94	2.73	2.56
$EC_{32}(n=185)$	19.22	8.53	5.90	4.67	3.94	3.45	3.10	2.83	2.62	2.45

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Figure 3: $EC_{32}(w)$ against η_1 when $n=5, 85,$ and $185,$ respectively.

Figure 4: $EC_{32}(w)$ against η_2 when $n=5, 85,$ and $185,$ respectively.

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Figure 5: $EC_{32}(w)$ against η_1 and η_2 when $n=50.$

From Sections 5.1, 5.2 and 5.3, it can be seen that the number of products, n , is very important as the expected costs are sensitive to it.

366 5. Conclusions

367 Conventional research on warranty claims simply assumes that claims are only due to product failures and
368 consumers will report claims upon product failure, which might not be true in reality. This paper models the
369 expected warranty claim costs when consumer behaviour is taken into account for products protected by non-
370 renewing warranty policy. The numerical examples in the paper show the relationships between parameters and
371 the expected costs. The paper also shows that the expected claim costs are sensitive to the number of products
372 sold.

373 With increasingly more accumulated warranty claim data, manufacturers should be able to develop more
374 accurate warranty claim models to predict the expected cost and the expected number of claims. Such models
375 should also include more relevant factors, such as failed but not reported phenomenon and non-failed but
376 reported claims, which might impact warranty claims.

377 Our future research includes the following issues.

- 378 • The probability of not detecting the cause of intermittent failures and the probability of failed but not
379 reported phenomenon were assumed to be dependent on the number of claims. Possible extensions are
380 to assume them to be associated with both product age and the number of intermittent failures reported.

- 381 • In this paper, only one cause of intermittent failures was considered. More than one cause of
382 intermittent failures should be studied.
- 383 • In the paper, when modelling the ability to detect the cause of intermittent failures, we assumed that
384 products were sold at the same date. However, products shipped to retailers might not be sold at the
385 same date. There might be delays between shipment dates and sales dates, known as sales delay, which
386 should be considered in our future work.
- 387 • The paper only considered repairable products with minimal repair. Further work should also analyse
388 warranty claims costs for non-repairable products or repairable products with different levels of
389 maintenance quality (see [33] for maintenance models, for example).
- 390 • The paper only considered non-renewing warranty policy. Other warranty policy can also be
391 considered.

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