Performance analysis of Mobile Content Delivery in Multiple Devices to Single Device Communication

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Abstract—In wireless communication networks, caching and delivering popular content via the device to device (D2D) communication has recently been proposed as an exciting and innovative technology in order to offload network data traffic. In this paper, a novel method of content delivery using multiple devices to the single device (MDSD) communication via D2D links is presented. An expression of the outage probability ($\mathbb{P}_{out}$) is analytically derived and validated by simulation to determine the success of the content delivery to the user equipment (UE). Zipf distribution with exponent shape parameter ρ is adopted to model the UE requests and content caching popularity which affects the achievable link data rate ($\mathcal{R}_a$). The results show that $\mathbb{P}_{out}$ decreases as the popularity of the content increases. Meanwhile, MDSD improves the UE experience in terms of $\mathbb{P}_{out}$ substantially compared to the single D2D link based method.

I. INTRODUCTION

The proliferation of smartphones has boosted wireless data traffic substantially during the last decade [1]. Due to the increase in wireless data traffic, the fourth-generations (4G) cellular systems have already reached its theoretical capacity [2]. Therefore, dealing with growing amount of data traffic is a critical issue, which needs to be solved for providing a high quality of services to the users. According to [1], the major contributors towards data traffic are videos, which accounts for more than a half of the total mobile data traffic, and is caused by duplicated requests for a few popular videos. For example, 10% of the videos in the Youtube account for nearly 80% of viewing [3]. This fact leads to an important solution to reduce the data traffic, by utilizing from the storage unit in a user equipment UE to store content temporarily and allowing other UEs to download the contents from the UE. By enabling a local user to communicate with others via the device to device (D2D) link, content caching is a useful method to offload the network data traffic, decreasing the average access latency and reduce the traffic load in the base station (BS).

D2D communication is an important innovation technology, due to the ability to exchange the data directly amongst devices in proximity without going through the BS [4], [5]. As the storage capability of the smartphones increases significantly with low cost, caching popular content in the mobile devices and using D2D communication for content delivery has been investigated as a promising way to enhance the user experience in terms of transmission delay, energy consumption, and throughput capacity [6], [7]. In [8], in a femtocell, one storage unit used to store content are considered as a helper for multiple UEs, where the connection between any UE and the helper is considered as D2D communication. The results showed that each UE throughput can be increased if there is a sufficient content reuse. In [9], a random and central caching placement methods were considered. The results demonstrated that the spectral efficiency can be enhanced up to two order of magnitude when the central caching method with D2D communication is used. In [10], an optimizing frequency reuse in order to reduce transmission power has been proposed. The whole cell was divided into small equally square clusters, and only one content can be received via one D2D link in one cluster to avoid intra-cluster interference. Since the assumptions in [10] were over simplified and did not consider the channel fading, this approach may not be practical. However, it reveals that the possibility of finding and downloading the desired content from neighbor UE using D2D communication is very unlikely in higher outage probability, due to the privacy concern, limited helpers holding the desired content, and the channel fading. These reasons motivate us to explore the transmission diversity (TD) in multiple devices to single device (MDSD) communication based D2D communication method. The impact of TD has been widely investigated in cellular networks in order to combat the effects of fading by transmitting the same data over a different antenna, i.e., maximum ratio transmission (MRT) [11]–[13]. In [14], the distribution of the signal to interference ratio (SIR) is derived by applying a Toeplitz matrix in which a multi-antenna small cell was considered. In [15], a closed-form expression for the distribution of the received signal from only two users has been derived, and an approximation for the summation of the received signal from multiple transmitters was provided. However, [15] only presented analytical results, for two user case by assuming the total interference as a Gaussian noise with fixed power value (variance). Moreover, to the best of knowledge, the distances r_i from the transmitters to the receiver in MRT technique has not been pointed out as a random in the literature (i.e. different distances).

This paper focuses on improving the performance of a reference UE in terms of outage probability of the content delivery, which is defined as the data rate $\mathcal{R}_a$ less than a target threshold value τ. In order to achieve this target, by adopting TD, an MDSD based method is proposed to cache and deliver contents in an environment where a high density of UEs appears, e.g. stadiums, and shopping centers. The main contributions of this paper are listed as follows.

1) Based on stochastic geometry, the outage probability for D2D communication is derived. Especially, closed-form expression are obtained when the path loss exponent $\alpha = 4$ and 2, respectively. In contrast to the literature, we proved that the total interference for limited area is not going to infinity ($\mathcal{I}_a \neq \infty$) when the condition of a Poisson point process (PPP) and $\alpha = 2$ applied.

2) The distribution of the received signals from multiple transmitters at different distances is derived. First, the probability density function (PDF) of the desired signal for a single link is defined as a special case of the Lomax distribution. Then, a Laplace transform is used to find the distribution for the summation of the received signals. A Bromwich integral and residue theorem are used to implement the inversion of the Laplace transform.

3) The results of the proposed method MDSD are compared with D2D method based on single transmitter, called single D2D based scheme. It is shown that the $\mathbb{P}_{out}$ can be reduced to 90% for MDSD compared to the single D2D based method.

The remainder of this paper is organized as follows. Section II describes the system model. In Section III, performance analysis is carried out, where the single integral expression of the received signal is derived. Results and discussions are presented in Section IV, while the paper is concluded in Section V.

II. ASSUMPTIONS AND SYSTEM MODEL

Consider a downlink cellular network, where the base stations (BSs) and user equipments (UEs) are randomly located in the system. Fig. 1 shows a part of the network, where N UEs are distributed within the radius d and modeled as a stationary homogeneous Poisson Point Process (PPP)
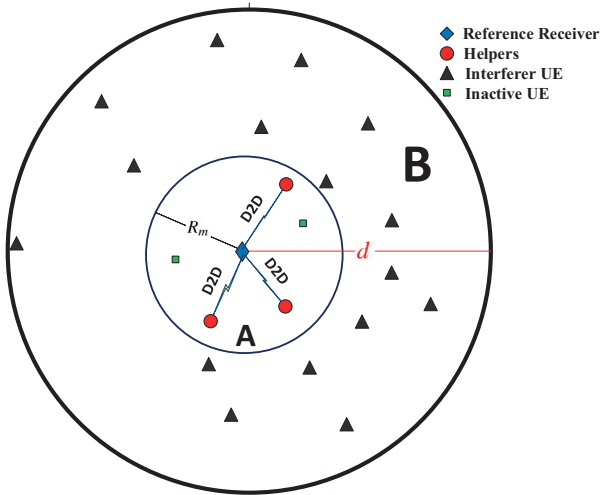

Figure 1: Depict the system model, where N user equipment distributed according to PPP. The diamond shape refers to the receiver user, the small filled circles shape represent the potential transmitters, the squares shape represent cellular user which is considered as idle, and the triangles shape refer to the cellular users that cause the interference to the reference user

Φ_λ of intensity λ in two-dimensional space R^2. In another word, the process $\Phi_\lambda = \{x_i\} \subset R^2$, where x_i is the ith (UE) node location which is i.i.d in the Euclidean plane, and $i = \{1, 2, \cdots, N\}$. λ is the expected number of users of PPP in a unit area. It is assumed that each UE has a cache unit, which is a part of storage device unit used to cache contents temporarily. For simplicity, it is assumed that the cache unit size is the same for all UEs, and there is at least one popular content is stored in each UE. It is assumed that there is a single reference UE as receiver located at the origin (o), supported by multiple transmitter devices called helpers (k_h), simultaneously within the area A of radius R_m. It is assumed that each UE in the system is a priori requested and cached its own desired content randomly and independently from a library of M different contents. The number of different content that cached by different UEs within the distance R_m denoted as L, where $L \leq M$. The main idea of the content caching and delivery method of MDSD is clarified as follows: if one UE requests a specific content l of interested, neighbors who have the desired content in their caches will serve the request via D2D links. Otherwise, the BS will serve the request. Therefore, the desired content may be received from multiple devices, and the data rate for receiving the content $\mathcal{R}_a$, can be improved to increase the successful ratio of the receiving requested content. UEs within R_m who do not have the desired content is considered to be inactive to the reference UE as shown in Fig. 1. It is assumed there is no interference inside A, and all signals received outside A, i.e. with distance to the reference UE in the range $(R_m, d]$ are considered as interference.

The most important common distribution used to model the request to a content l is Zipf distribution [16]. Since the UEs download cache the contents according to their interest, the content caching is also assumed to follow Zipf distribution as

$$\mathcal{W}_l = \frac{l^{-\rho}}{\zeta}, \quad \rho > 0, \quad 1 \leq l \leq \text{L} \tag{1}$$

where $\zeta = \sum_{l=1}^{L} l^{-\rho}$ is the normalizing constraint, $l \in \mathcal{L}$ is the index of a content cached, $\mathcal{L} = \{1, 2, 3, \cdots, L\}$ is the set of the total number of different contents that cached in UEs within A. When the value of ρ becomes large, only a small number of contents are most popular and account for most of the requests. On the other hand, when $\rho = 0$ the popularity of each content is the same, which means the contents are uniformly distributed. It is assumed that each device has a single Omni directional antenna and all UEs in the system have the same transmission power (unit power signal). The data rate $\mathcal{R}_a$ at the receiver is given by

$$\mathcal{R}_a = \log_2(1 + \text{SINR}) = \log_2\left(1 + \frac{\sum_{i=1}^{k_h} |h_i|^2 r_i^{-\alpha} P_t}{\sigma_n^2 + \mathcal{I}_{ag}}\right), \tag{2}$$

where P_t is the transmit power, r_i is the distance between reference UE and a serving helper i of a UE, k_h is the number of synchronized helpers that have the desired content l in their caches, and σ_n^2 is the additive white Gaussian noise power. α is

the path loss exponent depending on the carrier frequency and physical environment, which is approximated in the range of (1.6 - 6.5) [17]. It is assumed that a Rayleigh based small scale fading, where $|h_i|^2$ is the power gain following an exponential distribution with unit mean defined as $f_{|h_i|^2}(x) = \exp(-x)$. $\mathcal{I}_{ag}$ is the aggregation of the interference signals power coming from the outside area A and is given by

$$\mathcal{I}_{ag} = \sum_{j \in \Phi_\lambda / A} |g_j|^2 r_j^{-\alpha} P_t \tag{3}$$

where r_j is the distance from reference UE to the jth interferer UE and g_j is assumed Rayleigh channel fading coefficient with unit mean, $f_{|g_j|^2}(x) = \exp(-x)$.

III. PERFORMANCE ANALYSIS

In this section, the system performance is evaluated in terms of the outage probability $\mathbb{P}_{out}$. Outage happens if the desired content is not found in the cache of neighbors within the threshold distance R_m, or the received data rate $\mathcal{R}_a$ fails to bellow a given target threshold τ. $\mathbb{P}_{out}$ is evaluated in two cases, D2D and MDSD communication respectively as follow.

A. D2D communication

$\mathbb{P}_{out}$ is evaluated as a conditioning on the distance r between reference UE and the nearest helper. It is assumed that the desired content l is existing within the nearest UE. The distribution of the distance r is derived as [18].

$$f_r(r) = 2\pi\lambda e^{-2\pi\lambda r^2}. \tag{4}$$

Theorem 1. *Given the density of UEs λ, path loss exponent α, and the target data rate threshold τ, the outage probability of the D2D communication is given by*

$$\mathbb{P}_{out}^{D2D}(\lambda, \alpha, \tau) = 1 - \mathcal{W}_l \int_0^\infty \pi\lambda e^{-\pi\lambda v(1+\kappa(\tau,\alpha)) - \frac{2^\tau - 1}{\eta_o} v^{\alpha/2}} dv, \tag{5}$$

Proof. $\mathbb{P}_{out}^{D2D}$ is a complement of the coverage probability, which is defined as

$$\mathbb{P}_{out}^{D2D}(\lambda, \alpha, \tau) = 1 - \mathbb{E}_r\left[\mathbb{P}\left(\mathcal{R}_a^{D2D} > \tau\right), l \in \mathcal{L}\right], \tag{6}$$

Since the $\mathcal{R}_a^{D2D}$ and content popularity are independent events, (6) can be written as

$$\mathbb{P}_{out}^{D2D}(\lambda, \alpha, \tau) = 1 - \mathcal{W}_l \times \mathbb{E}_r\left[\mathbb{P}\left(\mathcal{R}_a^{D2D} > \tau\right)|r\right], \tag{7}$$

where $\mathcal{W}_l$ is defined in (1), and $\mathbb{E}_r(\cdot)$ is the expectation with respect to r. The probability that the received $\mathcal{R}_a^{D2D}$ exceeding a target threshold τ at distance r from the reference UE is given as [19]

$$\mathbb{E}_r[\mathbb{P}[\mathcal{R}_a^{D2D} > \tau]] = \int_{r>0} \mathbb{P}\left[\log_2(1 + \text{SINR}^{D2D}) > \tau|r\right] f_r(r)dr,$$

$$= 2\pi\lambda \int_{r>0} \mathbb{P}\left[\frac{|h|^2 r^{-\alpha} P_t}{\sigma_n^2 + \mathcal{I}_{ag}} > 2^\tau - 1|r\right] r e^{-\pi\lambda r^2} dr, \tag{8}$$

Since $|h|^2 \sim \exp(1)$, $\mathbb{P}[\cdot]$ in (8) is a probability function which is given by

$$\mathbb{P}\left[|h|^2 > (2^\tau - 1)r^\alpha \left(\frac{\sigma_n^2}{P_t} + \frac{\mathcal{I}_{ag}}{P_t}\right)|r\right]$$

$$= \mathbb{E}_{\mathcal{I}_n}\left[\mathbb{P}\left[|h|^2 > (2^\tau - 1)r^\alpha \left(\eta_o^{-1} + \mathcal{I}_n\right)|r\right]\right],$$

$$= \mathbb{E}_{\mathcal{I}_n}\left[e^{-(2^\tau - 1)r^\alpha \left(\eta_o^{-1} + \mathcal{I}_n\right)}\right],$$

$$= e^{-\frac{(2^\tau - 1)r^\alpha}{\eta_o}} \mathcal{L}_{\mathcal{I}_n}\left((2^\tau - 1)r^\alpha\right), \tag{9}$$

where $\eta o = \frac{P_t}{\sigma_n^2}$ is the signal to noise ratio, and $\mathcal{I}_n = \frac{\mathcal{I}_{ag}}{P_t}$ is the interference part. $\mathcal{L}_{\mathcal{I}_n}(\cdot)$ is the Laplace transform of the random variable $\mathcal{I}_n$ that is derived in appendix-A and defined as

$$\mathcal{L}_{\mathcal{I}_n}\left((2^\tau - 1)r^\alpha\right) = e^{-\pi\lambda r^2 \kappa(\tau, \alpha)}, \tag{10}$$

By plugging (10), (9), and (8) into (7), and changing variable $v = r^2$, then we get (5). ∎

For the generality of (5), in the following parts, a closed form expression is evaluated under different scenarios based on the path loss exponent α =4, and 2, respectively.

Lemma 1.1. *For the path loss exponent α=4, the outage probability, denoted as $\mathbb{P}_{out4}^{D2D}$, is given by*

$$\mathbb{P}_{out4}^{D2D}(\lambda, 4, \tau) = 1 - \mathcal{W}_l \pi\lambda \int_0^\infty e^{-\pi\lambda v(1+\kappa(\tau,4)) - \frac{\eta v^2}{\eta_o}} dv. \tag{11}$$

(11) has similar form to

$$\int_0^\infty e^{-\varrho x - \beta x^2} dx = \sqrt{\frac{\pi}{\beta}} Q\left(\frac{\varrho}{\sqrt{2\beta}}\right) e^{\frac{\varrho^2}{4\beta}}.$$

therefore (11) is defined as

$$\mathbb{P}_{out4}^{D2D} = 1 - \mathcal{W}_l \frac{\pi^{\frac{3}{2}}\lambda}{\sqrt{\frac{2^\tau - 1}{\eta_o}}} Q\left(\frac{\pi\lambda\left(1 + \kappa\left(\tau, 4\right)\right)}{\sqrt{2\beta}}\right) e^{\frac{(\pi\lambda(1+\kappa(\tau,4)))^2}{4\frac{2^\tau - 1}{\eta_o}}}, \tag{12}$$

where $Q(\cdot)$ is given by $Q(y) = \frac{1}{\sqrt{2\pi}} \int_y^\infty \exp(-v^2)dv$, $\kappa\left(\tau, 4\right) = \sqrt{2^\tau - 1}\left(\cot^{-1}(1/d^2) - \cot^{-1}\left(\sqrt{2^\tau - 1}\right)\right)$, and $\cot^{-1}(\cdot)$ is the inverse cotangent function.

Lemma 1.2. *For the path loss exponent α=2, the outage probability, denoted as $\mathbb{P}_{out2}^{D2D}$, is given by*

$$\mathbb{P}_{out2}^{D2D}(\lambda, 2, \tau) = 1 - \mathcal{W}_l \pi\lambda \int_0^\infty e^{-\pi\lambda v(1+\kappa(\tau,2)) - \frac{(2^\tau - 1)v}{\eta_o}} dv. \tag{13}$$

where $\kappa(\tau, 2)$ is defined as

$$\kappa(\tau, 2) = \int_1^d \frac{(2^\tau - 1)}{(2^\tau - 1) + y} dy = (2^\tau - 1)\log\left(\frac{2^\tau - 1 + d}{2^\tau}\right) \tag{14}$$

where $\log(\cdot)$ is the natural logarithmic function. By substituting (14) into (13), $\mathbb{P}_{out}$ is defined as

$$\mathbb{P}_{out2}^{D2D} = 1 - \frac{\mathcal{W}_l \pi\lambda}{\pi\lambda(1 + (2^\tau - 1)\log(\frac{(2^\tau - 1) + d}{2^\tau})) + \frac{(2^\tau - 1)}{\eta_o}}. \tag{15}$$

[20] proved that the interference is defective ($\mathcal{I}_{ag} = \infty$) in two-dimensional PPP when $\alpha = 2$. In contrast, the interference is not going to the infinity ($\mathcal{I}_{ag} \neq \infty$) for the limited area within the radius d, i.e. $\kappa(\tau, 2) \neq \infty \rightarrow \mathcal{L}_{\mathcal{I}_n}(\cdot) \neq 0$.

B. Multiple devices to single device (MDSD) communication

To compare with D2D communication, we suggest using MDSD based method in some environments, where the density of users is very high.

Theorem 2. *Given the density of UEs λ, path loss exponent $\alpha = 2$, and the target threshold τ, the outage probability of the MDSD is denoted as*

$$\mathbb{P}_{out}^{MD} = 1 - \sum_{K=1}^{N}\sum_{k_h=1}^{K}\binom{N}{K}\left(\frac{|A|}{|B|}\right)^K\left(1 - \frac{|A|}{|B|}\right)^{N-K}\binom{K}{k_h}(\mathcal{W}_l)^{k_h}$$

$$(1 - \mathcal{W}_l)^{K-k_h}\int_0^\infty \frac{e^{-x(k_h + \frac{(2^\tau - 1)}{\eta_o\varphi} + (2^\tau - 1)\nu)}}{\pi x \psi^{k_h}}\Omega_{k_h}(x)dx. \quad (16)$$

Proof. In MDSD, the received signal is the sum of the desired signals at the receiver which is expressed as

$$\mathbb{P}_{out}^{MD}(\alpha{=}2) = 1 - \mathbb{E}\left(\mathbb{P}\left[\frac{\sum_{i=1}^{k_h}|h_i|^2 r_i^{-\alpha}P_t}{\sigma_n^2 + \mathcal{I}_{ag}} > (2^\tau - 1)\right], l \in \mathcal{L}\right), \quad (17)$$

where the received $\mathcal{R}_a^{MD}$ for the content l is conditioned on k_h helpers within R_m. Therefore (17) is written as

$$\mathbb{P}_{out}^{MD}(\alpha{=}2) = 1 - \sum_{K=1}^{N}\sum_{k_h=1}^{K}\mathcal{H}_l\underbrace{\mathbb{P}\left[\mathcal{R}_a^{MD} > \tau\right]}_{(\star)}, \quad (18)$$

where $\mathcal{H}_l$ is the hit probability that there are a K UEs out of N inside A and k_h helpers out of K UEs holding a specific content l, which is denoted as

$$\mathcal{H}_l = \binom{N}{K}\left(\frac{R_m^2}{d^2}\right)^K\left(1 - \frac{R_m^2}{d^2}\right)^{N-K}\binom{K}{k_h}(\mathcal{W}_l)^{k_h}(1 - \mathcal{W}_l)^{K-k_h} \quad (19)$$

The probability of there are N UEs in the whole cell area B is follow a Poisson distribution as

$$\mathbb{P}(N \quad in \quad (B)) = \frac{(\lambda B)^N e^{-\lambda B}}{N!}. \quad (20)$$

$(\star)$ is the probability that the $\mathcal{R}_a^{MD}$ exceeds a target data rate threshold value τ as explained and proof below.

$$\mathbb{P}\left[\mathcal{R}_a^{MD} > \tau\right] = \mathbb{P}\left[\frac{\sum_{i=1}^{k_h}|h_i|^2 r_i^{-\alpha}P_t}{\sigma_n^2 + \mathcal{I}_{ag}} > (2^\tau - 1)\right],$$

$$= \mathbb{P}\left[\sum_{i=1}^{k_h}\frac{|h_i|^2}{r_i^\alpha} > (2^\tau - 1)\left(\frac{1}{\eta_o} + \mathcal{I}_n\right)\right], \quad (21)$$

By assuming that a reference user contact with the closest helper (not necessary closest UE), the PDF of the average distance r that given the desired content l existing within R_m defined as [21]:

$$f_{R_m}(r) = \frac{2\pi\lambda r \exp(-\pi\lambda r^2)}{1 - \exp(-\pi\lambda R_m^2)}, \quad 0 < r < R_m. \quad (22)$$

Letting $t_i = \frac{|h_i|^2}{r_i^\alpha}$, is the ratio of two random variables, and $Y = \sum_{i=1}^{k_h}t_i$. To find the distribution of t_i, the transformation of $w = r^\alpha$ is defined as

$$f_W(w) = \frac{2\pi\lambda}{\alpha\psi}w^{\frac{2-\alpha}{\alpha}}e^{-\pi\lambda w^{\frac{2}{\alpha}}} \quad (23)$$

where $\psi = 1 - e^{-\pi\lambda R_m^2}$. It is apparently that the distribution of t_i is a much complicated when $\alpha > 2$, that results in complexity to find the distribution of Y. It is assumed that $\alpha = 2$ in the following analysis for the sake of simplicity, which is also a typical value of path loss exponent in different environments [17]. The transformation of $w = r^2$ is defined as $f_W(w) = \frac{\pi\lambda}{\psi}e^{-\pi\lambda w}$. By solving the ratio of the two exponential random variables, the distribution of t_i is expressed as

$$f_T(t_i) = \frac{\lambda\pi}{\psi(\lambda\pi + t_i)^2}, \quad t_i > 0. \quad (24)$$

which is a special case of the Lomax distribution, defined as

$$f(x) = \frac{a\varphi^a}{(\varphi + x)^{a+1}}, \quad x > 0 \quad (25)$$

where a is a positive integer shape parameter, and $(\varphi > 0)$ is the scale parameter. In (24), the shape parameter $a = 1$, and scale parameter $\varphi = \lambda\pi$. The Laplace transform $f(s)$ is evaluated in order to find the distribution of the sum of (24) as

$$f(s) = \int_0^\infty \frac{\varphi e^{-st}}{\psi(\varphi + t)^2}dt = \frac{e^{\varphi s}E_2(\varphi s)}{\psi}, \quad (26)$$

where $E_a(\cdot)$ is the generalized exponential integral function [22], defined as

$$E_a(x) = \int_1^\infty \frac{e^{-xt}}{t^a}dt, \quad a = 1, 2, \cdots$$

For a complex s and $\Re(s) > 0$, the Laplace transform of Y is defined as

$$f_Y(s) = \int_0^\infty e^{-st}f_Y(t)dt = (f(s))^{k_h} = \left(\frac{e^{\varphi s}E_2(\varphi s)}{\psi}\right)^{k_h} \quad (27)$$

In order to find $f_Y(t)$, the inversion of $f_Y(s)$ is evaluated in Appendix-B. However, $f_Y(t)$ is the distribution of the summed received desired signals, which is defined as

$$f_Y(t) = \frac{1}{\pi\varphi\psi^{k_h}}\int_0^\infty \Omega_{k_h}(x)e^{-x\left(k_h + \frac{t}{\varphi}\right)}dx, \quad (28)$$

where $\Omega_{k_h}(x)$ is defined in Appendix-B, and the solution of (21) will be defined as

$$\mathbb{P}[\mathcal{R}_a^{MD} > \tau] = \frac{1}{\pi\varphi\psi^{k_h}}\int_V^\infty\int_0^\infty \Omega_{k_h}(x)e^{-x\left(k_h + \frac{t}{\varphi}\right)}dxdt,$$

$$= \int_0^\infty \frac{\Omega_{k_h}(x)}{\pi x \psi^{k_h}}e^{-x\left(k_h + \frac{V}{\varphi}\right)}dx,$$

$$= \int_0^\infty \frac{\Omega_{k_h}(x)e^{-x(k_h + \frac{(2^\tau - 1)}{\varphi\eta_o})}}{\pi x \psi^{k_h}}\mathcal{L}_{\mathcal{I}_n}\left(\frac{x(2^\tau - 1)}{\varphi}\right)dx, (29)$$

where $V = (2^\tau - 1)(\eta_o^{-1} + \mathcal{I}_n)$, $\mathcal{L}_{\mathcal{I}_n}\left(\frac{x(2^\tau - 1)}{\varphi}\right)$ is the Laplace transform of the aggregation interference solved in Appendix-C and defined as

$$\mathcal{L}_{\mathcal{I}_n}\left(\frac{x(2^\tau - 1)}{\varphi}\right) = e^{-x(2^\tau - 1)\log\left(\frac{x(2^\tau - 1) + \varphi d^2}{x(2^\tau - 1) + \varphi R_m^2}\right)} \quad (30)$$

where $\nu = \log\left(\frac{x(2^\tau - 1) + \varphi d^2}{x(2^\tau - 1) + \varphi R_m^2}\right)$. By substituting (30) into (29), and (29) into (18), we will get (16). ∎

IV. RESULTS AND DISCUSSION

In this section, the system performance in terms of $\mathbb{P}_{out}$ is numerically shown and validated by the simulation results. In the simulation, N UEs are generated according to the PPP with intensity λ. The top 10 contents ($L = 10$) are generated according to the Zipf distribution, and the desired content is assumed that the most popular one, i.e. $\mathcal{W}_1$. The radius of area B is set to $d = 100$ meter. The simulation results are carried out via a one million trials per point.

Fig. 2 illustrates the performance of the system when the single device to single device communication is considered. $\mathbb{P}_{out}$ versus a target threshold τ in the x-axis is shown. The results for a special case path loss exponent $\alpha = 4$ and $\alpha = 2$ are considered. The density of UEs is fixed to $\lambda = 0.15u/m^2$ and the popularity shape parameter ρ is fixed to 2. From this figure, it can be seen that a larger α leads to better performance, since the interference is much reduced when α increases compared to the signal received degradation. It is seen that the performance is enhanced roughly by 23% when $\alpha = 4$ and $\tau = 1$ bps/H$_z$ compared to $\alpha = 2$. The simulation results match well the analytical results. However, the results demonstrate that $\mathbb{P}_{out}$ is very high in D2D communication since it depends on τ and popularity of content at the same time.

Fig. 3 shows the analytical and simulation results of average $\mathbb{P}_{out}$ with respect to the threshold value τ. The parameters, ρ, λ, and ηo are fixed to 2, $0.15u/m^2$ and 20 dB respectively. The results are shown for different R_m (2-5 meter) in MDSD and the nearest distance in D2D. In this figure, the average outage probability of the UE in D2D communication (15) is compared with MDSD communication in (16), where $\alpha = 2$. It can be seen that the outage probability is decreased significantly when the MDSD based method is used instead of D2D based method, this is because the benefit from the transmission diversity which is used to combat the channel fading. Moreover, $\mathbb{P}_{out}$ decreases significantly when R_m increases in MDSD method since the number of helpers having the desired content increases, thereby it is probably increasing the chance of getting the desired content from neighbors. It can be observed that the performance gain is enhanced significantly. For example, for a target $\tau = 1$ bps/H$_z$, the performance gain is enhanced by 17% when $R_m = 2$ and 40% when $R_m = 3$ in MDSD compared to D2D, whereas the outage gain, will be around 90% for MDSD when $R_m = 4$ and target $\tau = 1.5$ bps/H$_z$ compared to D2D. However, the performance gain is improved significantly and the performance gap decreases when R_m increases.

The popularity shape parameter ρ is another factor playing an essential role in our system model. Fig. 4 depicts the average outage probability of the UE versus popularity shape parameter ρ for $\tau = 1.5$ bps/H$_z$, $\lambda = 0.15u/m^2$, and $\eta_o = 20$ dB. The curves are shown for MDSD in different R_m. It is seen that the outage probability decreases rapidly when the factor ρ increases from 0 to 1.5 and gradually when $\rho > 1.5$. Increasing ρ means that a less number of contents
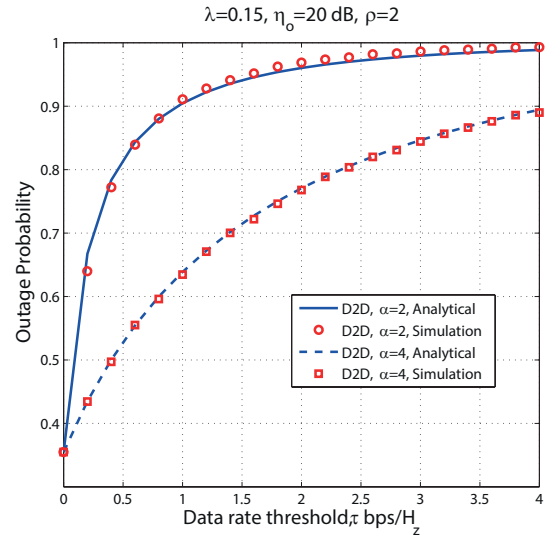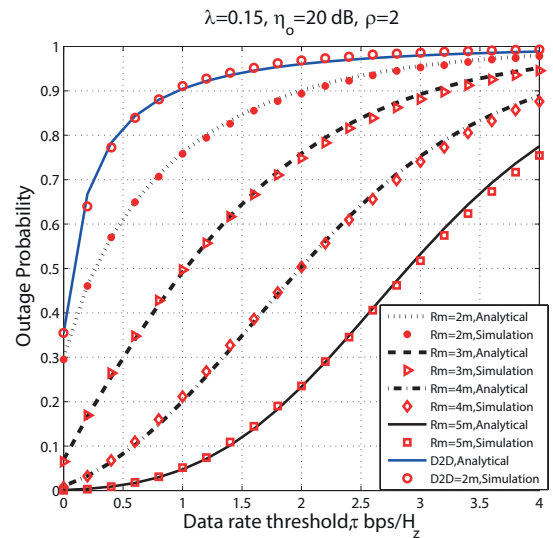

Figure 2: Outage probability vs. target threshold τ, D2D

Figure 3: Outage probability vs. target threshold value τ
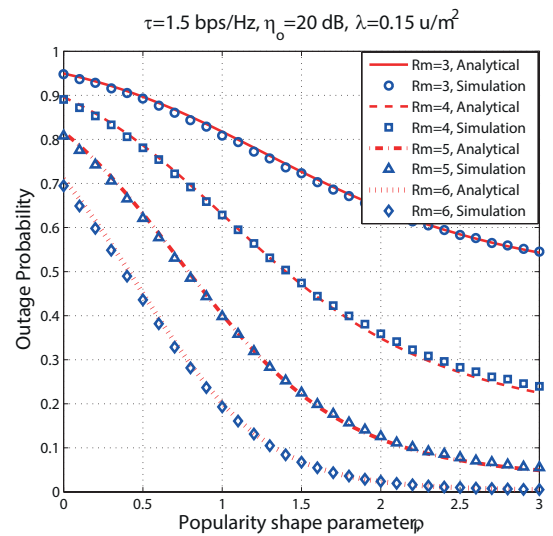

Figure 4: Outage probability vs Popularity shape parameter ρ

are more common requested, results in increasing the number of helpers that having the desired content. It can be seen that the performance gain is enhanced by approximately 25% when $\rho = 0$ and R_m increases from 3 to 6 meter, whereas the gain is enhanced by 60% for $\rho = 1$. In fact, when $\rho = 0$, all contents are uniformly distributed with equal probability and the outage probability will be higher than the Zipf distribution with shape parameter $\rho > 0$. Furthermore, as R_m increases, $\mathbb{P}_{out}$ decreases significantly since the number of UEs within A increases results in increasing the chance to finding and downloading the desired content from neighbors. However, the analytical results match well to simulation results, and the gain performance gap decreases when the distance R_m increases.

V. CONCLUSION

Content caching at the user terminal and using D2D communication is a promising way to enhance the performance of mobile networks in terms of latency, throughput capacity, energy saving and so on. We proposed a novel content delivery method in MDSD method based on D2D communication in order to enhance the system performance in terms of outage probability. Analysis is related to the popularity shape parameter ρ and the number of helpers having the desired content. A single integral expression for the summation of the desired received signal using Laplace transform and residue theorem was derived. An expression of the outage probability for D2D and MDSD based method using tools from stochastic geometry and point process theory was derived. The results showed that the performance was always improved when the popularity of contents depending on ρ increases, as well as the number of helpers is increased. Furthermore, it was shown that the analytical results match well to the Monte-Carlo simulation, and the performance improves significantly when the MDSD was used instead of D2D based method.

APPENDIX A
PROOF OF THE LAPLACE TRANSFORM IN THEOREM 1

Proof. The Laplace transform of the aggregation interference $\mathcal{I}_n$ can be expressed as

$$
\begin{aligned}
\mathcal{L}_{\mathcal{I}_n}(s) &= \mathbb{E}_{\mathcal{I}_n}\left[e^{-s\mathcal{I}_n}\right], \\
&= \mathbb{E}_{\Phi_\lambda,g}\left[e^{-s\sum_{j\in\Phi_\lambda/A}|g_j|^2 r_j^{-\alpha}}\right], \\
&\overset{(a)}{=} \mathbb{E}_{\Phi_\lambda,g}\left[\prod_{j\in\Phi_\lambda/A} e^{-s|g_j|^2 r_j^{-\alpha}}\right], \\
&\overset{(b)}{=} \mathbb{E}_{\Phi_\lambda}\left[\prod_{j\in\Phi_\lambda} \mathbb{E}_g\left[e^{-s|g_j|^2 r_j^{-\alpha}}\right]\right], \\
&\overset{(c)}{=} e^{-2\pi\lambda\int_r^\infty\left[1-\mathbb{E}_g\left[e^{-s|g_j|^2 v^{-\alpha}}\right]\right]v\,dv}. \quad (A.1)
\end{aligned}
$$

where (a) follows the properties of the exponential functions, (b) follows from the property of $|g_j|^2$ that is i.i.d in the PPP, and (c) follows from the definitions of probability generating functional (PGFL) [20], which state for function

$f(x)$ that $\mathbb{E}\left[\prod_{x\in\Phi}f(x)\right] = e^{-\lambda\int_{R^2}(1-f(x))dx}$. The expectation $\{\mathbb{E}_g(\cdot)\}$ in (A.1) is defined as

$$
\mathbb{E}_g\left[e^{-s|g_j|^2 r_j^{-\alpha}}\right] = \int_0^\infty e^{-s|g_j|^2 r_j^{-\alpha}}f(g)dg = \frac{1}{1+sr_j^{-\alpha}}, (A.2)
$$

Substituting (A.2) into (A.1) and put $s = (2^\tau - 1)r^\alpha$, we obtained

$$
\mathcal{L}_{\mathcal{I}_n}\left((2^\tau-1)r^\alpha\right) = e^{-2\pi\lambda\int_r^\infty \frac{(2^\tau-1)}{(2^\tau-1)+\left(\frac{v}{r}\right)^\alpha}v\,dv} \quad (A.3)
$$

Letting $y = \left(\frac{v}{r}\right)^2$, the integral is limited from 1 to ∞. Since the interference is for limited area within d, therefore the integral is limited to d. (A.3) becomes

$$
\mathcal{L}_{\mathcal{I}_n}\left((2^\tau-1)r^\alpha\right) = e^{-\pi\lambda r^2\int_1^d \frac{(2^\tau-1)}{(2^\tau-1)+y^{\frac{\alpha}{2}}}dy} = e^{-\pi\lambda r^2\kappa(\tau,\alpha)}, (A.4)
$$

where

$$
\kappa(\tau,\alpha) = \int_1^d \frac{(2^\tau-1)}{(2^\tau-1)+y^{\frac{\alpha}{2}}}dy \quad (A.5)
$$

∎

APPENDIX B
PROOF OF THE INVERSE LAPLACE TRANSFORM OF THEOREM 2

Proof. The inverse Laplace transform of (27) is defined as

$$
\begin{aligned}
f_Y(t) &= \mathcal{L}^{-1}(f_Y(s)) = \frac{1}{2j\pi}\int_{\epsilon-j\infty}^{\epsilon+j\infty} e^{st}f_Y(s)ds, \\
&= \frac{1}{2j\pi}\int_{\epsilon-j\infty}^{\epsilon+j\infty} e^{st}\left(\frac{e^{\varphi s}E_2(\varphi s)}{\psi}\right)^{k_h}ds. \quad (B.1)
\end{aligned}
$$

where $t > 0$, $j = \sqrt{-1}$ and the integration are done along the vertical $\Re(s) = \epsilon$ in the complex plane such that ϵ is greater than the real part of all singularities of $f_Y(s)$. A Bromwich contour shown in Fig. 5 is used in order to solve (B.1). The integrals along paths BCD and HIA go to zero as $R \to \infty$, also the integral along path EFG go to zero as $r \to 0$. A complex integral along path AB is evaluated by using the residue theorem as follow
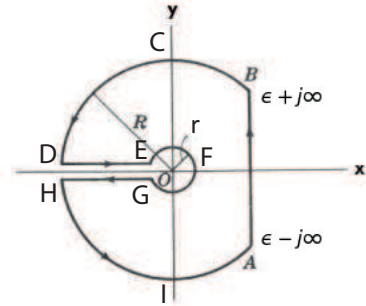

Figure 5: Bromwich contour

$$
\int_{AB} + \int_{DE} + \int_{GH} = \sum residues, \quad (B.2)
$$

Since the integrals along BCD, HIA, and EFG is approach to zero, thus

$$
\int_{AB} + \int_{DE} + \int_{GH} = 0, \quad \to \int_{AB} = -\int_{DE} - \int_{GH} \quad (B.3)
$$

which is give us the solution for (B.1). Now, the integrals along the paths DE and GH is defined as follow

A. Integral along DE

Letting $s = ve^{j\pi}$, v goes from R to r as s goes from $-R$ to $-r$, thus

$$\int_{DE} = \frac{1}{\psi^{k_h}} \int_{-R}^{-r} e^{s(t+k_h\varphi)} \left(E_2(\varphi s)\right)^{k_h} ds,$$

$$= \frac{1}{\psi^{k_h}} \int_{R}^{r} e^{-v(t+k_h\varphi)} \left(E_2(\varphi v e^{j\pi})\right)^{k_h} e^{j\pi} dv. \text{ (B.4)}$$

B. Integral along GH

Letting $s = ve^{-j\pi}$, v goes from r to R as s goes from $-r$ to $-R$, thus

$$\int_{GH} = \frac{1}{\psi^{k_h}} \int_{-r}^{-R} e^{s(t+k_h\varphi)} \left(E_2(\varphi s)\right)^{k_h} ds,$$

$$= \frac{1}{\psi^{k_h}} \int_{r}^{R} e^{-v(t+k_h\varphi)} \left(E_2(\varphi v e^{-j\pi})\right)^{k_h} e^{-j\pi} dv. \text{ (B.5)}$$

Substituting (B.5), (B.4) into (B.2), and after some manipulation (B.1) is expressed as

$$f_Y(t) = \frac{1}{2j\pi} \int_0^\infty \frac{e^{-v(t+k_h\varphi)}}{\psi^{k_h}} \left(\left(E_2(\varphi v e^{-j\pi})\right)^{k_h} - \left(E_2(\varphi v e^{j\pi})\right)^{k_h}\right) dv. \text{(B.6)}$$

From the definition (5.1.7) in [22], the generalized $E_a(\cdot)$ is written as

$$E_a(-v \pm j0) = E_a(-v) \mp j\pi \frac{v^{a-1}}{\Gamma(a)}. \tag{B.7}$$

where $\Gamma(\cdot)$ is a gamma function. By substituting (B.7) into (B.6), and setting the variable $x = \varphi v$ yields

$$f_Y(t) = \frac{-1}{2j\pi\varphi} \int_0^\infty \frac{e^{-x\left(k_h + \frac{t}{\varphi}\right)}}{\psi^{k_h}}$$
$$\left[\left(E_2(-x) - j\pi x\right)^{k_h} - \left(E_2(-x) + j\pi x\right)^{k_h}\right] dx. \tag{B.8}$$

From Binomial theory and for real a and b

$$(a-jb)^k - (a+jb)^k = -2j\sum_{n=0}^{k} \binom{k}{n} \sin\left(\frac{n\pi}{2}\right) a^{k-n} b^n. \tag{B.9}$$

where $\binom{k}{n} = \frac{k!}{n!(k-n)!}$ stands for binomial coefficient. $\Omega_{k_h}(x)$ is defined as

$$\Omega_{k_h}(x) = \sum_{n=0}^{k_h} \binom{k_h}{n} \sin\left(\frac{n\pi}{2}\right) (E_2(-x))^{k_h-n} (\pi x)^n. \text{(B.10)}$$

By substituting (B.10) into (B.8), we get the PDF of Y $(f_Y(t))$. ∎

APPENDIX C
PROOF OF THE LAPLACE TRANSFORM OF THEOREM 2

Proof. The Laplace transform of the aggregation interference $\mathcal{I}_n$ is defined as

$$\mathcal{L}_{\mathcal{I}_n}\left(\frac{x(2^\tau - 1)}{\varphi}\right) = \mathbb{E}_{\mathcal{I}_n}\left(e^{\frac{-x}{\varphi}(2^\tau - 1)\mathcal{I}_n}\right) \tag{C.1}$$

The same procedure in appendix-A is followed to evaluate $\mathcal{L}_{\mathcal{I}_n}\left(\frac{x(2^\tau - 1)}{\varphi}\right)$. By setting the limits of integral from R_m to d, (C.1) is expressed as

$$\mathcal{L}_{\mathcal{I}_n}(s) = e^{-2\pi\lambda \int_{R_m}^{d} \frac{s}{s+v^2} v dv} = e^{-\pi\lambda s \log\left(\frac{s+d^2}{s+R_m^2}\right)},$$

$$= e^{-x(2^\tau - 1)\log\left(\frac{x(2^\tau-1)+\varphi d^2}{x(2^\tau-1)+\varphi R_m^2}\right)} \tag{C.2}$$

∎

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