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Quantum speed-up capacity in different types of quantum channels for two-qubit open systems

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Abstract

A potential acceleration of a quantum open system is of fundamental interest in quantum computation, quantum communication, and quantum metrology. In this paper, we investigate on the “quantum speed-up capacity” which reveals the potential ability of a quantum system to be accelerated. We explore evolution of the speed-up capacity in different quantum channels for two-qubit states. We find although the dynamics of the capacity is variety in different kinds of channels, it is positive in most situations which are considered in the context except one. We give the reasons for the different features of the dynamics. Anyway, the speed-up capacity can be improved by memory effect. We find two ways which may be used to control the capacity in experiments: selecting an appropriate coefficient of an initial state or changing memory degree of environments.

Keywords: quantum speed-up capacity, quantum speed limit, two-qubit open systems, quantum channel, memory effect

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1 Introduction

Whether a quantum system has a potential capacity to be accelerated is an important question in a quantum process. A concept which may weigh this question is

$$Cap = \frac{(\tau - \tau_{QSL})}{\tau} \times 100\%, \tag{1}$$

where τ_{QSL} is a quantum speed limit(QSL) time[1-6] and τ is an actual evolution time. As the physical meaning of Eq. (1) will be revealed in next section, it gives a percentage of potential acceleration of a quantum system. It may play a decisive role in quantum computation[7-8], quantum communication[9-10], quantum optimal control[11-14] and quantum metrology[15-17].

With using Eq. (1), it is obvious that the QSL time which has been studied in recent years[18-25] need to be calculated to obtain the speed-up capacity. A unified lower bound of a QSL time involved Mandelstam-Tamm(MT)[26] and Margolus-Levitin(ML)[27] types in open systems has been derived by Deffner and Lutz[22]. It has also been confirmed that ML type bound based on the operator norm provides a sharpest bound of the QSL time in open systems and non-Markovianity leads to a smaller QSL time. The QSL time of a multi-qubit open system has now caught increasing attention. Since it has been found that the QSL time can be reduced with a special class of multi-qubit states in an amplitude-damping channel even in a memoryless environment[28], three interesting questions arise: 1) What is a common case for the speed-up capacity of multi-qubit open systems in different types of quantum channels? 2) Does memory effect play a same role to the speed-up capacity in different channels? 3) What will happen when the memory degree is changed? In this paper we consider these queries in two-qubit systems and show that even different quantum channels lead to different speed-up capacity, the capacity exists in most cases. We demonstrate that the capacity will be benefit from a memory environment. Moreover, two key factors which may be helpful to control the speed-up capacity of a quantum process in experiments are found.

The paper is structured as follows. In Sec 2 we interpret the physical meaning of the speed-up capacity and deal with the dynamics of multi-qubit open systems by Kraus operators. Evolution of the speed-up capacity of typical two-qubit states in different quantum channels without memory effect is explored and explained respectively in Sec 3. Memory and memory degree effect on the speed-up capacity are discussed in Sec 4. Finally, the results obtained in this work are summarized in Sec 5.

2 The quantum speed-up capacity and dynamics of multi-qubit open systems

With the form of Eq. (1), it can be seen that the speed-up capacity is just a ration of difference between τ and τ_{QSL} to τ. Since τ_{QSL} and τ are the minimal evolution time and the actual evolution time respectively, the difference between them represents a time length of which may be potentially reduced in the evolution. Therefore greater difference causes more potential time that may be speeded up. In this sense, Eq. (1) gives a maximal percentage of the evolution of a system, by which the process can be accelerated in theory with a given actual evolution time. This is the exact physical meaning of Eq. (1) as the definition of the quantum speed-up capacity. To obtain the capacity, the QSL time which can be derived by combining the results of MT and ML bounds[3-4,14,29] need to be calculated. A definition of the QSL time in open systems is[22,24,28,30-31]

$$\tau_{QSL} = \frac{\sin^2[B(\rho_0,\rho_\tau)]}{\min\{\Lambda_\tau^{op},\Lambda_\tau^{tr},\Lambda_\tau^{hs}\}}, \tag{2}$$

where $B(\rho_0,\rho_\tau) = \mathbf{arccos}(\sqrt{\langle\Psi_0|\rho_\tau|\Psi_0\rangle}$ denotes the Bures angle between an initial state $\rho_0=|\Psi_0\rangle\langle\Psi_0|$ and its target state ρ_τ, $\Lambda_\tau^{op,tr,hs} = \frac{1}{\tau}\int_0^\tau \|L_t(\rho_t)\|_{op,tr,hs}\,dt$ with L_t being a superoperator which satisfies $\frac{d\rho_t}{dt} = L_t(\rho_t)$. Here $\|A\|_{op} = \alpha_1$, $\|A\|_{tr} = \sum_i \alpha_i$ and $\|A\|_{hs} = \sqrt{\sum_i \alpha_i^2}$ are the operator norm, trace norm and the Hilbert-Schmidt norm, respectively. α_i is the singular value of A[32].

A popular and convenient description which indicates the dynamics of a state in a quantum channel is Kraus representation[33]. With this description, the evolution of a state ρ can be written in form of

$$\rho(t) = \sum_\mu K_\mu(t)\rho(0)K_\mu^\dagger(t), \tag{3}$$

where the operators K_μ are the so-called Kraus operators and satisfy $\sum_\mu K_\mu{}^\dagger K_\mu = \mathbf{1}$ for all t. When the system is composed of N subsystems with independent environment respectively, Eq. (3) is replaced by[34]

$$\rho(t) = \sum_{\mu\cdots\nu} K_\mu^1 \otimes \cdots \otimes K_\nu^N \rho(0) K_\mu^{1\dagger} \otimes \cdots \otimes K_\nu^{N\dagger} . \qquad (4)$$

By using Eq. (4), the evolution of a multi-qubit system can be evaluated.

3 Evolution of the speed-up capacity in different quantum channels

Now we focus on the evolution of the speed-up capacity of two-qubit states in different quantum channels where the N in Eq. (4) equals to 2. The operator norm which has been proved that provides a sharpest bound[22] is used here. Two classes of typical Bell-type initial state, $|\Psi_1\rangle = a|\mathbf{01}\rangle + \sqrt{\mathbf{1}-a^2}\,|\mathbf{10}\rangle$ and $|\Psi_2\rangle = a|\mathbf{11}\rangle + \sqrt{\mathbf{1}-a^2}\,|\mathbf{00}\rangle$ with coefficient $a \in [\mathbf{0},\mathbf{1}]$, are considered as the initial states respectively. The evolved state $\rho(t)$ is used as the target state to show the dynamics of the capacity.

3.1 Amplitude-damping channel

This channel represents the dissipative interaction between a qubit and its environment. The Hamiltonian model for the process can be written as follow[35]:

$$H_{AD} = \omega_0 \sigma_+ \sigma_- + \sum_k \omega_k a_k{}^\dagger a_k + \sum_k (\sigma_+ g_k a_k + \sigma_- g_k^* a_k{}^\dagger) , \qquad (5)$$

where $\sigma_\pm$ are the raising and lowering operators with ω_0 being the transition frequency of the qubit. Here ω_k denotes different field modes of the reservoir where $a_k(a_k{}^\dagger)$ is the annihilation(creation) operator and g_k is the coupling constant. A damped Jaynes-Cummings model is considered with

$$J(\omega) = \frac{\gamma_0 \lambda^2}{2\pi[(\omega_0 - \omega)^2 + \lambda^2]} , \qquad (6)$$

where λ defines the spectral width and γ_0 quantifies the coupling strength. The decoherence function of the model is

$$G(t) = e^{-\lambda t/2}[\cosh(\frac{dt}{2}) + \frac{\lambda}{d}\sinh(\frac{dt}{2})], \qquad (7)$$

where $d = \sqrt{\lambda^2 - 2\gamma_0\lambda}$. The environment is Markovian(memoryless) when $\gamma_0 < \lambda/2$, otherwise a non-Markovian(memory effect) environment is caused[36-40]. The Kraus operators of this model are given as

$$K_1(t)=\begin{pmatrix}1 & 0\\ 0 & \sqrt{p(t)}\end{pmatrix},\ K_2(t)=\begin{pmatrix}0 & \sqrt{1-p(t)}\\ 0 & 0\end{pmatrix}, \tag{8}$$

where the damping parameter $p(t)$ equals to $G^2(t)$. The evolution can be easily expanded to two-qubit systems by using Eq. (4).

The speed-up capacity of $|\Psi_1\rangle$ and $|\Psi_2\rangle$ as a function of the scaled time λt and coefficient a without memory effect is shown in Fig.1. It can be found that $|\Psi_1\rangle$ always has no speed-up capacity no matter how long it evolves. Yet $|\Psi_2\rangle$ has a nonzero speed-up capacity at the initial time (except $a=1$) and then it increases to an invariant value within a short time evolving. The capacity is in inverse proportion to coefficient a. These phenomena mean that $|\Psi_1\rangle$ has reached the best accelerated performance in this channel, while $|\Psi_2\rangle$ can obtain a further acceleration even when the environment is memoryless. It is an important character for state selecting in experiments.

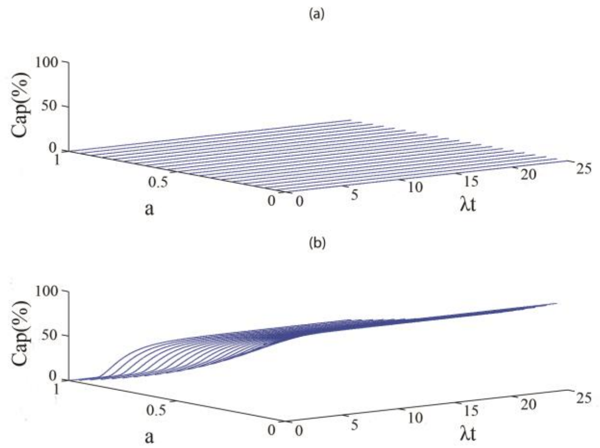

Fig.1 Evolution of the speed-up capacity of (a) $|\Psi_1\rangle$ and (b) $|\Psi_2\rangle$ in an amplitude-damping channel as a function of scaled time λt and coefficient a with $\lambda=50$.

3.2 Phase-damping channel

This process describes a pure dephasing type of interaction between a qubit and a bosonic reservoir. The Hamiltonian is written as follow[35]:

$$H_{PD}=\omega_0\sigma_z+\sum_k\omega_k a_k^{\dagger}a_k+\sum_k\sigma_z\left(g_k a_k+g_k^{*}a_k^{\dagger}\right). \tag{9}$$

A spectral density of an Ohmic-like form is considered here:

$$J(\omega)=\frac{\omega^s}{\omega_c^{s-1}}e^{-\omega/\omega_c}, \tag{10}$$

where s is the Ohmic parameter and ω_c is the cutoff frequency of the environment. By changing the relationship for s and constant 1, we obtain different Ohmic spectra which corresponds to sub-Ohmic environments($s<1$), Ohmic environments($s=1$), and super-Ohmic environments($s>1$), respectively. Besides, $s>2$ may cause a memory effect with zero T [41]. Kraus operators of this channel are given as

$$K_1(t)=\begin{pmatrix}1 & 0\\ 0 & p(t)\end{pmatrix},\ K_2(t)=\begin{pmatrix}0 & 0\\ 0 & \sqrt{1-p^2(t)}\end{pmatrix}, \tag{11}$$

where $p(t)$ is a dephasing parameter and can be calculated by

$$p(t)=\exp[-\int_0^t \gamma(t^{'})dt^{'}], \tag{12}$$

here $\gamma(t)$ is the dephasing rate and indicated as

$$\gamma(t)=\omega_c[1+(w_c t)^2]^{-s/2}\Gamma(s)\sin[s\arctan(\omega_c t)], \tag{13}$$

where $\Gamma(s)$ represents the Euler function. It also can be easily expanded to two-qubit systems by using Eq.(4).

Evolution of the speed-up capacity of $|\Psi_1\rangle$ and $|\Psi_2\rangle$ in this channel are much different from those in an amplitude-damping channel. A biggest distinction is that $|\Psi_1\rangle$ and $|\Psi_2\rangle$ do not have a different speed-up capacity anymore. As shown in Fig.2, the two classes of states have a same invariant speed-up capacity when coefficient a is same in a memoryless environment. There is a non-monotonic relationship between the capacity and coefficient a. This relationship also can be used to select an appropriate state for experiment aim.

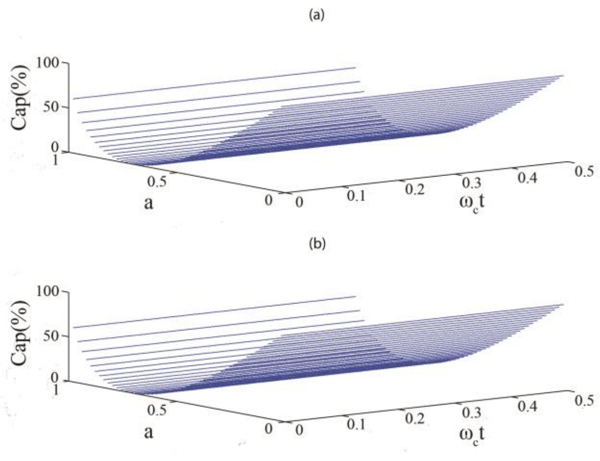

Fig.2 Evolution of the speed-up capacity of (a) $|\Psi_1\rangle$ and (b) $|\Psi_2\rangle$ in a phase-damping channel as a function of scaled time $\omega_c t$ and coefficient a with s=1, ω_c=1.

3.3 Bit flip, Phase flip, and Bit-phase flip channels

These three channels are all under the Markov approximation in which memory effect does not exist. The unified Lindblad operator of them in a single –qubit system is

$$\frac{d}{dt}\rho(t) = \gamma[\sigma_i \rho(t)\sigma_i - \rho(t)] , \qquad (14)$$

where γ is the time-independent dephasing rate and σ_i is the Pauli matrix with $i = x, y, z$ denote bit flip, phase-bit flip, and phase flip channels, respectively. The set of Kraus operators for each one of these channels are given as[34]

$$K_1(t) = \begin{pmatrix} \sqrt{1 - \frac{p(t)}{2}} & 0 \\ 0 & \sqrt{1 - \frac{p(t)}{2}} \end{pmatrix}, \quad K_2^i(t) = \sqrt{\frac{p(t)}{2}}\sigma_i , \qquad (15)$$

where $p(t) = 1 - \exp[-\gamma t]$.

In these three channels the speed-up capacity of $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are same. The evolution in a bit flip channel is shown in Fig.3. It can be seen that the capacity in this channel rises from zero to a very small value within a short time. The relationship between the finial value and coefficient a is still non-monotonic but different to it is in a phase-damping channel. The dynamics in a phase flip channel is similar to it is in a phase-damping channel as shown in Fig.4. This phenomenon is easy to be understood since it has been known that a phase-damping channel and a phase flip channel are exactly a same quantum operation[42]. In Fig.5 it is found that the speed-up capacity in a bit-phase flip channel is same as it is in a bit flip channel. This phenomenon can been easily confirmed in mathematics by using Eq. (2), (4) and (15).

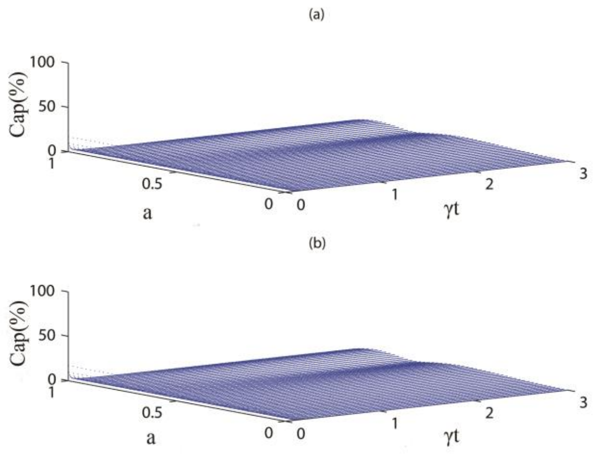

Fig.3 Evolution of the speed-up capacity of (a) $|\Psi_1\rangle$ and (b) $|\Psi_2\rangle$ in a bit flip channel as a function of scaled time γt and coefficient a with γ=10 .

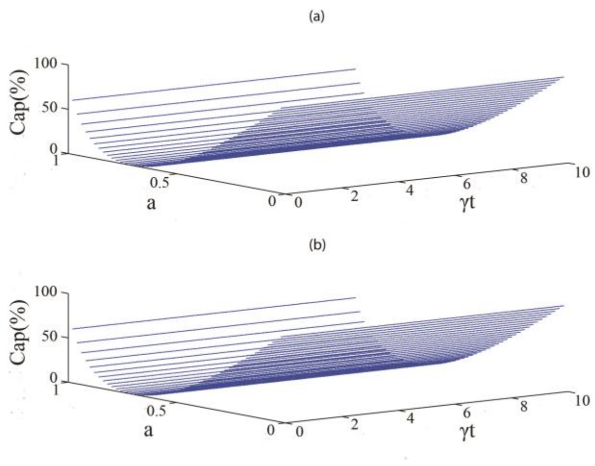

Fig.4 Evolution of the speed-up capacity of (a) $|\Psi_1\rangle$ and (b) $|\Psi_2\rangle$ in a phase flip channel as a function of scaled time γt and coefficient a with γ=10.

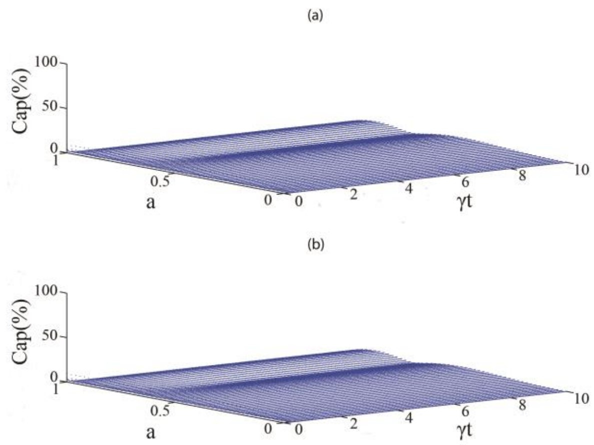

Fig.5 Evolution of the speed-up capacity of (a) $|\Psi_1\rangle$ and (b) $|\Psi_2\rangle$ in a bit-phase flip channel as a function of scaled time γt and coefficient a with γ=10.

3.4 Explaining for the different features of the speed-up capacity in different channels

Now we investigate on the reason of why the speed-up capacity has such different features in different channels. Two main different characters are interpreted here: 1) Why the capacity of $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are vastly different in an amplitude-damping channel but exactly the same in other channels we have considered? 2) Why the capacity is always exist in most situations we have considered and what conditions should be satisfied when the capacity is disappeared such as $|\Psi_1\rangle$ in an amplitude-damping channel? With answering to question 2), the results can be generalized to general scenarios.

Substituting Eq. (2) into Eq. (1), we have

$$C_{ap}=(1-\frac{\tau_{QSL}}{\tau})\times 100\%=(1-\frac{\sin^2[B(\rho_0,\rho_\tau)]}{\int_0^\tau \|L_t(\rho_t)\|_{op}\,dt})\times 100\%=(1-\frac{1-\langle\Psi_0|\rho_\tau|\Psi_0\rangle}{\int_0^\tau \|L_t(\rho_t)\|_{op}\,dt})\times 100\%. \quad (16)$$

The reduced density matrices of the states $|\Psi_1\rangle$ and $|\Psi_2\rangle$ at time t can be written as

$$\rho_1(t)=\begin{pmatrix}\rho_{11} & 0 & 0 & \rho_{14}\\ 0 & \rho_{22} & \rho_{23} & 0\\ 0 & \rho_{23}^* & \rho_{33} & 0\\ \rho_{14}^* & 0 & 0 & \rho_{44}\end{pmatrix},\quad \rho_2(t)=\begin{pmatrix}\rho_{11}^{'} & 0 & 0 & \rho_{14}^{'}\\ 0 & \rho_{22}^{'} & \rho_{23}^{'} & 0\\ 0 & \rho_{23}^{'*} & \rho_{33}^{'} & 0\\ \rho_{14}^{'*} & 0 & 0 & \rho_{44}^{'}\end{pmatrix}. \quad (17)$$

Then we can obtain $\langle\Psi_1|\rho_1(t)|\Psi_1\rangle=\alpha^2\rho_{22}+\alpha\sqrt{1-\alpha^2}(\rho_{23}+\rho_{23}^*)+(1-\alpha^2)\rho_{33}$ and $\langle\Psi_2|\rho_2(t)|\Psi_2\rangle=\alpha^2\rho_{44}^{'}+\alpha\sqrt{1-\alpha^2}(\rho_{14}^{'}+\rho_{14}^{'*})+(1-\alpha^2)\rho_{11}^{'}$. In phase-damping, Bit flip, Phase flip, and Bit-phase flip channels we find that $\rho_{11}=\rho_{33}^{'}$, $\rho_{22}=\rho_{44}^{'}$, $\rho_{33}=\rho_{11}^{'}$, $\rho_{44}=\rho_{22}^{'}$, $\rho_{14}=\rho_{23}^{'}$, $\rho_{23}=\rho_{14}^{'}$ are always satisfied during the evolution. Therefore $\langle\Psi_1|\rho_1(t)|\Psi_1\rangle$ always equals to $\langle\Psi_2|\rho_2(t)|\Psi_2\rangle$ in these channels. On the other hand, $L_t(\rho_1(t))$ and $L_t(\rho_2(t))$ have the same singular values[32,43] due to the symmetry between $\rho_1(t)$ and $\rho_2(t)$. This is the exact reason for why the capacity of $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are the same in these channels by using Eq. (16). When we come to the case that in an amplitude-damping channel, it is found that the symmetry between $\rho_1(t)$ and $\rho_2(t)$ is destroyed in diagonal elements. Accordingly, the capacity of $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are different in this channel.

Next we come to explain the second character. The definition of τ_{QSL} is originally derived from the inequality which has been obtained as the nonunitary generalization[22]

$$2\cos(B(\rho_0,\rho_\tau))\sin(B(\rho_0,\rho_\tau))\frac{dB(\rho_0,\rho_\tau)}{dt}\le\left|\langle\Psi_0|L_t(\rho_t)|\Psi_0\rangle\right|. \quad (18)$$

By using the relationship $\langle\Psi_0|L_t(\rho_t)|\Psi_0\rangle=tr\{L_t(\rho_t)\rho_0\}$ and the von Neumann trace inequality for operators[44,45], we get

$$2\cos(B(\rho_0,\rho_\tau))\sin(B(\rho_0,\rho_\tau))\frac{dB(\rho_0,\rho_\tau)}{dt}\le\|L_t(\rho_t)\|_{op}. \quad (19)$$

Integrating Eq. (19) over time, it is found that

$$\frac{\sin^2[B(\rho_0,\rho_\tau)]}{\int_0^\tau \|L_t(\rho_t)\|_{op}\,dt} \le 1\,. \tag{20}$$

By substituting Eq. (20) into Eq. (16), $C_{ap} \ge 0$ is obtained. This is the reason for why in most situations the capacity is positive. Obviously, the condition which causes the capacity equals to zero is the same condition which leads to equal sign in the von Neumann trace inequality. From this we can see that the evolution of state $|\Psi_1\rangle$ in an amplitude-damping channel reaches the condition and the capacity is disappeared in this case. Moreover, since Eq. (20) is a general equation which is independent of the values of the parameters of quantum channels and always set up in different dynamical processes, the results may be generalized to general occasions and more specific research will be proceeded in further work.

4 Memory and memory degree effect

In this section we will explore the scenarios in which the memory effect is emerged. How to distinguish whether the environment is a memory or memoryless one has been described in previous section. In an amplitude-damping channel we see things become a little different from previous when the system is affected by memory. As shown in Fig.6, $|\Psi_1\rangle$ still has no speed-up capacity at the beginning, but suddenly rises to an invariant value which has nothing to do with a. For $|\Psi_2\rangle$ the circumstance is similar to what happens in a memoryless environment, however it takes a shorter time to reach the invariant value which is higher than it is in a memoryless environment.

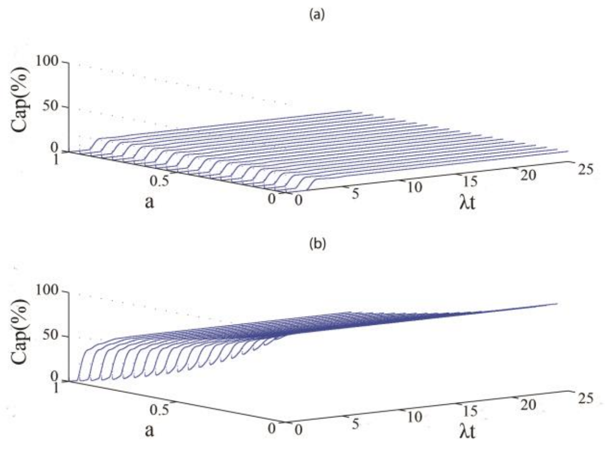

Fig.6 Evolution of the speed-up capacity of (a) $|\Psi_1\rangle$ and (b) $|\Psi_2\rangle$ in an amplitude-damping channel as a function of scaled time λt and coefficient a with λ=50.

Dynamics of the speed-up capacity in a phase-damping channel with memory effect is shown in Fig.7. Although it seems to be a same one to it is in a memoryless environment, it is found that the capacity is slightly rising to a higher value within a short time evolving when the environment is in memory as shown in Fig.8.

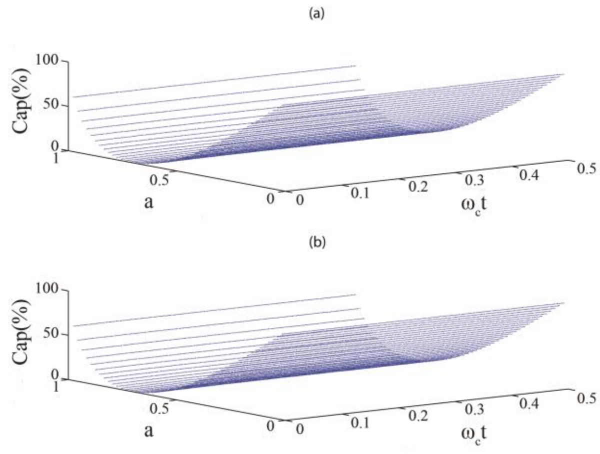

Fig.7 Evolution of the speed-up capacity of (a) $|\Psi_1\rangle$ and (b) $|\Psi_2\rangle$ in a phase-damping channel as a function of scaled time $\omega_c t$ and coefficient a with s=5, ω_c=1.

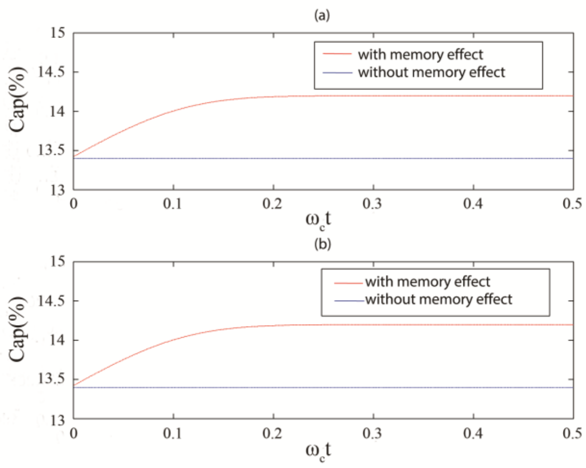

Fig.8 Contrast of the speed-up capacity of (a) $|\Psi_1\rangle$ and (b) $|\Psi_2\rangle$ between a memory and a memoryless environment in a phase-damping channel with $a = 0.5$.

Overall, despite the dynamics of the capacity is different in varieties of channels, it can draw a conclusion that memory effect causes more potentially accelerated ability in two-qubit open systems, which in fact is a powerful supplement to the result that non-Markoviantity may reduce the QSL time in a Jaynes-Cummings model of a single qubit open system[22]. It also testifies the generalization of the relationship between the non-Markovianity and the QSL time. Since the intrinsic reason of why quantum speedup connects directly with the non-Markovianity in single qubit open systems has been revealed in [46], our result about the relationship between the quantum speed-up capacity and the non-Markovianity in two-qubit open systems may also be explained by using the conclusion of [46].

Finally we deal with the question that what will happen if the memory degree is changed. We know that memory effect comes from non-Markovianity of the environment. So if the degree of non-Markovianity is higher, the memory degree is higher. It has been proved that the degree of non-Markovianity is in proportion to the coupling strength γ_0 in a damped Jaynes-Cummings model[37]. Therefore the memory degree is also in proportion to the γ_0 in a damped Jaynes-Cummings model. The speed-up capacity of $|\Psi_1\rangle$ and $|\Psi_2\rangle$ as a function of scaled time λt and γ_0 is shown in Fig.9. It is found that higher the memory degree leads to better speed-up capacity. Namely, more memory effect, more potential acceleration. It is remarkable that when γ_0 is high enough, there is a short fluctuation for $|\Psi_1\rangle$ before it reaches the invariant speed-up capacity. Since some methods which may control the speed of the evolution of single qubit systems in theory and experiments have been presented[46-57], our results may make some contribution to experiments in further work.

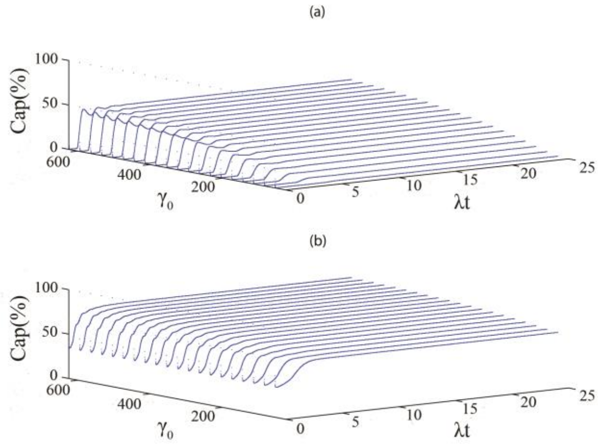

Fig.9 Evolution of the speed-up capacity of (a) $|\Psi_1\rangle$ and (b) $|\Psi_2\rangle$ in an amplitude-damping channel as a function of scaled time λt and the coupling strength γ_0 (in proportion to memory degree) with λ=50, $a = 0.5$.

5 Conclusion

In summary, we present a formula to feature the speed-up capacity of a quantum system. We find the capacity has different dynamics in varieties of channels and exists in most situations of two-qubit open systems. We interpret the characters of the capacity and demonstrate memory effect can always improve the capacity. We also find coefficient a of the initial state and the memory degree are two key factors which may useful in experiments to control the capacity.

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Reference

[1] Anandan J and Aharonov Y 1990 *Phys. Rev. Lett.* **65** 1697
[2] Levitin L B and Toffoli T 2009 *Phys. Rev. Lett.* **103** 160502
[3] Jones P J and Kok P 2010 *Phys. Rev. A* **82** 022107
[4] Zwierz M 2012 *Phys. Rev.A* **86** 016101
[5] Taddei M M, Escher B M, Davidovich L and de Matos Filho R L 2013 *Phys. Rev. Lett.* **110** 050402
[6] del Campo A, Egusquiza I L, Plenio M B and Huelga S F 2013 *Phys. Rev. Lett.* **110** 050403
[7] Lloyd S 2000 *Nature* (London) **406** pp.1047-1054
[8] Lloyd S 2002 *Phys. Rev. Lett.* **88** 237901
[9] Bekenstein J D 1981 *Phys. Rev. Lett.* **46** 623
[10] Yung M H 2006 *Phys. Rev. A* **74** 030303(R)
[11] Caneva T, Murphy M, Calarco T, Fazio R, Montangero S, Giovannetti V and Santoro G E 2009 *Phys. Rev. Lett.* **103** 240501
[12] Hegerfeldt G C 2013 *Phys. Rev. Lett.* **111** 260501
[13] Hegerfeldt G C 2014 *Phys. Rev. A* **90** 032110
[14] Deffner S 2014 *J. Phys. B* **47** 145502
[15] Giovanetti V, Lloyd S and Maccone L 2011 *Nat. Photonics* **5** pp222-229
[16] Chin A W, Huelga S F and Plenio M B 2012 *Phys. Rev. Lett.* **109** 233601
[17] Tsang M 2013 *New J. Phys.* **15** 073005
[18] Giovannetti V, Lloyd S and Maccone L 2003 *Phys. Rev. A* **67** 052109
[19] Luo S 2004 *Physica D* **189** pp1-7
[20] Deffner S and Lutz E 2013 *J. Phys. A* **46** 335302
[21]Russell B and Stepney S 2014 *Phys. Rev. A* **90** 012303
[22] Deffner S and Lutz E 2013 *Phys. Rev. Lett.* **111** 010402
[23] Latune C L, Escher B M, de Matos Filho R L and Davidovich L 2013 *Phys. Rev. A* **88** 042112
[24] Xu Z Y, Luo S, Yang W L, Liu C and Zhu S 2014 *Phys. Rev. A* **89** 012307
[25] Xu Z Y and Zhu S 2014 *Chin. Phys. Lett.* **31** 020301
[26] Mandelstam L and Tamm I 1945 *J. Phys.* (USSR) **9** pp249-254
[27] Margolus N and Levitin L B1998 *Physica D* **120** pp188-195
[28] Liu C, Xu Z Y and Zhu S 2015 *Phys. Rev. A* **91** 022102
[29] Pfeifer P and Frohlich J 1995 *Rev. Mod. Phys.* **67** 759
[30] Zhang Y J, Han W, Xia Y J, Cao J P and Fan H 2015 *Phys. Rev. A* **91** 032112
[31] Liu X, Wu W and Wang C 2017 *Phys. Rev. A* **95** 052118
[32] Bhatia R 1997 *Springer* Berlin
[33] Kraus K 1983 *Springer*, Berlin
[34] Salles A, de Melo F, Almeida M P, Hor-Meyll M, Walborn S P, Souto Ribeiro P H and Davidovich L 2008 *Phys. Rev. A* **78** 022322
[35] Fanchini F F, Karpat G, Castelano L K and Rossatto D Z 2013 *Phys. Rev. A* **88** 012105
[36] Breuer H P, Laine E M, Piilo J and Vacchini B 2016 *Rev. Mod. Phys.* **88** 021002
[37] Vasile R, Maniscalco S, Paris M G A, Breuer H P and Piilo J 2011 *Phys. Rev. A* **84** 052118
[38] Bylicka B, Johansson M and Acín A 2017 *Phys. Rev. Lett.* **118** 120501
[39] Ferialdi L 2017 *Phys. Rev. A* **95**, 020101(R)
[40] Sampaio R, Suomela S, Schmidt R and Ala-Nissila T 2017 *Phys. Rev. A* **95** 022120
[41] Haikka P, Johnson T H and Maniscalco S 2013 *Phys. Rev. A* **87** 010103(R)

[42] Nielsen M A and Chuang I L 2000 *Cambridge University Press* Cambridge
[43] Ali M, Rau A R P and Alber G 2010 *Phys. Rev. A* **81** 042105
[44] von Neumann J 1937 *Tomsk Univ. Rev.* **1** pp286-300
[45] Mirsky L 1975 *Monatshefte für Math.* **79** pp303-306
[46] Liu H B, Yang W L, An J H and Xu Z Y 2016 *Phys. Rev. A* **93** 020105(R)
[47] Cimmarusti A D, Yan Z Patterson, B D, Corcos L P, Orozco L A and Deffner S 2015 *Phys. Rev. Lett.* **114** 233602
[48] Zhang Y J, Han W, Xia Y J, Cao J P and Fan H 2015 *Phys. Rev. A* **91** 032112
[49] Noda S, Fujita M and Asano T 2007 *Nat. Photonics* **1** 449
[50] Porras D, Marquardt F, von Delft J and Cirac J I 2008 *Phys. Rev. A* **78** 010101(R)
[51] Dreisow F, Szameit A, Heinrich M, Pertsch T, Nolte S, Tünnermann A and Longhi S 2008 *Phys. Rev. Lett.* **101**, 143602
[52] Behzadi N, Ahansaz B, Ektesai A and Faizi E 2017 *Phys. Rev. A*. **95** 052121
[53] Li H, Yang M and Li D C 2016 *Chin. Phys. Lett.* **33** 50301
[54] Tong N H and Zheng D C 2017 *Chin. Phys. B* 26 060502
[55] Yang G, Jin J and Nie M 2017 *Chin. Phys. B* 26 040305
[56] Chen H, Li C Y and Li H R 2017 *Chin. Phys. B* 26 037105
[57] Zheng Q, Yang H Y and Zhi Q J 2017 *Chin. Phys. B* 26 010601